EFFECT OF RECTIFIED WAVES OF VOLTAGE UPON THE LOSSES AND EFFICIENCY IN DIRECT-CURRENT SHUNT MOTORS

by

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TABLE OF CONTENTS

Table of Contents (cont.)

Table of Contents (concl.)

SUMMARY OF STANDARD TESTS OF DIRECT-CURRENT SHUNT MOTORS

The Problem

This study is concerned with the problems arising when shunt motors are operated under rectified wave forms; wave forms which result from the operation of some device for blocking part of an alternating wave while passing another part. As a preliminary to taking up these problems, however, it seems well to review the standard tests of shunt motors in order to decide what tests to set up in investigation of the case using rectified waves.

Direct Loading Tests

A motor may be loaded by means of a brake or perhaps by belting to a generating device of some sort, the output of which is then dissipated in some controlled manner (1, 2, 3). In such a case, the output of the motor usually must be dissipated somewhere as waste power, and in the case of large machines this often renders the test impractical. Nevertheless, it is sometimes employed in practice on fairly large equipment (4).

If the motor is so loaded, it is a relatively simple matter to note the power supplied to the machine and the power delivered by the machine. The quotient:

$$\text{Efficiency} = \frac{P \text{ output}}{P \text{ input}}$$

will then yield the value for efficiency, while the difference:

$$P \text{ losses} = P \text{ input} - P \text{ output}$$

will give the value of the losses in the machine.

The values for efficiency and losses obtained by this means are perhaps not the most satisfactory possible for the reason that losses are obtained by taking the difference between two rather large numbers which are of roughly comparable size. In the case of a motor with 80 per cent efficiency at full load, therefore, if full-load power output were eight kilowatts (10.71 hp.), the input power would be ten kw. If, however, a ten per cent error were made in the determination of input power, so that power input was measured as nine kw. instead of ten, then the losses calculated by the above means would turn out to be one kw. instead of two kw. as calculated in the first case; an error of 50 per cent. The losses are important in calculation of motor behavior and any test which is so constituted that such a small error in determining other factors can throw such a large error into the calculation of losses is, to that extent, unsatisfactory.

Pump-back Tests

The pump-back test seems to have been designed primarily to avoid the necessity for dissipating great amounts of power during the testing of large machines (5, 6). It has a great added advantage of avoiding the trouble with loss calculation referred to above.

In testing a machine by this means, it is necessary to have two machines of comparable size. Then the one which it is desired to test as a motor is connected mechanically to the shaft of the other large machine. The other machine is connected as a generator supplying power to the first machine. Of course, the two machines will not run unless the losses inherent in the operation of both of them are supplied from some external source. This can be done by either electrical or mechanical means; i. e., a second generator may be connected in series with the machine being used as a generator in order to raise voltage without diminishing current from the large generator, thus adding power to the system, or a small motor may be belted to the common shaft of the two large machines, thus adding mechanical power to the system. In either case, the power added by the external source is equal to the total losses of the system. If the two large machines are identical or practically so, it is nearly correct to assume the power supplied to the system to be divided equally between the two machines. Then, if this power is designated as P supplied, it is found:

$$\text{Efficiency (motor)} = \frac{\text{P input (motor)} - \text{P losses (motor)}}{\text{P input (motor)}}$$

$$= \frac{\text{P input (motor)} - \frac{1}{2}\text{ P supplied}}{\text{P input (motor)}}$$

$$\text{P losses (motor)} = \tfrac{1}{2}\text{ P supplied}$$

Now, examining the effect of an error in determination of P input upon the accuracy of our calculations, it is seen that such an error will not affect the accuracy of P losses in the least, while errors in the determination of P losses will act only very slightly in throwing the value of efficiency off the true value. To illustrate this, the same motor may be assumed as in the previous example: 80 per cent efficiency, eight kw. rated output. If there is a ten per cent error in determining P losses which should be two kw. at rated output, so that P losses are found to be 2.2 kw., at the same time that P input is correctly read as ten kw., then

$$\text{Efficiency} = \frac{\text{P input} - \text{P losses}}{\text{P input}}$$

$$= \frac{10 - 2.2}{10}$$

$$= \frac{7.8}{10}$$

$$= 78 \text{ per cent}$$

which is an error of only two per cent resulting from an error in reading P losses of ten per cent. So it is seen that the same situation which leads to such definite error in determining losses in the case of direct loading, does not enter in in the case of pump-back tests in so far as the determination of losses is concerned, but leads to reduced error when finding efficiency by the pump-back method.

Unfortunately, it is not always possible to obtain two identical machines for testing and when this is impossible it is necessary to have exact data on the large machine being used as a generator as well as on the small machine with which losses are supplied.

Of course, it is not rigorously true that the losses, even in the case of two identical machines, are equal in the two machines. The losses in the generator can always be expected to be somewhat higher. Nevertheless, in consideration of the small effect which a discrepancy in P losses has upon efficiency, that assumption is often considered sufficiently close to the true condition for practical purposes.

Constant-loss Tests

A third general type of test which may be made is the constant-loss test (7, 8, 9). This test involves running the motor at no-load only and calculating the losses which would be present due to full-load currents. From this, using the expression:

$$\text{Efficiency} = \frac{\text{Power input} - \text{Power losses}}{\text{Power input}}$$

and taking Power input to be equal to line voltage times the sum of full-load armature current and field current, the losses are taken as the sum of the losses due to the full-load current flow. These losses are: I^2R in the armature and in the field, electrical losses in the brush contact, which are taken as the product of the two-volt brush drop and the full-load armature current, and a so-called "constant loss" which is taken as the total loss at rated field current and speed when motor is run at no-load with voltage equal to the expected back emf at full-load. This constant loss is then the total machine input under these conditions of no-load and rated quantities. This method has the distinct advantage over the previously mentioned one that no great quantity of power must be dissipated by a brake, driven generator, or other device. It has, however, the serious shortcoming that actual rated current is never driven through the components of the machine. Thus any effects which are in conflict with the theory will not be brought to light by this test. The most evident such difficulty is the question of the variation of resistance of the various windings with temperature. All resistances are adjusted mathematically to 75 degrees C., but if the motor is operated under conditions which give rise to a higher or lower temperature than this in practice, the motor will have different resistances at full-load than those used in calculating efficiency losses, etc., by this method. Another difficulty along this line is that the core loss will also vary with the temperature of the armature since the magnetization curve of the iron is different at high temperatures than at low. Specifically, the hysteresis loop will have a lesser area the higher the ambient temperature is made. Also, the resistance of the path taken by the eddy-currents in the core will change as temperature changes. Any test for core loss made at room temperature will be in error for this reason. Thus, since full-load current is not driven through the windings of the motor in such a test as this, the machine cannot be tested under actual operating temperature by this means, and a certain amount of approximation must be assumed in the results obtained. It has the great advantages over the other two testing methods described, however, that full-load power need not be dissipated during the test, and no other machines are needed to run the test.

STATEMENT OF ASSUMPTIONS MADE IN ABOVE TESTS

External Conditions

In all the tests outlined in the foregoing, the assumption is made that the impressed voltage is a perfectly uniform direct voltage; that it contains no components of other than zero frequency. This, of course, is not true in practice. One common source of rather considerable harmonic voltage is the phenomenon of commutation. In the dynamo laboratory at Kansas State College it is not uncommon to find harmonic voltages as high as five volts superimposed upon the 220 volts on the direct current bus. The frequency of this voltage is usually quite high, however--about 500 cycles per second--and its effect on direct current machinery seems to be negligible. Other assumptions made in connection with the external, impressed conditions under which these tests are assumed to be run are: constant room temperature, absence of strong stray magnetic fields, and, of course, no radiation or Maxwellian displacement current. All these assumptions, while valid enough in most practical cases, are possible sources of error.

Conditions Within the Motor

Further, it is assumed that the resistance of the motor armature windings as determined by the passage of a direct current while the machine is at standstill is equal to the effective resistance of the armature winding while the armature is in motion. This is a situation which might lead to rather serious errors for reasons which will be considered in greater detail later. Also, it is assumed that the core loss at constant field current is constant; the change in field flux resulting from armature reaction is often not considered. The voltage drop across the brushes (total of positive and negative brush drop) is assumed exactly equal to a constant two volts. It is assumed that the I^2R losses in all components of the machine vary strictly with the square of the value of the particular current involved; any change in effective resistance as a result of partial saturation of iron in the immediate vicinity of the component in question is therefore neglected.

A large number of assumptions along this line, all of which carry with them the possibility of introducing errors into the computation, could be cited. These will be thought to suffice for the purpose of emphasizing that tests in direct-current machines are by no means absolute. It is possible to examine any comprehensive test of a direct-current machine and point out assumptions which have introduced errors, however slight, into the work. The devising of a practical test consists largely in deciding which assumptions will introduce errors which are slight enough to be tolerated.

EXAMINATION OF STANDARD TESTS AS APPLIED TO TESTS ON NON-UNIFORM VOLTAGES

General Considerations

It is desirable to test a machine under conditions simulating as closely as possible those it may be expected to encounter in practical work. In this study, these conditions have been chosen as (1) constant field current, and (2) constant terminal voltage (average). In this way it is possible to avoid any influence which the voltage regulation of the source might otherwise exert on the situation and it is also possible to eliminate changes in power lost in the field rheostats. Furthermore, this closely counterfeits the practical situation encountered when a constant-speed (shunt) direct current motor is used to drive an erratic load.

Direct Loading Tests

With the above conditions, it is clearly possible to obtain exact data on efficiency and losses by means of direct loading. The disadvantages mentioned in the preceeding discussion remain, however.

Pump-back Tests

Since the source of the rectified wave is alternating, it is necessary to pump back energy as alternating current. In all practical applications, however, the alternating impressed voltage is furnished from a practically infinite bus at constant frequency. Thus, if the motor under test were loaded with an alternator, the output of which were then fed back into the ignitron, either the frequency would change as the motor speed changed, or else the motor would be held at constant speed throughout the test. Neither of these conditions is satisfactory. But it is possible to load the d. c. motor with a d. c. generator, and load the d. c. generator with another d. c. motor driving an alternator at a speed corresponding to line frequency. Then the output of this alternator may be fed into the ignitron completing the pump-back cycle. But this requires four rotating machines; each large enough to handle full-load motor output power. And even when the motor puts out full-load power, the power eventually fed back into the ignitron will be greatly diminished because of the losses of the other three machines. For instance, if both auxiliary d. c. machines have an efficiency over the range used of 80 per cent and the alternator has an efficiency of 90 per cent over this range, then when the motor under test puts out full rated power, only 57.5 per cent of rated power output is fed back into the ignitron; i. e., only about half the motor output power makes the complete circuit from motor output to motor input through this feedback circuit. If the motor itself is 80 per cent efficient at full rated output, then 46 per cent of the input power necessary to run the motor at full rated load arrives through the feedback circuit while about 54 per cent must be supplied externally. Clearly it would be much less trouble in most cases simply to dissipate all the output power and not bother with all the complications of the feedback circuit since such a small return is realized with the feedback connected.

Constant-loss Tests

It is realized at once that the use of this type of test is considerably more trouble in the case of a rectified wave than in the case of uniform direct voltage. This is true because in order to apply this type of test it is necessary to separate losses and losses are harder to identify in the case of a rectified wave form than in the case of uniform direct voltage.

For example, the losses in the field of the motor may be considered.

In the case of uniform direct current, the field loss is considered to be:

$$P\ \text{field} = I_f^2 \times R_f$$

But in the case of a rectified wave form, accepting the convention that the frequency of a component of voltage, current, or power is designated by subscripts in parentheses,

$$P\ \text{field} = I\ \text{field} \cdot V\ \text{field}$$

where I field • V field denotes the "dot product" or "inner product" of these two quantities and is defined by the following relation if multiples of 60 cycles are the only frequency components present besides the d. c. component.

$$(1) \quad V \cdot I = V_{(0)} I_{(0)} + V_{(60)} I_{(60)} + V_{(120)} I_{(120)} + \cdots + V_{(60\,n)} I_{(60\,n)} + \cdots$$

$$(n = 0, 1, 2, 3, 4, \ldots)$$

Now, this is not, in general, equal to the product of the square of any current (such as r. m. s.) and any resistance or effective resistance. This is true because the effective resistance of the winding varies widely with varying frequency. In fact, as Figure 1 shows, the effective resistance of a field winding (when not saturated) approximately doubles with an increase from zero frequency to 60 cycles, and continues to increase at about the same rate until about 200 cycles, beyond which point the data obtained does not permit accurate calculation of resistance. Appendix I contains data and computations on this point.

Also, it cannot readily be determined how much of this field power represents real I^2R loss in the winding and how much is really core loss caused by hysteresis and eddy currents in the iron of the field poles. Since the current flowing in the armature contains some alternating components, the armature reaction flux must be expected to contain some alternating components. This will have the dual effect of causing some hysteresis and eddy current disturbance in the iron of the field magnetic circuit, and inducing some alternating voltage in the field winding. The voltage thus induced may be expected to affect the magnitude of the losses previously mentioned. The field losses are thus seen to be much more complex in the present case than when uniform direct voltage is impressed. Data are given in Appendix VI and Appendix VII showing that all frequency components of loss

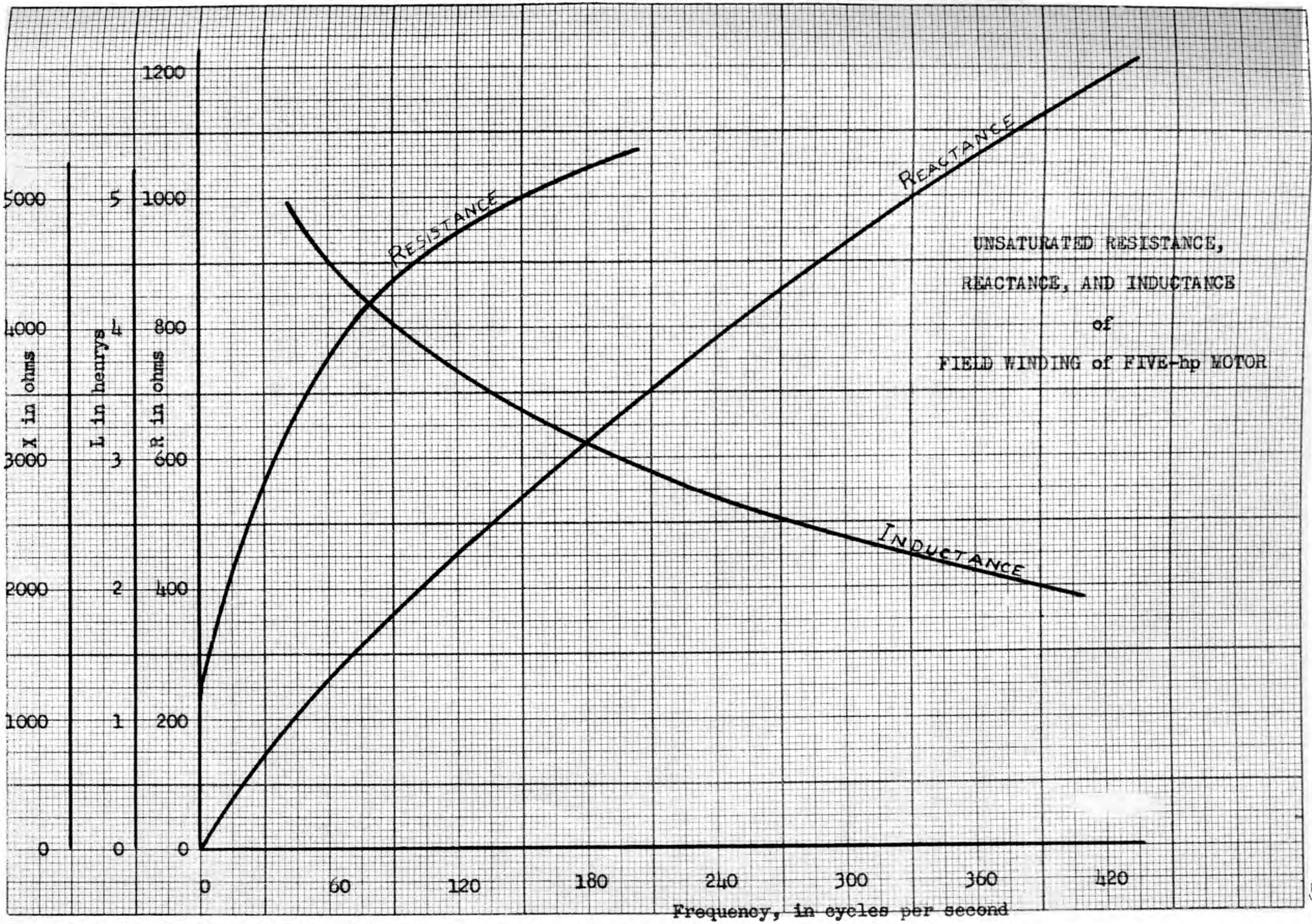

Fig. 1. Field winding resistance, reactance, and inductance.

in the field except

$$P_{(0)} = V_{(0)} I_{(0)}$$

can usually be neglected without serious error.

This is true mainly because the resistance and reactance of the field winding rise so rapidly with frequency, and the magnitudes of the components of voltage impressed fall so rapidly with frequency in ordinary cases, that all terms in equation (1) except the first are negligibly small. This is particularly true for the more usual practical case which is that of the three-phase ignitron rectifier in which case the lowest frequency component, exclusive of zero-frequency, is 180 cycles per second.

Turning to armature losses; in the uniform d. c. case, these consist of armature I^2R loss and armature core loss in the usual consideration. Other losses which intimately involve the armature are the brush contact loss, brush friction, bearing friction, and rotor windage loss.

An examination of armature I^2R losses along the lines outlined for field losses leads immediately to insuperable difficulties. The reason is this: in order to calculate armature losses for each component of voltage and current, it is necessary to know what current will result from each component of voltage. But in order to make such a calculation, it is necessary to know effective resistance and reactance of the armature to each frequency. The effective resistance might be found, but the question of reactance is more involved.

Voltage across an inductive circuit has been observed as equal to

$$V = N \frac{d\phi}{dt}$$

which may be written:

$$V = N \frac{d\phi}{di} \frac{di}{dt}$$

Now, the first part of this equation is designated as inductance, or, quantitively,

$$L = N \frac{d\phi}{di}$$

So, the inductance in each armature conductor could be calculated in terms of the current in each individual armature conductor quite easily. Now, if the current in each armature conductor were identical, the problem would be reducible to one which might be solved fairly easily. Since the d. c. machine uses a commutator, however, the current in each armature conductor is different than that in each other conductor, and, worst of all, these currents are all non-sinusoidal. Of course, the complications which arise from the fact that the core is saturated to some extent in practice have been neglected here, but even so, it can be seen that the problem is too complicated to permit of practical solution. Appendix II contains further discussion of the current in the individual armature conductors.

Furthermore, the core loss is affected by the fact that the flux is not constant but has an alternating component. Friction and windage losses, and brush contact losses are about the same in the two cases with the exception that in calculating the brush contact loss even with non-uniform currents, the average or zero frequency component of current should be used. Appendix III carries a proof of this statement.

STATEMENT OF POSSIBLE TEST FOR USE ON RECTIFIED WAVE FORMS

Basis of Machine Ratings

Practically all electrical apparatus is rated as to the maximum power which it can handle. In nearly all cases, this rating refers to the amount of power which can be handled by the piece of apparatus in question without exceeding safe operating temperature. Usually the temperature rise permitted for d. c. machines is 40 degrees, centigrade (10). If the ambient temperature is 40 degrees, centigrade, this means that the machine temperature is allowed to rise to 80 degrees, centigrade, but in order to take into account the fact that temperatures in the interior of the machine will exceed those on the surface, allowance of 25 degrees, centigrade is left in order to ensure that no part of the machine's insulation will reach a temperature in excess of 105 degrees, centigrade.

It is clear that if the machine were operated in a room whose ambient temperature significantly exceeded 40 degrees, centigrade (105 degrees, fahrenheit), the machine might be expected to overheat if its full name plate power were taken from the shaft. Such an installation was made about 15 years ago in a power plant in Lincoln, Nebraska; a 50-hp. motor was used to drive an induced-draft fan in the smoke-stack. The motor was located in a penthouse on the roof of the building proper, immediately over the boilers and in the same room as the economizers. In the heat of Lincoln summers, the temperature of this location greatly exceeds 105 degrees, fahrenheit; probably at times its temperature is in excess of 130 degrees, fahrenheit. In a very short time, therefore, the motor overheated and had to be overhauled. After a considerable amount of consultation it was decided to buy a new motor, also 50 hp., and put it on the opposite side of the fan, thus connecting 100 hp. to the fan. In the hottest seasons of the year, both these motors approach the maximum allowable temperature; i. e., handle the maximum safe amount of power, despite the fact that each is furnishing only about half its name-plate power output rating.

If a motor were located in a very cold place--for instance, a freezing room in a meat-packing plant--it would be possible to draw much more power from it than its name-plate rating without ill effects.

It is in order to inquire what causes the motor to heat. The answer to this question, of course, is that the various losses in the motor appear as heat which must be radiated from the motor, given to circulating water or air to be carried away by convection, or conducted to cooler parts of the motor or its framework. If a given machine is considered, then it is clear that if, in some way, the losses in the machine are increased while the output power is held constant, the heating in the machine will increase and the temperature will increase correspondingly.

The Problem in the Present Case

The problem with which this test is primarily concerned differs basically from that of a completely new machine in that here the concern is primarily with machines about which a good deal is already known; the efficiency, losses, speed regulation, etc., for a uniform voltage input, are determined already. The problem here is to find out something about the change in these quantities if a given non-uniform voltage is used.

There is an additional question which must be answered in view of the above discussion of ratings and their meaning. It is: to what extent must we change the rating of the machine in order to compensate for the increase in losses resulting from the use of non-uniform voltages?

If the results of tests indicate an approximate rule-of-thumb answer to the above question, it would be well to know it.

The nature of the wave form of impressed voltage is usually known or can be found. It will be assumed as per the previous statement that the average voltage can be assumed impressed alone across field. Since the average voltage is the same as the constant d. c. voltage in the uniform case and since this voltage is made equal to nominal line voltage in the non-uniform case, field losses may be assumed equal in the case of uniform and non-uniform voltage inputs.

It will be assumed, therefore, that the field flux is a constant and that, since speed is to be made equal in corresponding tests on the two wave forms, the rotational losses in the two cases are equal.

Power in the armature, then, may be measured in this fashion:

(1) measure d. c. armature current and voltage.

(2) measure a. c. (r. m. s.) armature current and voltage.

(3) measure power input to the armature by means of a wattmeter.

(4) measure magnitude of each component of voltage and of each component of current by means of a harmonic wave analyser or by a Fourier analysis of experimentally obtained waves.

(5) compare each d. c. voltage and current with the corresponding a. c. (r. m. s.) voltage or current. Find a. c. components of voltage and current from relations:

$$V_{ac} = \sqrt{V_{rms}^2 - V_{dc}^2}$$

$$I_{ac} = \sqrt{I_{rms}^2 - I_{dc}^2}$$

Take product of V_{ac} and I_{ac}.

(6) compare the product of d. c. voltage and current with the reading of the wattmeter. Note this difference.

(7) note that (5) represents the total a. c. volt-amperes, while difference (6) represents the

total a. c. real power. Divide (6) by (5) to obtain an "effective power factor" for the a. c. components of the wave.

(8) take the product of voltage and current components at each frequency; then operate with the effective power factor (7) to obtain an expression for "power" at each frequency.

(9) compare the sum of "powers" (8) with the total a. c. real power (6).

If (8) and (6) agree reasonably closely, the assumption implicit in step (7) that an "effective power factor" can be found which will give satisfactory results will have been established.

Then, if (8) and (6) agree reasonably well, it might be possible to obtain curves for effective impedance of the armature circuit at various frequencies which would enable one to calculate the sum of powers due to the a. c. components of the wave (8) without running the machine under the non-uniform impressed voltage.

This test procedure is, then, proposed as a possible method of arriving at reliable estimates of the extent to which losses will increase in a machine as a result of using non-uniform waves of impressed voltage.

It should be said that according to the assumptions made earlier in this proposed test procedure (i. e., that field flux is practically uniform even with strongly non-uniform impressed voltage) there can be no significant resultant torque in the motor as a consequence of these a. c. components of voltage and current in the armature. Hence, all a. c. power is loss power.

DISCUSSION OF WAVE FORMS LIKELY TO BE ENCOUNTERED IN PRACTICE

Half-wave Rectified Wave Form

Power supplies of this sort are used a good deal for very small motors, especially in the case of servomechanisms (11). It might be thought that the wave form resulting from the operation of a d. c. motor from a half-wave rectifier would be as shown in Fig. 2.

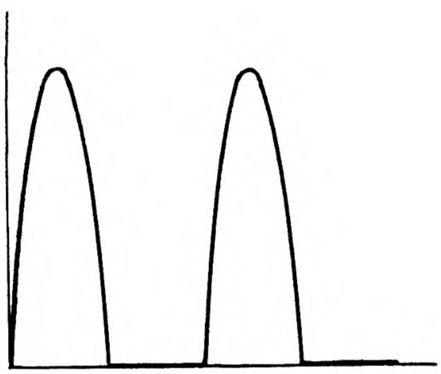

Fig. 2. Half-wave rectified wave (resistance load).

Fig. 3. Half-wave rectified wave (d. c. motor load).

It will immediately be realized that this is not the case, however, when it is remembered that the field flux remains substantially constant throughout the time of one cycle at usual supply frequencies, and therefore when the tube is extinguished the terminal voltage of the motor will jump discontinuously to the value of the speed voltage or back e. m. f. The true wave form for a half-wave rectified voltage with d. c. motor load is shown in Fig. 3.

The harmonic content of such a wave may be worked out; this is done in Appendix IV for a wave with the shape of Fig. 3.

Three-phase Rectified Wave Form

This is a wave form which is encountered often and can be considered to have the greatest harmonic content of any wave which is likely to be met with in power work. Most other power rectified wave forms either use filters, multi-phase transformer connections, or both, in order further to reduce the harmonic content of their output. Indeed as many as 72 phases have been used in ignitron rectifier sets for this reason.

In Fig. 4 is shown the wave form of voltage which occurs when a three-phase rectifier is loaded with a d. c. motor.

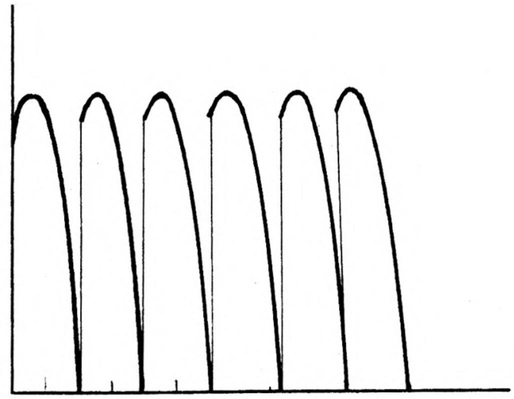

Figure 4. Three-phase rectified wave (d. c. motor load).

The harmonic content of this wave is worked out in Appendix IV also.

EXPERIMENTAL WORK TO DETERMINE VALIDITY OF METHOD SUGGESTED FOR USE ON RECTIFIED WAVE FORMS

General Discussion

Tests were made in considerable detail of a 5-hp. General Electric 220-volt motor and in somewhat less detail of a 15-hp. General Electric 220-volt motor. These machines were both tested by the method of direct loading with uniform direct current impressed. The 5-hp. machine was then tested by the method of direct loading with voltage impressed from the 3-phase ignitron in the laboratory, with voltage impressed from the same set but with only one phase connected, with voltage impressed from a d. c. generator in series with an alternator generating 60-cycle a. c. voltage, and with voltage impressed from a d. c. generator in series with an alternator generating 180-cycle a. c. voltage.

The 15-hp. machine was tested also with voltage impressed from the 3-phase ignitron.

Metering the necessary quantities made necessary a rather complicated wiring scheme despite the basic simplicity of the test. Plate I shows the 5-hp. machine ready for testing together with the various meters which were used. Plate II shows also the other equipment which was run in order to make the test.

Detailed discussion of problems met will be made as each test is taken up.

Tests of Five-Horsepower Motor

Test with Uniform Impressed Voltage. The test was made with motor terminal voltage adjusted continually to 220 volts in order to eliminate any influence which the voltage regulation of the source might otherwise have exerted on the test results. Power was dissipated by means of the eddy-current

EXPLANATION OF PLATE I

A. The 5-hp. motor being tested in this study.

G. E., 5-hp., 230-v, 20.5-a, 650-1950 rpm.

B. The special switching apparatus built for feeding any desired wave form to the Harmonic Wave Analyser and the Oscillograph.

C. Voltmeters.

D. Armature and Total Current and power meters.

E. Alternator used as load for 15-hp. motor. Also used as source for 60-cycle voltage in tests with d. c. plus 60-cycle impressed voltage.

F. The 15-hp. motor being tested in this study.

G. E., 15-hp., 220-v, 60-a, 600-1800 rpm.

G. Field current and power meters.

H. General Electrical Mechanical Oscillograph.

I. Hewlett-Packard Harmonic Wave Analyser.

J. Location of potentiometer controlling eddy-current brake

K. Spring balance showing thrust from eddy-current brake.

L. Eddy-current brake.

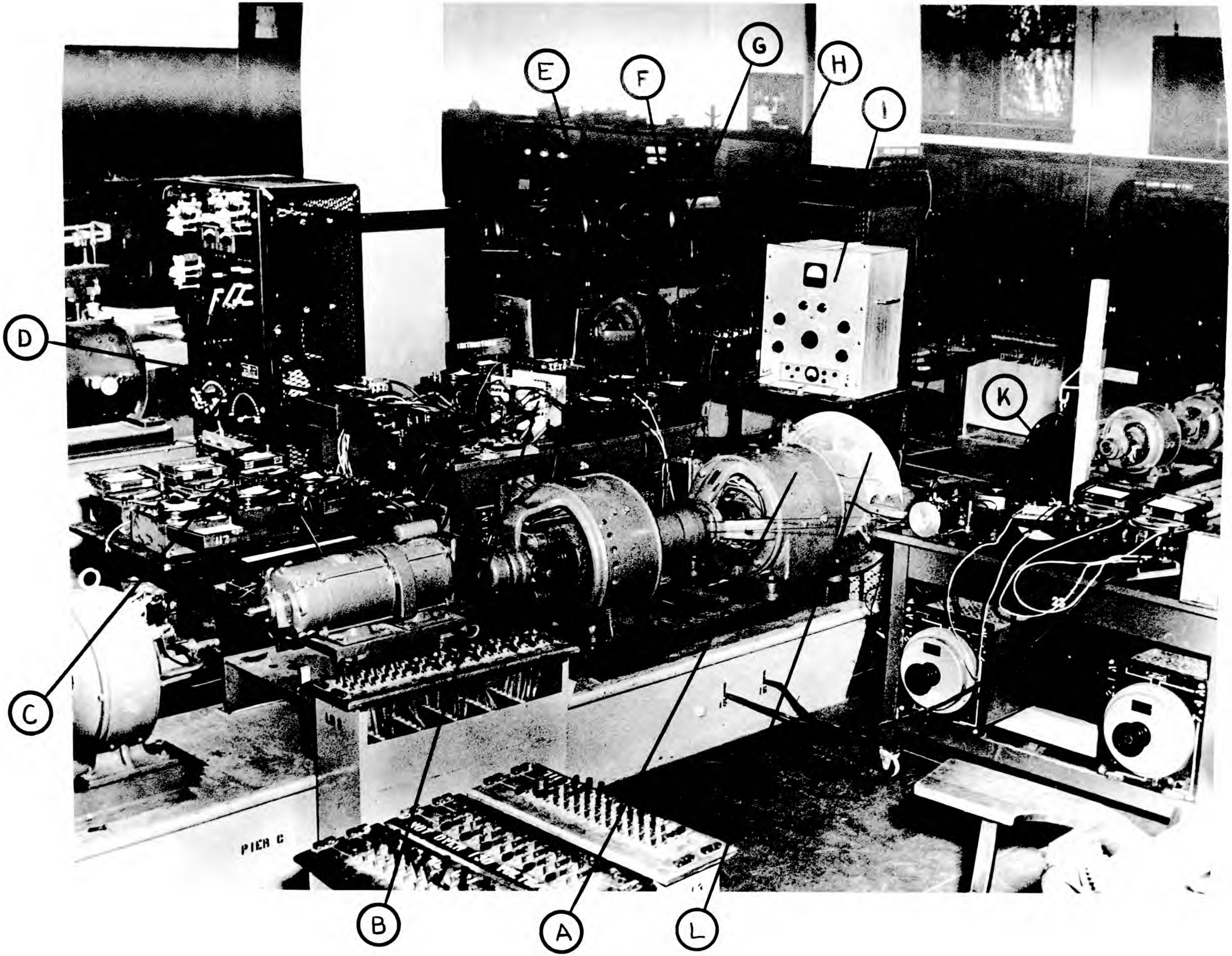

PLATE I

EXPLANATION OF PLATE II

A and B. Motor-generator set used to excite the eddy-current brake.

C. Switchboard.

D. Ignitron rectifier set.

E. Alternator used as load for 15-hp. motor and as source of 60-cycle component of d. c. plus 60-cycle waves.

F. Hewlett-Packard Harmonic Wave Analyser.

G. Harmonic Generator Set (out of picture).

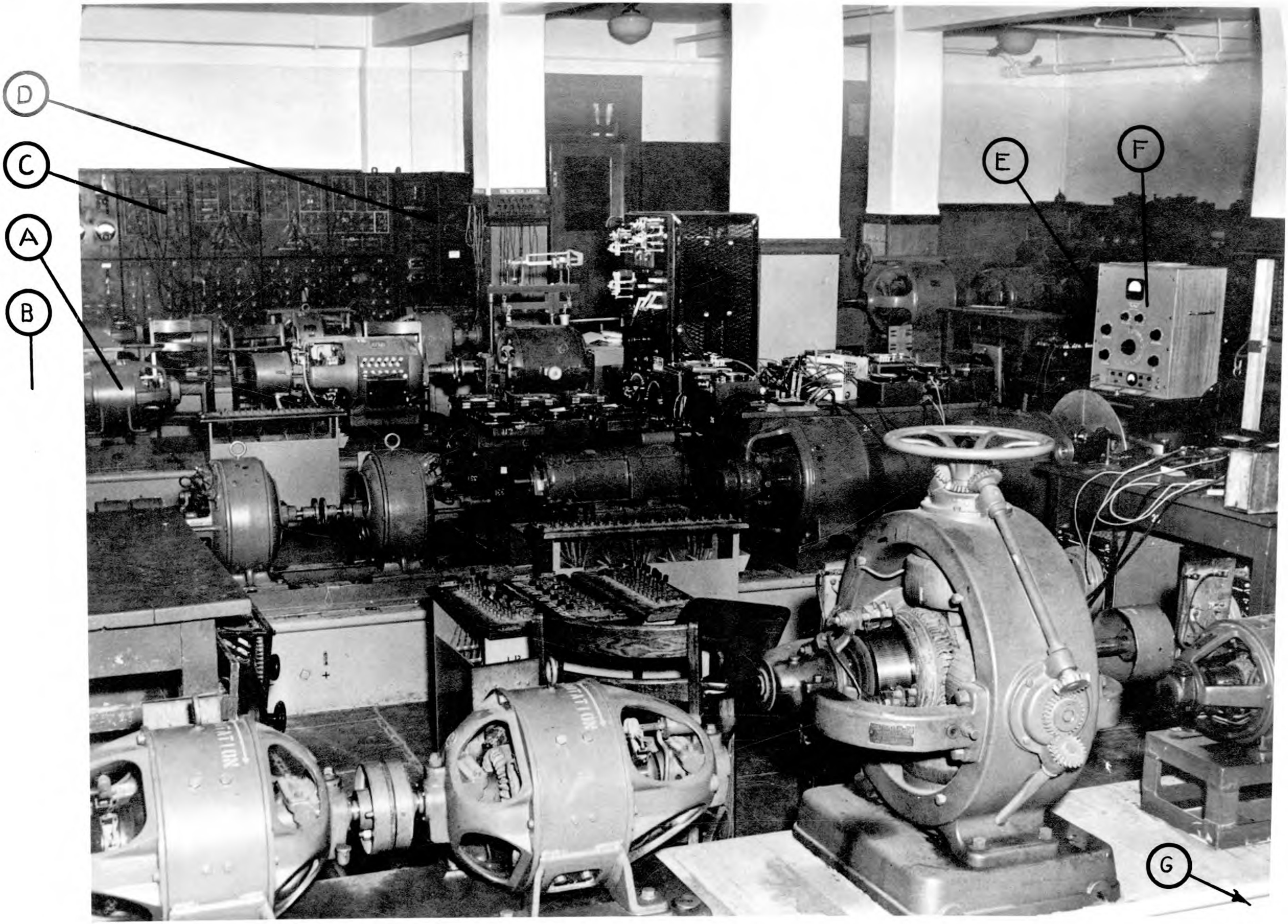

PLATE II

brake which can be seen in Plate I. Current for the eddy-current brake was supplied from a motor-generator set run for this purpose and was controlled by controlling the field excitation of this generator. The potentiometer which controlled the field excitation was located beside the motor being tested in order to expedite adjustments in load. Complete data and calculations for this test are given in Appendix V.

Test with Three-Phase Impressed Rectified Wave Form. Here, too, the terminal voltage was continually adjusted to 220 volts, average, with each change in load. Power was dissipated in the same way as in the above test. Quantities metered included field current and voltage, total current and voltage, armature current and voltage, and voltage across field rheostat. All quantities were measured (a) with direct current meters to obtain average values, (b) with alternating current meters to obtain r. m. s. values, and (c) in the case of full-load readings, provision was made to read frequency components of all voltages and currents, and to photograph wave forms of all voltages and currents. In addition, armature power, field power, and total power were read by means of wattmeters.

The provision for use of the Hewlett-Packard Harmonic Wave Analyser was made by building a special switching apparatus, by means of which it is possible to select the desired voltage or current signal and impress it across the Wave Analyser input terminals simply by throwing switches. The same apparatus is furnished with outputs which are designed for use with the General Electric Mechanical Oscillograph and which make possible the feeding of any voltage signal and any current signal to the two inputs of the Oscillograph simultaneously. The Oscillograph has built-in provision for photographing wave forms; several pictures of wave forms taken by this means appear in this work. The special switching appartus may be seen in Plate I.

Data and calculations for this test appear in Appendix VI.

Tests with Other Impressed Wave Forms. A test was made with impressed voltage supplied from the ignitron set with two of the three phases disconnected. The test results were not satisfactory because the current capacity of the set under such circumstances is inadequate to supply the motor load at heavy loads. In this test the motor operation became unstable and the motor stalled at about 25-30 per cent rated output. One reason why the ignitron set supplied insufficient current at relatively heavy motor loads is suggested in Appendix VII where the data and calculations for this test appear.

In order to compare operation of the motor when operated on single-phase ignitron output with something, a test was made using direct voltage generated in the ordinary way in series with 60-cycle alternating current. The terminal voltage of the motor was adjusted to the same value realized in the case of the single-phase ignitron and the strength of the 60-cycle component was made comparable to the r. m. s. value of all a. c. components of the single-phase wave form. A test was also made with average voltage held at 220 volts.

The same trouble was experienced in the former case as in the case of the single-phase ignitron; the motor stalled at about 25-30 per cent rated output which leads to the conclusion that the reason for stalling is primarily that the average voltage is too low for satisfactory operation. Details of the test, data, and computations are given in Appendix VII.

In order to furnish a basis for comparison with the three-phase ignitron test, the motor was run from a composite source which consisted of 220 volts direct voltage in series with a 180-cycle alternating voltage obtained from the Harmonic Generator in the E. E. Laboratory. Unfortunately, the current requirements of the motor exceeded the safe operating maximum current of the 180-cycle alternator at a rather low motor load. Consequently, these results are not very informative. The data and computations are given, however, in Appendix VII.

Tests of Fifteen-Horsepower Motor

<u>Test with Uniform Impressed Voltage</u>. Here, as in the previous case, the test was made by direct loading. The power

was dissipated by exciting the alternator which is coupled to the motor in question, and loading the alternator with rheostats. The problem of maintaining constant terminal voltage is a difficult one since none of the machines in the laboratory are large enough to furnish adequate current alone. The scheme finally used was this: the voltage from the powerhouse was adjusted to about 240 volts. Then the direct current bus in the laboratory was loaded with rheostats which, when all connected, drew about 100 amperes. This caused sufficient line drop in the power line between the powerhouse and the E. E. Laboratory to lower the motor terminal voltage in the laboratory to 220 volts. Then, as the motor's current was increased, the static load on the bus was decreased in order to maintain the bus voltage at 220 volts. Data and calculations are shown in Appendix VIII.

<u>Test under Three-Phase Rectified Wave Form</u>. Procedure was identical to that used in the case of the 5 hp. motor with the exception that simpler metering was used. Only total power input, d. c. voltage and current inputs, r. m. s. voltage and current inputs, speed, and power output from the alternator were measured. Power from the alternator was measured by the standard two-wattmeter method for metering in three-phase circuits. Since power output from the load alternator is known, it is possible to calculate power input to the alternator which is also power output from the motor, if

the alternator's efficiency is known at all loads. Data, computations, and results are given in Appendix IX.

COMPARISON AND EXAMINATION OF RESULTS

Comments on the Individual Tests

<u>Three-Phase Ignitron Test with Five-Horsepower Motor</u>. Data for this test are found in Appendix VI. Two values of "effective power factor" may be found. These are worked out in Appendix X.

The values thus found for "effective power factor" agree quite closely; one is 0.245, the other is 0.261.

Thus, the assumption suggested on page 22 is justified, and a workable value of "effective power factor" may be found.

Now, it is desirable to notice the fact which can best be verified by referring to the oscillograms in Appendix VI and Appendix VII that the shape of the wave form of impressed voltage goes through no significant change when load is varied between no-load and full-load. Most important, too, is the fact that the <u>shape of the current wave form also stays sensibly unchanged as load is changed</u>. This can readily be seen from Plates V and VI. This is extremely important since it means that losses due to a. c. currents do not change greatly with motor output. Instead, they are nearly constant with a constant impressed wave form of voltage. If this is the case, it at once suggests that it is possible to plot some sort of "effective impedance" or ratio of current to voltage versus frequency in a motor, independently of load. The losses can, perhaps, be found, then, which will be added to the motor when it is operated under the impressed voltage wave form in question. If this is done for the three-phase ignitron impressed voltage, the impedances given in Appendix X are obtained. The per cent impedances are shown in order to extend these results to motors of different ratings. First, it would be in order, however, to test these results on another wave form on this same motor. This is done in Appendix XI for the case of a d. c. generator in series with a 60-cycle alternator.

The results in Appendix XI show conclusively that these are "effective impedances" and "effective power factors" which may not be interpreted too generally. It has been shown that for the small 5-hp. motor this effective impedance and effective power factor are consistent, for impressed voltage from the three-phase ignitron set, but the assumption cannot be made on that basis that any wave form may be treated using the same "effective power factors" and "effective impedances". In other words, the losses of a given frequency in a d. c. motor depend not only upon the impressed voltage of that frequency, but also upon the strength <u>and</u> phase of impressed voltage of other frequencies.

It is possible, however, that the same impedances and power factors may be extended to cover the operation of other motors which are also running under impressed voltage from a three-phase ignitron set.

In Appendix XII, the necessary calculations are made for the 15-hp. motor operating under voltage from a three-phase ignitron set using the calculated "effective power factor" and "effective impedances".

As can be seen there, the effective impedance and effective power factor calculated for the 5-hp. motor give very tolerable results for the 15-hp. motor.

This has one very important meaning beyond the conclusion that the method can be used with three-phase ignitron wave forms on shunt motors of any size. It is that the losses due to the a. c. components of impressed voltage are proportional to the capacity of the motor and, hence, a reasonably satisfactory rule of thumb can be stated in terms of the percentage increase in losses of any shunt motor due to the use of a three-phase ignitron instead of a uniform voltage source. To obtain that rule of thumb, however, constructions according to the following method will be used.

Five-Horsepower Motor Tests

Uniform Voltage and Three-Phase Rectified Wave Form. In practice, the question which is of most importance in comparing these tests is this: how much must we decrease the output power in the case of the three-phase wave form in order to make the losses equal to the losses at full rated output with uniform direct voltage impressed?

It is convenient to investigate this question graphically. Fig. 5 shows the necessary construction. Fig. 5 is taken from Appendix VII where it occurs as page 89.

This construction is performed in the following steps:

1. Plot power losses versus power output on the same sheet for both the uniform direct voltage case and the case to be considered. This gives two curves of the shape of the curves in Fig. 5.
2. Draw a vertical line at the output power corresponding to rated output.
3. Note the point where the curve for power losses with uniform impressed voltage intersects this line. This point represents the permissable losses for this machine.
4. Draw a horizontal line through this point. Note the intersection between this line and

the curve for power losses in the case of the non-uniform voltage under consideration. The output power co-ordinate of this point is the maximum permissable load which can be put on the machine without overheating.

By this means, it is found that an allowance of ten per cent in power output will compensate for the increased losses due to the non-uniform wave. Actually, this has recently been investigated by Westinghouse engineers who used series motors instead of shunt and who found no significant difference (12).

One possible reason for the negative results obtained by the Westinghouse tests is that armature current, with all its harmonics, flows in the field of a series motor and a component of field flux is therefore set up which will tend to produce torque when acting with the component of armature current of like frequency. Thus, not all a. c. power is loss power. It is entirely possible that so little a. c. power is loss power that the resulting increase in P-loss would escape detection in a direct loading test which is the type which was used.

It should be noted that calculation of the increase in losses due to the addition of a. c. components to the input wave will not in general give definite information as to what

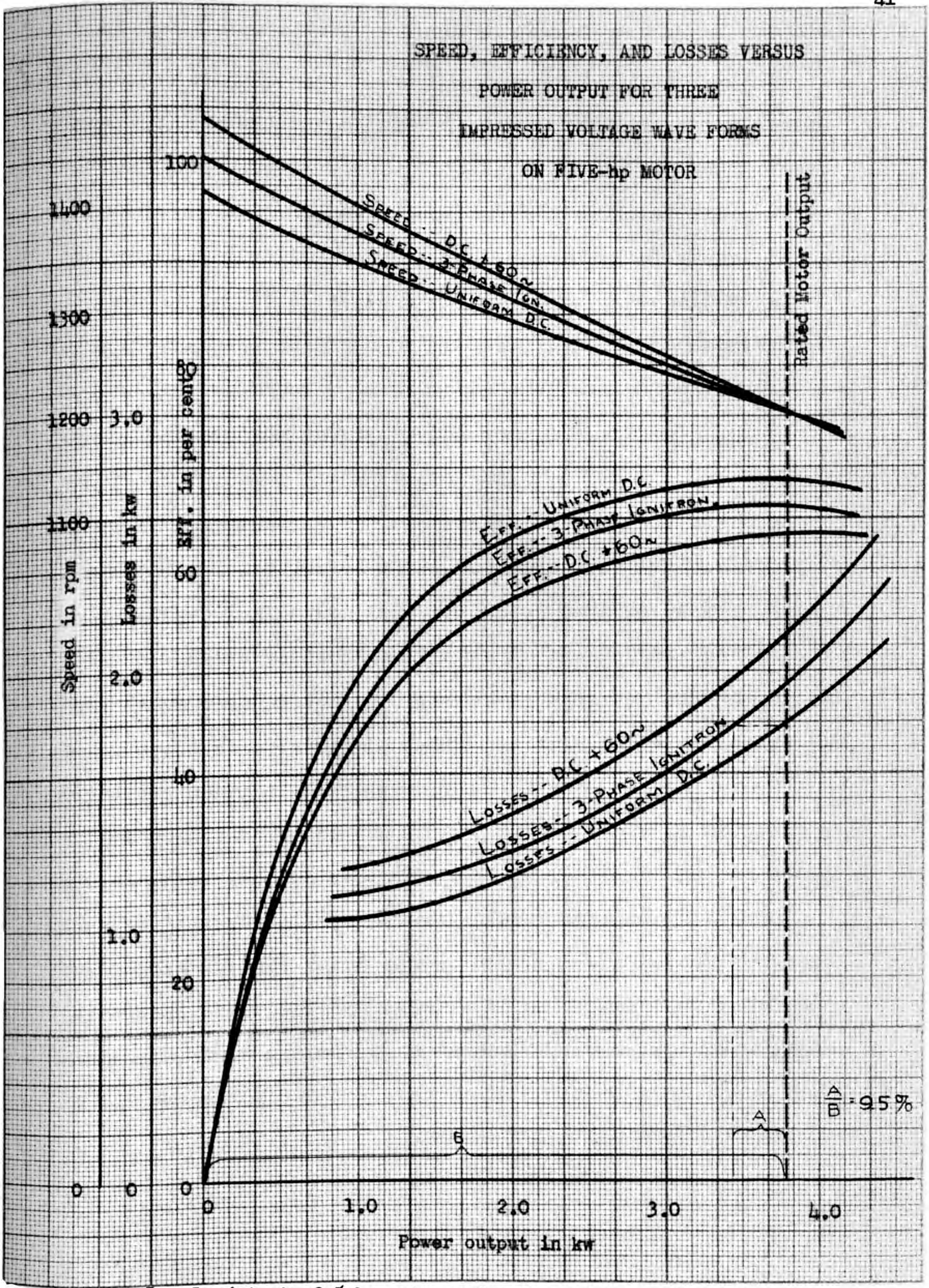

Fig. 5. Load test of 5-hp motor under three impressed voltage wave forms.

changes are necessary in output power in order to compensate for this increase in loss. Only by assuming something about the shape (i. e., slope) of the curve of losses versus output can we arrive at a theoretical result in this way. The difficulty is not as serious as it sounds, however; this is an approximate determination at best, and if the incremental losses due to the a. c. components are found, the loss curve can be assumed to conform to the condition:

$$P \text{ losses} = k \times (P \text{ output})$$

where k is a constant of proportionality. Since the loss curve is actually dished while the line resulting from the above assumption is not, the predicted decrease in output power necessary to decrease losses to their full-load, uniform voltage magnitude will be nearly equal to the adjustment which would be obtained by using the true curve of losses under the non-uniform impressed voltage.

A construction of the sort outlined above is carried out for the 5-hp. motor in Fig. 5; it is found that in order for the losses on three-phase ignitron operation to equal the losses at full rated output with uniform direct voltage impressed, only 90.5 per cent rated power may be taken from the motor, thus indicating a decrease in permissable output of 9.5 per cent.

Fifteen-Horsepower Motor Tests

It is immediately desirable to do the same thing for the large motor in order to check this value of power which may safely be taken from a shunt motor when it is operated on a three-phase ignitron. This is done in Fig. 6 which is the same as page 110 . The resulting decrease in permissable power output is found to be 9.83 per cent. It seems, then, that if a shunt motor is run on the output of a three-phase ignitron rectifier without filter, its rating must be reduced about ten per cent in order to ensure that overheating will not take place.

CONCLUSIONS

It was shown that knowledge of losses implies knowledge of efficiency. The attempt was made in this study to find a method which could be used to find out to what extent the losses in a direct current shunt motor increase when a non-uniform rectified wave form is impressed across the input to the motor.

A method was suggested whereby this might be done. It was found that that method would not give correct results for all wave forms, but could be used to find with acceptable

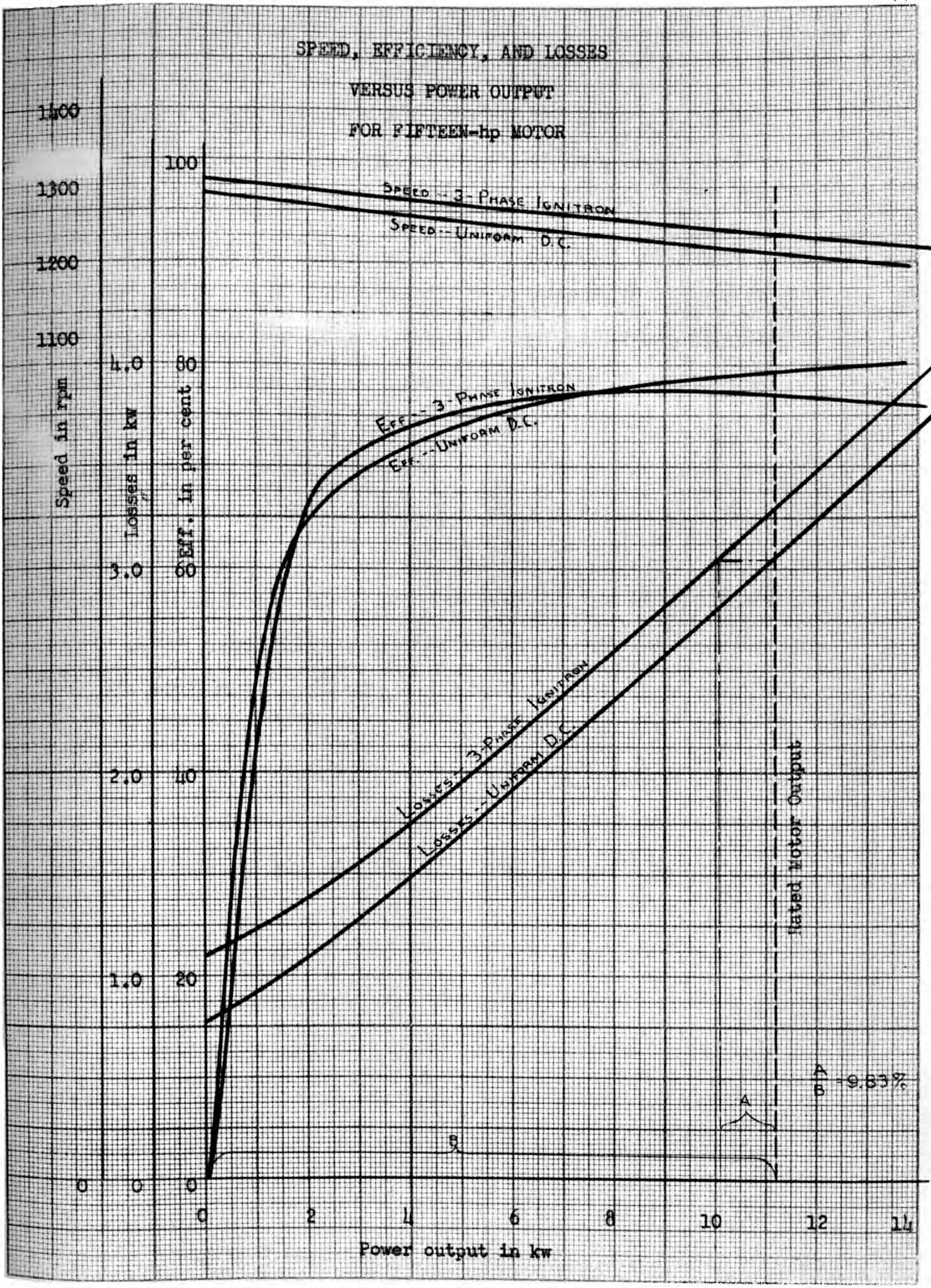

Fig. 6. Load test of 15-hp motor under two impressed voltage wave forms.

accuracy the increase in losses in different motors under the same rectified wave form; i. e., that given by an unfiltered three-phase ignitron set.

It was found further, that the increase in losses due to the presence of the alternating components in this impressed voltage is proportional to the rating of the machine. Hence, it is possible to state an approximate percentage increase in losses for all shunt motors. The figure suggested is ten per cent.

This means, of course, that if a motor is run from a three-phase ignitron set without filter, its true rating must be considered about 90 per cent of its name-plate rating.

Speed regulation was also examined, but no indication of the extent to which it depends upon wave form was found. Indications were that speed regulation may be expected to be greater for a rectified wave than for uniform impressed direct voltage, but the amount of that increase in regulation is less as the size of the machines is increased.

ACKNOWLEDGEMENT

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APPENDIX I

RESISTANCE, REACTANCE, AND INDUCTANCE OF SHUNT FIELD WINDING

Note on Meters

Since frequencies being measured are somewhat higher than those at which the laboratory meters are ordinarily used, it seems in order to inquire whether any error in readings is introduced by high frequencies.

To make correction for frequency, formulas are given in the carrying-cases of the instruments; a typical one is worked out here.

$$(2) \quad V \text{ true} = V \text{ read} \times (1 + .000,000,058\, f^2)$$

Now, if frequency is taken as 420 cycles, then f^2 is 172,400. When this is substituted into equation (2) above, the true voltage is obtained:

$$V \text{ true} = V \text{ read} \times (1 + .0098) = V \text{ read} \times (1 + .01)$$
$$= V \text{ read} \times (1.01)$$

So there is an error of not more than one per cent in reading any component of voltage with frequency lower than 420 cycles. This is sufficient accuracy so that it is not necessary to consider errors.

Tests of Shunt Field Winding

The shunt field of the small 5-hp. motor was tested, unsaturated, under impressed voltages from the harmonic generator in the E. E. Laboratory. Frequencies of 60-cycles, 120-cycles, 180-cycles, 300-cycles, and 420-cycles were thus obtained. Current, voltage, and power in the circuit of the shunt field were read; from these the reactance, resistance, and inductance were calculated.

Table 1.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment RESISTANCE, ETC. OF SHUNT FIELD WINDING **No.**

Test made by WAYNE B. SWIFT

(Leader) (Recorder) (Observers)

Rating of Apparatus Tested G. E. 5-hp, 230-v, 20.5-a **Lab. No.**

Date 4-6-50 **Wiring Checked by** **Data Checked by**

Meter No.	101	10C	219	66	356		
Scale Read					30w		
Current Range							
Voltage Range	.150	1	150	300			
	I_{ac}	I_{dc}	V_{ac}	V_{dc}	P	Freq.	VI_{ac}
1							
2	.076		150.5		6.8	52	11.44
3	.060		111.5		3.4	51	6.64
4	.071		144.5		5.8	58	10.27
5	.031		72.0		.9	119	2.24
6	.017		61.8		.3	180	1.05
7	.009		29			300	.26
8	.005		30			420	.15
9		.726		184		0	
10		.894		225		0	
11		.414		105		0	
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							
26							
27							
28							
29							
30							

	X	L	R
1			
2	1530	4.81	1178
3	1610	5.05	960
4	1678	4.59	1150
5	2125	2.85	937
6	3480	3.09	1040
7	3220	1.71	--
8	6010	2.28	--
9			253
10			252
11			253
12			
13			
14			
15			
16			
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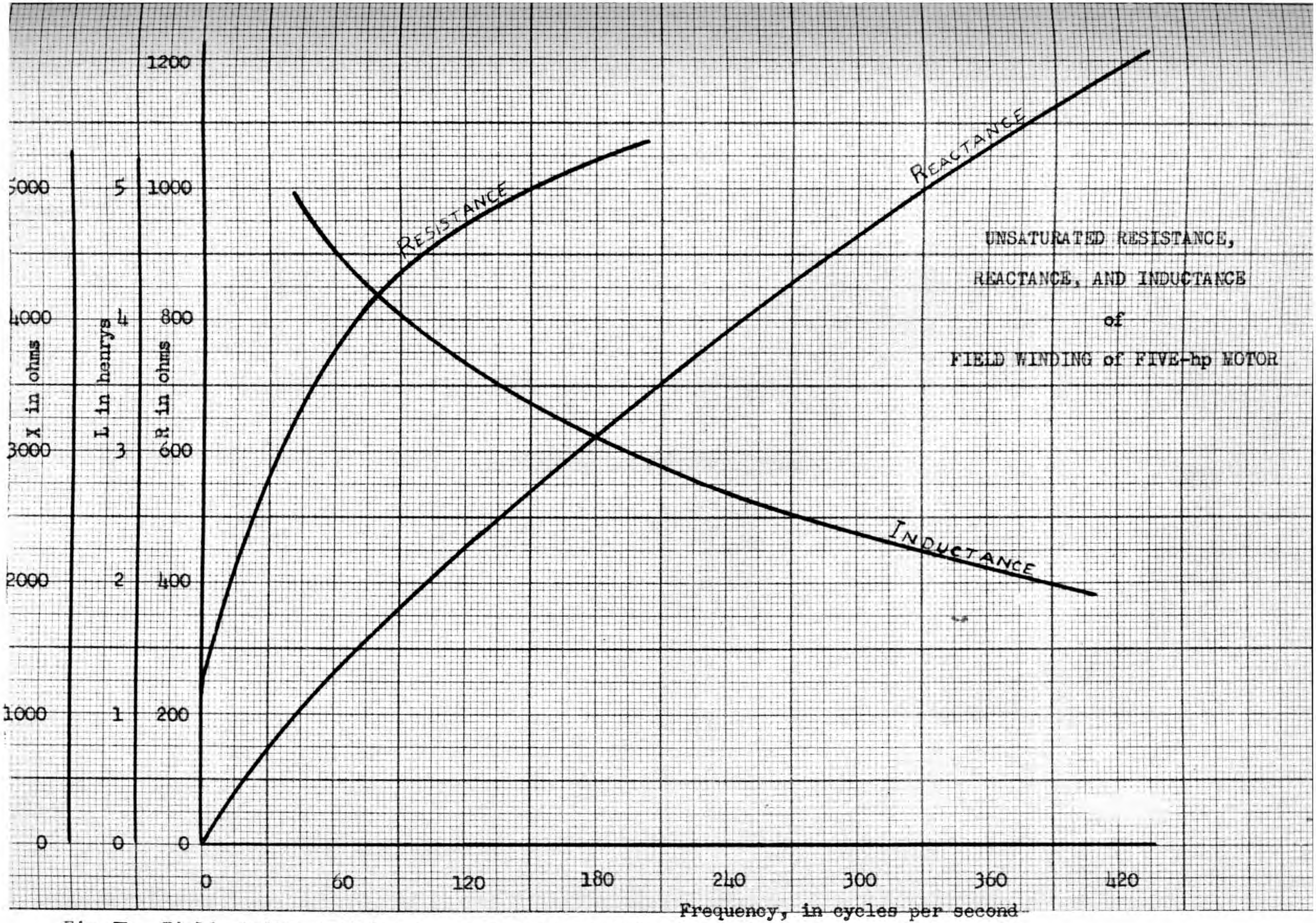


Fig. 7. Field winding resistance, reactance, and inductance.

APPENDIX II

THE CURRENT IN ARMATURE CONDUCTORS

For simplicity, it may be assumed that the complex wave of impressed voltage contains not an infinitude of frequency components, but only one. This frequency may be designated as f_1. It may also be assumed that the motor is rotating at a speed which induces an alternating voltage at a frequency f_2 in each individual armature conductor. Then, the current in an individual armature conductor will not vary at f_1 nor yet at f_2. It would completely change direction each 1/120th of a second if the motor were at rest and f_1 were 60-cycles. It would change direction each 1/40th of a second if f_1 were zero and the motor turned at a speed of 1200 rpm, assuming a 2-pole machine, which corresponds to 20 cycles per second.

In case the motor turned over once each time f_1 completed a full cycle, or, in the present case, at 3600 rpm, we find that the situation is different for each conductor in the armature.

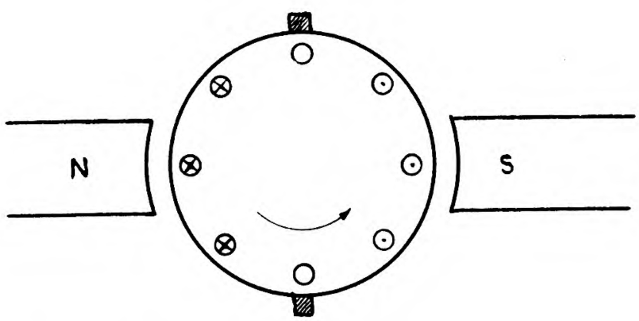


Fig. 8. Typical d. c. machine.

Whenever a conductor passes under a brush, the direction of current flowing in it is reversed. Thus, in an ordinary motor with uniform direct voltage impressed, current in each conductor is as shown in Fig. 9.

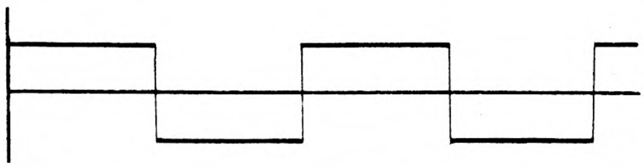


Fig. 9. Current in armature conductor.

Now, if the impressed voltage is not uniform, but is varying sinusoidally, the total current through the machine will also vary sinusoidally and the current in each conductor of the armature will therefore be the resultant of the wave of Fig. 9, and some sine wave of frequency f_1. This is shown in Fig. 10.

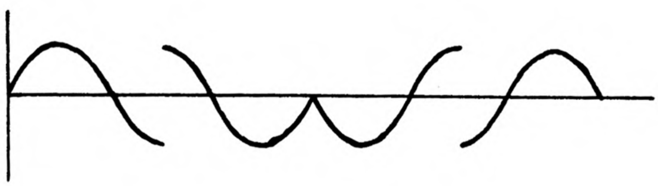


Fig. 10. Current in armature conductor.

The wave of current in each conductor of the armature thus contains numerous frequency components; the two frequencies f_1 and f_2, all odd harmonics of f_2, the sum and difference frequencies $(f_1 - f_2)$ and $(f_1 + f_2)$, and sum and difference frequencies representing the interaction of f_1 with each odd harmonic of f_2. Since none of these frequencies except f_1 are present in the input, it is necessary to consider the motor as a generator of all the other frequencies; a fact which greatly complicates the circuit analysis of the system. Also, even if it is assumed that the motor is driven at a speed for which f_2 equals f_1, these difficulties are not avoided, for although in some one coil or in two conductors the current will be as shown in Fig. 11, for all other conductors, commutation will <u>not</u> occur just as the total current passes through zero. Therefore, such a wave as Fig. 12 represents the current in most armature conductors, the angle θ representing the mechanical location on the armature of the conductor in question.

The consequence of this is that in any case, the current in any conductor of the armature of a direct current motor is a sharply varying affair with considerable discontinuous jumps (neglecting the inductance of that conductor).

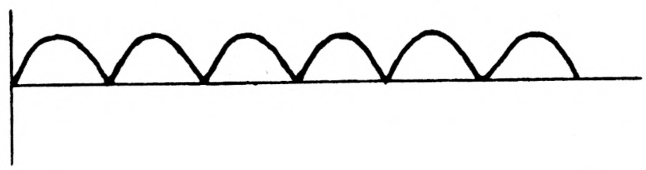


Fig. 11. Current in armature conductor.

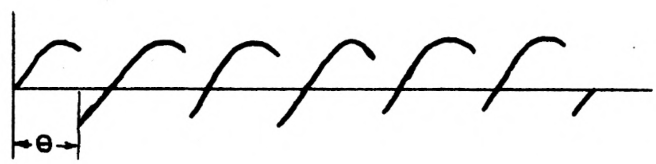


Fig. 12. Current in armature conductor.

Now, it can be seen from the above, that although we can find the inductance, yet, the transition from inductance to reactance, where the idea of reactance is as defined below, cannot be made.

This is true when reactance is thought of as something which, when divided into voltage, will give current as a result. The impressed voltage will, in general, contain no component at frequencies corresponding to the speed of commutation; yet currents will exist in the individual armature conductors at this frequency. The conclusion is that we must abandon the search for a value of armature reactance which will give the value of armature current at any frequency if divided into the voltage at that frequency.

It may be noted, however, that the degree of disturbance or the violence of fluctuation in current in each armature conductor is about the same whether in the uniform or the non-uniform case.

APPENDIX III

BRUSH CONTACT LOSS WITH NON-UNIFORM CURRENTS FLOWING

It is well known that in general

$$p = e \times i$$

where p, e, and i are all instantaneous.

If e is equal identically to a constant E, and i is any function, periodic with period "k" units of time, then

$$P \text{ average} = \frac{1}{k}\int_0^k E \times i \times dk$$

or

$$P \text{ average} = E \left\{ \frac{1}{k} \int_0^k i \times dk. \right\}$$

But

$$\frac{1}{k} \int_0^k i \times dk$$

is recognized as the average value of current. So

$$P \text{ average} = E \times I \text{ average}.$$

APPENDIX IV

THEORETICAL BREAKDOWN INTO FREQUENCY COMPONENTS OF THE TWO RECTIFIED WAVES CONSIDERED IN THIS STUDY

Single-Phase Ignitron Wave Form

In this analysis, each Fourier coefficient has been found by an individual integration. It is possible to make only one integration and derive thereby an algebraic expression for the coefficients A_n and B_n involving n. This method is used in calculation of the Fourier coefficients for the three-phase ignitron wave form (13, 14).

Despite the violence which such an assumption does to the mathematical nicety of the theory, A_0 will be assumed to be equal to the direct current, or zero-frequency component of the wave being analysed. The work in Churchill's book, loc. cit., assumes A_0 to be twice the zero-frequency component of the wave.

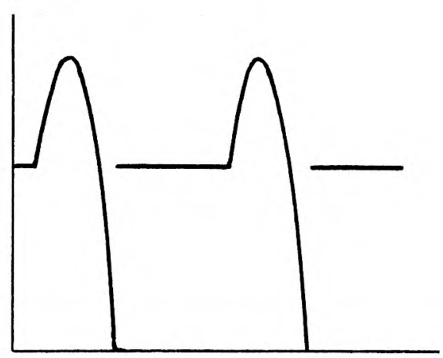


Fig. 13. Wave form from single-phase ignitron set.

Fig. 13 shows the voltage wave form actually encountered in running a motor from a single-phase ignitron supply.

To find harmonics, consider this wave as f (θ).

If impressed voltage is 220-v r. m. s., then, if V_{max} is 1.0, 220-v., instantaneous is 0.578. Average voltage (A_o) must then be 0.578.

Testing,

$$A_o = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\, d\theta = \frac{1}{2\pi}\quad \int_{.6175}^{\pi} \sin\theta\, d\theta + \int_{\pi}^{(2\pi + .6175)} .578\, d\theta$$

$$= \frac{1}{6.28}\left\{ - \left[\cos\pi - \cos .6175\right] + \left[\,6.8975 - 3.1416\right].578\right\}$$

$$= \frac{1}{6.28}\left\{\,1 + \left[.815 + 3.756\right] \times .578\right\}$$

$$= \frac{1}{6.28}\left\{1 + 2.64\right\} = 0.58$$

So the assumption of speed voltage is justified and it is safe to find other frequency components.

$$A = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin\theta\, d\theta$$

$$B = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos\theta\, d\theta$$

$$A_1 = \frac{1}{\pi}\left\{ \int_{\alpha}^{\pi} \sin^2 \theta \, d\theta + \int_{\pi}^{(2\pi + \alpha)} .578 \sin \theta \, d\theta \right\}, \alpha = .618$$

$$= \frac{1}{\pi}\left\{ \left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right|_{.618}^{3.14} - .578 \cos \theta \Big|_{3.14}^{6.28} \right.$$

$$\left. - .578 \cos \theta \Big|_{0}^{.618} \right.$$

$$= \frac{1}{3.14}\left\{ [1.57 - 0] - [.309 - .233] - .578 [1 + 1] \right.$$

$$\left. - .578 [.815 - 1] \right\}$$

$$= \frac{1}{3.14}\left\{ 1.57 - .309 + .233 - 1.156 + .1069 \right\}$$

$$= \frac{1}{3.14} \times .4449$$

$$= .1415$$

$$B_1 = \frac{1}{\pi}\left\{ \int_{\alpha}^{\pi} \sin \theta \cos \theta \, d\theta + \int_{\pi}^{6.28} .578 \cos \theta \, d\theta \right.$$

$$\left. + \int_{0}^{.618} .578 \cos \theta \, d\theta \right\}$$

$$= \frac{1}{3.14}\left\{ \left[\frac{1}{2} \sin^2 \theta \right|_{.618}^{3.14} + [.578 \sin \theta \Big|_{\pi}^{2\pi} + \Big|_{0}^{.618} \right.$$

$$= \frac{1}{3.14}\left\{- \frac{1}{2}\ (.578)^2 + .578\ (.578)\right\}$$

$$= \frac{1}{3.14}\left(\frac{.578^2}{2}\right) = .0589$$

$$A_2 = \frac{1}{\pi}\int_{o}^{2\pi} f\ (\theta)\ \sin 2\theta\ d\theta$$

$$= \frac{1}{\pi}\left\{ \int_{.618}^{3.14} \sin\theta\ \sin 2\theta\ d\theta + \int_{3.14}^{6.28} .578\ \sin 2\theta\ d\theta \right.$$

$$\left. + \int_{o}^{.618} .578\ \sin\theta\ d\theta \right\}$$

$$= \frac{1}{\pi}\left\{ \left[\frac{\sin\ (-\theta)}{-2} - \frac{\sin 30}{6}\ \right|_{.618}^{3.14} + \left[- \frac{.578}{2} \times \right.\right.$$

$$\left. \cos 2\theta \Big|_{3.14}^{6.28} \Big|_{o}^{.618} \right\}$$

$$= \frac{1}{\pi}\left\{ [\ o] - \left[\frac{-.578}{-2} - \frac{.960}{6}\right] + \left[\ \frac{-.578}{2}\ (.328 - 1)\right]\right\}$$

$$= \frac{1}{\pi}\left\{ - .289 + .160 + (-.289 \times -.672)\right\}$$

$$= \frac{1}{\pi}\ (.065) = .0207$$

$$B_2 = \frac{1}{\pi}\int_0^{2\pi} f\ (\theta)\ \cos\ 2\theta\ d\theta = \frac{1}{\pi}\int_{.618}^{3.14} \sin\ \theta\ \cos\ 2\theta\ d\theta$$

$$= \frac{1}{\pi}\left\{\left[-\ \frac{\cos\ -\theta}{-2} - \frac{\cos\ 3\theta}{6}\ \right|_{.618}^{3.14} + \frac{.578}{2}\left[\sin\ 2\theta\ \right|_{0}^{.618}\ \right\}$$

$$= \frac{1}{\pi}\left\{\left[-\ \frac{-1}{-2} - \frac{-1}{6}\right] - \left[-\ \frac{.815}{-2} - \frac{-.279}{6}\right]\right.$$

$$\left. + \left[\frac{.578}{2}\quad .944\right]\right\}$$

$$= \frac{1}{\pi}\left\{-\ .5 + .1677 - .4075 - .0465 + .2725\right\}$$

$$= \frac{1}{\pi}\ (-\ .5148)$$

$$= -\ .1638$$

Results are in the following table for fundamental and first harmonic of this wave.

Table 2. Fourier coefficients for single-phase ignitron wave.

Frequency	A	B
0	0.58	--
60	.1415	.0589
120	.0207	-.1638

The components represented by these coefficients are plotted in Fig. 14.

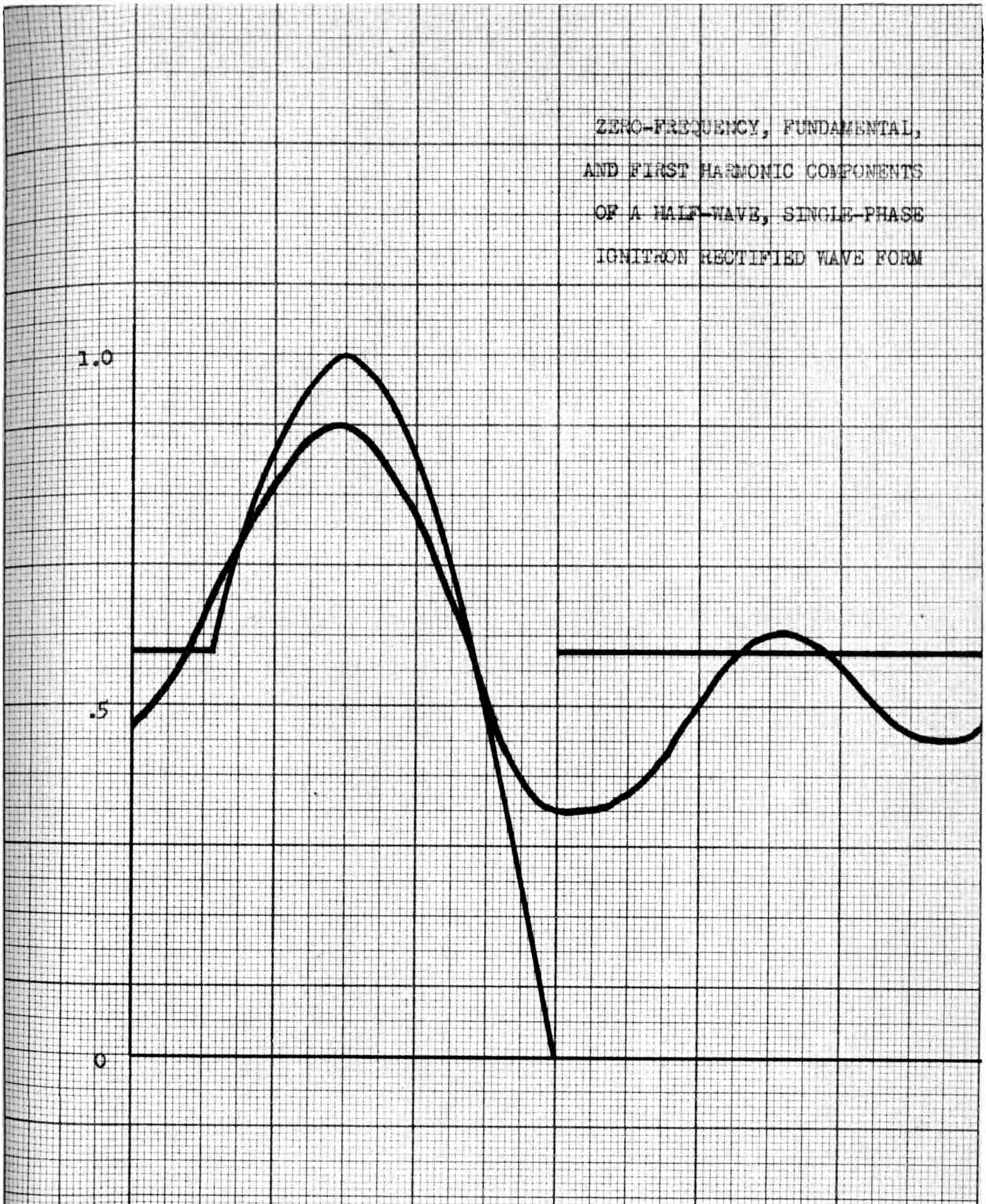


Fig. 14. Half-wave single-phase ignitron wave form.

Three-Phase Ignitron Wave Form

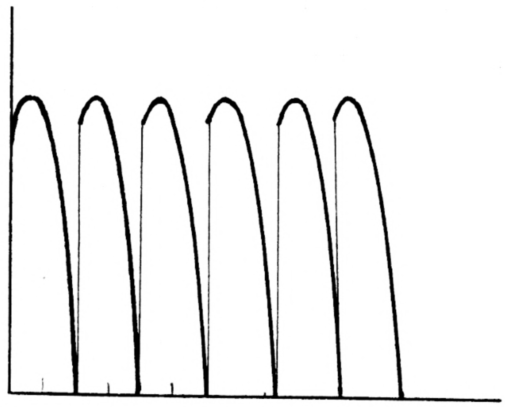


Fig. 15. Voltage wave form from three-phase ignitron set.

Fig. 15 shows the output of a three-phase ignitron set. Consider one cycle only, as in Fig. 16.

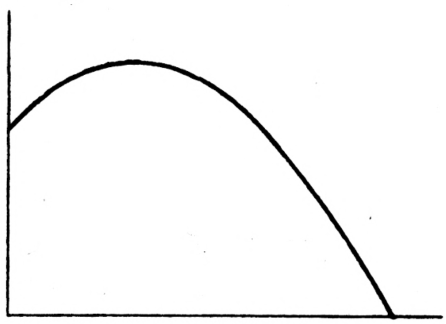


Fig. 16. Voltage wave form from three-phase ignitron set.

$$f(\theta) = \sin\left(\frac{\theta + \pi}{3}\right)$$

So

$$A_o = \frac{3}{2\pi}\int_0^{2\pi} \sin\left(\frac{\theta + \pi}{3}\right) \frac{1}{3} \; d\theta$$

$$= \frac{-3}{2\pi}\left[\cos \frac{\theta + \pi}{3}\right|_0^{2\pi} = -\frac{1}{2\pi}\left\{\cos \frac{3\pi}{3} - \cos\frac{\pi}{3}\right\}$$

$$= -\frac{3}{2\pi}\left\{(-1) - (\tfrac{1}{2})\right\} = -\frac{3}{2\pi}\left(-\frac{3}{2}\right) = \frac{9}{4\pi} = .715$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} \sin\frac{\theta + \pi}{3} \cos(n\theta)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi} \sin\left(\frac{\theta}{3} + \frac{\pi}{3}\right)\cos(n\theta)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi} \left[\sin\frac{\theta}{3}\cos\frac{\pi}{3} + \cos\frac{\theta}{3}\sin\frac{\pi}{3}\right]\cos n\theta \, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left[\frac{1}{2}\sin\left(\frac{1}{3}\theta\right)\cos(n\theta) + \frac{\sqrt{3}}{2}\cos\left(\frac{1}{3}\theta\right)\cos n\theta\right] d\theta$$

$$= \frac{1}{\pi}\left[\frac{1}{2}\left(-\frac{\cos\left(\frac{1}{3} - n\right)\theta}{2\left(\frac{1}{3} - n\right)} - \frac{\cos\left(\frac{1}{3} + n\right)\theta}{2\left(\frac{1}{3} + n\right)}\right|_0^{2\pi} +\right.$$

$$+\left(\frac{\sqrt{3}}{2}\left[\frac{\sin\left(\frac{1}{3}-n\right)\theta}{2\left(\frac{1}{3}-n\right)}+\frac{\sin\left(\frac{1}{3}+n\right)\theta}{2\left(\frac{1}{3}+n\right)}\right]\right)\Bigg|_{0}^{2\pi}$$

$$=\frac{1}{\pi}\left[\frac{1}{2}\left\{-\left(\frac{\cos\left(\frac{1}{3}-n\right)2\pi}{2\left(\frac{1}{3}-n\right)}+\frac{\cos\left(\frac{1}{3}+n\right)2\pi}{2\left(\frac{1}{3}+n\right)}\right)\right.\right.$$

$$\left.+\left(\frac{\cos 0}{2\left(\frac{1}{3}-n\right)}+\frac{\cos 0}{2\left(\frac{1}{3}+n\right)}\right)\right\}$$

$$+\frac{\sqrt{3}}{2}\left\{\left(\frac{\sin\left(\frac{1}{3}-n\right)2\pi}{2\left(\frac{1}{3}-n\right)}+\frac{\sin\left(\frac{1}{3}+n\right)2\pi}{2\left(\frac{1}{3}+n\right)}\right)\right.$$

$$\left.\left.-\left(\frac{\sin 0}{2\left(\frac{1}{3}-n\right)}+\frac{\sin 0}{2\left(\frac{1}{3}+n\right)}\right)\right\}\right]$$

$$=\frac{1}{2\pi}\left\{-\frac{\cos\left(\frac{2\pi}{3}-2\pi n\right)}{2\left(\frac{1}{3}-n\right)}-\frac{\cos\left(\frac{2\pi}{3}+2\pi n\right)}{2\left(\frac{1}{3}+n\right)}\right.$$

$$+\frac{1}{2\left(\frac{1}{3}-n\right)}+\frac{1}{2\left(\frac{1}{3}+n\right)}+\frac{\sqrt{3}\sin\left(\frac{2\pi}{3}-2\pi n\right)}{2\left(\frac{1}{3}-n\right)}$$

$$\left.+\frac{\sqrt{3}\sin\left(\frac{2\pi}{3}+2\pi n\right)}{2\left(\frac{1}{3}+n\right)}\right\}$$

Now, $\sin \phi = \sin(\phi + K \times 2\pi)$ where K = integer. So,

$$= \frac{1}{2\pi}\left(\left\{ - \frac{\left[\cos\frac{2\pi}{3}\right]\left[\left(\frac{1}{3}+n\right)+\left(\frac{1}{3}-n\right)\right]}{2\left(\frac{1^2}{3^2}-n^2\right)}\right\}\right.$$

$$\left. + \frac{\frac{1}{3}+n+\frac{1}{3}-n}{2\left(\frac{1^2}{3^2}-n^2\right)} + \frac{\left[\sqrt{3}\sin\left(\frac{2\pi}{3}\right)\right]\left[\frac{1}{3}+n+\frac{1}{3}-n\right]}{2\left(\frac{1^2}{3^2}-n^2\right)}\right)$$

$$= \frac{1}{2\pi}\left\{\frac{\left[-\left(-\frac{1}{2}\right)\times\frac{2}{3}\right]+\frac{2}{3}+\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{3}\right)}{2\left(\frac{1^2}{3}-n^2\right)}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{\frac{1}{3}+\frac{2}{3}+1}{2\left(\frac{1}{9}-n^2\right)}\right\} = \frac{2}{2\pi\left(\frac{1}{9}-n^2\right)2}$$

$$= \frac{1}{2\pi}\quad\frac{1}{\left(\frac{1}{9}-n^2\right)}$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi}\sin\frac{\theta+\pi}{3}\sin n\theta\, d\theta$$

Proceeding similarly, we obtain

$$= \frac{1}{\pi} \int_0^{2\pi} \left[\frac{1}{2} \sin \frac{1}{3}\theta \sin n\theta + \frac{\sqrt{3}}{2} \cos \frac{1}{3}\theta \sin n\theta \right] d\theta$$

$$= \frac{1}{\pi} \left[\frac{1}{2} \left\{ \left(\frac{\sin \left(\frac{1}{3} - n\right) 2\pi}{2\left(\frac{1}{3} - n\right)} - \frac{\sin \left(\frac{1}{3} + n\right) 2\pi}{2\left(\frac{1}{3} + n\right)} \right) \right. \right.$$

$$\left. - \left(\frac{\sin 0}{2\left(\frac{1}{3} - n\right)} - \frac{\sin 0}{2\left(\frac{1}{3} + n\right)} \right) \right\}$$

$$+ \frac{\sqrt{3}}{2} \left\{ \left(\frac{\cos \left(n - \frac{1}{3}\right) 2\pi}{2\left(\frac{1}{3} - n\right)} - \frac{\cos \left(n + \frac{1}{3}\right) 2\pi}{2\left(\frac{1}{3} + n\right)} \right) \right.$$

$$\left. \left. - \left(\frac{\cos 0}{2\left(\frac{1}{3} - n\right)} - \frac{\cos 0}{2\left(\frac{1}{3} + n\right)} \right) \right\} \right]$$

$$= \frac{1}{2\pi} \left[\left\{ \frac{\left(\sin\left(\frac{2\pi}{3} - 2\pi n\right)\right)\left(\frac{1}{3} + n\right) - \left(\sin\left(\frac{2\pi}{3} + 2\pi n\right)\right)\left(\frac{1}{3} - n\right)}{2\left(\frac{1^2}{3^2} - n^2\right)} \right\} \right.$$

$$+ \sqrt{3} \left\{ \frac{\left(\cos\left(-\frac{2\pi}{3} + 2n\right)\right)\left(\frac{1}{3} + n\right) - \left(\cos\left(\frac{2\pi}{3} + 2n\pi\right)\right)\left(\frac{1}{3} - n\right)}{2\left(\frac{1^2}{3^2} - n^2\right)} \right.$$

$$\left. \left. - \frac{\frac{1}{3} + n - \frac{1}{3} + n}{2\left(\frac{1^2}{3^2} - n^2\right)} \right\} \right]$$

$$= \frac{1}{2\pi}\left[\frac{\frac{\sqrt{3}}{2}\left(\left(\frac{1}{3}+n\right)-\left(\frac{1}{3}-n\right)\right)+\sqrt{3}\left[-\frac{1}{2}\left(\frac{1}{3}+n-\frac{1}{3}+n\right)\right]-2n}{2\left(\frac{1^2}{3^2}-n^2\right)}\right]$$

$$= \frac{1}{2\pi}\left[\frac{\frac{\sqrt{3}}{2}(2n)+\sqrt{3}\left(-\frac{2n}{2}\right)-2n}{2\left(\frac{1}{9}-n^2\right)}\right] = \frac{1}{2\pi}\left[\frac{2n\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}-1\right)}{2\left(\frac{1}{9}-n^2\right)}\right]$$

$$= \frac{1}{2\pi}\quad\frac{-n}{\frac{1}{9}-n^2} = -n\quad A_n$$

Now, making calculations in tabular form and remembering that $\frac{1}{2\pi} = .1593$, and $\frac{1}{9} = .1111$,

Table 3. Fourier coefficients for three-phase ignitron wave.

n	$\left(\frac{1}{9} - n^2\right)$	$\frac{1}{\frac{1}{9} - n^2}$	A_n	B_n
0			.715	----
1	- .889	-1.126	-.179	.179
2	- 3.889	- .257	-.0410	.082
3	- 8.889	- .1125	-.0179	.0537
4	-15.89	- .0630	-.0101	.0404
5	-24.9	- .0401	-.0064	.0320
6	-35.9	- .0279	-.0045	.0270

Table 3 (concl.)

n	$\left(\frac{1}{9} - n^2\right)$	$\frac{1}{\frac{1}{9} - n^2}$	A_n	B_n
7	-48.9	-.0205	-.0033	.0231
8	-63.9	-.0156	-.0025	.0200
			(Neglect all cosine terms in graph beyond n = 3)	(Neglect all sine terms in graph beyond n = 8)

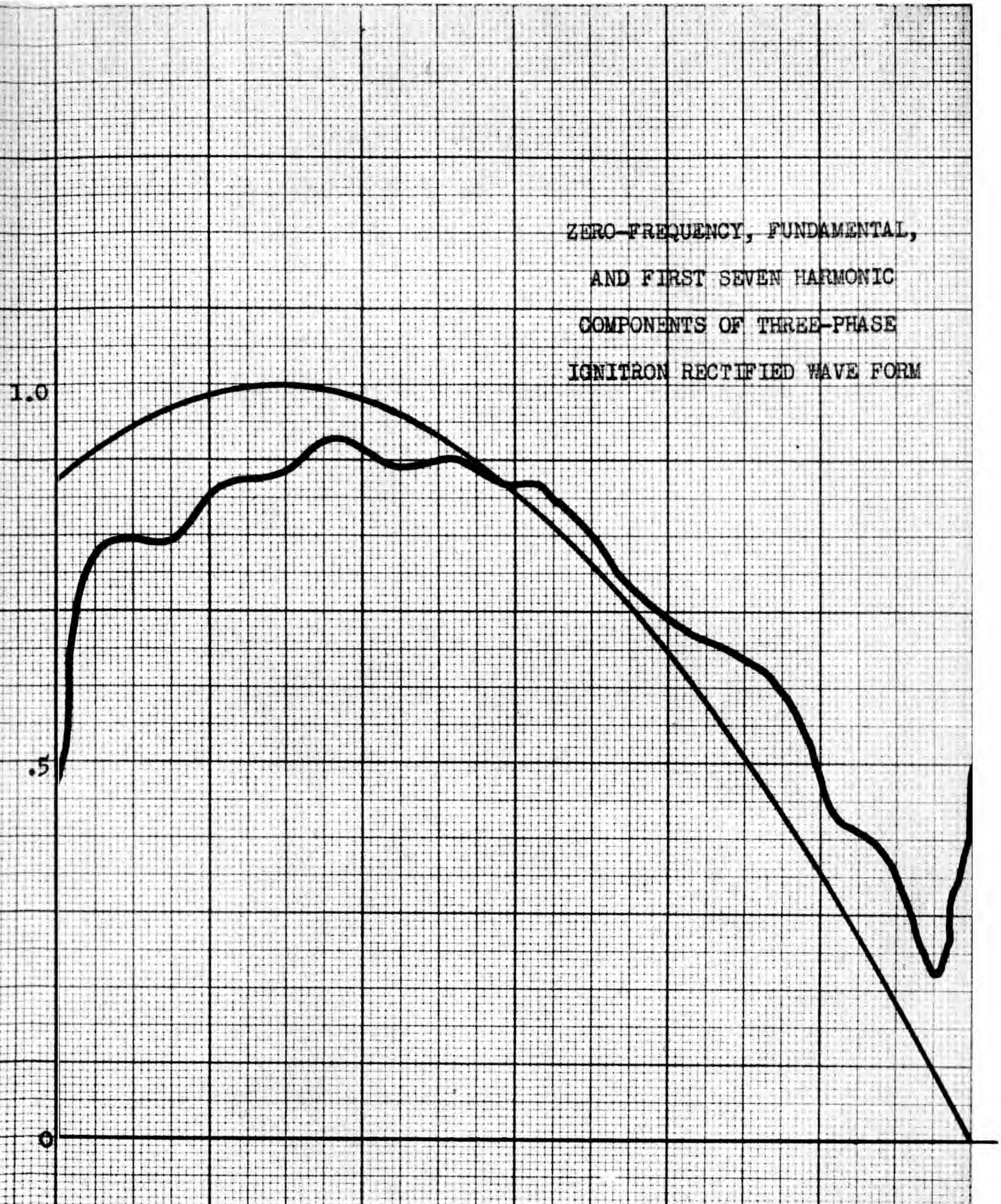


Fig. 17. Three-phase half-wave ignitron wave form.

APPENDIX V

LOAD TEST OF A FIVE-HORSEPOWER MOTOR UNDER UNIFORM DIRECT VOLTAGE IMPRESSED

Tests were made with source voltage adjusted at each step to the voltage of 220 volts, rated motor terminal voltage.

The wiring diagram was as shown in Fig. 18.

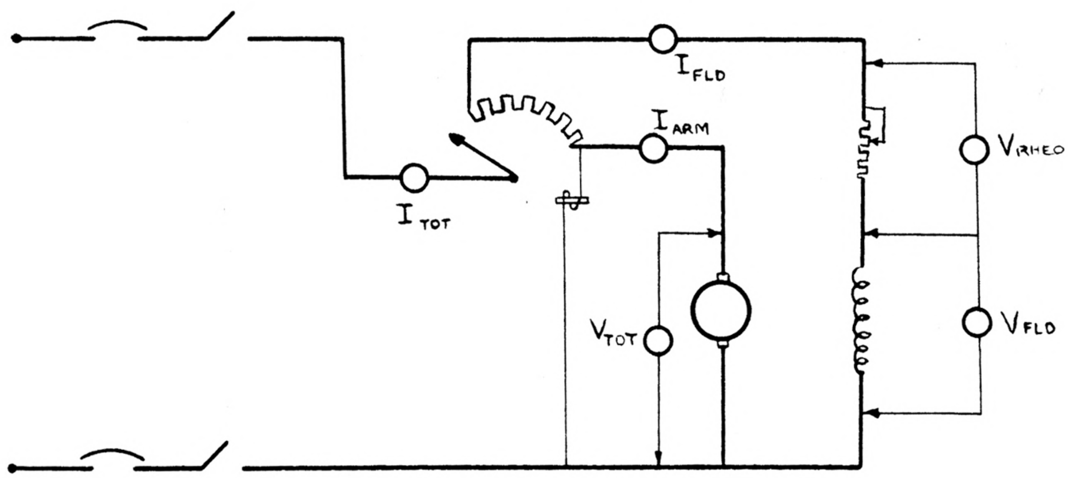


Fig. 18. Wiring diagram of a d. c. load test.

Data and calculations are shown on the following page.

Table 4.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment LOAD TEST--UNIFORM DIRECT VOLTAGE **No.**

Test made by WAYNE E. SWIFT
(Leader) (Recorder) (Observers)

Rating of Apparatus Tested G. E. 5-hp, 230-v, 20.5-a **Lab. No.**

Date 4-7-50 **Wiring Checked by** **Data Checked by**

	I_{fld}	I_{arm}	I_{tot}	V_{fld}	V_{rheo}	V_{tot}	VI_{tot}	P_{fld}	P_{arm}	P_{tot}	Speed	Load	P_{out}	Eff	P_{loss}
Meter No.	100	10A	10C	66	65	64		351 60w	330B 8kw	330A 8kw					
Scale Read															
Current Range	0.3	30	30	150	300	300									
Voltage Range															
1															
2	.19	27.2	27.4	60	161	220	6.04	9	6.16	6.16	1186	8.25	4.15	67.0	2.01
3	.19	25.1	25.2	60	160	220	5.54	9	5.64	5.64	1200	7.5	3.83	69.1	1.81
4	.19	18.5	18.6	60	159	218	4.05	9	4.08	4.08	1260	5.0	2.68	66.1	1.40
5	.19	12.6	12.8	60	160	219	2.80	9	2.80	2.80	1318	3.0	1.69	60.4	1.11
6	.19	8.6	8.8	60	161	220	1.94	9	1.96	1.96	1352	1.6	.92	47.5	1.04
7	.19	1.7	1.8	60	158	217	.39	9	.36	.36	1418	0	0	0	.36

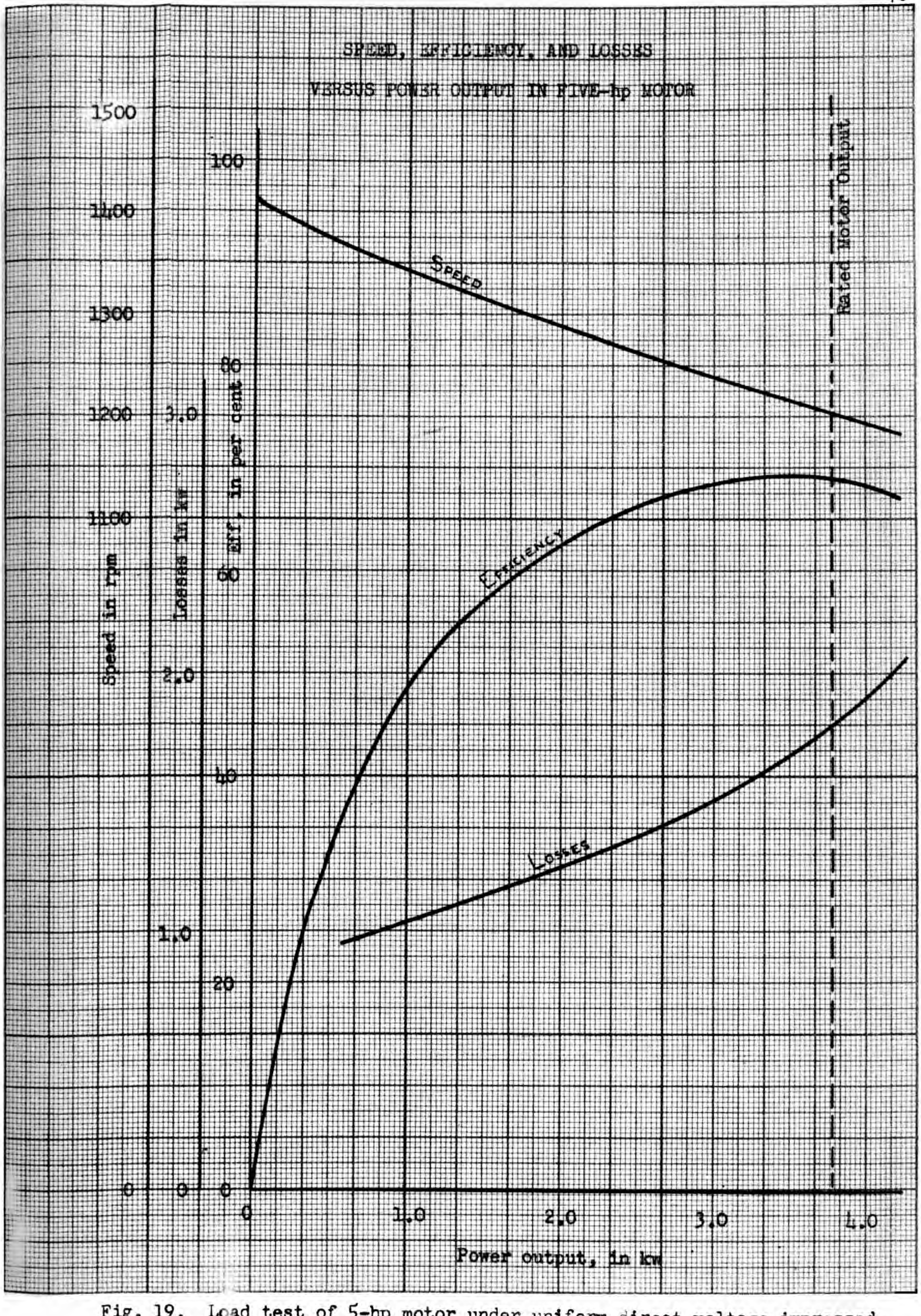


Fig. 19. Load test of 5-hp motor under uniform direct voltage impressed.

APPENDIX VI

LOAD TEST OF A FIVE-HORSEPOWER MOTOR UNDER IMPRESSED VOLTAGE WITH THREE-PHASE RECTIFIED WAVE FORM

In this case, two tests were made of the motor. One was with the ignitron setting adjusted in such a way as to give a terminal voltage of 220 volts at full rated load on the motor, and left at this setting throughout the test. This has some significance in the case of the ignitron because it leads to a condition of constant wave shape. When the firing angle of the ignitron is adjusted in order to adjust average voltage, the wave shape is altered somewhat and hence the harmonic structure of the wave is changed.

Even so, it was found that the adjustment of the ignitron firing control to maintain constant average motor terminal voltage caused only very slight changes in the shape of the wave impressed on the motor. The change in average voltage caused by the voltage regulation of the source, on the other hand, was very noticeable. It was decided, therefore, to consider only the test with maintained terminal voltage in analysing the results.

The question of exactly what are the wave forms of voltage and current throughout the machine is important. To answer the question, it is first important to remember that since the discussion involves a motor whose operation and ratings are already known in the d. c. case, we are primarily interested in the change in those characteristics. And most especially, we want to know how much the losses <u>at full-load</u> with whatever waves we have, exceed the losses at full-load with uniform d. c. impressed. Therefore, we are primarily interested in the wave forms of voltage and current in the motor when it is carrying approximately full-load.

These wave forms have been obtained and are shown in Plate III.

The wiring diagram for this test is shown in Plate IV. The physical setup in the laboratory is shown in Plates I and II.

Data are shown here; curves, however, are shown in the same sheets as the curves for the test under impressed voltage from a 220-volt d. c. generator in series with a 60-cycle alternator, page 89.

EXPLANATION OF PLATE III

(a) Armature current

(b) Armature voltage

(c) Field voltage

The wave form of total current is identical to (a).

The wave form of field current is a uniform direct current without harmonics.

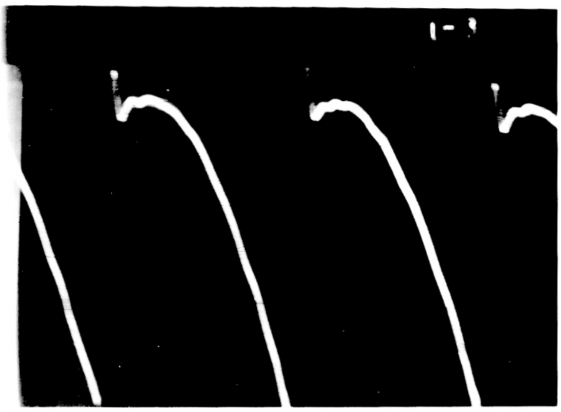
The wave form of total voltage is like (b).

The wave form of voltage across the field rheostat is shown in Fig. 20.

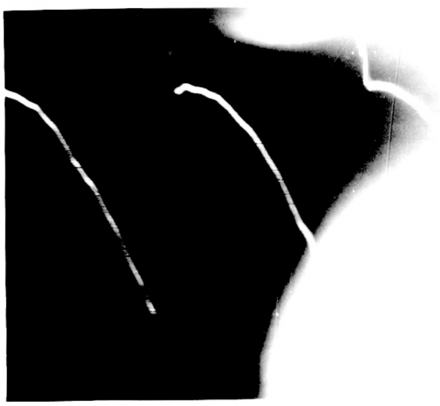
Fig. 20. Wave form of voltage across field rheostat.

PLATE III

(a)



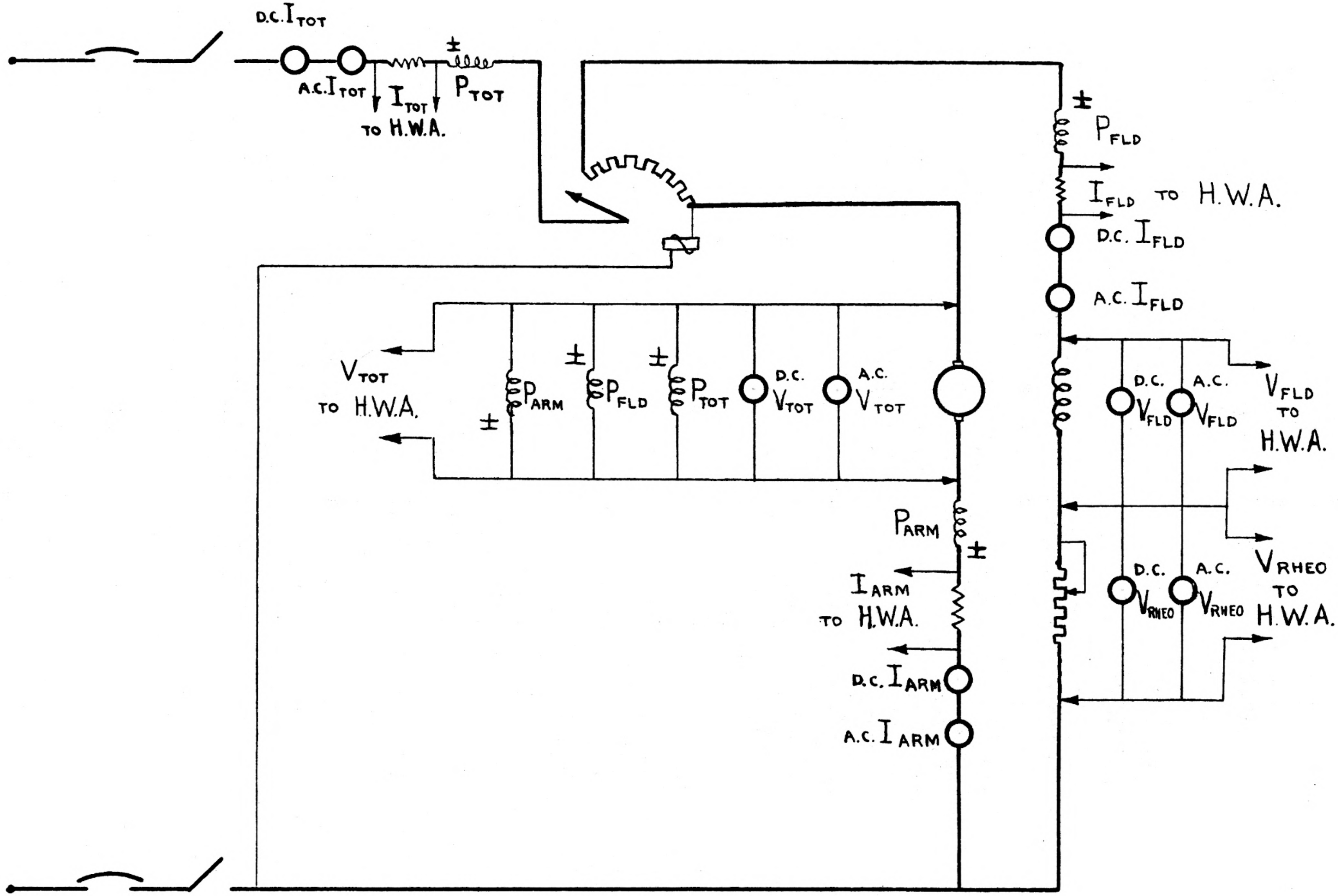
(b)



(c)

EXPLANATION OF PLATE IV

Wiring diagram for three-phase ignitron test.



D.C. I_{TOT}
A.C. I_{TOT}
I_{TOT}
TO H.W.A.
P_{TOT}
±
P_{FLD}
I_{FLD} TO H.W.A.
D.C. I_{FLD}
A.C. I_{FLD}
V_{TOT}
TO H.W.A.
P_{ARM}
P_{FLD}
P_{TOT}
D.C. V_{TOT}
A.C. V_{TOT}
D.C. V_{FLD}
A.C. V_{FLD}
V_{FLD} TO H.W.A.
P_{ARM}
I_{ARM}
TO H.W.A.
D.C. V_{RHEO}
A.C. V_{RHEO}
V_{RHEO} TO H.W.A.
D.C. I_{ARM}
A.C. I_{ARM}

PLATE IV

Table 5.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment LOAD TEST--3-ϕ IGN. RECTIFIED WAVE **No.**

Test made by WAYNE B. SWIFT

(Leader) (Recorder) (Observers)

Rating of Apparatus Tested G. E. 5-hp, 230-v, 20.5-a **Lab. No.**

Date 4-10-50 **Wiring Checked by** **Data Checked by**

	acI_{tot}	dcI_{tot}	acV_{tot}	dcV_{tot}	P_{tot}	$acVI_{tot}$	$dcVI_{tot}$
Meter No.	48	10C	219	71	330B		
Scale Read					8kw		
Current Range	40	30			40		
Voltage Range			300	300	200		
	29.5	28.9	247	220	6.60	7.29	6.35
	27.0	25.3	248	220	5.84	6.70	5.56
	21.8	20.3	249	220	4.60	5.43	4.46
	15.1	13.7	251	220	3.20	3.79	3.01
	10.0	8.5	247	219	2.00	2.47	1.86
	4.0	1.8	247	228	.48	.99	.41

Speed	Load	P_{out}	Eff	P_{loss}
1154	8.9	4.37	66.2	2.23
1200	7.5	3.82	65.4	2.02
1250	5.8	3.08	66.9	1.52
1327	3.3	1.93	60.2	1.27
1362	1.5	.87	43.5	1.13
1440	0	0	0	.48

HARMONIC ANALYSIS OF FULL-LOAD VOLTAGES AND CURRENTS

Frequency	180	360	540	720	900
I_{fld}	.0026	.0012	.0009	.0006	.0006
I_{arm}	7.10	1.44	.57	.37	.26
I_{tot}	7.35	1.44	.63	.39	.20
V_{fld}	66	30	19	14	11
V_{rheo}	2.2	1.0	.75	.50	.50
V_{tot}	95	41.8	26.5	17	13

APPENDIX VII

LOAD TEST OF A FIVE-HORSEPOWER MOTOR UNDER VARIOUS OTHER IMPRESSED VOLTAGE WAVE FORMS

Impressed Voltage Derived from D. C. Generator in Series with 60-Cycle Alternator

Wiring diagram for this test is the same as that for three-phase ignitron test, page 82.

Curves are shown in Fig. 21 which also shows curves for the operation of the motor under (a) uniform direct voltage input, and (b) three-phase ignitron input.

Wave forms actually impressed on the motor while it was carrying full rated load are shown in Plate V.

EXPLANATION OF PLATES V AND VI

(a) Armature current (full-load)

(b) Armature voltage

(c) Field voltage

The wave form of total current is like (a).

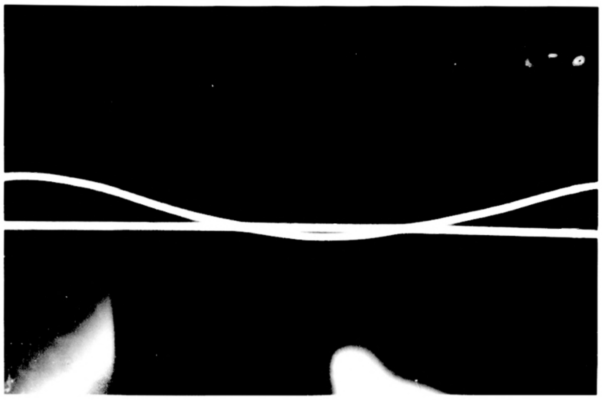
The wave form of field current is a uniform direct current without harmonics.

The wave form of total voltage is like (b).

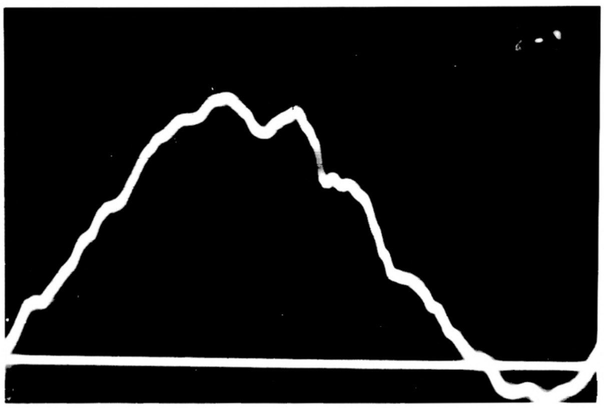
The wave form of voltage across the field rheostat is shown in Plate VI (a).

No-load armature current is shown in Plate VI (b), cf. Plate V (a). Note magnitude of a. c. component practically unchanged; reference simply shifted.

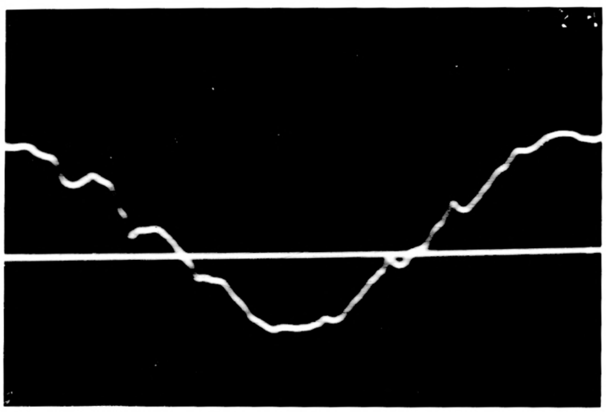
PLATE V



(a)

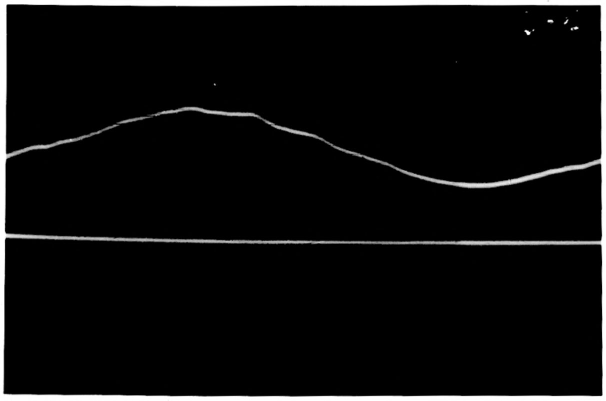


(b)

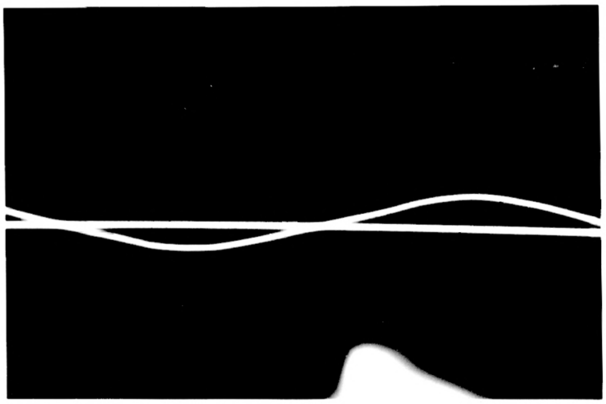


(c)

PLATE VI



(a)



(b)

Table 6.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment LOAD TEST--D. C. PLUS 60 CYCLES No.

Test made by WAYNE B. SWIFT (Leader) (Recorder) (Observers)

Rating of Apparatus Tested G. E. 5-hp, 230-v, 20.5-a Lab. No.

Date 5-29-50 Wiring Checked by Data Checked by

	acI_{tot}	dcI_{tot}	acV_{tot}	dcV_{tot}	P_{tot}	$dcVI_{tot}$	Load	Speed	P_{out}	Eff	P_{loss}
Meter No.	48	100	219	71	330B						
Scale Read					8kw						
Current Range	40	30			40						
Voltage Range			300	300	200						
Run # 1 -- With dcV_{tot} counterfeiting situation on single-phase ignitron											
	17.9	4.2		174	1.36		.1	1380	.059	4.32	1.30
	21.4	14.4		152	3.60		1.2	1200	.614	16.88	2.99
With further increases in load, motor stalled.											
Run # 2 -- With dcV_{tot} continually adjusted to 220 volts											
	32	27.7	231	220	6.68	6.10	8.5	1170	4.24	63.4	2.44
	29	24.6	230	218	5.88	5.36	7.3	1200	3.73	63.5	2.15
	24	19.6	230	220	4.68	4.27	5.5	1276	2.94	61.5	1.74
	15.8	10.5	234	220	2.76	2.29	2.5	1367	1.45	52.6	1.31
	9.4	1.9	234	220	.64	.41	0	1484	0	0	.64

HARMONIC ANALYSIS OF VOLTAGES AND CURRENTS UNDER CONDITIONS OF LINE 5 ABOVE

Frequency	60
I_{fld}	.0036
I_{arm}	11.4
I_{tot}	11.4
V_{fld}	58
V_{rheo}	27
V_{tot}	93

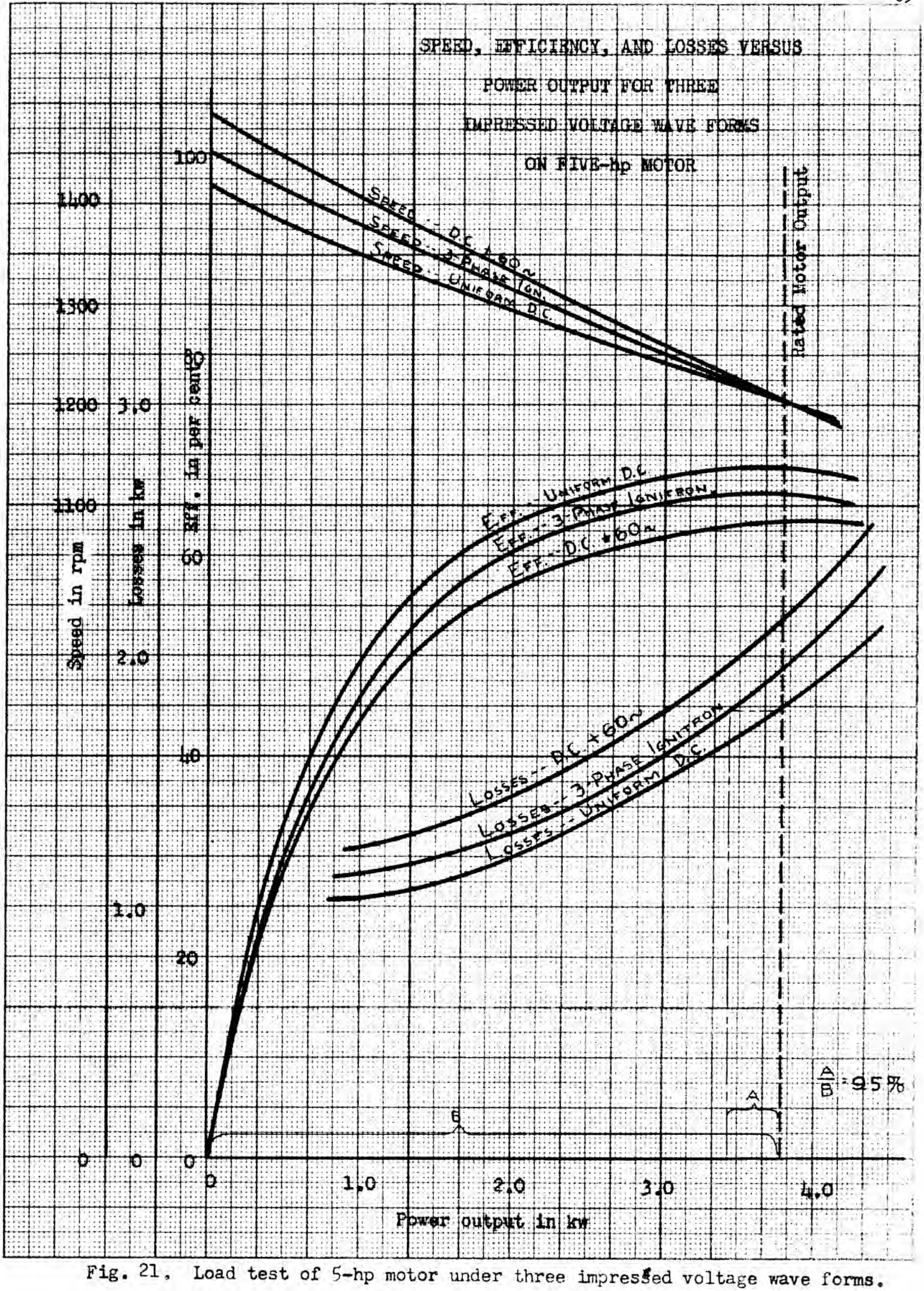


Fig. 21. Load test of 5-hp motor under three impressed voltage wave forms.

Impressed Voltage from Single-Phase Ignitron

The wiring diagram for this test is the same as that for the three-phase ignitron test, page 82.

Wave forms encountered are shown in Plates VII and VIII.

It was found to be impossible to carry load beyond about 25 to 30 per cent rated power output in this test. When that point was exceeded, the motor stalled. One factor, probably the controlling one, which leads to this situation was mentioned in the earlier discussion of this test. Another factor which also tends to limit the output of the machine, however, is inherent in the ignitron set. The basic circuit of the ignitron is shown in Fig. 22.

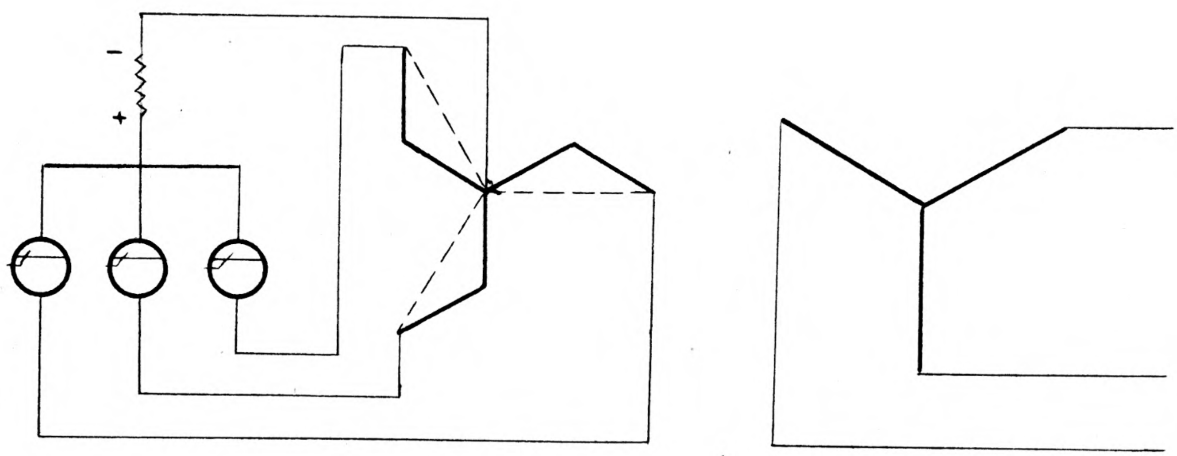


Fig. 22. Circuit of ignitron set.

It can be seen from the figure that the ignitron set includes a transformer which changes voltage from supply line voltage to some voltage suited to the ignitron tubes and the d. c. voltage desired at the output of the ignitron set. In order to eliminate any effects of saturation in the cores of the transformers which might result from the unidirectional currents which flow, a zig-zag transformer connection is used. Fig. 23 illustrates how, when the set is operating normally, this connection balances out any effect of the unidirectional character of the current.

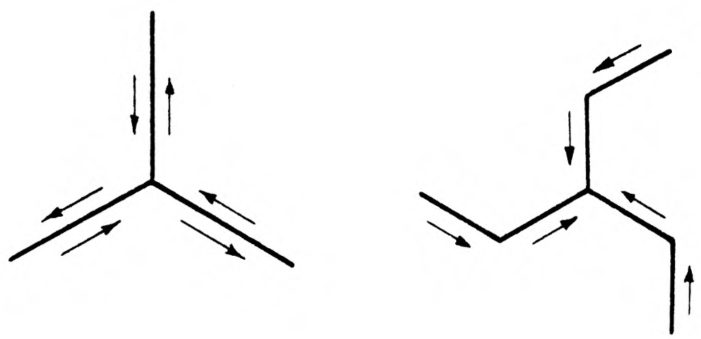


Fig. 23. Currents in ignitron set.

In the test on single-phase ignitron input, however, the normal connection is not used. Instead, two of the ignitrons are disconnected. The basic circuit is then as in Fig. 24.

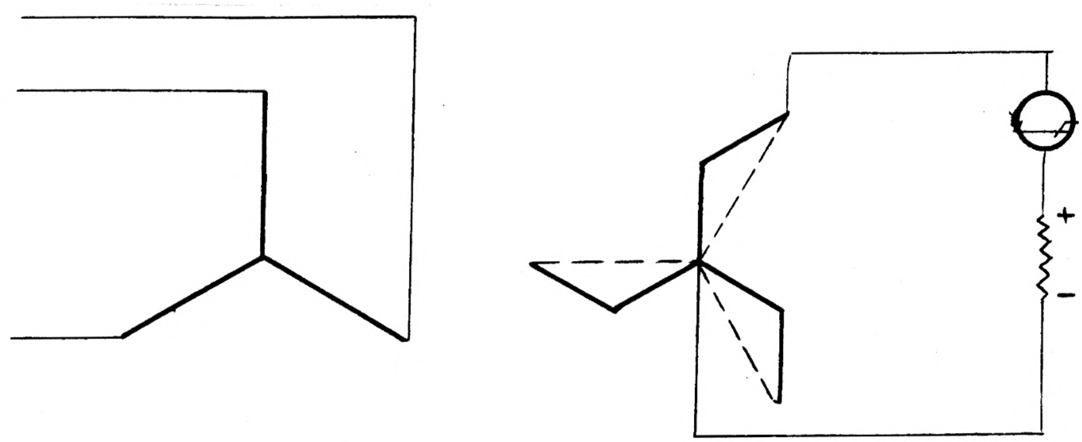


Fig. 24. Single-phase ignitron set.

If the circuit is as shown in Fig. 24, then examination of the current in the transformers shows that the effects of unidirectional currents are not balanced out, and some trouble with saturation in the cores may be encountered. This is shown in Fig. 25.

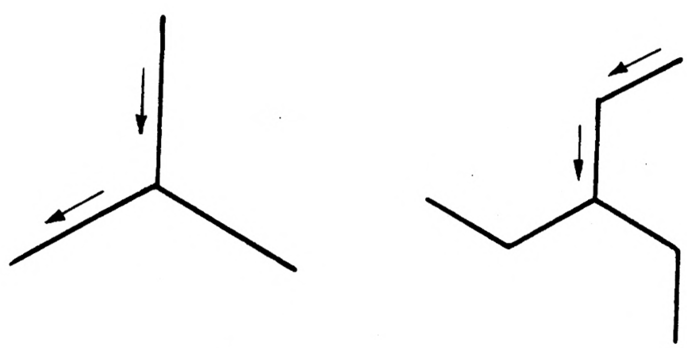


Fig. 25. Currents in single-phase ignitron set.

If the core is saturated, of course, the leakage reactance is effectively increased so less current may be expected to flow in the secondary which is the load circuit for the same load impedance and primary voltage. This will have the effect of reducing the secondary current necessary to trip the overload equipment in the primary circuit. Thus the maximum output current is effectively decreased.

EXPLANATION OF PLATES VII AND VIII

(a) No-load armature voltage.

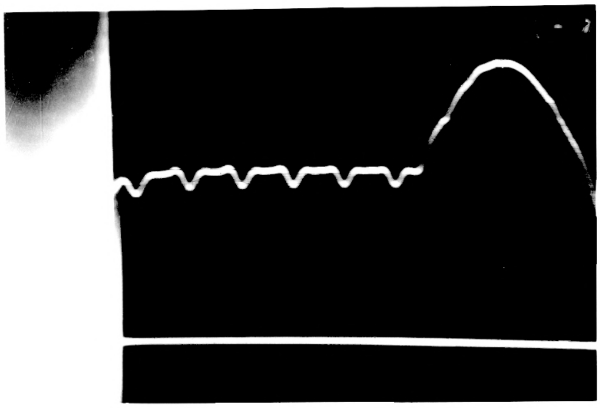
(b) Loaded armature voltage.

(c) Field rheostat voltage, both loaded and unloaded.

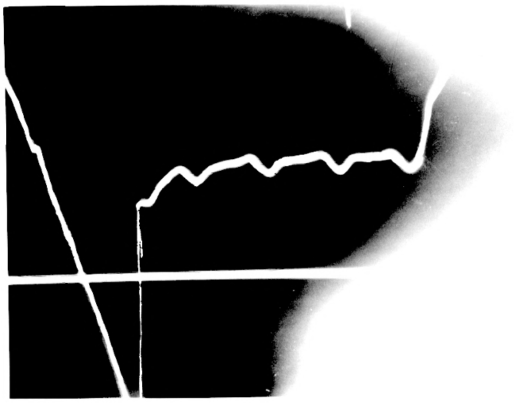
Total voltage like (a) and (b).

Total current, both loaded and unloaded, like (a) of Plate V. Armature current, both loaded and unloaded, shown in (a) of Plate V. Field current free from harmonics, both loaded and unloaded.

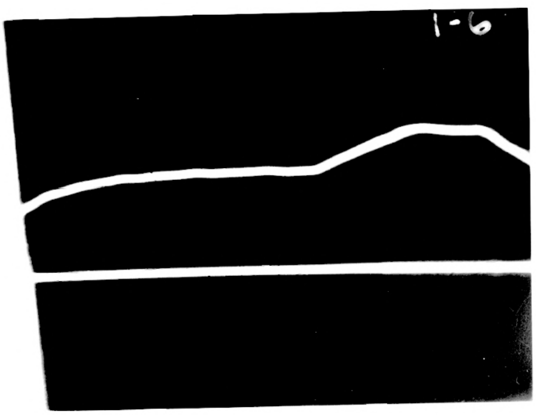
PLATE VII



(a)

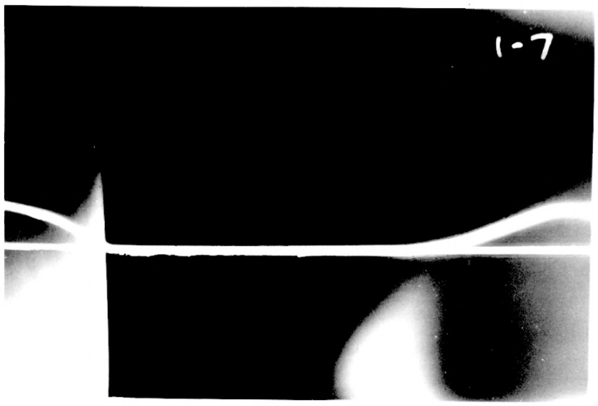


(b)



(c)

PLATE VIII



(a)

It was decided that the average terminal voltage changed so radically in magnitude as load was changed that no useful purpose would be served by plotting the results of the test. Data are given for the test, however. The information on harmonic structure of the wave reveals a very considerable change as load is increased on the motor. Attention is called to the two wave forms of terminal voltage shown in Plate VII. It is clear from them that a considerable change in harmonic structure must have taken place.

Table 7.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment LOAD TEST--1-ϕ IGN. RECTIFIED WAVE No.

Test made by WAYNE B. SWIFT (Leader) (Recorder) (Observers)

Rating of Apparatus Tested G. E. 5-hp, 230-v, 20.5-a Lab. No.

Date 4-27-50 Wiring Checked by Data Checked by

		1	2	3	4	5	6	7	8	9	10	11	12	13
	Meter No.	48		10C		219		71		330B				
	Scale Read									4kw				
	Current Range	40		30						20				
	Voltage Range					300		300		200				
		acI_{tot}		dcI_{tot}		acV_{tot}		dcV_{tot}		P_{tot}		Load		Speed
2		6.5		4.0				224		.804		.1		1540
3		17.4		11.7				160		2.10		1.2		1200
4		Any increase in load beyond this point caused motor to stall.												
6		HARMONIC ANALYSIS OF VOLTAGES AND CURRENTS												
8		Frequency			60		120		180		240		300	
10		(a) Conditions of line 2 above												
11		I_{fld}												
12		I_{arm}												
13		I_{tot}												
15		V_{fld}			20		25		15		6		8	
16		V_{rheo}			11.5		11		5		1.5		1.8	
17		V_{tot}			33		38		21		7.5		10.5	
19		(b) Conditions of line 3 above												
20		I_{fld}			.0025		.0022		.0007		.0006		.0004	
21		I_{arm}			7.8		4.5		1.5		.48		.36	
22		I_{tot}			7.9		4.5		1.5		.45		.30	
24		V_{fld}			41		39		12		14		7.4	
25		V_{rheo}			19		14		3		3		1.5	
26		V_{tot}			62		58		16		17.5		9.5	

	1	2	3	4	5
	P_{out}		Eff		P_{loss}
2	.066		8.15		.738
3	.614		29.3		1.49
8	360		420		
20	.0004		.0004		
21	.28		.14		
22	.27		.12		
24	8.2		4.9		
25	1.5		.5		
26	10.0		6.0		

Impressed Voltage from D. C. Generator in Series with 180-Cycle Alternator

Results from this test are not very satisfactory; as mentioned before, the current rating of the 180-cycle alternator did not permit testing at any loads beyond rather light ones. However, wave forms were obtained which tend to show the changes which take place as load is increased under such a wave form as this. They are shown in Plate IX and X.

Data are also presented for reference, but no curves are drawn because the data fall so far short of reaching full load on the motor.

EXPLANATION OF PLATES IX AND X

All Wave Forms with Approximately 50 Per Cent
Rated Load on Motor

(a) Armature current.

(b) Armature voltage.

(c) Field voltage.

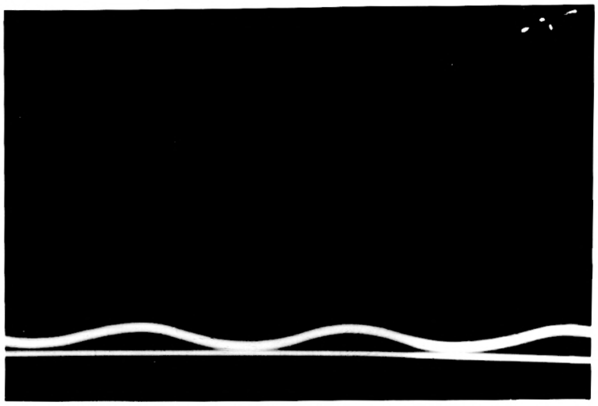
Total voltage is like (a).

Total current is like (b).

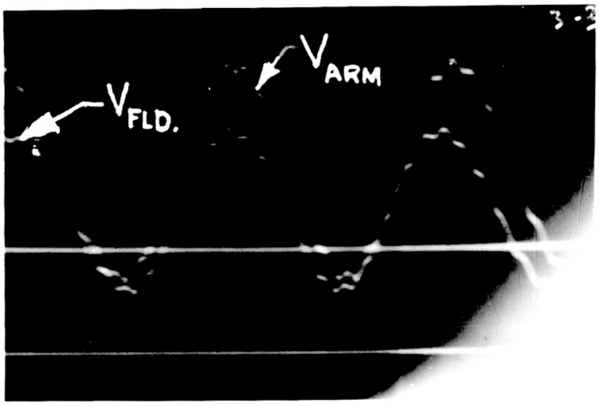
Field Current is free from harmonics.

(a) Voltage across field rheostat is shown in (a) of Plate X.

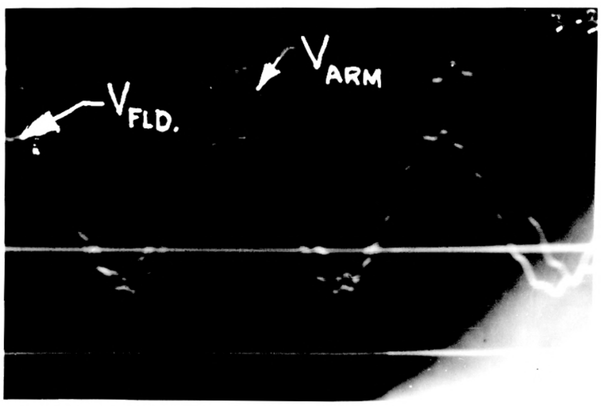
PLATE IX



(a)

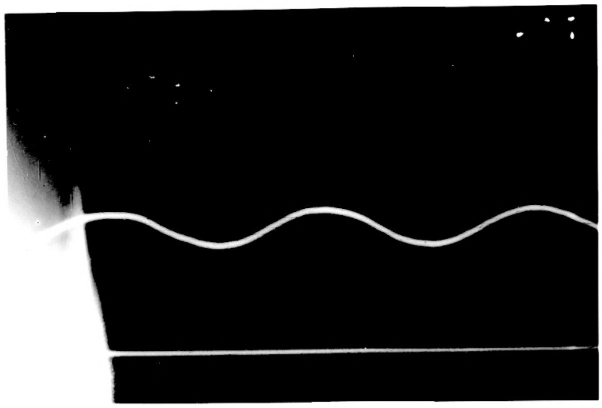


(b)



(c)

PLATE X



(a)

Table 8.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment LOAD TEST--D. C. PLUS 180 CYCLES No.

Test made by WAYNE B. SWIFT
(Leader) (Recorder) Observers)

Rating of Apparatus Tested G. E. 5-hp, 230-v, 20.5-a Lab. No.

Date 4-26-50 Wiring Checked by Data Checked by

	1	3	5	7	9
Meter No.	48	10C	219	71	330B
Scale Read					4kw
Current Range	40	30			20
Voltage Range			300	300	200

	acI_{tot}	dcI_{tot}	acV_{tot}	dcV_{tot}	P_{tot}	Load	Speed	P_{out}	Eff	P_{loss}
1										
2	4.70	2.1		230	.72	.1	1360	.058	8.0	.66
3	9.75	6.9		222	2.00	1.5	1290	.79	39.5	1.21
4	14.50	10.8		218	3.12	3.0	1280	1.57	50.4	1.55

5 Increases in load beyond this point caused current in the 180-cycle alternator to rise unduly

7 HARMONIC ANALYSIS OF VOLTAGES AND CURRENTS UNDER CONDITIONS OF LINE 5 ABOVE

9	Frequency	180
11	I_{fld}	.0036
12	I_{arm}	4.8
13	I_{tot}	4.9
15	V_{fld}	67
16	V_{rheo}	17
17	V_{tot}	88

APPENDIX VIII

LOAD TEST OF A FIFTEEN-HORSEPOWER MOTOR UNDER UNIFORM DIRECT VOLTAGE IMPRESSED

The motor was run under a constant terminal voltage of 220 volts supplied from the powerhouse. This voltage was maintained at constant 220 volts by adjusting the line drop in the line between the E. E. Laboratory and the powerhouse. This was done by varying a static rheostat load at the 220-volt bus in the E. E. Laboratory.

The motor was loaded by means of an alternator directly coupled to its shaft. The alternator was excited and was loaded by a system of rheostats. Load on the motor was then varied by varying the load on the alternator. Since data on the alternator was known from previous tests on an identical machine at the University of Nebraska; it is shown in Fig. 26; a curve was constructed which plotted power input to the alternator versus power output from the alternator. This curve is shown in Fig. 27. Power output from the alternator was read by wattmeters and then the power input to the alternator, which was also the power output from the motor, was read from this curve, Plate XII.

The wiring diagram of the motor is shown in Appendix IX, Fig. 28.

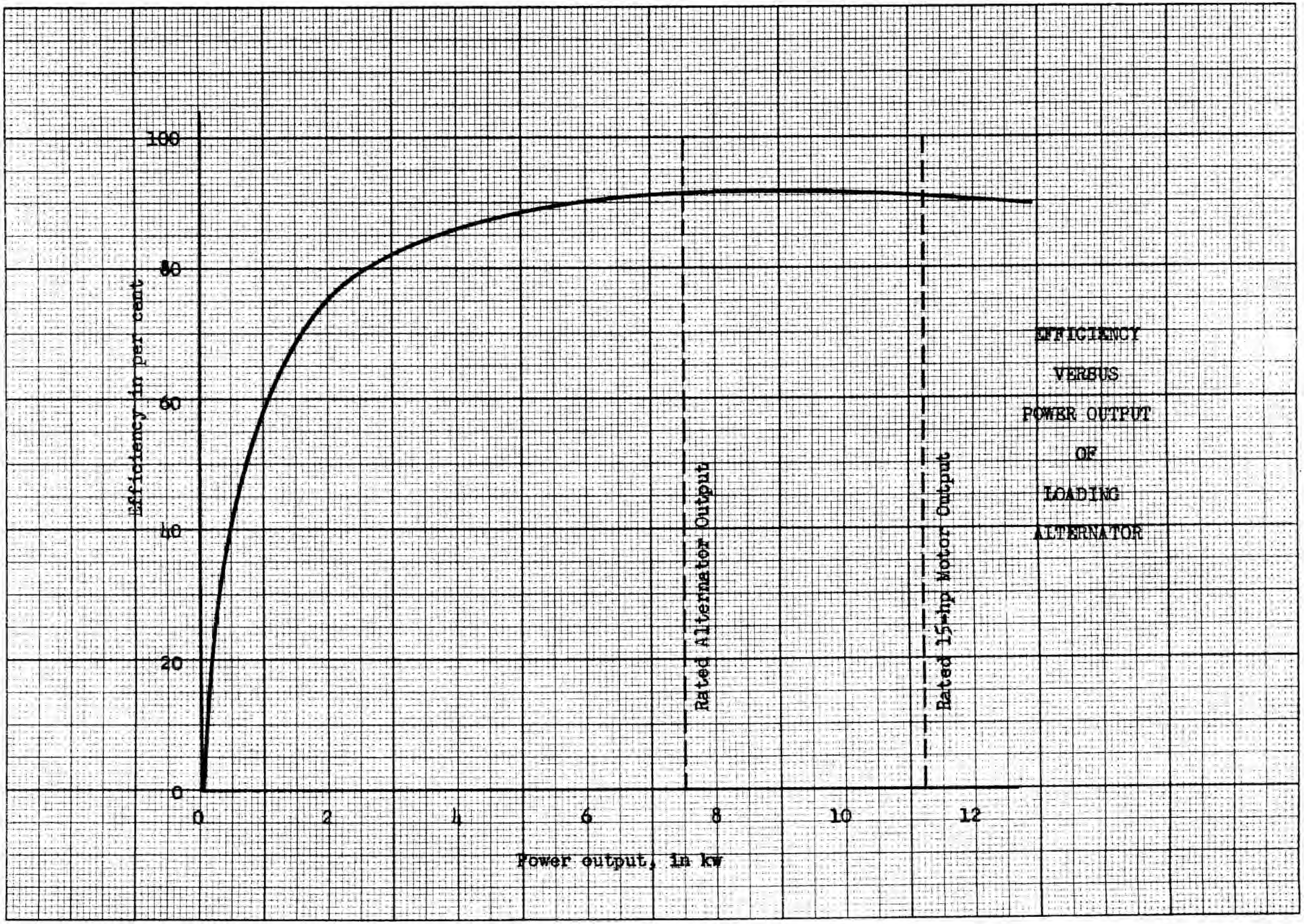


Fig. 26. Efficiency of loading alternator in test of 15-hp motor.

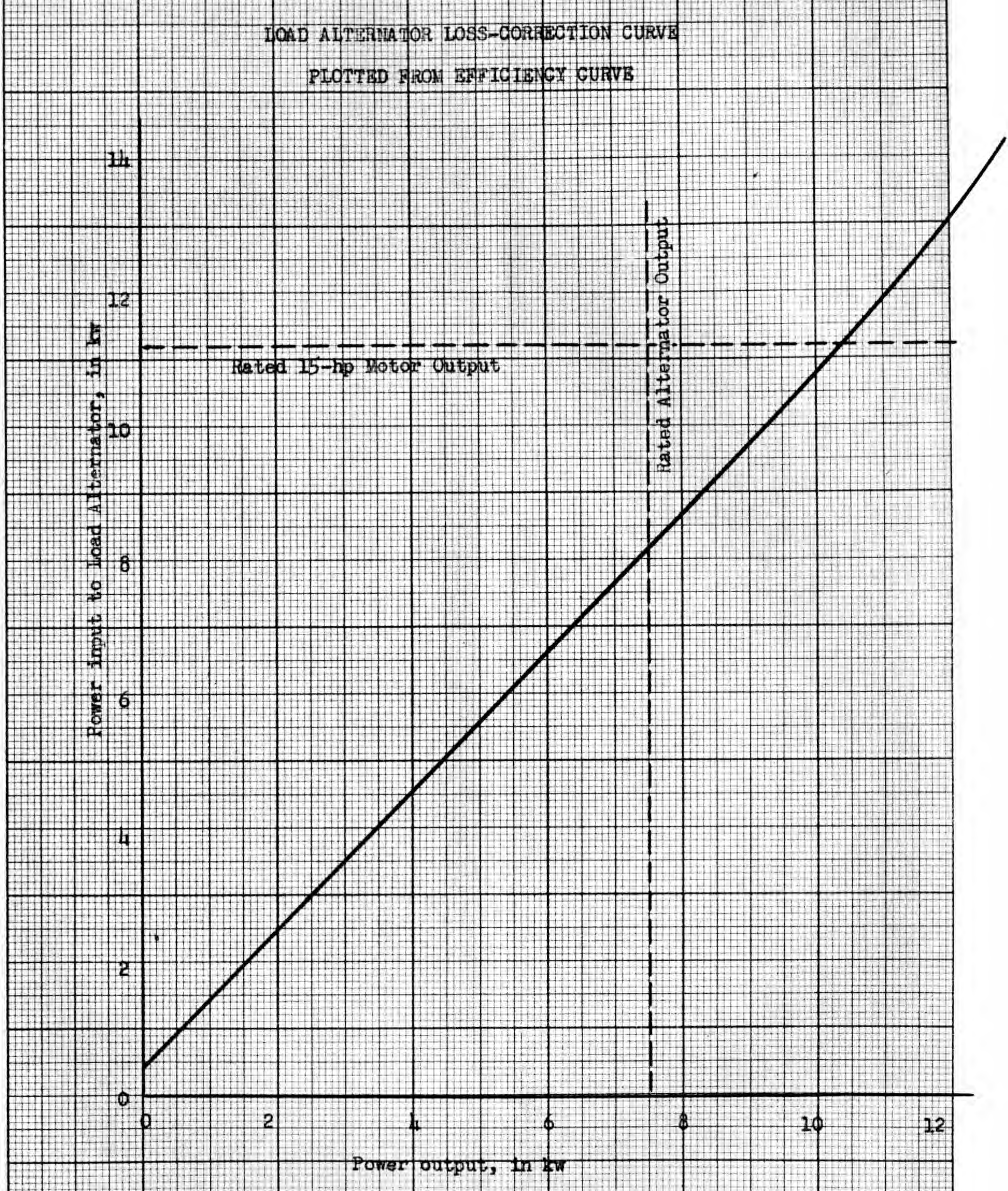


Fig. 27. Load alternator correction curve for 15-hp motor tests.

Table 9.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment LOAD TEST--UNIFORM DIRECT VOLTAGE **No.**

Test made by WAYNE B. SWIFT

(Leader) (Recorder) (Observers)

Rating of Apparatus Tested G. E. 15-hp, 220-v, 60-a **Lab. No.**

Date 6-24-50 **Wiring Checked by** **Data Checked by**

	1	2	3	4	5	6	7	8	9	10	11	12	13
Meter No.	100		66										
Scale Read													
Current Range	75												
Voltage Range			300										
	I_{tot}		V_{tot}		VI_{tot}		motor P_{out}		Speed		Eff		P_{loss}
1													
2	72.1		220		15.88		12.58		1200		79.3		3.30
3	57.8		219		12.65		9.95		1220		78.5		2.70
4	43.0		220		9.46		7.30		1242		77.1		2.16
5	30.0		220		6.60		4.82		1257		73.0		1.78
6	20.7		220		4.55		3.20		1266		70.4		1.35
7	13.1		220		2.88		1.88		1278		65.3		1.00
8	5.8		220		1.28		.40		1288		6.9		.88
9													
10													
11													
12													
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APPENDIX IX

LOAD TEST OF A FIFTEEN-HORSEPOWER MOTOR UNDER IMPRESSED VOLTAGE WITH THREE-PHASE RECTIFIED WAVE FORM

The wiring diagram for the motor during this test and the test discussed in Appendix VIII is shown in Fig. 28.

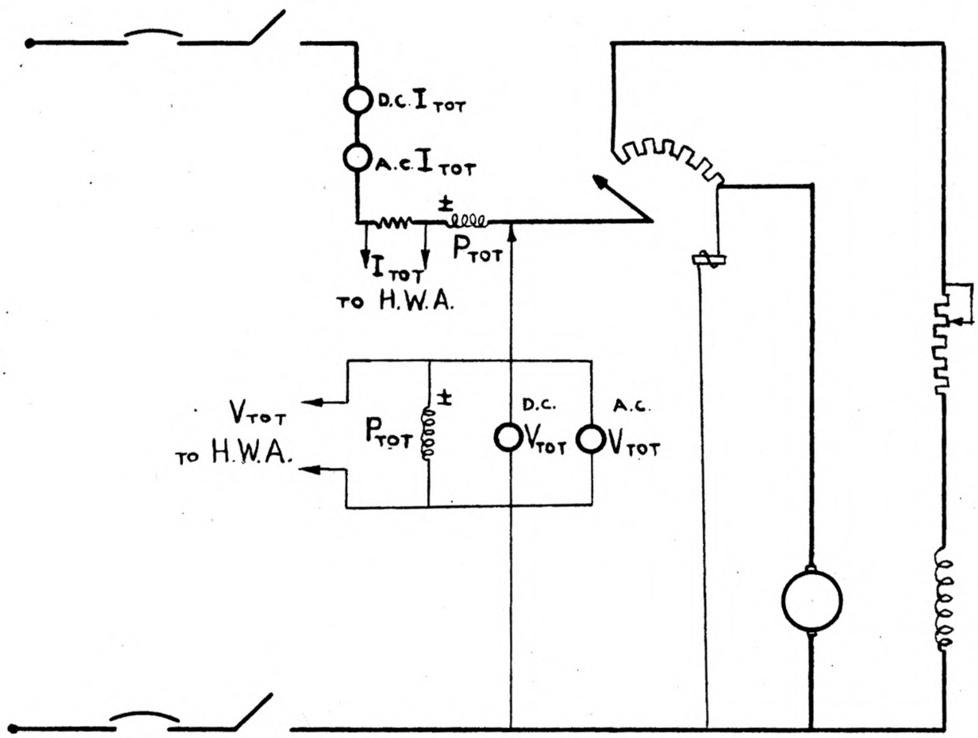

Fig. 28. Wiring diagram for tests of 15-horsepower motor with uniform direct voltage, and with three-phase ignitron wave form impressed.

Table 10.

ELECTRICAL ENGINEERING LABORATORY, KANSAS STATE COLLEGE

Name of Experiment LOAD TEST--3-φ IGN. RECTIFIED WAVE **No.**

Test made by WAYNE B. SWIFT
(Leader) (Recorder) (Observers)

Rating of Apparatus Tested G. E. 15-hp, 220-v, 60-a **Lab. No.**

Date 5-29-50 **Wiring Checked by** **Data Checked by**

	acI tot	dcI tot	acV tot	dcV tot	P tot	dcVI tot	motor P out	Speed	Eff	P loss
Meter No.	152	10C	219	71	329					
Scale Read					16kw					
Current Range	100	75			80					
Voltage Range			300	300	200					
1										
2	79.5	78 *	242	220	18.00	17.20	14.1	1210	76.1	4.1
3	67.0	65.5	243	220	15.00	14.50	11.5	1230	72.0	4.12
4	53.0	52.0	245	220	11.88	11.44	9.05	1240	76.7	2.67
5	36.0	35.0	247	220	8.00	7.70	5.88	1264	76.6	2.08
6	24.5	23.0	249	220	5.28	5.06	3.60	1282	73.1	1.58
7	--	6.5	244	220	1.68	1.43	.40	1300	25.6	1.16

* Estimated

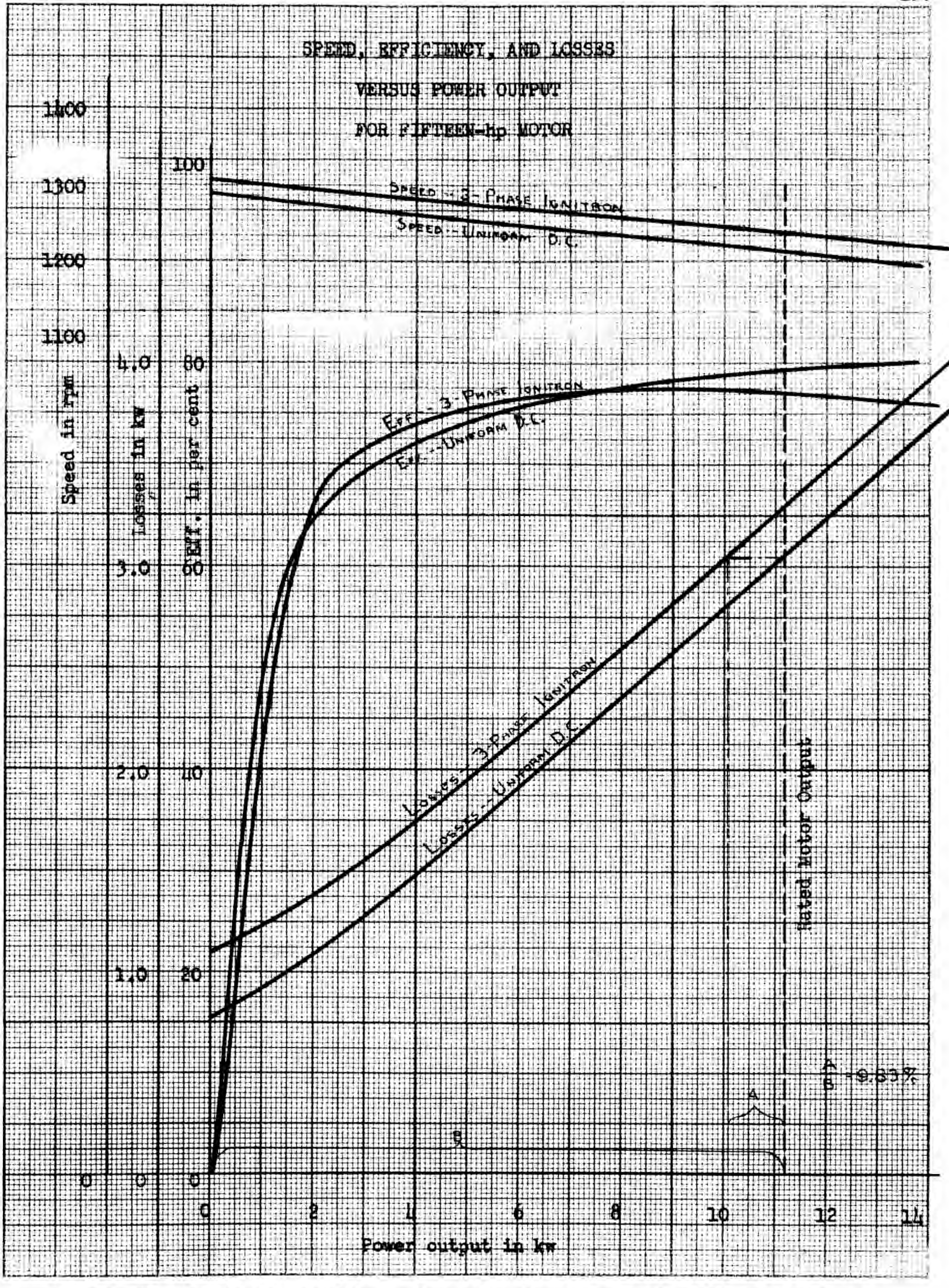

Fig.29. Load test of 15-hp motor under two impressed voltage wave forms.

APPENDIX X

COMPUTATION OF "EFFECTIVE POWER FACTOR" FROM APPENDIX IV

D. c. Power at full-load is

$$V\ \text{average} \times I\ \text{average} = 5560\ \text{watts.}$$

Total Power input is

$$P\ \text{total} = 5840\ \text{watts.}$$

Total a. c. Power is, then,

$$P\ \text{a. c.} = P\ \text{total} - P\ \text{d. c.} = 5840 - 5560 = 280\ \text{watts.}$$

Total a. c. volt-amperes are

$$V\text{-}A\ \text{a. c.} = V\ \text{a. c.} \times I\ \text{a. c.}$$

Now

$$V\ \text{a. c.} = \sqrt{V\ \text{rms}^2 - V\ \text{dc}^2} = \sqrt{248^2 - 220^2}$$

$$= 114.5\ \text{volts.}$$

$$I\ a.\ c. = \sqrt{I\ rms^2 - I\ dc^2} = \sqrt{27^2 - 25.3^2}$$

$$= 9.35 \text{ amperes.}$$

So a. c. volt-amperes are

$$V\ a.\ c. \times I\ a.\ c. = 114.5 \times 9.35 = 1071 \text{ v-a.}$$

"Effective Power Factor" is defined as

$$p.\ f.\ \text{"effective"} = \frac{P\ a.\ c.}{V\ a.\ c. \quad I\ a.\ c.} = \frac{280}{1071} = .261$$

To test this value of "effective power factor", any components of a. c. volt-amperes with magnitudes less than ten v-a will be neglected. This means that all components of frequency above 540 cycles will be neglected. Ten v-a is less than one-fourth of one per cent of full-load volt-amperes, so no great error will be introduced by dropping all smaller terms assuming the fourier series represented by these coefficients or components is a convergent one. It is, as is shown by the fact that the function to which it converges exists and has been used in the laboratory.

In Table 11, voltage and current components of the input voltage are shown. Also calculated are frequency components of a. c. volt-amperes.

Table 11. Components of voltage, current, and volt-amperes.

f	V	I	V-A
180	95.0	7.35	698
360	41.8	1.44	60
540	26.5	.63	17
Total a. c. volt-amperes			775

Calculating P a. c.,

$$
\begin{aligned}
P\ a.\ c. &= P\ \text{total} - P\ d.\ c. \\
&= 5{,}600 - 220 \times 24.6 \\
&= 5{,}600 - 5{,}410 \\
&= 190\ \text{watts}
\end{aligned}
$$

Then "effective power factor" is

$$
\begin{aligned}
p.\ f.\ \text{effective} &= \frac{P\ a.\ c.}{V\ a.\ c.\quad I\ a.\ c.} \\
&= \frac{190}{775} \\
&= .245
\end{aligned}
$$

Calculating "effective impedances", it is found that

Table 12. Components of impedance.

$Z_{(180)}=$ 12.9 ohms	=	115 per cent ohms
$Z_{(360)}=$ 29.0 ohms	=	259 per cent ohms
$Z_{(540)}=$ 42.0 ohms	=	375 per cent ohms

where 100 per cent Z is defined as that impedance which is required to limit current to full rated value with full rated voltage impressed.

APPENDIX XI

COMPUTATION OF THEORETICAL ALTERNATING COMPONENT OF CURRENT IN TEST OF FIVE-HORSEPOWER MOTOR UNDER IMPRESSED VOLTAGE FROM D. C. GENERATOR AND SIXTY-CYCLE ALTERNATOR IN SERIES

By plotting data in Table 12, Appendix X, it is found that

$$Z_{(60)} = 105 \text{ per cent} = 11.75 \text{ ohms}$$

From Appendix VII, at full-load

$$V\ a.\ c. = 68 \text{ volts (60-cycle component)}$$

Calculating theoretical I a. c. from above "effective impedance"

$$I\ a.\ c. = \frac{V\ a.\ c.}{Z_{(60)}} = \frac{68}{11.75} = 5.8 \text{ amperes.}$$

From Appendix VII, the I a. c. which actually flows is

I a. c. = 13.5 amperes.

The conclusion is necessarily that these "effective impedances" and "effective power factors" are of strictly limited usefulness.

APPENDIX XII

COMPUTATION OF THEORETICAL ALTERNATING COMPONENTS OF POWER IN TEST OF FIFTEEN-HORSEPOWER MOTOR UNDER IMPRESSED VOLTAGE WITH THREE-PHASE RECTIFIED WAVE FORM

For 15-hp. motor, 100 per cent impedance is

$$100 \text{ per cent } Z = \frac{220}{60}$$

$$= 3.67 \text{ ohms.}$$

So, following Table 12, Appendix X,

Table 13. Components of impedance for 15-hp. motor.

$Z_{(180)}$ =	115 per cent =	4.22 ohms
$Z_{(360)}$ =	259 per cent =	9.5 ohms
$Z_{(540)}$ =	375 per cent =	13.8 ohms

The wave form impressed is identical to that impressed on the five-hp. motor, so, from Appendix VI,

Table 14. Components of voltage.

$V_{(180)}$	= 95 volts
$V_{(360)}$	= 41.8
$V_{(540)}$	= 26.5

But from Appendix IV, it is found,

Table 15. Components of voltage.

$V_{(180)}$	= 89 volts
$V_{(360)}$	= 32
$V_{(540)}$	= 18

These two sets of values are fairly close, but since the values from Appendix VI represent actual measurements of the wave form as it is impressed across the ignitron output, it will be assumed that it is closer to the true voltage than the theoretical values from Appendix IV.

Then applying Ohm's law to the voltage components and using the "effective impedances" as in Table 14,

Table 16. Components of current and volt-amperes.

$I_{(180)}$	= 22.5 amperes
$I_{(360)}$	= 4.4 amperes
$I_{(540)}$	= 1.92 amperes
$V\text{-}A_{(180)}$	= 2140 v-a
$V\text{-}A_{(360)}$	= 184 v-a
$V\text{-}A_{(540)}$	= 51 v-a
Total $V\text{-}A_{a.c.}$	= 2375 v-a

Using the calculated "effective power factor",

$$P\ a.\ c. = (p.\ f.\ \text{effective})\ V\text{-}A_{a.c.}$$
$$= 0.25 \times 2375$$
$$= 594 \text{ watts}$$

From Appendix IX, the true a. c. power for this situation is measured as

$$P\ a.\ c. = P\ \text{input} - P\ d.\ c.$$
$$= 15{,}000 - 221 \times 65.5$$

$$= 15{,}000 - 14{,}440$$

$$= 560 \text{ watts}$$

This is excellent agreement--within six per cent. And as was shown early in this study, when the losses are determined with that much precision, results, using those values for losses, of calculations of efficiency will be very close.

APPENDIX XIII

SPEED REGULATION

Five-Horsepower Motor

<u>Uniform d. c. impressed voltage</u>. Reading from Fig. 19, substitute in definition of speed regulation:

$$\text{Speed Regulation} = \frac{\text{No-load speed - Full-load speed}}{\text{Full-load speed}}$$

$$= \frac{1415 - 1200}{1200} = \frac{215}{1200}$$

$$= 17.9 \text{ per cent}$$

<u>Three-phase ignitron impressed voltage</u>. Similarly, from Fig. 21,

$$\text{Speed Regulation} = \frac{1445 - 1200}{1200} = \frac{245}{1200}$$

$$= 20.4 \text{ per cent}$$

<u>Voltage from d. c. generator in series with 60-cycle alternator</u>. Similarly, from Fig. 21,

$$\text{Speed Regulation} = \frac{1485 - 1200}{1200} = \frac{285}{1200}$$

$$= 23.7 \text{ per cent}$$

Fifteen-Horsepower Motor

Uniform d. c. impressed voltage. Similarly, from Fig. 29,

$$\text{Speed Regulation} = \frac{1288 - 1210}{1210} = \frac{78}{1210}$$

$$= 6.45 \text{ per cent}$$

Three-phase ignitron impressed voltage. Similarly, from Fig. 29,

$$\text{Speed Regulation} = \frac{1300 - 1220}{1220} = \frac{80}{1220}$$

$$= 6.56 \text{ per cent}$$

Discussion of Above Results

Considering only the relation of speed regulation in the case of uniform voltage to regulation in the case of three-phase impressed wave forms, an increase in speed regulation of

$$\frac{20.4 - 17.9}{17.9} = 14 \text{ per cent}$$

Fourteen per cent is observed in the case of the small five-horsepower motor. In the case of the 15-hp. motor,

$$\frac{6.56 - 6.45}{6.45} = 1.7 \text{ per cent}$$