Efficient double auction mechanisms in the energy grid with connected and islanded microgrids

by

Mohammad Nazif Faqiry

B.S., Kabul University, 2007

M.S., Kansas State University, 2010

AN ABSTRACT OF A DISSERTATION

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Abstract

The future energy grid is expected to operate in a decentralized fashion as a network of autonomous microgrids that are coordinated by a Distribution System Operator (DSO), which should allocate energy to them in an efficient manner. Each microgrid operating in either islanded or grid-connected mode may be considered to manage its own resources. This can take place through auctions with individual units of the microgrid as the agents.

This research proposes efficient auction mechanisms for the energy grid, with islanded and connected microgrids. The microgrid level auction is carried out by means of an intermediate agent called an aggregator. The individual consumer and producer units are modeled as selfish agents. With the microgrid in islanded mode, two aggregator-level auction classes are analyzed: (*i*) price-heterogeneous, and (*ii*) price homogeneous.

Under the price heterogeneity paradigm, this research extends earlier work on the well-known, single-sided Kelly mechanism to double auctions. As in Kelly auctions, the proposed algorithm implements the bidding without using any agent level private information (i.e. generation capacity and utility functions). The proposed auction is shown to be an efficient mechanism that maximizes the social welfare, i.e. the sum of the utilities of all the agents. Furthermore, the research considers the situation where a subset of agents act as a coalition to redistribute the allocated energy and price using any other specific fairness criterion.

The price homogeneous double auction algorithm proposed in this research addresses the problem of price-anticipation, where each agent tries to influence the equilibrium price of energy by placing strategic bids. As a result of this behavior, the auction's efficiency is lowered. This research proposes a novel approach that is implemented by the aggregator, called virtual bidding, where the efficiency can be asymptotically maximized, even in the presence of price anticipatory bidders.

Next, an auction mechanism for the energy grid, with multiple connected microgrids is considered. A globally efficient bi-level auction algorithm is proposed. At the upper-level, the algorithm takes into account physical grid constraints in allocating energy to the microgrids. It is implemented by the DSO as a linear objective quadratic constraint

problem that allows price heterogeneity across the aggregators. In parallel, each aggregator implements its own lower-level price homogeneous auction with virtual bidding.

The research concludes with a preliminary study on extending the DSO level auction to multi-period day-ahead scheduling. It takes into account storage units and conventional generators that are present in the grid by formulating the auction as a mixed integer linear programming problem.

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Approved by:

Major Professor
Dr. Sanjoy Das

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Dedication

Dedicated to my dear parents Mohammad Ajan and Rabia Faqiry.

Chapter 1 - Introduction

Network economics has made automated trading feasible, where resources are exchanged for money through transactions that are done entirely through software *agents* and without the need for any human intervention. It is often the case that the agents participating in the trade of a resource are involved in direct competition with one another, where the objective of each agent is to maximize its own *payoff.* In these situations, the resource trade proceeds through an *auction* mechanism. In single-sided auctions, all agents are either *buyers* that compete to acquire a finite resource or *sellers* that compete to sell their goods. *Double auctions* are mechanisms that involve both buyers and sellers which simultaneously participate in the bidding process, and are allocated individual shares of the resource.

Recent technological advancements in communications and renewable energy generation have created much research interest in *energy auction algorithms* [1], [2]. In these mechanisms, the agents may represent individual domestic units within a microgrid, with energy representing the resource, and with PV-equipped homes experiencing a surplus of energy acting as sellers, and the remaining domestic units as buyers [3]. Being positioned closer to other consumer units, PV-equipped units are better placed to supply energy to the latter during exigent situations[4]. Complete isolation of a microgrid is an extreme example of such a case. Under these circumstances, the microgrid should allow bidirectional energy transactions between the units in the form of an auction mechanism that allows the buying and selling of electrical energy among individual agents. An agent may represent a single unit or an individual small scale microgrid involving a community of homes that collectively behave as a single unit in the ensuing auction[5]. These auction mechanisms shall be designed in such a way to provide incentives for the agents to participate. In other words, at the outcome of the auction both buying and selling agents receive a positive payoff. A buyer agent's payoff is typically the difference between its *utility* gained from consuming a certain amount of energy and the price that it has to pay in order to procure that energy. Likewise, a seller agent's payoff may be formulated as the sum of the monetary gain from supplying an amount of energy and utility it gains from retaining any surplus energy that is not traded.

An auction mechanism in which agents are priced differently is referred to as *price heterogeneous* auction. A subset of such auctions where all agents pay, or receive, equal per unit price is called *price uniform*, or price homogenous auction. The Kelly mechanism is an example of a price uniform auction which refers to a class of auction algorithms where agents are allowed to place individual bids on the resource, while a separate auctioneer that receives these bids, allocates the resource share of each bidding agent in proportion to the bid values that they have placed[6]. With a large number of agents, the *proportional allocation* mechanism has been shown to maximize the aggregate utilities of all agents, the latter commonly referred to as the *social welfare*(SW) [7]–[9]. SW maximizing mechanisms are also called *efficient* auctions.

Unfortunately, the underlying assumption for proportional allocation to be efficient is that the agents be *price takers*, i.e. ones that assume that the bids that they place do not influence the market price of the resource. While this is approximately true for auctions involving a large number of agents, in smaller auctions, agents are aware of their own *market power*, and accordingly, place strategic bids on the resource. Such a *price anticipatory* bidding results in a *loss in efficiency* where the resource allocation of the auction no longer maximizes the SW.

It must be noted that there are other efficient auctions that explicitly focus on eliciting truthful bidding from the bidders; the most significant ones being based on the well-known Vickrey-Clarke-Groves mechanism [10]. Furthermore, auctions designed to maximize the auctioneer's own earned revenue have been proposed in the vast body of literature on this subject. However, these mechanisms are not of direct relevance in this research, which focuses on efficient double auctions in grid-connected and islanded microgrids along with proportional allocation of energy among its agents.

1.1 Literature Review

Advances in communications and networking have made automated energy transactions via the grid feasible [1]. Consequently, grid auction design has been the subject of considerable recent research. Due to the complex nature of the problem, some recent work has focused on the application of nature inspired metaheuristics [11]–[16].Genetic algorithms, swarm intelligence, and hybrid approaches are popular choices for such applications[17].

Linear programming is another popular choice of algorithm. Discrete variables are handled by means of either tree-based search or relaxing and treating them as continuous ones. Unfortunately, these approaches entail the assumption of linearity and might not be the ideal choice of many grid auctions[18], [19].

Approaches for supply side auctions between power generation companies to sell energy at competitive prices have been the subject of much recent research [20],[21],[22],[23],[24],[25],[26],[27]. Typically, these approaches address generation scheduling and unit commitment whose treatment involves discrete design variables. Consequently, mixed integer linear programming has been extensively applied in such studies [20], [21], [25], [28].A two-stage bidding approach between the generation companies and a retailer that procures energy for distribution among consumers has been recently examined [21]. A large-scale day-ahead clearing scheme for the European market has been explored [25]. Mixed integer linear programming to minimize consumer price, while considering generator minimum up/down and ramp up/down times[28], and elsewhere, the presence of shiftable loads (i.e. loads which can be transferred across time slots) [20] have been investigated. Other supply side auctions make approximations in order to use linear programming [22]. One such study pertaining to the Brazilian energy grid, considers piecewise linear utility functions [22].Linearizing the constraints is used within a game-theoretic equilibrium formulation [24]. A large body of recent literature on energy auction algorithms model large utility company as the sellers [28],[29],[30],[21], [31]. Many of these studies consider objectives and/or constraints that are applicable only to energy trade, such as generation scheduling [28],[30], economic dispatch [31], and transmission losses [32]. A game-theoretic approach for decision making of storage units as seller agents in a smart grid for the maximum amount of energy to sell in the local market so as to maximize a utility that reflects the tradeoff between the revenues from energy trading and the accompanying costs has been studied [26]. A primal-dual approach to obtain optimal power flow is considered within a heterogeneous pricing framework [23]. Unfortunately, approaches where generating companies are involved in the bidding process are inapplicable within our context. Such auctions approach the problem primarily to establish a game-theoretic equilibrium, usually the Stackelberg equilibrium of a sequential game [21],[24],[27], [28].

Demand side auctions with the optimal procurement of energy among multiple buyers is another area of research activity [4],[13], [30],[33],[34],[35]. A simulation study using a Java based package (JADE) has been carried out [4]. Many of these studies investigate bidding across multiple time frames focusing on optimal operation of shiftable loads [35], [30]. An auction algorithm that incentivizes buyer participation is considered [34]. This study relies on historical data to penalize cheating behavior and ensure truthfulness. A limitation of this study is the underlying assumption of quadratic costs to make the problem formulation strictly convex.

Efficient auctions have begun to gain research attention in the present context energy trade. Several such investigations do not consider proportionally fair allocation of energy [36], [37], [26]. One recent study proposes a VCG-style auction with multiple sellers and a single demand response aggregator as the buyer [35]. A few other recent studies have also implemented VCG mechanism for energy allocation and trade [33], [38]. This mechanism is used for energy allocation between multiple buying agents [33]. However, the approach includes only a single seller; a bottleneck when the grid contains several PV equipped units. Another study that uses the VCG mechanism reports a double auction [39]. Some energy auction studies are designed for revenue maximization and are not SW maximizing (efficient) mechanisms [29], [31]. The cake-cutting algorithm has been applied to procure energy from multiple sellers and for a community of consumers acting in tandem as a single buyer [40]. A truthful buyers' auction that makes use of the Arrow-d'Aspremont-Gerard-Varet mechanism has been suggested [34].

While some existing approaches as well as this research explore Nash equilibrium, where all agents are assumed to act simultaneously [41],[35], [5], others use leader-follower games under Stackelberg equilibrium [36],[37],[42], [21]. This equilibrium concept is applicable when the energy market is modeled as an oligopoly with only a limited number of suppliers modeled as leader, or with the inclusion of an upper level agent such as an aggregator [36],[37],[42], [35]. This is in contrast to the present work that treats both buyers and sellers with equivalent market parity.

Proportional allocation in single-sided buyers' auctions is another line of research that has been rigorously analyzed [43],[44], [45]. It is shown that the auction is efficient under the assumption of price taking buyers. Furthermore, when the agents' utilities are strictly concave functions, the seminal study in [43] establishes a strong theoretical upper limit on the auction's loss of efficiency at $\frac{1}{4}$ of the maximum attainable SW. More recently, it has been shown that even when buyers are price anticipating, proportional allocation allows the mechanism to attain the best possible outcome [46]. Similar theoretical limits have been investigated for a more general class of auctions called smooth auctions with proportional allocation [47]. Theoretical properties of a situation with multiple sellers participating in a proportional allocating auction, and with inelastic (fixed) demand for the resource has been examined [44]. When the proportional allocation auction takes into account the costs of the network's links through which the resource (data) flows, the efficiency is shown to be at least $4\sqrt{2}–5$ of the SW maximum with price anticipating buyers and with convex costs [48]. In a separate study it has been shown that when the costs are linear, the mechanism's efficiency loss is lower bounded at ⅓ of the maximum value [49]. Unfortunately, all studies are based on the assumption that the utility functions' are convex. In a more general setting where this convexity assumption is not true, the mechanism's efficiency loss no longer enjoys a theoretical limit, and could in fact be arbitrarily large [9]. A few studies have proposed schemes to address an auction's efficiency loss arising from price anticipation. For instance an auction mechanism with price differentiation where each buyer has a different price, has been proposed [50]. This study also suggests a feedback control mechanism on the price vector that drives the auction to converge to the globally optimum SW.

Many research papers in the existing research on energy auctions report only single-sided ones [34],[41],[30],[35],[40],[5], [21]. However, there are some papers that do address some form of double auction [51], [39],[32],[29],[42],[31],[11],[26],[52], [53]. Unlike auctions between generation companies discussed previously that primarily aim to lower operation costs, the goal of these auction mechanisms is to optimize the distribution of energy within the customers in order to maximize the overall SW function (SWF) of the community, *i.e.* the aggregate utilities of all the units of the grid. One study that maximizes SWF, models its consumers as agents that collectively maximize the SWF [54]. This is an unrealistic approach for real world deployment where each agent adjusts is usage patterns only to maximize its individual payoff, *i.e.* the difference between its individual utility from consuming a certain amount of energy and the price it pays to procure the amount. Another study models the agents' utilities as quadratic functions [55]. A preliminary study on double energy auction based on the Kelly mechanism has been proposed [3]. Another double auction study that does not use proportional allocation addresses software issues rather than the auction [51]. Double auction mechanisms with proportional allocation have been studied [56]. However, this study considers only the case of price takers. None of these double energy auctions addresses the issue of price anticipatory behavior of the participating agents. To the best of the authors' knowledge, only one research in the literature on efficient energy auctions does examine the adverse effect of price anticipation [41]. This study is a single-sided auction with consumers of electricity acting as price anticipating buyers. Furthermore, none of the studies reported earlier addresses distribution grid physical system constraints during the auction process.

1.2 Contributions

This research proposes price heterogeneous and price uniform efficient double auctions for energy trade in both grid connected as well as islanded microgrids. Although this research considers energy as the traded resource, the underlying theoretical analysis is directly applicable to other divisible resource efficient auctions. The auctions include one set of agents as buyers, and another set as sellers. It assumes the presence of an aggregator as a separate mediating agent whose main role, unless indicated otherwise, is to (*i*) receive bids from agents (*ii*) allocate energy to the agents; (*iii*) iteratively converge to the market clearing equilibrium. The main contributions of this research under each setting are summarized in the following subsections. Details of the contributions in each of these subsections are provided in separate chapters throughout this dissertation.

1.2.1 Aggregator-level Price Heterogeneous Auction

This research is presented in chapter 2, which considers an isolated microgrid operated by an aggregator. Domestic units are modeled to play the role of buying and selling agents. The aggregator is modeled as an *impartial agent* that implements the auction in an iterative manner using public information declared by the agents during bidding. The physical grid constraints such as node voltage limits and transformer capacity has not been taken into account at this stage. The primary objective of this research was to devise a framework for a *price heterogeneous* double auction while preserving the private information of the trading agents.

The novel features of the proposed auction mechanism are as follows.

(*i*) It is designed to be *weakly budget balanced*, so that the total monetary reimbursement provided to the sellers in exchange for energy never exceeds the total revenue obtained from the buyers.

(*ii*) It performs the auction without requiring private information such as utility functions and generation capacities of the participating agents.

(*iii*) The resulting payoffs of all the participating agents are theoretically shown to be nonnegative so that the auction is *individually rational.*

(*iv*) It is designed to allow *in-auction* or *post-auction* energy and price redistribution in order to entertain the possibility of seller coordination using any other *fairness* criterion.

(*v*) It is SW maximizing for the set of buyers. When the supply is relatively small, the auction also maximizes the sellers' SW. Furthermore, when no extraneous fairness criterion is applied, even with enough supply, the auction is still able to attain the efficient allocation among all agents.

1.2.2 Aggregator-level Price Uniform Auction

This research is presented in chapter 3, which considers the same physical setting as before, i.e. an islanded microgrid, and proposes a *price homogenous* auction using *proportional allocation*. It addresses the problem of price anticipatory bidding that would arise in islanded microgrids with relatively small numbers of participating agents.

The novel features of the proposed auction mechanism are as follows.

(*i*) It has a *unique equilibrium* in spite of including both buying and selling agents and under price anticipatory conditions. This has been established through theoretical analysis.

(*ii*) It is implemented as a distributed iterative algorithm where selfish agents realistically simulate *price anticipation* using information from prior iterations, without using any knowledge from other agents.

(*iii*) It attains maximum *efficiency* even in the presence of *strategic bidders*. This new feature has been established theoretically and through simulation results.

(*iv*) It can incorporate a *selfish aggregator*, which seeks its own revenue in the form of a surcharge, leading to a bi-objective optimization framework. The existence of a non-singleton Pareto front has been analytically established.

1.2.3 Bi-level Energy Distribution Auction

This phase of the research is reported in chapter 4, which extends the auction mechanisms described above to multiple microgrids that are connected to the distribution system. In the presence of a fixed amount of supply from the wholesale market, it considers physical grid constraints during auction implementation.

The novel features of the proposed auction mechanism are as follows.

(*i*) It is *price heterogeneous* across microgrid, i.e. aggregators.

(*ii*) It allocates energy to microgrids in such a manner that under equilibrium conditions *global efficiency* is achieved.

(*iii*) It incorporates physical grid constraints such as distribution node voltage limits and substation transformer limits in the DSO level auction.
(*iv*) It uses a novel approach to eliminate the effect of *price anticipation* at the microgrid level and ensure convergence to the global efficient solution.
(*v*) It can be incorporated as distributed algorithm that complies with the physical topology of the distribution system and where the lower level bidding is carried out in parallel.

1.2.4 Multi-period price heterogeneous auction for distribution system operation

In order to generalize the above bi-level auction for *multi-period* operation, other physical aspects of the distribution grid such as inclusion of battery storage units, conventional generators, and price responsive loads have been taken into account at this stage and is presented in chapter 5. The aggregator has been considered as a load-serving unit that bids in the DSO-level auction on behalf of domestic unit by placing fixed and price responsive loads. In the presence of available supply from the wholesale market at Locational Marginal Price (LMP), i.e., the marginal price of serving the next MW of load, a *social surplus* maximizing auction is implemented among price responsive loads, conventional generators, and RES-charged battery storage units.

The novel features of the proposed auction mechanism are as follows.
(*i*) It is modeled to match the physical topology of the distribution system and proposes a multi-period day ahead auction due to inclusion of discrete devices such as battery storage units with temporal constraints using *mixed integer linear programming* (MILP).
(*ii*) It proposes auction clearing at distribution LMP (DLMP) that results to a price heterogeneous auction for market participants.
(*iii*) It provides a simulation based study of the dynamics of unit commitment, DLMP, and market participants' payments and reimbursements versus changes in LMP with and without penalties for DSO's deviation from its commitment in the wholesale market.

Chapter 2 - Aggregator-level Price Heterogeneous Energy Auction

In this chapter, a general-purpose double auction mechanism that is specifically relevant for energy transactions among competing buyers and sellers, i.e. agents, in a microgrid has been investigated. The proposed mechanism is modeled for rational agents with hidden private information, that are aiming to optimize their utility from consuming energy as well as their payoff in the transaction. This is achieved through the presence of a central auctioneer, i.e. an aggregator whose aim is to accomplish an individually rational, weakly budget balanced, and efficient (SW maximizing) trade among agents while not requiring their private information. These auction properties are achieved by the aggregator in an iterative process by considering a smartly selected SW maximizing surrogate objective function through which agents can take strategic decisions to maximize their utility and payoff. The sellers of energy are modeled to be prosumers that are equipped with PV panels as well as advanced smart meters and are willing to participate in selling their excess energy due to saturation in their energy consumption utility curves. Buyers of energy are assumed to be conventional consumers with smart meter capabilities to communicate with the aggregator and place bids to get optimal allocations. Buyers aim to maximize their energy usage utility less their payment amount whereas sellers maximize their own energy consumption utility plus the payment they receive for selling the rest of their generation. Furthermore, the mechanism allows the possibility of sellers' coalition where a fair redistribution of the energy supply is possible in a competitive market with less demand and more supply.

2.1 Introduction

Recent advances in automated network economics has led to a great deal of interest in network auctions where automated agents may buy or sell an arbitrary resource through a bidding process that is coordinated by a separate agent acting as the auctioneer. The individual agents are selfish, i.e. participate in the auction to maximize their individual payoffs. For buyers, the payoff is equal to the difference between the utility of the procured resource and the price paid, whereas for sellers it is equal to the monetary gain from the sale minus the net loss in utility. In contrast, the auctioneer may be altruistic, whose aim is not to maximize its own net revenue from the auction, but to redistribute the resource using some other established criteria, such as maximizing the SWF, i.e. the sum of all buyer and seller utilities.

The Kelly mechanism is a class of distributed auctions where the agents bid in terms of prices without having to disclose their utilities to the auctioneer [57]. Subsequently, this mechanism has been extended to various applications in communication networks [58], [59]. It has been applied to allocate transmission rates across multiple users in the internet, using the proportional fairness criteria [60]. The problem of power allocation among multiple users in a wireless network has been addressed in [61], where users participate simultaneously as buyers as well as sellers to cooperatively maximize their weighted sum rates. More recently, an auction mechanism for mobile data offloading involving mobile network operators as buyers and Wi-Fi or femtocell access points as buyers has been proposed [62]. In another study addressing a similar problem, a Kelly mechanism based auction algorithm is used within a leader-follower Stackelberg game framework [63].

The advent of alternative energy sources is causing a paradigm change in the operation of the energy grid [1]. It has shifted the generation of electricity away from a few large power plants towards several smaller individual units that are equipped with PV panels and other means to produce electricity from renewable sources. Although at present this energy is typically utilized to meet the individual units' own energy needs, it is envisaged that with greater penetration of PV-equipped homes in future, along with the development of more efficient solar panels, individual homes would be able to deliver energy to the grid [64]. Being positioned closer to other consumer units, these PV-equipped units are better placed to supply energy to the latter during exigent situations [4]. Complete isolation of a microgrid is an extreme example of such a case. Under these circumstances, the microgrid should allow bidirectional energy transactions between the units in the form of an auction mechanism that allows the buying and selling of electricity.

This chapter proposes an auction mechanism in the aggregator level containing several domestic units acting as agents and with no restrictions on the number of PV-equipped homes willing to sell energy to other units. In other words, the proposed auction is multi-agent, i.e. generalized enough to be applicable to systems involving multiple sellers as well as buyers, which are modeled as sets of distributed agents. In addition, it assumes that there exists an aggregator, which contains enough computational capabilities to act as an impartial auctioneer.

In contrast to most work on grid auctions which consider uniform pricing across all users in each time interval, this double auction uses price discrimination, where individual agents are priced heterogeneously. To the best of our knowledge, there are only a few papers that use discriminatory pricing [65],[23],[20], ; unfortunately none are applicable to the present context.

The proposed auction mechanism is weakly budget balanced, so that the total reimbursement provided to the sellers in exchange for energy never exceeds the total revenue obtained from the buyers. The payoff of each participating agent in the proposed auction is always guaranteed not to be lower than what its payoff would have been from non-participation in the trade. In other words, the auction is individually rational.

The proposed auction mechanism also allows separate and arbitrary utility functions for the agents, as long as they are monotonically increasing and concave. Moreover, the PV-equipped sellers have different maximum generation capacities. Although most proposed auction mechanisms make use of this information, in reality it must remain hidden from the aggregator. The proposed auction is able to attain the desirable outcome without the use of this information. Thus, it also works when user private utility functions are hidden. However, it must be noted that the auction may include an optional redistribution mechanism that may need access to such information [66], [67]. The redistribution option is included in the proposed mechanism in order to entertain the possibility of further agent coordination beyond the auction. This is when supply is high enough to meet the buyers' demand, leaving room for further bargaining with the sellers, who then opt to impose their own arbitrary fairness criteria to redistribute the allocated supply determined by the auction. Such a situation may not arise when there are relatively few sellers, as the outcome of the auction would involve selling their entire surplus energy.

With the increase in the penetration of renewable energy resources in the future, the proportion of sellers in the distribution system is expected to be high enough to warrant a separate fair redistribution mechanism. The reasons for such a redistribution are manifold. Agents may have to operate within a legal reimbursement framework [68], resolve conflicts of interest [69],[70],[71] , or other economic incentives [70],[72], [73]. The presence of storage devices either individually, or at a community level is another reason for further redistribution [69],[74],[73],[75],[76], [77]. Although, for simplicity, this research considers fair redistribution only for the sellers, the approach can readily be extended to include the buyers.

The proposed auction is SWF maximizing for the set of buyers and may be considered as an extension of the Kelly mechanism. However, as the approach is formulated to achieve several different fairness criteria for the sellers, it does not necessarily produce the SW maximizing allocation amongst the latter. When no extraneous fairness criterion is applied, the auction, by default, attains the efficient allocation among all agents. The tradeoff between fairness and efficiency is quantified in terms of the price of fairness. The fair redistribution mechanism may be incorporated either in-auction, or as a second stage, post-auction algorithm.

In order to demonstrate the efficacy of the proposed approach in real-world applications, simulations are carried out within an energy grid setting where the auction's resource is energy, with domestic users as buying and selling agents in a microgrid, or with microgrids themselves as agents within a network spanning a larger geographic area. It is important to note that the approach suggested in this research is general-purpose and can readily be applied to any other network domain with a tradeable divisible resource as long as domain-specific physical restrictions allow the auction to proceed. Hence, the simulations reported later assume that this is the case with energy related grid constraints, i.e. domain-specific physical constraints such as line, voltage, and transformer capacity limits, etc. do not affect the auction. Detailed, energy-specific auctions with physical constraints are studied in a later chapter.

In recent years, double auctions that involve both buyers and sellers of energy, with the latter being PV-equipped units rather than generation companies have begun to be examined[11],[78],[79],[2],[52], [53]. Unlike auctions between generation companies discussed previously that primarily aim to lower operation costs, the goal of these auction mechanisms is to optimize the distribution of energy within the customers in order to maximize the overall SWF of the community, i.e. the aggregate utilities of all the units of the grid. One study that maximizes SWF, models its consumers as agents that collectively maximize the SWF [54]. This is an unrealistic approach for real world deployment where each agent adjusts is usage patterns only to maximize its individual payoff, i.e. the difference between its individual utility from consuming a certain amount of energy and the price it pays to procure the amount. Another study models the agents' utilities as quadratic functions [55].

A few recent studies have implemented the VCG mechanism for energy allocation and trade [33], [38]. This mechanism is used for energy allocation between multiple buying agents [33]. However, the approach includes only a single seller; a bottleneck when the grid contains several PV equipped units.

For the reader's convenience, a list of notation and abbreviations for this chapter are provided in Appendix B.

2.3 Auction Framework

The microgrid consists of a set of buyer agents denoted as $\mathcal{D}$ and a set of seller agents denoted as $\mathcal{S}$. At the beginning of the iterative auction, the aggregator relays an initial price, which may reflect the actual price under non-isolated operation when the microgrid receives energy from the main grid. In order to ensure weak budget balance, the sellers can sell energy only at prices lower than or equal to p, whereas the buyers can procure energy at values higher than or equal to p.

Each seller responds to the aggregator by letting the latter know of the amount of energy a_j available for trade at a per unit price $p_j \leq p$. The energy a_j can never exceed its total energy generation g_j. Subsequently the auction proceeds in an iterative manner as shown in Figure 2.1.

An iteration of the proposed auction mechanism involves the following exchange of information. The aggregator computes the volume of energy $s_j \leq a_j$ that it is willing to procure from each seller, and separately d_i that it can deliver to each buyer. The aggregator optimization problem that is used to compute d_i and s_j for this task is addressed later. The buyer replies to the aggregator by placing a bid b_i in monetary units that it is willing to pay for d_i units of energy. Note that the condition $p\, d_i \leq b_i$ for weak budget balance is considered only by the aggregator. Simultaneously, the sellers return c_j, the per unit selling cost at which it is willing to supply the amount s_j.

As seen in Figure 2.1, private information is not provided to the aggregator. The underlying SW optimization problem that ensures efficiency incorporating both public and private data is discussed first.

2.3.1 Social Welfare Optimization Problem (SWOP)

The SWF that is maximized by SWOP consists of the total of all buyers' and sellers' utilities (u_i and v_j), summed separately as shown below, where for notational convenience, the arguments d_i and s_j within the function $\Theta(\bullet)$ hereafter refer to the demand and supply allocations for buyers and sellers.

Maximize w.r.t. d_i, s_j

$$\Theta(d_i, s_j) = \sum_{i \in \mathcal{D}} u_i(d_i) + \sum_{j \in \mathcal{S}} v_j(g_j - s_j)\,, \tag{2.1}$$

subject to,

$$pd_i \leq b_i; \quad \forall i \in \mathcal{D}, \tag{2.2}$$

$$s_j \leq a_j; \quad \forall j \in \mathcal{S}, \tag{2.3}$$

$$\sum_{i \in \mathcal{D}} d_i = \sum_{j \in \mathcal{S}} s_j\,. \tag{2.4}$$

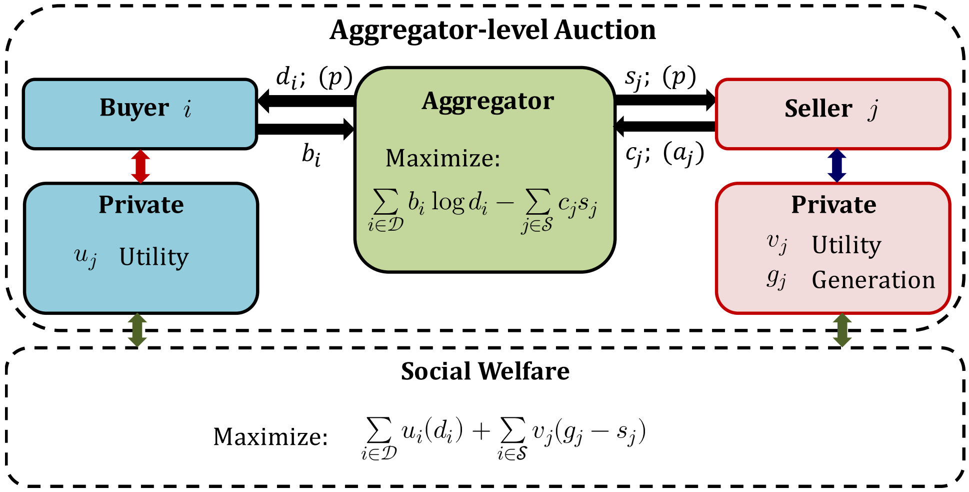

Figure 2.1: Schematic showing the flow of information between buying and selling agents and the aggregator. All parameters except those appearing within parenthesis are updated iteratively.

The first constraint in Eqn. (2.2) pertains to weakly budget balance for the buyers. The second constraint in Eqn. (2.3) ensures that the amount of energy that a seller exports to the microgrid never exceeds its declared availability. The last constraint in Eqn. (2.4) is present to ensure energy balance.

For any given values of the bids b_i and availabilities a_j given by the constraints in Eqns. (2.2) and (2.3), the SWOP defines a unique maximum at d_i^*, s_j^*. This follows from the fact that the SWOP objective function $\Theta(d_i, s_j)$ is the sum of strictly concave functions, and is strictly concave with all its constraints being linear. The Lagrangian function corresponding to the SWOP can be written as,

$$\mathcal{L}_\Theta(d_i, s_j, \lambda_i, \alpha_j, \mu) = \Theta(d_i, s_j) + \sum_{i\in\mathcal{D}} \lambda_i(pd_i - b_i) + \sum_{j\in\mathcal{S}} \alpha_j(s_j - a_j) + \mu\left(\sum_{i\in\mathcal{D}} d_i - \sum_{j\in\mathcal{S}} s_j\right), \quad (2.5)$$

resulting to the following equilibrium conditions,

$$pd_i^* \leq b_i, \quad (2.6)$$

$$\lambda_i^*(pd_i^* - b_i) = 0, \quad (2.7)$$

$$\alpha_j^*(s_j^* - a_j) = 0, \tag{2.8}$$

$$u_i'(d_i^*) + \lambda_i^* p + \mu^* = 0, \tag{2.9}$$

$$-v_j'(g_j - s_j^*) + \alpha_j^* - \mu^* = 0, \tag{2.10}$$

2.3.2 Aggregator Optimization Problem (AOP)

In order to achieve the SWOP objective, AOP is formulated as shown below.

Maximize w.r.t. d_i, s_j

$$\Phi(d_i, s_j) = \sum_{i \in \mathcal{D}} b_i \log d_i - \sum_{j \in \mathcal{S}} c_j s_j\,, \tag{2.11}$$

subject to constraints in Eqns. (2.2), (2.3), and (2.4) which are restated below,

$$pd_i \leq b_i; \quad \forall i \in \mathcal{D},$$

$$s_j \leq a_j; \quad \forall j \in \mathcal{S},$$

$$\sum_{i \in \mathcal{D}} d_i = \sum_{j \in \mathcal{S}} s_j.$$

It must be noted that the AOP formulation does not involve any hidden information from the buyers and sellers. For this reason, the objective that is maximized in AOP does not involve the agents' utility functions. Likewise, the second AOP constraint uses a_j instead of g_j, the latter being hidden from the aggregator.

The first term in the AOP objective function in Eqn. (2.11), which pertains to the buyers, is adapted from the Kelly mechanism. This mechanism is originally proposed for single sided network auctions [57] which also does not require hidden data. The Kelly mechanism has been studied in the context of communication networks [60],[80], [81]. The authors have suggested the use of such an auction for use in microgrid energy trade [3]. However, to the best of the authors' knowledge, its use in these auctions has not been examined so far elsewhere.

The second term in the AOP objective function in Eqn. (2.11), the summation of the monetary payment $c_j s_j$ given to each seller $j \in \mathcal{S}$, is the total sellers' reimbursement by the aggregator. In order to accommodate any desired fairness criteria for the sellers, this term has been cast as a linear function. Post-auction redistribution does not change the AOP objective as long as the reimbursed amount and the total volume of energy transacted do not change during the redistribution stage. For the same reason, in-auction redistribution can be readily incorporated within the proposed mechanism, simply by adding a weighted third term to the objective function.

For simplicity, this research takes into account only seller side fair redistribution. This setup may be viewed as one where the sellers have their own separate arrangement for fair redistribution [82],[83], [84] while the buyers are conventional consumers of energy. However, the framework can be readily extended to include coordinated buyers. This may be accomplished in a straightforward manner by incorporating another linear term in the AOP objective, similar to the second but with opposite sign.

The formulation in AOP offers the flexibility of any redistribution scheme among the sellers using any fairness criterion as long as the total energy volume S supplied by them remains equal to that delivered to the buyers and the total monetary amount reimbursed to them is fixed. All such solutions satisfying these conditions for s_j must be included in the set of optima of the AOP.

Figure 2.2 shows a graphical illustration of these considerations. Note that the optimum solution of the AOP for the buyers,d_i, is unique and coincides with that of the SWOP. The sellers' unique optimum solution of the SWOP is also optimal for the AOP. This solution can be made unique in the SWOP with the inclusion of a third convex term for fairness with a very small weight. Our simulations suggest that, when the auction proceeds without this third term, the auction arrives at the unique SWOP solution.

The Lagrangian of the AOP is defined as,

$$\begin{aligned}\mathcal{L}_{\Phi}(d_i, s_j, \gamma_i, \beta_j, \nu) = \Phi(d_i, s_j) + \sum_{i\in\mathcal{D}} \gamma_i(pd_i - b_i) \\ + \sum_{j\in\mathcal{S}} \beta_j(s_j - a_j) + \nu\left(\sum_{i\in\mathcal{D}} d_i - \sum_{j\in\mathcal{S}} s_j\right),\end{aligned} \tag{2.12}$$

resulting to the following equilibrium conditions,

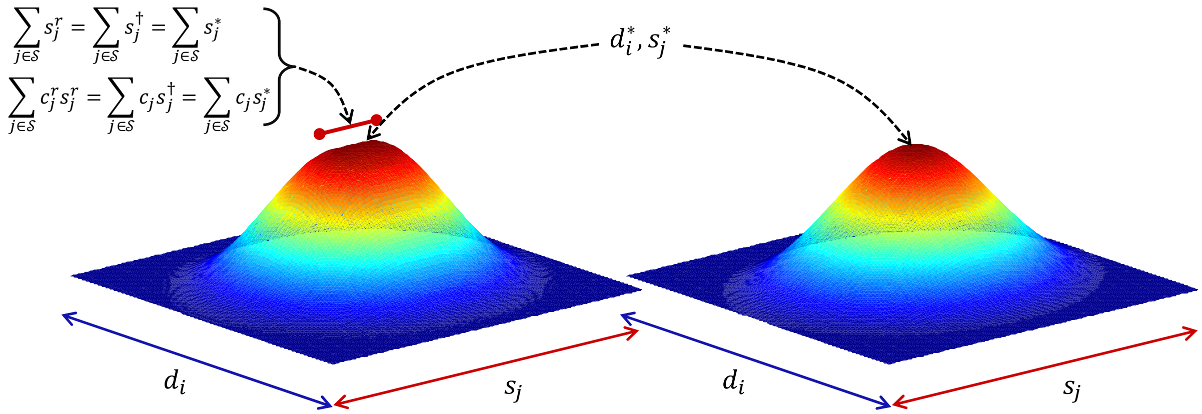

Figure 2.2 Schematic showing the optima defined by the AOP (left) and the SWOP (right). Both optima are unique with respect to the buyers and coincide $(d_i^\dagger, d_i^*)$. The SWOP's unique sellers solution (s_j^*) is also an optimal solution $(s_j^\dagger)$ of the AOP although the AOP admits other optima (s_j^r) depending on the fairness criterion as long as the constraints shown are satisfied.

$$pd_i^\dagger \leq b_i, \tag{2.13}$$

$$\gamma_i^\dagger \left(pd_i^\dagger - b_i\right) = 0, \tag{2.14}$$

$$\beta_j^\dagger \left(s_j^\dagger - a_j\right) = 0, \tag{2.15}$$

$$\frac{b_i}{d_i^\dagger} + \gamma_i^\dagger p + \nu^\dagger = 0, \tag{2.16}$$

$$-c_j + \beta_j^\dagger - \nu^\dagger = 0. \tag{2.17}$$

2.3.3 Buyer Optimization Problem

The buyer bids to maximize its own payoff π_i. This can be formulated as another problem that is carried out locally by the agent.

Maximize w.r.t. b_i

$$\pi_i = u_i(d_i) - b_i. \tag{2.18}$$

Differentiating w.r.t. d_i, yields the following,

$$u_i'(d_i) = \frac{\partial b_i}{\partial d_i}. \tag{2.19}$$

Upon receiving d_i from the aggregator, each buyer bids,

$$b_i = u_i'(d_i)d_i. \tag{2.20}$$

2.3.4 Seller Optimization Problem

At the beginning of the proposed iterative auction, the seller declares its availability a_j.The seller communicates the cost c_j at which it is willing to deliver the volume s_j of energy to the microgrid using the following problem formulation.

Maximize w.r.t. c_j

$$\pi_j = v_j(g_j - s_j) + c_j s_j. \tag{2.21}$$

When the seller does not overbid or underbid, this leads to the following cost-updating rule.

$$c_j = v_j'(g_j - s_j). \tag{2.22}$$

The reason why the seller does not overbid or underbid is as follows. Let us consider the case where $a_j > s_j$. Clearly, the seller j would not underbid by declaring a cost $c_j < v_j'(g_j - s_j)$ since the monetary payoff $c_j \Delta s_j$ obtained from this approach would be lower than the loss in utility $v_j(g_j - s_j - \Delta s_j)$. On the other hand, overbidding is not an optimal strategy since it would make $s_j = 0$. This can be seen by inserting the implicit constraint $s_j \geq 0$ to the AOP problem. In this case, the Lagrangian in Eqn. (2.12) becomes $\mathcal{L}_\Phi(d_i, s_j, \gamma_i, \beta_j, \nu) + \zeta s_j$ with the KKT conditions $\zeta \geq 0$ and $\zeta s_j = 0$ in addition to those of the AOP problem given by Eqns. (2.13) to (2.16) and with Eqn. (2.17) replaced with the equality $-c_j + \beta_j + \zeta - \nu = 0$. Since $\beta_j = 0$ when $a_j > s_j$, the equality reduces to $-c_j + \zeta - \nu = 0$. When the seller j does not overbid for a supply $s_j > 0$, it is seen that $\zeta = 0$ and $c_j = -\nu$. However if it overbids, then $c_j > -\nu$ whence $\zeta > 0$ so that$-c_j + \zeta - \nu = 0$, whence the new s_j is forced to be zero, removing the seller from the auction.

When $a_j = s_j$ a similar argument with ζ replaced with $\beta_j + \zeta$, indicating that the seller will neither overbid nor underbid.

2.4 Auction Properties Analysis

This section establishes various desirable features of the proposed energy double auction mechanism.

Proposition 2.1. The allocation $d_i^\dagger$ of each buyer i at the maximum of AOP is equal to the corresponding maximum d_i^* of SWOP, i.e. $d_i^\dagger = d_i^*$.

Proof of Proposition 2.1. From the assumption of strict concavity of any buyer's utility $u_i(\bullet)$, the function $u_i'(d_i)d_i$ given by the buyers' bid b_i in Eqn. (2.20) is strictly increasing. Since the buyer bid b_i remains unchanged for both allocations $d_i^\dagger$ and d_i^*, clearly $u_i'(d_i^\dagger)d_i^\dagger = u_i'(d_i^*)d_i^*$. Hence, it follows that $d_i^\dagger = d_i^*$.

■

Proposition 2.2. (*Quasi-efficiency*) The unique SWOP maximum at d_i^*, s_j^* satisfies the KKT conditions of AOP, so that,

$$(d_i^*, s_j^*) \in \underset{d_i, s_j}{\operatorname{argmax}}\, \Phi(d_i, s_j).$$

Proof of Proposition 2.2. From proposition-1, $d_i^\dagger = d_i^*$. Consider the case with $s_j^\dagger = s_j^*$. Letting $\gamma_i^\dagger = \lambda_i^*$, $\beta_j^\dagger = \alpha_j^*$, $\nu^\dagger = \mu^*$ and $c_j = -v_j'(g_j - s_j^*)$, Eqn. (2.13) – Eqn. (2.17) are satisfied. The statement of Proposition-2 follows immediately. Note that there may exist other values of $s_j^\dagger \neq s_j^*$ satisfying AOP's KKT conditions so that $\left(d_i^*, s_j^\dagger\right) \in \underset{d_i, s_j}{\operatorname{argmax}} \Phi(d_i, s_j)$. This extra degree of freedom offers the option of post-auction sellers' redistribution.

■

Proposition 2.3. (*Weak budget balance*) The proposed auction mechanism is weakly budget balanced.

Proof of Proposition 2.3. The net revenue remaining with the aggregator at the end of the auction is,

$$\pi_{agg} = \sum_{i \in \mathcal{D}} b_i - \sum_{j \in \mathcal{S}} c_j s_j^\dagger. \tag{2.23}$$

The statement implies that $\pi_{agg} \geq 0$. Hence, the following inequality must be established,

$$\sum_{i \in \mathcal{D}} b_i \geq \sum_{j \in \mathcal{S}} c_j s_j^\dagger. \tag{2.24}$$

The net revenue obtained from the buyers is the bids b_i summed over all buyers, $i \in \mathcal{D}$. Using Eqn. (2.13) the following inequality holds,

$$\sum_{i \in \mathcal{D}} b_i \geq \sum_{i \in \mathcal{D}} p d_i^\dagger. \tag{2.25}$$

From the energy balance constraint given by Eqn. (2.4) at the equilibrium, Eqn. (2.25) can be written as follow,

$$\sum_{i \in \mathcal{D}} b_i \geq \sum_{i \in \mathcal{D}} p d_i^\dagger = \sum_{j \in \mathcal{S}} p s_j^\dagger. \tag{2.26}$$

From Eqn. (2.22) it is seen that $v_j'(g_j - a_j) = c_j$. Since $c_j \leq p$, the inequality in Eqn. (2.26) can be rewritten as,

$$\sum_{i \in \mathcal{D}} b_i \geq \sum_{j \in \mathcal{S}} v_j'(g_j - a_j) s_j^\dagger. \tag{2.27}$$

Since $v_j'(g_j - s_j^\dagger) = c_j$ and $s_j^\dagger \leq a_j$, under the assumption that the utilities $v_j(\bullet)$ are concave, $v_j'(g_j - a_j) \geq v_j'(g_j - s_j^\dagger)$. Hence,

$$\sum_{j \in \mathcal{S}} v_j'(g_j - a_j) s_j^\dagger \geq \sum_{j \in \mathcal{S}} v_j'(g_j - s_j^\dagger) s_j^\dagger. \tag{2.28}$$

From Eqn. (2.27), Eqn. (2.28) , and Eqn. (2.22),

$$\sum_{i \in \mathcal{D}} b_i \geq \sum_{j \in \mathcal{S}} v_j'(g_j - s_j^\dagger) s_j^\dagger = \sum_{j \in \mathcal{S}} c_j s_j^\dagger. \tag{2.29}$$

As $\sum_{j \in \mathcal{S}} c_j s_j^\dagger$ is the reimbursement provided to sellers, the above inequality in Eqn. (2.29) implies that $\pi_{agg} \geq 0$.

■

Proposition 2.4. (*Individual Rationality*) The proposed auction mechanism is individually rational for all participating agents.

Proof of Proposition 2.4. This proposition will be established separately for the buyers and the sellers. Since the bidding strategy of every buyer i is to maximize its payoff $\pi_i = u_i(d_i) - b_i$ where $b_i = u_i'(d_i)d_i$, upon termination of the auction, i.e. at equilibrium, it is evident that,

$$d_i^\dagger = \text{argmax}(u_i(d_i) - u_i'(d_i)d_i). \tag{2.30}$$

Whence it follows that,

$$u_i(d_i^\dagger) - u_i'(d_i^\dagger)d_i^\dagger \geq u_i(0). \tag{2.31}$$

Since the utility of the buyer in the absence of any auction would have been $u_i(0)$, that is the right hand side of the inequality in Eqn. (2.31), it is concluded that the auction is individually rational for the buyers.

The payoff of each seller j after the auction terminates is $\pi_j^\dagger = v_j\left(g_j - s_j^\dagger\right) + c_j s_j^\dagger$. Since at $s_j^\dagger$, from Eqn. (2.22), $c_j = v_j'\left(g_j - s_j^\dagger\right)$, the payoff can be expressed as,

$$\pi_j^\dagger = v_j\left(g_j - s_j^\dagger\right) + v_j'\left(g_j - s_j^\dagger\right)s_j^\dagger. \tag{2.32}$$

Since the seller's strategy is to maximize its payoff, clearly $\pi_j^\dagger \geq v_j\left(g_j\right)$. From the Mean Value Theorem, there exists an $r_j \in \left(0, s_j^\dagger\right)$ such that,

$$v_j(g_j) = v_j\left(g_j - s_j^\dagger\right) + v_j'(g_j - r_j)s_j^\dagger. \tag{2.33}$$

From the concavity assumption of the utilities $v_j(\bullet)$, $v_j'\left(g_j - s_j^\dagger\right) \geq v_j'(g_j - r_j)$ so that using Eqn. (2.32) and Eqn. (2.33),

$$v_j\left(g_j - s_j^\dagger\right) + v_j'\left(g_j - s_j^\dagger\right)s_j^\dagger \geq v_j(g_j). \tag{2.34}$$

Since $v_j(g_j)$ represents the payoff of the seller j before the auction, from Eqn. (2.34), clearly the auction is individually rational for the sellers.

■

2.5 Fair Redistribution

This section addresses the problem of redistribution of the sellers' allocations using a predetermined fairness criterion. The vast literature of computational mechanism design defines several fairness criteria [85]. However, many such paradigms require sellers' hidden information, i.e. their utility functions, in their formulations, contradicting the underlying assumption of this research that the aggregator does not have access to the latter.

An in-auction implementation of any redistribution scheme can be readily accomplished by adding a fairness term to the AOP objective weighted infinitesimally as $\eta F(s_j^r)$, $\eta \ll 1$, so that the auction's properties outlined in the previous section are unaffected. Alternately, it can be implemented post-auction as a second stage of the overall mechanism, which is considered here. The redistribution must be carried out in such a manner that the total amount that the aggregator provides as reimbursement, R, to the sellers must remain unchanged. Hence, the redistribution algorithm follows the constraint below.

$$R = \sum_{j \in \mathcal{S}} c_j^r s_j^r = \sum_{j \in \mathcal{S}} c_j s_j^\dagger. \tag{2.35}$$

In a similar manner, the total energy S supplied by the sellers must remain fixed at that determined prior to redistribution. This is because, from energy balance in Eqn. (2.4), it must equal the total energy delivered to the buyers. Hence,

$$S = \sum_{j \in \mathcal{S}} s_j^r = \sum_{j \in \mathcal{S}} s_j^\dagger. \tag{2.36}$$

Lastly, the amount that each seller is allocated after redistribution should not exceed its declared availability, so that,

$$s_j^r \leq a_j; \quad j \in \mathcal{S}. \tag{2.37}$$

As a representative scheme, we focus on the maximum entropy redistribution [85]. The fair redistribution mechanism's using the maximum entropy criterion is given by,

$$F(s_j^r) = \sum_{j \in \mathcal{S}} \frac{s_j^r}{S} \log \frac{s_j^r}{S}. \tag{2.38}$$

With Eqns. (2.36) and (2.37) as constraints, maximizing $F(s_j^r)$ defines a fair redistribution optimization problem (FROP). The Lagrangian of the FROP is,

$$\begin{aligned} \mathcal{L}_F(s_j^r, \beta_j^r, \nu^r) = \sum_{j \in \mathcal{S}} \frac{s_j^r}{S} \log \frac{s_j^r}{S} + \sum_{j \in \mathcal{S}} \beta_j^r (s_j^r - a_j) \\ + \nu^r \left(\sum_{j \in \mathcal{S}} s_j^r - S \right) \end{aligned} \tag{2.39}$$

with the following equilibrium conditions,

$$\beta_j^r (s_j^r - a_j) = 0. \tag{2.40}$$

$$1 + \log \frac{s_j^r}{S} + S \beta_j^r + S \nu^r = 0. \tag{2.41}$$

This leads to solutions of the form, $s_j^r = Ke^{-S\beta_j^r}$, where $K = Se^{-1}e^{-S\nu^r}$. For all sellers with $s_j^r < a_j$, Eqn. (2.40) shows that $\beta_j^r = 0$, whence $s_j^r = K$. Since $\beta_j^r > 0$ for those sellers that reach their maximum availabilities, the inequality $s_j^r < K$ holds. The redistributed allocations can be stated succinctly as,

$$s_j^r = \min(a_j, K), \tag{2.42}$$

with the aggregate energy term constraint leading to the expression,

$$K = S - \sum_{s_j^r = a_j} s_j^r. \tag{2.43}$$

This reformulation of the FROP leads to the well-known *water filling algorithm* shown in Figure 2.3, and can be readily incorporated within the aggregator as an algorithm of computational complexity $O(|\mathcal{S}| \log|\mathcal{S}|)$.

The sellers per unit energy costs can be implemented in various ways. For instance, uniform pricing leads to,

$$c_j^r = \frac{1}{S} \sum_{j \in \mathcal{S}} c_j s_j^\dagger. \tag{2.44}$$

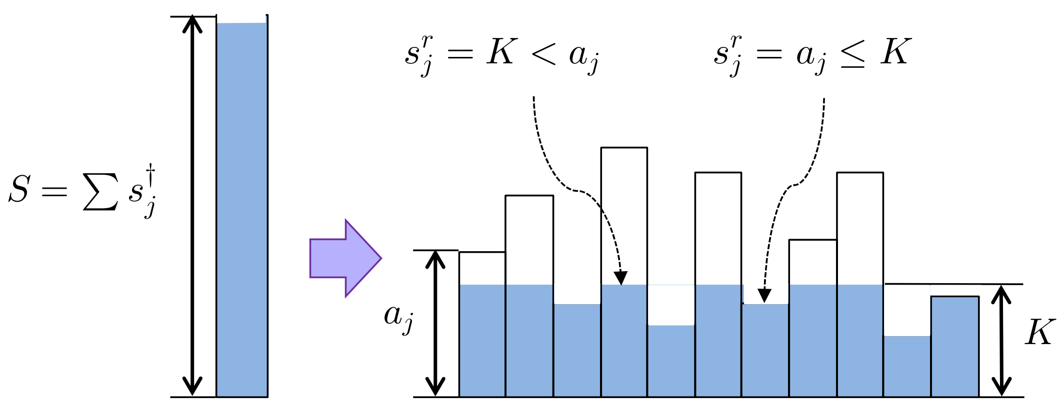

Figure 2.3: Illustration of the water-filling algorithm. The leftmost column represents the total amount of supply, which is redistributed to the columns in the right. In each column, the region shaded in blue (representing water), is the redistributed supply, s_j^r.

As mentioned earlier, this redistribution is accompanied by a loss in the overall SW that is expressed in terms of price of fairness, and is given by the following equation,

$$\kappa_F = \frac{\Theta\big(d_i^\dagger, s_j^\dagger\big) - \Theta\big(d_i^\dagger, s_j^r\big)}{\Theta\Big(d_i^\dagger, s_j^\dagger\Big)}. \tag{2.45}$$

2.6 Simulation Results

Several sets of simulations were performed to complement the theory. The auction in every case were initiated with a per unit market price of $p = 0.25$. Utilities of the buyers and sellers were assumed to follow logarithmic saturation curves according to Eqns. (2.46) and (2.47),

$$u_i(d_i) = x_i \log(y_i d_i + 1)\,, \tag{2.46}$$

$$v_j(g_j - s_j) = x_j \log(y_j(g_j - s_j) + 1)\,. \tag{2.47}$$

The quantities x_i, y_i, x_j and y_j were randomly generated for each agent from a uniform distribution centered at unity. The generations, g_j, for the sellers were also drawn in at random, uniformly in the interval $[2, 5]$.

In order to show that every individual agent is better off participating in the auction, i.e. the auction is individually rational, extensive simulations were performed to get an average seller and buyers' payoff under two cases of sellers with several cases of buyers as depicted in Figure 2.4. Notice that as the number of buyers in the auction increases, the average seller's payoff increases while that of the buyer decreases. For a given number of buyers, the payoff of an average seller is higher in the case of 10 sellers than that of 15 sellers and the average payoff of a buyer is lower in the case of 10 sellers than that of 15 sellers.

To illustrate that the auction allows price differentiation, with $c_i = b_i/d_i$ as the buyers' per unit energy price, Figure 2.5 and Figure 2.6 is presented to show the auction outcome for prices and allocations for two different cases of 5 sellers and 5 buyers (case I) and 5 sellers and 10 buyers (case II) representing two markets with low and high demand. Note that in case I all buyers pay the same minimum per unit price $p =$

0.25 (Figure 2.5) and receive nonzero allocations (Figure 2.6) as the number of buyers are lower. However, they are willing to pay different per unit prices more than $p = 0.25$ in case II as demand and the number of buyers is high. In case II, buyers who are willing to pay high per unit prices get non-zero allocations. For example, buyers 8, 9, and 10 that are not willing to pay higher per unit prices are allocated zero amounts. Note that in both cases, paying the highest per unit price does not mean getting the highest amount of allocation as every agents' utility curve is randomly generated resulting to different marginal utilities. This means that different agents marginal utilities reaches saturation at different prices after which they are not willing to increase or decrease their bids, as it is not profitable.

For the sellers in case I however, except seller 1, all other sellers are allocated lesser supply than their declared availabilities due to low demand in the market, i.e. they end up selling less than their declared availabilities as listed in table I. This is because the buyers' marginal utilities have reached down to saturation at the minimum buying price $p = 0.25$ and they are not allowed to purchase more due to the weakly budget balance constraint $pd_i \leq b_i$. Notice that, as there is more supply in the market in case I, the seller with the lowest selling price, i.e. seller 1, gets to sell all its declared availability.

Sellers 2 to 4 settle down at almost the same price as that of seller 1 and get to sell most of their declared availabilities whereas seller 5 does not sell any amount due to its high price. For case II however, as the number of buyers is high, the sellers get to sell all their declared availabilities at different per unit prices with seller 5 selling at the highest per unit price.

The aggregator's revenue π_{agg} given by Eqn. (2.23) in case I is 0.576 whereas it is 1.13 in case II showing the weakly budget balance property of the proposed double auction. This increase can be readily seen through the change in buyers' per unit prices from case I to case II when they increase from 5 to 10 buyers.

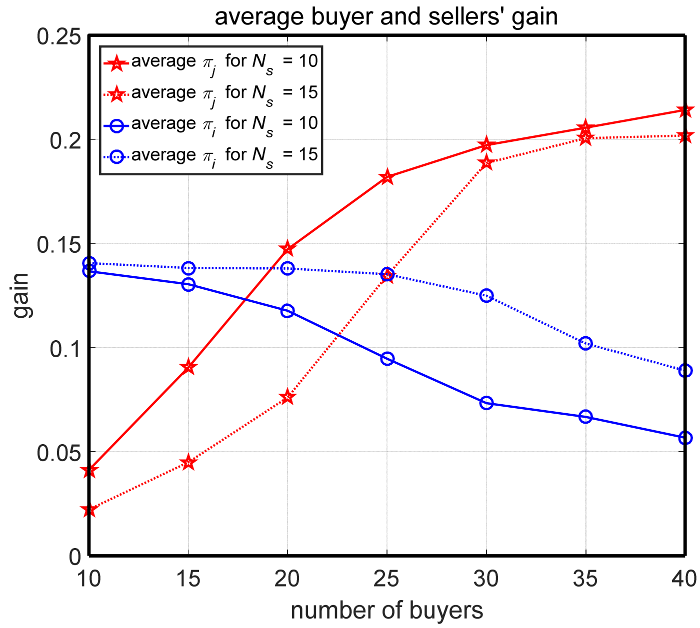

Figure 2.4: Average buyer and sellers' payoff for two cases of sellers with several buyers.

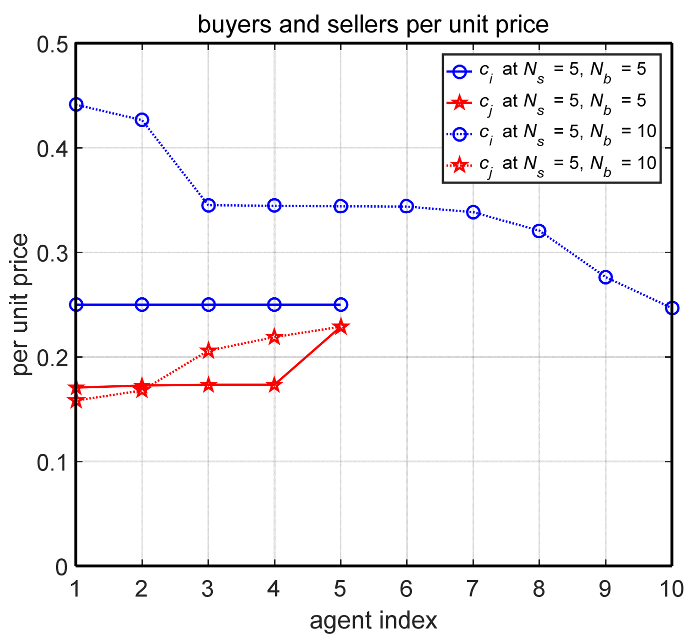

Figure 2.5: Buyers and sellers' per unit prices c_i and c_j for case I and case II.

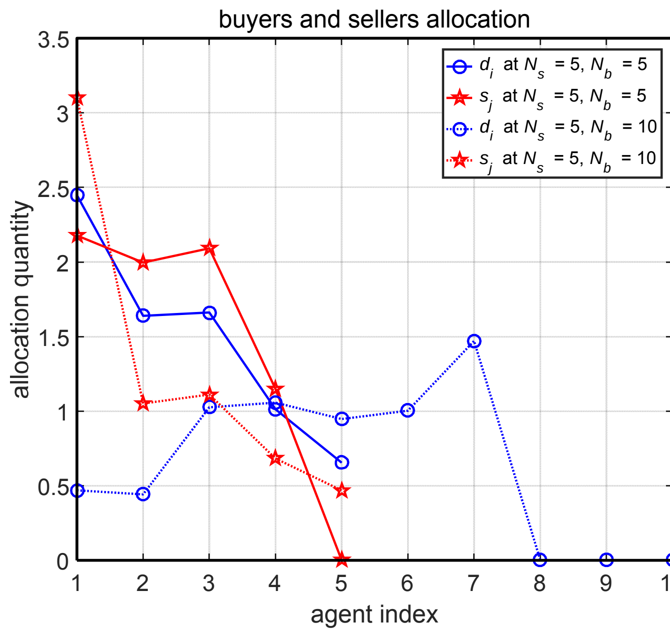

Figure 2.6: Buyers' allocation d_i and sellers' allocation s_j for $N_s = 5,\ N_b = 5$ (case I) and $N_s = 5,\ N_b = 10$ (case II).

Table 2.1: Outcome of the auction pertaining to the sellers, before and after redistribution.

Case	j	g_j	a_j	s_j	c_j	s_j^r	c_j^r
I	1	4.204	2.177	2.177	0.171	1.790	0.172
	2	3.205	2.022	1.997	0.173	1.790	0.172
	3	3.141	2.196	2.092	0.173	1.790	0.172
	4	4.526	1.889	1.149	0.173	1.790	0.172
	5	2.155	0.254	0.000	0.229	0.254	0.172
II	1	4.526	3.101	3.101	0.158	3.101	0.180
	2	2.155	1.052	1.052	0.168	1.052	0.180
	3	4.204	1.112	1.112	0.206	1.112	0.180
	4	3.141	0.683	0.683	0.219	0.683	0.180
	5	3.205	0.470	0.470	0.229	0.470	0.180

To present the effect of fair redistribution on the sellers' side, additional details of the above two case along with the fair redistribution outputs for the sellers are provided in Table 1.

One issue that is solved through a fair redistribution can be observed in case I. Note that sellers 2, 3, and 4 have submitted the same per unit price with their maximum available a_js for sale, however, they have been discriminated during allocation due to multiple optima in the AOP's objective as illustrated earlier in Figure 2.2. The water-filling algorithm discussed earlier is used for this purpose. This algorithm fairly redistributes the sellers' allocation, s_j^r with the new equally redistributed per unit price c_j^r. This clearly comes with a price, quantified earlier as the price of fairness in Eqn. (2.45), and is presented later in Figure 2.7.Notice that in case II, the distribution is already fair in allocations, i.e. $s_j = s_j^r$, as sellers supply at their declared availabilities due to high market demand and redistribution yields the same amounts. However, sellers are price discriminated due to different marginal utilities, which can be redistributed using uniform pricing at sellers consent.

The total sellers' and total buyers' welfare as well as the overall SW under 5 cases when no trade takes place, trade takes place, and when trade takes place and the aggregator redistributes the allocation for fairness purpose with the associated price of fairness, κ_F , is illustrated in Figure 2.7. Note that the SW is higher under trading than the case where no trade takes place, implying the benefit of the auction. Furthermore, the SW decreases after redistribution in the low demand case and is not affected in the high demand cases, where all the sellers sell their entire declared availabilities. The price of fairness is only non-zero when some of the sellers do not happen to sell their declared availabilities.

Lastly, to show that the auction is efficient, i.e. the AOP always attains the SW optimum, percent difference of the SW obtained by the AOP to that of the actual optimum SW has been recorded during each iteration for 4 different cases and has been depicted in Figure 2.8. As can be seen, the percent difference drops to almost zero within several iterations. Note that the AOP attains the actual SW optimum given that no in-auction fairness criterion is applied.

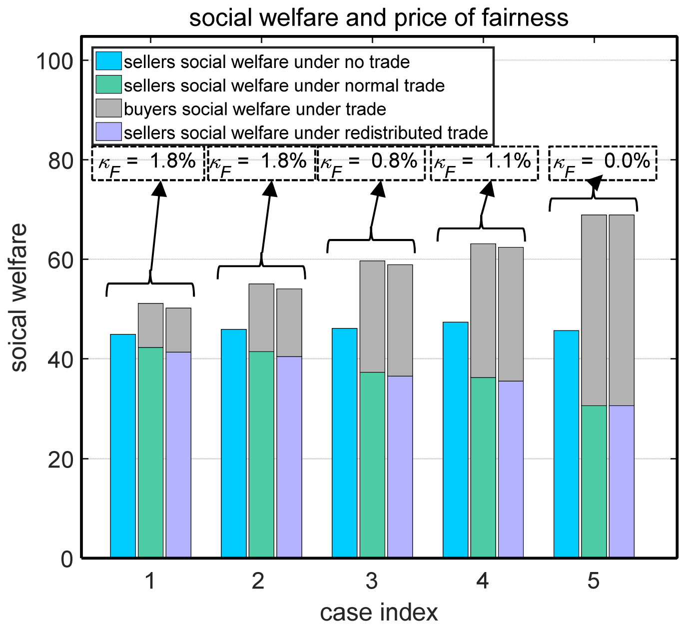

Figure 2.7: SW $\Theta(d_i^\dagger, s_j^\dagger)$ under 5 different cases (case 1: $N_s = 50,\ N_b = 20$, case 2: $N_s = 50$, $N_b = 30$, case 3: $N_s = 50,\ N_b = 50$, case 4: $N_s = 50,\ N_b = 60$, case 5: $N_s = 50,\ N_b = 100$) for no trading, trading, and trading with fair redistribution and the corresponding price of fairness κ_F in percent.

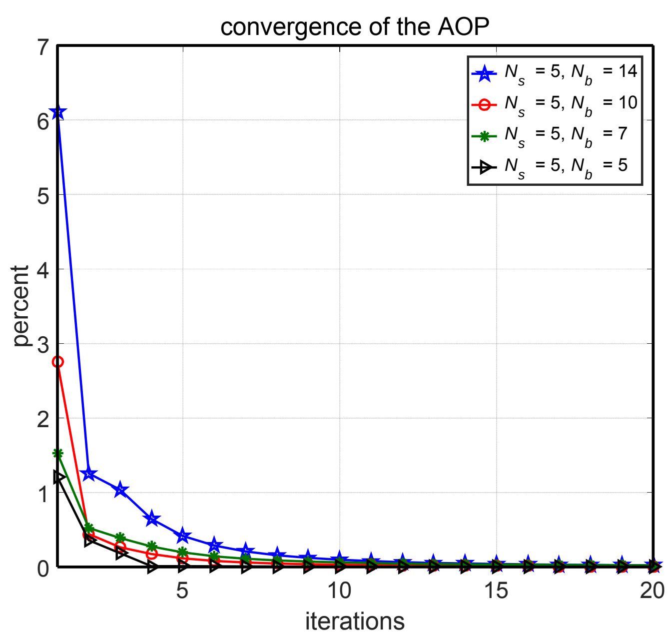

Figure 2.8: Difference between SWs attained by SWOP and AOP as a percentage of the latter

2.7 Conclusions

In this chapter a double sided, weakly budget balanced, individually rational, and efficient auction with hidden user information is presented, which can be applied to energy auctions in the grid. In the simulations reported earlier, an iteration of the double auction involved multiple steps of the underlying AOP algorithm in order to ensure that the allocations were close enough to the optima, before the aggregator allows rebidding. A regularization term weighted by a vanishingly small amount was introduced to AOP to let it converge to an optimum closest to the initial values. This was done to reduce the communication overhead that varies directly as the number of times the agents rebid. This approach differs from those taken elsewhere [62]. The implicit assumption in this research is that the aggregator possesses enough processing capabilities to implement an optimization algorithm. However it should be noted that this approach can be implemented in a distribute manner by using methods such as a dual decomposition algorithm which may increase the communication steps while reducing the aggregator's processing requirements [86].

Although not included in our earlier simulation results, it is possible to apply water-filling fairness criteria in-auction given that the agents' strategies are as given in sections 2.3.3 and 2.3.4 . The rationale is that given the simplicity of the water-filling algorithm, a post-auction implementation reduces the computational overload as opposed to an in-auction scheme where the maximum entropy criterion (Eqn. (2.38)) needs to be included as a third term in Eqn. (2.11) with a weight that is small enough not to affect the bidding process.

As mentioned earlier, it was observed that without any specific in-auction fairness, the AOP always converged to the SWOP solution. This was shown in Figure 2.8 as percent difference of the SW obtained by the AOP compared to the actual SW. As theoretical issues pertaining to this observation have not been addressed in this research, the authors do not recommend this approach unless a specific fairness criterion is needed. Future research directions may be directed towards extending the algorithm to guarantee convergence and fairness criterion.

Although simulations have been carried out within the context of energy trade, grid-specific engineering constraints have not been considered here. However, the core approach proposed here can be readily extended to meet such requirements. It should be noted that this scheme could be extended to handle multi-period operations. The presence of energy storage elements at the local agent level only changes the agents' biddings separately in each period, but not AOP. Conversely, the presence of a shared energy storage device with the aggregator would require an additional term and associated battery constraint in the AOP problem, while the agents place bids in the same manner as in this research. These extensions, which do not change the fundamental nature of the algorithm, can be easily addressed in subsequent research.

Given that in double sided auctions, it is impossible to simultaneously achieve perfect efficiency, budget balance, and individual rationality with incentive compatibility [87], [88], in this study the viability of a double sided individually rational, weakly budget balanced, quasi-efficient auction with agents not having to share private user information has been clearly established.

Chapter 3 - Aggregator-level Price Homogeneous Energy Auction

A distributed proportionally fair double-sided energy auction algorithm that can be implemented by an impartial aggregator, as well as a possible approach by which the agents may approximate price anticipation is investigated. Equilibrium conditions arising due to price anticipation is analyzed. A modified auction to mitigate the resulting loss in efficiency due to such behavior is suggested. This modified auction allows the aggregate SW of the agents to be arbitrarily close to that attainable with price taking agents. Next, equilibrium conditions when the aggregator collects a surcharge price per unit of energy traded is examined. A bi-objective optimization problem is identified that takes into account both the agents' SW as well as the aggregator's revenue from the surcharge. The results of extensive simulations, which corroborate the theoretical analysis, are reported.

3.1 Introduction

An efficient (SW maximizing), weakly budget balanced, price heterogeneous, and individually rational energy auction that is implemented through a profit seeking aggregator was introduced in chapter 2. The aggregator was designed to seek arbitrage opportunities, i.e. buy at a lower price and sell at a higher price. A predetermined fixed market price that was not being updated with iterations was initially sent to individual sellers that were willing to sell at less than or equal to this price. Buyers that were willing to buy at equal or greater than the assigned price were determined and a distributed double auction algorithm was implemented to maximize the SW indirectly.

In this chapter, we study a price uniform, i.e. all agents pay the same per unit price, distributed auction algorithm that is implemented by an impartial aggregator. The proposed auction algorithm is efficient, strongly budget balanced, price uniform, and individually rational. The market-clearing price is iteratively reached based on proportional allocation using the bids of the agents. The proposed mechanism does not require agents' generation and utility function information in order to maximize the SW.

For the sake of reader's convenience, a list of notation and abbreviations for this chapter are provided in Appendix B.

3.2 Auction Framework

3.2.1 Network Model

With energy as the resource involved in the trade, the network of agents in our model consists of a set $\mathcal{D}$ of buyers and another set $\mathcal{S}$ of sellers. Although grid energy auctions typically involve the presence of prosumers that buy and sell energy, we assume for simplicity that $\mathcal{D}$ and $\mathcal{S}$ are disjoint. The model also includes a separate entity, $\mathcal{A}$, the *aggregator* (or auctioneer) that is responsible for communicating with the other agents and implementing the auction. Unless otherwise indicated the aggregator acts as a selfless agent, requiring no separate parametrization of its own, in which case $\mathcal{A} = \emptyset$.

Each agent, whether a buyer or a seller, has its own utility representing the gain (in monetary units) it derives from consuming an amount of energy. The utility of a buyer $i \in \mathcal{D}$ is denoted as u_i, and that of a seller $j \in \mathcal{S}$, as v_j. As the sellers are capable of supplying energy, the model assumes that each has a fixed amount of energy g_j, called its generation that is available both for its own use and to sell.

The underlying physical network that implements the auction mechanism can be completely defined as the following 6-tuple Θ,

$$\Theta \triangleq (\mathcal{D}, \mathcal{S}, g_j, u_i, v_j, \mathcal{A}). \tag{3.1}$$

The mathematical treatment made throughout the rest of this paper is based on the following underlying assumptions.

(i) The utilities u_i and v_j are continuous, differentiable, monotonically increasing and strictly concave functions with non-negative arguments. In other words, $u_i', v_j' > 0$ and $u_i'', v_j'' < 0$ when the argument lies within the interval $(0, \infty)$.

(ii) There is at least one buyer and one seller, i.e. $\mathcal{D}, \mathcal{S} \neq \emptyset$, and furthermore that at least one buyer $i \in \mathcal{D}$ can obtain energy from some seller $j \in \mathcal{S}$ so that some trade takes place. This assumption can be summarized as follows.

$$\exists i \in \mathcal{D}, j \in \mathcal{S}, \ni \ u_i'(0) > v_j'(g_j). \tag{3.2}$$

3.2.2 Auction Process

The buyers and sellers' bidding processes are implemented as separate steps in the auction. Each buyer i receives from the aggregator its *demand* d_i, which is the amount of energy that is allocated for use. The buyer responds by communicating to the aggregator its *bid* b_i, which is the amount of money that it is willing to pay for it. Separately, each seller j receives a per unit *price* p of energy, and communicates back to the aggregator, its *availability* a_j that it is willing to supply.

The schematic in Figure 3.1 shows the layout of the entire auction process. The auction proceeds iteratively until termination when p converges to the market clearing price.

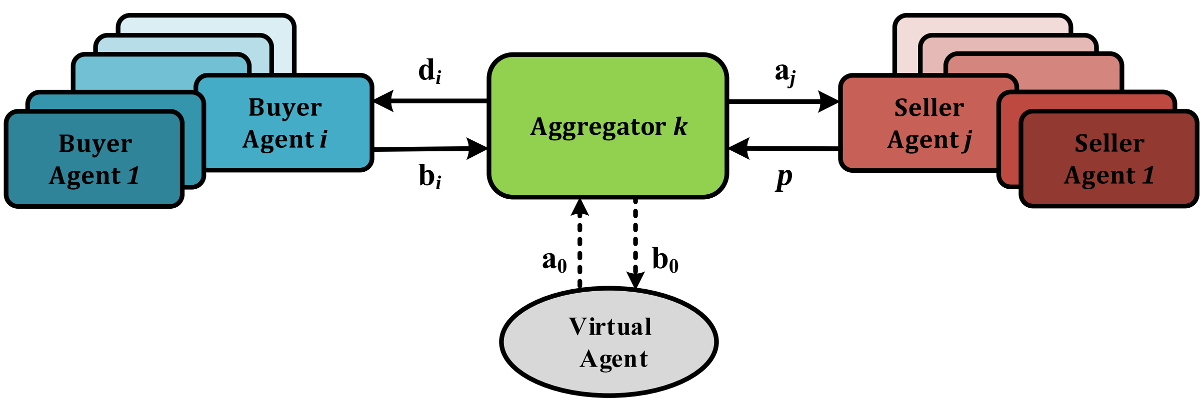

Figure 3.1: Schematic of the network model showing flow of bidding information during auction

3.3 Auction under Price Anticipation

3.3.1 Aggregator

It is assumed that there is no energy loss taking place during transmission. Thus, with the network operating under isolation as is also assumed in this section, the total amount of energy that is declared available by the sellers must be equal to the total amount demanded by the buyers, so that the following *energy balance* equation holds.

$$\sum_i d_i = \sum_j a_j . \tag{3.3}$$

In this section the aggregator is also assumed to be selfless and plays no additional role other than that specified earlier ($\mathcal{A} = \emptyset$), so that the money received as the total buyers' bids is exchanged for the total available energy sold by the sellers. Under these circumstances, the per unit price is given by,

$$p = \left(\sum_j a_j\right)^{-1} \sum_i b_i . \tag{3.4}$$

As the auction is based on proportional allocation of resources, the energy demand d_i that each buyer i receives from the aggregator must be proportional to its bid b_i so that,

$$d_i = \frac{b_i}{p}, \qquad \forall\ i \in \mathcal{D}. \tag{3.5}$$

3.3.2 Buyer

Each buyer i aims to maximize its payoff from the auction mechanism. Noting that it has to pay an amount b_i in order to receive energy d_i, it places a bid b_i in accordance with the following optimization problem.

Maximize w.r.t. b_i:

$$u_i(d_i) - b_i . \tag{3.6}$$

Proposition 3.1. The optimal bidding strategy of a buyer $i \in \mathcal{D}$ is,

$$b_i = d_i u_i'(d_i)(1 - \beta_i). \tag{3.7}$$

Here the quantity β_i is the market power of buyer i described later in this section.

Proof of Proposition 3.1. Let $A = \sum_j a_j$ and $B = \sum_i b_i$. The stationary condition of Eqn. (3.6) is obtained by differentiation with respect to the bid b_i as shown below.

$$u_i'(d_i) \frac{\partial d_i}{\partial b_i} = 1.$$

As buyer i is price anticipating, d_i is dependent on b_i through the price p. Hence replacing $\frac{\partial d_i}{\partial b_i}$appropriately using Eqn. (3.5) and applying the chain rule, we get

$$u_i'(d_i) \frac{1}{p} \left(1 - \frac{b_i}{p} \frac{\partial p}{\partial b_i}\right) = 1.$$

Using Eqn. (3.4) the above equality yields,

$$u_i'(d_i) \left(1 - \frac{b_i}{B}\right) = p.$$

Whence from Eqn. (12),

$$u_i'(d_i)(1 - \beta_i) = p.$$

Proposition-1 follows directly from the above and Eqn. (3.5) with $\beta_i = \frac{b_i}{B}$. More on this later.

■

3.3.3 Seller

Each seller j declares its availability a_j at price p to the aggregator to attain the maximum of its payoff, which is the sum of the money that it receives from selling energy as well as its own utility from consuming the remaining amount $g_j - a_j$ of energy. Noting that its availability cannot exceed its generation g_j, its participation in the auction is characterized by means of the following optimization problem.

Maximize w.r.t. a_j:

$$v_j(g_j - a_j) + pa_j. \tag{3.8}$$

Subject to:

$$a_j \leq g_j. \tag{3.9}$$

Proposition 3.2. The optimal bidding strategy of a seller $j \in \mathcal{S}$ is given by the expression below.

$$a_j = \min\{a_j^{\circ}, g_j\}, \tag{3.10}$$

where a_j° is the solution to the equation,

$$v_j'(g_j - a_j^{\circ}) = p(1 - \alpha_j). \tag{3.11}$$

The seller's market power α_j is described in the next section.

Proof of Proposition 3.2. Introducing the dual variable ρ_j, the Lagrangian of the problem defined in Eqns. (3.8) and (3.9) is,

$$\mathcal{L}_S(a_j, \rho_j) = v_j(g_j - a_j) + pa_j \ + \rho_j(a_j - g_j).$$

This yields the following complementary slackness and stationary conditions.

$$\rho_j(a_j - g_j) = 0,$$

$$v_j'(g_j - a_j) = p + a_j \frac{\partial p}{\partial a_j} + \rho_j.$$

Replacing $\frac{\partial p}{\partial a_j}$ in the stationary condition above appropriately using Eqn. (3.4),

$$v_j'(g_j - a_j) = p(1 - \alpha_j) + \rho_j.$$

When $a_j < g_j$, from the KKT condition $\rho_j(a_j - g_j) = 0$, we can write $\rho_j = 0$. With $\alpha_j = \frac{a_j}{A}$ and $\rho_j = 0$ the corresponding availability in $v_j'(g_j - a_j) = p(1 - \alpha_j) + \rho_j$ is equal to a_j° that solves Eqn. (3.11). The other situation in the KKT condition $\rho_j(a_j - g_j) = 0$ arises when $\rho_j < 0$, in which case the entire generated energy is declared available, i.e. $a_j = g_j$.

We can rewrite the above observations more concisely as,

$$\begin{cases} \rho_j = 0, & a_j < g_j \\ \rho_j < 0, & a_j = g_j. \end{cases}$$

■

3.3.4 Market power

The market power of an agent reflects its ability to influence the overall outcome of the auction. When the auction involves a large number of agents, an individual agent's action cannot exert a great deal of influence on the outcome; consequently the agent's market power is low. In the limiting case when there are an infinite number of agents, the market power approaches zero. It is under this limiting case that price taking conditions serves to approximate.

In the present case, the market power β_i of every buyer i and α_j that of sellers can be defined through separate expressions, given below,

$$\beta_i = \left(\sum\nolimits_{i'} b_{i'}\right)^{-1} b_i, \quad \forall\ i \in \mathcal{D}, \tag{3.12}$$

$$\alpha_j = \left(\sum\nolimits_{j'} a_{j'}\right)^{-1} a_j, \quad \forall\ j \in \mathcal{S}. \tag{3.13}$$

The level of awareness of each buyer or seller about the remaining agents can vary from complete unawareness (price taking) to full awareness of the others' bidding strategies (as required in Eqns. (3.12) and (3.13) above). A realistic scenario lies somewhere in between. In such a case, the iterative auction would allow the buyer or seller to approximate its market power from the information gleaned from previous iterations. The expressions below, which are derived from Eqns. (3.7) – (3.11) can be used as the means by which each buyer or seller can obtain such estimates. Superscripts $(k-1)$ and (k) have been introduced for clarity to indicate each iteration k and its immediately preceding iteration $k-1$.

$$\beta_i^{(k)} = 1 - \frac{b_i^{(k-1)}}{d_i^{(k-1)} u_i'\left(d_i^{(k-1)}\right)}, \tag{3.14}$$

$$\alpha_j^{(k)} = 1 - \frac{1}{p^{(k-1)}} \Big(v_j'\Big(g_j - a_j^{(k-1)}\Big) - \rho_j^{(k-1)} \Big). \tag{3.15}$$

Note that neither expression above incorporates quantities pertaining to the other agents present in the network.

The quantity ρ_j in Eqn. (3.15) is a dual variable obtained from the constrained optimization problem in Eqns. (3.8) and (3.9). It suffices to mention that $\rho_j = 0$ except in the case when the seller declares its entire generation as the availability, i.e. $a_j = g_j$. At the onset of the auction process ($k = 1$), when the agents lack prior information, the market powers may be initialized to zero so that the agents act as simple price takers.

3.3.5 Distributed Double Auction Algorithm

Before the bidding process takes place, there are several ways that the aggregator can initialize the auction variables p and the d_is. An effective way to minimize the number of steps would be to use stored historical data from previous rounds. Otherwise, the aggregator may use heuristic means to do so. In the most simplistic case, these variables may be assigned randomly. This initialization and the subsequent auction steps are outlined in Algorithm 1.

3.3.6 Equilibrium

The auction steps described earlier terminates when further updates of neither the price p nor any of the bids submitted by the agents to the aggregator are changed. This is when generalized Nash equilibrium [89] is established.

In order to characterize the equilibrium conditions under price anticipation, the functions $\pi_i(\bullet)$ and $\pi_j(\bullet)$ are introduced below,

$$\begin{aligned} \pi_i(d_i) = {} & \Big(1 - \Big(\sum\nolimits_j a_j\Big)^{-1} d_i\Big)\, u_i(d_i) \\ & + \Big(\sum\nolimits_j a_j\Big)^{-1} \int_0^{d_i} u_i(z)dz\,, \end{aligned} \tag{3.16}$$

$$\pi_j(g_j - a_j) = v_j(g_j - a_j)\left(\sum\nolimits_{j'} a_{j'}\right)\left(\sum\nolimits_{j' \neq j} a_{j'}\right)^{-1} - \left(\sum\nolimits_{j' \neq j} a_{j'}\right)^{-1} \int_0^{a_j} v_j(g_j - z)dz\,. \tag{3.17}$$

Algorithm 3.1 *Distributed Double Auction Algorithm*

Initialize $p^{(0)}$, $d_i^{(0)}$ $\forall\ i \in \mathcal{D}$

  // *Buyer* $i \in \mathcal{D}$: $\beta_i \leftarrow 0$, // *Seller* $j \in \mathcal{S}$: $\alpha_j \leftarrow 0$

 Set $k \leftarrow 1$

 While (termination criterion = 'F')

  Send $p^{(k)}$ *to sellers* $j \in \mathcal{S}$

  //*Seller* $j \in \mathcal{S}$ *bid*

  Receive $a_j^{(k)}$ *from sellers* $j \in \mathcal{S}$

  Send $d_i^{(k)}$ *to buyers* $i \in \mathcal{D}$

  //*Buyers* $i \in \mathcal{D}$ *bid*

  Receive $b_i^{(k)}$ *from buyers* $i \in \mathcal{D}$

 Increment $k \leftarrow k + 1$

  Obtain $d_i^{(k)}$

  Update $p^{(k)}$

  //*Buyers estimate* $\beta_i^{(k)}$, //*Sellers estimate* $\alpha_j^{(k)}$

 Evaluate termination criterion

End

The effect of price anticipation can be examined in terms of the following constrained optimization problem.

Maximize w.r.t. d_i, a_j

$$\mathit{\Pi}(d_i, a_j|\Theta) = \sum\nolimits_i \pi_i(d_i) + \sum\nolimits_j \pi_j(g_j - a_j)\,, \tag{3.18}$$

subject to constraints in Eqns. (3.3) and (3.9) which are restated below,

$$\sum\nolimits_i d_i = \sum\nolimits_j a_j,$$

$$a_j \leq g_j\ .$$

Denoting the solutions of the above maximization problem as $d_i^\dagger$ and $a_j^\dagger$, the SW is $U^\dagger \triangleq U\big(d_i^\dagger, a_j^\dagger \big| \Theta\big)$. Additionally, the maximum SW corresponding to the efficient solution is denoted as U^*.

Proposition 3.3. (i) There exists a unique equilibrium of the double auction where the demand d_i of each buyer $i \in \mathcal{D}$ and availability a_j of each seller $j \in \mathcal{S}$ is the solution to the optimization problem defined in Eqn. (3.18) with Eqns. (3.3) and (3.9) as constraints.

(ii) The SW attained under price anticipation is no greater than that attainable under price taking, i.e.

$$U^\dagger \leq U^*, \tag{3.19}$$

The above statement implies that there is a loss of efficiency due to price anticipation.

Proof of Proposition 3.3. Observe that, using Eqns. (3.3), (3.12) and (3.13), the derivatives of the functions defined earlier in Eqns. (3.16) and (3.17) are given by π_i' and π_j' in the following expressions,

$$\pi_i' = \frac{\partial}{\partial d_i}\pi_i = (1-\beta_i)u_i'(d_i),$$

$$\pi_j' = \frac{\partial}{\partial a_j}\pi_j = -\frac{1}{1-\alpha_j}v_j'(g_j - a_j).$$

From Eqn. (3.12) since $\beta_i > 0$, whenever $d_i > 0$, $\pi_i' > 0$. The factor $(1-\beta_i)$ in the expression for π_i' above is also strictly decreasing in b_i and hence d_i. Thus π_i' is also monotonically decreasing. Therefore π_i is a strictly concave function. In a similar manner, from Eqn. (3.13) it is clear that $\alpha_j < 1$ as long as $0 \leq a_j \leq g_j$, so that $\pi_j' < 0$ in the expression for π_j' above. Besides, as $\frac{1}{1-\alpha_j}$ is strictly increasing, the product is monotonically decreasing. Therefore π_j is strictly concave. Thus, there is a unique maximum of $\mathit{\Pi}\big(d_i, a_j \big| \Theta\big)$ as defined in Eqn. (3.18).

The Lagrangian of the problem defined in Eqn. (3.18), with Eqns. (3.3) and (3.9) acting as constraints, is given by $\mathcal{L}_{\mathit{\Pi}}$ in the following expression,

$$
\begin{aligned}
&\mathcal{L}_{\Pi}(d_i, a_j, \lambda_j, \mu) \\
&\qquad = \sum\nolimits_i \pi_i(d_i) + \sum\nolimits_j \pi_j(g_j - a_j) + \sum\nolimits_j \lambda_j(a_j - g_j) \\
&\qquad + \mu\left(\sum\nolimits_j a_j - \sum\nolimits_i d_i\right).
\end{aligned}
$$

The quantities μ and λ_j above are the dual variables introduced by the constraints in Eqns. (3.3) and (3.9). The primal conditions from Eqns. (3.3) and (3.9) must be satisfied. Furthermore, complementary slackness conditions yield,

$$\lambda_j(a_j - g_j) = 0.$$

From expressions given by π_i' and π_j' , the stationary conditions of the Lagrangian given by $\mathcal{L}_{\Pi}$ must satisfy,

$$
\begin{aligned}
&(1-\beta_i)u_i'(d_i) = \mu, \\
&v_j'(g_j - a_j) = (1-\alpha_j)(\lambda_j + \mu).
\end{aligned}
$$

From Eqns. (3.5), (3.7) and the stationary condition above for buyers, it is observed that $\mu = p$. Using Eqn. (3.5) it is seen that the buyer's bidding strategy defined in Eqn. (3.7) is satisfied.

When $a_j < g_j$, the complementary slackness condition shows that $\lambda_j = 0$. Replacing λ_j and μ with zero and p, Eqn. (3.11) is satisfied. On the other hand, when $a_j = g_j$, λ_j is set to an appropriate value. From the concavity assumption, $v_j'(g_j - a_j) < p(1-\alpha_j)$ so that $\lambda_j < 0$. We summarize these observations as follows.

$$
\begin{cases}
p = \mu \\
\lambda_j < 0 \ when\ a_j = g_j \\
\lambda_j = 0 \ when\ a_j < g_j.
\end{cases}
$$

From the above considerations, it is seen that both of the bottom two cases satisfy the seller's bidding strategy in Eqn. (3.10).

Eqn. (3.19) is trivially true since U^* is defined as the maximum SW, i.e. $U^* \triangleq \max\limits_{d_i, a_j} U(d_i, a_j | \Theta)$.

■

3.4 Double Auction under Price Taking

3.4.1 Efficient Solution

The efficient solution can be obtained from the following constrained optimization problem.

Maximize w.r.t. d_i, a_j

$$U(d_i, a_j|\Theta) = \sum_i u_i(d_i) + \sum_j v_j(g_j - a_j)\,, \tag{3.20}$$

subject to constraints in Eqns. (3.3) and (3.9) which are restated below,

$$\sum_i d_i = \sum_j a_j,$$

$$a_j \leq g_j, \quad \forall\ j \in \mathcal{S}.$$

For the sake of convenience, the buyer i's bidding strategy, which is that in Eqn. (3.7) with $\beta_i = 0$, is provided below.

$$b_i = d_i u_i{}'(d_i). \tag{3.21}$$

The seller j's strategy is determined according to Eqns. (3.10) and (3.11) where $\alpha_j = 0$, and given below,

$$a_j = \min\{a_j^\circ, g_j\}, \tag{3.22}$$

where a_j° is the solution to the equation,

$$v_j'(g_j - a_j^\circ) = p. \tag{3.23}$$

Proposition 3.4. Under the assumption that the buyers and sellers are price takers, the following statements are true for the double auction.

(i) The buyer and seller strategies are defined according to Eqns. (3.21), (3.22) and (3.23).
(ii) The equilibrium demand d_i^* of each buyer i and availability a_j^* of each seller j after the termination of the auction are unique solutions of Eqns. (3.3), (3.9) and (3.20).
(iii) There is no loss in efficiency, i.e.

$$U^* \triangleq U(d_i^*, a_j^* | \Theta) = \max_{d_i, a_j} U(d_i, a_j | \Theta)\,. \tag{3.24}$$

Thus, the unique equilibrium of the double auction is also the efficient solution.

Proof of Proposition 3.4. First, note that since the utilities are strictly concave, there is a unique optimum of the optimization problem defined in Eqn. (3.20) with constraints defined in Eqns. (3.3) and (3.9). The Lagrangian is given by $\mathcal{L}_U$ below,

$$\begin{aligned}
&\mathcal{L}_U(d_i, a_j, \lambda_j, \mu) \\
&\qquad = \sum\nolimits_i u_i(d_i) + \sum\nolimits_j v_j(g_j - a_j) + \sum\nolimits_j \lambda_j(a_j - g_j) \\
&\qquad + \mu \left(\sum\nolimits_j a_j - \sum\nolimits_i d_i\right).
\end{aligned}$$

The complementary slackness condition is,

$$\lambda_j(a_j - g_j) = 0,$$

and the stationary conditions for buyers and sellers are given below,

$$\begin{aligned}
&u_i'(d_i) = \mu, \\
&v_j'(g_j - a_j) = \lambda_j + \mu.
\end{aligned}$$

Comparing buyers stationary condition above with Eqn. (3.21), under proportional allocation in Eqn. (3.5), we see that $\mu = p$. Similarly, replacing $\frac{\partial p}{\partial a_j}$ in the stationary condition of sellers in proposition-2 with zero, the seller's KKT conditions satisfy the following,

$$\begin{aligned}
&\rho_j(a_j - g_j) = 0 \\
&v_j'(g_j - a_j) = p + \rho_j.
\end{aligned}$$

From comparing the two complementary slackness and stationary conditions of sellers, the following statements are true,

$$\begin{cases} p = \mu \\ \lambda_j < 0 \ when \ a_j = g_j \\ \lambda_j = 0 \ when \ a_j < g_j. \end{cases}$$

Statements (i) and (ii) follow from the above. Since the SW $\mathrm{U}(d_i, a_j | \Theta)$ is maximized, statement (iii) holds.

■

The market price at equilibrium from the auction is denoted as p^*. At equilibrium, the derivative of the utility function (also called *marginal utility*) of each buyer and that of each seller that is not trading its entire generation g_j is equal to the market price; and for traders that trade all of it, more than the price. Mathematically,

$$\begin{cases} u_i'(d_i^*) = p^* \\ v_j'(g_j - a_j^*) = p^* \ when \ a_j^* < g_j \\ v_j'(g_j - a_j^*) > p^* \ when \ a_j^* = g_j. \end{cases} \tag{3.25}$$

The equilibrium can be understood readily graphically as shown in Fig. 2(a), where in the x-axis, $l = \min_j v_j'(g_j)$, $m = \max_j v_j'(0)$, and $n = \max_i u_i'(0)$. Here, we define the aggregate *demand function* $D(p)$ as the total amount of energy delivered to the buyers as a function of the market price p. Likewise, we define the *availability function* $A(p)$ as the total availability declared by the suppliers as a function of p. Thus,

$$D(p) = \sum_i d_i \,, \tag{3.26}$$

$$A(p) = \sum_j a_j \,. \tag{3.27}$$

Proposition 3.5. Under the assumption of price taking, the following statements are true for the double auction.

($\boldsymbol{i}$) The availability function $A(p)$ is zero when $p \leq \min_j v_j'(g_j)$, monotonically increasing with price p in the interval $p \in \left(\min_j v_j'(g_j)\,, \max_j v_j'(0)\right)$ and constant when $p \geq \max_j v_j'(0)$. In other words,

$$\begin{cases} A(p) = 0, & p \leq \min_j v_j'(g_j) \\ A(p) \ mon. \ \ inc., & \min_j v_j'(g_j) < p < \max_j v_j'(0) \\ A(p) \ constant, & p \geq \max_j v_j'(0). \end{cases} \tag{3.28}$$

(***ii***) The demand function $D(p)$ is monotonically decreasing with price p in the interval $p \in \left(0, \max_i u_i'(0)\right)$ and zero when $p \geq \max_i u_i'(0)$.

$$\begin{cases} D(p)\ mon.\,dec., & p < \max_i u_i'(0) \\ D(p) = 0, & p \geq \max_i u_i'(0)\,. \end{cases} \tag{3.29}$$

(***iii***) At the unique equilibrium price p^*, $A(p^*) = D(p^*)$.

Proof of Proposition 3.5. For each seller j, from proof of proposition-2 it is seen that $\rho_j = 0$ when $a_j < g_j$. With $\alpha_j = 0$ under price taking, the stationary condition of a seller j is rewritten as,

$$v_j'(g_j - a_j) = p.$$

Hence,

$$a_j = g_j - v_j'^{-1}(p).$$

Since $v_j'' > 0$, a_j is strictly increasing in the interval $p \in \left(v_j'(g_j), v_j'(0)\right)$. Moreover as $a_j \in [0, g_j]$ and as Eqn.(3.27) shows, $A(p)$ is the sum of all a_js, statement (i) follows.

For each buyer i, from proof of proposition-1, with $\beta_i = 0$,

$$u_i'(d_i) = p.$$

Hence,

$$d_i = u_i'^{-1}(p).$$

Since $u_i'' > 0$, d_i is strictly decreasing with p in the interval $p \in \left(0, u_i'(0)\right)$. Moreover $d_i = 0$ when $p \geq u_i'(0)$. Hence, statement (ii) follows directly from Eqn. (3.26) where $D(p)$ is expressed as the sum of all d_is.

From Eqn. (3.2) , there exists a non-empty interval $p \in \left(\min_j v_j'(g_j)\,, \max\left(\max_i u_i'(0)\,, \max_j v_j'(0)\right)\right)$ within which $D(p)$ is monotonically decreasing or zero and $A(p)$ is monotonically increasing or fixed at a positive value. Thus there is a unique p^* such that $A(p^*) = D(p^*)$.

■

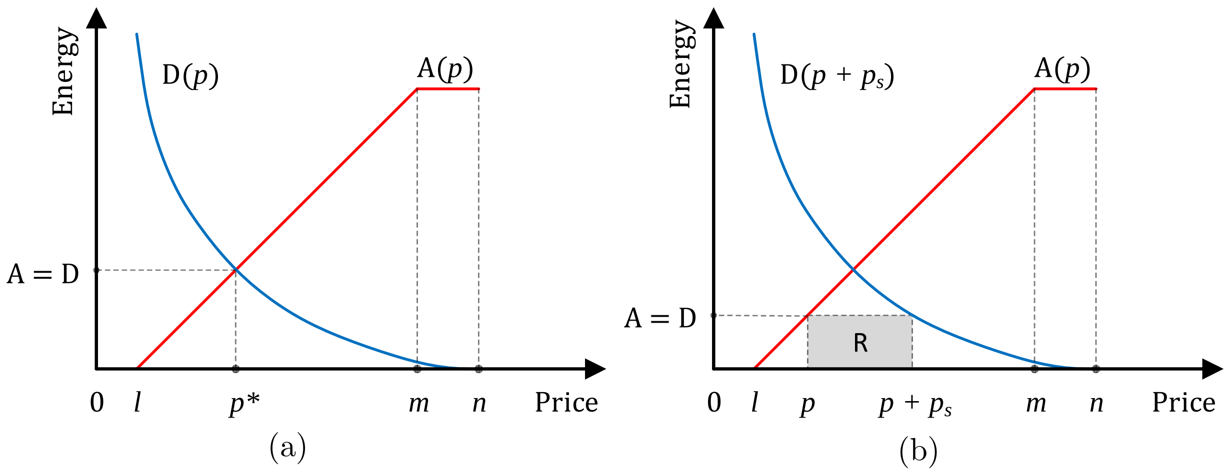

Figure 3.2: Plots of the $A(p)$ and $D(p)$ as functions of p. (a) Equilibrium conditions under price taking. (b) The addition of surcharge price with revenue R being the area shaded in grey.

3.4.2 Virtual Bidding

In general, the loss of efficiency, when d_i is the demand of each buyer i and a_j is the allocation of each seller j, can be expressed as follows.

$$L_{\Theta}(d_i, a_j) = \frac{U(d_i^*, a_j^* | \Theta) - U(d_i, a_j | \Theta)}{U(d_i^*, a_j^* | \Theta)}. \tag{3.30}$$

The loss that takes place when the agents participate in the auction as price anticipators is $L_{\Theta}(d_i^{\dagger}, a_j^{\dagger})$. This section shows how the basic proportional allocation double auction mechanism can be extended to mitigate the loss of efficiency.

In order to minimize the loss L_{Θ}, a *virtual agent* can be introduced to the network defined earlier in Eqn. (3.1). The virtual agent, which is indexed with the subscript '0', participates in the auction simultaneously as a buyer and a seller with arbitrarily large availability a_0. As the virtual agent is incorporated within the aggregator, we let $\mathcal{A} = \{a_0\}$.

Since the virtual agent does not have its own generation, it buys back the amount of energy a_0 declared as its availability at the market price defined in Eqn. (3.4), so that,

$$b_0 = pa_0. \tag{3.31}$$

Proposition 3.6. As the virtual agent's availability a_0 increases, the loss in efficiency L_Θ from price anticipation decreases. In the limiting case,

$$\lim_{a_0 \to \infty} L_\Theta(d_i^\dagger, a_j^\dagger) = 0. \tag{3.32}$$

Since the inclusion of virtual bidding allows the auction to behave like a price taking mechanism, for the remainder of this section we assume that the buyers and sellers behave as price takers.

Proof of Proposition 3.6. Since from Eqn. (3.31) $b_0 = pa_0$, the expression for the price in Eqn. (3.4) is replaced with,

$$p = \Big(a_0 + \sum\nolimits_j a_j\Big)^{-1} \Big(b_0 + \sum\nolimits_i b_i\Big).$$

With the addition of virtual bidding, Eqns. (3.12) and (3.13) pertaining to market powers are rewritten as,

$$\beta_i = \Big(b_0 + \sum\nolimits_{i'} b_{i'}\Big)^{-1} b_i, \quad \forall\ i \in \mathcal{D},$$

$$\alpha_j = \Big(a_0 + \sum\nolimits_{j'} a_{j'}\Big)^{-1} a_j, \quad \forall\ j \in \mathcal{S}.$$

The above two expressions show that β_i and α_j monotonically decrease with increasing a_0. From the expressions π_i' and π_j' in the proof of proposition-3, it follows that $\frac{\partial}{\partial d_i}\pi_i$ and $\frac{\partial}{\partial a_j}\pi_j$ monotonically approach u_i' and v_j'. As we have shown that π_i and π_j are strictly concave, it follows that they increase monotonically with increasing values of a_0; whence from Eqn. (3.18), $U^\dagger$ also increases monotonically. In the limiting case, $\lim\limits_{b_0 \to \infty} \beta_i = 0$, and $\lim\limits_{a_0 \to \infty} \alpha_j = 0$; whereupon it follows that $\lim\limits_{a_0 \to \infty} U^\dagger = U^*$. Simultaneously, from Eqn.

(3.30), the loss of efficiency decreases monotonically towards zero. This shows that inserting the virtual bidder into the auction allows the auction to simulate as price-taking.

■

3.4.3 Surcharge

A situation where the aggregator, $\mathcal{A}$, is no longer a strictly selfless enabler in the auction process but also has its own incentive to implement the mechanism by levying a *surcharge* price p_s per unit of energy traded is investigated in this section. Thus in the model in Eqn. (3.1) the aggregator now includes the surcharge, which we indicate by letting it be given by $\mathcal{A} = \{a_0 \to \infty, p_s\}$. The total revenue earned by the aggregator from the auction with the introduction of surcharge is given by,

$$R = p_s \sum\nolimits_j a_j. \tag{3.33}$$

The expression for price in Eqn. (3.4) is modified to account for surcharge as follows,

$$\sum\nolimits_i b_i = (p + p_s) \sum\nolimits_j a_j. \tag{3.34}$$

With proportional allocation, the demand d_i that each buyer i receives is given by the following expression that replaces the earlier Eqn. (3.5),

$$d_i = \frac{b_i}{p_s + p}. \tag{3.35}$$

Figure 3.2(b) illustrates the effect of the surcharge. Eqn. (3.35) shows that the buyers purchase energy at an effective per unit price of $p_s + p$ which is higher than p that the sellers receive per unit of energy traded. The volume of energy traded is equal to $D(p_s + p) = A(p)$, which is lower than $A(p^*) = D(p^*)$.

It is shown that the price-taking auction is the solution to the following constrained optimization problem.

Maximize w.r.t. d_i, a_j

$$\begin{aligned} \Omega(d_i, a_j | \Theta) = \sum\nolimits_i u_i(d_i) + \sum\nolimits_j v_j(g_j - a_j) \\ -p_s \sum\nolimits_j a_j, \end{aligned} \tag{3.36}$$

subject to constraints in Eqns. (3.3) and (3.9) restated below,

$$\sum_i d_i = \sum_j a_j,$$

$$a_j \leq g_j.$$

Proposition 3.7. Under the assumption of price taking, the following statements are true with surcharge price $p_s > 0$.

(*i*) The buyer and seller strategies are defined according to Eqns. (3.21), (3.22) and (3.23).
(*ii*) The equilibrium demand d_i of each buyer i and availability a_j of each seller j of the auction are unique solutions of Eqns. (3.3), (3.9) and (3.36).
(*iii*) There exists a Pareto front where any increase in revenue R is associated with a simultaneous decrease in the SW in Eqn. (3.20).
(*iv*) There exists an optimal surcharge p_s^{OPT} that maximizes the aggregator $\mathcal{A}$'s revenue R.

Proof of Proposition 3.7. The equilibrium of the optimization problem defined in Eqn. (3.36) is unique because the addition of the linear term involving p_s does not alter the concavity property.

The Lagrangian of the problem defined in Eqn. (3.36), with Eqns. (3.3) and (3.9) as constraints, is given by,

$$\begin{aligned}
&\mathcal{L}_{\Omega}(d_i, a_j, \lambda_j, \mu) \\
&\qquad = \sum_i u_i(d_i) + \sum_j v_j(g_j - a_j) - p_s \sum_j a_j \\
&\qquad + \sum_j \lambda_j (a_j - g_j) + \mu \left(\sum_j a_j - \sum_i d_i \right).
\end{aligned}$$

The stationary conditions satisfy,

$$\lambda_j (a_j - g_j) = 0,$$

$$u_i'(d_i) = \mu + p_s,$$

$$v_j'(g_j - a_j) = \lambda_j + \mu.$$

Analogous to the reasoning provided in the proof of Proposition-4, it can be established that at equilibrium we must have,

$$\begin{cases} p = \mu \\ \lambda_j < 0 \; when \; a_j = g_j \\ \lambda_j = 0 \; when \; a_j < g_j. \end{cases}$$

This establishes the statements (*i*) and (*ii*).

From Eqn. (3.33), when $p_s = 0$, the revenue $R = 0$. However, from the assumption in Eqn. (3.2), the aggregate A is nonzero. Thus, the SW is at the unique maximum $U^* > 0$. Increasing p_s monotonically increases the revenue R and monotonically decreases the SW U. This shows the existence of a non-singleton Pareto front as claimed in (*iii*).

It can be readily inferred from Figure 3.2(b) that for a sufficiently large value of p_s, the aggregate demand is zero, so that the volume of energy traded is zero and $R = 0$. In fact the upper limit of p_s is defined as,

$$p_s < \max_i u_i'(0) - \min_j v_j'(g_j).$$

It is concluded that, there is an optimal p_s that maximizes the aggregator's revenue R verifying the claim in statement (*iv*).

■

3.5 Simulation Results

In order to compliment the theoretical considerations in the earlier sections, several sets of simulations were carried out. A total of five scenarios were considered, where the number of buyers and sellers were $|\mathcal{D}| = 2, |\mathcal{S}| = 3$, $|\mathcal{D}| = 2, |\mathcal{S}| = 6$, $|\mathcal{D}| = 2, |\mathcal{S}| = 10$, $|\mathcal{D}| = 3, |\mathcal{S}| = 2$ and $|\mathcal{D}| = 4, |\mathcal{S}| = 4$. In order to analyze the effect of price anticipation, the total number of agents were made relatively small in comparison to other simulation studies. Moreover, the first three scenarios contain only 2 sellers. This reflects the situation is a realistic microgrid, where the number of PV-equipped units is usually lower than the number of those without it. The fourth and fifth scenarios were added to explore the performance of the double auction under other potential situations.

The utilities of the buyers and sellers assumed to follow logarithmic saturation curves according to Eqns. (2.46) and (2.47) with s_j replaced by a_j. The quantities x_i, y_i, x_j and y_j were different for each agent, and were generated randomly from a uniform distribution centered at unity. The generations, g_j, for the sellers were also drawn in at random, uniformly in the interval $[g_{\min}, g_{\max}]$.

The first set of simulations was performed to examine the effect of price anticipation of the buyers and sellers upon the double auction. The results of this study are shown in Figure 3.3. It can be seen that in each case there is a reduction in the SW due to price anticipation. Further detailed analysis shows that when considered separately, while the SW of the buyers reduces due to price anticipation, the SW of the sellers is increased. This is because Propositions 1 and 2 indicate that price anticipation $(\beta_i, \alpha_j > 0)$ causes the values of d_i and a_j to be lower than with price taking $(\beta_i, \alpha_j = 0)$. Consequently, the volume of energy being traded is also less so that the surplus amount of energy $g_j - a_j$ remaining with each seller j is higher, which also increases its utility $v_j(g_j - a_j)$.

Although, due to price anticipation, the SW in a double auction is lower than its optimal value, the utilities of the sellers change in the opposite direction. Thus, an observation made from this study is that the effect of price anticipatory agents in double-auctions is less severe in comparison to single-sided auctions.

The effect of virtual bidding was investigated through a second set of simulations. The results of these simulations are provided in Figure 3.4, separately for each of the five scenarios. It can be seen that the loss in efficiency L_Θ approaches zero as a_0 increases towards $a_0 \to \infty$.

This observation holds true for each of the five scenarios that were simulated, and is consistent with Proposition-6.

In order to examine the role of the surcharge price p_s on the double auction, a set of simulations were carried out for each of the five scenarios described earlier. As price-taking conditions are assumed, the agents' market powers were always set at $\beta_i = 0, \alpha_j = 0$, throughout the iterative mechanism.

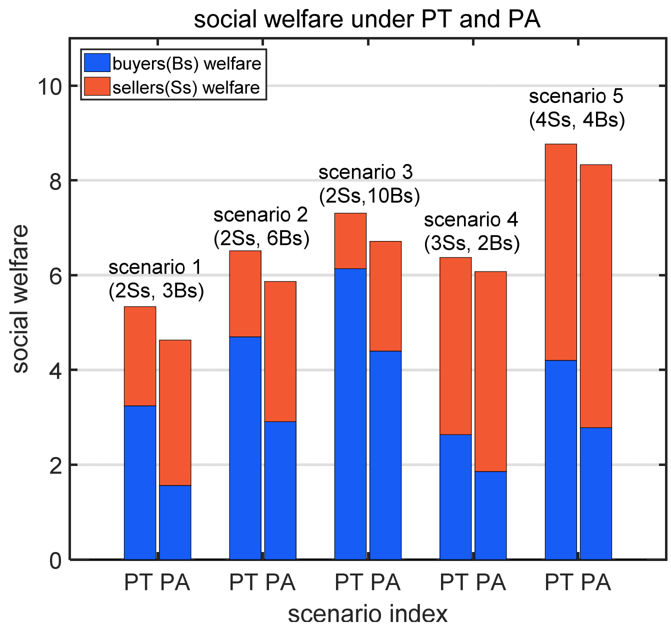

Figure 3.3: SW U under price taking (PT) and price anti-cipation (PA) for each scenario.

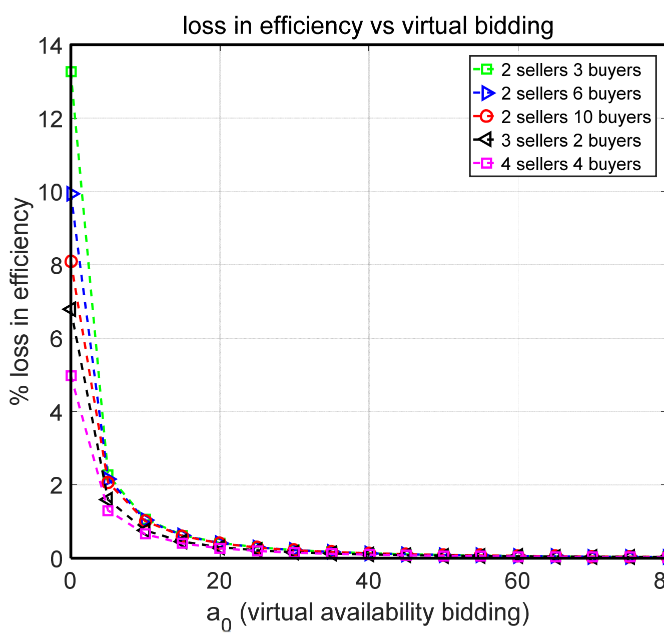

Figure 3.4: Loss in efficiency L_{Θ} as a function of a_0 for each scenario.

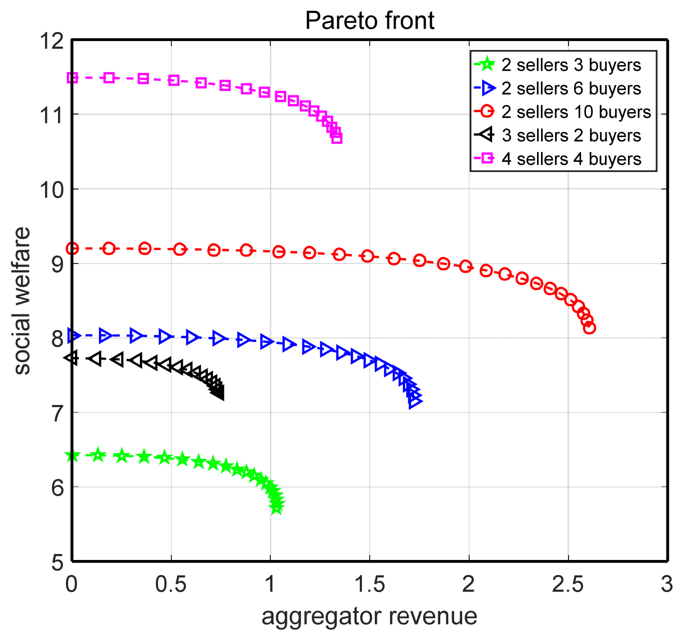

Figure 3.5: Pareto front of the revenue R and SW U with varying surcharge p_s, for each scenario.

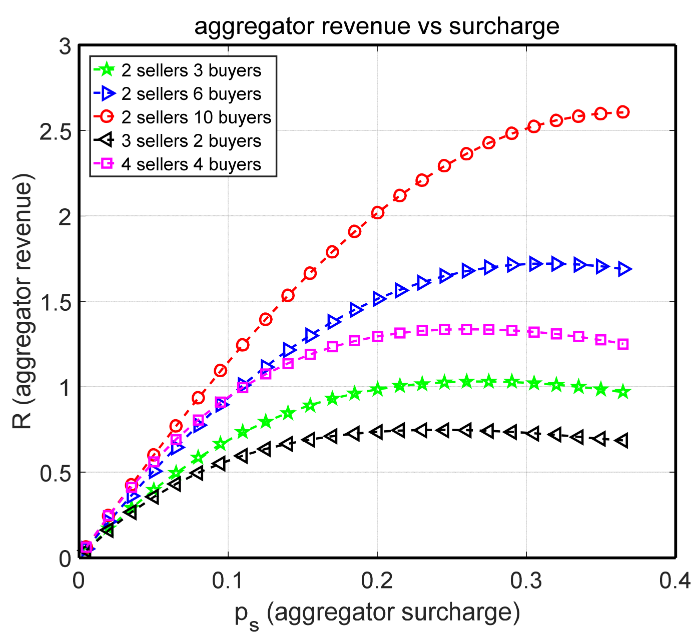

Figure 3.6: Aggregator's revenue R as a function of surcharge price p_s, for each scenario.

The auction was simulated until equilibrium for different values of the surcharge price p_s. Figure 3.5 shows the Pareto front discussed in the claim (*iii*) in Proposition-7. In each scenario, the extreme left ends of the fronts correspond to $p_s = 0$ so that the SW is maximum $U = U^*$, while the aggregator's revenue $R = 0$.

As p_s progressively increases until p_s^{OPT}, so does R, while U decreases. The right ends of the Pareto fronts correspond to $p_s = p_s^{\mathrm{OPT}}$. When p_s exceeds p_s^{OPT}, both U and R decrease, which is not shown in Figure 3.5.

Figure 3.6 shows how the aggregator's revenue R varies with surcharge p_s. In each scenario, R increases with p_s until it reaches its maximum when the surcharge is p_s^{OPT}. In all but one scenario, the revenue R can be seen to decrease beyond its corresponding maximum. These results are consistent with claim (*iv*) of Proposition-7.

3.6 Conclusions

The distributed double auction algorithm in Section 3.3 can be implemented readily by the aggregator, even in the presence of a virtual agent or with surcharge pricing. The algorithm can optionally consider price-anticipatory agents. A possible method by which real world agents may use information gathered from earlier iteration to imitate price anticipation has been suggested. It is shown that with price anticipating agents, the double auction's equilibrium coincides with that of a constrained optimization problem whose objective function Π is different from the SWF U, resulting in a loss of efficiency. It is shown that when the aggregator incorporates a virtual agent that is simultaneously both a buyer and a seller, can minimize the loss of efficiency, so that the double auction can reach the efficient equilibrium.

A generalized auction scenario where the aggregator receives a surcharge price is investigated where in the limiting case; the aggregator may act as a selfish agent trying to maximize its revenue R from the auction. With the SW U and revenue R as independent objectives, a bi-objective framework for the double auction mechanism is suggested.

The theoretical analysis has been supplemented by several simulations. The results of the simulations are in complete agreement with the theory.

Chapter 4 - Bi-level Energy Distribution Auction

In this chapter, a bi-level energy distribution mechanism that achieves *global efficient solution* is proposed. In the presence of a fixed amount of supply from the wholesale market, the objective of the DSO is to maximize global SW and distribute the available energy to the aggregators that value it the most, while considering the physical grid constraints. The DSO SW optimization problem (DSWOP) is decomposed into a *master* and a *sub-problem*. The master problem is solved by an iterative *DSO-level auction* (DLA) among aggregators by solving a decomposed problem with linear objective function and linear and quadratic constraints. In parallel, the sub-problem is solved by an *aggregators-level auction* (ALA) through a *homogenously priced auction* among its consumers and prosumers. The proposed bi-level auction mechanism is shown to be globally efficient and achieve DSWOP equilibrium conditions.

4.1 Introduction

The proliferation of Renewable Energy Resources (RES) at the distribution level is reshaping the market structure of the Distribution System Operators (DSO). The electricity sector has devolved from a highly regulated system operated by vertically integrated utilities to a relatively decentralized system based more fully on market valuation and allocation mechanisms [90]. RES owners such as PV equipped homes are anticipated to participate in such mechanisms more strategically while seeking profit [78], [79]. DSOs on the other hand, are expected to leverage the available local resources in order to capture additional value by optimizing the system for least cost operation while maintaining the physical system operation constraints [91]. One of the key challenges for efficient energy distribution mechanisms is its design in such a way that can motivate active participation of customers [37]. Without active participation of customers in such energy distribution mechanisms, the benefits of smart grid will not be fully realized [92]. Therefore, efficient mechanisms that ensure the optimal operation within distribution system limitations and constraints while maintaining incentives for customers to participate are needed.

In this chapter, an iterative bi-level energy distribution mechanism that converges to a globally social optimal solution while maintaining physical system constraints and providing incentives for customers to participate without asking for their private information such as utility functions and generation capacities is proposed. The inner level auction, referred to as ALA, is conducted by a local aggregator residing at each distribution node among downstream consumers and prosumers in order to provide maximization of their profit and achieve equilibrium conditions. The upper level auction, referred to as DLA, is implemented by the DSO among aggregators competing for the share of energy that the DSO receives from the wholesale market. An aggregator can act as a seller or buyer depending on the demand and supply availability of its own customers, i.e. consumers and prosumers, in order to achieve equilibrium conditions. The goal of the DSO's auction is to optimally allocate its committed, or estimated, energy purchase from the wholesale market among competing aggregators, while maximizing the SW and maintaining system physical constraints such as voltage, line, and transformer limits.

For the sake of reader's convenience, a list of notation and abbreviations for this chapter are provided in Appendix B.

4.2 Auction Framework

Consider a radial distribution network as shown in Figure 4.2 with $\mathcal{N}$ denoting the set of nodes excluding root. Let $\mathcal{A}$ show the set of aggregators in the network with one physical node's customers served by only one aggregator. An aggregator residing on a single distribution node $k, k \in \mathcal{N}$, serves a set of $\mathcal{N}_B^k$ buyer agents and $\mathcal{N}_S^k$ seller agents with PV generation. Each buyer i, and seller j has hidden utility functions $u_i^k(d_i^k)$, and $v_j^k(g_j^k - s_j^k)$ that shows the amount of satisfaction they derive from consuming electrical energy d_i^k and $(g_j^k - s_j^k)$. Here, d_i^k is the demand of buyer i, s_j^k is the supply, and g_j^k is the PV generation of the j^{th} seller within k^{th} aggregator. The sum of these utility functions is the SW function (SWF) of the customers served by aggregator $k,\ k \in \mathcal{A}$. The aggregator's responsibility is to ensure supply demand balance while maximizing the SWF of its customers without access to their hidden utility functions and generation amounts.

As stated earlier, the proposed bi-level auction mechanism is implemented in two levels among aggregators residing on distribution nodes by the DSO (DLA) and among consumers and prosumers residing on a lateral feeder connected to a distribution node by the aggregators (ALA). In the ALA, each agent's objective is to maximize its own profit by participation in the auction. Each aggregator's objective is to maximize its customers' SW. The aggregators do this through participation in the DLA by competing with other aggregators to in order to get their optimal supply share of real power p_k. The DSO's objective is to implement the DLA iteratively until equilibrium is established and the maximum global SW is attained.

During each iteration of the DLA, aggregator k receives real power supply p_k, implements its ALA and submits its' per unit price c_k for p_k. Similar to the price homogenous auction in chapter 3, the price c_k is obtained as the market-clearing price of the ALA. Given the new set of prices c_k for each aggregator $k \in \mathcal{A}$, the DSO reruns the DLA to find the new supply p_k, while maintaining physical system constraints such as voltage limits, line flow limits, and substation transformer capacity. This procedure continues until convergence is achieved.

4.3 DSO-level Auction

4.3.1 Distribution System Constraints

The DLA proceeds while considering physical distribution grid constraints such as per unit (pu) voltage magnitudes at each node, line MVA limits, and substation transformer capacity. In this research, it is assumed that distribution system is balanced and all quantities and amounts can be represented per phase at each consumer or prosumer node.

In the single branch radial distribution system shown in Figure 4.1 the real and reactive power flow at each branch $k+1$ (branch going to node $k+1$) and the corresponding node $k+1$ voltage are described by the DistFlow Equations from [93], [94].

$$P_{k+1} = P_k - r_k \frac{P_k^2 + Q_k^2}{V_k^2} - p_k, \tag{4.1}$$

$$Q_{k+1} = Q_k - x_k \frac{P_k^2 + Q_k^2}{V_k^2} - q_k, \tag{4.2}$$

$$V_{k+1}^2 = V_k^2 - 2(r_k P_k + x_k Q_k) + (r_k^2 + x_k^2) \frac{P_k^2 + Q_k^2}{V_k^2}. \tag{4.3}$$

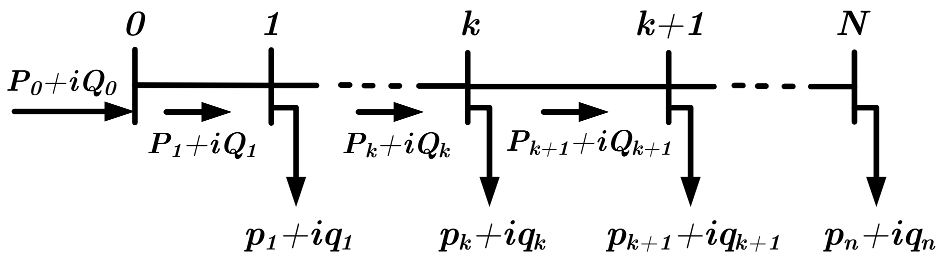

Figure 4.1. Schematic diagram of a unidirectional single branch radial distribution system with N nodes excluding the root node.

Here, V_k is node k's per unit voltage magnitude, (r_k, x_k) is the line resistance/reactance and (P_k, Q_k) is the line real/reactive power flow of the line $(u(k), k)$ where $u(k)$ is the immediate upstream node to node k. The net real and reactive power injection into node k is denoted by (p_k, q_k). This is the load of node k minus generation of k provided by a DG or the sum of demand minus the sum of supply of an aggregator. In this research the simplified version of the DistFlow equations in Eqns. (4.1)–(4.3) have been used. The simplified DistFlow equations have been extensively used in the literature [93],[94],[95],[96],[97] and are given by Eqns. (4.4) – (4.6) below,

$$P_{k+1} = P_k - p_k, \tag{4.4}$$

$$Q_{k+1} = Q_k - q_k, \tag{4.5}$$

$$V_{k+1} = V_k - \frac{r_k P_k + x_k Q_k}{V_0}, \tag{4.6}$$

where V_0 is the root node voltage. For a radial distribution feeder such as the one shown in Figure 4.2, with $\Delta V_k = \frac{r_k P_k + x_k Q_k}{V_0}$, the DistFlow equations for node k can be written as follow,

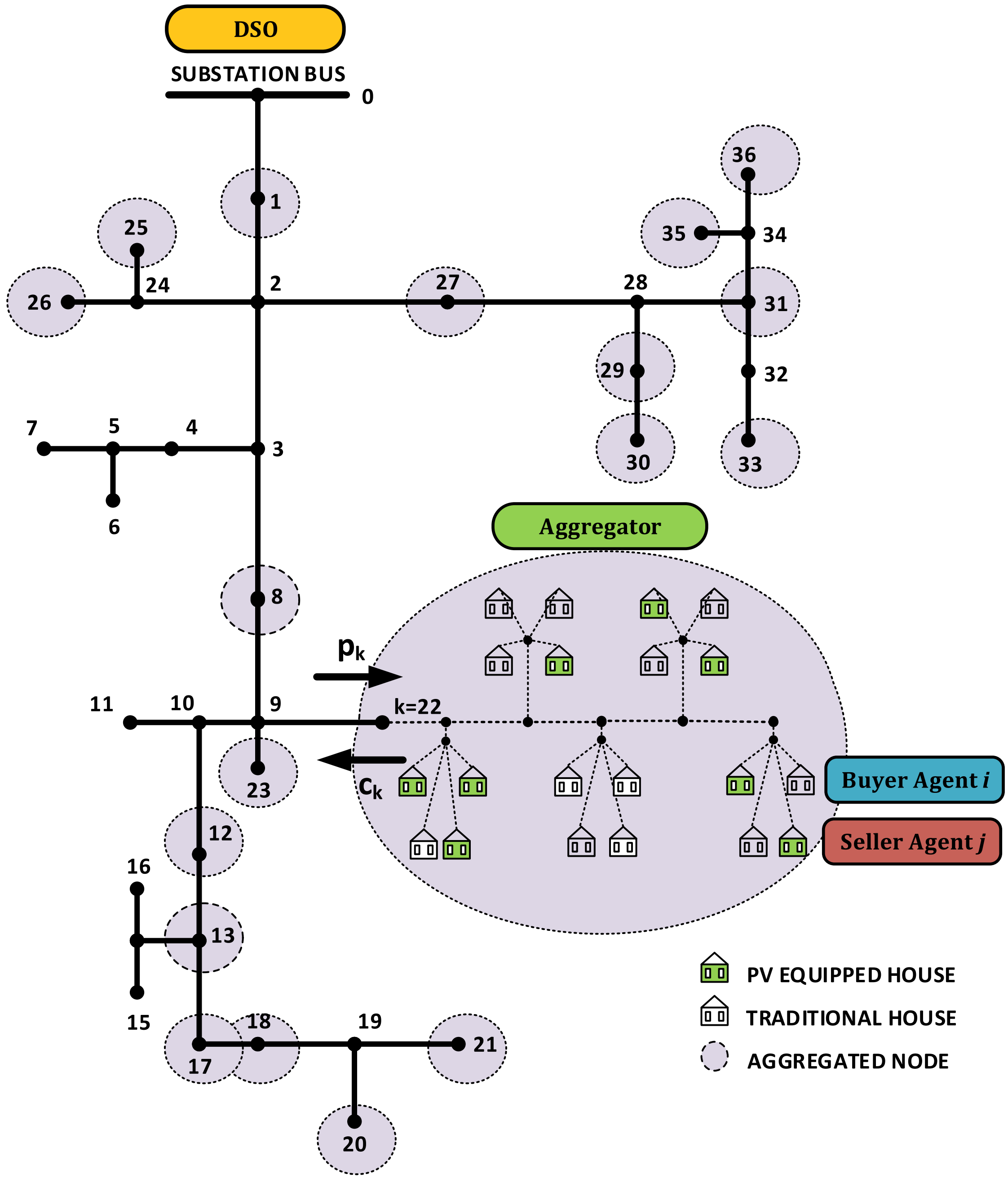

Figure 4.2. Radial distribution feeder with DSO and aggregators forming the upper level and aggregators with home level agents forming the lower level auction participants.

$$P_k = p_k + \sum_{l \in \mathcal{D}(k)} p_l\,, \tag{4.7}$$

$$Q_k = q_k + \sum_{l \in \mathcal{D}(k)} q_l\,, \tag{4.8}$$

$$V_k = V_0 - \sum_{l \in \mathcal{U}(k)} \Delta V_l\,. \tag{4.9}$$

Here, $\mathcal{D}(k)$ and $\mathcal{U}(k)$ are the sets of downstream and both immediate and separated upstream nodes of k. For example, in the radial distribution system shown in Figure 4.2, $\mathcal{D}(31) = \{32,33,34,35,36\}$ and $\mathcal{U}(31) = \{1,2,27,28,31\}$. For the sake of compactness, we deviate from previous chapters' notation and use vector-matrix notation. Let

$$\begin{aligned}
&\mathbf{p} = [p_k]_{k\in\mathcal{A}},\\
&\mathbf{q} = [q_k]_{k\in\mathcal{A}},\\
&\boldsymbol{P} = [P_k]_{k\in\mathcal{N}},\\
&\boldsymbol{Q} = [Q_k]_{k\in\mathcal{N}},\\
&\boldsymbol{V} = [V_k]_{k\in\mathcal{N}},\\
&\Delta\boldsymbol{V} = [\varDelta V_k]_{k\in\mathcal{N}}\\
&\mathbf{r} = [r_k]_{k\in\mathcal{N}},\\
&\mathbf{x} = [x_k]_{k\in\mathcal{N}}.
\end{aligned}$$

The following system architecture matrices are defined,

$$[\mathbf{A}]_{kl} = \begin{cases} 1 & k = aggregator\ l \\ 0 & otherwise, \end{cases} \tag{4.10}$$

$$[\mathbf{D}]_{kl} = \begin{cases} 1 & l \in \mathcal{D}(k)\ or\ k = l \\ 0 & otherwise, \end{cases} \tag{4.11}$$

$$[\mathbf{U}]_{kl} = \begin{cases} 1 & l \in \mathcal{U}(k) \\ 1 & l = k \\ 0 & otherwise. \end{cases} \tag{4.12}$$

In the above equations, $\mathbf{A}$ is a $N \times A$ and $\mathbf{D}, \mathbf{U}$ are $N \times N$ matrices associated with the spatial topology of the network. The matrix $\mathbf{A}$ is the node-aggregator matrix that has an entry of unity ('1') at every column on the row, i.e. node, where it resides. The matrices $\mathbf{D}$ and $\mathbf{U}$ corresponds to the descendant and ancestor nodes. Every row (node) of $\mathbf{D}$ and $\mathbf{U}$ has a one where the corresponding column (node) is its descendant or ancestor, and a zero elsewhere. Note that $\mathbf{D} = \mathbf{U}^{\mathbf{T}}$.

The simplified DistFlow equations given by Eqns. (4.7) – (4.9) are thus given by the following equations,

$$\boldsymbol{P} = \mathbf{DAp}, \tag{4.13}$$

$$\boldsymbol{Q} = \mathbf{DAq}, \tag{4.14}$$

$$\boldsymbol{V} = V_0 \mathbf{1}_N - \mathbf{U}\Delta\boldsymbol{V}, \tag{4.15}$$

where,

$$\Delta\boldsymbol{V} = \frac{1}{V_0}(\mathbf{r} \circ \boldsymbol{P} + \mathbf{x} \circ \boldsymbol{Q}). \tag{4.16}$$

It is also assumed in this research that homes are furnished with smart meters as well as inverters[98], [99] if sellers, and are capable of communicating their real power supply/demand and as well as their reactive power supply/demand to/from the grid. Every home may have a different power factor, but known during each iteration of the ALA.

In order to account for physical system constraints such as node voltages, in addition to c_k, the DLA also requires the aggregators to return θ_k, the fraction of p_k that upon multiplication gives their reactive power supply/demand q_k, i.e. $q_k = \theta_k p_k$. With $\boldsymbol{\theta} = [\theta_k]_{k \in \mathcal{A}}$ and $\mathbf{q} = \boldsymbol{\theta} \circ \mathbf{p}$ as elementwise (Hadamard) product of $\boldsymbol{\theta}$ and $\mathbf{p}$**,** using Eqn. (4.16), DistFlow equations in Eqns. (4.13) –(4.14) (4.15) can be written as follow,

$$\boldsymbol{P} = \mathbf{DAp}, \tag{4.17}$$

$$\boldsymbol{Q} = \mathbf{DA}(\boldsymbol{\theta} \circ \mathbf{p}), \tag{4.18}$$

$$\boldsymbol{V} = V_0 \mathbf{1}_N - \frac{1}{V_0}\mathbf{U}\big(\mathbf{r} \circ \mathbf{DAp} + \mathbf{x} \circ \mathbf{DA}(\boldsymbol{\theta} \circ \mathbf{p})\big). \tag{4.19}$$

In the above version of simplified DistFlow equations, the real/reactive branch flows and node voltages are entirely given as a function of nodes real power injection $\mathbf{p}$, substation per unit voltage V_0 and distribution system topology. With the latter two known to the DSO, it can implement the DLA to determine $\mathbf{p}$ by using Eqns. (4.17) – (4.19) to set up its physical system constraints.

The physical system constraints of the DLA are described below. The node voltage constrains are dealt with first. Using Eqn. (4.19), and $\delta = 0.05$, the voltages must satisfy the following for all N nodes,

$$\mathbf{1}_N - \boldsymbol{\delta} \leq V_0 \mathbf{1}_N - \frac{1}{V_0}\mathbf{U}\big(\mathbf{r} \circ \mathbf{DAp} + \mathbf{x} \circ \mathbf{DA}(\boldsymbol{\theta} \circ \mathbf{p})\big) \leq \mathbf{1}_N + \boldsymbol{\delta}. \tag{4.20}$$

Letting

$$\mathbf{M}_P = \frac{1}{V_0}\mathbf{Ur} \circ \mathbf{DA},$$

$$\mathbf{M}_Q = \tfrac{1}{V_0}\boldsymbol{Ux} \circ \boldsymbol{DA},$$

$$\mathbf{M} = \mathbf{M}_P + \mathbf{M}_Q \ diag\boldsymbol{\theta},$$

$$\underline{\mathbf{l}} = (V_0 - 1)\mathbf{1}_N - \boldsymbol{\delta},$$

$$\bar{\mathbf{l}} = (V_0 - 1)\mathbf{1}_N + \boldsymbol{\delta}.$$

Eqn. (4.20) can be written as,

$$\underline{\mathbf{l}} \leq \mathbf{Mp} \leq \bar{\mathbf{l}}. \tag{4.21}$$

In Eqn. (4.21), $\mathbf{M}$ shows the sensitivity matrix of size $N \times A$. The (i, k) entry in $\mathbf{M}$ shows the effect of real power injection of the k^{th} aggregator on node i's voltage.

The substation transformer capacity constraint is formulated next. In the DLA, since a real power amount P_0 is assigned in the wholesale market auction, it is necessary that the sum of real power injection to all aggregator buses must not exceed this amount. Furthermore, the reactive power flowing into the distribution system through the substation transformer must not result in overloading it. Denoting the MVA limit of the transformer by MVA_{tr}^2, the reactive power capacity of the transformer is given by,

$$Q_0 = \sqrt{MVA_{tr}^2 - P_0^2}.$$

Thus, the following must be added to the DSO's physical system constraints,

$$\mathbf{1}_A^T\mathbf{p} = P_0, \tag{4.22}$$

$$\boldsymbol{\theta}^T\mathbf{p} \leq Q_0. \tag{4.23}$$

The line flow constraints are formulated as follow. Since each line segment has an MVA limit, the following constraint shall be met for all branches, i.e. $\forall k$,

$$P_k^2 + \ Q_k^2 \leq \overline{S}_k^2.$$

Let $\mathbf{E}_k$ be a matrix of size $N \times N$ with a single one at the (k, k) location and zero elsewhere. The above line flow constraint can be rewritten in terms of all nodes real power injection $\mathbf{p}$ follow,

$$\boldsymbol{P}^T\mathbf{E}_\mathrm{k}\boldsymbol{P} + \ \boldsymbol{Q}^T\mathbf{E}_\mathrm{k}\boldsymbol{Q} \leq \overline{S}_k^2.$$

Replacing $\boldsymbol{P}$ and $\boldsymbol{Q}$ using Eqns.(4.17) and (4.18) and simplifying further yields,

$$\mathbf{p}^T(\mathbf{A}^T\mathbf{D}^T\boldsymbol{E}_k\mathbf{D}\mathbf{A} + \ diag\boldsymbol{\theta}\mathbf{A}^T\mathbf{D}^T\mathbf{E}_k\mathbf{D}\mathbf{A}diag\boldsymbol{\theta})\mathbf{p} \leq \overline{S}_k^2$$

Letting,

$$\mathbf{Z}_k = \mathbf{A}^T\mathbf{D}^T\mathbf{E}_k\mathbf{D}\mathbf{A} + \ diag\boldsymbol{\theta}\mathbf{A}^T\mathbf{D}^T\mathbf{E}_k\mathbf{D}\mathbf{A}diag\boldsymbol{\theta},$$

the line flow constraint can be completely written in terms of node real power injections as follow,

$$\mathbf{p}^T\mathbf{Z}_k\mathbf{p} \leq \overline{S}_k^2 \tag{4.24}$$

4.3.2 DSO Social Welfare Optimization Problem (DSWOP)

The goal of DSO is to maximize the global SW subject to physical system constraints. Mathematically speaking,

Maximize w.r.t. $[\mathbf{d}^k]_{k\in\mathcal{A}}, [\mathbf{s}^k]_{k\in\mathcal{A}}, \mathbf{p}$

$$\Omega([\mathbf{d}^k]_{k\in\mathcal{A}}, [\mathbf{s}^k]_{k\in\mathcal{A}}, \mathbf{p}) = \sum_{k\in\mathcal{A}} \Theta_k(\mathbf{d}^k, \mathbf{s}^k), \tag{4.25}$$

subject to,

$$\mathbf{1}_A^T\mathbf{p} = P_0, \tag{4.26}$$

$$\boldsymbol{\theta}^T\mathbf{p} \leq Q_0, \tag{4.27}$$

$$\mathbf{M}\mathbf{p} \geq \underline{\mathbf{l}}, \tag{4.28}$$

$$\mathbf{M}\mathbf{p} \leq \bar{\mathbf{l}}, \tag{4.29}$$

$$\mathbf{p}^T\mathbf{Z}_k\mathbf{p} \leq \overline{S}_k^2 \tag{4.30}$$

$$\mathbf{p} = \left[\mathbf{1}_{N_B^k}^T\mathbf{d}^k - \mathbf{1}_{N_S^k}^T\mathbf{s}^k\right]_{k\in\mathcal{A}}, \tag{4.31}$$

$$\mathbf{s}^k \leq \mathbf{g}^k. \tag{4.32}$$

In the above DSWOP the objective function in Eqn. (4.25), i.e. Θ_k, is the SW of each aggregator k given below,

$$\Theta_k(\mathbf{d}^k, \mathbf{s}^k|p_k) = \mathbf{1}_{N_B^k}^T\mathbf{u}^k + \mathbf{1}_{N_S^k}^T\mathbf{v}^k,$$

where,

$$\mathbf{u}^k = [u_i^k(d_i^k)]_{i\in\mathcal{N}_B^k},$$

$$\mathbf{v}^k = [v_j^k(g_j^k - s_j^k)]_{i\in\mathcal{N}_S^k}.$$

The Constraints in Eqns. (4.26) and (4.27) are to ensure that substation transformers are not overloaded. Node voltage limits and line flow constraints are given by Eqns. (4.28) – (4.30). Node power balance constraints are ensured by Eqn. (4.31)(4.33). The constraint in Eqn. (4.32) is to guarantee that sellers supply must not exceed their generation for all aggregators.

The DSWOP given by Eqns. (4.25) – (4.32) is a function of $\mathbf{d}^k, \mathbf{s}^k$, and **p.** The only constraint that couples $(\mathbf{d}^k, \mathbf{s}^k)$ and $\mathbf{p}$ is given by Eqn. (4.31). Thus, the DSWOP can be decomposed as explained next.

4.3.3 DSWOP Decomposition

The Lagrangian of the DSWOP to be maximized by relaxing the problem is,

$$\begin{aligned}\mathfrak{L}([\mathbf{d}^k]_{k\in\mathcal{A}}, [\mathbf{s}^k]_{k\in\mathcal{A}}, \mathbf{p}) = &\sum_{k\in\mathcal{A}} \Theta_k(\mathbf{d}^k, \mathbf{s}^k)\\ &+\alpha(\mathbf{1}_A^T\mathbf{p} - P_0) - \beta(\boldsymbol{\theta}^T\mathbf{p} - Q_0)\\ &+\underline{\boldsymbol{\zeta}}^T(\mathbf{Mp} - \underline{\mathbf{l}}) - \overline{\boldsymbol{\zeta}}^T(\mathbf{Mp} - \bar{\mathbf{l}}) - \overline{\boldsymbol{\xi}}^T\left(\mathbf{p}^T\mathbf{Z}_k\mathbf{p} - \overline{S}_k^2\right)\\ &-\boldsymbol{\lambda}^T\left(\left[\mathbf{1}_{N_B^k}^T\mathbf{d}^k - \mathbf{1}_{N_S^k}^T\mathbf{s}^k\right]_{k\in\mathcal{A}} - \mathbf{p}\right)\\ &-\sum_{k\in\mathcal{A}}\boldsymbol{\gamma}_k^T(\mathbf{s}^k - \mathbf{g}^k).\end{aligned} \tag{4.33}$$

This can be rewritten as,

$$\mathfrak{L}([\mathbf{d}^k]_{k\in\mathcal{A}}, [\mathbf{s}^k]_{k\in\mathcal{A}}, \mathbf{p}) \;= L(\mathbf{p}) + \sum_{k\in\mathcal{A}} \mathcal{L}_k(\mathbf{d}^k, \mathbf{s}^k)\,, \tag{4.34}$$

where,

$$\begin{aligned}L(\boldsymbol{p}) = &\boldsymbol{\lambda}^T\mathbf{p} + \alpha(\mathbf{1}_A^T\mathbf{p} - P_0) - \beta(\boldsymbol{\theta}^T\mathbf{p} - Q_0)\\ &+\underline{\boldsymbol{\zeta}}^T(\mathbf{Mp} - \underline{\mathbf{l}}) - \overline{\boldsymbol{\zeta}}^T(\mathbf{Mp} - \bar{\mathbf{l}}) - \overline{\boldsymbol{\xi}}^T\left(\mathbf{p}^T\mathbf{Z}_k\mathbf{p} - \overline{S}_k^2\right),\end{aligned} \tag{4.35}$$

$$\mathcal{L}_k(\mathbf{d}^k, \mathbf{s}^k) = \Theta_k(\mathbf{d}^k, \mathbf{s}^k) - \boldsymbol{\gamma}_k^T(\mathbf{s}^k - \mathbf{g}^k)$$
$$-\lambda_k \left(\mathbf{1}_{N_B^k}^T \mathbf{d}^k - \mathbf{1}_{N_S^k}^T \mathbf{s}^k - p_k\right). \tag{4.36}$$

In Eqns. (4.35) and (4.36)(4.38), a copy of $\boldsymbol{\lambda}^{\mathrm{T}}\mathbf{p}$ is retained in $L(\mathbf{p})$ and $\sum_{k\in\mathcal{A}} \mathcal{L}_k(\mathbf{d}^k, \mathbf{s}^k)$ so that the problem can be decoupled and solved distributively[100]. The dual variable $\boldsymbol{\lambda}$ is obtained from the aggregators and is considered fixed in order to solve Eqn. (4.35) for $\mathbf{p}$. Similarly, in Eqn. (4.36) p_k is considered fixed and every aggregator solves for $\mathbf{d}^{\mathrm{k}}, \mathbf{s}^{\mathrm{k}}$ and returns λ_k as its market equilibrium price. The aggregator level auction described later incorporates a mechanism called virtual bidding, as a result of which it can be assumed that $\nabla_{\mathbf{d}^k}\lambda_k = \mathbf{0}$ and $\nabla_{\mathbf{s}^k}\lambda_k = \mathbf{0}$ so that the true λ_k is returned in order to ensure solving DSWOP via this decomposition method.

The basic idea of decomposition is to decompose the original large problem into distributively solvable sub-problems which are then coordinated by a high-level master problem by means of some kind of signaling [100]. An illustration of this technique is shown in Figure 4.3. In this case, the DSO implements the DLA to optimize $L(\mathbf{p})$ in Eqn.(4.35) that is only a function of $\mathbf{p}$. This can be formulated in terms of the following optimization problem with linear objective and linear and quadratic constraints.

Maximize w.r.t. $\mathbf{p}$

$$\boldsymbol{\lambda}^T\mathbf{p}, \tag{4.37}$$

subject to,

$$\mathbf{1}_A^T\mathbf{p} = P_0, \tag{4.38}$$

$$\boldsymbol{\theta}^T\mathbf{p} \le Q_0, \tag{4.39}$$

$$\mathbf{M}\mathbf{p} \ge \underline{\mathbf{l}}, \tag{4.40}$$

$$\mathbf{M}\mathbf{p} \le \bar{\mathbf{l}}, \tag{4.41}$$

$$\mathbf{p}^T\mathbf{Z}_k\mathbf{p} \le \overline{S}_k^2\,. \tag{4.42}$$

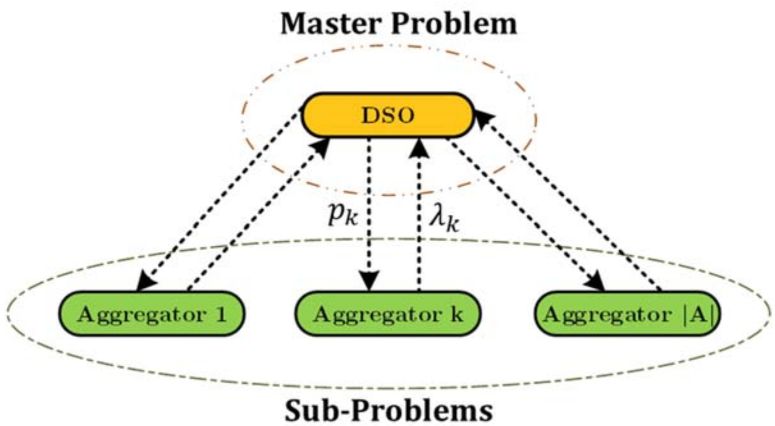

Figure 4.3. Illustration of the decomposition method. Master problem is solved by DSO to determine p_k and sub-problem is solved by the aggregators to determine λ_k.

For the sub-problems, each aggregator k needs to optimize $\mathcal{L}_k(\mathbf{d}^{\boldsymbol{k}}, \mathbf{s}^{\boldsymbol{k}})$ in Eqn. (4.36) that is a function of only $\mathbf{d}^{\boldsymbol{k}}$ and $\mathbf{s}^{\boldsymbol{k}}$ when it obtains p_k from the DSO. It needs to solve the following sub-problem in parallel and returns λ_k as the Lagrange multiplier of the energy balance constraint.

Maximize w.r.t. $\mathbf{d}^k, \mathbf{s}^k$

$$\Theta_k(\mathbf{d}^k, \mathbf{s}^k),$$

subject to,

$$p_k = \mathbf{1}^T_{N_B^k}\mathbf{d}^k - \mathbf{1}^T_{N_S^k}\mathbf{s}^k,$$

$$\mathbf{s}^k \leq \mathbf{g}^k.$$

Since Θ_k is unknown, each aggregator implements the following auction algorithm instead to achieve the goal.

4.4 Aggregator-level Auction

Aggregator level auction in Chapter 3 with virtual bidding has been used for the design of lower level auction. As shown in Figure 4.4, each aggregator implements a lower level proportional allocation auction in order to solve the sub-problem and return λ_k.

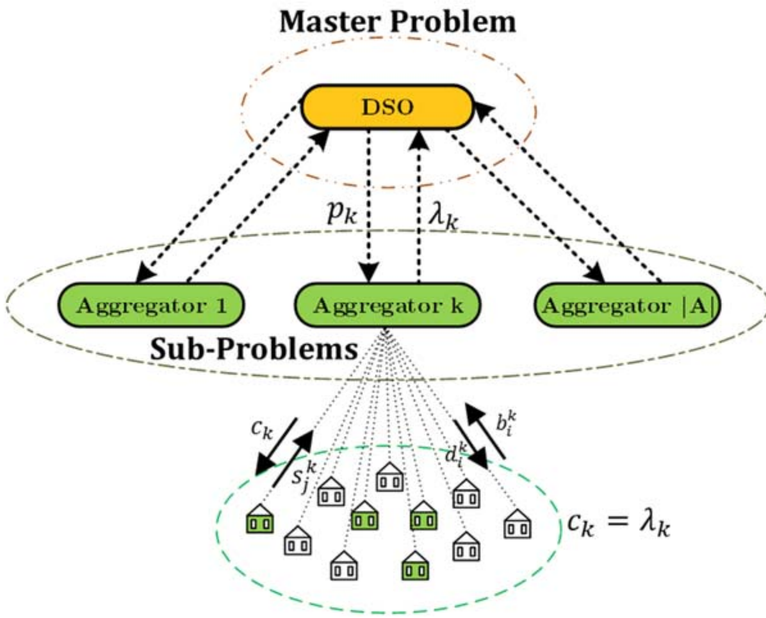

Figure 4.4. Sub-problem auction illustration that achieve SW maximization of aggregators.

4.4.1 Aggregator Social Welfare Optimization Problem (ASWOP)

The goal of each aggregator is to solve each sub-problem as presented in section 4.3.3 re-written below,

Maximize w.r.t. $\mathbf{d}^k, \mathbf{s}^k$

$$\Theta_k(\mathbf{d}^k, \mathbf{s}^k) = \mathbf{1}^T_{N_B^k}\mathbf{u}^k + \mathbf{1}^T_{N_S^k}\mathbf{v}^k, \tag{4.43}$$

subject to,

$$p_k = \mathbf{1}^T_{N_B^k}\mathbf{d}^k - \mathbf{1}^T_{N_S^k}\mathbf{s}^k, \tag{4.44}$$

$$\mathbf{s}^k \leq \mathbf{g}^k. \tag{4.45}$$

The Lagrangian is given by Eqn. (4.36) and is rewritten below,

$$\mathcal{L}_k(\mathbf{d}^k, \mathbf{s}^k) = \Theta_k(\mathbf{d}^k, \mathbf{s}^k) - \boldsymbol{\gamma}_k^T(\mathbf{s}^k - \mathbf{g}^k)$$
$$-\lambda_k\left(\mathbf{1}^T_{N_B^k}\mathbf{d}^k - \mathbf{1}^T_{N_S^k}\mathbf{s}^k - p_k\right).$$

Differentiating the Lagrangian above yields,

$$\nabla_{\mathbf{d}^k} \mathcal{L}_k = \nabla_{\mathbf{d}^k} \mathbf{u}^k - \lambda_k \mathbf{1}_{N_B^k}, \tag{4.46}$$

$$\nabla_{\mathbf{s}^k} \mathcal{L}_k = \nabla_{\mathbf{s}^k} \mathbf{v}^k - \boldsymbol{\gamma}_k + \lambda_k \mathbf{1}_{N_B^k}. \tag{4.47}$$

At equilibrium condition, the following is true,

$$p_k = \mathbf{1}_{N_B^k}^T \mathbf{d}^k - \mathbf{1}_{N_S^k}^T \mathbf{s}^k, \tag{4.48}$$

$$\nabla_{\mathbf{d}^k} \mathbf{u}^k = \lambda_k \mathbf{1}_{N_B^k}, \tag{4.49}$$

$$\nabla_{\mathbf{s}^k} \mathbf{v}^k = \boldsymbol{\gamma}_k - \lambda_k \mathbf{1}_{N_S^k}, \tag{4.50}$$

where Eqns.(4.48) – (4.50) represent *energy*, *demand*, and *supply equilibria*.

It has been proved below that *energy equilibrium* is established by the ALA and that the *demand* and *supply equilibria* conditions are satisfied under the assumption of *virtual bidding*, since the buying and selling agents adopt *price-taking* strategies. Furthermore, as the utilities $\mathbf{u}^k$ and $\mathbf{v}^k$ are assumed to be strictly concave and monotonic, the dual variable, λ_k and the uniform per unit energy price, c_k established through the auction process described next are equal. For further details, the reader is referred to chapter 3 and [79].

4.4.2 Aggregator-level Auction (ALA) Algorithm

The aggregator implements the auction algorithm 4.1 during each iteration of DLA in order to get c_k, i.e. λ_k, and establish energy, demand, and supply equilibria.

Virtual bidding is used to show that ALA achieves energy equilibrium. Virtual bidding was dealt in detail in section 3.4.2 where it was shown that the addition of virtual bidder that acts both as a seller and buyer simultaneously decreases the market power of the buyers and sellers resulting in eliminating loss in efficiency. Here a more direct proof is shown below which also takes into account the energy that the aggregator receives from the DSO. Let $\forall k \in \mathcal{A}, c_k^0$ and c_k be the market equilibrium price under price-taking and price-anticipating conditions. The virtual supply, s_0, of the virtual bidder, is kept arbitrarily high and is sold to the aggregator at c_k^0. Under these circumstances,

$$
\begin{aligned}
\lim_{s_0\to\infty} c_k &= \lim_{s_0\to\infty} \frac{c_k^0 s_0 + \mathbf{1}_{N_B^k}^T \mathbf{b}^k}{p_k + s_0 + \mathbf{1}_{N_S^k}^T \mathbf{s}^k} \\
&= c_k^0 \\
&= \frac{\mathbf{1}_{N_B^k}^T \mathbf{b}^k}{p_k + \mathbf{1}_{N_S^k}^T \mathbf{s}^k}.
\end{aligned}
$$

Algorithm 4.1 *ALA Algorithm*

Receive from DSO p_k
Initialize c_k
Repeat
  Send c_k *to sellers*
  Receive $\mathbf{s}^k$ *from sellers*
  $\mathbf{d}^k \leftarrow \frac{1}{c_k}\mathbf{b}^k$
  Send $\mathbf{d}^k$ *to buyers*
  Receive $\mathbf{b}^k$ *from buyers*
  $c_k^0 \leftarrow \frac{\mathbf{1}_{N_B^k}^{\mathrm{T}} \mathbf{b}^k}{p_k + \mathbf{1}_{N_S^k}^{\mathrm{T}} \mathbf{s}^k}$
  $c_k \leftarrow \frac{c_k^0 s_0 + \mathbf{1}_{N_B^k}^{\mathrm{T}} \mathbf{b}^k}{p_k + s_0 + \mathbf{1}_{N_S^k}^{\mathrm{T}} \mathbf{s}^k}$
Until equilibrium
Send c_k *to DSO*

Likewise,

$$
\begin{aligned}
\lim_{s_0\to\infty} \frac{\partial}{\partial b_i^k} c_k &= \lim_{s_0\to\infty} \frac{1}{p_k + s_0 + \mathbf{1}_{N_S^k}^T \mathbf{s}^k} \frac{\partial c_k^0}{\partial b_i^k} \\
&= 0, \\
\lim_{s_0\to\infty} \frac{\partial}{\partial s_j^k} c_k &= \lim_{s_0\to\infty} \frac{1}{p_k + s_0 + \mathbf{1}_{N_S^k}^T \mathbf{s}^k} \left(1 - \frac{c_k^0 s_0 + \mathbf{1}_{N_B^k}^T \mathbf{b}^k}{p_k + s_0 + \mathbf{1}_{N_S^k}^T \mathbf{s}^k}\right) \frac{\partial c_k^0}{\partial s_j^k} \\
&= 0.
\end{aligned}
$$

Henceforth, it is assumed that the aggregator simulate the ALA with a high s_0 so that buyer and seller agents act as price takers. Under these circumstances, the following hold,

$$c_k = \frac{\mathbf{1}_{N_B^k}^T \mathbf{b}^k}{p_k + \mathbf{1}_{N_S^k}^T \mathbf{s}^k}, \tag{4.51}$$

$$\frac{\partial}{\partial b_i^k} c_k = 0, \tag{4.52}$$

$$\frac{\partial}{\partial s_j^k} c_k = 0. \tag{4.53}$$

Proportionally fair allocation of energy achieves energy balance, as shown below. Since by proportional allocation,

$$\mathbf{d}^k = \tfrac{1}{c_k} \mathbf{b}^k, \tag{4.54}$$

the buyers bid in Eqn. (4.54) is $\mathbf{b}^k = c_k \mathbf{d}^k$. Replacing $\mathbf{b}^k$ in Eqn. (4.51) and solving for p_k results,

$$p_k = \mathbf{1}_{N_B^k}^T \mathbf{d}^k - \mathbf{1}_{N_S^k}^T \mathbf{s}^k. \tag{4.55}$$

This proves that ALA achieves the *energy balance equilibrium* condition of ASWOP given by Eqn. (4.48). Eqn.(4.55) shows that the difference in total demand of buyers and total supply of sellers is fed by the DSO's injection p_k.

In addition to energy balance, budget balance is also ensured by the ALA. Notice that from Eqn. (4.51)**,** for an aggregator k,

$$c_k p_k + c_k \mathbf{1}_{N_S^k}^T \mathbf{s}^k = \mathbf{1}_{N_B^k}^T \mathbf{b}^k. \tag{4.56}$$

The first and second term in the LHS in Eqn. (4.56) is the reimbursement to DSO and sellers respectively, whereas the term on the RHS is the payment by the buyers. This shows that the ALA is strongly budget balanced.

4.4.2 Buyer Problem

The buyers' problem is given in Eqn. (3.6) and restated here for an aggregator k.

Maximize w.r.t. b_i^k

$$u_i^k(d_i^k) - b_i^k.$$

The ALA achieves demand equilibrium as follow. The stationary conditions of the buyer problem is

$$\frac{\partial}{\partial b_i^k} u_i^k(d_i^k) - 1 = 0.$$

Using chain rule yields,

$$\frac{\partial u_i^k}{\partial d_i^k} \frac{\partial d_i^k}{\partial b_i^k} - 1 = 0.$$

Using Eqn. (4.54) for buyer i,

$$\frac{\partial u_i^k}{\partial d_i^k} \frac{\partial}{\partial b_i^k} \left(\frac{b_i^k}{c_k} \right) - 1 = 0,$$

$$\Rightarrow \frac{\partial u_i^k}{\partial d_i^k} \frac{1}{c_k} \left(1 - \frac{b_i^k}{c_k} \frac{\partial}{\partial b_i^k} c_k \right) - 1 = 0.$$

Under virtual bidding assumption from Eqn. (4.52), $\frac{\partial c_k}{\partial b_i^k} = 0$ so that,

$$\frac{\partial u_i^k}{\partial d_i^k} = c_k. \tag{4.57}$$

Since $c_k = \lambda_k$ (chapter 3 and [79]) it is seen that virtual bidding allows *demand equilibrium* condition given by Eqn. (4.49) to be satisfied.

4.4.3 Seller Problem

The seller problem is given by Eqns. (3.8) and (3.9) restated below.

Maximize w.r.t. s_j^k

$$v_j^k(g_j^k - s_j^k) + c_k s_j^k,$$

subject to,

$$s_j^k \le g_j^k.$$

The Lagrangian of the above problem is given by,

$$L_j(s_j^k) = v_j^k(g_j^k - s_j^k) + c_k s_j^k - \gamma_j^k(s_j^k - g_j^k).$$

The supply equilibrium is achieved by ALA as shown below. At equilibrium,

$$\frac{\partial}{\partial s_j^k} L_j(s_j^k) = 0$$

$$\Rightarrow \frac{\partial}{\partial s_j^k} \upsilon_j^k(g_j^k - s_j^k) + c_k + s_j^k \frac{\partial c_k}{\partial s_j^k} - \gamma_j^k = 0.$$

Under the assumption of virtual bidding in Eqn. (4.53), $\frac{\partial c_k}{\partial s_j^k} = 0$ so that,

$$\frac{\partial}{\partial s_j^k} \upsilon_j^k = \gamma_j^k - c_k. \tag{4.58}$$

Since $c_k = \lambda_k$ (chapter 3 and [79]) it is seen that virtual bidding allows *supply equilibrium* condition given by Eqn. (4.50) to be satisfied.

The proposed bi-level auction converges to the global efficient solution if it is decomposed appropriately as explained above. For proof of convergence, interested readers are referred to [100], [101], [102].

4.5 Simulation Results

The simulation results reported in this section corroborates the theory in this chapter. A modified IEEE 37 node system with a base value of 100kVA is used to simulate the bi-level mechanism. Seventeen aggregators with different number of buyers and sellers were generated and assigned to the nodes with load as labeled in Figure 4.5. The number of customers in each aggregator are listed in Table 4.1 resulting to 589 agents with 340 buyers and 249 sellers. The utilities of buyers and sellers were assumed to follow logarithmic saturation curves according to Eqns. (2.46) and (2.47) . The quantities x_i, y_i, x_j and y_j in these equations were different for each agent, and were generated randomly and adjusted so that marginal utilities are scaled to reasonable per unit prices . The generation g_j for sellers were also drawn at random, uniformly in the interval $[0.05,\ 0.5]$ pu.

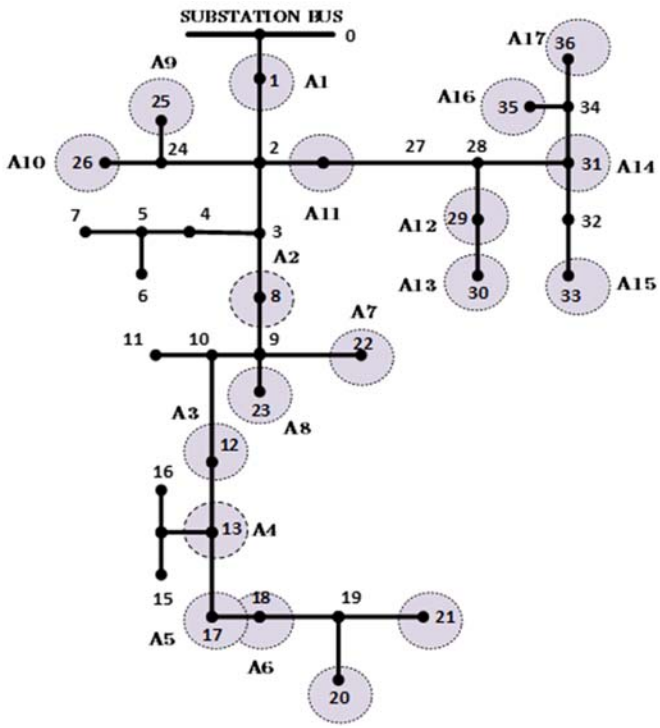

Figure 4.5. Modified IEEE 37 node system with aggregator numbering.

Table 4.1. Number of seller and buyer agents in each aggregator

Aggregator	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Node	1	8	12	13	17	18	22	23	25	26	27	29	30	31	33	35	36
N_B	11	5	22	11	22	30	50	5	22	33	11	11	5	33	22	11	33
N_S	22	20	22	22	2	22	5	20	2	2	22	11	20	22	2	22	11

Three different scenarios were generated to investigate how the amount of injection P_0 from the substation node influences the distribution of energy and its associated cost to each aggregator. These scenarios (labelled I, II, and III) were generated with the following values of the injected power, $P_0 = 0.01$ pu (low wholesale market supply), $P_0 = 10$ pu, and $P_0 = 22$ pu(high wholesale market supply). The DLA mechanism is solved using Matlab's fmincon function at each iteration and the outcome under each scenario are depicted in Figures 4.6, 4.7, and 4.8. For the ALAs, only the auction outcomes of aggregator 5 and 13 under the two extreme scenarios, when P_0 is 0.01 and 22 pu, is reported as tabulated in Table 4.2 and 4.3.

Figures 4.6(a), 4.7(a), and 4.8(a), shows the energy injection p_k to each aggregator k and cost c_k that they pay. The energy p_k and its' per unit cost c_k refers to the efficient solution of the DLA and ALA under equilibrium. In other words, the amounts p_k depicted for each scenario are the solutions at equilibrium when the global SW given by Eqn. (4.25) has stabilized at its maximum, which is shown in part (b) of each figure. Furthermore, the costs c_k are the stable market price of the ALA at equilibrium and is different for each aggregator, making the DLA price heterogeneous. The node pu voltages are shown in part (c) and each line segments real, reactive, and apparent power flows, within the given line MVA limits are shown in part (d) of each figure.

Note that the amount of extraction/injection from/to each aggregator node depends on the available wholesale market supply P_0, the number of buyers and sellers, sellers' generation capacity g_j^k, and marginal utilities $u_i'^k$ and $v_j'^k$ of each aggregator k. Aggregators with deficit are assigned positive p_k while those with surplus are assigned negative p_k, i.e. they supply to the network.

In general, aggregators with available surplus energy and lower equilibrium price c_k supply more p_k to the rest of the network while those that have higher c_k supply less. Similarly, aggregators with deficit energy are assigned more p_k if its equilibrium price c_k is higher. For example in part (a) of all three scenarios in figures 4.6(a), 4.7(a), and 4.8(a), consider aggregator 1 and 13 located on nodes 1 and 30. Aggregator 1 has 22 sellers and 11 buyers whereas 13 has 20 sellers and 5 buyers. Due to low number of buyers in aggregator 13, i.e. low demand, and slightly lower equilibrium price c_k, it is assigned to supply a higher p_k compared to aggregator 1. A similar outcome can be observed for injection by the grid to aggregators with high number of buyers and less number of sellers.

Figures 4.6(b), 4.7(b), 4.8(b) depict the convergence of the bi-level mechanism to the global SW under each scenario. As the amount of wholesale market injection P_0 increases from 0.01 pu in scenario I to 22 pu in scenario III, the global SW increases and stabilizes at higher values. This is because, as seen from part (a) of each figure, with increasing P_0; the prices go down, positive injections increase and negative injections decrease, as more sellers in each aggregator decide to consume their generation rather than selling it. As an interesting example, aggregator 3 with 22 buyers and 22 sellers on node 12 supplies to the grid in the first two scenarios when prices are relatively high. However, it receives energy back from the grid in scenario III when price is low, resulting to a higher SW as defined to be the sum of utilities of buyers plus that of sellers.

Figures 4.6(c, d), 4.7(c, d), and 4.8(c, d) show that when P_0 increases, node voltages decrease and line flows increase. Also, from scenario I to III, negative line flows decrease and more positive line flows appear. For example, the flow in line segment going from node 2 to node 27 is negative in scenario I and positive in II, and III. This is because of the scarcity of supply in scenario I, and because aggregators 9 and 10 has high number of buyers than sellers (see Table 4.1), energy needs to be injected to these nodes. This energy is supplied from aggregator 1 and 2 as well as other aggregators in the right hand side branch from node 2. However, in scenarios II and III the demand of these nodes are supplied by aggregator 1 as well as by the substation injection as the flows in line segment 2 to 27 and 2 to 3 become positive.

In order to illustrate the result of ALAs, auction outcome for aggregator 13 and 5 for the 2 extreme scenarios, I and III, are presented in Tables 4.2 and 4.3. The left column in each table contains aggregator 13's and the right column shows that of aggregator 5. In scenario I, aggregator 13 exports 4.241 pu to the grid (p_{13}), which is equal to the sum of supplies ($\sum_j s_j^{13}$) by the sellers minus sum of demand of buyers ($\sum_i d_i^{13}$). The equilibrium price c_{13} is 0.522 at which the buyers marginal utilities stabilize, i.e. $u_i'^{13} = 0.522$. In the case of sellers, only those with marginal utilities $v_j'^{13}$ equal to c_{13} get to supply a nonzero s_j^{13}. As seen, seller number 10 and 11 with $v_j'^{13} > c_{13}$ have zero s_j^{13}, which means that they decide to consume all their generation g_j^{13} themselves.

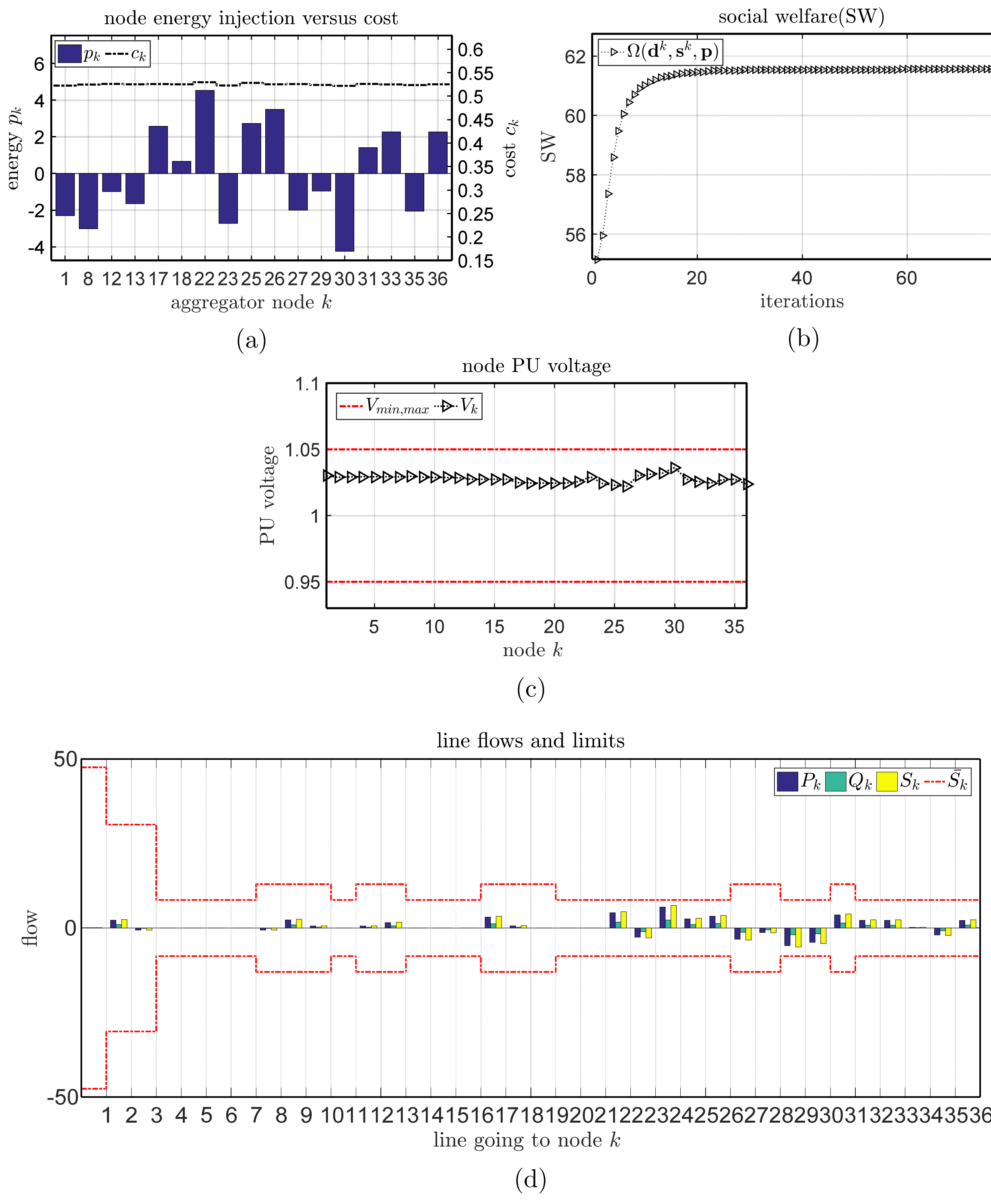

Figure 4.6. DLA outcome for scenario I, i.e. $P_0 = 0.01$ pu. (a) Node injections and prices. (b) Global SW convergence. (c) Node pu voltages. (d) Real, reactive, and apparent pu line flows.

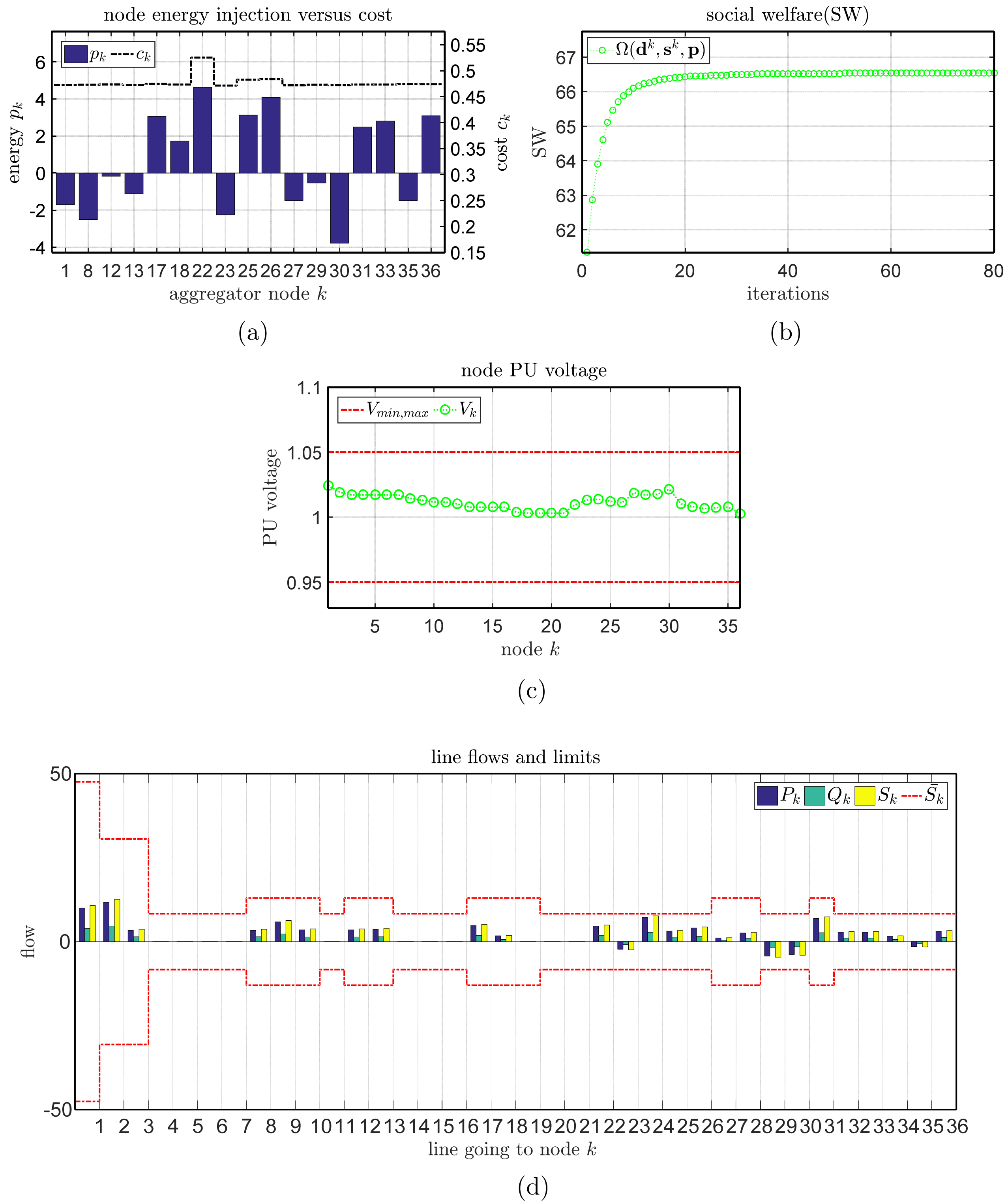

Figure 4.7. DLA outcome for scenario II, i.e. $P_0 = 10$ pu. (a) Node injections and prices. (b) Global SW convergence. (c) Node pu voltages. (d) Real, reactive, and apparent pu line flows.

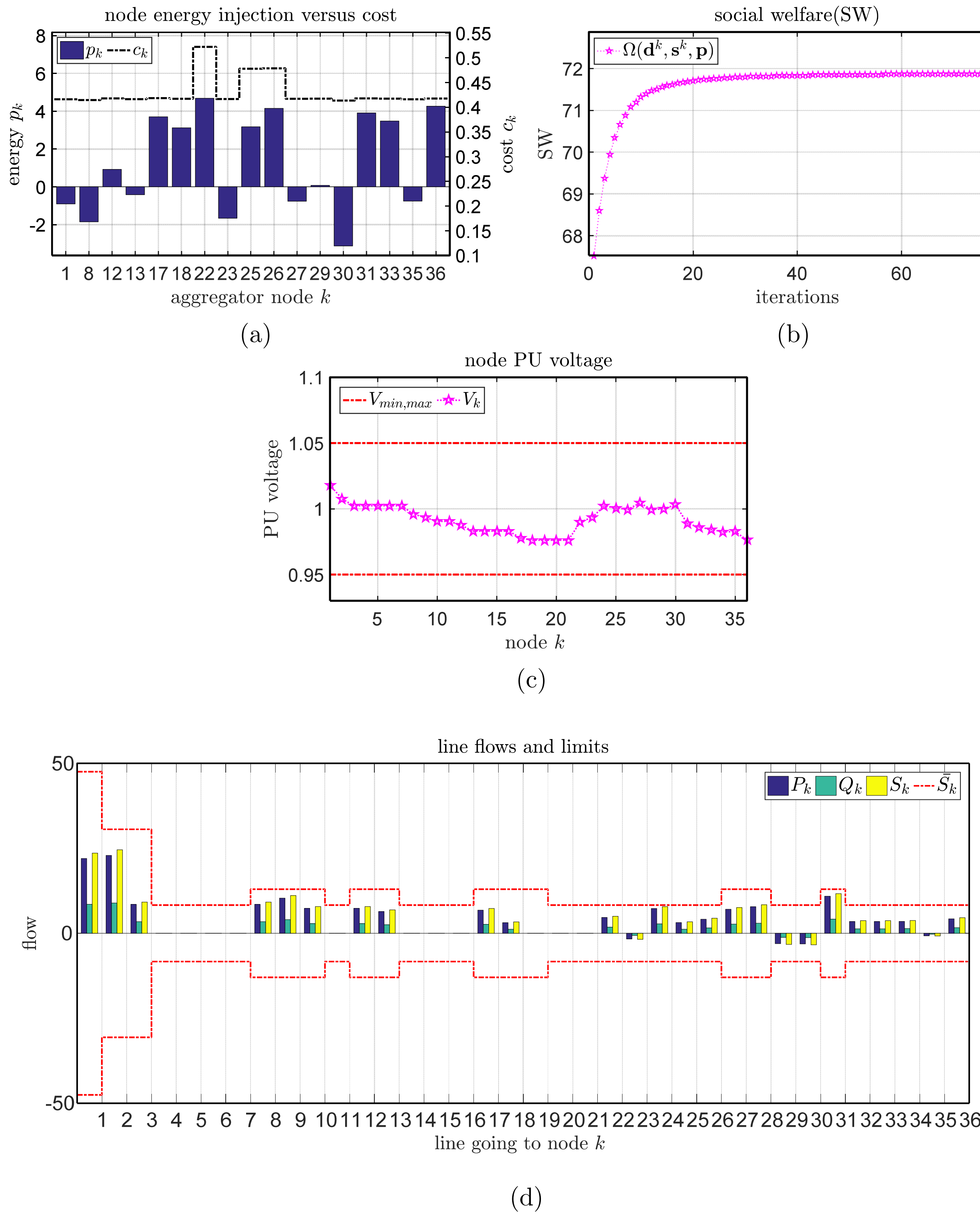

Figure 4.8. DLA outcome for scenario III, i.e. $P_0 = 22$. (a) Node injections and prices. (b) Global SW convergence. (c) Node pu voltages. (d) Real, reactive, and apparent pu line flows.

Table 4.2. ALA outcome for aggregator 13 and 5 under scenario I.

Aggregator 13 $\left(\mathcal{N}_B^{13}=5, \mathcal{N}_S^{13}=20\right)$			Aggregator 5 $(\mathcal{N}_B^{5}=22, \mathcal{N}_S^{5}=2)$		
p_{13} (pu)	c_{13} (\$/kWhr)	V_{13} (pu)	p_5 (pu)	c_5 (\$/kWhr)	V_5 (pu)
-4.241	0.522	1.036	2.572	0.526	1.025
d_i^{13}(pu)	c_i^{13} (\$/kWhr)	$u_i'^{13}$(\$/kWhr)	d_i^{5}(pu)	c_i^{5} (\$/kWhr)	$u_i'^{5}$(\$/kWhr)
0.152	0.522	0.522	0.022	0.526	0.526
0.132	0.522	0.522	0.156	0.526	0.526
0.155	0.522	0.522	0.135	0.526	0.526
0.037	0.522	0.522	0.123	0.526	0.526
0.114	0.522	0.522	0.138	0.526	0.526
g_j^{13}(pu)	s_j^{13}(pu)	$v_j'^{13}$(\$/kWhr)	0.155	0.526	0.526
0.190	0.041	0.522	0.147	0.526	0.526
0.414	0.310	0.522	0.092	0.526	0.526
0.482	0.419	0.522	0.086	0.526	0.526
0.497	0.361	0.522	0.145	0.526	0.526
0.342	0.216	0.522	0.128	0.526	0.526
0.488	0.356	0.522	0.134	0.526	0.526
0.303	0.173	0.522	0.146	0.526	0.526
0.473	0.318	0.522	0.114	0.526	0.526
0.425	0.315	0.522	0.095	0.526	0.526
0.057	0.000	0.697	0.157	0.526	0.526
0.131	0.000	0.589	0.093	0.526	0.526
0.333	0.289	0.522	0.095	0.526	0.526
0.402	0.307	0.522	0.101	0.526	0.526
0.385	0.230	0.522	0.119	0.526	0.526
0.421	0.303	0.522	0.106	0.526	0.526
0.372	0.372	0.516	0.157	0.526	0.526
0.422	0.361	0.522	g_j^{15}(pu)	s_j^{15}(pu)	$v_j'^{15}$(\$/kWhr)
0.159	0.020	0.522	0.163	0.070	0.526
0.319	0.179	0.522	0.160	0.010	0.526
0.271	0.260	0.522			

Similar observations for aggregator 13 can be made in scenario III. However now the price stabilizes at 0.413 at which it exports lower p_{13} , and buyers buy more. Notice that this time, in addition to sellers 10, and 11, sellers 1 and 18 also declare zero supply. Aggregator 5 on the other hand, stabilizes at $c_5 > c_{13}$ in both scenarios due to higher number of buyers. Buyers buy less in scenario I than in III. Due to the same reasons described above, both sellers supply in scenario I whereas only seller 1 supplies in III.

Table 4.3. ALA outcome for aggregator 13 and 5 under scenario III.

Aggregator 13 $\left(\mathcal{N}_B^{13}=5,\mathcal{N}_S^{13}=20\right)$			Aggregator 5 $(\mathcal{N}_B^{5}=22,\mathcal{N}_S^{5}=2)$		
p_{13} (pu)	c_{13} (\$/kWhr)	V_{13} (pu)	p_5 (pu)	c_5 (\$/kWhr)	V_5 (pu)
-3.131	0.413	1.003	3.700	0.418	0.978
d_i^{13}(pu)	c_i^{13} (\$/kWhr)	$u_i'^{13}$(\$/kWhr)	d_i^5(pu)	c_i^5 (\$/kWhr)	$u_i'^5$(\$/kWhr)
0.202	0.413	0.413	0.071	0.418	0.418
0.183	0.413	0.413	0.205	0.418	0.418
0.205	0.413	0.413	0.184	0.418	0.418
0.088	0.413	0.413	0.172	0.418	0.418
0.165	0.413	0.413	0.187	0.418	0.418
g_j^{13}(pu)	s_j^{13}(pu)	$v_j'^{13}$(\$/kWhr)	0.204	0.418	0.418
0.190	0.000	0.430	0.196	0.418	0.418
0.414	0.260	0.413	0.141	0.418	0.418
0.482	0.369	0.413	0.134	0.418	0.418
0.497	0.310	0.413	0.194	0.418	0.418
0.342	0.165	0.413	0.177	0.418	0.418
0.488	0.306	0.413	0.183	0.418	0.418
0.303	0.123	0.413	0.195	0.418	0.418
0.473	0.268	0.413	0.163	0.418	0.418
0.425	0.264	0.413	0.143	0.418	0.418
0.057	0.000	0.697	0.206	0.418	0.418
0.131	0.000	0.589	0.142	0.418	0.418
0.333	0.239	0.413	0.143	0.418	0.418
0.402	0.257	0.413	0.149	0.418	0.418
0.385	0.180	0.413	0.167	0.418	0.418
0.421	0.253	0.413	0.155	0.418	0.418
0.372	0.324	0.413	0.206	0.418	0.418
0.422	0.311	0.413	g_j^5(pu)	s_j^5(pu)	$v_j'^5$(\$/kWhr)
0.159	0.000	0.472	0.163	0.021	0.418
0.319	0.128	0.413	0.160	0.000	0.499
0.271	0.210	0.413			

Note that the equilibrium solution of one aggregator is directly dependent on that of others. During each iteration of the DLA, optimal energy p_k is allocated based on declared aggregator prices c_k in the previous iteration. Each aggregator's c_k is altered by this new p_k as it changes its supply amount based on which buyers and sellers buy and sell in the ALA. The result is a new set of equilibrium prices c_k that upon communication may change the whole energy distribution at the DLA. Upon repetition of this process, the DLA achieves global SW where in every aggregator, agents' marginal utilities becomes equal to its price when they trade. During each iteration, DLA also considers distribution grid constraints such as node voltages that needs to be within specified range.

4.6 Conclusions

In order to solve DSWOP, a globally efficient bi-level energy auction mechanism that is implemented by DSO at the upper and aggregators at the lower level is proposed. The auction is shown to converge and attain global efficient solution by decomposing DSWOP into a master and sub-problem that upon solution, solves DSWOP. The master problem is shown to be a linear objective with linear and quadratic constraints formed by the physical grid constraints. The sub-problem is modeled to be the aggregator SW maximization problem that is solved through the proposed auction mechanism in chapter 3 in the presence of price anticipatory bidders and when customers' utility functions are hidden from the aggregator. Simulation results confirm the theory in the proposed mechanism.

Chapter 5 - Multi-period Price Heterogeneous Auction for Distribution System Operation

Energy distribution in current electricity market proceeds in a centralized manner where participants, i.e. generation companies, load serving entities, bid their supply-demand curves, and the Independent System Operator (ISO) schedules generation based on unit commitment on a day-ahead basis. Furthermore, the bidding process is not iterative and the per-unit price of energy is determined based on the Locational Marginal Price (LMP), which is equal to the Lagrange multiplier of the energy balance constraint at every bus.

In this chapter, a multi-period double energy auction in the distribution system in the same setting as the current electricity market is studied. The distribution system involves dispatch-able generation units, renewable generation units supported by battery storage systems (BSSs), fixed loads, price responsive loads, and supply from the Wholesale Market (WSM) at LMP. The auction is implemented within a Distribution System Operator (DSO) premises using Mixed Integer Linear Programming (MILP). The proposed auction is cleared at the Distribution LMP (DLMP) and is observed to be weakly budget balanced if no penalty is applied for DSO's deviation from its original commitment with the WSM. Furthermore, the dynamics of LMP and DLMP, and their effect on distribution market participants scheduled quantities as well as the WSM supply to the distribution system is investigated.

This chapter will provide the necessary background to facilitate the reader with logic of the proposed future research in the next chapter.

5.1 Introduction

The design of DSO's market has acquired a significant research momentum due to increasing penetration of renewable energy resources and the accompanying net load volatility. As an intermediate market operator, the DSO may use forecasted and (or) historical load and system distributed generation (DG) data to bid in the WSM. The independent system operator (ISO) clears WSM, announces the DSO's LMP and scheduled energy amount. In such a situation, once DSO collects the information of its scheduled power at the LMP, it may implement a DAM double auction in its own service territory to seek further efficient resource allocation and maximize SW. The service area under the control of the DSO can be comprised of various loads and generation units. The generation units in the network can be of dispatch-able and non-dispatch-able kind. Non-dispatch-able units that are mainly renewable energy resource such as wind and solar are intermittent and causes uncertainty while weakening the classical demand price correlation [103], [104]. However, these restrictions can be alleviated by channeling intermittent renewable generation output power through battery storage systems (BSSs) [74], [105].

While distinctive models for DSO are proposed by various researchers in the electricity market [106], [107], [108], [109] a broader model that can handle involvement of all kinds of market participants has not yet been developed. Distribution market clearing and payment mechanisms are still open questions that are yet to be answered with viable and sound assumptions.

Seen hierarchically, the distribution service territory starts at the bus where the utility company can bid in the WSM through DSO. A sub-transmission network then distributes the power to different distribution buses (D-buses), i.e. substations [106]. Each D-bus serves smaller substations at medium voltage that may cover a smaller geographic area or a community at low voltage. In the low voltage distribution system, different aggregator models including those in the previous chapters have been proposed in the literature that may be incentivized to aggregate classical fixed as well as price responsive loads and bid in the distribution market [110], [111], [79].

This chapter proposes a multi-period DAM energy auction for DSO given the LMP and its commitment with the WSM based on forecasted or historical data. The goal of the DSO is to implement a double auction post WSM commitment to maximize the SW and the surplus of the market participants such as DGs, BSSs, and price responsive loads by scheduling the least cost generation and serving loads with highest bids.

For the sake of reader's convenience, a list of notation and abbreviations for this chapter are provided in Appendix B.

5.2 Auction Framework

In the proposed model when the DSO receives its committed energy information and the LMP of the ISO, it asks for bids from the generator units, BSSs, and load aggregators in the low voltage distribution system. Generator units are assumed to submit a three-segment bid and amount as well as their ramp up/down rates and startup/shut down costs. Load aggregators are assumed to be capable of dividing the aggregated load to a fixed load segment amount that needs to be served at all times and a two segment bid and amount for its price responsive part. The fixed segment part of the load is expected to be served at the DLMP and does not accompany any monetary bid value. This is because not all loads are price responsive, i.e. a high portion of the load is price inelastic and needs to be served at all times. We assume that renewable energy resources at the distribution level are coupled with BSSs for a smooth participation in the auction, and declare their selling price as well as their unit characteristics to the DSO for optimal operation. BSSs have the potential to help integrate deeper penetrations of renewable energy onto electricity grids and deliver efficient, low-cost, fundamental electricity-grid services [74]. It is considered in this paper that the BSSs are backed up and charged only by its own renewable energy resource(s). The proposed model can be easily extended for idle BSSs that can be charged or discharged by the grid in the DSO's optimization problem by adding an extra set of constraints.

Equipped with the aforementioned considerations, the DSO runs the energy auction and clears the market by providing power to the successful biding parties at DLMP. The concept of DLMP was initially proposed by [112] and has been used by many researchers for distribution system congestion management, market clearing, and loss minimization [104],[107],[113],[114],[115]. DLMP shows the true marginal cost of supplying the next increment of load at a distribution bus. Although in this study we do not consider any distribution line congestion or voltage constraints, the true value of a unit of energy at the DSO bus and at D-buses differs from LMP due to different bids of market participants at those buses. As stated earlier, the DLMP is attained as the Lagrange multiplier of the energy balance constraint at each D-bus. The model proposed here can be easily extended to include line or transformer congestions through Shift Factors or Power Transfer Distribution Factor using DC power flow, a viable approximation at the sub-transmission level of the distribution system. A second approach that seems more promising in smaller distribution systems is the application of simplified DistFlow equations to account for line or transformer overloading as well as the D-bus voltage constraints [94], [93], [95],[96],[97]. The simplified DistFlow equations can be implemented in the DSO's optimization problem as a set of linear constraints.

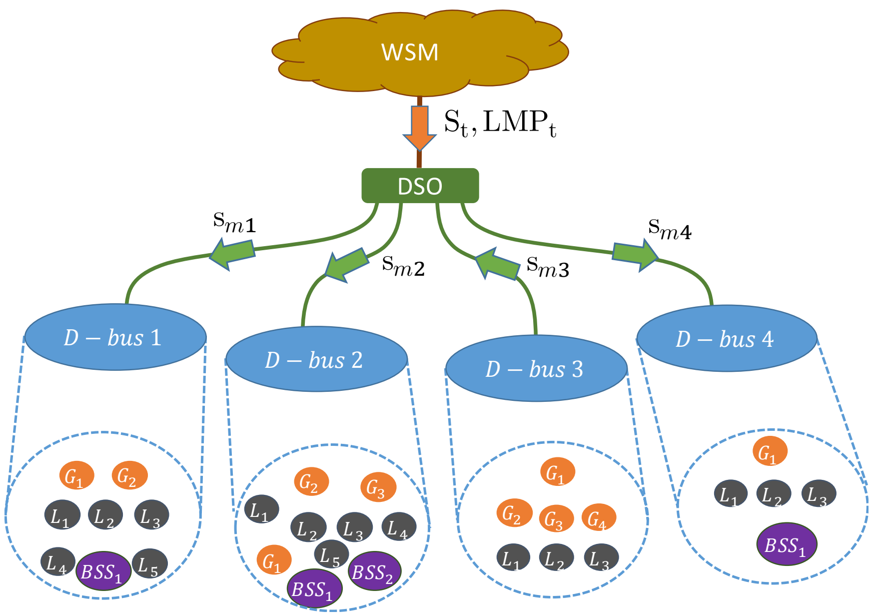

Figure 5.1: System architecture with four distribution buses.

Figure 5.1 depicts a sample hypothetical distribution system architecture with four distribution buses (D-buses) fed by sub-transmission lines from the main DSO bus in a radial configuration. Each D-bus is considered to include dispatch-able generation units (denoted by G), BSSs charged by renewable resources, and several aggregated loads (denoted by L). The DSO receives the committed supply of S_t (its demand at WSM) at LMP from the ISO and runs a day−ahead auction to determine its own unit commitment and supply distribution while maximizing the overall system SW.

The SW maximization of the system can be modeled as in Eqn. (5.1) subject to the system constraints in Eqns. (5.2) – (5.30) .

Maximize w.r.t. $\mathrm{dx},\ \mathrm{px}, y, z, \mathrm{e}, \mathrm{s}$

$$
\begin{aligned}
f = & \sum_t \sum_m \sum_l \sum_{r>1} \mathrm{CL}_{mlrt}\ \mathrm{dx}_{mlrt} - \sum_t \sum_m \sum_g \sum_q \mathrm{CG}_{mgq}\ \mathrm{px}_{mgqt} \\
& - \sum_t \sum_m \sum_g\ \mathrm{STC}_{mg}\ y_{mgt} - \sum_t \sum_m \sum_g \mathrm{SDC}_{mg}\ z_{mgt} \\
& - \sum_t \sum_m \sum_b \mathrm{CB}_{mb}\ \mathrm{e}_{mbt} - \sum_t \sum_m \mathrm{LMP}_t\ \mathrm{s}_{mt}, \qquad (5.1)
\end{aligned}
$$

subject to constraints in Eqns. (5.2) – (5.30) $\forall\ m \in \mathcal{M},\ \forall\ g \in \mathcal{G},\ \forall\ b \in \mathcal{B},\ \forall\ l \in \mathcal{L},\ \forall\ t \epsilon \mathcal{T}$,

$$\textstyle\sum_m \mathrm{s}_{mt} \leq \mathrm{S}_t \qquad \forall t, \qquad (5.2)$$

$$\sum_g \mathrm{p}_{mgt} + \sum_b \mathrm{e}_{mbt} + \mathrm{s}_{mt} = \sum_l \mathrm{d}_{mlt} \qquad \forall m, \forall t. \qquad (5.3)$$

The first term in the RHS of Eqn. (5.1) pertains to the price responsive loads and allocate to those loads that have the highest bids. Note that, as the loads are assumed to submit their fixed load segment without any monetary bid and two price responsive segments with monetary bids, the fixed segment is excluded from the first term, i.e. $r > 1$. The second, third, and fourth terms in the RHS of Eqn. (5.1) relate to generators and allocate generation to those units that have bid the least amount and incur smaller start up and shut down costs. The fifth term in the RHS of Eqn. (5.1) is modeled to pick BSSs with lowest bid and the sixth term shows how much to take from the ISO's committed supply. The objective function in Eqn. (5.1) picks energy sellers with least marginal cost and energy buyers with highest valuation in order to maximize surplus and result an efficient allocation.

Eqn. (5.2) indicates that the sum of supplies channeled through the DSO into each D-bus shall not exceed the committed schedule to the ISO. Similarly, Eqn. (5.3) ensures that the demand of each individual D-bus is supplied by the allocated portion of supply from WSM, and the supply of the generation and BSS units in that bus.

$$\mathrm{D}_{mlt} \ \ = \ \sum_r \mathrm{dx}_{mlrt} \qquad \forall l, \forall m, \forall t, \qquad (5.4)$$

$$0 \leq \mathrm{dx}_{mlrt} \leq \mathrm{DX}_{mlr}^{max} \qquad \forall r, \forall l, \forall m, \forall t, \tag{5.5}$$

$$\mathrm{D}_{ml}^{\min} \leq \mathrm{d}_{mlt} \leq \mathrm{D}_{ml}^{\max} \qquad \forall l, \forall m, \forall t, \tag{5.6}$$

$$\mathrm{p}_{mgt} = \sum_{q} \mathrm{p}_{mgqt} \qquad \forall g, \forall m, \forall t, \tag{5.7}$$

$$0 \leq \mathrm{p}_{mgqt} \leq \mathrm{PX}_{mgq}^{max} \qquad \forall q, \forall g, \forall m, \forall t, \tag{5.8}$$

$$\mathrm{P}_{mg}^{\min}\, i_{mgt} \leq \mathrm{p}_{mgt} \leq \mathrm{P}_{mg}^{\max}\, i_{mgt} \qquad \forall g, \forall m, \forall t. \tag{5.9}$$

Eqn. (5.4) shows that the total demand of a load is equal to the demand of the fixed segment ($r = 1$) plus the demand of its responsive segments ($r > 1$). Eqns. (5.5) and (5.6) indicates the segment limits and total demand limits of each load. Eqn. (5.7), signifies that power generated by a generation unit is equal to the aggregated segment generation of that unit. Eqn. (5.8) assures that the power generated at each segment by a generation unit does not violate the predefined lower and upper limits of generation in that segment. If committed, the total power generated by a dispatch-able unit shall lie within its lower and upper generation limits, as indicated by Eqn. (5.9).

$$\mathrm{p}_{mgt} - \mathrm{p}_{mg(t-1)} \leq \mathrm{RU}_{mg} \qquad \forall g, \forall m, \forall t, \tag{5.10}$$

$$\mathrm{p}_{mg(t-1)} - \mathrm{p}_{mgt} \leq \mathrm{RD}_{mg} \qquad \forall g, \forall m, \forall t, \tag{5.11}$$

$$0 \leq su_{mgt} \leq |\mathcal{T}|\; i_{mgt} \qquad \forall g, \forall m, \forall t, \tag{5.12}$$

$$(|\mathcal{T}| + 1) i_{mgt} - |\mathcal{T}| \leq su_{mgt} - su_{mg,\, t-1} \leq 1 \qquad \forall g, \forall m, \forall t, \tag{5.13}$$

$$su_{mgt} \geq \mathrm{MUTG}_{mg}\, z_{mg,t+1} \qquad \forall g, \forall m, \forall t, \tag{5.14}$$

$$0 \leq sd_{mgt} \leq |\mathcal{T}|\; (1 - i_{mgt}) \qquad \forall g, \forall m, \forall t, \tag{5.15}$$

$$1 - (|\mathcal{T}| + 1) i_{mgt} \leq sd_{mgt} - sd_{mg,\; t-1} \leq 1 \qquad \forall g, \forall m, \forall t, \tag{5.16}$$

$$sd_{mgt} \geq \mathrm{MDTG}_{mg}\, y_{mg,t+1} \qquad \forall g, \forall m, \forall t, \tag{5.17}$$

$$i_{mgt} - i_{mgt-1} = y_{mgt} - z_{mgt} \qquad \forall g, \forall m, \forall t, \tag{5.18}$$

$$y_{mgt} + z_{mgt} \leq 1 \qquad \forall g, \forall m, \forall t, \tag{5.19}$$

The ramp up and ramp down constraints of each individual generation unit is ensured by Eqns. (5.10) and (5.11). Furthermore, Eqns. (5.12) – (5.19) shows the MILP formulation for minimum uptime and minimum down time constraints of the generation units.

$$\mathrm{E}^{\min} j_{mbt} \le \mathrm{e}_{mbt} \le \mathrm{E}^{\max} j_{mbt} \qquad \forall b, \forall m, \forall t, \tag{5.20}$$

$$\mathrm{C}^{\min} \le \mathrm{c}_{mbt} \le \mathrm{C}^{\max} \qquad \forall b, \forall m, \forall t, \tag{5.21}$$

$$\mathrm{c}_{mbt} = \mathrm{c}_{mb(t-1)} - \mathrm{e}_{mbt} \qquad \forall b, \forall m, \forall t, \tag{5.22}$$

$$0 \le dc_{mbt} \le |\mathcal{T}|\ j_{mbt} \qquad \forall b, \forall m, \forall t, \tag{5.23}$$

$$(|\mathcal{T}|+1) j_{mbt} - |\mathcal{T}| \le dc_{mbt} - dc_{mb,t-1} \le 1 \qquad \forall b, \forall m, \forall t, \tag{5.24}$$

$$dc_{mbt} \ge \mathrm{MDTB}_{mb} u_{mb,t+1} \qquad \forall b, \forall m, \forall t, \tag{5.25}$$

$$0 \le cc_{mbt} \le |\mathcal{T}|\ (1 - j_{mbt}) \qquad \forall b, \forall m, \forall t, \tag{5.26}$$

$$1 - (|\mathcal{T}|+1) j_{mbt} \le cc_{mbt} - cc_{mb,t-1} \le 1 \qquad \forall b, \forall m, \forall t, \tag{5.27}$$

$$cc_{mbt} \ge \mathrm{MDTB}_{mb} v_{mb,t+1} \qquad \forall b, \forall m, \forall t, \tag{5.28}$$

$$j_{mbt} - j_{mbt-1} = v_{mbt} - u_{mbt} \qquad \forall b, \forall m, \forall t, \tag{5.29}$$

$$u_{mbt} + v_{mbt} \ \le 1 \qquad \forall b, \forall m, \forall t. \tag{5.30}$$

In order to increase the life expectancy of the BSS, minimum and maximum amount of energy withdrawal from these units are bounded by the given limits in Eqn. (5.20). Likewise, Eqn. (5.21) keeps the BSS safe from overcharging and deep discharging. Eqn. (5.22) indicates the BSSs' charging state update. Consecutive minimum discharging and charging hours' constraints of BSSs are represented by Eqns. (5.23) – (5.30).

5.3 Simulation Results

The tabulated data in this section pertains to market participants bidding information and serves as basis for the simulation reported. Table 5.1 shows each BSS unit bidding information submitted to the DSO. The bidding information for each generator unit is summarized in Table 5.2 and the hourly WSM LMP and supply into the DSO market is shown in Table 5.3. Loads bidding information has been considered to vary at every timeslot and is provided in Appendix A.

Table 5.1: Bidding information of each BSS at each D-bus

Bus	Unit	(C^{min}, C^{max}) (MWh)	(E^{min}, E^{max}] (MWh/hr.)	(MDTB, MCTB) (hr.)	CB ($/MWh)
1	BSS1	(1 – 10)	(0.4 – 2)	(3 – 6)	35
2	BSS1	(1 – 8)	(0.4 – 2)	(3 – 6)	33
2	BSS2	(1 – 10)	(0.4 – 2)	(3 – 6)	36.5
4	BSS1	(1 – 8)	(0.4 – 2)	(2 – 6)	34

Table 5.2: Segment generation and unit price for each D-bus

(Bus, Unit)	(PX_1^{max}, CX_1) (MW,$)	(PX_2^{max}, CX_2) (MW,$)	(PX_3^{max}, CX_3) (MW,$)	(RU,RD) (MW/h)	(STC, SDC ($)
(1,G1)	(1.5, 36.7)	(2.5, 39.3)	(1, 42)	(2.5, 2.5)	(75, 60)
(1,G2)	(1.6, 34.8)	(2, 37.8)	(1.4, 40.5)	(2.5, 2.5)	(60, 60)
(2,G1)	(1.5, 30)	(1.7, 33)	(1.8, 39)	(2.5, 2.5)	(45, 54)
(2,G2)	(1.4, 36.9)	(1.8,39.6)	(1.8,43.8)	(2.5, 2.5)	(51, 45)
(2,G3)	(1, 34.5)	(1.5, 36)	(0.5, 39.6)	(3, 3)	(84, 45)
(3,G1)	(1.2, 29.4)	(1.8, 30.6)	(2, 34.5)	(2.5, 2.5)	(0, 0)
(3,G2)	(1.8, 32.1)	(1.45, 32.6)	(1.75, 34.5)	(2.5, 2.5)	(45, 51)
(3,G3)	(0.8, 35.7)	(1.7, 37.5)	(0.5, 40.5)	(3, 3)	(60, 48)
(3,G4)	(0.95, 36.3)	(1.1, 37.5)	(0.95, 40.5)	(3, 3)	(0 ,0)
(4,G1)	(1.9, 37.5)	(1.7, 41.4)	(1.4, 44.5)	(2.5, 2.5)	(10, 10)

Table 5.3: Supply at the DSO bus from the ISO at the LMP

LMP and ISO supply in 24hr scheduling horizon						
t	1	2	3	4	5	6
LMP	22.07	24.83	24.83	23.45	24.83	24.83
S_t	29.04	32.67	32.67	30.855	32.67	32.67
t	7	8	9	10	11	12
LMP	26.21	27.59	28.97	30.35	33.11	30.35
S_t	34.485	36.3	38.115	39.93	43.56	39.93
t	13	14	15	16	17	18
LMP	27.59	26.21	26.48	25.38	27.04	30.35
S_t	36.3	34.485	34.848	33.396	35.574	39.93
t	19	20	21	22	23	24
LMP	31.73	34.49	31.73	28.97	26.21	23.45
S_t	41.475	45.375	41.745	38.115	34.485	30.855

The MILP model presented in Eqns. (5.1) – (5.30) is coded in GAMS and solved using CPLEX solver for 24-hour horizon. The simulation summarized in the upcoming figures present the validity of the theory in the model. Several sets of analysis were further conducted to see the effect of LMP on the auction outcome. Simulation results indicates that loads, generators and BSS are very responsive to changes in LMP at the DSO bus at a given amount of supply by the WSM. At low LMPs, more from the WSM is allocated to loads due to high local generation bids. As the LMP increases, more internal generation at each D-bus is scheduled. A similar effect is observed on serving the price responsive loads.

Figure 5.2 depicts LMP versus DSO's supply allocation to each D-bus as a portion of the total committed schedule that it gets from WSM during 24h scheduling horizon. Notice that supply to each D-bus is very responsive to the changes in LMP. When LMP is low, more power will start to flow to the loads from WSM. However, during peak LMP hours the power flow from DSO drops significantly, to the extent that D-bus number three feeds power back into the distribution network, i.e. other D-bus. This is because D-bus 3 has more cheap generation and loads in other D-buses are ready to purchase at higher price than that of its own local loads.

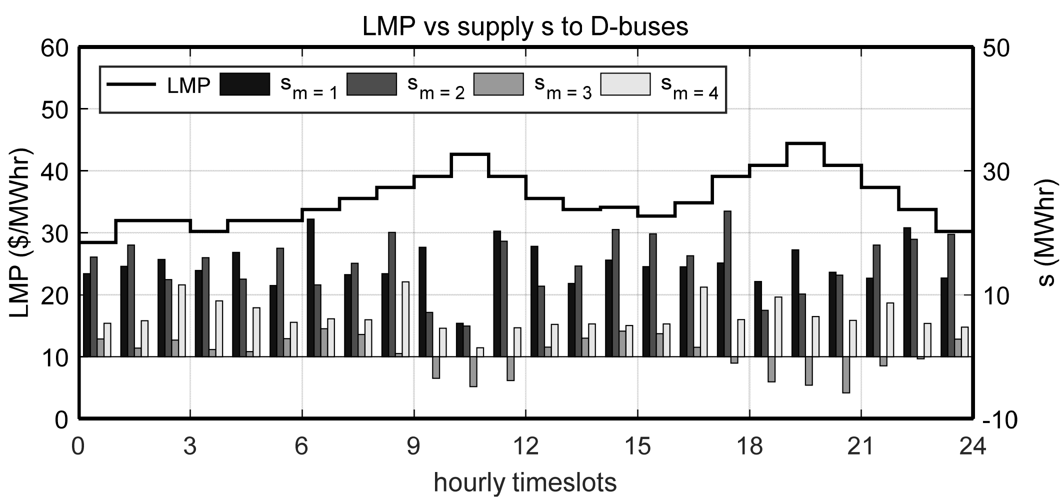

Figure 5.2: Hourly DSO supply to each D-bus vs. LMP.

Figure 5.3 shows LMP versus each D-buses' internal generation. The internal generation at each D-bus increases with increase in LMP.

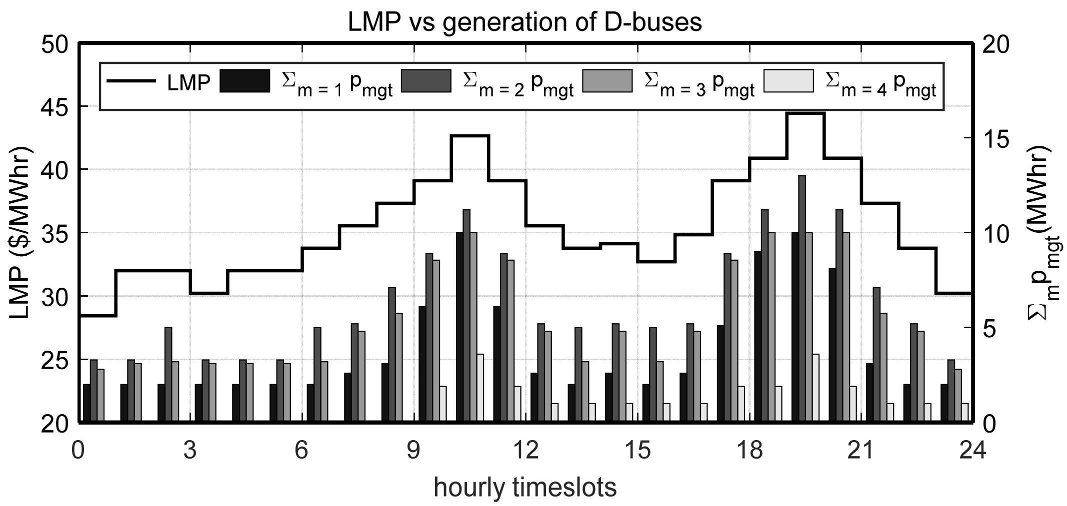

Figure 5.3: Hourly D-bus generation vs. LMP.

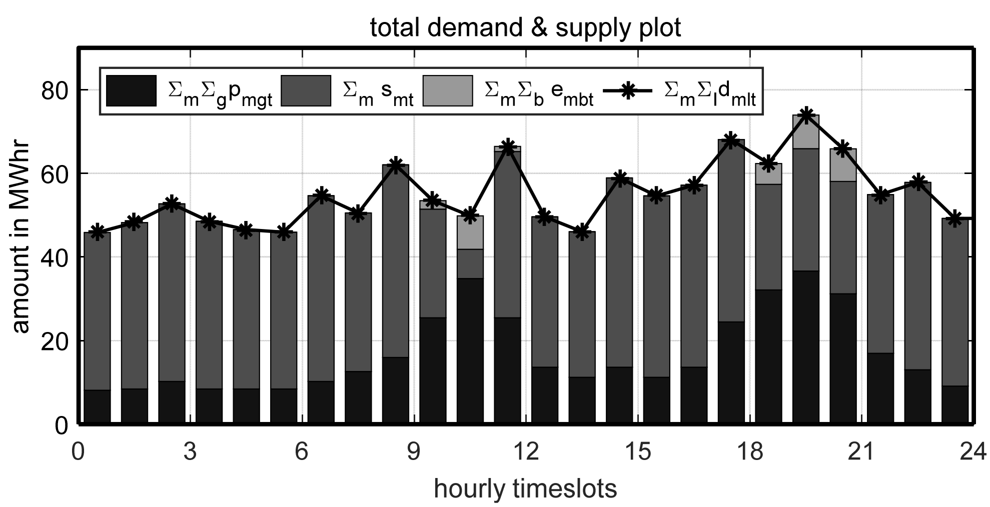

Figure 5.4: Total demand and supply by the DSO, BSS and generators.

Figure 5.4 shows total power demand equals the total supply, which consists of supply from the WSM, internal generation and supply from BSSs. Notice that the BSSs are scheduled only during peak LMP hours for at least three consecutive hours due to their minimum discharging time constraints.

Figure 5.5 depicts the LMP values versus BSSs' commitment considering its charging/discharging limits. Note that all BSSs are set to 6 hours of minimum charging and three minimum discharging hours except the minimum discharge time of BSS at D-bus 4. Note that the plot meets all the requirements set in the constraints in Eqns. (5.20) – (5.30). BSSs are scheduled during high price hours with more power scheduled at peak LMPs than its neighboring hours. In addition, the sum of assigned energy during scheduling horizon does not exceed each BSSs capacity.

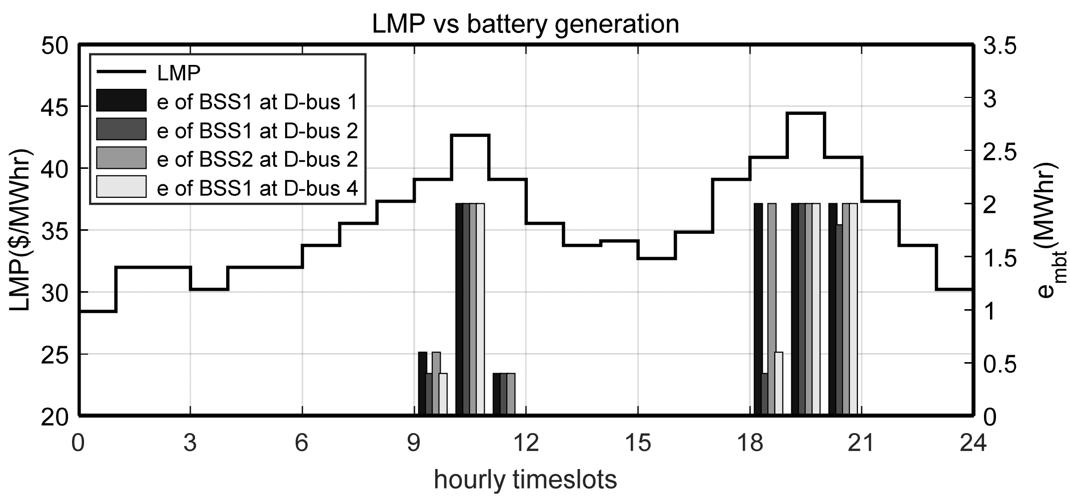

Figure 5.5: Supply from BSSs as DLMP changes.

The plot in Figure 5.6 shows scheduled behavior of the second BSS at D-bus 2 for two different DLMPs along with its declared selling price depicted with the horizontal line. As seen, the BSS is only scheduled when DLMP is higher than its bid. The scheduled amount of power withdrawal from this BSS is also higher where the difference between its bid and the DLMP is higher. A similar observation is made when generation units behavior were studied.

In order to study the dynamics of DLMP versus LMP, Figure 5.7 and Figure 5.8 were plotted. Figure 5.7 shows the DLMP behavior during the scheduling horizon for two

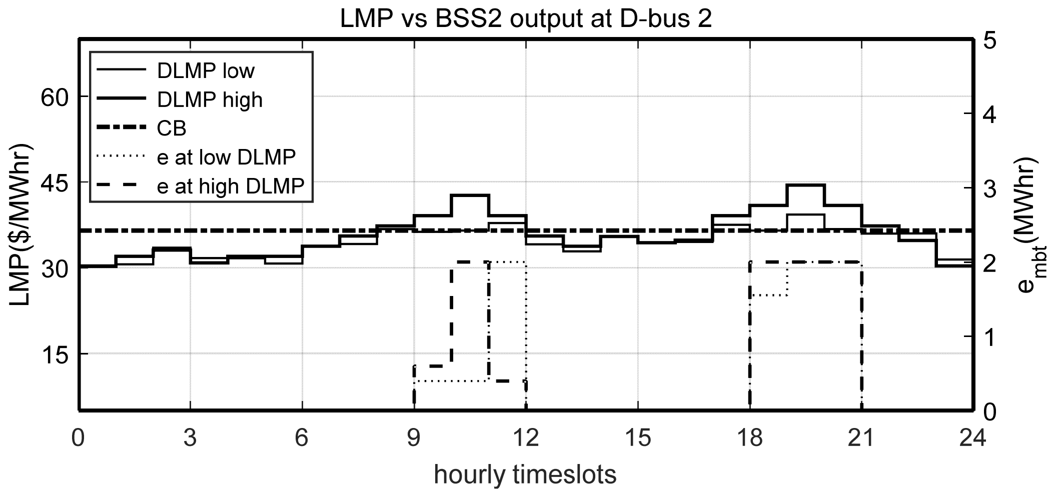

Figure 5.6: Supply from the BSS at two different DLMPs.

different set of LMPs. When LMP is low compared to the bids of the generators, loads and BSSs, DLMP deviates significantly and becomes higher than LMP. In the case when LMP increases, the DLMP also increases and overlaps with LMP in most hours. Note that further increase in LMP will cause DLMP to equal LMP in all hours. This is because at LMPs higher than the generator bids, all generators are scheduled to serve the fixed and or price responsive loads and serving any extra MWhr will incur a cost equal to LMP. Figure 5.8 illustrates this concept further by showing the 24-h LMP average for 10 scenarios by increasing the LMP with a fixed percentage at each scenario. As expected, at low LMPs the entire committed supply of the WSM is injected into the D-buses, whereas lower amounts are drawn when LMP increases.

The simulation results reported here assumes that no penalty is incurred by the DSO for deviating from what was committed to ISO. The DSO's objective function in Eqn. (5.1) can be modified by adding a penalty function $\phi(\gamma)$ to account for penalty incurred due to any deviation from original commitment.

$$\phi(\gamma) = -\gamma \left(\sum_{t} \mathrm{S}_t - \sum_{t} \sum_{m} \mathrm{s}_{mt} \right) \tag{5.31}$$

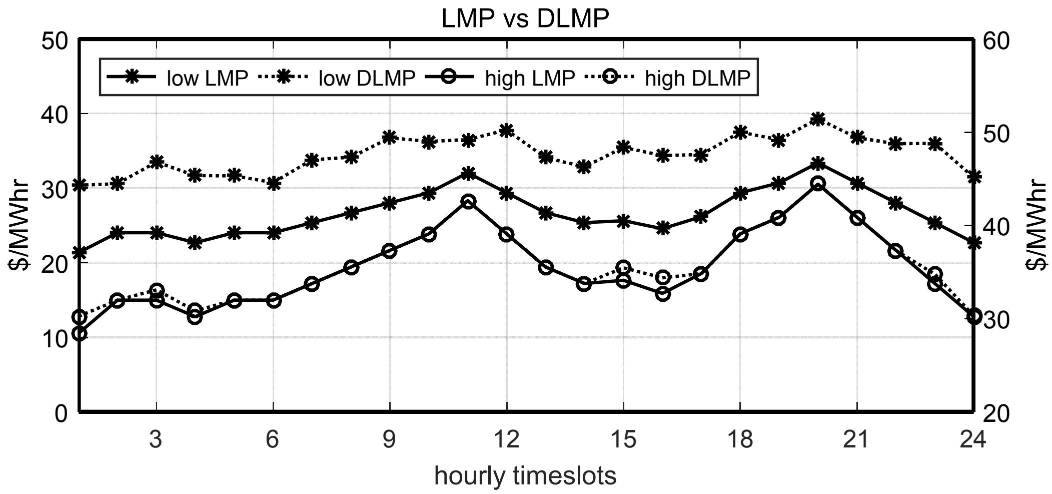

Figure 5.7: LMP versus DLMP for 24-hour horizon.

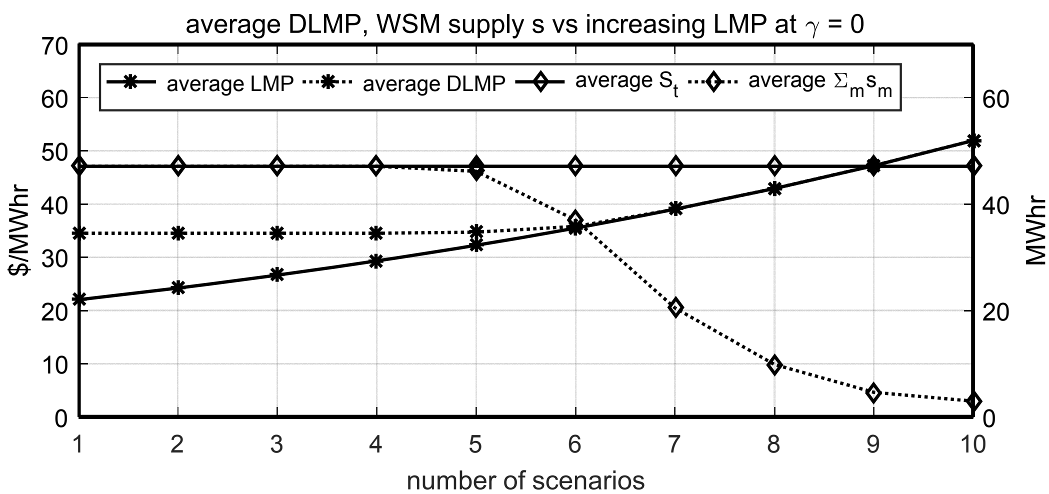

Figure 5.8: Average LMP vs DLMP and total supply to DSO.

Note that any deviation will not cause extra congestion in the WSM transmission system, as DSO will draw less power than what was originally committed due to constraint in Eqn. (5.2). To show the effect of applying penalty, the total deviation over the scheduling horizon from original commitment was plotted as a function of γ in Figure 5.9 for three different LMPs. Notice that for higher LMPs, a higher penalty is required to make the deviation zero. This means that if LMP is high, more internal generation at D-buses and BSSs will be committed, and it takes a higher penalty to force power injection from the WSM market in order to make the deviation zero.

Budget dynamics of the 10 increasing LMP scenarios for a fixed supply S_t from WSM, without any penalty applied ($\gamma = 0$), is shown in Figure 5.10. At lower LMPs, during scenarios one to five, the DSO makes money. It sells energy at higher DLMP while buying it at lower LMP. As the LMP is increased further, less power is purchased from the WSM and DLMP approaches LMP. As a result, the DSO's revenue drops down to zero after the fifth scenario. Note that, if DSO is penalized for any deviation, it loses a monetary amount equal to $\phi(\gamma)$ given by Eqn. (5.31). This is because deviation occurs at higher LMPs after scenario five when DSO's revenue is zero with no penalty($\gamma = 0$). In such a scenario, the DSO has to bear the deviation cost $\phi(\gamma)$.

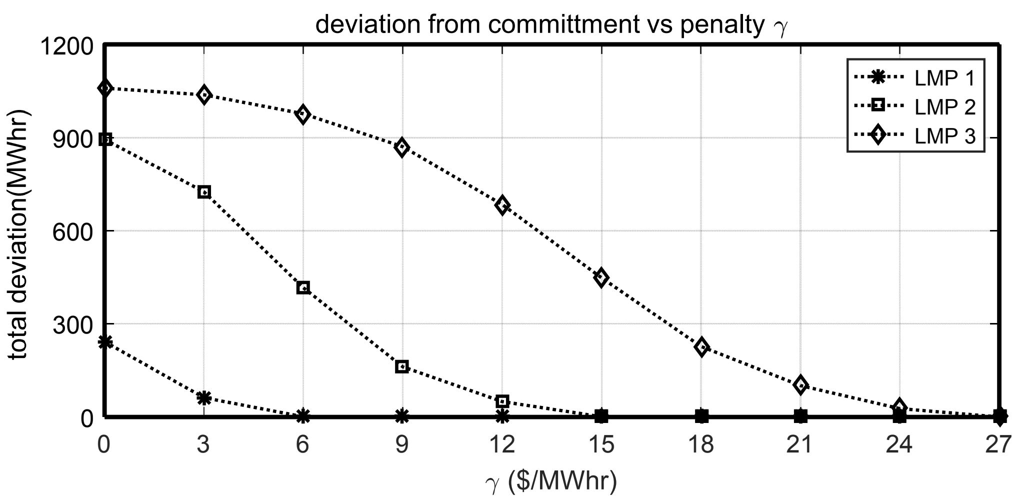

Figure 5.9: Deviation from ISO supply vs penalty for three different LMPs, with LMP1(t) < LMP 2(t) < LMP 3(t).

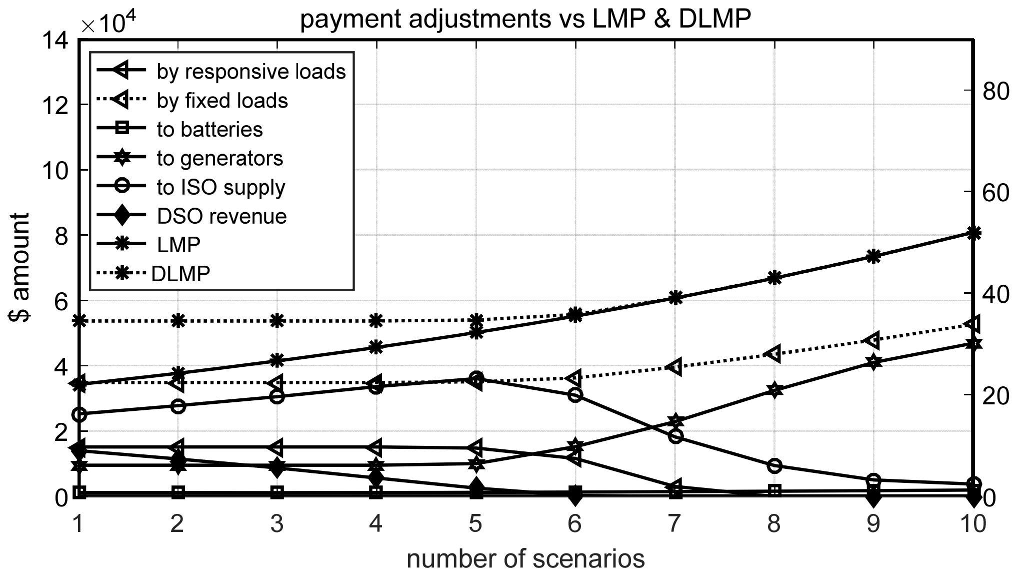

Figure 5.10: Payments and reimbursements.

5.4 Conclusions

In this chapter, distribution system operator's day−ahead market auction in the presence of distribution level conventional generation units, renewable energy resources coupled with BSSs, and loads with fixed and price responsive segments using MILP were modeled and studied. The DSO is considered to have prior agreement to purchase a fixed committed amount from the WSM. The DSO uses MILP to optimally schedule its available resources and maximize the SWF. By clearing the auction at DLMP, the dynamics of DLMP versus LMP and their effect on the outcome of the auction and the resulting payments were studied. Simulation results shows that, if DSO is not penalized for deviating from its committed schedule with the WSM, the auction is always weakly budget balanced. The DSO only makes money when LMP is cheaper at a given fixed supply from the ISO.

Chapter 6 - Conclusion and Future Work

In this chapter, the research reported in chapters 2 to 5 are summarized and possible future directions are suggested. Chapter-specific conclusions were presented earlier in their respective chapters.

6.1 Conclusion

This dissertation studies efficient energy distribution mechanisms in islanded and grid-connected microgrids in the form of double auctions. An aggregator is assigned to each microgrid to implement price-heterogeneous or price-homogenous auctions among selfish buyers and sellers of energy with hidden information. Then, a bi-level auction implemented by the DSO among aggregators in the form of linear programming, and by the aggregators among buyer and seller agents in the form of proportional allocation, is proposed. In the DSO level auction, physical grid constraints such as node voltages and substation transformer capacity has been incorporated. The proposed bi-level auction mechanism is designed to be price-heterogeneous at the upper and price-homogenous in the lower level. Finally, a DSO level auction for multi-period day-ahead auction that includes supply from wholesale market, battery storage units, conventional generators, and price responsive loads has been proposed. The proposed multi-period double auction is modeled to maximize SW and is modeled as a mixed-integer linear programming in order to perform unit commitment and account for inter-temporal constraints. Key features of the research done in this dissertation are summarized next.

Traditional single-sided efficient resource auctions were extended to efficient price heterogeneous and price-homogenous double-sided auctions in microgrids with energy being the resource under trade. Both auctions were designed under the assumption that agents' utility functions remain hidden from the auctioneer, i.e. aggregator. Weak and strongly budget balance cases were studied by imposing it as constraint in the price-heterogeneous auction and by levying a surcharge in the case of price-homogenous auction.

With agents bidding strategically, equilibrium conditions were exploited. A possible way that the agents can exercise market power by strategic bidding, i.e. price anticipation, and influence the market price was explored. Agents market powers were quantified in terms of their bids and the loss in efficiency due to strategic bidding was investigated. A novel approach to mitigate the loss in efficiency in terms of virtual bidding was proposed.

Next, the case where microgrids are connected to the distribution system has been investigated. The DSO has been considered to distribute its fixed committed energy from the wholesale market to aggregators serving individual microgrids in which PV equipped sellers and buyers reside. The mechanism was formulated in terms of a bi-level auction, with upper and lower levels being DLA and ALA, which aims to maximize global SW. The problem was decomposed into a master and a sub-problem. The DLA was shown to be a linear programming that solves the master problem subject to physical distribution grid constraints. The sub-problem was solved using ALA with the help of virtual bidding in order to exercise price taking. It was shown that the proposed bi-level auction algorithm achieves global efficient solution and maximizes global SW.

Finally, a multi-period DSO auction implemented among RES-charged BSSs, conventional generators, and price responsive loads was proposed. The DSO's goal in this case is to implement a centralized social surplus maximizing auction at the presence of bids and asks of the participating agents and its commitment with the wholesale market at LMP. It does so by running a price based unit commitment and schedules the least cost generators and BSSs while serving high value loads. The market is cleared at each distribution bus DLMP. The dynamics of DLMP versus LMP under the cases when DSO deviates from its commitment due to cheaper local generation were analyzed.

6.2 Future Work

A major portion of the research reported in this dissertation is for single-period efficient energy distribution auctions. Chapters 2 and 3 discuss more theory and mechanism design aspects of these efficient energy auctions while chapters 4 includes theory as well as its application in distribution grid with physical constraints. With chapter 4 being single-period, chapter 5 provides a preliminary study towards multi-period auctions including units with temporal constraints. Thus, the following research directions and extensions are proposed in different aspects of the research provided in each chapter.

Extension of the efficient bi-level auction algorithm proposed in chapter 4 to multi-period efficient auction with battery storage units, conventional generators, and price responsive loads in the DSO and aggregator levels is a tempting research direction one can take. This would make the problem similar to a decentralized unit commitment problem. A few recent research papers that try to address the problem of decentralized unit commitment each with specific methods of its own are reported in [116], [117], [118]. None of these papers takes physical system constraints such as distribution node voltages into account, which makes the problem more realistic for distribution system. Future studies can also be carried out to model the multi-period auction in a game theoretic setting and investigate general market equilibrium conditions [89].

In energy auctions, typically, the distribution nodal prices, i.e. DLMPs are given by the Lagrange multipliers of the energy balance constraints that is solved by the central operator, i.e. DSO [112]. A constraint violation adds up to the nodal price, resulting to different nodal price than what is on the substation node. With further penetration of RES based units into the distribution system and the emergence of price responsive aggregators, a theoretical analysis of nodal pricing that would take into account all physical grid constraints are missing in the literature and is another line of research for those who are interested in theoretical analysis. Few attempts in this line of research are published in [119], [120], [114], [121]. Considering the nonlinear nature of AC optimal power flow (ACOPF), one possible way to model the physical constraints is the use of simplified DistFlow equations [93], [94] that can be set as linear constraints to the DSO's optimization problem.

There are several ways that learning algorithms can be implemented in DLA and ALA auctions reported in chapter 2 - 5 by using machine learning and artificial intelligence. Supervised machine learning algorithms [122] to estimate the bid prices and amounts from historical bidding and web weather data can either reduce or eliminate the computations involved during auction. Self-organization and distributed clustering algorithms [123], [124] can be used to study coalition formation. Reinforcement learning [125] can be utilized to dynamically schedule battery storage units and conventional generators.

Cooperative agent strategies that was briefly addressed in chapter 2 in the context of fair redistribution can be investigated further [126], [127], [128]. The existing body of literature on cooperative game theory can be utilized to investigate cooperative behavior within domestic units for better utilization of available resources (e.g. to increase SW) by aggregators so that a set of domestic units form a stable coalition.

Dynamic game theory can be used to examine energy distribution using Stackelberg game [129], [130], [35]. Recent theoretical research have studied inverse Stackelberg game where the upper level agent elicit desired behavior among lower level agent [131], [132].

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Appendix A - Tables

{D-bus, load, seg}	**load bids for 24-hours for their price responsive segments**																							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
{1,1,1}	25.7	33.5	31.2	29.9	25.7	25.7	34.6	25.6	30.6	35.4	35.9	44.2	33.6	27.2	25.9	27.9	30.9	43.0	34.8	35.5	37.7	30.3	28.7	31.9
{1,1,2}	24.7	29.1	28.3	29.0	23.6	22.1	33.4	24.2	29.1	29.9	34.3	39.6	32.5	25.4	25.3	24.4	26.8	39.7	32.2	31.7	33.8	26.2	24.9	26.3
{1,2,1}	25.9	32.4	28.8	35.8	31.7	35.4	36.6	28.5	31.2	40.0	34.2	43.0	32.0	30.9	30.1	26.2	35.8	36.8	37.2	42.0	37.3	34.4	31.5	24.8
{1,2,2}	23.0	28.5	25.5	31.9	28.4	31.1	33.4	22.5	29.9	39.4	33.5	42.6	29.6	28.2	27.9	21.6	32.1	32.2	31.6	36.2	36.2	33.5	27.4	24.3
{1,3,1}	24.0	33.5	32.1	28.0	24.8	30.6	29.0	36.5	33.0	42.0	40.0	36.0	31.4	24.5	32.8	35.1	35.7	40.7	33.2	46.0	33.9	37.0	26.7	32.6
{1,3,2}	22.0	31.0	30.4	26.8	19.9	28.0	23.7	34.1	28.4	39.6	35.1	31.4	29.1	23.2	28.1	29.4	33.7	36.7	30.6	41.0	29.3	36.0	21.5	26.6
{1,4,1}	29.6	30.4	27.8	30.2	33.2	26.3	29.0	30.7	28.4	40.6	37.3	36.4	37.1	32.8	35.9	35.8	34.1	33.6	35.7	49.5	35.8	35.1	34.8	28.6
{1,4,2}	26.1	25.5	22.5	24.3	33.2	21.1	25.3	24.8	25.2	37.7	32.5	35.0	34.1	27.4	32.4	30.7	29.6	30.1	34.2	45.5	35.3	31.4	30.8	24.3
{1,5,1}	30.4	23.3	34.7	24.7	35.6	27.4	33.8	30.4	33.7	33.2	39.3	44.8	39.0	23.7	35.1	28.1	32.7	33.3	45.2	44.5	44.5	31.7	38.4	26.8
{1,5,2}	27.5	19.4	28.6	23.8	26.9	25.0	34.0	31.0	29.9	31.1	34.0	38.4	34.6	24.5	35.4	22.0	30.4	32.5	38.2	44.4	45.1	30.5	38.6	22.7
{2,1,1}	21.8	27.9	32.1	27.7	31.1	28.7	30.1	25.8	36.0	37.6	43.1	41.2	30.8	31.1	33.5	37.5	25.7	36.7	41.8	43.2	40.1	39.3	37.5	30.3
{2,1,2}	18.4	22.8	30.0	25.0	30.8	27.6	26.2	23.8	30.6	36.9	37.2	38.0	26.6	25.2	31.7	35.0	22.9	32.1	36.8	42.6	39.1	37.2	37.1	27.2
{2,2,1}	30.8	34.7	32.4	30.8	23.8	24.5	29.1	35.8	35.6	30.7	41.7	43.5	29.4	34.0	37.1	33.3	35.0	43.9	35.0	39.6	38.2	43.3	32.3	31.4
{2,2,2}	30.2	32.9	29.7	30.2	17.8	22.5	27.3	35.4	33.8	30.4	38.7	38.9	25.6	33.5	36.7	28.6	29.6	40.7	34.3	34.6	36.2	41.6	27.8	31.4
{2,3,1}	24.9	34.5	34.5	31.7	26.5	25.6	28.0	35.2	33.4	39.0	38.2	43.3	29.4	36.3	32.8	27.4	31.6	44.2	33.3	39.1	36.6	34.1	30.7	32.9
{2,3,2}	24.5	34.0	29.7	26.0	22.4	24.8	23.7	34.5	32.7	35.2	36.3	39.4	24.9	32.8	28.3	26.0	27.2	38.4	28.1	38.6	34.4	31.9	26.6	29.3
{2,4,1}	24.2	30.3	27.6	26.8	26.3	32.9	28.4	32.6	38.5	30.8	36.2	38.4	30.1	32.8	36.6	34.4	32.7	42.6	35.2	48.4	44.4	37.4	36.0	28.0
{2,4,2}	18.4	27.7	23.5	22.3	23.7	29.0	27.8	27.0	37.4	29.2	31.4	35.5	25.5	30.5	35.0	34.1	28.7	40.0	32.5	44.8	44.0	35.5	31.4	23.8
{2,5,1}	30.6	26.8	33.0	34.6	25.3	23.0	30.9	37.8	40.4	37.2	41.7	42.0	34.5	31.1	37.2	36.8	36.1	40.2	41.4	43.8	41.4	36.8	27.5	35.4
{2,5,2}	29.3	26.4	27.5	30.4	21.9	21.1	29.9	34.1	34.4	36.2	40.1	39.6	34.0	27.0	34.8	30.9	33.7	36.5	40.4	41.5	40.4	32.2	22.3	33.3
{3,1,1}	28.7	28.1	32.6	28.2	25.3	33.7	36.1	40.4	41.1	36.3	30.8	31.3	33.9	25.0	40.4	35.2	35.0	39.9	42.6	35.2	30.8	26.0	33.1	26.3
{3,1,2}	24.2	23.7	28.2	27.5	21.2	30.9	34.8	39.8	36.2	35.2	29.8	27.3	28.5	21.9	36.2	34.2	29.3	36.7	38.6	35.0	26.0	21.5	32.3	23.1
{3,2,1}	31.4	26.8	35.0	26.5	31.6	27.8	31.8	25.9	29.5	36.3	40.2	38.0	37.2	28.1	32.2	32.1	26.2	37.2	39.9	36.5	34.0	32.9	34.4	31.5
{3,2,2}	29.1	21.9	33.1	21.6	26.9	22.7	28.7	22.1	23.8	33.7	39.8	32.8	33.4	26.0	26.2	30.7	22.3	33.6	37.6	35.6	33.9	30.4	33.3	27.1
{3,3,1}	22.2	33.3	28.4	31.4	25.9	24.4	32.9	31.1	42.1	35.5	48.5	41.2	34.9	39.1	26.7	27.5	28.9	41.8	36.1	45.0	30.6	31.5	26.4	26.5
{3,3,2}	19.9	29.6	24.9	28.2	24.2	22.9	30.2	29.7	37.2	29.6	48.3	38.0	34.4	34.2	20.7	27.1	23.3	41.7	32.0	40.3	27.4	26.2	21.0	22.8
{4,1,1}	29.0	27.5	36.6	35.0	36.0	30.3	24.9	30.5	40.9	35.2	37.6	36.9	28.5	28.3	35.3	30.3	39.9	32.3	41.7	46.9	40.8	38.1	30.1	29.3
{4,1,2}	25.7	24.0	33.5	34.5	31.6	24.3	22.7	24.7	38.8	29.9	34.8	34.4	27.2	27.6	33.5	25.9	35.2	28.2	41.6	41.9	35.3	33.5	29.9	27.0
{4,2,1}	19.4	23.3	36.7	26.4	31.4	30.7	30.2	29.6	38.0	29.2	41.3	31.9	33.6	32.9	31.9	28.5	38.3	39.0	35.2	49.1	44.0	32.7	28.0	27.8
{4,2,2}	17.4	22.0	33.6	20.9	27.6	30.1	27.8	29.2	35.0	26.6	35.3	27.1	30.7	27.6	31.0	26.1	32.7	33.5	30.9	45.3	41.9	27.1	27.3	23.4
{4,3,1}	28.9	29.2	30.8	20.6	31.8	26.5	29.4	31.9	38.5	38.8	40.1	42.3	30.7	35.3	33.3	28.2	32.7	40.5	35.3	43.7	32.1	32.1	29.6	28.8
{4,3,2}	27.2	27.8	26.5	16.8	28.2	22.5	29.2	29.8	35.8	37.4	35.8	37.2	29.0	30.9	32.5	23.2	31.9	37.0	33.1	38.9	29.1	29.2	24.3	26.6

{D-bus, load, seg}	load amount for 24-hours for their fixed and two price responsive segments																						
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
{1.1.1}	2.4	2.7	2.7	2.6	2.7	2.7	2.9	3.1	3.2	3.4	3.7	3.4	3.1	2.9	2.9	2.8	3	3.4	3.5	3.8	3.5	3.2	2.9	2.6
{1.1.2}	0.7	0.8	0.8	0.8	0.8	0.8	0.9	0.9	1	1	1.1	1	0.9	0.9	0.9	0.8	0.9	1	1.1	1.1	1.1	1	0.9	0.8
{1.1.3}	0.9	1	1	0.9	1	1	1	1.1	1.1	1.2	1.3	1.2	1.1	1	1	1	1	1.2	1.2	1.3	1.2	1.1	1	0.9
{1.2.1}	3.1	3.3	3.5	3.7	3.5	3.3	3.6	3.7	4.3	4.3	4.4	4.6	4	3.6	3.7	3.4	3.9	4.2	4.9	4.9	4.5	4.2	3.6	3.2
{1.2.2}	0.8	0.8	0.9	0.9	0.9	0.8	0.9	0.9	1.1	1.1	1.1	1.2	1	0.9	0.9	0.8	1	1.1	1.2	1.2	1.1	1.1	0.9	0.8
{1.2.3}	1.4	1.5	1.6	1.7	1.6	1.5	1.6	1.7	1.9	1.9	2	2.1	1.8	1.6	1.7	1.5	1.8	1.9	2.2	2.2	2	1.9	1.6	1.4
{1.3.1}	0.1	0.3	0.2	0	0.2	0.2	0.2	0.1	0.2	0.4	0.1	0.1	0.2	0.3	0.1	0	0.3	0.2	0.3	0.4	0	0.2	0.1	0.1
{1.3.2}	1.9	1.6	1.4	2.1	1.6	1.6	2.2	1.7	1.4	1.7	1.4	2	1.5	1.4	1.7	1.3	1.8	1.8	2.2	1.8	1.7	1.7	2.1	1.5
{1.3.3}	1.5	1.5	1.9	1.4	1.8	1.3	1.4	1.9	1.9	1.9	1.2	1.8	1.6	1.2	1.2	1.1	2	1.8	1	2	1.8	1.9	1.7	1.7
{1.4.1}	1.6	1.4	1.6	1.6	1.3	1.3	1.9	1.5	2.2	1.9	1.7	1.8	1.4	1.6	1.5	2	1.6	1.8	1.6	2.1	1.9	2	1.8	1.2
{1.4.2}	1.1	1	1.1	1.1	0.9	0.9	1.3	1	1.6	1.3	1.2	1.3	1	1.1	1.1	1.4	1.1	1.3	1.1	1.5	1.4	1.4	1.3	0.9
{1.4.3}	1.2	1	1.2	1.2	0.9	1	1.4	1.1	1.7	1.4	1.3	1.4	1.1	1.2	1.1	1.5	1.2	1.3	1.2	1.6	1.5	1.5	1.4	0.9
{1.5.1}	4.5	5.5	5.4	5.3	5.2	5.1	5.6	5.8	6.6	6.5	7.6	6.4	5.9	5.5	5.8	5.7	5.5	6.7	7.1	7.3	7.2	6.1	5.7	5.4
{1.5.2}	3.6	4.4	4.3	4.3	4.1	4.1	4.5	4.7	5.3	5.2	6.1	5.1	4.8	4.4	4.7	4.5	4.4	5.4	5.7	5.9	5.7	4.9	4.5	4.3
{1.5.3}	3.4	4.1	4	4	3.9	3.8	4.2	4.4	4.9	4.8	5.7	4.8	4.5	4.1	4.4	4.2	4.2	5.1	5.3	5.5	5.4	4.6	4.2	4
{2.1.1}	1.7	1.7	1.7	1.9	2.1	2.1	2.3	2.1	1.7	2.4	2.6	2.6	1.8	2	2.1	1.8	2.3	2.5	2.4	2.2	2.6	2.1	2.3	1.4
{2.1.2}	1.1	1.1	1.1	1.3	1.4	1.3	1.5	1.4	1.1	1.6	1.7	1.7	1.2	1.3	1.4	1.1	1.5	1.6	1.5	1.5	1.7	1.4	1.5	0.9
{2.1.3}	1.3	1.3	1.3	1.5	1.6	1.5	1.7	1.6	1.3	1.8	2	2	1.3	1.5	1.6	1.3	1.7	1.9	1.8	1.7	1.9	1.6	1.7	1
{2.2.1}	3.2	3.7	3.4	3.5	3.5	3.7	3.7	3.8	4.1	4.5	4.8	4.8	3.9	3.7	3.7	3.8	4.3	4.3	4.3	5	4.3	4.6	3.5	3.7
{2.2.2}	1.6	1.8	1.7	1.7	1.8	1.9	1.8	1.9	2.1	2.2	2.4	2.4	1.9	1.9	1.8	1.9	2.1	2.1	2.2	2.5	2.2	2.3	1.7	1.8
{2.2.3}	1.9	2.2	2	2.1	2.1	2.2	2.2	2.3	2.5	2.7	2.9	2.9	2.3	2.2	2.2	2.3	2.6	2.6	2.6	3	2.6	2.7	2.1	2.2
{2.3.1}	1	0.8	1	0.7	1	1.2	0.7	1.2	0.9	1.1	0.9	0.8	1	0.9	1	0.8	1.3	1.1	1.1	1.1	0.9	1.2	0.7	1
{2.3.2}	2	1.5	2.1	1.5	1.6	1.7	1.5	2	2.1	1.9	1.9	1.7	1.5	1.9	1.9	2	1.3	1.5	1.8	1.2	1.4	1.5	1.9	1.6
{2.3.3}	1.7	1.5	2	1.4	2.4	2.3	2.2	2.1	1.5	2.2	2.2	2.3	2	1.8	2.3	2.1	1.8	1.6	2	1.7	2.1	2.2	1.4	2.2
{2.4.1}	6.4	6.3	6.1	6.5	6.3	6.1	6.5	6.2	6.3	6.2	6.7	6.3	6.4	6.1	6.2	6.6	6.7	6.4	6.6	6.4	6.4	6.2	6.6	6.2
{2.4.2}	4.8	4.7	4.6	4.9	4.7	4.6	4.9	4.6	4.7	4.6	5	4.7	4.8	4.6	4.7	5	5	4.8	5	4.8	4.8	4.6	4.9	4.6
{2.4.3}	4.4	4.3	4.2	4.5	4.4	4.2	4.5	4.2	4.3	4.2	4.6	4.3	4.4	4.2	4.3	4.6	4.6	4.4	4.6	4.4	4.4	4.3	4.5	4.3
{2.5.1}	3.1	3.1	3.3	3.2	2.9	3.2	3.4	3.3	3.3	2.9	3.5	3.4	3.5	3.2	3.4	3.3	3.2	3.4	3.3	3.1	3.1	3.1	3.4	3.1
{2.5.2}	1.7	1.7	1.8	1.8	1.6	1.8	1.9	1.8	1.8	1.6	1.9	1.8	1.9	1.7	1.9	1.8	1.7	1.9	1.8	1.7	1.7	1.7	1.9	1.7
{2.5.3}	1.1	1.1	1.1	1.1	1	1.1	1.2	1.1	1.2	1	1.2	1.2	1.2	1.1	1.2	1.1	1.1	1.2	1.2	1.1	1.1	1.1	1.2	1.1
{3.1.1}	1	0.9	1.3	0.9	1.1	1.1	0.8	1.2	1.1	1.6	1.9	1.2	1.4	1.4	1.4	1.1	0.8	1.1	1	1.6	1.4	1.6	1.5	0.9
{3.1.2}	2.1	2.1	2.4	1.8	1.9	2.6	1.7	2	2.1	1.9	1.6	1.9	2.7	1.9	2.1	2.5	2.2	2	2.5	1.7	2.6	2.3	2.2	2.4
{3.1.3}	2.6	2.1	2	2.4	1.9	2.3	2.4	2.4	2.3	2.3	2.1	1.8	1.9	2.7	2.4	2.1	2.5	2.3	2.3	2	2.4	2.1	2	2.3
{3.2.1}	3.8	3.6	3.3	3.6	3.6	3.3	3.6	3.7	3.4	3.8	3.4	3.8	3.8	3.8	3.5	3.7	3.9	3.5	3.3	3.9	3.3	3.4	3.5	3.6
{3.2.2}	1.9	1.8	1.7	1.8	1.8	1.6	1.8	1.9	1.7	1.9	1.7	1.9	1.9	1.9	1.7	1.9	1.9	1.7	1.7	1.9	1.6	1.7	1.8	1.8
{3.2.3}	1.5	1.4	1.3	1.4	1.4	1.3	1.4	1.5	1.4	1.5	1.4	1.5	1.5	1.5	1.4	1.5	1.6	1.4	1.3	1.5	1.3	1.3	1.4	1.4
{3.3.1}	1	1.5	1.3	1.3	1.3	1.1	1.2	1.1	1.2	1.6	1	1.2	1.2	1.6	1.5	1.6	1.4	1.5	1.2	1.5	1.5	1.3	1.5	1.4
{3.3.2}	0.4	0.5	0.4	0.5	0.4	0.4	0.4	0.4	0.4	0.6	0.3	0.4	0.4	0.6	0.5	0.6	0.5	0.5	0.4	0.5	0.5	0.4	0.5	0.5
{3.3.3}	0.6	0.9	0.8	0.8	0.8	0.6	0.7	0.6	0.7	1	0.6	0.7	0.7	0.9	0.9	1	0.8	0.9	0.7	0.9	0.9	0.8	0.9	0.8
{4.1.1}	3.3	3.4	3.8	3.1	3.3	3.7	3.9	3.6	3.9	4.4	5.2	4.4	4	4	3.8	3.6	4.2	4.5	4.9	4.7	4.7	4.5	3.8	3.4
{4.1.2}	2.1	2.2	2.5	2	2.1	2.4	2.5	2.4	2.5	2.9	3.4	2.8	2.6	2.6	2.5	2.3	2.7	2.9	3.2	3	3	2.9	2.5	2.2
{4.1.3}	1.3	1.4	1.5	1.3	1.3	1.5	1.6	1.5	1.5	1.8	2.1	1.7	1.6	1.6	1.5	1.4	1.7	1.8	2	1.9	1.9	1.8	1.5	1.4
{4.2.1}	1.9	2	1.8	2.2	2.1	1.8	2.1	1.8	2	2	1.9	2	2.1	1.7	1.7	2.1	1.9	1.9	1.6	1.8	2.2	2.2	2	1.9
{4.2.2}	1.1	1.2	1.1	1.3	1.3	1.1	1.3	1.1	1.2	1.2	1.1	1.2	1.2	1	1	1.3	1.1	1.2	1	1.1	1.3	1.3	1.2	1.2
{4.2.3}	0.9	1	0.9	1.1	1	0.9	1.1	0.9	1	1	0.9	1	1	0.8	0.8	1.1	1	1	0.8	0.9	1.1	1.1	1	1
{4.3.1}	0.2	0.4	0	0.4	0.4	0.1	0.2	0.5	0.4	0.4	0	0.1	0.2	0.3	0.6	0.6	0.6	0.6	0.4	0.5	0.5	0.1	0.6	0.4
{4.3.2}	0.3	0.5	0	0.5	0.6	0.1	0.2	0.8	0.6	0.6	0	0.1	0.3	0.4	0.8	0.8	0.9	0.8	0.5	0.7	0.7	0.1	0.8	0.6
{4.3.3}	0.4	0.7	0.1	0.8	0.8	0.2	0.3	1.1	0.8	0.8	0	0.2	0.4	0.5	1.1	1.2	1.2	1.2	0.8	1	1	0.1	1.2	0.8

Appendix B - Nomenclature

Notation for Chapter 2

Abbreviations

SW	Social Welfare
AOP	Aggregator Optimization Problem
SWOP	SW Optimization Problem
FROP	Fair Redistribution Optimization Problem
KKT	Karush-Kuhn-Tucker
RHS	Right Hand Side
LHS	Left Hand Side

Nomenclature

$\mathcal{D}$	Set of buyer agents
$\mathcal{S}$	Set of seller agents
N_b	Number of buyer agents, where $N_b = \lvert\mathcal{D}\rvert$
N_s	Number of seller agents, where $N_s = \lvert\mathcal{S}\rvert$
i	Index of a buyer, where $i \in \mathcal{D}$
j	Index of a seller, where $j \in \mathcal{S}$
u_i	Utility function of the i^{th} buyer
u_i'	Marginal utility of the i^{th} buyer
g_j	Generation capacity of the j^{th} seller
v_j	Utility function of the j^{th} seller
v_j'	Marginal utility of the j^{th} seller
d_i	Demand delivered to the i^{th} buyer
b_i	Buying price bid placed by the i^{th} buyer
c_i	Buying per unit price payed by the i^{th} buyer

a_j	Availability declared by j^{th} seller
s_j	Supply amount assigned to the j^{th} seller
c_j	Minimum per unit selling price by the j^{th} seller
p	The minimum per unit buying and maximum per unit selling price of energy
Θ	SWOP objective function
$\mathcal{L}_\Theta$	Lagrangian of SWOP
λ_i	Dual variable in $\mathcal{L}_\Theta$, corresponding to $pd_i \leq b_i$
α_j	Dual variable in $\mathcal{L}_\Theta$, corresponding to $s_j < a_j$
μ	Dual variable in $\mathcal{L}_\Theta$, corresponding to $\sum_{i\in\mathcal{D}} d_i = \sum_{j\in\mathcal{S}} s_j$
d_i^*	d_i at equilibrium as the efficient solution of SWOP
s_j^*	s_j at equilibrium as the efficient solution of SWOP
λ_i^*	λ_i at equilibrium in SWOP
α_j^*	α_j at equilibrium in SWOP
μ^*	μ at equilibrium in SWOP
Φ	AOP objective function
$\mathcal{L}_\Phi$	Lagrangian of AOP
γ_i	Dual variable in $\mathcal{L}_\Phi$, corresponding to $pd_i \leq b_i$
β_j	Dual variable in $\mathcal{L}_\Phi$, corresponding to $s_j < a_j$
ν	Dual variable in $\mathcal{L}_\Phi$, corresponding to $\sum_{i\in\mathcal{D}} d_i = \sum_{j\in\mathcal{S}} s_j$
$d_i^\dagger$	d_i at equilibrium as the solution of AOP
$s_j^\dagger$	s_j at equilibrium as the solution of AOP
$\gamma_i^\dagger$	γ_i at equilibrium in AOP
$\beta_j^\dagger$	β_j at equilibrium in AOP
$\nu^\dagger$	ν at equilibrium in AOP
ζ	Dual variable of constraint $s_j \geq 0$ in seller's problem
π_i	Buyer's payoff from the auction
π_j	Seller's payoff from the auction
π_{agg}	Aggregator's benefit

F	Fairness term function
η	Fairness term coefficient
$\mathcal{L}_{\mathrm{WF}}$	Lagrangian of FROP
s_j^r	The sellers' redistributed supply
S	Sum of the redistributed supply of all sellers
R	Total sellers' revenue
c_j^r	The sellers' redistributed selling price
β_j^r	Dual variable in $\mathcal{L}_{\mathrm{WF}}$ for $s_j^r \leq a_j$
ν^r	Dual variable in $\mathcal{L}_{\mathrm{WF}}$ for $\sum_{j \in \mathcal{S}} s_j^r = S$
K	Solution constant for FROP
κ_F	Price of fairness

Notation for Chapter 3

Abbreviations

SW	Social Welfare
PT	Price Taking
PA	Price Anticipation
RHS	Right Hand Side
LHS	Left Hand Side

Nomenclature

Θ	Network model representation
$\mathcal{D}$	Set of buyer agents
$\mathcal{S}$	Set of seller agents
N_B	Number of buyer agents, where $N_B = \lvert D \rvert$
N_S	Number of seller agents, where $N_S = \lvert \mathcal{S} \rvert$
i	Index of a buyer, where $i \in \mathcal{D}$
j	Index of a seller, where $j \in \mathcal{S}$
k	Index of iteration
u_i	Utility function of the i^{th} buyer
u_i'	Marginal utility of the i^{th} buyer
g_j	Generation capacity of the j^{th} seller
v_j	Utility function of the j^{th} seller
v_j'	Marginal Utility of the j^{th} seller
$\mathcal{A}$	A set representing the aggregator parameters
d_i	Demand delivered to the i^{th} buyer
b_i	Buying price bid placed by the i^{th} buyer
b_0	Buying price bid placed by the virtual agent
a_j	Availability declared by j^{th} seller
a_0	Availability declared by the virtual agent

p	The per unit market price of energy
p_s	Per unit surcharge price by the aggregator
a_j°	Solution to seller j's problem for availability corresponding to price p
β_i	Market power of the i^{th} buyer
α_j	Market power of the j^{th} seller
π_i	Equivalent utility function of the i^{th} buyer under price anticipation
π_j	Equivalent utility function of the j^{th} seller under price anticipation
Π	SW maximization problem objective function under price anticipation
$d_i^\dagger$	SW maximizing solution under price anticipation for the i^{th} buyer
$a_j^\dagger$	SW maximizing solution under price anticipation for the j^{th} seller
$U^\dagger$	SW attained under price anticipation
d_i^*	Efficient (price taking) solution for the i^{th} buyer
a_j^*	Efficient (price taking) solution for the j^{th} seller
U^*	Maximum attainable SW corresponding to the efficient solution.
p^*	The market price under equilibrium
p_s^{OPT}	Optimal per unit surcharge price by the aggregator
D	Aggregate demand function
A	Aggregate availability function
L_Θ	Loss in efficiency
R	Aggregator revenue function
Ω	SW maximization objective function with aggregator surcharge p_s
B	Sum of bids of all the buyers.
$\mathcal{L}_S$	Lagrangian corresponding to the seller's optimization problem
ρ_j	Dual variable in $\mathcal{L}_S$ corresponding to the constraint $a_j < g_j$ in SOP
$\mathcal{L}_\Pi$	Lagrangian of SW maximization problem under PA
$\mathcal{L}_{\text{U}}$	Lagrangian of SW maximization problem under PT
$\mathcal{L}_\Omega$	Lagrangian of SW maximization problem under PT with surcharge
λ_j	Dual variable in $\mathcal{L}_\Pi$, $\mathcal{L}_{\text{U}}$ and $\mathcal{L}_\Omega$ corresponding to constraint $a_j < g_j$
μ	Dual variable in $\mathcal{L}_\Pi$,$\mathcal{L}_{\text{U}}$,and $\mathcal{L}_\Omega$ corresponding to constraint $\sum_j a_j = \sum_i d_i$

Notation for Chapter 4

Abbreviations

DSO	Distribution System Operator
RES	Renewable Energy Resources
ALA	Aggregator Level Auction
DLA	DSO Level Auction
PU	Per Unit
DSWOP	DSO SW Optimization Problem
LM	Lagrange Multiplier
RHS	Right Hand Side
LHS	Left Hand Side

Nomenclature

$\mathcal{N}$	Set of nodes, excluding root
N	Number of distribution nodes, where $N = \lvert\mathcal{N}\rvert$
$\mathcal{A}$	Set of nodes with aggregators
A	Number of aggregators, where $A = \lvert\mathcal{A}\rvert$
δ	Maximum allowable PU voltage deviation
P_0	PU active power from root, i.e. substation
Q_0	PU reactive power from root
V_0	PU voltage at root
k, l	The k^{th}, l^{th} nodes $k, l \in \mathcal{N}$
$\mathcal{D}(k)$	Set of downstream nodes of node k
$\boldsymbol{u}(k)$	Index of immediate upstream node of node k
$\mathcal{U}(k)$	Index of all upstream nodes of node k, $k \in \mathcal{U}(k)$
r_k	PU resistance of line $(\boldsymbol{u}(k), k)$
x_k	PU reactance of line $(\boldsymbol{u}(k), k)$
p_k	PU active power injected into node k

q_k	PU reactive power injected into node k
P_k	PU active power flowing through line $(\boldsymbol{u}(k), k)$
Q_k	PU reactive power flowing through line$(\boldsymbol{u}(k), k)$
ΔV_k	PU voltage drop through line$(\boldsymbol{u}(k), k)$
V_k	PU voltage at node k
c_k	The per unit price of node k when $k \in \mathcal{A}$
$\mathcal{N}_B^k$	Set of buyers at node k, $k \in \mathcal{A}$
$\mathcal{N}_S^k$	Set of sellers at node k
i	The i^{th} buyer
j	The j^{th} seller
g_j^k	The maximum supply in KW-hr from the j^{th} seller
d_i^k	The demand in KW-hr delivered to the i^{th} buyer
s_j^k	The supply availability in KW-hr by the j^{th} seller
u_i^k	Utility of buyer agent i
v_j^k	Utility of seller agent j
$u_i'^k$	Marginal utility of buyer agent i
$v_j'^k$	Marginal utility of seller agent j
$\mathbf{d}^k$	Vector of d_i^k of the k^{th} aggregator of size $\mathcal{N}_B^k \times 1$
$\mathbf{s}^k$	Vector of s_j^k of the k^{th} aggregator of size $\mathcal{N}_S^k \times 1$
$\mathbf{u}^k$	Vector of u_i^k of the k^{th} aggregator of size $\mathcal{N}_B^k \times 1$
$\mathbf{v}^k$	Vector of v_j^k of the k^{th} aggregator of size $\mathcal{N}_S^k \times 1$
$\mathbf{c}$	Vector of c_k of size $A \times 1$
$\mathbf{p}$	Vector of p_k of size $A \times 1$
$\mathbf{q}$	Vector of q_k of size $A \times 1$
$\boldsymbol{\theta}$	Vector of θ of size $A \times 1$
$\boldsymbol{P}$	Vector of P_k of size $N \times 1$
$\boldsymbol{Q}$	Vector of Q_k of size $N \times 1$
$\boldsymbol{V}$	Vector of V_k of size $N \times 1$
$\boldsymbol{\delta}$	Vector of δ of size $N \times 1$

$\underline{\mathbf{l}}$	Vector of lower limits of size $N \times 1$
$\bar{\mathbf{l}}$	Vector of lower limits of size $N \times 1$
$\overline{S}_k$	Vector of line MVA limits of size $N \times 1$
$\Delta \boldsymbol{V}$	Vector of ΔV of size $N \times 1$
$\mathbf{r}$	Vector of r_k of size $N \times 1$
$\mathbf{x}$	Vector of x_k of size $N \times 1$
$\mathbf{A}$	Node-Aggregator Incidence matrix of size $N \times A$
$\mathbf{D}$	Node-Node Descendant matrix of size $N \times N$
$\mathbf{U}$	Node-Node Ancestor matrix of size $N \times N$
$\mathbf{M}$	Node Voltage - Aggregator power injection sensitivity matrix of size $N \times A$
$\mathbf{M}_P$	Dummy matrix for P of size $N \times A$
$\mathbf{M}_Q$	Dummy matrix for Q of size $N \times A$
$\mathbf{E}_k$	Matrix of size $N \times N$ with 1 at the (k, k) location
$\mathbf{Z}_k$	Symmetric matrix
$\mathit{\Omega}$	DSWOP objective function
$\mathfrak{L}$	DSWOP Lagrangian function
$\mathcal{L}$	Decomposed Lagrangian function for master problem
L	Decomposed Lagrangian function for sub-problem
α	DSWOP LM for constraint $\mathbf{1}_A^{\mathrm{T}}\mathbf{p} = P_0$
β	DSWOP LM for constraint $\boldsymbol{\theta}^{\mathrm{T}}\mathbf{p} \leq Q_0$
$\underline{\boldsymbol{\zeta}}$	DSWOP vector of LMs for constraint $\mathbf{Mp} \geq \underline{\mathbf{l}}$
$\overline{\boldsymbol{\zeta}}$	DSWOP vector of LMs for constraint $\mathbf{Mp} \leq \bar{\mathbf{l}}$
$\overline{\boldsymbol{\xi}}$	DSWOP vector of LMs for constraint $\mathbf{p}^{\mathrm{T}}\mathbf{Z}_k\mathbf{p} \leq \overline{\mathrm{S}}_k^2$
$\boldsymbol{\lambda}$	DSWOP vector of LMs for constraint $\mathbf{p} = \left[\mathbf{1}_{N_B^k}^{\mathrm{T}}\mathbf{d}^k - \mathbf{1}_{N_S^k}^{\mathrm{T}}\mathbf{s}^k\right]_{k \in \mathcal{A}}$
γ_k	DSWOP LM for constraint $\mathbf{s}^k \leq \mathbf{g}^k$

Notation for Chapter 5

Abbreviations

DSO	Distribution System Operator
BSS	Battery Storage System
MILP	Mixed Integer Linear Programming
LMP	Locational Marginal Pricing
DLMP	Distribution-LMP
DAM	Day-ahead Market
WSM	Wholesale Market
ISO	Independent System Operator
DG	Distributed Generation
RES	Renewable Energy Resources
RHS	Right Hand Side
LHS	Left Hand Side

Nomenclature

m	Distribution bus index
g	Generators index
b	BSS index
l	Load index
q	Generation segments index
r	Load segments index
t	Timeslot index
$\mathcal{M}$	Set of distribution buses
$\mathcal{G}$	Set of generator units
$\mathcal{B}$	Set of battery storages
$\mathcal{L}$	Set of price responsive loads
$\mathcal{T}$	Set of time slots

f	Objective function indicating SW
ϕ	Deviation from commitment penalty function
px	Segment generation
p	Generation output
e	BSS energy output
s	Supply from WSM
dx	Segment load
d	Total demand of load
c	BSS state of charge
i	Commitment state of generator units
j	Commitment state of BSS units
y	Startup indicator of generator units
z	Shut down indicator of generator units
sd	Shut down counter for generator
su	Start-up counter for generator
u	BSS charging indicator
v	BSS discharging indicator
cc	Charging counter for BSS
dc	Discharging counter for BSS
S	Total fixed supply allocated by the ISO to DSO
$\mathrm{PX}^{\max}$	Maximum segment generation in each segment
$\mathrm{P}^{\min}$	Minimum generation of a generator unit
$\mathrm{P}^{\max}$	Maximum generation of a generator unit
CX	Segment generation cost of a generator unit
CB	Selling cost of BSS energy
CG	Selling cost of Generator energy
CL	Buying cost of load
STC	Start-up cost of a generator unit
SDC	Shut down cost of a generator unit
RU	Rump up rate of a generator unit

RD	Rump down rate a generator unit
MDTG	Minimum down time of a generator unit
MUTG	Minimum up time of a generator unit
MDTB	Minimum discharge time of a BSS
MCTB	Minimum charge time of a BSS
E^{min}	Minimum BSS energy withdraw amount
E^{max}	Maximum BSS energy withdraw amount
C^{min}	Minimum state of charge of BSS
C^{max}	Maximum state of charge of BSS