DIGITAL IMPLEMENTATION AND PARAMETER TUNING OF ADAPTIVE NONLINEAR DIFFERENTIAL LIMITERS

by

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Abstract

It has been shown that the performance of communications systems can be severely limited by non-Gaussian and impulsive interference from a variety of sources. The non-Gaussian nature of this interference provides an opportunity for its effective mitigation by nonlinear filtering. In this thesis, we describe blind adaptive analog nonlinear filters, referred to as Adaptive Nonlinear Differential Limiters (ANDLs), that are characterized by several methodological distinctions from the existing digital solutions. When ANDLs are incorporated into a communications receiver, these methodological differences can translate into significant practical advantages, improving the receiver performance in the presence of non-Gaussian interference. A Nonlinear Differential Limiter (NDL) is obtained from a linear analog filter by introducing an appropriately chosen feedback-based nonlinearity into the response of the filter, and the degree of nonlinearity is controlled by a single parameter. ANDLs are similarly controlled by a single parameter, and are suitable for improving quality of non-stationary signals under time-varying noise conditions. ANDLs are designed to be fully compatible with existing linear devices and systems (i.e., ANDLs behavior is linear in the absence of impulsive interference), and to be used as an enhancement, or as a simple low-cost alternative, to state-of-the-art interference mitigation methods. We provide an introduction to the NDLs and illustrate their potential use for noise mitigation in communications systems. We also develop a digital implementation of an ANDL. This allows for rapid prototyping and performance analysis of various ANDL configurations and use cases.

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Acknowledgments

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Finally, I wish to dedicate this thesis to my parents Susan Newth and Robert Scutti, and my brother Peter Scutti. They have provided me with their encouragement and support throughout my life and education.

Chapter 1

Introduction

Communication systems form the backbone of the modern digital world. We live in a dense soup of electromagnetic signals carrying information between smartphones, WiFi enabled devices, and even our appliances. A direct consequence is that additional steps must be taken in order to mitigate interference from nearby devices.

This problem is growing larger each day. The Internet of Things (IoT) is defined by the ITU as a global infrastructure for the information society, enabling advanced services by interconnecting (physical and virtual) things based on existing and evolving interoperable information and communication technologies. The development of this infrastructure will trigger the deployment of billions of additional devices competing for spots in the finite amount of spectrum available. Gartner, Inc. predicts that over 20 billion IoT devices will be in use by 2020 [2].

1.1 Background

Noise is a fundamental consideration in the design of any communication system and comes in different forms. Thermal noise cannot be entirely eliminated because it arises as a physical consequence of electronics. Another major form of noise is man-made or technogenic noise.

As can be seen in figure 1.1 the prevalence of such noise varies with location and frequency band and is intensifying with the proliferation of handheld electronic devices especially in densely populated areas [1].

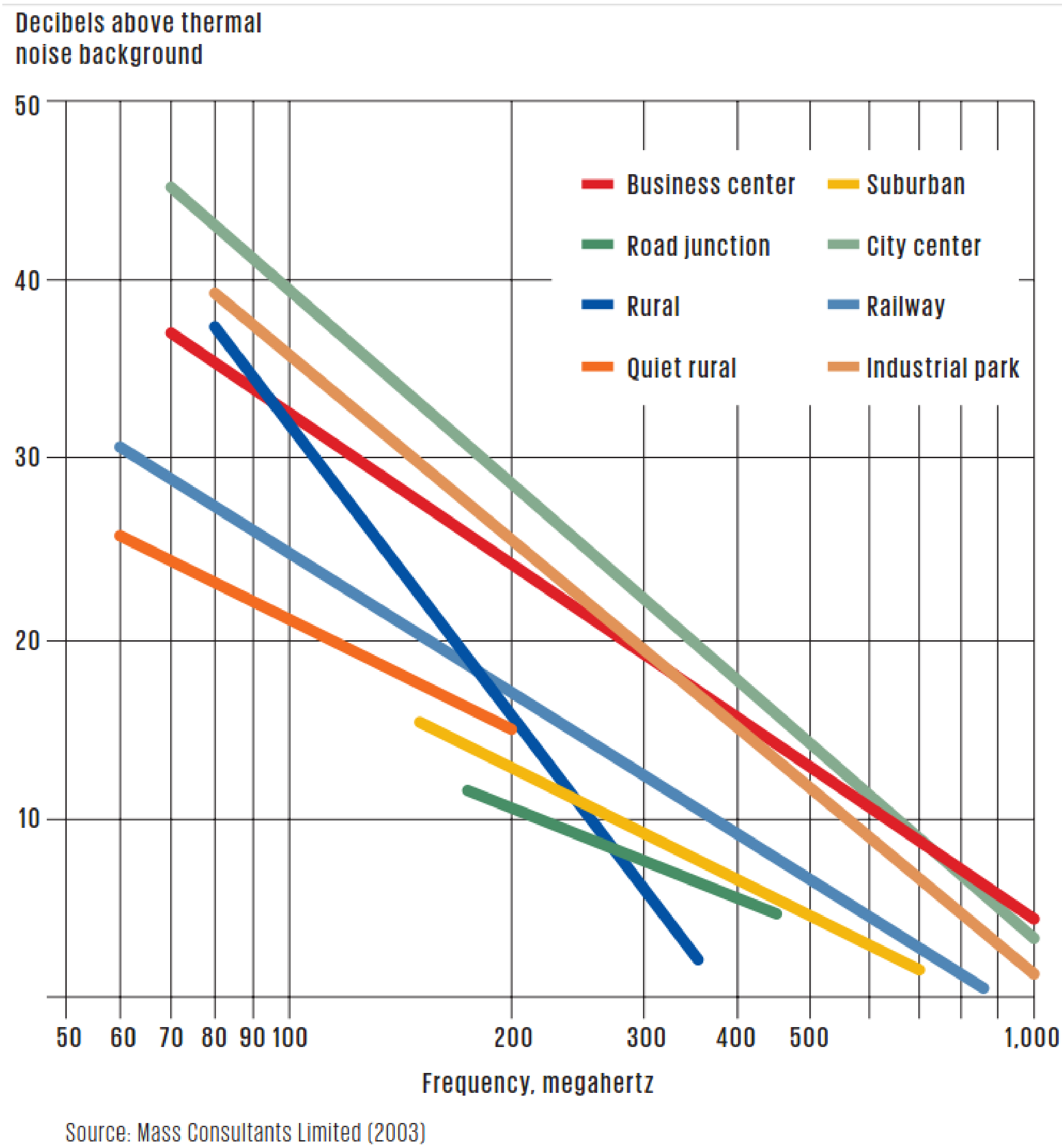


Figure 1.1: *Noise Above Thermal Background vs Frequency[1]*

Besides having different sources, technogenic noise varies from thermal noise in several ways. Because it arises from purposely designed signals, man-made noise tends to follow some type of pattern as opposed to being purely random like thermal noise [3]. The most relevant distinction to this work is the difference in amplitude distribution. This provides an opportunity for recognizing the noise and mitigating it. Linear filters are capable of discrimination between wanted and unwanted signals along only one dimension: spectral content. A consequence of this is that any noise energy residing in the passband of interest cannot be removed by linear filters, regardless of properties such as temporal structure. On the other hand, nonlinear filters can be designed to exploit additional differences between signal and noise in order to selectively remove noise energy and increase signal-to-noise ratio (SNR) in the passband of interest. This enables increases in channel capacity in the presence of technogenic noise. An illustration of this capability is shown in figure 1.2. A particular type of nonlinear filter referred to as an Adaptive Nonlinear Differential Limiter (ANDL) (that was proposed in [4, 5, 6] and discussed in the following sections) can be substituted in place of certain linear filters in the front-end of a receiver (such as the anti-aliasing filter which precedes an ADC) and can preserve channel capacity in the presence of technogenic noise or interference.

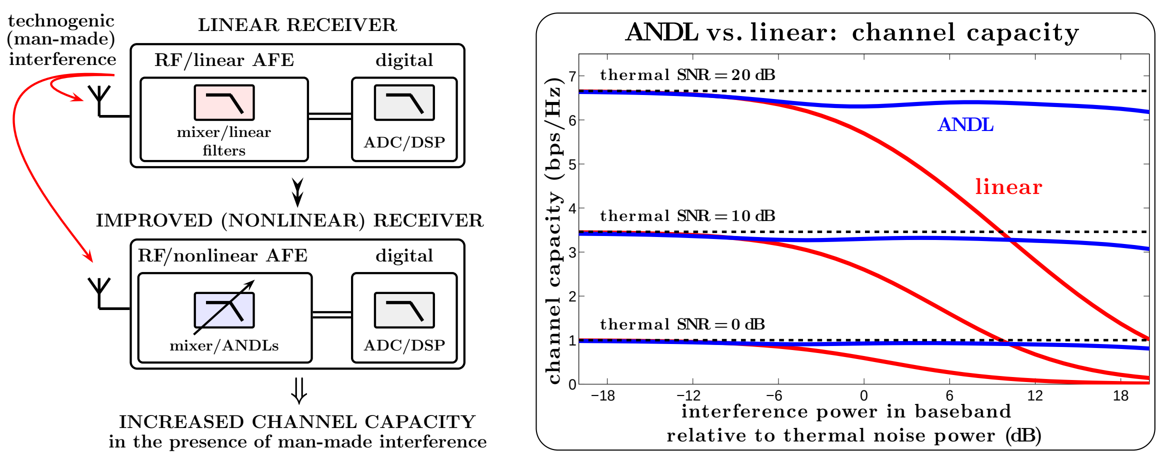


Figure 1.2: *Improved Receiver*

As highlighted earlier, technogenic noise originates from various sources that span the electronic world we live in. Previously conducted channel surveys [7, 8, 9, 10] and also theoretical models [5, 11, 12, 13, 14] show that such interference often manifests itself as non-Gaussian and, in particular, *impulsive* noise, and can in many instances dominate over the thermal noise [5, 7, 8, 10, 11]. Traditional methods are often impractical or ineffective at combating such noise due to its wide variety of sources. Therefore, there exists a compelling need to investigate techniques that avoid as many assumptions about the properties of such noise as possible. A "blind" nonlinear filtering approach such as the ANDLs discussed in this thesis can offer a generalized solution.

1.2 Prior Work

Other nonlinear techniques for mitigating non-Gaussian, impulsive noise have been proposed. Many of these approaches rely on assumptions about the underlying distribution of the noise. Using these assumptions, along with digital processing, statistical models can be fit to the noise. For example, the α-stable [15] and Middleton class A, B, and C [16] distributions are commonly used to model the interference in wired [17] and wireless [18] communications. In the context of powerline communications (PLC), [19] and [20] also pursue a model-based approach to mitigate cyclostationary impulse noise. However, the model-based nature of these techniques has some inherent disadvantages. Model fitting can be time and computationally intensive. Furthermore, a good fit may not always exist in which case performance can suffer under model mismatch.

Methods that are less dependent on a particular choice of noise model have been researched as well. Those include receiver designs based on flexible classes of distributions (e.g. myriad filter [21, 22], Normal Inverse Gaussian (NIG) [23]), or directly on the log-likelihood ratio shape (e.g. soft limiter [24], hole puncher [25], and p-norm [23, 26]). A more direct technique for designing a receiver independent of specific noise distribution assumptions (henceforth referred to as a *blind* receiver) is to apply a memoryless nonlinearity to the input signal after digital conversion, as in [27]. This can be shown to be locally optimal in low-SNR conditions [28, 29]. In [30], samples were either clipped or blanked according to amplitude. The determination of the clipping threshold was based in detection theory and relies only on an approximation of impulse arrival rate. Other methods for applying nonlinearities have been investigated in [31]. In addition to clipping, noise can be estimated and subtracted, as in [32].

A fundamental limitation of these nonlinear techniques is that they can only act on signals after digital conversion, or in other words: pass through an analog-to-digital converter (ADC). The ADC imposes a bandwidth limitation based on sampling rate and therefore requires a lowpass anti-aliasing filter. This filter will spread impulsive noise energy in the time domain after which it is too late to deal with effectively [4, 5, 6]. Increasing the sampling rate will decrease this effect. However, this comes at the cost of memory and computational resources. These limitations make digital techniques unsuited for real-time implementation and treatment of non-stationary noise.

1.3 Contributions of Thesis

In this thesis, we describe blind analog nonlinear filters, referred to as Nonlinear Differential Limiters (NDLs), and demonstrate their noise mitigation abilities in communications systems. We then develop a digital implementation of an Adaptive NDL (ANDL) using a conventional Digital Signal Processor (DSP) for use in audio signal processing. The digital platform allows for fast and flexible prototyping as well as filter parameter tuning. Finally we investigate methods for tuning the parameters of the ANDL in order to optimize performance.

Due to its relatively simple implementation, an ANDL can be used either as a standalone inexpensive impulsive noise reduction tool, or it can be used in combination with other interference mitigation methods. When used alone, ANDLs can provide improvement in the overall signal quality ranging from “no harm” for low noise conditions (the ANDL behaves as a linear filter in the absence of technogenic noise) to over 10 dB improvement in the overall passband SNR for high-power noise with a strong impulsive component. Quantitative results can be difficult to predict for nonlinear techniques. Performance may vary significantly based on particular conditions and design choices. These include the signal and noise compositions encountered in practice, the particular choice of NDL, and also the linear pre-filtering stage (the noise figure of the receiver amplifier for example). However, the NDLs' ability to disproportionally reduce the PSD of impulsive noise in the signal passband creates an attractive opportunity for noise mitigation in communications systems.

Chapter 2

NDL Basics

In this section we introduce the NDL with a brief theoretical description. Further information about different NDL configurations and more detailed discussions can be found in [4, 33].

2.1 NDL as an Approximation of a Median Filter

Advancements in technology have made processing power very inexpensive. This has paved the way for digital filtering techniques to become widely used, often in place of analog solutions. These digital methods allow for much greater adaptability, but come at the cost of complexity in both hardware and software. In addition, real time processing of signals is constrained by the rate at which signals can be sampled and the rate at which those samples can be processed. In communications systems, this restricts digital processing to the baseband leaving RF filtering to analog solutions.

One strategy for creating digital filters is to emulate existing analog techniques. Once a signal has been sampled, a processor can perform mathematical operations on the set of samples that recreate how an analog filter would physically respond to such a signal. Many frequency selective digital filters are designed in this way. However, digital filters are not constrained to this approach and digital filters without analog counterparts are also useful. For example, a processor can sort the samples and implement signal processing techniques based on order statistics such as in a median filter.

In a digital system, a median, or more generally, a quantile filter can be implemented simply by sorting collected samples. Sorting can be computationally expensive, but the operation is straightforward. An analog realization of a quantile filter is less obvious. The NDL is one such realization. It relies on a transformation of signals into normalized continuous scalar fields with the mathematical properties of distribution functions.

Consider a discriminator which outputs a 1, 0, or $\frac{1}{2}$ depending on whether the signal $x(t)$ is less than, greater than, or equal to some displacement variable D respectively. Mathematically, this can be described with the Heaviside function:

$$\Theta[D - x(t)] = \begin{cases} 1 & \text{if } x(t) < D \\ \frac{1}{2} & \text{if } x(t) = D \cdot \\ 0 & \text{if } x(t) > D \end{cases} \tag{2.1}$$

The time average of equation 2.1 is the fraction of time that the signal $x(t)$ remains below D. On the time interval $[0, T]$ this can be expressed as:

$$\Phi(D) = \frac{1}{T}\int_0^T \Theta[D - x(t)]dt. \tag{2.2}$$

Note that $\Phi(D)$ shares some properties with cumulative distribution functions. It is bounded $[0, 1]$. It is a monotonically increasing function of D. Also, it yields the probability that $x(t)$ has a value *less* than or equal to D for any instant of time on the fixed time interval $[0, T]$. We can use a generalized time-windowing function $w(t)$ and obtain Φ as a function of time as well as D. In other words we move from a fixed time window, to a sliding time window. For a time window function such that $w(t) \geq 0$ and $\int_{-\infty}^{\infty} w(s)ds = 1$ the distribution function is the following convolution integral:

$$\Phi(D,t) = \int_{-\infty}^{\infty} w(t-s)\Theta[D-x(s)]ds = w(t) * \Theta[D-x(t)]. \tag{2.3}$$

The function in equation 2.3 behaves like the time-localized CDF of $x(t)$. If we set $\Phi(D,t) = q$, for some constant $q \in (0,1)$ then the value of D satisfying the equation is the output of a quantile filter of order q. We can define the output more generally as:

$$\Phi(D_q(t),t) = q, \quad 0 < q < 1. \tag{2.4}$$

As a function of two variables, equation 2.4 can be graphed in three dimensions as is shown in figure 2.1. Along the D axis, Φ is the time-localized CDF of $x(t)$. Along the t axis, the CDF changes with time. The constant q defines a plane normal to the q-axis. The intersection of this plane with $\Phi(D,t)$ defines a level curve. The value of D corresponding to the level curve is the output of a quantile filter of order q and follows $D_q(t)$.

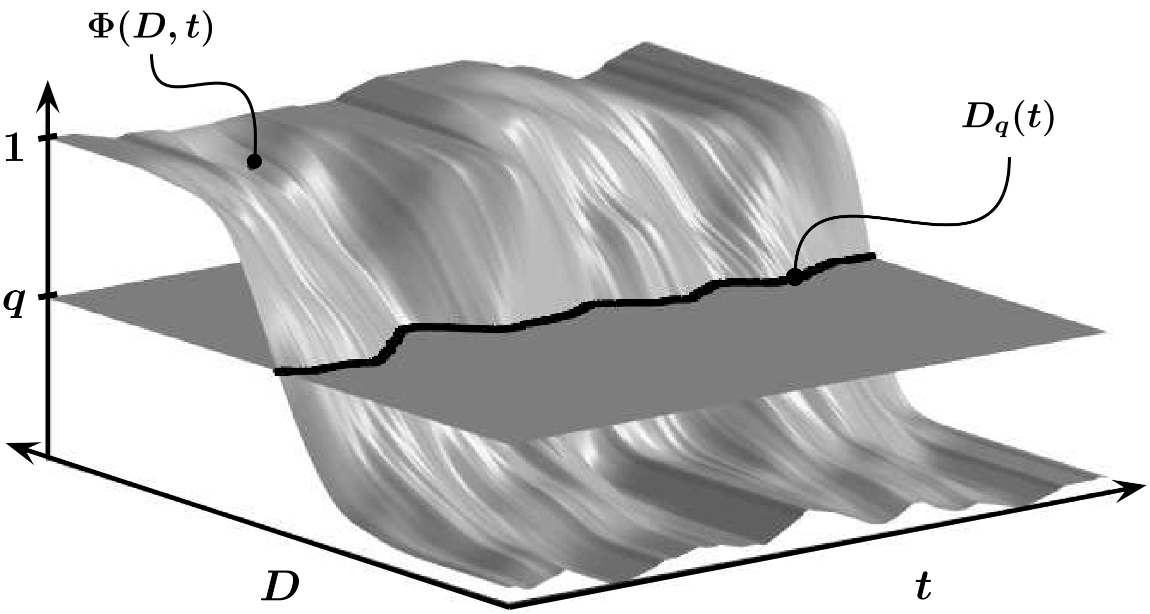


Figure 2.1: *Level Curve*

An explicit differential equation for a level curve can be obtained through differentiating equation 2.4 with respect to time:

$$\frac{d\Phi}{dt} = \frac{\partial \Phi}{\partial D_q}\frac{dD_q}{dt} + \frac{\partial \Phi}{\partial t} = 0, \tag{2.5}$$

And then solving equation 2.5 for the derivative of $D_q(t)$ to obtain the following equation:

$$\frac{dD_q}{dt} = -\frac{\partial \Phi/\partial t}{\partial \Phi/\partial D_q}. \tag{2.6}$$

The solution to the differential equation 2.6 will follow the output of a quantile filter, but only if we select the proper initial condition $D_q(t_0)$ where $\Phi[D_q(t_0), t_0] = q$. Interpreting a quantile filter as the solution to a differential equation may be useful for finding an analog implementation because many differential equations can be solved with analog circuits.

It is useful now to consider separately the numerator and denominator of the differential equation 2.6. They are the two partial derivatives of the bi-variate function $\Phi(D, t)$. The partial derivative with respect to time t is:

$$\frac{\partial \Phi}{\partial t} = \frac{dw(t)}{dt} * \Theta[D - x(t)], \tag{2.7}$$

which is the windowed time average of the threshold distribution (windowed instantaneous slope along the t-axis in figure 2.1). The partial derivative with respect to the threshold D is:

$$\frac{\partial \Phi}{\partial D} = w(t) * \frac{d}{dD}\Theta[D - x(t)], \tag{2.8}$$

and can be interpreted using Φ's similarity to a distribution function. Its derivative is analogous to the time-localized amplitude probability density of signal $x(t)$. Thus equation 2.8 is the threshold density of $x(t)$ in the moving window $w(t)$. Recall that the discriminator function is the Heaviside function as in equation 2.1 whose derivative is known as the Dirac δ-function. We can expand equation 2.8 as follows:

$$\frac{\partial \Phi}{\partial D} = \phi(D,t) = \int_{-\infty}^{\infty} w(t-s)\delta[D-x(s)]ds = w(t) * \delta[D-x(t)]. \tag{2.9}$$

Up until this point we have not taken into consideration the limitations of a practical measurement device. Equation 2.1 is valid only for an *ideal* discriminator which is capable of infinitely fine comparisons between D and $x(t)$. A realistic choice of discriminator function, referred to as $\mathcal{F}$, would approximate the Heaviside function, but cannot contain a step discontinuity. A function that goes monotonically from 0 to 1 over a finite width of ΔD is one such choice of realistic discrimination function. We will denote such a function $\mathcal{F}_{\Delta D}$. The derivative of this function is finite everywhere. We denote the derivative as $f_{\Delta D}$. These two functions can replace $\Theta[D]$ and $\delta(D)$ respectively.

Another concern is the dependence on choice of initial condition. We would like a solution that converges independent of initial conditions. One way to accomplish this is to add a term proportional to $q - \Phi(D_q, t)$ to the right hand side of equation 2.6 so that we get:

$$\frac{dD_q}{dt} = -\frac{\partial \Phi(D_q,t)/\partial t}{\phi(D_q,t)} + v[q - \Phi(D_q,t)], \quad v > 0. \tag{2.10}$$

The added term ensures convergence because $\Phi(D,t)$ is a monotonically increasing function of D for all t and any initial condition. Therefore, when the output of the median filter is below D_q, the additional term will be positive, and work to increase the output. When the output is above D_q the opposite is true. The parameter v is the characteristic convergence speed and has units of 'threshold per time'.

At this point it is still unclear how a device capable of solving the equation might be designed. However, through the consideration of additional physical properties of such a filter, we can obtain a practical implementation. We will consider filtering effects present in a physical measurement system.

First is the effect of the finite bandwidth of a connecting cable. This does not need to be a physical cable. It could also represent the bandwidth of something like a connecting trace on a circuit board. Either way, we will model the bandwidth restriction as a low pass filter. Electronic filters can be described in terms of a unit step response H_τ. The simplest low pass filter is a first order filter described by the following equation:

$$H_\tau = \Theta(t)(1 - e^{-t/\tau}), \tag{2.11}$$

where τ is the characteristic response time. We choose such a response to model the resistance and capacitance of a connecting cable such that $RC = \tau$. We may also use the derivative of the step response, which is the impulse response:

$$h_\tau = \frac{dH_\tau}{dt} = \Theta(t)e^{-\frac{t}{\tau} - ln\tau}. \tag{2.12}$$

The second filtering effect is from the designed impulse response of the device. We will denote this as w_T. The combined impulse response of the entire measurement system is then the convolution of w_T with equation 2.11:

$$w(t) = h_\tau(t) * w_T(t). \tag{2.13}$$

We have called this overall response $w(t)$ because it is the same time-windowing function introduced in equation 2.3. Recall that the time derivative of this windowing function is part of the differential equation 2.8 for the quantile filter. The derivative with respect to time of the windowing function is:

$$\frac{dw}{dt} = \frac{dh_\tau}{dt} * w_T(t) = \frac{1}{\tau}[\delta(t) - h_\tau(t)] * w_T(t) = \frac{1}{\tau}[w_T(t) - w(t)]. \tag{2.14}$$

We can then substitute equation 2.14 into equation 2.7 and obtain:

$$\frac{\partial \Phi}{\partial t} = \frac{dw(t)}{dt} * \mathcal{F}_{\Delta D}[D - x(t)] = \frac{1}{\tau} w_T(t) * w_T(t) * \mathcal{F}_{\Delta D}[D - x(t)]. \tag{2.15}$$

We substitute equation 2.7 into equation 2.6 to get:

$$\frac{dD_q}{dt} = \frac{\Phi(D_q,t) - w_T(t) * \mathcal{F}_{\Delta D}[D_q - x(t)]}{\tau h_\tau(t) * w_T(t) * f_{\Delta D}[D_q - x(t)]}. \tag{2.16}$$

Convergence to the output of a quantile filter of order q is obtained by substituting $\Phi(D_q, t)$ with q. Which yields the result:

$$\frac{dD_q}{dt} = \frac{q - w_T(t) * \mathcal{F}_{\Delta D}[D_q - x(t)]}{\tau h_\tau(t) * w_T(t) * f_{\Delta D}[D_q - x(t)]}. \tag{2.17}$$

This is equivalent to choosing a speed of convergence of

$$v = \frac{1}{\tau h_\tau(t) * w_T(t) * f_{\Delta D}[D_q - x(t)]}, \tag{2.18}$$

And substituting into equation 2.10.

At this point we still have a generalized description of a practical quantile filter. We would like to create something analogous to a first order median ($q = \frac{1}{2}$) filter, but with nonlinear properties such that we obtain an approximation to an exact median filter. First we will define a discriminator function and it's derivative (called a probe) as a ramp from $-\alpha$ to α as illustrated in figure 2.2. We call α a resolution parameter because 2α gives the smallest distance between two values that our discriminator function can resolve. Note that as $\alpha \to 0$ the discriminator converges to the Heaviside function.

We define $\chi(t)$ as the output of an exact median filter. τ_0 is the time constant of the measurement device which we take to have an exponential response ($w_T(t)$).

$$\dot{\chi}(t) = \lim_{\alpha \to 0} \frac{\frac{1}{2} - \mathcal{F}_{2\alpha}[\chi(t) - x(t)]}{\int_{-\infty}^{t} exp(\frac{s-t}{\tau_0}) f_{2\alpha}[\chi(t) - x(t)]}. \tag{2.19}$$

Note that the denominator of equation 2.19 is problematic due to the convolution integral. However, outside the interval $[-\alpha, \alpha]$, the probe function $f_{2\alpha}(\chi(t) - x(t))$ is equal to zero and can be ignored. Inside the interval the probe takes a value of $\frac{1}{2\alpha}$. This can be brought outside the integral. Finally, the exponential will contribute most of it's value near

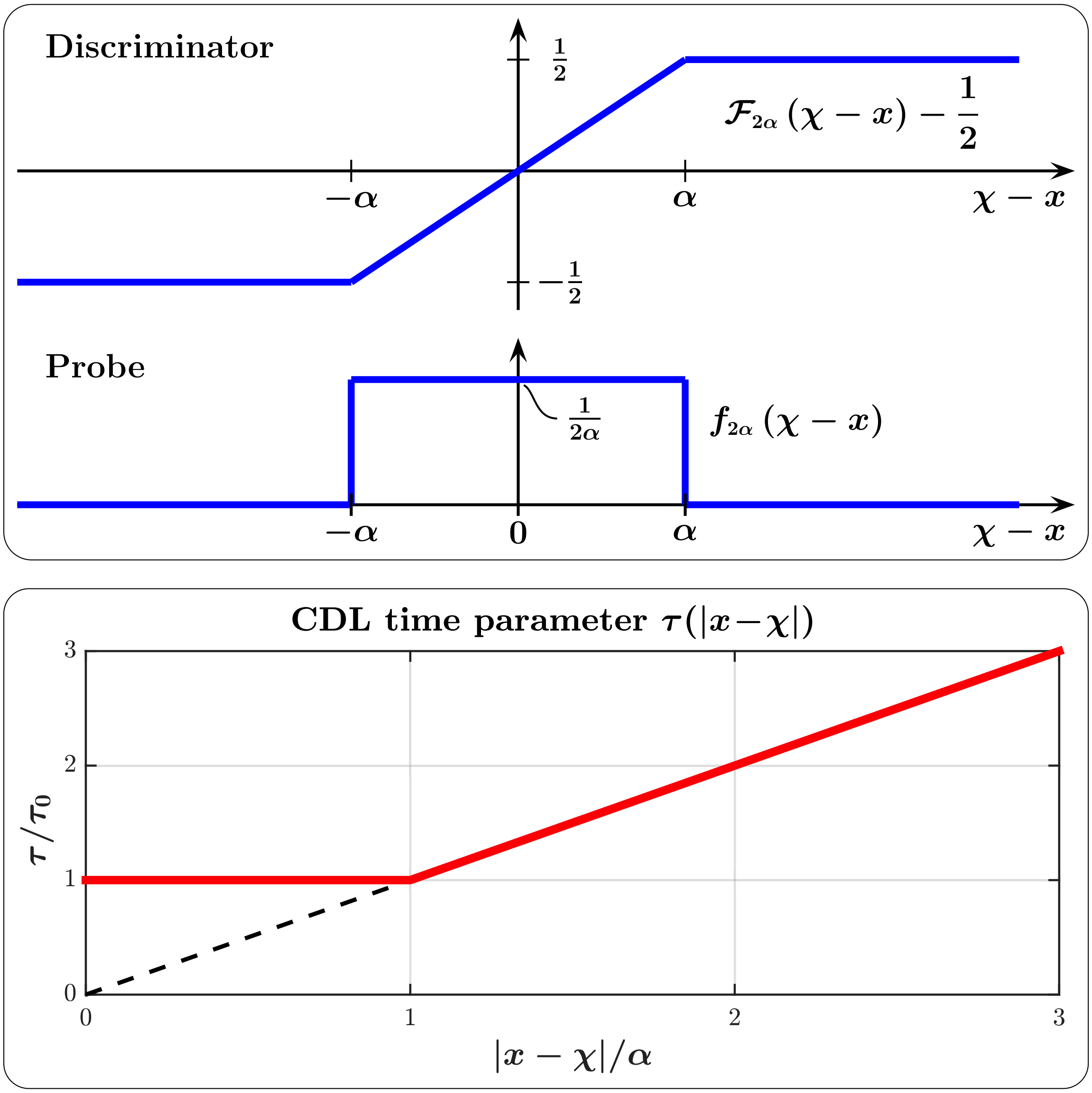


Figure 2.2: *Discriminator and Probe Functions*

$s = t$. For sufficiently large α we can make the simplifying assumption that the integral evaluates to τ_0 because most of the area under the decaying exponential will lay within the limits of the integral. Therefore the denominator of equation 2.19 is replaced with the constant $\frac{\tau_0}{2\alpha}$. We are left with a piecewise defined differential equation. We can restate this equation as:

$$\chi = x - \tau(|x-\chi|)\dot{\chi}, \tag{2.20}$$

where $\tau = \tau(|x-\chi|)$ is given as:

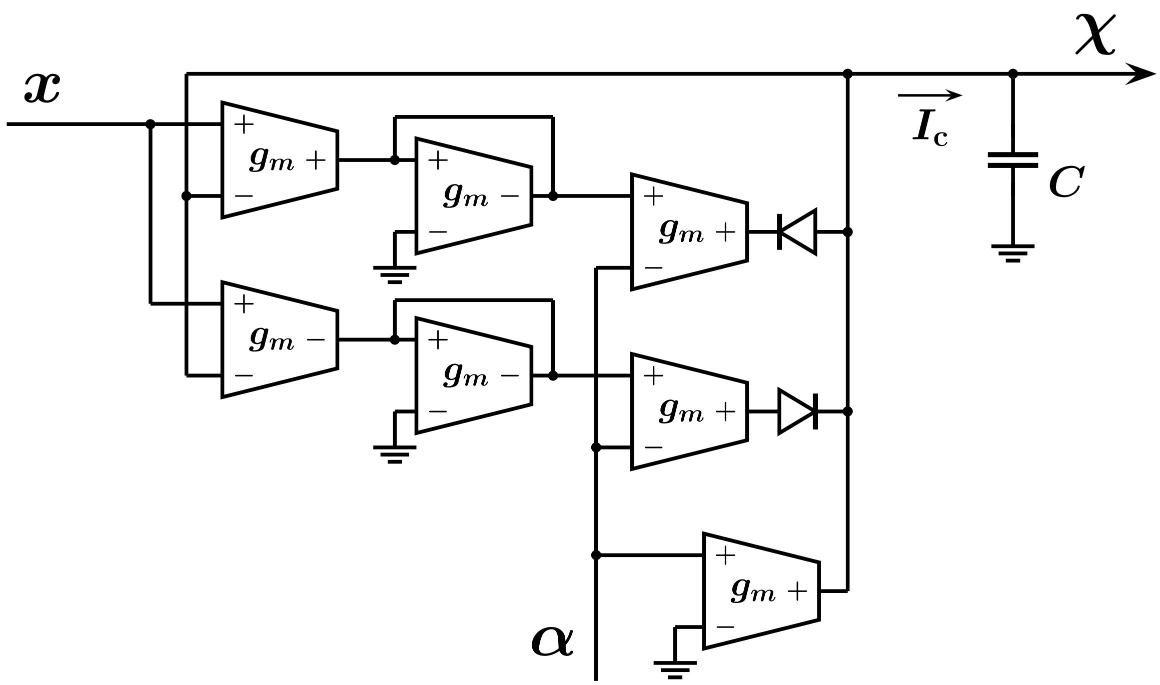
$$\tau(|x-\chi|) = \tau_0 \times \begin{cases} 1 & \text{for } |x-\chi| \leq \alpha \\ \frac{|x-\chi|}{\alpha} & \text{else} \end{cases}. \tag{2.21}$$

In this form we can make sense of the filter in a more intuitive way. We can think about what happens if we use the filter to remove unwanted impulsive outliers from some underlying desired signal. First consider the case where a signal without impulsive outliers is passed into the filter. The difference between the input and output of the filter will be small and thus the filter will behave like a linear lowpass filter with time constant τ_0. However if an outlier enters the filter, the difference between input and output will become large enough for τ to enter its non-constant region. As the difference gets larger, so does τ. A larger τ means that the output of the filter reacts more slowly to changes in the input. Therefore, for the duration of the impulsive spike, the output is not permitted to vary as much as it would in a linear filter. Furthermore, the effect is proportional to the size of the impulsive spike. Larger spikes will induce a greater reduction in rate of change of the output value of the filter. Note that the filter does not simply clip or soft-limit the entire signal. Instead it limits its output based on how far and fast the input deviates from it.

A final consideration that we need to make involves choice of the resolution parameter α. As previously noted, as $\alpha \to 0$ the filter converges to a median filter. However, imposing a strict median filter will end up being detrimental to our signal of interest. On the other extreme, as $\alpha \to \infty$ the CDL converges to a linear filter. There will be some intermediate value of $\alpha = \alpha_0$ for which SNR is maximized but the exact value is dependent on the specific signal and noise mixture. We will revisit choice of α in chapter 3.

2.2 Analog Circuit Implementation

An NDL can be implemented using an arrangement of standard electronic amplifiers along with diodes. Figure 2.3 provides an idealized example of proposed implementation of a 1st order CDL based on operational transconductance ampliers (OTAs). Transconductance cells based on the metal-oxide-semiconductor (MOS) technology represent an attractive technological platform for implementation of such active nonlinear filters as NDLs, and for their incorporation into IC-based signal processing systems (see, for example, [34, 35]). NDLs based on transconductance cells offer simple and predictable design, easy incorporation into ICs based on the dominant IC technologies, small size (10 to 15 small transistors are likely to use less silicon real estate on an IC than a real resistor), and may be usable from the low audio range to gigahertz applications.



$$I_c = \alpha\, g_m \times \begin{cases} \frac{x-\chi}{\alpha} & \text{for } |x-\chi| \le \alpha \\ \operatorname{sign}(x-\chi) & \text{otherwise} \end{cases}$$

Figure 2.3: *OTA-based 1st Order CDL*

The capability for analog implementation is a key advantage of the NDL in that it decouples bandwidth from processor speed. The NDL relies on acting on an input signal with a bandwidth as wide as possible. The wider bandwidth enables impulsive noise to appear as narrowly as possible. Any reduction in input bandwidth will inevitably lead to a spreading of impulsive noise energy across time, thus corrupting other parts of the signal that may have been salvaged otherwise. Processing in the digital domain provides flexibility, but it imposes strict limits on maximum bandwidth available for processing. In the analog domain, maximum bandwidth is a function of the tolerances with which we can construct a device.

In the next chapters we will focus on optimal choice of resolution parameter α. First for stationary signal+noise distributions in chapter 3 and then for non-stationary signal+noise distributions in chapter 4.

Chapter 3

Choice of Resolution Parameter for Stationary Signals

In the previous chapter we introduced a particular type of NDL as a first order approximation of a median filter. A first order filter of this type can be described with two equations (2.20 and 2.21). This leaves two remaining variables to describe a particular filter. The time constant τ_0 and resolution parameter α. In this chapter we will examine the choice of resolution parameter α and its implications for stationary signal and noise distributions. That is, signals whose statistical properties do not vary with time. We wish to choose a value for α such that the signal of interest may pass through the NDL, but at the same time impulsive noise is mitigated as much as possible. The NDL behaves either as a median filter ($\alpha \to 0$) or as a linear filter ($\alpha \to \infty$) which implies that somewhere between these two extremes is an optimal value for the resolution parameter. Here we take optimal to mean that some measure of signal quality (SNR, bit error rate, etc.) is maximized. Our goal is to find the value of α corresponding to this maximum. The stationary assumption of signal and noise implies that once an optimal value for α is obtained, it will remain optimal and therefore: $\alpha(t) = \alpha_0$.

3.1 Resolution Parameter in the Time Domain

We cannot find a frequency response of an NDL because it operates nonlinearly on signals. However, the effect of resolution parameter choice can be better understood by examining its effect on some simple reference signals. A linear first order filter will respond to an impulse with an exponential decay. The NDL does not have an impulse response and cannot be analyzed this way due to its nonlinear properties. This can be seen by considering equation 2.21. Scaling the time constant to infinity leads to a filter with infinitesimal bandwidth. However, we can consider its response to a series of boxcar pulses with decreasing widths and therefore increasing bandwidths.

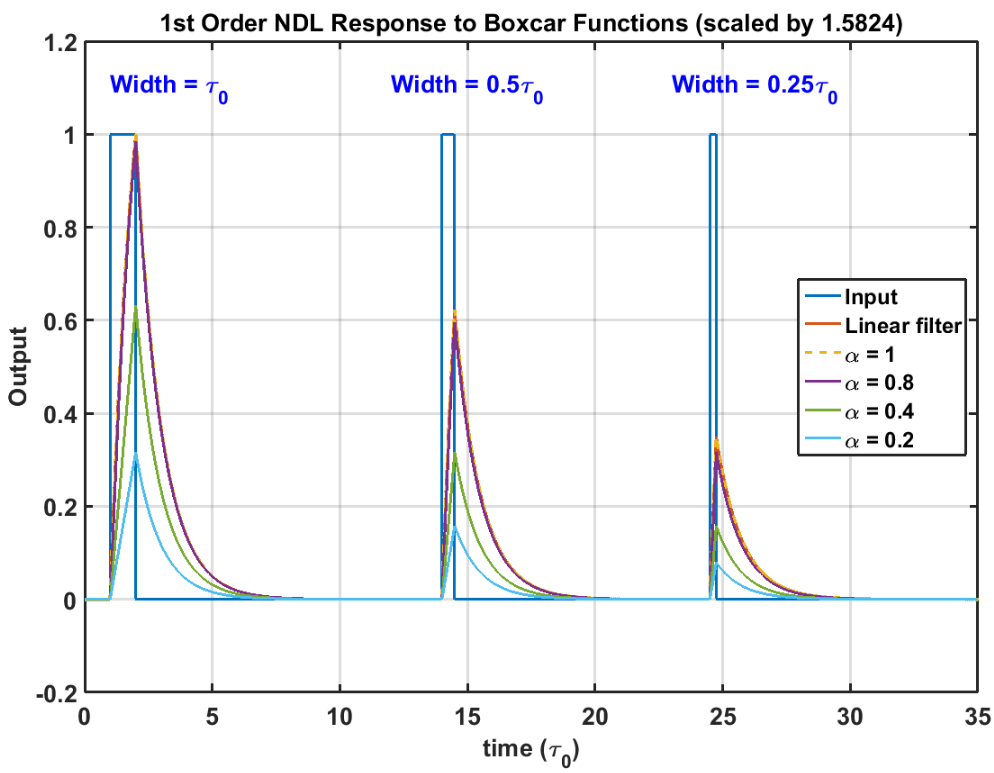


Figure 3.1: *1st Order Boxcar Responses of NDL*

If we input a unit impulse to an NDL with $\alpha = 1$ we expect an identical output to a linear filter due to equation 2.21 (the difference between input and output of the filter does not exceed 1) and this is indeed what we see. Smaller values of α yield proportionally smaller first order decaying responses. If we repeat the test again with a 2nd order NDL, we get a slightly different result. The impulse response is again a scaled version of the linear filter's response (exponentially decaying sinusoid), however the scaling is different. Instead of scaling directly with α, the responses are scaled to α^2.

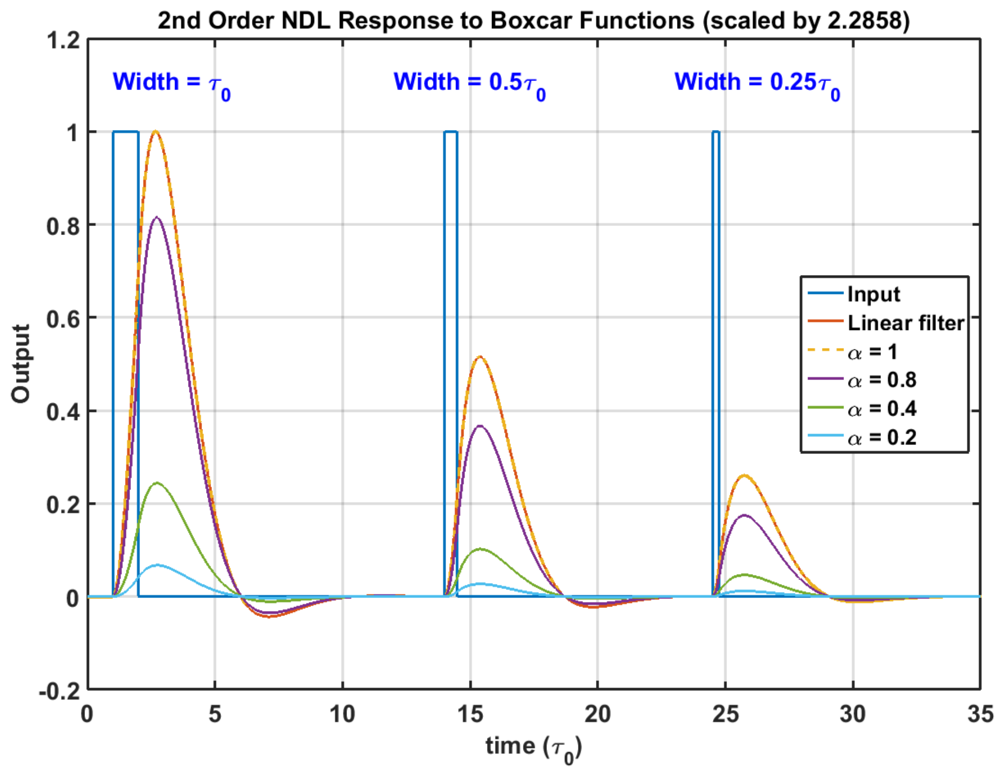


Figure 3.2: *2nd Order Boxcar Response of NDL*

An interesting tradeoff of the higher order filters is that even though they are more effective at reducing the overall energy of the impulse spike, they tend to spread energy more through time. The energy of the first order response is concentrated near the original impulse spike. This can be a crucial consideration when trying to minimize phase distortion.

An NDL does not have a unit impulse response, but it does have a unit step response. Once again we can consider equations 2.20 and 2.21 to understand what it will look like. Because the step function contains only a single change of value and the NDL is a type of lowpass filter, the output should begin at 0 and end at 1. It should behave nonlinearly at first while $|x - \chi| > \alpha$ and then behave linearly as χ converges to 1. The step response can therefore be divided into two regions. First: one where the output varies from the input by an amount greater than α, and second: one where it does not. The duration of the first region will depend on the size of α. A smaller α means a more slowly reaction output, and therefore more time spent in the first region.

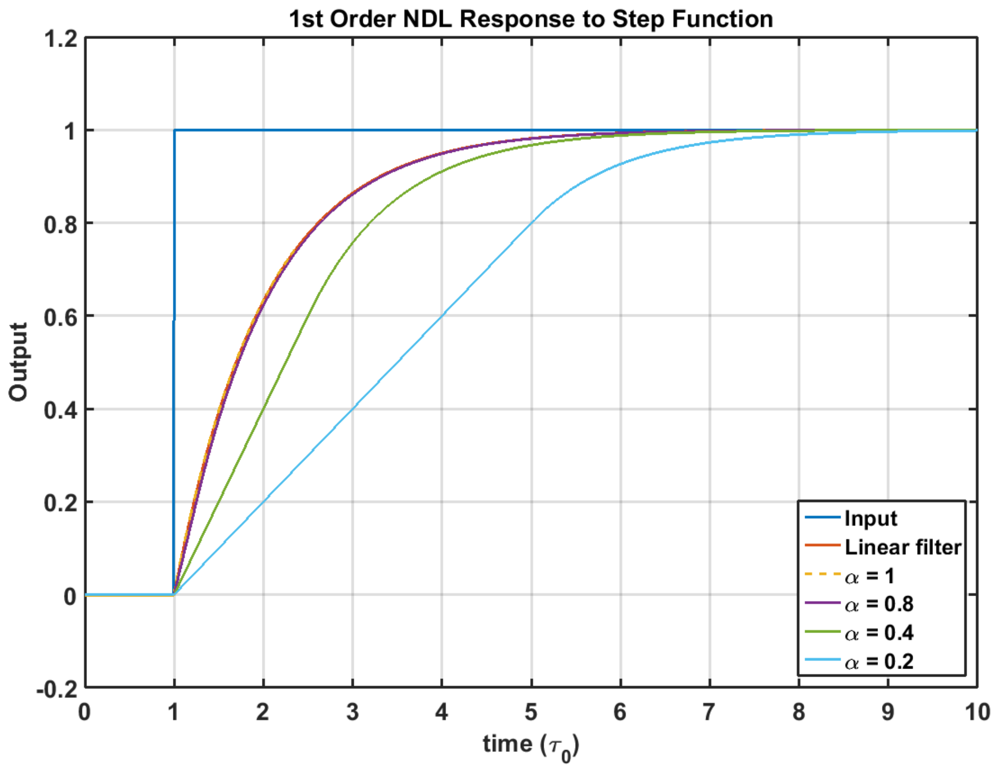


Figure 3.3: *1st Order Step Response of NDL*

If we examine the first order NDL's step response we can see that initially, the output ramps linearly toward 1 and the slope of the ramp decreases with α. After the ramp, the response transitions into that of a linear 1st order lowpass filter and decays exponentially toward 1. We can also examine the step response of a 2nd order NDL.

Instead of a linear ramp in the first region, the output follows a parabolic path. The response then transitions into the damped sinusoidal response of a 2nd order linear filter in the second region.

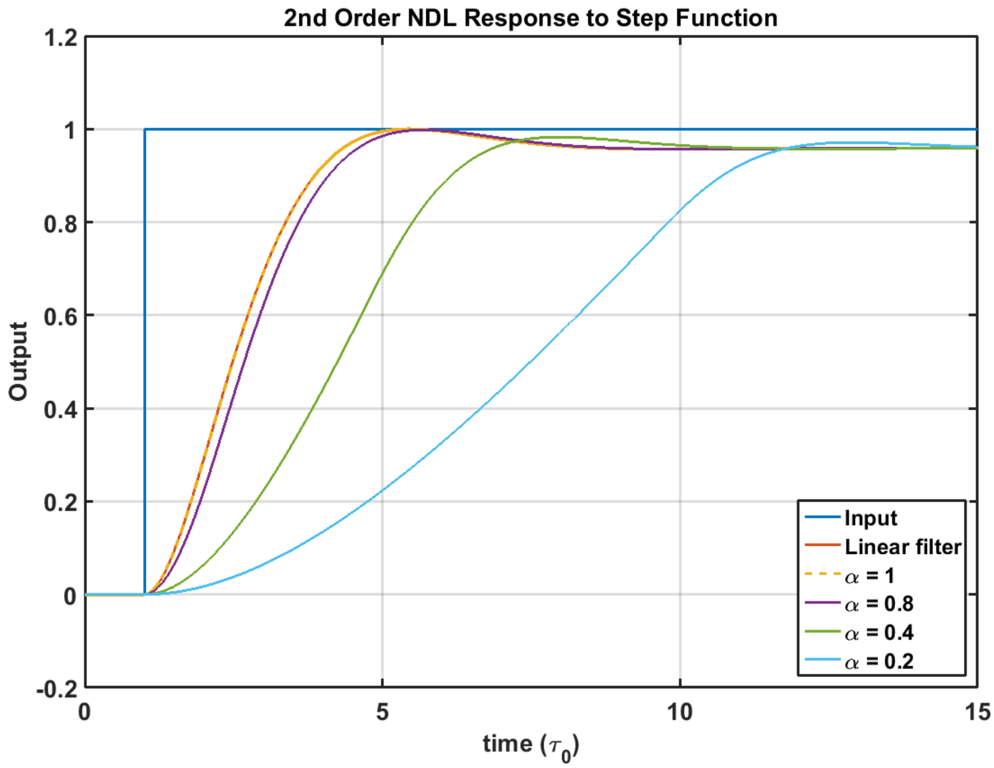


Figure 3.4: *2nd Order Step Response of NDL*

3.2 Choice of Resolution Parameter α

We previously stated that the an optimal value of α is one that will maximize some measure of signal quality such as SNR. A useful way to think about choice of resolution parameter is to treat 2α as the width of an interquantile range filter. An interquantile range filter can be used to remove impulsive noise from a signal of interest because impulsive noise power is concentrated in bursts with short timescales and large amplitudes. The NDL works as an approximation of this type of filter. Practically, we want to define the interquantile range as narrowly as possible without causing more than some maximum level of tolerated distortion in the signal of interest. This will yield maximum attenuation to any noise that enters the system.

In chapter 1 we illustrated how an NDL could be used in place of an anti-aliasing filter, or as a front end filter in general. We will demonstrate resolution parameter tuning through a qualitative example of this type of application. In order to obtain results that are generally applicable to any particular modulation or communication scheme we will not simulate an entire transmit and receive chain. Instead we will simulate signal and noise and directly compute SNR improvement after applying an NDL filter. In order to demonstrate the benefits of using NDLs in as real a setting as possible we use noise models from a setting where impulsive noise is a recognized and well studied issue.

3.3 Application to Powerline Communications

One target application area for NDLs is in Powerline Communications (PLC). In PLC, the electrical power distribution grid is used as a channel for communications. The benefit of doing this is to provide a communication medium for various sensors and control equipment required for the deployment of various smart grid technologies. With PLC, utilities eliminate the need to run additional wires and instead re-use the existing power distribution lines. However, the long lengths of unshielded wire act as antennas and large amounts of radiated noise infiltrates the channel. Also the switching noise from electrical devices creates large transient voltage spikes. Studies of the noise present in PLC channels have found that much of the noise is impulsive in nature. The mitigation of such noise is therefore an area of interest.

3.3.1 Description of Model

We are interested in a generic evaluation of performance invariant to the choice of modulation scheme. Therefore we shall use white Gaussian noise bandlimited to $42 - 89$kHz (used by the PRIME specification in the CENELEC A band [36]) as the signal of interest. The noise will consist of a mixture of three components:

1. A Gaussian component with power spectral density decaying at approximately 30dB per 1 MHz.

2. Cyclostationary exponentially decaying noise with a repetition period of twice the AC frequency and with a duration ranging from about $100\mu - 10ms$.

3. Impulsive bursts (duration of several μs) with normally distributed amplitudes with an expected arrival time of $100\mu s$.

This is a generalized model for the type of noise typically encountered in a PLC channel. In the following demonstration, the power of first noise component is held constant while the impulsive components are increased. The power ratio of cyclostationary to impulsive burst noise is also held constant. This means that the overall SNR of the input signal decreases, while the impulsivity of the noise mixture increases.

The NDL filter used consists of a CDL stage with a time constant $\tau_0 \approx 0.9\mu s$ followed by a linear 2nd order filter stage with the same time constant $\tau = tau_0$ and quality factor of Q=1. Overall this gives us a 3rd order NDL (so-called because it converges to a 3rd order Butterworth filter) with a cutoff frequency of 178kHz as $\alpha \to \infty$.

We wish to vary the resolution parameter and observe the SNR of the filtered signal in order to identify a maximum SNR. We will do this by applying a multiplicative constant, referred to as Gain G, to some measure of central tendency of the signal that we use as a base value of α_0. From this we obtain $\alpha(t) = G * \alpha_0$. The measure used in this case is the median of the absolute value of the signal. We choose the median in order to minimize the effect of impulsive outliers. Any signal centered at zero with peak to peak width less than or equal to 2α will pass through the NDL linearly. This implies that $G \approx 2$ will yield near-optimal results.

The solid lines in figure 3.5 show the SNR of the output of the NDL while the dashed lines show the SNR of the output of an equivalent Butterworth filter. This figure demonstrates the NDL's ability to preserve performance in the presence of impulsive noise. Furthermore, it can be seen that the NDL converges to the same performance as a linear filter as $\alpha \to \infty$ which does no "harm" to the signal when impulsive noise is not present. The NDL is most effective at providing SNR improvement over linear filters when impulsive noise is most

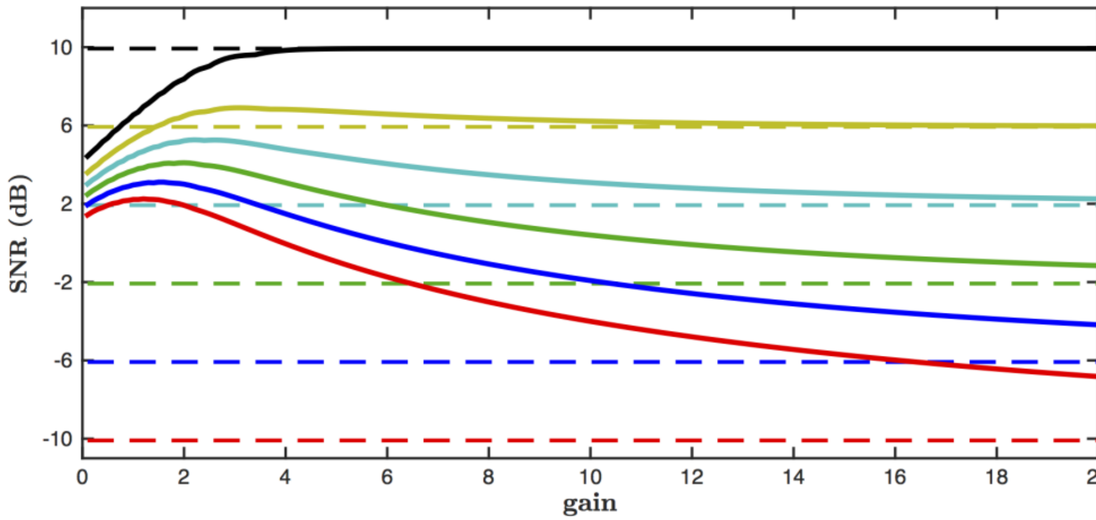


Figure 3.5: *SNR Improvement in PLC Channel*

prevalent. In the worst case example of -10dB SNR, the NDL improves performance by more than 12dB over the Butterworth filter.

We can examine the effect of different noise compositions in figure 3.6 by fixing the resolution parameter to correspond to maximum SNR. Then we vary the total noise power holding the relative levels of thermal and impulsive noise constant. We do this for three different noise compositions each with different levels of impulsivity. The black trace shows SNR for an equivalent linear filter. Performance of the linear filter is invariant to noise composition and varies only with total noise power. The different noise compositions are as follows:

- Green: Narrow ($\approx 100\mu s$) cyclostationary noise bursts
- Blue: Mid-range ($\approx 500\mu s$) cyclostationary noise bursts
- Red: Wide ($\approx 2.5ms$) cyclostationary noise bursts
- Black: Linear filter SNR

In figure 3.6 we can see the effect of varying noise impulsivity. In the top subfigure, we

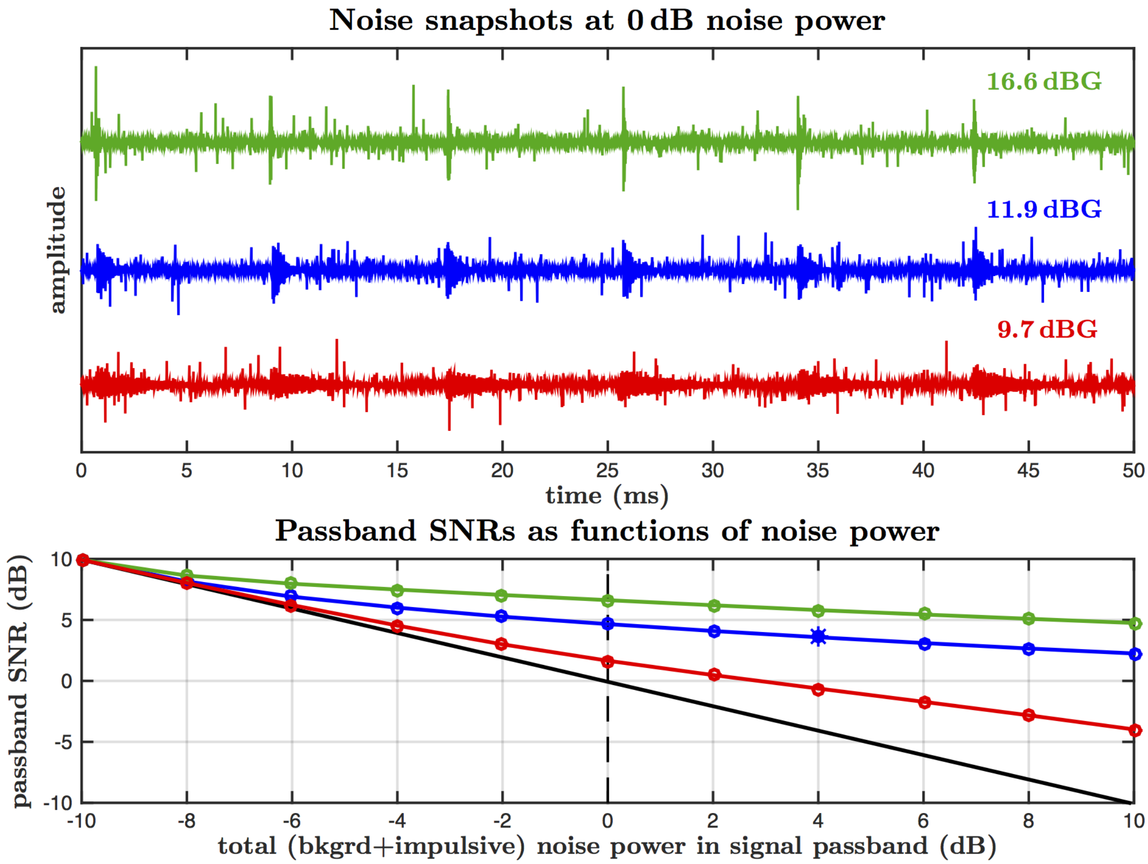


Figure 3.6: *PLC SNR versus Noise Composition*

examine the signal traces and see that when noise is more impulsive, spikes of noise arrive less frequently, but tend to be larger in amplitude. In the bottom subfigure, the NDL is shown to preserve channel integrity in the presence of impulsive noise. The effectiveness of the NDL increases when the noise is more impulsive, but performance will never decrease compared to a linear filter as long as α is chosen properly. Impulsivity or peakedness is quantified in terms of kurtosis, the fourth centralized moment. The units (dBG) are decibels relative to Gaussian.

$$\text{Kurtosis(x) (dBG)} = 10\log_{10}\left[\frac{\text{Kurtosis(x)}}{\text{Kurtosis(y) where y} \sim \mathcal{N}(0, Var(x))}\right] \tag{3.1}$$

Figure 3.7 shows a trace extracted from the signal before (red line) and after (blue line)

NDL filtering. The green line is affected by thermal (Gaussian) noise only. In the absence of impulsive noise, the green trace is not altered by the NDL. However, impulsive noise bursts are mitigated by the NDL.

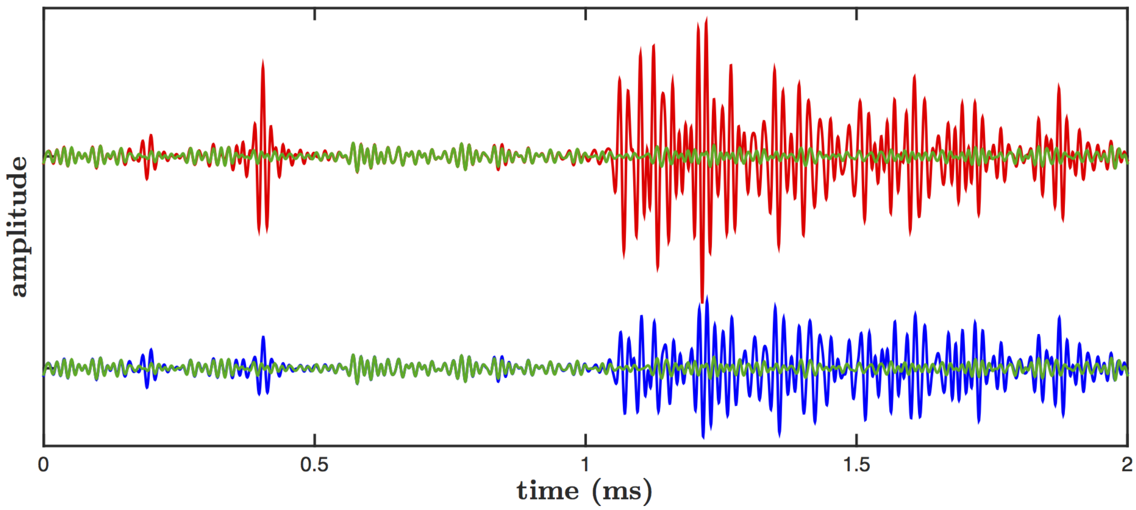


Figure 3.7: *PLC Channel Filtered Signal Traces*

In this chapter we have illustrated how NDLs can be configured to act as a type of interquantile range filter. This makes them useful for filtering unwanted, impulsive outliers from signals which is an important application in areas such as PLC. In the next chapter we will investigate how to choose the resolution parameter α under non-stationary noise and signal conditions and construct an Adaptive NDL or ANDL.

Chapter 4

Choice of Resolution Parameter for Non-Stationary Signals

Most communications systems rely on the assumption of stationarity of the underlying signal and noise distributions. However, there are situations where this assumption does not hold. Speech signals, for example, can be highly non-stationary. In chapter 3 we considered applying an NDL in a stationary signal and noise environment. In this chapter we will consider the more difficult problem of choosing the resolution parameter in a non-stationary or dynamic environment. In doing so we introduce another variant of the NDL, the Adaptive NDL or ANDL. Additionally, we will no longer rely on the assumption that we have access to a pre-calculated measure of central tendency. The principal difference between the NDL and ANDL is that we no longer provide a value for the resolution parameter α to the filter. Instead, the filter determines the value of α independently.

4.1 The Adaptive NDL

When working with nonlinear filters, it is not always straightforward to derive an optimal choice of design parameters. However, we can come up with some design guidelines which will help us design a more effective filter. The primary parameter of interest with regard to the ANDL is the resolution parameter α. For the ANDL, this parameter is a function of time ($\alpha = \alpha(t)$) in order to accommodate the changing properties of the signal and noise. Our general strategy for determining α is to generate a value from the input signal. For stationary, or nearly stationary, signal and noise distributions we can keep a piece-wise constant value of α over intervals shorter than or equal to the coherence time T_C of the channel. If T_C is large and values of α remain optimal for a relatively long time, then spending resources optimizing α becomes justifiable. For instance, computing the SNR over a range of values as was done in chapter 3 can take a long time, but we may gain more capacity due to SNR improvement than is consumed by the search process. The expression for α can be expressed as

$$\alpha(t) = \alpha_i \quad iT_C \leq t < (i+1)T_C \quad i = 0, 1, 2, \ldots \quad . \tag{4.1}$$

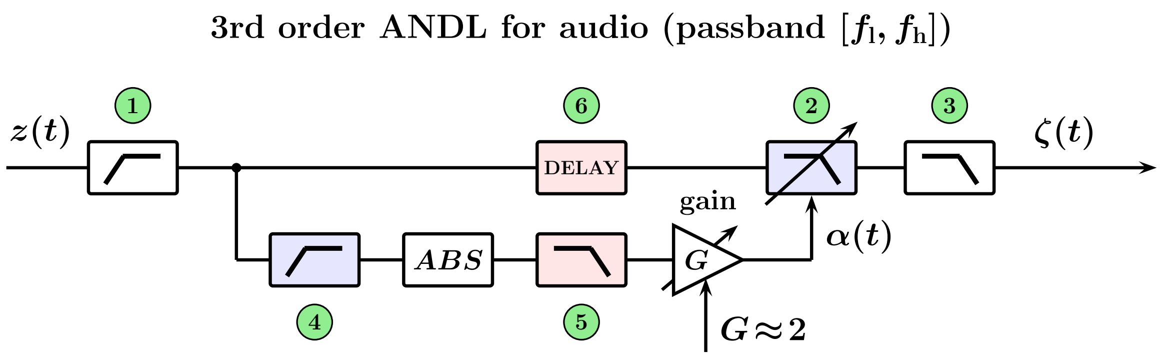
It may also be the case that the signal has too short a coherence time in order to make this approach feasible. In this case we estimate α through the use of a sub-circuit.

4.1.1 ANDL Subcircuit Design

The sub-circuit will output a time-varying, localized estimate of central tendency which we can scale with gain G to obtain α. Figure 4.1 illustrates how an ANDL can be constructed from an NDL with additional components used to generate an α estimate. This particular ANDL operates as a bandpass filter for use in removing impulsive noise from audio signals.

Though this ANDL is targeted for audio applications, the design will be extendible to other frequency ranges. Audio will provide a useful test case because of its relatively low bandwidth requirements and because filtering can be readily demonstrated by playing the audio back.

The ANDL is composed of several different subsections configured as shown in figure 4.1.



(1) 1st order highpass with $\tau = \frac{1}{2\pi f_l}$

(2) 1st order CDL with $\tau_0 = \frac{1}{2\pi f_h}$

(3) 2nd order lowpass with $\tau = \tau_0$ and $Q = 1$

(4) 1st order highpass with $\tau = \tau_0$

(5) 2nd order lowpass with $\tau = \tau_B \gg \tau_0$ and $Q = 1/\sqrt{3}$ (Bessel)

(6) Delay $\tau_d = \tau_B \sqrt{3}$

Figure 4.1: *Block Diagram of 3rd Order Digital ANDL*

Overall, the ANDL behaves as a bandpass filter. The low and high corner frequencies of the ANDL are chosen as f_l an f_h respectively. A time-varying signal $z(t)$ is the filter's input and $\zeta(t)$ is its output. We call this a 3rd order ANDL because it uses a first order CDL (2) cascaded with a linear 2nd order filter (3) for an overall 3rd order response as a low pass filter.

Starting at the input, there is a linear high pass filter (1) with a corner frequency equal to the lower corner frequency of the overall bandpass response of the ANDL. After this first filter, the signal chain splits into two branches. The lower branch will be used to generate a time varying value for α, the resolution parameter, and the upper branch will be the signal path. The signal path contains a delay line (6) in order to compensate for the for the delay imposed by filters (4) and (5), but is otherwise a the same NDL seen in chapter 3.

Let's examine the α estimation branch. We wish to find a time varying value of α to be used by the CDL (3). In general, we would like a value of α that is sensitive only to the instantaneous energy of the signal content in $z(t)$, not the noise. The α parameter should become larger when there is a great amount of signal energy present. This will allow the signal to pass through the filter in its linear mode more easily, but ideally keep α as small as possible in order to most effectively mitigate impulsive noise.

The first component of this branch is a first order highpass filter 4. We use this filter as a differentiator in order to get a measure of the peak-to-peak amplitude of the incoming signal. The filter is first order so that the signal attenuation in the passband is minimized. Next, we take the absolute value in order to remove dependence on polarity. Recall that α is strictly positive.

The final block in the α determination branch is a 2nd order low pass filter 5 with a time constant of τ_B and $Q = \frac{1}{\sqrt{3}}$. This filter will perform a time-windowed average. We use a Bessel filter in order to get a flat delay spread across all frequencies. τ_B can be chosen according to the stationarity of the impulsive noise. A smaller time constant will allow α to adapt quickly to changes in the amount of signal energy, whereas a larger time constant will allow for a better estimation of the average signal energy level across longer timescales. We wish for α to be invariant to impulsive noise spikes. The averaging

The final subsection of the α determination branch is an amplifier. We will use this to adjust α by a multiplicative gain factor G. In general G may be a function of time, but we will consider it to be a constant. In order to find a reasonable value for G we can use an example. Let's consider what happens when we input a signal without noise that is Normally distributed and bandlimited to fall within the passband of the ANDL. The distribution of the signal is invariant when filtered linearly due to the properties of Gaussian random variables. Also, the highpass filter removes any DC offset and centers the distribution at zero. The next operation performed is to take an absolute value. If our input distribution is some random variable X, then we have now obtained another random variable $Y = |X|$. Y is distributed as a half-normal random variable. The next filter is a lowpass averaging filter that will output a time windowed expectation of Y. This gives us a way to relate to the standard deviation σ of the input signal X as shown in equation 4.2:

$$\sigma_X(t) = \sqrt{\frac{\pi}{2}}\mu_Y(t). \tag{4.2}$$

We wish to set the resolution parameter as small as possible while still allowing most of the signal to pass through unchanged. Recall that 2α is the width of the interquantile filter that the NDL approximates. If we set the range as equal to 4σ, or 2σ about the mean, then 95% of the signal should pass through unchanged. We can solve for a gain that ensures this like so:

$$\alpha(t) = 2\sigma_X(t) = 2\sqrt{\frac{\pi}{2}}\mu_Y(t) \approx 2.5\mu(t) \Rightarrow G = 2.5. \tag{4.3}$$

Next we will verify our reasoning with some simulations of the ANDL.

4.2 ANDL Simulations

We would like to confirm the theory discussed in the previous section with a simulation of the ANDL. We will start with a test of the α estimation subcircuit.

4.2.1 Pure Signal Input

From the relationship between the the mean of the half-normal and standard deviation of the corresponding normal distribution shown in 4.2, we know that for an input that is distributed normally with a standard deviation of $\sqrt{\frac{\pi}{2}}$, the subcircuit should output a value of unity. The speed at which the subcircuit converges to this value is a function of the time constant of filter 5, τ_B. In figure 4.2 we plot the convergence of the α estimate to unity for three values of τ_B.

The estimate indeed converges to unity. The speed of convergence is governed by τ_B. Larger values of τ_B have slower convergence times, but are more stable once they have settled on a value. In any case, care must be taken to set τ_B according to the specific application.

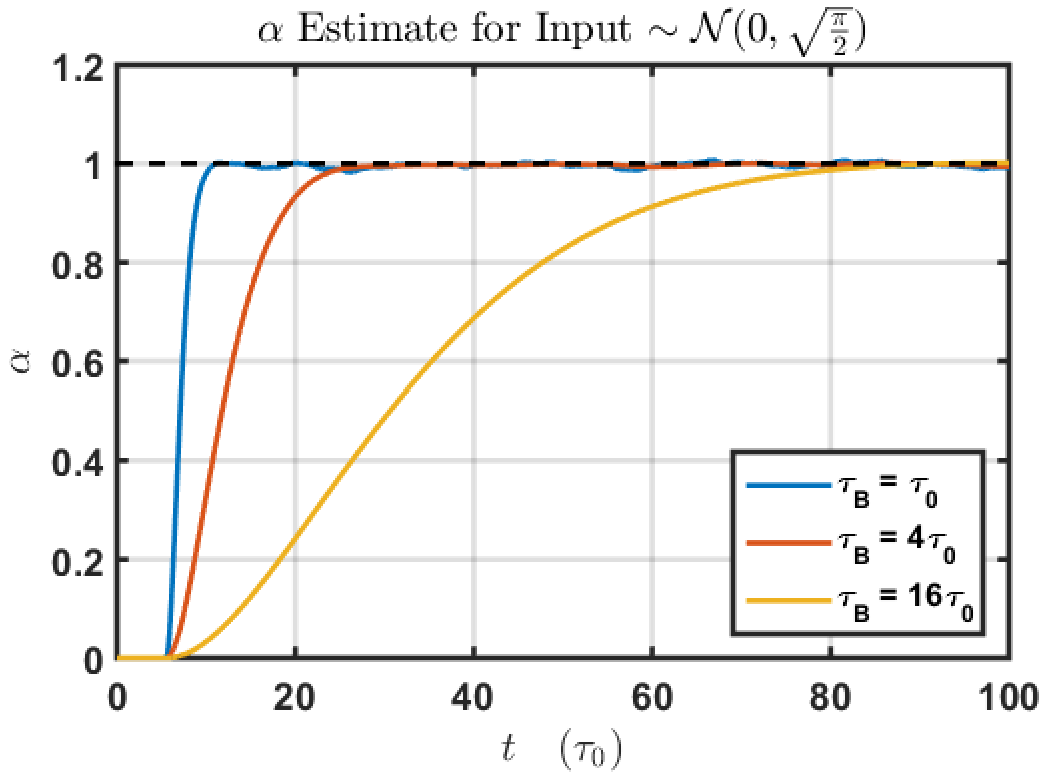


Figure 4.2: α *Estimate Convergence for Pure Signal Input*

For instance, when filtering a speech signal, the choice of τ_B would be a trade-off between being fast enough to transition from silence to speech, and long enough to average across individual phonemes.

4.2.2 Signal and Thermal Noise Input

If unity is the ideal value set when we consider the signal by itself, we can evaluate how much the measure of α is offset when corrupted by noise. We cannot discriminate the signal from thermal noise as they are identically distributed. However, we can compare how much the α estimate is affected by thermal noise versus impulsive noise. Ideally, the estimate should be less affected by impulsive noise.

The addition of thermal noise with power equivalent to the power of the signal increases the estimate of α by roughly 1.4 times the baseline of 1 as seen in figure 4.3. A larger value

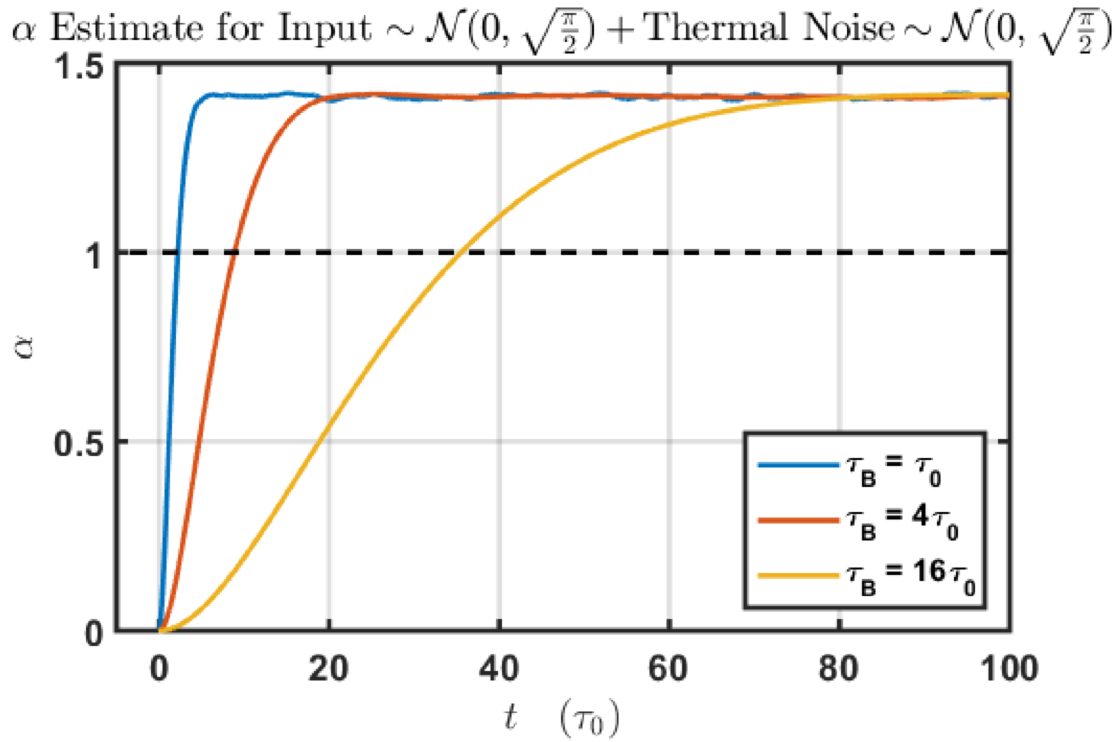


Figure 4.3: *α Estimate Convergence for Signal+Thermal Noise Input*

will allow both the signal and noise to pass through the ANDL as if it were a linear filter which is ideal because we cannot discriminate between the two.

4.2.3 Signal and Noise Input

We can also observe what happens to the estimate when we input a signal that is corrupted by impulsive noise.

We observe in figure 4.4 that the estimate is less affected for impulsive noise and only increases to about 1.1 times the baseline value. Another observation is that for small τ_B, the impulsive noise spikes cause more deviations about the average.

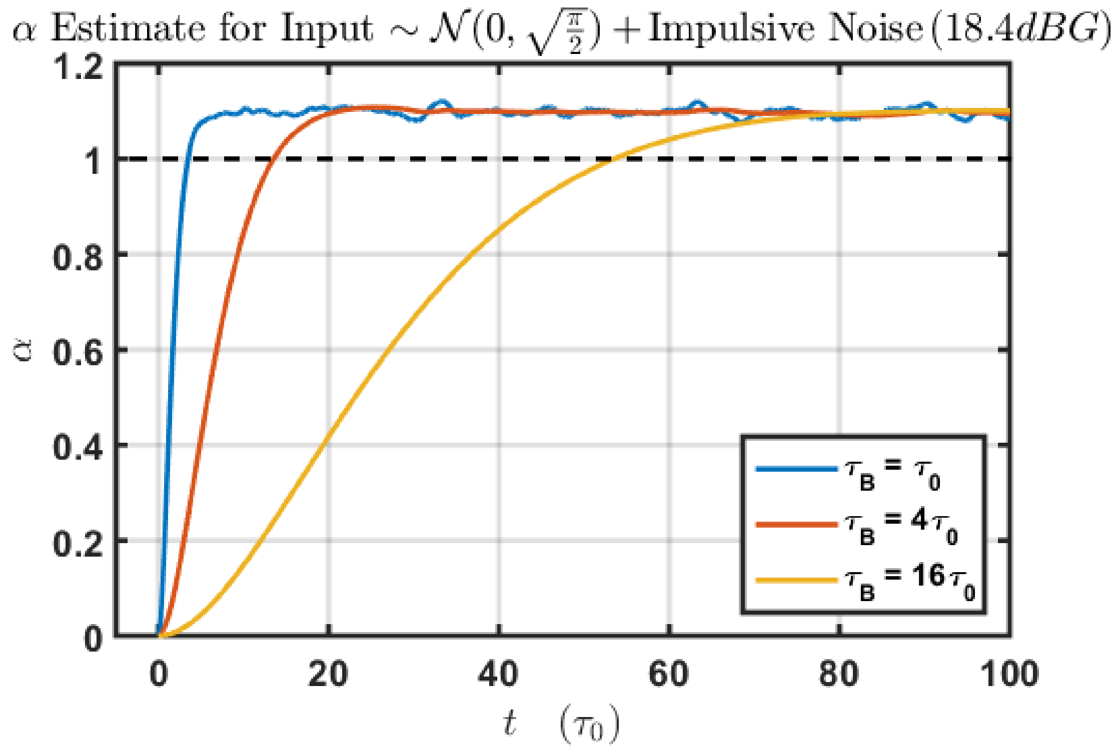


Figure 4.4: *α Estimate Convergence for Signal+Impulsive Noise Input*

4.2.4 Speech Signal Input

We will use speech as an example of a non-stationary signal input that can be encountered in the real world. The most basic unit of speech is a syllable and for this example we will look at the syllable "ta". The time constant of filter 5 controls the width of the averaging window for the α estimation sub-circuit. If this window is too large, α will not respond to the change in signal energy level fast enough and the signal will be distorted. We wish to make the window as large as possible in order to minimize the sensitivity to impulsive outliers. However, the larger the window is, the longer it will take the sub-circuit to "notice" that signal is present. For a speech signal this will translate to the beginning of sounds being distorted. In the following simulations we use a time constant of 0.25ms for the Bessel filter.

We have chosen to plot the α estimate against the absolute value of the signal in figure 4.5. The estimate should form an envelope for the signal magnitude. The delay line aligns the two traces.

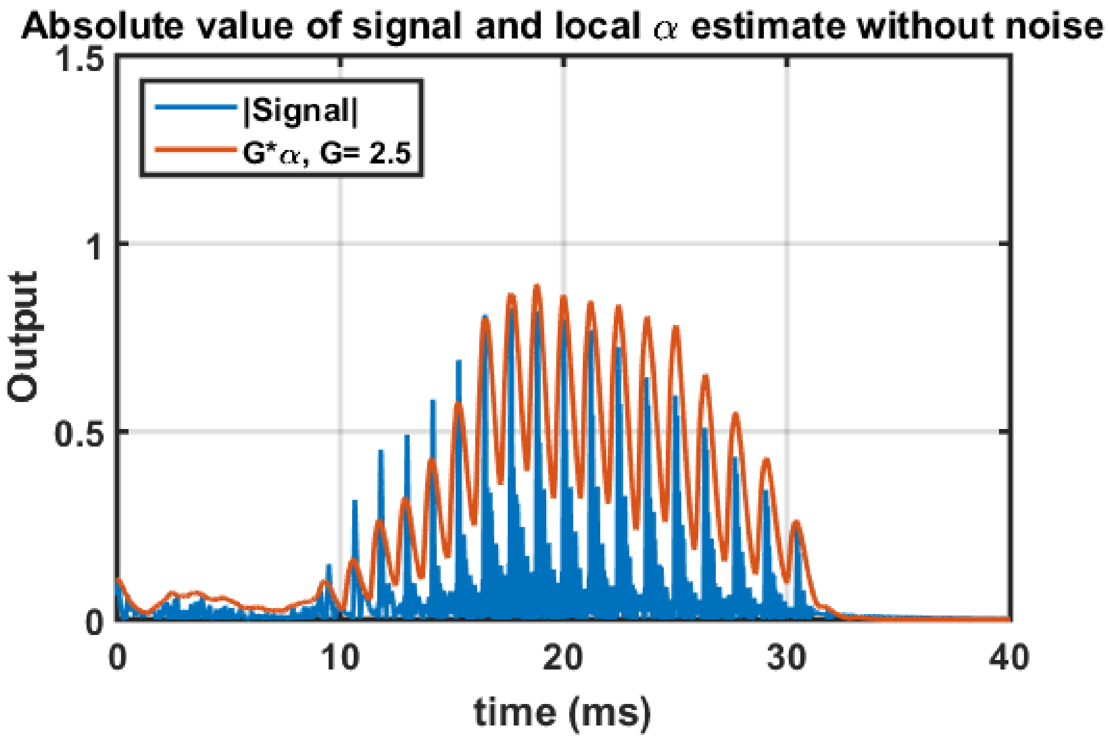


Figure 4.5: *Resolution Parameter Estimate for Speech Signal*

The subcircuit is working nearly ideally. The orange trace forms an envelope for the blue trace meaning that the voice signal is processed linearly, but also that the ANDL is on the threshold of its nonlinear regime. The next plot shows the same signal except with impulsive noise added.

We cannot perfectly eliminate the effect of noise on the α estimate. However, the blue trace remains below the orange trace in the absence of an impulsive noise spike. Also, the noise spikes invariably protrude beyond the value of α. This means that they will be filtered nonlinearly by the ANDL.

In this chapter we have proposed an extension to the NDL for use in non-stationary environments. We call this the ANDL because it can adapt to changing signal parameters with a time-varying estimate of the resolution parameter. In the next chapter we implement the ANDL on a digital signal processing platform.

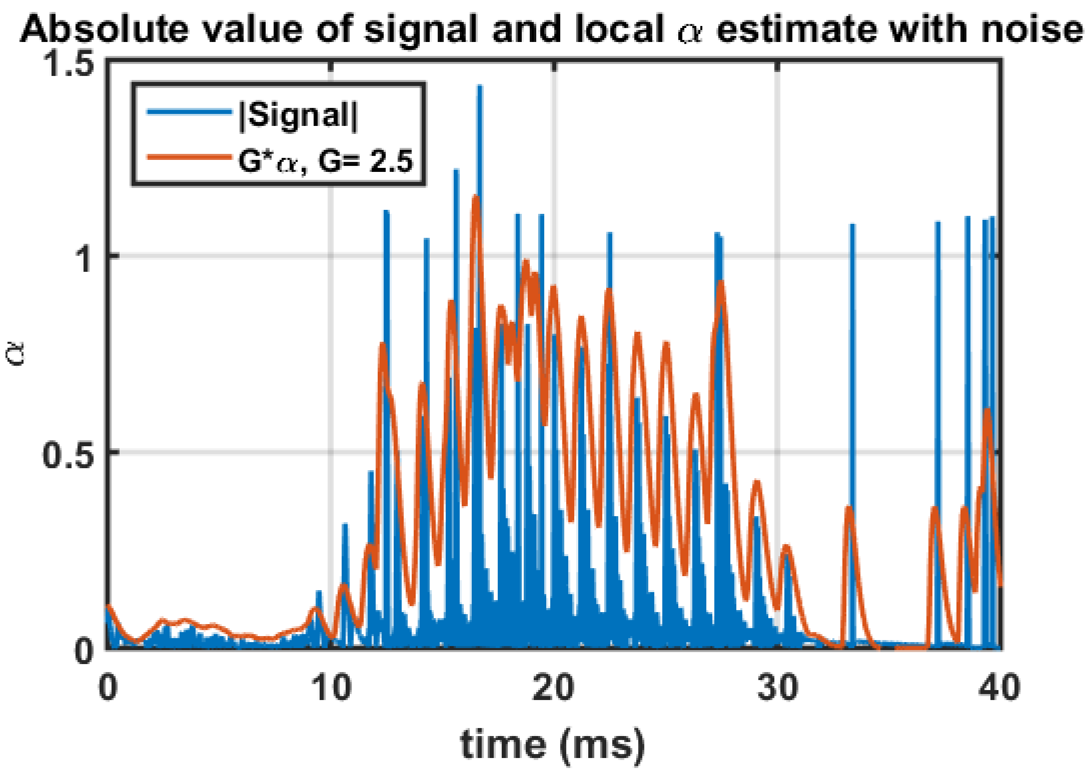


Figure 4.6: *Resolution Parameter Estimate for Speech Signal and Noise*

Chapter 5

A Digital Implementation of the ANDL for Audio

In this chapter we develop a digital implementation of an ANDL filter. Specifically we will develop the filter on the ADSP-21469 EZ-Board along with the Blackfin/SHARC USB EZ-Extender board as our hardware platform. CrossCore Studio will be used as our development environment.

A digital implementation provides the opportunity for rapid prototyping. Filter parameters can be reconfigured at will which enables performance evaluation to be done quickly and efficiently versus creating new analog designs each time we wish to alter the filter. The key disadvantage is the bandwidth requirement for the ANDL. As mentioned previously, the ANDL operates most effectively when the input signal is of as wide a bandwidth as possible. In digital systems, typically the smallest bandwidth possible is used in order to avoid additional unnecessary computations. The ANDL will function best when acting on a signal sampled at roughly an order of magnitude greater than the highest frequency in the passband of the signal. In practical systems, it may not be feasible to increase the sampling rate by this wide of a margin. However, this does not mean that digital ANDLs can only be practical in a laboratory environment. We can take advantage of a trade-off between speed and precision. DSP hardware can operate at higher sampling rates if the precision of each sample is reduced. Interpolation and decimation filters allow conversion between high rate, low precision and vice-versa. Therefore up-sampling ahead of the ANDL and down-sampling afterward provides a way to mitigate this drawback of the digital ANDL implementation. Furthermore, the price of processing power continues to decline.

5.1 Hardware Platform Description

The ADSP-21469 EZ-Board is a debug/evaluation board for the ADSP-21469 SHARC DSP. Some relevant technical specifications of the ADSP-21469 are its maximum clock performance of 450MHz and support for 32-bit floating point arithmetic. The processor also has dedicated hardware for FIR and IIR filtering and circular buffering along with other hardware features for DSP applications. The expansion interface provides a direct connection to many of the DSP's signals. Through this interface we can connect a USB peripheral in the form of an EZ-Extender Board. This board allows us to interface with a PC through USB at high speeds and is used for reading and writing data. A picture of the board can be seen in figure 5.1.

5.2 Software Platform Description

The coding environment used is Analog Device's CrossCore Embedded Studio (CCES). CCES is an Eclipse based IDE that can be used to write and debug code for the ADSP-21469. Through the addition of Board Support Packages, useful example programs can be added and modified to suit our design goals. CCES also allows for simple porting of code between different Analog Devices processors through configuration of the compiler.

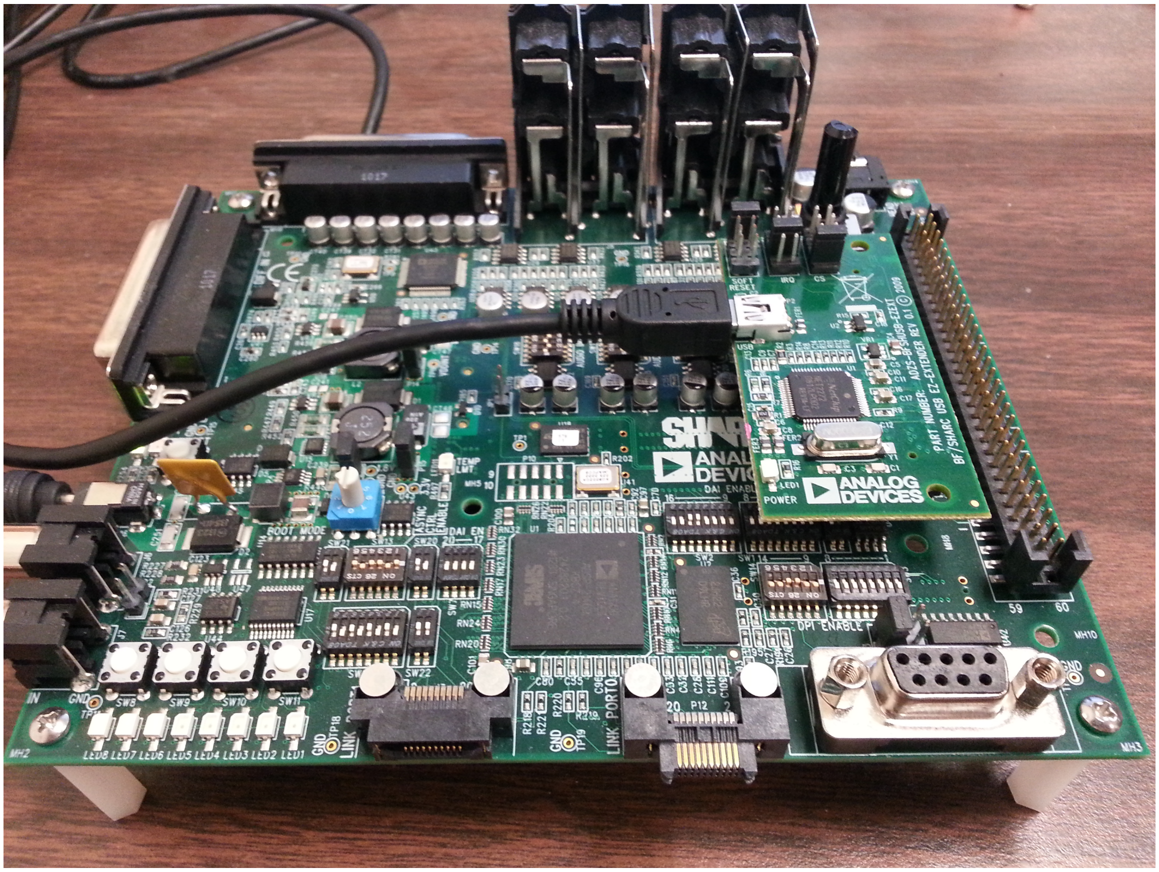


Figure 5.1: *SHARC DSP Development Board with USB Extender*

5.3 Block Component Design

We would like to implement the ANDL filter introduced in chapter 4. From the block diagram of the ANDL in figure 4.1, we can see that we have several linear filters to implement. We have chosen to use IIR filters as they are less resource intensive than FIR filters. In total we will use four linear filters. Two of these will be first order highpass filters (1, 4) and two will be second order lowpass filters (3, 5). Finally the first order CDL 2 will be derived from a first order lowpass linear filter.

We can derive a first order high pass filter from a first order low pass filter with the following transformation:

$$y_{HP} = x - y_{LP} \tag{5.1}$$

With this, we have simplified our design task to the design of only two types of filters, a first order highpass and a second order lowpass filter.

5.3.1 Filter Derivations

We have chosen to implement the linear filter subsections of the ANDL as first and second order infinite impulse response (IIR) filters. IIR filters can be described in terms of a difference equation such as:

$$y[n] = b_0x[n] + b_1x[n-1] + ... + b_Px[n-P] - a_1y[n-1] - a_2y[n-2] - ... - a_Qy[n-Q] \tag{5.2}$$

Where the order of the filter is $max(P, Q)$. Therefore we need to determine the filter coefficients to design the filter.

Blocks 1, 2, and 4 of the ANDL will be based on a first order filter. We start from a first order differential equation for a low pass filter:

$$y = x - \tau\dot{y} \tag{5.3}$$

Next we discretize the equation:

$$y_{i-1} = x_{i-1} - \tau\dot{y}_{i-1} \tag{5.4}$$

Once we have the discretized form, we can substitute the derivative terms out with a first order finite difference equation. This gives us an approximation of the derivative around the sample, and converges to the true derivative as $dt \to 0$. In a digital system, dt is equal to the sampling period. We cannot make dt equal to 0, but as long as it is about an order of magnitude shorter than τ, the approximation will be good enough.

$$\dot{y}_i = \frac{y_i - y_{i-1}}{dt} \tag{5.5}$$

After the substitution we can solve for y_i in order to obtain a difference equation in standard form:

$$y_i = \frac{dt}{\tau + dt} x_i + \frac{\tau}{\tau + dt} y_{i-1} \tag{5.6}$$

This concludes the design of the first order highpass filters. Next we follow the same procedure to design the second order low-pass filters in the ANDL implementation. We start with a second order low-pass differential equation:

$$y = x - \tau \dot{y} - (\tau Q)^2 \ddot{y} \tag{5.7}$$

We then discretize:

$$y_{i-1} = x_{i-1} - \tau \dot{y}_{i-1} - (\tau Q)^2 \ddot{y}_{i-1} \tag{5.8}$$

In addition to the finite difference formula for a first derivative, we also need a finite difference for a second derivative. We will use the central difference formula.

$$\ddot{y}_i = \frac{y_{i+1} - 2y_i + y_{i-1}}{dt^2} \tag{5.9}$$

After substitution and solving for y_i we arrive at a difference equation for a 2nd order low pass IIR filter:

$$y_i = \frac{1}{\frac{\tau^2}{dt^2} + \frac{\tau}{2Qdt}} \left[x_{i-1} + y_{i-1} \left(\frac{2\tau^2}{dt^2} - 1 \right) + y_{i-2} \left(\frac{\tau}{2Qdt} - \frac{\tau^2}{dt^2} \right) \right] \tag{5.10}$$

The final block we need to implement in the ANDL is the nonlinear CDL. Because we are working in a digital system, it is simple to implement a simple conditional statement comparing $|z - \zeta|$ and α. Based on the comparison, the CDL can do one of two things: if $|z - \zeta| \leq \alpha$ the CDL will simply proceed as if it is a normal filter; otherwise the filter will recompute its coefficients with a new value for the time constant: $\tau = \tau_0 \frac{|x-\zeta|}{\alpha}$.

With these equations we now have everything we need in order to implement the ANDL.

5.4 Results

In order to evaluate how well our ANDL implementation performs, we will compare to our previous implementation in MATLAB. The key difference between the two implementations is that the DSP implementation operates on a sample-by-sample basis whereas in the MATLAB implementation the entire signal is sent through the ANDL one block or operation at a time. Because the implementations are fundamentally different, we cannot expect their respective outputs to agree exactly, but they should perform very similarly.

Feeding input signals to the ANDL is done through a Windows command prompt application. This application establishes a USB connection with the DSP through the Extender board and allows for file I/O through the C standard library. It also allows use to display some useful information.

The program is pointed at an input file. The file is read into the program sample by sample and an output file is created sample by sample. In this way the program is easily adaptable to run in real time instead of with static input files.

The input files used will be generated using MATLAB. This way we can compare the two implementations using the exact same input signals. It is possible to extract the signals from the ANDL at various points between the different blocks in both implementations and view them.

In figure 5.3 it can be seen that the DSP and MATLAB implementations of the ANDL perform nearly identically. The output of each implementation with $\alpha \to \infty$ is shown as a dashed line to compare compare to the performance of a linear filter.

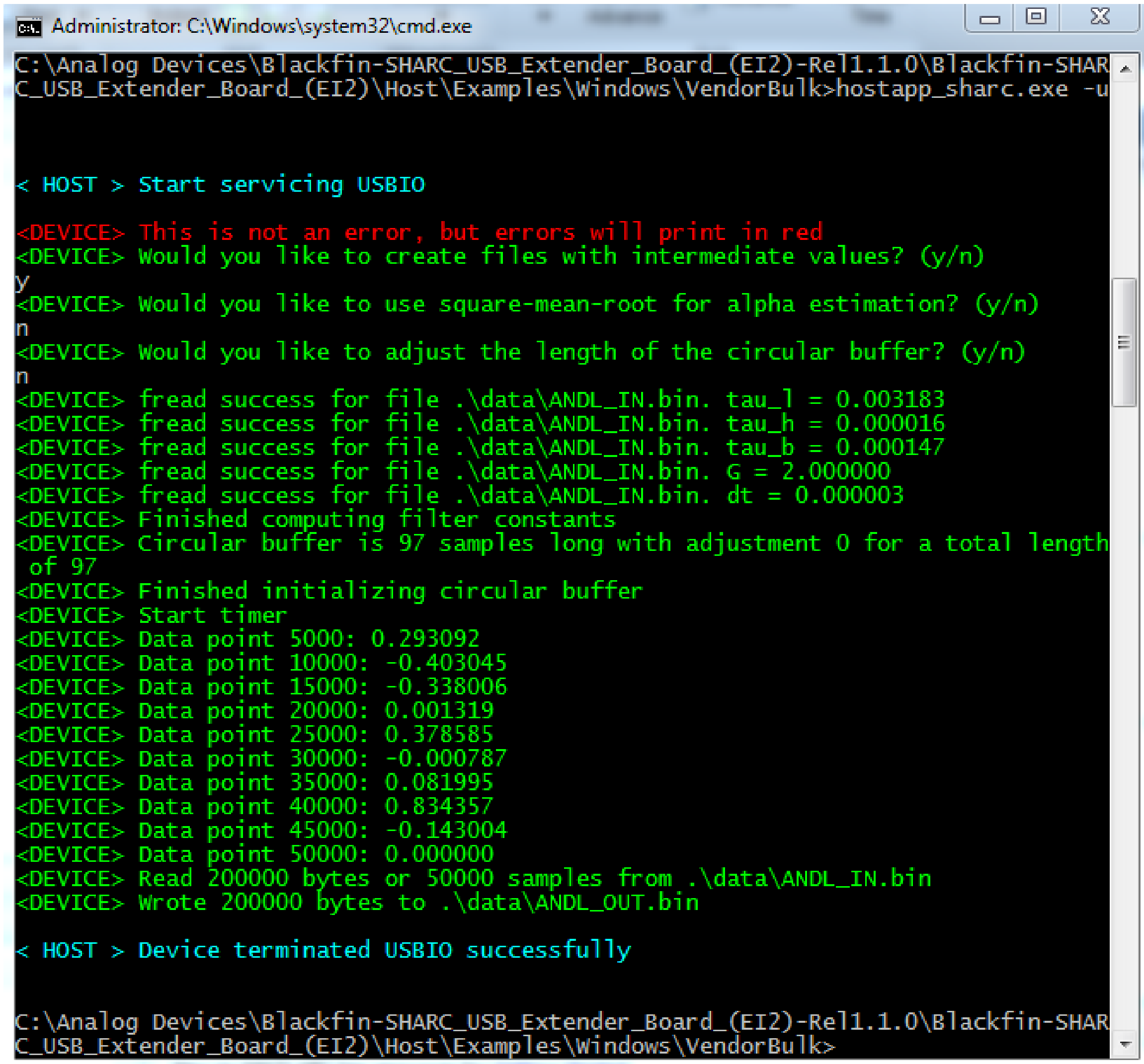


Figure 5.2: *Command Prompt Output*

In this chapter we have implemented the ANDL on a digital signal processing platform. We have also compared the performance of this implementation of the ANDL to the MATLAB implementation introduced in chapter 3. In the next chapter we will discuss some areas we have identified for further research.

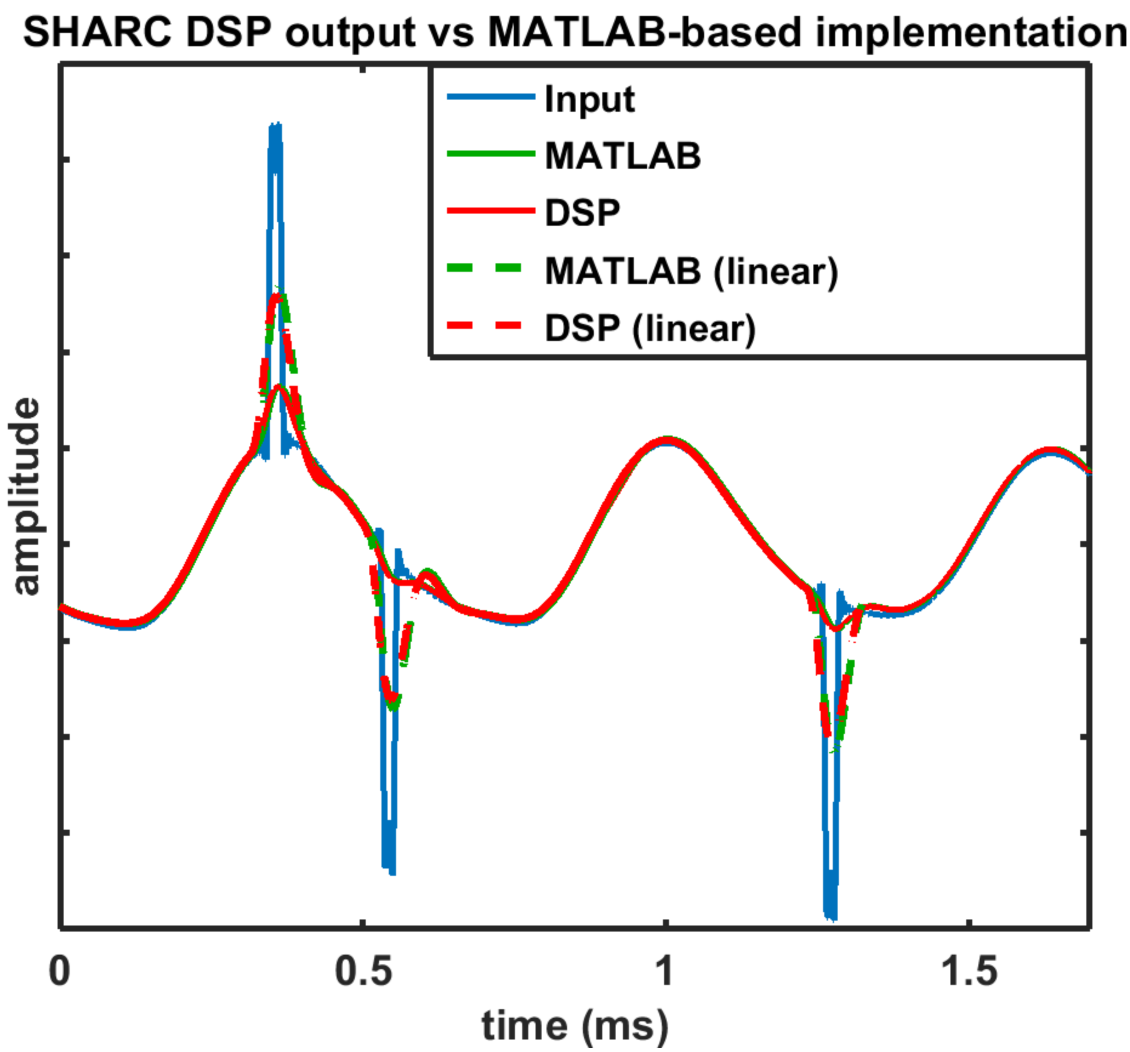


Figure 5.3: *Digital vs MATLAB Implementation*

Chapter 6

Conclusion and Future Work

In this thesis, we describe blind adaptive nonlinear filters referred to as Adaptive Nonlinear Differential Limiters (ANDLs) and demonstrate that they can be used to recover communication channel capacity in the presence of impulsive noise. The ANDL can be deployed with flexibility as either a standalone tool, or as part of a larger overall noise mitigation strategy. On their own, ANDLs offer SNR improvement that ranges from "no harm" in low-noise conditions to over 10dB in highly impulsive noise environments.

6.1 Summary of Key Contributions

This thesis provides a theoretical introduction to NDL filters in chapter 2. We show that the NDL is an analog implementation of an approximation of a median filter. Then we describe strategies for resolution parameter tuning for both stationary and non-stationary signal+noise environments in chapter 3 and chapter 4 respectively. Additionally, the tuning of additional parameters related to the resolution parameter estimation sub-circuit is discussed in chapter 4.

This thesis introduces a digital implementation of an ANDL in chapter 5. The advantages of a digital implementation include rapid prototyping ability and increased flexibility of deployment using off-the-shelf digital signal processors instead of specialized analog circuitry. The digital ANDL also introduces a sample-wise scheme for generating output values.

6.2 Future Work

There is room to expand upon the work presented in this thesis. We identify two major areas for further research.

6.2.1 Investigation of BER in an OFDM System

The first is more detailed numerical simulations. The work presented in chapter 3 took a general approach, but there are advantages to performing a simulation of a more specific system. In particular we are interested in developing:

- A time-domain simulation of OFDM modulated communication system would allow for metrics of signal quality beyond SNR to be extracted such as bit-error-rate (BER).
- Investigation of the effect that NDL filtering has on signal phase. This is crucial for most common modulation schemes including QAM and OFDM.
- Real-time adjustment of resolution parameter in order to maximize a feedback variable such as SNR or BER. Determination of resources required to keep NDL in optimal operating region for feasibility analysis.

Performing these investigations would further the development of NDL technology for use in practical communications systems.

6.2.2 Application of Digital Implementation to Real Time Signals

The second area to expand on is the work in chapter 5 where we implemented a digital ANDL. Signal processing in a digital environment presents unique challenges of minimizing sampling rate requirements and processor cycles needed per sample. A digital system also allows for much more flexibility in terms of sample manipulation. Some research ideas that we would be interested in pursuing in this area are:

- Investigation of the effect of multi-rate processing. The digital ANDL relies on a data rate of about an order of magnitude beyond the Nyquist frequency. We propose that the NDL filtering is performed after the high-rate sigma-delta encoder and prior to downsampling to a lower rate, but higher precision signal.

- Modification of the existing digital implementation into a fully real-time system. The current implementation requires an existing file on a PC in order to perform filtering. With a modification of how samples are input to the filter, a real-time implementation is possible.

- Experiment with different types of α estimation sub-circuits. Digital processing allows for many non-linear operations to be performed at similar speeds. These operators can provide additional resistance to outliers and therefore better approximation of signal energy. One example is a square-mean-root filter.

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