

Tests of redshift-space distortions models in configuration space for the analysis of the BOSS final data release

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ABSTRACT

Observations of redshift-space distortions in spectroscopic galaxy surveys offer an attractive method for observing the build-up of cosmological structure, which depends both on the expansion rate of the Universe and our theory of gravity. In preparation for analysis of redshift-space distortions from the Baryon Oscillation Spectroscopic Survey (BOSS) final data release, we compare a number of analytic and phenomenological models, specified in configuration space, to mock catalogues derived in different ways from several N -body simulations. The galaxies in each mock catalogue have properties similar to those of the higher redshift galaxies measured by BOSS but differ in the details of how small-scale velocities and halo occupancy are determined. We find that all of the analytic models fit the simulations over a limited range of scales while failing at small scales. We discuss which models are most robust and on which scales they return reliable estimates of the rate of growth of structure: we find that models based on some form of resummation can fit our N -body data for BOSS-like galaxies above $30 h^{-1}$ Mpc well enough to return unbiased parameter estimates.

Key words: gravitation – galaxies: haloes – galaxies: statistics – cosmological parameters – large-scale structure of Universe.

1 INTRODUCTION

The evolution of our Universe appears to be well described by Einstein’s theory of General Relativity. The large-scale structure within it is understood as the consequence of gravitational instability, which amplifies primordial fluctuations laid down at early times. Within this paradigm, the growth of large-scale structure is driven by the motion of matter and inhibited by the cosmological expansion. Once the content, spatial geometry and initial perturbations in the Universe are specified, Einstein’s theory makes precise predictions for the expansion rate and, simultaneously, the rate of growth of large-scale structure. These predictions can be compared with observations made in galaxy redshift surveys to test the theory and to provide constraints on the parameters of our cosmological

model. We can measure the velocity field from maps of galaxies in such surveys because the galaxy redshifts, from which distances are inferred, include components from both the Hubble flow and peculiar velocities from the comoving motions of galaxies (see Hamilton 1998 for a review). Thus, even though the statistics of the galaxy distribution are isotropic, redshift surveys exhibit an anisotropic distribution. The anisotropy encodes information about the build-up of structure and provides a sharp test of the theory (see e.g. Berlind, Narayanan & Weinberg 2001; Zhang et al. 2007; Guzzo et al. 2008; Jain & Zhang 2008; Nesperis & Perivolaropoulos 2008; Percival & White 2008; McDonald & Seljak 2009; Song & Koyama 2009; Song & Percival 2009; White, Song & Percival 2009; Song et al. 2010, 2011; Zhao et al. 2010, for recent studies).

The measured anisotropy of the clustering of galaxies seen in redshift surveys combines virial motions within haloes on small scales (Jackson 1972) and supercluster infall on large scales (Kaiser 1987). Over the past two decades, the measurement of these effects

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has become increasingly precise (Cole, Fisher & Weinberg 1995; Peacock et al. 2001; Percival et al. 2004; da Angela et al. 2008; Guzzo et al. 2008; Okumura et al. 2008; Blake et al. 2011; Samushia, Percival & Raccanelli 2012; Reid et al. 2012; Blake et al. 2013; de la Torre et al. 2013; Samushia et al. 2013; Bel et al. 2014; Samushia et al. 2014; Sanchez et al. 2014; Tojeiro et al. 2014). Reliably extracting cosmological information from such precise measurements requires models which are accurate, to the level of the data uncertainties, on the scales over which they are fitted. In this paper, we compare how well a number of different models recover the growth rate in mock catalogues made from N -body simulations, in order to delineate the range of validity of the models in preparation for using them to analyse data from the final release of data from the Baryon Oscillation Spectroscopic Survey (BOSS; Dawson et al. 2013), which is part of Sloan Digital Sky Survey III (SDSS-III; Eisenstein et al. 2011). To this end, we have developed a number of mock galaxy catalogues with clustering properties similar to those of the higher redshift BOSS galaxies. While having similar clustering and number density, the mock catalogues differ in detail and have been generated to test various aspects of the redshift-space distortion (RSD) models.

Our focus in this paper is on a set of analytic and phenomenological models described in configuration space (and described further in Section 2, see also Okumura & Jing 2011; Bianchi et al. 2012; de la Torre & Guzzo 2012; Gil-Marín et al. 2012 and references below). Recent, complementary, tests of models in Fourier space have been presented in Jennings, Baugh & Pascoli (2011), Blake et al. (2011), Kwan, Lewis & Linder (2012), Okumura, Seljak & Descjacques (2012), Zheng et al. (2013), Okumura et al. (2014), Beutler et al. (2014) and N -body-based models for fitting the data to smaller scales have been presented in Reid et al. (2014). The outline of the paper is as follows. In Section 2, we introduce the models we will be considering and the parameters upon which they depend. In Section 3, we describe the N -body simulations and mock catalogues which we shall use as a reference for the models to fit. Section 4 describes our methodology and we discuss the implications in Section 5.

2 MODELS

In this section, we describe the models which we shall fit to the N -body data, with a focus on models which make predictions in configuration space (recent, complementary tests of models in Fourier space have been presented in the papers cited above). The first two models are ‘dispersion models’, which Scoccimarro (2004) showed inherently assume an unphysical pairwise velocity distribution. The second two models express the redshift-space correlation function as an integral of the real-space correlation function times a pairwise velocity distribution function.

Throughout, we adopt the standard ‘plane-parallel’ or ‘distant-observer’ approximation, in which the line-of-sight (LOS) direction to each object is taken to be the fixed direction \hat{z} . This has been shown to be a good approximation at the level of current observational error bars (e.g. fig. 10 of Samushia et al. 2012 or fig. 8 of Yoo & Seljak 2014).

2.1 The Eulerian dispersion model

The simplest model for RSD combines the supercluster infall enhancement of Kaiser (1987) with an independent small-scale suppression to account for the virial motions of satellites in haloes (the finger-of-god effect). In Fourier space, the assumption of an

exponential pairwise velocity distribution with a scale-independent width, along with the linear theory relation for supercluster infall, leads to (Peacock 1992; Park et al. 1994; Peacock & Dodds 1994; Ballinger, Peacock & Heavens 1996; Hamilton 1998; Hatton & Cole 1999)

$$P(k, \mu) = P_r(k) \frac{(b + f\mu^2)^2}{1 + k^2 \mu^2 \sigma_E^2} \quad (1)$$

where σ_E is a free parameter to be fit to the data, b is the large-scale (assumed scale independent) bias and $f = d \ln \delta / d \ln a \approx \Omega_m^{0.55}(z)$ is the growth rate of perturbations. A motivation for the exponential form of the pairwise velocity distribution can be found in White (2001) and Seljak (2001), and expressions for the Legendre moments of equation (1) can be found in Cole et al. (1995). The exact form of the small-scale suppression and the value of σ_E are strongly dependent on the galaxy population being modelled (Jing & Börner 2004; Li et al. 2007). Sometimes a model with $(1 + k^2 \mu^2 \sigma_E^2 / 2)^2$ in the denominator is used; this agrees with equation (1) for low k . An alternative formulation is to take the high- k damping term as a Gaussian, $\exp[-k^2 \mu^2 \sigma_E^2]$. Again the expressions match for low k .

In equation (1), we can use the linear theory power spectrum, or we can use a model for the non-linear power spectrum (but ignoring non-linear bias and the generic perturbation theory differences in the non-linear evolution of velocity and density). In the study of Blake et al. (2011), equation (1) with a non-linear power spectrum calculated as in Smith et al. (2003) performed very well against data when fitted in Fourier space. We shall consider both the linear and non-linear forms, referring to the former as the Eulerian dispersion model (EDM) and the latter as the non-linear dispersion model (NDM) (following Peebles 1980; Fisher 1995). Tinker, Weinberg & Zheng (2006) and Tinker (2007), which are developments of the work in Hatton & Cole (1999), discuss a number of improvements to models like this based on fits to numerical simulation results. We shall not consider these improvements, since the original form is the most widely used.

The correlation function, which shall be the primary focus of this paper, is the Fourier transform of this power spectrum. If we expand the correlation function in Legendre polynomials, L_ℓ ,

$$\xi(s, \hat{s} \cdot \hat{z}) = \sum_\ell \xi_\ell(s) L_\ell(\hat{s} \cdot \hat{z}), \quad (2)$$

where s is the redshift-space coordinate, then

$$\xi_\ell(s) = (2\ell + 1) i^\ell \int \frac{d^3k}{(2\pi)^3} P(k, \mu) L_\ell(\mu) j_\ell(ks), \quad (3)$$

where j_ℓ is a spherical Bessel function of order ℓ . We shall restrict our attention to the lowest non-vanishing moments, $\ell = 0$ and 2.

This model has three free parameters, which we can take to be either $b\sigma_8, f\sigma_8$ and σ_E or b, f and σ_E depending on whether we wish to treat the linear theory power spectrum amplitude as known.

We note that the model of Chuang et al. (2013) is almost the same as the EDM, differing only in the modelling of the acoustic peak region. Since the acoustic feature has very little weight in our fits, we expect that our discussion of the EDM will apply approximately to this model as well.

2.2 Perturbation theory inspired model

We also consider a model inspired by, but not directly derived from, perturbation theory in combination with the reasoning described above. We call this model the perturbation-inspired model (PIM). It

was used to interpret the BOSS data in Sanchez et al. (2013, 2014). This model assumes

$$P(k, \mu) = \left(\frac{b + f\mu^2}{1 + k^2 f^2 \sigma_p^2 \mu^2} \right)^2 \left[e^{-k^2 \sigma_p^2} P_{\text{lin}}(k) + A P_{22}(k) \right], \quad (4)$$

where b, f, σ_p and A are free parameters and P_{22} is the standard second-order, mode-coupling term in Eulerian perturbation theory (Peebles 1980; Juszkiewicz 1981; Vishniac 1983; Goroff et al. 1986; Makino, Sasaki & Suto 1992; Jain & Bertschinger 1994). For further details on, and tests of, the model see Sanchez et al. (2013, 2014). In comparison with the other models, note the factors of f in the prefactor, the squaring of the finger-of-god term, the double duty played by the parameter σ_p and the isotropic nature of the broadening of the acoustic peak (i.e. the exponential multiplying P_{lin}). As it stands, this model has one additional parameter (A) that can be varied in the fit beyond the large-scale bias, b and σ_p .

2.3 The Gaussian streaming model

The ‘Gaussian streaming model’ (GSM) was developed in Reid & White (2011) and Reid et al. (2012), where it was shown to fit the monopole and quadrupole of the correlation functions of mock galaxies with a large-scale bias $b \simeq 2$ to the per cent level on scales above $25 h^{-1}$ Mpc. This model has been used to interpret the clustering of galaxies measured in BOSS by Reid et al. (2012) and Samushia et al. (2013, 2014). The GSM is inspired by the work of Peebles (1980) and Fisher (1995). The pairwise velocity distribution is different for components along and perpendicular to the (real-space) pair separation vector $r\hat{r}$, with the mean pairwise velocity $v_{12}(r)\hat{r}$ oriented along \hat{r} . The mean and variance of the pairwise velocity distribution for the LOS velocity component therefore depend on $\mu_r = \hat{r} \cdot \hat{z}$. The GSM approximates the pairwise velocity distribution as Gaussian and enforces pair conservation by integrating over all possible real-space LOS pair separations y that appear at redshift-space separation s_{\parallel} . Specifically, we assume that the redshift-space halo correlation function is

$$1 + \xi^s(s_{\perp}, s_{\parallel}) = \int \frac{dy}{\sqrt{2\pi}\sigma_{12}} [1 + \xi(r)] \exp \left\{ -\frac{[s_{\parallel} - y - \mu_r v_{12}]^2}{2\sigma_{12}^2} \right\}, \quad (5)$$

where $\xi(r)$, v_{12} and σ_{12} are to be provided from an analytic theory. In its basic form (Reid & White 2011; Reid et al. 2012), integrated Lagrangian perturbation theory with scale dependent but local Lagrangian bias (Matsubara 2008b) is used for the real-space correlation function of haloes, while the halo infall velocity and dispersion computed in standard perturbation theory with scale-independent bias. In order to go from haloes to galaxies, Reid et al. (2012) showed that it suffices to introduce a single additional parameter, σ_{FOG} , akin to the σ_E in equation (1). This is taken to be an isotropic, scale-independent dispersion which is added in quadrature to σ_{12} and modifies the scale dependence of the quadrupole moment on small scales.

2.4 The Lagrangian streaming model

The Lagrangian streaming model (LSM) is a hybrid model based upon the work in Reid & White (2011), Reid et al. (2012), Carlson, Reid & White (2013), Wang, Reid & White (2013) and White (2014). It uses the Zel’dovich approximation (Zel’dovich 1970) with a model for local Lagrangian bias introduced by Matsubara (2008a,b, 2011) to predict the real-space correlation function of biased tracers. This model predicts the real-space statistics

very well, compared to N -body simulations, but fares less well for the redshift-space statistics of biased tracers, particularly the quadrupoles. For this reason, the real-space correlation function is ‘convolved’ with a Gaussian as in equation (5), except that the dispersion is computed from Lagrangian rather than Eulerian perturbation theory and we shall denote it as σ_L . As discussed extensively in Reid & White (2011), the measured quadrupole is very sensitive to the pairwise velocity, v_{12} . For this reason, we use the one-loop expansion for v_{12} rather than just the lowest order (i.e. Zeldovich) term. [Comparison of this model with the full one-loop calculation in Wang et al. (2013) indicates that the differences are at the per cent level on scales larger than $20 h^{-1}$ Mpc, except around the acoustic peak. Since the acoustic peak carries little weight in our fits, we can treat the LSM and the model described in Wang et al. (2013) as essentially identical, though the LSM is computationally easier.] As in the GSM, an additional parameter needs to be included to model fingers of god. We considered two approaches: an additional, isotropic, component to the Gaussian as in the GSM and an exponential form for the extra dispersion. In general, we find that the exponential form produces almost identical results to the Gaussian form. Since the latter is easier to implement, we shall use it throughout.

In principle, the LSM depends on several parameters, but we choose to fix the bias parameters using the peak-background split expressions as described in White (2014). We have found that allowing the second bias parameter to float in the fits produces almost the same correlation functions as fixing it to the peak-background expression, so we work with the reduced parameter space. For a fixed linear power spectrum then, the final model depends on a single bias parameter, f and σ_L ; i.e. the same number of parameters as the Eulerian model above.

2.5 Comparison

Fig. 1 compares the monopole and quadrupole moments of the redshift-space correlation function of haloes for the different theoretical models described above. We used the linear theory power spectrum appropriate to the T1 simulations (see Section 3) and a large-scale bias of $b = 2$ for all models, and have varied the finger-of-god parameters, σ_i , from $0.5 h^{-1}$ Mpc to $5 h^{-1}$ Mpc to illustrate their effects. Note that the σ_i have a much larger effect on the quadrupole than the monopole, altering small scales more than large. For such highly biased galaxies, the effect of the finger-of-god modelling becomes comparable to the observational uncertainty for BOSS-like surveys below about $30 h^{-1}$ Mpc. The broadening of the acoustic peak in the Lagrangian models compared to the Eulerian models is clearly evident.

3 SIMULATIONS

In order to validate these different models, we use mock catalogues derived from N -body simulations. Such validations have been performed for many of the models, often in the paper in which they were introduced; however, our goal is to enable a side-by-side comparison using a common set of simulations, and to test the models for mock galaxies with properties similar to those of BOSS galaxies at $z \simeq 0.5$. We have created these mock catalogues from a number of different simulations, with differing cosmologies, codes and mock catalogue generation methods (see Table 1 for a summary). While each is comparable to the BOSS data, the differing assumptions provide a good check of the models’ ability to handle the complexities of how real galaxies trace the cosmic web.

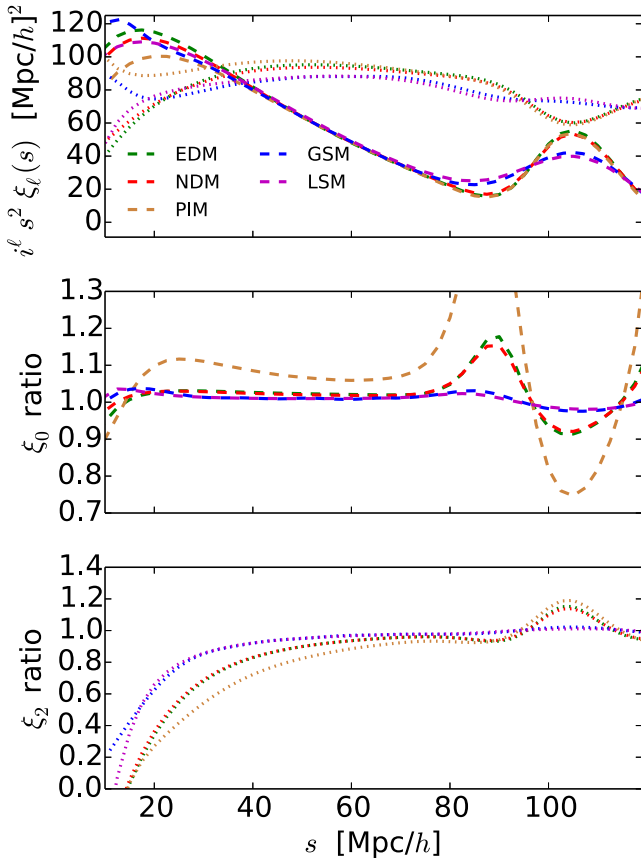


Figure 1. A comparison of the theoretical models described in Section 2. In the top panel, the predictions for the monopole moment of the correlation function (multiplied by s^2 to reduce the dynamic range) are shown as dashed lines while those for the quadrupole moment are shown as dotted lines. We have taken the finger-of-god parameter, σ , to be small ($0.5 h^{-1}$ Mpc) to highlight the differences in the perturbative part of the models. In the lower panels, we plot the ratio of the monopole and quadrupole moments with $\sigma = 5 h^{-1}$ Mpc to those with $\sigma = 0.5 h^{-1}$ Mpc to show the effects of varying σ . Generally, higher σ goes with a sharper ‘break’ at small s in the quadrupole and lower clustering at small s in the monopole but σ has a much larger effect on the quadrupole than the monopole, and alters small scales more than large. For such highly biased galaxies, the effect of the finger-of-god modelling becomes comparable to the observational uncertainty for BOSS-like surveys below about $30 h^{-1}$ Mpc. The broadening of the acoustic peak in the Lagrangian models compared to the Eulerian models is clearly evident. For the PIM, we have set $A = 1$.

3.1 TREEPM

The first set of simulations which we use (labelled T0 below) are those used in White et al. (2011), Reid & White (2011), Wang et al. (2013) and White (2014). The catalogues are based on 20 realizations of the Λ cold dark matter (Λ CDM) model with $\Omega_m = 0.274$ and $h = 0.7$, each employing 1500^3 particles in a periodic box of side length $1500 h^{-1}$ Mpc for a total volume of $68 h^{-1}$ Gpc³. The simulations were run with the TREEPM code of White (2002), and the mock catalogues are described further in White et al. (2011). Briefly, haloes were found using the friends-of-friends algorithm and populated with galaxies resembling those of BOSS using a halo occupation distribution (HOD). Each central galaxy was placed at the minimum of the halo potential and given a velocity equal to that of the halo centre-of-mass, while halo particles were picked at random to model satellite galaxies.

Table 1. A summary of information about the simulations used in this paper (see the text for further discussion). Each is of the Λ CDM family with the indicated values of the matter density (Ω_m) and Hubble parameter (h). We use a single redshift output to build the mock catalogues, which is near $z = 0.55$. The growth rate at this redshift, $f = d \ln D / d \ln a \approx \Omega_m^{0.55}$ where $D(a)$ is the linear theory growth factor. The normalization of the linear theory power spectrum at $z = 0.55$ is given in terms of $\sigma_8(z = 0.55)$, denoted $\sigma_8^{(0.55)}$. Finally, we give the average number density, in $10^{-4} h^{-3} \text{Mpc}^3$, and the total volume, in $h^{-3} \text{Gpc}^3$, used in the computation of ξ_ℓ . For model T1, we have three variants (00, 01 and 10) which are described further in the text.

Name	Ω_m	h	$f^{(0.55)}$	$\sigma_8^{(0.55)}$	\bar{n}	Vol
T0	0.274	0.70	0.74	0.61	3.6	68
T1	0.292	0.69	0.76	0.62	4	26
MD	0.307	0.68	0.77	0.61	4	16
QPM	0.290	0.70	0.76	0.61	4.1	268

A second set of simulations (labelled T1), run with the same code, is also used (these simulations have also been used in Reid et al. 2014). This set has less volume but higher mass and force resolution and a different cosmology. This set consists of 10 realizations of a Λ CDM model with $\Omega_m = 0.292$ and $h = 0.69$, each employing 2048^3 particles in a periodic box of side length $1380 h^{-1}$ Mpc. At each output of the simulation, two sets of halo catalogues were constructed. The first is based on the friends-of-friends algorithm as above. The second uses a spherical overdensity criterion. As before, an HOD was used to generate mock galaxies. For the friends-of-friends haloes, central and satellite galaxies were placed in the halo as above, except for one of the simulations (labelled 01) the satellite velocities were boosted by 25 per cent while in the base simulation (labelled 00) they were not. For the spherical overdensity catalogue (labelled 10), the central galaxies were placed at rest compared to the inner region of the halo, rather than the halo centre-of-mass. Since massive haloes contain significant kinematic substructure, these prescriptions can differ by a fair fraction of the halo velocity dispersion, and generate differing amount of small-scale RSD. Taken together, these simulations explore a range of cosmologies and galaxy velocity prescriptions, as will be seen in the clustering statistics. We generated some other variants of these models, but in all cases the results were consistent with those we explore in more detail below so we focus on the models listed from now on.

Especially for the higher resolution simulations, the box size is not optimal for resolving features (such as the baryon acoustic peak) at $> 100 h^{-1}$ Mpc. Since $100 h^{-1}$ Mpc is about 10 per cent of the side length of the periodic box, the missing long-wavelength modes have an impact on the correlation function and the large-scale velocities which act to broaden the acoustic peak. Fortunately, most of the RSD signal comes from smaller scales so this is not a major concern for the purposes of this work.

Due to the combination of large volume and high force and mass resolution, we shall take the T1 simulations as our fiducial choice in the figures below, and we will comment specifically when the results from the other simulations show a qualitatively different behaviour. We shall also use the SO catalogue (denoted 10 above) as our fiducial model, as this catalogue provides a very good match to the projected correlation function and monopole and quadrupole moments of the redshift-space correlation function of CMASS galaxies in BOSS (Reid et al. 2014).

3.2 Big MultiDark

A further set of mock catalogues were generated from the ‘Big MultiDark’ (hereafter BigMD or MD) simulation. This simulation employed 3840^3 particles in a periodic box of side length $2500 h^{-1}$ Mpc. The cosmology was close to the best-fitting Planck cosmology (Planck Collaboration 2014), with $\Omega_m = 0.307$. Unlike the halo occupation-based methods described above, these catalogues were produced by halo abundance matching as described in Nuzza et al. (2013), with galaxies associated with substructures. This has a more complex, and possibly realistic, velocity distribution than the HOD models above. Unfortunately, the total volume of this simulation is only slightly larger than our fiducial mock survey (see below) so significant finite-volume ‘noise’ remains.

3.3 Quick particle mesh

Later, we will use a set of *approximate* mock catalogues, which were run in order to generate covariance matrices for the BOSS measurements via Monte Carlo techniques (see White, Tinker & McBride 2014, for further discussion). These mock catalogues use low-mass and force resolution particle-mesh simulations employing 1280^3 particles in a $2560 h^{-1}$ Mpc box run with large time steps. At select times the particles, and their local density smoothed on $2 h^{-1}$ Mpc scales, were dumped and these particles were then sampled (with a density-dependent probability) to form a set of mock haloes which are then populated using an HOD (White et al. 2014). These approximate mock catalogues contain roughly the right amount of monopole and quadrupole power, but deviate from the high-resolution simulations on scales smaller than $30\text{--}50 h^{-1}$ Mpc. We shall use these very large volume simulations, in a differential manner, to test the impact of observing strategy on the recovered cosmological parameters.

4 COMPARISON

4.1 Ideal data

We begin by asking to what extent each of our models can recover the simulation parameters when applied to ‘ideal’ data. To this end, we generate correlation functions from fixed-time outputs of the simulations assuming no observational non-idealities. For the TREEPM and QPM runs, we use the periodic outputs and generate a mean and variance for each point from the independent runs. For the BigMD run, we divide the $2500 h^{-1}$ Mpc box into its eight octants and compute the correlation function in each octant, using the octants to determine the mean and variation on the parameters.

We show the monopole and quadrupole moments of the mock galaxies for a selection of the catalogues in Fig. 2. Note that the small-scale behaviour of the quadrupole moment depends upon the model chosen for the satellite and central velocities, while the amplitude of the monopole moment is largely set by mimicking the BOSS data. The range of slopes and behaviours reflects the differences in the underlying cosmology, linear power spectrum and bias prescription and makes for a good test of the theoretical models.

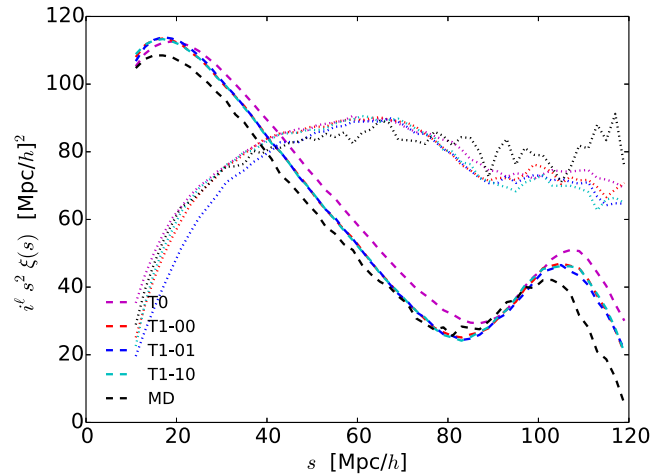


Figure 2. The monopole and quadrupole moments of the correlation function, times s^2 to reduce the dynamic range in the plot, for the simulations described in the text (see also Table 1). In each case, the mean value of the statistic is plotted, in bins of width $2 h^{-1}$ Mpc, and error bars have been suppressed for clarity. Models T0 and T1 are described in Section 3.1, while MD is described in Section 3.2. Model T1 has three variants, which differ in terms of the halo finder and whether a velocity bias is assumed for the satellites.

In order to provide fits to the data, we need a covariance matrix. We use the linear theory expressions¹ (Bernstein 1994; Eisenstein & Zaldarriaga 2001; Cohn 2006; Huff et al. 2007)

$$\text{Cov} [\xi_{\ell_1}(s_1), \xi_{\ell_2}(s_2)] = \frac{2}{V} i^{\ell_1 + \ell_2} (2\ell_1 + 1)(2\ell_2 + 1) \int \frac{d^3k}{(2\pi)^3} \times P^2(k, \mu) L_{\ell_1}(\mu) L_{\ell_2}(\mu) j_{\ell_1}(ks_1) j_{\ell_2}(ks_2) \quad (6)$$

in order to generate a correlation matrix (sometimes referred to as a reduced covariance matrix). Comparison of the linear theory correlation matrix with that produced from the QPM mock catalogues shows that they are quite similar in structure (see also Reid et al. 2012). The correlation matrix is independent of the volume of the survey or any scaling of the bins (e.g. whether it is $s^2 \xi_0$ or ξ_0 that is being constrained). The covariance matrix we use is then obtained from the linear theory correlation matrix by multiplying each row and column of the correlation matrix by the standard deviation per bin, measured from the independent simulations or octants, scaled (with error $\propto V^{-1/2}$) as if for a $5 (h^{-1} \text{Gpc})^3$ volume, roughly the volume expected for high- z galaxies in the final BOSS data set. This ensures that we can assess sub- σ systematics with respect to the BOSS data but we are not simply fitting noise from the finite number of mock catalogues [since the volume of the mocks exceeds $5 (h^{-1} \text{Gpc})^3$ in all cases]. By matching the error bars to those expected from BOSS, we can also assess how degeneracies and nuisance parameters affect the fits in a way relevant to BOSS. In preparing these matrices and ξ_ℓ measurements, we cover $10 < s < 120 h^{-1}$ Mpc using $2 h^{-1}$ Mpc bins.² We work with the first two even multipole moments, ξ_0 and ξ_2 . In order to drop points at low or high s , we simply add a large number to the nominal error bar before computing the covariance matrix and its inverse. This provides a simple way of restricting the fitting range

¹The formula given assumes infinitesimal bins. We additionally account for the finite bin width by integrating $s^2 j_\ell$ over the bin.

²We have tested that increasing the bin width to $4 h^{-1}$ Mpc does not alter the results.

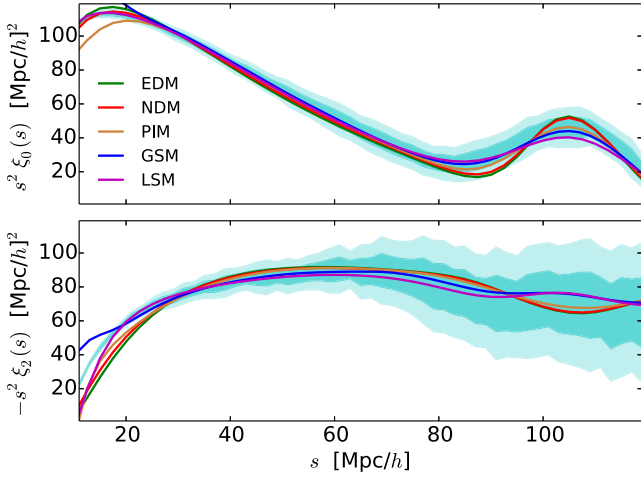


Figure 3. The best-fitting models to the T1 simulation at $z \simeq 0.55$. Each model was fit to the data in the range $30 < s < 120 h^{-1}$ Mpc, holding the linear theory power spectrum and f fixed at the appropriate values for this simulation. The shaded bands around the simulation results indicate the adopted 1 and 2σ errors, derived from the diagonal elements of the covariance matrix. The error bars are highly correlated. The sharper acoustic peak in the monopole of the Eulerian models than the Lagrangian models is expected, as the Eulerian models do not account for non-linear broadening of the peak. Some of the modes which are responsible for this broadening are not captured by the T1 simulations, so the N -body peak is not as broad as it should be. The models are quite similar in their monopole predictions at intermediate scales however and the large scales carry little weight in the fit.

which requires minimal changes to the code (though it does not fully remove the influence of the small scale points, which requires zeroing the correlations, but when the model is close to the data the difference is small and the approximation does not qualitatively affect our conclusions).

We begin by fitting the models with the ‘correct’ linear theory power spectrum and cosmology (although we allow f to float in order to see if we recover the correct value). Since the distance scale and the shape of the power spectrum are not allowed to vary, this test is quite constraining. There is no ‘slop’ from other parts of the theory to hide a bad fit. Our nominal fitting range for most models is $30 < s < 120 h^{-1}$ Mpc. For the BigMD simulation, the internal error estimate from the eight octants becomes unreliable at the largest scale so we truncate the fit at $100 h^{-1}$ Mpc. For the EDM and NDM models, we wish to avoid fitting the acoustic peak region entirely, since the shape of the peak is much sharper than seen in the N -body simulations (where the peak is broadened by non-linear evolution). As the peak height can be lowered, in these models, by increasing f the fits become highly unstable to large f if the peak is included in the fit. Thus, we include only $s < 80 h^{-1}$ Mpc for these two models. Note that the acoustic peak region is not very important for this test as we are holding the cosmology and linear power spectrum fixed, and the relative error on the large-scale points is much larger than those at smaller scales. For a more general fit, information from the acoustic peak would need to be included in some manner (e.g. as a prior on distance).

Figs 3 and 4 show how well the models fit the monopole and quadrupole moments measured from the T1 and BigMD simulation at $z \simeq 0.55$ – 0.56 . (The situation is qualitatively very similar for the other mock catalogues.) The behaviour of the models near the acoustic peak has been discussed previously. Some of the modes which are responsible for the broadening of the acoustic peak at

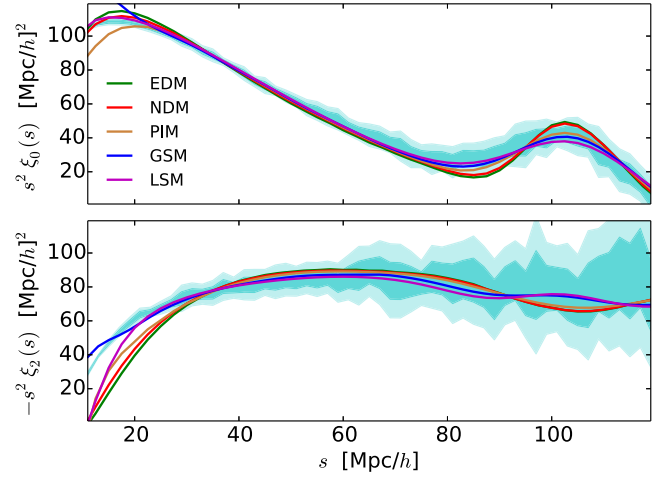


Figure 4. As in Fig. 3 but for the BigMD simulation at $z \simeq 0.56$. The error bars are highly correlated and the noise from the smaller total volume is apparent.

$s \simeq 110 h^{-1}$ Mpc are not captured by the T1 simulations, so the N -body peak is not as broad as it should be. The simulation is well converged at intermediate scales, however, and the large scales carry little weight in the fit. The noise from finite volume is evident in the BigMD simulation.

Note that the Eulerian models predict a higher (i.e. more negative) quadrupole at large scales ($40 h^{-1}$ Mpc $< s < 80 h^{-1}$ Mpc) with a sharper ‘break’ at small scales than the Lagrangian models, and in particular they underpredict the N -body data on scales below $30 h^{-1}$ Mpc. On the tens of Mpc scales which dominate the fits the GSM and LSM agree very well and agree quite well with the N -body data. All of the models predict very similar behaviour for the monopole over the range $30 h^{-1} < s < 70 h^{-1}$ Mpc and in particular the N -body monopole is well matched by all of the models in this range.

We can look for any biases or instabilities in the fit using a Monte Carlo Markov Chain. We run several such chains with a parallel implementation of the affine-invariant ensemble sampler described in Goodman & Weare (2010). To begin, we allow f , the bias and the finger-of-god parameter to vary at fixed linear power spectrum, with infinitely wide, flat priors.³

The marginalized posterior distribution for f is shown in Fig. 5 for the models and simulations (Fig. 6 highlights the T1 simulations not shown here). The linear theory power spectrum has been held fixed and the other two parameters, bias and finger-of-god dispersion, have been marginalized over. We also show the profile likelihood (Wilks 1938), i.e. the value of the best fit at fixed f , allowing the large-scale bias and finger-of-god parameter to vary. This statistic provides information on the goodness of fit which is not affected by parameter volume effects. As we shall see, the agreement between the posterior and profile likelihood is reasonably good in all cases indicating that our posterior likelihoods are not unduly misleading due to volume effects, but we shall return to this point below.

The models all fare reasonably well for all simulations, but biases are visible at the 1σ level for several models. While they fare well for the BigMD simulation, the EDM and NDM models are biased slightly low for T0 and T1-10. This is a consequence of the mismatch

³Except for the GSM model where we imposed a prior $\sigma < 10 h^{-1}$ Mpc for numerical reasons.

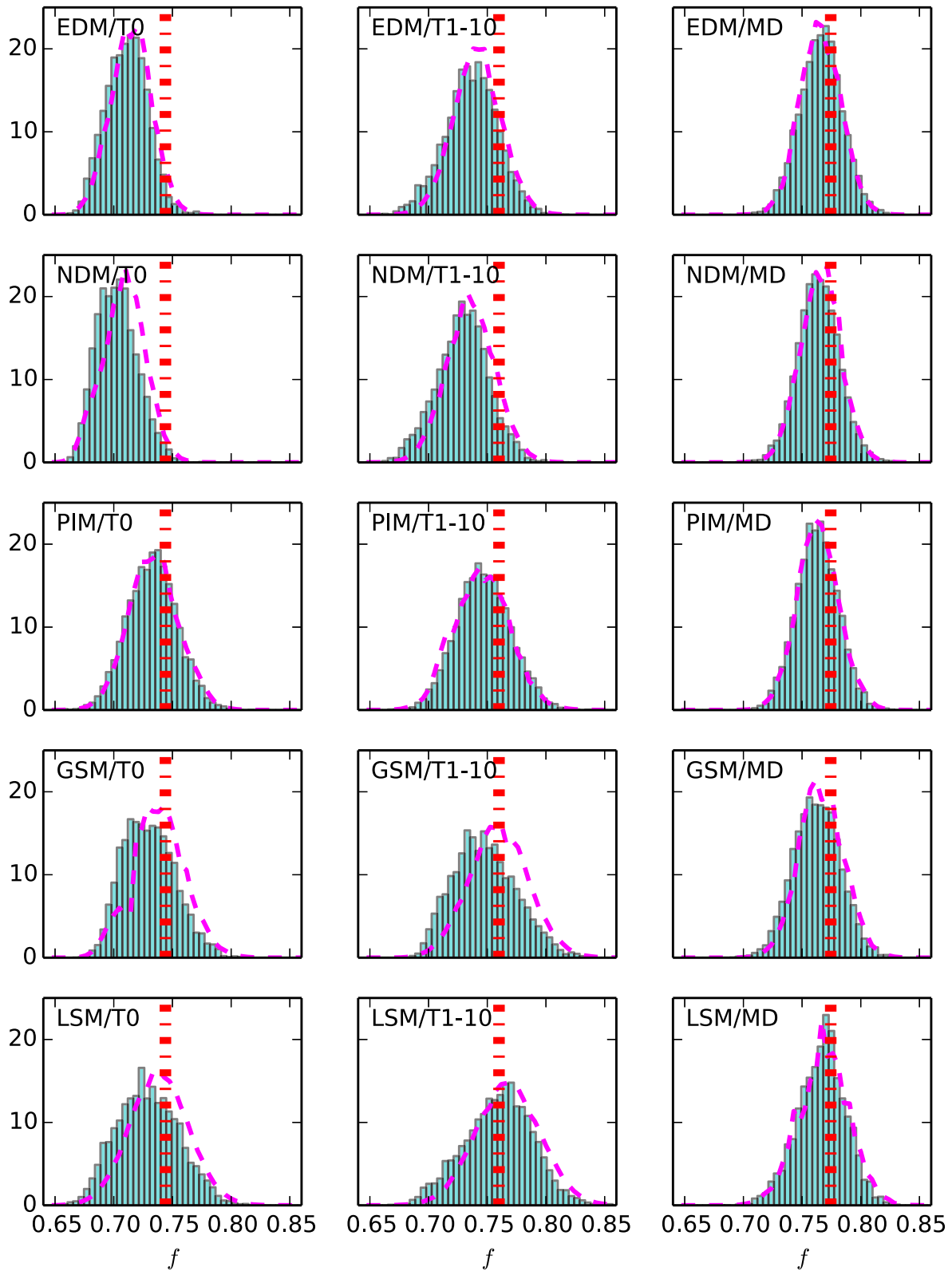


Figure 5. The profile likelihood (dashed lines) and the marginalized posterior distribution of f (filled histogram; from a Monte Carlo Markov Chain), for the models described in Section 2 and the simulations described in Section 3. Scales larger than $30 h^{-1}$ Mpc have been fit, as described in the text. The vertical line marks the value of f appropriate to each cosmology.

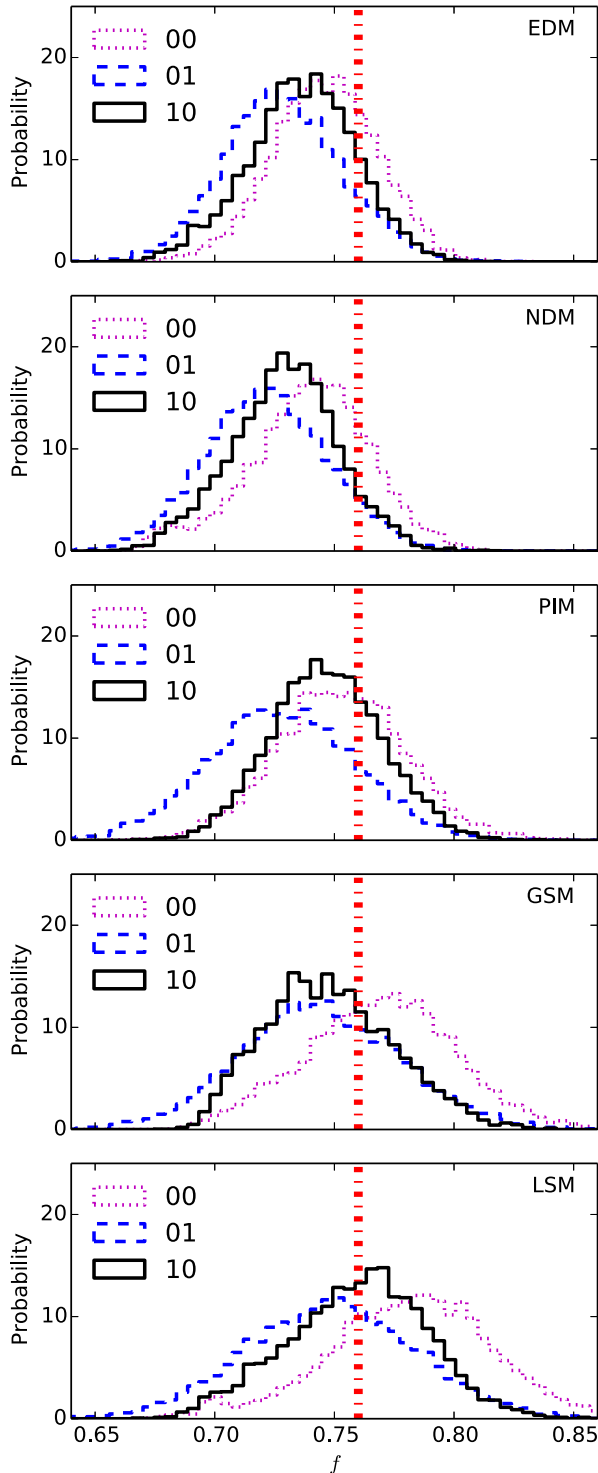


Figure 6. The marginalized posterior distribution of f for models built upon the T1 simulation, which have different prescriptions for how central and satellite velocities are assigned and different halo finding schemes (the numbering is described in Section 3: 00 uses the friends-of-friends halo centre-of-mass velocity for the central galaxy, 01 enhances satellite velocities by 25 per cent and 10 is our fiducial model with SO haloes). As in the earlier figures, the vertical line marks the value of f appropriate to this cosmology at $z \simeq 0.55$ and the data were fit over the range $30 < s < 120 h^{-1}$ Mpc. As the T1 simulations cover about five times our fiducial $5 (h^{-1} \text{Gpc})^3$, we expect shifts of around 0.5σ .

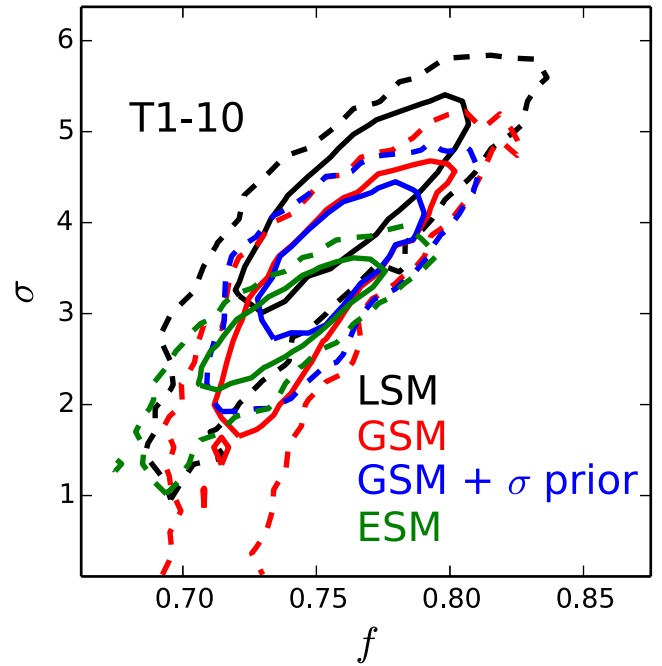


Figure 7. The marginalized joint posterior distribution of f and σ for the T1-10 mock for the LSM (black), GSM (red) and EDM (green) models. In each case, there is a positive correlation between f and σ . If we apply the prior on σ advocated in Reid et al. (2012) for the GSM model, low values of σ are disfavoured (blue). The central value of f shifts by $\sim 0.3\sigma$ and the uncertainty on f is reduced by 25 per cent after the application of the σ prior.

in the quadrupole (compared to the other theories) seen in Fig. 1. The PIM performs well for all of the simulations and the Lagrangian models do not show evidence for bias for any of these simulations either: the distribution of f values inferred from the fit is very well centred on the correct value for most simulations.

Since we are fitting our models to correlation functions measured from finite simulation volumes, we expect some scatter between the peak of the likelihood and the ‘true’ values, even in the case of a perfect model. The size of the typical offset should roughly be $\sqrt{V_5}\sigma_f$, where V_5 is the total volume of the simulations in units of our fiducial volume, $5 h^{-3} \text{Gpc}^3$.

For the T1 simulation, we generated three catalogues which differ in the way dark matter haloes are identified and populated with mock galaxies (see Section 3.1). We compare the f distributions in Fig. 6. Again the simpler Eulerian models tend to lie slightly low while the other models tend to perform better. This is quite a significant test of our simple, one-parameter finger-of-god prescription. As shown in Fig. 2, the shape of the quadrupole at small scales is different between the catalogues and yet the value of f is well recovered in all cases. This indicates that our simple one-parameter finger-of-god prescription is adequate to describe the effects of such galaxy motions on scales above $30 h^{-1}$ Mpc (see also Fig. 7).

In all of the above comparisons, the posterior distribution of f for the EDM and NDM models lie low compared to the true value of f in the T1 simulation. We now examine how the fits behave as we vary the minimum scale in the fit, s_{\min} . Fig. 8 shows the posterior distribution of f for the minimum s of both the monopole and quadrupole varying s_{\min} from 20 to $40 h^{-1}$ Mpc (the larger value is the range over which they have typically been used; Chuang et al. 2013; Sanchez et al. 2013, 2014).

We see the expected behaviour, an increasing bias but tighter distribution as we lower s_{\min} . For all models but LSM, the bias at

$s_{\min} = 20 h^{-1}$ Mpc is significant. While LSM appears to fare well in this test, the bias for this model can rise to 1σ for other simulations in the T1 series (specifically model 01, with enhanced satellite galaxy velocities). Increasing s_{\min} to $24 h^{-1}$ Mpc when fitting the LSM returns unbiased fits for all catalogues, with the exception of T1-00. For this model, the likelihood is broad and centred roughly 1σ higher than the ‘true’ value in the simulation. The difference in χ^2 between the best-fitting value of f and the ‘true’ value is small however ($\Delta\chi^2 < 1$) suggesting that the model does not rule out the correct value of f . For this reason, we see no evidence that the LSM cannot be used down to $\simeq 25 h^{-1}$ Mpc.

As we increase s_{\min} , we see generally less bias and a wider posterior distribution, though the bias remains significant for EDM and NDM even at $40 h^{-1}$ Mpc. The behaviour of the LSM is at first sight surprising, because the bias appears to increase with increasing s_{\min} . This can be traced to the positive correlation between f and the finger-of-god parameter, σ , shown in Fig. 7 (see also the discussion in Reid et al. 2012). As we increase s_{\min} , the finger-of-god parameter becomes less constrained, and newly allowed low values of σ correspond to lower values of f . The peak of the marginal distribution thus shifts to lower f . If we had a well motivated prior for σ , this degeneracy would be less important. For instance, Reid et al. (2014) propose a Gaussian prior on σ^2 around $14 \pm 5 (h^{-1} \text{ Mpc})^2$. As shown in Fig. 7, this prior disfavors low values of σ , corresponding to models in which the galaxies have small intrahalo velocities inconsistent with the observed anisotropic clustering on small scales. Application of this prior shifts the central marginalized value of f upwards by $\sim 0.3\sigma$ (thus removing the small bias evident in Figs 6 and 5) and reduces its uncertainty by 25 per cent. Since the other mock catalogues have purposely not been tuned to match the observed small-scale anisotropic clustering, the proposed prior is not applicable to those mock samples and we have not attempted to use it.

The tests presented above are quite stringent, as they allow only a minimum number of parameters to vary. It is also of interest to ask how well the models perform when we allow the linear theory power spectrum and cosmological parameters to vary. In Fig. 9, we show the marginalized posterior for f from the LSM model, allowing the linear theory power spectrum to vary within a CMB prior as in Samushia et al. (2014). Specifically, we allow $\omega_c \equiv \Omega_c h^2$, $\omega_b \equiv \Omega_b h^2$, n_s , $f\sigma$ and the large-scale bias to vary with a prior on ω_c , ω_b and n_s . We hold h and σ_8 fixed at their fiducial values. As discussed extensively in Reid et al. (2012) and Samushia et al. (2013, 2014) within the Λ CDM family, current CMB data tightly constrain the linear theory power spectrum so that allowing it to float or holding it fixed return almost the same marginalized constraints on f . This is clearly illustrated by Fig. 9.

4.2 Observational effects

The above tests have been performed on ‘ideal’ measurements, involving constant time outputs, with constant number density done in periodic boxes. We now turn to a discussion of how departures from this ideal situation affect measurements of the growth rate.

In this section, we make use of the QPM mock catalogues, in a differential fashion. We compare the ξ_ℓ measured on the constant time, uniform density, periodic QPM boxes to that computed on catalogues which have been projected on to the sky and observed under various constraints. We shall call these latter catalogues sky mocks. Throughout, we shall work with the average of 100 sky mocks, sufficient to look for effects at much less than the expected

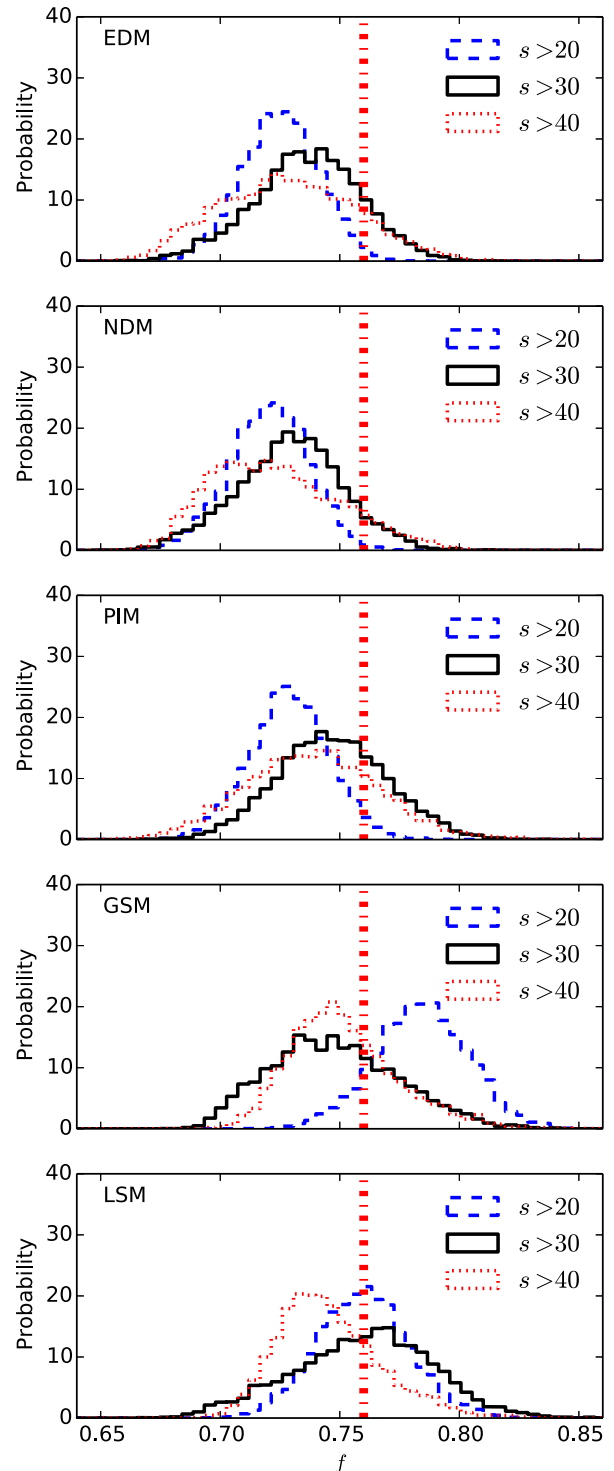


Figure 8. The marginalized posterior distribution of f as a function of the minimum scale in the fit (in h^{-1} Mpc) for each theoretical model. Again the T1-10 simulation is used and the vertical line marks the value of f appropriate to this cosmology at $z \simeq 0.55$. A similar behaviour is seen for the other simulations (see the text).

statistical error of BOSS. We now show that the effects on ξ_ℓ (and the covariance matrix) at the relevant scales are unimportant.

First, we compare the average correlation function of the sky mocks to that of the periodic mocks, assuming no fibre collisions, redshift failures or systematics. The major differences are from the

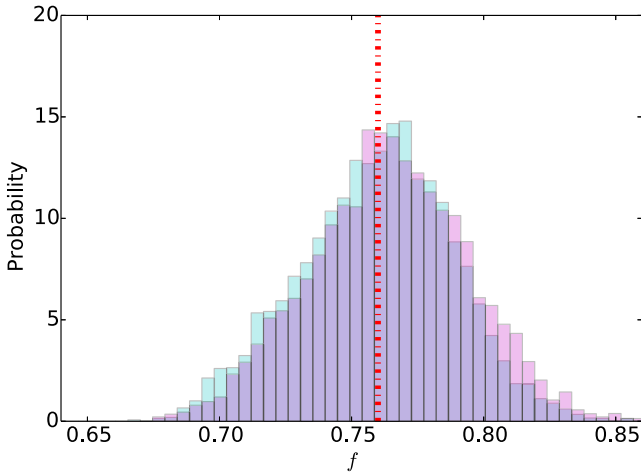


Figure 9. The marginalized posterior distribution of f for the fiducial cosmology allowing the linear theory power spectrum (and cosmological parameters) to vary within a CMB prior (magenta histogram) compared with the case of $P_{\text{lin}}(k)$ fixed (cyan histogram). Within the Λ CDM family current, CMB data tightly constrain the linear theory power spectrum, so that the constraints with fixed $P_{\text{lin}}(k)$ or with a CMB prior are almost identical. Again the vertical line marks the value of f appropriate to this cosmology at $z \simeq 0.55$.

shape of $\bar{n}(z)$ and (to a lesser extent) wide-angle RSD. We convert angles and redshifts to distances using the proper cosmology for the simulations, use the correlation function estimator of Landy & Szalay (1993) and assign redshifts to random points by ‘shuffling’ the data redshifts, which is the technique used in most of the BOSS papers to date. We use 20 times as many randoms as data points.

The average ξ_ℓ from the sky and periodic mocks differ by a small but measurable amount, even on large scales. This major driver of this difference is the redshift distribution chosen for the random points. This has been studied before, most recently by Samushia et al. (2012). Those authors showed that our approach gives the smallest error for a wide-angle survey like BOSS, but this error is not negligible. Luckily, the error is very small (less than 1 per cent) on the smallest scales and grows more slowly than the statistical error as we move to larger scales. Since our measurements are heavily weighted to small scales, this effect is actually less important for RSD than for e.g. anisotropic fitting of the baryon acoustic oscillation peak. Over the range 20–60 h^{-1} Mpc the effect is always less than 2.5 per cent, in both the monopole and quadrupole moments of the redshift-space correlation function. This is much less than our assumed statistical errors and comparable to the systematics from the modelling.

Next we introduce ‘fibre collisions’. To mimic this observational effect, we divide the mock galaxies into those which have a neighbour within 62 arcsec and those which do not. We measure from the BOSS sample, as a function of position on the sky, the fraction of close pairs where one galaxy precluded the other getting a redshift. For each mock pair at the same sky position, we randomly remove one galaxy from the pair, increasing the weight of its neighbour by 1. If a galaxy has more than one near neighbour, we choose one at random. This is only an approximation to the fibre assignment scheme employed by BOSS on the real data (Blanton et al. 2003), but it captures the main features of the effect.

In common with earlier work (see e.g. Reid et al. 2014, for a recent study), we find that fibre collision correction using nearest neighbours has little impact on the correlation function on large scales. The monopole of the redshift-space correlation function is

affected by less than 1 per cent on scales above $10 h^{-1}$ Mpc while the quadrupole is affected by less than 1 per cent above $30 h^{-1}$ Mpc and only by 2 per cent at $20 h^{-1}$ Mpc. Both of these errors are comfortably below the statistical and modelling error and can be ignored.

Next we discuss the impact of evolution across the sample, and the approximation inherent in interpreting our measurement as ξ_ℓ ‘at’ a given redshift.

4.3 Lightcone evolution

If the number density is slowly varying with redshift over the scales which dominate our fits,⁴ the correlation function we measure is (Matarrese et al. 1997; White, Martini & Cohn 2008)

$$\xi_{\ell,\text{obs}}(s) \simeq \frac{\int dz (dN/dz)^2 (H/\chi^2) \xi_\ell(s, z)}{\int dz (dN/dz)^2 (H/\chi^2)}, \quad (7)$$

where dN/dz is the redshift distribution of the sample, H is the Hubble parameter at redshift z and χ is the comoving angular diameter distance to redshift z .

To get a sense for the amount by which evolution could change the shape of the correlation functions, we assumed a Gaussian dN/dz , centred at $z = 0.55$ and with varying widths. Using the LSM as ‘truth’, and the cosmology of the BigMD simulation, we computed the integral and compared it to $\xi(r, z_{\text{eff}})$, where z_{eff} is evaluated by an integral similar to equation (7) but with z in the numerator rather than $\xi(s, z)$. Fig. 10 shows the range of values of the moments of the correlation function take on across the redshift range for two different assumptions about the bias evolution: passive evolution (Fry 1996) and constant bias. We expect the former to be a better approximation for BOSS galaxies, and we see predicts more constant clustering with redshift. Fig. 11 compares the average correlation function multipoles to those evaluated at z_{eff} . The ratio is very close to scale invariant, and within a few per cent of unity even for quite broad redshift windows (for comparison the dN/dz for the BOSS CMASS sample has approximately 10 per cent width). This suggests that we can safely interpret the measurements of the correlation function as at an ‘effective’ redshift with little impact on the cosmology.

5 DISCUSSION

A key probe of cosmological expansion and our theory of gravity is the growth of large-scale structure, as revealed in the clustering of galaxies observed in large redshift surveys. A growing understanding of the process of structure formation allows us to compare well-motivated models to data from ever larger surveys to provide tight constraints on deviations from the standard model of cosmology. In this paper, we have compared the predictions from a number of phenomenological models for the anisotropic clustering of galaxies measured in configuration space against a series of mock catalogues designed to mimic the high-redshift sample from BOSS.

Even dealing with ‘ideal’ data, we have found that none of the proposed models fit the mock data on all scales, but they all do a good job of fitting the monopole and quadrupole moments of ξ in the range $40 < s < 80 h^{-1}$ Mpc. Dispersion models based on the simplest versions of Eulerian perturbation theory do not provide a good fit to the baryon acoustic peak feature of BOSS-like galaxies at

⁴We have explicitly checked that this is true for the BOSS DR12 data set by counting galaxy pairs and comparing to equation (7).

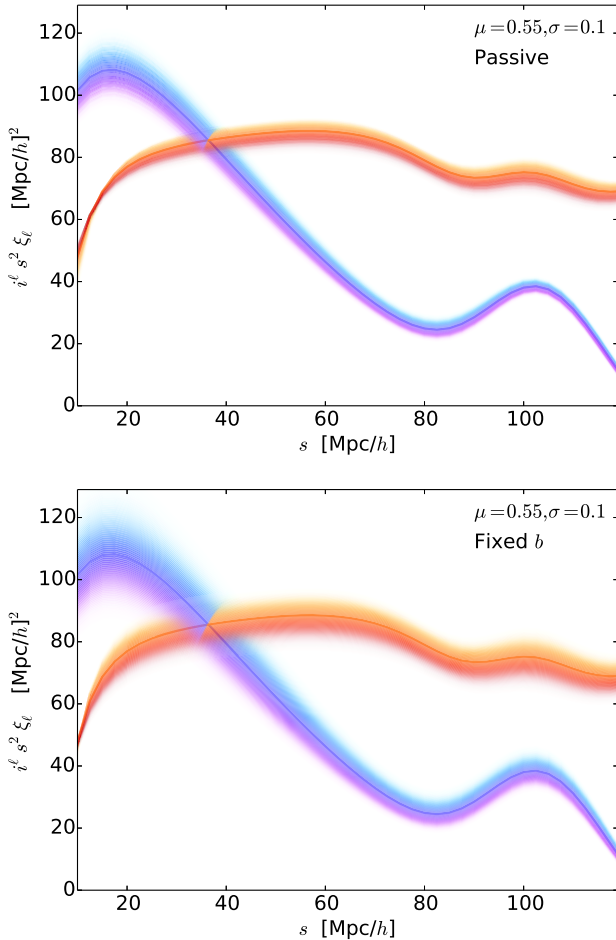


Figure 10. The evolution of the correlation function multipoles with redshift, as determined by the LSM for the cosmology of the BigMD simulation. In each panel, the colour scale indicates the redshift while the transparency scales with the weight in equation (7). The upper panel shows the result for a Gaussian dN/dz with the indicated mean and dispersion assuming passive evolution ($b - 1 \propto D^{-1}$; Fry 1996) while the lower panel assumes fixed b . The former is a better approximation to the evolution of the clustering of BOSS galaxies (White et al. 2011).

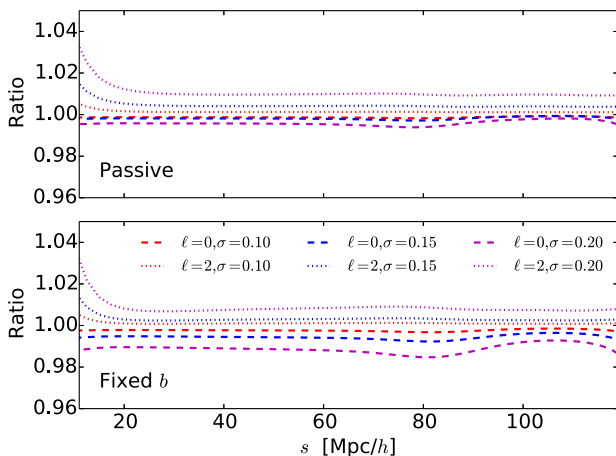


Figure 11. The ratio of the redshift-averaged correlation function multipoles to the model evaluated at z_{eff} , i.e. using a constant-time approximation, for Gaussian dN/dz with the listed mean and standard deviation (for the $\sigma = 0.2$ case we have set $dN/dz = 0$ for $z < 0.05$). In all cases, the ratio is nearly scale independent and close to unity.

$z \simeq 0.5$, but fare well on the shape of the monopole and quadrupole on intermediate scales. They tend to slightly underpredict the growth rate of structure when fit down to $20\text{--}30 h^{-1}$ Mpc scales.

More sophisticated models, and models based on Lagrangian perturbation theory, such as the GSM and LSM (Section 2), provide unbiased fits to the data on scales above $25\text{--}30 h^{-1}$ Mpc. In particular, the LSM is straightforward and fast to compute and provides a reliable model for fitting BOSS-like galaxies at $z \simeq 0.5$.

Including non-idealities such as fibre collisions or lightcone evolution does not appreciably modify these conclusions. Using standard corrections for fibre collisions and interpreting the light-cone data as a measurement at an effective redshift introduces errors which are small compared to other sources of uncertainty.

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REFERENCES

- Ballinger W. E., Peacock J. A., Heavens A. F., 1996, *MNRAS*, 282, 877
 Bel J. et al., 2014, *A&A*, 563, 37
 Berlind A., Narayanan V. K., Weinberg D. H., 2001, *ApJ*, 549, 688.
 Bernstein G. M., 1994, *ApJ*, 424, 569
 Beutler F. et al., 2014, *MNRAS*, 443, 1065
 Bianchi D., Guzzo L., Branchini E., Majerotto E., de la Torre S., Marulli F., Moscardini L., Angulo R. E., 2012, *MNRAS*, 427, 2420
 Blake C. et al., 2011, *MNRAS*, 415, 2876
 Blake C. et al., 2013, *MNRAS*, 436, 3089
 Blanton M. R., Lin H., Lupton R., Maley F., Young N., Zehavi I., Loveday J., 2003, *AJ*, 125, 2276
 Carlson J., Reid B. A., White M., 2013, *MNRAS*, 429, 1674
 Chuang C. H. et al., 2013, *MNRAS*, 433, 3559
 Cohn J. D., 2006, *New. Astron.*, 11, 226
 Cole S., Fisher K., Weinberg D. H., 1995, *MNRAS*, 275, 512
 da Angela J. et al., 2008, *MNRAS*, 383, 565
 Dawson K. et al., 2013, *AJ*, 145, 10
 de la Torre S., Guzzo L., 2012, *MNRAS*, 427, 327
 de la Torre S. et al., 2013, *A&A*, 557, 54
 Eisenstein D. J., Zaldarriaga M., 2001, *ApJ*, 546, 2
 Eisenstein D. J. et al., 2011, *AJ*, 142, 72
 Fisher K. B., 1995, *ApJ*, 448, 494
 Fry J. N., 1996, *ApJ*, 461, L65
 Gil-Marín H., Wagner C., Verde L., Porcini C., Jimenez R., 2012, *J. Cosmol. Astropart. Phys.*, 11, 029
 Goodman J., Wearé J., 2010, *Commun. Appl. Math. Comput. Sci.*, 5, 65
 Goroff M. H., Grinstein B., Rey S.-J., Wise M. B., 1986, *ApJ*, 311, 6
 Guzzo L. et al., 2008, *Nature*, 451, 541
 Hamilton A. J. S., 1998, in Hamilton D., ed., *Astrophysics and Space Science Library*, Vol. 231: The Evolving Universe. Kluwer, Dordrecht, p. 185
 Hatton S. J., Cole S., 1999, *MNRAS*, 310, 113
 Huff E., Schulz A. E., White M., Schlegel D. J., Warren M. S., 2007, *Astropart. Phys.*, 26, 351
 Jackson J., 1972, *MNRAS*, 156, 1

- Jain B., Bertschinger E., 1994, *ApJ*, 431, 495
 Jain B., Zhang P., 2008, *Phys. Rev. D*, 78, 063503
 Jennings E., Baugh C. M., Pascoli S., 2011, *MNRAS*, 410, 2081
 Jing Y. P., Börner G., 2004, *ApJ*, 617, 782
 Juszkiewicz R., 1981, *MNRAS*, 197, 931
 Kaiser N., 1987, *MNRAS*, 227, 1
 Kwan J., Lewis G. F., Linder E. V., 2012, *ApJ*, 748, 78
 Landy S. D., Szalay A. S., 1993, *ApJ*, 412, 64
 Li C., Jing Y. P., Kauffmann G., Börner G., Xi K., Wang L., 2007, *MNRAS*, 376, 984
 McDonald P., Seljak U., 2009, *J. Cosmol. Astropart. Phys.*, 10, 7
 Makino N., Sasaki M., Suto Y., 1992, *Phys. Rev. D*, 46, 585
 Matarrese S., Coles P., Lucchin F., Moscardini L., 1997, *MNRAS*, 286, 115
 Matsubara T., 2008a, *Phys. Rev. D*, 77, 063530
 Matsubara T., 2008b, *Phys. Rev. D*, 78, 083519
 Matsubara T., 2011, *Phys. Rev. D*, 83, 083518
 Nesseris S., Perivolaropoulos L., 2008, *Phys. Rev. D*, 77, 023504
 Okumura T., Jing Y., 2011, *ApJ*, 726, 5
 Okumura T. et al., 2008, *ApJ*, 676, 889
 Okumura T., Seljak U., Desjacques V., 2012, *J. Cosmol. Astropart. Phys.*, 11, 014
 Okumura T., Seljak U., Vlah Z., Desjacques V., 2014, *J. Cosmol. Astropart. Phys.*, 05, 003
 Park C., Vogeley M. S., Geller M. J., Huchra J. P., 1994, *ApJ*, 431, 569
 Peacock J. A., 1992, *MNRAS*, 258, 581
 Peacock J. A., Dodds S., 1994, *MNRAS*, 267, 1020
 Peacock J. A. et al., 2001, *Nature*, 410, 169
 Peebles P. J. E., 1980, *The Large-Scale Structure of the Universe*. Princeton Univ. Press, Princeton
 Percival W. J., White M., 2008, *MNRAS*, 393, 297
 Percival W. J. et al., 2004, *MNRAS*, 353, 1201
 Planck Collaboration XVI, 2014, *A&A*, 571, A16
 Reid B. A., White M., 2011, *MNRAS*, 417, 1913
 Reid B. A. et al., 2012, *MNRAS*, 426, 2719
 Reid B. A., Seo H.-J., Leauthaud A., Tinker J. L., White M., 2014, *MNRAS*, 444, 476
 Samushia L., Percival W., Raccanelli A., 2012, *MNRAS*, 420, 2102
 Samushia L. et al., 2013, *MNRAS*, 429, 1514
 Samushia L. et al., 2014, *MNRAS*, 439, 3504
 Sanchez A. et al., 2013, *MNRAS*, 433, 1202
 Sanchez A. et al., 2014, *MNRAS*, 440, 2692
 Scoccimarro R., 2004, *Phys. Rev. D*, 70, 083007
 Seljak U., 2001, *MNRAS*, 325, 1359
 Smith R. E. et al., 2003, *MNRAS*, 341, 1311
 Song Y.-S., Koyama K., 2009, *J. Cosmol. Astropart. Phys.*, 1, 48
 Song Y.-S., Percival W. J., 2009, *J. Cosmol. Astropart. Phys.*, 10, 4
 Song Y. S., Sabiu C. G., Nichol R. C., Miller C. J., 2010, *J. Cosmol. Astropart. Phys.*, 1, 25
 Song Y. S., Sabiu C. G., Kayo I., Nichol R. C., 2011, *J. Cosmol. Astropart. Phys.*, 5, 20
 Tinker J. L., 2007, *MNRAS*, 374, 477
 Tinker J. L., Weinberg D. H., Zheng Z., 2006, *MNRAS*, 368, 85
 Tojeiro R. et al., 2014, *MNRAS*, 440, 2222
 Vishniac E., 1983, *MNRAS*, 203, 345
 Wang L., Reid B. A., White M., 2013, *MNRAS*, 437, 588
 White M., 2001, *MNRAS*, 321, 1
 White M., 2002, *ApJS*, 579, 16
 White M., 2014, *MNRAS*, 439, 3630
 White M., Martini P., Cohn J. D., 2008, *MNRAS*, 390, 1179
 White M., Song Y.-S., Percival W., 2009, *MNRAS*, 397, 1348
 White M. et al., 2011, *ApJ*, 728, 126
 White M., Tinker J. L., McBride C. K., 2014, *MNRAS*, 437, 2594
 Wilks A. S. S., 1938, *Ann. Math. Stat.*, 1, 60
 Yoo J., Seljak U., 2014, preprint ([arXiv:1308.1093](https://arxiv.org/abs/1308.1093))
 Zel'dovich Y., 1970, *A&A*, 5, 84
 Zhang P., Liguoi M., Bean R., Dodelson S., 2007, *Phys. Rev. Lett.* 99, 1302
 Zhao G. B., Giannantonio T., Pogosian L., Silvestri A., Bacon D. J., Koyama K., Nichol R. C., Song Y. S., 2010, *Phys. Rev. D*, 81, 103510
 Zheng Y., Zhang P., Jing Y., Lin W., Pan J., 2013, *Phys. Rev. D*, 88, 103510

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