THE RELATIONSHIP BETWEEN PRIVATE ECONOMIC GROWTH AND PUBLIC NONMILITARY INFRASTRUCTURE CAPITAL STOCK: AN EMPIRICAL STUDY OF THE U.S. ECONOMY

by

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AN ABSTRACT OF A DISSERTATION

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Abstract

This dissertation has focused primarily on the relationship between aggregate private output and a measure of the public fixed capital stock for the U.S. economy using two different approaches for the years 1947-2005. The study starts with a brief survey of the existing literature on the relationship between private output and public capital and continues with an analysis of data on some macroeconomic variables related to private output and public capital. It employs a production function approach to provide empirical estimates and analyze its econometric problems, and continues with a vector autoregression (VAR) model. It uses two criteria, the Akaike Information Criterion and the Schwartz Bayesian Criterion, to compare the performance of the two models tested.

There are several differences between this study and the existing literature. The most important difference is that each of the other studies uses only a single approach to analyze the relationship between the public capital stock and private economic growth while this study uses two different methodologies to analyze the same relationship and tests the two models using the same aggregate macroeconomic annual data on the U.S. economy from 1947 to 2005. This study represents the first attempt to provide estimates of the elasticities of private output with respect to the private capital stock, private labor stock, public nonmilitary capital stock, and public core infrastructure capital stock by employing two different approaches so that the comparison of the elasticities resulting from the two different approaches can be most meaningful. Moreover, this study also represents the first attempt to provide estimates of the marginal products of the above four inputs. Second, the studies that employ a production function approach are *ad hoc* and so is the production function approach of this study, but the production function approach section of this study is the only one having an explicit capital evolution equation for both the private and the public capital stock. All of the other studies using annual data use aggregate macroeconomic data on related variables for less than thirty years while this study employs aggregate data from 1947 to 2005 (fifty nine years). Lastly, the other production function studies are incomplete in the sense that they either do not attempt to deal with some major econometric problems such as a common trend (resulting in a spurious correlation) and the direction of the causation or when they do

acknowledge major econometric problems, they do not do anything to correct them. This study, on the other hand, will try to detect major econometric problems. Once the problem is detected, the study will employ measures to deal with the problem.

Major findings of this study are as follows. First, the causation runs from the public fixed capital stock to private output rather than in the other direction. Second, most of the studies in the existing literature report a positive impact of the private fixed capital stock on private output that is too small to be credible, whereas they report a positive impact of the public fixed capital stock on private output that is too large to be credible. However, the estimates of this study suggest not only a positive impact of the public capital stock on private output that seems credible but also a positive and very large impact of the private capital stock on private output. Third, the results of several joint hypothesis tests conducted show that there is enough sample evidence to claim that not only that the private sector operates under constant returns to scale in all inputs, private and public, for the years 1947-2005 but also that the private fixed capital stock is more important to the aggregate private production process than either of the two measures of the public fixed capital stock.

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Major findings of this study are as follows. First, the causation runs from the public fixed capital stock to private output rather than in the other direction. Second, most of the studies in the existing literature report a positive impact of the private fixed capital stock on private output that is too small to be credible, whereas they report a positive impact of the public fixed capital stock on private output that is too large to be credible. However, the estimates of this study suggest not only a positive impact of the public capital stock on private output that seems credible but also a positive and very large impact of the private capital stock on private output. Third, the results of several joint hypothesis tests conducted show that there is enough sample evidence to claim that not only that the private sector operates under constant returns to scale in all inputs, private and public, for the years 1947-2005 but also that the private fixed capital stock is more important to the aggregate private production process than either of the two measures of the public fixed capital stock.

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I. Introduction

Since the seminal work of Solow (1956) and Swan (1956), macroeconomists have tried to understand the determinants of economic growth for both developed and developing countries. After the slowdown in the growth rate of both labor productivity and multifactor productivity (MFP) of both the U.S. economy and the economies of many other developed countries in the 1970s and 1980s, some macroeconomists attributed the slowdown to a large decrease in per capita research and development expenditures, a big increase in energy prices, high costs of social regulations, inefficient uses of the private capital stock, the changing composition of the work force, or other related phenomena. The behavior of the public capital stock was seldom considered as one of the potential factors in explaining the productivity slowdown even though macroeconomists knew that the public infrastructure capital stock was one of the important inputs in the production process of the aggregate output.

In a series of papers in published 1989, Aschauer first noted the empirical fact in the U.S. economy (and also in some other developed economies) that public infrastructure investment decreased in the late 1960s and early 1970s and the trend growth rate of both the labor productivity and MFP decreased a little later, in about 1973. He then tried to exploit that fact by concluding that the slowdown in the economic growth of the U.S. economy was mostly due to the decrease in the public infrastructure investments. His papers (1989a, 1989b, 1989c, 1990a, 1990b) received an unusual amount of attention not only from macroeconomists but also from some policy makers, and the empirical relationship between output and public infrastructure capital stock became one of the favorite topics for econometric research in the early 1990s. In less than a decade after his papers appeared in the literature, at least 50 other studies tried to explain the productivity slowdown using a similar method with different data and research techniques.

The public *fixed* capital stock of an economy can be defined as large physical capital-intensive infrastructure such as streets, highways, mass transit and airports, power and communication systems, and water and sewer lines, and the rules, regulations, and the institutions that create a physical environment in which the whole economy can efficiently operate.[1] Since creating such an environment requires very large capital-intensive natural monopolies, most of the infrastructure capital stock here in the U.S. and elsewhere is publicly owned. Also, since the main objective of the public infrastructure capital stock is to create a suitable environment in which the whole economy can efficiently operate, the public infrastructure capital stock is one of the primary determinants of the extent to which households and firms make the long-term investments in physical capital, skills, and technology to achieve long-run economic objectives.

By creating a suitable environment in which the economy can efficiently operate, the public infrastructure capital stock increases the productivity of the private sector and hence lowers per unit costs and increases aggregate private output. In addition to this direct effect, the public capital stock can indirectly affect the aggregate private output by changing either private fixed investment or interest rates. The indirect effects are discussed in Akkina & Celebi (2002) and Cain (1997). The magnitude of the direct and indirect effects of the public infrastructure capital stock on aggregate private output depends also on whether public capital is financed by borrowing from the public, by taxes, or by a reduction in other public expenditure categories.

The main objective of this dissertation is to analyze the relationship between the private sector output and the public infrastructure capital stock. The plan of the dissertation is as follows. A brief survey of the existing literature on the relationship between private output and public capital is given in Section II. Section III provides an analysis of data on some macroeconomic variables related to private output and public capital of the U.S. economy. Two methodologies, a production function approach and a vector autoregression (VAR) model, will be used to analyze the relationship between private sector output and the public infrastructure capital stock throughout the dissertation. Section IV discusses a production function approach together with its empirical estimates and econometric problems to analyze the relationship between the private sector output and the public infrastructure capital stock. A VAR model is employed in Section V to explain the relationship among related macroeconomic variables, including the relationship between the private sector output and the public infrastructure capital stock. A comparison of the two approaches tested is provided in Section VI. Concluding remarks are made in Section VII.

There are several differences between this study and the existing literature. The most important difference is that each of the other studies uses only a single approach to analyze the relationship between the public capital stock and private economic growth while this study uses two different methodologies— a production function approach and a VAR model— to analyze this relationship and test the two models using the same aggregate macroeconomic annual data on the U.S. economy. This study represents the first attempt to provide estimates of the elasticities of private output with respect to the private capital stock, private labor stock, public nonmilitary capital stock, and public core infrastructure capital stock by employing two different approaches so that the comparison of the elasticities resulting from the two different approaches can be most meaningful. The same data on the U.S. economy from 1947 to 2005 are used in both approaches. Moreover, this study also represents the first attempt to provide estimates of the marginal products of the above four inputs by employing two different approaches so that the comparison of the marginal products of the four inputs resulting from the two different approaches can be most meaningful.

The studies employing a production function approach are *ad hoc* and so is the production function model approach of this study, but the production function approach section of this study is the only one having an explicit capital evolution equation for both the private capital stock and the public capital stock. All of the other studies using annual data use aggregate macroeconomic data on related variables for less than thirty years, while this study employs aggregate data from 1947 to 2005 (fifty nine years), a substantially longer time period. Lastly, the other production function studies are incomplete in the sense that they either do not attempt to deal with some major econometric problems such as a common trend (resulting in a spurious correlation) and the direction of the causation between the dependent variable and some of the independent variables or when they do acknowledge major econometric problems, they do not do anything to correct them. This study, on the other hand, will try to detect major econometric problems. Once the problem is detected, the study will employ measures to deal with the problem.

II. A Brief Survey of the Existing Literature on the Relationship between Private Output and Public Infrastructure Capital Stock

All studies modeling the relationship between the public infrastructure capital stock or public nonmilitary infrastructure capital stock and some measures of the private sector aggregate output after Aschauer's aforementioned papers can be classified into six groups. We will review each in turn.

The first approach is called *the production function approach*. Most of the macroeconomists following this approach employ a generalized Cobb-Douglas production function with either constant returns to scale in private inputs or across all inputs. They assume Hicks-neutral technological progress and estimate the aggregate production function in one of the following two forms.

(1) $\ln Y_t = \ln A_t + a\ln K_t + b\ln L_t + c\ln G_t$ or

(2) $\ln(Y_t / K_t) = \ln A_t + b\ln(L_t / K_t) + c\ln(G_t / K_t)$,

where Y_t is a measure of the real aggregate private sector output in time t, A_t is multifactor productivity, K_t is the aggregate private sector fixed capital stock used to produce Y_t, L_t is the private sector labor force used to produce Y_t, G_t is the public sector non-military fixed capital stock, a, b, and c are parameters to be estimated, where ln stands for the natural logarithm and the sub index t indicates time. Aschauer (1989a), together with some others, replaces $\ln A_t$ with a constant and a trend variable and adds the manufacturing sector's capacity utilization rate to the model as a proxy representing the aggregate capacity utilization rate of the private sector to control for the influence of the business cycle. Since the models are estimated in double log forms, the estimated coefficients are the elasticities of real aggregate private sector output with respect to the corresponding variables. For example, the estimate of c, denoted by $\hat{\mathbf{c}}$, is the elasticity of real private output with respect to the public fixed capital stock, indicating the percentage change in private output when the public capital stock increases by one percent.

While most of the studies use a Cobb-Douglas specification, a few studies use a more general translog function. Table 1 summarizes some of the studies using the production function approach. As can be seen from the table, $\hat{\mathbf{c}}$ varies between -0.1 and

0.7 and is found to be statistically insignificant in several studies. The large variation in the $\hat{c}$ across studies indicates a lack of robustness of these estimates. Some of the higher estimates imply a very large positive effect of public capital on private output, an effect that may not be credible. For example, Aschauer's first estimate (1989a) of 0.39 means a 1 percent increase in the public nonmilitary infrastructure capital stock results in a 0.39 percent increase in real private sector output. Given the size of the public capital stock and private output, this figure implies that the marginal product of public capital is 60 percent, that is, a 1 dollar increase in public capital results in a 0.6 dollars increase in private output. For the same study, the marginal product of private capital is 30 percent.

Table 1: Production Function Estimates of the Elasticity of Private Output with respect to Public Capital (*)

Study	Scope	Specification	Data	$\hat{c}$(**)
Ratner (1983)	National	Cobb-Douglas; log level	Time series, 1949-73	0.06
Costa et al. (1987)	48 states	Translog; level	Cross-section, 1972	0.19-0.26
Aschauer (1989a)	National	Cobb-Douglas; log level	Time series, 1949-85	0.39
Ram & Ramsey (1989)	National	Cobb-Douglas; log level	Time series, 1949-85	0.24
Aschauer (1989c)	G-7	Cobb-Douglas; delta log	Panel data, 1966-85	0.34-0.73
Merriman (1990)	48 states	Translog; level	Cross-section, 1972	0.20
Munnell (1990a)	National	Cobb-Douglas; log level	Time series, 1949-87	0.31-0.39
Aaron (1990)	National	C-D; log level, delta log	Time series, 1952-85	not robust
Munnell & Cook (1990)	48 states	Cobb-Douglas; log level	Pooled cross-sec, 1970-86	0.15
Aschauer (1990b)	50 states	Cobb-Douglas; log level	C-sect. averaged, 1965-83	0.055-0.11
Eisner (1991)	48 states	C-D & translog, log level	Pooled cross-sec,1970-86	0.17
Ford & Poret (1991)	National	Cobb-Douglas; delta log	Time series, 1957-89	0.39-0.54
Tatom (1991)	National	Cobb-Douglas; delta log	Time series, 1949-89	insignificant
Hulten&Schwab (1991)	National	C-D; log level, delta log	Time series, 1949-85	insignificant
Garcia-Mila et al. (1992)	48 states	Cobb-Douglas; log level	Panel data, 1969-82	0.04-0.05
Holtz-Eakin (1992)	48 states	Cobb-Douglas; log level	Panel data, 1969-86	insignificant
Munnell (1993)	48 states	Cobb-Douglas; log level	Pooled cross-sec,1970-86	0.14-0.17
Finn (1993)	National	Cobb-Douglas; delta log	Time series, 1950-89	0.16
Eisner (1994)	National	Cobb-Douglas; log level	Time series, 1961-91	0.27
Pinnoi (1994)	48 states	Translog; level	Panel data, 1970-86	-0.11-0.08
Baltagi & Pinnoi (1995)	48 states	Cobb-Douglas; log level	Panel data, 1970-86	insignificant
Sturm & DeHaan (1995)	National	C-D; log level, delta log	Time series, 1949-85	0.41, insigni
Everaert&Heylen (2004)	Belgian	Translog; level	Panel data, 1965-96	0.31
Kamps (2004a)	22 OECD	Cobb-Douglas, log level	Panel data, 1960-2001	0.22

(*): Table 1 is obtained from Table 14.1 of Sturm et al. (1998, pp.384-86) and Table A2 of Romp & de Haan (2005, pp.60-1).
(**): $\hat{c}$: Output elasticity of public capital.

Aschauer (1990a, p. 16), looking at his parameter estimates, concludes that "*increases in GNP resulting from increased public infrastructure spending are estimated to exceed those from private investment by a factor of between two and five.*" It is difficult to believe that public capital is more productive at the margin than private capital in the private production process.

There are several advantages as well as disadvantages of the production function methodology. The advantages are as follows. It is easy to estimate the model and the model is based on the most widely used production function, an aggregate Cobb-Douglas production function. It is flexible in the sense that any variable that is believed to influence private output can be added to the model. Because of this flexibility, Aschauer (1989a) included the manufacturing sector's capacity utilization rate as a proxy representing the total capacity utilization rate of the private sector. Tatom (1991) included not only the manufacturing sector's capacity utilization rate multiplicatively but also energy prices. These two modifications have been heavily criticized by others.

The disadvantages are as follows. First, the wide range of the parameter estimates makes the estimates less credible. Second, most of the models either produce statistically inconsistent estimates or implausibly high marginal products of public capital. Third, all models assume that the causation runs from the public capital stock to private output. What if a causal influence runs in the other direction? The majority of the models do not even check to see whether the principal direction of causation might run from private output to public capital investment. Fourth, data could be non-stationary or private output and the public capital stock could be integrated of order-one processes, yielding spurious correlations. Fifth, by using an aggregate Cobb-Douglas production function, this approach imposes a unitary elasticity of substitution between private inputs, which cannot necessarily be reconciled with the facts.

To eliminate various problems with the production function approach, some studies employ *the cost function approach* by assuming that private firms either minimize their total cost or maximize their total profit subject to certain constraints. For example, Cohen & Paul (2004), Conrad & Seitz (1992 and 1994), Lynde & Richmond (1992 and 1993a), Moreno et al. (2003), and Nadiri & Mamuneas (1994a and 1994b) assume that private firms try to minimize their cost (C) subject to a given level of output (Y_t) by using exogenously determined input prices (p_t^i) and level of technology (A_t) and by choosing the quantity of inputs (q_t^i) with the assumption that the use of the services of public capital stock (G_t) is free for the private firms as follows.

(3) $\quad$ Minimize $C(p_t^i, q_t^i, A_t, G_t) = p_t^1 q_t^1 + \ldots + p_t^n q_t^n \qquad \text{s.t.} \qquad Y_t = f(q_t^i, A_t, G_t)$

Some other studies such as Deno (1988) and Lynde & Richmond (1993b) assume that private firms try to maximize their profits (π) subject to a given production function (that is, the way to combine the inputs to produce the maximum amount of output) by choosing the amounts of private inputs and output (Y_t) by using exogenously determined input prices (p_t^i), the price of output (p_t^Y) and level of technology (A_t) as follows.

(4) $\quad$ Maximize $\pi\,(p_t^Y, p_t^i, q_t^i, A_t, G_t) = p_t^Y\, Y_t - (p_t^1 q_t^1 + \ldots + p_t^n q_t^n)$ s.t. $Y_t = f(q_t^i, A_t, G_t)$

Given the cost or profit function and certain regularity conditions, one can derive a unique production function by applying duality theory. The dual function satisfying either Equation (3) or Equation (4) can be approximated by a second-order Taylor approximation like the transcendental logarithmic (translog) or the generalized Leontief function. If a translog form is used, then the first order conditions result in input share equations; and if a generalized Leontief form is used, then the first order conditions result in input demand equations. The first order conditions and the cost or profit function can be estimated separately by the OLS estimation method or together by a system estimator to increase efficiency. In addition to estimating several elasticities, the cost function approach can estimate the shadow value of public capital as a proxy for its unknown market price to determine whether there is insufficient or excessive public capital.

When private firms optimize, they take into consideration two variables, the level of technology and the services of public capital stock. All of the studies mentioned with the exception of the studies done by Nadiri and Mamuneas (1994a and1994b) approximate the level of technology by a time trend, while Nadiri and Mamuneas use publicly financed R&D expenditures. All of the studies assume that private firms do not pay for the services of the public infrastructure capital stock; in other words, they assume that the services of public capital stock enter into the private production process as unpaid factors of production. Other studies such as Conrad & Seitz (1994), Deno (1988), and

Nadiri and Mamuneas (1994b) adjust the public capital stock by the capacity utilization rate by multiplying the public capital variable by the manufacturing sector's capacity utilization rate to reflect its usage by the private sector.

The research conducted by Nadiri and Mamuneas (1994a and 1994b) are unique in the sense that they disaggregate the public sector capital stock into two parts, infrastructure capital stock and publicly financed R&D investment, and they estimate the effects of the public sector capital stock on private productivity at a much more disaggregated industry level using data on twelve two–digit U.S. manufacturing industries. They treat the two types of public capital as unpaid factors in the private production process and jointly estimate the cost and input share equations to determine the effects of not only the public infrastructure capital stock but also publicly financed R&D expenditures on the cost structure of the private sector.

To compare the estimated results of this approach with those of the production function approach, they either apply Shephard's Lemma to the minimized cost function or Hotelling's Lemma to the maximized profit function to obtain the firm's supply functions. For example, the derivative of the optimized profit function with respect to the output price gives the firm's net supply function and the derivative of the optimized profit function with respect to the *i*-th input price gives the negative of the firm's demand for the *i*-th input, which can then be used to calculate output elasticities for public capital.

The estimated results for a sample of studies using the cost function approach are provided in Table 2. The results show that increases in the public infrastructure capital stock either decreases private sector costs or increases private sector profits and thus positively affects private sector real output. Notably, the estimated effects are substantially smaller than the ones reported by Aschauer and the other production function approach studies that yield very high output elasticities of public capital.

The cost function approach employs a very flexible functional form and hence imposes minimal restrictions on the production structure, while the production function approach imposes unitary elasticity of substitution between private inputs. The flexible functional form comes with a significant disadvantage, however, in that it requires a considerable amount of information. For example, if the production process requires *n*

Table 2: A Sample of Cost/Profit Function Approach Studies (*)

Study	Data	Specification	Public capital variables	Conclusion
Cohen & Morrison Paul (2004)	USA, states 1982-96	cost: generalized Leontief	highway capital constructed using perpetual inventory met	infrastructure investment reduces own costs and increases cost-reducing effect of adjacent states
Conrad & Seitz (1992)	Germany, panel, 4 sectors, 1961-88	cost: translog	core infrastructure	cost elasticities: from -0.22 to -0.07 largest inpact in trade $ tranport sector
Conrad & Seitz (1994)	Germany, panel, 3 sectors, 1961-88	cost: translog	use (CU * G) to measure flow of G's services	increase in G reduces costs, productivity slowdown is partially a result of slowdown of the growth of G
Deno (1988)	USA, panel, 36 SMSA's, 1970-78	profit: translog	core or total, (G* m), m: % of population employed in manuf.	output elasticity: 0.69 G more effective in declining regions
Lynde & Richmond (1992)	USA, time series, 1958-89, nonfinancial corporate sector	cost: translog	total, federal, state and local	G has a positive marginal product, but statistical analysis indicates problems in interpreting the estimates
Lynde & Richmond (1993a)	U.K., time series, 1966:I-90:II, Manufacturing	cost: translog	total	output elasticity: 0.20. Higher G in the 1980s could have increased labor productivity by 0.5 percent per year
Lynde & Richmond (1993b)	USA, time series, 1958-89, nonfinancial corporate sect.	profit: translog	total	output elasticity: 0.20. Public capital per private labor can explain 40 percent of productivity slowdown
Moreno et al. (2003)	Spain, regions and sectors, 1980-91	cost: translog	infrastructure	public and private investments both increase efficiency
Morrison & Schwartz (1996a)	USA, panel, 6 New England States: CT, ME, NH, VT, MA, RI manufacturing	cost: generalized Leontief	motorways, water, and sewers	private capital is more valuable for society than public capital, and public investment is warranted if public policy is ineffective at increasing private investment
Nadiri & Mamuneas (1994a)	USA, panel, 12 manufacturing sectors	cost: generalized Cobb-Douglas	total and R&D use (CU * G)	cost elasticities: from -.0.21 to -0.11 G and R&D are not major contributors to MFP but the contribution of G to MFP is twice as large as that of R&D
Nadiri & Mamuneas (1994b)	USA, panel, 12 manufacturing Sectors	cost: generalized Cobb-Douglas	total and R&D; use of (CU * G) does not change results	cost elasticities: from -.0.21 to -0.10 private capital and public R&D both have higher rates of return than G

(*): Table 2 is obtained from Table 14.2 of Sturm et al. (1998, pp.387-94) and Table A3 of Romp & de Haan (2005, p.62).

private inputs, then in addition to the constant term and the coefficients of some possible dummy variables, $2n+ n (n-1)/2$ coefficients need to be estimated. The high number of coefficients to be estimated substantially decreases the degrees of freedom. The cost function approach also gives any indirect effect of public capital on private output. For example, if public capital and some of the private inputs are substitutes, then firms can adjust their demand for those inputs under the cost function approach. But several time series problems such as multicollinearity between second order terms which are cross-products of the inputs and nonstationarity of time series still remain.

The third approach employs some form of the vector autoregression (VAR) model or vector error correction model (VECM). Some studies such as McMillin & Smyth (1994), Otto & Voss (1996), and Pereira et al. (1999 and 2003) employ a VAR model while some others such as Clarida (1993), Everaert (2003) and Kamps (2004b) use a VECM. The studies using *the VAR model approach* usually have a system of four simultaneous equations whose dependent variables are private output, private capital stock, private labor, and public capital stock and whose independent variables are the

Table 3: A Sample of VAR/VECM Studies Measuring the Effects of Public Capital Stock on Related Macroeconomic Variables (*)

Study	Data	Model	Variables (**)	Conclusions
Clarida (1993)	USA, France, Germany, UK	VECM	MFP, G	MFP and public capital are cointegrated but direction of causality is unclear
McMillin & Smyth (1994)	USA, 1952-90	VAR, levels & 1st differences	H/K, P^E/P^Y, G/K, inflation	No significant effect of public capital
Otto & Voss (1996)	Australia, 1959:III-92:II	VAR	Y,K,G,H	No relationship between public capital and labor or output. Private capital affects public capital positively
Pereira et al. (1999)	USA, 1956-89	VAR, 1st differences log levels	Y,K,G,L	Public capital is productive but a lot less than suggested by Aschauer (1989a)
Pereira et al. (2003)	Spain (regional and national)	VAR, 1st differences log levels	Y,K,G,L	Positive and significant long-run effects on output, employment, and private capital
Everaert (2003)	Belgian regions, 1953-96	VECM	Y,K,G	Output elasticity of public capital = 0.14 Output elasticity of private capital= 0.4
Kamps (2004b)	22 OECD countries, 1960-2001	VECM	Y,K,G,L	For most of the countries, there is a positive and significant effect on growth

(*): Table 3 is obtained from Table 14.3 of Sturm et al. (1998, p.395) and Table A4 of Romp & de Haan (2005, pp.63-4).
(**): MFP: multifactor productivity; G: public capital stock; K: private capital; L: private labor; P^E/P^Y: relative price of energy; H/K: hours of work per unit of private capital; Y: private sector GDP.

lagged values of all dependent variables. If required, deterministic variables such as a constant or a trend are also included. The most notable advantage of the VAR model is that it does not impose any identifying conditions from economic theory nor *a priori* causality direction. One important difficulty with the VAR model approach is that since it does not reveal the underlying production process, it is not easy to obtain estimates of the output elasticity for public capital. One can obtain those estimates via the impulse response functions, and they provide estimates of the long run effects of different exogenous shocks.

Table 3 above summarizes some of the studies using the VAR model. As can be seen from the table, public capital positively affects private output but again the estimated effects are substantially smaller than those reported by Aschauer and the other production function studies.

As mentioned at the beginning of this study, Aschauer first observed an empirical fact about the U.S. economy that public nonmilitary infrastructure investments decreased in the late 1960s and early 1970s and that both labor productivity growth and MFP growth decreased somewhat later in about 1973. He then attempted to exploit that fact by concluding that the slowdown in the economic growth of the U.S. economy was mostly due to the decrease in the public nonmilitary infrastructure investments. After observing the data, he found very high correlations between private output and the public capital stock and was convinced that the causation ran from the public capital stock to private output rather than the other way around. He then explicitly claimed that the public capital stock in the U.S. was and had been below the optimal level, suggesting that public policy makers needed to increase the level of the public infrastructure capital stock not only because of the statistically significant and quantitatively very high positive effects of public capital on private output but also because of the fact that the level of the public capital was and had been below the optimal level.

Gramlich (1994, p. 1181) discusses four ways of determining whether a shortage of public infrastructure capital stock exists, of which one is termed "*engineering assessments of infrastructure needs*." Following Gramlich, I refer to the fourth group of the studies on the relationship between private output and public capital *the engineering needs assessment approach*.

Gramlich (1994, p. 1181) refers to several "engineering needs assessment" studies[2] conducted by several public agencies. He criticizes the studies by saying that they arbitrarily chose an initial period and assumed that the actual capital stock was equal to the desired level in that period. Then they measured the desired investment based on the condition of and need for capital facilities. The total shortage over time then was obtained by calculating the cumulative sum of the differences between the desired level of investment and the actual level of investment every year. He also claims that the studies lacked economic reasoning in that they assumed fixed proportions during the entire time period and therefore did not allow for any adjustment for either excessive or underutilized initial capital.

Gramlich provides a detailed discussion of the above studies involving highways and streets. As can be seen from Tables A6, A7, and A8 in Appendix B of this dissertation, highways and streets were the largest and the most volatile component of the public infrastructure capital stock from 1925 to 2005. Using the U.S. Congressional Budget Office estimates, Gramlich then concludes that there was a small highway infrastructure gap in the early 1980s but by 1987 there was almost no spending gap at all. He also refers to the needs assessments for water and sewer systems and concludes that the shortage of the public infrastructure capital stock was small in the 1980s. Therefore, even though the engineering needs assessment approach studies indicate that the actual level of the public infrastructure capital stock was below the optimum level in the 1980s, the shortage apparently was considerably smaller than the one suggested by Aschauer.

The fifth approach is called *cross-country regression models* based on endogenous growth models. Unlike the previous approaches that employ mainly a time-series framework, the fifth approach relating private output to public capital uses a cross-country regression. Endogenous growth models assign a central role to the capital formation; in other words, the key component of endogenous growth models is capital formation containing not only physical capital but also human capital, infrastructure capital, and knowledge capital. In a cross-country regression model study such as Barro (1989, 1991, and 2001), Easterly & Rebelo (1993), Lucas (1988), Mankiw et al. (1992), Rebelo (1991), and Romer (1986, 1989, and 1990), the growth model of a measure of private output reduces to a single equation around the steady state as follows.

(5) $\Delta \ln (Y/L)_{0,T} = a + b(Y/L)_0 + c(K^G/Y)_{0,T}$ + (a set of conditional variables),

where $(Y/L)_{0,T}$ is the average output per unit of labor over a time interval [0, T], a catch-up variable $(Y/L)_0$ is the initial level of real output per unit of labor, and $(K^G/Y)_{0,T}$ is the average rate of real public capital as the percentage of GDP over the time interval [0, T]. The set of conditional variables here contains measures of changing composition of human capital such as averages of primary and/or secondary enrolments, measures of political stability such as political assassinations, coups, and war casualties, measures of economic freedom, or government consumption as a percentage of GDP.

When the model is empirically estimated, K^G is replaced with I^G, public investment, because data on public capital are either difficult to obtain or are not available in some countries, especially in most of the developing countries. Tables 14.4 and 14.5 of Sturm et al. (1998, pp. 395-96) and Table A5 of Romp & de Ha an (2005, p. 64) provide the estimated results for several cross-country regression models. The estimated coefficients of public investment or public capital (or the marginal productivity of public capital) are negative, ambiguous, or insignificant or positive and statistically significant. When they are positive and statistically significant, their magnitudes are considerably smaller than the ones suggested by either Aschauer or the other production function studies.

Cross-country regression models have been criticized on several grounds such as omitted variables, reverse causation, “arbitrary” sample selections and measurement errors. Moreover, since these models are single-equation models, they imply only one endogenous variable while economic theory might indicate more than one endogenous variable in the system.

The last group of studies relating private output to public capital employs *structural models*. Here, a set of reduced-form equations that result from a small macroeconomic model of the economy is simultaneously estimated. The main difference between this approach and the VAR approach is that the structural models employ appropriate theories of economics while the VAR model is a statistical approach not derived from economic theory. In a structural model, the economy is described by a set of simultaneous equations, each of which is grounded in economic theory. From the set of the equations of the macroeconomic model, a set of reduced form equations is derived, that is, one equation for each endogenous variable relating the endogenous variable to all exogenous variables in the system. The set of reduced form equations is then estimated.

Sturm et al. (1998, pp. 377-79) discuss the estimated results of several structural models. The estimated effects of public capital on private output are either unrealistically high (for example, the marginal product of public investment in the long run was found to be 3.7 in one of the studies)[3] or positive but very small.

This section can be summarized as follows. Most of the studies discussed so far have found a statistically significant and positive relationship between public infrastructure capital and private economic growth, while very few studies have found either a negative or positive and statistically insignificant relationship between the two. Therefore, we can convincingly accept that the public capital stock affects private output in a positive and statistically significant way. However, even though there is no consensus in the literature about the magnitude of the positive effect of public capital on private output, most researchers believe that Aschauer's findings appreciably overestimate the true magnitude. Moreover, most of the studies on the U.S. economy confirm that there has been a shortage of public infrastructure capital in the United States, but the shortage is substantially smaller than the one suggested by Aschauer.

III. An Analysis of Data on some Macroeconomic Variables related to Private Real Output and Public Nonmilitary Infrastructure Capital Stock of the U.S. Economy

We believe that we are unlikely to achieve good and sustainable future economic growth without knowing not only the determinants of economic growth in the past and present but also how robust the economic growth has been. Hence, it is very important to analyze both the historical determinants of the growth rate of the U.S. economy and how good the economic growth has been so that we might do something to make it even better in the future. Therefore, the main purpose of this section is to empirically analyze the data on private economic growth and public infrastructure development in the United States, together with the data on some other related macroeconomic variables to determine what, if anything, can be done to achieve strong and sustainable economic growth in the future.

Discussions of not only the infrastructure development but also well-written histories of the U.S. economy can be found in Abramovitz and David (2000), Brownlee (2000), Cain (1997), Denison (1962, 1974, and 1985), Engerman and Gallman (eds., 2000), Field (2003), Gordon (1999), Jorgenson (1990), Jorgenson and Griliches (1967), Kendrick (1961), Seely (1993), Stafford (1999), and Temin (2000).

No invention has ever had as large an effect on the development and economic growth of the U.S. economy as have railroads. Since the impact of the railroads on the nineteenth-century U.S. economic growth was so big, many researchers such as the ones mentioned in Cain (1997) and Field (2003) used only the railroads to explain the spectacular economic development of the U.S. economy from 1830 until the first quarter of the twentieth century.

The United States began operating its first railroad, the Baltimore and Ohio, in 1830 and experienced railroad mileage exceeding that of railroads in France, the German States, and the United Kingdom combined by the Civil War. By 1860, more than $1 billion was invested in railroads while only little more than $200 million was invested in canals. By the end of the third quarter of the nineteenth century, the nation was tied together by railroads and canals, but the primary explanation of the spectacular economic growth was the construction of the railroads.[4]

By the end of the nineteenth century, the United States had become a large-industrialized common market as a result of large spending on infrastructure development. Due to large investments in railroads and canals, transportation costs decreased substantially. Therefore, the private sector was able to exploit economies of scale to lower overall production costs because of the enlarged common market. According to Cain (1997, p.132), wagon rates varied between 30 and 70 cents per ton-mile by the early 1820s. For example, the rate on the Erie Canal in 1817 was 19.1 cents per ton-mile. However, railroad freight rates decreased to 2 cents per ton-mile and passenger rates decreased to less than 1 cent per mile by the end of the nineteenth century. Large investments in railroad and canal infrastructures resulted in large decreases in transportation costs, which in turn resulted in a large increase in private manufacturing production. For example, private manufacturing production increased from 5 percent of total production in 1799 to nearly 50 percent of total production by 1899.[5]

The private railroad sector continued to dominate the United States economy throughout the nineteenth century until the late 1930s in a way that no single sector has done either before or since. In 1941, for example, private fixed capital in railroads was 26.9 percent of the total private nonresidential fixed assets.[6] Starting at the beginning of the twentieth century, the role previously played by railroad and canal construction was gradually taken over by large public investments in core infrastructure capital stock including education, health care, highways and streets, airports, power and transportation systems, and water and sewer lines. By the early 1970s, The United States had almost completed the core public infrastructure system that it currently has. As will be discussed in detail below, the all-time high economic growth rate of the United States economy from 1933 to 1973 was a direct result of both large investments on public core infrastructure systems and more productive inventions of the late nineteenth and early twentieth centuries.

Until 1994, an aggregate Hicks-Neutral Cobb-Douglas production function of the form

(6) $Y_t = A_t K_t^a L_t^{(1-a)}$,

where Y_t is some measure of real aggregate output in time t and K_t and L_t are the real capital and labor stocks, respectively, used to produce that output, was used to calculate the annual growth rate of multifactor productivity (MFP) as follows.

$$(7) \qquad g_t^A = g_t^Y - a\, g_t^K - (1\text{-}a)\, g_t^L,$$

where g_t stands for the annual growth rate of the corresponding variable between the time periods t-1 and t and where a is the elasticity of private output with respect to the private capital used to produce that output, i.e., when private capital increases by one percent, private output increases by an a percent. For the U.S. economy, the estimates of a have been between 0.3 and 0.4, and the most commonly used value of a is 0.32.

After the pioneering works of Denison (1962, 1974, and 1985), Griliches (1960), Jorgenson (1990), and Jorgenson and Griliches (1967), the U.S. Bureau of Labor Statistics (BLS) adopted the Jorgenson framework, described in Jorgenson and Griliches (1967) and Jorgenson (1990), suggesting changing composition adjustments in both labor and capital inputs in 1994 for all BLS publications on MFP growth over the period since 1948. The BLS now first considers the composition of capital and labor inputs and then adjusts the amounts according to their changing compositions. The BLS, for example, adjusts labor hours by considering the changing educational and demographic characteristics of the labor force, and then uses the composition-adjusted quantities to find the corresponding annual growth rates.

Before analyzing the recent data on private economic growth and public infrastructure capital, we emphasize an historical fact that has not properly been appreciated in the economic literature until recently. Following Aschauer, the economics literature has treated the economic slowdown of the years 1973-1995 as a deviation from the historical trend and singled this period out for explanation. In other words, the economic literature has assumed that the growth rates observed until 1973 were normal ones and thus needed no explanation. As can be seen from the tables below, the growth rates of GDP, private GDP, labor productivity and MFP before 1933 and after 1973 are very similar and relatively small in size compared to the relatively high growth rates of GDP, private GDP, labor productivity and MFP between 1933 and 1973. Therefore, unlike the researchers following Aschauer we believe that the time period over which the low growth observed does not need much explanation but the 40-year time period over which high growth observed needs special attention.

We claim that in addition to the Great Depression and World War II, the more productive nature of the inventions of the late nineteenth and early twentieth centuries can explain the relatively high growth rate of the 40-year time period. As well said by Gordon (1999, p.127), *"the inventions of the late nineteenth century and early twentieth century were more fundamental creators of productivity than the electronic/internet era of today."*

The earlier inventions can be classified into six groups as follows. *Public infrastructure* including water, sewer, highways, and airports; *electricity* including electric motors, electric lights, and consumer appliances; *rearranging molecules* including petrochemicals, plastics, and pharmaceuticals; *internal-combustion engines* including all motor vehicles providing highway, railway, and airway transportations; *new means of exchange and trade* including stock markets and other financial markets, supermarkets, and all other big grocery and convenient retail stores; and *communications and entertainment systems* including telephones, radios, televisions, and movies. The six groups of early inventions are believed to have had a larger overall positive effect on productivity growth than today's VCRs, DVDs, or internet. Although DVDs and VCRs, for example, can combine television and movies, neither DVDs nor VCRs are capable of producing the magnitude of the effects of either televisions or movies on productivity growth.

After emphasizing the fact that the U.S. economy experienced significantly higher growth rates in all of the real macroeconomic variables over the years 1933-1973 than either before 1933 or after 1973, we therefore need mostly to explain the years 1933-73. Moreover, some recent studies such as Bernstein, Mamuneas, and Pashardes (2004), Jorgenson (2001), and Litan and Rivlin (2001) claim that the U.S. economy started to experience a productivity speed-up in 1995 as a result of significant technological progress in information technologies in particular and in internet and internet-related industries in general. They claim that improved information technologies and significant developments in internet-related industries have lowered per unit cost in almost every industry and therefore increased multifactor productivity everywhere since they have been widely used everywhere, "*leading to a* ***permanent*** *improvement in growth prospects*" (Jorgenson (2001), p.1, quotation from Alan Greenspan) of the U.S. economy. They claim that the new era starting in 1995 seems to be very similar to the years 1933-1973.

It is important to emphasize that after examining the data carefully we believe that even though the U.S. economy has experienced a productivity speed-up since 1995, there is little evidence to conclude that the new era will be similar to the years 1933-73 since the growth rates of real variables over the years 1995-2005 are smaller than the growth rates of the corresponding variables over the years 1933-1973. Furthermore, productivity decelerated in the past two years (2005:1-2007:1) to a growth rate of about 1.5 per cent per year. We therefore claim that after considering the last 200 years of the U.S. economy it is reasonable to conclude that the growth rates of the real variables over the years 1933-73 are significantly larger than those before 1933 and after 1973.

Having discussed important stylized facts about the U.S economy, we proceed to empirically analyze the data of the U.S. economy on private economic growth, public infrastructure development, and related variables in detail. The methodology that we use to calculate the annual growth rate of a discrete variable throughout this dissertation together with the methodology that we use to divide a given time interval among several mutually exclusive sub intervals is explained in Appendix A. The definitions of the variables that we use throughout this study, the sources of the raw macroeconomic data, and the raw data are provided in Appendix B.

As can be seen from Table A1 in Appendix B and Table 4 below, The United States nominal gross domestic product (nominal GDP) grew from $104 current-billion in 1929 to $12,456 current-billion in 2005, an annual growth rate of 6.5 percent. In current-year prices, the U.S. GDP increased by a factor of 120 in 76 years. Over the same time period, nominal GDP per capita increased from $851 to $42,022 (a factor of 49), an annual growth rate of 5.3 percent; nominal private GDP increased from $88 billion to $9,447 billion (a factor of 107), an annual growth rate of 6.3 percent; and nominal nonresidential private GDP increased from $84 billion to $8,676 billion (a factor of 103), an annual growth rate of 6.3 percent. From 1948 to 2005, nominal GDP per labor hour increased from \$2.2 to \$48.5 (a factor of 22), an annual growth rate of 5.6 percent over 57 years.

Considering the years 1929-2005, in terms of output, employment, labor productivity, and MFP, the worst time interval was from 1929 to 1933 over which GDP, GDP per capita, private GDP, and nonresidential private GDP decreased by 14.1%, 14.8%, 15.8%, and 15.1% per year, respectively. On the other hand, the most technologically progressive years of the U.S. economy were 1933-1944, over which all measures of output grew fastest compared to any other time interval during the last two centuries. For example, nominal GDP, nominal GDP per capita, private GDP, and nonresidential private GDP grew by 13.2%, 12.2%, 12.7%, and 12.8% per year, respectively. These findings are in agreement with the findings of the other tables in this section and in Appendix B, implying that the Great Depression and the Second World War had a positive effect on any measure of output, labor productivity, and, in turn, MFP.[7]

Table 4: Annual Growth Rates of Nominal GDP, Private GDP, and Private Nonresidential GDP

Year	Nominal GDP			Nominal Private GDP	Nominal Nonresidential Private GDP
	Levels	per capita	per labor hour		
1929-1933	-14.10	-14.76	--	-15.75	-15.06
1933-1944	13.16	12.17	--	12.73	12.78
1944-1949	3.99	2.44	--	5.82	4.55
1949-1973	7.09	5.53	5.66	6.77	6.74
1973-2005	7.11	5.99	5.66	7.07	7.04
1929-1941	1.69	0.92	--	1.58	1.64
1933-1973	8.33	6.92	--	8.25	8.09
1948-1973	6.76	5.20	5.49	6.40	6.39
1973-2005	7.11	5.99	5.66	7.07	7.04
1948-2005	6.96	5.65	5.59	6.78	6.75
1929-2005	6.50	5.27	--	6.34	6.29

Several studies such as Abramovitz & David (2000), Cain (1997), Denison (1974), Field (2003), Gordon (1999), Kendrick (1961), and Seely (1993) have shown that the large investments in railroads and canals were the primary reasons for the development of the U.S. economy during the nineteenth and early twentieth centuries. The primary reasons for relatively high output growth, labor productivity growth, and for MFP growth of the U.S. economy during the years 1905-1973 were large public infrastructure investments and new discoveries adopted or implemented by both the private sector and the government. In addition to those primary reasons, the Great Depression and the Second World War were the other primary reasons for the all-time high growth rates of the golden years 1933-1944.

During the Depression years, output and employment collapsed in every sector but they increased substantially in the research and development (R&D) sector. For example, Field (2003, p.1406) refers to Mowery's study by observing that employment of research scientists and engineers increased by 72.9 percent from 1929 to 1933 and that R&D employment in the U.S. manufacturing sector almost tripled from 10,918 to 27,777 from 1933 to 1940. Firms invested heavily in R&D to survive, but as a result of a large decrease in households' income, aggregate demand fell and therefore new techniques and discoveries created during the years 1929-1933 remained either unexploited or only partially unexploited until 1933. Those new techniques created but insufficiently exploited in the early 1930s were being used after 1933 and hence contributed positively and substantially to all measures of output, labor productivity, and MFP throughout the 1930s and the 1940s.

As can be seen from Cain (1997), Field (2003), Gordon (1999), and Seely (1993), there is now an emerging consensus among researchers that the years 1905-1973 witnessed growth rates in all measures of output, labor productivity, and MFP that are higher than those of the years both before 1905 and after 1973. This empirical fact can also be clearly seen from the tables of both this section and Appendix B and is definitely true for the years 1933-1973. For example, nominal GDP, GDP per capita, private GDP, and nonresidential private GDP grew by 8.3%, 6.9%, 8.3%, and 8.1% per year from 1933 to 1973, respectively, each of which is larger than the corresponding growth rates of 7.1%, 6.0%, 7.1%, and 7.1% per year from 1973 to 2005. We verify this empirical fact using real variables below.

As can be seen from both Table A2 in Appendix B and Table 5 below, from 1929 to 2005 real GDP increased from $865 billion to $11,049 billion (a factor of 13), an annual growth rate of 3.4 percent in 76 years; real GDP per capita increased from $7,105 to $37,275 (a factor of 5), an annual growth rate of 2.2 percent; real private GDP increased from $575 billion to $8,600 billion (a factor of 15), an annual growth rate of 3.6 percent; real nonresidential GDP increased from $516 billion to $7,992 billion (a factor of 16), an annual growth rate of 3. 7 percent; and real GDP per labor hour

Table 5: Annual Growth Rates of Real GDP, Private GDP, and Private Nonresidential GDP

	Real GDP			Real	Real
Year	Levels	per capita	per labor hour	Private GDP	Nonresidential Private GDP
1871-1891 (*)	--	--	--	4.41	--
1891-1913 (*)	--	--	--	4.43	--
1913-1928 (*)	--	--	--	3.11	--
1929-1933	-7.42	-8.13	--	-9.43	-7.74
1933-1944	9.96	9.00	--	9.70	9.89
1944-1949	-1.98	-3.44	--	0.99	-0.70
1949-1973	4.15	2.64	2.76	4.22	4.22
1973-1995	2.84	1.84	1.24	3.12	3.30
1995-2005	3.24	2.00	2.34	3.73	3.60
1929-1941	2.84	2.06	--	2.58	2.85
1933-1944	**9.96**	**9.00**	**--**	**9.70**	**9.89**
1933-1973	**4.92**	**3.56**	**--**	**5.29**	**5.11**
1995-2005	**3.24**	**2.00**	**2.34**	**3.73**	**3.60**
1996-2005	3.19	1.92	2.31	3.63	3.52
1933-1973	4.92	3.56	--	5.29	5.11
1948-1973	3.96	2.44	2.72	4.00	4.03
1948-2005	3.40	2.13	2.07	3.61	3.67
1973-1995	2.84	1.84	1.24	3.12	3.30
1973-2005	2.96	1.89	1.57	3.31	3.39
1995-2005	3.24	2.00	2.34	3.73	3.60
1929-2005	3.41	2.20	--	3.62	3.67

(*): Taken from Gordon (1999, p.124).

increased from $13.4 in 1948 to $43.1 in 2005 (a factor of 3), an annual growth rate of 2.1 percent in 57 years. Both real GDP and real GDP per capita decreased over two sub intervals; they decreased by 7.4% and 8.1% per year from 1929 to 1933, respectively, and by 2.0% and 3.4% per year from 1944 to 1949.

The United States declared war on Japan and Germany in December of 1941 and real GDP increased by the all time high growth rate of 18.5% from 1941 to 1942, and the growth rates of all measures of output remained very high throughout the war years because of enormous increases in military expenditures. Total federal military expenditures reached $22.9 billion in 1942—an amount equal to 14.1% of the same year's total GDP—, $63.4 billion in 1943— an amount equal to 31.9% of the same year's GDP—, $76.0 billion in 1944— an amount equal to 34.6% of the same year's GDP—, and $80.5 billion in 1945— an amount equal to 36.1% of the same year's GDP. Active-duty military personnel reached its all time high of 12.1 million in 1945.[8] Starting in 1946, federal military spending decreased substantially and never returned to its pre-war share of the nation's GDP.

It is clear from the tables of this section and the tables in Appendix B that the United States experienced all time high growth rates of all measures of output and of related macroeconomic variables during the golden era years 1933-1944; for example, real GDP, real GDP per capita, real private GDP, and real nonresidential private GDP increased by 10.0 percent, 9.0 percent, 9.7 percent, and 9.9 percent per year, respectively, each of which was more than twice its corresponding value over any other sub interval either before 1933 or after1944.

Let us now examine the growth rate of real private GDP more closely since we have data since 1871. As can be seen from Table 5 above, real private GDP increased by 4.4 percent per year from 1871 to 1891, by 4.4 percent per year from 1891 to 1913, by 3.1 percent per year from 1913 to 1928, but *decreased* by 9.4 percent per year from 1929 to 1933 and then *increased by 9.7 percent* per year from 1933 to 1944, by 1.0 percent per year from 1944 to 1949, by 4.2 percent per year from 1949 to 1973, by 3.1 percent per year from 1973 to 1995, and by 3.7 percent per year from 1995 to 2005. The years 1871-2005 were divided among nine sub intervals and the growth rate of real private GDP over one of the nine sub intervals, the golden era, of 9.7 percent per year was more than double the highest growth rate of any of the other eight sub intervals. Moreover, real private GDP increased by 5.3 percent per year from 1933 to 1973, an annual growth rate that was higher than that of any of the time intervals before 1905 and after 1973. This, therefore, supports the emerging consensus that the U.S. economy experienced unusual growth rates during the years 1905-1973. This experience requires further explanation. Furthermore, the annual growth rates of all measures of real output over the years 1933-1973 were greater than those over any other sub intervals either before 1933 or after 1973.

It is important to emphasize that the data on real output and on related productivity measures support the statement that early inventions described above were more beneficial to economic growth than the electronic/internet era of recent years. For example, although the annual growth rate of real private output of 3.3 percent over the years 1973-2005 was close to the growth rate of 4.4 percent over the years 1891-1913 or to the growth rate of 4.4 percent over the years 1871-1891, the former was smaller than the latter, implying that the early inventions were more beneficial than the recent ones.

Furthermore, as can be seen from Table 5 above and Tables 6 and 11 below, the U.S. economy has experienced a productivity speed-up since 1995, but contrary to Bernstein, Mamuneas, and Pashardes (2004), Jorgenson (2001), and Litan and Rivlin (2001) there is little evidence to conclude that the new era will be similar to the years 1933-73. Let us examine this claim further.

As can be seen from Tables 6 and 11 below, Jorgenson (2001) claims that in the early 1990s a large decrease in the prices of the output of the information technology industries resulted in a large, economy-wide expension of use of that output, which in turn resulted in a large increase in the growth rates of GDP, MFP, and labor productivity beginning in 1995. By referring to several recent studies, Litan and Rivlin (2001, p.314) claim that the internet has increased productivity growth by 0.2 to 0.4 percent per year. This has occurred via three channels: i) by significantly reducing the cost of production and the cost of distribution, ii) by increasing management efficiency, and iii) "*by increasing competition, making prices more transparent, and broadening markets for buyers and sellers, which puts pressure on suppliers to adopt techniques that translate* *into cost savings."* They claim that the internet will have a sustained favorable effect on productivity growth in the future.

We agree with the claim that improved and affordable information technologies and significant developments in internet and related industries have lowered per unit cost in almost every sector and therefore positively contributed to the growth rates of real GDP, MFP, and labor productivity in a significant way. This trend can be clearly seen from Tables 5, 6, and the others. We, on the other hand, do not agree with the claim that new era is comparable to the years 1933-1973. As can be clearly seen from Table 5, real GDP grew by 4.9 percent per year over the years 1933-1973, while it grew by 3.2 percent per year over the years 1995-2005. Real GDP per capita grew by 3.6 percent per year over the years 1933-1973, whereas it grew by 2.0 percent per year over the years 1995-2005; real private GDP increased by 5.3 percent per year over the years 1933-1973, while it increased by 3.7 percent per year over the years 1995-2005; and real nonresidential

Table 6: Breakdown of the Contributions to the Growth Rates of Real GDP, MFP, and Labor Productivity (*)

	1948-1999	1948-1973	1973-1990	1990-1995	1995-1999
(1): GDP growth rates	**3.46**	**3.99**	**2.86**	**2.36**	**4.08**
Contribution of IT to (1)	0.40 (12%)	0.20 (5%)	0.46 (16%)	0.57 (24%)	1.18 (29%)
Contribution of NIT to (1)	3.06 (88%)	3.79 (95%)	2.40 (84%)	1.79 (76%)	2.91 (71%)
(2): MFP growth rates	**0.61**	**0.92**	**0.25**	**0.24**	**0.75**
Contribution of IT to (2)	0.16 (26%)	0.06 (7%)	0.19 (76%)	0.25 (104%)	0.50 (67%)
Contribution of NIT to (2)	0.45 (74%)	0.86 (93%)	0.06 (24%)	-0.01 (-4%)	0.25 (33%)
(3): ALP growth rates	**2.09**	**2.82**	**1.26**	**1.19**	**2.11**
Contr. of capital deepening to (3)	1.13 (54%)	1.45 (51%)	0.79 (63%)	0.64 (53%)	1.24 (59%)
Contr. of labor quality to (3)	0.34 (16%)	0.46 (16%)	0.22 (17%)	0.32 (27%)	0.12 (6%)
Contribution of MFP to (3)	0.61 (29%)	0.92 (33%)	0.25 (20%)	0.24 (20%)	0.75 (35%)

(*): Obtained from Tables 6, 7, and 8 of Jorgenson (2001, pp. 19, 23, and 25).
GDP = gross domestic product, IT = information technology, NIT = non-information technology
MFP = multifactor productivity, and ALP = average labor productivity.

private GDP grew by 5.1 percent per year over the years 1933-1973, whereas it grew by 3.6 percent per year over the years 1995-2005. These findings are in agreement with the other tables in this study.

Table A3 in Appendix B provides the annual growth rates of real GDP, private sector real GDP, MFP, and related variables for the years 1929-2005, and Figure 1, Figure 2, and Figure 3 illustrate some of these. Since the growth rates calculated between subsequent years are very much subject to short run business cycle fluctuations, figures utilizing those growth rates might not reveal the true relationships among the variables to the visual examination because short run fluctuations can cause too much noise.

To eliminate the noise and to capture the true long run relationship among the variables of the interest, we calculated the growth rates compounded annually and referred the compound growth rates *the long run growth rates* given in Table 7, Table 9, and Table 11, and used some of them to draw the other figures of this section. Figures 4A, 4B, 4C, and 4D use Table 7, while Figures 5A, 5B, 5C, and 5D use Table 9. The main difference between Table 7 and Table 9 is the following. Table 7 contains the sub intervals taken from Table 1 of Gordon (1999, p.124) while Table 9 contains another possible set of sub intervals that can be obtained by examining the same data somewhat differently. It is important to remind the reader that the sub intervals of the two tables are obtained using the same approach, letting the data reveal sub intervals. The purpose of Table 9 is to show the reader not only that two researchers can choose different sub intervals even when they use the same approach —here letting the data reveal sub intervals— but also how sensitive the outcomes can be if one chooses a different set of sub intervals. In other words, no matter what one does, the choice of a set of sub intervals remains somewhat arbitrary.

Table 8 and Table 10 provide correlation coefficients between the variable of interest — here either the long run annual growth rate of public nonmilitary fixed capital stock or the long run annual growth rate of public core infrastructure capital stock— and the long run annual growth rates of MFP, labor productivity, real private GDP per labor hour, real private GDP, real GDP, and real GDP per capita. The correlation coefficients provided in Table 8 are calculated from the growth rates given in Table 7 while the correlation coefficients provided in Table 10 are calculated from the growth rates given in Table 9.

Figure 1: Growth Rates of Real GDP, Real GDP Per Capita, and Real Public Nonmilitary Fixed Capital

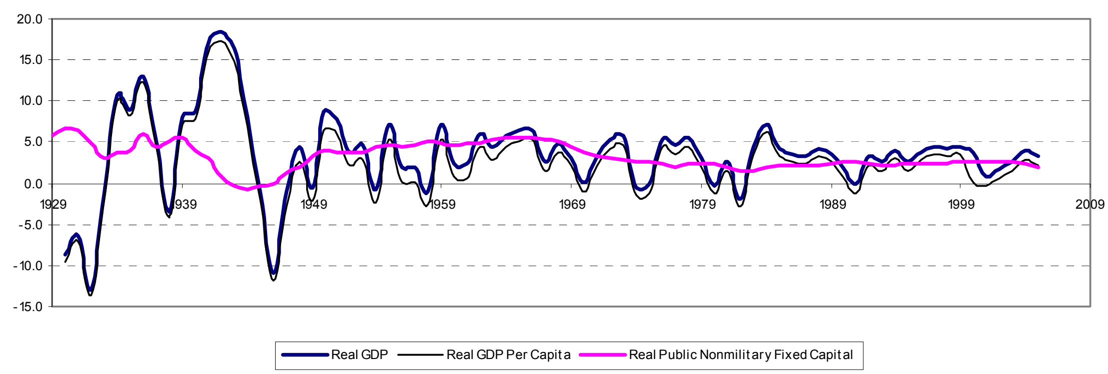


Figure 2: Growth Rates of Real Private GDP, Public Core Infrastructure Capital, and Multifactor Productivity

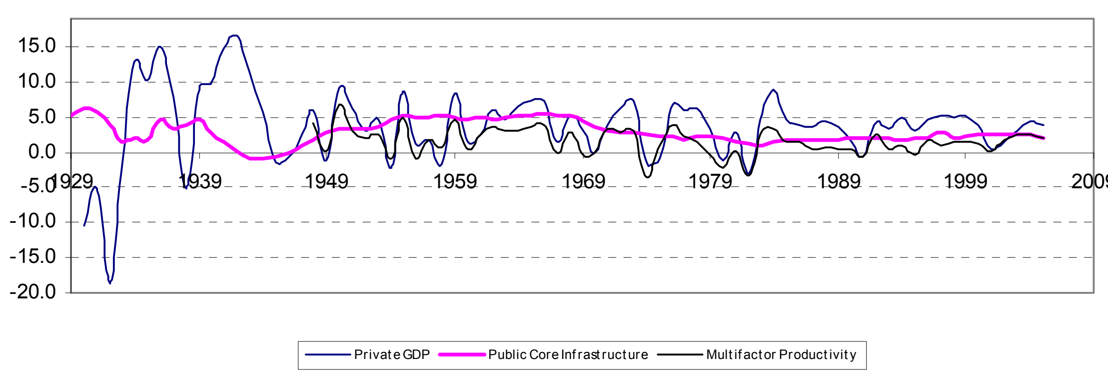
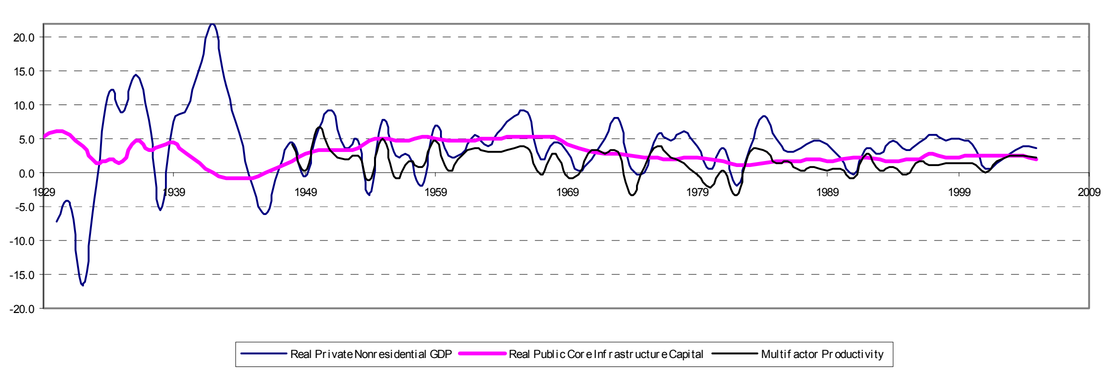


Figure 3: Growth Rates of Real Private Nonresidential GDP, Real Public Core Infrastructure Capital, and Multifactor Productivity



As can be seen from Table 8 and Table 10, the correlation coefficients between the long run annual growth rate of public core infrastructure capital and the long run annual growth rate of MFP, labor productivity, real private GDP per labor hour, and real private GDP are very high. For example, reading from Table 8 they are 0.88, 0.86, 0.83, and 0.69, respectively. They are even higher if one restricts the overall time interval to be the years 1929-1996. In this case, they are 0.94, 0.95, 0.98, and 0.69, respectively,

Table 7: Annual Growth Rates of Private Sector Productivity and Related Measures (*)

Time Interval	Real GDP			Public Sector		Real Private GDP		
	Levels	per capita	per labor hour	Non-military infra-structure capital	Core Infra-struc-ture capital	per labor hour	GDP	MFP
1929-1948	3.43	2.43	--	3.11	2.19	--	3.66	--
1948-1964	3.83	2.10	2.88	4.47	4.43	3.23	3.66	2.33
1964-1972	4.00	2.84	2.40	4.49	4.40	3.01	4.29	1.60
1972-1979	3.36	2.34	1.29	2.45	2.29	1.64	3.84	0.93
1979-1988	2.99	2.04	1.39	1.99	1.59	1.68	3.24	0.53
1988-1996	2.68	1.64	1.32	2.37	1.98	1.71	3.04	0.66
1996-2005	3.19	1.92	2.31	2.48	2.47	2.99	3.63	1.57

(*): The time intervals are taken from Gordon (1999, p.124).

over the restricted time interval. A correlation coefficient must lie inside the closed interval [-1,1], and if it is 1, then the correlation between the two variables is said to be *positive* and *perfect*. Hence, any values close to 1 represent high and positive correlations between the two variables and here we have many between 0.9 and 1.

Table 8: Correlation Coefficients between the Variable of Interest and the Others (*)

	Public nonmilitary fixed capital		Public core infrastructure capital	
	1929-2005	1929-1996	1929-2005	1929-1996
MFP	0.831	0.932	0.876	0.941
Real GDP per labor hour	0.823	0.953	0.861	0.948
Real Private GDP per labor hour	0.783	0.983	0.830	0.976
Real Private GDP	0.671	0.696	0.686	0.691
Real GDP	0.898	0.897	0.878	0.876
Real GDP per capita	0.591	0.560	0.516	0.509

(*): This table uses the long run annual growth rates given in Table 7.

Similar but somewhat weaker relationships exist between the long run annual growth rate of the public nonmilitary capital stock and the long run annual growth rates of MFP, labor productivity, real private GDP per labor hour, real private GDP, real GDP, and real GDP per capita. They are 0.83, 0.82, 0.78, and 0.67, respectively, over the unrestricted time interval and 0.93, 0.95, 0.98, and 0.70, respectively, over the restricted time interval.

As can be seen from Table 11 below, the most technologically progressive years of the U.S. economy were 1933-1944. Since the BLS' calculation of MFP using the Jorgenson framework goes back only to 1948, we could not obtain the growth rate of MFP from 1933 to 1973. We have it calculated from 1948 to 1973 which is 2.2 percent per year, that is, MFP grew by 2.2 percent per year from 1948 to 1973. It is important to remind the reader that the 2.2 percent annual growth rate is larger than that of any other time interval. Furthermore, as can be seen from the measures of output variables for which we have data for the years 1933-48, we would have had a bigger growth rate of MFP—a rate much bigger than 2.2 percent per year— for the years 1933-1973 if we had had data on MFP for the years 1933-1947. It is also clear that not only do the annual growth rates of MFP prior to 1929 and after 1973 exhibit similar patterns, but they also are much smaller than the ones between 1948 and 1973.

As stated above, Figures 4A, 4B, 4C, and 4D are obtained from Table 7. It is very clear from Figure 4A that there is a very high positive correlation not only between the long run annual growth rate of public core infrastructure capital stock and the long run annual growth rate of MFP, but also between that of public core infrastructure capital stock and those of real GDP per labor hour and of real private GDP per labor hour. The correlation is very high from 1929 until 1996, and gets weaker thereafter.

The only difference between Figure 4A and Figure 4B is that Figure 4A uses the long run annual growth rate of public core infrastructure capital stock while Figure 4B uses that of public nonmilitary fixed capital stock. Reading from Figure 4B, one can readily see the high and positive correlation between the long run annual growth rate of public nonmilitary fixed capital stock and the long run annual growth rates of MFP, real GDP per labor hour, and of private real GDP per labor hour, but the correlation is somewhat weaker than the one between the three variables and public core infrastructure capital stock.

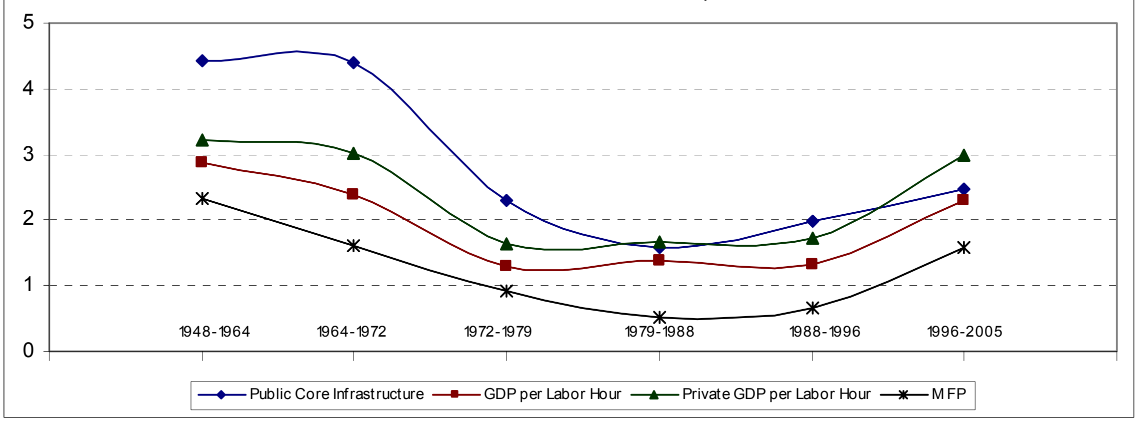


Figure 4A: Annual Growth Rates of Real GDP per Labor Hour, Private GDP per Labor Hour, MFP, and Public Core Infrastructure Capital

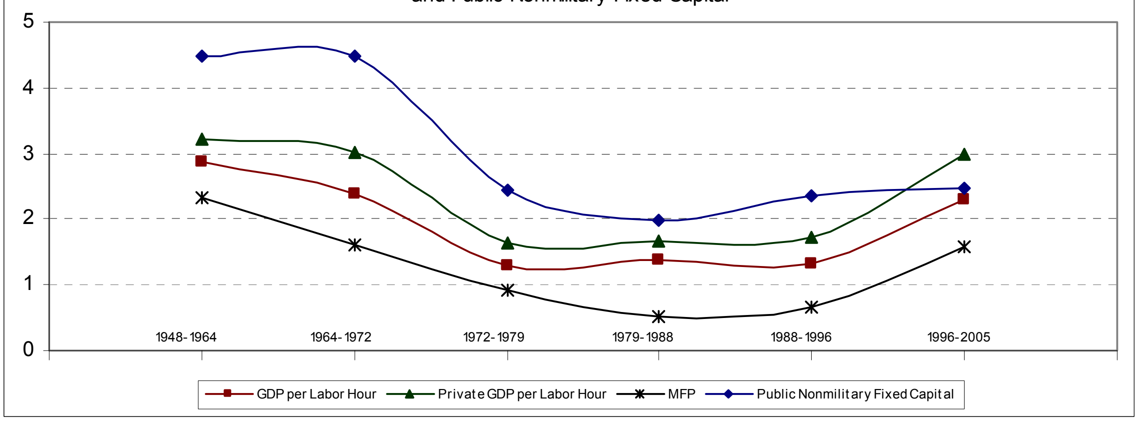


Figure 4B: Annual Growth Rates of Real GDP per Labor Hour, Private GDP per Labor Hour, MFP, and Public Nonmilitary Fixed Capital

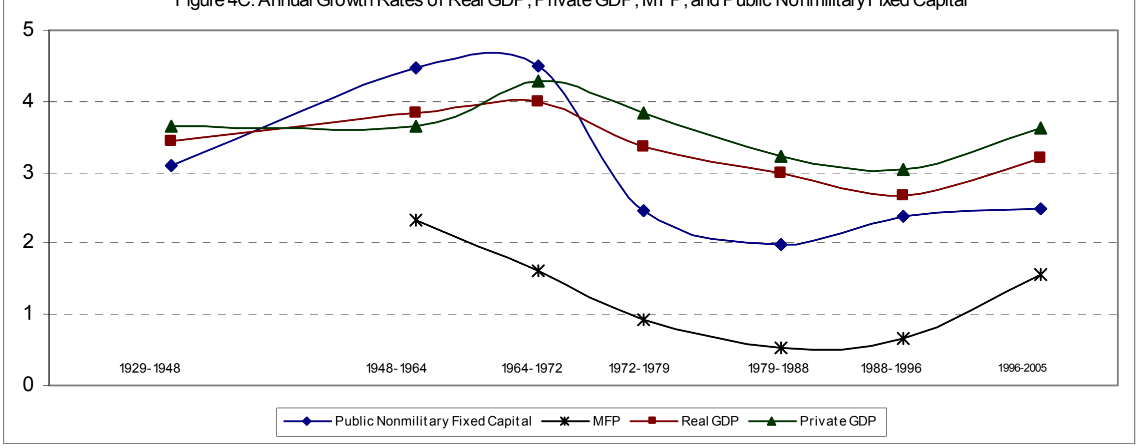


Figure 4C: Annual Growth Rates of Real GDP, Private GDP, MFP, and Public Nonmilitary Fixed Capital

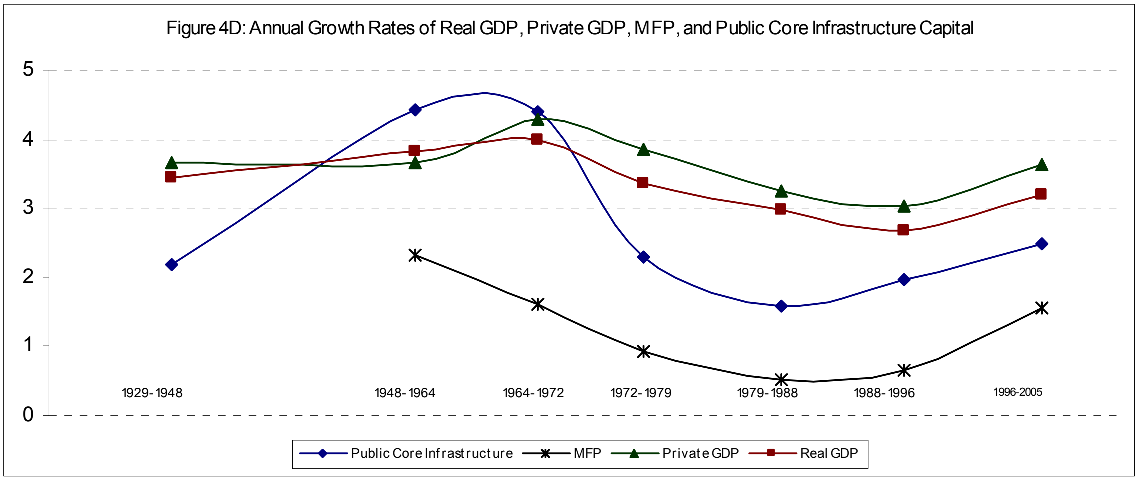


Figure 4C shows a positive and high correlation between the long run annual growth rate of public nonmilitary fixed capital stock and those of MFP, real GDP, and of private GDP from 1929 to 1996 and a lower correlation thereafter. Figure 4D exhibits a similar but somewhat stronger relationship between the long run annual growth rate of public core infrastructure capital stock and those of MFP, real GDP, and of real private GDP. It is very clear from both Figures 4A-4D and Table 8 that there is a very high correlation between the variable of interest and the selected variables, meaning that when the variable of interest goes up, the selected variables go up too and vice versa.

Figures 5A, 5B, 5C, and 5D are obtained from Table 9. Figure 5A has exactly the same variables as Figures 4A, Figure 5B has exactly the same variables as Figures 4B, Figure 5C has exactly the same variables as Figures 4C, and Figure 5D has exactly the same variables as Figures 4D. Since the time intervals of Table 7 capture the long run relationship between the variable of interest—here either the long run annual growth rate of public core infrastructure capital stock or the long run annual growth rate of public nonmilitary fixed capital stock— and the long run annual growth rates of MFP, labor productivity, and of private GDP per labor hour a little better than the time intervals of Table 9 do, Figures 4A, 4B, 4C, and 4D do a better job in capturing the long run relationship between the variable of interest and the selected three variables than Figures 5A, 5B, 5C, and 5D do. However, Figures 5A, 5B, 5C, and 5D are still showing high and positive correlations between the long run annual growth rate of the variable of interest and those of the three selected variables.

Table 9: Annual Growth Rates of Private Sector Productivity and Related Measures

Time Interval	Real GDP			Public Sector		Real Private GDP		
	Levels	per capita	per labor hour	Non-military infra-structure capital	Core Infra-struc-ture capital	per labor hour	GDP	MFP
1948-1964	3.83	2.10	2.88	4.47	4.43	3.23	3.66	2.33
1964-1973	4.20	3.06	2.39	4.30	4.22	3.01	4.61	1.76
1973-1982	2.00	1.00	0.94	2.22	2.02	1.07	2.03	-0.19
1982-1990	4.02	3.06	1.62	2.13	1.66	2.12	4.50	1.39
1990-2000	3.27	2.01	1.67	2.41	2.17	2.13	3.84	0.95
2000-2005	2.39	1.39	2.50	2.45	2.50	3.27	2.68	1.87

It is important to emphasize that the positive correlation coefficients between the long run annual growth rate of public nonmilitary fixed capital stock and the long run annual growth rates of real private GDP, real GDP, and real GDP per capita given by Table 10 are larger than the positive correlation between the long run annual growth rate of public core infrastructure capital stock and the long run annual growth rates of the same variables given again by Table 10, which is a contrary to the consensus among the researchers since public core infrastructure capital stock is expected to have a positive but quantitatively larger effect on real private GDP, real GDP, and on real GDP per capita than public nonmilitary fixed capital stock does.

Table 10: Correlation Coefficients between the Variable of Interest and the Others (*)

	Public nonmilitary fixed capital		Public core infrastructure capital	
	1948-2005	**1948-1990**	**1948-2005**	**1948-1990**
MFP	0.644	0.770	0.654	0.730
Real GDP per labor hour	0.744	0.910	0.781	0.889
Real Private GDP per labor hour	0.638	0.869	0.681	0.841
Real Private GDP	0.401	0.338	0.310	0.281
Real GDP	0.580	0.542	0.487	0.486
Real GDP per capita	0.374	0.308	0.277	0.245

(*): This table uses the data given in Table 9.

Figure 5A: Annual Grow th Rates of Real GDP per Labor Hour, Private GDP per Labor Hour, MFP, and Public Core Infrastructure Capital

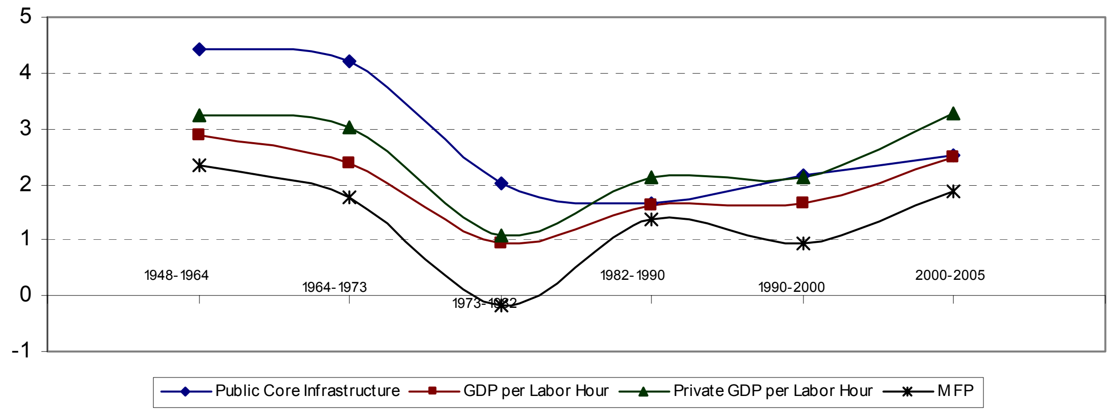


Figure 5B: Annual Grow th Rates of Real GDP per Labor Hour, Private GDP per Labor Hour, MFP, and Public Nonmilitary Fixed Capital

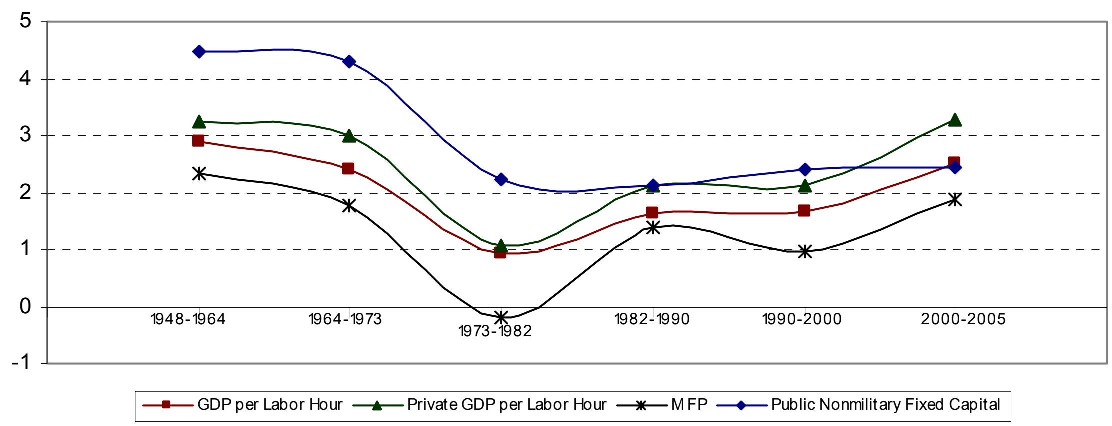
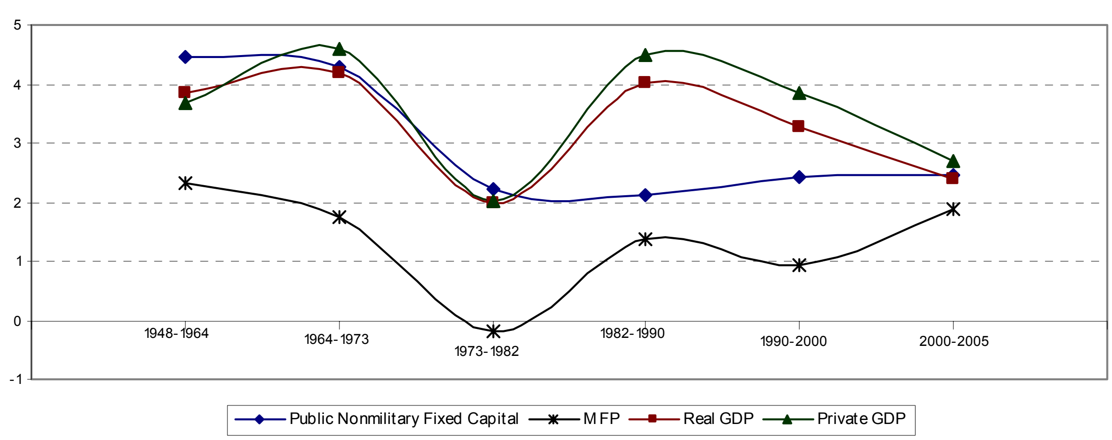
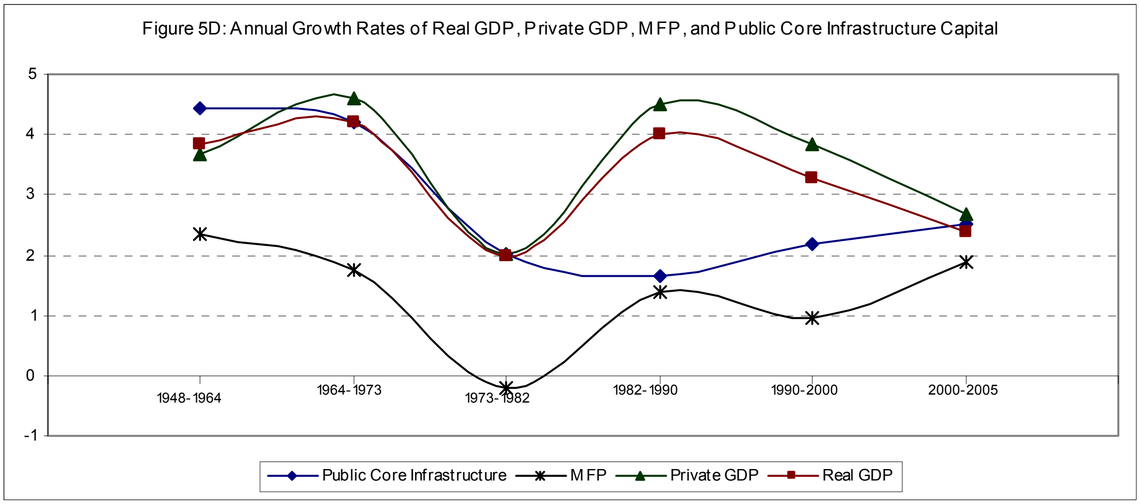


Figure 5C: Annual Growth Rates of Real GDP, Private GDP, MFP, and Public Nonmilitary Fixed Capital





We would like to emphasize one more issue before discussing the public capital stock. Field (2003, p.1399) advances the hypothesis that "*the years 1929-1941 were, in the aggregate, the most technologically progressive of any comparable period in U.S. economic history*" and discusses it throughout his paper. Our findings, on the other hand, suggest that the most productive years were 1933-1944, and we believe that Field could have ended up with our years if he had not chosen business-cycle peaks for both beginning and end points of his sub intervals.

Several researchers have shown that public infrastructure investments have both direct and indirect positive and quantitatively large effects on any measure of output, including any portion of output produced by the private sector.[9] Several researchers have also shown that both the private rate of return and the social rate of return on *initial* public infrastructure expenditures have been positive and very high, implying that the initial public spending on infrastructure development is warranted.[10] After the initial expenditures, diminishing returns commence, yet the rate of return still remains high[11] but not for a long time. This suggests that after the initial investment the profitability of public investment projects decreases sharply, making it less beneficial sequentially to undertake the subsequent projects. For example, Gramlich (1994, p.1184) refers to a 1988 Congressional Budget Office report on individual highway investment projects indicating that new urban construction projects were estimated to have a real rate of

Table 11: Annual Growth Rates of Private Sector Productivity and Related Measures

Time Interval	Real GDP			Private Sector		Public Sector		Real Private GDP			
	Levels	per capita	per labor hour	Labor input	Capital input	Non-military infra-structure capital	Core Infra-structure capital	per labor hour	per unit of capital	GDP	MFP
1871-1891 (*)	--	--	--	3.56	4.48	--	--	--	--	4.41	0.21
1891-1913 (*)	--	--	--	2.92	3.85	--	--	--	--	4.43	0.86
1913-1928 (*)	--	--	--	1.42	2.21	--	--	--	--	3.11	1.01
1929-1933	-7.42	-8.13	--	--	-0.31	5.24	4.31	--	--	-9.43	1.01 (*)
1933-1944	9.96	9.00	--	--	0.34	3.22	2.06	--	--	9.70	--
1944-1949	-1.98	-3.44	--	--	3.15	1.20	0.92	3.60	-2.89	0.99	2.05
1949-1973	4.15	2.64	2.76	0.98	3.65	4.46	4.42	3.29	0.45	4.22	2.19
1973-1995	2.84	1.84	1.24	1.54	2.76	2.21	1.88	1.54	-0.66	3.12	0.56
1995-2005	3.24	2.00	2.34	0.72	2.81	2.48	2.43	3.06	-0.55	3.73	1.55
1929-1941	2.84	2.06	--	--	0.30	4.73	3.47	--	--	2.58	2.27 (**)
1933-1944	**9.96**	**9.00**	**--**	**--**	**0.34**	**3.22**	**2.06**	**--**	**--**	**9.70**	**--**
1933-1973	**4.92**	**3.56**	**--**	**--**	**2.67**	**3.71**	**3.32**	**--**	**--**	**5.29**	**--**
1995-2005	**3.24**	**2.00**	**2.34**	**0.72**	**2.81**	**2.48**	**2.43**	**3.06**	**-0.55**	**3.73**	**1.55**
1933-1973	4.92	3.56	--	--	2.67	3.71	3.32	--	--	5.29	--
1948-1973	3.96	2.44	2.72	0.80	3.64	4.41	4.36	3.31	0.31	4.00	2.15
1948-2005	3.40	2.13	2.07	1.07	3.15	3.22	3.06	2.57	-0.21	3.61	1.43
1973-1995	2.84	1.84	1.24	1.54	2.76	2.21	1.88	1.54	-0.66	3.12	0.56
1973-2005	2.96	1.89	1.57	1.28	2.77	2.30	2.05	1.98	-0.63	3.31	0.85
1995-2005	3.24	2.00	2.34	0.72	2.81	2.48	2.43	3.06	-0.55	3.73	1.55
1929-2005	3.41	2.20	--	--	2.55	3.19	2.84	--	--	3.62	--

(*): Taken from Gordon (1999, pp.124-5).

(**): Taken from Field (2003, p.1404).

return of 15 percent. The rate of return on the maintenance of existing highway conditions meeting minimum standards was estimated to be 35 percent, while the rate of return on the maintenance of current highway conditions that does not meet minimum standards was estimated to be only 5 percent. Moreover Cain (1997, p.130) reports that Fogel (1960) estimated the private rate of return on the Union Pacific Railroad to be 11.6 percent and the social rate to be 29.9 percent, which Fogel considered to be consistent with other investments.[12] Hence, even considering only the direct effects of public infrastructure investments, researchers have concluded that initial and a few subsequent public infrastructure expenditures are warranted.

Akkina and Celebi (2002) and Cain (1997, pp.132-5) discuss the indirect benefits and costs of public infrastructure investments and conclude that the indirect effects of public infrastructure investments on private economic growth have also been positive and very high. For example, public infrastructure investments in transportation have lowered transportation costs substantially and therefore created a large nationwide common market allowing the private economy to exploit economies of scale.

Table A4 in Appendix B provides the current-cost net stock of public fixed assets in both current year prices and chained 2000 prices for the years 1925-2005. The values for each year are the total cost the government would pay to rebuild the entire existing capital stock in that year. Table 12 below provides the growth rates of current-cost real net stock of public fixed assets compounded annually over several sub intervals and Table A5 in Appendix B gives the breakdown of public fixed assets as the shares of total public fixed assets for the years 1925-2005. Federal government real net stock of military capital increased from $70 billion in 1929 to $884 billion in 2005 (a factor of 13), an annual growth rate of 3.4%. The share of military capital stock in the total increased slightly from 13.0 percent to 13.1 percent.

It is interesting to emphasize that the military capital stock did not increase significantly from 1925 until 1940 but increased slightly from $70 billion in 1929 to $80 billion in 1940. The United States declared war on Japan and Germany in 1941 and the real military capital stock increased from $80 billion in 1940 to $154 billion in 1941 (an increase of 92.5 percent), then increased very sharply to $415 billion in 1942, to $753 billion in 1943, to $1,041 billion in 1944, and to $1,142 billion in 1945. In five years the real military capital stock increased by 1,327.5 percent, an increase that may not have been observed in the history of mankind. It is also noteworthy that the military capital stock decreased sharply after the War was over—decreased to $990 billion in 1946, to $847 billion in 1947, to $729 billion in 1948, to $665 billion in 1949, and to $601 billion in 1950— but never returned even close to the pre-war level. It remained at $600 billion level for a while and then fluctuated over time with a positive time trend.

The federal government real net stock of nonmilitary fixed capital increased from $30 billion in 1929 to $547 billion in 2005 (a factor of 18), an annual growth rate of 3.9 percent. On the other hand, the share of military capital stock in total increased substantially from 6.4 percent in 1929 to 8.2 percent in 2005. It grew at an annual rate of 5.0 percent over the years 1933-1973 and at an annual rate of 1.7 percent over the years 1973-2005.

Table 12: Annual Growth Rates of Current-Cost Real Net Stock of Public Fixed Assets

Year	Federal		State / Local	Total Non-military	Total
	Military	Nonmilitary			
1929-1933	-1.38	11.00	4.85	5.24	4.50
1933-1944	28.43	10.33	2.39	3.22	10.15
1944-1949	-8.59	1.71	1.11	1.20	-3.46
1949-1973	0.77	3.33	4.64	4.46	3.23
1973-2005	0.32	1.72	2.37	2.30	1.92
1929-1941	6.79	12.38	3.96	4.73	5.15
1933-1973	6.42	5.00	3.57	3.71	4.21
1948-1973	0.36	3.35	4.58	4.41	3.02
1973-2005	0.32	1.72	2.37	2.30	1.92
1948-2005	0.34	2.43	3.33	3.22	2.40
1929-2005	3.39	3.91	3.13	3.19	3.25

State and local government real net stock of fixed capital increased from $470 billion in 1929 to $4,890 billion in 2005 (a factor of 10), an annual growth rate of 3.1 percent. It grew by an annual growth rate of 3.6 percent over the years 1933-1973 while it grew by an annual growth rate of 2.4 percent over the years 1973-2005. However, the share of state and local government real net stock of fixed capital in total decreased from 80.6 percent in 1929 to 78.7 percent in 2005. Moreover, the share of state and local government real net stock of fixed capital in total nonmilitary fixed capital stock decreased from 92.7 percent in 1929 to 90.6 percent in 2005.

It is important to remind the reader that even though a very small portion of state government fixed capital stock is for military purposes, it is assumed that all of the state government capital stock is nonmilitary not only because data on the breakdown do not exist but also because we are aware of the fact that the military portion has historically been sufficiently small that it can be safely ignored.

Total nonmilitary real net stock of public fixed capital increased from $500 in 1929 to $5,437 in 2005 (a factor of 11), an annual growth rate of 3.2%. It grew by an annual growth rate of 3.7% over the years 1933-1973 and by 2.3% over the years 1973-2005. As can be seen from Table A4 in Appendix B, the annual growth rate of public nonmilitary fixed capital stock was above 4% until 1970 except for the years 1943-1949 during which the growth rate was either negative or very small. After 1970, it gradually declined and remained around 2.5% throughout the 1980s and 1990s. As observed above, the slowdown in the annual growth rate of public nonmilitary fixed capital stock was first noticed and then invoked by Aschauer to explain the slowdown in the annual growth rate of both overall labor productivity and MFP.

The slowdown in the annual growth rate of the real net public nonmilitary fixed capital stock can be better seen from Table 12 above. The annual growth rate on average was 5.2% over the years 1929-1933; it decreased to 3.2% over the years 1933-1944 and to 1.2% over the years 1944-1949, then increased to 4.5% over the years 1949-1973, and then decreased sharply to 2.3% over the years 1973-2005. The decline in the annual growth rate of real net public nonmilitary fixed capital stock from the sub interval 1949-1973 to the last sub interval 1973-2005 was 48.4 percent, meaning that the growth rate almost fell in half.

Table A6 in Appendix B gives current-cost net stock of state and local governments' fixed assets in billions of current year dollars. The values for each year are the total cost in billions of the same year's prices that the state and local governments would need to pay to rebuild the entire existing capital stock in that year. Table A7 in Appendix B, on the other hand, provides the same values in billions of chained 2000 year's prices. The values for each year are the total cost in billions of chained 2000 year's prices that the state and local governments would need to pay to rebuild the entire existing capital stock in that year. Table 13 below provides the growth rates of current-cost net real stock of state and local governments' fixed assets compounded annually over some sub intervals.

As can be seen from Table A7 in Appendix B, the state and local governments' total fixed assets is first divided into two subgroups, equipment/software and structures, and then the subgroup structures is also divided into two subgroups, core infrastructure and other structures. One of the variables of interest has been total public core infrastructure capital stock, containing fixed assets devoted to education, health care, highways and streets, power and transportation systems, and sewer and water lines. The values that Table A7 in Appendix B reports as the real net total public core infrastructure capital stock underestimate the true values somewhat since the federal government also has fixed capital stock devoted to education, health care, highways and streets, power and transportation systems, and sewer and water lines. But because data on the breakdown of, say, federal government's fixed assets on education as military education and nonmilitary education do not exist and because the total amount of federal government's fixed assets devoted to education is small, we simply assume that all of the federal government's fixed capital devoted to education is devoted to military education. In other words, we have to assume that only state and local governments own core infrastructure capital stock.

Table 13: Annual Growth Rates of Current-Cost Net Real Stock of State and Local Governments' Fixed Assets

Year	Equipment and Software	Core Infrastructure	Other Structures
1929-1933	4.89	4.31	18.36
1933-1944	-0.87	2.06	7.94
1944-1949	3.93	0.92	2.47
1949-1973	7.65	4.42	5.71
1973-2005	5.52	2.05	3.31
1929-1941	2.52	3.47	13.36
1933-1973	4.77	3.32	5.90
1948-1973	7.72	4.36	5.73
1973-2005	5.52	2.05	3.31
1948-2005	6.48	3.06	4.36
1929-2005	5.09	2.84	5.42

State and local governments' fixed assets in the form of equipment and software increased from $5.6 billion in 1929 to $245 billion in 2005 (a factor of 44), an annual growth rate of 5.1%. This enormous increase was mostly due to the recent widespread use of computer equipment and software since the level increased by nearly tenfold from $5.6 billion to $56 billion over the years 1929-1978. State and local governments' total core infrastructure capital stock increased from $456 billion in 1929 to $3,825 billion in 2005 (a factor of 8.5), an annual growth rate of 2.8% and total other infrastructure capital stock increased from $15 billion in 1929 to $821 billion in 2005 (a factor of 55), an annual growth rate of 5.4%.

From 1929 to 2005, state and local governments' core infrastructure capital stock on education increased from $132 billion to $1,040 (a factor of 8) with a compound annual growth rate of 2.8%. The core capital stock for health care increased from $20 billion to $131 billion (a factor of 6.6), an annual growth rate of 2.5%. The core infrastructure capital stock for highways and streets increased from $190 billion to $1,529 billion (a factor of 8), an annual growth rate of 2.8%. The core infrastructure capital stock for power and transportation systems increased from $35 billion to $485 billion (a factor of 14), an annual growth rate of 3.5%. Moreover, the core infrastructure capital stock for water and sewer lines increased from $79 billion to $640 billion (a factor of 8), an annual growth rate of 2.8%. The five components of state and local governments' core infrastructure capital stock increased by compound annual growth rates of 3.5%, 2.6%, 3.3%, 3.6%, and 3.0%, respectively, over the years 1933-1973 and by compound annual growth rates of 1.9%, 1.9%, 1.7%, 3.2%, and 2.5%, respectively, over the years 1973-2005. The real net total core infrastructure capital stock increased by compound rates of 3.3% and 2.1% over the intervals 1933-1973 and 1973-2005, respectively. It is again clear that the decline in the growth rates from 1933-1973 to 1973-2005 is substantial.

Table A8 in Appendix B gives the breakdown of state and local government's fixed assets as the shares of the state and local government's total fixed assets. The share of equipment and software increased from 1.7% of the total in 1925 to 3.9% of the total in 2005. Similarly, the share of other structures increased substantially from 3.4% in 1925 to 17.0% of the total in 2005, whereas the share of total core infrastructure capital stock declined sharply from 95% of the total in 1925 to 79% of the total in 2005. The portion of the total devoted to education, initially 20% of the total, fluctuated slightly over time, and ended up being 21% of the total. Moreover, it remained relatively constant over the entire time interval. However, the portion devoted to highways and streets, the largest and most volatile component of the total, initially 54% of the total, remained around 50% of the total throughout the 1930s and 1940s, decreased to about 40% of the total during the 1950s, 1960s, and 1970s and to about 33% of the total thereafter. Considering the years 1925-2005, highways and streets constituted somewhat more than a half of the total public core infrastructure capital stock during the first two decades and about one third of the total during the last two decades. The shares of education and water and sewer lines in the total fluctuated slightly, but exhibited almost no trend over the entire time interval 1925-2005. The shares of health care and highways and streets, however, declined while the shares of power and transportation systems rose substantially from 5.1% of the total in 1925 to 9.9% of the total in 2005.

Table A9 in Appendix B gives current-cost net stock of private fixed assets in billions of both current-year prices and chained 2000 year prices for the years 1925-2005. The values for each year are the total cost in billions of either the same year's prices or the chained 2000 year's prices that the private sector would need to pay to rebuild the entire existing capital stock in that year. Table 14 below gives the growth rates of real net nonresidential, residential, and total stock of private fixed assets compounded

Table 14: Annual Growth Rates of Current-Cost Net Stock of Real Private Fixed Assets

Year	Nonresidential	Residential	Total
1929-1933	-0.18	0.00	-0.31
1933-1944	-0.01	0.51	0.34
1944-1949	2.92	2.79	3.15
1949-1973	3.50	3.62	3.65
1973-2005	2.82	2.64	2.77
1929-1941	0.11	0.50	0.30
1933-1973	2.45	2.65	2.67
1948-1973	3.47	3.62	3.64
1973-2005	2.82	2.64	2.77
1948-2005	3.10	3.07	3.15
1929-2005	2.47	2.50	2.55

annually over several sub intervals. Over the years 1929-2005, nonresidential real net stock of private fixed assets increased from $1,804 billion to $11,485 billion (a factor of 6.4), an annual growth rate of 2.5 percent, residential real net stock of private fixed assets increased from $1,884 billion to $12,344 billion (a factor of 6.6), an annual growth rate of 2.5 percent, and real net stock of total private fixed assets increased from $3,506 billion to $23,832 billion (a factor of 6.7), an annual growth rate of 2.6 percent.

Over the sub intervals 1933-1973 and 1973-2005, the real net stock of total private fixed assets increased by an annual growth rate of 2.7% and 2.8%, respectively; the real net stock of total private residential fixed assets increased by an annual growth rate of 2.7% and 2.6%; and the real net stock of total private nonresidential fixed assets increased by an annual growth rate of 2.5% and 2.8%. It is important to remind the reader that the only macroeconomic variable mentioned so far whose annual growth rate over the years 1973-2005 was larger than its annual growth rate over the years 1933-1973 is the real net stock of private nonresidential, residential, and total fixed assets.

IV. The Relationship between Private Output and Public Nonmilitary Fixed Capital: A Production Function Approach

In this section, we will develop a neoclassical growth model incorporating the public infrastructure capital stock as a component of the aggregate private production function. We assume that the aggregate private production technology is characterized by a Harrod-neutral technological process as follows.

(8) $Y_t = F(K_t, A_tL_t, G_t)$,

where Y_t is a measure of real aggregate output of goods and services of the private sector, K_t is a measure of the private capital stock, A_t is a measure of "knowledge" or the "effectiveness of labor" or Harrod-neutral technical change, L_t is a measure of aggregate employment of labor services of the private sector, G_t is a measure of the public sector nonmilitary capital stock, and the subscript t stands for time periods. The productivity measure is implicitly assumed to be not only a function of aggregate shocks, Z_t, but also time independent of the aforementioned variables. Assuming a generalized aggregate Cobb-Douglas production technology with constant returns to scale across all inputs, private and public, we have

(9) $Y_t = K_t^a G_t^b (A_t L_t)^{1-a-b}$,

where the productivity measure and the private labor services are assumed to grow exponentially over time with constant growth rates m and n, respectively, as follows.

(10) $A_t = A_0 e^{mt}$ and $L_t = L_0 e^{nt}$.

We assume that the government provides the services of the public capital stock to the private sector without charging user fees and finances the public capital stock through taxes. The rationale behind the existence of the public capital stock is that either the private firms are unable— for example, economies of scale in the production process may require a very large scale that cannot be afforded by any private firm— or unwilling to own the public capital stock or the nation can be better off if the private firms do not own the public capital stock.

Constant returns to scale across all factors in the production process together with the two assumptions, private factors are paid according to their marginal products and public factors are free, implies that private output will not be fully distributed among the private factors of the production. However, if we assume constant returns to scale over only the private inputs together with the neoclassical assumption that the private factors are paid according to their marginal products, then private output would be exhaustively distributed among the private inputs.

We assume that a constant fraction, s_k, of the aggregate private output is devoted to the private capital stock and that the private capital stock depreciates over time with a geometric depreciation rate, d. Then, the private capital stock evolves over time according to the following equation.

(11) $\dot{K}_t = s_k Y_t - d\, K_t$,

where the dot above the letter stands for the time derivative of the corresponding variable in the continuous time case and the change in the corresponding variable relative to the previous period in the discrete time case. By assuming that a constant fraction, s_g, of private output is devoted to the public capital stock and that the public capital stock depreciates over time with the same geometric depreciation rate, d, we can then obtain the public capital evolution equation as follows.

(12) $\dot{G}_t = s_g Y_t - d\, G_t$.

Dividing Equation (9) by effective labor, $A_t L_t$, we can get private output per effective labor, y_{et}, in intensive form as follows.

(13) $y_{et} = k_{et}^{a}\, g_{et}^{b}$,

where k_{et} stands for private capital per unit of effective of labor and g_{et} stands for public capital per unit of effective labor. Dividing Equation (11) by K_t and substituting Equation (9) into the resulting equation, we have

(14) $\dot{K}_t / K_t = s_k\, k_{et}^{a-1}\, g_{et}^{b} - d$.

Taking the natural logarithm of the definition of k_{et}, differentiating both sides of the resulting equation with respect to time, and substituting Equation (14) into the resulting equation, we have the evolution equation for the private capital per unit of effective labor while the economy is in the transition toward its steady state as follows.

(15) $\dot{k}_{et} = s_k\, k_{et}^{a}\, g_{et}^{b} - (d+m+n)\, k_{et}$.

Dividing Equation (12) by G_t and substituting Equation (9) into the resulting equation, we have

(16) $\dot{G}_t / G_t = s_g k_{et}^{a} g_{et}^{b-1} - d$.

Taking the natural logarithm of the definition of g_{et}, differentiating both sides of the resulting equation with respect to time, and substituting Equation (16) into the resulting equation, we have the public capital per unit of effective labor evolution equation as follows.

(17) $\dot{g}_{et} = s_g k_{et}^{a} g_{et}^{b} - (d+m+n) g_{et}$.

Taking the natural logarithm of Equation (13), differentiating both sides of the resulting equation with respect to time, and substituting Equations (15) and (17) into the resulting equation, we have the private output per unit of effective labor evolution equation as follows.

(18) $\dot{y}_{et} = (a\, s_k k_{et}^{a-1} g_{et}^{b} + b\, s_g k_{et}^{a} g_{et}^{b-1}) k_{et}^{a} g_{et}^{b} - (a+b)(d+m+n) k_{et}^{a} g_{et}^{b}$.

IV.1. Steady State Analysis

As the economy is in transition toward its steady state, the private capital per unit of effective labor, the public capital per unit of effective labor, and private output per unit of effective labor evolve over time according to the system of the differential equations (15), (17), and (18). The system of Equations (15), (17), and (18) does not have a set of closed form solutions. However, the system can numerically be solved for given values of the parameters a, b, d, m, n, s_k, and s_g. Although the system cannot be solved explicitly while the economy is in transition toward its steady state, it can easily be solved once the economy reaches its steady state.

The steady state is reached when $\dot{k}_{et} = \dot{g}_{et} = \dot{y}_{et} = 0$. Therefore, substituting zero into Equations (15), (17), and (18) for $\dot{k}_{et}$, $\dot{g}_{et}$, and $\dot{y}_{et}$, dropping the subscript t from the variables (since once the economy reaches its steady state, the values of k, g, and y will not change over time) and solving the system of the resulting three equations for k, g, and y, we have the steady state values of the private capital per unit of effective labor, the public capital per unit of effective labor, and private output per unit of effective labor, respectively, as follows.

(19) $k_e^* = s_k^{[(1-b) / (1-a-b)]} s_g^{[b / (1-a-b)]} (d+m+n)^{[1 / (a+b-1)]}$,

(20) $g_e^* = s_g^{[(1-a) / (1-a-b)]} s_k^{[a / (1-a-b)]} (d+m+n)^{[1 / (a+b-1)]}$,

(21) $y_e^* = s_k^{[a / (1-a-b)]} s_g^{[b / (1-a-b)]} (d+m+n)^{[(a+b) / (a+b-1)]}$.

Let us now examine the effects of the investment rates in private and public capital, the depreciation rate, the productivity growth rate, and the population growth rate on the steady state values of the private capital per unit of effective labor, public capital per unit of effective labor, and on private output per unit of effective labor once the economy reaches its steady state. The effects of any permanent change in the fraction of private output devoted to private capital on k_e^*, g_e^*, and y_e^* are given by $[(1-b)/(1-a-b)]*[k_e^*/s_k]$, $[a/(1-a-b)]*[g_e^*/s_k]$, and $[a/(1-a-b)]*[y_e^*/s_k]$, respectively. The effects of any permanent change in the fraction of private output in terms of taxes paid to the government devoted to public capital on k_e^*, g_e^*, and y_e^* are given by $[b/(1-a-b)]*[k_e^*/s_g]$, $[(1-a)/(1-a-b)]*[g_e^*/s_g]$, and $[b/(1-a-b)]*[y_e^*/s_g]$, respectively. It is clear from these expressions that any increase in either s_k or s_g results in an increase in k_e^*, g_e^*, and y_e^* if the aggregate production function exhibits decreasing returns to scale in both private and public capital.

The effects of any change in either the deprecation rate, the productivity growth rate, or the population growth rate on the variables of interest are given by $[1/(a+b-1)]*[k_e^*/(d+m+n)]$, $[1/(a+b-1)]*[g_e^*/(d+m+n)]$, and $[(a+b)/(a+b-1)]*[y_e^*/(d+m+n)]$, respectively, each of which is negative if the aggregate production function exhibits decreasing returns to scale in both private and public capital. Therefore, any permanent increase in either d, m, or in n will result in a decrease in k_e^*, g_e^*, and y_e^* if aggregate production technology exhibits decreasing returns to scale in K and G.

It is reasonable to assume that a+b is less than one for the U.S. economy, where a is the elasticity of an aggregate measure of private output with respect to private capital and where b is the elasticity of private output with respect to an aggregate measure of public capital. Therefore, to increase the steady state values of the private capital per unit of effective labor, the public capital per unit of effective labor, or the private output per unit of effective labor, public policy makers should not only increase the fraction of taxes devoted to public capital formation but also adopt policies encouraging the private sector to increase the fraction of private output devoted to private capital formation.

IV.2. Preliminary Empirical Estimates

Some of the time series studies using production function models report a very large effect of the public nonmilitary fixed or core infrastructure capital stock on private output. In most cases, the claimed impact is too large to be credible. For example, Munnell's (1990a) and Holtz-Eakin's (1988) estimates suggest that a one percent increase in the public fixed capital stock results in a 0.34 percent increase in aggregate private output. Their estimates also suggest that the marginal product of public fixed capital is 0.6 while the marginal product of private fixed capital is 0.3, i.e., the marginal product of public fixed capital exceeds that of private fixed capital by a factor of two. Aschauer (1990a, p.16) claims an even larger impact of the public nonmilitary fixed capital stock by saying that "*increases in GNP resulting from increased public infrastructure spending are estimated to exceed those from private investment by a factor of between two and five.*"

Our estimates given below also suggest a statistically significant and quantitatively positive and large impact of both the public nonmilitary fixed capital stock and the public core infrastructure capital stock on both aggregate real private output and aggregate real nonresidential private output. Before discussing our findings in detail, we would like to emphasize three distinguishing features of them.

First, as explained before, the economics literature has reached the consensus that the public fixed capital stock and private output are positively correlated. The consensus is not so clear on the direction of the causation. Whereas most of the researchers believe that causation runs from the public capital stock to private output, some argue that the causation might run the other way around, from private output to the public fixed capital stock. Our findings, however, support the claim that the causation runs from the public fixed capital stock to private output rather than the other way around.

Second, the economics literature is far away from reaching a consensus on the magnitude of the positive effect of the public fixed capital stock on private output. On one hand, most of the studies discussed earlier estimate a very large positive magnitude of the impact of the public fixed capital stock on private output. On the other hand, they estimate a relatively small magnitude of the impact of the private fixed capital stock on private output. However, our estimates, on one hand, show that the magnitudes of the positive impact of the public fixed capital stock on private output are smaller than those given above or discussed earlier. On the other hand, our findings suggest that the magnitudes of the positive effect of the private fixed capital stock on private output are *considerably* greater than those given above or reported earlier. In other words, most of the studies report a positive impact of the private fixed capital stock on private output that is too small to be credible, while they report a positive impact of the public fixed capital stock on private output that is too large to be credible. However, our estimates, on average, suggest not only a positive impact of the public capital stock on private output that is credible but also a positive and credible impact of the private capital stock on private output.

Third, it is well known that time series data on private output, private capital, and public capital have a unit root most of the time, thereby yielding a spurious regression. Therefore, any estimation using the data with a common trend can yield test statistics such as calculated t-statistics, F-statistics, or adjusted R^2, that overestimate not only the true relationships between private output and each of the independent variables but also the explanatory power of the model. To overcome this problem once the problem is detected, the model should be estimated in the form of first differences. In other words, since time series data on aggregate private output, private capital, and public capital are not stationary in that they drift over time, it is essential to remove the common trend to eliminate spurious correlations by first-differencing the data and then using the data in terms of first differences to capture the true relationship between the variable of interest and each of the independent variables. Our estimates given in Tables 15A-15D below suffer from the common trend, while our estimates given in Tables 17A and 17B below overcome this problem through first differencing, thus revealing the true relationships between the variable of interest and the independent variables.

Technology can be modeled in an aggregate Cobb-Douglas production function in three different but equivalent ways in terms of measuring the magnitudes of the impact of the public fixed capital stock on any measure of the private sector output as follows.
A Hicks-neutral technological progress of the form $Y_t = A_t K_t^a G_t^b L_t^c$,
a capital augmenting technological progress of the form $Y_t = (A_t K_t)^a G_t^b (A_t L_t)^c$,
or a Harrod-neutral technological progress of the form $Y_t = K_t^a G_t^b (A_t L_t)^c$, where c is the elasticity of private output with respect to private labor and the other variables are as defined above. In terms of measuring the magnitudes of the impact of any measure of the public fixed capital stock on any measure of private output, it does not really make any difference which form is used. The main difference between the above three forms and Equation (9) is that the above three forms do not impose any restriction on the returns to scale in the production process, while Equation (9) imposes a constant returns to scale across all inputs, private and public, on the production process.

As explained at the beginning of Section IV of this dissertation, the services of public capital stock enters into a production function type model as unpaid factors of production. Some researchers, such as Aschauer (1989a, 1989b, 1989c, 1990a, and 1990b) and Tatom (1991), enter the manufacturing sector's capacity utilization rate denoted by CU into the aggregate production function multiplicatively as a proxy representing the aggregate capacity utilization rate of the private sector by assuming that the use of the services of the public capital stock by the private sector is proportional to the private sector's capacity utilization rate. In other words, the private sector's use of the public capital stock depends primarily on not only the state of the economy but also the stage of the business cycle. Therefore, adjusting the use of the services of public fixed capital stock by CU can be modeled as one of the three ways, none of which is better than the other in terms of measuring the magnitude of the effects of the public capital stock on private output
$Y_t = A_t K_t^a (CU_tG_t)^b L_t^c$, $Y_t = (A_t K_t)^a (CU_tG_t)^b (A_t L_t)^c$, or $Y_t = K_t^a (CU_tG_t)^b (A_t L_t)^c$.
None of the last three forms impose any restriction on the production process other than the unitary elasticity of substitution of one input —the private capital input, the private labor input, or the public capital input— for another input in the production process.

Most of the time, there is *a priori* reason to believe that time series data on private output, public capital, and private capital have a common trend. Therefore, at the estimation stage the technology variable A_t is replaced with a constant term and a time trend variable in the expectation that the time trend variable would at least partially remove the biases imparted by the common trend from the estimated results. Combining all of the above, the following six equations will be estimated using aggregate annual data for the years 1947-2005.

(22) $\ln(Y_t) = e_1 + e_2\, T + e_3 \ln(CU_t) + e_K \ln(K_t) + e_L \ln(L_t) + e_G \ln(G_t) + e_4\, D + U_{1t},$

(23) $\ln(Y_t) = e_1 + e_2\, T + e_K \ln(K_t) + e_L \ln(L_t) + e_G \ln(CU_t * G_t) + e_3\, D + U_{2t},$

(24) $\ln(Y_t / K_t) = e_1 + e_2\, T + e_3 \ln(CU_t) + (1-e_K) \ln(L_t / K_t) + e_G \ln(G_t / L_t) + e_4\, D + U_{3t},$

(25) $\ln(Y_t / K_t) = e_1 + e_2\, T + (1-e_K) \ln(L_t / K_t) + e_G \ln(CU_t * G_t / L_t) + e_3\, D + U_{4t},$

(26) $\ln(Y_t / L_t) = e_1 + e_2\, T + e_3 \ln(CU_t) + e_K \ln(K_t / L_t) + e_G \ln(G_t / L_t) + e_4\, D + U_{5t},$

(27) $\ln(Y_t / L_t) = e_1 + e_2\, T + e_K \ln(K_t / L_t) + e_G \ln(CU_t * G_t / L_t) + e_3\, D + U_{6t},$

where Y is the aggregate real private output in billions of chained 2000 dollars, K is the year-end estimate of the aggregate private real fixed capital stock in billions of chained 2000 dollars, L is the aggregate number of private total labor hours in billions of hours, G is the year-end estimate of the aggregate public real nonmilitary fixed capital stock in billions of chained 2000 dollars, CU is the manufacturing sector's total capacity utilization rate, D is an intercept dummy variable measuring the effects of the September 11 terrorist attack and the ongoing U.S. war against global terrorism together with the U.S. invasion of Afghanistan and the U.S. invasion of Iraq, e_K is the elasticity of private output with respect to private capital input, e_L is the elasticity of private output with respect to private labor input, e_G is the elasticity of private output with respect to public capital input, T is a time trend, ln stands for the natural logarithm, the subscript t stands for time, and U_{it} are independent identically distributed error terms with zero mean and constant variance.

Equations (22) and (23) do not constrain the returns to scale in the aggregate production process, while equations (24), (25), (26), and (27) assume an aggregate production technology for the U.S. economy that exhibits constant returns to scale across all inputs, including private capital, private labor, and public capital. The manufacturing sector's capacity utilization rate serves as a proxy representing the aggregate capacity utilization rate of the private sector and enters into equations (22), (24), and (26) as a measure to capture the effects of the business cycle on real private output, while it enters into equations (23), (25), and (27) multiplicatively together with the public capital stock input to let the private sector adjust the use of the services of the public capital stock to different stages of the business cycle.

Three different values —year-end values, mid-year values, and beginning-year values— of the measures of both the private and the public capital stocks are used throughout the estimation stage. In other words, each equation with the same variables is estimated three times, and each time one of the three estimates of private and public capitals is used. The use of the mid-year estimates is believed to underestimate the true amount of private and public capital available to the private sector in that year while the use of the end-year values is believed to overestimate the true amount of the private and public capitals available to the private sector. Therefore, the true impact is believed to be somewhere in between the two. Beginning-year values, however, are employed for a totally different reason. The main purpose of the use of the beginning-year estimates is to see whether there exists a causal influence running from private output to public capital formation.

The rationale underlying the use of the beginning-year values of the public capital stock can be better explained as follows. The claim can be made that the direction of causation runs from high levels of private output to the greater amount of the public capital stock and not the other way around. One way to test whether this claim is true is to regress the public capital stock variable on private output variable to see whether the estimated coefficient of private output variable is statistically significant. Another way is to use the beginning-year estimates of an appropriate measure of the public capital stock to see whether the results are similar to the ones obtained using either the mid-year values or the end-year values of the appropriate measure of the public capital stock variable. If the results are similar, then there is no reason to expect the existence of the reverse causation since there is no a priori reason justified by economic theories to claim that the level of public capital stock measured at the end of year t can be determined by the level of private output measured at the and of year t+1. If the results are statistically significantly different, then the first way should be used to see whether the reverse causation exists.

We now discuss our estimated results, econometric problems, and our solutions to the econometric problems as follows. Using Equations (22) and (23), the aggregate private sector real output (Y) in billions of chained 2000 dollars is in double-logarithmic forms regressed on the following explanatory variables using annual data for the years 1947-2005. The equations are estimated by the ordinary least squares (OLS) method. The independent variables are a constant and a time trend (T), the manufacturing sector's total capacity utilization rate (CU) as a proxy representing the aggregate capacity utilization rate of the private sector aimed at controlling for the effects of business cycle on private real output when used separately or allowing the private sector to adjust the use of the public fixed capital when used multiplicatively with either G or G_{ci}, the aggregate private sector net stock of real fixed assets (K) in billions of chained 2000 dollars, the aggregate private sector total labor hours (L) in billions of hours, and either the aggregate public sector net stock of nonmilitary real fixed capital (G) in billions of chained 2000 dollars or the aggregate public sector net stock of real core infrastructure capital (G_{ci}) in billions of chained 2000 dollars, and an intercept dummy variable (D) aimed at controlling for the exogenous shocks to the private economy since 2001. The estimated results are given in Table 15A below.

We now discuss the estimation results. As can be seen from Table 15A, extremely high values of the adjusted R^2 combined with very high values of the calculated standard F-statistics *seem* to indicate that every estimated equation successfully explains almost all of the total variation in the aggregate private real output. The high values of the calculated standard t-statistics *seem* to indicate not only that every explanatory variable included is a relevant one but also that every estimated coefficient is statistically significant at almost all conventional significance levels (α = 0.10, 0.05, 0.025, or even 0.01). Moreover, all of the estimated coefficients have the expected signs.

However, the low values of the calculated Durbin-Watson test-statistic show that the least squares residuals are strongly auto-correlated. The very high values of the adjusted R^2 and highly significant t-statistics accompanied with low values of the calculated Durbin-Watson test-statistic suggest that the existence of spurious regressions is very likely.

Table 15A: Empirical Estimates of Production Function Model with Level Data (*)

Dependent Variable: ln Y with unrestricted input elasticities

Equat.	const.	time	ln CU	ln K	ln L	ln G	ln G_{ci}	ln CU*G	ln CU*G_{ci}	D	adj R^2	D-W	F
	A) end-year values of public and private capitals												
1.1	-1.88	0.008	0.29	0.294	0.649	0.322				0.066	0.999	1.16	13677
	(-2.1)	(2.66)	(4.57)	(2.02)	(7.34)	(4.28)				(5.88)			
1.2	-1.74	0.009	0.307	0.315	0.647		0.275			0.064	0.999	1.1	12913
	(-1.8)	(2.91)	(4.73)	(2.03)	(6.76)		(3.79)			(5.52)			
1.3	-0.26	0.01		0.282	0.665				0.293	0.064	0.999	1.09	15770
	(-0.3)	(3.17)		(2.99)	(10.44)				(9.67)	(5.60)			
1.4	-0.59	0.008		0.322	0.631			0.305		0.067	0.999	1.17	16671
	(-0.7)	(2.69)		(3.62)	(10.4)			(10.01)		(5.99)			
	B) mid-year values of public and private capitals												
1.5	-1.33	0.011	0.321	0.241	0.648		0.299			0.064	0.999	1.08	12276
	(-1.3)	(3.18)	(4.87)	(1.53)	(7.14)		(4.29)			(5.30)			
1.6	0.205	0.011		0.218	0.659				0.311	0.064	0.999	1.07	15006
	(0.24)	(3.50)		(2.25)	(10.1)				(10.19)	(5.41)			
1.7	-0.17	0.009		0.261	0.626			0.325		0.067	0.999	1.16	15916
	(-0.2)	(2.99)		(2.86)	(9.98)			(10.64)		(5.81)			
1.8	-1.47	0.009	0.303	0.217	0.647	0.348				0.066	0.999	1.13	13087
	(-1.5)	(2.97)	(4.73)	(1.47)	(7.68)	(4.8)				(5.64)			
	C) beginning-year values of public and private capitals												
1.9	-1.31	0.01	0.334	0.207	0.616	0.339				0.066	0.999	1.11	12249
	(-1.3)	(3.06)	(5.21)	(1.44)	(7.73)	(5.07)				(5.39)			
1.10	-1.15	0.011	0.348	0.223	0.622		0.295			0.065	0.999	1.06	11592
	(-1.1)	(3.27)	(5.32)	(1.47)	(7.35)		(4.65)			(5.14)			
1.11	0.609	0.012		0.17	0.644				0.321	0.064	0.999	1.03	14117
	(0.66)	(3.71)		(1.72)	(9.39)				(10.45)	(5.19)			
1.12	0.227	0.01		0.211	0.613			0.338		0.067	0.999	1.12	15017
	(0.26)	(3.22)		(2.25)	(9.35)			(10.93)		(5.54)			

(*): Equations are estimated using the OLS estimation method with aggregate annual data from 1947 to 2005. Calculated t-statistics are given in parentheses. Y=real private GDP, CU=capacity utilization rate in manufacturing sector, K=real total private capital, L=total private business labor hours, G=real total public nonmilitary fixed capital, G_{ci}=real total public core infrastructure capital, D=dummy variable taking the values of 1 for the years 2001-2005 and 0 for the other years.

Let us now discuss the estimated results in more detail. It is important to remind the reader that intercept dummy (binary or dichotomous) variables take just two values, 1 or 0, to indicate the presence or absence of a unique event. The unique event in our equations is a structural change in the private sector resulting from the 9/11 terrorist attack, the ensuing U.S. war on global terrorism, and U.S. military action in both Afghanistan and Iraq.

The estimated coefficient of the intercept dummy variable aimed at measuring the effects of the aforementioned exogenous shocks on the aggregate private economy for the years 2001-2005 in every estimated equation reported in the following tables is found to be positive and statistically significant at even a 0.1 percent significance level, suggesting that the aggregate private real output function has shifted outward in a parallel manner since 2001. The positive sign might reflect the "military" nature of the aggregate private economy. The positive and highly statistically significant estimated coefficient of the dummy variable in each equation should be interpreted as the average annual increase in the aggregate private real output resulting from the aforementioned exogenous shocks for the years 2001-2005 holding the values of all the other independent variables constant at their sample averages. For example, the estimated coefficient of 0.066 of the dummy variable in Equation 1.1 means that the aggregate private real output increased by $66 million per year for the years 2001-2005 as a result of the aforementioned exogenous shocks to the private economy while holding the values of the other independent variables constant at their sample averages.

Whenever the manufacturing sector's capacity utilization rate as a proxy representing the aggregate capacity utilization rate of the private sector is used as an explanatory variable, its estimated coefficient is found to be positive, quantitatively very large, and statistically highly significant in every equation. The estimated coefficient is the elasticity of private real output with respect to the private sector's total capacity utilization rate. As can be seen from Table 15A, the average of that elasticity is 0.32, meaning that a one percent increase in the capacity utilization rate resulted in a 0.32 percent increase in the private real output while holding the values of the other explanatory variables constant at their sample averages.

From now on instead of interpreting an estimated coefficient of a single equation, we will interpret the average of the estimates of that coefficient from the table whose results we are discussing. The estimated elasticities of private output with respect to private fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15A and given in Table16 below. The averages of those elasticities are 0.247, 0.639, 0.330, and 0.299, respectively. They should be interpreted as the percentage increases in private output when the private capital, private labor, public nonmilitary fixed capital, or public core infrastructure capital increases by one percent, respectively, ceteris paribus.

The estimated marginal products of private capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15A and given in Table16 below. The averages of those marginal products are 0.079, 17.24, 0.489, and 0.613, respectively. They should be interpreted as the increases in private output when the private capital, private labor hours, public nonmilitary fixed capital, or public core infrastructure capital each increases by one unit —here the unit of measurement of private labor is an hour while that of each of the others is a dollar—, respectively, holding the values of the other explanatory variables constant at their sample averages. In other words, the marginal product of private capital of 0.079 means that private output increases by almost 8 cents when private capital increases by one dollar while holding the use of other inputs constant at their sample averages. However, the marginal products of public nonmilitary fixed capital and public core infrastructure capital of 0.489 and 0.613 mean that private output increases by 49 cents and 61 cents when public nonmilitary fixed capital and public core infrastructure capital each increases by one dollar, respectively, while holding the use of the other inputs constant at their sample averages. It is important to note that the former increase is too low to be credible while the latter increases are too high to be credible. The marginal product of private labor is 17.24, meaning that private output increases by $17.24 when one additional labor hour is employed while holding the use of the other inputs constant at their sample averages.

Five features are common to almost all of the tables below and therefore worth noting. First, the estimated elasticities of private output with respect to the two measures of public capital are smaller than the ones discussed at the beginning of this subsection. Second, the estimated elasticity of private output with respect to private capital is substantially greater than the ones mentioned above or reported in the literature. Third, the estimated elasticity of private output with respect to private capital is still smaller than the estimated elasticity of private output with respect to the two measures of public capital, but the difference is not as large as claimed by most of the aforementioned studies. Fourth, the estimated elasticity of private output with respect to private labor is in agreement with the estimates done by the U.S. Bureau of Labor Statistics. Fifth, the estimated elasticity of private output with respect to public nonmilitary fixed capital is larger than the estimated elasticity of private output with respect to public core infrastructure capital even though the opposite result was expected. Moreover, the estimated results will improve appreciably, starting with Table 17A once the econometric problems are addressed.

Using Equations (24) and (25), the aggregate private sector real output per unit of private fixed capital (Y/K) is in double-logarithmic forms regressed on the following explanatory variables using annual data for the years 1947-2005. The equations are estimated by the ordinary least squares (OLS) method. The explanatory variables used are a constant and a time trend (T), the manufacturing sector's total capacity utilization rate (CU), the aggregate private sector total labor hours per unit of private capital (L/K), the aggregate public sector net stock of nonmilitary real fixed capital per unit of private labor hours (G/L) or the aggregate public sector net stock of real core infrastructure capital per unit of private labor hours (G_{ci}/L), and a dummy variable (D) aimed at measuring the exogenous shocks to the private economy since 2001. The estimation results are provided in Table 15B below.

As can be seen from Table 15B, high values of the adjusted R^2 combined with very high values of the calculated standard F-statistics again *seem* to indicate that each estimated equation successfully explains more than 95 percent of the total variation in the aggregate private real output per unit of private fixed capital. The high values of the calculated standard t-statistics *seem* to show that not only every explanatory variable included is relevant but also every estimated coefficient is statistically significant at almost all conventional significance levels (α = 0.10, 0.05, 0.025, or even 0.01). Moreover, all of the estimated coefficients have the expected signs. However, the low values of the calculated Durbin- Watson test-statistic show that the least squares residuals are strongly auto-correlated. Very high values of the adjusted R^2 and highly significant

Table 15B: Empirical Estimates of Production Function Model with Level Data

Dependent Variable: ln (Y / K) with constant returns to scale across K, L, and G

Equat.	const.	time	ln CU	ln L/K	ln G/L	ln G_{ci}/L	ln CU*G_{ci}/L	ln CU*G/L	D	adj R^2	D-W	F
	A) end-year values of public and private capitals											
1.13	-0.3	0.014	0.308	0.816	0.297				0.054	0.958	1.08	267
	(-0.5)	(11.0)	(4.71)	(5.78)	(3.86)				(5.38)			
1.14	-0.28	0.015	0.32	0.794		0.26			0.053	0.957	1.03	256
	(-0.5)	(9.37)	(4.82)	(5.36)		(3.52)			(5.14)			
1.15	1.36	0.015		0.857			0.292		0.051	0.957	1.02	325
	(7.02)	(18.5)		(13.3)			(9.4)		(5.24)			
1.16	1.14	0.014		0.827				0.303	0.053	0.959	1.08	339
	(6.68)	(19.3)		(13.8)				(9.71)	(5.53)			
	B) mid-year values of public and private capitals											
1.17	-0.1	0.015	0.307	0.892	0.34				0.055	0.957	1.06	256
	(-0.2)	(11.9)	(4.7)	(6.62)	(4.61)				(5.51)			
1.18	-0.08	0.015	0.321	0.865		0.297			0.055	0.954	1.02	244
	(-0.14)	(10.3)	(4.81)	(6.15)		(4.21)			(5.31)			
1.19	1.46	0.016		0.891			0.31		0.054	0.955	1.01	310
	(7.72)	(19.1)		(14.1)			(10.1)		(5.46)			
1.20	1.24	0.014		0.861				0.323	0.056	0.957	1.08	326
	(7.42)	(19.8)		(14.7)				(10.4)	(5.74)			
	C) beginning-year values of public and private capitals											
1.21	-0.19	0.014	0.329	0.891	0.341				0.058	0.954	1.06	239
	(-0.37)	(13.0)	(5.12)	(7.36)	(5.07)				(5.66)			
1.22	-0.18	0.015	0.343	0.863		0.297			0.058	0.951	1.02	227
	(-0.32)	(11.4)	(5.24)	(6.91)		(4.68)			(5.5)			
1.23	1.51	0.016		0.908			0.32		0.057	0.952	0.99	289
	(8.25)	(19.5)		(14.8)			(10.4)		(5.58)			
1.24	1.3	0.014		0.88				0.336	0.058	0.955	1.07	305
	(7.96)	(20.1)		(15.4)				(10.8)	(5.83)			

t-statistics accompanied with low values of the calculated Durbin-Watson test-statistic suggest that spurious regressions are very likely.

The estimated elasticities of private output with respect to private fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15B and provided in Table16 below. The averages of those elasticities are 0.142, 0.549, 0.323, and 0.291, respectively. They should be interpreted as the percentage increases in private output when the private capital, private labor, public nonmilitary fixed capital, or public core infrastructure capital each increases by one percent, respectively, while holding the values of the other explanatory variables constant at their sample averages.

The estimated marginal products of private capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15B and given in Table16 below. The averages of those marginal products are 0.046, 14.84, 0.484, and 0.581, respectively. These numbers should be interpreted as the increases in private output when the private capital, private labor hour, public nonmilitary fixed capital, or public core infrastructure capital each increases by one unit, respectively, holding the values of the other explanatory variables constant at their sample averages. In other words, the marginal product of private capital of 0.046 means that private output increases by almost 5 cents when private capital increases by one dollar, ceteris paribus. However, the marginal products of public nonmilitary fixed capital and public core infrastructure capital of 0.48 and 0.58 mean that private output increases by 48 cents and 58 cents when public nonmilitary fixed capital and public core infrastructure capital each increases by one dollar, respectively, ceteris paribus. It is important to mention that the former increase is implausibly low, while the latter increases are implausibly high. The marginal product of private labor is 14.84, meaning that private output increases by $14.84 when one additional labor hour is employed while holding the other inputs constant at their sample averages.

Using Equations (24) and (25) again, the aggregate private nonresidential real output per unit of private nonresidential fixed capital (Y_{nr}/K_{nr}) is in double-logarithmic forms regressed on the following explanatory variables using annual data for the years 1947-2005. The equations are estimated by the ordinary least squares (OLS) method. The explanatory variables used are a constant and a time trend (T), the manufacturing sector's total capacity utilization rate (CU), the aggregate private sector labor hours per unit of private nonresidential fixed capital (L/K_{nr}), the aggregate public sector net stock of nonmilitary real fixed capital per unit of private labor hours (G/L) or the aggregate public sector net stock of real core infrastructure capital per unit of private labor hours (G_{ci}/L), and an intercept dummy variable (D) aimed at measuring the exogenous shocks to the private economy since 2001. The estimated results are provided in Table 15C below.

As can be seen from Table 15C, high values of the adjusted R^2 combined with very high values of the calculated standard F-statistics again *seem* to indicate that every estimated equation successfully explains more than 96 percent of the total variation in the aggregate private nonresidential real output per unit of private nonresidential real fixed capital using the variations in the explanatory variables. The high values of the calculated standard t-statistics *seem* to show that not only that every explanatory variable included is relevant but also that every estimated coefficient is statistically significant at almost all conventional significance levels (α = 0.10, 0.05, 0.025, or even 0.01). Moreover, all of the estimated coefficients have the expected signs. However, the low values of the calculated Durbin-Watson test-statistic show that the least squares residuals are strongly auto-correlated. Very high values of the adjusted R^2 and highly significant t-statistics accompanied by low values of the calculated Durbin-Watson test-statistic suggest that spurious regressions are very likely.

The estimated elasticities of private nonresidential real output with respect to private nonresidential fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15C and given in Table16 below. The averages of those elasticities are 0.149, 0.546, 0.311, and 0.299, respectively. They should be interpreted as the percentage increases in the private nonresidential real output when the private nonresidential fixed capital, private labor, public nonmilitary fixed capital, or public core infrastructure capital each increases by one percent, respectively, while holding the values of the other explanatory variables constant at their sample averages.

The estimated marginal products of private nonresidential fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15C and provided in Table16 below. The averages of those marginal products are 0.098, 14.77, 0.488, and 0.622, respectively. These numbers should be interpreted as the increases in the private nonresidential real output when the private nonresidential fixed capital, private labor hour, public nonmilitary fixed capital, or public core infrastructure capital each increases by one unit, respectively, holding the values of the other explanatory variables constant at their sample averages. In other words, the marginal product of private nonresidential fixed capital is 0.098, meaning that private nonresidential real output increases by almost 10 cents when private nonresidential fixed capital increases by one dollar, ceteris paribus. However, the marginal products of public nonmilitary fixed capital and public core

Table 15C: Empirical Estimates of Production Function Model with Level Data (*)

Dependent Variable: $\ln (Y_{nr} / K_{nr})$ with constant returns to scale across K, L, and G

Equat.	const.	time	ln CU	ln L/K_{nr}	ln G/L	ln G_{ci}/L	ln CU*G_{ci}/L	ln CU*G/L	D	adj R^2	D-W	F
	A) end-year values of public and private capitals											
1.25	1.07	0.014		0.784				0.286	0.041	0.963	0.93	382
	(8.21)	(17.9)		(14.2)				(11.4)	(4.38)			
1.26	1.24	0.016		0.809			0.274		0.039	0.961	0.86	362
	(8.65)	(17.7)		(13.7)			(11.0)		(4.03)			
1.27	0.16	0.016	0.233	0.874	0.33				0.038	0.963	0.87	304
	(0.3)	(10.7)	(3.15)	(6.88)	(5.62)				(3.56)			
1.28	0.17	0.016	0.251	0.847		0.29			0.037	0.961	0.83	285
	(0.28)	(9.6)	(3.29)	(6.36)		(5.14)			(3.39)			
	B) mid-year values of public and private capitals											
1.29	1.15	0.014		0.82				0.307	0.042	0.962	0.93	366
	(8.88)	(18.2)		(14.9)				(12.1)	(4.4)			
1.30	1.33	0.016		0.846			0.291		0.04	0.959	0.85	345
	(9.31)	(18.1)		(14.4)			(11.6)		(4.08)			
1.31	0.41	0.016	0.218	0.96	0.371				0.037	0.962	0.83	295
	(0.74)	(11.4)	(2.9)	(7.65)	(6.42)				(3.6)			
1.32	0.41	0.017	0.237	0.93		0.326			0.037	0.959	0.79	273
	(0.7)	(10.2)	(3.04)	(7.04)		(5.86)			(3.43)			
	C) beginning-year values of public and private capitals											
1.33	1.21	0.015		0.847				0.321	0.044	0.959	0.9	344
	(9.48)	(18.5)		(15.6)				(12.6)	(4.43)			
1.34	1.4	0.016		0.873			0.303		0.042	0.957	0.83	325
	(9.95)	(18.5)		(15.1)			(12.1)		(4.18)			
1.35	0.43	0.016	0.225	0.987	0.385				0.039	0.96	0.8	279
	(0.81)	(12.2)	(3.01)	(8.39)	(7.07)				(3.81)			
1.36	0.44	0.017	0.245	0.958		0.339			0.039	0.96	0.76	257
	(0.78)	(11.1)	(3.16)	(7.77)		(6.5)			(3.67)			

(*): Y_{nr} = real private non-residential output, K_{nr} = real private non-residential capital stock, and the other variables are defined earlier.

infrastructure capital of 0.488 and 0.622 mean that private nonresidential real output increases by 49 cents and 62 cents when public nonmilitary fixed capital and public core infrastructure capital each increases by one dollar, respectively, ceteris paribus. It is important to note that the former increase is still too low to be credible while the latter increases are too high to be credible. The marginal product of private labor is 14.77, meaning that private nonresidential real output increases by $14.77 when one additional labor hour is employed while holding the use of the other inputs constant at their sample averages.

Using Equations (26) and (27) again, the aggregate private nonresidential real output per unit of private labor (Y_{nr}/L) is in double-logarithmic forms regressed on the following explanatory variables using annual data for the years 1947-2005. The equations are estimated by the ordinary least squares (OLS) method. The explanatory variables used are a constant and a time trend (T), the manufacturing sector total capacity utilization rate (CU), the aggregate private nonresidential fixed capital per unit of private labor (K_{nr}/L), the aggregate public sector net stock of nonmilitary real fixed capital per unit of private labor (G/L) or the aggregate public sector net stock of real core infrastructure capital per unit of private labor (G_{ci}/L), and a dummy variable (D) aimed at measuring the exogenous shocks to the private economy since 2001. The estimated results are provided in Table 15D below.

As can be seen from Table 15D, again very high values of the adjusted R^2 statistics combined with very high values of the calculated standard F-statistics *seem* to indicate that every estimated equation highly successfully explains more than 99 percent of the total variation in the aggregate private nonresidential real output per unit of private labor using the variations in the explanatory variables. The high values of the calculated standard t-statistics *seem* to show that not only that every explanatory variable included is relevant but also that every estimated coefficient is highly statistically significant at almost all conventional significance levels $\alpha = 0.10, 0.05, 0.025$, or even 0.01. Moreover, all of the estimated coefficients have the expected signs. However, the low values of the calculated Durbin-Watson test-statistic show that the least squares residuals are strongly auto-correlated. Very high values of the adjusted R^2 and highly significant t-statistics accompanied by low values of the calculated Durbin-Watson test-statistic suggest that spurious regressions are very likely.

The estimated elasticities of private nonresidential real output with respect to private nonresidential fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15D and provided in Table16 below. The averages of those elasticities are 0.162, 0.539, 0.312, and 0.289, respectively. These numbers should be interpreted as the percentage increases in the private nonresidential real output when the private nonresidential fixed capital, private labor, public nonmilitary fixed capital, or public core infrastructure capital each increases by one percent, respectively, while holding the values of the other explanatory variables constant at their sample averages.

Table 15D: Empirical Estimates of Production Function Model with Level Data

Dependent Variable: $\ln(Y_{nr}/L)$ with constant returns to scale across K, L, and G

Equat.	const.	time	ln CU	ln K_{nr}/L	ln G/L	ln G_{ci}/L	ln CU*G_{ci}/L	ln CU*G/L	D	adj R^2	D-W	F
	A) end-year values of public and private capitals											
1.37	0.17	0.016	0.251	0.153		0.29			0.037	0.998	0.84	6709
	(0.28)	(9.6)	(3.3)	(1.15)		(5.14)			(3.39)			
1.38	0.16	0.015	0.233	0.126	0.33				0.038	0.998	0.87	7144
	(0.3)	(10.7)	(3.15)	(0.99)	(5.62)				(3.56)			
1.39	1.06	0.014		0.217				0.289	0.041	0.998	0.93	8981
	(8.21)	(17.9)		(3.91)				(11.4)	(4.38)			
1.40	1.24	0.016		0.191			0.274		0.039	0.998	0.86	8534
	(8.65)	(17.7)		(3.23)			(11.0)		(4.03)			
1.41	0.165	0.016	0.251	0.152		0.29			0.037	0.998	0.83	6709
	(0.28)	(9.6)	(3.3)	(1.15)		(5.14)			(3.39)			
	B) mid-year values of public and private capitals											
1.42	0.4	0.016	0.218	0.042	0.37				0.038	0.998	0.83	7040
	(0.7)	(11.4)	(2.89)	(0.33)	(6.4)				(3.61)			
1.43	0.41	0.017	0.237	0.071		0.325			0.037	0.998	0.79	6528
	(0.69)	(10.2)	(3.04)	(0.54)		(5.85)			(3.45)			
1.44	1.15	0.014		0.181				0.306	0.042	0.998	0.92	8735
	(8.87)	(18.2)		(3.29)				(12.1)	(4.41)			
1.45	1.33	0.016		0.155			0.29		0.04	0.998	0.85	8246
	(9.3)	(18.1)		(2.64)			(11.6)		(4.1)			
	C) beginning-year values of public and private capitals											
1.46	1.21	0.015		0.152				0.321	0.044	0.998	0.9	8346
	(9.48)	(18.5)		(2.8)				(12.6)	(4.42)			
1.47	1.39	0.016		0.127			0.303		0.042	0.998	0.83	7887
	(9.95)	(18.5)		(2.18)			(12.1)		(4.17)			

The estimated marginal products of private nonresidential fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 15D and given in Table16 below. The averages of those marginal products are 0.106, 14.57, 0.532, and 0.601, respectively. The marginal product of private nonresidential fixed capital is 0.106, meaning that private nonresidential real output increases by almost 11 cents when private nonresidential fixed capital increases by one dollar, ceteris paribus. However, the marginal products of public nonmilitary fixed capital and public core infrastructure capital of 0.532 and 0.601 mean that private nonresidential real output increases by 53 cents and 60 cents when public

Table 16: Estimated Private Output Elasticities and Marginal Products (*)

Equation	$\hat{e}_K$	$\hat{e}_L$	$\hat{e}_G$	$\hat{e}_{Gci}$	MPK	MPL	MPG	MPG_{ci}
Y 1.1	0.294	0.649	0.322		0.093	17.5	0.423	
Y 1.2	0.315	0.647		0.275	0.099	17.5		0.494
Y 1.3	0.282	0.665		0.293	0.089	17.9		0.654
Y 1.4	0.322	0.631	0.305		0.102	17.0	0.499	
Y 1.5	0.241	0.648		0.299	0.076	17.5		0.544
Y 1.6	0.218	0.659		0.311	0.069	17.8		0.703
Y 1.7	0.261	0.626	0.325		0.082	16.9	0.539	
Y 1.8	0.217	0.647	0.348		0.070	17.5	0.464	
Y 1.9	0.207	0.616	0.339		0.067	16.6	0.458	
Y 1.10	0.223	0.622		0.295	0.073	16.8		0.544
Y 1.11	0.170	0.644		0.321	0.055	17.4		0.736
Y 1.12	0.211	0.613	0.338		0.069	16.5	0.553	
Y 1.13	0.184	0.519	0.297		0.058	14.0	0.391	
Y 1.14	0.206	0.534		0.260	0.065	14.4		0.467
Y 1.15	0.143	0.565		0.292	0.045	15.3		0.651
Y 1.16	0.173	0.524	0.303		0.055	14.2	0.496	
Y 1.17	0.108	0.552	0.340		0.035	14.9	0.453	
Y 1.18	0.135	0.568		0.297	0.043	15.3		0.540
Y 1.19	0.109	0.581		0.310	0.035	15.7		0.701
Y 1.20	0.139	0.538	0.323		0.045	14.5	0.536	
Y 1.21	0.109	0.550	0.341		0.036	14.9	0.461	
Y 1.22	0.137	0.566		0.297	0.045	15.3		0.548
Y 1.24	0.120	0.544	0.336		0.039	14.7	0.565	
average	**0.197**	**0.596**	**0.326**	**0.295**	**0.063**	**16.1**	**0.487**	**0.598**
Y_{nr} 1.25	0.216	0.498	0.286		0.141	13.5	0.468	
Y_{nr} 1.26	0.191	0.535		0.274	0.124	14.5		0.612
Y_{nr} 1.27	0.126	0.544	0.330		0.082	14.7	0.434	
Y_{nr} 1.28	0.153	0.557		0.290	0.100	15.1		0.521
Y_{nr} 1.29	0.180	0.513	0.307		0.119	13.9	0.509	
Y_{nr} 1.30	0.154	0.555		0.291	0.102	14.9		0.658
Y_{nr} 1.33	0.153	0.526	0.321		0.103	14.2	0.540	
Y_{nr} 1.34	0.127	0.570		0.303	0.085	15.4		0.695
Y_{nr} 1.36	0.042	0.619		0.339	0.028	16.7		0.625
Y_{nr} 1.37	0.153	0.557		0.290	0.100	15.1		0.521
Y_{nr} 1.38	0.126	0.544	0.330		0.082	14.7	0.434	
Y_{nr} 1.39	0.217	0.494	0.289		0.141	13.3	0.645	
Y_{nr} 1.40	0.191	0.535		0.274	0.125	14.5		0.612
Y_{nr} 1.41	0.152	0.558		0.290	0.100	15.1		0.521
Y_{nr} 1.44	0.181	0.513	0.306		0.120	13.9	0.508	
Y_{nr} 1.45	0.155	0.555		0.290	0.102	14.9		0.656
Y_{nr} 1.46	0.152	0.527	0.321		0.102	14.2	0.540	
Y_{nr} 1.47	0.127	0.570		0.303	0.085	15.4		0.695
average	**0.155**	**0.543**	**0.311**	**0.294**	**0.102**	**14.7**	**0.510**	**0.612**

(*):Obtained from Tables 15. $\hat{e}_K$, $\hat{e}_L$, $\hat{e}_G$, **and** $\hat{e}_{Gci}$ are estimated elasticities of private output w.r.t. private capital and labor, and public nonmilitary and core infrastructure capital, respectively, and **MPK, MPL, MPG**, and **MPG_{ci}** are corresponding marginal products.

nonmilitary fixed capital and public core infrastructure capital each increases by one dollar, respectively, ceteris paribus. It is important to note that the former increase is still too low to be credible while the latter increases are too high to be credible. The marginal product of private labor is 14.57, meaning that private nonresidential real output increases by $14.57 when one additional labor hour is employed while holding the use of the other inputs constant at their sample averages.

We have used the word "*seem*" several times above to refer to the fact that when very high values of the adjusted R^2 and calculated standard F-statistics and highly significant t-statistics are accompanied with low values of the calculated Durbin-Watson test-statistic, the relationships found are less likely valid. Put differently, the equations are most likely mis-specified. A mis-specified model can, in general, be a result of i) *omitted variables* in which a relevant variable(s) is (are) missing, ii) *extraneous variables* in which an irrelevant variable(s) is (are) included, or iii) *strongly auto-correlated residuals* when the impacts of contemporaneous shocks to the dependent variable do not largely dissipate within the same time period or a combination of the above three problems.

IV.3. Econometric Problems and Their Solutions

We would like now not only to discuss some *econometric problems* associated with the estimated results obtained using level data but also to address them once the problems are detected. It is well known that macroeconomic level data result from uncontrolled experiments carried out by households, firms, and the government and therefore they can move together in a systematic way. When this happens, the macroeconomic variables are said to be collinear and the level data containing several variables might suffer from multi-collinearity, which can be a serious problem if the data are not rich enough to isolate the explanatory variables' individual effects from one another with a desired degree of precision even though the least squares estimator is still the best linear unbiased (BLUE) estimator. When an explanatory variable does not exhibit enough variation within the sample data, we can have a collinearity problem resulting from the linear association between that variable and the constant term if the constant is used and therefore it will be difficult to detect the impact of that variable on the dependent variable or to isolate the impact of that variable on the dependent variable from the impacts of the other explanatory variables.

One way to detect whether the data suffer from possible collinearity problems is to use sample correlation coefficients between pairs of explanatory variables to measure linear associations between them. If aggregate macroeconomic time series data are used, then a correlation coefficient between two explanatory variables of greater than 0.8 indicates a potentially harmful collinearity problem. Sample correlation coefficients, however, can not help to detect collinearity or multi-collinearity problems resulting from linear associations between an explanatory variable and the linear combination of the other explanatory variables. In this case, the suspected explanatory variable(s) should be regressed on the other explanatory variables called an auxiliary regression(s), and if R^2 statistics from the auxiliary regression(s) is (are) high or the sum of squared errors is low, then the data are said to suffer from collinearity or multi-collinearity problems. Our level data suffer from multi-collinearity problems since, for example, sample correlation coefficients between K and G_{ci} and K and T are -0.83 and -.082, respectively, in Equation 1.10. The corresponding equation using the first differenced data is Equation 2.13 in which the sample correlation coefficient between K and G_{ci} is -0.46, indicating that the new data no longer have the corresponding collinearity problem.

The calculated Durbin-Watson test-statistics, denoted by either D-W or d throughout this dissertation, reported in Tables 15A-15D, indicate that the error terms in every equation are first order positively and strongly correlated. Let us start discussing this in more detail by assuming that the error term in each equation can be best modeled by $u_t = ru_{t-1} + v_t$, where the new random component v_t is assumed to be independent identically distributed random variable with zero mean and constant variance. The autoregressive parameter r then determines how quickly the impact of a contemporaneous shock to the aggregate private real economy dissipates. We are assuming for now that the absolute value of r is less than one and therefore the first-order autoregressive error process is stationary. It is important to remind the reader that the larger the absolute value of r, the bigger the carryover from one period to the next.

The reported calculated Durbin-Watson test-statistic can be approximated by

$d = [[\sum_{t=2}^{n}(\hat{u}_t - \hat{u}_{t-1})^2]/\sum_{t=1}^{n}\hat{u}_t^2$ which is approximately equal to $2(1-\hat{r})$, where the $\hat{u}_t$ are the estimated least squares residuals given by the column vector $\mathbf{\hat{u}}_{nx1} = \mathbf{y}_{nx1} - \mathbf{X}_{nxk}\,\mathbf{\hat{a}}_{kx1}$, where **û** is an nx1 column vector of the residuals, **y** is an nx1 column vector of the values of the dependent variable, **X** is an nxk matrix of the values of the explanatory variables including the constant, and **â** is an kx1 column vector of the estimated coefficients. If $\hat{r} = 0$, then d is approximately equal to 2, indicating that the model errors are not auto-correlated. However, if $\hat{r} = 1$, then d is approximately equal to zero and thus any low value of d implies that errors are positively correlated. (If $\hat{r} = -1$, then d is approximately equal to 4 and hence any value of d close to 4 implies that errors are negatively correlated.)

To test for the existence of positive first-order auto-correlated errors, the null and alternative hypotheses are set as follows.[13] H_0: r = 0 vs. H_1: r > 0. If $d > d_u$, we fail to reject the null hypothesis and conclude that the sample data support the claim that no positive autocorrelation exists; if $d < d_L$, then we reject the null hypothesis and conclude that the sample data suggest that first-order autocorrelation exists; and if $d_L < d < d_u$, then the Durbin-Watson bounds test is inconclusive and hence another method should be used. Since at a five percent significance level the calculated Durbin-Watson test statistic for every equation reported in Tables 15A-15D is less than the lower bound of the critical test-statistic (for example, $d_L = 1.452$ when n = 55 and k = 4, $d_L = 1.414$ when n = 55 and k = 5, … , $d_L = 1.334$ when n = 55 and k = 7), we reject the null hypothesis and conclude that the sample data support the existence of a first-order positive autoregressive error process in every model.

Sampling properties of the ordinary least squares (OLS) estimator under auto-correlated errors are discussed in Appendix C below. It is important to remember that when errors are auto-correlated, the OLS estimator does not use the correct estimated covariance matrix to find the standard errors of the estimated coefficients. Therefore, it might overstate or understate the true sampling variability of the estimated coefficients and hence confidence intervals might be too wide or too narrow. Furthermore, hypothesis tests might reject a correct null hypothesis H_0 less or more often than suggested by the significance level of the test and therefore hypothesis tests can be misleading.

When one trended and non-stationary macroeconomic time series is regressed on another trended and non-stationary time series using the least squares estimator, the estimated results can be artificial and very much misleading. Granger and Newbold (1974) point out[14] that when high R^2 and significant t-statistics are accompanied with low d's, the estimated results might "look good" but the results have no real meaning because test statistics are not using the right estimated covariance matrix of the least squares estimates. Because of the artificial and misleading natures of such regressions, Granger and Newbold (1974) call them "*spurious regressions*," a name that has been used subsequently.

When some of the explanatory variables are not stationary in general or are random walks in particular, then the matrix **X'X**/n does not converge to any limiting value and therefore the least squares estimator can not produce consistent results, and the usual inference procedures fail. To show this, Granger and Newbold (1974, pp. 116-17) conduct several Monte Carlo experiments in which several non-stationary series y_t, x_{1t}, x_{2t}, x_{3t}, x_{4t}, and x_{5t} containing 50 observations each are generated as *independent random walks*. Then, they regress y_t on each x_{it} separately and find that $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$, $\hat{a}_4$, and $\hat{a}_5$ are each found to be statistically *significant* 76, 78, 93, 95, and 96 percent of the time, respectively, with the average values of d of 0.32, 0.46, 0.55, 0.74, and 0.88 while in fact no significant relationship existed. They then repeat the experiment using not level values but first differences and find that $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$, $\hat{a}_4$, and $\hat{a}_5$ are each found to be statistically *insignificant* 92, 96, 98, 90, and 94 percent of the time with d = 2.00, 1.99, 1.91, 2.01, and 1.99, respectively. Granger and Newbold (1974, pp. 116-17), therefore, conclude that "*…if one's variables are random walks or near random walks and one includes in regression equations variables which should in fact not be included, then **it will be the rule** rather than the exception to find spurious relationships. It is also clear that a high value for R^2 or adjusted R^2, combined with a low value of d, is **no indication of a true relationship.** …If a regression equation relating economic variables is found to have strongly autocorrelated residuals, equivalent to a low Durbin-Watson value, the*

***only conclusion that can be reached is that the equation is mis-specified**, whatever the value of R^2 observed."*

A mis-specified equation can, in general, be the result of i) omitted variable model in which a relevant variable(s) is(are) missing, ii) extraneous variable model in which an irrelevant variable(s) is(are) included, or iii) *strongly* auto-correlated residuals when the impacts of contemporaneous shocks to the variable of interest do not largely dissipate within the same time period or a combination of the three. Once the model is found to be a mis-specified equation, then either the common trend needs to be removed, a lagged dependent variable can be used as an explanatory variable if justified, or the relevant assumption of the estimation method is modified to allow a simple first-order autoregressive form for the residuals. For example, instead of using the ordinary least squares estimator, one can use an estimated generalized least squares estimator to produce the "true" covariance matrix of the estimates.

In the presence of integrated time series of order one or more, conventional t and F test-statistics do not follow standard t- and F-distributions when a null hypothesis is true. Therefore, the standard critical values are not appropriate or biased. Moreover, the least squares estimator cannot produce consistent estimates. Hence, one must de-trend the series before the estimation stage. The de-trending can be done in two ways, i) estimation of time-trend regressions or ii) differencing series once if it is integrated of order one or twice if it is integrated of order two, and so on and so forth.

If the trended series is a function of time of the form $z_t = a_1 + a_2T + u_t$, where u_t are the error terms, then the estimated least squares residuals $\hat{u}_t = z_t – \hat{a}_1 – \hat{a}_2T$ form a corresponding de-trended series called *trend stationary* (TS) processes. Conversely, if z_t is in fact generated by a random walk process with a drift of the form $z_t = z_{t-1} + a_2 + u_t$, then the de-trending can be achieved by first-differencing of the form $\Delta z_t = z_t – z_{t-1} = a_2 + u_t$ called *difference stationary* (DS) processes. In other words, a trend stationary series can be transformed into a stationary series by removing the deterministic time trend while a difference stationary series can be transformed into a stationary series by differencing. Therefore, it is important to determine how the trended series z_t is generated so that one can use the appropriate de-trending process. It is important to emphasize that if z_t is in fact a random walk with a drift and we use the estimated least squares residuals to form a new series that we believe to be stationary, the new series will not be stationary. Similarly, if z_t is in fact a trend stationary process and we use differencing to generate a new series that we believe to be stationary, the new series will not be stationary.

Dickey and Fuller (1979 and 1981) develop a test called the Dickey-Fuller test. This test was used, for example, by Nelson and Plosser (1982) to determine if a time series is a difference stationary or a trend stationary process. We will use a similar procedure to determine whether the aggregate private real output series, the aggregate private real fixed capital series, the aggregate private total labor hour series, the aggregate public sector real nonmilitary fixed capital series, and the aggregate public sector real core infrastructure capital series for the years 1947-2005 are trend stationary or difference stationary processes as follows.

Let $z_t = a_1 + a_2T + a_3z_{t-1} + u_t$, where u_t is an independent identically distributed random variable with zero mean and constant variance, i.e., where $u_t \sim iid(0, \sigma^2)$. If $a_2 = 0$ and $a_3 = 1$, then z_t reduces to $z_t = a_1 + z_{t-1} + u_t$, which is a random walk with a drift, i.e., a difference stationary process. Conversely, if a_3 is less than one in absolute value, then z_t is a trend stationary process. Therefore, to determine whether z_t is a trend or difference stationary process, we need to test a joint null hypothesis of the form H_0: $a_2 = 0$ and $a_3 = 1$.

To test the joint null hypothesis H_0: $a_2 = 0$ and $a_3 = 1$, Dickey and Fuller (1981) determine the distribution of the standard F-statistic under the assumption that the null hypothesis is true and report the critical values, called the Dickey-Fuller critical values thereafter, in Table VI of the same study (1981, p. 1063). To illustrate the use of Table VI, Dickey and Fuller (1981, p. 1070) then study the logarithm of the quarterly Federal Reserve Board Production Index (1950:1 through 1977:4) by assuming that the time series is adequately represented by a model of the form $z_t = a_1 + a_2T + a_3z_{t-1} + a_4(z_{t-1} - z_{t-2}) + u_t$, where u_t are independent identically distributed $(0, \sigma^2)$ random variables.

To determine whether the aggregate private real output series, the aggregate private real fixed capital series, the aggregate private total labor hour series, the aggregate public sector real nonmilitary fixed capital series, and the aggregate public sector real core infrastructure capital series for the years 1947-2005 are trend stationary or difference stationary processes, we have first estimated an unrestricted model of the form $z_t - z_{t-1} = a_1 + a_2T + (a_3-1)z_{t-1} + a_4(z_{t-1} - z_{t-2}) + u_t$ and a restricted model of the form

$z_t - z_{t-1} = b_1 + b_2(z_{t-1} - z_{t-2}) + u_t$, where u_t are independent identically distributed random variables with zero mean and constant variance, by the ordinary least squares estimator. We have then used the sum of squared errors from the restricted least squares estimation, denoted by SSE_R, and the sum of squared errors from the unrestricted least squares estimation, denoted by SSE_U, to conduct the Dickey-Fuller tests to test the joint null hypothesis of the form H_0: $a_2 = 0$ and $a_3 = 1$.

We have also used an extension of the procedure described in the above paragraph suggested by Dickey and Pantula (1987) of the form $\Delta^2 z_t = a_1 + a_2 T + (a_3\text{-}1)\, \Delta z_{t-1} + a_4 \Delta^2 z_{t-1} + u_t$, where u_t are iid ~ $(0, \sigma^2)$, to determine whether the aforementioned aggregate macroeconomic variables are integrated of order one, denoted by I(1), processes containing exactly one unit root each or integrated of order two, denoted by I(2), processes containing two unit roots each. We have found that all of the aforementioned aggregate macroeconomic variables are difference stationary processes. The aggregate private real output series, the aggregate private total labor hour series, and the aggregate public sector real core infrastructure capital series are I(1) processes while the aggregate public sector real nonmilitary fixed capital series is a I(2) process. The aggregate private real fixed capital series, on the other hand, is either I(1) or I(2) process depending on how much autocorrelation in the residuals can be tolerated. Here are the details of our results.

$\Delta\hat{Y}_t = 2.022 + 2.685\ T + 0.005\ Y_{t-1} + 0.213\ \Delta Y_{t-1}$, $SSE_U = 451058$, D-W=1.90, adj.R^2 =0.36
(0.08)   (0.86)   (0.20)   (1.54)

$\Delta\hat{Y}_t = 67.77 + 0.499\ \Delta Y_{t-1}$, $SSE_R = 562048$, D-W = 1.99, adj.R^2 = 0.23
(3.34)   (4.15)

$u = [(SSE_R - SSE_U) / J] / [SSE_U / (n\text{-}3\text{-}k)] = [(562048 - 451058)/ 2] / (451058/ 52) = 6.39$, where J is the number of restrictions in the joint null hypothesis, n is the number of observations, and k is the number of estimated coefficients in the unrestricted model. When n = 50, the 90, 95, 97.5, and 99 per cent points of the distributions given in Table VI of Dickey and Fuller (1981, p. 1063) are 5.61, 6.73, 7.81, and 9.31, respectively. Therefore, since the calculated Dickey-Fuller test-statistic u = 6.39 is less than the critical Dickey-Fuller test-statistics $u_{\alpha,J,(n-3-k)} = u_{\alpha,2,50}$ at almost all conventional significance levels α = 0.05, 0.025, or 0.01, we fail to reject the joint null hypothesis and hence conclude that the aggregate private real output series for the years 1947-2005 is an integrated of order one difference stationary process. Thus, to avoid spurious results, one should neither de-trend the series using a time trend regression discussed above nor use the un-differenced level values in a regression.

It is very important to take a moment here to remind the reader that failing to reject the joint null hypothesis does not necessarily mean that the joint null hypothesis is true since there is a chance of a Type II error, accepting the null hypothesis when in fact the alternative hypothesis is true. Similarly, rejecting the joint null hypothesis does not necessarily mean that the general alternative hypothesis is true since there is a chance of a Type I error, rejecting the null hypothesis when in fact the null hypothesis is true.

$$\underset{}{\Delta \ln \hat{K}_t} = \underset{(0.7)}{0.09} + \underset{(0.4)}{0.0002T} - \underset{(-0.58)}{0.009 \ln \hat{K}_{t-1}} + \underset{(5.7)}{0.616\, \Delta \ln \hat{K}_{t-1}}, \quad SSE_U = 0.00099, \text{ D-W} = 1.60, \text{ adj.}R^2 = 0.58$$

$$\Delta \ln \hat{K}_t = \underset{(2.82)}{0.0079} + \underset{(8.46)}{0.738\, \Delta \ln \hat{K}_{t-1}}, \quad SSE_R = 0.00107, \text{ D-W} = 1.65, \text{ adj.}R^2 = 0.56$$

Even though the calculated Dickey-Fuller test-statistic u = 2.02 is less than the critical Dickey-Fuller test-statistic $u_{\alpha,2,50}$ at all conventional significance levels α = 0.10, 0.05, 0.025, or 0.01, not rejecting the joint null hypothesis here does not necessarily mean that the aggregate private real fixed capital series is an integrated of order one difference stationary process since the calculated Durbin-Watson test-statistic falls into the region where the Durbin-Watson bounds test is inconclusive, i.e., $d_L = 1.45 < 1.60 < 1.68 = d_U$. Therefore, we have applied the procedure suggested by Dickey and Pantula (1987) to test whether it is an integrated of order two or higher process as follows.

$$\Delta^2 \ln \hat{K}_t = \underset{(4.04)}{0.02} - \underset{(-2.8)}{0.0001T} - \underset{(-4.15)}{0.469\, \Delta \ln \hat{K}_{t-1}} + \underset{(2.1)}{0.27\, \Delta^2 \ln \hat{K}_{t-1}}, \quad SSE_U = 0.0009, \text{ D-W} = 1.72, \text{ adj.}R^2 = 0.21$$

$$\Delta^2 \ln \hat{K}_t = \underset{(-0.243)}{-0.0002} + \underset{(0.287)}{0.038\, \Delta^2 \ln \hat{K}_{t-1}}, \quad SSE_R = 0.0011, \text{ D-W} = 1.72, \text{ adj.}R^2 = -0.02$$

Since the calculated Dickey-Fuller test-statistic u = 5.56 is less than the critical Dickey-Fuller test-statistic $u_{\alpha,2,50}$ at all conventional significance levels α = 0.10, 0.05, 0.025, or 0.01, we fail to reject the joint null hypothesis H_0: $a_2 = 0$, $a_3 = 1$ and therefore conclude that the aggregate private real fixed capital stock series for the years 1947-2005 is an integrated of order two difference stationary process.

$$\Delta \hat{L}_t = \underset{(3.21)}{1.002} + \underset{(3.26)}{0.003}\, T - \underset{(-3.19)}{0.222}\, \hat{L}_{t-1} + \underset{(1.79)}{0.226}\, \Delta \hat{L}_{t-1},\ SSE_U = 0.02664,\ D\text{-}W = 1.87,\ adj.R^2 = 0.14$$

$$\Delta \hat{L}_t = \underset{(2.57)}{0.009} + \underset{(1.20)}{0.159}\, \Delta \hat{L}_{t-1},\ SSE_R = 0.03199,\ D\text{-}W = 1.87,\ adj.R^2 = 0.01$$

Since the calculated Dickey-Fuller test-statistic u = 5.02 is less than the critical Dickey-Fuller test-statistic $u_{\alpha,2,50}$ at all conventional significance levels α = 0.10, 0.05, 0.025, or 0.01, we fail to reject the joint null hypothesis H_0: $a_2 = 0$, $a_3 = 1$ and therefore conclude that the aggregate private sector total labor hour series for the years 1947-2005 is an integrated of order one difference stationary process.

$$\Delta \hat{G}_t = \underset{(3.22)}{12.17} + \underset{(2.01)}{1.19}\, T - \underset{(-1.96)}{0.015}\, \hat{G}_{t-1} + \underset{(19.95)}{0.932}\, \Delta \hat{G}_{t-1},\ SSE_U = 2169,\ D\text{-}W=1.01,\ adj.R^2 =0.94$$

$$\Delta \hat{G}_t = \underset{(0.99)}{6.697} + \underset{(26.6)}{0.935}\, \Delta \hat{G}_{t-1},\ SSE_R = 2337,\ D\text{-}W = 0.96,\ adj.R^2 = 0.94$$

Very high adjusted R^2 and highly significant t-statistics combined with a low value of the calculated Durbin-Watson test-statistics are clear indications of the existence of a second unit root. Thus, we have applied the procedure suggested by Dickey and Pantula (1987) to test whether the series is an integrated of order two process by estimating the following equations.

$$\Delta^2 \hat{G}_t = \underset{(1.61)}{5.39} + \underset{(-1.39)}{0.084}\, T - \underset{(-1.81)}{0.096}\, \Delta \hat{G}_{t-1} + \underset{(4.25)}{0.536}\, \Delta^2 \hat{G}_{t-1},\ SSE_U= 1649,\ D\text{-}W=1.77,\ adj.R^2 =0.27$$

$$\Delta^2 \hat{G}_t = \underset{(0.55)}{0.442} + \underset{(4.12)}{0.507}\, \Delta^2 \hat{G}_{t-1},\ SSE_R = 1829,\ D\text{-}W = 1.72,\ adj.R^2 = 0.23$$

Since the calculated Dickey-Fuller test-statistic u = 2.84 is less than the critical Dickey-Fuller test-statistic $u_{\alpha,2,50}$ at all conventional significance levels α = 0.10, 0.05, 0.025, or 0.01, we fail to reject the joint null hypothesis H_0: $a_2 = 0$, $a_3 = 1$ and therefore conclude that the aggregate public sector real nonmilitary fixed capital stock series for the years 1947-2005 is an integrated of order two difference stationary process.

$$\Delta \hat{G}_{ci,t} = \underset{(3.37)}{17.37} + \underset{(2.63)}{1.68}\, T - \underset{(-2.59)}{0.031}\, \hat{G}_{ci,t-1} + \underset{(17.62)}{0.925}\, \Delta \hat{G}_{ci,t-1}, \quad SSE_U = 2124,\ D\text{-}W = 2.01,\ adj.R^2 = 0.88$$

$$\Delta \hat{G}_{ci,t} = \underset{(1.12)}{5.93} + \underset{(26.1)}{0.912}\, \Delta \hat{G}_{ci,t-1}, \quad SSE_R = 2403,\ D\text{-}W = 1.82,\ adj.R^2 = 0.88$$

Since the calculated Dickey-Fuller test-statistic u = 3.42 is less than the critical Dickey-Fuller test-statistic $u_{\alpha,2,50}$ at all conventional significance levels α = 0.10, 0.05, 0.025, or 0.01, we fail to reject the joint null hypothesis H_0: $a_2 = 0$, $a_3 = 1$ and therefore conclude that the aggregate public sector real core infrastructure capital stock series for the years 1947-2005 is an integrated of order one difference stationary process.

Many macro econometric studies have employed co-integration since 1987 when Engle and Granger (1987) introduced the concept of co-integration because the ordinary least squares regression of a co-integrated dependent variable on the other co-integrated explanatory variables can produce "super consistent" estimates. Stock (1987) proves that if one non-stationary but co-integrated dependent variable is regressed on non-stationary but co-integrated explanatory variables using the ordinary least squares estimator, then the estimated coefficients can converge to their true values faster than if stationary data are used to carry out the ordinary least squares estimation. Therefore, if a set of variables is non-stationary, we need to check whether there exists a linear combination of these variables that is stationary, i.e., we need to check whether these variables are co-integrated. If they are co-integrated, then it is preferable to use the non-stationary data in the regression process not only to obtain "super consistent" estimates but also to avoid possible mis-specification error since if a linear combination is already stationary, then differencing or de-trending the data might result in a mis-specification error. Consequently, it is important to take a moment here now to discuss the concept of co-integration and tell the reader that we have considered the use of this tool in our regression analysis.

Engle and Granger (1987) produce the following definition of co-integration. The components of the vector $\mathbf{X}_t = (\mathbf{X}_{1t}, \mathbf{X}_{2t}, \ldots, \mathbf{X}_{kt})'$ are said to be co-integrated of order m, n, denoted by $\mathbf{X}_t \sim CI(m, n)$, if i) every component of $\mathbf{X}_t$ is integrated of order m and ii) there exists a vector $\mathbf{b} = (b_1, b_2, \ldots, b_k)$ such that $\mathbf{b}\mathbf{X}_t = b_1\mathbf{X}_{1t} + b_2\mathbf{X}_{2t} + \ldots + b_k\mathbf{X}_{kt}$ is integrated of order (m-n), where n > 0.

The definition implies the following three important observations. First, every variable in the set must be integrated of the same order. That is, if every variable in the set is not integrated of the same order, then the variables cannot be co-integrated. Therefore, the necessary but not sufficient condition for a set of variables to be co-integrated is that every variable in the set must be integrated of the same order. Second, co-integration refers to a linear combination of non-stationary variables integrated of the same order and therefore the co-integration vector **b** is not unique since $\mathbf{b}\mathbf{X}_t = b_1\mathbf{X}_{1t} + b_2\mathbf{X}_{2t} + \ldots + b_k\mathbf{X}_{kt}$ implies $a\mathbf{b}\mathbf{X}_t = ab_1\mathbf{X}_{1t} + ab_2\mathbf{X}_{2t} + \ldots + ab_k\mathbf{X}_{kt}$ for any nonzero a, meaning that if **b** is a co-integrating vector, then so is a**b**. Third, if $\mathbf{X}_t$ represents k non-stationary variables integrated of the same order, then there can at most be k-1 linearly independent co-integrating vectors, and the number of co-integrating vectors is called the co-integrating rank of $\mathbf{X}_t$.

We have found that the aggregate private real output series, the aggregate private sector total labor hour series, and the aggregate public sector core infrastructure capital stock series are integrated of order one processes, whereas the aggregate private sector real fixed capital stock series and the aggregate public sector real nonmilitary fixed capital stock series are integrated of order two processes. Therefore, the necessary condition for the set of five macroeconomic variables to be co-integrated is not met. Consequently, we did not need to check whether there existed a linear combination of these non-stationary variables that was stationary since we knew that such linear combination did not exist.

We have re-estimated Equations (22) and (23) using the ordinary least squares estimator with first-differenced data and the estimated results are provided in Table 17A below. As can be seen from Table 17A, relatively high values of the adjusted R^2 combined with very high values of the calculated standard F-statistics clearly show that every estimated equation successfully explains, on average, more than 69 percent of the total variation in the aggregate private real output using the variations in the explanatory variables. The high values of the calculated standard t-statistics clearly show that not only that virtually every explanatory variable included is relevant but also that virtually every estimated coefficient is highly statistically significant at almost all conventional significance levels. Moreover, all of the estimated coefficients have their expected signs.

The values of the calculated Durbin-Watson test-statistic show that the least squares residuals are *not* auto-correlated for any equation that does not contain the dummy variable except Equation 2.3 where the calculated Durbin-Watson test-statistic falls into the region where the Durbin-Watson bounds test is inconclusive. However, six

Table 17A: Empirical Estimates of Production Function Model with First Differences (*)

Dependent Variable: ln (Δ Y) with unrestricted input elasticities

Equat.	const.	Δ CU	Δ L	ln ΔK	ln ΔG	ln ΔG_{ci}	ln $\Delta CU{*}G_{ci}$	ln ΔCU*G	D	adj R^2	D-W	F
	A) end-year values of public and private capitals											
2.1	-3.1	0.158	0.212	0.941	0.372					0.686	2.25	33
	(-1.9)	(3.18)	(3.91)	(2.11)	(0.88)							
2.2	-0.7		0.297	0.626			0.252		1.25	0.716	2.32	38
	(-0.4)		(5.38)	(1.89)			(2.2)		(2.44)			
2.3	-0.26	0.111	0.296	0.433	0.346					0.717	2.42	30
	(-.14)	(2.19)	(4.89)	(.93)	(.86)							
2.4	-0.37	0.114	0.287	0.625	0.129				1.31	0.714	2.39	30
	(-.2)	(2.25)	(4.8)	(1.56)	(.39)				(2.57)			
	B) mid-year values of public and private capitals											
2.5	-2.1	0.156	0.242	0.495	0.734					0.676	2.21	31
	(-1.3)	(3.05)	(4.7)	(1.12)	(1.68)							
2.6	-2.2	0.159	0.238	0.747		0.468				0.669	2.21	30
	(-1.4)	(3.1)	(4.58)	(1.97)		(1.3)						
2.7	-2.5		0.248	0.921			0.346			0.664	2.18	39
	(-1.5)		(5.31)	(3.26)			(3.19)					
2.8	0.61		0.332	0.408			0.204		1.48	0.703	2.31	35
	(0.3)		(6.27)	(1.27)			(1.79)		(2.85)			
2.9	-2.6		0.232	0.895				0.381		0.685	2.32	43
	(-1.6)		(5.09)	(3.28)				(3.8)				
2.10	0.21		0.307	0.433				0.266	1.38	0.721	2.41	38
	(.12)		(6.05)	(1.42)				(2.59)	(2.8)			
	C) beginning-year values of public and private capitals											
2.11	-1.3	0.158	0.25	0.397	0.698					0.665	2.16	30
	(-0.8)	(3.04)	(5.02)	(0.98)	(1.88)							
2.12	-1.5	0.162	0.252	0.609		0.49				0.658	2.15	29
	(-0.9)	(3.07)	(5.0)	(1.7)		(1.54)						
2.13	1.34	0.103	0.328	0.196	0.304				1.52	0.703	2.32	29
	(.75)	(3.02)	(6.15)	(.55)	(1.01)				(3.02)			
2.14	-1.3		0.291	0.758			0.247			0.631	2.21	34
	(-0.8)		(6.33)	(2.65)			(2.53)					
2.15	-2.4		0.217	0.781				0.506		0.702	2.25	47
	(-1.6)		(4.76)	(3.05)				(4.61)				
2.16	0.2		0.287	0.376				0.369	1.31	0.733	2.3	41
	(0.12)		(5.69)	(1.32)				(3.18)	(2.7)			

(*): Δ = first differences and the other variables are defined earlier.

equations contain the dummy variable and the calculated Durbin-Watson test-statistics for these equations fall into the open interval (2.28, 2.59) where the Durbin-Watson bounds test is inconclusive, but four of the six calculated Durbin-Watson test-statistics are 2.30, 2.31, 2.32, and 2.32, which are clearly very close to the lower bound of the inconclusive region. Moreover, the other two, 2.39 and 2.41, are closer to the lower bound of the inconclusive interval than the upper bound. Therefore, it is safe to conclude that first-differencing of data has produced reliable and valid results.

As can be seen from Table 17A, four equations are estimated using the end-year estimates of the values of private and public fixed capital, six equations are estimated using the mid-year estimates of the corresponding values, and six equations are estimated using the beginning-year estimates of the corresponding values. Table 17A together with Table 18 below supports the claim that the causation runs from the public capital stock to private output, not the other way around.

Let us take a moment here to briefly describe the methodology that has been used throughout this dissertation to calculate the elasticities and the marginal products. Let Y be given by $Y = A_1^u A_2^w$, where u and w are the elasticities of Y with respect to A_1 and A_2, respectively, which is not linear in u and w and therefore cannot be estimated by the ordinary least squares (OLS) estimator to obtain the estimates for u and w. The double-natural-logarithmic transformation of the given relationship, however, produces a new relationship $\ln Y = u\ln A_1 + w\ln A_2$, which is linear in u and w and therefore can be estimated by the OLS estimator to obtain estimates for u and w. Then the OLS estimates $\hat{u}$ for u and $\hat{w}$ for w will be the estimated elasticities of Y with respect to A_1 and A_2, respectively. The estimated marginal products of A_1, denoted by MPA_1, and the estimated marginal products of A_2, denoted by MPA_2, then can be found by $MPA_1 = \hat{u}(\bar{Y}/\bar{A}_1)$ and $MPA_2 = \hat{w}(\bar{Y}/\bar{A}_2)$, respectively, where the "bar" above the variable stands for the sample average of the corresponding variable.

Logarithmic transformation produces a desirable result here, but it can sometimes come with a heavy price since the logarithmic operator is only defined on positive real numbers. When the natural logarithm of first-differenced values of a macroeconomic variable is used, then sometimes too much information can be lost because of the fact that the logarithmic operator is not defined on the negative real numbers. If the model cannot afford to lose that much information and there is no a better solution, then the model can be estimated of the form $\ln(\Delta Y_t) = u\ln(\Delta A_{1,t}) + w(\Delta A_{2,t})$. Then, the estimated elasticities of Y with respect to A_1 and A_2 will be given by $\hat{u}$ and $\hat{w}(\overline{\Delta A_2})$, respectively, and the marginal products of A_1 and A_2 will be given by $\hat{u}(\overline{\Delta Y} / \overline{\Delta A_1})$ and $\hat{w}(\overline{\Delta Y})$, respectively.

The estimated elasticities of private real output with respect to private fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 17A and provided in Table18 below. The averages of those elasticities are 0.603, 0.401, 0.411, and 0.335, respectively. These numbers should be interpreted as the percentage increase in the private real output when the private fixed capital, private labor, public nonmilitary fixed capital, or public core infrastructure capital each increases by one percent, respectively, while holding the values of the other explanatory variables constant at their sample averages. Several observations are noteworthy here. First, the two measures of public fixed capital are no longer more important to the private production than the private fixed capital stock since a ten percent increase in any of the two measures of public capital results in a four percent increase in private output while a ten percent increase in the private fixed capital stock results in a six percent increase in private output. Second, the estimated elasticity of private output with respect to private labor is less than the estimates reported in the literature or found by the U.S. Bureau of Labor Statistics. Third, the estimated elasticity of private output with respect to private capital is larger than any other estimates in the literature that we are aware of.

The estimated marginal products of private fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 17A and provided in Table18 below. The averages of those marginal products are 0.230, 34.60, 0.745, and 0.929, respectively. Each of them should be interpreted as the increase in private output when the private fixed capital stock, private labor hours, public nonmilitary fixed capital stock, or public core infrastructure capital stock each increases by one unit, respectively, ceteris paribus. In other words, the marginal product of private capital of 0.230 means that private output increases by 23 cents when the private fixed capital stock increases by one dollar while holding the values of the other explanatory variables constant at their sample averages. It is very important to emphasize that this finding seems to be credible. Similarly, the marginal products of public nonmilitary fixed capital and public core infrastructure capital of 0.745 and 0.929 mean that private real output increases by 75 cents and 93 cents, respectively, when public nonmilitary fixed capital and public core infrastructure capital each increases by one dollar, ceteris paribus. Although the marginal product of private fixed capital is less than that of public nonmilitary fixed capital which is less than that of public core infrastructure capital, this does not mean that public core infrastructure capital is more important to the private production than public nonmilitary fixed capital and so on; this outcome is an expected result of the *diminishing marginal productivity of each input*. In other words, since the private fixed capital stock is, on average, 4.2 times bigger than the public nonmilitary fixed capital stock, it might be plausible to have an outcome indicating that the marginal product of public nonmilitary capital stock is 3.2 times bigger than that of private fixed capital stock. Similarly, since the private fixed capital stock is, on average, 5.7 times larger than the public core infrastructure capital stock, it might be reasonable to obtain an outcome implying that the marginal product of public core infrastructure capital is 4 times bigger than that of the private fixed capital stock. The marginal product of private labor is 34.6, meaning that private output increases by $34.60 when one additional labor hour is employed while holding the use of the other inputs constant at their sample averages.

Equations (22) and (23) are re-estimated using the ordinary least squares estimator with first-differenced data for the private nonresidential real output and the estimated results are provided in Table 17B below. Table 17B contains the *best* estimated results of this section, in which high values of the adjusted R^2 combined with very high values of the calculated standard F-statistics clearly show that every estimated equation successfully explains, on average, more than 73 percent of the total variation in the aggregate private nonresidential real output. The high values of the calculated standard t-statistics clearly show that not only that every explanatory variable included is relevant but also that every estimated coefficient of those explanatory variables is statistically significant at rigorous significance levels. Furthermore, all of the estimated coefficients have their expected signs. Moreover, six of the nine estimated equations do not contain the dummy variable and the calculated Durbin-Watson test-statistics for them are 1.97, 1.99, 2.02, 2.04, 2.08, and 2.09, each of which is almost 2.00 which is the result that can be obtained when the autoregressive parameter r in the first-order representation of the residuals $u_t = ru_{t-1} + v_t$, where $v_t \sim iid(0, \sigma^2)$, is zero. However, three equations contain the dummy variable and the calculated Durbin-Watson test-statistics for them are 2.34, 2.37, and 2.40, which fall into the open interval (2.28, 2.59) where the Durbin-Watson bounds test is inconclusive, but they are closer to the lower bound of the inconclusive region than the upper bound. Therefore, it is safe to conclude that first-differencing of data has revealed reliable and valid relationships.

Table 17B: Empirical Estimates of Production Function Model with First Differences

Dependent Variable: $\ln(\Delta Y_{nr})$ with unrestricted input elasticities

Equation	constant	ΔL	$\ln \Delta K_{nr}$	$\ln(\Delta CU*G_{ci})$	$\ln(\Delta CU*G)$	D	adj R^2	D-W	F
	A) end-year values of public and private capitals								
2.17	-1.4	0.201	0.823		0.332		0.733	2.09	54
	(-1.3)	(4.99)	(4.13)		(3.64)				
2.18	0.24	0.278	0.556		0.199	1.26	0.781	2.37	53
	(0.22)	(6.57)	(2.85)		(2.21)	(3.6)			
2.19	-0.99	0.227	0.812	0.259			0.707	2.02	48
	(-0.86)	(5.53)	(3.85)	(2.72)					
	B) mid-year values of public and private capitals								
2.20	-0.72	0.244	0.741		0.256		0.712	2.08	49
	(-0.67)	(6.33)	(3.77)		(2.98)				
2.21	0.87	0.314	0.47		0.131	1.36	0.769	2.4	49
	(0.83)	(8.04)	(2.48)		(1.57)	(3.83)			
2.22	-0.48	0.259	0.731	0.218			0.696	1.99	45
	(-0.43)	(6.61)	(3.6)	(2.35)					
	C) beginning-year values of public and private capitals								
2.23	-0.83	0.242	0.708		0.333		0.728	2.04	53
	(-0.82)	(6.35)	(3.85)		(3.57)				
2.24	0.65	0.308	0.472		0.193	1.28	0.777	2.34	51
	(0.64)	(7.89)	(2.64)		(2.08)	(3.62)			
2.25	0.04	0.294	0.667	0.147			0.684	1.97	43
	(0.04)	(7.94)	(3.34)	(1.82)					

The estimated elasticities of private nonresidential real output with respect to private nonresidential fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 17B and given in Table18 below. The averages of those elasticities are 0.664, 0.391, 0.241, and 0.208, respectively. Several things are worth noting here. First, the two measures of public fixed capital are no longer more important to the private non-residential output than the private nonresidential fixed capital. Second, the estimated elasticity of private nonresidential real output with respect to private labor is less than those reported in the

Table 18: Estimated Private Output Elasticities and Marginal Products with First Differences of Data (*)

Equation	$\hat{e}_K$	$\hat{e}_L$	$\hat{e}_G$	$\hat{e}_{Gci}$	MPK	MPL	MPG	MPG_{ci}
ΔY 2.1	0.941	0.315	0.372		0.355	27.2	0.616	
ΔY 2.2	0.626	0.442		0.252	0.236	38.2		0.772
ΔY 2.3	0.433	0.441	0.346		0.164	38.1	0.572	
ΔY 2.4	0.625	0.427	0.129		0.235	36.9	0.213	
ΔY 2.5	0.495	0.360	0.734		0.189	31.1	1.220	
ΔY 2.6	0.747	0.354		0.468	0.285	30.6		1.130
ΔY 2.7	0.921	0.360		0.346	0.351	31.1		1.070
ΔY 2.8	0.408	0.493		0.204	0.156	42.6		0.631
ΔY 2.9	0.895	0.345	0.381		0.341	29.8	0.812	
ΔY 2.10	0.433	0.457	0.266		0.165	39.5	0.443	
ΔY 2.11	0.397	0.373	0.698		0.153	32.2	1.170	
ΔY 2.12	0.609	0.375		0.490	0.235	32.4		1.200
ΔY 2.13	0.196	0.488	0.304		0.076	42.2	0.513	
ΔY 2.14	0.758	0.433		0.247	0.292	37.4		0.773
ΔY 2.15	0.781	0.323	0.506		0.301	27.9	1.090	
ΔY 2.16	0.376	0.427	0.369		0.145	36.9	0.796	
average	**0.603**	**0.401**	**0.411**	**0.335**	**0.230**	**34.6**	**0.745**	**0.929**
ΔY_{nr} 2.17	0.823	0.299	0.332		0.605	24.1	0.652	
ΔY_{nr} 2.18	0.556	0.414	0.199		0.409	33.3	0.391	
ΔY_{nr} 2.19	0.812	0.338		0.259	0.597	27.2		0.738
ΔY_{nr} 2.20	0.741	0.363	0.256		0.549	29.2	0.507	
ΔY_{nr} 2.21	0.470	0.466	0.131		0.348	37.6	0.260	
ΔY_{nr} 2.22	0.731	0.385		0.218	0.542	31.0		0.627
ΔY_{nr} 2.23	0.708	0.359	0.333		0.527	28.9	0.669	
ΔY_{nr} 2.24	0.472	0.459	0.193		0.351	36.9	0.388	
ΔY_{nr} 2.25	0.667	0.438		0.147	0.497	35.2		0.428
average	**0.664**	**0.391**	**0.241**	**0.208**	**0.492**	**31.5**	**0.478**	**0.598**

(*):Obtained from Tables 17A and 17B. $\hat{e}_K$, $\hat{e}_L$, $\hat{e}_G$, and $\hat{e}_{Gci}$ are estimated elasticities of private output with respect to private capital, labor, public nonmilitary, and core infrastructure capital, respectively, and **MPK, MPL, MPG**, and **MPG_{ci}** are corresponding marginal products.

literature or found by the U.S. Bureau of Labor Statistics. Third, the estimated elasticity of private nonresidential output with respect to private nonresidential fixed capital is larger than any other estimates in the literature that we are aware of.

The estimated marginal products of private nonresidential fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital are calculated from the estimated results of Table 17B and given in Table18 above. The averages of those marginal products are 0.492, 31.50, 0.478, and 0.598, respectively, meaning that private nonresidential real output increases by 49 cents, \$31.50, 48 cents, and by 60 cents when private nonresidential fixed capital, private labor, public nonmilitary fixed capital, and public core infrastructure capital each increases by \$1, 1 hour, \$1, and by \$1, respectively, ceteris paribus. We believe that these findings tell the whole story about the aggregate private production process and seem to be credible.

Recall that Equation 2.3 did not contain the dummy variable but the calculated Durbin-Watson test-statistic still fell into the inconclusive region. Recall also that we did find that both the private fixed capital series and the public nonmilitary fixed capital series were I(2) difference stationary processes. Therefore, we have re-estimated Equations 2.3 and 2.4 twice each, once with the double-differenced data on public nonmilitary fixed capital and single-differenced data on the other variables and another time with double-differenced data on the two I(2) series and single-differenced data on the other explanatory variables as follows.

$$\ln(\Delta\hat{Y}_t) = \underset{(-2.1)}{-3.6} + \underset{(3.3)}{0.173}\,\Delta CU_t + \underset{(3.52)}{0.187}\,\Delta L_t + \underset{(4.28)}{1.32}\ln(\Delta K_t) - \underset{(-0.57)}{22.5}\ln(\Delta^2 G_t) \quad \text{adj } R^2 = 0.69,\ \text{D-W} = 2.17,\ F = 33$$

$$\ln(\Delta\hat{Y}_t) = \underset{(-0.48)}{-0.95} + \underset{(2.2)}{0.12}\,\Delta CU_t + \underset{(4.4)}{0.27}\,\Delta L_t + \underset{(2.3)}{0.81}\ln(\Delta K_t) - \underset{(-0.21)}{7.7}\ln(\Delta^2 G_t) + \underset{(2.5)}{1.3}D \quad \text{adj } R^2 = 0.72,\ \text{D-W} = 2.33,\ F = 30$$

$$\ln(\Delta\hat{Y}_t) = \underset{(24.9)}{4.1} + \underset{(1.2)}{0.07}\,\Delta CU_t + \underset{(2.9)}{0.18}\,\Delta L_t + \underset{(3.0)}{128}\ln(\Delta^2 K_t) - \underset{(-1.2)}{48.9}\ln(\Delta^2 G_t) \quad \text{adj } R^2 = 0.65,\ \text{D-W} = 1.26,\ F = 27$$

$$\ln(\Delta\hat{Y}_t) = \underset{(24.6)}{3.7} + \underset{(0.41)}{0.02}\,\Delta CU_t + \underset{(4.8)}{0.26}\,\Delta L_t + \underset{(3.3)}{118}\ln(\Delta^2 K_t) - \underset{(-0.32)}{11}\ln(\Delta^2 G_t) + \underset{(4.6)}{1.8}\,D \quad \text{adj } R^2 = 0.75,\ \text{D-W} = 1.89,\ F = 34$$

Double-differencing results in losing too much valuable information and therefore the estimated coefficients not only become insignificant but also have unexpected signs.

Therefore, we conclude that double-differencing here cannot assist in finding the true relationships.

We have also performed several joint hypothesis tests using both the level and first-differenced data. First, we might be interested in determining whether the aggregate private output production technology exhibits i) increasing returns to scale in private inputs K and L or ii) increasing returns to scale in all inputs K, L, and G or G_{ci}. To determine these, we have conducted several joint hypothesis tests and reported the test results in Table 19 below, calculated from Table A11 in Appendix B. For example, by testing the null hypothesis of the form H_0: $e_K + e_L \leq 1$ against the alternative hypothesis of the form H_1: $e_K + e_L > 1$ for Equation 1.1, we ask whether the difference of 0.057 between the hypothesized value of 1 and the sum of the estimated elasticities of 0.943 is due to chance or whether the difference is statistically significantly negative, suggesting increasing returns to scale in K and L. Since the calculated test-statistic, given by $t = (\hat{e}_K + \hat{e}_L - 1) / (\hat{Var}[\hat{e}_K] + \hat{Var}[\hat{e}_L] + 2 * \hat{Cov}[\hat{e}_K, \hat{e}_L])^{0.5}$, is $t = -0.433 < 1.671 = t_{0.05, 52}$, we fail to reject the null hypothesis at a 5 percent significance level and conclude that the non-negative difference of 0.057 is due to chance and there is no sample evidence suggesting increasing returns to scale in K and L. In other words, there is enough sample evidence to claim that the null hypothesis is compatible with the sample data.

By testing the null hypothesis of the form H_0: $e_K + e_L + e_G \leq 1$ against the alternative hypothesis of the form H_1: $e_K + e_L + e_G > 1$, we ask whether the difference of –0.264 between the hypothesized value of 1 and the sum of the estimated elasticities of 1.264 is due to chance or whether the difference is statistically significantly negative suggesting increasing returns to scale in K, L, and G or G_{ci}. The calculated test-statistic, $t = (\hat{e}_K+\hat{e}_L+\hat{e}_G-1) / (\hat{Var}[\hat{e}_K]+\hat{Var}[\hat{e}_L]+\hat{Var}[\hat{e}_G] + 2*(\hat{Cov}[\hat{e}_K,\hat{e}_L]+\hat{Cov}[\hat{e}_K,\hat{e}_G]+\hat{Cov}[\hat{e}_L,\hat{e}_G]))^{0.5}$, is $t = 2.209 > t_{0.05, 52}$ and therefore we reject the null hypothesis at a 5 percent significance level in favor of the alternative and conclude that the negative difference of –0.264 is *not* due to chance. In other words, the result of this joint hypothesis test seems to indicate that the alternative hypothesis is compatible with the sample data suggesting that the aggregate private sector seems to operate under increasing returns to scale in K, L, and G or G_{ci} for the years 1947-2005.

Table 19: Joint Hypothesis Testing of Increasing Returns to Scale (*)

Equation	degrees of freedom (df)	H_0: $e_K+e_L\leq1$ vs H_1: $e_K+e_L>1$ calculated t-statistic (**)	conclusion	H_0: $e_K+e_L+e_G\leq1$ vs H_1: $e_K+e_L+e_G>1$ calculated t-statistic (***)	conclusion
Y 1.1	52	-0.433	fail to reject H_0	2.209	reject H_0
Y 1.2	52	-0.280	fail to reject H_0	1.930	reject H_0
Y 1.3	53	-0.426	fail to reject H_0	1.985	reject H_0
Y 1.4	53	-0.389	fail to reject H_0	2.194	reject H_0
Y 1.5	52	-0.765	fail to reject H_0	1.466	fail to reject H_0
Y 1.6	53	-0.945	fail to reject H_0	1.480	fail to reject H_0
Y 1.7	53	-0.896	fail to reject H_0	1.718	reject H_0
Y 1.8	52	-0.968	fail to reject H_0	1.705	reject H_0
Y 1.9	52	-1.187	fail to reject H_0	1.226	fail to reject H_0
Y 1.10	52	-1.012	fail to reject H_0	1.029	fail to reject H_0
Y 1.11	53	-1.356	fail to reject H_0	1.003	fail to reject H_0
Y 1.12	53	-1.326	fail to reject H_0	1.240	fail to reject H_0

(**): $\mathbf{t} = (\hat{e}_K+\hat{e}_L-1) / (Var[\hat{e}_K]+Var[\hat{e}_L]+2*Cov[\hat{e}_K, \hat{e}_L])^{0.5}$ and rejection region for H_0 is $\mathbf{t > t_{.05,df}}$

(***): $\mathbf{t} = (\hat{e}_K+\hat{e}_L+\hat{e}_G-1) / (Var[\hat{e}_K]+Var[\hat{e}_L]+Var[\hat{e}_G]+2*(Cov[\hat{e}_K,\hat{e}_L]+Cov[\hat{e}_K,\hat{e}_G]+Cov[\hat{e}_L,\hat{e}_G]))^{0.5}$

Equation	degrees of freedom (df)	H_0: $e_K+e_L\leq1$ vs H_1: $e_K+e_L>1$ calculated t-statistic (**)	conclusion	H_0: $e_K+e_L+e_G\leq1$ vs H_1: $e_K+e_L+e_G>1$ calculated t-statistic (***)	conclusion
ΔY 2.1	54	0.622	fail to reject H_0	2.177	reject H_0
ΔY 2.2	54	0.234	fail to reject H_0	0.972	fail to reject H_0
ΔY 2.3	53	-0.302	fail to reject H_0	0.699	fail to reject H_0
ΔY 2.4	53	0.147	fail to reject H_0	0.551	fail to reject H_0
ΔY 2.5	54	-0.345	fail to reject H_0	2.020	reject H_0
ΔY 2.6	54	0.284	fail to reject H_0	0.853	fail to reject H_0
ΔY 2.7	55	1.034	fail to reject H_0	2.160	reject H_0
ΔY 2.8	54	-0.341	fail to reject H_0	0.317	fail to reject H_0
ΔY 2.9	55	0.910	fail to reject H_0	2.254	reject H_0
ΔY 2.10	54	-0.395	fail to reject H_0	0.506	fail to reject H_0
ΔY 2.11	54	-0.585	fail to reject H_0	1.639	fail to reject H_0
ΔY 2.12	54	-0.046	fail to reject H_0	1.601	fail to reject H_0
ΔY 2.13	53	-0.941	fail to reject H_0	-0.038	fail to reject H_0
ΔY 2.14	55	0.674	fail to reject H_0	1.461	fail to reject H_0
ΔY 2.15	55	0.408	fail to reject H_0	2.258	reject H_0
ΔY 2.16	54	-0.740	fail to reject H_0	0.567	fail to reject H_0

(**): $\mathbf{t} = (\hat{e}_K+\hat{e}_L-1) / (Var[\hat{e}_K]+Var[\hat{e}_L]+2*Cov[\hat{e}_K, \hat{e}_L])^{0.5}$ and rejection region for H_0 is $\mathbf{t > t_{.05,df}}$

(***): $\mathbf{t} = (\hat{e}_K+\hat{e}_L+\hat{e}_G-1) / (Var[\hat{e}_K]+Var[\hat{e}_L]+Var[\hat{e}_G]+2*(Cov[\hat{e}_K,\hat{e}_L]+Cov[\hat{e}_K,\hat{e}_G]+Cov[\hat{e}_L,\hat{e}_G]))^{0.5}$

(*):Obtained from Table A11. $\hat{e}_K$, $\hat{e}_L$, and $\hat{e}_G$ are estimated elasticities of private output with respect to private capital, labor, and public fixed capital (either nonmilitary or core infrastructure capitals), respectively.

As can be seen from Table 19 above, we have tested the null hypothesis of the form H_0: $e_K + e_L \leq 1$ against the alternative hypothesis of the form H_1: $e_K + e_L > 1$ twenty eight times, and in each case we are unable to reject the null hypothesis. Consequently, we conclude that the sample data suggest that the aggregate private sector is not operating under increasing returns to scale in K and L for the years 1947-2005. On the other hand, we have tested the null hypothesis of the form H_0: $e_K + e_L + e_G \leq 1$ against the alternative hypothesis of the form H_1: $e_K + e_L + e_G > 1$ again 28 times and rejected the null hypothesis 11 times. Since we reject the null hypothesis 39 percent of the time and fail to reject the null 61 percent of the time, we conclude that the sample data support the claim that the aggregate private sector is less likely operating under increasing returns to scale in K, L, and G or G_{ci}.

Second, recall that Equations (22) and (23) impose unitary elasticity of substitution of one input —the private capital input, the private labor input, or the public capital input— for another on the aggregate private production process. In addition to this restriction, Equations (24), (25), (26), and (27) impose constant returns to scale in all inputs K, L, G or G_{ci} on the aggregate private production process. Thus, we might be interested in determining whether the second restriction —the restriction that the aggregate private sector is operating under constant returns to scale in all inputs K, L, and G or G_{ci}— is a reasonable assumption. To determine this, we have tested the null hypothesis of the form H_0: $e_K + e_L + e_G = 1$ against the alternative hypothesis of the form H_1: $e_K + e_L + e_G \neq 1$ twelve times and reported the test results in Table 20 below. Under the null hypothesis that all of the J restrictions (here we have only one restriction) are true, the calculated test-statistic, given by $F = [(SSE_R - SSE_U) / J] / [SSE_U / (n - k)]$, has a standard $F_{\alpha, J, n-k}$ distribution and therefore we reject the null hypothesis if the calculated F-statistic exceeds the critical value of $F_{\alpha, J, n-k}$ statistic. Using this criterion, we reject the null hypothesis of the form H_1: $e_K + e_L + e_G \neq 1$ two times at a 5 percent significance level and fail to reject the null hypothesis 10 times. In other words, the calculated test-statistic F has been less than the critical test-statistic $F_{.05, 1, n-k}$ ten out of twelve times and therefore we fail to reject the null hypothesis 83 percent of the time at a 5 percent significance level. As a result, we conclude that there is enough sample evidence suggesting that the restriction that the aggregate private sector operates under constant returns to scale in all inputs K, L, G, or G_{ci} for the years 1947-2005 is in fact *not* false.

Table 20: Joint Hypothesis Testing of Constant Returns to Scale Across All Inputs (*)

H_0: $e_K+e_L+e_G=1$ vs H_1: $e_K+e_L+e_G\neq1$

Unrestricted Model Equation	SSE_U	n-k	Restricted Model Equation	SSE_R	calculated F-statistic (**)	conclusion (***)
Y 1.1	0.01386	52	**Y/K 1.13**	0.01516	4.85	**reject H_0**
Y 1.2	0.01468	52	**Y/K 1.14**	0.01574	3.74	fail to reject H_0
Y 1.3	0.01471	53	**Y/K 1.15**	0.01582	4.01	fail to reject H_0
Y 1.4	0.01391	53	**Y/K 1.16**	0.01519	4.89	**reject H_0**
Y 1.5	0.01545	52	**Y/K 1.18**	0.01611	2.24	fail to reject H_0
Y 1.6	0.01545	53	**Y/K 1.19**	0.01612	2.27	fail to reject H_0
Y 1.7	0.01457	53	**Y/K 1.20**	0.01534	2.81	fail to reject H_0
Y 1.8	0.01449	52	**Y/K 1.17**	0.01535	3.08	fail to reject H_0
Y 1.9	0.01548	52	**Y/K 1.21**	0.01593	1.52	fail to reject H_0
Y 1.10	0.01636	52	**Y/K 1.22**	0.01673	1.20	fail to reject H_0
Y 1.11	0.01643	53	**Y/K 1.23**	0.01675	1.04	fail to reject H_0
Y 1.12	0.01544	53	**Y/K 1.24**	0.01591	1.59	fail to reject H_0

(*):Obtained from Tables 15A and 15B. **e_K, e_L, and e_G** are elasticities of private output with respect to private capital and labor, and public fixed capital (either nonmilitary or core infrastructure capitals), respectively. **SSE_U** and **SSE_R** are the sums of squared errors from the unrestricted and restricted least squares estimations, respectively, and **n** and **k** are number of observations and estimated coefficients in the unrestricted model.

(**): $F=[(SSE_R - SSE_U) / J] / [SSE_U / (n - k)]$, where J is the number of linear restrictions.

(***): Under the null hypothesis that all of the J restrictions are true, the calculated F-statistic has an $F_{J,n-k}$ distribution and therefore the null hypothesis is rejected if the calculated F-statistic exceeds an appropriate critical value from this distribution.

Lastly, some researchers claim that any of the two measures of the public fixed capital stock is more important to (or productive in) the aggregate private production process than the private fixed capital stock, whereas other researchers claim that the private fixed capital stock is more important to (or productive in) the private production process than any of the two measures of the public fixed capital stock (or if not more important than the public core infrastructure capital stock, the private capital stock is at least more important than the public nonmilitary fixed capital stock). Therefore, we might also be interested in determining which one of these two claims is reasonable. We have tested each claim 28 times using either the level or first-differenced data and reported the test results in Table 21 below.

Table 21: Joint Hypothesis Testing of Relative Importance of Private or Public Capital (*)

Equation	degrees of freedom (df)	H_0: e_G-e_K≤0 vs H_1: e_G-e_K>0 calculated t-statistic (**)	conclusion	H_0: e_K-e_G≤0 vs H_1: e_K-e_G>0 calculated t-statistic (***)	conclusion
Y 1.1	52	0.131	fail to reject H_0	-0.131	fail to reject H_0
Y 1.2	52	-0.181	fail to reject H_0	0.181	fail to reject H_0
Y 1.3	53	0.096	fail to reject H_0	-0.096	fail to reject H_0
Y 1.4	53	-0.157	fail to reject H_0	0.157	fail to reject H_0
Y 1.5	52	0.264	fail to reject H_0	-0.264	fail to reject H_0
Y 1.6	53	0.802	fail to reject H_0	-0.802	fail to reject H_0
Y 1.7	53	0.583	fail to reject H_0	-0.583	fail to reject H_0
Y 1.8	52	0.616	fail to reject H_0	-0.616	fail to reject H_0
Y 1.9	52	0.651	fail to reject H_0	-0.651	fail to reject H_0
Y 1.10	52	0.348	fail to reject H_0	-0.348	fail to reject H_0
Y 1.11	53	1.288	fail to reject H_0	-1.288	fail to reject H_0
Y 1.12	53	1.140	fail to reject H_0	-1.140	fail to reject H_0
(**): $\mathbf{t} = (\hat{e}_G - \hat{e}_K) / (Var[\hat{e}_G]+Var[\hat{e}_K]-2*Cov[\hat{e}_G,\hat{e}_K])^{0.5}$ and rejection region for H_0 is $\mathbf{t > t_{.05,df}}$ (***): $\mathbf{t} = (\hat{e}_K - \hat{e}_G) / (Var[\hat{e}_K]+Var[\hat{e}_G]-2*Cov[\hat{e}_K,\hat{e}_G])^{0.5}$					
ΔY 2.1	54	-0.697	fail to reject H_0	0.697	fail to reject H_0
ΔY 2.2	54	-1.185	fail to reject H_0	1.185	fail to reject H_0
ΔY 2.3	53	-0.109	fail to reject H_0	0.109	fail to reject H_0
ΔY 2.4	53	-0.770	fail to reject H_0	0.770	fail to reject H_0
ΔY 2.5	54	0.288	fail to reject H_0	-0.288	fail to reject H_0
ΔY 2.6	54	-0.895	fail to reject H_0	0.895	fail to reject H_0
ΔY 2.7	55	-1.980	fail to reject H_0	1.980	**reject H_0**
ΔY 2.8	54	-0.674	fail to reject H_0	0.674	fail to reject H_0
ΔY 2.9	55	-1.816	fail to reject H_0	1.816	**reject H_0**
ΔY 2.10	54	-0.567	fail to reject H_0	0.567	fail to reject H_0
ΔY 2.11	54	0.420	fail to reject H_0	-0.420	fail to reject H_0
ΔY 2.12	54	-0.197	fail to reject H_0	0.197	fail to reject H_0
ΔY 2.13	53	0.190	fail to reject H_0	-0.190	fail to reject H_0
ΔY 2.14	55	-1.758	fail to reject H_0	1.758	**reject H_0**
ΔY 2.15	55	-1.023	fail to reject H_0	1.023	fail to reject H_0
ΔY 2.16	54	-0.026	fail to reject H_0	0.026	fail to reject H_0
(**): $\mathbf{t} = (\hat{e}_G - \hat{e}_K) / (Var[\hat{e}_G]+Var[\hat{e}_K]-2*Cov[\hat{e}_G,\hat{e}_K])^{0.5}$ and rejection region for H_0 is $\mathbf{t > t_{.05,df}}$ (***): $\mathbf{t} = (\hat{e}_K - \hat{e}_G) / (Var[\hat{e}_K]+Var[\hat{e}_G]-2*Cov[\hat{e}_K,\hat{e}_G])^{0.5}$					

(*):Obtained from Table A11. $\hat{e}_K$, $\hat{e}_L$, and $\hat{e}_G$ are estimated elasticities of private output with respect to private capital, labor, and public fixed capital (either nonmilitary or core infrastructure capitals), respectively.

By testing the null hypothesis of the form H_0: $e_G - e_K \leq 0$ against the alternative hypothesis of the form H_1: $e_G - e_K > 0$, we ask whether any of the two measures of the public fixed capital stock is more important than the private fixed capital stock to the aggregate private production process. We have tested this claim 28 times and could not reject the null hypothesis even once. Since the calculated test-statistic, given by $t = (\hat{e}_G - \hat{e}_K) / (\hat{Var}[\hat{e}_G] + \hat{Var}[\hat{e}_K] - 2*\hat{Cov}[\hat{e}_G, \hat{e}_K])^{0.5}$, has always been less than the corresponding critical test-statistic $t_{0.05,\, n\text{-}k}$, every time we fail to reject the null hypothesis at a 5 percent significance level and conclude that there is no sample evidence suggesting that any of the two measures of the public fixed capital stock is more important than the private fixed capital stock. In other words, there is enough sample evidence to claim that the null hypothesis is compatible with the sample data, meaning that the private fixed capital stock is more important.

By testing the null hypothesis of the form H_0: $e_K - e_G \leq 0$ against the alternative hypothesis of the form H_1: $e_K - e_G > 0$, we ask whether the private fixed capital stock is more important than any of the two measures of the public fixed capital stock to the aggregate private production process. We have tested this claim again 28 times and rejected the null hypothesis three times, that is, 11 percent of the time. Since the calculated test- statistic, given by $t = (\hat{e}_K - \hat{e}_G) / (\hat{Var}[\hat{e}_K] + \hat{Var}[\hat{e}_G] - 2*\hat{Cov}[\hat{e}_K, \hat{e}_G])^{0.5}$, has been greater than the corresponding critical test-statistic $t_{0.05,\, n\text{-}k}$ three times, we reject the null hypothesis 11 percent of the time at a 5 percent significance level in favor of the alternative hypothesis and conclude that there is little sample evidence suggesting that the private fixed capital stock is more important than any of the two measures of the public fixed capital stock. Combining the results of the two tests reported in Table 21, it is reasonable to conclude that there is *no* sample evidence supporting the claim that any of the two measures of the public fixed capital is more important to the aggregate private production process than the private fixed capital stock, whereas there is *little* evidence suggesting that the private fixed capital stock is more important to (or productive in) the aggregate private production process than any of the two measures of the public fixed capital stock.

V. The Relationship between Private Output and Public Nonmilitary Fixed Capital: A Vector Autoregression Model

When we are certain which variables are exogenous and which variables are endogenous, then we can use a macroeconomic structural model which yields a simultaneous set of reduced-form equations. The number of simultaneous equations in the reduced-form set is equal to the number of endogenous variables in the model, meaning that the primitive system can be solved for endogenous variables to get one equation for each endogenous variable explaining that variable as a function of the exogenous variables in the system, the lag values of the endogenous variables, and the disturbance terms. The set of reduced-form equations can then be simultaneously estimated using an appropriate estimator method. Here not only the inclusion of the variables but also the nature of the variables (that is, whether the variables are endogenous or exogenous) is dictated by the relevant theories of economics.

When we are *not* certain which variables are endogenous and which variables are exogenous, then we can treat every variable symmetrically by letting the time path of every variable in the model be affected by the present and past realizations of every variable in the system. Here, only the inclusion of the variables needs to be justified by relevant economic theories. The lag length, however, must be justified by the statistical significance of the last lag. In other words, when we are not certain which variables are endogenous, it is reasonable to use a vector autoregression (VAR) model of an appropriate order. Therefore, we use a VAR model to analyze the relationships among four aggregate macroeconomic variables that we use in Section IV of this study. Since we use annual data, we expect a first-order VAR to sufficiently explain the relationships among private output, private capital, private labor, and a measure of public capital.

V.1. A First-Order Structural Vector Autoregression Model

We model the aggregate private economy using a first-order structural VAR model as follows.

$$
(28)\quad \begin{cases} y_t = b_{10} - b_{12}\, k_t - b_{13}\, l_t - b_{14}\, g_t + c_{11}\, y_{t-1} + c_{12}\, k_{t-1} + c_{13}\, l_{t-1} + c_{14}\, g_{t-1} + u_{yt}, \\ k_t = b_{20} - b_{21}\, y_t - b_{23}\, l_t - b_{24}\, g_t + c_{21}\, y_{t-1} + c_{22}\, k_{t-1} + c_{23}\, l_{t-1} + c_{24}\, g_{t-1} + u_{kt}, \\ l_t = b_{30} - b_{31}\, y_t - b_{32}\, k_t - b_{34}\, g_t + c_{31}\, y_{t-1} + c_{32}\, k_{t-1} + c_{33}\, l_{t-1} + c_{34}\, g_{t-1} + u_{lt}, \\ g_t = b_{40} - b_{41}\, y_t - b_{42}\, k_t - b_{43}\, l_t + c_{41}\, y_{t-1} + c_{42}\, k_{t-1} + c_{43}\, l_{t-1} + c_{44}\, g_{t-1} + u_{gt}, \end{cases}
$$

where y_t, k_t, l_t, and g_t are stationary processes representing aggregate private output, private fixed capital, private labor, and a measure public fixed capital in time t, respectively, and where u_{yt}, u_{kt}, u_{lt}, and u_{gt} are corresponding uncorrelated white-noise disturbances with constant variances $\sigma^2{}_y$, $\sigma^2{}_k$, $\sigma^2{}_l$, and $\sigma^2{}_g$. The coefficients ($-b_{12}$), ($-b_{13}$), and ($-b_{14}$) are the contemporaneous effects of a unit change in k_t, l_t, and g_t on y_t, respectively. Similarly, the coefficients ($-b_{41}$), ($-b_{42}$), and ($-b_{43}$) are the contemporaneous effects of a unit change in y_t, k_t, and l_t on g_t, respectively. If, for example, $b_{12} \neq 0$, then u_{kt} has an indirect contemporaneous effect on y_t, and if $b_{21} \neq 0$, then u_{yt} is said to have an indirect contemporaneous effect on k_t. Equation (28) can also be written in a compact form as follows.

(29) $\mathbf{BX}_t = \mathbf{B}_0 + \mathbf{CX}_{t-1} + \mathbf{U}_t$, where

(30) $\mathbf{B} = [1, b_{12}, b_{13}, b_{14}; b_{21}, 1, b_{23}, b_{24}; b_{31}, b_{32}, 1, b_{34}; b_{41}, b_{42}, b_{43}, 1]$, $\mathbf{X}_t = [y_t; k_t; l_t; g_t]$, $\mathbf{B}_0 = [b_{10}; b_{20}; b_{30}; b_{40}]$, $\mathbf{C} = [c_{11}, c_{12}, c_{13}, c_{14}; c_{21}, c_{22}, c_{23}, c_{24}; c_{31}, c_{32}, c_{33}, c_{34}; c_{41}, c_{42}, c_{43}, c_{44}]$, $\mathbf{X}_{t-1} = [y_{t-1}; k_{t-1}; l_{t-1}; g_{t-1}]$, and $\mathbf{U}_t = [u_{yt}; u_{kt}; u_{lt}; u_{gt}]$,

where hereafter the rows of each matrix are separated from each other by semicolons and the elements within each row are separated from each other by commas. The composition of the column vector $\mathbf{X}_t$, that is, the variables to be included in $\mathbf{X}_t$, is not only justified by economic theories but also by one of our objectives. It is important to remind the reader that one of our objectives is to build models explaining private economic growth. Another objective is to compare the performance of the two models to determine the superior one. Since we use the four variables in our first model, we have to use them here too. Therefore, we included the four variables in $\mathbf{X}_t$.

V.2. A First-Order Vector Autoregression Model in Standard Form

Multiplying both sides of Equation (29) by the inverse of the square matrix **B**, we obtain a first-order vector autoregression (VAR) model in standard form describing the aggregate U.S. private economy as follows.

(31) $\mathbf{X}_t = \mathbf{B}^{-1}\mathbf{B}_0 + \mathbf{B}^{-1}\mathbf{C}\mathbf{X}_{t-1} + \mathbf{B}^{-1}\mathbf{U}_t$, where $\mathbf{B}^{-1} = [1/\det(\mathbf{B})]*\mathrm{Adj}(\mathbf{B})$, where

$\det(\mathbf{B}) = 1 + b_{12}[b_{23}b_{31}(1-b_{24})-b_{21}] + b_{31}[b_{13}(b_{24}b_{42}-1)+b_{14}(b_{43}-b_{23}b_{42})] + b_{32}[b_{21}(b_{13}-b_{14}b_{43})+b_{24}b_{43}-b_{23}] + b_{34}[b_{43}(1+b_{12}b_{21})+b_{23}b_{42}] + b_{41}[b_{13}(b_{34}-b_{24}b_{32})+b_{23}(b_{14}b_{32}-b_{12}b_{34})+b_{12}b_{24}-b_{14}] + b_{42}[b_{21}(b_{14}-b_{13}b_{34})-b_{24}]$ and

$\mathrm{Adj}(\mathbf{B}) = [1-b_{34}b_{43}+b_{23}(b_{34}b_{42}-b_{32})+b_{24}(b_{32}b_{43}-b_{42}),\ b_{12}(b_{34}b_{43}-1)+b_{32}(b_{13}-b_{14}b_{43})+b_{42}(b_{14}-b_{13}b_{34}),\ b_{12}(b_{23}-b_{24}b_{43})+b_{13}(b_{24}b_{42}-1)+b_{14}(b_{43}-b_{24}b_{42}),\ b_{12}(b_{24}-b_{23}b_{34})+b_{13}(b_{34}-b_{24}b_{32}) + b_{14}(b_{23}b_{32}-1);\ b_{21}(b_{34}b_{43}-1)+b_{31}(b_{23}-b_{24}b_{43})+b_{41}(b_{24}-b_{23}b_{34}),\ 1-b_{34}b_{43}+b_{31}(b_{14}b_{43}-b_{13})+b_{41}(b_{13}b_{34}-b_{14}),\ b_{13}(b_{21}-b_{24}b_{41})+b_{23}(b_{14}b_{41}-1)+b_{43}(b_{24}-b_{14}b_{21}),\ b_{21}(b_{14}-b_{13}b_{34})+b_{23}(b_{34}-b_{14}b_{31})+b_{24}(b_{13}b_{31}-1);\ b_{21}(b_{32}-b_{34}b_{42})+b_{31}(b_{24}b_{42}-1)+b_{41}(b_{34}-b_{24}b_{32}),\ b_{12}(b_{31}-b_{34}b_{41})+b_{32}(b_{14}b_{41}-1)+b_{42}(b_{34}-b_{14}b_{31}),\ 1-b_{24}b_{42}+b_{12}(b_{24}b_{41}-b_{21})+b_{14}(b_{21}b_{42}-b_{41}),\ b_{31}(b_{14}-b_{12}b_{24})+b_{32}(b_{24}-b_{14}b_{21})+b_{34}(b_{12}b_{21}-1);\ b_{21}(b_{42}-b_{32}b_{43})+b_{31}(b_{43}-b_{23}b_{42})+b_{41}(b_{23}b_{32}-1),\ b_{12}(b_{41}-b_{31}b_{43})+b_{32}(b_{43}-b_{13}b_{41})+b_{42}(b_{13}b_{31}-1),\ b_{41}(b_{13}-b_{12}b_{23})+b_{42}(b_{23}-b_{13}b_{21})+b_{43}(b_{12}b_{21}-1),\ 1-b_{23}b_{32}+b_{12}(b_{23}b_{31}-b_{21})+b_{13}(b_{21}b_{32}-b_{31})]$.

Equation (31) can also be written in a more compact form as follows.

(32) $\mathbf{X}_t = \mathbf{A}_0 +\mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{W}_t$, where $\mathbf{A}_0 = \mathbf{B}^{-1}\mathbf{B}_0$, $\mathbf{A}_1 = \mathbf{B}^{-1}\mathbf{C}$, and $\mathbf{W}_t = \mathbf{B}^{-1}\mathbf{U}_t$.

Also, $\mathrm{Cov}(\mathbf{W}_t) = [\sigma^2_1, \sigma_{12}, \sigma_{13}, \sigma_{14};\ \sigma_{21}, \sigma^2_2, \sigma_{23}, \sigma_{24};\ \sigma_{31}, \sigma_{32}, \sigma^2_3, \sigma_{34};\ \sigma_{41}, \sigma_{42}, \sigma_{43}, \sigma^2_4]$, where $\sigma^2_i = \mathrm{Var}(w_{it})$ and $\sigma_{ij} = \mathrm{Cov}(w_{it}, w_{jt})$ for all i = 1, 2, 3, 4 and all j = 1, 2, 3, 4. Similarly, Equation (32) can easily be generalized to produce a VAR model of order p in standard form for any positive integer $p \geq 2$ as follows.

(33) $\mathbf{X}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{A}_2\mathbf{X}_{t-2} + \ \dots + \mathbf{A}_p\mathbf{X}_{t-p} + \mathbf{W}_t$.

The variables in the column vector $\mathbf{X}_t$ are determined by the production function model that we used in Section IV of this dissertation and justified by economic theories. At the estimation stage, the lag length p will be determined by statistical significance.

Equation (33) can be estimated by the ordinary least squares (OLS) estimator. Moreover, the OLS estimates are both consistent and asymptotically efficient. Although the error terms are correlated since $Cov(w_{it}, w_{jt}) \neq 0$, we still use the OLS estimator to estimate Equation (33) because seemingly unrelated regressions (SUR) cannot improve the efficiency of the estimates. A SUR method could improve the efficiency of the estimates if the explanatory variables in every equation in Equation (33) were *not* the same.

Equation (32) can also be written in a vector moving average (VMA) form as

$$(34) \qquad \mathbf{X}_t = \overline{X} + \sum_{i=0}^{\infty} A_1^i W_{t-i} ,$$

where the column vector $\overline{X}$ contains the sample averages of the four variables used. It is instructive to rewrite Equation (34) in terms of the disturbance terms of Equation (29) as.

$$(35) \qquad \mathbf{X}_t = \overline{X} + \sum_{i=0}^{\infty} M_i U_{t-i} \text{, where } \mathbf{M}i = \mathbf{A}^i{}_1\mathbf{B}^{-1}.$$

Equation (35) is the VMA representation of the four-variable first-order VAR in standard form, which is an essential part of Sims's (1980) methodology in the sense that it enables us to trace out the time path of the various shocks (here u_{yt}, u_{kt}, u_{lt}, and u_{gt}) to the variables contained in the VAR system (here y_t, k_t, l_t, and g_t). Therefore, the VMA representation of the VAR model can be used to find the impact multipliers and the other multipliers to construct the impulse response functions.

Here the coefficients $m_{jk}(0)$ are the impact multipliers, the coefficients $m_{jk}(1)$ are one-period multipliers, the coefficients $m_{jk}(2)$ are two-period multipliers, and so forth. For example, the coefficient $m_{12}(0)$ is the instantaneous impact of a unit change in u_{kt} on y_t. If level data are used in the estimation stage, then the coefficient $m_{12}(0)$ is the marginal product of private fixed capital; and if data in double logarithmic form are used, then the coefficient $m_{12}(0)$ is the elasticity of private output with respect to private fixed capital. Similarly, the coefficient $m_{14}(0)$ is the instantaneous impact of a unit change in u_{gt} on y_t, the marginal product of public fixed capital if level data are used. Therefore, the impact multipliers can be used to find the elasticities of private output with respect to the private capital, private labor, and public fixed capital and the corresponding marginal products.

Similarly, the coefficients $m_{11}(1)$, $m_{12}(1)$, $m_{13}(1)$, and $m_{14}(1)$ are the one-period responses of unit changes in u_{yt-1}, u_{kt-1}, u_{lt-1}, and u_{gt-1} on y_t, respectively. If we update the model by one period, then the coefficients $m_{11}(1)$, $m_{12}(1)$, $m_{13}(1)$, and $m_{14}(1)$ also give the effects of a unit change in u_{yt}, u_{kt}, u_{lt}, and u_{gt} on y_{t+1}, respectively.

Here the sixteen set of coefficients $m_{11}(i)$, $m_{12}(i)$, …, $m_{14}(i)$, $m_{21}(i)$, …, $m_{24}(i)$, $m_{31}(i)$, …, $m_{34}(i)$, $m_{41}(i)$, …, $m_{44}(i)$ are called the *impulse response functions*, each of which can be plotted against i (here i stands for time lags) to visually represent the behavior of the four variables in response to the various shocks, here u_{yt}, u_{kt}, u_{lt}, and u_{gt}. For example, plotting $m_{14}(0)$, $m_{14}(1)$, …, $m_{14}(i)$, … against i yields the graph of the relationship between private output and public fixed capital over time. Moreover, the cumulative sum of the effects of u_{gt} on the y_t sequence after n periods is given by

$\sum_{i=0}^{n} m_{14}(i)$, which is the *long run multiplier* for large n.

So far we have assumed that the y_t, k_t, l_t, and g_t sequences are stationary processes, which we know from Section IV of this study is not the case. Do we need to worry about this? After reporting the estimated results obtained from the level data and then examining them carefully, we will decide whether we need to take additional actions such as using first differences. For now we should note that Sims (1980) and Sims, Stock, and Watson (1990) suggest that differencing need *not* be used even when the variables are integrated of order one. They claim that the main objective of a VAR analysis is to determine the interrelationships among the variables in the system. In other words, they claim that the main objective is not to obtain estimates of the parameters. They object to differencing because they believe that differencing “throws away” information that is essential to capture the true long-run co-movements in the data. For the same reason, they suggest that the data need not be detrended even when the variables are trend-stationary processes. They further claim that a trending variable can be well approximated by a unit root plus drift in a VAR analysis.

V.3. Empirical Estimates of A VAR Model

An estimated n-variable VAR model in standard form is under-identified in the sense that knowledge of the various a_{ij} and knowledge of the covariance matrix of the disturbance terms in the standard form are necessary but *not* sufficient to identify the primitive system. For example, estimates of $\mathbf{A}_1$ and Cov($\mathbf{W}_t$) in Equation (32) are not sufficient to identify the primitive system given in Equation (28). Therefore, researchers must impose additional restrictions on the four-variable VAR system to identify impulse responses.

One possible way to impose additional restrictions is to use the Choleski decomposition as follows. By setting $b_{21} = b_{31} = b_{32} = b_{41} = b_{42} = b_{43} = 0$ in the primitive system, we obtain the matrix **B** whose lower triangular entries are all zero. Recall that Equation (32) implies that $\mathbf{W}_t = \mathbf{B}^{-1}\mathbf{U}_t$, which yields the following four equations when combined with the above restriction.

$$(36)\quad \begin{cases} w_{yt} = u_{yt} - b_{12}\,u_{kt} + (b_{12}b_{23} - b_{13})\,u_{lt} + [b_{12}(b_{24} - b_{23}b_{34}) + b_{13}b_{34} - b_{14}]\,u_{gt}, \\ w_{kt} = u_{kt} - b_{23}\,u_{lt} + (b_{23}b_{34} - b_{24})\,u_{gt}, \\ w_{lt} = u_{lt} - b_{34}\,u_{gt}, \text{ and} \\ w_{gt} = u_{gt}. \end{cases}$$

Equation (36) implies that not only knowledge of the values of w_{yt}, w_{kt}, w_{lt}, and w_{gt} sequences but also knowledge of the correlation coefficients between w_{yt} and w_{kt}, between w_{yt} and w_{lt}, between w_{yt} and w_{gt}, between w_{kt} and w_{lt}, between w_{kt} and w_{gt}, and between w_{lt} and w_{gt} allows for the calculations of u_{yt}, u_{kt}, and u_{lt} sequences. Recall, for example, that the correlation coefficient between w_{yt} and w_{kt} is equal to the covariance between w_{yt} and w_{kt} divided by the product of the variances of w_{yt} and w_{kt}. The result can easily be generalized to the n-variable case. Exact identification of an n-variable VAR requires $[(n^2\text{-}n)/2]$ restrictions since there are n regression residuals and n structural shocks. Since the Choleski decomposition is triangular and since there are $[(n^2\text{-}n)/2]$ entries in either the upper or the lower triangular part of a square matrix of dimension n, the Choleski decomposition imposes exactly $[(n^2\text{-}n)/2]$ restrictions by setting all of either the upper or the lower triangular entries of the square matrix **B** to be equal to zero.

Another way to identify the impulse responses is to obtain an estimate of the matrix **B** and substitute it into Equation (35). This can be done by estimating Equation (28) using the OLS estimator. Therefore, to obtain the impact multipliers and the other multipliers, we will first estimate Equation (28) to get an estimate of **B**. Then we will

Table 22: Empirical Estimates of VAR Model with Level Data (*)

Eq.	Dep.Var	const.	Y_{t-1}	K_{t-1}	L_{t-1}	G_{t-1}	G_{cit-1}	$(CU*G)_{t-1}$	$(CU*G_{ci})_{t-1}$	adj R^2	D-W	F
3.1	Y_t	372	1.01	0.042	-4.83	-0.04				0.998	1.47	7929
		(1.67)	(14.4)	(0.91)	(-1.8)	(-0.4)						
	K_t	-224	0.165	0.853	4.61	0.37				0.997	0.91	7321
		(-1.7)	(4.73)	(29.9)	(2.76)	(5.61)						
	L_t	5.08	-0.003	0.001	0.919	0.002				0.989	1.43	1353
		(0.65)	(-1.1)	(0.61)	(9.59)	(0.51)						
	G_t	79.9	0.058	-0.03	-0.24	1.06				0.999	0.25	98259
		(2.04)	(4.73)	(-3.8)	(-0.5)	(55.7)						
3.2	Y_t	410	0.99	0.06	-5.24		-0.1			0.998	1.45	8009
		(1.82)	(13.4)	(1.3)	(-1.9)		(-0.9)					
	K_t	-271	0.198	0.859	4.87		0.419			0.999	0.97	7815
		(-2.0)	(4.39)	(32.2)	(2.93)		(5.81)					
	L_t	5.75	-0.003	0.002	0.91		0.001			0.989	1.42	1347
		(0.72)	(-1.1)	(0.95)	(9.41)		(0.19)					
	G_t	105	0.07	-0.03	-0.66		1.06			0.999	0.41	73955
		(3.34)	(6.36)	(-4.4)	(-1.7)		(63.7)					
3.3	Y_t	338	1.01	0.04	-4.28				-0.06	0.998	1.45	7964
		(1.59)	(15.1)	(1.29)	(-1.7)				(-0.64)			
	K_t	39	0.14	0.94	0.71				0.27	0.999	0.86	8345
		(0.29)	(3.15)	(51.8)	(0.43)				(4.95)			
	L_t	6.6	-0.003	0.001	0.896				0.003	0.989	1.45	1366
		(0.89)	(-1.1)	(1.19)	(9.89)				(0.87)			
	G_t	561	0.01	0.05	-6.6				0.9	0.991	1.64	1709
		(3.75)	(0.21)	(2.24)	(-3.6)				(14.8)			
3.4	Y_t	334	1.02	0.03	-4.29			-0.02		0.998	1.47	7920
		(1.55)	(15.6)	(1.11)	(-1.6)			(-0.33)				
	K_t	99	0.12	0.94	0.09			0.22		0.999	0.81	8915
		(0.72)	(2.71)	(52.8)	(0.06)			(4.73)				
	L_t	7.5	-0.003	0.001	0.88			0.003		0.989	1.46	1381
		(1.01)	(-1.1)	(1.09)	(9.7)			(1.17)				
	G_t	690	0.007	0.06	-8.3			0.904		0.992	1.62	1962
		(3.34)	(0.12)	(2.28)	(-3.3)			(13.4)				

(*): Equations are estimated using the OLS estimation method with aggregate annual data from 1947 to 2005. Calculated t-statistics are given in parentheses. Y=real private GDP, CU=capacity utilization rate in manufacturing sector, K=real total private capital, L=total private business labor hours, G=real total public nonmilitary fixed capital, G_{ci}=real total public core infrastructure capital.

Table 23: Empirical Estimates of VAR Model with Level Data and Nonresidential Output (*)

Eq.	Dep.Var	const.	Y_{nrt-1}	K_{nrt-1}	L_{t-1}	G_{t-1}	G_{cit-1}	$(CU*G)_{t-1}$	$(CU*G_{ci})_{t-1}$	adj R^2	D-W	F
4.1	$Y_{nr,t}$	155	1.02	0.15	-2.03			0.039		0.998	1.52	9705
		(0.81)	(17.2)	(0.32)	(-0.86)			(0.73)				
	$K_{nr,t}$	-350	-0.004	0.919	4.63			0.154		0.999	0.66	71177
		(-3.5)	(-0.12)	(37.8)	(3.73)			(5.43)				
	L_t	7.47	-0.003	0.002	0.883			0.004		0.989	1.47	1386
		(0.96)	(-1.2)	(1.2)	(9.3)			(1.6)				
	G_t	759	0.014	0.112	-9.07			0.942		0.992	1.64	1928
		(3.48)	(0.21)	(2.15)	(-3.4)			(15.5)				
4.2	$Y_{nr,t}$	142	1.02	0.019	-1.86				0.032	0.998	1.51	9649
		(0.74)	(16.8)	(0.41)	(-0.79)				(0.48)			
	$K_{nr,t}$	-393	0.013	0.91	5.06				0.195	0.999	0.71	74139
		(-4.0)	(0.42)	(37.3)	(4.2)				(5.7)			
	L_t	6.5	-0.003	0.002	0.896				0.004	0.989	1.46	1366
		(0.83)	(-1.06)	(1.18)	(9.44)				(1.25)			
	G_t	603	0.019	0.08	-7.06				0.935	0.991	1.66	1669
		(3.79)	(0.37)	(2.05)	(-3.6)				(17.1)			
4.3	$Y_{nr,t}$	121	1.02	0.018	-1.55		0.02			0.998	1.49	9618
		(0.59)	(15.9)	(0.28)	(-0.63)		(0.25)					
	$K_{nr,t}$	-650	0.05	0.812	8.37		0.269			0.999	0.86	87953
		(-6.7)	(1.66)	(26.1)	(7.2)		(7.0)					
	L_t	3.8	-0.003	0.002	0.933		0.003			0.989	1.44	1342
		(0.46)	(-1.01)	(0.69)	(9.27)		(0.78)					
	G_t	94	0.07	-0.06	-0.51		1.05			0.999	0.48	83237
		(3.02)	(7.33)	(-5.5)	(-1.36)		(84.6)					
4.4	$Y_{nr,t}$	111	1.02	0.01	-1.44	0.034				0.998	1.51	9673
		(0.57)	(16.6)	(0.14)	(-0.61)	(0.49)						
	$K_{nr,t}$	-581	0.034	0.816	7.7	0.212				0.999	0.42	7854
	(mid-year)	(-7.6)	(1.44)	(33.1)	(8.5)	(8.0)						
	L_t	3.3	-0.003	0.002	0.941	0.003				0.989	1.45	1365
		(0.42)	(-1.14)	(0.52)	(9.9)	(1.11)						
	G_t	108	0.067	-0.06	-0.61	1.05				0.999	0.25	7854
	(mid-year)	(2.95)	(5.84)	(-4.7)	(-1.4)	(81.5)						
4.5	$Y_{nr,t}$	102	1.02	0.001	-1.31	0.04				0.998	1.51	9663
		(0.51)	(16.7)	(0.02)	(-0.54)	(0.56)						
	$K_{nr,t}$	-625	0.022	0.81	8.24	0.23				0.999	0.79	83873
		(-6.3)	(0.76)	(25.0)	(6.9)	(6.7)						
	L_t	3.16	-0.003	0.001	0.942	0.003				0.989	1.45	1357
		(0.38)	(-1.1)	(0.41)	(9.4)	(1.11)						
	G_t	65	0.063	-0.06	-0.07	1.05				0.999	0.29	8324
		(1.64)	(5.2)	(-4.5)	(-0.14)	(74.4)						

(*): Equations are estimated using the OLS estimation method with aggregate annual data from 1947 to 2005. Calculated t-statistics are given in parentheses. Y_{nr} = real private nonresidential GDP, K_{nr} = real private sector nonresidential capital stock, and the other variables are defined earlier.

estimate Equation (32) to get an estimate of $\mathbf{A}_1$ and then substitute the estimates of **B** and $\mathbf{A}_1$ into Equation (35) to get impulse responses. From the estimated impulse response functions, one can calculate the three marginal products and the three elasticities of private output. The impact multipliers obtained from the estimation using the level data for the private output equation are in fact the three marginal products. Moreover, the corresponding elasticities can easily be calculated by the procedure used in Section IV of this dissertation.

Using aggregate private real output, Equation (32) is estimated by the OLS estimator with level data, and the estimated results are provided in Table 22 above. The corresponding impact multipliers and the other multipliers constituting the corresponding impulse response functions are reported in Table 25A below. Using aggregate private nonresidential real output, Equation (32) is estimated by the OLS estimator with level data, and the estimated results are given in Table 23 above. The corresponding impact multipliers and the other multipliers constituting the corresponding impulse response functions are reported in Table 25B below.

As can be seen from Table 22 and Table 23 above, the adjusted R^2 statistic in each estimated equation is virtually 1.00, indicating that virtually all variation in the dependent variable seems to be explained. The very high calculated F-statistic seems to indicate that every equation seems to have a very high explanatory power. However, the calculated Durbin-Watson test statistics reported are very low, indicating that the error terms are strongly and positively correlated. Since not only that high adjusted R^2, high calculated F-statistics, and significant t-statistics are accompanied with low calculated Durbin-Watson test-statistics but also that impulse response functions reported in Table 25A and Table 25B do not diminish rapidly over time, the estimated results might "look good" but the results have no real meaning. Therefore, Equation (32) is re-estimated with first differences of log-level data, and the estimated results are reported in Table 24 below.

The impact multipliers and the other multipliers constituting the impulse response functions are calculated from Table 24 and reported in Table 25C below. As can be seen from Tables 24 and 25C, the estimated results reveal the true relationships among the four aggregate macroeconomic variables well. To see this, let us compare the impulse response functions reported in Tables 25A, 25B, and 25C. The impulse response functions reported in Tables 25A and 25B do not diminish fast enough over time, indicating that not only the impacts of a contemporaneous shock but also the impacts of a previous shock in one of the four variables on the other variables do not dissipate fast enough over time. On the other hand, the impulse response functions reported in Table 25C diminish rapidly over time, indicating that not only the impacts of a

Table 24: Empirical Estimates of VAR Model with First Differences of Log-Levels(*)

Eq.	Dep.Var	const.	$\Delta(\ln Y_{t-1})$	$\Delta(\ln K_{t-1})$	$\Delta(\ln L_{t-1})$	$\Delta(\ln G_{t-1})$	$\Delta(\ln G_{cit-1})$	adj R^2	D-W	F
5.1	$\Delta(\ln Y_t)$	0.05	0.288	-1.12	-0.352	0.451		0.03	1.99	1.41
		(2.64)	(1.16)	(-1.50)	(-1.16)	(1.11)				
	$\Delta(\ln K_t)$	0.007	0.034	0.612	-0.026	0.111		0.58	1.71	19.9
		(2.55)	(0.91)	(5.33)	(-0.57)	(1.81)				
	$\Delta(\ln L_t)$	0.033	0.351	-1.204	-0.051	0.114		0.07	1.96	1.92
		(2.09)	(1.71)	(-1.93)	(-0.21)	(0.34)				
	$\Delta(\ln G_t)$	-0.002	0.027	0.119	-0.041	0.914		0.93	0.88	178
		(-0.74)	(0.96)	(1.42)	(-1.18)	(20.1)				
5.2	$\Delta(\ln Y_t)$	0.051	0.314	-0.964	-0.402		0.258	0.02	2.00	1.21
		(2.66)	(1.27)	(-1.27)	(-1.33)		(0.71)			
	$\Delta(\ln K_t)$	0.008	0.037	0.619	-0.029		0.094	0.57	1.72	19.7
		(2.66)	(0.99)	(5.38)	(-0.64)		(1.68)			
	$\Delta(\ln L_t)$	0.033	0.361	-1.128	-0.072		0.034	0.06	1.96	1.89
		(2.08)	(1.77)	(-1.81)	(-0.29)		(0.11)			
	$\Delta(\ln G_t)$	-0.002	0.041	0.141	-0.063		0.903	0.93	1.28	175.9
		(-0.94)	(1.32)	(1.51)	(-1.70)		(19.9)			
Eq.	Dep.Var	const.	$\Delta(\ln Y_{nrt-1})$	$\Delta(\ln K_{nrt-1})$	$\Delta(\ln L_{t-1})$	$\Delta(\ln G_{t-1})$	$\Delta(\ln G_{cit-1})$	adj R^2	D-W	F
5.3	$\Delta(\ln Y_{nr,t})$	0.048	-0.177	-0.969	0.321	0.665		0.04	1.88	1.45
		(3.22)	(-0.64)	(-2.03)	(1.05)	(1.92)				
	$\Delta(\ln K_{nr,t})$	0.006	-0.066	0.618	0.164	0.191		0.68	1.83	30.3
		(2.21)	(-1.28)	(7.01)	(2.89)	(2.96)				
	$\Delta(\ln L_t)$	0.038	-0.203	-1.07	0.483	0.264		0.08	1.85	2.16
		(2.83)	(-0.83)	(-2.51)	(1.76)	(0.85)				
	$\Delta(\ln G_t)$	0.001	0.025	-0.021	-0.019	0.956		0.92	0.85	169
		(0.72)	(0.74)	(-0.34)	(-0.52)	(22.1)				

(*): Equations are estimated using the OLS estimation method with first differences of log-levels of aggregate annual data (that is, with the growth rates of the original variables) from 1947 to 2005. Calculated t-statistics are given in parentheses. Δ stands for "the change in" and ln stands for the natural logarithm. The other variables are defined earlier.

contemporaneous shock but also the impacts of a previous shock in one of the four variables on the other variables dissipate rapidly.

Table 25A: Impulse Response Functions of the Estimated VAR Models (*)

Eq.	Dep.V	Shocks	0	1	2	3	4	5	6	7	8	9	10	long-run
3.1	Y	Y	4.11	3.66	3.35	3.13	2.99	2.91	2.87	2.88	2.93	3.00	3.11	34.9
		K	3.32	2.93	2.64	2.43	2.28	2.18	2.11	2.08	2.08	2.10	2.15	26.3
		L	93.8	78.7	67.0	57.7	50.5	44.7	40.1	36.3	33.3	30.8	28.8	562
		G	0.12	0.15	0.17	0.20	0.22	0.23	0.24	0.25	0.25	0.24	0.22	2.3
	K	Y	1.91	2.87	3.58	4.10	4.49	4.77	4.97	5.11	5.23	5.31	5.39	47.7
		K	2.66	3.32	3.78	4.08	4.25	4.34	4.36	4.33	4.27	4.18	4.07	43.6
		L	51.1	76.4	94.2	106.0	113.2	116.8	117.5	116.0	112.7	108.1	102.3	1114
		G	-0.3	0.09	0.45	0.81	1.16	1.51	1.85	2.18	2.49	2.80	3.09	16.2
	L	Y	0.12	0.10	0.08	0.07	0.06	0.05	0.04	0.03	0.03	0.02	0.02	0.6
		K	0.12	0.09	0.08	0.07	0.06	0.05	0.05	0.04	0.03	0.03	0.02	0.6
		L	3.89	3.34	2.92	2.58	2.32	2.10	1.92	1.76	1.63	1.50	1.39	25.4
		G	-0.02	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.0
	G	Y	0.03	0.17	0.28	0.36	0.42	0.47	0.51	0.54	0.58	0.61	0.65	4.6
		K	-0.1	0.03	0.07	0.09	0.10	0.09	0.07	0.05	0.03	0.01	0.00	0.5
		L	-1.4	1.48	2.97	3.42	3.08	2.13	0.74	-0.98	-2.96	-5.10	-7.34	-3.9
		G	1.05	1.13	1.21	1.28	1.35	1.41	1.47	1.52	1.56	1.60	1.62	15.2
3.2	Y	Y	4.39	3.82	3.42	3.14	2.95	2.83	2.76	2.74	2.74	2.77	2.82	34.4
		K	4.49	3.88	3.43	3.12	2.89	2.74	2.64	2.58	2.55	2.55	2.57	33.4
		L	89.5	72.5	59.7	50.0	42.6	36.9	32.3	28.7	25.8	23.4	21.4	483
		G_{ci}	0.14	0.12	0.11	0.10	0.09	0.08	0.06	0.04	0.01	-0.01	-0.05	0.7
	K	Y	2.17	3.36	4.23	4.87	5.33	5.67	5.91	6.08	6.21	6.30	6.38	56.5
		K	3.35	4.48	5.26	5.79	6.13	6.33	6.42	6.43	6.39	6.30	6.18	63.1
		L	50.9	79.2	98.4	111	118	121	121	118	114	108	100	1139
		G_{ci}	-0.4	0.07	0.47	0.85	1.23	1.59	1.94	2.28	2.60	2.90	3.18	16.8
	L	Y	0.12	0.10	0.09	0.07	0.07	0.06	0.05	0.05	0.05	0.04	0.04	0.7
		K	0.15	0.12	0.11	0.10	0.09	0.08	0.07	0.07	0.06	0.06	0.06	1.0
		L	3.73	3.21	2.83	2.55	2.34	2.18	2.06	1.96	1.87	1.79	1.71	26.2
		G_{ci}	-0.02	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.0
	G_{ci}	Y	0.01	0.16	0.26	0.32	0.36	0.39	0.40	0.41	0.41	0.41	0.42	3.6
		K	-0.1	0.04	0.09	0.11	0.10	0.07	0.02	-0.02	-0.07	-0.12	-0.17	0.0
		L	-1.1	0.9	1.5	0.9	-0.4	-2.3	-4.8	-7.5	-10.6	-13.8	-17.1	-54.4
		G_{ci}	1.04	1.13	1.21	1.29	1.35	1.41	1.45	1.49	1.51	1.52	1.52	14.9
	Y	Y	5.27	4.61	4.13	3.79	3.56	3.42	3.36	3.35	3.39	3.46	3.57	41.9
		K	2.71	2.38	2.14	1.97	1.85	1.78	1.75	1.74	1.76	1.79	1.85	21.7
		L	97.6	80.4	67.5	57.8	50.8	45.8	42.3	40.2	39.2	39.1	39.7	600
		$CU*G_{ci}$	2.74	2.29	1.95	1.69	1.50	1.36	1.25	1.18	1.13	1.10	1.09	17.3
	K	Y	2.51	3.95	4.88	5.44	5.74	5.86	5.85	5.78	5.67	5.57	5.48	56.7
		K	2.32	2.98	3.39	3.61	3.71	3.71	3.65	3.56	3.45	3.34	3.24	37.0
		L	50.5	79.3	95.8	103.1	104.0	100.1	93.1	84.2	74.2	63.9	53.8	902

3.3		**CU*G_{ci}**	1.24	2.29	3.00	3.44	3.70	3.80	3.80	3.72	3.59	3.42	3.23	35.2
	L	**Y**	0.15	0.13	0.12	0.10	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.9
		K	0.08	0.07	0.07	0.06	0.05	0.05	0.04	0.03	0.03	0.02	0.02	0.5
		L	3.98	3.55	3.17	2.82	2.49	2.18	1.88	1.60	1.34	1.08	0.84	24.9
		CU*G_{ci}	0.09	0.08	0.07	0.07	0.06	0.06	0.05	0.15	0.04	0.04	0.03	0.7
	CU*G_{ci}	**Y**	2.82	1.67	0.81	0.18	-0.27	-0.58	-0.77	-0.88	-0.91	-0.87	-0.79	0.4
		K	1.38	0.79	0.35	0.02	-0.20	-0.37	-0.48	-0.54	-0.57	-0.56	-0.52	-0.7
		L	58.6	29.6	7.4	-9.3	-21.8	-31.0	-37.4	-41.5	-43.8	-44.6	-44.1	-178
		CU*G_{ci}	2.53	1.76	1.16	0.69	0.32	0.03	-0.18	-0.34	-0.47	-0.56	-0.62	4.3
3.4	**Y**	**Y**	5.20	4.64	4.22	3.91	3.69	3.55	3.48	3.48	3.52	3.61	3.74	43.0
		K	2.29	2.05	1.87	1.73	1.64	1.58	1.55	1.54	1.56	1.60	1.65	19.1
		L	99.9	84.4	72.1	62.4	54.9	49.3	45.4	42.8	41.5	41.1	41.7	635
		CU*G	2.14	1.85	1.62	1.43	1.28	1.17	1.08	1.02	0.98	0.95	0.94	14.5
	K	**Y**	2.37	3.68	4.53	5.03	5.30	5.40	5.39	5.32	5.24	5.16	5.12	52.5
		K	2.09	2.58	2.87	3.02	3.07	3.05	2.99	2.91	2.82	2.74	2.68	30.8
		L	50.1	76.8	92.0	98.6	98.9	94.8	87.7	79.0	69.5	59.9	50.9	858
		CU*G	0.89	1.66	2.20	2.54	2.73	2.82	2.82	2.77	2.68	2.56	2.44	26.1
	L	**Y**	0.15	0.13	0.12	0.10	0.08	0.07	0.06	0.05	0.03	0.02	0.02	0.8
		K	0.47	0.06	0.05	0.05	0.04	0.03	0.03	0.02	0.02	0.01	0.01	0.8
		L	4.04	3.62	3.21	2.80	2.41	2.04	1.68	1.35	1.04	0.76	0.50	23.5
		CU*G	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.5
	CU*G	**Y**	3.84	2.38	1.27	0.45	-0.12	-0.51	-0.74	-0.84	-0.83	-0.73	-0.57	3.6
		K	1.54	0.92	0.44	0.09	-0.15	-0.31	-0.41	-0.45	-0.45	-0.41	-0.34	0.5
		L	82.4	44.6	15.4	-6.6	-22.9	-34.4	-42.0	-46.4	-48.2	-47.8	-45.6	-151
		CU*G	2.66	1.89	1.27	0.79	0.41	0.13	-0.08	-0.23	-0.34	-0.40	-0.43	5.7

(*): Obtained from Table 22.

Let us now interpret the estimated results reported in Tables 24 and 25C. Table 25C gives the impulse response functions of three estimated sets of equations. Since each set is estimated using a first-order VAR with first-differences of log-level data, the impact multipliers listed in the column titled "**0**" show the change in the contemporaneous growth rates of the corresponding variable when the growth rate of a contemporaneous shock increases by one. The column titled "1" is called one-period multipliers, measuring the change in the growth rate of the corresponding variable in time t+1 when the growth rate of a shock in one of the four variables increases by one in time t. The values in the other columns should be interpreted in the same way.

The values reported in the last column are called the long-run multipliers. Let us discuss the long-run multipliers in more detail. We will interpret the first four values of the last column. When the growth rate of private output increases by one in time t, then the growth rate of private output will increase by 4.5 over the subsequent ten years, ceteris paribus. Similarly, when the growth rate of the private fixed capital stock increases by one in time t, then the growth rate of private output will increase by 14.8 over the subsequent ten years, ceteris paribus. And, when the growth rate of the private total labor hours increases by one in time t, then the growth rate of private output will increase by 3.6 over the subsequent ten years, ceteris paribus. Finally, when the growth

Table 25B: Impulse Response Functions of the Estimated VAR Models (*)

Eq.	Dep.Var	Shocks	0	1	2	3	4	5	6	7	8	9	10	longrun
4.1	Y_{nr}	Y_{nr}	6.07	6.00	5.92	5.84	5.77	5.71	5.67	5.66	5.68	5.73	5.81	63.9
		K_{nr}	1.63	1.61	1.58	1.56	1.54	1.52	1.51	1.50	1.51	1.52	1.53	17.0
		L	98.7	95.8	92.5	89.0	85.7	82.6	79.9	77.6	75.8	74.6	73.9	926.1
		CU*G	1.96	1.96	1.96	1.95	1.94	1.92	1.91	1.90	1.90	1.91	1.92	21.2
	K_{nr}	Y_{nr}	1.75	3.39	4.50	5.20	5.57	5.71	5.66	5.50	5.25	4.96	4.66	52.2
		K_{nr}	1.57	1.84	2.02	2.11	2.14	2.12	2.07	2.00	1.91	1.82	1.73	21.3
		L	28.7	61.3	82.9	95.7	101.8	102.8	100.1	94.7	87.6	79.5	70.8	905.8
		CU*G	0.35	1.10	1.65	2.04	2.30	2.46	2.55	2.58	2.56	2.52	2.46	22.6
	L	Y_{nr}	0.21	0.19	0.17	0.15	0.13	0.11	0.09	0.07	0.05	0.04	0.03	1.2
		K_{nr}	0.05	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.3
		L	4.53	4.09	3.65	3.20	2.76	2.34	1.95	1.57	1.23	0.92	0.64	26.9
		CU*G	0.07	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.03	0.03	0.02	0.5
	CU*G	Y_{nr}	5.32	3.34	1.84	0.75	0.00	-0.48	-0.74	-0.82	-0.76	-0.59	-0.33	7.5
		K_{nr}	0.91	0.52	0.23	0.03	-0.10	-0.18	-0.21	-0.21	-0.19	-0.14	-0.07	0.6
		L	93.5	51.5	19.6	-4.1	-21.0	-32.3	-39.0	-42.2	-42.4	-40.3	-36.4	-93.0
		CU*G	2.85	2.07	1.46	0.98	0.62	0.36	0.18	0.06	0.00	-0.01	0.00	8.6
4.2	Y_{nr}	Y_{nr}	6.02	5.90	5.82	5.75	5.70	5.68	5.68	5.70	5.75	5.82	5.91	63.7
		K_{nr}	1.82	1.80	1.78	1.76	1.75	1.75	1.75	1.75	1.77	1.78	1.81	19.5
		L	99.2	95.6	92.4	89.5	87.0	84.9	83.1	81.8	80.8	80.2	79.9	954.2
		CU*G_{ci}	2.52	2.49	2.46	2.44	2.43	2.43	2.42	2.43	2.44	2.46	2.48	27.0
	K_{nr}	Y_{nr}	1.80	3.55	4.73	5.50	5.94	6.14	6.17	6.07	5.89	5.66	5.41	56.9
		K_{nr}	1.61	1.98	2.23	2.37	2.44	2.46	2.43	2.38	2.30	2.22	2.13	24.6
		L	30.0	65.1	88.5	102.8	110.3	112.6	111.2	107.0	100.9	93.6	85.6	1007.5
		CU*G_{ci}	0.53	1.54	2.27	2.79	3.15	3.37	3.50	3.56	3.56	3.53	3.46	31.3
	L	Y_{nr}	0.21	0.19	0.17	0.15	0.13	0.12	0.10	0.09	0.07	0.06	0.05	1.3
		K_{nr}	0.06	0.06	0.05	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02	0.4
		L	4.57	4.13	3.71	3.32	2.94	2.59	2.25	1.93	1.63	1.34	1.07	29.5
		CU*G_{ci}	0.09	0.09	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.7
	CU*G_{ci}	Y_{nr}	3.82	2.31	1.19	0.38	-0.18	-0.55	-0.76	-0.85	-0.83	-0.74	-0.58	3.2
		K_{nr}	0.82	0.47	0.20	0.01	-0.11	-0.20	-0.25	-0.27	-0.27	-0.24	-0.20	0.0
		L	68.4	35.9	11.4	-6.8	-19.9	-28.9	-34.7	-37.9	-39.0	-38.4	-36.4	-126.2
		CU*G_{ci}	2.70	1.93	1.33	0.87	0.53	0.27	0.09	-0.03	-0.10	-0.14	-0.15	7.3
	Y_{nr}	Y_{nr}	5.09	4.94	4.86	4.82	4.81	4.83	4.88	4.94	5.01	5.10	5.20	54.5
		K_{nr}	2.48	2.42	2.38	2.36	2.35	2.36	2.37	2.39	2.41	2.44	2.48	26.4
		L	96.4	92.0	88.9	86.9	85.6	84.8	84.4	84.2	84.3	84.6	85.0	957.0

4.3		**G_{ci}**	0.13	0.18	0.24	0.29	0.34	0.40	0.46	0.51	0.57	0.62	0.68	4.4
	K_{nr}	**Y_{nr}**	1.57	3.06	4.13	4.87	5.38	5.70	5.89	5.97	5.97	5.92	5.83	54.3
		K_{nr}	1.78	2.33	2.71	2.94	3.08	3.13	3.12	3.07	2.98	2.86	2.73	30.7
		L	31.1	67.3	93.2	111.1	122.9	130.0	133.5	134.1	132.4	128.8	123.8	1208.3
		G_{ci}	-0.3	-0.2	0.05	0.25	0.47	0.71	0.95	1.20	1.46	1.73	1.99	8.4
	L	**Y_{nr}**	0.18	0.15	0.14	0.12	0.11	0.10	0.09	0.09	0.08	0.07	0.07	1.2
		K_{nr}	0.09	0.08	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03	0.6
		L	4.48	3.99	3.61	3.31	3.07	2.86	2.68	2.51	2.35	2.19	2.02	33.1
		G_{ci}	-0.02	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.0
	G_{ci}	**Y_{nr}**	0.01	0.20	0.31	0.38	0.41	0.42	0.42	0.41	0.41	0.42	0.44	3.8
		K_{nr}	-0.05	-0.01	-0.01	-0.03	-0.06	-0.09	-0.13	-0.17	-0.20	-0.22	-0.23	-1.2
		L	-1.3	1.57	2.51	2.05	0.60	-1.54	-4.12	-6.95	-9.88	-12.8	-15.5	-45.3
		G_{ci}	1.05	1.14	1.22	1.30	1.37	1.44	1.50	1.55	1.59	1.62	1.64	15.4
4.4	**Y_{nr}**	**Y_{nr}**	4.20	4.07	3.99	3.93	3.91	3.91	3.92	3.96	4.00	4.06	4.13	44.1
		K_{nr}	0.68	0.67	0.66	0.66	0.65	0.65	0.64	0.64	0.64	0.64	0.64	7.2
		L	88.5	84.3	81.1	78.6	76.7	75.3	74.2	73.4	72.7	72.3	72.1	849.0
		G	0.08	0.14	0.19	0.25	0.31	0.36	0.42	0.47	0.53	0.58	0.63	4.0
	K_{nr}	**Y_{nr}**	0.43	1.63	2.48	3.07	3.46	3.70	3.82	3.86	3.84	3.77	3.67	33.7
		K_{nr}	1.08	1.06	1.03	0.98	0.91	0.84	0.76	0.68	0.60	0.51	0.42	8.9
		L	84.2	41.4	64.9	81.2	91.9	98.2	101.1	101.3	99.5	96.0	91.2	950.7
		G	-0.2	-0.1	0.11	0.29	0.47	0.67	0.88	1.08	1.30	1.51	1.72	7.8
	L	**Y_{nr}**	0.14	0.12	0.10	0.09	0.08	0.07	0.06	0.05	0.04	0.04	0.03	0.8
		K_{nr}	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.1
		L	4.10	3.61	3.21	2.88	2.60	2.36	2.13	1.93	1.73	1.53	1.34	27.4
		G	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.0
	G (mid-year)	**Y_{nr}**	0.00	0.17	0.29	0.37	0.42	0.47	0.50	0.54	0.59	0.64	0.72	4.7
		K_{nr}	-0.1	-0.1	-0.1	-0.12	-0.15	-0.17	-0.19	-0.20	-0.21	-0.22	-0.21	-1.7
		L	-1.6	1.30	2.56	2.62	1.81	0.43	-1.31	-3.24	-5.21	-7.09	-8.78	-18.5
		G	1.04	1.11	1.18	1.25	1.32	1.37	1.43	1.47	1.51	1.53	1.55	14.8
4.5	**Y_{nr}**	**Y_{nr}**	4.77	4.67	4.59	4.55	4.52	4.51	4.53	4.55	4.59	4.64	4.71	50.6
		K_{nr}	1.86	1.81	1.78	1.75	1.72	1.70	1.69	1.68	1.68	1.68	1.68	19.0
		L	92.7	89.1	86.3	84.0	82.1	80.5	79.1	77.9	76.9	76.1	75.4	900.2
		G	0.23	0.30	0.37	0.43	0.50	0.57	0.64	0.70	0.77	0.83	0.90	6.2
	K_{nr}	**Y_{nr}**	1.39	2.62	3.50	4.11	4.50	4.73	4.84	4.84	4.78	4.66	4.51	44.5
		K_{nr}	1.56	1.85	2.03	2.13	2.16	2.13	2.06	1.96	1.84	1.69	1.54	21.0
		L	28.2	60.1	82.9	98.7	108.9	114.8	117.2	116.9	114.3	110.1	104.5	1056.6
		G	-0.2	-0.1	0.15	0.35	0.56	0.78	1.01	1.25	1.48	1.72	1.95	9.0
	L	**Y_{nr}**	0.16	0.14	0.13	0.11	0.10	0.09	0.08	0.07	0.06	0.05	0.04	1.0
		K_{nr}	0.06	0.06	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.4
		L	4.30	3.84	3.46	3.14	2.87	2.62	2.39	2.18	1.98	1.78	1.58	30.1
		G	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.0
	G	**Y_{nr}**	0.03	0.24	0.38	0.47	0.53	0.57	0.59	0.62	0.64	0.68	0.72	5.5
		K_{nr}	-0.1	-0.1	-0.1	-0.05	-0.07	-0.10	-0.13	-0.15	-0.17	-0.18	-0.19	-1.2
		L	-1.7	2.12	4.01	4.50	3.96	2.68	0.91	-1.16	-3.38	-5.61	-7.74	-1.4
		G	1.06	1.15	1.23	1.30	1.38	1.45	1.51	1.57	1.62	1.66	1.69	15.6

(*): Obtained from Table 23.

rate of the public nonmilitary fixed capital stock increases by one in time t, then the growth rate of private output will increase by 3.2 over the subsequent ten years, ceteris paribus. It is clear from the above interpretations that the estimated impact of the private fixed capital stock on private real output is substantially larger than that of the public nonmilitary fixed capital stock. The estimates reported in Table 25C, however, are not in agreement with the estimated impacts of private labor on private output reported in the literature. The estimates given in Table 25C are considerably smaller than those reported in the literature.

Table 25C: Impulse Response Functions of the Estimated VAR Models(*)

Eq.	Dep.Var	Shocks	0	1	2	3	4	5	6	7	8	9	10	longrun
5.1	Y	Y	6.88	-0.83	-1.31	-0.34	0.01	0.01	0.00	0.01	0.02	0.03	0.03	4.51
		K	25.2	-4.6	-5.4	-1.3	0.09	0.11	0.07	0.12	0.17	0.20	0.20	14.84
		L	7.0	-1.4	-1.5	-0.4	0.00	-0.01	-0.03	-0.01	0.00	0.00	0.00	3.62
		G	-0.02	1.02	0.53	0.25	0.22	0.23	0.22	0.21	0.20	0.19	0.18	3.23
	K	Y	0.90	0.65	0.35	0.20	0.15	0.12	0.09	0.08	0.07	0.06	0.06	2.73
		K	4.43	3.08	1.73	1.09	0.82	0.67	0.56	0.49	0.45	0.42	0.39	14.13
		L	0.97	0.64	0.30	0.16	0.01	0.07	0.04	0.03	0.02	0.02	0.02	2.28
		G	0.14	0.27	0.34	0.35	0.35	0.35	0.35	0.34	0.33	0.32	0.31	3.45
	L	Y	5.08	1.07	-1.12	-0.81	-0.31	-0.14	-0.11	-0.09	-0.07	-0.05	-0.04	3.41
		K	19.8	2.5	-5.4	-3.6	-1.5	-0.8	-0.6	-0.5	-0.4	-0.3	-0.3	8.98
		L	6.5	0.9	-1.3	-0.8	-0.3	-0.1	-0.1	-0.1	-0.04	-0.02	-0.01	4.62
		G	-1.7	0.05	0.16	-0.09	-0.20	-0.21	-0.21	-0.21	-0.21	-0.21	-0.20	-3.06
	G	Y	0.00	0.07	0.08	0.13	0.16	0.18	0.18	0.18	0.18	0.18	0.17	1.51
		K	0.26	0.63	0.72	0.94	1.10	1.17	1.18	1.17	1.15	1.13	1.10	10.55
		L	-0.16	-0.11	-0.10	-0.04	0.00	0.02	0.03	0.04	0.04	0.04	0.04	-0.20
		G	1.29	1.26	1.21	1.16	1.11	1.07	1.03	1.00	0.97	0.94	0.91	11.95
5.2	Y	Y	6.77	-0.72	-1.27	-0.29	0.05	0.02	0.00	0.00	0.01	0.18	0.01	4.76
		K	25.8	-4.2	-5.4	-1.1	0.24	0.12	0.00	0.04	0.09	0.11	0.12	15.82
		L	6.36	-1.26	-0.14	-0.25	0.05	0.00	-0.02	-0.01	0.00	0.00	0.00	4.74
		G_{ci}	0.05	0.68	0.29	0.10	0.09	0.11	0.10	0.09	0.09	0.08	0.08	1.76
	K	Y	0.88	0.65	0.35	0.21	0.15	0.12	0.09	0.08	0.07	0.06	0.06	2.72
		K	4.49	3.17	1.78	1.12	0.85	0.68	0.57	0.49	0.45	0.41	0.39	14.40
		L	0.88	0.60	0.28	0.15	0.10	0.07	0.05	0.03	0.02	0.02	0.02	2.22
		G_{ci}	0.11	0.21	0.26	0.28	0.28	0.27	0.27	0.26	0.25	0.24	0.24	2.67
	L	Y	4.96	1.08	-1.07	-0.78	-0.28	-0.12	-0.11	-0.09	-0.08	-0.06	-0.05	3.40
		K	20.1	2.8	-5.3	-3.5	-1.4	-0.7	-0.6	-0.6	-0.5	-0.4	-0.4	9.58
		L	5.89	0.86	-1.19	-0.73	-0.21	-0.07	-0.07	-0.05	-0.04	-0.02	-0.02	4.35
		G_{ci}	-1.16	0.01	0.03	-0.16	-0.23	-0.23	-0.22	-0.22	-0.21	-0.21	-0.20	-2.80
		Y	0.00	0.08	0.06	0.12	0.18	0.20	0.21	0.21	0.02	0.20	0.19	1.47

	G_{ci}	K	0.27	0.64	0.68	0.98	1.22	1.32	1.33	1.32	1.30	1.27	1.23	11.56
		L	-0.13	-0.11	-0.12	-0.05	0.00	0.33	0.04	0.05	0.05	0.05	0.05	0.16
		G_{ci}	1.19	1.17	1.11	1.05	1.00	0.96	0.92	0.89	0.86	0.82	0.79	10.76
	Y_{nr}	Y_{nr}	4.49	-0.33	-0.57	-0.55	-0.34	-0.15	-0.03	0.01	0.02	0.01	0.00	2.56
		K_{nr}	16.3	-1.7	-3.4	-3.1	-1.9	-0.8	-0.2	0.1	0.13	0.08	0.03	5.50
		L	6.3	-0.3	-1.1	-1.2	-0.8	-0.4	-0.11	0.00	0.02	0.01	0.00	2.59
		G	-0.60	0.41	0.63	0.55	0.39	0.26	0.17	0.14	0.13	0.13	0.13	2.34
	K_{nr}	Y_{nr}	0.65	0.67	0.44	0.20	0.04	-0.02	-0.04	-0.04	-0.03	-0.01	-0.01	1.85
		K_{nr}	3.8	3.8	2.5	1.1	0.21	-0.19	-0.29	-0.25	-0.18	-0.11	-0.08	10.22
		L	1.13	1.30	0.89	0.40	0.07	-0.09	-0.14	-0.13	-0.10	-0.08	-0.06	3.19
5.3		G	-0.08	-0.06	0.09	0.24	0.33	0.36	0.35	0.34	0.31	0.29	0.28	2.45
	L	Y_{nr}	3.49	0.07	-0.61	-0.66	-0.44	-0.20	-0.04	0.02	-0.04	0.04	0.02	1.65
		K_{nr}	15.5	0.1	-3.7	-3.8	-2.4	-1.1	-0.22	0.17	0.28	0.24	0.16	5.32
		L	6.5	0.59	-1.10	-1.30	-0.89	-0.41	-0.08	0.08	0.13	0.12	0.09	3.72
		G	-1.7	-0.29	0.13	0.11	-0.04	-0.19	-0.28	-0.31	-0.31	-0.29	-0.27	-3.40
	G	Y_{nr}	-0.04	0.00	-0.02	-0.03	-0.04	-0.04	-0.04	-0.03	-0.03	-0.03	-0.03	-0.33
		K_{nr}	-0.14	-0.09	-0.21	-0.26	-0.28	-0.27	-0.26	-0.24	-0.22	-0.21	-0.20	-2.38
		L	-0.19	-0.17	-0.20	-0.22	-0.22	-0.21	-0.20	-0.19	-0.18	-0.17	-0.17	-2.12
		G	1.14	1.11	1.08	1.04	1.00	0.96	0.92	0.88	0.84	0.81	0.77	10.55

(*): Obtained from Table 24.

V.4. Granger Causality

An n-variable VAR model can theoretically be constructed with each equation in the model containing exactly p lags for any positive integer $n \geq 2$ and for any positive integer $p \geq 1$ so that the OLS estimator produces consistent and asymptotically efficient estimates of the coefficients in the system. Each equation in the system can be allowed to have different number of lags, however. In this case, the system is called *near-VAR* and seemingly unrelated regressions (SUR) yield consistent and asymptotically efficient estimates of the coefficients, not the OLS estimator. To preserve the symmetry of the system in order to use the OLS estimator efficiently, when one new variable is added to the model, the degrees of freedom decreases by p, and when one new lag is added to the system, the degrees of freedom decreases by n. Hence, the degrees of freedom erodes quickly and substantially when either the number of variables in the system or the number of lags increases. Therefore, we need to be very conservative in determining these numbers. Economics should be employed judiciously to choose the variables that have significant and quantitatively large effects on one another, and significance tests should be used to determine the last lag whose coefficients are mostly statistically significant.

An n-variable VAR with p lags in standard form can be written as

(37) $[x_{1t}; x_{2t}; \ldots; x_{nt}] = [a_{10}; a_{20};\ldots; a_{n0}] + [A_{11}(L), A_{12}(L), \ldots, A_{1n}(L);\ A_{21}(L), A_{22}(L),\ldots, A_{2n}(L); \ldots; A_{n1}(L), A_{n2}(L), \ldots, A_{nn}(L)] * [x_{1t-1}; x_{2t-1}; \ldots; x_{nt-1}] + [w_{1t}; w_{2t}; \ldots; w_{nt}]$,

where the terms w_{it} are white-noise disturbances that might be correlated with one another and where $A_{ij}(L)$ represent polynomials in the lag operator L. For example,
$A_{11}(L) = a_{11}(1) + a_{11}(2)L + a_{11}(3)L^2 + \ldots + a_{11}(p)^{Lp-1}$,
$A_{12}(L) = a_{12}(1) + a_{12}(2)L + a_{12}(3)L^2 + \ldots + a_{12}(p)L^{p-1}$,
and so on. It is clear that the polynomials $A_{ij}(L)$ are all degrees of (p-1) each since every equation in the model has exactly p lags to preserve the symmetry of the system to use the OLS estimator efficiently.

We might want to know whether the current and past realizations of a variable in a VAR model can help to improve the forecast of another variable in the system. Granger (1969) describes a procedure, known as the "*Granger-causality*" test thereafter, that can be used to determine this. Let us describe the test in general for an n-variable VAR model with p lags given in Equation (37). The first equation of the system is
$x_{1t} = a_{10} + [a_{11}(1)x_{1t-1} + a_{11}(2)x_{1t-2} + \ldots + a_{11}(p)x_{1t-p}] + [a_{12}(1)x_{2t-1} + a_{12}(2)x_{2t-2} + \ldots + a_{12}(p)x_{2t-p}] + \ldots + [a_{1n}(1)x_{nt-1} + a_{1n}(2)x_{nt-2} + \ldots + a_{1n}(p)x_{nt-p}]$.
We say that the variable x_{2t} does *not* Granger-cause the variable x_{1t} if all of the coefficients in the polynomial $A_{12}(L)$ are zero, that is, if $a_{12}(1) = a_{12}(2) = \ldots = a_{12}(p) = 0$. A standard F-test can be used to test these joint hypotheses. In particular, in a four-variable VAR model with one lag given in Equation (32), we say that private output does *not* Granger-cause public capital if the coefficient $a_{41} = 0$. This single hypothesis can be tested by a standard t-test.

It is important to emphasize that the Granger-causality test is neither a cause-effect test nor a test for exogeneity. Rather, it is a predictability test. In other words, for public capital to be exogenous, it is necessary that it *not* be affected by the contemporaneous values of private output, private capital, and private labor. However, the Granger-causality test only checks the effects of the past values of private output on the current value of the public capital. Therefore, the Granger-causality test only measures whether the present and past realizations of private output help to forecast the future values of public capital.

As can be seen from Tables 22, 23, and 24, we have estimated the first-order VAR model twelve times and the standard t-test shows that five out of twelve times (42 percent of the time) the estimated coefficient of the private output in the public capital equation is statistically different from zero while seven out of twelve times (58 percent of the time) the estimated coefficient of the private output in the public capital equation is not found to be statistically different from zero. Therefore, there is little evidence to claim that private output Granger-causes public capital. In other words, there is evidence to conclude that the claim that "private output does *not* Granger-cause public capital" is compatible with the sample data for the aggregate U.S. private economy for the years 1947-2005.

VI. A Comparison of the Production Function and Vector Autoregression Approaches

A key research question concerns how well each model fits the data. Since R^2 and the average of the sum of squared errors are two commonly used measures of goodness-of-fit in ordinary least squares, we say the estimated model is a good fit if either R^2 is relatively high or the average of the sum of squared errors is relatively low. The fit, however, improves even when an *irrelevant* explanatory variable is added to the model. To overcome this problem, we can use the adjusted R^2 since adding an irrelevant regressor to the model results in an increase in R^2 but a decrease in the adjusted R^2. In other words, the adjusted R^2 increases if and only if the added regressor is a relevant one. Since the reported adjusted R^2 has always been relatively high in every estimated equation, it is safe to conclude that every estimated equation fits the data well.

We also want to know which model fits the data better. There are several selection criteria that can be used to determine which model fits the data better. Among all, the two most commonly used selection criteria are the Akaike Information Criterion, denoted by AIC and given by [AIC = N ln(sum of squared errors) + 2k], and the Schwartz Bayesian Criterion, denoted by SBC and given by [SBC = N ln(sum of squared errors) + k ln(N)], where N is the number of *usable* observations, k is the number of estimated parameters including the constant term, and ln stands for the natural logarithm. It is important to remember that when lagged variables are used, some observations are lost. Since N enters into the two formulas, it is essential to compare models having the same number of *usable* observations; otherwise we end up with comparing the performance of the models over *different* sample periods.

When we compare the performance of two models, say model 1 and model 2, we say model 1 fits the data better than model 2 does if the AIC (or SBC) for model 1 is smaller than that for model 2. It is important to emphasize that when we use any of the two selection criteria to compare the performance of alternative models, the models must be estimated over the same sample period to be directly comparable.

When we add a regressor that does not have any explanatory power to the model, then the two selection criteria increase but since ln(N) is always greater than two, the

SBC always selects a more parsimonious model than does the AIC, meaning that the marginal cost of adding explanatory variables is greater with the SBC than with the AIC.

Of the two selection criteria, the SBC has superior large sample properties and is asymptotically consistent, while the AIC works better with small samples and is biased toward selecting an over-parameterized model. We should be confident in our results if the two selection criteria select the same model. If, on the other hand, they select different models, then we need to proceed with caution. We already know that the AIC tends to select an over-parameterized model and therefore we need to check whether every explanatory variable has a statistically significant effect. We also know that the SBC tends to select the more parsimonious model and hence we need to check whether the residuals are white noise.

We have calculated the AIC and the SBC for corresponding estimated models of both the production function approach and the vector autoregression (VAR) model. The two calculated selection criteria are given in Table 26 below. As can be seen from the

Table 26: Model Selection(*)

VAR Model			Production Function Approach			Selected
Equation	AIK	SBC	Equation	AIK	SBC	Model(**)
3.1	488.9	498.8	1.1	-413.9	-400.3	1.1
3.2	488.4	498.2	1.2	-410.9	-397.3	1.2
3.3	488.7	498.5	1.3	-422.1	-410.2	1.3
3.4	488.9	498.8	1.4	-424.8	-413.1	1.4
4.1	472.4	482.3	1.35	-419.4	-407.6	1.35
4.2	472.7	482.5	1.26	-421.7	-411.8	1.26
4.3	472.9	482.7	1.28	-418.7	-406.9	1.28
4.4	472.6	482.5	1.31	-421.3	-409.5	1.31
4.5	472.6	482.5	1.27	-422.1	-410.3	1.27
5.1	-360.5	-357.7	2.3	-8.2	3.6	5.1
5.2	-359.8	-357.1	2.4	-7.7	4.2	5.2
5.3	-369.3	-366.8	2.24	-37.7	-27.7	5.3

(*): AIC = N ln(sum of squared errors) + 2k and SBC = N ln(sum of squared errors + k ln(N), where N is the number of *usable* observations and k is the number of parameters estimated including the constant term.

(**): When model 1 and model 2 are compared, we say model 1 fits the data better than model 2 does if the AIC (or SBC) for model 1 is smaller than that for model 2.

table, the two selection criteria clearly show that the production function approach fits the level data better than does the first-order VAR model, whereas the VAR model fits the differenced data better than the production function approach. The calculated values of the two selection criteria for the nine estimated equations of the production function approach are considerably smaller than those for the VAR model, indicating that the production function approach works a lot better with the level data. However, the calculated values of the two selection criteria for the VAR model using first differences of log-level data are substantially smaller than those for the production function approach using the natural logarithm of the first differences of level data, indicating that the VAR model works with first differences of level data better.

VII. Concluding Remarks

We have focused primarily on the relationship between aggregate private output and a measure of the public fixed capital stock for the U.S. economy for the years 1947-2005. Our major findings can be summarized in this conclusion.

First, our in-depth analyses of U.S. aggregate macroeconomic variables related to private sector economic growth and the public sector nonmilitary fixed capital stock or core infrastructure capital stock have shown that the years 1973-1995 were not really significant deviations from the natural historical trend of private economic growth as claimed in some of the recent economic literature that followed Aschauer's seminal works. Some of the research has either assumed or concluded that the growth rates observed until 1973 were in agreement with the historical trend of the U.S. private economic growth and therefore needed little explanation. The economic slowdown of the years 1973-1995, however, was thought to be a significant deviation from the historical path and therefore necessitates further explanation. Our in-depth analyses, on the other hand, have shown that the years 1933-1973 were deviations from the historical trend of private economic growth in the sense that the average annual growth rates of private real output and related macroeconomic variables were significantly greater than those observed either before 1933 or after 1973. The relatively high growth rates over the forty-year time period were mostly the result of the highly productive nature of the inventions of the late nineteenth and early twentieth centuries, large investments in core infrastructure, the Great Depression and World War II. Furthermore, the U.S. private economy experienced all-time high growth rates in all measures of output and related macroeconomic variables during the golden era years of 1933-1944. Considering any comparable periods, the golden era years 1933-44 were, without question, the most technologically progressive period in the history of the U.S. economy.

Second, the U.S. private economy experienced a productivity speed-up after 1995 as a result of the fact that improved and affordable information technologies and significant developments in Internet and information-technology related industries have enhanced efficiency. The recent developments in information technologies and internet related industries have lowered the cost of production and distribution, increased management efficiency and market competition, and broadened markets for buyers and sellers and therefore positively contributed to the growth rates of real GDP, MFP, and labor productivity in a significant way. And yet, there is still little evidence to conclude that the new era will be similar to the years 1933-1944.

Third, we have empirically tested two models, a production function approach and a VAR model, to measure the effects of public fixed capital and private output on each other. Some researchers have claimed that the causation runs from private output to public fixed capital while most of the researchers have found that the causation runs in the other direction. We have found little evidence to support the claim that the causation runs from private output to public capital. Our estimates show that the claim that the causation runs from public fixed capital to private output is generally compatible with the sample data.

Fourth, the economics literature is far from reaching a consensus on the magnitude of the positive effect of the public fixed capital stock on private output. Most of the studies in the existing literature report an impact of the private fixed capital stock on private output that is too small to be credible, whereas they report an impact of the public fixed capital stock on private output that is too large to be credible. Our estimates, on the other hand, suggest not only a positive impact of the public fixed capital stock on private output that seems credible but also a positive and quantitatively very large and credible impact of the private fixed capital stock on private output. Our estimates also show that, contrary to the existing economics literature, there is sufficient evidence to support the claim that the private fixed capital stock is more important at the margin to the aggregate private production process than the public nonmilitary fixed capital stock or the public core infrastructure capital stock.

Fifth, we have tested the two models using both level data and first differences of log-level data and found that the ordinary least squares estimates using the level data have such econometric problems as multi-collinearity and spurious regressions. We have conducted several Dickey-Fuller tests and augmented Dickey-Fuller tests and found that the aggregate private real output series, the aggregate private sector total labor hour series, and the aggregate public sector core infrastructure capital stock series are integrated of order one processes, whereas the aggregate private sector real fixed capital stock series and the aggregate public sector real nonmilitary fixed capital stock series are integrated of order two processes. Therefore, after determining that the variables are difference-stationary processes, we have estimated the production function model using the natural logarithm of the first differences of level data and the first-order VAR model using the first-differences of log-level data. As a result, the goodness-of-fit has increased substantially, yielding the true relationships among the variables.

Sixth, we have tested the two models using both level data and first differences of log-level data and found that the production function approach fits the level data better than does the first-order vector autoregression (VAR) model, while the VAR model fits the first differences of the log-level data better than the production function approach. We believe that the long-run multipliers derived from the estimated impulse response functions of the VAR model describe the relationships among private output, private fixed capital, private labor, and public fixed capital well.

Lastly, we have conducted several joint hypothesis tests and found that there is sufficient sample evidence to support the claim that not only that the private sector operates under constant returns to scale in all inputs, private and public, for the years 1947-2005, but also that the private fixed capital stock is more important to the aggregate private production process than either of the two measures of the public fixed capital stock.

Notes

1. The definition is my own.
2. Gramlich (1994, p. 1181) refers to the following "engineering needs assessment" studies.
    - A) Association of General Contractors (May 1983), *"America's Infrastructure: A Plan to Rebuild."*
    - B) Federal Highway Administration (1993), *"The Status of the Nation's Highways, Bridges, and Transit: Conditions and Performance,"* U.S. Department of Transportation, Federal Highway Administration, Washington, DC.
    - C) National Council on Public Works Improvement (1988), *"Fragile Foundations: A Report on America's Public Works."*
    - D) U.S. Congressional Budget Office (Apr. 1983), *"Public Works Infrastructure: Policy Considerations for the 1980s,"* U.S. GPO, Washington, DC.
    - E) U.S. Congressional Budget Office (Sept. 1988), *"New Directions for the Nation's Public Works,"* U.S. GPO, Washington, DC.
    - F) U.S. Congressional Budget Office (Aug. 1993), *"The Growth of Federal User Charges,"* U.S. GPO, Washington, DC.
    - G) U.S. Department of Transportation (1990), *Moving America: New Directions, New Opportunities,"* Washington, DC.
3. Westerhout, E.W.M and J. van Sinderen (1994), *The Influence of Tax and Expenditure Policies on Economic Growth in the Netherlands: An Empirical Analysis,"* De Economist, 142: 43-61.
4. See Cain (1997, pp.123-4) and Field (2003, pp.1407-8).
5. See Cain (1997, p.132).
6. See Field (2003, p.1407).
7. The title of the Field's (2003) article is *"The most technologically progressive decade of the century"* and throughout his article he claims that "*the years 1929-1941 were, in the aggregate, the most technologically progressive of any comparable period in U.S. economic history,"* Field (2003, p.1399). Field came up with the years 1929-1941 as the best years because he used the business-cycle peaks for the beginning and end points of his sub intervals. If he had let the data reveal the best years, then he would in fact have concluded that the best years of the U.S. economic history, the golden years, were 1933-1944.
8. Field (2003, p.1403).
9. To find a list of those researchers, see Akkina and Celebi (2002).
10. See Cain (1997), Fernald (1999), Gramlich (1994), and Munnell (1990a, 1990b, 1992, and 1993).
11. Construction of the interstate highway system of the United States peaked in the late 1950s and early 1960s and was largely completed by 1973. Fernald (1999) studies the effect of building roads on the relative productivity performance of U.S. industries from 1953 to 1989. He concludes that road building produced a one-time, unrepeatable productivity boost in the 1950s and 1960s. In other words, he tries to show that for certain public infrastructure capital stock, here roads, the rate of return is very high until the required level is reached, after which diminishing returns set in, lowering the rate of return very sharply. Once the required level is reached, then new roads should not be built, instead the existing ones should properly be maintained. He concludes that the U.S. does not have a big shortage of roads but it has a big shortage of maintenance. However, some other researchers such as Cain (1997) and Gramlich (1994) believe that after the initial investment, diminishing returns set in slowly, still leaving a high rate of return.
12. Fogel, R.W. (1960), *"The Union Pacific Railroad: A Case in Premature Enterprise,"* The Johns Hopkins Press, Baltimore.
13. To test for the existence of *negative* first-order autocorrelated errors, set the null and alternative hypotheses as follows. H_0: $r = 0$ vs. H_1: $r < 0$. If $d < 4 – d_u$, then we fail to reject the null hypothesis and conclude that the sample data suggest that no negative autocorrelation exists; if $d > 4 - d_L$, we reject the null hypothesis and conclude that the sample data support the existence of first-order negative autocorrelation; and if $4 – d_u < d < 4 – d_L$, then the Durbin-Watson bounds test is inconclusive and thus another method should be used.
14. Dickey and Fuller (1979, 1981, and 1987), Engle and Granger (1987), Granger (1981 and 1986), and Stock (1987) are some of the other studies trying to make the same point.

Appendix A. Methodology Used to Calculate Growth Rates and Methodology Used to Divide a Given Time Interval into Several Sub Intervals

The methodology that we used to calculate the annual growth rate of any discrete variable over a closed time interval throughout this dissertation will now be explained. The formula $\mathbf{A_t = A_i\,(1+r)^{(t-i)}}$ is used to find the value of A in time t using the value of A in time i together with the annual growth rate r based on the assumption that compounding takes place only once within a given time period. In this case, if A_t, A_i, t, and i are known, then r is given by $e^{\{[\ln(A_t / A_i)]/(t-i)\}} - 1$. If it is reasonable to assume that the compounding takes place infinitely many times within any given time period, then the appropriate formula $\mathbf{A_t = A_i\, e^{r(t-i)}}$ is used, yielding r as $\mathbf{\ln(A_t / A_i) / (t - i)}$. Since we believe that for any macroeconomic variable it is not reasonable to assume that the compounding can take place infinitely many times within a given time period, we used the former to find the growth rates of all variables compounded annually given in the tables of this dissertation.

Second, some of the tables of this dissertation contain only the growth rates of some macroeconomic variables compounded annually. It is clear that a given time interval can be divided among several mutually exclusive sub intervals in a number of ways. Since different sub intervals have different initial and terminal time periods and since the growth rate of a variable compounded annually depends on the values of the variable at the initial and terminal time periods, different sub intervals can obviously yield different annual growth rates. It is also clear that the choice of sub intervals is somewhat arbitrary. To either eliminate or decrease the arbitrariness, some researchers let the data reveal the sub intervals, whereas some other researchers first justify some rules and then use those rules to determine sub intervals. For example, Gordon (1999) lets the data reveal the sub intervals while Field (2003, pp.1402-3) suggests the use of the following rule.

"*Perhaps the most critical imperative in analyses of productivity trends is that comparisons be made between years in which the economy is at similar stages of the business cycle. In the expansion phase of a cycle, as output increases, input hours go up, but often only with a lag. Cyclical recovery, in and of itself, will commonly lead to an* *acceleration in productivity growth rates that slows as the expansion nears its end, and a measurement from trough to peak, for example, may tell us little about long-term trends. The most straightforward way to avoid the contamination of cyclical effects is to choose business-cycle peaks for both beginning and end points of comparison."*

When the time interval to be divided among several sub intervals is very long, then the rule suggested by Field can not serve its purpose well since, for example, the unemployment rate at one business-cycle peak can be quite different from the unemployment rate at another business-cycle peak. Therefore, following the former group, we let the data mostly reveal sub intervals. In other words, to find, for example, a time period over which growth rates were the highest, we tried different initial and terminal points and chose the ones yielding highest growth rates.

Appendix B. Definitions of the Variables, Data Sources, and the Raw Data

The raw data used in this dissertation are produced by two main federal agencies, the Bureau of Economic Analysis (BEA, a branch of the U.S. Department of Commerce) and the Bureau of Labor Statistics (BLS, a branch of the U.S. Department of Labor). All but the second column of Table A10 of the raw data used here are posted on the worldwide websites of the BEA and BLS. The second column of Table A10 contains total economy labor hours received via an email from BLS. Each table of this dissertation employing the raw data provides the source of the data in a footnote of the table. Here, we would like to mention two things common to all tables.

First, the worldwide website of each federal agency contains several tables with the same title but different time intervals. Sometimes it is even possible to find several tables with both the same title and the same time interval but with different datum entries. Most of the time, the tables with the same title do not have any posted information that can be used to distinguish one from another. This makes it a very time consuming process to decide which tables we should be using. Each agency has also posted a long list of its employees in charge of answering questions asked by the public. The lists contain the names, phone numbers, and email addresses of the employees in charge. Even when you call to seek some help from those employees to decide which tables you need to use, most of the time you see that the employees in charge are not aware of the details of tables posted on the websites. Therefore, we have spent a substantial amount of time to determine which tables we need to use.

Second, you start with say the website of the BEA and spend some time to determine the tables that you need to use. Then you go to the website of the BLS to find the corresponding data there, but you see that the BLS does not have the corresponding data for some of the variables/tables that you chose before. Then you go back to the website of the BEA to start all over or vice versa. This also makes it a very time consuming process to see what kind of data available.

Data on the manufacturing sector's total capacity utilization rate for the years 1959-2005 are taken from Table *B-54." Capacity utilization rates, 1959-2005,"* which is from Appendix B of the 2006 *Economic Report of the President*. These data are produced by the Board of Governors of the Federal Reserve System and posted on the following website of the U.S. Government Printing Office (GPO Access) http://www.gpoaccess .gov/eop/tables06.html under *"Production and Business Activity."* Capacity utilization rate data are manufacturing sector total series including manufacturing as defined in the North American Industry Classification System (NAICS) plus those industries —logging, newspaper, periodical, book and directory-publishing— that have traditionally been considered to be manufacturing and included in the industrial sector. Manufacturing sector's total capacity utilization rate data for the years 1947-1958 are produced by us as eight-year moving backward average of the data for the years 1959 and thereafter. For example, the 1958 year value is the simple average value of the values of the years 1959-1966, and the 1957 year value is the simple average value of the values of the years 1958-1965, and so on.

Level data on the aggregate private sector total labor hours are produced by the U.S. Bureau of Labor Statistics (BLS) and sent to us via an attachment of an email by Mr. Shawn Sprague, whose email address is Sprague.Shawn@bls.gov and whose phone number is (202)691-5612, upon our request. His above contact information is obtained from the BLS website and he has been contacted to get the level data on labor hours since the BLS does not publish the level data.

In our regression analysis, we use the aggregate private sector total labor hours in billions of hours which is the difference between the aggregate business sector total labor hours in billions of hours and the aggregate government enterprises total labor hours in billions of hours. These data were received via an attachment of an email. The BLS does not publish the level data but it does publish the index data. Therefore, the level data received via an attachment of an email are first verified by the customized index data posted on the following website of the BLS http://data.bls.gov/PDQ/outside.jsp?survey=pr and then are used in our regression analysis. For example, from the above website, you can get customized index data with the base year of 1992, i.e., 1992=100, on aggregate total business sector labor hours for the years 1947-2005, which we have done. Then we created another index series for the aggregate total business sector labor hours with the same base year from the level data received via an attachment of an email and then compared the two index series to see whether we have an exact match. After verifying the exact match, we have used the level data received via an attachment of an email in our regression analysis.

Data on two measures of aggregate private sector real output are obtained from the corresponding nominal data and the corresponding quantity index data produced by the U.S. Bureau of Economic Analysis (BEA) and posted on its following two websites http://www.bea.gov/bea/dn/nipaweb/SelectTable.asp?Selected=N and http://www.bea.gov/bea/dn2/gdpbyind_data.htm as follows.

One of the lines under "value added (millions of dollars)" of this website of the BEA (http://www.bea.gov/bea/pn/GDPbyInd_VA_NAICS_47to97R.xls) contains federal government enterprises' nominal value added and another line from the same section contains state & local government enterprises' nominal value added for the years 1977-1997. One of the lines under "chain-type quantity indexes for value added (2000=100)" contains corresponding quantity indexes for the federal government enterprises' value added and another line from the same section contains corresponding quantity indexes for the state & and local government enterprises' value added for the years 1977-1997. One of the lines under "value added (millions of dollars)" of this website of the BEA (http://www.bea.gov/bea/pn/GDPbyInd_VA_NAICS_1998-2005.xls) contains federal government enterprises' nominal value added and another line from the same section contains state & local government enterprises' nominal value added for the years 1998-2005. One of the lines under "chain-type quantity indexes for value added (2000=100)" contains corresponding quantity indexes (2000=100) for the federal government enterprises' value added and another line from the same section contains corresponding quantity indexes (2000=100) for the state & and local government enterprises' value added for the years 1998-2005. Two new chain-type quantity indexes for value added (2000=1.00) for the years 1977-1997 and 1998-2005 are found by dividing the values of the above quantity indexes by 100. Federal government enterprises' real GDP series for the years 1977-2005 is then found by multiplying the 2000 year value of the federal government enterprises' nominal GDP by the corresponding chain-type new quantity index (2000=1.00) series. Similarly, state & local government enterprises' real GDP series for the years 1977-2005 is found by multiplying the 2000 year value of state & local government enterprises' nominal GDP by the corresponding chain-type new quantity index (2000=1.00) series. Total government enterprises' real GDP series for the years 1977-2005 is then found by adding the two real series. Despite our great effort, we could not find data on total government enterprises' real GDP for the years 1947-1976 and therefore we have calculated the missing year values as the weighted moving backward average of the values of the years 1977-2005.

Total business sector real GDP are taken form line 2 of "Table 1.3.6. Real Gross Value Added by Sector, Chained Dollars" which is customized to contain the years 1947-2005 from this website http://www.bea.gov/bea/dn/nipaweb/TableView.asp?SelectedTable=25&FirstYear=2004&LastYear=2006&Freq=Qtr of the BEA. The total private sector *real* annual GDP series for the years 1947-2005 is then found by subtracting the total government enterprises real GDP series from the total business sector real GDP series.

Total private sector residential real GDP series for the years 1947-2005 is found by multiplying the 2000 year value of the nominal gross private residential investment taken from "Table 1.1.5. Gross Domestic Product (Billions of dollars)" posted on this website http://www.bea.gov/bea/dn/nipaweb/TableView.asp?SelectedTable=5&FirstYear=2004&LastYear=2006&Freq=Qtr of the BEA by the corresponding quantity index (2000=1.00) line obtained by dividing the corresponding index line by 100 of "Table 1.1.3. Real Gross Domestic Product, Quantity Indexes (2000=100)" posted on the website http://www.bea.gov/bea/dn/nipaweb/TableView.asp?SelectedTable=3&FirstYear=2004&LastYear=2006&Freq=Qtr of the BEA. Total private sector nonresidential *real* annual GDP series for the years 1947-2005 is then found by subtracting the total private sector residential real GDP series from the total private sector real GDP series.

Data on two measures of the private sector aggregate real fixed capital stock for the years 1947-2005 are calculated as follows. A new quantity index (2000=1.00) series is obtained by dividing "private fixed assets" line by 100 of "Table 2.2. Chain-Type Quantity Indexes for Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type (2000=100)" posted on http://www.bea.gov/national/FA2004/TableView.asp?SelectedTable=19&FirstYear=2000&LastYear=2005&Freq=Year website of the BEA. "Table 2.1. Current-Cost Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type" posted on the website http://www.bea.gov/national/FA2004/TableView.asp?SelectedTable=18&FirstYear=2000&LastYear=2005&Freq=Year of the BEA gives the breakdown of the year-end estimates of the private sector fixed assets by type in billions of current-year dollars. The aggregate private sector *real* net fixed capital stock for the years 1947-2005 is then obtained by multiplying the 2000 year nominal value of "private fixed assets" by the new quantity index (2000=1.00) series.

"Nonresidential equipment and software" line of "Table 2.2. Chain-Type Quantity Indexes for Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type (2000=100)" for the years 1947-2005 is divided by 100 to get a new quantity index (2000=1.00) series. The year-end estimates of the private sector nonresidential equipment and software real fixed assets series for the years 1947-2005 is then calculated by multiplying the 2000 year nominal value of the private sector nonresidential equipment and software fixed assets taken from the "nonresidential equipment and software" line of "Table 2.1. Current-Cost Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type" by the corresponding new quantity index (2000=1.00) line.

"Nonresidential structures" line of "Table 2.2. Chain-Type Quantity Indexes for Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type (2000=100)" for the years 1947-2005 is divided by 100 to get a new quantity index (2000=1.00) series. The year-end estimates of the private sector nonresidential structures real fixed assets series for the years 1947-2005 is then calculated by multiplying the 2000 year nominal value of the private sector nonresidential structures fixed assets taken from the "nonresidential structures" line of "Table 2.1. Current-Cost Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type" by the corresponding new quantity index (2000=1.00) line. The sum of the two series, the year-end estimates of the private sector nonresidential equipment and software real net fixed assets series and the year-end estimates of the private sector nonresidential structures real net fixed assets series, then gives the year-end estimates of the private sector nonresidential *real* net fixed assets for the years 1947-2005.

The mid-year estimates of the aggregate private sector real fixed capital stock series for the years 1947-2005 are obtained by assigning the simple average value of the values of the two subsequent years, say year t-1 and year t, of the year-end estimate series of the aggregate private sector real fixed capital stock to the value of year t of the year-end estimate series. The beginning-year estimates of the aggregate private sector real fixed capital stock series for the years 1947-2005 are found by replacing the value of year t of the year-end estimate series of the aggregate private sector real fixed capital stock with the value of year t-1 of the year-end estimate series. The mid-year and beginning-year estimates of the private sector nonresidential real fixed capital stock are calculated in a similar fashion.

Data on two measures of the public sector aggregate real fixed capital stock for the years 1947-2005 are found as follows. "Table 7.2A. Chain-Type Quantity Indexes for Net Stock of Government Assets, 1925-1996, (2000=100)" posted on the website http://www.bea.gov/national/FA2004/TableView.asp?SelectedTable=36&FirstYear=1991&LastYear=1996&Freq=Year of the BEA gives the year-end estimates of the breakdown of public fixed assets quantity indexes for the years 1925-1996. Table 7.2B posted on the website http://www.bea.gov/national/FA2004/TableView.asp?SelectedTable=31&FirstYear=2000&LastYear=2005&Freq=Year of the BEA gives the same thing for the years 1997-2005. Several lines for the years 1947-2005 from Table 7.2A and the corresponding lines for the years 1997-2005 from Table 7.2B are combined to create another quantity index (2000=100) table, each of whose entries is then divided by 100 to get a new quantity index (2000=1.00) table for the selected lines.

"Table 7.1A. Current-Cost Net Stock of Government Fixed Assets, 1925-1996 (Billions of dollars; yearend estimates)" posted on the website http://www.bea.gov/national/FA2004/TableView.asp?SelectedTable=35&FirstYear=1991&LastYear=1996&Freq=Year of the BEA gives the year-end estimates of the nominal values of the breakdown of the public sector fixed capital stock for the years 1925-1996. Table 7.1B posted on the website http://www.bea.gov/national/FA2004/TableView.asp?SelectedTable=30&FirstYear=2000&LastYear=2005&Freq=Year of the BEA gives the same thing for the years 1997-2005. Selected lines corresponding to the selected quantity index lines for the years 1947-1996 from Table 7.1A and the corresponding lines for the years 1997-

2005 from Table 7.1B are combined to create a table of the year-end estimates of the nominal values of the public sector fixed capital stock in billions of dollars.

The year-end estimates of the aggregate public sector real net total fixed capital stock for the years 1947-2005 are found by multiplying the 2000 year nominal value of the public sector total fixed assets taken from the "government fixed assets" line of Table 7.1B by the entries of the corresponding line of the new quantity index (2000=1.00) table. As explained before, the public sector total military fixed capital stock is assumed to contain only federal government military capital stock. In other words, state & local governments are assumed to own no military fixed capital stock. Therefore, the year-end estimates of the aggregate public sector real net military fixed capital stock for the years 1947-2005 are found by multiplying the 2000 year nominal value of the public sector total military fixed assets taken from the "national defense" line of Table 7.1B by the entries of the corresponding line of the new quantity index (2000=1.00) table. The year-end estimates of the aggregate public sector *real* net nonmilitary fixed capital stock for the years 1947-2005 are then calculated by subtracting the year-end estimates of the aggregate public sector real net military fixed capital stock from the year-end estimates of the aggregate public sector real net total fixed capital stock.

As explained before, the federal government is assumed to own no core infrastructure capital stock. In other words, the public sector core infrastructure capital stock is assumed to contain only state & local government core infrastructure capital stock. Therefore, our year-end estimates of the aggregate public sector real net core infrastructure capital stock for the years 1947-2005 underestimate the true values. Moreover, even though Line 53, "other structures," of Tables 7.1A, 7.1B, 7.2A, and 7.2B consists primarily of electric and gas facilities, transit systems, and airfields, we will assume that it contains only core infrastructure net capital stock in power and transportation systems and therefore we overestimate core infrastructure capital stock in power and transportation systems. Thus, considering these two opposite effects, our year-end estimates of the aggregate public sector real net core infrastructure capital stock for the years 1947-2005 likely underestimate the true values somewhat and be calculated as follows.

The year-end estimates of the public core infrastructure real net capital stock in *education* for the years 1947-2005 are found by multiplying the 2000 year nominal value of the state & local governments "educational" fixed capital stock taken from Line 46 of Table 7.1B by the new quantity index (2000=1.00) line obtained by dividing by 100 of the corresponding quantity index (2000=100) line of Tables 7.2A and 7.2B.

The year-end estimates of the public core infrastructure real net capital stock in *health care* for the years 1947-2005 are calculated by multiplying the 2000 year nominal value of the state & local governments' "hospital" fixed capital stock taken from Line 47 of Table 7.1B by the new quantity index (2000=1.00) line obtained by dividing by 100 of the corresponding quantity index (2000=100) line of Tables 7.2A and 7.2B.

The year-end estimates of the public core infrastructure real net capital stock in *highways and streets* for the years 1947-2005 are obtained by multiplying the 2000 year nominal value of the state & local governments' "highways and streets" fixed capital stock taken from Line 49 of Table 7.1B by the new quantity index (2000=1.00) line obtained by dividing by 100 of the corresponding quantity index (2000=100) line of Tables 7.2A and 7.2B.

The year-end estimates of the public core infrastructure real net capital stock in *power and transportation systems* for the years 1947-2005 are found by multiplying the 2000 year nominal value of the state & local governments' "other structures" fixed capital stock taken from Line 53 of Table 7.1B by the new quantity index (2000=1.00) line obtained by dividing by 100 of the corresponding quantity index (2000=100) line of Tables 7.2A and 7.2B.

The year-end estimates of the public core infrastructure real net capital stock in *sewer lines* for the years 1947-2005 are calculated by multiplying the 2000 year nominal value of the state & local governments' "sewer systems structures" fixed capital stock taken from Line 51 of Table 7.1B by the new quantity index (2000=1.00) line obtained by dividing by 100 of the corresponding quantity index (2000=100) line of Tables 7.2A and 7.2B. The year-end estimates of the public core infrastructure real net capital stock in *water lines* for the years 1947-2005 are found by multiplying the 2000 year nominal value of the state & local governments' "water supply facilities" fixed capital stock taken from Line 52 of Table 7.1B by the new quantity index (2000=1.00) line obtained by dividing by 100 of the corresponding quantity index (2000=100) line of Tables 7.2A and 7.2B. The sum of the all real capital stocks given in this paragraph then gives our year-end estimates of the aggregate public sector *real* net core infrastructure capital stock for the years 1947-2005.

The mid-year estimates of the aggregate public sector real net fixed nonmilitary capital stock series for the years 1947-2005 are obtained by assigning the simple average value of the values of the two subsequent years, say year t-1 and year t, of the year-end estimate series of the aggregate public sector real fixed nonmilitary capital stock to the value of year t of the year-end estimate series. The beginning-year estimates of the aggregate public sector real fixed nonmilitary capital stock series for the years 1947-2005 are found by replacing the value of year t of the year-end estimate series of the aggregate public sector real fixed nonmilitary capital stock with the value of year t-1 of the year-end estimate series. The mid-year and beginning-year estimates of the public sector real net core infrastructure capital stock are calculated similarly.

Table A1: Nominal GDP, Private GDP, and Nonresidential Private GDP (*)

(Billions of Dollars)

Year	GDP						Private GDP				Private Nonresid.GDP	
			(**)		(***)				(***)			
	levels	%Δ	$	%Δ	$	%Δ	levels	%Δ	$	%Δ	levels	%Δ
1929	104	--	851	--	--	--	88	--	--	--	84	--
1930	91	-12.0	741	-12.9	--	--	76	-13.5	--	--	74	-12.3
1931	77	-16.1	617	-16.8	--	--	62	-18.1	--	--	61	-17.9
1932	59	-23.3	470	-23.8	--	--	46	-25.9	--	--	46	-25.0
1933	56	-3.9	449	-4.5	--	--	44	-4.0	--	--	44	-3.7
1934	66	17.0	522	16.3	--	--	53	19.3	--	--	52	18.9
1935	73	11.1	576	10.3	--	--	60	12.6	--	--	58	12.1
1936	84	14.3	654	13.6	--	--	68	14.5	--	--	67	14.1
1937	92	9.7	713	9.0	--	--	76	11.5	--	--	74	11.2
1938	86	-6.3	663	-7.0	--	--	70	-8.6	--	--	68	-8.8
1939	92	7.1	704	6.2	--	--	75	8.2	--	--	72	7.1
1940	101	10.0	767	8.9	--	--	84	11.5	--	--	81	11.3
1941	127	25.0	950	23.8	--	--	107	26.6	--	--	102	27.0
1942	162	27.8	1201	26.4	--	--	133	25.0	--	--	131	27.9
1943	199	22.7	1452	21.0	--	--	155	16.8	--	--	154	17.6
1944	220	10.7	1588	9.3	--	--	166	6.8	--	--	165	6.9
1945	223	1.5	1594	0.4	--	--	164	-1.2	--	--	162	-1.4
1946	222	-0.4	1572	-1.4	--	--	175	6.6	--	--	167	2.9
1947	244	9.9	1694	7.8	--	--	201	14.8	1.9	--	189	12.9
1948	269	10.2	1836	8.4	2.19	--	225	12.0	2.2	11.3	209	10.9
1949	267	-0.7	1792	-2.4	2.22	1.6	220	-2.0	2.2	1.3	206	-1.7
1950	294	9.9	1929	7.7	2.40	8.0	245	11.1	2.4	9.5	224	9.0
1951	339	15.5	2191	13.5	2.62	9.2	281	14.7	2.7	11.2	262	17.0
1952	358	5.6	2274	3.8	2.74	4.5	292	4.0	2.8	4.0	274	4.3
1953	379	5.9	2369	4.1	2.87	4.7	309	5.6	2.9	4.3	289	5.7
1954	380	0.3	2333	-1.5	2.96	3.3	306	-0.9	3.0	2.6	285	-1.6
1955	415	9.0	2500	7.1	3.13	5.6	335	9.5	3.1	5.5	310	8.8
1956	438	5.5	2590	3.6	3.25	3.7	352	5.0	3.2	3.4	328	5.9
1957	461	5.4	2681	3.5	3.45	6.3	369	4.9	3.5	6.5	347	5.7
1958	467	1.3	2672	-0.4	3.62	5.0	368	-0.1	3.6	4.7	346	-0.2
1959	507	8.4	2849	6.6	3.79	4.6	401	8.9	3.8	4.6	373	7.8
1960	526	3.9	2914	2.3	3.91	3.2	413	2.9	3.9	2.8	387	3.6
1961	545	3.5	2965	1.8	4.07	4.2	424	2.8	4.1	4.4	398	2.9
1962	586	7.5	3139	5.9	4.27	4.9	456	7.5	4.3	5.6	427	7.4
1963	618	5.5	3264	4.0	4.47	4.6	480	5.1	4.5	4.4	448	4.7
1964	664	7.4	3458	5.9	4.68	4.7	516	7.6	4.7	4.5	482	7.6
1965	719	8.4	3701	7.0	4.90	4.8	561	8.8	4.9	5.2	527	9.4
1966	788	9.6	4008	8.3	5.19	5.8	614	9.5	5.3	6.7	582	10.5
1967	833	5.7	4190	4.5	5.42	4.5	643	4.7	5.5	5.1	611	4.9
1968	910	9.3	4534	8.2	5.82	7.3	702	9.1	5.9	7.5	663	8.5
1969	985	8.2	4858	7.1	6.15	5.6	756	7.8	6.2	5.1	714	7.7
1970	1039	5.5	5065	4.3	6.60	7.3	789	4.3	6.6	6.5	747	4.7
1971	1127	8.5	5428	7.2	7.20	9.0	854	8.3	7.2	8.6	798	6.8
1972	1238	9.9	5900	8.7	7.69	6.9	941	10.2	7.7	6.8	871	9.2

1973	1383	11.7	6525	10.6	8.33	8.2	1060	12.7	8.4	8.5	985	13.1
1974	1500	8.5	7014	7.5	9.00	8.1	1146	8.0	9.0	7.9	1080	9.6
1975	1638	9.2	7586	8.1	10.1	12.4	1246	8.8	10.3	13.8	1183	9.6
1976	1825	11.4	8372	10.4	11.0	8.3	1398	12.2	11.1	8.5	1316	11.2
1977	2031	11.3	9221	10.2	11.8	7.5	1567	12.1	12.0	7.8	1456	10.7
1978	2295	13.0	10309	11.8	12.7	7.9	1783	13.8	13.0	8.2	1652	13.4
1979	2563	11.7	11390	10.5	13.8	8.8	2000	12.2	14.1	8.5	1859	12.6
1980	2790	8.8	12276	7.8	15.1	9.1	2156	7.8	15.3	8.8	2033	9.3
1981	3128	12.1	13633	11.1	16.9	11.9	2419	12.2	17.1	11.4	2296	13.0
1982	3255	4.0	14050	3.1	17.8	5.6	2477	2.4	17.9	4.9	2372	3.3
1983	3537	8.7	15128	7.7	19.0	6.7	2699	9.0	19.2	7.1	2547	7.4
1984	3933	11.2	16678	10.3	20.1	5.9	3019	11.8	20.3	5.6	2838	11.5
1985	4220	7.3	17738	6.4	21.1	4.9	3231	7.0	21.2	4.6	3043	7.2
1986	4463	5.7	18585	4.8	22.1	4.5	3405	5.4	22.1	4.6	3185	4.7
1987	4740	6.2	19561	5.3	22.8	3.4	3602	5.8	22.7	2.7	3368	5.8
1988	5104	7.7	20875	6.7	23.9	4.6	3874	7.6	23.8	4.7	3635	7.9
1989	5484	7.5	22220	6.4	25.0	4.6	4163	7.4	24.9	4.7	3923	7.9
1990	5803	5.8	23265	4.7	26.4	5.6	4379	5.2	26.4	5.9	4155	5.9
1991	5996	3.3	23781	2.2	27.6	4.8	4478	2.3	27.6	4.8	4273	2.8
1992	6338	5.7	24854	4.5	29.2	5.6	4743	5.9	29.3	6.1	4506	5.5
1993	6657	5.0	25829	3.9	30.0	2.6	4997	5.4	30.1	2.6	4731	5.0
1994	7072	6.2	27171	5.2	30.9	3.0	5339	6.8	30.9	2.8	5037	6.5
1995	7398	4.6	28153	3.6	31.5	2.1	5592	4.7	31.5	1.9	5289	5.0
1996	7817	5.7	29477	4.7	32.9	4.4	5943	6.3	33.0	4.6	5609	6.1
1997	8304	6.2	31016	5.2	34.0	3.2	6352	6.9	34.1	3.4	6003	7.0
1998	8747	5.3	32361	4.3	35.0	3.1	6703	5.5	35.2	3.5	6317	5.2
1999	9268	6.0	33989	5.0	36.4	3.9	7115	6.1	36.7	4.0	6690	5.9
2000	9817	5.9	34788	2.4	38.1	4.6	7534	5.9	38.4	4.9	7087	5.9
2001	10128	3.2	35523	2.1	39.7	4.4	7709	2.3	40.2	4.7	7240	2.2
2002	10470	3.4	36355	2.3	41.6	4.8	7904	2.5	42.3	5.2	7400	2.2
2003	10961	4.7	37685	3.7	43.8	5.2	8273	4.7	44.6	5.4	7701	4.1
2004	11713	6.9	39885	5.8	46.3	5.6	8864	7.1	47.2	5.8	8189	6.3
2005	12456	6.3	42022	5.4	48.5	4.9	9447	6.6	49.6	5.0	8676	6.0

(*): The raw data are obtained from the following tables of the Bureau of Economic Analysis and the labor data are obtained from the Bureau of Labor Statistics and given in Appendix B. *Table 1.1.5. Gross Domestic Product, Table 1.3.5. Gross Value Added by Sector*, and several old tables.

(**): Per capita variables.

(***): Per labor hour variables.

Table A2: Real GDP, Private Real GDP, and Nonresidential Private Real GDP (*)

(Billions of Chained 2000 Dollars)

Year	GDP						Private GDP				Private Nonresid.GDP	
			(**)		(***)				(***)			
	levels	%Δ	$	%Δ	$	%Δ	levels	%Δ	$	%Δ	levels	%Δ
1929	865	--	7105	--	--	--	575	--	--	--	516	--
1930	791	-8.6	6424	-9.6	--	--	515	-10	--	--	479	-7.1
1931	740	-6.4	5965	-7.2	--	--	488	-5.2	--	--	458	-4.4
1932	644	-13	5156	-14	--	--	397	-19	--	--	382	-17
1933	636	-1.3	5061	-1.9	--	--	387	-2.7	--	--	374	-2.0
1934	704	10.8	5572	10.1	--	--	436	12.8	--	--	419	12.0
1935	767	8.9	6027	8.2	--	--	481	10.2	--	--	456	9.0
1936	867	13.0	6768	12.3	--	--	553	15.1	--	--	522	14.5
1937	911	5.1	7072	4.5	--	--	596	7.8	--	--	562	7.7
1938	880	-3.4	6776	-4.2	--	--	565	-5.2	--	--	531	-5.6
1939	951	8.1	7264	7.2	--	--	619	9.5	--	--	571	7.5
1940	1034	8.8	7827	7.7	--	--	680	9.8	--	--	625	9.6
1941	1211	17.1	9079	16.0	--	--	780	14.8	--	--	723	15.6
1942	1435	18.5	10644	17.2	--	--	911	16.7	--	--	881	22.0
1943	1671	16.4	12220	14.8	--	--	1013	11.2	--	--	996	13.0
1944	1807	8.1	13053	6.8	--	--	1070	5.7	--	--	1055	6.0
1945	1786	-1.1	12766	-2.2	--	--	1054	-1.6	--	--	1036	-1.8
1946	1589	-11	11241	-12	--	--	1046	-0.7	--	--	972	-6.2
1947	1575	-0.9	10924	-2.8	--	--	1074	2.7	10.4	--	979	0.7
1948	1643	4.4	11206	2.6	13.4	--	1138	5.9	11.0	5.3	1024	4.6
1949	1635	-0.5	10957	-2.2	13.6	1.8	1125	-1.2	11.2	2.2	1019	-0.5
1950	1777	8.7	11672	6.5	14.5	6.8	1229	9.3	12.1	7.8	1085	6.4
1951	1915	7.7	12365	5.9	14.8	1.9	1306	6.3	12.4	3.0	1186	9.3
1952	1988	3.8	12620	2.1	15.2	2.8	1346	3.1	12.8	3.0	1228	3.5
1953	2080	4.6	12982	2.9	15.7	3.4	1409	4.6	13.2	3.3	1286	4.7
1954	2065	-0.7	12669	-2.4	16.1	2.3	1376	-2.3	13.4	1.1	1243	-3.3
1955	2213	7.1	13336	5.3	16.7	3.7	1496	8.8	14.0	4.8	1342	8.0
1956	2256	1.9	13356	0.1	16.7	0.3	1516	1.3	14.0	-0.2	1374	2.4
1957	2301	2.0	13380	0.2	17.2	2.9	1543	1.7	14.4	3.3	1409	2.6
1958	2279	-1.0	13033	-2.6	17.7	2.6	1518	-1.6	14.9	3.1	1383	-1.9
1959	2441	7.1	13728	5.3	18.3	3.4	1647	8.5	15.5	4.2	1478	6.9
1960	2502	2.5	13847	0.9	18.6	1.7	1673	1.6	15.7	1.4	1516	2.6
1961	2560	2.3	13936	0.6	19.1	3.0	1714	2.5	16.4	4.1	1557	2.7
1962	2715	6.1	14556	4.4	19.8	3.4	1815	5.9	17.0	4.0	1643	5.5
1963	2834	4.4	14976	2.9	20.5	3.5	1901	4.7	17.7	4.0	1708	4.0
1964	2999	5.8	15627	4.3	21.1	3.1	2023	6.4	18.3	3.4	1819	6.5
1965	3191	6.4	16423	5.1	21.8	2.9	2171	7.3	19.0	3.8	1973	8.5
1966	3399	6.5	17293	5.3	22.4	2.9	2331	7.4	19.9	4.6	2151	9.0
1967	3485	2.5	17536	1.4	22.7	1.4	2366	1.5	20.3	1.9	2191	1.9
1968	3653	4.8	18199	3.8	23.4	3.0	2487	5.1	21.0	3.6	2289	4.4
1969	3765	3.1	18578	2.1	23.5	0.6	2564	3.1	21.1	0.5	2360	3.1
1970	3772	0.2	18395	-1.0	24.0	2.0	2562	-0.1	21.6	2.0	2370	0.4
1971	3899	3.4	18774	2.1	24.9	3.8	2663	3.9	22.5	4.2	2418	2.0
1972	4105	5.3	19557	4.2	25.5	2.5	2830	6.3	23.2	3.0	2542	5.1

1973	4342	5.8	20488	4.8	26.1	2.5	3035	7.2	23.9	3.3	2749	8.1
1974	4320	-0.5	20199	-1.4	25.9	-0.9	2991	-1.4	23.5	-1.5	2764	0.6
1975	4311	-0.2	19962	-1.2	26.6	2.7	2961	-1.0	24.4	3.6	2763	0.0
1976	4541	5.3	20826	4.3	27.3	2.4	3163	6.8	25.2	3.3	2918	5.6
1977	4751	4.6	21570	3.6	27.5	1.1	3355	6.1	25.7	2.0	3058	4.8
1978	5015	5.6	22531	4.5	27.8	0.8	3566	6.3	26.0	1.1	3250	6.3
1979	5173	3.2	22987	2.0	27.9	0.5	3685	3.3	26.0	0.0	3381	4.0
1980	5162	-0.2	22716	-1.2	27.9	0.0	3640	-1.2	25.9	-0.3	3400	0.6
1981	5292	2.5	23061	1.5	28.6	2.3	3745	2.9	26.4	2.1	3524	3.6
1982	5189	-1.9	22400	-2.9	28.4	-0.5	3636	-2.9	26.3	-0.6	3456	-1.9
1983	5424	4.5	23199	3.6	29.2	2.7	3834	5.4	27.2	3.6	3579	3.6
1984	5814	7.2	24652	6.3	29.8	2.1	4176	8.9	28.0	2.9	3883	8.5
1985	6054	4.1	25444	3.2	30.3	1.8	4373	4.7	28.7	2.3	4076	5.0
1986	6264	3.5	26084	2.5	31.0	2.3	4539	3.8	29.5	3.0	4205	3.2
1987	6475	3.4	26725	2.5	31.2	0.7	4703	3.6	29.7	0.5	4362	3.7
1988	6743	4.1	27578	3.2	31.6	1.1	4910	4.4	30.2	1.6	4572	4.8
1989	6981	3.5	28285	2.6	31.8	0.8	5090	3.7	30.5	1.0	4763	4.2
1990	7113	1.9	28514	0.8	32.3	1.7	5169	1.5	31.1	2.2	4870	2.2
1991	7101	-0.2	28162	-1.2	32.7	1.3	5132	-0.7	31.7	1.7	4862	-0.2
1992	7337	3.3	28772	2.2	33.8	3.2	5341	4.1	33.0	4.3	5034	3.5
1993	7533	2.7	29225	1.6	33.9	0.3	5514	3.2	33.2	0.5	5181	2.9
1994	7836	4.0	30103	3.0	34.2	0.9	5789	5.0	33.5	1.0	5425	4.7
1995	8032	2.5	30566	1.5	34.2	0.1	5963	3.0	33.6	0.2	5610	3.4
1996	8329	3.7	31407	2.8	35.1	2.4	6238	4.6	34.6	2.9	5856	4.4
1997	8704	4.5	32507	3.5	35.6	1.5	6571	5.3	35.2	1.9	6182	5.6
1998	9067	4.2	33544	3.2	36.3	2.0	6892	4.9	36.2	2.8	6474	4.7
1999	9470	4.4	34729	3.5	37.2	2.4	7249	5.2	37.3	3.1	6806	5.1
2000	9817	3.7	34788	0.2	38.1	2.4	7534	3.9	38.4	2.9	7087	4.1
2001	9891	0.8	34691	-0.3	38.8	1.9	7568	0.5	39.5	2.8	7119	0.5
2002	10049	1.6	34894	0.6	40.0	3.0	7685	1.5	41.2	4.2	7215	1.3
2003	10301	2.5	35417	1.5	41.2	3.0	7928	3.2	42.8	3.9	7419	2.8
2004	10704	3.9	36449	2.9	42.3	2.7	8278	4.4	44.1	3.1	7718	4.0
2005	11049	3.2	37275	2.3	43.1	1.8	8600	3.9	45.1	2.4	7992	3.6

(*): The raw data are obtained from the following tables of the Bureau of Economic Analysis and the labor data are obtained from the Bureau of Labor Statistics and given in Appendix B. *Table 1.1.5. Gross Domestic Product; Table 1.3.5. Gross Value Added by Sector; Table 1.1.3. Real Gross Domestic Product, Quantity Indexes; Table 1.3.6. Real Gross Value Added by Sector, Chained (2000) Dollars;* and several old tables.

(**): Per capita variables.

(***): Per labor hour variables.

Table A3: Growth Rates of Private Sector Productivity and Related Measures (*)

Year	Real GDP		Real Private Sector			Public Sector		Real Private GDP				
	Real GDP	Real GDP per capita	Labor Input (**)	Capital Services (**)	Capital per labor hour (**)	Non-military infrastructure capital	Core Infrastructure capital	per labor hour	per unit of capital	Levels	Combined labor, capital (**)	Multi-factor productivity
1929	--	--	--	2.9	--	5.8	5.3	--	--	--	--	--
1930	-8.6	-9.6	--	1.5	--	6.7	6.3	--	-12.9	-10	--	--
1931	-6.4	-7.2	--	0.0	--	6.4	5.7	--	-6.6	-5.2	--	--
1932	-13	-14	--	-1.3	--	4.8	3.8	--	-18.6	-19	--	--
1933	-1.3	-1.9	--	-1.4	--	3.0	1.5	--	-1.4	-2.7	--	--
1934	10.8	10.1	--	-0.9	--	3.8	2.0	--	14.4	12.8	--	--
1935	8.9	8.2	--	-0.3	--	4.0	1.8	--	11.2	10.2	--	--
1936	13.0	12.3	--	0.5	--	6.0	4.7	--	15.4	15.1	--	--
1937	5.1	4.5	--	1.1	--	4.5	3.2	--	7.2	7.8	--	--
1938	-3.4	-4.2	--	0.2	--	5.0	4.0	--	-6.3	-5.2	--	--
1939	8.1	7.2	--	0.8	--	5.5	4.6	--	9.3	9.5	--	--
1940	8.8	7.7	--	1.5	--	4.0	2.8	--	8.9	9.8	--	--
1941	17.1	16.0	--	1.9	--	3.0	1.5	--	13.1	14.8	--	--
1942	18.5	17.2	--	-0.2	--	0.9	0.0	--	14.5	16.7	--	--
1943	16.4	14.8	--	-0.8	--	-0.2	-0.8	--	11.4	11.2	--	--
1944	8.1	6.8	--	-0.2	--	-0.8	-0.9	--	6.5	5.7	--	--
1945	-1.1	-2.2	--	0.7	--	-0.4	-0.7	--	-1.4	-1.6	--	--
1946	-11	-12	--	3.2	--	-0.1	-0.1	--	-1.4	-0.7	--	--
1947	-0.9	-2.8	--	4.1	--	1.2	0.9	--	-0.5	2.7	--	--
1948	4.4	2.6	0.6	4.4	--	2.0	1.7	5.3	1.8	5.9	1.8	4.1
1949	-0.5	-2.2	-3.3	2.8	6.5	3.4	2.9	2.2	-5.3	-1.2	-1.4	0.2
1950	8.7	6.5	2.0	3.6	2.3	3.9	3.5	7.8	5.7	9.3	2.5	6.8
1951	7.7	5.9	3.2	3.7	0.6	3.7	3.4	3.0	1.8	6.3	3.3	3.0
1952	3.8	2.1	0.1	3.3	3.3	3.6	3.4	3.0	-0.7	3.1	1.1	1.9
1953	4.6	2.9	1.3	3.5	2.2	3.6	3.7	3.3	1.2	4.6	2.0	2.6
1954	-0.7	-2.4	-3.4	3.3	7.0	4.4	4.8	1.1	-5.6	-2.3	-1.3	-1.0
1955	7.1	5.3	3.9	3.4	-0.3	4.6	5.1	4.8	5.2	8.8	3.7	5.1
1956	1.9	0.1	1.5	3.7	2.1	4.5	4.9	-0.2	-2.5	1.3	2.2	-0.9
1957	2.0	0.2	-1.5	3.3	4.9	4.6	4.9	3.3	-1.9	1.7	0.0	1.7
1958	-1.0	-2.6	-4.6	2.6	7.6	5.0	5.2	3.1	-4.8	-1.6	-2.4	0.7
1959	7.1	5.3	4.1	3.3	-0.8	4.8	5.0	4.2	5.8	8.5	3.9	4.6
1960	2.5	0.9	0.5	2.8	2.8	4.7	4.7	1.4	-1.7	1.6	1.2	0.4
1961	2.3	0.6	-1.6	2.9	4.6	4.9	4.9	4.1	-0.6	2.5	-0.2	2.7
1962	6.1	4.4	1.8	3.3	1.5	5.0	4.7	4.0	2.9	5.9	2.3	3.6
1963	4.4	2.9	0.7	3.6	2.9	5.3	5.0	4.0	1.3	4.7	1.6	3.1
1964	5.8	4.3	2.9	4.0	1.0	5.5	5.2	3.4	2.8	6.4	3.2	3.2
1965	6.4	5.1	3.2	4.6	1.2	5.7	5.3	3.8	3.2	7.3	3.7	3.6
1966	6.5	5.3	2.6	5.6	2.9	5.6	5.4	4.6	2.9	7.4	3.6	3.8
1967	2.5	1.4	-0.2	5.8	6.2	5.3	5.3	1.9	-2.7	1.5	1.7	-0.2
1968	4.8	3.8	1.6	4.2	2.4	5.1	5.3	3.6	1.4	5.1	2.4	2.7
1969	3.1	2.1	2.9	4.7	2.1	4.3	4.3	0.5	-0.7	3.1	3.5	-0.4

1970	0.2	-1.0	-1.7	4.5	6.8	3.7	3.7	2.0	-3.9	-0.1	0.1	-0.2
1971	3.4	2.1	-0.8	4.1	4.6	3.3	3.2	4.2	0.5	3.9	0.7	3.2
1972	5.3	4.2	3.4	3.9	0.6	3.0	2.9	3.0	2.6	6.3	3.6	2.7
1973	5.8	4.8	3.5	5.6	1.9	2.9	2.7	3.3	3.0	7.2	4.2	3.0
1974	-0.5	-1.4	0.7	5.2	5.2	2.7	2.5	-1.5	-5.6	-1.4	2.0	-3.4
1975	-0.2	-1.2	-4.4	3.4	8.2	2.5	2.3	3.6	-4.3	-1.0	-2.0	1.0
1976	5.3	4.3	2.9	3.0	-0.1	2.4	2.2	3.3	4.3	6.8	2.9	3.9
1977	4.6	3.6	4.0	3.7	-0.3	2.0	1.9	2.0	3.2	6.1	3.9	2.2
1978	5.6	4.5	5.2	4.3	-0.8	2.3	2.2	1.1	2.7	6.3	4.9	1.4
1979	3.2	2.0	3.0	5.3	1.9	2.4	2.3	0.0	-0.6	3.3	3.7	-0.4
1980	-0.2	-1.2	-0.6	5.2	6.2	2.3	2.1	-0.3	-5.0	-1.2	1.1	-2.3
1981	2.5	1.5	1.5	5.1	4.3	1.9	1.6	2.1	-0.2	2.9	2.6	0.3
1982	-1.9	-2.9	-1.4	4.2	6.7	1.5	1.2	-0.6	-5.6	-2.9	0.4	-3.3
1983	4.5	3.6	2.2	2.9	1.2	1.5	1.1	3.6	3.3	5.4	2.4	3.0
1984	7.2	6.3	6.0	5.0	-0.8	1.8	1.4	2.9	6.5	8.9	5.7	3.2
1985	4.1	3.2	2.6	5.0	2.6	2.1	1.6	2.3	1.5	4.7	3.3	1.4
1986	3.5	2.5	1.2	4.3	3.6	2.2	1.7	3.0	0.5	3.8	2.2	1.6
1987	3.4	2.5	3.2	3.6	0.6	2.2	1.7	0.5	0.6	3.6	3.3	0.3
1988	4.1	3.2	3.5	3.7	0.9	2.2	1.9	1.6	1.6	4.4	3.6	0.8
1989	3.5	2.6	3.1	3.7	1.0	2.3	1.8	1.0	0.9	3.7	3.3	0.4
1990	1.9	0.8	0.0	3.1	3.6	2.6	2.0	2.2	-1.0	1.5	1.0	0.5
1991	-0.2	-1.2	-1.2	2.6	5.0	2.5	2.1	1.7	-2.9	-0.7	0.0	-0.7
1992	3.3	2.2	1.0	2.2	2.4	2.5	2.1	4.3	2.4	4.1	1.4	2.7
1993	2.7	1.6	2.9	2.9	0.2	2.2	1.9	0.5	1.5	3.2	2.9	0.3
1994	4.0	3.0	4.5	3.4	-0.5	2.1	1.8	1.0	2.8	5.0	4.1	0.9
1995	2.5	1.5	2.8	4.2	1.4	2.3	1.9	0.2	0.6	3.0	3.3	-0.3
1996	3.7	2.8	2.1	4.4	2.7	2.5	2.0	2.9	1.9	4.6	2.8	1.8
1997	4.5	3.5	3.9	5.1	1.7	2.4	2.8	1.9	2.4	5.3	4.3	1.0
1998	4.2	3.2	2.3	5.6	3.5	2.4	2.1	2.8	1.7	4.9	3.4	1.5
1999	4.4	3.5	2.7	6.2	4.0	2.6	2.4	3.1	1.8	5.2	3.8	1.4
2000	3.7	0.2	1.1	6.0	4.9	2.6	2.5	2.9	0.5	3.9	2.6	1.3
2001	0.8	-0.3	-1.4	4.4	6.8	2.6	2.7	2.8	-2.9	0.5	0.3	0.2
2002	1.6	0.6	-1.3	2.6	5.2	2.7	2.7	4.2	-1.1	1.5	-0.1	1.6
2003	2.5	1.5	-0.1	2.1	2.7	2.6	2.7	3.9	1.0	3.2	0.6	2.6
2004	3.9	2.9	1.5	2.5	1.2	2.4	2.5	3.1	2.2	4.4	1.8	2.6
2005	3.2	2.3	1.5	2.5	1.0	2.0	2.1	2.4	1.5	3.9	1.8	2.1

(*): The raw data are obtained from the Bureau of Economic Analysis and the Bureau of Labor Statistics. The level variables are in billions of chained 2000 dollars.

(**): The values for the years 1949-2004 are taken from *Table 3. Private business sector: Productivity and related measures* of the Bureau of Labor Statistics; and the other values are calculated by us.

Table A4: Current-Cost Net Stock of Public Fixed Assets (*)

(Yearend Estimates)

Year	Billions of current dollars							Billions of Chained 2000 dollars						
	Federal		State/local		Total Nonmil		Total	Federal		State/local		Total Nonmil		Total
	Militar	Nonmil	Level	%Δ	Level	%Δ		Militar	Nonmil	Level	%Δ	Level	%Δ	
1925	7	2	29	--	32	--	38	77	27	374	--	401	--	467
1926	7	2	30	3.8	33	3.5	39	75	27	394	5.5	422	5.2	485
1927	6	2	31	3.6	34	3.4	40	74	28	418	6.2	446	5.9	506
1928	6	3	32	3.2	35	3.3	41	72	29	444	6.1	473	5.9	530
1929	5	3	33	1.9	36	2.0	41	70	30	470	5.9	500	5.8	554
1930	5	3	32	-2.1	35	-2.0	40	69	32	502	6.8	534	6.7	585
1931	4	3	29	-9.0	32	-8.6	36	68	34	534	6.3	568	6.4	618
1932	4	3	29	-1.0	32	-0.9	36	67	38	558	4.4	596	4.8	643
1933	4	3	34	17.9	37	18.7	42	66	45	569	2.0	614	3.0	661
1934	5	4	37	8.8	41	10.2	46	66	54	583	2.5	637	3.8	684
1935	5	5	39	4.3	44	6.3	49	65	67	595	2.1	662	4.0	709
1936	5	6	43	10.3	49	11.9	54	67	78	624	4.9	702	6.0	750
1937	5	7	45	4.2	52	5.5	57	69	86	647	3.7	734	4.5	784
1938	6	8	46	2.9	54	3.9	59	70	95	676	4.4	771	5.0	822
1939	6	8	48	3.7	56	4.1	62	72	102	711	5.2	813	5.5	866
1940	7	9	52	9.0	61	9.3	68	80	109	736	3.5	845	4.0	905
1941	14	11	60	16.0	71	15.7	85	154	121	750	1.9	871	3.0	1013
1942	38	11	67	12.0	79	11.3	117	415	127	752	0.2	879	0.9	1296
1943	68	12	69	1.6	81	2.8	149	753	132	745	-1	877	0	1636
1944	90	13	67	-2.3	80	-1.5	170	1041	133	737	-1	870	-1	1913
1945	104	14	70	3.9	84	5.3	188	1142	136	730	-1	867	0	2009
1946	105	17	78	12.1	95	13.6	200	990	135	731	0.1	866	0	1859
1947	101	20	93	19.8	113	18.8	214	847	137	740	1.2	876	1.2	1730
1948	91	22	102	8.9	123	9.0	215	729	139	754	2.0	894	2.0	1636
1949	81	22	97	-4.6	119	-3.7	200	665	145	779	3.3	924	3.4	1604
1950	80	23	109	12.8	132	11.2	212	601	151	809	3.8	960	3.9	1579
1951	90	24	123	12.4	147	11.1	237	626	156	839	3.8	996	3.7	1641
1952	98	26	130	5.9	156	6.5	254	676	162	870	3.7	1032	3.6	1727
1953	105	26	129	-1.1	155	-1.2	259	729	165	905	4.0	1069	3.6	1818
1954	114	26	135	4.4	161	4.1	275	760	168	949	4.9	1117	4.4	1896
1955	119	28	149	11.1	178	10.3	297	774	170	997	5.1	1168	4.6	1962
1956	130	30	169	13.3	199	12.0	329	786	174	1046	4.9	1220	4.5	2026
1957	135	31	177	4.8	209	4.9	343	795	177	1099	5.0	1276	4.6	2091
1958	140	32	190	6.8	222	6.3	361	808	182	1158	5.4	1340	5.0	2168
1959	143	33	196	3.3	229	3.3	372	832	187	1218	5.1	1405	4.8	2257
1960	146	35	205	4.8	240	4.7	385	847	194	1277	4.9	1470	4.7	2337
1961	151	36	217	6.0	254	5.9	404	869	202	1341	5.0	1543	4.9	2432
1962	159	39	233	7.1	272	7.2	431	892	212	1407	4.9	1620	5.0	2531
1963	161	43	248	6.6	291	6.9	452	901	226	1480	5.2	1706	5.3	2626
1964	163	46	265	6.7	311	6.8	474	902	241	1559	5.3	1800	5.5	2722
1965	165	51	288	8.8	339	9.0	504	896	258	1644	5.4	1902	5.7	2816
1966	171	56	318	10.5	374	10.5	545	895	275	1734	5.5	2009	5.6	2922
1967	181	60	348	9.3	408	9.0	589	900	286	1830	5.5	2115	5.3	3033
1968	187	66	386	11.0	452	10.6	639	893	293	1929	5.4	2222	5.1	3132

1969	196	72	436	12.9	508	12.4	703	882	299	2021	4.7	2319	4.3	3216
1970	208	78	497	14.1	575	13.2	782	868	303	2101	4.0	2404	3.7	3286
1971	215	85	548	10.2	633	10.1	848	841	307	2176	3.6	2483	3.3	3338
1972	232	91	601	9.7	692	9.3	923	818	312	2245	3.2	2557	3.0	3387
1973	251	99	689	14.7	788	14.0	1039	799	317	2312	3.0	2630	2.9	3440
1974	286	118	873	26.7	991	25.7	1277	788	323	2378	2.8	2701	2.7	3500
1975	296	130	915	4.7	1045	5.5	1341	780	328	2441	2.6	2769	2.5	3561
1976	324	138	955	4.5	1094	4.7	1418	776	334	2501	2.5	2834	2.4	3621
1977	337	149	1021	6.9	1170	7.0	1507	772	339	2553	2.1	2892	2.0	3673
1978	361	163	1124	10.0	1287	10.0	1648	767	347	2613	2.4	2960	2.3	3735
1979	400	184	1293	15.0	1477	14.7	1877	767	353	2677	2.5	3030	2.4	3804
1980	439	209	1501	16.1	1710	15.8	2149	769	360	2740	2.3	3100	2.3	3875
1981	465	233	1664	10.8	1896	10.9	2361	773	369	2790	1.8	3159	1.9	3938
1982	489	246	1756	5.5	2002	5.6	2491	782	375	2831	1.5	3206	1.5	3994
1983	520	252	1779	1.3	2031	1.4	2551	798	382	2873	1.5	3255	1.5	4059
1984	548	262	1840	3.4	2102	3.5	2650	819	390	2925	1.8	3315	1.8	4140
1985	570	272	1924	4.6	2197	4.5	2767	848	398	2987	2.1	3385	2.1	4241
1986	602	283	2051	6.6	2334	6.3	2936	882	405	3056	2.3	3461	2.2	4351
1987	626	296	2171	5.9	2467	5.7	3093	916	414	3125	2.3	3539	2.2	4463
1988	668	310	2274	4.7	2584	4.7	3251	939	420	3198	2.3	3618	2.2	4565
1989	704	327	2397	5.4	2725	5.5	3429	957	427	3274	2.4	3701	2.3	4667
1990	735	344	2522	5.2	2866	5.2	3601	975	436	3360	2.6	3796	2.6	4780
1991	766	357	2600	3.1	2956	3.2	3722	985	445	3447	2.6	3892	2.5	4885
1992	800	369	2716	4.5	3085	4.4	3885	990	456	3532	2.5	3988	2.5	4986
1993	823	386	2855	5.1	3241	5.1	4064	985	466	3611	2.2	4077	2.2	5069
1994	857	404	3035	6.3	3438	6.1	4295	975	473	3689	2.2	4162	2.1	5143
1995	865	426	3213	5.9	3639	5.9	4505	963	483	3774	2.3	4257	2.3	5224
1996	867	448	3366	4.8	3815	4.8	4682	953	500	3863	2.4	4363	2.5	5319
1997	868	466	3567	5.9	4033	5.7	4901	936	508	3961	2.5	4468	2.4	5406
1998	872	484	3745	5.0	4229	4.9	5101	920	516	4060	2.5	4576	2.4	5498
1999	891	508	4000	6.8	4508	6.6	5399	908	525	4171	2.7	4695	2.6	5603
2000	896	529	4288	7.2	4817	6.8	5713	896	529	4288	2.8	4817	2.6	5713
2001	904	543	4533	5.7	5076	5.4	5980	886	531	4411	2.9	4943	2.6	5828
2002	914	556	4796	5.8	5352	5.4	6266	880	535	4540	2.9	5076	2.7	5956
2003	928	571	5009	4.4	5580	4.3	6508	878	539	4668	2.8	5206	2.6	6085
2004	985	608	5635	12.5	6243	11.9	7228	881	543	4786	2.5	5329	2.4	6211
2005	1033	648	6226	10.5	6874	10.1	7907	884	547	4890	2.2	5437	2.0	6324

(*) The raw data are obtained from the following tables of the Bureau of Economic Analysis.

Table 7.1A. Current-Cost Net Stock of Government Fixed Assets, 1925-1996; Table 7.1B. Current-Cost Net Stock of Government Fixed Assets 1997-2005; Table 7.2A. Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets, 1925-1996; Table 7.2B. Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets, 1997-2005.

Table A5: Breakdown of Public Fixed Assets As Shares of the Total

Year	Federal Military %	Federal Military %Δ	Federal Nonmilitary %	Federal Nonmilitary %Δ	State and Local (**) %	State and Local (**) %Δ	State and Local (***) %	State and Local (***) %Δ	Total (*) Nonmilitary %	Total (*) Nonmilitary %Δ
1925	17.8	--	6.3	--	76.0	--	92.4	--	82.2	--
1926	16.8	-5.2	6.1	-2.3	77.0	1.4	92.6	0.3	83.2	1.1
1927	15.8	-6.5	6.0	-2.0	78.3	1.6	92.9	0.3	84.3	1.3
1928	14.3	-9.3	6.2	2.6	79.6	1.7	92.8	-0.1	85.7	1.7
1929	13.0	-9.1	6.4	3.5	80.6	1.4	92.7	-0.2	87.0	1.5
1930	12.1	-6.9	6.5	2.8	81.1	0.6	92.5	-0.2	87.7	0.7
1931	12.2	0.8	6.9	5.7	81.2	0.1	92.1	-0.4	88.1	0.5
1932	11.8	-3.5	7.0	1.1	81.2	0.1	92.1	-0.1	88.2	0.2
1933	10.5	-10.5	7.7	9.3	81.8	0.7	91.4	-0.7	89.5	1.4
1934	10.2	-2.7	8.7	13.8	81.0	-0.9	90.3	-1.3	89.8	0.3
1935	9.9	-3.7	10.3	17.8	79.7	-1.7	88.6	-1.9	89.9	0.2
1936	9.6	-2.7	11.4	11.4	79.0	-0.9	87.3	-1.4	90.4	0.5
1937	9.4	-1.6	12.4	8.5	78.0	-1.3	86.3	-1.2	90.4	0.0
1938	9.3	-1.6	13.2	6.1	77.5	-0.6	85.5	-0.9	90.7	0.4
1939	9.3	-0.4	13.5	2.3	77.3	-0.3	85.2	-0.4	90.7	0.0
1940	9.8	5.5	13.6	1.0	76.8	-0.6	84.9	-0.2	90.4	-0.4
1941	16.3	67.3	12.4	-8.7	71.2	-7.2	85.1	0.2	83.7	-7.4
1942	32.8	101.0	9.7	-22.3	57.6	-19.1	85.6	0.6	67.3	-19.6
1943	45.8	39.6	8.3	-14.0	45.9	-20.4	84.7	-1.1	54.2	-19.4
1944	53.1	15.9	7.5	-9.3	39.4	-14.2	83.9	-0.9	46.9	-13.4
1945	55.4	4.4	7.6	1.5	36.9	-6.3	82.8	-1.3	44.5	-5.1
1946	52.4	-5.4	8.7	13.7	38.9	5.4	81.7	-1.3	47.6	6.8
1947	47.2	-10.0	9.3	6.8	43.5	11.9	82.4	0.8	52.8	11.0
1948	42.5	-10.0	10.2	9.4	47.3	8.8	82.3	-0.1	57.5	8.9
1949	40.5	-4.7	11.0	7.9	48.5	2.4	81.6	-0.9	59.5	3.4
1950	37.6	-7.2	10.8	-1.6	51.7	6.6	82.7	1.4	62.5	5.0
1951	38.1	1.3	10.1	-6.7	51.9	0.5	83.8	1.3	62.0	-0.8
1952	38.5	1.1	10.3	2.2	51.2	-1.3	83.3	-0.6	61.5	-0.8
1953	40.4	4.9	9.9	-3.5	49.7	-3.1	83.4	0.1	59.6	-3.1
1954	41.5	2.9	9.6	-3.2	48.9	-1.6	83.6	0.3	58.5	-1.9
1955	40.2	-3.2	9.5	-1.3	50.3	3.0	84.2	0.7	59.8	2.3
1956	39.6	-1.5	9.0	-5.3	51.4	2.2	85.2	1.2	60.4	1.0
1957	39.2	-0.9	9.1	1.1	51.7	0.5	85.1	-0.1	60.8	0.6
1958	38.7	-1.5	8.9	-2.0	52.4	1.5	85.5	0.5	61.3	1.0
1959	38.4	-0.7	8.9	0.6	52.7	0.4	85.5	0.0	61.6	0.4
1960	37.8	-1.6	9.0	0.3	53.3	1.1	85.6	0.1	62.2	1.0
1961	37.2	-1.4	9.0	0.5	53.8	0.9	85.7	0.1	62.8	0.9
1962	36.8	-1.1	9.1	1.4	54.1	0.5	85.6	-0.1	63.2	0.7
1963	35.6	-3.3	9.5	3.6	54.9	1.6	85.3	-0.3	64.4	1.9
1964	34.4	-3.3	9.7	2.3	55.9	1.7	85.2	-0.1	65.5	1.8
1965	32.8	-4.7	10.0	3.8	57.1	2.3	85.1	-0.2	67.2	2.5
1966	31.3	-4.5	10.3	2.5	58.4	2.2	85.0	-0.1	68.7	2.2
1967	30.7	-2.1	10.3	-0.3	59.1	1.2	85.2	0.2	69.3	0.9

1968	29.3	-4.4	10.3	0.1	60.4	2.2	85.5	0.3	70.7	1.9
1969	27.8	-5.1	10.2	-0.7	62.0	2.6	85.9	0.5	72.2	2.1
1970	26.5	-4.6	9.9	-2.7	63.5	2.5	86.5	0.7	73.5	1.8
1971	25.4	-4.5	10.0	0.6	64.7	1.8	86.6	0.1	74.6	1.6
1972	25.1	-1.0	9.8	-1.7	65.1	0.6	86.9	0.3	74.9	0.3
1973	24.1	-4.0	9.5	-2.9	66.4	2.0	87.4	0.6	75.9	1.3
1974	22.4	-7.1	9.2	-3.4	68.4	3.1	88.1	0.8	77.6	2.3
1975	22.1	-1.3	9.7	5.6	68.2	-0.3	87.5	-0.7	77.9	0.4
1976	22.9	3.5	9.8	0.4	67.4	-1.2	87.3	-0.2	77.1	-1.0
1977	22.3	-2.3	9.9	1.3	67.8	0.6	87.3	-0.1	77.7	0.7
1978	21.9	-2.1	9.9	0.3	68.2	0.6	87.3	0.0	78.1	0.6
1979	21.3	-2.6	9.8	-1.3	68.9	1.0	87.6	0.3	78.7	0.7
1980	20.4	-4.1	9.7	-0.7	69.8	1.4	87.8	0.3	79.6	1.1
1981	19.7	-3.7	9.8	1.4	70.5	0.9	87.7	-0.1	80.3	0.9
1982	19.6	-0.2	9.9	0.3	70.5	0.0	87.7	0.0	80.4	0.1
1983	20.4	3.8	9.9	0.1	69.7	-1.1	87.6	-0.1	79.6	-0.9
1984	20.7	1.4	9.9	0.1	69.4	-0.4	87.5	-0.1	79.3	-0.4
1985	20.6	-0.4	9.8	-0.5	69.5	0.2	87.6	0.1	79.4	0.1
1986	20.5	-0.5	9.6	-2.1	69.9	0.4	87.9	0.3	79.5	0.1
1987	20.2	-1.3	9.6	-0.8	70.2	0.5	88.0	0.2	79.8	0.3
1988	20.5	1.5	9.5	-0.4	69.9	-0.4	88.0	0.0	79.5	-0.4
1989	20.5	0.0	9.5	0.3	69.9	0.0	88.0	0.0	79.5	0.0
1990	20.4	-0.5	9.5	0.0	70.0	0.2	88.0	0.0	79.6	0.1
1991	20.6	0.8	9.6	0.3	69.8	-0.3	87.9	-0.1	79.4	-0.2
1992	20.6	0.1	9.5	-0.8	69.9	0.1	88.0	0.1	79.4	0.0
1993	20.3	-1.7	9.5	0.0	70.3	0.5	88.1	0.1	79.7	0.4
1994	20.0	-1.5	9.4	-1.1	70.6	0.6	88.3	0.2	80.0	0.4
1995	19.2	-3.8	9.5	0.6	71.3	1.0	88.3	0.0	80.8	0.9
1996	18.5	-3.6	9.6	1.3	71.9	0.8	88.2	-0.1	81.5	0.9
1997	17.7	-4.3	9.5	-0.7	72.8	1.2	88.4	0.2	82.3	1.0
1998	17.1	-3.5	9.5	-0.3	73.4	0.9	88.6	0.1	82.9	0.8
1999	16.5	-3.5	9.4	-0.8	74.1	0.9	88.7	0.2	83.5	0.7
2000	15.7	-4.9	9.3	-1.7	75.1	1.3	89.0	0.3	84.3	1.0
2001	15.1	-3.6	9.1	-1.9	75.8	1.0	89.3	0.3	84.9	0.7
2002	14.6	-3.5	8.9	-2.3	76.5	1.0	89.6	0.3	85.4	0.6
2003	14.3	-2.3	8.8	-1.1	77.0	0.6	89.8	0.2	85.7	0.4
2004	13.6	-4.4	8.4	-4.1	78.0	1.3	90.3	0.5	86.4	0.7
2005	13.1	-4.1	8.2	-2.5	78.7	1.0	90.6	0.3	86.9	0.7

(*): Yearend Estimates. (**): As a percentage of the Total. (***): As a % of the Nonmilitary Total.

Table A6: Current-Cost Net Stock of State and Local Governments' Fixed Assets (*)

(Billions of Dollars--Yearend Estimates)

Year	Equipm. and Software		Structures															
			Core												Other	
			Educational		Health Care		Highways Streets		Power, Transport.		Sewer, Water		Total		Structures	
	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ
1925	0.5	--	5.8	--	0.9	--	16	--	1.5	--	3.7	--	28	--	1.0	--
1926	0.5	0	6.1	5	0.9	0	16	1.3	1.6	6.7	3.9	5.4	29	2.9	1.1	10
1927	0.7	40	6.3	3	1.0	11	16	1.3	1.7	6.2	4.1	5.1	29	2.8	1.3	18
1928	0.8	14	6.5	3	1.1	10	16	0.6	1.9	12	4.4	7.3	30	3.1	1.4	8
1929	0.8	0	6.5	0	1.1	0	16	0.6	2.0	5.3	4.6	4.5	31	1.3	1.5	7
1930	0.8	0	6.0	-8	1.1	0	16	-2	2.1	5.0	4.6	0.0	30	-2	1.5	0
1931	0.9	13	5.3	-12	1.0	-9	14	-11	2.1	0.0	4.2	-9	27	-10	1.5	0
1932	0.9	0	4.8	-9	1.0	0	15	2.8	2.2	4.8	4.1	-2	27	0	1.3	-13
1933	0.9	0	5.1	6	1.1	10	18	23	2.9	32	4.6	12	32	19	1.5	15
1934	0.9	0	5.4	6	1.1	0	20	8.8	3.2	10	5.1	11	35	8.5	1.8	20
1935	0.9	0	5.6	4	1.2	9	21	5.1	3.1	-3	5.4	5.9	36	4.3	1.9	6
1936	0.9	0	6.5	16	1.4	17	22	4.8	3.7	19	6.2	15	40	9.7	2.4	26
1937	1.0	11	7.2	11	1.5	7	21	-1	4.1	11	6.7	8.1	41	3.5	2.7	13
1938	1.0	0	7.4	3	1.6	7	22	1.4	4.2	2.4	7.0	4.5	42	2.4	3.0	11
1939	1.1	10	7.8	5	1.7	6	22	1.4	4.3	2.4	7.2	2.9	43	2.6	3.5	17
1940	1.2	9	8.4	8	1.8	6	24	10	4.6	7.0	7.6	5.6	47	8.4	4.1	17
1941	1.2	0	8.9	6	2.0	11	30	25	5.0	8.7	8.2	7.9	54	17	4.6	12
1942	1.2	0	9.4	6	2.1	5	37	20	4.9	-2	8.3	1.2	61	13	5.0	9
1943	1.1	-8	10	3	2.2	5	37	0.8	4.4	-10	8.9	7.2	62	1.3	5.4	8
1944	1.1	0	10	4	2.3	5	34	-7	4.4	0	9.5	6.7	60	-3	5.4	0
1945	1.1	0	11	10	2.6	13	34	0.6	4.9	11	9.5	0.0	62	3.3	6.1	13
1946	1.2	9	13	17	3.1	19	37	8.5	5.5	12	11	15	70	12	7.0	15
1947	1.5	25	17	28	4.0	29	44	17	6.4	16	12	11	83	19	8.9	27
1948	1.8	20	18	7	4.2	5	47	7.6	7.2	13	14	16	90	9	9.6	8
1949	1.8	0	17	-3	4.2	0	43	-9	6.6	-8	14	2.1	85	-5	9.7	1
1950	2.3	28	20	13	4.8	14	48	12	6.9	5	16	13	96	12	11	15
1951	2.5	9	23	16	5.6	17	54	12	7.8	13	17	5.6	108	12	13	15
1952	2.7	8	24	5	5.9	5	57	5	8.3	6.4	19	8.2	114	5.8	14	7
1953	2.8	4	25	5	6.0	2	53	-8	8.0	-4	20	7.6	112	-2	15	5
1954	3.2	14	26	5	6.1	2	55	4.6	7.7	-4	21	5.0	116	4.0	15	6
1955	3.8	19	30	14	6.6	8	61	11	8.2	6.5	23	9.6	129	11	17	10
1956	4.0	5	34	14	7.3	11	70	15	9.3	13	26	11	147	14	19	12
1957	4.2	5	36	6	7.4	1	73	3.0	9.7	4.3	28	8.6	153	4.6	20	5
1958	4.5	7	39	7	7.7	4	79	8.3	10	5.2	29	4.3	164	6.8	21	7
1959	4.7	4	42	9	8.2	6	79	0.5	11	3.9	29	-1	168	2.6	23	8
1960	5.1	9	44	5	8.4	2	82	4.3	11	4.7	30	4.9	176	4.6	24	6
1961	5.5	8	47	6	8.7	4	88	6.7	12	6.3	31	4.3	186	5.9	26	7
1962	5.9	7	49	6	8.9	2	96	9.0	13	6.8	33	5.4	199	7.1	28	7
1963	6.4	8	52	5	9.1	2	103	7.9	14	7.9	35	5.5	213	6.6	29	5
1964	7.1	11	56	8	9.6	5	109	5.5	15	8.8	37	5.5	226	6.4	31	8
1965	7.8	10	61	8	10	4	120	9.7	16	11	40	7.9	246	8.8	34	8

1966	8.8	13	67	11	11	6	133	11	18	11	43	7.6	272	10	38	11
1967	9.8	11	74	9	11	6	146	9.5	21	13	46	6.8	297	9	41	10
1968	11	10	84	14	13	12	158	8.4	23	12	51	11	328	10	47	15
1969	12	12	95	13	14	12	178	13	27	17	55	9	369	13	55	15
1970	14	12	107	12	16	11	208	17	31	15	62	11	422	14	61	12
1971	15	7	119	11	17	12	222	6.8	34	11	71	16	463	10	70	14
1972	16	9	131	10	19	10	241	8.8	38	9	78	10	507	9	78	12
1973	18	14	147	12	21	12	284	18	44	16	86	10	582	15	89	14
1974	23	27	173	18	25	18	378	33	58	33	110	27	744	28	107	19
1975	26	14	183	6	27	8	375	-1	63	8	123	12	771	3.6	118	10
1976	29	11	197	7	30	10	370	-1	67	7	133	8	797	3.4	130	10
1977	32	10	216	10	33	12	375	1.4	72	7	147	11	843	5.9	146	13
1978	36	12	240	11	38	13	396	5.6	81	13	167	14	923	9.4	166	13
1979	40	13	272	13	43	15	458	16	96	18	193	15	1061	15	192	16
1980	46	15	304	12	49	13	552	21	113	18	218	13	1235	16	220	14
1981	51	11	328	8	54	9	632	14	130	15	228	4.9	1371	11	242	10
1982	55	7	344	5	57	6	658	4.2	140	8.0	244	6.8	1443	5.2	259	7
1983	58	7	354	3	59	4	636	-3	145	3.3	254	4.0	1448	0.4	272	5
1984	63	9	370	4	62	5	634	0	150	3.4	272	7.1	1487	2.7	289	6
1985	70	10	383	4	65	4	664	4.8	157	4.5	279	2.6	1547	4.0	307	6
1986	78	11	402	5	68	5	717	8.0	164	4.9	292	4.7	1643	6.2	331	8
1987	84	8	419	4	71	5	762	6.3	175	6.3	306	4.8	1733	5.5	355	7
1988	92	10	439	5	75	5	777	2.0	186	6.2	327	6.8	1803	4.0	379	7
1989	103	11	462	5	78	5	809	4.1	197	6.1	342	4.8	1887	4.7	408	7
1990	115	12	481	4	81	4	850	5.1	207	5.1	355	3.6	1973	4.6	434	6
1991	123	8	493	2	82	2	870	2.4	214	3.4	365	2.8	2024	2.6	452	4
1992	131	6	515	5	86	4	891	2.3	222	3.7	393	7.7	2107	4.1	478	6
1993	139	6	544	6	91	6	922	3.5	232	4.4	418	6.4	2207	4.7	510	7
1994	147	6	579	6	96	6	984	6.8	244	5.4	436	4.2	2340	6.0	548	7
1995	157	7	608	5	100	4	1054	7.0	258	5.4	458	5.1	2477	5.9	579	6
1996	163	4	642	6	104	4	1106	5.0	263	2.2	473	3.2	2589	4.5	615	6
1997	168	3	697	9	105	1	1177	6.4	350	33	497	5.0	2825	9.1	573	-7
1998	177	5	749	7	110	5	1220	3.7	365	4.3	512	3.1	2956	4.6	612	7
1999	188	6	807	8	116	5	1301	6.6	385	5.4	543	6.1	3152	6.6	660	8
2000	199	6	868	7	122	5	1399	7.5	412	7.1	579	6.6	3380	7.2	709	7
2001	208	4	937	8	128	5	1462	4.5	432	4.7	610	5.4	3568	5.6	757	7
2002	217	4	1003	7	133	4	1530	4.6	462	7.1	638	4.6	3765	5.5	813	7
2003	225	4	1062	6	139	5	1557	1.8	490	5.9	671	5.2	3919	4.1	865	6
2004	236	5	1189	12	152	9	1794	15	558	14	741	10	4434	13	965	12
2005	246	4	1323	11	165	9	2033	13	617	10	783	5.6	4921	11	1059	10

(*) The raw data are obtained from the following tables of the Bureau of Economic Analysis: *Table 7.1A. Current-Cost Net Stock of Government Fixed Assets, 1925-1996; Table 7.1B. Current-Cost Net Stock of Government Fixed Assets, 1997-2005; Table 7.2A. Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets, 1925-1996; Table 7.2B. Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets, 1997-2005.*

Table A7: Current-Cost Net Stock of State and Local Governments' Fixed Assets (*)

(Billions of 2000 Dollars--Yearend Estimates)

Year	Equipment and Software		Structures															
			Core												Other Structures	
			Educational		Health Care		Highways and Streets		Power and Transport.		Sewer and Water		Total			
	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ	Level	%Δ
1925	3.1	--	111	--	15	--	153	--	26	--	63	--	369	--	8	--
1926	3.6	17	117	5	16	6	160	5	27	6	67	6	388	5	9	10
1927	4.4	20	122	4	17	7	169	6	30	11	72	7	410	6	10	13
1928	5.0	14	127	4	19	9	179	6	33	7	76	6	433	6	12	18
1929	5.6	13	132	4	20	8	190	6	35	6	79	4	456	5	15	22
1930	6.0	6	137	4	22	9	204	8	37	6	84	6	485	6	18	22
1931	6.4	8	142	3	24	9	219	7	40	9	88	5	512	6	22	22
1932	6.8	5	143	1	25	7	231	6	42	5	90	3	532	4	26	17
1933	6.8	1	142	-1	26	3	238	3	43	1	91	1	540	2	29	12
1934	6.8	-1	143	1	27	2	245	3	43	1	93	2	551	2	33	13
1935	6.7	0	145	1	27	1	250	2	44	3	95	2	561	2	36	9
1936	6.7	-1	151	5	28	4	260	4	48	9	99	5	587	5	40	11
1937	6.9	3	154	2	29	3	270	4	51	5	102	3	606	3	44	11
1938	7.2	5	158	2	30	4	283	5	54	6	106	4	630	4	48	10
1939	7.7	6	165	5	32	6	295	4	57	6	110	4	659	5	55	13
1940	7.8	1	165	0	32	1	307	4	59	3	114	3	677	3	62	14
1941	7.6	-3	165	0	33	0	314	2	60	2	116	2	687	1	67	8
1942	7.2	-5	164	0	32	0	314	0	60	0	116	1	687	0	69	3
1943	6.6	-7	162	-1	32	0	312	-1	60	-1	116	0	682	-1	69	-1
1944	6.2	-7	160	-1	33	1	309	-1	59	-1	115	-1	676	-1	68	-1
1945	5.8	-6	158	-1	33	1	306	-1	59	-1	115	0	671	-1	67	-1
1946	5.8	0	156	-1	33	1	307	0	59	0	115	0	670	0	68	2
1947	6.2	7	156	0	33	-1	310	1	60	1	117	2	676	1	70	3
1948	6.8	11	159	2	33	1	315	2	60	1	120	3	688	2	72	3
1949	7.5	9	164	4	35	5	323	2	61	1	124	3	707	3	76	6
1950	8.1	8	172	5	38	6	333	3	62	1	127	3	732	3	81	6
1951	8.6	7	181	5	40	6	341	3	63	1	131	3	756	3	87	7
1952	9.2	7	190	5	42	5	351	3	63	1	135	3	782	3	92	6
1953	9.9	7	200	5	43	3	363	4	64	1	139	3	811	4	97	6
1954	11	8	214	7	44	3	381	5	65	2	144	3	849	5	102	5
1955	11	5	230	7	46	3	401	5	67	2	149	4	893	5	108	5
1956	12	7	244	6	47	2	421	5	69	4	155	4	936	5	113	5
1957	13	10	260	6	48	2	442	5	71	3	161	4	982	5	119	5
1958	14	8	276	6	49	3	467	6	74	4	167	3	1033	5	126	6
1959	15	8	289	5	51	3	493	6	78	5	173	4	1085	5	133	6
1960	17	9	303	5	52	2	518	5	83	6	179	4	1136	5	140	5
1961	18	7	319	5	53	2	546	5	88	6	186	4	1191	5	148	6
1962	19	7	333	5	54	2	574	5	92	5	194	4	1247	5	158	6
1963	21	9	350	5	55	2	605	5	97	6	202	4	1310	5	167	6
1964	23	10	369	5	56	2	636	5	104	7	212	5	1377	5	176	6
1965	25	9	391	6	57	2	667	5	112	7	222	5	1450	5	186	6

1966	27	9	417	7	59	2	700	5	119	6	232	4	1527	5	196	6
1967	29	8	446	7	60	3	732	5	128	8	241	4	1609	5	209	6
1968	32	7	474	6	63	4	765	4	138	8	253	5	1693	5	223	6
1969	34	7	497	5	65	4	794	4	148	7	263	4	1767	4	239	7
1970	36	7	516	4	67	3	820	3	157	6	271	3	1831	4	252	6
1971	38	6	533	3	69	3	846	3	164	4	278	3	1890	3	267	6
1972	41	7	548	3	71	3	869	3	171	4	286	3	1944	3	279	5
1973	44	8	564	3	73	2	888	2	178	4	294	3	1997	3	290	4
1974	47	8	580	3	75	3	902	2	187	5	304	3	2048	3	301	4
1975	50	6	594	2	78	4	916	2	193	3	314	3	2095	2	314	4
1976	52	4	605	2	81	4	928	1	201	4	325	3	2141	2	326	4
1977	54	3	612	1	84	3	940	1	211	5	333	3	2180	2	337	3
1978	56	4	619	1	86	3	956	2	223	6	344	3	2227	2	348	3
1979	58	4	625	1	88	2	975	2	235	6	354	3	2278	2	359	3
1980	61	5	631	1	91	2	991	2	247	5	366	3	2325	2	372	4
1981	63	3	634	1	92	2	1001	1	260	5	375	2	2362	2	383	3
1982	65	4	636	0	94	2	1007	1	270	4	384	2	2390	1	394	3
1983	69	5	637	0	95	1	1015	1	279	3	390	2	2416	1	405	3
1984	73	7	639	0	96	1	1029	1	287	3	398	2	2450	1	418	3
1985	80	8	643	1	97	1	1047	2	296	3	406	2	2489	2	433	4
1986	86	8	649	1	98	1	1065	2	304	3	417	3	2533	2	451	4
1987	92	7	654	1	99	1	1083	2	311	2	429	3	2577	2	468	4
1988	99	7	662	1	100	1	1103	2	318	2	442	3	2625	2	485	4
1989	108	9	672	1	101	1	1122	2	324	2	454	3	2672	2	504	4
1990	117	9	683	2	102	1	1144	2	331	2	467	3	2727	2	525	4
1991	124	6	697	2	104	2	1165	2	339	2	480	3	2785	2	545	4
1992	130	5	713	2	106	2	1186	2	346	2	492	3	2844	2	563	3
1993	136	4	727	2	108	2	1208	2	354	2	502	2	2899	2	581	3
1994	143	5	740	2	109	1	1231	2	361	2	510	2	2952	2	598	3
1995	151	5	759	3	111	1	1252	2	369	2	518	2	3009	2	617	3
1996	157	4	780	3	112	1	1274	2	375	2	528	2	3070	2	638	3
1997	165	5	783	0	119	6	1320	4	385	3	549	4	3156	3	641	0
1998	176	6	807	3	120	1	1345	2	392	2	559	2	3222	2	663	3
1999	187	7	836	4	121	1	1372	2	400	2	569	2	3298	2	685	3
2000	199	6	868	4	122	1	1399	2	412	3	579	2	3380	2	709	4
2001	210	6	902	4	123	1	1430	2	424	3	590	2	3469	3	732	3
2002	221	5	937	4	125	1	1458	2	438	3	603	2	3561	3	758	4
2003	229	4	973	4	127	2	1485	2	455	4	616	2	3656	3	783	3
2004	237	3	1008	4	129	2	1509	2	471	3	629	2	3746	2	804	3
2005	245	3	1040	3	131	2	1529	1	485	3	640	2	3825	2	821	2

(*) The raw data are obtained from the following tables of the Bureau of Economic Analysis: *Table 7.1A. Current-Cost Net Stock of Government Fixed Assets, 1925-1996; Table 7.1B. Current-Cost Net Stock of Government Fixed Assets, 1997-2005; Table 7.2A. Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets, 1925-1996;* and *Table 7.2B. Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets, 1997-2005.*

Table A8: Breakdown of State and Local Governments' Fixed Assets as Shares of the Total

(Yearend Estimates)

Year	Equipment and Software		Structures													
			Core												Other Structures	
			Educational		Health Care		Highways and Streets		Power and Transport.		Sewer and Water		Total			
	%	%Δ	%	%Δ	%	%Δ	%	%Δ	%	%Δ	%	%Δ	%	%Δ	%	%Δ
1925	1.7	--	20	--	3.1	--	54	--	5.1	--	13	--	95	--	3.4	--
1926	1.7	-3	20	2.0	3.0	-3.0	53	-1.8	5.3	3.5	13	2.3	95	-0.2	3.7	6.7
1927	2.2	35	20	-0.7	3.2	6.9	52	-2.6	5.4	2.2	13	1.1	94	-1.1	4.2	14
1928	2.5	10	20	-0.3	3.4	6.3	50	-2.8	5.9	8.0	14	3.7	93	-0.4	4.3	4.0
1929	2.4	-2	20	-1.5	3.3	-1.5	50	-0.9	6.1	3.7	14	3.0	93	-0.2	4.6	5.5
1930	2.5	2	19	-5.7	3.4	2.2	50	0.3	6.5	7.3	14	2.2	93	-0.2	4.7	2.2
1931	3.1	24	18	-2.9	3.4	-0.1	49	-2.4	7.2	9.9	14	0.3	92	-1.1	5.1	9.9
1932	3.1	1	17	-8.5	3.4	1.0	51	3.9	7.6	5.8	14	-1.4	92	0.7	4.5	-12
1933	2.6	-15	15	-9.9	3.2	-6.7	53	4.4	8.5	12	13	-4.9	93	0.6	4.4	-2.2
1934	2.4	-8	15	-2.7	3.0	-8.1	53	0.1	8.6	1.4	14	1.9	93	-0.3	4.8	10
1935	2.3	-4	14	-0.6	3.1	4.6	53	0.7	8.0	-7.1	14	1.5	93	0.0	4.9	1.2
1936	2.1	-9	15	5.2	3.3	5.8	51	-5.0	8.6	8.2	14	4.1	92	-0.5	5.6	15
1937	2.2	7	16	6.3	3.4	2.8	48	-5.4	9.2	6.3	15	3.7	92	-0.6	6.1	8
1938	2.2	-3	16	-0.1	3.5	3.6	47	-1.5	9.2	-0.5	15	1.5	91	-0.5	6.5	8
1939	2.3	6	16	1.6	3.6	2.5	46	-2.2	9.0	-1.3	15	-0.8	90	-1.0	7.4	13
1940	2.3	0	16	-1.2	3.5	-2.9	47	0.9	8.9	-1.9	15	-3.2	90	-0.6	7.9	7.4
1941	2.0	-14	15	-8.7	3.3	-4.2	50	7.9	8.3	-6.3	14	-7.0	90	0.6	7.6	-3.3
1942	1.8	-11	14	-5.7	3.1	-6.2	54	7.6	7.3	-12	12	-9.6	91	0.5	7.4	-2.9
1943	1.6	-10	14	1.5	3.2	3.1	54	-0.8	6.4	-12	13	5.5	91	-0.3	7.9	6.3
1944	1.6	2	15	6.6	3.4	7.0	51	-5.1	6.6	2.4	14	9.3	90	-0.3	8.1	2.4
1945	1.6	-4	16	5.6	3.7	8.7	49	-3.3	7.0	7.0	14	-3.9	90	-0.7	8.8	8.6
1946	1.5	-3	17	4.6	4.0	6.5	48	-3.1	7.1	0.3	14	2.5	89	-0.2	9.0	2.5
1947	1.6	4	18	7.3	4.3	7.7	47	-1.9	6.9	-2.8	13	-7.3	89	-0.7	9.5	6.2
1948	1.8	10	18	-2.1	4.1	-3.6	46	-1.2	7.1	3.3	14	6.3	89	-0.1	9.4	-0.9
1949	1.9	5	18	1.9	4.3	4.9	44	-4.1	6.8	-3.9	15	7.1	88	-0.7	10.0	5.9
1950	2.1	13	18	0.4	4.4	1.3	44	-0.4	6.3	-7.3	15	0.4	88	-0.5	10.2	2.4
1951	2.0	-3	18	3.0	4.6	3.8	44	0.0	6.3	0.5	14	-6.1	87	-0.2	10.5	2.4
1952	2.1	2	18	-1.0	4.5	-0.5	44	-0.6	6.4	0.4	14	2.1	87	-0.2	10.6	1.0
1953	2.2	5	19	5.8	4.7	2.8	41	-6.9	6.2	-2.6	15	8.7	87	-0.9	11.3	6.2
1954	2.4	9	19	0.8	4.5	-2.6	41	0.1	5.7	-7.8	16	0.6	86	-0.4	11.4	1.0
1955	2.5	7	20	2.4	4.4	-2.6	41	0.2	5.5	-4.1	15	-1.4	86	0.0	11.2	-1.1
1956	2.4	-7	20	0.9	4.3	-2.5	42	1.3	5.5	0.0	15	-1.8	87	0.4	11.1	-1.3
1957	2.4	0	20	1.1	4.2	-3.2	41	-1.7	5.5	-0.4	16	3.7	86	-0.1	11.2	0.6
1958	2.4	0	20	0.1	4.1	-2.6	41	1.4	5.4	-1.6	15	-2.3	86	0.0	11.1	-0.2
1959	2.4	1	21	5.1	4.2	3.1	40	-2.7	5.4	0.6	15	-4.2	86	-0.6	11.7	4.6
1960	2.5	4	22	0.4	4.1	-2.3	40	-0.5	5.4	-0.1	15	0.1	86	-0.2	11.8	0.9
1961	2.5	2	21	-0.3	4.0	-2.3	40	0.6	5.4	0.3	14	-1.6	86	-0.1	11.8	0.6
1962	2.5	0	21	-1.4	3.8	-4.5	41	1.8	5.4	-0.3	14	-1.5	86	0.0	11.8	-0.1
1963	2.6	2	21	-1.2	3.7	-4.0	42	1.3	5.5	1.3	14	-1.0	86	0.1	11.7	-1.0
1964	2.7	4	21	1.5	3.6	-1.2	41	-1.1	5.6	2.0	14	-1.2	85	-0.3	11.9	1.4
1965	2.7	1	21	-0.7	3.5	-4.3	42	0.8	5.7	1.8	14	-0.8	85	0.0	11.8	-0.5

1966	2.8	2	21	0.6	3.3	-4.1	42	0.6	5.7	0.4	13	-2.7	85	-0.1	11.8	0.3
1967	2.8	2	21	0.1	3.2	-3.3	42	0.3	5.9	3.1	13	-2.2	85	-0.1	11.9	0.3
1968	2.8	-1	22	2.4	3.2	0.6	41	-2.4	6.0	1.1	13	0.0	85	-0.5	12.3	3.4
1969	2.8	-1	22	0.5	3.2	-0.8	41	-0.2	6.2	3.6	13	-3.2	85	-0.3	12.5	2.0
1970	2.7	-2	21	-1.6	3.1	-2.9	42	2.3	6.2	0.7	12	-2.3	85	0.3	12.4	-1.4
1971	2.6	-3	22	1.0	3.2	1.2	40	-3.1	6.3	1.0	13	4.9	85	-0.5	12.8	3.7
1972	2.6	-1	22	0.3	3.2	0.7	40	-0.8	6.3	-0.3	13	0.2	84	-0.3	13.0	1.9
1973	2.6	-1	21	-2.3	3.1	-2.8	41	2.8	6.4	1.5	13	-3.8	84	0.2	12.9	-0.8
1974	2.6	0	20	-6.6	2.9	-6.6	43	4.8	6.7	4.9	13	0.4	85	0.9	12.2	-5.8
1975	2.9	9	20	1.1	3.0	2.7	41	-5.2	6.9	2.9	13	6.9	84	-1.1	12.8	5.4
1976	3.0	6	21	2.8	3.1	4.9	39	-5.5	7.1	2.9	14	3.4	83	-1.1	13.6	5.7
1977	3.1	3	21	2.7	3.3	5.2	37	-5.2	7.0	-0.2	14	3.5	83	-1.0	14.3	5.4
1978	3.2	1	21	0.9	3.3	2.3	35	-4.0	7.2	2.6	15	3.8	82	-0.6	14.8	3.0
1979	3.1	-2	21	-1.6	3.3	-0.1	35	0.4	7.4	2.3	15	0.0	82	0.0	14.8	0.7
1980	3.1	-1	20	-3.8	3.3	-2.5	37	3.9	7.5	1.9	15	-2.6	82	0.3	14.6	-1.4
1981	3.1	0	20	-2.6	3.2	-1.3	38	3.2	7.8	3.6	14	-5.4	82	0.1	14.5	-0.7
1982	3.1	2	20	-0.7	3.2	0.8	37	-1.3	8.0	2.3	14	1.2	82	-0.3	14.7	1.4
1983	3.3	6	20	1.8	3.3	2.9	36	-4.5	8.1	2.0	14	2.7	81	-0.9	15.3	3.7
1984	3.4	5	20	0.9	3.4	1.6	34	-3.7	8.1	-0.1	15	3.5	81	-0.7	15.7	2.9
1985	3.6	5	20	-1.0	3.4	-0.5	35	0.2	8.1	0.0	14	-1.9	80	-0.5	16.0	1.5
1986	3.8	4	20	-1.7	3.3	-1.6	35	1.3	8.0	-1.6	14	-1.7	80	-0.4	16.1	1.1
1987	3.8	2	19	-1.3	3.3	-1.1	35	0.4	8.0	0.4	14	-1.0	80	-0.3	16.3	1.3
1988	4.0	5	19	0.0	3.3	-0.1	34	-2.7	8.2	1.4	14	1.9	79	-0.7	16.7	2.2
1989	4.3	6	19	-0.3	3.2	-0.8	34	-1.3	8.2	0.6	14	-0.6	79	-0.7	17.0	1.9
1990	4.5	6	19	-1.0	3.2	-1.3	34	-0.1	8.2	-0.1	14	-1.5	78	-0.6	17.2	1.2
1991	4.7	4	19	-0.6	3.2	-1.2	33	-0.7	8.2	0.3	14	-0.3	78	-0.5	17.4	1.1
1992	4.8	2	19	0.1	3.2	-0.2	33	-2.0	8.2	-0.7	14	3.1	78	-0.4	17.6	1.2
1993	4.9	1	19	0.4	3.2	0.4	32	-1.5	8.1	-0.7	15	1.2	77	-0.4	17.9	1.5
1994	4.9	0	19	0.1	3.2	-0.3	32	0.5	8.1	-0.8	14	-1.9	77	-0.2	18.0	1.0
1995	4.9	1	19	-0.8	3.1	-1.9	33	1.1	8.0	-0.5	14	-0.7	77	0.0	18.0	-0.1
1996	4.8	-1	19	0.8	3.1	-0.7	33	0.2	7.8	-2.4	14	-1.5	77	-0.2	18.3	1.3
1997	4.7	-3	20	2.5	2.9	-4.6	33	0.4	9.8	25	14	-0.9	79	3.0	16.1	-12
1998	4.7	0	20	2.3	2.9	0.1	33	-1.3	9.8	-0.6	14	-1.9	79	-0.4	16.3	1.8
1999	4.7	-1	20	0.9	2.9	-1.4	33	-0.1	9.6	-1.3	14	-0.7	79	-0.2	16.5	1.0
2000	4.6	-1	20	0.3	2.8	-1.9	33	0.3	9.6	-0.1	13	-0.6	79	0.0	16.5	0.2
2001	4.6	-1	21	2.2	2.8	-1.1	32	-1.2	9.5	-1.0	13	-0.3	79	-0.1	16.7	1.0
2002	4.5	-1	21	1.1	2.8	-1.5	32	-1.1	9.6	1.3	13	-1.1	79	-0.2	17.0	1.5
2003	4.5	-1	21	1.4	2.8	0.2	31	-2.6	9.8	1.4	13	0.8	78	-0.4	17.3	1.9
2004	4.2	-7	21	-0.6	2.7	-2.8	32	2.4	9.9	1.4	13	-1.8	79	0.6	17.1	-1
2005	3.9	-6	21	0.8	2.7	-1.7	33	2.6	9.9	0.0	13	-4.4	79	0.5	17.0	-1

Table A9: Current-Cost Net Stock of Private Fixed Assets (*)

(Yearend Estimates)

Year	Billions of current dollars						Billions of Chained 2000 dollars					
	Nonresidential		Residential		Total		Nonresidential		Residential		Total	
	Levels	%Δ	Levels	%Δ	Levels	%Δ	Levels	%Δ	Levels	%Δ	Levels	%Δ
1925	115	--	101	--	216	--	1588	--	1651	--	3085	--
1926	119	3.8	104	3.7	224	3.8	1643	3.5	1723	4.3	3203	3.8
1927	122	2.2	107	2.6	229	2.4	1693	3.0	1788	3.8	3308	3.3
1928	125	2.7	116	8.0	241	5.2	1742	2.9	1847	3.3	3407	3.0
1929	125	0.1	120	3.5	245	1.7	1804	3.5	1884	2.0	3506	2.9
1930	119	-4.6	114	-4.6	234	-4.6	1846	2.3	1897	0.7	3556	1.5
1931	107	-10.7	94	-18.2	200	-14.4	1848	0.1	1903	0.3	3558	0.0
1932	99	-7.4	84	-10.2	183	-8.7	1822	-1.4	1895	-0.4	3513	-1.3
1933	100	1.5	92	9.2	192	5.1	1792	-1.7	1884	-0.6	3463	-1.4
1934	102	1.4	93	1.1	194	1.1	1771	-1.2	1877	-0.3	3432	-0.9
1935	101	-0.3	94	1.6	196	0.7	1759	-0.7	1879	0.1	3421	-0.3
1936	112	10.4	105	11.1	217	10.7	1764	0.3	1887	0.4	3440	0.5
1937	117	4.5	111	6.4	228	5.4	1784	1.1	1897	0.6	3478	1.1
1938	116	-1.1	113	1.6	229	0.3	1782	-0.1	1908	0.6	3486	0.2
1939	116	0.7	117	3.2	233	1.9	1786	0.2	1934	1.3	3515	0.8
1940	124	6.3	128	9.7	252	8.0	1802	0.9	1965	1.6	3568	1.5
1941	140	12.9	140	9.1	279	11.0	1827	1.4	1999	1.7	3635	1.9
1942	149	6.5	151	7.7	299	7.1	1816	-0.6	2005	0.3	3629	-0.2
1943	151	1.4	164	8.6	314	5.0	1794	-1.2	1999	-0.3	3601	-0.8
1944	152	0.9	175	6.9	327	4.0	1790	-0.3	1993	-0.3	3594	-0.2
1945	167	10.0	184	5.5	352	7.6	1812	1.2	1986	-0.3	3620	0.7
1946	206	23.4	227	23.2	433	23.3	1870	3.2	2038	2.6	3734	3.2
1947	248	20.2	267	17.7	515	18.9	1938	3.6	2113	3.7	3887	4.1
1948	272	9.7	288	7.7	560	8.7	2012	3.8	2205	4.3	4059	4.4
1949	277	1.8	303	5.0	579	3.5	2067	2.7	2286	3.7	4197	3.4
1950	314	13.2	340	12.3	653	12.7	2131	3.1	2406	5.2	4381	4.4
1951	342	9.0	369	8.6	711	8.8	2201	3.3	2500	3.9	4545	3.7
1952	360	5.4	388	5.1	748	5.2	2264	2.9	2589	3.6	4697	3.3
1953	376	4.4	402	3.7	778	4.0	2338	3.2	2682	3.6	4863	3.5
1954	386	2.6	423	5.3	809	4.1	2404	2.8	2783	3.8	5025	3.3
1955	425	10.2	456	7.7	881	8.9	2483	3.3	2905	4.4	5222	3.9
1956	470	10.6	478	4.8	948	7.6	2573	3.6	3012	3.7	5416	3.7
1957	502	6.8	494	3.3	997	5.1	2661	3.4	3107	3.2	5597	3.3
1958	512	1.8	509	3.0	1021	2.4	2722	2.3	3201	3.1	5743	2.6
1959	533	4.3	531	4.3	1065	4.3	2793	2.6	3330	4.0	5933	3.3
1960	545	2.1	552	3.9	1097	3.0	2874	2.9	3444	3.4	6119	3.1
1961	560	2.8	572	3.6	1132	3.2	2950	2.7	3556	3.3	6298	2.9
1962	580	3.6	593	3.7	1173	3.6	3040	3.0	3681	3.5	6507	3.3
1963	601	3.6	608	2.5	1209	3.0	3133	3.1	3826	3.9	6739	3.6
1964	637	5.9	657	8.1	1294	7.0	3248	3.7	3980	4.0	7007	4.0
1965	682	7.1	698	6.2	1379	6.6	3400	4.7	4125	3.6	7311	4.3
1966	743	8.9	754	8.1	1497	8.5	3574	5.1	4248	3.0	7624	4.3
1967	803	8.1	806	6.8	1609	7.5	3729	4.4	4363	2.7	7906	3.7
1968	883	9.9	890	10.5	1773	10.2	3887	4.2	4500	3.1	8209	3.8

1969	972	10.1	952	7.0	1925	8.6	4059	4.4	4640	3.1	8533	3.9
1970	1070	10.1	1008	5.9	2078	8.0	4215	3.8	4765	2.7	8821	3.4
1971	1178	10.1	1133	12.4	2312	11.2	4358	3.4	4941	3.7	9138	3.6
1972	1283	8.9	1268	11.9	2551	10.3	4517	3.7	5155	4.3	9513	4.1
1973	1447	12.8	1452	14.6	2900	13.7	4716	4.4	5364	4.1	9930	4.4
1974	1763	21.8	1653	13.8	3416	17.8	4898	3.9	5510	2.7	10271	3.4
1975	1943	10.2	1792	8.4	3735	9.3	5023	2.5	5624	2.1	10513	2.4
1976	2127	9.5	1989	11.0	4116	10.2	5151	2.6	5783	2.8	10802	2.8
1977	2366	11.2	2321	16.7	4687	13.9	5308	3.0	5993	3.6	11176	3.5
1978	2687	13.6	2677	15.3	5364	14.5	5515	3.9	6218	3.7	11619	4.0
1979	3103	15.5	3108	16.1	6211	15.8	5753	4.3	6428	3.4	12080	4.0
1980	3563	14.8	3505	12.8	7068	13.8	5971	3.8	6570	2.2	12447	3.0
1981	4033	13.2	3770	7.6	7803	10.4	6205	3.9	6688	1.8	12807	2.9
1982	4288	6.3	3941	4.5	8229	5.5	6390	3.0	6766	1.2	13070	2.1
1983	4417	3.0	4100	4.0	8516	3.5	6543	2.4	6918	2.3	13372	2.3
1984	4680	6.0	4331	5.6	9011	5.8	6775	3.5	7107	2.7	13799	3.2
1985	4944	5.6	4565	5.4	9509	5.5	7028	3.7	7298	2.7	14247	3.2
1986	5180	4.8	4930	8.0	10110	6.3	7228	2.8	7524	3.1	14672	3.0
1987	5462	5.4	5249	6.5	10711	5.9	7412	2.5	7755	3.1	15083	2.8
1988	5817	6.5	5574	6.2	11392	6.4	7603	2.6	7977	2.9	15497	2.7
1989	6168	6.0	5884	5.5	12052	5.8	7796	2.5	8185	2.6	15901	2.6
1990	6500	5.4	6111	3.9	12611	4.6	7986	2.4	8362	2.2	16266	2.3
1991	6633	2.0	6248	2.2	12881	2.1	8112	1.6	8508	1.7	16535	1.6
1992	6839	3.1	6600	5.6	13439	4.3	8223	1.4	8685	2.1	16822	1.7
1993	7162	4.7	7005	6.1	14167	5.4	8379	1.9	8889	2.3	17189	2.2
1994	7551	5.4	7506	7.1	15057	6.3	8555	2.1	9117	2.6	17604	2.4
1995	7955	5.3	7840	4.5	15794	4.9	8786	2.7	9335	2.4	18068	2.6
1996	8347	4.9	8271	5.5	16618	5.2	9055	3.1	9579	2.6	18595	2.9
1997	8819	5.6	8731	5.6	17549	5.6	9370	3.5	9824	2.6	19171	3.1
1998	9320	5.7	9300	6.5	18621	6.1	9727	3.8	10096	2.8	19812	3.3
1999	9861	5.8	9987	7.4	19847	6.6	10106	3.9	10386	2.9	20489	3.4
2000	10514	6.6	10676	6.9	21190	6.8	10514	4.0	10676	2.8	21190	3.4
2001	11020	4.8	11465	7.4	22485	6.1	10795	2.7	10962	2.7	21756	2.7
2002	11330	2.8	12193	6.4	23523	4.6	10960	1.5	11266	2.8	22226	2.2
2003	11692	3.2	13225	8.5	24917	5.9	11109	1.4	11606	3.0	22716	2.2
2004	12533	7.2	14660	10.9	27193	9.1	11283	1.6	11974	3.2	23262	2.4
2005	13544	8.1	15800	7.8	29344	7.9	11485	1.8	12344	3.1	23832	2.5

(*) The raw data are obtained from the following tables of the Bureau of Economic Analysis: *Table 2.1. Current-Cost Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type* and *Table 2.2. Chain-Type Quantity Indexes for Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type*

Table A10: Total Labor Hours (Billions of Hours) (*)

Year	Total Economy (Unpublished)	Business Sector (**)	Government Enterprises	Private Business Sector (***)	Nonfarm Business Sector	Private Households	Nonprofit Organizations	Nonmilitary Government	Armed Forces	Total Government
1947	N/A	104.595	1.341	103.254	86.862	2.261	3.025	10.743	N/A	N/A
1948	123.029	105.366	1.467	103.899	88.247	2.099	3.112	11.103	2.818	13.921
1949	120.262	102.008	1.537	100.471	84.836	1.980	3.275	11.430	3.106	14.536
1950	122.437	103.417	1.528	101.889	87.558	2.216	3.413	11.751	3.167	14.918
1951	129.471	106.721	1.606	105.115	91.681	2.177	3.528	12.631	6.020	18.651
1952	130.794	106.965	1.752	105.213	92.691	2.072	3.642	12.949	6.919	19.868
1953	132.262	108.302	1.725	106.577	95.035	2.098	3.782	12.986	6.818	19.804
1954	128.397	104.672	1.730	102.942	91.806	1.923	3.993	13.127	6.413	19.540
1955	132.610	108.559	1.753	106.806	95.500	2.179	4.264	13.501	5.861	19.362
1956	134.816	110.210	1.773	108.437	97.859	2.275	4.407	14.202	5.493	19.695
1957	133.609	108.649	1.799	106.850	97.311	2.226	4.563	14.658	5.312	19.970
1958	128.961	103.777	1.861	101.916	93.235	2.259	4.781	15.021	4.985	20.006
1959	133.653	108.100	1.990	106.110	97.560	2.237	5.065	15.434	4.807	20.241
1960	134.620	108.276	2.035	106.241	98.116	2.250	5.545	15.872	4.714	20.586
1961	133.732	106.668	2.086	104.582	97.072	2.172	5.597	16.504	4.876	21.380
1962	137.110	108.575	2.107	106.468	99.204	2.157	5.918	17.175	5.392	22.567
1963	138.205	109.313	2.135	107.178	100.322	2.130	6.122	17.614	5.160	22.774
1964	141.793	112.467	2.169	110.298	104.000	2.103	6.085	18.180	5.126	23.306
1965	146.675	116.284	2.253	114.031	108.024	2.042	6.314	19.168	5.119	24.287
1966	151.839	119.313	2.310	117.003	111.762	1.973	6.580	20.436	5.847	26.283
1967	153.555	118.982	2.384	116.598	111.738	2.085	6.848	21.574	6.449	28.023
1968	156.344	120.767	2.432	118.335	113.711	1.993	7.186	22.259	6.570	28.829
1969	160.142	123.838	2.491	121.347	116.978	1.910	7.526	22.861	6.499	29.360
1970	157.345	121.383	2.592	118.791	115.130	1.807	7.606	23.291	5.851	29.142
1971	156.648	121.062	2.626	118.436	114.917	1.737	7.641	23.705	5.131	28.836
1972	160.957	124.846	2.628	122.218	118.617	1.706	7.897	24.652	4.484	29.136
1973	166.083	129.619	2.685	126.934	123.432	1.681	8.023	25.214	4.232	29.446
1974	166.698	129.858	2.788	127.070	123.612	1.452	8.257	25.885	4.033	29.918
1975	161.969	124.237	2.790	121.447	118.339	1.370	8.508	26.714	3.930	30.644
1976	166.638	128.359	2.789	125.570	122.548	1.561	8.679	26.976	3.851	30.827
1977	172.487	133.285	2.746	130.539	127.384	1.744	8.872	27.489	3.842	31.331
1978	180.595	140.113	2.830	137.283	134.025	1.719	9.050	28.700	3.842	32.542
1979	185.458	144.846	2.926	141.920	138.810	1.634	9.310	28.846	3.747	32.593
1980	184.958	143.574	2.978	140.596	137.684	1.520	9.677	29.392	3.773	33.165
1981	185.317	144.555	2.922	141.633	138.663	1.496	9.984	28.438	3.765	32.203
1982	182.574	141.298	2.967	138.331	135.630	1.482	10.436	28.447	3.879	32.326
1983	185.848	143.777	2.981	140.796	138.273	1.445	10.801	28.845	3.963	32.808
1984	195.198	152.184	3.119	149.065	146.692	1.549	11.012	29.518	4.054	33.572
1985	199.659	155.721	3.178	152.543	150.559	1.549	11.324	30.168	4.075	34.243
1986	202.005	156.956	3.228	153.728	151.725	1.543	11.831	30.797	4.106	34.903
1987	207.459	161.644	3.240	158.404	156.321	1.553	12.379	31.039	4.084	35.123
1988	213.657	166.080	3.323	162.757	160.803	1.579	12.932	32.301	4.088	36.389
1989	219.559	170.468	3.396	167.072	165.217	1.542	13.680	33.179	4.086	37.265
1990	219.938	169.476	3.475	166.001	164.566	1.503	14.696	33.786	3.952	37.738
1991	216.853	165.479	3.483	161.996	160.582	1.418	15.445	34.123	3.871	37.994
1992	217.036	165.088	3.398	161.690	160.210	1.516	15.610	34.635	3.585	38.220
1993	222.108	169.558	3.475	166.083	164.900	1.504	15.997	35.199	3.325	38.524
1994	229.040	176.286	3.600	172.686	170.855	1.361	16.322	35.516	3.155	38.671

1995	234.644	181.147	3.671	177.476	175.540	1.353	16.756	36.055	3.004	39.059
1996	237.565	184.034	3.674	180.360	178.665	1.297	16.749	36.278	2.879	39.157
1997	244.500	190.203	3.729	186.474	184.931	1.243	17.157	36.806	2.820	39.626
1998	249.762	193.930	3.733	190.197	188.870	1.328	17.982	37.486	2.769	40.255
1999	254.660	197.811	3.718	194.093	193.047	1.232	18.316	38.311	2.709	41.020
2000	257.891	199.761	3.755	196.006	194.951	1.124	18.836	39.224	2.701	41.925
2001	254.859	195.405	3.817	191.588	190.957	1.097	19.415	40.029	2.729	42.758
2002	251.447	190.517	3.820	186.697	186.025	1.141	20.074	40.761	2.774	43.535
2003	250.270	189.250	3.814	185.436	184.956	1.131	20.237	40.673	2.794	43.467
2004	253.156	191.623	3.814	187.809	187.418	1.187	20.470	40.878	2.813	43.691
2005	256.604	194.382	3.799	190.583	190.144	1.235	20.727	41.232	2.827	44.059

(*) Source: Various tables of the Bureau of Labor Statistics. The sum of labor hours from Jan.1 to Dec.31 of each year.

(**): GDP-Households-Institutions-General Government.

(***): Business Sector-Government Enterprises.

Table A11: Estimated Elasticities of Private Output and Their Variances and Covariances (*)

Equation	$\hat{e}_K$	$\hat{e}_L$	$\hat{e}_G$	Var[$\hat{e}_K$]	Var[$\hat{e}_L$]	Var[$\hat{e}_G$]	Cov[$\hat{e}_K$,$\hat{e}_L$]	Cov[$\hat{e}_K$,$\hat{e}_G$]	Cov[$\hat{e}_L$,$\hat{e}_G$]
Y 1.1	0.294	0.649	0.322	0.0211	0.0078	0.0057	-0.0058	-0.0093	0.0050
Y 1.2	0.315	0.647	0.275	0.0241	0.0092	0.0053	-0.0074	-0.0097	0.0054
Y 1.3	0.282	0.665	0.293	0.0089	0.0041	0.0009	0.0013	-0.0016	0.0007
Y 1.4	0.322	0.631	0.305	0.0079	0.0037	0.0009	0.0015	-0.0014	0.0006
Y 1.5	0.241	0.648	0.299	0.0247	0.0082	0.0049	-0.0060	-0.0093	0.0046
Y 1.6	0.218	0.659	0.311	0.0093	0.0043	0.0009	0.0016	-0.0016	0.0007
Y 1.7	0.261	0.626	0.325	0.0083	0.0039	0.0009	0.0018	-0.0014	0.0006
Y 1.8	0.217	0.647	0.348	0.0218	0.0071	0.0053	-0.0046	-0.0091	0.0043
Y 1.9	0.207	0.616	0.339	0.0208	0.0063	0.0045	-0.0025	-0.0079	0.0032
Y 1.10	0.223	0.622	0.295	0.0230	0.0072	0.0040	-0.0034	-0.0079	0.0034
Y 1.11	0.170	0.644	0.321	0.0097	0.0047	0.0009	0.0022	-0.0015	0.0007
Y 1.12	0.211	0.613	0.338	0.0087	0.0043	0.0010	0.0023	-0.0014	0.0006
average	**0.247**	**0.639**	**0.314**	**0.0157**	**0.0059**	**0.0029**	**-0.0016**	**-0.0052**	**0.0025**
					(**)		**(***)**		**(****)**
ΔY 2.1	0.941	0.315	0.372	0.1999	0.0065	0.1795	-0.0184	-0.1437	0.0108
ΔY 2.2	0.626	0.442	0.252	0.1098	0.0068	0.0131	-0.0159	0.0116	-0.0063
ΔY 2.3	0.433	0.441	0.346	0.2172	0.0081	0.1618	-0.0258	-0.1275	0.0092
ΔY 2.4	0.625	0.427	0.129	0.1620	0.0079	0.1121	-0.0220	-0.0702	0.0052
ΔY 2.5	0.495	0.360	0.734	0.1977	0.0059	0.1920	-0.0132	-0.1490	0.0069
ΔY 2.6	0.747	0.354	0.468	0.1439	0.0060	0.1303	-0.0118	0.0885	0.0056
ΔY 2.7	0.921	0.360	0.346	0.0800	0.0048	0.0118	-0.0055	0.0037	-0.0044
ΔY 2.8	0.408	0.493	0.204	0.1034	0.0062	0.0129	-0.0127	0.0123	-0.0061
ΔY 2.9	0.895	0.345	0.381	0.0744	0.0046	0.0101	-0.0047	0.0022	-0.0041
ΔY 2.10	0.433	0.457	0.266	0.0936	0.0057	0.0106	-0.0108	0.0087	-0.0053
ΔY 2.11	0.397	0.373	0.698	0.1635	0.0055	0.1377	-0.0071	-0.1068	0.0013
ΔY 2.12	0.609	0.375	0.490	0.1279	0.0056	0.1011	-0.0075	-0.0679	0.0020
ΔY 2.13	0.196	0.488	0.304	0.1299	0.0063	0.0917	-0.0117	-0.0507	-0.0006
ΔY 2.14	0.758	0.433	0.247	0.0823	0.0046	0.0096	-0.0033	0.0037	-0.0036
ΔY 2.15	0.781	0.323	0.506	0.0658	0.0046	0.0121	-0.0027	0.0028	-0.0048
ΔY 2.16	0.376	0.427	0.369	0.0818	0.0056	0.0134	-0.0083	0.0102	-0.0063
average	**0.603**	**0.401**	**0.382**	**0.1271**	**0.0059**	**0.0750**	**-0.0113**	**-0.0357**	**-0.00004**

(*):Obtained from Tables 15A and 17A. $\hat{e}_K$, $\hat{e}_L$, and $\hat{e}_G$ are estimated elasticities of private output with respect to private capital and labor, and public fixed capital (either nonmilitary or core infrastructure capitals), respectively.

(**): If lnY=a+b*lnK+c*L+…, then $\hat{e}_L=\hat{c}*L$ and therefore $Var[\hat{e}_L]=E[\hat{e}_L^2]-(E[\hat{e}_L])^2=E[L^2 * \hat{c}^2]-(E[L*\hat{c}])^2=L^2 * Var[\hat{c}]$.

(***): $Cov[\hat{e}_K,\hat{e}_L]=E[(\hat{e}_K-E[\hat{e}_K])*(\hat{e}_L-E[\hat{e}_L])]=L*E[(\hat{e}_K-E[\hat{e}_K])*(\hat{c}-E[\hat{c}])]=L*Cov[\hat{e}_K,\hat{c}]$.

(****):$Cov[\hat{e}_L,\hat{e}_G]=E[(\hat{e}_L-E[\hat{e}_L])*(\hat{e}_G-E[\hat{e}_G])]=L*E[(\hat{e}_G-E[\hat{e}_G])*(\hat{c}-E[\hat{c}])]=L*Cov[\hat{e}_G,\hat{c}]$.

Appendix C. Sampling Properties of the Ordinary Least Squares Estimator under Auto-Correlated Errors

Let us discuss the sampling properties of the least squares estimator **a** = (**X'X**)$^{-1}$**X'Y** for **A** under auto-correlated errors. Under auto-correlated errors, our complete statistical model can be summarized as follows since E[$u_t u_s$] is no longer zero for all t ≠ s, where E stands for the expected value. (If E[$u_t u_s$] were zero for all t ≠ s, then Cov[**u**] = E[**uu'**] = $\sigma^2 I_n$.) **y** = **XA**+**u**, E[**u**] = **0**, Cov[**u**] = E[**uu'**] = **w**. Consequently, E[**a**] = E[(**X'X**)$^{-1}$**X'Y**] = E[(**X'X**)$^{-1}$ **X'**(**XA**+**u**)] = E[**A**+(**X'X**)$^{-1}$**X'u**] = E[**A**]+ (**X'X**)$^{-1}$**X'**E[**u**] = **A**+(**X'X**)$^{-1}$**X'0** = **A**. Thus, the least squares estimator remains *unbiased*; however, as can be seen from the below demonstration, it loses *efficiency*. Recall that

a – **A** = (**X'X**)$^{-1}$**X'Y – A** = (**X'X**)$^{-1}$**X' (XA**+**u**) – **A =** (**X'X**)$^{-1}$**X'u.**

Cov[**a**] = E[(**a**-E[**a**])(**a**-E[**a**])'] = E(**a**-**A**)(**a**-**A**)'] = E[(**X'X**)$^{-1}$ **X'u**((**X'X**)$^{-1}$**X'u**)'] = E[(**X'X**)$^{-1}$**X'uu'X**(**X'X**)$^{-1}$] = (**X'X**)$^{-1}$**X'**E[**uu'**]**X**(**X'X**)$^{-1}$ = (**X'X**)$^{-1}$ **X'wX**(**X'X**)$^{-1}$ which is (**X'X**)$^{-1}$ **X'wX**(**X'X**)$^{-1}$ if Cov[**u**] = **w** and which is σ^2(**X'X**)$^{-1}$ if Cov[**u**] = $\sigma^2 I_n$.

Under auto-correlated errors, the covariance matrix of the least squares estimator is (**X'X**)$^{-1}$ **X'wX**(**X'X**)$^{-1}$ but we mistakenly assume that it is σ^2(**X'X**)$^{-1}$ and therefore use the wrong covariance matrix for **a** to make inferences as follows. We compute the least squares estimate of σ^2 as σ^2 **=** (**y**-**Xa**)'(**y**-**Xa**) / (n-k) and mistakenly assume that the standard errors of the estimated coefficients are the square roots of the main diagonal entries of the estimated covariance matrix $\hat{\sigma}^2$ (**X'X**)$^{-1}$ while the true sampling variability of the least squares estimates is given by the true estimated covariance matrix (**X'X**)$^{-1}$ **X'wX**(**X'X**)$^{-1}$. Hence, the standard errors taken from the wrong estimated covariance matrix might overstate or understate the true sampling variability of the estimated coefficients and therefore confidence intervals might be too wide or too narrow. Moreover, hypothesis tests might reject a correct null hypothesis less or more often than suggested by the significance level of the test. Therefore, even though the least squares estimator still produces unbiased estimates, it is no longer efficient. Furthermore, the standard errors are no longer appropriate. They yield too narrow or too wide confidence intervals, and hypothesis tests are invalid.

References

Aaron, H.J. (1990), "Discussion" in A.H. Munnell (ed.), *Is There a Shortfall in Public Capital Investment?* Federal Reserve Bank of Boston, Conference Series No.34, Boston.

Abramovitz, Moses and Paul A. David (2000), "American Macroeconomic Growth in the Era of Knowledge-Based Progress: The Long Run Perspective," in Stanley L. Engerman and Robert E. Gallman (eds.) (2000), The Cambridge Economic History of the United States, Vol. III, pp.1-92, Cambridge University Press, Cambridge.

Akkina, K.R. and M.A. Celebi (2002), "The Determinants of Private Fixed Investment and the Relationship between Public and Private Capital Accumulation in Turkey," *Pakistan Development Review*, Vol. 41, No. 3, pp. 243-54.

Aschauer, D.A. (1989a), "Is Public Expenditure Productive?", *Journal of Monetary Economics*, Vol. 23, pp. 177-200.

Aschauer, D.A. (1989b), "Does Public Capital Crowd Out Private Capital?", *Journal of Monetary Economics*, Vol. 24, pp. 171-88.

Aschauer, D.A. (1989c), "Public Investment and Productivity Growth in the Group of Seven," Federal Reserve Bank of Chicago, *Economic Perspectives*, Vol. 13, pp. 17-25.

Aschauer, D.A. (1990a), "Public Investment and Private Sector Growth," Washington, D.C.: Economic Policy Institute.

Aschauer, D.A. (1990b), "Why Is Infrastructure Important?", in A.H. Munnell (ed.), Is There a Shortfall in Public Capital Investment?, Federal Reserve bank of Boston, Boston.

Aschauer, D.A. (2000a), "Public Capital and Economic Growth: Issues of Quantity, Finance, and Efficiency," *Economic Development and Cultural Change*, Vol. 48, No. 1, pp. 391-406.

Aschauer, D.A. (2000b), "Do States Optimize?" Public Capital and Economic Growth, *The Annals of Regional Science*, Vol. 34, pp. 343-63.

Baltagi, B.H. and N. Pinnoi (1995), "Public Capital Stock and State Productivity Growth: Further Evidence from an Error Components Model," *Empirical Economics*, Vol. 20, pp. 351-59.

Barro, R.J. (1989), "A Cross-Country Study of Growth, Saving, and Government," *NBER Working Paper* No. 2855.

Barro, R.J. (1991), "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics*, Vol.106, pp. 407-43.

Barro, R.J. (2001), "Human Capital and Growth," *American Economic Review*, Vol. 91, No. 2, pp. 12-17.

Berndt, E.R. and R.W. Crandall et al. (1980), "The Decline in Productivity Growth," Federal Reserve Bank of Boston, *Conference Series No. 22*, Boston.

Bernstein, Jeffrey I., T.P. Mamuneas, and P.Pashardes (2004), "Technical Efficiency and U.S. Manufacturing Productivity Growth," *Review of Economics and Statistics*, Vol. 86, No. 1, pp. 402-12.

Bougheas, S., P.O. Demetriades, and T.P. Mamuneas (2000), "Infrastructure, Specialization, and Economic Growth," *Canadian Journal of Economics*, Vol. 33,

No. 2, pp. 506-22.
Brownlee, W. Elliot (2000), "The Public Sector," in Stanley L. Engerman and Robert E. Gallman (eds.) (2000), The Cambridge Economic History of the United States, Vol. III, pp. 969-1013, Cambridge University Press, Cambridge.
Cadot, O., L.H. Roller, and A. Stephan (1999), "A Political Economy Model of Infrastructure Allocation: An Empirical Assessment," *CEPR Discussion Paper* No. 2336.
Cadot, O., L.H. Roller, and A. Stephan (2002), "Contribution to Productivity or Pork Barrel? The Two Faces of Infrastructure Investment," *WZB Discussion Paper* No. 02-09.
Calderon, C. and L. Serven (2002), "The Output Cost of Latin America's Infrastructure Gap," *Central Bank of Chile Working Paper* No.186.
Cain, L.P. (1997), "Historical Perspective on Infrastructure and U.S. Economic Development," *Regional Science and Urban Economics*, Vol. 27, pp. 117-38.
Canning, D. (1998), "A Database of World Infrastructure Stocks, 1950-1995," *World Bank Economic Review*, Vol. 12, pp. 529-47.
Canning, D. and P. Pedroni (1999), "Infrastructure and Long Run Economic Growth," Mimeo.
Caselli, F. and W.J. Coleman II (2002), "The U.S. Technology Frontier," *American Economic Review*, Vol. 92, No. 2, pp. 148-52.
Clarida, R.H. (1993), "International Capital Mobility, Public Investment and Economic Growth," *NBER Working Papers 4506*.
Cohen, Jeffrey P. and Catherine J. Morrison Paul (2004), "Public Infrastructure Investment, Interstate Spatial Spillovers and Manufacturing Costs," *The Review of Economics and Statistics*, Vol. 86, No. 2, pp. 551-60.
Conrad, Klaus and Helmut Seitz (1992), "The Public Capital Hypothesis: The Case of Germany," *Recherches Economiques de Louvain*, Vol.58, pp.309-27.
Conrad, Klaus and Helmut Seitz (1994), "The Economic Benefits of Public Infrastructure," *Applied Economics*, Vol. 26, pp. 303-11.
Costa, J. da Silva, R.W. Ellson, and R.C. Martin (1987), "Public Capital, regional Output, and Developments: Some Empirical Evidence," *Journal of Regional Science*, Vol. 27, pp. 419-37.
Demetriades, P.O. and T.P. Mamuneas (2000), "Intertemporal Output and Employment Effects of Public Infrastructure Capital: Evidence from 12 OECD Economies," *Economic Journal*, Vol. 110, pp. 687-712.
Denison, E. F. (1974), "Accounting for United States Economic Growth, 1929-1969," *Brookings Institution Press*, Washington, DC.
Deno, Kevin T. (1988), "The Effect of Public Capital on U.S. Manufacturing Activity: 1970 to 1978," *Southern Economic Journal*, Vol.55, No. 2, pp.400-11.
Dessus, S. and R. Herrera (2000), "Public Capital and Growth Revisited: A Panel Data Assessment," *Economic Development and Cultural Change*, Vol. 48, No.2, pp. 407-18.
Dickey, David A. and Wayne A. Fuller (1979), "Distribution of the Estimates for Autoregressive Time Series with Unit Root," *Journal of American Statistical Association*, Vol. 74, pp. 427-31.
Dickey, David A. and Wayne A. Fuller (1981), "Likelihood Ratio Statistics for

Autoregressive Time Series with a Unit Root," *Econometrica*, Vol. 49, No. 4, pp. 1057-1072.

Dickey, David A. and S. Pantula (1987), "Determining the Order of Differencing in Autoregressive Processes," *Journal of Business and Economic Statistics*, Vol. 15, pp. 455-61.

Dickey, David A., D. W. Jansen, and D. L. Thornton (1991), "A Primer on Cointegration with an Application to Money and Income," *Federal Reserve Bank of St. Louis Review*, Vol. 73, pp. 58-78.

Easterly, W. and S. Rebelo (1993), "Fiscal Policy and Economic Growth," *Journal of Monetary Economics*, Vol.32, pp. 417-58.

Eisner, R. (1991), "Infrastructure and Regional Economic Performance: Comment," *New England Economic Review*, Sep/Oct: 47-58.

Eisner, R. (1994), "Real Government Saving and the Future," *Journal of Economic Behavior and Organization*, Vol. 23, pp. 111-26.

Engerman, Stanley L. and Robert E. Gallman (eds.) (2000), "The Cambridge Economic History of the United States," Volume III, The Twentieth Century, Cambridge University Press, Cambridge.

Engle, Robert F. and Clive W.J. Granger (1987), "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, Vol. 55, No. 2, pp. 251-76.

Esfahani, H. and M.T. Ramires (2003), "Institutions, Infrastructure, and Economic Growth," *Journal of Development Economics*, Vol. 70, pp. 443-77.

Everaert, G. (2003), "Balanced Growth and Public Capital: An Empirical Analysis with I(2) Trends in Capital Stock data," *Economic Modelling*, Vol. 26, pp. 741-63.

Everaert, G. and F. Heylen (2004), "Public capital and Long-term labor Market Performance in Belgium," *Journal of policy Modelling*, Vol. 26, pp. 95-112.

Fernald, J.G. (1999), "Roads to Prosperity? Assessing the Link between Public Capital and Productivity," *American Economic Review*, Vol. 89, No. 3, pp. 619-38.

Field, A.J. (2003), "The Most Technologically Progressive Decade of the Century," *American Economic Review*, Vol. 93, No.4, pp. 1399-413.

Finn, M. (1993), "Is All Government Capital Productive?," Federal Reserve Bank of Richmond, *Economic Quarterly*, Vol. 79, pp. 53-80.

Ford, R. and P. Poret (1991), "Infrastructure and Private-Sector Productivity," *OECD Economic Studies*, No. 17, pp. 63-89.

Fuhrer, J.C. and J.S. Little (eds.) (1996), "Technology and Growth," *Federal Reserve Bank of Boston, Conference Series No. 40*, Boston.

Garcia-Mila, T. and T.J. McGuire (1992), "The Contribution of Publicly Provided Inputs to States' Economies," *Regional Science and Urban Economics*, No. 22, pp. 229-41.

Gonzalo, Jesus and C.W.J. Granger (1995), "Estimation of Common Long-Memory Components in Cointegrated Systems," *Journal of Business and Economic Statistics*, Vol. 13, pp.27-35.

Gordon, Robert J. (May 1999), "U.S. Economic Growth since 1870: One Big Wave?" *American Economic Review*, Vol. 89, No. 2, pp. 123-28.

Gramlich, E.M. (1994), "Infrastructure Investment: A review Essay," *Journal of Economic Literature*, Vol.32, pp. 1176-96.

Granger, Clive William John (1969), "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," *Econometrica*, Vol.37, pp.424-38.
Granger, Clive William John (1980), "Testing for Causality: A Personal Viewpoint," *Journal of Economic Dynamics and Control*, Vol.2, pp. 329-52.
Granger, Clive William John (1981), "Some Properties of Time Series Data and Their Use in Econometric Model Specification," *Journal of Econometrics*, Vol. 16, pp. 121-30.
Granger, Clive William John (1986), "Developments in the Study of Cointegrated Economic Variables," *Oxford Bulletin of Economics and Statistics*, Vol.48, pp. 213-28.
Granger, Clive William John (1988), "Some Recent Development in a Concept of Causality," *Journal of Econometrics*, Vol.39, pp.199-211.
Granger, Clive William John (2004), "Time Series Analysis, Cointegration, and Applications," *American Economic Review*, Vol. 94, No. 3, pp. 421-25.
Granger, Clive William John and A. A. Weiss (1983), "Time Series Analysis of Error-Correction Models," in S. Karlin, T. Amemiya, and L.A. Goodman (eds.) (1983), Studies in Econometrics: Time Series and Multivariate Statistics, Academic Press, New York, pp. 255-78.
Granger, Clive William John and Niels Haldrup (1997), "Separation in Cointegrated Systems and Persistent-Transitory Decompositions," *Oxford Bulletin of Economics and Stattistics*, Vol.59, pp. 449-64.
Granger, Clive William John and Norman Swanson(1996), "Further Developments in the Study of Cointegrated Variables," *Oxford Bulletin of Economics and Statistics*, Vol. 58, pp. 374-86.
Granger, Clive William John and P. Newbold (1974), "Spurious Regressions in Econometrics." *Journal of Econometrics*, Vol.2, pp. 111-20.
Hall, R.E. and C.J. Jones (Feb. 1999), "Why Do Some Countries Produce so much more Output per Worker than Others?," *Quarterly Journal of Economics*, pp. 83-116.
Holtz-Eakin, Douglas (1988), "Private Output, Government Capital, and the Infrastructure 'Crisis'," Discussion Paper Series No.394, Columbia University, New York.
Holtz-Eakin, Douglas (1992), "Public Sector Capital and the Productivity Puzzle," *NBER Working Paper 4122*, published in Review of Economics and Statistics, (1994) Vol. 76, pp. 12-21.
Holtz-Eakin, Douglas and A.E. Schwartz (1995), "Infrastructure in a Structural Model of Economic Growth," *Regional Science and Urban Economics*, Vol. 25, pp.131-51.
Hulten, C.R. and R.M. Schwab (1991), "Is There too Little Public Capital?," Conference Paper, American Enterprise Institute.
Jones, C.I. (2002), "Sources of U.S. Economic Growth in a World of Ideas," *American Economic Review*, Vol. 92, No. 1, pp. 220-39.
Jorgenson, Dale W. (2001), "Information Technology and the U.S. Economy," *American Economic Review*, Vol. 91, No. 1, pp. 1-32.
Kamps, Christophe (2004a), "New Estimates of Government Net Capital Stocks for 22 OECD Countries 1960-2001," *IMF Working Paper No. 04/67*.
Kamps, Christophe (2004b), "The Dynamic Effects of Public Capital: VAR Evidence for 22 OECD Countries," *Kiel Institute of World Economics Working Paper No.*

1224.
Kendrick, John (1961), *Productivity Trends in the United States*, Princeton University Press, Princeton.
Kim, E. (1998), "Economic Gain and Loss from Public Infrastructure Investment," *Growth and Change*, Vol. 29, pp. 445-69.
Levine, R. (1997), "Financial Development and Economic Growth: Views and Agenda," *Journal of Economic Literature*, Vol. 35, pp. 688-726.
Ligthart, J.E. (2002), "Public Capital and Output Growth in Portugal: An Empirical Analysis," *European Review of Economics and Finance*, Vol.1, No. 2, pp.3-30.
Litan, Robert E. and Alice M. Rivlin (2001), "Projecting the Economic Impact of the Internet," *American Economic Review*, Vol. 91, No. 2, pp.313-17.
Lucas, R.E. (1988), "On the Mechanics of Economic Development," *Journal of Monetary Economics*, Vol. 22, pp. 3-42.
Lucas, R.E. (2003), "Macroeconomic Priorities," *American Economic review*, Vol. 93, No. 1, pp. 1-14.
Lynde, Catherine and James Richmond (1992), "The Role of Public Capital in Production," *The Review of Economics and Statistics*, Vol. 74, No. 1, pp.37-44.
Lynde, Catherine and James Richmond (1993a), "Public Capital and Total Factor Productivity," *International Economic Review*, Vol.34, pp.401-14.
Lynde, Catherine and James Richmond (1993b), "Public Capital and Long Run Costs In U.K. Manufacturing," *Economic Journal*, Vol.103, pp.880-93.
Mankiw, N. G., D. Romer, and D.N. Weil (May 1992), "A Contribution to the Empirics Of Economic Growth," *Quarterly Journal of Economics*, pp. 407-37.
McMillin, W.D. and D.J. Smyth (1994), "A Multivariate Time Series Analysis of the United States Aggregate Production Function," *Empirical Economics*, Vol. 19, pp. 659-73.
Merriman, D. (1990), "Public Capital and Regional Output. Another Look at Some Japanese and American Data," *Regional Science and Urban Economics*, Vol. 20, pp. 437-58.
Moreno, Rosina, Enrique Lopez-Bazo, and Manuel Arts (2003), "On the Effectiveness of Private and Public Capital," *Applied Economics*, Vol. 35, pp.727-40.
Morrison, Catherine J. and Amy Ellen Schwartz (1996a), "Public Infrastructure, Private Input Demand and Economic Performance in New England Manufacturing," *Journal of Business & Statistics*, Vol. 14, No. 1, pp.91-101.
Morrison, Catherine J. and Amy Ellen Schwartz (1996b), "State Infrastructure and Productive Performance," *American Economic Review*, Vol. 86, No. 5, pp. 1095-1111.
Munnell, A.H. (1990a), "Why Has Productivity Growth Declined? Productivity and Public Investment," *New England Economic Review*, Jan/Feb, pp. 2-22.
Munnell, A.H.(ed.) (1990b), "Is There a Shortfall in Public Capital Investment?," Federal Reserve Bank of Boston, Conference Series No.34, Boston.
Munnell, A.H. (1992), "Policy Watch. Infrastructure Investment and Economic Growth," *Journal of Economic Perspectives*, Vol. 6, pp. 189-98.
Munnell, A.H. (1993), "An Assessment of Trends in and Economic Impacts of Infrastructure Investment," in Infrastructure Policies for the 1990s, OECD, Paris.
Munnell, A.H. and L.M. Cook (1990), "How Does Public Infrastructure Affect Regional

Economic Performance?," in A.H. Munnell (ed.), Is There A Shortfall in Public Capital Investment?, Federal Reserve Bank of Boston, Conference Series No. 34, Boston.

Nadiri, M. Ishaq and Theofanis P. Mamuneas (1994a), "The Effects of Public Infrastructure and R&D Capital on the Cost Structure and Performance of U.S. Manufacturing Industries," *The Review of Economics and Statistics*, Vol. 76, No. 1, pp. 22-37.

Nadiri, M. Ishaq and Theofanis P. Mamuneas (1994b), "Infrastructure and Public R&D Investments, and the Growth of Factor Productivity in U.S. Manufacturing Industries," *NBER Working Paper No. 4845*.

Nelson, Charles R. and Charles I. Plosser (1982), "Trends and Random Walks in Macroeconomic Time Series," *Journal of Monetary Economics*, Vol. 10, pp. 139-62.

Otto, G. and G.M. Voss (1996), "Public Capital and Private Production in Australia," *Southern Economic Journal*, Vol. 62, pp. 723-38.

Pereira, A.M. and R. Flores de Frutos (1999), "Public Capital Accumulation and Private Sector Performance," *Journal of Urban Economics*, Vol. 46, No. 2, pp. 300-22.

Pereira, A.M. and O. Roca-Sagales (2003), "Spillover Effects of Public Capital Formation: Evidence from the Spanish Regions," *Journal of Urban Economics*, Vol. 53, No. 2, pp. 238-256.

Perotti, R. (1996), "Growth, Income Distribution, and Democracy: What the Data say," *Journal of Economic Growth*, No. 1, pp. 149-87.

Peterson, G.E. (1990), "Is Public Infrastructure Undersupplied?," in A.H. Munnell (ed.), Is There a Shortfall in Public Capital Investment?, Federal Reserve bank of Boston, Conference Series No.34, Boston.

Peterson, G.E. (1991), "Historical Perspective on Infrastructure Investment: How Did We Get Where We Are?," American Enterprise Institute Discussion Paper.

Phillips, P.C.B. (1986), "Understanding Spurious Regressions in Econometrics," *Journal Of Econometrics*, Vol. 33, pp. 311-40.

Pinnoi, N. (1994), "Public Infrastructure and Private Production. Measuring Relative Contributions," *Journal of Economic Behavior and Organization*, Vol. 23, pp. 127-48.

Prescott, E.C. (1998), "Lawrence R. Klein Lecture 1997 Needed: A Theory of Total Factor Productivity," *International Economic Review*, Vol. 39, No. 3, pp. 525-51.

Ram, R. and D.D. Ramsey (1989), "Government Capital and Private Output in the United States. Additional Evidence," *Economics Letters*, Vol. 30, pp. 223-26.

Ratner, J.B. (1983), "Government Capital and the Production Function for U.S. Private Output," *Economics Letters*, Vol. 13, pp. 213-17.

Rebelo, S. (1991), "Long Run Policy Analysis and Long Run Growth," *Journal of Political Economy*, Vol. 99, pp. 500-21.

Romer, P.M. (1989), "Human Capital and Growth: Theory and Evidence," *NBER Working Paper No. 3173*.

Romer, P.M. (1990), "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, No. 5, pp.71-102.

Romer, P.M. (1994), "The Origins of Endogenous Growth," *Journal of Economic Perspectives*, Vol. 8, No. 1, pp. 3-22.

Romp, Ward and Jakob de Haan (2005), "Public Capital and Economic Growth: A Critical Survey," *EIB Papers*, Vol. 10, No. 1, pp. 40-70.

Seely, Bruce (1993), "A Republic Bound Together," *The Wilson Quarterly*, Winter.

Sims, Christopher (1980), "Macroeconomics and Reality," *Econometrica*, Vol. 48, No. 1, pp.1-49.

Sims, C., J. Stock, and M. Watson (1990), "Inference in Linear Time Series Models with Some Unit Roots," *Econometrica*, Vol. 58, pp. 113-44.

Solow, R.M. (1956), "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, Vol. 70, No. 1, pp. 65-94.

Solow, R. M. (1957), "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, Vol. 39, No. 3, pp. 312-20.

Stafford, Frank P. (May 1999), "Economic Growth: How Good Can It Get?" *American Economic Review*, Vol. 89, No. 2, pp. 40-44.

Stock, James H. (1987), "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors," *Econometrica*, Vol. 55, pp. 1035-1056.

Sturm, J.E. and Jakob de Haan (1995), "Is Public Expenditure really Productive? New Evidence for the U.S. and the Netherlands," *Economic Modelling*, 12: pp. 60-72.

Sturm, J.E., G.H. Kuper, and J. de Haan (1998), "Modelling Government Investment and Economic Growth on a Macro Level: A Review" in S. Brakman, H. van Ees, and S.K. Kuipers (eds.), *Market Behavior and Macroeconomic Modelling*, MacMillan Press Ltd., London, UK.

Sturm, J.E., J. Jacobs, and P. Groote (1999), "Output Effects of Infrastructure Investment in the Netherlands, 1853-1913," *Journal of Macroeconomics*, Vol. 21, No. 2, pp. 355-80.

Summers, R. and A. Heston (1991), "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988," *Quarterly Journal of Economics*, Vol. 106, No. 2, pp. 327-68.

Swan, T. (1956), "Economic Growth and Capital Accumulation," *Economic Record*, Vol. 32, pp. 334-61.

Tatom, J.A. (1991), "Public Capital and Private Sector Performance," *Federal Reserve Bank of St. Louis Review*, Vol. 73, pp. 3-15.

Temin, Peter (2000), "The Great Depression," in Stanley L. Engerman and Robert E. Gallman (eds.) (2000), The Cambridge Economic History of the United States, Vol. III, pp. 301-29, Cambridge University Press, Cambridge.

Temple, J (1999), "The New Growth Evidence," *Journal of Economic Literature*, Vol. 37, pp. 112-56.

World Bank (1994), Annual Report, World Bank.