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WIND MODELS AND OPTIMUM SELECTION OF WIND TURBINE SYSTEMS

by

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1. INTRODUCTION

Because today's industrial society consumes energy at such an alarmingly high rate, domestic supplies of fossil fuels such as natural gas and oil, which are non-renewable resources, have become severely depleted. This high consumption rate is the cause of the current energy dilemma facing the world. The skyrocketing cost of these fossil fuels has forced a re-evaluation of raw energy sources used to generate electricity. Nuclear fuel is a possible candidate to replace fossil fuel for large-scale power generator. However, public fear of nuclear energy and its consequences has proved to be a formidable barrier to the development of nuclear power. Hence, other means to generate power have been investigated. To many, renewable energy resources such as solar and wind energy are especially attractive. In addition, to low input energy costs afforded by solar and wind energy systems, these resources are available to all countries and consequently need not be imported from other lands. Such resources could help make the U.S. and other industrialized nations less dependent on expensive foreign oil.

Potentially available wind power, a form of solar power, in the Continental U.S., the Aleutian Arc, and the Eastern Seaboard has been estimated to be 10^5 GW [1]. This is more than thirty times the estimated total annual average power requirements of the U.S. by 1980.

Because of this large potential and the recent large increase in traditional energy costs, the development of various wind energy systems is currently receiving wide attention. However, wind power is not a recent development, but has been used by man throughout history. As far back as 200 B.C. wind turbines were used to grind grain. From then up until the present day, wind turbines have been used by many societies to supply energy for a variety of industrial activities. In the mid-19th century wind power was used in rural America to pump water and generate electricity until the 1930's when the Rural Electrification Administration displaced these units with more reliable centralized electric power. Thus, small wind systems became a second choice for power generation.

Past wind systems have not been limited to serving small local power requirements. A central station wind-powered electrical system was conceived by Palmer Putnam and built by the S. Morgan Smith Company of York, Pennsylvania during the early 1940s [1]. This system was rated at 1250 kW and fed power into the electrical network of the Central Vermont Public Service Company. The unit operated intermittently until March, 1945. At this time, one of the blades broke off near the hub where a known weakness had been identified but had not been corrected because of the wartime material shortages. However, after a comprehensive economic analysis, it was discovered that the unit, even if repaired, could not compete effectively, at that time, with conventional electrical generation plants. Hence, although the project was a technical success, economics forced the plant to be abandoned. Other similar endeavors in Denmark, Russia, France, England, and Cermany during this same period of time suffered the same economic fate [1].

Currently, the rising cost of traditional energy sources has kindled new interest in wind power. In September 1975 under a Federal wind energy program, the Energy Research and Development Administration (ERDA) began testing a wind turbine generator rated at 100 kW in 18 mph winds (the Plum Brook unit of the National Aeronautics and Space Administration (NASA)). Through this program, the problems associated with large wind turbine generators are to be investigated. These include reducing capital costs, eliminating television interference, and improving aesthetic appearances [2]. Although large wind turbines may offer the potential of lower capital costs per installed kilowatt, small wind turbines of comparatively simpler design are already on the market and do not face the problems of television interference or aesthetic impact. Consequently, wind power generation faces an economy of scale, i.e., whether to install one large central station unit or use several small units to serve particular load demands.

Whichever size units are chosen to be used, areas of high wind power potential must be identified. Reed [3] has compiled extensive power calculations from historical meteorological data tapes for many locations. However, because of the wind's inherent variability, the power output from wind turbines is often highly variable from day to day and even from minute to minute. Consequently, there have been many studies to try to characterize this variability analytically so as to be able to predict the energy that can be extracted by a particular wind turbine. Justus, et. al., [4], made use of the Weibull distribution in modeling wind speed distributions and computing total energy production for two different central station power units, a 100 kW wind turbine with characteristics similar to the NASA experimental unit and a hypothetical 1 MW unit, at various sites throughout the United States. Johnson [5], too, used the Weibull distribution in analyzing wind turbine performance at specific Kansas locations. In addition, Hennessey [6] incorporated the Weibull distribution in his study of computing mean power densities at several locations; but he solved for the Weibull distribution's parameters using a matching-moments method, whereas Justus and Johnson estimate the parameters by a linear least squares technique. Corotis [7] also investigated the Weibull distribution using the matching-moments parameter estimation method for computing the available power in the wind as a function of wind speed. He also examined the use of the Rayleigh distribution, which is a special case of the Weibull distribution involving the estimation of only one parameter. Cliff [8] also employed the Rayleigh distribution to estimate the average annual output of a wind turbine. Finally, Kaminsky [9] analyzed four different distribution functions, log-normal, gamma, Weibull, and Rayleigh, and compared how well each of these distributions fit given wind speed data. Kaminsky solved for the parameters of the above distributions by using maximum likelihood estimators for all except the log-normal distribution for which he employed a matching-moments estimation. However, common to all these studies that analyze wind turbine performance by using an analytical distribution to characterize wind speed data, it is assumed that all the generated electricity is used by the electrical network, i.e., the wind turbine is a replacement for conventional generators and no generated energy is wasted.

Obermeier [10] studied the prospect of using wind power to meet particular load demands. He investigated the use of wind turbines in current commercial production and his criterion for an "optimal" wind turbine and battery size was one which will supply the entire demand, i.e., the customer does not need to purchase any electricity. However, there was no consideration of the cost to the customer of such an optimal system. Because economics plays a key role in the determination of energy policies, another criterion for choosing an "optimal" wind turbine size would be to test many different sized wind systems and see which one would save the user the most money in comparison to purchasing all the demanded electricity from a utility.

Based upon these previous wind power and feasibility studies, the scope of this work was two-fold. First, various analytical models of wind speed distributions were investigated. Several of the more common methods of solving for the parameters of the Weibull distribution were studied. In addition, the beta distribution was introduced and investigated to determine how well it could represent wind speed data. Two goodness of fit tests were performed on each analytical model to determine the appropriateness of each model in describing observed wind speed distributions. This study of the representation of observed wind speed distributions by analytically fit functions is presented in Chapter 2. In the second phase of this work a methodology was developed whereby an appropriate analytical wind speed model was used to compute an economically optimal wind turbine system to serve a particular load.

The details of this optimization methodology and an investigation of the sensitivity of the optimally sized wind turbine generator system to the problem parameters is presented in Chapter 3.

2. ANALYTICAL REPRESENTATIONS OF WIND SPEED DISTRIBUTIONS

2.1 Introduction

To evaluate accurately the energy potential of a wind turbine generator system (WTGS) for a given application and geographical area, it is first necessary to have sufficient information about the wind speeds likely to be encountered by a WTGS at a specific location. Wind speed characteristics may vary widely for different geographical regions as well as locally as a result of local terrain features. Moreover, the wind speeds vary throughout the day as well as with the season. Ideally, one would like to have the average wind speed distribution for times throughout a day and for days throughout the year at a specific site. For more detailed analyses, more detailed information about the wind speed characteristics may be needed, e.g., standard deviations of the average wind speed, wind duration (or persistence) characteristics, and gust characteristics. Such characterizations of the wind speeds may be obtained from historical wind velocity data which have been collected at many locations in the United States and other countries over a period of many years. In their most basic form, these data represent measured values of wind speeds (either "instantaneous" or averaged) at periodic times throughout the day for every day of the year. Because of the large amount of information, these historical wind records are most often stored on magnetic tape which can readily be processed by computers. From the historical records a great deal of information about the average characteristics of the wind at the specific location can be computed if a sufficient number of years has been included in the data records.

For the evaluation of a WTGS at a specific site, the most fundamental wind speed information needed is the average speed distribution for various times throughout a day in any season (or month). From the meteorological wind data tapes, various wind speed frequency (or probability) distributions have been obtained by averaging the observed wind speed data from various intervals throughout the day for each month. Extensive compilations of such frequency distributions for many locations in the United States have been published [11,12]. With such averaged wind speed distributions the potential of a given site for generating power can be evaluated accurately.

However, even the use of these historical averaged wind speed distributions (usually presented as histograms) still involves extensive data manipulation. Moreover, if the average speed distributions have been generated from only a few years of historical data, the resultant frequency distribution may still differ significantly from the actual wind frequency distribution at the site as a consequence of large wind speed variations. For the analysis of wind potential, it is often easier to use a smooth, analytical, functional representation of the observed wind frequency. The use of such analytical functions to represent the wind speed distribution allows a significant simplification in the calculation of energy generated from a WTGS by allowing much analytical simplification in the analysis and, thereby, requiring considerably fewer numerical computations. Furthermore, the use of analytical functional representations of the wind speed distributions tends to smooth out data variations resulting from insufficient experimental data. Strict use of the experimental data may not produce an accurate representation of the actual wind speed distribution as a result of statistical fluctuations in the data, whereas the analytical function representation may yield a more accurate wind speed representation by averaging over the statistical fluctuations. Of course, the function used to represent the wind data must be shown to be very representative of actual wind speed distributions. Finally, most experimental wind speed distributions are presented as histograms with relatively large velocity intervals or bins. This "binning" procedure of the historical measurements produces at best an approximate representation of the wind speed distribution, which, in reality is a continuous distribution. Thus, the use of analytical functions to represent the wind speed distributions resembles the smooth and continuous nature of actual wind speed distributions as well as preserving the distribution character and accuracy of representing the average wind speeds.

For modeling accurately power generation over the speed range in which the WTGS power output varies rapidly with wind speed, the use of a continuous wind speed model is conceptually more appealing than the use of a discrete frequency distribution. The WTGS power output varies, ideally, with the cube of the wind speed. The discrete frequency distribution of wind speeds obtained from historical wind data usually contains only two or three intervals over the rapidly varying (with the cube of the wind speed) transition range (from cut-in to rated speeds) of the WTGS response.

Consequently, there has been much effort in the past few years to find various simple functions which can represent accurately actual wind speed distributions as well as to develop techniques for finding the parameters of such analytical representations, which fit the observed wind data [4,5,6,7,8,9,12]. In the first phase of this work, several techniques for fitting two functions (the Weibull and beta distributions), which are well suited to represent wind speed distributions, are examined and two tests (chi-square and power ratio), are developed to indicate the accuracy of the resulting fits.

2.2 Description of Wind Speeds

For analysis of WTGS performance, in addition to the WTGS response to wind speeds, only information about the distribution of speeds for a given daily time interval is needed. The directional dependence of wind speeds is not a concern in this study. Wind speed distributions can be described by either of the following two distributions:

(i) $f(v) \equiv$ "probability density function". The quantity $f(v)dv$ is the probability that the wind is in the interval dv about speed v and is normalized such that

$$\int_0^\infty f(v)dv = 1. \qquad (2.2\text{-}1)$$

(ii) $F(v) \equiv \int_0^v f(v')dv'$ = "cumulative distribution" function, i.e., the probability that the wind speed is less than v. (2.2-2)

Historical wind speed recordings are not continuous, but rather the data are grouped into discrete speed intervals called wind speed bins. In such a discrete representation, the possible range of wind speeds, from zero to high storm speeds, is divided into n contiguous subintervals of widths Δv_i bounded by speeds v_i and v_{i+1}. In most compilations of wind speed data, the *frequency distribution* of wind speeds in each discrete subinterval is given, i.e., the probability P_i (or fraction of the total number of observations) in which the wind speed was observed to be in the i-th speed subinterval ($v_i<v<v_{i+1}$), is given:

$$P_i \equiv \int_{v_i}^{v_{i+1}} f(v)\, dv. \qquad (2.2\text{-}3)$$

An example of a frequency distribution compiled from historical records is given in Table 2.2-1. From such frequency data, a discretized form of the probability density function can be constructed as

$$f_i = \frac{P_i}{\Delta v_i}\,,\ v_i < v < v_{i+1}\,. \qquad (2.2\text{-}4)$$

Also, the wind speed data can be represented by a discrete form of the cumulative distribution function,

$$F_i \equiv \sum_{i=1}^{j} P_i,\ j=1,\ldots,n. \qquad (2.2\text{-}5)$$

The corresponding probability density and cumulative distribution

Table 2.2-1. Sample Wind Speed Frequency Distribution Showing "Binning" of Wind Speed Data (From Ref. 11).

DATA PROCESSING DIVISION
ETAC, USAF
ASHEVILLE, N. C. 28801

SURFACE WINDS

PERCENTAGE FREQUENCY OF WIND
DIRECTION AND SPEED
(FROM HOURLY OBSERVATIONS)

14944 (STATION) SIOUX FALLS S DAK WBAS (STATION NAME) 48-64 (YEARS) OCT (MONTH)

ALL WEATHER (CLASS) 1200-1400 (HOURS (L S T))

CONDITION

SPEED (KNTS) DIR.	1 - 3	4 - 6	7 - 10	11 - 16	17 - 21	22 - 27	28 - 33	34 - 40	41 - 47	48 - 55	≥56	%	MEAN WIND SPEED
N	.4	1.1	2.5	3.3	.9	.3	.1					8.5	11.1
NNE	.1	.4	.9	1.9	.6							3.9	11.7
NE	.1	.4	1.1	.4	.1							2.0	8.8
ENE	.1	.3	.6	.9	.1							2.0	10.0
E	.1	.4	.8	.2								1.5	8.0
ESE	.1	.4	1.0	.8	.2							2.5	10.2
SE	.1	.7	1.4	2.3	.9	.1	.1					5.6	12.1
SSE	.1	.8	2.5	4.6	1.6	.3						9.9	12.4
S	.4	1.5	3.4	6.1	2.5	.3	.1					14.4	12.3
SSW	.2	.6	2.7	2.8	1.0							7.4	11.3
SW	.4	1.1	1.8	2.0	.2							5.5	9.5
WSW	.1	.6	2.0	1.3	.3	.2						4.4	10.7
W	.1	.5	1.3	1.1	.8	.3	.1	.1		.1		4.3	13.4
WNW	.2	.3	2.0	2.3	1.7	.8	.2					7.5	14.2
NW	.1	.6	1.6	2.5	2.6	1.5	.3					9.2	15.8
NNW	.2	.8	1.6	3.0	1.9	1.1	.1	.1				8.8	14.4
VARBL													
CALM												2.3	..
	3.0	10.5	27.3	35.6	15.3	4.8	.9	.1		.1		100.0	12.1

TOTAL NUMBER OF OBSERVATIONS 1581

functions of the data in Table 2.2-1 are shown in Figs. 2.2-1 and 2.2-2, respectively. The mean wind speed, μ, and the dispersion (or variance), σ^2, of the wind speeds at any particular time are calculated from the continuous speed distributions as

$$\mu = \int_0^\infty vf(v)dv, \tag{2.2-6}$$

and

$$\sigma^2 = \int_0^\infty (v-\mu)^2 f(v)dv. \tag{2.2-7}$$

From the discrete representation, i.e., the experimental data, the mean and variance can be estimated by

$$\bar{v} = \sum_{i=1}^{n} P_i \, v_{i+\frac{1}{2}} \, , \tag{2.2-8}$$

and

$$s^2 = \sum_{i=1}^{n} (v_{i+\frac{1}{2}} - \bar{v})^2 P_i, \tag{2.2-9}$$

where n is the total number of speed subintervals, and $v_{i+\frac{1}{2}}$ is the midpoint of wind speed subinterval i.

2.3 Weibull Probability Density Function

The two-parameter Weibull distribution [4,5] is the most widely used distribution to characterize wind data because of the simplicity of its cumulative distribution function. The Weibull distribution is given as

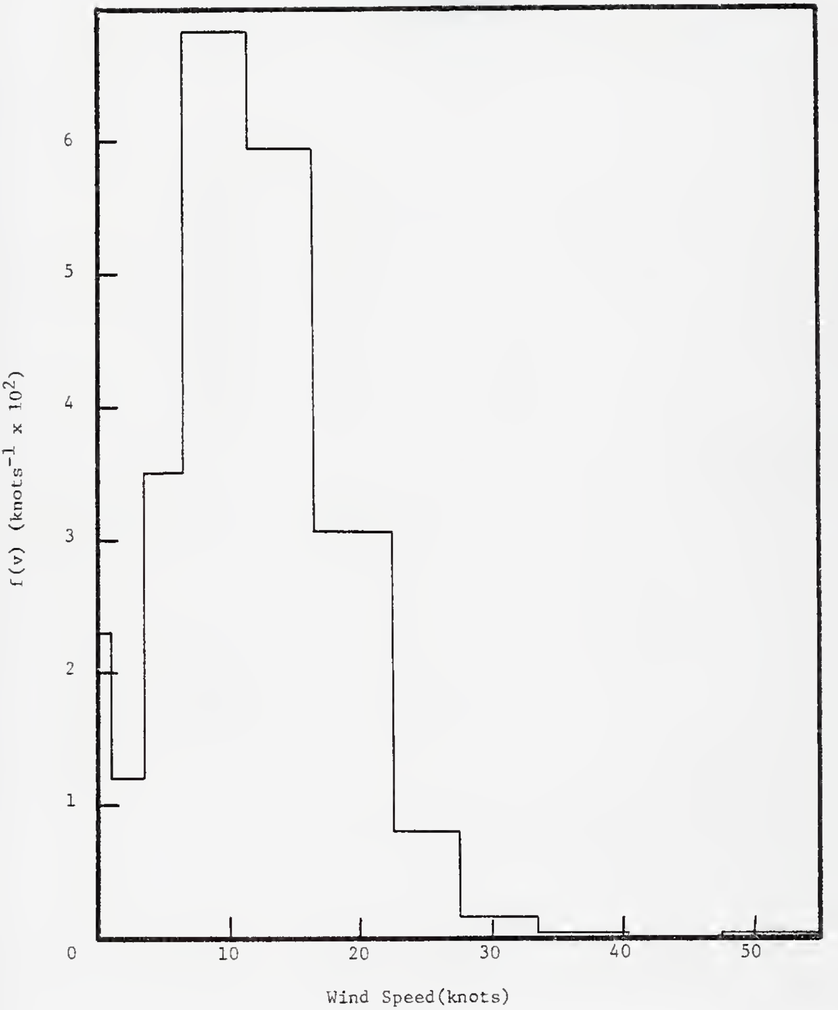

Fig. 2.2-1. Probability density function for data in Table 2.2-1.

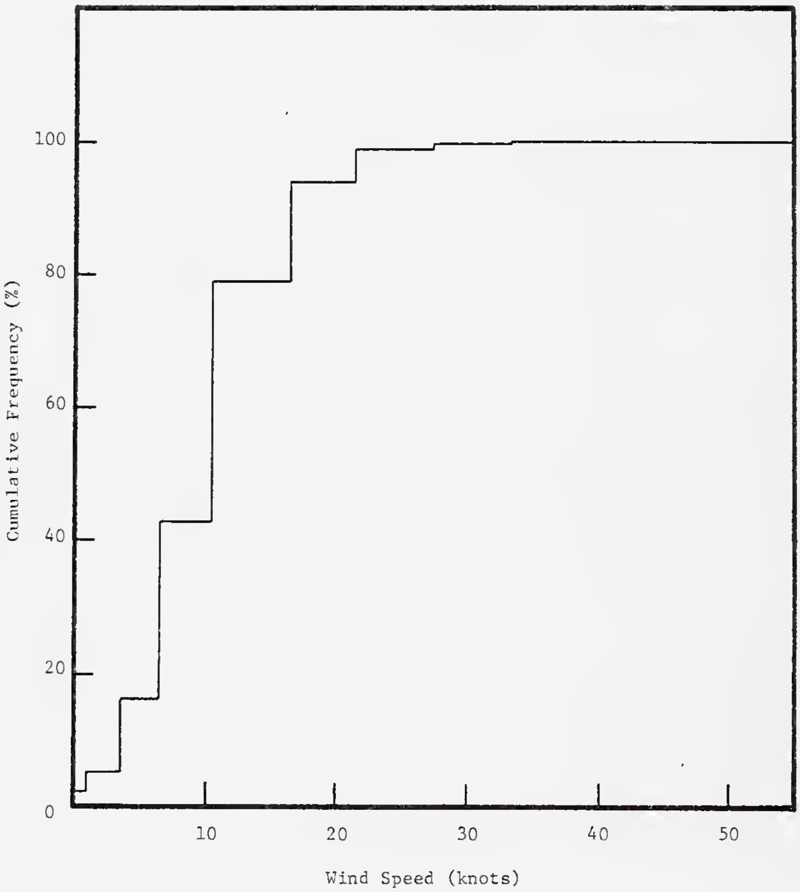

Fig. 2.2-2. Cumulative distribution function for data in Table 2.2-1.

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} \exp\left(-\left(\frac{v}{c}\right)^{k}\right), \quad v,k,c>0, \tag{2.3-1}$$

where v is the wind speed, c is the *scale parameter* and k is the *shape parameter*. The corresponding cumulative distribution is given by

$$\int_0^v f(v)\, dv = F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^k\right]. \tag{2.3-2}$$

The mean and variance of the Weibull distribution, which will be needed later for fitting wind data, are [4]:

$$\mu = c\Gamma\left(1 + \frac{1}{k}\right), \tag{2.3-3}$$

and

$$\sigma^2 = c^2\Gamma\left(1 + \frac{2}{k}\right) - c^2\Gamma^2\left(1 + \frac{1}{k}\right), \tag{2.3-4}$$

where $\Gamma(x)$ is the gamma function, defined by

$$\Gamma(x) \equiv \int_0^\infty y^{x-1}\, e^{-y}\, dy, \quad x>0. \tag{2.3-5}$$

The effect of variation in the shape parameter, k, upon f(v) is illustrated in Fig. 2.3-1. In this case the scale parameter, c, is set to unity; however, f(v) can be obtained for other values of c from these graphs by simply dividing the ordinate by c and multiplying the abscissa by c. This adjustment preserves the requirement of unit area under the curve [5]. For values of k between zero and unity, the distribution has a mode at zero and is monotonically decreasing (ex-

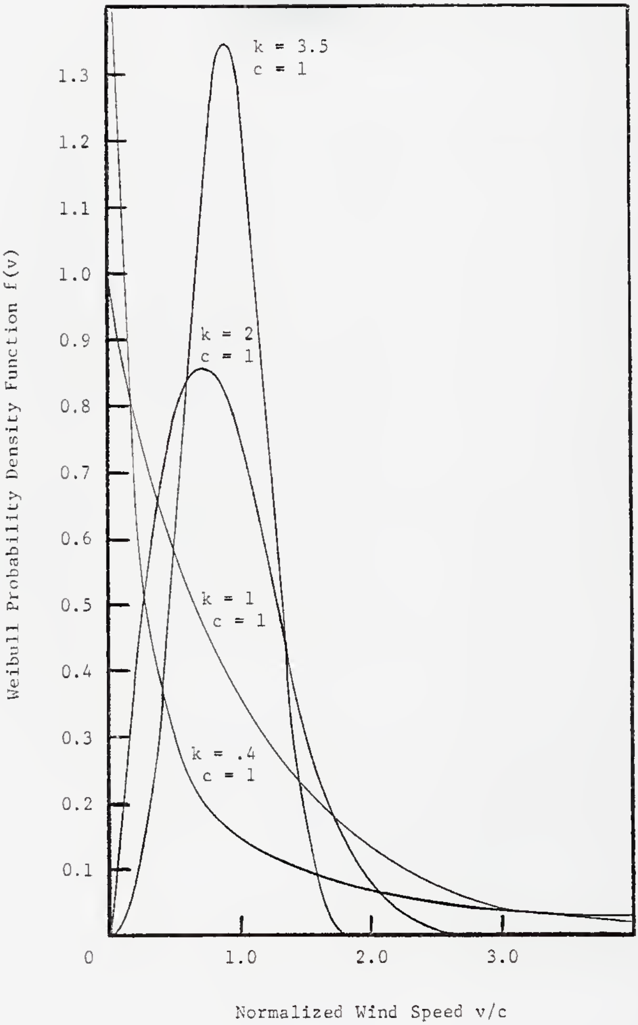

Fig. 2.3-1. The standard form of the Weibull distribution.

potential-shaped). For k equal to unity, the distribution is purely exponential. For v greater than 3.5, the distribution becomes approximately normal [6].

The Rayleigh distribution is a special case of the Weibull distribution. In the Rayleigh distribution, the shape parameter, k, is fixed at a value equal to 2. Consequently, the Rayleigh distribution is a single parameter distribution depending solely upon the value of the mean of the distribution.

2.3.1 Representation of Wind Speeds by the Weibull Distribution

(a) Double Logarithmic Transformation Least Squares (DLTLS) Method

Several methods to estimate the unknown parameters, c and k, of the Weibull distribution have been proposed [4,5,6,7,12]. One method is to perform a least squares fit of the observed data to the double logarithm of the observed cumulative distribution function. The cumulative Weibull distribution function is a more tractable form than its probability density function. The cumulative distribution function is linearized by taking the logarithm twice of each side as shown below [5]

$$\ln[-\ln(1-F_j)] = k \ln v_{i+\frac{1}{2}} - k \ln c \,. \tag{2.3-6}$$

This result is of the linear form

$$y = ax + b, \tag{2.3-7}$$

where

$$y = \ln[-\ln(1 - F_j)],$$

$$x = \ln v_{i+\frac{1}{2}}$$

$$a = k,$$

and $$b = -k \ln c$$

Hence, a least squares procedure can be used on Eq. (2.3-7) to yield estimates of the parameters a and b, which in turn can be used to form estimates of k and c. Thus, use of the observed cumulative distribution function, F_j, with a standard linear least squares method gives estimates of the parameters c and k. To avoid the case where F_j is exactly unity (i.e., when j is equal to n), a modification of Eq. (2.3-6) has been used [13], namely

$$\ln[-\ln(1.0001-F_j)] = k \ln v_{i+\frac{1}{2}} - k \ln c. \tag{2.3-8}$$

Because of the addition of the small amount in the left hand side of Eq. (2.3-8), each value is slightly in error. Alternately, the final data point, F_n, can be omitted and Eq. (2.3-8) used with no modification.

The least squares estimates a and b (Eq. (2.3-7)) are given by [14]:

$$a = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i x_i y_i - \sum_{i=1}^{n} w_i x_i \sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i x_i^2 - (\sum_{i=1}^{n} w_i x_i)^2}, \tag{2.3-9}$$

and

$$b = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i} - \frac{a \sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}, \tag{2.3-10}$$

where w_i is the weighting factor for speed subinterval i.

Often the weighting factors for the least squares technique are assumed to be unity, i.e., each point has the same effect on the least squares estimators. Johnson [13] reports that weighting factors of unity cause all speed subintervals to have the same effect on the least squares estimators, whereas some speed subintervals actually represent more observations than others. Consequently, in order for the speed subintervals with the most observations to have more effect on the least squares estimators, each speed subinterval can be weighted with the probability, P_i, of a wind speed occurring in that i-th speed subinterval. Thus, the resulting Weibull distribution will fit the speed subintervals which have the most observations better than the Weibull distribution obtained by using unit weighting factors. However, this weighting scheme is strictly judgmental and not statistically justified.

Weighting factors which are inversely proportional to the variance of the residuals between the observed data and the expected value (as predicted by the functional form used to model the data) yield least squares estimators, which are of minimum variance [15]. Hence, these are the best estimators, in the least squares sense. The weighting factors are not independent of the parameters of the model; however, it is normally assumed that the variance of the residual is caused only by the variance in the observed data. Thus, the weighting factors are commonly taken as the reciprocal of the variance of the observed data. In this case (finding least squares estimates for a and b of Eq. (2.3-7)), the weighting factors should be the reciprocals of the variance of the y values, i.e., the reciprocal of the logarithm of the negative logarithm of unity minus the cumulative distribution of the observed data! However, because the variance of the observed data is unknown, the statistically correct weighting scheme is unable to be performed.

Irrespective of which weighting factors are used, the above linear least squares method minimizes the sum of the errors associated with the linear approximation of the doubly logarithmic transformed cumulative distribution function, i.e., the fit parameters are chosen so as to minimize

$$E_1 = \sum_{i=1}^{n} \{(k \ln v_{i+\frac{1}{2}} - k \ln c) - \ln[-\ln(1 - F_i)]\}^2. \qquad (2.3\text{-}11)$$

However, these methods do not guarantee that the sum of the squared errors of the actual cumulative distribution,

$$E_0 = \sum_{i=1}^{n} \{(1 - \exp[-[\frac{v_{i+\frac{1}{2}}}{c}]^k]) - F_i\}^2, \qquad (2.3\text{-}12)$$

is a minimum.

To illustrate this point, suppose the wind data, which are shown in Table 2.3-1, are used. A double logarithmic transformed least squares fit to the cumulative Weibull distribution yields k equal to 1.42 and c equal to 6.20 so that the fitted distribution can be written as,

$$f(v) = 0.229 \left(\frac{v}{6.2}\right)^{0.42} \exp\left[-\left(\frac{v}{6.2}\right)^{1.42}\right] . \qquad (2.3\text{-}13)$$

Table 2.3-1. Hypothetical Wind Speed Data to be Fit by a Weibull Distribution Using the Method of Least Squares.

Speed Subinterval (knots)	Subinterval midpoint (knots)	P_i	f_i	F_i
0-2	1	0.092	0.0460	0.092
2-6	4	0.2288	0.0572	0.3208
6-12	9	0.441	0.0735	0.7618
12-18	15	0.1758	0.0293	0.9376
18-24	21	0.0624	0.0104	1.0

The error parameter E_0, using this result in Eq. (2.3-12), is 6.8×10^{-3}, while E_1, i.e., the sum of the squared differences between the doubly logarithmic transform of the cumulative distribution associated with Eq. (2.3-13) and the given data, is 0.491. However, a Weibull distribution can be found which passes through all the data points except the third speed subinterval. The parameters of this distribution are k equal to 1.5 and c equal to 10.0 while written explicitly

$$f(v) = 0.15 \left(\frac{v}{10.0}\right)^{0.5} \exp\left[-\left(\frac{v}{10.0}\right)^{1.5}\right] . \qquad (2.3\text{-}14)$$

The error parameter, E_0, using this result in Eq. (2.3-12), is now smaller with a value of 0.166×10^{-3}, while E_1 increases to 3.08. The fit of Eq. (2.3-14) to the data is considerably improved, although that of the doubly logarithmic transform of the cumulative Weibull is poorer than in the first case (Eq. (2.3-13)). Hence, the DLTLS estimation described in this section assures the minimization of E_1, but not necessarily the minimization of E_0!

(b) Matching-Moments Method

Another procedure to obtain estimates of c and k is by a matching-moments method. The sample mean and variance of the wind data, given by Eqs. (2.2-8) and (2.2-9), are set equal to the mean and variance of the Weibull probability density function, Eqs. (2.3-3) and (2.3-4), to obtain

$$\bar{v} = \mu = c\Gamma(1 + \frac{1}{k}), \tag{2.3-15}$$

$$s^2 = \sigma^2 = c^2[\Gamma(1 + \frac{2}{k}) - \Gamma^2(1 + \frac{1}{k})]. \tag{2.3-16}$$

From these equations, c can be expressed in terms of k as

$$c = \frac{\bar{v}}{\Gamma(1 + \frac{1}{k})}. \tag{2.3-17}$$

Substitution of c from Eq. (2.3-17) into Eq. (2.3-16) yields an equation for k, namely,

$$\frac{\Gamma(1 + \frac{2}{k})}{\Gamma^2(1 + \frac{1}{k})} - \frac{\bar{v}^2 + s^2}{\bar{v}^2} = 0. \tag{2.3-18}$$

To solve Eq. (2.3-18) for k, a numerical procedure for finding the roots of a nonlinear equation must be used. In this study, the Mueller's iteration method (an elegant successive bisection technique [30]) is preferred over a technique such as Newton's method, because the former does not require the calculation of derivatives which can become cumbersome when dealing with gamma functions. Once k is found, c can be determined by substitution of k into Eq. (2.3-17). If calculations are to be performed by hand, a crude estimate of k can be made by using Kotel'nikov's nomogram [6]. The value for c can then be obtained by substitution of k into Eq. (2.3-17).

The matching-moments technique is very flexible in that only the sample data mean and variance are needed to solve for k and c. Consequently, diurnal effects on the wind distribution can be studied with some ease. Since diurnal variations are generally given in terms of their effect on mean seasonal wind speeds, the resulting frequency distribution is easily computed. However, a numerical method or Kotel'-nikov's nomogram approximation is needed to solve for k; this numerical computation is an unattractive feature of this method.

2.4 Beta Probability Density Function

The second probability density function proposed for the modeling of wind data is the two-parameter beta distribution,

$$f(v) = \begin{cases} \dfrac{1}{v_{max}\, B(\alpha,\beta)} \left(\dfrac{v}{v_{max}}\right)^{\alpha-1} \left(1 - \dfrac{v}{v_{max}}\right)^{\beta-1}, & 0 \le v \le v_{max} \\ 0, & \text{otherwise} \end{cases} \qquad (2.4\text{-}1)$$

where

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} ,$$

α and β are positive parameters, and v_{max} is the maximum speed for which the beta distribution is defined, i.e., maximum observed wind speed.

Variation of the parameters α and β causes the beta distribution to assume many shapes as illustrated in Fig. 2.4-1. For α less than β, the beta distribution is positively skewed, i.e., there is a longer "tail" to the right of the maximum than to the left. For α greater than β the distribution is negatively skewed and for α equal to β, the distribution is symmetric about the value $\frac{1}{2}\, v_{max}$. In the special case of both α and β equal to unity, the beta distribution is uniform over its entire defined wind speed range.

The mean and variance of the beta distribution, which will be needed in the following subsection for fitting wind data, are

$$\mu = \frac{\alpha}{\alpha + \beta} \; v_{max} \, , \tag{2.4-2}$$

$$\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \; v_{max}^2 \; . \tag{2.4-3}$$

2.4.1 Representation of Wind Speeds by the Beta Distribution

To use the beta distribution to model given wind speed data, a method must be found to obtain values for the parameters α and β of the beta distribution. Techniques such as a non-linear least squares or maximum likelihood estimates can be used to find estimates of α and β. However, the above techniques require numerical procedures in the computation of α and β.

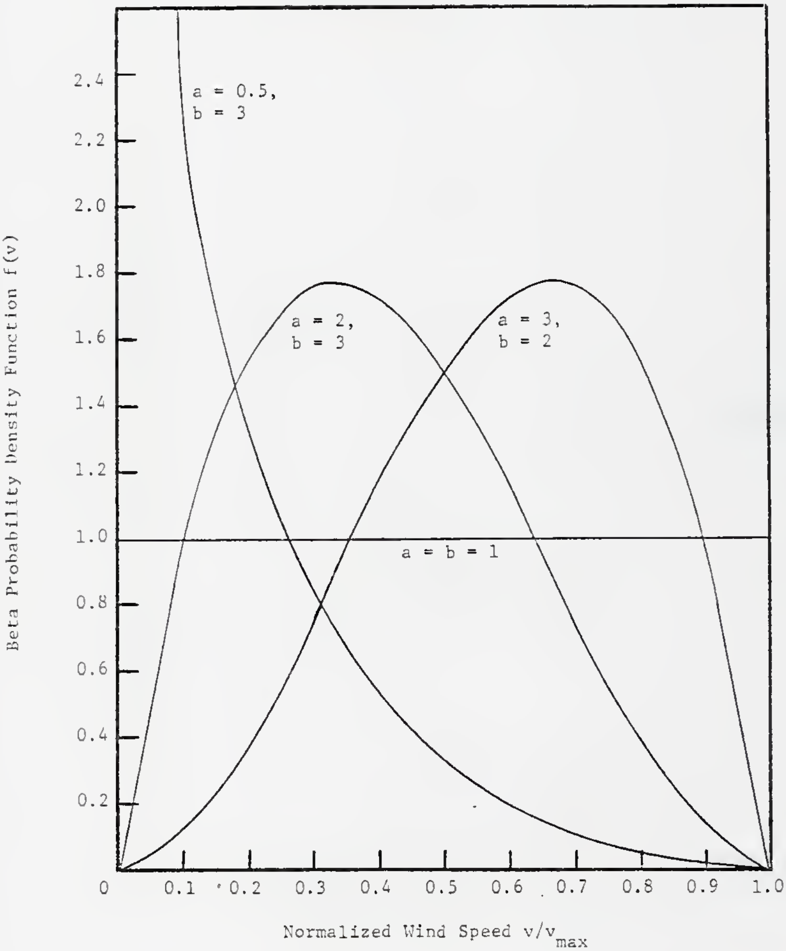

Fig. 2.4-1. The standard form of the beta distribution.

Another method to solve for α and β is by the matching-moments technique. As was done when solving for the parameters of the Weibull distribution by matching-moments, the sample data mean, Eq. (2.2-8), and variance, Eq. (2.2-9), are set equal to the beta distribution's mean (Eq. (2.4-2)) and variance (Eq. (2.4-3)). The resulting equations can be solved simultaneously for α and β to yield

$$\alpha = \frac{\bar{v}}{v_{max}} \left[\frac{\bar{v}(v_{max} - \bar{v})}{s^2} - 1\right], \tag{2.4-4}$$

$$\beta = \frac{v_{max} - \bar{v}}{\bar{v}} \alpha . \tag{2.4-5}$$

It should be emphasized that α and β can be obtained analytically.

The beta distribution is a particularly attractive distribution for wind modeling because it can assume many shapes and because it is defined only over a finite speed interval. These two properties help the beta distribution to resemble the given wind data more closely since all real wind frequency distributions are zero beyond some maximum wind speed, v_{max}. Furthermore, the parameters of the beta distribution are more easily obtained by matching-moments than those for the Weibull fit since a numerical (iterative) solution is not required for the beta distribution.

2.5 Description of Goodness of Fit Tests

Once the parameters of the desired distribution are computed, the accuracy of these distributions to represent the wind speed data must be verified. Two tests, a chi-squared (χ^2) test and a *power ratio* test, to judge the goodness of fit are developed in this section.

(a) χ^2 Test

The first test is the χ^2 test which is based on the following statistic [16]:

$$\chi^2 = \sum_{i=1}^{n} \frac{[NP_i^{obs} - NP_i]^2}{NP_i} , \tag{2.5-1}$$

where P_i^{obs} is the observed frequency of occurrence of wind speeds in subinterval i obtained from recorded data, while P_i is the expected frequency as predicted by the analytical model (i.e., as computed from Eq. (2.2-3). The symbol N represents the total number of all wind speed observations used to obtain the observed frequency distribution.

To determine the significance of the χ^2 test, the number of *degrees of freedom,* equal to the number of speed subintervals, n, minus the number of different independent linear restrictions imposed on the observations, must be determined. For the present case, one restriction, i.e., the loss of one degree of freedom, is due to the fact that the probability in the last speed subinterval is determined after the probabilities in the first n-1 speed subintervals are known. Furthermore, an additional constraint (loss of one degree of freedom) results from each independent parameter, e.g., $\bar{v}$ and s, which allows the determination of α and β, of a distribution estimated from the data [16]. Thus, the number of degrees of freedom, ν, is given by,

$$\nu = n - 1 - e, \tag{2.5-2}$$

where n is the number of speed subintervals and e is the number of parameters estimated from the data.

The number of degrees of freedom and the calculated value of χ^2 are used to determine the level of significance, i.e., the probability that chance would allow a value of χ^2 as large or larger than the one calculated. This significance level can be found from either standard tables or numerically integrating the incomplete gamma function, which the χ^2 distribution follows. Thus, if the calculated χ^2 is larger than the theoretical χ^2, the hypothesis that the analytical distribution, with parameters estimated from the data, represents the wind speed data is rejected.

In the computation of χ^2, it is imperative that NP_i^{obs} is the number of observations and not a probability percentage. The χ^2 test is invalid if probability percentages are used unless the sample size is exactly 100 [16]. The sample size is crucial because as noted from Eq. (2.5-1), the χ^2 value is a function of the sample size. Hence, use of the incorrect sample size changes the χ^2 value and consequently changes the significance level of the χ^2 test which could result in an erroneous conclusion about the fit of data to the analytical distributions.

Also, when performing the χ^2 test, wind speed subintervals should be grouped such that each wind speed subinterval for the analytical distribution contains at least a single observation [17]. With this adjustment, large contributions to χ^2 from wind speed subintervals with few observations are avoided. Table 2.5-1 illustrates the effect of grouping data. This example shows that the statistic χ^2 can be reduced by a factor of 2 if the data are grouped properly, i.e., combining wind speed subintervals so that every subinterval has at least a single observation. It must be noted, though, that grouping the intervals at the tails so that the expected observations are much greater than unity causes the χ^2 test to loose its power. This is because grouping may cover up the most distinct differences where the two distributions differ the most [17].

(b) Power Ratio Test

In the second test, the power available in the wind for a given analytical distribution is computed and compared to the power available as computed from the histogram of data from which the analytical model was obtained. The ratio of these two power calculations is defined to be the *power ratio*. A power ratio of unity indicates power calculations using the observed wind speed distribution and the fitted analytical distribution yield identical results. Consequently, this test shows the accuracy of the analytical fit over the entire speed range of interest for wind turbines rather than comparing accuracies at selected intervals.

To compute this power ratio, an expression for average wind power must be obtained. Using the analytical distribution, the average power output of a WTGS, $\overline{P}_{fit}$, is given by the equation

Table 2.5-1. Tabulation of χ^2 Values Illustrating Effect of Grouping Data.

Wind Speed Subinterval (knots)	NP_i^{obs}	NP_i	Contribution to χ^2 (ungrouped)	Contribution to χ^2 (grouped)
0.0 - 1.0	61	56.3	0.392	0.392
1.0 - 3.5	293	305.8	0.552	0.552
3.5 - 6.5	452	434.5	0.747	0.747
6.5 -10.5	517	498.6	0.679	0.679
10.5 -16.5	452	455.5	0.020	0.020
16.5 -21.5	143	162.7	2.45	2.45
21.5 -27.5	52	58.8	1.02	1.02
27.5 -33.5	10	6.53	0.253	*1.16
33.5 -40.5	2	0.27	11.1	
Totals	1982	1982	17.2	7.02

*last two subintervals combined

$$\overline{P}_{fit} = \int_0^\infty R(v)f(v)dv, \tag{2.5-3}$$

where R(v) is the *response function* of the wind turbine generator, i.e., the power obtained from a WTGS when the wind has a speed v, and f(v) is the wind speed probability density function to be tested. To calculate the power available when using the discrete probability density function, f_i, the integral in Eq. (2.5-3) must be divided up and summed over each of n speed subintervals with f(v) equal to f_i for v between v_i and v_{i+1}. Hence, when using the discrete probability density function to compute the average power output of a WTGS, $\overline{P}_{obs}$, the following equation is used:

$$\overline{P}_{obs} = \sum_{i=1}^{n} f_i \int_{v_i}^{v_{i+1}} R(v)\, dv. \tag{2.5-4}$$

To evaluate the above integrals, the WTGS response function, R(v), must first be selected. Several response function models have been proposed. Justus, et. al., [4] suggest a quadratic polynomial. The model used for such a fit was NASA's Plum Brook WTGS. The parameters of this unit are a rated power output of 100 kW, a *cut-in speed* (a wind below which the generator produces no power) of 8 mph, a *rated speed* of 18 mph, and a *furling speed* (a wind speed above which the generator turns off to prevent damage to the system) of 60 mph. The response function is described by

$$R(v) = \begin{cases} 0 & v \le v_c \\ A + Bv + Cv^2, & v_c < v \le v_r \\ P_r, & v_r < v \le v_{furl} \\ 0, & v > v_{furl} \end{cases} \tag{2.5-5}$$

where v_c is the cut-in speed, v_r is the rated speed, v_{furl} is the furling speed, and P_r is the rated speed. The coefficients A, B, and C are determined from the following conditions

$$\left.\begin{aligned} A + Bv_c + Cv_c^2 &= 0 \\ A + Bv_r + Cv_r^2 &= P_r \\ A + Bv_o + Cv_o^2 &= P_r \left(\frac{v_o}{v_r}\right)^3 \end{aligned}\right\}, \tag{2.5-6}$$

where

$$v_o = \frac{v_c + v_r}{2} .$$

The response function chosen for use in this study is a more idealized one and is based upon the theoretical power in the wind due to the mass and velocity of air molecules. The total power available from the motion of air with speed v through a cross-sectional area A is given by [5]

$$P_w = \frac{1}{2} \rho A v^3, \tag{2.5-7}$$

where P_w is the power in watts, ρ is the density of air in kg/m^3, A is the exposed area in m^2, and v is the wind speed in m/s. The important feature of this relation is that the available power varies with the cube of the wind speed.

The wind turbine response function used in this study was assumed to follow a cubic relation also. Fig. 2.5-1 shows this idealized WTGS response function. Below v_c (cut-in speed), the wind turbine produces effectively zero power due to electrical and mechanical losses. Between v_c and v_r (rated speed), power proportional to the cube of the wind speed is produced. Once the rated speed is reached, the wind turbine reaches its rated power output, P_r. For speeds greater than v_r, a control system varies the blade pitch so that the generator capacity is not exceeded and the power ouput is maintained at its rated power. However, if the wind ever exceeds v_{furl}, the WTGS is shut down or furled in order to prevent damage to the system. This idealized model is expressed by the following equation:

$$R(v) = \begin{cases} 0 & , v \leq v_c, \\ P_r\left(\frac{v}{v_r}\right)^3 & , v_c < v \leq v_r, \\ P_r & , v_r < v \leq v_{furl}, \\ 0 & , v > v_{furl} . \end{cases} \qquad (2.5\text{-}8)$$

Because storm level winds are usually suppressed in reported wind speed data, the speed v_{furl} is set equal to the v_{max} which is the maximum speed reported in the wind speed data. Furthermore, unless specified, the cut-in speed is given by the equation

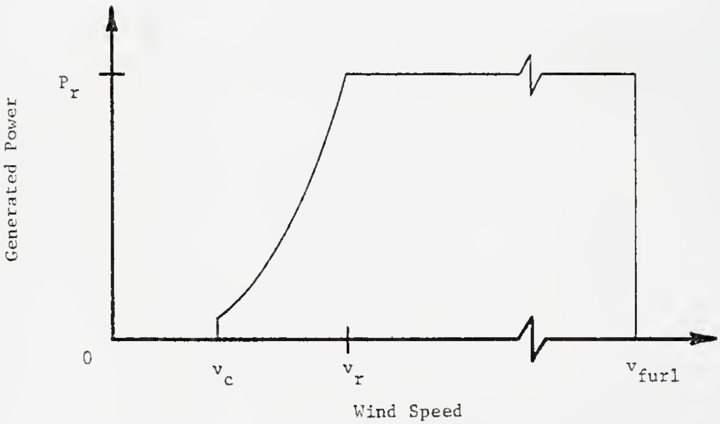

Fig. 2.5-1. An idealized WTGS response function.

$$v_c = 0.46416\ v_r. \tag{2.5-9}$$

This relation implies that the WTGS is idle when the output power is less than 10% of the rated power for the WTGS.

Substitution of the WTGS response function given in Eq. (2.5-8) into Eq. (2.5-3) yields an explicit form for the average power output of a WTGS using the analytical distribution as follows

$$\bar{P}_{fit} = \int_{v_c}^{\gamma} P_r \left(\frac{v}{v_r}\right)^3 f(v)\ dv + \int_{v_r}^{v_{max}} P_r\ f(v)\ dv, \tag{2.5-10}$$

where $\gamma = \min\ (v_r,\ v_{max})$.

If, in any of the integrals in Eq. (2.5-10), the upper limit is less than the lower limit, the integral is assumed to be zero. Because the above integrals cannot be evaluated analytically when the analytical distribution is used, a numerical technique is required. Standard Gauss-Legendre quadrature was used (see Section 3.2-4).

When using the WTGS response function (Eq. (2.5-8)) to compute the average power output of a WTGS for the discrete wind speed distribution from Eq. (2.5-4), care must be taken to use the proper form of the WTGS response function depending upon the magnitude of the speeds in a particular speed subinterval. For example, if the rated speed occurs in the middle of a speed subinterval, the speeds in the subinterval less than or equal to v_r follow the cubic power relationship given in Eq. (2.5-8) while the speeds greater than v_r follow the constant power relationship.

Consequently, computation of the power ratio requires calculation of the average power using both the analytical and discrete probability density functions. The ratio of these two results gives an indication of the closeness of fit between the analytical and discrete wind speed probability density functions for wind power calculation. A value of unity means there is an excellent fit, while values significantly less than or greater than unity indicate an under- or overestimation of the available wind power, respectively.

2.6 Results

To solve for the parameters of the fitting distributions and to evaluate the goodness of fit tests, a computer routine, CURVEFIT, was developed. A listing and explanation are contained in Appendix A. Wind speed data were obtained from the National Climatic Center, Asheville, North Carolina for seventeen sites throughout the United States. A map showing the locations of the sites is given in Fig. 2.6-1. These sites are representative of the many possible wind conditions and power densities found throughout the United States. Table 2.2-1 is a typical listing of the "binned" wind speed data obtained from the National Climatic Center. For each of the seventeen sites, wind speed data were given in eight, three-hour intervals throughout a day for a particular month. Data for the months of January, April, July, and October were chosen in this study to simulate the four seasons of the year. Consequently, 544 (= 8 x 4 x 17) observed wind speed distributions were chosen and fit by analytical distributions. Although anemometer heights

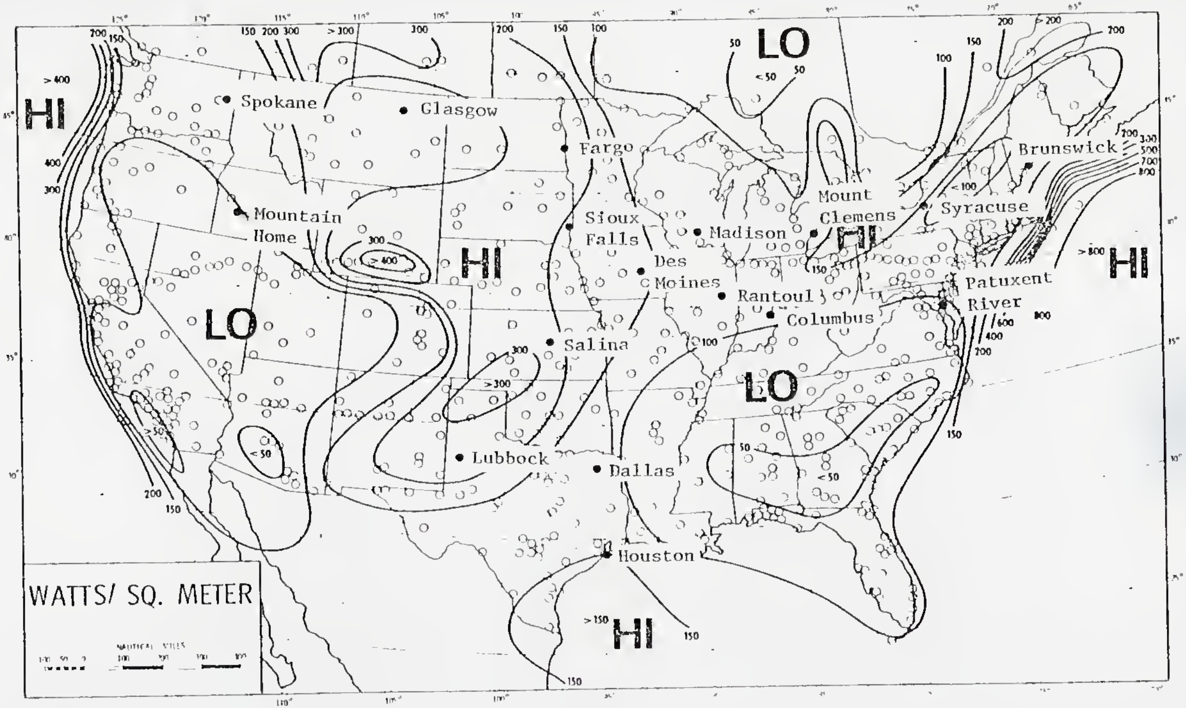

Fig. 2.6-1. Average annual wind power densities for the United States. Circles indicate meteorological stations. Dark circles indicate locations of wind speed data analyzed in this study. Contour lines designate areas of equal power density (From Ref. 3).

for the recorded wind speed data typically varied during the recording period at individual observation stations, no attempt could be made to adjust for these variations. This is because complete historical records of anemometer heights at each location used were unavailable. However, anemometer heights were roughly from 7 to 10 meters and consequently, any changes incurred because of varying anemometer heights should be minimal.

Example results from the CURVEFIT program are given in Tables 2.6-1 and 2.6-2. Two tables are used to present the results from each location. The first table lists the parameters for each of the analytical distributions calculated from the wind speed data while the second table lists the results of the goodness of fit tests run on each analytical distribution. In Table 2.6-1, the first line gives the mean, standard deviation, and the fit parameters for the wind speed distribution data from Syracuse, New York, in the month of January during the first three hours of a typical day (beginning at midnight).

The parameters given in Table 2.6-1 can be used to plot the probability density functions. Figure 2.6-2 shows the four analytical fits whose parameters are given in the first line of Table 2.6-1 together with the actually observed distribution.

Table 2.6-2 presents the goodness of fit statistics for the cases presented in Table 2.6-1. This table lists the χ^2 values for each of the four analytical distributions. The integer in parentheses is the number of degrees of freedom associated with the χ^2 value for that

Table 2.6-1. A Sample Output Table from the CURVEFIT Routine Showing the Parameters of the Analytical Distributions.

PARAMETERS OF VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT SYRACUSE, NEW YORK

MONTH	TIME (HRS)	MEAN SPEED (KNOTS)	STD. DEV. (KNOTS)	WEIBULL DISTRIBUTION PARAMETERS LST. SQS.-UNWTD. K	C	LST. SQS.-WTD. K	C	MATCHING-MOMENTS K	C	BETA DISTRIBUTION PARAMETERS ALPHA	BETA
1	0- 3	8.979	5.818	1.328	7.387	1.336	7.682	1.578	10.002	1.632	5.728
1	3- 6	8.884	5.907	1.352	7.623	1.357	7.605	1.535	9.868	1.547	5.503
1	6- 9	8.733	5.756	1.297	7.096	1.304	7.412	1.549	9.710	1.590	5.784
1	9-12	9.648	5.535	1.443	8.229	1.437	8.396	1.671	10.800	1.775	5.676
1	12-15	10.623	5.901	1.547	9.230	1.619	9.473	1.870	11.965	2.129	5.987
1	15-18	9.734	5.640	1.641	9.072	1.613	8.563	1.784	10.941	2.023	6.393
1	18-21	8.806	5.591	1.360	7.367	1.378	7.504	1.614	9.829	1.724	6.205
1	21-24	9.117	5.706	1.429	7.816	1.418	7.835	1.639	10.190	1.753	6.035
4	0- 3	8.166	4.571	1.421	6.845	1.462	6.948	1.690	9.149	1.797	5.576
4	3- 6	8.047	4.929	1.425	6.623	1.442	6.811	1.679	9.011	1.785	5.645
4	6- 9	8.827	5.162	1.409	7.098	1.461	7.547	1.766	9.916	1.890	5.283
4	9-12	10.993	5.749	1.597	9.563	1.711	9.780	1.999	12.404	2.393	6.423
4	12-15	12.127	5.738	2.097	11.657	1.993	11.066	2.234	13.692	2.830	6.620
4	15-18	11.958	5.515	1.834	11.512	2.022	10.793	2.299	13.498	3.266	9.707
4	18-21	9.215	5.408	1.456	8.500	1.571	8.042	1.760	10.351	2.147	8.918
4	21-24	8.271	5.397	1.352	7.063	1.369	6.995	1.566	9.206	1.665	6.488
7	0- 3	5.636	3.576	1.316	4.429	1.358	4.479	1.615	6.292	1.770	6.867
7	3- 6	5.473	3.424	1.319	3.988	1.314	4.277	1.640	6.117	1.650	4.833
7	6- 9	6.463	3.996	1.408	5.163	1.412	5.258	1.661	7.231	1.766	5.750
7	9-12	8.361	4.249	1.724	6.929	1.796	7.163	2.064	9.439	2.390	5.472
7	12-15	9.654	4.394	1.933	8.826	2.107	8.494	2.334	10.895	3.148	7.776
7	15-18	9.615	4.271	1.819	8.652	2.127	8.408	2.393	10.847	3.326	8.262
7	18-21	6.859	3.890	1.509	6.156	1.632	5.738	1.827	7.719	2.268	8.809
7	21-24	5.634	3.528	1.340	4.487	1.389	4.492	1.639	6.298	1.823	7.075
10	0- 3	6.781	4.282	1.305	5.119	1.314	5.495	1.623	7.574	1.643	5.019
10	3- 6	6.744	4.275	1.303	5.069	1.308	5.466	1.617	7.529	1.634	5.027
10	6- 9	6.785	4.553	1.181	5.367	1.247	5.459	1.519	7.527	1.682	8.356
10	9-12	8.753	4.992	1.509	6.885	1.494	7.407	1.816	9.847	1.778	3.808
10	12-15	9.958	5.158	1.623	8.740	1.721	8.738	2.020	11.238	2.565	7.866
10	15-18	8.953	4.759	1.747	7.999	1.742	7.776	1.963	10.098	2.326	6.377
10	18-21	7.179	4.700	1.347	6.456	1.420	5.973	1.561	7.988	1.743	8.088
10	21-24	7.206	4.822	1.217	6.300	1.346	5.941	1.524	7.998	1.743	9.746

Table 2.6-2. A Sample Output Table from the CURVEFIT Routine Showing the Results of the Goodness of Fit Tests.

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT SYRACUSE, NEW YORK

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(6) 304.	(6) 227.	(6) 2.96	(5) 7.07	0.589	0.636	0.976	0.997
1	3- 6	(6) 226.	(5) 231.	(6) 9.90	(5) 31.4	0.650	0.645	1.01	1.03
1	6- 9	(5) 315.	(6) 231.	(6) 6.96	(5) 8.90	0.577	0.629	0.971	0.994
1	9-12	(5) 289.	(6) 242.	(6) 4.77	(5) 7.80	0.612	0.639	0.973	0.989
1	12-15	(6) 313.	(5) 265.	(6) 9.25	(5) 22.4	0.644	0.666	0.994	0.999
1	15-18	(5) 175.	(5) 314.	(6) 22.9	(5) 26.4	0.705	0.627	0.993	1.00
1	18-21	(5) 293.	(5) 258.	(6) 8.95	(5) 12.4	0.589	0.607	0.965	0.981
1	21-24	(5) 268.	(5) 259.	(6) 10.2	(5) 9.84	0.606	0.613	0.966	0.982
4	0- 3	(5) 315.	(5) 296.	(5) 14.9	(4) 25.8	0.574	0.578	0.991	1.02
4	3- 6	(4) 345.	(4) 281.	(5) 3.77	(4) 9.00	0.529	0.561	0.967	0.991
4	6- 9	(5) 447.	(5) 305.	(5) 27.6	(4) 32.6	0.527	0.588	0.962	0.984
4	9-12	(6) 369.	(5) 327.	(6) 31.5	(5) 52.3	0.634	0.648	0.983	0.982
4	12-15	(5) 208.	(5) 344.	(6) 16.2	(5) 17.7	0.758	0.689	0.996	0.990
4	15-18	(6) 249.	(5) 387.	(5) 26.0	(5) 62.0	0.747	0.658	0.986	0.970
4	18-21	(6) 187.	(5) 286.	(5) 12.1	(5) 14.6	0.724	0.612	0.995	0.991
4	21-24	(5) 221.	(5) 245.	(5) 4.83	(5) 13.4	0.619	0.598	0.985	1.00
7	0- 3	(3) 422.	(3) 412.	(4) 29.9	(3) 45.4	0.466	0.446	0.980	0.999
7	3- 6	(3) 674.	(3) 453.	(3) 47.5	(3) 57.5	0.346	0.450	0.952	0.977
7	6- 9	(4) 423.	(4) 373.	(4) 13.7	(4) 25.5	0.443	0.467	0.955	0.981
7	9-12	(4) 547.	(4) 449.	(4) 10.0	(4) 14.9	0.461	0.488	0.950	0.965
7	12-15	(4) 296.	(4) 561.	(4) 11.6	(4) 20.8	0.624	0.534	0.978	0.972
7	15-18	(4) 398.	(4) 631.	(4) 39.7	(4) 84.0	0.626	0.525	0.984	0.975
7	18-21	(4) 256.	(3) 430.	(4) 37.6	(4) 46.6	0.644	0.451	0.997	1.00
7	21-24	(3) 402.	(3) 423.	(4) 27.2	(3) 41.3	0.473	0.431	0.978	0.995
10	0- 3	(4) 534.	(4) 352.	(4) 33.0	(4) 40.1	0.432	0.522	0.948	0.985
10	3- 6	(4) 541.	(4) 346.	(4) 37.3	(4) 40.1	0.425	0.524	0.946	0.983
10	6- 9	(5) 360.	(5) 325.	(5) 33.9	(4) 37.7	0.577	0.551	0.970	0.985
10	9-12	(4) 656.	(4) 390.	(4) 41.9	(4) 23.7	0.449	0.544	0.927	0.971
10	12-15	(5) 359.	(5) 371.	(5) 21.9	(5) 42.7	0.609	0.590	0.961	0.956
10	15-18	(4) 289.	(4) 385.	(5) 9.57	(4) 10.5	0.587	0.545	0.970	0.977
10	18-21	(5) 184.	(4) 350.	(5) 39.6	(4) 40.4	0.707	0.535	1.02	1.04
10	21-24	(5) 245.	(4) 287.	(5) 30.6	(5) 45.8	0.731	0.554	1.01	1.02

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

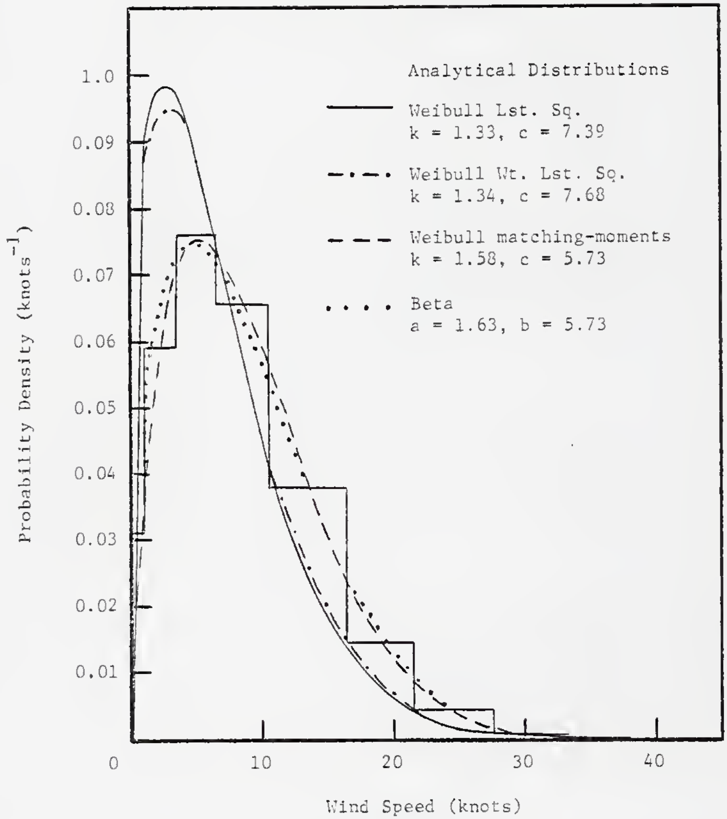

Fig. 2.6-2. Comparison of analytical distributions to observed distribution for Syracuse, New York, data (January, hours 1 - 3).

particular analytical fit. The final four columns show the power ratio computed for each analytical fit. The rated power (100 kW), cut-in speed (8 mph), and rated speed (18 mph) used were those of the NASA experimental wind turbine generator at Plum Brook, Ohio.

(a) Results of χ^2 Test

The results of the χ^2 test reveal rather large values for all analytical fits. For the Weibull distributions using the least squares technique to estimate parameters, all χ^2 values are in excess of 180. This is quite large considering the χ^2 value at the 0.995 confidence level with ten degrees of freedom is 25.2 and all tabulated χ^2 values have fewer than ten degrees of freedom. Hence, there is less than a 0.5% chance that the least squares fitted distributions describe the given data. This result was not totally unexpected because, as shown earlier, the least squares estimation of the Weibull distribution parameters does not guarantee the best fit to the data.

However, large χ^2 values were also obtained for the matching-moments estimation of both the Weibull and beta distribution parameters. Over 93% did not pass the non-significance test at the 0.005 significance level. But there are some χ^2 values which indicate a good fit of the analytical functions to the data (see Table 2.6-2 in which the analysis of the fits to the wind data from Syracuse, New York is shown). In the examples of Table 2.6-2, about half of the matching-moments Weibull distribution pass the χ^2 test at the 0.005 significance level, and of those all except one pass the test at significance levels less than or equal to 0.01. For the beta distribution, only 31% pass the test at a significance level of 0.005 or less. Examination of the remaining tables in Appendix B shows other isolated cases where the χ^2 values for both the Weibull and beta distribution, using the matching-moments parameter estimation, yield significant results. In addition, there are sporadic cases where the beta distribution gives the largest χ^2 value of all of the analytical fits.

Similar results were obtained by Kaminsky for all the analytical distributions he used to describe observed wind speed data [9], i.e., the χ^2 values were typically very large. However, of the four distributions (log-normal, gamma, Weibull, and Rayleigh) he used to characterize wind speed distributions, the lowest χ^2 values were obtained with the gamma and Weibull distributions.

Because the χ^2 test merely sums the square of the deviations between the data and the analytical fit at each speed subinterval, the contributions to the χ^2 value from each of these speed subintervals was investigated. In one case, for the April wind speed data from Columbus, Indiana during hours nine through twelve, both the Weibull and beta distributions using matching-moments estimation yield χ^2 values greater than the other Weibull distributions. A breakdown of the contributions to the χ^2 value by speed subinterval is shown in Table 2.6-3.

For the Weibull and beta distributions using matching-moments estimation, 98% of the total contribution to the final χ^2 value arises from the first two speed subintervals, or speeds between 0 and 3.5

Table 2.6-3. List of Contributions to χ^2 Value for all Analytical Wind Speed Distributions Computed for Columbus, Indiana, for Fourth Month, Hours 9-12.

	Contribution to χ^2			
Speed Subinterval (knots)	Weibull-Lst Sqs.	Weibull-wtd. Lst Sqs.	Weibull-Matching-Moments	Beta
0 - 1.0	1.06	0.0279	582	842
1.0 - 3.5	337	298	68.9	70.1
3.5 - 6.5	56.2	38.1	1.33	3.11
6.5 - 10.5	90.5	94.9	12.3	15.3
10.5 - 16.5	210	138	2.29	1.18
16.5 - 21.5	117	45.6	0.247	0.113
21.5 - 27.5	0.020	6.73	4.65	3.94*
27.5 - 33.5	1.71	6.56	0.324	
χ^2 Total	813	628	672	936
Degrees of Freedom	5	5	5	5

*Last two subintervals combined.

knots. The remaining speed subintervals contribute little to the final χ^2 value. On the other hand, the other two Weibull distributions (obtained by least squares fits) show rather uniform contributions from each speed subinterval to the total χ^2 value. There are no speed subintervals which contribute such a large amount to the final value. This is typical of most of the remaining data, i.e., the deviations between the Weibull and beta distribution using the matching-moments estimation techniques and the actual wind speed distribution occur in the first two speed subintervals, whereas in the two least square fits to the Weibull distribution, an equally large amount is contributed to the χ^2 statistic by all speed subintervals. Consequently, this indicates that the matching-moments estimations of the parameters of the Weibull and beta distributions fit the intermediate and high wind speed subintervals much more closely than at the low speed end of the distribution. It is the middle and upper speed subintervals which are important in the analysis of a WTGS and for which the matching-moments technique produces good fits with the Weibull and beta distributions.

(b) Results of Power Ratio Test

The power ratio values were also obtained for all 544 wind distributions fit by the Weibull and beta distributions. In addition to use of the cut-in and rated speeds of the NASA Plum Brook WTGS (a cut-in speed of 8 mph and a rated speed of 18 mph), rated speeds of 12, 15, 21, and 24 mph were also used. Rated power was held constant at 100 kW. Cut-in speeds for the latter four cases were given by Eq. (2.5-9).

The mean power ratios obtained from the wind speed data at the seventeen sites for each WTGS size are tabulated in Table 2.6-4. As explained earlier, 544 observed wind speed distributions were used to calculate the power ratios for each WTGS size. From this table it can be seen that parameter estimations obtained by unweighted and weighted least squares of the doubly logarithmic transformed cumulative Weibull distribution grossly underestimate the power available, whereas the matching-moments parameter estimations of both the Weibull and beta distributions yield values very close to unity.

Frequency plots showing the distributions of the power ratio values about their calculated mean are shown in Figs. 2.6-3 to 2.6-12. Because of the large dispersion of power ratios about the mean value for the Weibull distributions which were obtained by the least squares estimators, their power ratios are grouped into wider class intervals in order to yield a smoother distribution.

Since the power ratios obtained from both the beta and Weibull distributions using matching-moments parameter estimation are so close to the expected value of unity, it is reasonable to test these values and ascertain whether there is a significant difference. Consequently, a t test [18] is applied.

A t test is used to determine if there is a significant difference between an observed mean and a theoretical mean or between two observed means. Philosophically, one establishes the hypothesis that the two means (observed and theoretical or two observed means) are equal or that the

Table 2.6-4. List of Power Ratio Means and Standard Deviations for Various Analytical Distributions and Generator Rated Speeds.

Method For Calculation of Power Ratio	Rated Speed v_r (mph)	Power Ratio Mean $\overline{r}$	Sample Std. Dev. s_r
Weibull- lst. sq.	12	0.589	0.115
" " "	15	0.556	0.109
" " "	18	0.529	0.108
" " "	21	0.519	0.113
" " "	24	0.519	0.118
Weibull-wt. lst. sq.	12	0.664	0.0754
" " " "	15	0.648	0.0607
" " " "	18	0.636	0.0730
" " " "	21	0.648	0.105
" " " "	24	0.671	0.159
Weibull- matching-moments	12	0.982	0.0355
" " "	15	0.988	0.0376
" " "	18	0.975	0.0427
" " "	21	0.986	0.0475
" " "	24	0.998	0.0647
Beta	12	0.992	0.0232
"	15	1.01	0.0311
"	18	1.001	0.0331
"	21	1.01	0.0342
"	24	1.02	0.047

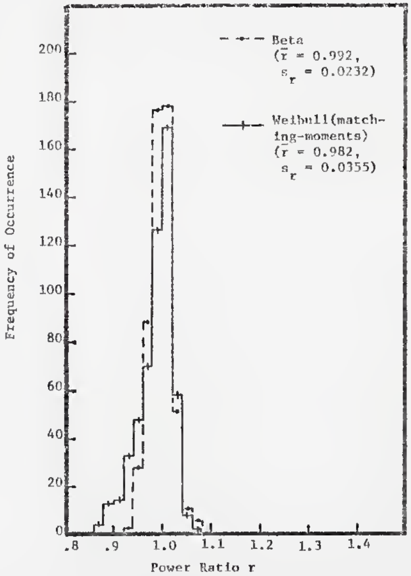

Fig. 2.6-3. Distribution of power ratio values using beta and Weibull (matching-moments) distributions for a WTGS with a rated speed of 12 mph and a rated power of 100 kW.

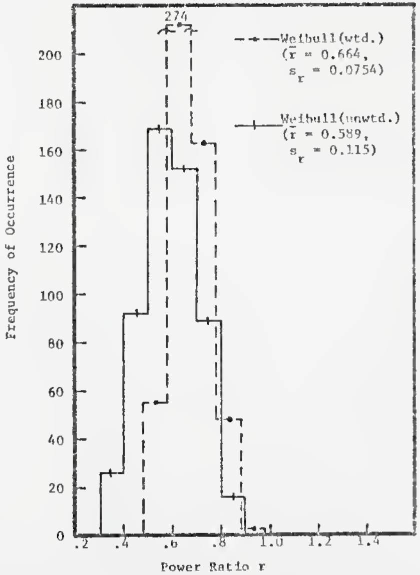

Fig. 2.6-4. Distribution of power ratio values using two least squares(weighted and unweighted) Weibull distributions for a WTGS with a rated speed of 12 mph and a rated power of 100 kW.

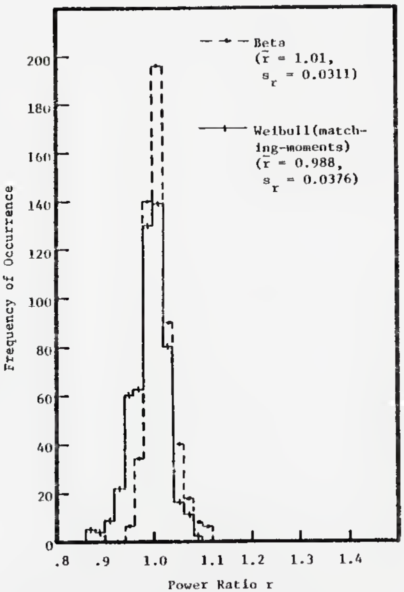

Fig. 2.6-5. Distribution of power ratio values using beta and Weibull (matching-moments) distributions for a WTGS with a rated speed of 15 mph and a rated power of 100 kW.

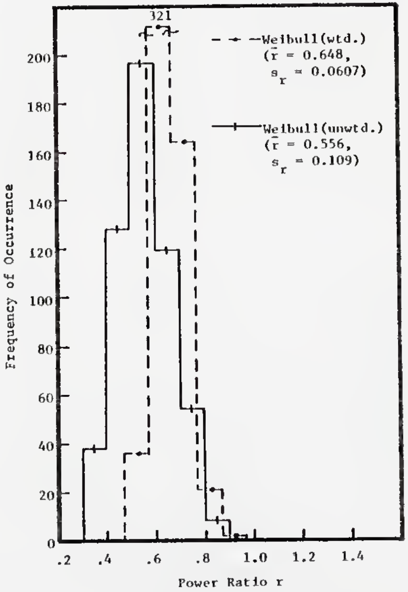

Fig. 2.6-6. Distribution of power ratio values using two least squares (weighted and unweighted) Weibull distributions for a WTGS with a rated speed of 15 mph and a rated power of 100 kW.

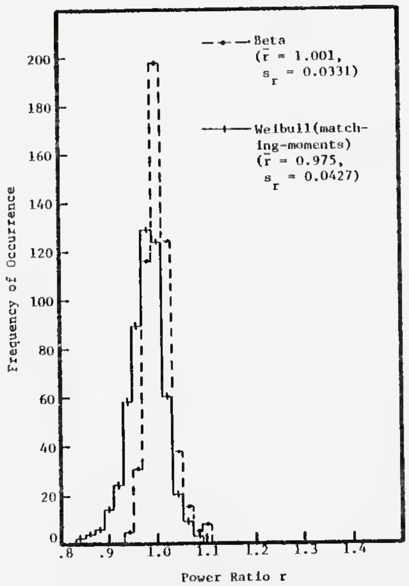

Fig. 2.6-7. Distribution of power ratio values using beta and Weibull (matching-moments) distributions for a WTGS with a rated speed of 18 mph and a rated power of 100 kW.

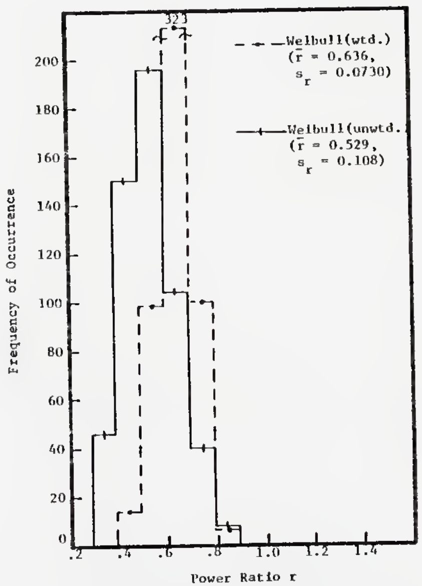

Fig. 2.6-8. Distribution of power ratio values using two least squares (weighted and unweighted) Weibull distributions for a WTGS with a rated speed of 18 mph and a rated power of 100 kW.

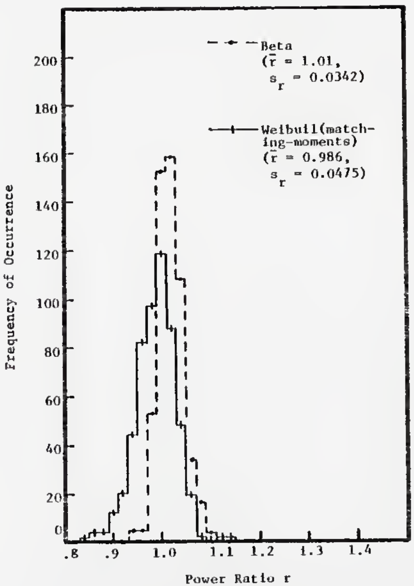

Fig. 2.6-9. Distribution of power ratio values using beta and Weibull (matching-moments) distributions for a WTGS with a rated speed of 21 mph and a rated power of 100 kW.

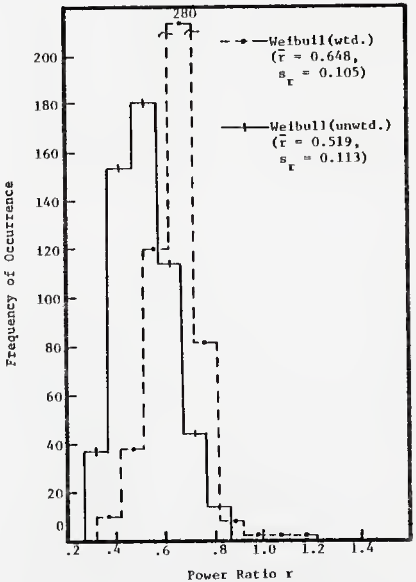

Fig. 2.6-10. Distribution of power ratio values using two least squares (weighted and unweighted) Weibull distributions for a WTGS with a rated speed of 21 mph and a rated power of 100 kW.

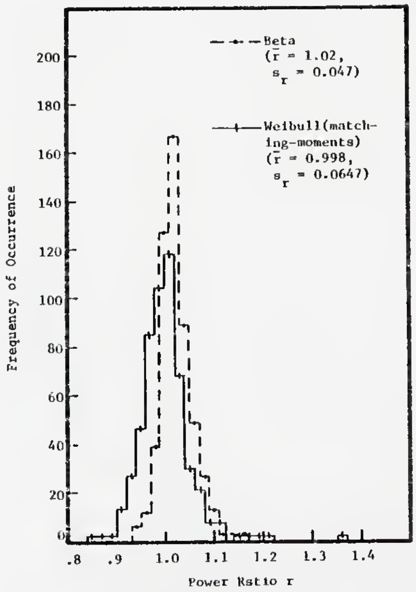

Fig. 2.6-11. Distribution of power ratio values using beta and Weibull (matching-moments) distributions for a WTGS with a rated speed of 24 mph and a rated power of 100 kW.

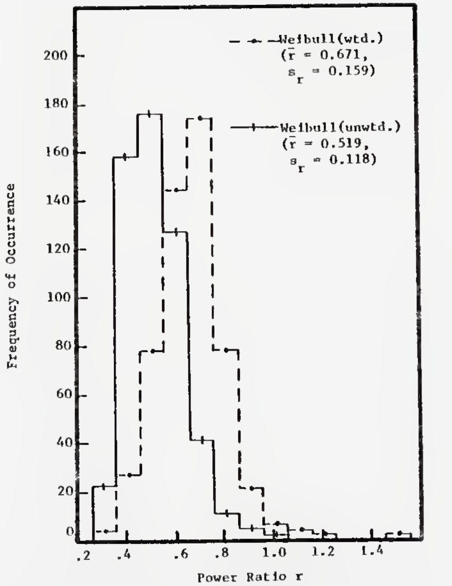

Fig. 2.6-12. Distribution of power ratio values using two least squares (weighted and unweighted) Weibull distributions for a WTGS with a rated speed of 24 mph and a rated power of 100 kW.

samples used to calculate the observed means are from the same population. The calculated t value is compared to a t value (called the critical t value) at a specified level of significance, i.e., to the integral of the t distribution with an integration limit set such that the area of integration is equal to unity minus the level of significance. Thus, the level of significance is the probability that chance will allow a t value equal to or greater than the critical t value.

The t statistic is defined by [18]

$$t = \frac{|\bar{r} - \mu|}{[s_r^2/N]^{\frac{1}{2}}} ,$$

where $\bar{r}$ is the sample mean, μ is the mean which $\bar{r}$ is to be tested against (equal to unity for the sample variance, and N is the sample size), and s_r is the standard deviation of the sample values from the sample mean.

Table 2.6-5 lists the results of this test for the power ratio sample means. The t value at the 0.001 significance level with an infinite number of degrees of freedom (each mean power ratio mean was computed from 544 wind speed distributions) is 3.291, which means that there is less than a 0.1% probability that a random sample of power ratios drawn from a population with a mean of μ will yield a t value of 3.291 or greater. All but two power ratio sample means have t values larger than 3.291; thus, it can be stated that the true mean of the power ratio using a particular analytical distribution is something other than unity at the 0.001 level of significance.

Table 2.6-5. List of t Values for Comparison of Power Ratio Means of Beta and Weibull (Matching-Moments) Distributions to Unity.

Method of Power Ratio Calculation	Rated Speed v_r (mph)	Power Ratio Mean $\bar{r}$	t Statistic	t From Table (Confidence Level = 0.001, Degrees of Freedom = 544 $\sim \infty$)
Weibull- Matching-Moments	12	0.982	12.2	3.291
" "	15	0.988	7.38	3.291
" "	18	0.975	13.7	3.291
" "	21	0.986	6.92	3.291
" "	24	0.992	2.96	3.291
Beta	12	0.992	8.24	3.291
"	15	1.01	6.52	3.291
"	18	1.00	0.705	3.291
"	21	1.01	9.89	3.291
"	24	1.02	11.2	3.291

Although the t test does not confirm the suspicion that the power ratio values for the distributions using matching-moments estimators have a mean of unity, a significant difference can be seen between the power ratios for the distributions using matching-moments estimators and those distributions using least squares estimators. The latter distributions yielded power ratio values ranging from .29 to 2.2 with most values being between .4 and .8 while the former distributions were consistently in the .85 to 1.4 range with 90% of the values between .90 and 1.1. Since the speeds at which the wind turbine generators produce significant power correspond to the intermediate and high speed range, and since the best power ratio values are obtained from the matching-moments estimation of the Weibull and beta distribution parameters, the conclusion drawn from the χ^2 test that the matching-moments estimators yield distributions which accurately fit the intermediate and high speed subintervals is substantiated. Furthermore, because a wind turbine cannot use low speeds (wind speeds below the cut-in speed), analytical models which accurately characterize intermediate and high speed subintervals are most desirable.

Finally, the t test was performed again to investigate whether there is a significant difference between the mean values of the power ratios of the beta and Weibull distributions, whose parameters were obtained by the matching-moments technique. In this case since the means of two distributions are compared against each other, rather than comparing one mean with a predetermined value, a slightly different form of Eq. (2.6-2) is used, namely [19]

$$t = \frac{|\bar{r}_1 - \bar{r}_2|}{s_d} \quad , \tag{2.6-3}$$

where $$s_d = s_c \left(\frac{N_1 + N_2}{N_1 + N_2}\right)^{1/2} \quad ,$$

$$s_c = \left(\frac{s_1^{\,2}\ (N_1 - 1 + s_2^{\,2}\ (N_2 - 1)}{(N_1 - 1) + (N_2 - 1)}\right)^{1/2} ,$$

N_1 and N_2 are the sample sizes of each distribution to be compared, and s_1 and s_2 are the standard deviations of each distribution. The quantity s_c is called the *pooled estimate* of the population standard deviation. The quantity s_d is then just the difference in the standard deviations of the two means.

The results are shown in Table 2.6-6. The t values are all much greater than the t value at a significance level of 0.001 and an infinite number of degrees of freedom. Consequently, it can be stated that the two distributions do yield significantly different power ratio values and hence the two distributions are different. From Table 2.6-4, it can be seen the beta distribution usually overestimates the available power since the power ratios corresponding to this distribution are greater than unity whereas the Weibull distribution using matching-moments estimators underestimates the available power for all wind turbine sizes considered.

In summary, analysis of 544 observed wind speed distributions by the routine CURVEFIT shows that either the Weibull or beta distributions obtained by the matching-moments technique gives an excellent fit to

Table 2.6-6. List of t Values for Comparison of Power Ratio Mean of Beta Distribution to Power Ratio Mean of Weibull (Matching-Moments) Distribution.

Rated Speed v_r (mph)	s_c	s_d	t	t from table (significance levels = 0.01 degrees of freedom = 1086 $\sim \infty$)
12	0.0300	0.00182	5.66	3.291
15	0.0345	0.00209	9.86	3.291
18	0.0382	0.00232	11.25	3.291
21	0.0585	0.00355	8.06	3.291
24	0.0566	0.00343	8.95	3.291

observed windspeed data. Although most χ^2 values for both distributions are too large to accept either distribution as a good fit at even the 0.005 significance level, they are far better representations of the data than the Weibull distributions obtained by using a least squares estimation of the parameters. These unexpectedly large values of χ^2 for the Weibull and beta distributions, whose parameters are estimated by the matching-moments technique, results from discrepancies in the fit in the very low speed subintervals. The remaining wind speed subintervals are very accurately fit. This conclusion is supported further by near unity results for the power ratio test, which places large emphasis on intermediate and high speeds. Hence, for the speed subintervals of interest, the Weibull or beta distribution derived from the matching-moments method are generally very representative of the wind speed data.

3. WIND TURBINE GENERATOR OPTIMIZATION

3.1 Introduction

An optimum wind turbine generator system (WTGS) depends upon many factors, such as its intended application, wind characteristics, economic considerations, aesthetic aspects, system reliability, and practical constraints placed upon its design, location, building materials, etc. Consequently, the term *optimum* can assume many meanings. An optimum system may be one which is completely autonomous, i.e., capable of generating the demanded power without the need of any back-up system. Still another interpretation may require that the amount of time the wind machine is down for repairs is to be minimized. For this case the optimum system would be one which makes use of the most reliable components in the manufacture of the wind system.

In this time of increasing energy awareness, wind power is being looked upon to generate electricity on both a central station and a local, decentralized level. As noted in Chapter 1, some technical and economical problems still plague the large, central station units. Small wind turbines, on the other hand, have produced power successfully for many years. In contrast to both fossil and nuclear fuels, wind power is both a relatively clean and renewable source of energy. However, for wind generated electricity to make a significant impact as a decentralized power source, it must be shown to be capable of producing energy competitively with more conventional sources of power. For decentralized production, economic optimization becomes a key factor. Like all economic decisions, the desirability of one alternative over another is based upon the alternative that saves the user the most money. For a WTGS to be economically viable it must be demonstrated that it can save the user money, otherwise it is wiser economically to continue to purchase all of the demanded power from the utility. To compute the economic savings afforded by use of a certain size WTGS in a particular application, the power output from the wind system (given wind speed data at the location where the WTGS will be used) must be matched with the load demands of the application to see how much of the load the wind system can supply. The remainder of the demand load must be provided by a back-up system or purchased from an energy utility. In addition, the capital cost of the WTGS and its amortization becomes an economic factor to be considered in such an analysis.

Most previous economic studies on the use of a WTGS to generate power have simply examined the total amount of energy generated with the assumption that the generated power can always be used. However, for decentralized applications the demand power requirements will have to be matched to the power production of the WTGS both of which will vary throughout the day and from season to season. Very little work has been done to date on examining the economics of matching a dedicated WTGS to a given demand load. Developmental Planning and Research Associates (DPRA) of Manhattan, Kansas, has recently performed pioneering work in this area [12]. Their study entailed the development of a methodology to determine the national impact that economically optimum sized WTGSs could have in various agricultural applications. However, this investigation did not examine the sensitivity of their economic optimization procedure to changes in the model parameters, e.g., mean wind speed, variations of wind speeds about this mean, and variations in load demand. Consequently, in the second phase of this work, the methodology for economic optimization of a WTGS given wind and load distribution is examined and the results of sensitivity studies on the optimization model are reported. In addition, this methodology is used for a realistic example to determine the WTGS size needed in order to realize maximum savings over the purchase of all of the required power. Also, the effect of detailed wind speed and load information on the optimum WTGS size is illustrated by this realistic example.

Two major assumptions have been made about the application of a WTGS to a dedicated load in this study. First, the WTGS was assumed to generate AC electric power compatable with that supplied by the utility grid. Furthermore, the WTGS was connected to the electric grid in such a manner that the demanded electric power which cannot be supplied by the WTGS was purchased from the electric utility. This implies the use of an interfacing system between the WTGS and the power grid so that a blending of wind generated and utility power can occur. This system will keep the WTGS in synchronization with the utility grid as well as monitor whether power is to be taken from the grid or dumped onto the grid depending upon the demanded load and the output of the WTGS. Consequently, power supplied from the WTGS will be indistinguishable to the user from power supplied by the utility grid. Second, there was no energy storage capability associated with the WTGS. Hence, whenever the wind was of sufficient strength to generate power above that demanded by the load, the excess was either wasted or sold to the utility or another customer.

3.2 Optimization Methodology

The methodology used in the present optimization study involves the development of a series of procedures or modules. First, the power output of a given sized WTGS for a particular distribution of wind speeds must be matched with the given demand load. Because demand loads and wind speeds typically vary throughout a day and throughout days in a year, load demand and wind speed profiles must be given both for various times throughout a day and for many typical days (or seasons) throughout a year. This detail in the wind and load data is necessary to compute accurate values for the WTGS performance values, e.g., the generated, purchased, and excess power. Second, once these WTGS performance values are calculated, the savings in purchased energy costs plus the profit from selling excess power (if any) must be compared with the costs associated with the installation of a WTGS, i.e., capital and maintenance costs, to find if the given WTGS size yields a net savings. Third, other WTGS sizes, i.e., WTGSs with different rated powers and speeds, must be evaluated in order to find the WTGS size which affords the maximum economic savings. If too small a WTGS is chosen the energy production will be insignificant compared to the cost of the WTGS. Similarly, if too large a system is chosen, the WTGS costs will overshadow any energy savings. Consequently, a key element of the methodology is a search technique to optimize the savings achieved by installation of a WTGS.

Because of the inherent structure of the methodology and the large number of different sizes of WTGSs which will have to be tried, this study lends itself to a computer algorithm. A flow diagram for such a computer algorithm is shown in Fig. 3.2-1. The algorithm consists of combining the results of several modules, each module models some aspect of necessary information needed for the computation of the size parameters for the optimal WTGS.

For the computer algorithm to give accurate estimates of the WTGS needed to supply a particular load with electrical power, the input data must be sufficiently detailed. This includes specifying the wind speeds and load demand requirements for several time intervals throughout each day and for typical days throughout a year. The "typical" days used throughout a year will be termed *seasons*. Within a given season, wind speeds will fluctuate from day to day for a particular daily time interval, and representation of the wind speed for this interval by a constant value for every day in the season is not at all realistic. The wind speed in any daily time interval must be characterized by a wind speed distribution. Such wind speed distribution information is available for many locations throughout the United States [11,12]. Although load demands will also fluctuate within a time interval, detailed information about these fluctuations is generally unavailable. Consequently, the load demand was assumed constant for every time interval. This assumption of constant load demands within a time interval is not altogether inaccurate because for residences with an established living pattern or an enterprise with well-defined energy applications, the load will vary only slightly

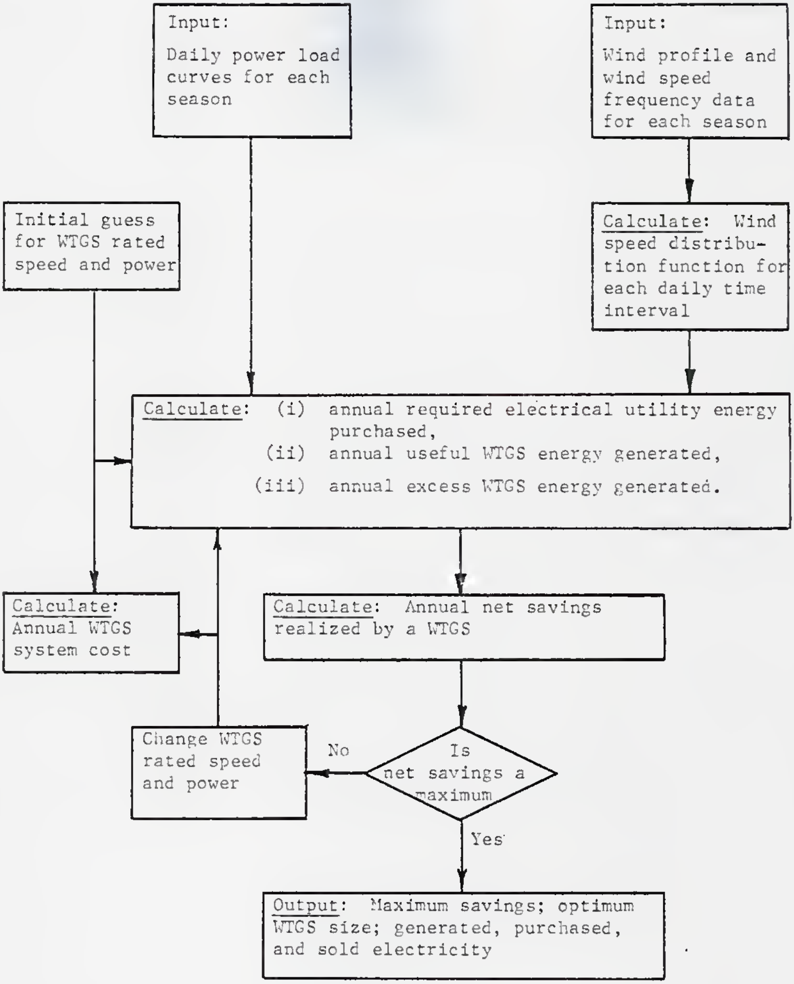

Fig. 3.2-1. Block diagram of optimization methodology.

from day to day in a season for any daily time period. Finally, the purchase and selling price (if any) of electricity must be established.

With the optimization methodology and the type of input data specified, models for each of the program modules must be constructed. In the following sections, the models for each program module used in this investigation are described in detail.

3.2.1 Wind Models and Data Requirements

The most fundamental wind data needed for this optimization study are the wind speed distributions expected for a given daily time interval and for a given season. If detailed wind speed data are available, they can be used directly as a discrete form of the probability density function or modeled by an analytical distribution. Because the analytical wind speed distribution is conceptually more appealing, since in reality the wind behaves in a smoothly varying manner, and is mathematically easier to handle, the Weibull and beta distribution functions were examined in Chapter 2. From this study it was found that if the two parameters of either of these analytical representations are estimated by a matching-moments technique, accurate characterizations of actual wind speed distributions can be obtained. The use of analytical distributions to represent the wind data greatly reduces the amount of data required for each daily time interval and for each season since only the two parameters of the distributions need be specified.

Since many wind speed distributions are needed to characterize the variability of the wind, the following terminology convention is adopted for the remainder of this report. The term $f^{ij}(v)$ denotes the wind speed distribution, either discrete or continuous, in the i-th daily time interval of the j-th season. The number of time intervals and seasons chosen is arbitrary and depends upon the variability of the wind throughout the day and the year at the site chosen for the WTGS, i.e., one should use many intervals if the wind distribution changes rapidly throughout the day or throughout the year.

Often, detailed wind speed data, usually in the form of magnetic computer tapes, are available only for a limited number of locations. However, the required wind speed distribution functions can be synthesized in an approximate manner from less detailed meteorological data than are readily available. For such a synthesis, two pieces of wind data are required for each location, namely (i) the wind speed frequency distribution averaged over each month or season, and (ii) an average wind speed distribution as a function of time of day for each month or season. The overall distribution of frequency of wind speeds at any time of day is determined primarily by the local weather patterns of fronts and other slowly varying meteorological phenomena. Thus, the relatively short-time diurnal variation, which are caused primarily by solar heating effects, can be expected to affect the average speed at any time of day but not the overall shape of the wind frequency distribution for any daily time interval. Hence, an approximate method which can be used to generate the required distribution $f^{ij}(v)$ is to assume its overall shape (i.e., variance) is the same as the distribution of wind speeds in the j-th season but with the mean shifted so that the mean speed corresponds to the mean speed observed in the i-th time interval of the j-th season. Consequently, only two parameters are needed to compute the necessary wind speed distributions: (i) the mean speed in the i-th time interval of the j-th season, and (ii) the seasonal variance of wind speeds about the seasonal mean wind speed. Hence, with the mean speed and variance determined for each daily time interval, the matching-moments technique can be used to obtain the parameters of an analytical distribution for each daily time interval. This synthesis technique, while quite straightforward, is only approximate and should not be used if wind speed distribution data are available for each daily time interval for each season.

3.2.2 WTGS Response Model

The response function for the WTGS used in this optimization study is an idealized one which was discussed in Chapter 2, namely

$$R(v) = \begin{cases} 0 & , v \leq v_c, \\ P_r(\frac{v}{v_r})^3 & , v_c < v \leq v_r, \\ P_r & , v_r < v \leq v_{furl}, \\ 0 & , v > v_{furl}. \end{cases} \qquad (2.5\text{-}8)$$

As noted earlier, v_{furl} is set equal to v_{max}, which is the maximum speed included in the wind speed distribution with storm conditions excluded. Also, cut-in speed, v_c is set equal to 0.46416 v_r. This assumes that the WTGS is idle when the output power is less than 10% of the rated power for the WTGS.

The response function, R(v), gives the net electrical power output of the WTGS exposed to a steady wind speed v. Hence, all efficiency and performance factors of the WTGS are taken into account by this function and no explicit description is needed for the generic type of wind turbine or electrical conversion system used, e.g., blade diameter, number of blades, aerodynamic efficiency, drive train efficiency, and inverter efficiency. To describe completely the response of the WTGS, only a pair of parameters, P_r and v_r, are needed to describe the entire system response.

No allowance is made for the separate effects of wind gusts on the WTGS. In effect it is assumed that the wind speed distributions, $f^{ij}(v)$, have been determined from measurements made with instruments which have the same dynamic response time constants as the idealized WTGS.

In most realistic systems, there is also a furling speed above which the WTGS is feathered or shutdown to prevent damage from high winds. However, for most wind speed distribution data, abnormally high wind conditions are suppressed. By suppression of such storm level winds from wind speed data, the existence of a furling speed has effectively been incorporated by setting $f^{ij}(v)$ to zero for v greater than v_{max}. Hence, v_{max} becomes the furling speed of the model WTGS response function.

The assumed response function used in this study resembles the basic features of many response functions of actual wind systems. However, the methodology developed in this study is sufficiently general that the response function for any particular WTGS could be readily substituted.

3.2.3 Matching Load to Available Power

Once the wind speed model and WTGS response model are specified, the output power that a particular size WTGS can generate from the given wind speed conditions can be matched with the demand load. The three power calculations needed in the optimization analysis, are the annual required electric utility energy purchased, the annual useful WTGS energy generated, and the annual excess WTGS energy generated. In the following subsections the formulas used to calculate each of the above power calculations are derived.

(a) Purchased Electrical Energy

If the WTGS is too small or the wind not of sufficient strength to handle the load power requirements, electrical power must be purchased from the utility to supplement the power generated by the WTGS. Hence, the amount of power that must be purchased over the i-th daily time interval in the j-th season is

$$P_b^{ij} = [P_d^{ij} - R(v)]\, H[P_d^{ij} - R(v)], \qquad (3.2\text{-}1)$$

where

$$H[x] = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

P_d^{ij} is the electrical power demand for the i-th daily time interval in the j-th season, and R(v) is the WTGS electrical power output when the wind speed is v. The unit step function, H[x], is needed to describe the case when the power demand is less than the WTGS power generated, i.e., when no purchased power is needed.

Because the output response of the WTGS depends upon the wind frequency distribution, the power that must be purchased from the utility is averaged with the wind frequency distribution to compute the average or expected purchased power for the i-th daily time interval in the j-th season. This purchased power is given by

$$\overline{P}_b^{ij} = \int_0^\infty [P_d^{ij} - R(v)]\ f^{ij}(v)\ H[P_d^{ij} - R(v)]dv. \qquad (3.2\text{-}2)$$

Since high wind speeds are suppressed, the highest wind speed observed at a location, v_{max}, can replace the upper limit in the above equation. Also, since R(v) is equal to zero when v is less than v_c, Eq. (3.2-2) becomes

$$\overline{P}_b^{ij} = \begin{cases} \int_0^{v_{max}} P_d^{ij}\ f^{ij}(v)dv - \int_{v_c}^{v_{max}} R(v)\ f^{ij}(v)dv, & P_r \leq P_d^{ij} \qquad (3.2\text{-}3a) \\ \int_0^{\delta} P_d^{ij}\ f^{ij}(v)dv - \int_{v_c}^{\varepsilon} R(v) f^{ij}(v)dv, & P_r > P_d^{ij} \qquad (3.2\text{-}3b) \end{cases}$$

where

$$\delta = \begin{cases} v_c, & v_c > v_d, \\ \min\ (v_d, v_{max}), & v_c \leq v_d, \end{cases}$$

$$\varepsilon = \min(v_d,\ v_{max}),$$

P_r is the rated power of the WTGS, and v_d (which is less than v_r) is the speed at which the WTGS first produces the demanded power. The first term on the right hand side in Eq. (3.2-3a) can be simplified by noting

$$\int_0^{v_{max}} f^{ij}(v)dv \begin{cases} = 1 \text{ for beta distributions and actual} \\ \quad \text{wind histograms,} \\ \approx 1 \text{ for Weibull distributions.} \end{cases} \tag{3.2-4}$$

Therefore,

$$\int_0^{v_{max}} P_d^{ij}\ f^{ij}(v)dv \ = \ P_d^{ij} \int_0^{v_{max}} f^{ij}(v)dv = P_d^{ij}. \tag{3.2-5}$$

The upper limits on the integrals of Eq. (3.2-3b) are different than those of Eq. (3.2-3a) because when P_r is greater than P_d^{ij}, there exists a demand speed, v_d, such that $P(v_d)$ is equal to P_d^{ij}. The speed v_d is calculated by solving Eq. (2.5-8) for v_d, with $P(v_d)$ equal to P_d^{ij}, namely

$$v_d = \left(\frac{P_d^{ij}}{P_r}\right)^{1/3} \tag{3.2-6}$$

Two sets of limits are needed in the first integral of Eq. (3.2-3b) to account for the two possible relationships between v_c and v_d. When v_c is greater than v_d, the wind speed is sufficiently high (according to Eq. (3.2-6)) to cause the WTGS to produce power but is still below the cut-in speed and the WTGS remains idle. Thus power must be purchased until v is greater than v_c. When v_c is less than or equal to v_d, power is only purchased until the wind speed reaches v_d and the WTGS produces enough power to satisfy the demand load. For wind speeds greater than v_d, the WTGS generates excess power and hence no power needs to be purchased. If in any of the above integrals the lower limit is greater than the upper limit, the entire integral is set to zero.

Finally, subsitution of the explicit form for P(v), Eq. (2.5-8), into Eqs. (3.2-3a) and (3.2-3b) yields

$$\overline{P}_b^{ij} = \begin{cases} P_d^{ij} - P_r \int_{v_c}^{\gamma} (\frac{v}{v_r})^3 f^{ij}(v)dv - P_r \int_{v_r}^{v_{max}} f^{ij}(v)dv, \; P_r \leq P_d^{ij} & (3.2\text{-}7a) \\ \int_0^{\delta} P_d^{ij} f^{ij}(v)dv - P_r \int_{v_c}^{\varepsilon} (\frac{v}{v_r})^3 f^{ij}(v)dv, \; P_r > P_d^{ij} & (3.2\text{-}7b) \end{cases}$$

where

$$\gamma = \min(v_r, v_{max}).$$

The average power output of the WTGS, defined by

$$\overline{P}^{ij} = \int_{v_c}^{v_{max}} R(v) \; f^{ij}(v)dv, \qquad (3.2\text{-}8)$$

cannot always be used to estimate the expected power purchased which intuitively one might expect to be simply the difference between the demand and average WTGS output power, i.e., $\overline{P}_b^{ij} = P_d^{ij} - \overline{P}^{ij}$. Such a procedure may produce an erroneous result for the amount of purchased power. Substitution of Eq. (3.2-8) for the second integral in Eq. (3.2-3a) yields

$$\overline{P}_b^{ij} = P_d^{ij} - \overline{P}^{ij} \ , \ P_r \leq P_d^{ij} \qquad (3.2\text{-}9)$$

which is the intuitive result. However, if Eq. (3.2-8) is used in Eq. (3.2-3b), the result is

$$\overline{P}_b^{ij} = P_d^{ij} - \overline{P}^{ij} + \int_{\delta}^{\zeta} [P(v) - P_d^{ij}] \ f^{ij}(v)dv, \qquad (3.2\text{-}10)$$

where

$$\zeta = \max(v_d, v_{max}).$$

Thus, for the case v_r greater than v_d,

$$\overline{P}_b^{ij} > P_d^{ij} - \overline{P}^{ij}. \qquad (3.2\text{-}11)$$

Hence, the purchased power would be underestimated when only the average power output of the WTGS and the demanded power are used.

Finally, because utility costs are based on energy consumption, the energy used in every time interval is found by multiplying the expected power purchased in that interval by the duration of the time interval, Δt_i. Then this quantity is summed over all time intervals in the typical day of a season. The total annual purchased energy is found by multiplying the daily consumption by the number of days in a season, d_j, and summing over all seasons. Hence, the expected total annual purchased energy, E_b, is given by

$$E_b = \sum_{j=1}^{s} d_j \sum_{i=1}^{t} \bar{P}_b^{ij} \, \Delta t_i, \qquad (3.2\text{-}12)$$

where s is the number of seasons in a year and t is the number of daily intervals.

(b) Generated Electrical Energy

The annual amount of electrical energy generated by the WTGS and used by the load, E_g, can be computed from the result of the previous section as the difference between the total demand and total purchased electrical energy, i.e.,

$$E_g = \sum_{j=1}^{s} d_j \sum_{i=1}^{t} P_d^{ij} \, \Delta t_i - E_b. \qquad (3.2\text{-}13)$$

(c) Surplus Electrical Energy

Whenever the WTGS produces more power than is demanded, the excess must be stored, sold, or wasted. Storage has not been considered in this study. For the i-th daily time interval in the j-th season the amount of surplus power is

$$\bar{P}_s^{ij} = [R(v) - P_d^{ij}] \, H[R(v) - P_d^{ij}]. \qquad (3.2\text{-}14)$$

Again, the unit step function is needed so when R(v) is less than P_d^{ij}, no surplus power is generated.

Averaging this quantity with the wind speed probability density function for the i-th daily time interval in the j-th season gives the expected surplus power (upon substitution for R(v))

$$\overline{P}_s^{ij} = \begin{cases} P_r \int_{\eta}^{\theta} \left(\frac{v}{v_r}\right)^3 f^{ij}(v)dv + P_r \int_{v_r}^{v_{max}} f^{ij}(v)dv - P_d^{ij} \int_{\eta}^{v_{max}} f^{ij}(v)\,dv, \; P_r > P_d^{ij} & (3.2\text{-}15a) \\ 0\,, \quad P_r \le P_d^{ij} & (3.2\text{-}15b) \end{cases}$$

where $\eta = \max(v_d, v_c)$,

and $\theta = \min\,(v_r, v_{max})$.

The lower limit, $\max(v_d, v_c)$, of two integrals in Eq. (3.2-15a) is required since there can be no surplus power generated until v_c has been reached.

As was true in the calculation of the amount of purchased electricity, the use of only an average power output from the WTGS and the demand power can give misleading results for the average surplus power. This derivation is as follows

$$\overline{P}_s^{ij} = \int_{\eta}^{v_{max}} f^{ij}(v)[R(v) - P_d^{ij}]dv \qquad (3.2\text{-}16a)$$

$$= \int_{0}^{v_{max}} f^{ij}(v)[R(v) - P_d^{ij}]dv - \int_{v_c}^{\eta} f^{ij}(v)\,[R(v) - P_d^{ij}]dv \qquad (3.2\text{-}16b)$$

$$= \overline{P}^{ij} - P_d^{ij} + \int_{v_c}^{\eta} f^{ij}(v)[P_d^{ij} - R(v)]dv \qquad (3.2\text{-}16c)$$

$$> \overline{P}^{ij} - P_d^{ij} . \tag{3.2-16d}$$

Hence, the amount of surplus power which is calculated by taking the difference between the average WTGS output and the demand power will always be underestimated.

The annual amount of excess WTGS energy, E_s, is given by

$$E_s = \sum_{j=1}^{s} d_j \sum_{i=1}^{t} \overline{P}_s^{ij} \Delta t_i . \tag{3.2-17}$$

3.2.4 Evaluation of Integrals for P_b^{ij} and P_s^{ij}

When a beta or Weibull probability density function is substituted in Eqs. (3.2-7a), (3.2-7b), and (3.2-15a) the integrals cannot be evaluated analytically; a numerical technique must be used. The method chosen in this study is Gauss-Legendre quadrature. The quadrature formula for a range (a,b) may be written as [20]

$$\int_a^b g(x)dx \simeq \left(\frac{b - a}{2}\right) \sum_{i=1}^{L} w_i \, g\left(\frac{z_i(b - a) + b + a}{2}\right), \tag{3.2-18}$$

where L is the number of quadrature points used, w_i is the weighting factor at point i, and z_i is i-th quadrature ordinate. Extensive tables of z_i and w_i have been compiled [21].

Because the beta distribution was used in most of the sensitivity studies of this chapter to characterize the necessary wind speed distributions, the following scheme was used to determine a quadrature order that would minimize computational effort yet maintain accuracy of integral evaluation. If g(x) is a polynomial function, then Gauss-Legendre quadrature will evaluate a polynomial of the order 2L + 1 or less exactly. Since the beta distribution is a polynomial function of order $\alpha + \beta - 2$, if α and β are integer parameter values greater than unity, integrals of the form $\int_{v_1}^{v_2} f^{ij}(v)dv$ can be evaluated exactly by a quadrature order of $\frac{1}{2}(\alpha + \beta - 3)$ and integrals of the form $\int_{v_1}^{v_2} (\frac{v}{v_r})^3 f^{ij}(v)dv$ can be evaluated by a quadrature order of $\frac{1}{2}(\alpha + \beta)$. However, for most wind speed distributions, the parameters of the beta distribution, α and β, are non-integer values, but by choosing the quadrature order to be the nearest integer greater than $\frac{1}{2}(\alpha + \beta)$, it can be expected that the numerical evaluation of the integrals would still give accurate, although not exact, results. However, for most beta distributions, a quadrature order of six was found to be adequate to cover most values of α and β encountered in the fitted wind distribution; but, before using any quadrature order, the range of expected α and β values should be computed and if large values are indicated, a higher quadrature order should be used for those cases so accurate results can be obtained.

If the discrete form of the probability density function is used in Eqs. (3.2-7a), (3.2-7b), or (3.2-15a), the integrals can be evaluated exactly since the discrete distribution is a constant over each speed subinterval. Consequently, these equations must be divided into n speed subintervals and integrated between the endpoints since the discrete distribution assumes a different but constant value in every speed sub-interval. The integrations over every speed subinterval are summed to yield the integration over the entire speed range. This summation can be written as follows

$$\sum_{k=1}^{n} f_k^{ij} \int_{v_k}^{v_{k+1}} Q(v)dv, \qquad (3.2\text{-}19)$$

where n is the number of speed intervals, $f_k^{ij}(v)$ is the value at the k-th speed subinterval of the probability density function for the i-th time interval in the j-th season, and Q(v) is either the WTGS response function, R(v), corresponding to the speed subinterval to be integrated or the demand power, P_d^{ij}, for the i-th time interval in the j-th season. It should be noted that Q(v) is a polynomial (or constant) that can be integrated analytically.

Therefore, both analytical and discrete probability density functions can be used to evaluate the integrals that give the purchased and surplus electricity. This use of both types of distributions gives flexibility to the optimization program in that the effect of the distribution type on the optimum WTGS can be determined.

3.2.5 WTGS Cost Model

Because WTGS electrical components and other major system components have not been produced in significant quantitites, the costs of various sized wind turbines are difficult to determine. The costs vary with the size and type of rotor, gear mechanism, inverter, and tower as well as the type of feathering device used to prevent damage to the WTGS at high wind speeds. Furthermore, control units which automatically blend WTGS output power with utility power to meet the demand load have not been commercially developed. A compilation of costs of WTGSs already in production has been made by DPRA [12] using data reported by both Obermeier [10] and Rosen, et. al. [22]. The reported values are for WTGSs with a rated speed of 25 mph and are shown in Fig. 3.2-2 in the form of cost per kilowatt (\$/kW) as a function of rated power (in kW). As can be seen from Fig. 3.2-2, the data from Rosen, et. al., yields slightly lower capital costs than the data from Obermeier.

To interpolate between data points to obtain costs for various other sizes of WTGSs, a quadratic polynomial has been fit to the more optimistic values provided by Rosen, et al. The equation for this cost model is [12]

$$\ln\left(\frac{\$}{\mathrm{kW}}\right) = 7.73971 - 0.46578[\ln(P_r)] + 0.02573[\ln(P_r)^2], \qquad (3.2\text{-}20)$$

$$P_r \geq 1 \text{ kW},\ v_{ref} = 25 \text{ mph}.$$

where v_{ref} is the *reference rated speed* for all WTGSs used to obtain this result. For a WTGS rated below 1 kW, the slope of Eq. (3.2-20) at P_r equal to 1 kW is used to give the simple linear cost model

$$\frac{\$}{\mathrm{kW}} = 2297.8\ (P_r)^{-0.46578},\ P_r < 1 \text{ kW}, \qquad (3.2\text{-}21)$$

$$v_{ref} = 25 \text{ mph}.$$

Because these cost models assume a constant rated speed (25 mph) and because the optimization methodology varies the rated speed in order to find the optimum combination of rated power and rated speed, a conversion factor is needed so that the values obtained from the above cost models can be applied to other rated speeds. Eldridge [1]

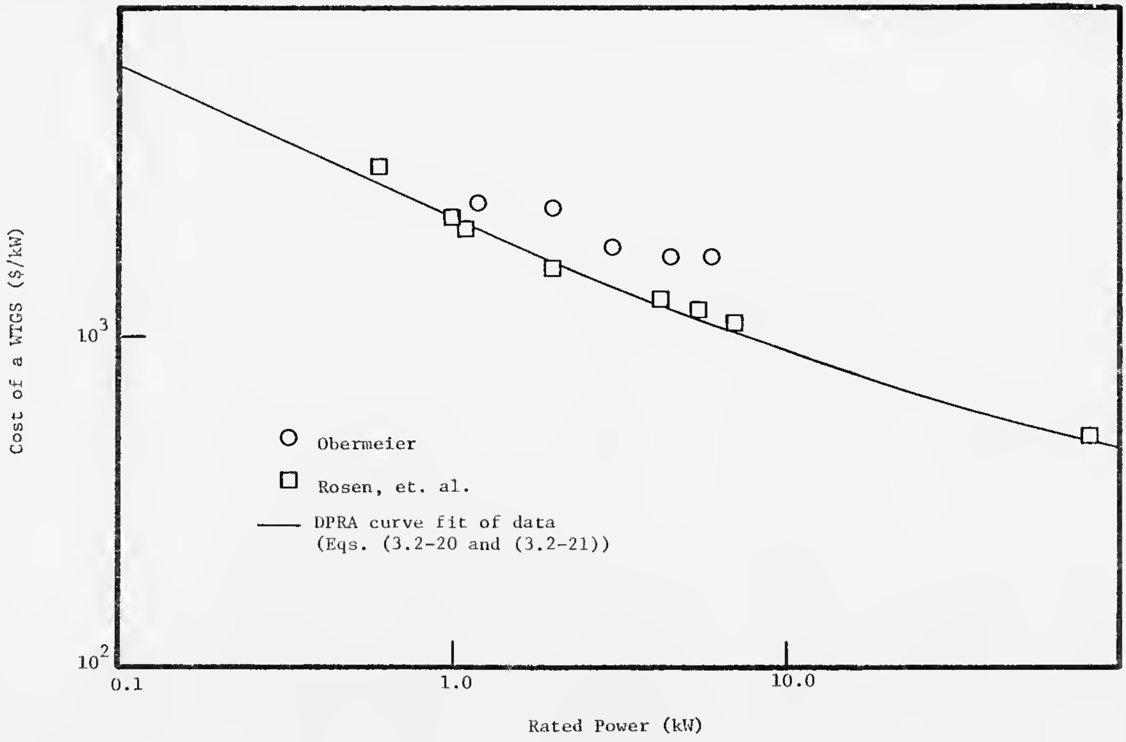

Fig. 3.2-2. Cost of a WTGS versus rated power.

reports that capital costs of a WTGS decrease as the rated speed increases since, for a given output capacity of a WTGS, the size and weight of the blades, rotor, gear train, and other system components will generally decrease when the WTGS is designed for higher rated speeds. This is due to the fact that power derived from the wind varies linearly with the area swept by the rotor but varies with the cube of the wind speed. Hence, for a given rated power, area decreases considerably if rated speed increases. Using data from references 10 and 22, DPRA has derived an expression correcting costs ($/kW) for different rated speeds. For a WTGS with a rated speed different from that used to derive Eqs. (3.2-20) and (3.2-21), the cost can be corrected for this change in rated speed by multiplying the capital cost ($/kW) by $(v/v_{ref})^{-2}$.

Other costs associated with the WTGS are those of operation and maintenance. Obermeier [10] reports that annual operation and maintenance costs are a fixed percentage of the capital cost of the WTGS and depend on the type of use as well as the durability of system components. Maintenance costs of one percent for factory produced wind generators, three percent for partly owner assembled systems, and five percent for home built systems are typical. In this study, a three percent annual maintenance cost was assumed.

Although the cost model described above and used in this study is based on rather uncertain data and many different types of WTGS, the model is felt to approximate expected costs of WTGS given a sufficiently large degree of production. However, the optimization procedure is written such that any other cost model could be readily substituted for the one used in this study.

3.2.6 Calculation of Annual Net Savings

The net financial savings, or burden if money stands to be lost, realized by the installation of a WTGS is determined by adding together the money saved on purchased electrical energy costs and made by selling surplus electricity (if any), and subtracting from this value the costs associated with a WTGS. The following equation expresses this relationship

$$A = (E_g)(C_b) + (E_s)(C_s) - AC_{WTGS} - AOM, \qquad (3.2\text{-}22)$$

where A is the annual net savings, E_g is the electrical energy (kWh) generated in a year by the WTGS and which is used, E_s is the surplus electrical energy generated in a year by the WTGS which is wasted or sold to another user (e.g., utility or other enterprise), C_b is the cost ($/kWh) of purchasing electrical energy from the utility, C_s is the price for which surplus WTGS electricity can be sold (if wasted, C_s equals zero), AC_{WTGS} is the effective annual cost ($) of the WTGS, and AOM is the effective annual cost for operating the WTGS.

In the previous section, capital cost models were given in order to compute the cost of installing a particular size WTGS. Because the WTGS represents such a large investment, the capital cost is amortized over a period of m years, at a yearly interest rate i_r. The fraction of the initial investment required each year for the amortization is called the *capital recovery factor* (CRF) and is given by [23]

$$CRF = \frac{i_r (1 + i_r)^m}{(1 + i_r)^m - 1} \quad . \tag{3.2-23}$$

The capital cost, C_c, of the WTGS is multiplied by the CRF to obtain the annual net cost, A, i.e.

$$A = (C_c)(CRF). \tag{3.2-24}$$

In this study, the interest rate was assumed to be 10% per year with a 20 year WTGS life expectancy. No salvage value was assumed for the WTGS at the end of the life expectancy. The annual operation and maintenance costs for the WTGS are assumed to be three percent of the capital cost of the WTGS.

3.2.7 Optimization Technique

Because the purpose of this optimization study was to maximize the annual net savings described by Eq. (3.2-22), a technique to find the maximum of this *objective function* was needed. The net annual savings depends upon the size of the WTGS through its rated power and rated speed, and hence the problem becomes a two-parameter optimization problem. An algorithmic technique such as the method of steepest descent could be used to find the maximum of the objective function, $A(P_r, v_r)$. However, because this method requires evaluation of the first derivatives of the objective function, its use was precluded in the present study because of the complexities in calculating the derivatives of the generated, purchased, and surplus powers. Hence, a search technique is more appropriate since such a method is merely an organized procedure that chooses points so that the contours of the two-dimensional space are scanned in order to find the point that yields the maximum value of the objective function. The technique chosen for this study is the sequential simplex pattern search method [24,25,26]. The computer subroutine used to calculate the maximum by the simplex technique was adapted from a program written by Lai [27].

3.3 Sensitivity of the Optimally Sized WTGS to Problem Parameters

The computer code BLOHARD was written to carry out the optimization methodology described in the previous sections of this chapter. This routine is based on a similar code developed by DPRA in their study of wind applications in agriculture [12]. A listing, explanation, and sample output of routine BLOHARD are contained in Appendix C. The program BLOHARD was used to investigate the sensitivity of the optimally sized WTGS to various problem parameters such as diurnal variations in load, load size, diurnal variations in wind speed, variance in wind speed distributions, variations in mean wind speed, and credit given for surplus electricity. Furthermore, the effects of seasonal changes in the wind speed distributions and load demands are also examined.

3.3.1 Model Wind and Load Profiles

So that the sensitivity of the optimization procedure to the problem parameters can be analyzed, idealized models for both the wind speed and the demand load must be developed. Crawford, et.al. [28] report that due to solar effects, mean wind speeds are lowest just after midnight, rise gradually after sunrise reaching a peak in the middle afternoon, and then fall gradually after sunset back to the low value. The wind speed data, which were analyzed in the previous chapter, and were obtained from the National Climatic Center, Asheville, North Carolina, for 17 locations throughout the United States, exhibit such diurnal fluctuations. One simple model which exhibits such diurnal variations is a sinusoidal variation about some mean daily or seasonal speed of the form

$$v(t) = \overline{v} - a \cos\left(\frac{2\pi t}{24} - \frac{\pi}{4}\right), \qquad (3.3\text{-}1)$$

where v(t) is the average or expected speed at time t (in hours beginning at midnight), $\overline{v}$ is the mean daily speed, and "a" is the amplitude of the variation about the mean speed. Eq. (3.3-1) reaches the minimum at 3 hours and the maximum at 15 hours, which approximately correspond to the observations made by Crawford, et.al.

To generate analytical representations of such model wind speed distributions, e.g., wind speed distributions for eight, three-hour intervals in the day, the average value of Eq. (3.3-1) was calculated for every daily interval of interest. This procedure yields an average speed for every daily time interval. To obtain a distribution of speeds about this mean value, for each daily time interval a dispersion or variance of speeds about each interval's mean speed needs to be specified.

For the model winds used in this study the variance was assumed to vary linearly with the interval's mean speed,

$$s_i^2 = c\,\overline{v}_i, \qquad (3.3\text{-}2)$$

where s_i^2 is the wind speed variance for the i-th time interval of the day, $\overline{v}_i$ is the average speed for the i-th daily time interval, and c is the constant of proportionality between the mean speed and variance, called the *coefficient of variation.*

Once the mean and variance of the wind speed for a particular daily time period are calculated from Eqs. (3.3-1) and (3.3-2), a model wind speed distribution for that time period can be obtained by using a beta function representation whose parameters are chosen such that the required mean and variance are realized. In Chapter 2 it was shown that such a matching-moments technique was both analytically very simple as well as yielding beta distributions which modeled actual wind speed data very accurately. To obtain model wind speed profiles for use in the sensitivity analyses, each day was divided into eight time intervals or periods each of three hours duration. Diurnal fluctuations of 10% and 25% of the daily mean wind speed were chosen (i.e., a equal to 0.1 $\overline{v}$ and 0.25 $\overline{v}$, respectively in Eq. (3.3-1)). Corotis [7] reports that diurnal variations of about 10% are representative of the range of fluctuations encountered in a season with relatively constant winds and a value of about 25% is typical for a season with rather gusty winds. The diurnal fluctuations as a function of daily time interval for these two cases are listed in Table 3.3-1. So that the effect of both a large and small variance in the average speed for every daily time interval can be studied, coefficients of variation, c, equal to

Table 3.3-1. Average Speed as a Function of Time Interval. Mean Daily Wind Speed is 10 Knots for Both Cases.

Time Interval (hrs)	Small wind fluctuation 10% of mean speed (knots)	Large wind fluctuation 25% of mean speed (knots)
0 - 3	9.10	7.75
3 - 6	9.10	7.75
6 - 9	9.63	9.07
9 - 12	10.4	10.9
12 - 15	10.9	12.3
15 - 18	10.9	12.3
18 - 21	10.4	10.9
21 - 24	9.63	9.07

1 and 4 (corresponding to relatively steady and gusty wind conditions, respectively) were selected. Finally, a maximum wind speed of 50 knots was used for the v_{max} term in the beta distribution. Normally, speeds above 50 knots are considered storm speeds for which a WTGS would not be operated.

Table 3.3-2 assigns a wind model number to the different permutations of the two diurnal variations paired with the two coefficients of variation to be studied. To study the effect of seasonal mean wind speed on the optimum WTGS, one additional case was run. The seasonal mean speed was doubled to 20 knots and combined with the large diurnal variation and large coefficient of variation to form a model characteristic of high speed, gusty wind conditions.

A second necessary component of the sensitivity studies is the construction of model demand load profiles. DPRA [12] and Obermeier [10] report that typically two load demand peaks are reached throughout the day. In this study, the greatest load demands were assumed to occur between the hours of 9 and 12 and again from hours 15 to 18. Load demands were assumed to be constant throughout every three-hour period, i.e., there is no distribution in the demand about the given load for a particular time interval. Three cases were considered as being representative of the various types of load demands that may occur throughout the day; a *flashy load* (one with variations in the mean daily load of 100%), a *smooth load* (one with variations in the mean daily load of 50%), and a *constant load* (no variation throughout the day). Table 3.3-3 lists the daily variations in each model load as a function of time interval. The

Table 3.3-2. Wind Model Numbers Assigned to the Various Combinations of Diurnal Fluctuations and Coefficients of Variation.

Wind Model #	Diurnal Fluctuation	Coefficient of Variation
1	25% of seasonal mean speed	4
2	25% of seasonal mean speed	1
3	10% of seasonal mean speed	4
4	10% of seasonal mean speed	1
5	25% of seasonal mean speed (20 knots)	4

Table 3.3-3. Model Load Variations as a Function of Time Interval for Flashy, Smooth, and Constant Loads. (All Loads are normalized to a daily average of unit demand (1 kW)).

Time Interval (hrs)	Flashy Load	Smooth Load	Constant Load
1 - 3	0.0	0.5	1.0
3 - 6	0.5	0.75	1.0
6 - 9	1.0	1.0	1.0
9 - 12	2.0	1.5	1.0
12 - 15	1.0	1.0	1.0
15 - 18	2.0	1.5	1.0
18 - 21	1.0	1.0	1.0
21 - 24	0.5	0.75	1.0

load profiles in this table were normalized to an average demand of one unit (1 kW); consequently, for a given mean load, the values in the table are multiplied by the desired mean daily load demand. Mean daily load demands of 5 kW and 35 kW were chosen as values representative of a small residential load and a residential load combined with a farm load (or possibly a combination of several residential loads), respectively.

3.3.2 Results of the Model Case Studies

The model wind distributions and model demand load profiles discussed in the previous section, were used to determine the sensitivity of the economically optimum WTGS to various types of winds and loads. In particular, 14 combinations of the model wind speed distributions with model loads were used. These combinations or *cases* are defined in Table 3.3-4. In Cases 1 through 12 the model wind profile and load curve were assumed to hold for the entire year, i.e., only a single season was considered. For Case 13 two seasons were used for the year while Case 14 used four seasons with actual wind speed distributions and load profiles characteristic of a Kansas farming operation. Although the single season cases are unrealistic, they will tend to accentuate any peculiar features affecting the selection of the optimum WTGS. Consequently, these single season cases are useful in identifying general trends in optimum WTGS size without specifying large amounts of wind speed or load demand data. However, the interaction of seasonal variations in input wind speed and load data is important when examining the competitiveness of wind energy with conventional energy sources; hence, more

Table 3.3-4. Case Numbers Assigned to Various Combinations of Wind and Load Models. (Surplus electrical energy is assumed to have no value unless otherwise noted).

Case Number	Wind Model (see Table 3.3-2)	Type of Load (see Table 3.3-3)
1	1	Constant - 5 kW average
2	1	Smooth - 5 kW average
3	1	Flashy - 5 kW average
4	5	Smooth - 5 kW average
5	1	Smooth - 5 kW average Credit for surplus @ 2¢/kWh
6	4	Flashy - 35 kW average
7	1	Flashy - 35 kW average
8	3	Flashy - 35 kW average
9	1	Smooth - 35 kW average
10	1	Smooth - 5 kW average Credit for surplus @ 2¢/kWh
11	2	Flashy - 35 kW average
12	5	Smooth - 35 kW average
13	3 (season #1) 1 (season #2)	Two season problem: Smooth - 10 kW average Smooth - 20 kW average
14	Actual seasonal wind data for Wichita, KS	Four Season Problem: Loads typical of winter wheat/sorghum farm operation

detailed information is needed. Cases 13 and 14 investigate this interaction of seasonal variations and their effect upon the optimum WTGS size.

A summary of results for these 14 cases is presented in Tables 3.3-5 through 3.3-7 for various assumed costs of utility supplied electricity. The results in these tables will be referred to throughout the remainder of this section and are presented here for convenience. The optimum WTGS size, i.e., rated power and rated speed, is listed along with the amount of electricity generated and used, the amount of electricity which had to be purchased from the utility, and the amount of surplus electricity which is wasted or sold to other users. The column headed "% self-sufficiency" gives the percentage of the entire energy demand that is generated by the WTGS. Finally, the annual net savings of the optimum WTGS is listed. In the following subsections the results of these case studies are discussed.

(a) Effect of Load Variations on Optimum WTGS

Cases 1,2,3,7 and 9 best illustrate the effect of the three different model load profiles upon the optimum WTGS. Figs. 3.3-1 and 3.3-3 show that the optimum rated power is the same as the maximum load demand if the cost of utility power is sufficiently high. However, the rated speed increases while approaching the breakeven value, i.e., the point at which costs of installing a WTGS equal the fuel costs saved, but as purchased electrical costs decrease further the rated power may begin to decrease rapidly. This behavior is expected because as the rated speed is increased, initial or capital costs decrease by approximately the square of

Table 3.3-5. Optimum WTGS Size and Output Characteristics for 5 kW Average Load Examples - Cost of Electricity = \$0.15/kWh.

Case Number	Optimum Pr (kW)	Generator Vr (knots)	Energy Generated and Used (kWh)	Purchased Energy (kWh)	Surplus Energy (kWh)	% self-Sufficiency	Annual Net Savings (\$)
1	5.0	15.4	1.56×10^4	2.82×10^4	0.0	35.7	637
2	7.5	16.9	1.67×10^4	2.71×10^4	3.5×10^3	38.0	680
3	10.0	18.4	1.62×10^4	2.76×10^4	7.03×10^3	36.9	573
4	7.5	18.9	3.33×10^4	1.05×10^4	1.21×10^4	76.2	3544
5	11.9	18.3	1.79×10^4	2.59×10^4	1.00×10^4	40.8	780

Table 3.3-6. Optimum WTGS Size and Output Characteristics for 5 and 35 kW Average Load Examples - Cost of Electricity = $0.105/kWh.

Case Number	Optimum Generator Pr (kW)	Optimum Generator Vr (knots)	Energy Generated and Used (kWh)	Purchased Energy (kWh)	Surplus Energy (kWh)	% self-Sufficiency	Annual Net Savings ($)
1	5.00	19.0	1.09×10^4	3.29×10^4	0.0	25.0	27.60
2	7.48	20.9	1.17×10^4	3.21×10^4	1.76×10^3	26.6	29.67
3	9.91	23.0	1.08×10^4	3.30×10^4	3.46×10^3	25.1	-50.36
4	7.56	20.8	3.13×10^4	1.25×10^4	1.08×10^4	71.7	2081
5	8.86	20.5	1.32×10^4	3.06×10^4	3.36×10^3	30.1	71.43
6	70.0	13.0	1.99×10^5	1.08×10^5	1.03×10^5	64.8	6220
7	70.0	16.5	1.32×10^5	1.75×10^5	6.42×10^4	43.0	4825
8	70.0	16.6	1.22×10^5	1.84×10^5	7.05×10^4	39.9	3961
9	52.5	15.0	1.37×10^5	1.70×10^5	3.30×10^4	44.6	5557
10	104	16.8	1.49×10^5	1.58×10^5	1.35×10^5	48.6	6531
11	70.0	13.1	2.08×10^5	9.88×10^4	8.44×10^4	67.8	7531
12	52.5	17.5	2.44×10^5	6.28×10^4	9.32×10^4	79.5	19090
13	25.1	17.89	4.55×10^4	8.59×10^4	1.56×10^4	34.6	1125
14	25.6	16.1	6.21×10^4	9.42×10^4	2.10×10^4	39.7	1978

Table 3.3-7. Optimum WTGS Size and Output Characteristics for 35 kW Average Load Examples- Cost of Electricity = $0.065/kWh.

Case Number	Optimum Generator Pr (kW)	Optimum Generator Vr (knots)	Energy Generated and Used (kWh)	Purchased Energy (kWh)	Surplus Energy (kWh)	% Self-Sufficiency	Annual Net Savings ($)
6	37.2	14.3	1.10×10^5	1.97×10^5	1.99×10^4	35.8	-455
7	57.1	20.8	8.02×10^4	2.26×10^5	2.31×10^4	26.1	321
8	35.1	19.6	5.94×10^4	2.47×10^5	1.20×10^4	19.4	4.84
9	52.5	19.1	9.60×10^4	2.10×10^5	1.70×10^4	31.5	811
10	70.4	19.0	1.11×10^5	1.96×10^5	4.24×10^4	36.0	1215
11	54.8	15.1	1.40×10^5	1.67×10^5	2.84×10^4	45.6	127
12	52.5	20.0	2.25×10^5	8.15×10^4	7.87×10^4	73.4	9667
13	WTGS not Feasible						
14	20.0	20.4	3.59×10^4	1.21×10^5	5.84×10^3	23.0	-90.34

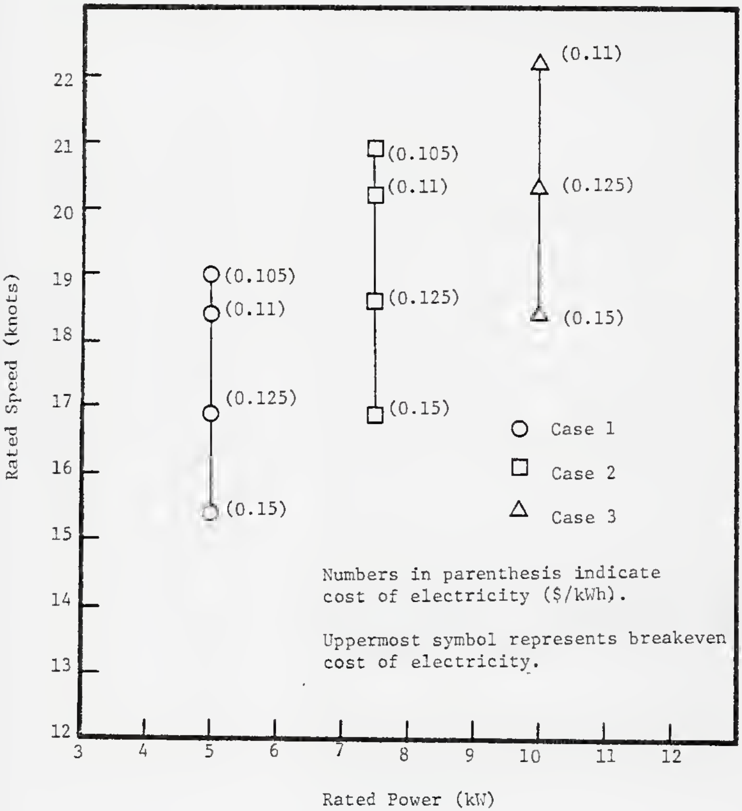

Fig. 3.3-1. Size of optimum WTCS as a function of cost of electricity - Cases 1,2, and 3.

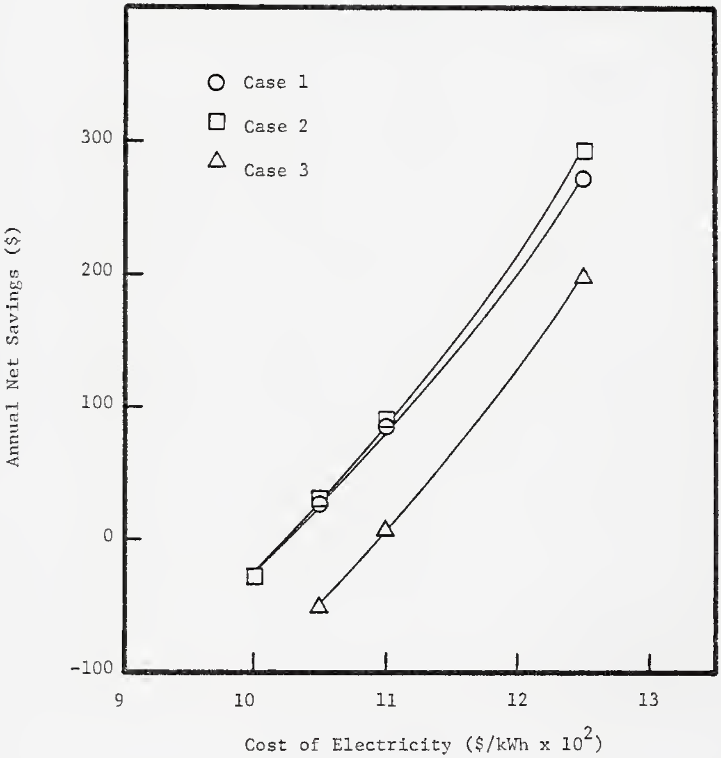

Fig. 3.3-2. Annual net savings versus cost of electricity for optimum WTCS - Cases 1, 2, and 3.

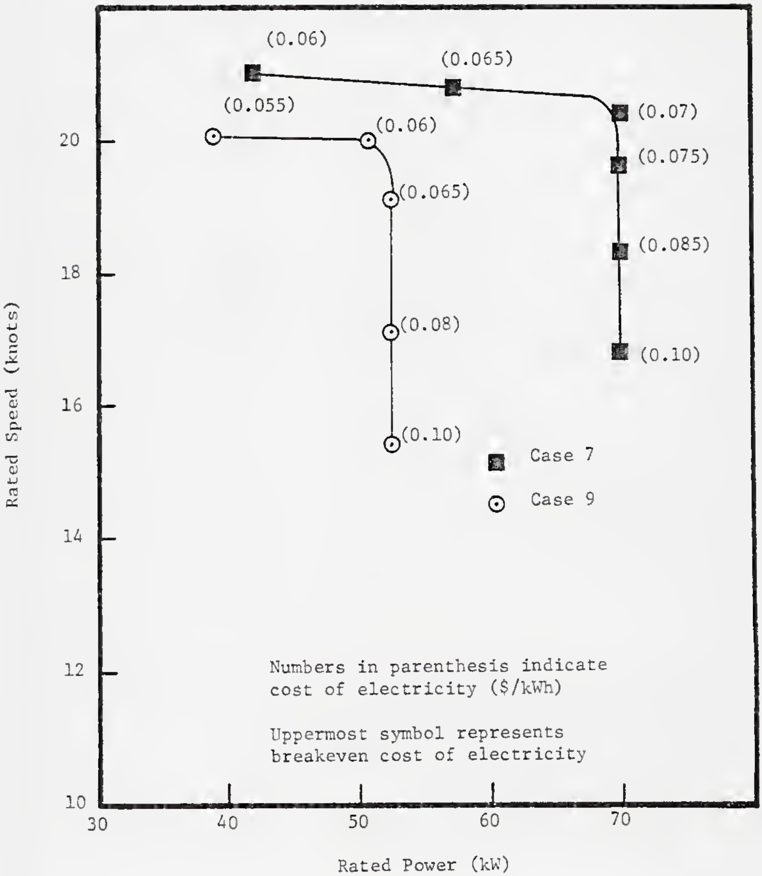

Fig. 3.3-3. Size of optimum WTGS as a function of cost of electricity - Cases 7 and 9.

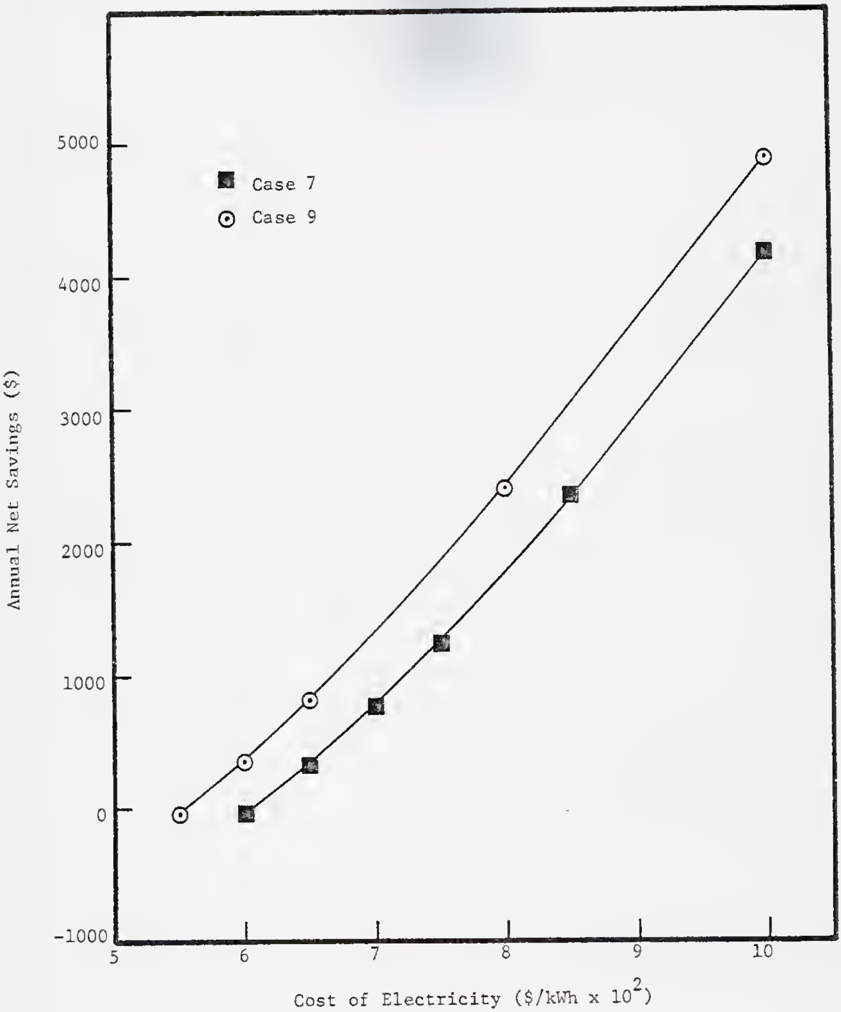

Fig. 3.3-4. Annual net savings versus cost of electricity for optimum WTGS - Cases 7 and 9.

the ratio of the speed change due to the decrease in the WTGS rotor size. Capital costs are also reduced if the rated power decreases, however, in the 5 kW case, the saving achieved is apparently not significant enough to compensate for the loss in generation capacity by using a smaller rated power. But for the kW mean load demand cases, generator rated power decreases sharply as the breakeven value is approached. Initially, as the cost of purchased electricity decreases, rated speed increases to decrease capital costs. However, an upper limit on the rated speed is eventually reached, so the rated power decreases while the rated speed stays relatively constant. As electrical costs decrease further, the WTGS becomes economically infeasible and the optimization program converges to a "zero-cost" size WTGS, i.e., one with a vanishingly small rated power with an exceedingly large rated speed. Such a zero cost WTGS is one that is physically vanishingly small.

From Figs. 3.3-2 and 3.3-4 it can be seen that the type of model load has a relatively small effect on the breakeven cost of purchased electricity (less than 1¢/kWh difference). At all costs of purchased electricity that yield a positive annual net savings, slightly more savings are achieved by the smooth load than either the flashy or constant load. Furthermore, self-sufficiency is about the same for all three loads at any given cost of purchased electricity. Consequently, for the assumed diurnal load fluctuations, only a small increase in annual net savings and decrease in the breakeven value is achieved by attempting to smooth the variations in load demand. In fact, a constant load fared slightly worse than a smooth load as far as annual net savings and self-sufficiency are concerned. This intuitively unexpected result is a consequence of the diurnal variations in the model winds used for these examples which are inphase with the variations in the demand load. If the model load fluctuations occurred in the time intervals with the lower mean speeds or out-of-phase with the model wind, the constant load model can be expected to exhibit a larger annual net savings and a lower breakeven value.

One peculiar result found in this and all sensitivity studies in this section was for electrical costs slightly below breakeven, an optimum WTGS was found by BLOHARD that yielded an annual net savings that was negative, i.e, a loss. This result is caused by the simplex optimization technique converging upon a local maximum that has an associated negative savings rather than converging to the global maximum of a zero-size WTGS with an annual net savings of zero. However, as the electrical costs decrease further below the breakeven cost, the optimization routine converges to the expected annual net savings of zero. Consequently, such negative results for net savings indicate that if the optimum WTGS must be used at a particular electrical cost where an annual net loss is found, this system is the best one to use, since this system will lose the least amount of money.

Furthermore, the simplistic nature of the curves, i.e., horizontal or vertical trajectories, shown in the plots of rated speed and rated power as a function of cost of electricity, is a consequence of using a single season to describe an entire year. As more wind speed and load data are supplied so that seasonal fluctuations can be characterized, trajectories are obtained that exhibit a complex variation with rated speed and power. However, sensitivity analyses using a single season of data are instructive in that general tendencies in the optimum WTGS size can be identified.

(b) Effect of Average Load Size on the Optimum WTGS

The effect of average load size on the optimum WTGS is best seen from Figs. 3.3-2 and 3.3-4 which show annual net savings as a function of cost of purchased electricity. Larger average loads have approximately a 1.5 times reduction in the breakeven electrical cost for the same wind and normalized load models. Breakeven costs for the 35 kW average load range between 5.5¢/kWh to 6.5¢/kWh while the 5 kW average loads have breakeven costs of about 10.5¢/kWh. This variation with average load is expected since larger WTGSs are less expensive on a per unit capacity basis than are smaller systems. Consequently, because of the large net savings potential, large loads are more attractive to wind energy applications. However, as the WTGS rated power increases, a value of rated power is reached whereby the cost function assumed in Eq. (3.2-20) is no longer valid. This is due to the requirement of more expensive and sophisticated control systems and other components for a large WTGS that are not needed for a small WTGS. Hence, for very large loads, the WTGS cost function should be modified for large units to reflect the expected increase in per unit capacity costs with size.

Besides lower breakeven electrical costs, the cases with large average load demands exhibit a greater self-sufficiency at a specified electrical cost than the lower average load demand, given the same wind models. For example, Cases 2 and 9, 3 and 7, and 5 and 10 are the same wind models and normalized load models with the only difference lying in the average load demand. From Table 3.3-6, which shows the results of the optimum WTGS for the same cost of purchased electricity, the self-sufficiency factor is at least 17% greater for the large average load demand than for the small one. Hence, this too shows larger average loads are better candidates for wind energy applications than small average loads.

As the breakeven electrical cost is approached, the rated power for the cases with the large average load demand is very sensitive to slight changes in electrical costs. From this behavior, it can be inferred that large loads do not lose a significant amount of generation capacity by reducing the generator rated power in order to reduce capital costs.

(c) Effect of Diurnal Variations on the Optimum WTGS

Cases 7 and 8 demonstrate the effect of diurnal variations on the optimum WTGS. From the results shown in Fig. 3.3-6, there is only a slight effect on annual net savings; the case with larger diurnal variations saves more money than the case with small diurnal variations. However, the differences in annual net savings become negligible with

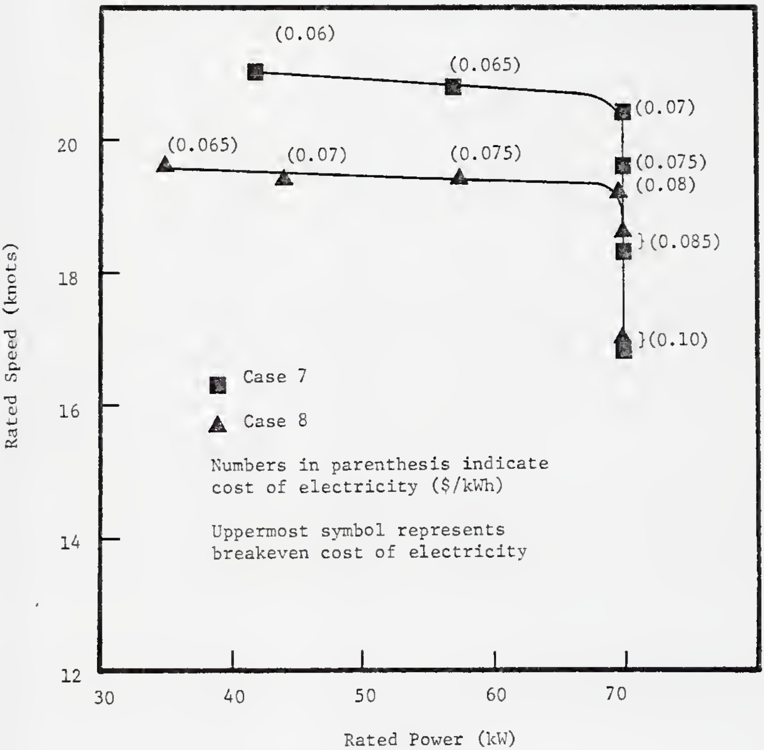

Fig. 3.3-5. Size of optimum WTGS as a function of cost of electricity - Cases 7 and 8.

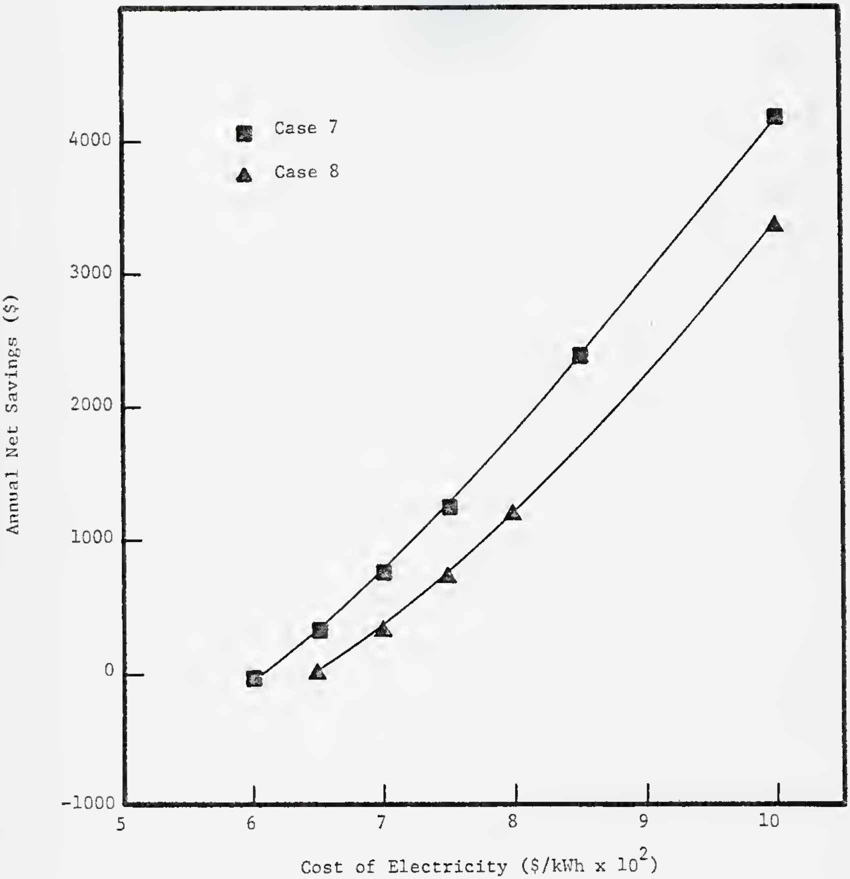

Fig. 3.3-6. Annual net savings versus cost of electricity for optimum WTGS - Cases 7 and 8.

decreasing electrical costs and both cases have comparable breakeven costs, i.e., less than 0.5¢/kWh difference. Figure 3.3-5 indicates that the optimum WTGS size is the same for both cases up to a certain electrical energy cost. Below this critical cost, the case with the small diurnal variations shows sharp reductions in the rated power as electrical costs decrease whereas the case with large diurnal variations remains at the same rated power and just increases the rated speed. As breakeven is approached, both systems have reduced power ratings, but the case with large diurnal variations has higher rated speeds.

An explanation of this observation is that there is less variability in the wind speeds that occur during the daily wind speed peak in the case with a small diurnal variation than the case with a large diurnal variation. This occurs because the variance of wind models used is assumed to be directly proportional to the average wind speed for any daily interval (see Eq. (3.3-2)). Since the daily wind speed peak coincides with the larger load demands for these model cases, there is a greater probability of higher wind speeds occurring in the daily wind speed peak of the wind model with the larger diurnal variations. Hence, because of this greater probability of higher speeds, the rated speed of the optimum WTGS can be increased (so as to lower capital costs), and still provide a sufficient amount of electricity to try to cover the peak load demand, rather than lowering capital costs by decreasing the rated power and thereby decreasing the capability of the WTGS to supply the peak load demand. However, this probability of higher than average wind speeds does not occur when there is only a small diurnal variation;

hence, the rated power must be reduced instead, so that capital costs can be lowered and an economically feasible system obtained.

In addition to having only a slight effect on the annual net savings, diurnal variations affect the self-sufficiency of the WTGS only slightly. Winds with large diurnal variations have a somewhat larger self-sufficiency than winds with small diurnal variations about the same seasonal mean wind speed. This effect is seen by examining the results of Cases 7 and 8 listed in Tables 3.3-6 and 3.3-7. Consequently, little effect on the optimum WTGS is observed by diurnal variations if the overall mean wind speed remains constant.

(d) Effect of Wind Speed Fluctuations on the Optimum WTGS

The effects of changes in the variance of the wind speed distribution of speeds about the average speed in a particular time interval has also been investigated. In the model wind profiles, coefficients of variation with values of four and unity are used. Cases 6 and 8 and Cases 7 and 11 are excellent examples because the only difference between these pairs is in the values used for the coefficients of variation in the wind profiles. Figure 3.3-7 shows that for large electrical costs, the annual net savings for the cases with small variances and either large or small diurnal variations are quite large. However, as the cost of electricity decreases, the curves drop off sharply, ending up with breakeven values that are nearly the highest of all cases studied. Figure 3.3-8 shows the effects differences in the variances of wind

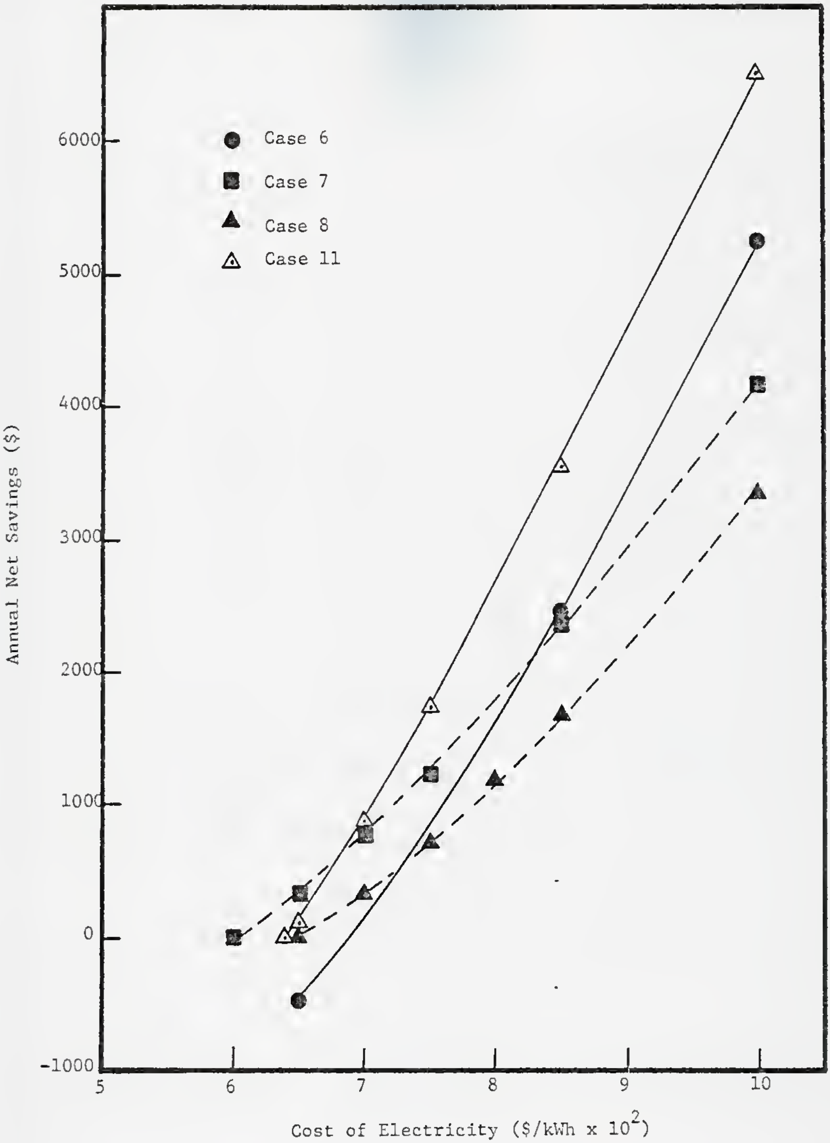

Fig. 3.3-7. Annual net savings versus cost of electricity for optimum WTGS - Cases 6, 7, 9, and 11.

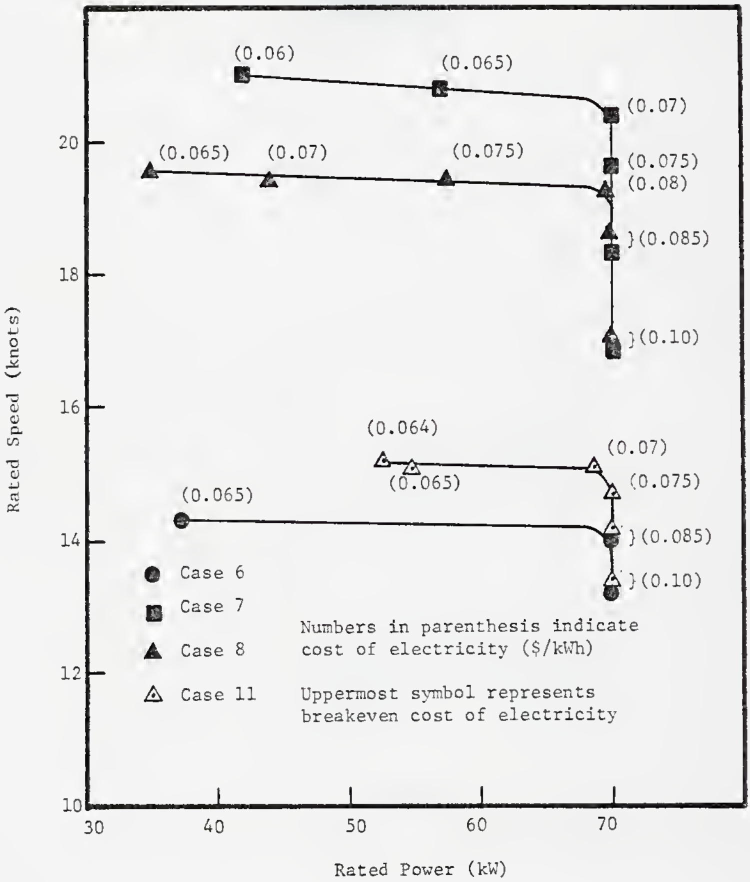

Fig. 3.3-8. Size of optimum WTGS as a function of cost of electricity - Cases 6, 7, 8, and 11.

speed distributions have on the size of the optimum WTGS. As expected, for sufficiently high electrical costs, the rated power attains the value of the largest load demanded in the season. The cases with a small coefficient of variation exhibit much lower rated speeds (and have higher capital costs). Again, with decreasing electrical costs, the rated speed increases while rated power remains constant, along with the characteristic sharp break as rated speed stays constant while rated power decreases rapidly. However, the range of rated speeds at which the rated power remains constant is significantly smaller in the cases with the small coefficient of variation than the cases with the large coefficient of variation.

These differences between the cases caused by different wind speed variances can best be explained by noting that the wind speed distribution with a coefficient of variation of unity is very highly peaked around the average value. Because of this small variation in wind speeds about the mean, there is a relatively small probability of wind speeds much greater than one standard deviation beyond the mean speed. Hence, although the optimum rated speed increases with decreasing cost of purchased electricity, too much of an increase in rated speed would place the rated speed value at a point on the tail of the wind speed distribution where the wind has only a slight probability of occurring. Since very little power could be generated by increasing the rated speed, the rated power must be reduced instead in order to lower the capital costs of the WTGS. Therefore, the WTGS becomes infeasible at a much lower rated speed for a wind speed distribution with a small coefficient of variation than for a distribution with a large coefficient of variation because the system's rated capacity must be reduced sooner when the wind speed distribution has a smaller coefficient of variation.

Although the breakeven costs are higher for a wind model with a small coefficient of variation, the self-sufficiency is enhanced. Tables 3.3-6 and 3.3-7 show that the cases under study have values of self-sufficiency that differ from each other by at least 15% for equivalent costs of electricity. Hence, although breakeven costs are higher, wind speeds with distributions that are grouped very closely about the mean speed can produce much greater amounts of power than wind speed distributions that are highly dispersed about the same mean speed.

(e) Effect of Doubling Mean Wind Speed on the Optimum WTGS

To study the effect of variations in the seasonal mean speed upon the optimum WTGS, a model wind speed was doubled from 10 knots to 20 knots. The wind model used had both the large diurnal variations and the large coefficient of variation, while the model load chosen was the flashy load with average load demands of both 5 kW and 35 kW.

The results for Cases 4 and 12 (shown in Fig. 3.3-9) indicate that annual net savings and breakeven costs are dramatically increased and decreased, respectively. In increasing the mean wind speed from 10 to 20 knots, the breakeven value for the 5 kW average load model was reduced to about 2.75¢/kWh from the previous value of 10.5¢/kWh and for the 35 kW average load model, the breakeven value was reduced to 1.5¢/kWh from the previous value of 6.0¢/kWh. Tables 3.3-5 through 3.3-7

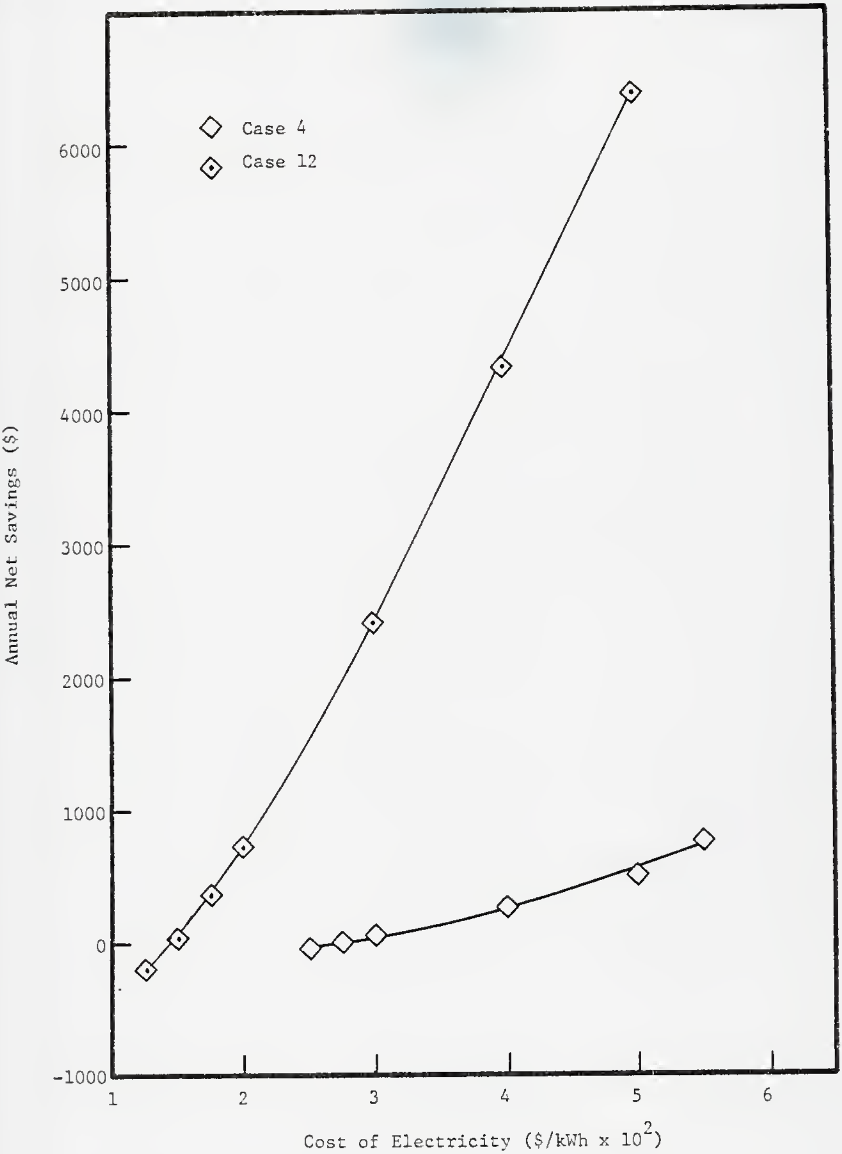

Fig. 3.3-9. Annual net savings versus cost of electricity for optimum WTGS - Cases 4 and 12.

show that self-sufficiency was increased also. Even at the breakeven value, the optimum WTGS for both the large and small average load demand has a self-sufficiency in excess of 35%. Figure 3.3-10 shows that results similar to the 10 knot seasonal mean speed cases (Cases 2 and 10) are obtained, i.e., rated power achieves the value of the maximum load demand and with decreasing electrical costs the rated speed increases until breakeven is reached (like the previous 5 kW mean load demand cases) or the rated speed increases to a certain point and then stays constant while the rated power decreases until breakeven is reached (similar to the previous 35 kW mean load demand cases). Furthermore, the range of speeds at which the rated power of the WTGS remains constant is much greater than the corresponding case with a 10 knot mean wind speed. This is to be expected since for the same coefficient of variation, a mean speed of 20 knots produces a greater variability of wind speeds about this mean. Hence, because there is a greater probability of higher wind speeds occurring for a mean wind speed of 20 knots than for one of 10 knots, the rated speed of the optimum WTGS can be increased to lower capital costs of the WTGS before reaching the point where rated power must be decreased to lower capital costs.

The investigation of the effect of increases in seasonal mean speeds on the optimum WTGS is important because seasonal mean speeds generally increase with the height of a WTGS above ground [3]. Much more power can be generated by increasing the height of the WTGS above ground. However, because increasing the tower height increases the cost of the

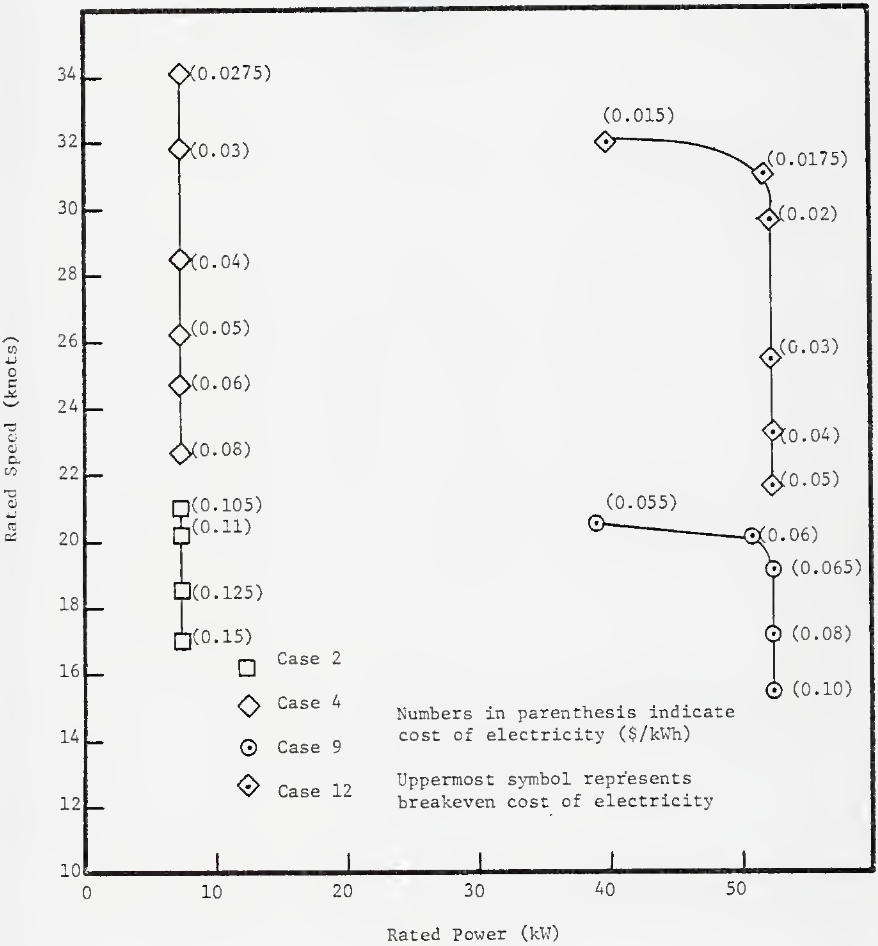

Fig. 3.3-10. Size of optimum WTGS as a function of cost of electricity - Cases 2, 4, 9, and 12.

WTGS, there will be a tradeoff in increased power production and increased tower capital costs.

(f) Effect of Credit for Surplus Electricity on the Optimum WTGS

Of considerable interest in the development of the WTGS is the effect of the utility giving credit for any surplus energy that is fed back into the utility grid. A credit of 2¢/kWh was chosen since any credit given by the utility in the near future can be expected to be considerably less than the price that is charged to purchase electricity. This expected small credit is due to the fact that costs of purchased electricity reflect the capital costs of the utility, amortization of the transmission and distribution systems, costs of protective systems, etc., in addition to actual fuel costs. Since any power fed into the utility grid by the WTGS will save the utility only the cost of fuel, the low credit value is reflective of this partial savings. This credit was applied to both the 5 kW and 35 kW average demand load models.

The effects of credit for surplus electricity are best seen by contrasting Cases 2 and 5 and Cases 9 and 10. The only difference between each of these cases is that Cases 5 and 10 receive credit for surplus electricity. The results obtained for the optimum WTGS are tabulated in Tables 3.3-5 through 3.3-7. As can be seen, considerably larger optimal rated powers are attained in order to generate more surplus energy. However, the rated speeds remain about the same as in the cases where no credit was given. Furthermore, only a moderate increase in self-sufficiency is achieved if credit is given for surplus electricity.

Figures 3.3-12 and 3.3-13 show that there is only a slight increase in the annual net savings but very little effect in the breakeven cost. Finally, as electrical costs decrease, Figs. 3.3-11 and 3.3-14 show that both rated power and rated speed decrease in a smooth but complex manner, unlike the sharp breaks in the trajectories of rated speed versus rated power in the cases where no credit is given for surplus energy.

Consequently, for the expected low value of credit received for surplus power, the effect on the optimum WTGS is slight. Although more net savings are achieved at every cost of purchased electricity, there are only slight improvements in the breakeven values and self-sufficiencies. Unless larger credits are given for generation of surplus energy by a WTGS, it is conjectured that more effective use of this surplus energy can be made by storing the excess, e.g., in batteries.

(g) Effect of Seasonal Variations on the Optimum WTGS

(i) Two Seasons:

In the first multiple season case study, two seasons of load and wind data were used to characterize the entire year. For the first season, wind model 3 supplies power to a smooth load with a 10 kW seasonal average and for the second season, wind model 1 supplies power to a smooth load with a 20 kW seasonal average. Figure 3.3-15 shows the results of this two season optimization problem along with the results of each season run as a single season case, i.e., used for the entire year. As can be seen, for high costs of purchased electricity, the rated power of the optimum WTGS is the same as the maximum load demand

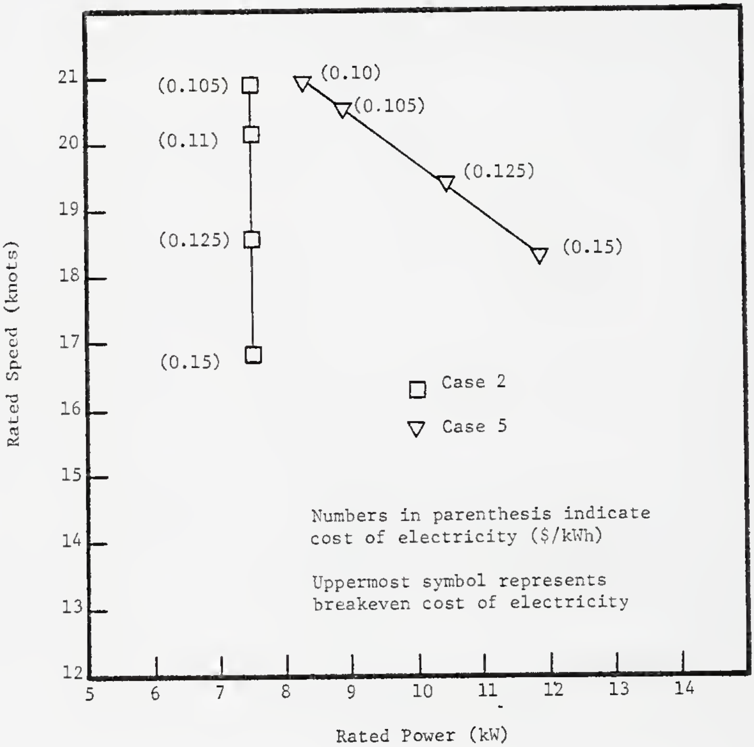

Fig. 3.3-11. Size of optimum WTGS as a function of cost of electricity - Cases 2 and 5.

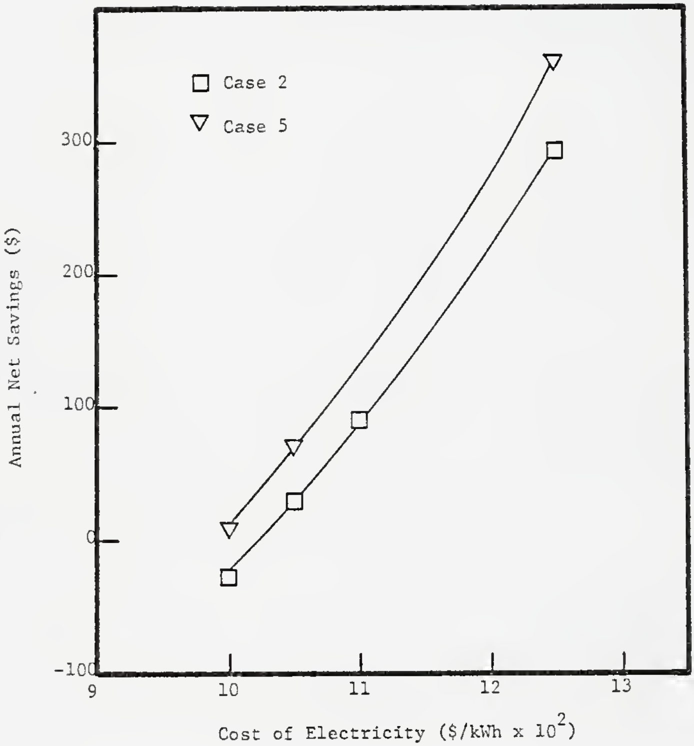

Fig. 3.3-12. Annual net savings versus cost of electricity for optimum WTGS - Cases 2 and 5.

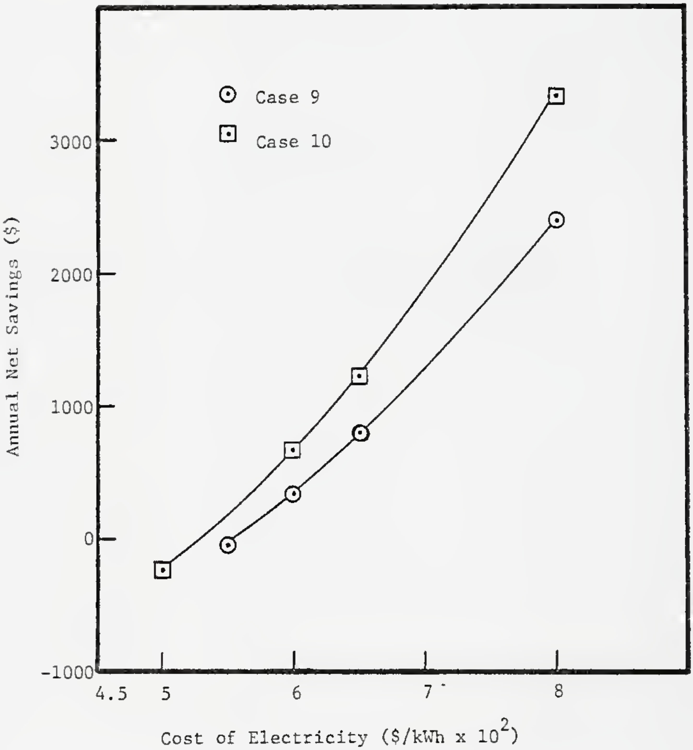

Fig. 3.3-13. Annual net savings versus cost of electricity for optimum WTGS - Cases 9 and 10.

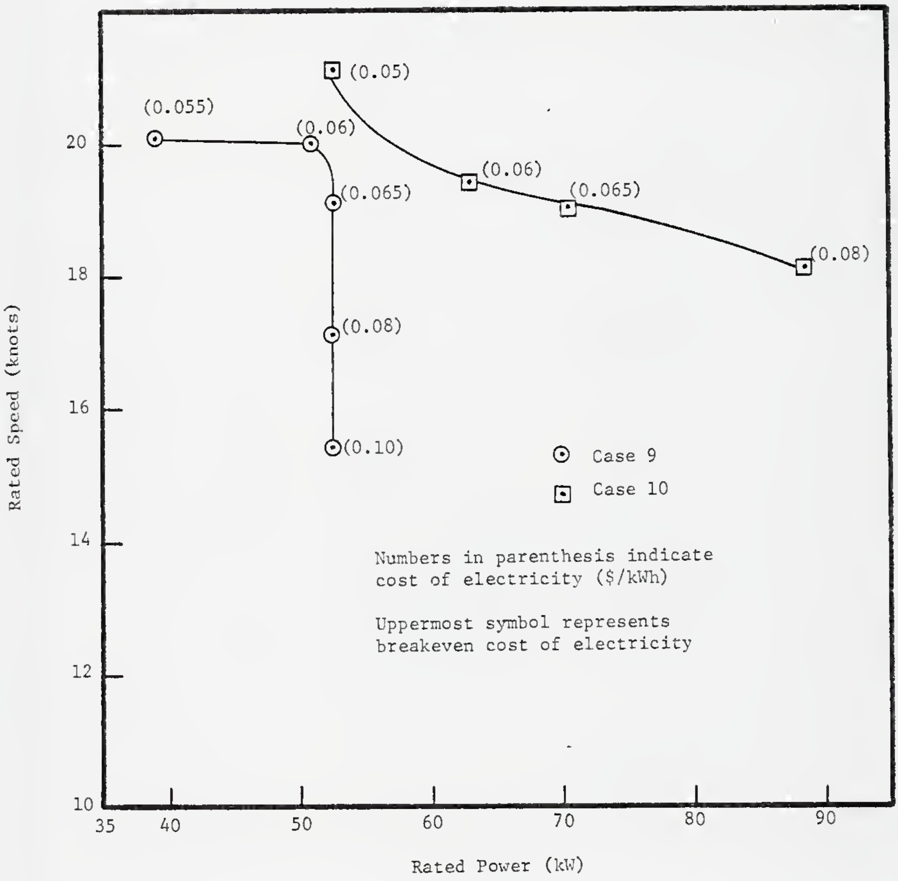

Fig. 3.3-14. Size of optimum WTGS as a function of cost of electricity - Cases 9 and 10.

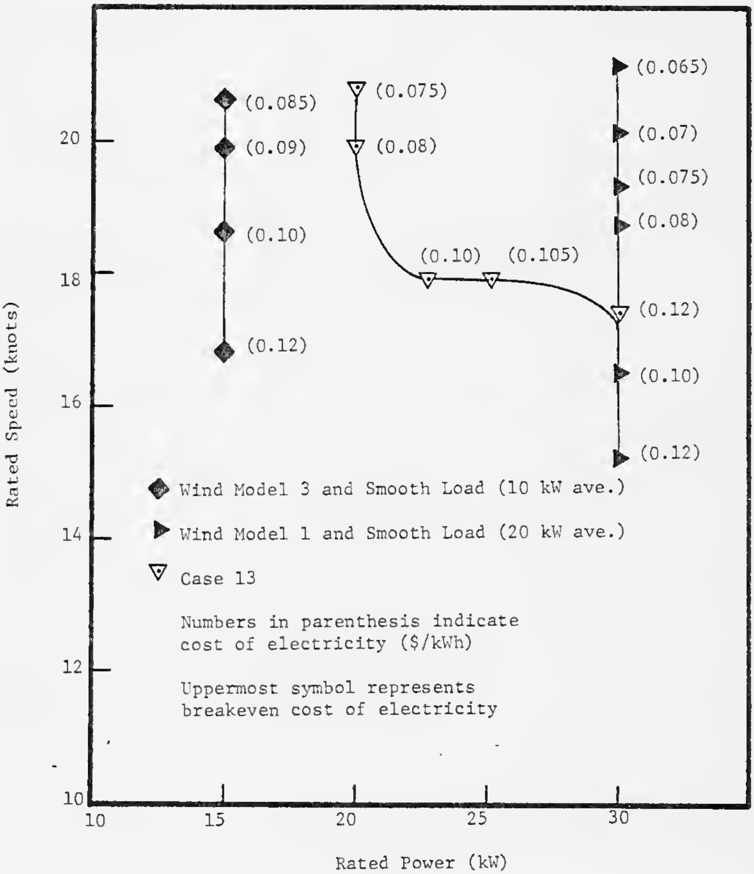

Fig. 3.3-15. Size of optimum WTGS as a function of cost of electricity - Case 13.

for the entire year. As the cost of electricity decreases the optimal rated speed increases while holding the rated power constant in order to decrease capital costs. Again, the tendency is to try to cover as much of the peak demand as possible. However, if this cannot be achieved, the rated speed remains constant while rated power decreases so as to decrease capital costs. But because there are two seasons with different average load demands, there exists another rated power at which the WTGS can sufficiently supply the peak of the lesser load demand as well as supply an adequate portion of the greater load demand, though it cannot cover all of the maximum demand load. Hence, as the cost of electricity approaches breakeven, the size of the WTGS remains at this lower value of rated power while rated speed continues to increase.

Figure 3.3-16 shows that the annual net savings for the two season case lies between the values obtained when each season characterizes an entire year. Furthermore, the breakeven cost is approximately the average value of the individual seasons when each season is used for the entire year. Consequently, the effect of seasonal variations on the optimum net savings is that of averaging the results obtained from the optimization procedure when each season is run separately.

Although Fig. 3.3-15 still exhibits the step-like characteristic of the single season models, it must be kept in mind that the seasonal load variation also exhibited these characteristics, i.e., the two seasonal load profiles differ by a factor of two. In realistic cases, the seasonal wind and load changes are not as pronounced, but vary

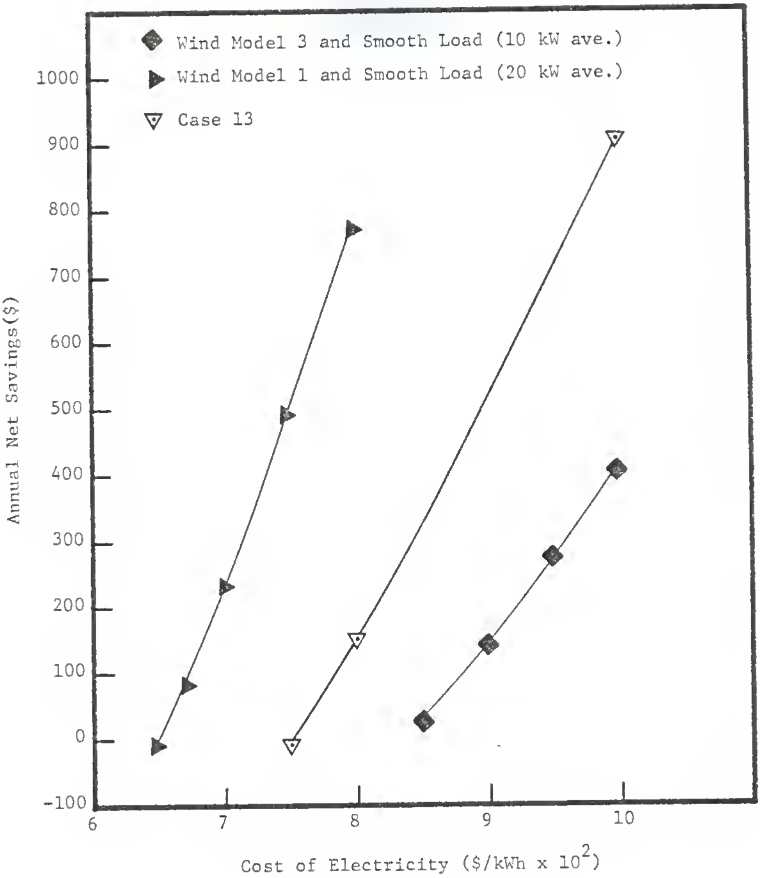

Fig. 3.3-16. Annual net savings versus cost of electricity for optimum WTGS - Case 13.

smoothly from season to season. Hence, as more seasons are added it can be expected that the trajectory of the WTGS size as a function of cost of electricity will vary in a smooth but more complex manner.

(ii) Four Seasons:

A four season case was studied to see if the results obtained from the idealized model wind speed and load models could be used to predict the behavior of realistic wind speed and load profiles. A load model representative of a Kansas winter wheat-sorghum farm was chosen. Wind speed data for Wichita, Kansas was obtained from the National Climatic Center and are listed in Table 3.3-8. These wind speed data are assumed to be applicable to winter wheat-sorghum operations near this city. Load information for the residence and farm was obtained from DPRA [12]. Both the residence and farm load are shown in Tables 3.3-9 and 3.3-10. The residential load is based on a totally electric home for a family of four under a normal daily living pattern. The only significant farm load comes from aeration fans which are used in the wheat storage bins to prevent spoilage.

The discrete wind speed distributions in every three-hour interval are modeled by the beta distribution using the method of matching-moments. The parameters of the approximating beta distribution of the wind distributions for each daily and seasonal time interval are tabulated in Table 3.3-11. As a comparison of the effect of using approximating analytical distributions, the discrete wind speed distributions were also used in the optimization routine for this realistic case. The results of

Table 3.3-8. Wind Speed Distribution Data for Wichita, Kansas (From Ref. 12).

Season	Time Interval (hrs)	Probability (%) of Wind in Speed Group (knots) 0 - 1	1 - 3	3 - 6	7 - 10	11 - 16	17 - 21	22 - 27	28 - 33	34 - 40	41 - 47
Fall	0 - 3	9.6	8.2	11.4	30.5	24.7	7.3	1.8	0.5	0.0	0.0
	3 - 6	10.5	8.1	18.6	30.3	24.0	6.4	1.9	0.2	0.0	0.0
	6 - 9	7.4	8.2	18.8	29.9	25.6	7.6	2.1	0.4	0.0	0.0
	9 - 12	2.7	5.5	13.2	26.6	33.7	13.2	4.4	0.7	0.0	0.0
	12 - 15	2.3	4.8	11.1	25.6	32.4	15.47	6.4	1.5	0.2	0.0
	15 - 18	2.8	4.6	13.7	26.7	30.5	14.8	6.2	0.7	0.0	0.0
	18 - 21	9.9	8.9	18.1	27.7	26.0	7.9	1.5	0.0	0.0	0.0
	21 - 24	10.9	7.5	17.3	28.6	26.2	7.0	2.2	0.3	0.0	0.0
Winter	0 - 3	6.3	7.5	17.5	28.7	24.9	11.0	3.4	0.7	0.0	0.0
	3 - 6	7.1	7.7	19.4	26.4	25.9	9.3	3.8	0.4	0.0	0.0
	6 - 9	6.9	8.2	17.8	26.1	26.6	10.6	3.2	0.6	0.0	0.0
	9 - 12	3.7	6.2	14.5	25.6	29.8	14.7	4.6	0.9	0.0	0.0
	12 - 15	2.2	5.3	12.4	24.4	31.4	16.4	6.9	1.0	0.0	0.0
	15 - 18	2.3	4.8	14.2	27.0	31.3	14.2	5.6	0.6	0.0	0.0
	18 - 21	6.3	8.3	20.0	27.8	23.3	10.7	3.4	0.2	0.0	0.0
	21 - 24	6.4	7.4	17.5	29.0	23.6	12.0	3.8	0.3	0.0	0.0
Spring	0 - 3	5.9	6.2	15.1	26.5	26.8	13.1	5.7	0.7	0.0	0.0
	3 - 6	5.2	5.4	15.5	26.5	30.2	12.0	4.3	0.8	0.1	0.0
	6 - 9	4.3	4.4	12.8	24.1	35.1	13.9	4.5	0.9	0.0	0.0
	9 - 12	1.2	2.5	7.9	20.3	36.1	19.4	9.9	2.3	0.4	0.0
	12 - 15	0.8	2.3	7.9	19.8	33.3	20.4	11.5	3.4	0.5	0.1
	15 - 18	1.0	2.4	8.5	20.4	33.1	19.0	11.8	3.3	0.5	0.0
	18 - 21	3.2	4.8	14.7	27.5	29.0	13.9	5.6	1.1	0.2	0.0
	21 - 24	5.6	6.3	16.5	27.8	25.8	11.7	5.1	1.2	0.0	0.0
Summer	0 - 3	5.5	8.5	21.3	36.5	22.1	5.1	1.0	0.0	0.0	0.0
	3 - 6	8.3	10.8	23.1	34.9	18.7	4.1	0.1	0.0	0.0	0.0
	6 - 9	5.2	7.7	22.3	36.2	22.0	5.6	1.0	0.0	0.0	0.0
	9 - 12	3.0	4.7	16.3	34.5	30.3	8.7	2.3	0.2	0.0	0.0
	12 - 15	2.6	4.1	16.1	33.7	29.0	11.5	2.8	0.2	0.0	0.0
	15 - 18	1.8	4.4	14.7	32.4	31.9	11.6	3.0	0.2	0.0	0.0
	18 - 21	2.4	4.6	16.4	36.1	30.0	9.0	1.3	0.1	0.1	0.0
	21 - 24	4.6	7.6	20.7	36.6	23.3	6.2	0.8	0.2	0.0	0.0

Table 3.3-9. Typical Residence Load for Family of Four on Western Kansas Winter Wheat-Sorghum Farm (From Ref. 12).

	Time Interval (hrs.)							
Season	0 - 3	3 - 6	6 - 9	9 - 12	12 - 15	15 - 18	18 - 21	21 - 24
	Load Demand (kW)							
Fall	5.96	3.77	15.6	8.48	6.47	7.53	15.9	12.6
Winter	19.7	20.0	31.8	22.2	20.2	21.0	36.9	28.6
Spring	11.5	12.0	21.1	11.2	9.22	4.78	15.9	12.6
Fall	7.91	5.72	19.9	12.8	17.8	16.13	24.5	18.9

Table 3.3-10. Typical Load for Kansas Winter Wheat-Sorghum Farm (From Ref. 12).

	Time Interval (hrs.)							
Season	0 - 3	3 - 6	6 - 9	9 - 12	12 - 15	15 - 18	18 - 21	21 - 24
	Load Demand (kW)							
Fall	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
Winter	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
Spring	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Fall	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 3.3-11. Parameters of Beta Distribution for Wichita, Kansas Wind Speed Data.

Seaon	Time Interval (hrs)	α	β	V_{max} (knots)
Fall	0 - 3	1.32	3.73	33.5
	3 - 6	1.56	4.42	33.5
	6 - 9	1.67	4.38	33.5
	9 - 12	2.14	4.13	33.5
	12 - 15	2.28	5.21	40.5
	15 - 18	2.00	3.72	33.5
	18 - 21	1.41	2.94	27.5
	21 - 24	1.53	4.14	33.5
Winter	0 - 3	1.59	3.72	33.5
	3 - 6	1.54	3.75	33.5
	6 - 9	1.56	3.71	33.5
	9 - 12	1.85	3.64	33.5
	12 - 15	2.00	3.50	33.5
	15 - 18	2.09	3.49	33.5
	18 - 21	1.57	3.88	33.5
	21 - 24	1.60	3.74	33.5
Spring	0 - 3	1.61	3.34	33.5
	3 - 6	1.99	5.44	40.5
	6 - 9	2.05	3.88	33.5
	9 - 12	2.68	5.07	40.5
	12 - 15	2.81	6.34	47.5
	15 - 18	2.45	4.45	40.5
	18 - 21	2.07	5.20	40.5
	21 - 24	1.55	3.32	33.5
Summer	0 - 3	1.88	4.14	27.5
	3 - 6	1.74	4.46	27.5
	6 - 9	1.90	4.12	27.5
	9 - 12	2.41	5.49	33.5
	12 - 15	2.36	5.08	33.5
	15 - 18	2.51	5.20	33.5
	18 - 21	2.88	8.61	40.5
	21 - 24	2.17	5.98	33.5

the BLOHARD routine for both the approximating beta and the actual discrete wind speed distributions are listed in Table 3.3-12 for various costs of purchased electricity. There is little difference between the optimum WTGS for either wind speed distribution. For low electrical energy costs, an optimum WTGS, which yields a negative annual net savings, is found. This occurrence indicates a local optimum for which the amount of money lost by using a WTGS is minimized although the true optimum would be a zero-size WTGS.

From Fig. 3.3-17, which shows the WTGS size plotted as a function of electrical energy costs, the shape of the trajectory has many characteristics of those found for the single and dual season idealized wind speed and load models. For very high costs of electricity, the rated power of the optimum WTGS tends toward the maximum load for the year. With decreasing electrical costs, rated power decreases somewhat but rated speed increases at a faster rate. Finally, as electrical costs approach breakeven, the rated power begins to decrease at a faster rate than that at which the rated speed increases. Although this realistic case does not exhibit the dramatic step-like changes obtained for the model wind speed and load profiles, the overall behavior is still quite similar to these model cases.

Breakeven costs can be determined from a plot of annual net savings versus cost of electricity as shown in Fig. 3.3-18. For this farming enterprise, the cost of electricity must be about 6.8¢/kWh in order to offset the installation cost by the savings in purchased electricity.

Table 3.3-12. Optimum WTGS Size and Output Characteristics for Kansas Winter Wheat-Sorghum Farm.

Type of Wind Speed Distribution Used	Cost of Electricity (\$/kWh)	Optimum P_r (kW)	Gen. V_r (knots)	Elect. Gend. & Used (kWh)	Elect. needed (kWh)	Surplus elect. (kWh)	Annual Net savings (\$)	% Self-sufficiency
Beta	0.12	26.7	15.4	6.73×10^4	8.89×10^4	2.56×10^4	2950	43.1
	0.10	25.5	16.6	5.98×10^4	9.64×10^4	1.97×10^4	1673	38.3
	0.08	24.5	18.4	4.97×10^4	1.07×10^5	1.35×10^4	569	31.8
	0.07	23.3	19.6	4.26×10^4	1.14×10^5	9.80×10^3	104	27.3
	0.06	16.3	21.2	2.79×10^4	1.28×10^5	2.78×10^3	-253	17.8
Discrete	0.10	25.6	16.2	6.21×10^4	9.41×10^4	2.06×10^4	1722	39.7
	0.08	24.7	18.1	5.13×10^4	1.05×10^5	1.40×10^4	581	32.8
	0.06	16.3	21.0	2.85×10^4	1.28×10^5	2.87×10^3	-263	18.3

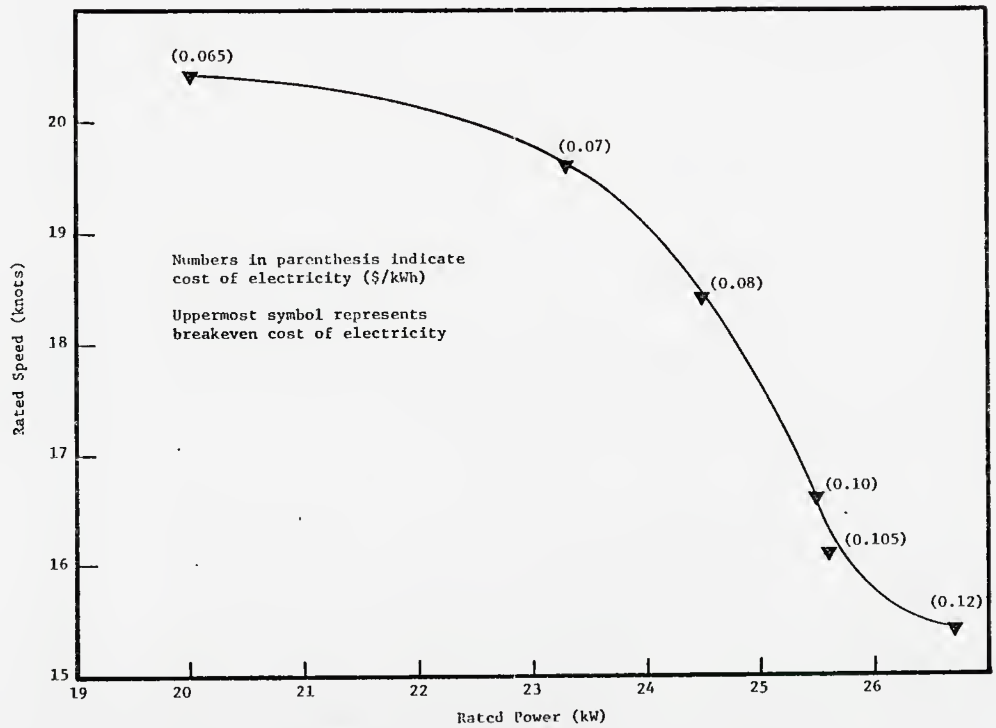

Fig. 3.3-17. Size of optimum WTGS as a function of cost of electricity - Case 14.

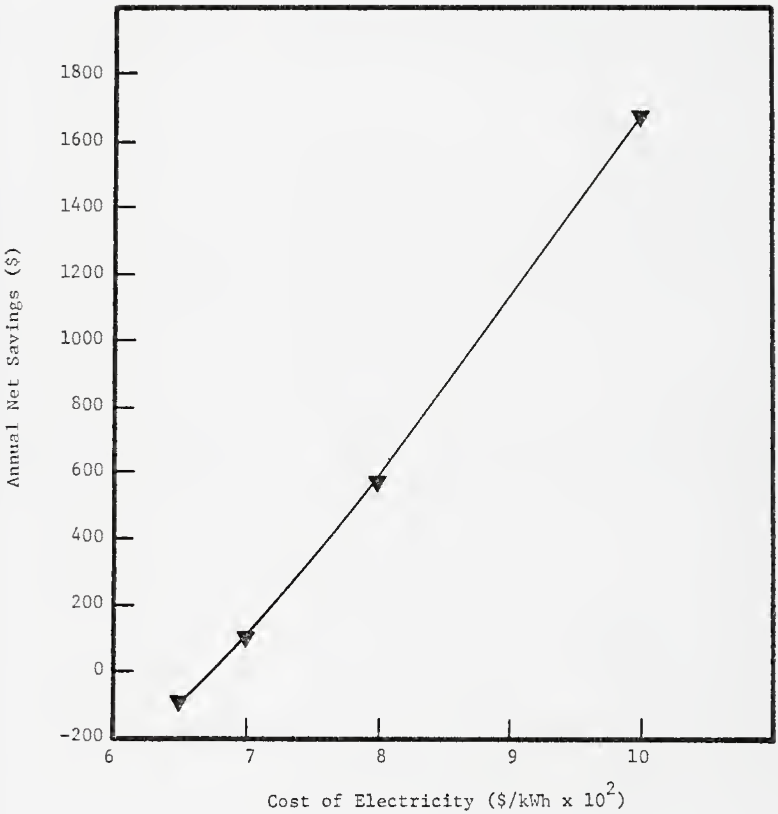

Fig. 3.3-18. Annual net savings versus cost of electricity for optimum WTGS - Case 14.

This value is subject to change with changes in the interest rate and the lifespan of the WTGS. As mentioned earlier, an interest rate of 10% per year and a lifetime of 20 years were assumed. These values are typical for today's economy and used in order to get an indication of the feasibility of the WTGS. For more accurate results, more precise values of interest rates and useful life costs should be used, as well as more accurate cost models for the type of WTGS to be installed.

For this realistic example and with the stated economic assumptions, the breakeven cost of electricity is well above the current price charged for purchasing electricity. However, conclusions as to the feasibility of a WTGS are dependent upon the technical and economic assumptions made. A lower cost model for the WTGS will definitely alter the results towards making the WTGS economically more attractive. Similar results would be obtained if low interest loans become a reality (e.g., through government subsidies). Decisions concerning these economic parameters and questions must be made before any definite statement is made as to the economic feasibility of the WTGS.

Although the results for this realistic case show the WTGS to be currently economically infeasible, a more important result is the observation that wind speed and load data must be supplied on a multiple season basis in order to get an accurate value for the optimum size WTGS. As less data are supplied, the more inaccurate is the optimization procedure and the calculated optimization results. The interaction of one season with another is as important as the interaction of diurnal variations. Therefore, for the optimization methodology to yield accurate results, the input data should be as complete as possible if the methodology described in this chapter is to be used for analysis of actual wind energy applications.

4. CONCLUSIONS AND RECOMMENDATIONS

In this research, two important aspects of the extraction of energy from the wind were analyzed. First, as an aid to help predict the power available from the wind, analytical wind speed distribution models were examined along with methods to estimate the model parameters. Three techniques were presented for the estimation of the parameters of a modeling Weibull distribution, which is the most commonly used analytical representation of observed wind speed data. In addition, the beta distribution was introduced as an alternative wind speed distribution model. Two goodness of fit tests were performed on each analytical distribution to test the appropriateness of each model in describing observed wind speed distributions. Second, a methodology was described, whereby, given wind speed distribution models for a particular location and power demand data for a particular energy-consuming enterprise, the economically optimal size WTGS could be determined such that the net annual economic savings realized with the WTGS are maximized. It was assumed that the WTGS produced utility-compatible electric power, and furthermore was connected into the utility grid so that when the WTGS could not generate all the required power, the deficit could be purchased from the power grid.

It has been shown that for parameter estimation of the Weibull distribution, the matching-moments technique yielded a Weibull distribution that represents the wind speed data much more closely than least squares fitting techniques. From the results of the analyses of 544 observed wind speed distributions, the χ^2 goodness of fit test indicated little as to the accuracy of either of the parameter estimation techniques (matching-moments or least square) in modeling most of the observed wind speed distributions. However, the large χ^2 values obtained for approximating Weibull distributions fit by the matching-moments method were found to be almost totally a result of poor fits at very low speeds, i.e., below four knots. At intermediate and high wind speeds (the regions most important for a WTGS), the Weibull matching-moments fits generally produced excellent approximations of the observed distributions. The beta distribution, whose parameters were also estimated by a matching-moments technique, exhibited a similar result - poor fits at very low speeds but excellent fits at intermediate and high speeds.

The second procedure developed to test the fitted Weibull and beta distributions, the power ratio test, confirmed the observation that the matching-moments parameter estimation technique represented the wind speeds greater than four knots very accurately. With this test, the ratio was calculated of the power obtained from a given size WTGS when the analytical (fit) wind speed distribution was used to the power generated by the same WTGS when the discrete (observed) wind speed distribution was used. Application of this test to 544 observed wind speed distributions showed that the matching-moments technique yielded Weibull and beta distributions with power ratios that were very close to the ideal value of unity. The least squares techniques, however, produced Weibull distributions that, when the power ratio test was performed, yielded discrepanies of as much as 70% from unity. Consequently, for both the Weibull and beta distributions, the matching-moments technique of parameter estimation provided distributions that accurately model observed wind speed distributions.

It was also seen that because only two wind speed statistics, i.e., the mean wind speed and variance of wind speeds about this mean are needed to calculate the parameters of the Weibull or beta distribution using the matching-moments technique, the need for detailed historical wind speed information is eliminated. Since detailed meteorological wind speed distribution data are not readily available for most locations, much simpler and less time-consuming measurements or analysis of meteorological data tapes to obtain the mean wind speed and the variance of wind speeds need be performed. Although both Weibull and beta distributions give accurate fits of the wind speed distributions when the parameters are estimated by the matching-moment technique, the beta distribution is particularly attractive since its two parameters can be calculated directly from the mean and variance of the wind speed. Calculation of the Weibull parameters, on the other hand, requires the numerical solution of transcendental equations.

From the sensitivity studies performed on the optimization methodology in part two of this work, it was seen, that in addition to supplying detailed wind speed distributions and load demands to characterize accurately diurnal variations in wind speeds and load requirements, seasonal variations need to be represented also. If only a single season is used, the optimal WTGS size as a function of cost of purchased electricity does not vary smoothly as expected, but exhibits very rapid changes in size. However, as more seasons are added to the analysis, the trajectories of the optimal WTGS size as a function of cost of electricity vary in a much smoother fashion. Besides needing detailed diurnal wind speed distribution and load demand requirements, seasonal variations in these two inputs must also be characterized. However, single season sensitivity analyses are useful in identifying the general trends of the optimization methodology.

Of all the sensitivity studies analyzed, the parameter that had the largest effect on the optimum WTGS was, as expected, the mean speed at the WTGS site. For even the single season case, locations that have mean speeds of 20 knots were capable of producing electricity on a competitive basis with utility supplied energy. Another parameter which affects significantly the size of the optimum WTGS was the size of the load served by the WTGS. For large loads, more money can be saved from the installation of an optimum WTGS, and consequently, power can be produced more cheaply. This preference for larger loads is a direct result of the lower unit capacity costs for larger WTGSs inherent in the WTGS cost model. Although the unit capacity cost ($/kWh) generally decreases as the rated power of a WTGS increases, a power level is eventually reached ($\sim$ 40-60 kW) above which this observation is no longer true and the cost ($/kW) actually increases. This is caused by the fact that for large rated powers, the complexity and sophistication of the WTGS must increase compared to smaller machines in order to handle the large amount of power. Consequently, this transition power must be clearly noted so that a different cost model can be assumed to account for this added system complexity.

Another parameter that was shown to have only a slight effect on the optimum WTGS was the relative magnitude of the peaks in the load demand. As long as the load peaks coincide or are inphase with the daily wind speed peaks, only marginally more net savings and lower breakeven costs are achieved if the load varies smoothly about an average wind speed than a load with large or flashy variations about the same average wind speed. However, a constant load demand throughout the day has lower net savings and a higher breakeven cost than the smoothly varying load with the same average load demand as the constant load. Hence, load peaks whether large or small, which are inphase with the wind speed peaks tend to favor the development of wind power by lowering the breakeven value.

The relative magnitude of diurnal wind speed variations were found to affect the size of the optimum WTGS to a lesser degree. The optimum WTGS produces a greater net savings and lower breakeven costs for a wind with large diurnal variations than a wind with small diurnal variations about the same daily mean speed. As was true in the load variation cases, as long as the peaks in the wind speed and load variations are inphase, the magnitude of either do not affect the optimum WTGS significantly.

The variance of wind speeds about a mean wind speed exhibited a detrimental and also an advantageous effect depending upon the cost of purchased electricity. Wind speed distributions with small variances, i.e., highly peaked around a mean speed, had large net savings for high costs of purchased electricity, but the net savings decreased sharply with decreasing purchased energy costs. The breakeven value was higher for wind speed distributions with a small variance than the distributions with a large variance. However, the self-sufficiency of the optimal WTGS was enhanced when the location site had wind speed distribution with a small variance. Consequently, if a greater self-sufficiency is required, sites with almost constant speeds should be chosen, whereas, if net savings is to be maximized, sites which have wind speed distributions with large variances are more attractive.

Finally, the investigation of the effect of credit for surplus electricity on the optimum WTGS showed the optimum rated power was most affected. The WTGS rated power increased greatly compared to similar cases which received no credit for surplus power. This increase is reasonable since as more surplus power is generated, more money is made. For the small credits which are likely to be given (if any credit is given at all), no significant effect on annual net savings, self-sufficiency, or breakeven value is realized. However, as noted earlier, if the WTGS rated power increases beyond a certain critical value, a higher cost model needs to be applied. Hence, this greater WTGS cost may overshadow any profit from selling surplus power.

Many areas for further study have emerged during the course of this work. For instance, when modeling observed wind speed distributions, the effect of bin size (speed subintervals) used in the histogram of observed wind speed distributions should be investigated. This study used rather large speed subintervals when characterizing wind speed data. To reduce the bin size, analysis of the meteorological data tapes would be required. Furthermore, the bin size can be reduced only so far before statistical fluctuations (caused by the paucity of wind speed data) will begin to mask actual detail in the wind speed distribution. The whole area of extracting the most information about the wind speed distribution from finite amounts of observed data is a very important one if accurate analysis of wind energy is desired.

Although only two analytical distributions, the Weibull and beta, were examined in this study, many more can be tried. For example, the beta-prime distribution could be used as a fitting distribution. The beta distribution used in this study is defined over a finite interval, while the beta-prime distribution is defined over the entire positive axis [29]. Other distributions worthy of further study are the gamma distribution [9] and the single parameter Rayleigh distribution [7,8,9].

The optimization methodology investigated in this work considered only several basic features of WTGS to obtain an optimum match between wind energy production and demand load. Extensions to this model could be made to incorporate many additional features. For example, a logical extension would be to assume a distribution function about a given mean value which would give the variation of the demand load rather than assuming a constant or mean load during every time interval. Such a change would require only slight modifications in the optimization methodology.

An additional sensitivity study would involve the investigation of load management. In this study, the load demand peaks were assumed *a priori*, and no attempt was made to manage the load, within given constraints, to improve further the savings afforded by the optimum WTGS. Such load management as part of the optimization procedure would require the inclusion of linear programming techniques into the present methodology.

Because the availability of low cost loans would affect the feasibility of a WTGS, the exact nature of such loans is yet another parameter which could be incorporated easily into the objective function of the present methodology. Similarly, much more detailed economic models for WTGS costs, amortization incentives, tax credits, surplus WTGS energy credits, etc. should be investigated. Such investigation would require only minor modifications of the present methodology which has been written in a highly modularized fashion to facilitate alterations in components of the methodology. As previously noted, the high wind speeds favor the economic viability of a WTGS. Because wind speeds generally increase with height above ground [3], a WTGS at a greater height could potentially produce larger amounts of power. However, as WTGS height increases, the tower cost also increases. Consequently, with a cost model for tower costs and a model for increasing wind speed with height, the optimization methodology could be easily modified to include WTGS height as a variable to be optimized in the selection of the economically optimal WTGS.

This study assumed no storage of excess power. Conceivably the inclusion of battery storage could improve the economic attractiveness of wind systems, and the effects of battery storage should be investigated. Because the optimum amount of battery storage capacity can be expected to be very sensitive to the amount of surplus energy generated, much more detailed wind speed information is needed so that an accurate estimate of optimal capacity can be made. This would require either model wind speed data for every day in the year or correlations which describe how wind speeds depend on earlier wind speeds.

Finally, this same optimization methodology used in this work can be applied directly to solar energy studies of electrical energy generation. The wind speed distributions would be replaced by solar insolation distributions, i.e., the probability a particular amount of radiant energy will strike a surface in a given time interval. The WTGS response function can be replaced by the solar cell response to the radiant energy that strikes it. If solar collectors are used to produce thermal energy, then storage capacity would have to be included in the optimization procedure (analogous to battery storage for the wind energy problem). Hence, the optimization methodology used in this study can be expected to be applicable to the selection of optimal solar energy systems which maximize the system parameters so as to produce the maximum economic savings.

In conclusion, this research was two-fold. The first part studied the modeling of observed wind speed distributions by analytical distributions. The second part applied the wind speed models to match the available wind power from a WTGS with the load demand requirements to compute the size of an optimal WTGS, i.e., a WTGS that saves the user the most amount of money by replacing normally purchased power with that generated by a WTGS. This type of detailed investigation is needed in order to determine the impact wind generated power can have on future energy policies.

5. ACKNOWLEDGEMENTS

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7. APPENDICES

APPENDIX A

The Program CURVEFIT

The code CURVEFIT calculates the parameters of analytical wind model distributions (the three Weibull distributions and the beta distribution) by fitting these distributions to observed discrete wind speed distributions. In addition, the χ^2 and power ratio statistics are generated as a measure of the goodness of fit of the analytical distributions. The program input requires the roots and weights of any even order Gauss-Legendre quadrature desired. For the computation of the power ratio, the parameters of the WTGS (i.e., rated power, rated speed, cut-in speed) are needed. Finally, the boundaries of the wind speed subintervals used in the discrete distribution are required along with the frequency of wind speed observations in each subinterval and the total number of wind speed observations made. Detailed input requirements are described by comment cards in the FORTRAN IV program listing which is included in this Appendix.

Estimation of the parameters of the fitting distributions is performed according to the methods described in subsections 2.3.1 and 2.4.1. The numerical procedure used to solve Eq. (2.3-18) for the parameters of the Weibull distribution using the matching-moments method is the subroutine RTMI [30]. The subroutines WBLFIT and F compute the initial points for the Mueller's iteration technique used in RTMI.

The calculations required for the χ^2 and power ratio tests described in Section 2.5 are performed by the subroutine CHISQ and POWER, respectively. Any Weibull or beta distribution values needed by CHISQ and POWER are computed by the subprograms WB or FI, respectively. The necessary integrals are computed by the subroutine GAUSS which uses a specified even order Gauss-Legendre quadrature.

The output of CURVEFIT is either a table of model distribution parameter values or a table of goodness of fit statistics (similar to Tables 2.6-1 and 2.6-2) depending upon the values of program option parameters. The program is written in FORTRAN IV for the Kansas State University ITEL Advanced System 5 (which is equivalent operationally to an IBM 370/158 system). Liberal use of comment cards and variable names with high mneomic content assist the user in deciphering the program logic.

```
              C******************************* CURVEFIT ************************************
              C*                                                                           *
              C* THIS PROGRAM COMPUTES THE PARAMETERS OF THE WEIBULL DISTRIBUTIONS(LEAST    *
              C* SQUARES, WEIGHTED LEAST SQUARES, AND MATCHING-MOMENTS TECHNIQUES) AND THE *
              C* BETA DISTRIBUTION(MATCHING-MOMENTS TECHNIQUE).  CHI-SQUARE AND POWER RATIO *
              C* STATISTICS ARE ALSO COMPUTED.                                             *
              C*                                                                           *
              C*                                                                           *
              C* INPUT DATA:                                                               *
              C*                                                                           *
              C*   CARD 1    FORMAT (I2)                                                   *
              C*       NHALF = THE HALF VALUE OF THE EVEN ORDER GAUSS-LEGENDRE QUADRATURE   *
              C*               USED TO EVALUATE THE NECESSARY INTEGRALS                    *
              C*                                                                           *
              C*   CARD 2    FORMAT (4G20.7)                                               *
              C*       ROOT(I) = QUADRATURE ORDINATES(ONLY POSITIVE VALUES)                *
              C*                 (MAY BE MANY CARDS)                                       *
              C*                                                                           *
              C*   CARD 3    FORMAT (4G20.7)                                               *
              C*       WEIGHT(I) = QUADRATURE WEIGHTS                                      *
              C*                 (MAY BE MANY CARDS)                                       *
              C*                                                                           *
              C*   CARD 4    FORMAT (4I5)                                                  *
              C*       INN = TOTAL NUMBER OF POSSIBLE SPEED SUBINTERVALS(USUALLY 11)       *
              C*       IPR = RATED POWER OF WIND TURBINE(KW)                               *
              C*       IVR = RATED SPEED OF WIND TURBINE(MPH)                              *
              C*       IVC = CUT-IN SPEED OF WIND TURBINE(MPH)                             *
              C*                                                                           *
              C*   CARD 5    FORMAT(12F5.2)                                                *
              C*       IVINT(I) = ENDPOINTS OF WIND SPEED SUBINTERVALS(KNOTS)              *
              C*                                                                           *
              C*   CARD 6    FORMAT(20A4)                                                  *
              C*       TITLE = TITLE CARD FOR WIND SPEED DATA SET ANALYZED                 *
              C*               (MAY BE MANY CARDS)                                         *
              C*                                                                           *
              C*   CARD 7    FORMAT(I2,I3,10I4,I8,I5)                                      *
              C*       MONTH = MONTH FROM WHICH WIND SPEED DATA IS OBTAINED                *
              C*       NTIME = DAILY TIME PERIOD FROM WHICH WIND SPEED DATA IS OBTAINED    *
              C*       IFREQ(I) = FREQUENCY(X 1000) OF OBSERVATIONS IN I-TH SPEED SUBINTERVAL*
              C*                (BEGINNING WITH FREQUENCY IN 2ND SPEED SUBINTERVAL)       *
              C*       NUMMY = SPACE FOR DATA IDENTIFICATION PURPOSES(CAN ALSO BE USED TO   *
              C*               ADD TWO MORE SPEED SUBINTERVALS)                            *
              C*       NOBS = TOTAL NUMBER OF WIND SPEED DATA OBSERVATIONS                 *
              C*              (MAY BE MANY CARDS)                                          *
              C*                                                                           *
              C* WRITTEN BY L. A. POCH, KANSAS STATE UNIVERSITY, DECEMBER 1977.            *
              C*                                                                           *
              C*****************************************************************************
0001                REAL*4 FREQ(41),TITLE(40),VINT(41),Y(41),X(41),V(41),FREQN(41),
                   1 F(41),ROOT(20),WEIGHT(20),FREY(41)
0002                REAL*4 MMEAN,IVINT(41),K,MEAN
0003                INTEGER*4 IFREQ(41),DF1,DF2,DF3,DF4
0004                COMMON/LINK1/K,C,AA,BB,VMAX,VRATED,FCTR
0005                COMMON/LINK3/NHALF,ROOT,WEIGHT
0006                EXTERNAL WB,FI,V3FI,V3W8
              C*** READ IN GAUSS-LEGENDRE QUADRATURE ORDINATES AND WEIGHTS
0007                READ(5,113)NHALF
0008            113 FORMAT(I2)
```

```
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 0009                REAO(5,12)(ROOT(I),I=1,NHALF)
 0010                REAO(5,12)(WEIGHT(I),I=1,NHALF)
 0011             12 FORMAT(4G20.7)
                C*** READ IN NUMBER OF SPEED SUBINTERVALS AND WINO TURBINE SPECIFICATIONS
 '0012               READ(5,2)INN,IPR,IVR,IVC
 0013              2 FORMAT(4I5)
 0014                NN=INN+1
 0015                PRATEO=IPR
                C*** CONVERT MPH TO KNOTS
 0016                VRATED=IVR/1.15
 0017                VCUTIN=IVC/1.15
                C*** READ IN ENOPOINTS OF SPEED SUBINTERVALS AND TITLE
 0018                READ(5,1)(IVINT(I),I=1,NN)
 0019              1 FORMAT(12F5.2)
 0020              3 READ(5,100) TITLE
 0021            100 FORMAT(20A4)
 0022                WRITE(6,110)(TITLE(I),I=1,33)
 0023            110 FORMAT('1'////,33A4)
 0024                PRINT 1001
                C*** REMOVE COMMENT TO LIST CHI-SQUARE ANO POWER RATIO TABLE
                C1001 FORMAT('0',118('-'))
 0025           1001 FORMAT('0',113('-'))
 0026                PRINT 102
 0027            102 FORMAT(T52,'WEIBULL OISTRIBUTION PARAMETERS',T98,'BETA OISTRIBUTIO
                    1N'/' MONTH  TIME   MEAN SPEED   STD. OEV.   LST. SOS.-UNWTO.   LST
                    2. SQS.-WTD.   MATCHING-MOMENTS         PARAMETERS'/T9,'(HRS)   (KNO
                    3TS)       (KNOTS)',3(8X,'K        C '),8X,'ALPHA     BETA')
                C*** REMOVE COMMENT TO LIST CHI-SQUARE AND POWER RATIO TABLE
                C 102 FORMAT(T27,'RESULTS OF CHI-SQUARED TEST',T81,'RESULTS OF POWER RAT
                C     1IO TEST='/T24,'WEIBULL DISTRIBUTION',T79,'WEIBULL DISTRIBUTION'/
                C     2 ' MONTH  TIME    LST. SQS.    LST. SQS.     MATCHING-',7X,'BETA',
                C     3 9X,'LST. SQS.   LST. SQS.   MATCHING-',7X,'BETA'/
                C     4T9,'(HRS)    (UNWTD.)',6X,'(WTD.)',6X,'MOMENTS',5X,'OISTRIBUTION',
                C     5 6X,'(UNWTD.)',5X,'(WTO.)',4X,'MOMENTS',5X,'OISTRIBUTION')
 0028                PRINT 1001
 0029             99 N=INN
                C*** READ IN FRECUENCY OF OBSERVATIONS
 0030                REAO(5,101,END=98)MONTH,NTIME,(IFREQ(I),I=2,N),NUMMY, NOBS
 0031            101 FORMAT(I2,I3,10I4,I8,I5)
                C*** IF MORE THAN ONE LOCATION IS TO BE ANALYZEO A BLANK CARD FOR OATA CARD 7
                C    WILL CAUSE THE PROGRAM TO INCREMENT TO THE TITLE CARO OF THE NEXT OATA SET
 0032                IF(NOBS .NE. 0) GO TO 13
                C*** REMOVE COMMENT TO LIST CHI-SQUARE AND POWER RATIO TABLE
                C    PRINT 997,IPR,IVR,IVC
                C 997 FORMAT('-*  POWER RATIO COMPUTEO FOR RATEO POWER = ',I3,
                C    1 ' KW, RATED SPEEO = ',I2,' MPH, CUT-IN SPEEO = ',I2,' MPH'/
                C    2 ' ',118('-'))
 0033                GO TO 3
 0034             13 DO 172 I=1,NN
 0035            172 VINT(I)=IVINT(I)
 0036                OO 401 I=2,N
 0037            401 FREQ(I)=IFREQ(I)/1000.
 0038                SUM=0.0
                C*** COMPUTE FREQUENCY OF OBSERVATIONS IN INITIAL SPEEO SUBINTERVAL
 0039                DO 54 I=2,N
 0040.            54 SUM=SUM+FREQ(I)
 0041                FREQ(1)=1.0-SUM
 0042                SUM=SUM+FREQ(1)
```

```
      C*** COMPUTE DISCRETE CUMULATIVE DISTRIBUTION FUNCTION
            SUM1=0.0
            DO 41 I=1,N
            FREQ(I)=FREQ(I)/SUM
            SUM1=SUM1+FREQ(I)
         41 F(I)=SUM1
            DO 130 I=1,N
            IF(F(I) .GE. .999999)GO TO 131
        130 CONTINUE
        131 N=I
            VMAX=VINT(N+1)
            SUM2=0.0
      C*** COMPUTE DISCRETE PROBABILITY DENSITY FUNCTION
            DO 46 IK=1,N
            SUM2=SUM2+FREQ(IK)
            F(IK)=SUM2
            Y(IK)=VINT(IK+1)-VINT(IK)
            FREQN(IK)=FREQ(IK)/Y(IK)
         46 V(IK)=0.5*(VINT(IK+1)+VINT(IK))
      C*** COMPUTE MEAN, VARIANCE, AND STANDARD DEVIATION OF WIND DATA
            MEAN=0.0
            VAR=0.0
            DO 30 I=1,N
         30 MEAN=MEAN+V(I)*FREQ(I)
            DO 31 I=1,N
         31 VAR=VAR + (V(I)-MEAN)**2*FREQ(I)
            STDEV=SQRT(VAR)
      C*** COMPUTE WIND TURBINE GENERATOR POWER USING DISCRETE PROBABILITY DENSITY
      C     FUNCTION
            HPOWER=0.0
            DO 300 I=1,N
            IF(VINT(I+1) .LT. VCUTIN) GO TO 300
            IF(VINT(I) .GT. VRATED) GO TO 304
            IF(VINT(I) .LT. VCUTIN .AND. VINT(I+1) .GT. VRATED) GO TO 302
            IF(VINT(I) .LT. VCUTIN .AND. VINT(I+1) .LT. VRATED) GO TO 303
            IF(VINT(I) .GT. VCUTIN .AND. VINT(I+1) .LT. VRATED) GO TO 305
            IF(VINT(I) .GT. VCUTIN .AND. VINT(I+1) .GT. VRATED) GO TO 333
            PRINT 332,I
        332 FORMAT('0INTERVAL DOES NOT FIT ANY CATEGORY',5X,'INTERVAL=',I5)
            GO TO 300
        333 HPOWER=HPOWER+(0.25*(VRATED**4-VINT(I)**4)/VRATED**3 +
           1 VINT(I+1) - VRATED)*FREQN(I)
            GO TO 300
        302 HPOWER=HPOWER + (0.25*(VRATED**4 - VCUTIN**4)/VRATED**3 +
           1 VINT(I+1) - VRATED)*FREQN(I)
            GO TO 300
        303 HPOWER=HPOWER + 0.25*FREQN(I)*(VINT(I+1)**4 - VCUTIN**4)/VRATED**3
            GO TO 300
        304 HPOWER=HPOWER + FREQN(I)*(VINT(I+1)-VINT(I))
            GO TO 300
        305 HPOWER=HPOWER + 0.25*FREQN(I)*(VINT(I+1)**4 - VINT(I)**4)/
           1 VRATED**3
        300 CONTINUE
            HPOWER=PRATED*HPOWER
      C
      C*** WEIBULL FITTING
      C*** LEAST SQUARES SOLUTION FOR THE WEIBULL FIT,
            SUMX=0.0
```

```
      SUMY=0.0
      SUMXY=0.0
      SUMX2=0.0
      N1=N
      DO 20 I=1,N1
      Y(I)=ALOG(-ALOG(1.0001-F(I)))
      X(I)=ALOG(V(I))
      SUMX2=SUMX2 + X(I)**2
      SUMX=SUMX + X(I)
      SUMY=SUMY + Y(I)
   20 SUMXY=SUMXY + X(I)*Y(I)
      K=(SUMXY - SUMX*SUMY/N1)/(SUMX2 - SUMX**2/N1)
      B=(SUMY - K*SUMX)/N1
      C=EXP(-B/K)
      UWTK=K
      UWTC=C
      MMEAN=C*GAMMA(1.0+1.0/K)
      VVAR=C**2*GAMMA(1.0+2.0/K) - MMEAN**2
      FCTR=1.0
C*** COMPUTE CHI-SQUARE AND POWER RATIO STATISTICS
      CALL CHISQ(VINT,FREQ,WB,NOBS,CHI21,N,2,OF1)
      CALL POWER(PRATED,HPOWER,VRATED,VCUTIN,VMAX,V3WB,WB,WP1,CAPFC1)
C
C***  WEIBULL FIT WITH A LEAST SQUARES WEIGHTED BY OBSERVED FREQUENCY
      SUMX=0.0
      SUMY=0.0
      SUMXY=0.0
      SUMX2=0.0
      DO 60 I=1,N
      Y(I)=ALOG(-ALOG(1.0001-F(I)))
      X(I)=ALOG(V(I))
      SUMX2=SUMX2 + FREQ(I)*X(I)**2
      SUMX=SUMX + FREQ(I)*X(I)
      SUMY=SUMY + FREQ(I)*Y(I)
   60 SUMXY=SUMXY + FREQ(I)*X(I)*Y(I)
      K=(SUMXY - SUMX*SUMY)/(SUMX2 - SUMX**2)
      B=SUMY - K*SUMX
      C=EXP(-B/K)
      WTK=K
      WTC=C
      MMEAN=C*GAMMA(1.0+1.0/K)
      VVAR=C**2*GAMMA(1.0+2.0/K) - MMEAN**2
      FCTR=1.0
C*** COMPUTE CHI-SQUARE AND POWER RATIO STATISTICS
      CALL CHISQ(VINT,FREQ,WB,NOBS,CHI22,N,2,OF2)
      CALL POWER(PRATED,HPOWER,VRATED,VCUTIN,VMAX,V3WB,WB,WP2,CAPFC2)
C
C***  CALCULATION BY MATCHING MEAN AND VARIANCE TO WEIBULL DISTRIBUTION
      CALL WBLFIT(MEAN,VAR,C,K,IER)
      IF(IER.NE.0) GO TO 35
      XMMK=K
      XMMC=C
      MMEAN=C*GAMMA(1.0+1.0/K)
      VVAR=C**2*GAMMA(1.0+2.0/K) - MMEAN**2
      FCTR=1.0
C*** COMPUTE CHI-SQUARE AND POWER RATIO STATISTICS
      CALL CHISQ(VINT,FREQ,WB,NOBS,CHI23,N,2,OF3)
      CALL POWER(PRATED,HPOWER,VRATED,VCUTIN,VMAX,V3WB,WB,WP3,CAPFC3)
```

```
               GO TO 50
            35 PRINT 36,IER
            36 FORMAT(' FIT BY MATCHING MEAN AND VARIANCE COULD NOT BE ACHIEVED.
              1  RETURN ERROR CODE=',I3)
      C
      C*** FIT DATA TO A BETA DISTRIBUTION BY MATCHING MEAN AND VARIANCE
            50 VMAX=VINT(N+1)
               AA=(MEAN/VMAX)*(MEAN*(VMAX-MEAN)/VAR - 1.0)
               BB=(VMAX-MEAN)*AA/MEAN
               MMEAN=AA*VMAX/(AA+BB)
               VVAR=AA*BB*VMAX**2/((AA+BB)**2*(AA+BB+1.0))
               FCTR=GAMMA(AA + BB)/(VMAX*GAMMA(AA)*GAMMA(BB))
      C*** COMPUTE CHI-SQUARE AND POWER RATIO STATISTICS
               CALL CHISQ(VINT,FREQ,F1,NOBS,CHI24,N,2,DF4)
               CALL POWER(PRATED,HPOWER,VRATED,VCUTIN,VMAX,V3FI,FI,WP4,CAPFC4)
               NTIME=NTIME+1
               NTIME1=NTIME-3
      C*** REMOVE COMMENT TO LIST CHI-SQUARE AND POWER RATIO TABLE
      C        PRINT 160,MONTH,NTIME1,NTIME,DF1,CHI21,DF2,CHI22,DF3,CHI23,DF4
      C       1 ,CHI24,CAPFC1,CAPFC2,CAPFC3,CAPFC4
      C 160 FORMAT(I4,3X,I2,'-',I2,3X,4('(',I2,')',G9.3),5X,G10.3,2X,G10.3,1X,
      C       1 G10.3,4X,G10.3)
               PRINT 160,MONTH,NTIME1,NTIME,MEAN,STDEV,UWTK,UWTC,WTK,WTC,XMMK,
              1  XMMC,AA,BB
           160 FORMAT(I4,3X,I2,'-',I2,4X,F7.3,6X,F7.3,1X,3(3X,F7.3,1X,F7.3),3X,
              1 2F9.3)
               GO TO 99
      C*** REMOVE COMMENT TO LIST CHI-SQUARE AND POWER RATIO TABLE
      C     98 PRINT 97,IPR,IVR,IVC
      C     97 FORMAT('-*  POWER RATIO COMPUTED FOR RATED POWER = ',I3,
      C       1 ' KW, RATED SPEED = ',I2,' MPH, CUT-IN SPEED = ',I2,' MPH'/
      C       2 ' ',118('-')/'1')
            98 PRINT 97
            97 FORMAT(' ',113('-')/'1')
               STOP
               END
```

```
FORTRAN IV G LEVEL  21                                OATE = 78135          23/28/52

0001           FUNCTION GLQUAD (A,B,FN)
         C*** GAUSS-LEGENDRE QUADRATURE OF FUNCTION FN OVER INTERVAL (A,B)
         C*** INTEGRAL IS SET TO ZERO IF LOWER LIMIT LARGER THAN UPPER LIMIT
0002           REAL*4 ROOT(20),WEIGHT(20)
0003           COMMON/LINK3/NHALF,ROOT,WEIGHT
0004           GLQUAD=0.0
0005           IF (A.GE.B) RETURN
0006           BA=0.5*(B-A)
0007           AB=0.5*(A+B)
0008           DO 10 I=1,NHALF
0009        10 GLQUAD=GLQUAD+WEIGHT(I)*(FN(AB+BA*ROOT(I))+FN(AB-BA*ROOT(I)))
0010           GLQUAD=BA*GLQUAD
0011           RETURN
0012           END


0001           FUNCTION FI(V)
         C*** SUBROUTINE CALCULATES VALUES OF EITHER (V/VRATED)***3*BETA OR JUST BETA
         C    DISTRIBUTION
0002           COMMON/LINK1/K,C,AIJ,BIJ,VMAXIJ,VRATED,FCTR
0003           FI=(V/VMAXIJ)**(AIJ-1.)*(1.-V/VMAXIJ)**(BIJ-1.)
0004           FI=FCTR*FI
0005           RETURN
0006           ENTRY V3FI(V)
0007           V3FI=(V/VRATED)**3*(V/VMAXIJ)**(AIJ-1.)*(1.-V/VMAXIJ)**(BIJ-1.)
0008           V3FI=FCTR*V3FI
0009           RETURN
0010           END


         C
0001           SUBROUTINE POWER(PRATED,HPOWER,VRATED,VCUTIN,VMAX,V3FC,FC,GEN,
              1 CAPFC)
         C*** SUBROUTINE COMPUTES GENERATED WIND TURBINE POWER FROM ANALYTICAL
         C    DISTRIBUTION.  ALSO COMPUTES POWER RATIO
0002           EXTERNAL V3FC,FC
0003           GEN=PRATED*(GLQUAD(VCUTIN,AMIN1(VRATED,VMAX),V3FC)+
              1 GLQUAD(VRATED,VMAX,FC))
0004           CAPFC=GEN/HPOWER
0005           RETURN
0006           END
```

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```
C
      SUBROUTINE CHISC(Z,FREQ,FT,J,CHI,N,ESTPAR,DF)
C***  SUBROUTINE COMPUTES CHI-SQUARE VALUE
C     ESTPAR = NO. OF PARAMETERS OF A OISTRIBUTICN ESTIMATEO FROM THE DATA.
C     DF = DEGREES CF FREEOCM OF CHI-SQUARE CISTRIBUTICN
      REAL*4 Z(41),FREO(41),FCT(41),FREY(41)
      INTEGER*4 DF,ESTPAR
      EXTERNAL FT
      CHI=0.D
      OC 400 I=1,N
      FREY(I)=FREC(I)
  400 FCT(I)=GLOUAO(Z(I),Z(I+1),FT)
      INOEXI=0
      INOEX2=0
      DC 401 I1=1,N
      JJ=N+1-I1
      XNUM=J*FCT(JJ)
      IF(XNUM .GE. 1.0) GO TO 401
      IF(JJ .EO. 1) GO TO 402
      INDEX2=INOEX2+1
      FCT(JJ-1)=FCT(JJ-1)+FCT(JJ)
      FREY(JJ-1)=FREY(JJ-1)+FREY(JJ)
      NNN=N-INDEX1-INOEX2
      GC TC 401
  402 FREY(2)=FREY(1)+FREY(2)
      FCT(2)=FCT(1)+FCT(2)
      INDEX1=INOEX1+1
      NNN=N-INDEX1-INDEX2
      DO 403 III=1,NNN
      FCT(III)=FCT(III+1)
  403 FREY(III)=FREY(III+1)
  401 CONTINUE
      NN=N-INOEX1-INDEX2
      DC 405 I=1,NN
      CHI2=(FCT(I)-FREY(I))**2/FCT(I)
      CHI2=CHI2*J
  405 CHI=CHI+CHI2
      IF(INDEX1 .GE. 1)INOEX1=INOEX1+1
      IF(INDEX2 .GE. 1)INOEX2=INOEX2+1
      OF=NN-1-ESTPAR
      RETURN
      ENO
```

FORTRAN IV G LEVEL 21 WBLFIT DATE = 78135 23/28/52

```
      SUBROUTINE WBLFIT(MEAN,SIG2,C,K,IER)
C*** SUBROUTINE COMPUTES STARTING POINTS FOR MUELLER'S ITERATION
      REAL*4 K,MEAN
      COMMON/WBL/ALPHA
      EXTERNAL F
      IER=0
      ALPHA=     1.0+SIG2/MEAN**2
      IF (ALPHA.GT.0.0) GO TO 12
      PRINT 13, ALPHA
   13 FORMAT(' ALPHA IS NEGATIVE=',G15.7,'   NO SOLUTION POSSIBLE')
      IER=4
      RETURN
C
C*** CALCULATE INITIAL STARTING POINTS WHICH BRACKET SOLUTION FOR K
C*** SEARCH FOR THE LEFT HAND STARTING POINT
   12 DO 10 I=1,100,2
      X=1.0001/I
      AA=F(X)
      IF(AA.GT.0.0) GO TO 15
   10 CONTINUE
      PRINT 11,X,AA,ALPHA
   11 FORMAT(' COULD NOT FIND LEFT HAND STARTING POINT OR RAN OUT OF ITE
     1RATIONS',/' X=',G12.4,' F(X)=',G12.4,'  ALPHA=',G12.4)
      IER=3
      RETURN
   15 XL=X
C*** SEARCH FOR RIGHT HAND STARTING POINT
      DO 20 I=1,50
      X=XL+I
      AA=F(X)
      IF (AA.LT.0.0) GO TO 25
   20 CONTINUE
      PRINT 21,X,AA,ALPHA
   21 FORMAT(' COULD NOT FIND RIGHT HAND STARTING POINT OR RAN OUT OF IT
     1ERATIONS',/' X=',G12.4,' F(X)=',G12.4,'  ALPHA=',G12.4)
      IER=3
      RETURN
   25 XR=X
C
C*** SOLVE FOR C AND K
      CALL RTMI(K,FN,F,XL,XR,0.000001,100,IER)
      C=MEAN/GAMMA(1.0+1.0/K)
      RETURN
      END
```

FORTRAN IV G LEVEL 21 WB DATE = 78135 23/28/52

```
      FUNCTION WB(V)
C*** SUBROUTINE CALCULATES VALUES OF EITHER (V/VRATED)***3*WEIBULL OR JUST
C    WEIBULL DISTRIBUTION
      COMMON/LINK1/AIJ,BIJ,AA,BB,VMAXIJ,VRATED,FCTR
      XN=(V/BIJ)**AIJ
      IF(XN .GT. 170.) GO TO 370
      WB=(AIJ/BIJ)*(V/BIJ)**(AIJ-1)*EXP(-XN)
      RETURN
      ENTRY V3WB(V)
      XN=(V/BIJ)**AIJ
      IF(XN .GT. 170.) GO TO 370
      V3WB=(V/VRATED)**3*(AIJ/BIJ)*(V/BIJ)**(AIJ-1)*EXP(-XN)
      GO TO 371
  370 WB=0.0
      V3WB=0.0
  371 RETURN
      END
```

```
      REAL FUNCTION F*4(X)
      COMMON/WBL/ALPHA
      REAL*4 FF,XX
C*** EXTERNAL FUNCTION NEEDED BY SUBROUTINE RTMI
      XX=X
      FF=GAMMA(1.0+2.0/XX)/(GAMMA(1.0+1.0/XX))**2
      F=FF-ALPHA
      RETURN
      END
```

```
FORTRAN IV G LEVEL  21                MAIN              DATE = 78135          23/28/52

              C
              C     ...............................................................
              C
              C        SUBROUTINE RTMI
              C
              C        PURPOSE
              C           TO SOLVE GENERAL NONLINEAR EQUATIONS OF THE FORM FCT(X)=0
              C           BY MEANS OF MUELLER'S ITERATION METHOD.
              C
              C        USAGE
              C           CALL RTMI (X,F,FCT,XLI,XRI,EPS,IEND,IER)
              C           PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT.
              C
              C        DESCRIPTION OF PARAMETERS
              C           X      - RESULTANT ROOT OF EQUATION FCT(X)=0.
              C           F      - RESULTANT FUNCTION VALUE AT ROOT X.
              C           FCT    - NAME OF THE EXTERNAL FUNCTION SUBPROGRAM USED.
              C           XLI    - INPUT VALUE WHICH SPECIFIES THE INITIAL LEFT BOUND
              C                    OF THE ROOT X.
              C           XRI    - INPUT VALUE WHICH SPECIFIES THE INITIAL RIGHT BOUND
              C                    OF THE ROOT X.
              C           EPS    - INPUT VALUE WHICH SPECIFIES THE UPPER BOUND OF THE
              C                    ERROR OF RESULT X.
              C           IEND   - MAXIMUM NUMBER OF ITERATION STEPS SPECIFIED.
              C           IER    - RESULTANT ERROR PARAMETER CODED AS FOLLOWS
              C                     IER=0 - NO ERROR,
              C                     IER=1 - NO CONVERGENCE AFTER IEND ITERATION STEPS
              C                             FOLLOWED BY IEND SUCCESSIVE STEPS OF
              C                             BISECTION,
              C                     IER=2 - BASIC ASSUMPTION FCT(XLI)*FCT(XRI) LESS
              C                             THAN OR EQUAL TO ZERO IS NOT SATISFIED.
              C
              C        REMARKS
              C           THE PROCEDURE ASSUMES THAT FUNCTION VALUES AT INITIAL
              C           BOUNDS XLI AND XRI HAVE NOT THE SAME SIGN. IF THIS BASIC
              C           ASSUMPTION IS NOT SATISFIED BY INPUT VALUES XLI AND XRI, THE
              C           PROCEDURE IS BYPASSED AND GIVES THE ERROR MESSAGE IER=2.
              C
              C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
              C           THE EXTERNAL FUNCTION SUBPROGRAM FCT(X) MUST BE FURNISHED
              C           BY THE USER.
              C
              C        METHOD
              C           SOLUTION OF EQUATION FCT(X)=0 IS DONE BY MEANS OF MUELLER'S
              C           ITERATION METHOD OF SUCCESSIVE BISECTIONS AND INVERSE
              C           PARABOLIC INTERPOLATION, WHICH STARTS AT THE INITIAL BOUNDS
              C           XLI AND XRI. CONVERGENCE IS QUADRATIC IF THE DERIVATIVE OF
              C           FCT(X) AT ROOT X IS NOT EQUAL TO ZERO. ONE ITERATION STEP
              C           REQUIRES TWO EVALUATIONS OF FCT(X). FOR TEST ON SATISFACTORY
              C           ACCURACY SEE FORMULAE (3,4) OF MATHEMATICAL DESCRIPTION.
              C           FOR REFERENCE, SEE G. K. KRISTIANSEN, ZERO OF ARBITRARY
              C           FUNCTION, BIT, VOL. 3 (1963), PP.205-206.
              C
              C     ...............................................................
              C
0001                SUBROUTINE RTMI(X,F,FCT,XLI,XRI,EPS,IEND,IER)
              C
              C
```

```
C     PREPARE ITERATION
      IER=0
      XL=XLI
      XR=XRI
      X=XL
      TOL=X
      F=FCT(TOL)
      IF(F)1,16,1
    1 FL=F
      X=XR
      TOL=X
      F=FCT(TOL)
      IF(F)2,16,2
    2 FR=F
      IF(SIGN(1.,FL)+SIGN(1.,FR))25,3,25
C
C     BASIC ASSUMPTION FL*FR LESS THAN 0 IS SATISFIED.
C     GENERATE TOLERANCE FOR FUNCTION VALUES.
    3 I=0
      TOLF=100.*EPS
C
C
C     START ITERATION LOOP
    4 I=I+1
C
C     START BISECTION LOOP
      DO 13 K=1,IEND
      X=.5*(XL+XR)
      TOL=X
      F=FCT(TOL)
      IF(F)5,16,5
    5 IF(SIGN(1.,F)+SIGN(1.,FR))7,6,7
C
C     INTERCHANGE XL AND XR IN ORDER TO GET THE SAME SIGN IN F AND FR
    6 TOL=XL
      XL=XR
      XR=TOL
      TOL=FL
      FL=FR
      FR=TOL
    7 TOL=F-FL
      A=F*TOL
      A=A+A
      IF(A-FR*(FR-FL))8,9,9
    8 IF(I-IEND)17,17,9
    9 XR=X
      FR=F
C
C     TEST ON SATISFACTORY ACCURACY IN BISECTION LOOP
      TOL=EPS
      A=ABS(XR)
      IF(A-1.)11,11,10
   10 TOL=TOL*A
   11 IF(ABS(XR-XL)-TOL)12,12,13
   12 IF(ABS(FR-FL)-TOLF)14,14,13
   13 CONTINUE
C     END OF BISECTION LOOP
C
```

```
FORTRAN IV G LEVEL  21                 RTMI              DATE = 78135          23/28/52

              C      NO CONVERGENCE AFTER IEND ITERATION STEPS FOLLOWED BY IEND        RTMI1170
              C      SUCCESSIVE STEPS OF BISECTION OR STEADILY INCREASING FUNCTION     RTMI1180
              C      VALUES AT RIGHT BOUNDS. ERROR RETURN.                             RTMI1190
 0045                IER=1                                                             RTMI1200
 0046             14 IF(ABS(FR)-ABS(FL))16,16,15                                       RTMI1210
 0047             15 X=XL                                                              RTMI1220
 0048                F=FL                                                              RTMI1230
 0049             16 RETURN                                                            RTMI1240
              C                                                                        RTMI1250
              C      COMPUTATION OF ITERATED X-VALUE BY INVERSE PARABOLIC INTERPOLATIONRTMI1260
 0050             17 A=FR-F                                                            RTMI1270
 0051                DX=(X-XL)*FL*(1.+F*(A-TOL)/(A*(FR-FL)))/TOL                       RTMI1280
 0052                XM=X                                                              RTMI1290
 0053                FM=F                                                              RTMI1300
 0054                X=XL-DX                                                           RTMI1310
 0055                TOL=X                                                             RTMI1320
 0056                F=FCT(TOL)                                                        RTMI1330
 0057                IF(F)18,16,18                                                     RTMI1340
              C                                                                        RTMI1350
              C      TEST ON SATISFACTORY ACCURACY IN ITERATION LOOP                   RTMI1360
 0058             18 TOL=EPS                                                           RTMI1370
 0059                A=ABS(X)                                                          RTMI1380
 0060                IF(A-1.)20,20,19                                                  RTMI1390
 0061             19 TOL=TOL*A                                                         RTMI1400
 0062             20 IF(ABS(DX)-TOL)21,21,22                                           RTMI1410
 0063             21 IF(ABS(F)-TOLF)16,16,22                                           RTMI1420
              C                                                                        RTMI1430
              C      PREPARATION OF NEXT BISECTION LOOP                                RTMI1440
 0064             22 IF(SIGN(1.,F)+SIGN(1.,FL))24,23,24                                RTMI1450
 0065             23 XR=X                                                              RTMI1460
 0066                FR=F                                                              RTMI1470
 0067                GO TO 4                                                           RTMI1480
 0068             24 XL=X                                                              RTMI1490
 0069                FL=F                                                              RTMI1500
 0070                XR=XM                                                             RTMI1510
 0071                FR=FM                                                             RTMI1520
 0072                GO TO 4                                                           RTMI1530
              C      END OF ITERATION LOOP                                             RTMI1540
              C                                                                        RTMI1550
              C                                                                        RTMI1560
              C      ERROR RETURN IN CASE OF WRONG INPUT DATA                          RTMI1570
 0073             25 IER=2                                                             RTMI1580
 0074                RETURN                                                            RTMI1590
 0075                END                                                               RTMI1600
```

APPENDIX B

χ^2 and Power Ratio Tables

This appendix lists the remainder of the tables showing the χ^2 and power ratio statistics for the remaining 16 locations. The table format is identical to the format used in Table 2.6-2.

Table B.1. Sample Output Tables from the CURVEFIT Routine Showing the Results of the Goodness of Fit Tests.

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT BRUNSWICK, MAINE

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(5) 806.	(5) 574.	(5) 449.	(5) 258.	0.403	0.662	0.901	0.982
1	3- 6	(5) 670.	(5) 454.	(5) 328.	(5) 161.	0.432	0.662	0.906	1.00
1	6- 9	(5) 698.	(5) 465.	(5) 341.	(5) 157.	0.413	0.652	0.891	0.985
1	9-12	(6) 946.	(6) 613.	(6) 542.	(5) 357.	0.419	0.638	0.887	0.927
1	12-15	(6) 758.	(6) 547.	(6) 423.	(5) 381.	0.499	0.660	0.942	0.962
1	15-18	(5) 700.	(5) 401.	(5) 220.	(5) 137.	0.454	0.640	0.936	0.988
1	18-21	(5) 883.	(5) 627.	(5) 511.	(5) 296.	0.402	0.652	0.898	0.972
1	21-24	(4) 853.	(4) 565.	(4) 467.	(4) 203.	0.395	0.602	0.874	1.00
4	0- 3	(5) 691.	(5) 494.	(5) 281.	(4) 176.	0.406	0.682	0.922	0.985
4	3- 6	(5) 711.	(5) 537.	(5) 329.	(4) 214.	0.410	0.693	0.926	0.993
4	6- 9	(5) 674.	(5) 414.	(5) 236.	(5) 138.	0.429	0.653	0.925	0.987
4	9-12	(5) 501.	(5) 329.	(5) 102.	(5) 131.	0.566	0.648	0.995	1.02
4	12-15	(5) 465.	(5) 369.	(5) 66.7	(5) 127.	0.614	0.643	0.999	1.00
4	15-18	(5) 471.	(5) 419.	(5) 104.	(5) 197.	0.625	0.633	1.01	1.01
4	18-21	(5) 458.	(4) 361.	(5) 84.8	(4) 109.	0.547	0.581	0.999	1.02
4	21-24	(4) 670.	(4) 460.	(4) 241.	(4) 154.	0.412	0.643	0.932	1.02
7	0- 3	(3) 960.	(3) 637.	(3) 346.	(3) 194.	0.292	0.680	0.931	1.00
7	3- 6	(3) 947.	(3) 665.	(3) 369.	(3) 216.	0.300	0.721	0.948	1.01
7	6- 9	(3) 981.	(3) 633.	(3) 314.	(3) 192.	0.321	0.609	0.926	0.999
7	9-12	(4) 755.	(4) 506.	(4) 159.	(4) 193.	0.443	0.536	0.962	0.980
7	12-15	(4) 709.	(3) 627.	(4) 204.	(4) 437.	0.557	0.514	0.986	0.986
7	15-18	(4) 661.	(3) 630.	(4) 175.	(4) 336.	0.559	0.523	0.993	0.993
7	18-21	(4) 630.	(4) 495.	(4) 159.	(3) 191.	0.473	0.512	0.978	0.994
7	21-24	(3) 823.	(3) 514.	(3) 209.	(3) 128.	0.312	0.590	0.921	0.991
10	0- 3	(4) 049.	(4) 625.	(4) 439.	(4) 238.	0.362	0.674	0.895	1.00
10	3- 6	(5) 767.	(5) 604.	(5) 406.	(4) 246.	0.401	0.718	0.916	0.994
10	6- 9	(4) 738.	(4) 471.	(4) 291.	(4) 126.	0.361	0.628	0.877	0.985
10	9-12	(5) 729.	(5) 481.	(5) 266.	(5) 220.	0.459	0.633	0.945	0.981
10	12-15	(5) 526.	(5) 412.	(5) 151.	(4) 195.	0.584	0.607	1.00	1.01
10	15-18	(4) 444.	(4) 401.	(4) 106.	(4) 138.	0.589	0.578	1.02	1.03
10	18-21	(5) 492.	(5) 359.	(5) 115.	(4) 90.4	0.459	0.638	0.963	1.02
10	21-24	(4) 889.	(4) 566.	(4) 403.	(4) 200.	0.326	0.629	0.860	0.965

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT PATUXENT RIVER, MARYLAND

MONTH	TIME (HRS)	RESULTS OF CHI-SQUARED TEST: WEIBULL DISTRIBUTION LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	RESULTS OF POWER RATIO TEST*: WEIBULL DISTRIBUTION LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(5) 781.	(5) 568.	(5) 407.	(5) 282.	0.513	0.668	0.959	1.02
1	3- 6	(6) 808.	(6) 633.	(6) 470.	(5) 355.	0.513	0.695	0.961	1.00
1	6- 9	(6) 851.	(6) 647.	(6) 453.	(5) 370.	0.499	0.688	0.951	0.990
1	9-12	(5) 776.	(5) 479.	(5) 310.	(5) 201.	0.521	0.665	0.960	1.01
1	12-15	(6) 594.	(6) 369.	(6) 132.	(5) 135.	0.582	0.681	0.992	1.01
1	15-18	(6) 711.	(6) 506.	(6) 254.	(5) 263.	0.545	0.690	0.985	1.01
1	18-21	(6) 815.	(6) 614.	(6) 433.	(5) 305.	0.492	0.691	0.947	1.00
1	21-24	(5) 937.	(5) 616.	(5) 499.	(5) 285.	0.469	0.655	0.934	1.00
4	0- 3	(5) 907.	(5) 726.	(5) 528.	(5) 497.	0.503	0.664	0.969	1.00
4	3- 6	(5)0.107E 04	(5) 860.	(5) 723.	(5) 586.	0.457	0.655	0.937	0.979
4	6- 9	(6) 799.	(7) 691.	(6) 490.	(5) 570.	0.573	0.679	0.977	0.982
4	9-12	(6) 615.	(6) 465.	(5) 243.	(5) 347.	0.613	0.664	1.00	1.00
4	12-15	(5) 636.	(5) 480.	(5) 254.	(5) 345.	0.624	0.668	1.01	1.02
4	15-18	(5) 660.	(5) 481.	(5) 249.	(5) 323.	0.606	0.660	1.01	1.02
4	18-21	(6) 755.	(6) 607.	(6) 391.	(5) 417.	0.561	0.678	0.994	1.01
4	21-24	(5) 991.	(5) 796.	(5) 650.	(5) 567.	0.494	0.666	0.971	1.01
7	0- 3	(3)0.142E 04	(3) 952.	(3) 589.	(3) 423.	0.331	0.567	0.921	0.985
7	3- 6	(4)0.113E 04	(4) 790.	(4) 442.	(4) 351.	0.367	0.627	0.936	0.982
7	6- 9	(4)0.118E 04	(4) 858.	(4) 488.	(4) 459.	0.398	0.617	0.958	0.988
7	9-12	(4) 875.	(4) 636.	(4) 236.	(4) 283.	0.457	0.542	0.974	0.992
7	12-15	(4) 850.	(4) 650.	(4) 263.	(4) 348.	0.499	0.540	0.987	0.998
7	15-18	(5) 772.	(4) 638.	(4) 256.	(4) 382.	0.571	0.564	1.00	1.00
7	18-21	(5) 917.	(5) 816.	(4) 374.	(4) 427.	0.579	0.625	1.04	1.06
7	21-24	(4)0.114E 04	(5)0.100E 04	(4) 553.	(4) 652.	0.516	0.632	1.00	1.01
10	0- 3	(5) 953.	(5) 786.	(5) 589.	(5) 429.	0.472	0.682	0.954	1.02
10	3- 6	(5)0.101E 04	(5) 769.	(5) 592.	(5) 410.	0.440	0.663	0.927	0.986
10	6- 9	(5) 848.	(5) 630.	(5) 404.	(5) 317.	0.480	0.669	0.956	1.01
10	9-12	(6) 621.	(5) 511.	(5) 229.	(5) 279.	0.615	0.651	1.02	1.03
10	12-15	(6) 571.	(5) 505.	(5) 198.	(5) 264.	0.720	0.636	1.04	1.03
10	15-18	(5) 717.	(5) 613.	(5) 309.	(5) 390.	0.608	0.650	1.02	1.03
10	18-21	(5)0.110E 04	(5) 932.	(5) 660.	(4) 534.	0.459	0.692	0.961	1.01
10	21-24	(5)0.104E 04	(5) 890.	(5) 656.	(5) 539.	0.469	0.688	0.963	1.01

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT DALLAS, TEXAS

		RESULTS OF CHI-SQUARED TEST WEIBULL DISTRIBUTION				RESULTS OF POWER RATIO TEST* WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(4) 798.	(4) 450.	(4) 286.	(4) 115.	0.447	0.600	0.908	1.00
1	3- 6	(4) 835.	(4) 485.	(4) 323.	(4) 151.	0.440	0.606	0.913	1.00
1	6- 9	(5) 885.	(5) 533.	(5) 314.	(5) 220.	0.450	0.637	0.935	0.975
1	9-12	(5)0.102E 04	(5) 557.	(5) 342.	(5) 279.	0.486	0.640	0.952	0.976
1	12-15	(5) 874.	(5) 501.	(5) 266.	(5) 226.	0.542	0.657	0.963	0.982
1	15-18	(5) 843.	(5) 468.	(5) 205.	(5) 188.	0.530	0.648	0.964	0.982
1	18-21	(5) 792.	(5) 441.	(5) 209.	(5) 149.	0.460	0.637	0.943	0.982
1	21-24	(5) 938.	(5) 516.	(5) 350.	(5) 196.	0.435	0.634	0.918	0.968
4	0- 3	(5) 886.	(5) 464.	(5) 259.	(5) 160.	0.476	0.633	0.933	0.966
4	3- 6	(4)0.102E 04	(4) 605.	(4) 469.	(4) 222.	0.442	0.582	0.903	0.972
4	6- 9	(5) 839.	(5) 438.	(5) 181.	(5) 151.	0.488	0.633	0.953	0.976
4	9-12	(5) 829.	(5) 482.	(5) 188.	(5) 216.	0.583	0.675	0.984	0.992
4	12-15	(5) 626.	(5) 450.	(5) 36.1	(5) 58.6	0.628	0.684	0.994	0.997
4	15-18	(5) 758.	(5) 470.	(5) 118.	(5) 146.	0.599	0.680	0.992	0.997
4	18-21	(5) 690.	(5) 409.	(5) 152.	(5) 180.	0.535	0.634	0.972	0.985
4	21-24	(5) 800.	(5) 489.	(5) 332.	(5) 232.	0.501	0.648	0.945	0.981
7	0- 3	(4)0.100E 04	(4) 548.	(4) 209.	(4) 215.	0.426	0.557	0.952	0.976
7	3- 6	(4)0.105E 04	(4) 591.	(4) 269.	(4) 253.	0.408	0.567	0.946	0.976
7	6- 9	(4) 904.	(4) 544.	(4) 165.	(4) 181.	0.450	0.560	0.967	0.992
7	9-12	(4) 707.	(4) 556.	(4) 81.9	(4) 122.	0.519	0.547	0.989	1.01
7	12-15	(4) 656.	(4) 626.	(4) 96.5	(4) 141.	0.538	0.532	1.00	1.01
7	15-18	(5) 671.	(4) 581.	(4) 158.	(4) 314.	0.586	0.553	0.993	0.988
7	18-21	(4) 780.	(4) 613.	(4) 200.	(4) 305.	0.518	0.538	0.991	0.997
7	21-24	(4) 962.	(4) 584.	(4) 237.	(4) 313.	0.446	0.540	0.957	0.970
10	0- 3	(4) 859.	(4) 516.	(4) 268.	(4) 136.	0.411	0.611	0.914	0.999
10	3- 6	(4) 981.	(4) 587.	(4) 327.	(4) 184.	0.384	0.616	0.914	0.990
10	6- 9	(4) 955.	(4) 622.	(4) 326.	(4) 210.	0.407	0.621	0.930	0.998
10	9-12	(4) 874.	(4) 504.	(4) 169.	(4) 134.	0.475	0.595	0.952	0.999
10	12-15	(4) 679.	(4) 472.	(4) 33.1	(4) 59.3	0.529	0.587	0.985	1.02
10	15-18	(5) 644.	(5) 480.	(5) 108.	(4) 156.	0.563	0.594	0.988	0.995
10	18-21	(5) 720.	(5) 526.	(5) 188.	(4) 206.	0.503	0.608	0.985	1.01
10	21-24	(4) 958.	(4) 538.	(4) 316.	(4) 170.	0.403	0.606	0.913	0.986

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT HOUSTON, TEXAS

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(4) 0.114E 04	(4) 758.	(4) 475.	(4) 286.	0.369	0.633	0.907	0.979
1	3- 6	(4) 0.112E 04	(4) 785.	(4) 513.	(4) 304.	0.382	0.648	0.914	0.996
1	6- 9	(4) 0.109E 04	(4) 717.	(4) 426.	(4) 259.	0.386	0.628	0.915	0.986
1	9-12	(4) 0.105E 04	(4) 660.	(4) 352.	(4) 358.	0.469	0.595	0.957	0.988
1	12-15	(4) 861.	(4) 583.	(4) 232.	(4) 289.	0.537	0.592	0.979	0.996
1	15-18	(4) 869.	(4) 606.	(4) 198.	(4) 200.	0.521	0.564	0.979	0.991
1	18-21	(4) 782.	(4) 509.	(4) 161.	(4) 158.	0.453	0.595	0.970	1.02
1	21-24	(4) 0.106E 04	(4) 716.	(4) 429.	(4) 275.	0.395	0.638	0.926	0.996
4	0- 3	(4) 0.149E 04	(4) 960.	(4) 739.	(4) 462.	0.320	0.607	0.877	0.946
4	3- 6	(4) 0.131E 04	(4) 940.	(4) 683.	(4) 425.	0.352	0.648	0.901	0.979
4	6- 9	(5) 0.126E 04	(5) 0.100E 04	(5) 753.	(4) 652.	0.417	0.649	0.928	0.961
4	9-12	(5) 0.106E 04	(5) 738.	(5) 528.	(4) 0.121E 04	0.532	0.586	0.969	0.964
4	12-15	(5) 863.	(5) 673.	(5) 429.	(4) 0.115E 04	0.602	0.609	0.992	0.904
4	15-18	(5) 789.	(4) 831.	(4) 288.	(3) 43.2	0.626	0.560	0.989	0.974
4	18-21	(5) 891.	(4) 689.	(4) 317.	(4) 542.	0.531	0.551	0.975	0.975
4	21-24	(5) 0.118E 04	(5) 859.	(5) 559.	(4) 481.	0.400	0.618	0.914	0.945
7	0- 3	(3) 962.	(4) 841.	(4) 442.	(3) 326.	0.476	0.988	1.15	1.15
7	3- 6	(4) 938.	(6) 843.	(4) 482.	(3) 374.	0.691	1.18	1.31	1.33
7	6- 9	(5) 0.125E 04	(6) 0.110E 04	(5) 644.	(4) 519.	0.547	0.849	1.06	1.10
7	9-12	(4) 0.108E 04	(4) 794.	(4) 316.	(3) 421.	0.438	0.514	0.963	0.974
7	12-15	(4) 796.	(4) 633.	(4) 212.	(4) 333.	0.540	0.542	0.983	0.985
7	15-18	(4) 0.112E 04	(4) 835.	(4) 464.	(4) 821.	0.496	0.527	0.974	0.978
7	18-21	(4) 0.102E 04	(4) 816.	(4) 368.	(3) 445.	0.474	0.547	0.987	1.00
7	21-24	(5) 0.112E 04	(5) 0.107E 04	(4) 475.	(3) 398.	0.795	0.854	1.28	1.30
10	0- 3	(4) 977.	(4) 845.	(4) 507.	(4) 393.	0.428	0.768	0.996	1.10
10	3- 6	(4) 0.122E 04	(4) 924.	(4) 608.	(4) 390.	0.336	0.758	0.935	1.01
10	6- 9	(3) 0.137E 04	(3) 933.	(3) 616.	(3) 344.	0.310	0.612	0.891	1.00
10	9-12	(4) 950.	(4) 592.	(4) 200.	(4) 247.	0.453	0.545	0.960	0.977
10	12-15	(4) 779.	(4) 573.	(4) 84.9	(4) 131.	0.498	0.533	0.973	0.984
10	15-18	(4) 732.	(4) 636.	(4) 172.	(4) 386.	0.575	0.522	0.981	0.974
10	18-21	(4) 985.	(4) 741.	(4) 296.	(3) 261.	0.405	0.626	0.979	1.01
10	21-24	(4) 0.102E 04	(4) 745.	(4) 413.	(4) 260.	0.355	0.730	0.946	1.01

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT DES MOINES, IOWA

		RESULTS OF CHI-SQUARED TEST WEIBULL DISTRIBUTION				RESULTS OF POWER RATIO TEST* WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(5) 451.	(5) 384.	(5) 149.	(5) 225.	0.653	0.647	1.02	1.02
1	3- 6	(5) 449.	(5) 394.	(5) 148.	(5) 210.	0.660	0.649	1.03	1.02
1	6- 9	(5) 394.	(5) 377.	(5) 128.	(5) 173.	0.674	0.654	1.03	1.03
1	9-12	(5) 467.	(5) 321.	(5) 76.9	(5) 100.	0.611	0.673	1.01	1.03
1	12-15	(6) 468.	(6) 325.	(6) 103.	(5) 191.	0.635	0.683	0.997	0.992
1	15-18	(5) 519.	(5) 345.	(5) 129.	(5) 176.	0.602	0.668	0.998	1.01
1	18-21	(6) 423.	(5) 428.	(5) 155.	(5) 210.	0.687	0.657	1.04	1.03
1	21-24	(6) 472.	(5) 393.	(5) 163.	(5) 242.	0.652	0.656	1.02	1.02
4	0- 3	(6) 374.	(6) 334.	(6) 135.	(5) 186.	0.696	0.674	1.01	1.01
4	3- 6	(6) 465.	(6) 395.	(5) 195.	(5) 314.	0.683	0.668	1.01	1.00
4	6- 9	(6) 473.	(6) 349.	(6) 174.	(5) 255.	0.649	0.690	1.00	1.00
4	9-12	(6) 444.	(6) 300.	(6) 106.	(6) 210.	0.682	0.736	0.998	0.994
4	12-15	(7) 378.	(7) 275.	(7) 91.3	(7) 232.	0.728	0.765	0.993	0.983
4	15-18	(7) 289.	(6) 295.	(6) 58.4	(6) 127.	0.772	0.773	1.00	0.993
4	18-21	(6) 313.	(5) 321.	(6) 84.6	(5) 91.8	0.710	0.699	1.02	1.02
4	21-24	(6) 364.	(5) 329.	(6) 125.	(5) 167.	0.711	0.680	1.02	1.01
7	0- 3	(4) 827.	(4) 637.	(4) 259.	(3) 316.	0.474	0.547	0.991	1.01
7	3- 6	(4) 757.	(4) 651.	(4) 268.	(4) 348.	0.576	0.572	1.02	1.03
7	6- 9	(4) 890.	(4) 631.	(4) 250.	(4) 349.	0.437	0.549	0.961	0.978
7	9-12	(5) 611.	(4) 503.	(4) 148.	(4) 346.	0.597	0.547	0.971	0.959
7	12-15	(5) 624.	(4) 487.	(4) 135.	(4) 291.	0.576	0.565	0.980	0.975
7	15-18	(5) 585.	(4) 513.	(4) 113.	(4) 255.	0.577	0.553	0.983	0.977
7	18-21	(4) 440.	(3) 504.	(4) 66.7	(4) 110.	0.616	0.461	0.986	0.982
7	21-24	(5) 735.	(4) 671.	(4) 243.	(4) 377.	0.725	0.522	1.04	1.03
10	0- 3	(5) 791.	(4) 743.	(4) 409.	(4) 579.	0.647	0.583	1.05	1.05
10	3- 6	(5) 735.	(4) 667.	(4) 338.	(4) 494.	0.622	0.578	1.04	1.04
10	6- 9	(5) 598.	(4) 553.	(4) 256.	(4) 420.	0.687	0.597	1.04	1.02
10	9-12	(6) 562.	(5) 403.	(5) 172.	(5) 368.	0.612	0.648	0.993	0.985
10	12-15	(6) 548.	(6) 382.	(6) 205.	(6) 588.	0.647	0.685	0.989	0.976
10	15-18	(5) 633.	(5) 404.	(5) 204.	(5) 277.	0.565	0.645	0.984	0.991
10	18-21	(5) 663.	(4) 645.	(4) 281.	(4) 374.	0.660	0.579	1.05	1.05
10	21-24	(5) 660.	(4) 654.	(4) 319.	(4) 547.	0.738	0.576	1.05	1.03

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT COLUMBUS, INDIANA

		RESULTS OF CHI-SQUARED TEST WEIBULL DISTRIBUTION				RESULTS OF POWER RATIO TEST* WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(4) 522.	(4) 390.	(4) 326.	(4) 215.	0.443	0.601	0.926	0.995
1	3- 6	(5) 636.	(5) 463.	(5) 410.	(4) 372.	0.435	0.630	0.926	0.954
1	6- 9	(4) 758.	(4) 573.	(4) 481.	(4) 399.	0.416	0.609	0.935	0.975
1	9-12	(4)0.101E 04	(4) 748.	(4) 670.	(4) 559.	0.438	0.591	0.935	0.972
1	12-15	(5) 917.	(5) 703.	(5) 627.	(4) 769.	0.492	0.615	0.956	0.966
1	15-18	(5) 898.	(5) 714.	(4) 579.	(4) 792.	0.498	0.612	0.969	0.975
1	18-21	(5) 857.	(5) 733.	(5) 552.	(4) 539.	0.474	0.670	0.973	0.999
1	21-24	(5) 899.	(5) 721.	(5) 564.	(4) 508.	0.433	0.658	0.941	0.972
4	0- 3	(4) 698.	(4) 622.	(4) 562.	(4) 466.	0.417	0.631	0.945	0.989
4	3- 6	(4) 661.	(4) 508.	(4) 525.	(4) 394.	0.411	0.642	0.943	1.00
4	6- 9	(5) 696.	(5) 573.	(5) 512.	(4) 546.	0.472	0.647	0.966	0.983
4	9-12	(5) 814.	(5) 628.	(5) 672.	(4) 936.	0.526	0.634	0.980	0.984
4	12-15	(6) 907.	(6) 741.	(5) 973.	(5)0.211E 04	0.545	0.631	0.972	0.962
4	15-18	(5) 791.	(5) 569.	(5) 510.	(4) 929.	0.545	0.620	0.985	0.983
4	18-21	(5) 782.	(5) 656.	(4) 432.	(4) 551.	0.526	0.626	0.995	1.00
4	21-24	(4)0.104E 04	(4) 812.	(4) 632.	(4) 510.	0.391	0.629	0.932	0.977
7	0- 3	(3)0.105E 04	(3) 949.	(3) 579.	(3) 452.	0.343	0.772	1.08	1.11
7	3- 6	(3) 886.	(3) 766.	(3) 467.	(3) 346.	0.332	0.787	1.04	1.09
7	6- 9	(3)0.118E 04	(3) 929.	(3) 599.	(3) 435.	0.316	0.685	0.981	1.03
7	9-12	(3)0.135E 04	(3)0.108E 04	(3) 809.	(3) 596.	0.349	0.603	0.938	1.00
7	12-15	(4)0.115E 04	(4) 887.	(4) 672.	(4) 701.	0.408	0.607	0.956	0.978
7	15-18	(4)0.104E 04	(4) 831.	(4) 568.	(4) 644.	0.440	0.603	0.978	0.996
7	18-21	(3)0.134E 04	(3)0.112E 04	(3) 635.	(3) 547.	0.367	0.664	1.02	1.05
7	21-24	(3)0.122E 04	(4)0.107E 04	(4) 625.	(3) 500.	0.399	0.871	1.08	1.10
10	0- 3	(4) 789.	(4) 739.	(4) 534.	(3) 431.	0.375	0.741	0.978	1.03
10	3- 6	(4) 707.	(4) 725.	(4) 499.	(4) 400.	0.433	0.756	1.01	1.09
10	6- 9	(4) 888.	(4) 797.	(4) 608.	(4) 444.	0.383	0.696	0.940	1.01
10	9-12	(4)0.110E 04	(4) 769.	(4) 626.	(4) 487.	0.392	0.613	0.927	0.973
10	12-15	(4)0.106E 04	(4) 770.	(4) 677.	(4) 630.	0.441	0.596	0.946	0.976
10	15-18	(4)0.106E 04	(4) 772.	(4) 577.	(4) 605.	0.414	0.589	0.952	0.974
10	18-21	(3)0.121E 04	(3)0.103E 04	(3) 664.	(3) 522.	0.369	0.645	0.971	1.04
10	21-24	(4)0.129E 04	(4)0.117E 04	(4) 824.	(4) 641.	0.358	0.753	0.966	1.02

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT RANTOUL, ILLINOIS

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(5) 963.	(5) 753.	(5) 528.	(5) 494.	0.530	0.668	0.989	1.02
1	3- 6	(5) 998.	(5) 818.	(5) 563.	(5) 576.	0.536	0.661	0.992	1.02
1	6- 9	(5)0.104E 04	(5) 732.	(5) 400.	(5) 469.	0.531	0.637	0.982	1.00
1	9-12	(6)0.101E 04	(6) 728.	(5) 440.	(5) 791.	0.593	0.648	0.990	0.984
1	12-15	(6) 859.	(6) 621.	(5) 293.	(5) 589.	0.625	0.663	1.00	0.992
1	15-18	(6) 860.	(6) 641.	(5) 295.	(5) 457.	0.609	0.651	1.00	1.00
1	18-21	(6)0.105E 04	(6) 793.	(6) 482.	(5) 559.	0.545	0.663	0.980	0.950
1	21-24	(5)0.115E 04	(5) 809.	(5) 528.	(5) 461.	0.509	0.657	0.970	1.00
4	0- 3	(6)0.103E 04	(6) 895.	(5) 659.	(5) 773.	0.548	0.668	0.992	1.00
4	3- 6	(5) 938.	(5) 741.	(5) 520.	(5) 558.	0.527	0.656	0.985	1.01
4	6- 9	(5)0.103E 04	(5) 722.	(5) 477.	(5) 629.	0.569	0.644	0.985	0.995
4	9-12	(6) 774.	(6) 563.	(6) 186.	(5) 429.	0.664	0.705	1.00	0.993
4	12-15	(6) 627.	(6) 508.	(6) 65.8	(6) 133.	0.694	0.738	1.01	1.00
4	15-18	(6) 782.	(6) 569.	(6) 279.	(5) 574.	0.664	0.703	1.01	0.999
4	18-21	(5)0.118E 04	(5) 893.	(5) 639.	(5) 698.	0.543	0.654	0.998	1.02
4	21-24	(6)0.117E 04	(6) 976.	(6) 707.	(5) 813.	0.544	0.669	0.986	0.995
7	0- 3	(3)0.167E 04	(3)0.115E 04	(3) 614.	(3) 402.	0.308	0.611	0.948	1.00
7	3- 6	(3)0.180E 04	(3)0.124E 04	(3) 644.	(3) 413.	0.305	0.605	0.952	1.01
7	6- 9	(4)0.196E 04	(4)0.149E 04	(4) 743.	(4) 781.	0.396	0.563	0.959	0.979
7	9-12	(5)0.153E 04	(4)0.125E 04	(4) 580.	(4) 997.	0.535	0.522	0.979	0.976
7	12-15	(5)0.126E 04	(4)0.100E 04	(4) 405.	(4) 723.	0.542	0.534	0.979	0.976
7	15-18	(5)0.133E 04	(4)0.106E 04	(4) 441.	(4) 847.	0.538	0.521	0.974	0.970
7	18-21	(4)0.177E 04	(4)0.141E 04	(4) 649.	(3) 620.	0.422	0.584	0.987	1.01
7	21-24	(3)0.217E 04	(3)0.146E 04	(3) 771.	(3) 536.	0.293	0.612	0.961	1.00
10	0- 3	(4)0.123E 04	(4) 970.	(4) 546.	(4) 419.	0.424	0.657	0.965	1.03
10	3- 6	(5)0.108E 04	(6) 964.	(5) 521.	(4) 480.	0.523	0.708	1.00	1.04
10	6- 9	(5)0.136E 04	(5)0.104E 04	(5) 567.	(4) 511.	0.449	0.644	0.955	0.987
10	9-12	(5)0.101E 04	(5) 804.	(5) 308.	(5) 633.	0.604	0.589	0.987	0.976
10	12-15	(5)0.110E 04	(5) 753.	(5) 341.	(5) 562.	0.558	0.616	0.984	0.986
10	15-18	(4)0.131E 04	(4) 862.	(4) 357.	(4) 418.	0.476	0.581	0.967	0.995
10	18-21	(5)0.116E 04	(5) 977.	(5) 441.	(4) 475.	0.534	0.642	1.03	1.05
10	21-24	(4)0.142E 04	(4)0.104E 04	(4) 602.	(4) 464.	0.400	0.640	0.946	0.996

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT SALINA, KANSAS

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(5) 625.	(5) 435.	(5) 298.	(5) 175.	0.462	0.666	0.935	1.01
1	3- 6	(6) 530.	(7) 444.	(6) 283.	(6) 230.	0.526	0.707	0.955	0.990
1	6- 9	(6) 561.	(6) 416.	(6) 258.	(5) 188.	0.499	0.696	0.958	1.00
1	9-12	(5) 625.	(5) 408.	(5) 287.	(5) 178.	0.496	0.659	0.948	1.00
1	12-15	(6) 550.	(6) 344.	(6) 189.	(5) 192.	0.552	0.679	0.975	0.987
1	15-18	(5) 566.	(5) 379.	(5) 222.	(5) 209.	0.542	0.662	0.984	1.01
1	18-21	(4) 671.	(4) 425.	(4) 266.	(4) 163.	0.452	0.625	0.937	1.02
1	21-24	(5) 701.	(5) 459.	(5) 331.	(5) 204.	0.451	0.656	0.931	0.989
4	0- 3	(7) 555.	(7) 455.	(6) 327.	(6) 314.	0.553	0.699	0.972	0.989
4	3- 6	(5) 716.	(5) 573.	(5) 504.	(5) 347.	0.494	0.659	0.951	1.01
4	6- 9	(6) 698.	(6) 568.	(6) 529.	(5) 450.	0.521	0.676	0.957	0.979
4	9-12	(6) 556.	(6) 360.	(6) 284.	(6) 364.	0.613	0.704	0.989	0.990
4	12-15	(7) 474.	(7) 314.	(6) 206.	(6) 372.	0.654	0.726	0.996	0.989
4	15-18	(6) 426.	(6) 324.	(6) 213.	(6) 323.	0.680	0.723	1.01	1.01
4	18-21	(6) 546.	(6) 414.	(6) 256.	(5) 285.	0.580	0.688	1.00	1.01
4	21-24	(6) 661.	(6) 520.	(6) 415.	(5) 368.	0.511	0.689	0.968	0.992
7	0- 3	(5) 797.	(5) 636.	(5) 452.	(4) 429.	0.450	0.646	0.949	0.976
7	3- 6	(5) 754.	(5) 615.	(5) 404.	(4) 358.	0.443	0.677	0.956	0.990
7	6- 9	(4) 881.	(4) 661.	(4) 527.	(4) 398.	0.416	0.622	0.933	0.985
7	9-12	(5) 701.	(5) 538.	(5) 438.	(4) 549.	0.522	0.635	0.984	0.993
7	12-15	(4) 743.	(4) 543.	(4) 466.	(4) 459.	0.505	0.605	0.972	0.999
7	15-18	(4) 724.	(4) 512.	(4) 443.	(4) 429.	0.509	0.605	0.969	0.996
7	18-21	(5) 531.	(5) 513.	(4) 319.	(4) 707.	0.670	0.595	0.998	0.979
7	21-24	(5) 716.	(5) 618.	(4) 404.	(4) 464.	0.515	0.648	0.998	1.01
10	0- 3	(6) 766.	(6) 681.	(5) 502.	(5) 457.	0.491	0.698	0.964	0.953
10	3- 6	(5) 847.	(5) 680.	(5) 524.	(4) 389.	0.429	0.683	0.941	0.995
10	6- 9	(5) 782.	(5) 590.	(5) 456.	(5) 333.	0.440	0.674	0.943	0.996
10	9-12	(5) 615.	(5) 412.	(5) 307.	(5) 226.	0.538	0.663	0.966	1.00
10	12-15	(5) 607.	(5) 376.	(5) 269.	(5) 210.	0.567	0.677	0.985	1.01
10	15-18	(5) 630.	(5) 428.	(5) 336.	(5) 304.	0.550	0.666	0.983	1.01
10	18-21	(6) 661.	(6) 619.	(5) 394.	(5) 390.	0.568	0.710	1.01	1.04
10	21-24	(6) 679.	(6) 591.	(5) 404.	(5) 374.	0.511	0.698	0.973	1.00

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT LUBBOCK, TEXAS

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(6) 876.	(7) 921.	(5) 711.	(5) 758.	0.578	0.701	1.00	1.01
1	3- 6	(5)0.101E 04	(5) 973.	(5) 778.	(4) 673.	0.497	0.681	0.981	1.02
1	6- 9	(5)0.118E 04	(5)0.104E 04	(5) 784.	(5) 653.	0.471	0.692	0.969	1.02
1	9-12	(6) 984.	(6) 781.	(6) 642.	(5) 647.	0.543	0.681	0.978	0.993
1	12-15	(7) 954.	(7) 763.	(7) 762.	(6) 948.	0.594	0.701	0.983	0.982
1	15-18	(6) 992.	(6) 780.	(6) 766.	(6) 823.	0.569	0.680	0.975	0.983
1	18-21	(6)0.106E 04	(6)0.101E 04	(5) 757.	(5) 897.	0.550	0.678	0.996	1.00
1	21-24	(5)0.106E 04	(5) 996.	(5) 887.	(4) 930.	0.473	0.662	0.973	0.992
4	0- 3	(6) 629.	(7) 645.	(6) 626.	(5) 825.	0.610	0.681	1.00	0.997
4	3- 6	(6) 805.	(6) 765.	(5) 644.	(5) 744.	0.555	0.675	0.995	1.00
4	6- 9	(6)0.103E 04	(6) 925.	(6) 805.	(5) 927.	0.544	0.672	0.982	0.991
4	9-12	(7) 801.	(7) 640.	(6) 633.	(6)0.128E 04	0.661	0.707	1.00	0.992
4	12-15	(7) 709.	(7) 566.	(7) 525.	(6) 774.	0.673	0.727	1.00	0.997
4	15-18	(7) 793.	(7) 635.	(7) 682.	(6)0.100E 04	0.660	0.726	1.00	0.997
4	18-21	(6) 762.	(6) 650.	(6) 613.	(6) 779.	0.624	0.695	1.00	1.01
4	21-24	(7) 672.	(7) 646.	(6) 547.	(5) 710.	0.642	0.690	1.02	1.02
7	0- 3	(5) 904.	(6) 949.	(5) 828.	(4) 944.	0.516	0.687	0.981	0.990
7	3- 6	(4)0.130E 04	(4)0.115E 04	(4) 821.	(4) 706.	0.404	0.687	0.974	1.01
7	6- 9	(4)0.137E 04	(4)0.106E 04	(4) 827.	(4) 635.	0.385	0.648	0.934	0.987
7	9-12	(4)0.160E 04	(4)0.115E 04	(4)0.115E 04	(4)0.146E 04	0.437	0.550	0.943	0.956
7	12-15	(4)0.146E 04	(4)0.111E 04	(4) 987.	(4)0.143E 04	0.460	0.541	0.952	0.961
7	15-18	(4)0.123E 04	(4) 954.	(4) 768.	(4)0.141E 04	0.511	0.537	0.972	0.975
7	18-21	(4)0.110E 04	(4) 894.	(4) 743.	(4)0.109E 04	0.507	0.570	0.982	0.991
7	21-24	(5) 881.	(6) 886.	(5) 625.	(4) 743.	0.582	0.691	1.03	1.04
10	0- 3	(5) 931.	(5) 951.	(5) 789.	(4) 736.	0.478	0.690	0.981	1.01
10	3- 6	(4)0.118E 04	(4) 965.	(4) 726.	(4) 535.	0.379	0.670	0.937	0.999
10	6- 9	(4)0.130E 04	(4)0.102E 04	(4) 742.	(4) 518.	0.384	0.665	0.930	0.999
10	9-12	(5)0.115E 04	(5) 873.	(5) 746.	(5) 921.	0.515	0.638	0.981	0.991
10	12-15	(6)0.123E 04	(6)0.104E 04	(5)0.101E 04	(5)0.178E 04	0.546	0.629	0.976	0.970
10	15-18	(5)0.119E 04	(5) 927.	(5) 839.	(4)0.112E 04	0.519	0.619	0.971	0.977
10	18-21	(4)0.127E 04	(4)0.109E 04	(4) 828.	(4) 867.	0.442	0.629	0.976	0.999
10	21-24	(5)0.109E 04	(5)0.108E 04	(4) 916.	(4)0.101E 04	0.486	0.674	0.986	1.00

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT FARGO, NORTH DAKOTA

MONTH	TIME (HRS)	RESULTS OF CHI-SQUARED TEST: WEIBULL DISTRIBUTION LST. SQS. (UNWTD.)	WEIBULL DISTRIBUTION LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	RESULTS OF POWER RATIO TEST*: WEIBULL DISTRIBUTION LST. SQS. (UNWTD.)	WEIBULL DISTRIBUTION LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(7) 184.	(6) 223.	(6) 28.0	(6) 32.7	0.766	0.724	1.02	1.01
1	3- 6	(7) 217.	(6) 255.	(6) 53.8	(6) 70.2	0.763	0.716	1.02	1.01
1	6- 9	(6) 291.	(6) 240.	(6) 37.7	(6) 61.2	0.678	0.712	1.01	1.02
1	9-12	(6) 242.	(6) 213.	(6) 36.9	(6) 52.8	0.717	0.733	1.02	1.01
1	12-15	(6) 317.	(6) 219.	(6) 17.8	(6) 31.9	0.685	0.739	1.00	1.00
1	15-18	(6) 381.	(6) 251.	(6) 46.2	(6) 56.8	0.662	0.729	1.01	1.01
1	18-21	(6) 309.	(6) 237.	(6) 35.1	(6) 42.7	0.674	0.719	1.02	1.02
1	21-24	(6) 251.	(6) 224.	(6) 45.0	(6) 57.7	0.714	0.727	1.02	1.02
4	0- 3	(6) 355.	(6) 260.	(6) 76.4	(5) 123.	0.659	0.704	1.01	1.01
4	3- 6	(6) 371.	(6) 241.	(6) 69.9	(5) 99.4	0.644	0.711	1.01	1.01
4	6- 9	(6) 368.	(6) 229.	(6) 47.7	(6) 50.1	0.667	0.742	1.01	1.02
4	9-12	(6) 214.	(6) 202.	(7) 7.90	(6) 0.20	0.795	0.806	1.01	1.01
4	12-15	(7) 245.	(7) 217.	(7) 13.7	(6) 11.8	0.798	0.826	1.01	1.01
4	15-18	(6) 314.	(6) 215.	(6) 45.1	(6) 59.6	0.734	0.798	1.00	1.00
4	18-21	(6) 170.	(6) 389.	(6) 47.2	(6) 56.2	0.800	0.758	1.02	1.01
4	21-24	(6) 291.	(6) 260.	(6) 68.5	(5) 90.2	0.698	0.718	1.02	1.01
7	0- 3	(6) 398.	(5) 344.	(5) 124.	(5) 192.	0.769	0.613	1.03	1.01
7	3- 6	(6) 413.	(5) 359.	(5) 128.	(5) 208.	0.673	0.594	1.00	0.990
7	6- 9	(5) 522.	(5) 328.	(5) 120.	(4) 137.	0.503	0.611	0.967	0.983
7	9-12	(5) 403.	(5) 282.	(5) 48.4	(5) 82.3	0.590	0.639	0.992	1.00
7	12-15	(5) 344.	(5) 294.	(5) 24.1	(5) 61.6	0.641	0.657	0.992	0.985
7	15-18	(6) 265.	(5) 399.	(5) 62.1	(5) 102.	0.767	0.665	1.02	1.00
7	18-21	(6) 282.	(4) 409.	(5) 101.	(5) 104.	0.814	0.611	1.04	1.02
7	21-24	(5) 393.	(4) 332.	(4) 85.6	(4) 118.	0.585	0.590	1.01	1.01
10	0- 3	(6) 252.	(5) 297.	(6) 58.5	(5) 74.3	0.711	0.695	1.03	1.03
10	3- 6	(6) 413.	(6) 284.	(6) 117.	(5) 164.	0.617	0.679	0.998	0.998
10	6- 9	(6) 280.	(5) 252.	(6) 40.8	(5) 47.5	0.679	0.692	1.02	1.02
10	9-12	(6) 361.	(6) 223.	(6) 26.9	(6) 32.4	0.672	0.743	1.00	1.00
10	12-15	(6) 346.	(6) 222.	(6) 45.3	(6) 66.8	0.703	0.769	1.00	1.00
10	15-18	(6) 190.	(6) 306.	(6) 39.1	(6) 38.2	0.788	0.765	1.02	1.01
10	18-21	(6) 267.	(5) 298.	(6) 52.2	(5) 68.5	0.699	0.695	1.02	1.02
10	21-24	(6) 296.	(5) 274.	(6) 66.2	(5) 81.5	0.692	0.701	1.02	1.02

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT GLASGOW, MONTANA

MONTH	TIME (HRS)	RESULTS OF CHI-SQUARED TEST: WEIBULL DISTRIBUTION LST. SQS. (UNWTD.)	WEIBULL DISTRIBUTION LST. SQS. (WTD.)	WEIBULL DISTRIBUTION MATCHING-MOMENTS	BETA DISTRIBUTION	RESULTS OF POWER RATIO TEST*: WEIBULL DISTRIBUTION LST. SQS. (UNWTD.)	WEIBULL DISTRIBUTION LST. SQS. (WTD.)	WEIBULL DISTRIBUTION MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(6) 245.	(6) 207.	(5) 126.	(5) 131.	0.589	0.681	0.996	1.02
1	3- 6	(5) 297.	(5) 212.	(5) 145.	(4) 127.	0.497	0.650	0.964	0.996
1	6- 9	(5) 311.	(5) 259.	(5) 189.	(4) 160.	0.525	0.659	0.979	1.02
1	9-12	(6) 305.	(6) 274.	(5) 203.	(5) 257.	0.579	0.675	0.989	0.990
1	12-15	(5) 282.	(5) 183.	(5) 116.	(5) 111.	0.531	0.662	0.995	1.02
1	15-18	(5) 292.	(5) 239.	(5) 165.	(4) 215.	0.554	0.649	0.982	0.985
1	18-21	(5) 314.	(5) 222.	(5) 154.	(4) 157.	0.490	0.640	0.967	0.987
1	21-24	(6) 246.	(6) 210.	(5) 115.	(5) 123.	0.610	0.668	1.00	1.03
4	0- 3	(4) 268.	(4) 198.	(4) 117.	(4) 119.	0.524	0.631	1.00	1.06
4	3- 6	(5) 251.	(5) 190.	(5) 111.	(4) 96.6	0.516	0.659	0.977	1.02
4	6- 9	(5) 214.	(5) 141.	(5) 59.7	(5) 64.9	0.546	0.652	0.983	1.01
4	9-12	(6) 243.	(6) 175.	(6) 124.	(5) 160.	0.610	0.683	0.986	0.988
4	12-15	(6) 214.	(6) 161.	(6) 115.	(6) 229.	0.657	0.700	0.993	0.984
4	15-18	(6) 145.	(6) 128.	(6) 48.0	(5) 57.5	0.698	0.716	1.01	1.01
4	18-21	(5) 208.	(5) 154.	(5) 70.1	(4) 74.4	0.575	0.643	1.00	1.03
4	21-24	(5) 273.	(5) 203.	(5) 131.	(4) 126.	0.511	0.644	0.974	1.00
7	0- 3	(4) 309.	(5) 291.	(4) 161.	(4) 172.	0.585	0.671	1.05	1.08
7	3- 6	(4) 325.	(4) 254.	(4) 150.	(4) 125.	0.443	0.661	0.978	1.04
7	6- 9	(4) 331.	(4) 241.	(4) 145.	(4) 132.	0.471	0.625	0.977	1.02
7	9-12	(4) 338.	(4) 241.	(4) 171.	(4) 174.	0.512	0.607	0.980	1.01
7	12-15	(5) 268.	(5) 195.	(5) 98.5	(4) 127.	0.564	0.629	1.01	1.02
7	15-18	(5) 253.	(5) 182.	(5) 90.5	(5) 151.	0.574	0.633	0.982	0.979
7	18-21	(6) 244.	(6) 213.	(5) 117.	(5) 146.	0.634	0.662	1.00	1.00
7	21-24	(5) 261.	(4) 228.	(4) 107.	(4) 129.	0.669	0.617	1.05	1.06
10	0- 3	(4) 340.	(4) 278.	(4) 187.	(4) 151.	0.464	0.662	0.979	1.05
10	3- 6	(4) 371.	(4) 303.	(4) 210.	(4) 172.	0.452	0.647	0.968	1.03
10	6- 9	(5) 317.	(6) 293.	(5) 195.	(4) 194.	0.536	0.690	0.990	1.01
10	9-12	(5) 256.	(5) 188.	(5) 116.	(5) 130.	0.571	0.654	0.997	1.02
10	12-15	(6) 208.	(6) 174.	(5) 115.	(5) 154.	0.675	0.707	1.03	1.03
10	15-18	(5) 206.	(5) 157.	(5) 67.2	(5) 80.0	0.633	0.684	1.02	1.03
10	18-21	(4) 310.	(4) 283.	(4) 162.	(4) 177.	0.554	0.641	1.03	1.05
10	21-24	(5) 335.	(5) 321.	(4) 200.	(4) 227.	0.594	0.684	1.04	1.05

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT SIOUX FALLS, SOUTH DAKOTA

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(5) 402.	(5) 224.	(5) 43.9	(5) 28.8	0.499	0.624	0.947	0.990
1	3- 6	(5) 449.	(5) 255.	(5) 95.3	(5) 62.6	0.490	0.638	0.945	0.993
1	6- 9	(5) 377.	(5) 215.	(5) 48.2	(5) 40.7	0.515	0.644	0.959	1.01
1	9-12	(6) 392.	(6) 249.	(6) 75.5	(5) 93.4	0.564	0.652	0.971	0.979
1	12-15	(6) 333.	(5) 255.	(6) 38.7	(5) 59.1	0.641	0.682	1.00	1.00
1	15-18	(5) 325.	(5) 266.	(5) 47.4	(5) 73.9	0.638	0.672	1.01	1.02
1	18-21	(5) 426.	(5) 271.	(5) 88.3	(5) 97.4	0.537	0.627	0.986	1.01
1	21-24	(5) 416.	(5) 260.	(5) 95.0	(5) 94.4	0.532	0.648	0.980	1.01
4	0- 3	(6) 458.	(6) 318.	(6) 189.	(5) 197.	0.551	0.675	0.974	0.988
4	3- 6	(6) 458.	(6) 313.	(6) 185.	(5) 180.	0.545	0.681	0.977	0.994
4	6- 9	(6) 566.	(6) 373.	(6) 272.	(5) 283.	0.530	0.658	0.950	0.959
4	9-12	(6) 308.	(6) 226.	(6) 58.6	(6) 106.	0.698	0.747	1.01	1.00
4	12-15	(6) 362.	(6) 230.	(6) 85.9	(6) 149.	0.694	0.760	1.00	0.997
4	15-18	(6) 359.	(6) 235.	(6) 44.7	(6) 79.1	0.681	0.750	1.01	1.00
4	18-21	(5) 233.	(5) 243.	(6) 28.5	(5) 38.8	0.682	0.681	1.01	1.01
4	21-24	(5) 452.	(5) 268.	(5) 120.	(5) 103.	0.535	0.656	0.978	1.01
7	0- 3	(4) 758.	(4) 445.	(4) 198.	(4) 150.	0.355	0.563	0.913	0.952
7	3- 6	(4) 708.	(4) 449.	(4) 207.	(4) 172.	0.366	0.582	0.930	0.965
7	6- 9	(5) 548.	(5) 430.	(4) 171.	(4) 250.	0.533	0.572	0.954	0.955
7	9-12	(5) 501.	(4) 421.	(4) 116.	(4) 378.	0.583	0.538	0.959	0.944
7	12-15	(5) 669.	(4) 451.	(4) 205.	(4) 498.	0.521	0.571	0.958	0.953
7	15-18	(4) 541.	(4) 369.	(4) 62.1	(4) 85.0	0.520	0.571	0.967	0.982
7	18-21	(4) 443.	(4) 349.	(4) 25.3	(4) 34.2	0.473	0.500	0.960	0.975
7	21-24	(5) 372.	(5) 299.	(4) 61.4	(4) 85.3	0.561	0.571	0.971	0.979
10	0- 3	(5) 493.	(5) 313.	(5) 111.	(4) 98.7	0.466	0.613	0.949	0.979
10	3- 6	(5) 536.	(5) 360.	(5) 178.	(4) 133.	0.455	0.632	0.936	0.976
10	6- 9	(5) 641.	(5) 406.	(5) 252.	(4) 193.	0.429	0.625	0.921	0.957
10	9-12	(5) 493.	(5) 289.	(5) 119.	(5) 112.	0.549	0.650	0.971	0.990
10	12-15	(7) 444.	(6) 325.	(6) 215.	(6) 672.	0.665	0.689	0.991	0.974
10	15-18	(6) 336.	(5) 283.	(5) 78.4	(5) 153.	0.662	0.671	1.01	1.00
10	18-21	(4) 432.	(4) 300.	(4) 67.2	(4) 94.4	0.512	0.573	0.989	1.02
10	21-24	(4) 551.	(4) 326.	(4) 119.	(4) 91.4	0.454	0.593	0.947	0.997

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT MADISON, WISCONSIN

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(6) 656.	(6) 471.	(6) 319.	(5) 307.	0.484	0.654	0.942	0.960
1	3- 6	(6) 647.	(6) 472.	(6) 313.	(5) 289.	0.482	0.653	0.936	0.957
1	6- 9	(5) 631.	(5) 424.	(5) 243.	(5) 178.	0.470	0.634	0.935	0.979
1	9-12	(6) 543.	(6) 345.	(5) 126.	(5) 191.	0.538	0.625	0.956	0.957
1	12-15	(5) 417.	(5) 352.	(5) 37.7	(5) 91.3	0.602	0.628	0.983	0.989
1	15-18	(5) 273.	(5) 347.	(5) 43.4	(5) 67.3	0.677	0.627	1.01	1.00
1	18-21	(5) 443.	(5) 291.	(5) 95.3	(5) 127.	0.543	0.643	0.980	1.02
1	21-24	(6) 479.	(6) 337.	(6) 143.	(5) 169.	0.540	0.644	0.950	0.956
4	0- 3	(5) 592.	(5) 387.	(5) 246.	(5) 149.	0.471	0.647	0.932	0.987
4	3- 6	(5) 783.	(5) 504.	(5) 402.	(5) 221.	0.414	0.610	0.885	0.943
4	6- 9	(5) 867.	(5) 613.	(5) 605.	(5) 432.	0.457	0.607	0.911	0.945
4	9-12	(5) 416.	(5) 323.	(5) 13.0	(5) 23.5	0.646	0.695	0.998	1.00
4	12-15	(5) 422.	(5) 359.	(5) 29.6	(5) 56.3	0.665	0.700	0.991	0.992
4	15-18	(6) 397.	(5) 327.	(5) 57.8	(5) 143.	0.663	0.693	0.995	0.987
4	18-21	(5) 416.	(5) 306.	(5) 102.	(5) 155.	0.601	0.646	1.01	1.02
4	21-24	(5) 536.	(5) 382.	(5) 220.	(5) 193.	0.521	0.663	0.972	1.02
7	0- 3	(6) 496.	(5) 412.	(4) 127.	(4) 126.	0.742	0.666	1.07	1.09
7	3- 6	(4) 579.	(5) 449.	(4) 179.	(4) 139.	0.432	0.659	0.960	0.993
7	6- 9	(4) 796.	(4) 525.	(4) 251.	(4) 223.	0.396	0.582	0.939	0.972
7	9-12	(4) 529.	(4) 408.	(4) 99.4	(4) 207.	0.548	0.549	0.975	0.972
7	12-15	(4) 474.	(4) 457.	(4) 74.2	(4) 118.	0.569	0.569	0.998	1.01
7	15-18	(4) 452.	(4) 517.	(4) 43.8	(4) 77.1	0.555	0.536	0.989	0.993
7	18-21	(4) 448.	(4) 418.	(4) 87.4	(4) 137.	0.578	0.504	0.990	0.992
7	21-24	(4) 590.	(4) 459.	(4) 140.	(3) 124.	0.455	0.576	0.998	1.03
10	0- 3	(4) 739.	(4) 438.	(4) 239.	(4) 132.	0.376	0.607	0.905	0.974
10	3- 6	(4) 761.	(4) 465.	(4) 234.	(4) 141.	0.369	0.590	0.903	0.962
10	6- 9	(5) 738.	(5) 476.	(5) 273.	(4) 229.	0.416	0.611	0.917	0.948
10	9-12	(5) 372.	(4) 362.	(5) 52.6	(5) 124.	0.647	0.617	0.994	0.982
10	12-15	(5) 178.	(4) 397.	(5) 16.5	(4) 10.9	0.797	0.653	1.01	0.985
10	15-18	(5) 246.	(4) 403.	(5) 32.3	(5) 53.1	0.701	0.586	1.00	0.987
10	18-21	(5) 490.	(5) 389.	(5) 149.	(4) 186.	0.570	0.[illegible]20	0.990	0.998
10	21-24	(4) 729.	(4) 412.	(4) 209.	(4) 111.	0.385	0.[illegible]90	0.902	0.968

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT MOUNT CLEMENS, MICHIGAN

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(6) 954.	(6) 803.	(6) 541.	(5) 517.	0.547	0.674	0.973	0.996
1	3- 6	(6) 914.	(6) 744.	(6) 506.	(5) 476.	0.537	0.689	0.983	1.01
1	6- 9	(6) 970.	(6) 779.	(6) 548.	(5) 527.	0.525	0.672	0.966	0.986
1	9-12	(6) 946.	(6) 749.	(6) 541.	(5) 723.	0.573	0.662	0.991	0.993
1	12-15	(6) 806.	(6) 612.	(5) 381.	(5) 667.	0.623	0.660	1.00	0.998
1	15-18	(6) 813.	(6) 650.	(5) 340.	(5) 533.	0.623	0.646	0.998	0.995
1	18-21	(7) 767.	(7) 612.	(6) 346.	(6) 461.	0.593	0.673	0.980	0.986
1	21-24	(7) 819.	(7) 686.	(6) 427.	(6) 487.	0.578	0.684	0.976	0.988
4	0- 3	(4)0.110E 04	(4) 787.	(4) 450.	(4) 341.	0.437	0.623	0.941	1.01
4	3- 6	(5)0.112E 04	(5) 850.	(5) 561.	(4) 457.	0.447	0.659	0.949	0.990
4	6- 9	(5)0.107E 04	(5) 811.	(5) 569.	(5) 519.	0.481	0.645	0.952	0.985
4	9-12	(5) 737.	(5) 554.	(5) 268.	(5) 406.	0.606	0.648	1.01	1.02
4	12-15	(6) 512.	(5) 648.	(5) 193.	(5) 256.	0.724	0.609	1.03	1.02
4	15-18	(6) 507.	(5)0.109E 04	(5) 245.	(5) 341.	0.755	0.672	1.03	1.02
4	18-21	(5) 919.	(5) 666.	(5) 386.	(4) 592.	0.550	0.606	0.984	0.986
4	21-24	(6) 892.	(6) 731.	(5) 404.	(5) 510.	0.565	0.647	0.997	1.00
7	0- 3	(3)0.142E 04	(3) 951.	(3) 505.	(3) 310.	0.314	0.644	0.940	1.00
7	3- 6	(4)0.120E 04	(4) 866.	(4) 454.	(3) 301.	0.356	0.714	0.957	1.00
7	6- 9	(4)0.140E 04	(4)0.102E 04	(4) 566.	(4) 468.	0.373	0.646	0.952	0.989
7	9-12	(4)0.107E 04	(4) 754.	(4) 312.	(4) 394.	0.461	0.547	0.970	0.986
7	12-15	(4) 868.	(4) 740.	(4) 200.	(4) 335.	0.536	0.526	0.990	0.996
7	15-18	(4)0.111E 04	(4) 850.	(4) 338.	(4) 654.	0.500	0.512	0.979	0.980
7	18-21	(4)0.151E 04	(4)0.112E 04	(4) 620.	(3) 837.	0.423	0.527	0.957	0.966
7	21-24	(4)0.155E 04	(4)0.124E 04	(4) 680.	(3) 643.	0.404	0.645	0.984	1.01
10	0- 3	(5)0.103E 04	(5) 800.	(5) 483.	(4) 363.	0.428	0.686	0.944	0.993
10	3- 6	(5) 988.	(5) 763.	(5) 445.	(4) 317.	0.425	0.687	0.940	0.995
10	6- 9	(5)0.117E 04	(5) 883.	(5) 572.	(4) 428.	0.407	0.660	0.920	0.967
10	9-12	(4)0.104E 04	(4) 648.	(4) 351.	(4) 283.	0.459	0.594	0.946	0.988
10	12-15	(5) 780.	(5) 552.	(5) 223.	(4) 328.	0.570	0.620	0.995	0.999
10	15-18	(5) 777.	(5) 600.	(5) 219.	(4) 313.	0.571	0.593	0.997	1.00
10	18-21	(5)0.105E 04	(6) 909.	(5) 514.	(4) 610.	0.540	0.653	0.992	1.00
10	21-24	(5) 997.	(5) 801.	(5) 451.	(4) 381.	0.472	0.668	0.967	1.01

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT MOUNTAIN HOME, IDAHO

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(4) 943.	(4) 564.	(4) 436.	(4) 155.	0.365	0.594	0.849	0.979
1	3- 6	(4) 883.	(4) 547.	(4) 418.	(4) 134.	0.376	0.583	0.843	0.986
1	6- 9	(4) 896.	(4) 511.	(4) 387.	(4) 104.	0.367	0.574	0.832	0.976
1	9-12	(4) 943.	(4) 566.	(4) 440.	(4) 123.	0.398	0.559	0.846	0.979
1	12-15	(5) 933.	(5) 567.	(5) 433.	(5) 195.	0.443	0.620	0.896	0.961
1	15-18	(5) 850.	(5) 468.	(5) 272.	(5) 116.	0.428	0.622	0.897	0.959
1	18-21	(5) 859.	(5) 503.	(5) 334.	(5) 142.	0.379	0.629	0.870	0.951
1	21-24	(5) 950.	(5) 600.	(5) 477.	(5) 220.	0.370	0.636	0.861	0.950
4	0- 3	(5) 585.	(5) 413.	(5) 286.	(5) 133.	0.464	0.651	0.907	1.03
4	3- 6	(6) 632.	(6) 453.	(6) 266.	(5) 178.	0.481	0.694	0.945	1.00
4	6- 9	(6) 734.	(6) 504.	(6) 352.	(6) 218.	0.468	0.670	0.920	0.973
4	9-12	(6) 750.	(6) 460.	(6) 345.	(6) 201.	0.530	0.666	0.938	0.965
4	12-15	(6) 750.	(6) 406.	(6) 274.	(6) 159.	0.528	0.688	0.959	0.989
4	15-18	(5) 695.	(5) 397.	(5) 279.	(5) 127.	0.544	0.666	0.955	1.01
4	18-21	(6) 598.	(6) 415.	(6) 235.	(5) 214.	0.558	0.695	0.989	1.01
4	21-24	(6) 535.	(6) 403.	(6) 202.	(5) 148.	0.515	0.706	0.977	1.04
7	0- 3	(5) 760.	(5) 566.	(5) 311.	(4) 188.	0.420	0.693	0.932	1.01
7	3- 6	(4) 830.	(4) 601.	(4) 309.	(4) 203.	0.411	0.665	0.944	1.03
7	6- 9	(4) 885.	(4) 584.	(4) 380.	(4) 182.	0.399	0.631	0.898	1.01
7	9-12	(5)0.107E 04	(5) 748.	(5) 517.	(5) 357.	0.421	0.636	0.916	0.966
7	12-15	(5) 753.	(5) 550.	(5) 254.	(4) 194.	0.475	0.662	0.963	1.01
7	15-18	(6) 780.	(6) 567.	(6) 337.	(5) 329.	0.514	0.661	0.957	0.976
7	18-21	(5)0.105E 04	(5) 746.	(5) 565.	(5) 632.	0.499	0.620	0.957	0.971
7	21-24	(5) 793.	(5) 722.	(5) 383.	(4) 374.	0.557	0.698	1.05	1.10
10	0- 3	(5) 720.	(5) 589.	(5) 355.	(5) 254.	0.459	0.710	0.945	1.05
10	3- 6	(5) 677.	(5) 548.	(5) 266.	(4) 190.	0.495	0.695	0.981	1.06
10	6- 9	(5) 756.	(5) 621.	(5) 325.	(4) 230.	0.493	0.699	0.981	1.05
10	9-12	(6) 777.	(6) 517.	(6) 304.	(5) 211.	0.488	0.661	0.931	0.966
10	12-15	(6) 641.	(6) 399.	(6) 204.	(6) 99.6	0.500	0.671	0.931	0.984
10	15-18	(5) 623.	(5) 401.	(5) 186.	(5) 152.	0.516	0.662	0.953	1.04
10	18-21	(5) 671.	(5) 540.	(5) 228.	(4) 181.	0.512	0.697	1.01	1.09
10	21-24	(5) 551.	(5) 426.	(5) 221.	(5) 146.	0.462	0.706	0.944	1.08

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

Table B.1. (cont'd)

GOODNESS OF FIT STATISTICS FOR VARIOUS ANALYTICAL FITS TO OBSERVED WIND SPEED DISTRIBUTIONS AT SPOKANE, WASHINGTON

		RESULTS OF CHI-SQUARED TEST				RESULTS OF POWER RATIO TEST*			
		WEIBULL DISTRIBUTION				WEIBULL DISTRIBUTION			
MONTH	TIME (HRS)	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION	LST. SQS. (UNWTD.)	LST. SQS. (WTD.)	MATCHING-MOMENTS	BETA DISTRIBUTION
1	0- 3	(5) 323.	(5) 236.	(5) 43.8	(4) 73.5	0.537	0.619	1.01	1.08
1	3- 6	(5) 335.	(5) 219.	(5) 29.2	(4) 44.2	0.499	0.607	0.981	1.04
1	6- 9	(5) 299.	(5) 218.	(5) 25.7	(4) 46.6	0.512	0.589	0.973	1.04
1	9-12	(5) 369.	(5) 248.	(5) 58.6	(5) 72.6	0.510	0.611	0.972	1.03
1	12-15	(5) 311.	(5) 239.	(5) 31.7	(4) 48.0	0.547	0.597	0.990	1.03
1	15-18	(5) 323.	(5) 251.	(5) 43.6	(4) 68.7	0.519	0.578	0.987	1.04
1	18-21	(4) 382.	(4) 247.	(4) 38.1	(4) 96.2	0.469	0.570	0.971	1.07
1	21-24	(5) 375.	(5) 265.	(5) 75.4	(4) 92.2	0.523	0.637	1.00	1.07
4	0- 3	(4) 443.	(4) 288.	(4) 47.3	(4) 56.9	0.453	0.547	0.966	1.00
4	3- 6	(4) 373.	(4) 302.	(4) 36.9	(4) 58.7	0.486	0.518	0.985	1.01
4	6- 9	(4) 372.	(4) 373.	(4) 28.0	(4) 35.9	0.516	0.518	0.981	0.999
4	9-12	(4) 322.	(4) 325.	(5) 22.7	(4) 22.3	0.592	0.593	0.993	1.00
4	12-15	(4) 336.	(4) 279.	(5) 9.39	(5) 9.44	0.584	0.613	0.987	0.998
4	15-18	(4) 292.	(4) 326.	(5) 20.4	(4) 27.2	0.604	0.596	0.999	1.01
4	18-21	(4) 307.	(4) 283.	(4) 39.8	(4) 47.6	0.535	0.526	0.978	0.997
4	21-24	(4) 348.	(4) 294.	(4) 68.3	(4) 85.7	0.553	0.571	1.01	1.03
7	0- 3	(3) 676.	(3) 417.	(3) 50.8	(3) 43.3	0.308	0.432	0.933	0.954
7	3- 6	(3) 654.	(3) 411.	(3) 38.8	(3) 39.8	0.325	0.419	0.934	0.956
7	6- 9	(3) 267.	(2) 483.	(3) 47.2	(3) 43.3	0.563	0.393	0.966	0.965
7	9-12	(3) 347.	(3) 440.	(4) 12.8	(3) 17.5	0.490	0.441	0.959	0.964
7	12-15	(3) 344.	(3) 387.	(4) 9.46	(4) 18.7	0.502	0.476	0.966	0.977
7	15-18	(3) 395.	(3) 397.	(4) 20.9	(4) 27.1	0.489	0.478	0.963	0.973
7	18-21	(4) 240.	(3) 548.	(4) 118.	(3) 81.0	0.750	0.409	1.03	1.03
7	21-24	(4) 428.	(3) 406.	(3) 57.8	(3) 84.0	0.594	0.447	0.990	0.999
10	0- 3	(4) 458.	(4) 257.	(4) 42.8	(4) 42.3	0.412	0.528	0.947	0.983
10	3- 6	(5) 302.	(4) 292.	(4) 46.4	(4) 56.3	0.780	0.534	1.05	1.06
10	6- 9	(4) 266.	(4) 296.	(4) 29.8	(4) 39.3	0.664	0.508	1.01	1.01
10	9-12	(5) 186.	(4) 373.	(5) 88.1	(5) 79.6	0.857	0.558	1.05	1.05
10	12-15	(4) 249.	(4) 292.	(5) 29.2	(4) 33.5	0.575	0.548	0.986	1.01
10	15-18	(4) 453.	(4) 272.	(4) 20.2	(4) 27.1	0.423	0.523	0.951	0.998
10	18-21	(4) 462.	(4) 296.	(4) 52.7	(4) 49.0	0.392	0.539	0.948	0.990
10	21-24	(4) 465.	(4) 300.	(4) 66.1	(4) 62.1	0.417	0.571	0.962	1.01

* POWER RATIO COMPUTED FOR RATED POWER = 100 KW, RATED SPEED = 18 MPH, CUT-IN SPEED = 8 MPH

APPENDIX C

The Program BLOHARD

The purpose of this program is to find the economically optimum WTGS to serve a particular demand load given sufficient wind speed data. There are two versions of BLOHARD which differ only in the input wind speed data. Version A uses the observed wind speed data in its calculations, while version B assumes the wind speed distributions are given by a beta distribution. The program is written in FORTRAN IV for use on the Kansas State University ITEL AS/5 System (equivalent operationally to an IBM 370/158). Because of the liberal amount of comment cards in the listing which follows and the variable names of high mnemoic content, only a brief modular description of the program is given. Following the program listing is a sample output.

BLOHARD maximizes the objective function of Eq. (3.2-22) according to the methodology described in Section 3.2. The subroutine OBJN performs the calculations necessary for the optimization methodology. The WTGS cost model is calculated in the subroutine MONEY. Although only one cost function is specified, MONEY is capable of using another cost function of the same form as the functions given by Eqs. (3.2-20) and (3.2-21). The calculations required in the simplex search pattern are performed by the subroutine SIMPLEX. Finally, the BREZE subroutine determines the maximum and minimum power requirements and maximum wind speed of the input data.

```
C*********************************** BLOHARD ***********************************
C*                                                                              *
C* THIS PROGRAM MATCHES DAILY WIND SPEED DISTRIBUTIONS TO THE REQUIRED DAILY   *
C* LOAD DEMAND.  THE WTGS IS OPTIMIZED TO FIND THE BEST SIZED SYSTEM(I.E.      *
C* OPTIMUM RATED POWER AND RATED SPEED) SO AS TO MAXIMIZE THE ELECTRICAL        *
C* SAVINGS.  BETA DISTRIBUTION OR OBSERVED WIND SPEED DISTRIBUTION CAN BE USED *
C*                                                                              *
C*                                                                              *
C* INPUT DATA:  (VERSION A - BETA DISTRIBUTION AS WIND SPEED MODEL)             *
C*                                                                              *
C*   CARD 1    FORMAT (I2)                                                      *
C*       NHALF = THE HALF VALUE OF THE EVEN ORDER GAUSS-LEGENDRE QUADRATURE     *
C*               USED TO EVALUATE THE NECESSARY INTEGRALS.                      *
C*                                                                              *
C*   CARD 2    FORMAT (4G20.0)                                                  *
C*       ROOT(I) = QUADRATURE ORDINATES(ONLY POSITIVE VALUES)                   *
C*                 (MAY BE MANY CARDS)                                          *
C*                                                                              *
C*   CARD 3    FORMAT (4G20.0)                                                  *
C*       WEIGHT(I) = QUADRATURE WEIGHTS                                         *
C*                   (MAY BE MANY CARDS)                                        *
C*                                                                              *
C*   CARD 4    FORMAT (5I5,2G10.0)                                              *
C*       NDINT = NUMBER OF DAILY SUBINTERVALS FOR WIND AND LOAD DATA            *
C*       NYINT = NUMBER OF SEASONS FOR WHICH WIND AND LOAD DATA ARE GIVEN       *
C*       NYEARS = NUMBER OF YEARS WTGS IS AMORTIZED                             *
C*       INT = YEARLY INTEREST RATE                                             *
C*       ICOST = NUMBER OF WTGS COST FUNCTION USED(TWO ARE POSSIBLE)            *
C*       VREF(1) = REFERENCE RATED SPEED OF FIRST WTGS COST FUNCTION(KNOTS)     *
C*       VREF(2) = REFERENCE RATED SPEED OF SECOND WTGS COST FUNCTION(KNOTS)    *
C*                                                                              *
C*   CARD 5    FORMAT (8G10.0)                                                  *
C*       Z(1) = INITIAL SIMPLEX POINT RATED POWER(KW)                           *
C*       Z(2) = INITIAL SIMPLEX POINT RATED SPEED(KNOTS)                        *
C*       EPSI = CONVERGENCE CRITERION OF SIMPLEX TECHNIQUE                      *
C*       FRACT1 = FRACTION OF LOAD MAXIMUM POWER DEMAND USED AS STEP SIZE IN    *
C*                SIMPLEX TECHNIQUE                                             *
C*       FRACT2 = FRACTION OF MAXIMUM WIND SPEED USED AS STEP SIZE IN SIMPLEX   *
C*                TECHNIQUE                                                     *
C*                                                                              *
C*   CARD 6    FORMAT (20A4)                                                    *
C*       TITLE = TITLE CARD FOR PROBLEM TO BE ANALYZED                          *
C*                                                                              *
C*   CARD 7    FORMAT (8G10.0)                                                  *
C*       MEAN = MEAN WIND SPEED IN M-TH DAILY INTERVAL OF MM-TH SEASON(KNOTS)   *
C*       VAR = VARIANCE IN WIND SPEEDS IN M-TH DAILY INTERVAL OF MM-TH SEASON   *
C*             (KNOTS**2)                                                       *
C*       VMAX(M,MM) = MAXIMUM WIND SPEED IN M-TH DAILY INTERVAL OF MM-TH SEASON*
C*                    (KNOTS)                                                   *
C*                    (MAY BE MANY CARDS)                                       *
C*                                                                              *
C*   CARD 8    FORMAT (8G10.4)                                                  *
C*       PLOAD(I,J) = AVERAGE POWER REQUIREMENT FOR THE I-TH DAILY INTERVAL IN *
C*                    J-TH SEASON(KW).  (MAY BE MANY CARDS)                     *
C*                                                                              *
C*   CARD 9    FORMAT (2G10.0)                                                  *
C*       $BUY = COST OF PURCHASED ELECTRICITY($/KWH)                            *
C*       $SEL = VALUE OF ELECTRICITY SOLD($/KWH)                                *
```

```
             C*                                                                          *
             C*                                                                          *
             C* INPUT DATA:  (VERSION B - OBSERVED WIND SPEED DISTRIBUTION USED)         *
             C*                                                                          *
             C*   CARD 1     FORMAT (I3)                                                 *
             C*        INN = TOTAL NUMBER OF POSSIBLE SPEED SUBINTERVALS(USUALLY 11)     *
             C*                                                                          *
             C*   CARD 2     FORMAT (12F5.2)                                             *
             C*        IVINT(I) = ENDPOINTS OF WIND SPEED SUBINTERVALS(KNOTS)            *
             C*                                                                          *
             C*   CARD 3     FORMAT (5I5,2G10.0)                                         *
             C*              (SAME AS CARD 4 - VERSION A)                                *
             C*                                                                          *
             C*   CARD 4     FORMAT (8G10.0)                                             *
             C*              (SAME AS CARD 5 - VERSION A)                                *
             C*                                                                          *
             C*   CARD 5     FORMAT (20A4)                                               *
             C*              (SAME AS CARD 6 - VERSION A)                                *
             C*                                                                          *
             C*   CARD 6     FORMAT (I2,I3,10I4,I8,I5)                                   *
             C*        MONTH = MONTH FROM WHICH SPEED DATA IS OBTAINED                   *
             C*        NTIME = DAILY TIME PERIOD FROM WHICH WIND SPEED DATA IS OBTAINED  *
             C*        IFREQ(I) = FREQUENCY(X 1000) OF OBSERVATIONS IN I-TH SPEED SUBINTERVAL*
             C*                   (BEGINNING WITH FREQUENCY IN 2ND SPEED SUBINTERVAL)    *
             C*        NUMMY = SPACE FOR DATA IDENTIFICATION PURPOSES(CAN ALSO BE USED TO ADD*
             C*                TWO MORE SPEED SUBINTERVALS)                              *
             C*        NOBS = TOTAL NUMBER OF WIND SPEED DATA OBSERVATIONS               *
             C*               (MAY BE MANY CARDS)                                        *
             C*                                                                          *
             C*   CARD 7     FORMAT (8G10.4)                                             *
             C*              (SAME AS CARD 8 - VERSION A)                                *
             C*                                                                          *
             C*   CARD 8     FORMAT (2G10.0)                                             *
             C*              (SAME AS CARD 9 - VERSION A)                                *
             C*                                                                          *
             C* WRITTEN BY L. A. POCH, KANSAS STATE UNIVERSITY, JANUARY 1978             *
             C*                                                                          *
             C****************************************************************************
             C
0001               IMPLICIT REAL*8(A-H,O-Z,$)
0002               REAL*8 FREQ(41),V(41),F(91),TITLE(20),IVINT(41),
                 X FCTR(8,4),A(8,4),B(8,4),Z(2),STEP(2),PLOAD(8,4),VMAX(8,4)
0003               REAL*8 Y(41),ROOT(20),WEIGHT(20),X(41),MN(8,4),VR(8,4),VREF(2)
0004               REAL*8 MMEAN,MAXSAV,MEAN,K
0005               INTEGER*4 IFREQ(21)
0006               COMMON/LINK1/A,B,PLOAD,FCTR,VMAX,VREF,SDYS,HRS,$BLY,$SEL,CRF,
                 1 VCUTIN,NYINT,NDINT,ICOST
0007               COMMON/LINK2/AIJ,BIJ,VMAXIJ,VRATED,PRATED,FRACT3,FRACT4
0008               COMMON/LINK3/ROOT,WEIGHT,NHALF
0009               REAL*8 FREQN(12,8,4),VINT(12,8,4),MIM(8,4)
0010               COMMON/LINK4/FREQN,VINT,MIM
             C*** REMOVE COMMENT IF VERSION A IS TO BE USED AND ADD COMMENT TO APPROPRIATE
             C     VERSION B CARDS
             C      READ(5,113)NHALF
             C  113 FORMAT(I2)
             C      READ(5,12)(ROOT(I),I=1,NHALF)
             C      READ(5,12)(WEIGHT(I),I=1,NHALF)
             C   12 FORMAT(4G20.0)
```

```
C      READ 10,NOINT,NYINT,NYEARS,INT,ICCST,VREF(1),VREF(2)
C   10 FCRMAT(515,2G10.0)
C      READ 1,Z(1),Z(2),EPSI,FRACT1,FRACT2
C  111 READ(5,100) TITLE
C  100 FCRMAT(20A4)
C      WRITE(6,110) TITLE
C  110 FCRMAT('1',20A4)
C      OC 170 MM=1,NYINT
C      OO 170 M=1,NOINT
C      READ1,MEAN,VAR,VMAX(N,MM)
C    1 FORMAT(8G10.0)
C*** FIT OATA TO A BETA OISTRIBUTION BY MATCHING MEAN AND VARIANCE
C      A(M,MM)=(MEAN/VMAX(M,MM))*(MEAN*(VMAX(M,MM)-MEAN)/VAR-1.000)
C      B(M,MM)=(VMAX(M,MM)-MEAN)*A(M,MM)/MEAN
C      VVAR=A(M,MM)*B(M,MM)*VMAX(M,MM)**2/((A(M,MM)+8(M,MM))**2*(A(N,MM)
C     X +B(M,MM)+1.000))
C      MMEAN=A(M,MM)*VMAX(M,MM)/(A(M,MM)+B(M,MM))
C      FCTR(M,MM)=OGAMMA(A(M,MM)+B(M,MM))/(VMAX(M,MM)*OGAMMA(A(M,MM))
C     X *OGAMMA(8(M,MM)))
C*** BEGIN VERSION B
       READ(5,2) INN
     2 FORNAT(I3)
       NN=INN+1
       READ(5,11)(IVINT(1),I=1,NN)
    11 FORMAT(12F5.2)
       READ 10,NOINT,NYINT,NYEARS,INT,ICOST,VREF(1),VREF(2)
    10 FORMAT(5I5,2G10.0)
       READ 1,Z(1),Z(2),EPS1,FRACT1,FRACT2
     1 FCRMAT(8G10.0)
   111 READ(5,100) TITLE
   100 FCRMAT(20A4)
       WRITE(6,110) TITLE
   110 FCRMAT('1',20A4)
       DC 170 MM=1,NYINT
       OO 170 M=1,NOINT
    99 N=INN
       OC 172 I=1,NN
   172 VINT(I,M,MM)=IVINT(I)
       VMAX(M,MM)=IVINT(NN)
       READ(5,101)MONTH,NTIME,(IFREQ(1),I=2,N),NUNNY, NOBS
   101 FORMAT(I2,I3,10I4,I8,I5)
       DC 401 I=2,N
   401 FREQ(1)=OFLOAT(IFREQ(I))/1000.DO
       SUM=0.000
       DC 54 I=2,N
    54 SUM=SUM+FREQ(I)
       FREQ(1)=1.000-SUN
       SUM=SUM+FREQ(1)
       SUM1=0.000
       DC 41 I=1,N
       SUM1=SUM1+FREQ(1)
    41 F(I)=SUMI
       OC 130 I=1,N
       IF(F(I) .GE. 9.9999990-01) GO TO 131
   130 CCNTINUE
   131 N=I
       MIM(M,MM)=N
       SUM2=0.000
```

```
      DO 46 IK=1,N
      SUM2=SUM2+FREQ(IK)
      F(IK)=SUM2
      Y(IK)=VINT(IK+1,M,MM)-VINT(IK,M,MM)
      V(IK)=0.500*(VINT(IK+1,M,MM)+VINT(IK,M,MM))
   46 FREQN(IK,M,MM)=FREQ(IK)/Y(IK)
C
C*** CALCULATE MEAN AND VARIANCE OF WIND DATA
      MEAN=0.000
      VAR=0.000
      DO 30 I=1,N
   30 MEAN=MEAN+V(I)*FREQ(I)
      DO 31 I=1,N
   31 VAR=VAR + (V(I)-MEAN)**2*FREQ(I)
      MN(M,MM)=MEAN
      VR(M,MM)=VAR
C*** END VERSION B
  170 CONTINUE
C
C*** READ IN LOAD DATA
      HRS=24.000/NOINT
      SDYS=365.000/NYINT
      SHRS=24.000*SDYS
      DO 21 J=1,NYINT
   21 READ 112, (PLOAD(I,J),I=1,NOINT)
  112 FORMAT(8G10.4)
      PRINT 22,HRS
   22 FORMAT(//'0AVERAGE VELOCITY FOR ENTERPRISE DURING EACH',G10.3,
     1 'HOUR INTERVAL BEGINNING AT MIDNIGHT (IN KNOTS):')
      DO 26 J=1,NYINT
   26 PRINT 16,J,(MN(I,J),I=1,NOINT)
      PRINT 23, HRS
   23 FORMAT(    //'0ELECTRICAL POWER DEMAND FOR ENTERPRISE DURING EACH'
     1,G10.3,'HOUR INTERVAL BEGINNING AT MIDNIGHT (IN KW):')
      DO 25 J=1,NYINT
   25 PRINT 16,J,(PLOAD(I,J),I=1,NOINT)
   16 FORMAT(' SEASON',I2,':   ',10G11.4,/(13X,10G11.4))
C
C*** CALCULATE AVERAGE POWER NEEDS
      WAV=0.000
      PAV=0.000
      DO 300 J=1,NYINT
      DO 300 I=1,NOINT
      WAV=WAV+MN(I,J)
  300 PAV=PAV+ PLOAD(I,J)
      WAV=WAV/(NOINT*NYINT)
      PAV=PAV/(NOINT*NYINT)
      PRINT 311,PAV,WAV
  311 FORMAT(//'0AVERAGE POWER REQUIREMENT=',G12.5,'KW.',5X,
     1 'AVERAGE WIND SPEED=',G12.5,'KNOTS')
      CALL BREZE(PLOAD,NOINT,NYINT,PMAX,PMIN,VMAX,VMAXX)
      PRINT 34, PMAX,PMIN
   34 FORMAT(' EXTREMA OF POWER REQUIREMENTS ARE',2G15.8,'KW.')
 1111 READ(5,350,END=98) $BUY,$SEL
  350 FORMAT(2G10.0)
      PRINT 35,$BUY,$SEL
   35 FORMAT(//' ASSUMED COST OF ELECTRICITY = ',G10.3,' ($/KWH)'/
     1 ' VALUE OF ELECTRICITY SOLD = ',G10.3,' ($/KWH)')
```

```
FORTRAN IV G LEVEL  21                    MAIN              DATE = 78135          23/29/01

              C
              C** CALCULATE OPTIMUM SAVINGS AND WTGS PARAMETERS
 0096              PRINT 33
 0097           33 FORMAT(//'OGEN. RATING    VRATED    VCUTIN                   ANN. WIND
                  XCOST    GEN. ELECT.    PURCH. ELECT.    SCLD/WASTED    ANN. NET SA
                  XVINGS',/,5X,'(KW)',T18,'(KTS)',T28,'(KTS)',                T53,'($)',
                  X T69,'(KWH)',T85,'(KWH)',T101,'(KWH)',T120,'($)')
 0098              SPCAF=(1.D0+INT/100.D0)**NYEARS
 0099              CRF=SPCAF*INT/100.D0/(SPCAF-1.D0)
 0100              Z(1)=Z(1)
 0101              Z(2)=Z(2)
 0102              STEP(1)=PMAX*FRACT1
 0103              STEP(2)=VMAXX*FRACT2
 0104              CALL SIMPX(Z,MAXSAV,2,STEP,100,100, EPSI ,1.000,0.500,2.000)
              C
              C*** PRINT FINAL ANSWERS
 0105              VCUTIN=4.6415890-01*Z(2)
 0106              MAXSAV=-MAXSAV
 0107              PRINT 44,Z(1),Z(2),VCUTIN,MAXSAV
 0108           44 FORMAT('0',//'OOPTIMUM GENERATOR SIZE=',G10.3,'KW',5X,'RATED SPEED
                  1=',G10.3,'(KTS)',5X,'CUT-IN SPEED=',G10.3,'(KTS)',5X,
                  2 /' MAXIMUM SAVINGS ($)=',G15.8)
 0109              GO TO 1111
 0110           98 STOP
 0111              END
```

```
      C
            SUBROUTINE OBJN(X,NETSAV,N)
      C*** SUBROUTINE COMPUTES VALUES NEEDED IN ECONOMIC OBJECTIVE FUNCTION
            IMPLICIT REAL*8(A-H,O-Z,$)
            REAL*8 X(2),A(8,4),B(8,4),PLOAD(8,4),FCTR(8,4),DEFICT(8,4),
           1 BUY(8,4),SEL(8,4),EXCESS(8,4),GEN(8,4),VMAX(8,4),VREF(2)
            REAL*8 NETSAV,LIMIT
            COMMON/LINK1/A,B,PLOAD,FCTR,VMAX,VREF,SDYS,HRS,$BUY,$SEL,CRF,
           1 VCUTIN,NYINT,NOINT,ICOST
            COMMON/LINK2/AIJ,BIJ,VMAXIJ,VRATED,PRATED,FRACT3,FRACT4
            REAL*8 FRECN(12,8,4),VINT(12,8,4),MIM(8,4)
            COMMON/LINK4/FRECN,VINT,MIM
            PRATED=X(1)
            VRATED=X(2)
            IF(VRATED .GT. 0.D0 .AND. PRATED .GT. 0.D0) GO TO 10
            NETSAV=1.D010
            RETURN
         10 VCUTIN=4.6415890-01*VRATED
            SUMBUY=0.D00
            SUMSEL=0.D00
            SUMGEN=0.D00
      C*** REMOVE COMMENT IF VERSION A IS TO BE USED AND ADD COMMENT TO APPROPRIATE
      C     VERSION B CARDS
      C     EXTERNAL FI,V3FI
      C     DO 40 I=1,NOINT
      C     DO 40 J=1,NYINT
      C     AIJ=A(I,J)
      C     BIJ=B(I,J)
      C     VMAXIJ=VMAX(I,J)
      C     IF(PRATED .GT. PLOAD(I,J)) GO TO 20
      C     BUY(I,J)    =PLOAD(I,J)-PRATED*FCTR(I,J)*(GLQUAD(VCUTIN,DMIN1
      C    1 (VRATED,VMAXIJ),V3FI)+GLQUAD(VRATED,VMAXIJ,FI))
      C     SEL(I,J)=0.D00
      C     GO TO 30
      C  20 VO=VRATED*(PLOAD(I,J)/PRATED)**3.3333333333333330-01
      C     LIMIT=DMIN1(VO,VMAXIJ)
      C     IF(VO .LT. VCUTIN)LIMIT=DMAX1(VO,VCUTIN)
      C     IF(VCUTIN .GE. VMAXIJ) LIMIT=VMAXIJ
      C     BUY(I,J)    =FCTR(I,J)*(PLOAD(I,J)*GLQUAD(0.D00,LIMIT,FI)
      C    1 -PRATED*GLQUAD(VCUTIN,DMIN1(VO,VMAXIJ),V3FI))
      C     SEL(I,J)    =FCTR(I,J)*(PRATED*(GLQUAD(DMAX1(VO,VCUTIN),DMIN1
      C    1 (VRATED,VMAXIJ),V3FI)
      C    2 +GLQUAD(VRATED,VMAXIJ,FI))-PLOAD(I,J)*GLQUAD(DMAX1(VO,VCUTIN),
      C    3 VMAXIJ,FI))
      C  30 GEN(I,J)=PLOAD(I,J)-BUY(I,J)
      C     SUMBUY=SUMBUY+BUY(I,J)
      C     SUMSEL=SUMSEL+SEL(I,J)
      C  40 SUMGEN=SUMGEN+GEN(I,J)
      C*** BEGIN VERSION B
            DO 306 M=1,NOINT
            DO 306 MM=1,NYINT
            MI=MIM(M,MM)
            HBUY=0.D00
            HSEL=0.D00
            IF(PRATED .GT. PLOAD(M,MM)) GO TO 320
            DO 300 I=1,MI
            IF(VINT(I+1,M,MM) .LE. VCUTIN)GO TO 300
            IF(VINT(I,M,MM) .GE. VRATED) GO TO 304
```

```
0027              IF(VINT(I,M,MM) .LE. VCUTIN .AND. VINT(I+1,M,MM) .GE. VRATED)
                 1 GO TO 302
0028              IF(VINT(I,M,MM) .LT. VCUTIN .AND. VINT(I+1,M,MM) .LT. VRATED)
                 1 GO TO 303
0029              IF(VINT(I,M,MM) .GE. VCUTIN .AND. VINT(I+1,M,MM) .LE. VRATED)
                 1 GO TO 305
0030              IF(VINT(I,M,MM) .GT. VCUTIN .AND. VINT(I+1,M,MM) .GT. VRATED)
                 1GO TO 333
0031              PRINT 32,I,M,MM,PRATED,VRATED
0032           32 FORMAT('0INTERVAL DOES NOT FIT ANY CATEGORY',/,3I5,2G15.7)
0033              GO TO 300
0034          333 HBUY=HBUY+(0.25D0*(VRATED**4-VINT(I,M,MM)**4)/VRATED**3+
                 1 VINT(I+1,M,MM)-VRATED)*FREQN(I,M,MM)
0035              GO TO 300
0036          302 HBUY=HBUY+(0.25D0*(VRATED**4-VCUTIN**4)/VRATED**3+VINT(I+1,M,MM)-
                 1 VRATED)*FREQN(I,M,MM)
0037              GO TO 300
0038          303 HBUY=HBUY+.25D0*FREQN(I,M,MM)*(VINT(I+I,M,MM)**4-VCUTIN**4)/VRATED
                 1 **3
0039              GO TO 300
0040          304 HBUY=HBUY+FREQN(I,M,MM)*(VINT(I+1,M,MM)-VINT(I,M,MM))
0041              GO TO 300
0042          305 HBUY=HBUY+0.25D0*FREQN(I,M,MM)*(VINT(I+1,M,MM)**4-
                 1 VINT(I,M,MM)**4)/VRATED**3
0043          300 CONTINUE
0044              BUY(M,MM)=PLOAD(M,MM)-HBUY*PRATED
0045              SEL(M,MM)=HSEL
0046              GO TO 330
0047          320 VD=VRATED*(PLOAD(M,MM)/PRATED)**3.333333D-01
0048              DO 301 I=1,M1
0049              IF(VINT(I+1,M,MM) .LE. VCUTIN) GO TO 311
0050              IF(VINT(I,M,MM) .GE. VRATED) GO TO 318
0051              IF(VINT(I,M,MM) .LE. VCUTIN .AND. VINT(I+1,M,MM) .GE. VD .AND.
                 1 VINT(I+1,M,MM) .LE. VRATED) GO TO 312
0052              IF(VINT(I,M,MM) .LT. VCUTIN .AND. VINT(I+1,M,MM) .LE. VD)
                 1 GO TO 313
0053              IF(VINT(I,M,MM) .GT. VCUTIN .AND. VINT(I+1,M,MM) .LE. VD)
                 1 GO TO 314
0054              IF(VINT(I,M,MM) .LE. VD .AND. VINT(I+1,M,MM) .LE. VRATED)
                 1 GO TO 319
0055              IF(VINT(I,M,MM) .LT. VCUTIN .AND. VINT(I+1,M,MM) .GT. VD .AND.
                 1 VINT(I+1,M,MM) .GT. VRATED) GO TO 321
0056              IF(VINT(I,M,MM) .GT. VD .AND. VINT(I+1,M,MM) .LE. VRATED)
                 1 GO TO 315
0057              IF(VINT(I,M,MM) .GT. VD .AND. VINT(I+1,M,MM) .GT. VRATED)
                 1 GO TO 316
0058              IF(VINT(I,M,MM) .LT. VD .AND. VINT(I+1,M,MM) .GT. VRATED)
                 1 GO TO 317
0059              PRINT 31,I,M,MM,PRATED,VRATED,VD
0060           31 FORMAT('0INTERVAL DOES NOT FIT ANY CATEGORY',/,3I5,3G15.7)
0061              GO TO 301
0062          321 HBUY=HBUY+FREQN(I,M,MM)*(PLOAD(M,MM)*(DMAX1(VD,VCUTIN)-
                 1 VINT(I,M,MM))-PRATED*0.25D0*(DMAX1(VD,VCUTIN)**4-VCUTIN**4)/
                 2 VRATED**3)
0063              HSEL=HSEL+FREQN(I,M,MM)*(PRATED*(0.25D0*(VRATED**4-
                 1DMAX1(VD,VCUTIN)**4)/VRATED**3+VINT(I+1,M,MM)-VRATED)-PLOAD(M,MM)*
                 2 (VINT(I+1,M,MM)-DMAX1(VD,VCUTIN)))
0064              GO TO 301
```

```
FORTRAN IV G LEVEL  21                  OBJM              DATE = 78135          23/29/01

0065        311 HBUY=HBUY+FREQN(I,M,MM)*PLOAD(M,MM)*(VINT(I+1,M,MM)-VINT(I,M,MM))
0066            GO TO 301
0067        312 HBUY=HBUY+FREQN(I,M,MM)*(PLOAD(M,MM)*(DMAX1(VD,VCUTIN)-
               1 VINT(I,M,MM))-PRATED*0.25D0*(DMAX1(VCUTIN,VD)**4-VCUTIN**4)/
               2 VRATED**3)
0068            HSEL=HSEL+FREQN(I,M,MM)*(PRATED*0.25D0*(VINT(I+1,M,MM)**4-
               1 DMAX1(VD,VCUTIN)**4)/VRATED**3-PLOAD(M,MM)*(VINT(I+1,M,MM)-
               2 DMAX1(VD,VCUTIN)))
0069            GO TO 301
0070        313 HBUY=HBUY+FREQN(I,M,MM)*(PLOAD(M,MM)*(VINT(I+1,M,MM)-VINT(I,M,MM))
               1 -PRATED*0.25D0*(VINT(I+1,M,MM)**4-VCUTIN**4)/VRATED**3)
0071            GO TO 301
0072        314 HBUY=HBUY+FREQN(I,M,MM)*(PLOAD(M,MM)*(VINT(I+1,M,MM)-VINT(I,M,MM))
               1 -PRATED*0.25D0*(VINT(I+1,M,MM)**4-VINT(I,M,MM)**4)/VRATED**3)
0073            GO TO 301
0074        315 HSEL=HSEL+FREQN(I,M,MM)*(PRATED*0.25D0*(VINT(I+1,M,MM)**4-
               1 VINT(I,M,MM)**4)/VRATED**3-PLOAD(M,MM)*(VINT(I+1,M,MM)-
               2 VINT(I,M,MM)))
0075            GO TO 301
0076        316 HSEL=HSEL+FREQN(I,M,MM)*(PRATED*(.25D0*(VRATED**4-VINT(I,M,MM)**4)
               1 /VRATED**3+VINT(I+1,M,MM)-VRATED)-PLOAD(M,MM)*(VINT(I+1,M,MM)-
               2 VINT(I,M,MM)))
0077            GO TO 301
0078        317 HBUY=HBUY+FREQN(I,M,MM)*(PLOAD(M,MM)*(VD-VINT(I,M,MM))-PRATED*
               1 0.25D0*(VD**4-VINT(I,M,MM)**4)/VRATED**3)
0079            HSEL=HSEL+FREQN(I,M,MM)*(PRATED*(.25D0*(VRATED**4-VD**4)/VRATED**3
               1 +VINT(I+1,M,MM)-VRATED)-PLOAD(M,MM)*(VINT(I+1,M,MM)-VD))
0080            GO TO 301
0081        318 HSEL=HSEL+FREQN(I,M,MM)*(PRATED-PLOAD(M,MM))*(VINT(I+1,M,MM)-
               1 VINT(I,M,MM))
0082            GO TO 301
0083        319 HBUY=HBUY+FREQN(I,M,MM)*(PLOAD(M,MM)*(VD-VINT(I,M,MM))-PRATED*
               1 0.25D0*(VD**4-VINT(I,M,MM)**4)/VRATED**3)
0084            HSEL=HSEL+FREQN(I,M,MM)*(PRATED*0.25D0*(VINT(I+1,M,MM)**4-VD**4)/
               1 VRATED**3-PLOAD(M,MM)*(VINT(I+1,M,MM)-VD))
0085        301 CONTINUE
0086            SEL(M,MM)=HSEL
0087            BUY(M,MM)=HBUY
0088        330 GEN(M,MM)=PLOAD(M,MM)-BUY(M,MM)
0089            SUMBUY=SUMBUY+BUY(M,MM)
0090            SUMSEL=SUMSEL+SEL(M,MM)
0091        306 SUMGEN=SUMGEN+GEN(M,MM)
          C*** END VERSION B
0092            SEAHRS=SDYS*HRS
0093            SUMBUY=SUMBUY*SEAHRS
0094            SUMSEL=SUMSEL*SEAHRS
0095            SUMGEN=SUMGEN*SEAHRS
0096            CALL MONEY(PRATED,VRATED,VREF,  ANCCST,CM,CRF,ICOST)
0097            NETSAV=-SUMGEN*$BUY+ANCCST-SUMSEL*$SEL+CM
0098            ANS=-NETSAV
0099            PRINT 43,PRATED,VRATED,VCUTIN,ANCCST,SUMGEN,SUMBUY,SUMSEL,ANS
0100         43 FORMAT(3X,3G11.5,11X,G13.6,5X,G13.6,4X,G13.6,2X,G13.6,5X,G13.6)
0101            RETURN
0102            END
```

```
      C
            SUBROUTINE BREZE(PLOAD,NDINT,NYINT,PMAX,PMIN,VMAX,VMAXX)
      C*** SUBROUTINE CALCULATES THE MAXIMUM AND MINIMUM POWER REQUIREMENTS
      C    AND MAXIMUM WIND SPEED
            IMPLICIT REAL*8(A-H,O-Z)
            REAL*8 PLOAD(8,4),VMAX(8,4)
            PMAX=0.0D0
            VMAXX=0.0D0
            PMIN=1.0D13
            DO 10 J=1,NYINT
            DO 10 I=1,NDINT
            A=PLOAD(I,J)
            B=VMAX(I,J)
            IF(A.GT.PMAX) PMAX=A
            IF(B .GT. VMAXX) VMAXX=B
         10 IF(A.LT.PMIN) PMIN=A
            RETURN
            END
```

```
      C
            SUBROUTINE MONEY(PRATED,VRATED,VREF,  ANCOST,CM,CRF,ICOST)
      C*** SUBROUTINE CALCULATES COST OF WTGS (CAN ADD ADDITIONAL COST FUNCTION IF
      C    DESIRED
            IMPLICIT REAL*8(A-H,O-Z)
            DIMENSION VREF(2)
              REAL*8 COEF(4,2)/2297.8D0,-0.465782D0,7.739710D0,2.57327D-2,
           1 4*0.0D0/
            IF (PRATED.GE.1.0D0) GO TO 10
            UNTCST=COEF(1,ICOST)*PRATED**COEF(2,ICOST)
            GO TO 20
         10 PRATLN=DLOG(PRATED)
            UNTCST=DEXP(COEF(3,ICOST)+COEF(2,ICOST)*PRATLN+COEF(4,ICOST)*
           X  PRATLN**2)
         20 COST=PRATED*UNTCST*(VREF(ICOST)/VRATED)**2
            ANCOST=COST*CRF
            CM=0.0300*COST
            RETURN
            END
```

```
      C
            FUNCTION GLQUAD (A,B,FN)
      C*** GAUSS-LEGENDRE QUADRATURE OF FUNCTION FN OVER INTERVAL (A,B)
      C*** INTEGRAL IS SET TO ZERO IF LOWER LIMIT LARGER THAN UPPER LIMIT
            IMPLICIT REAL*8(A-H,O-Z)
            REAL*8 ROOT(20),WEIGHT(20)
            COMMON/LINK3/ROOT,WEIGHT,NHALF
            GLQUAD=0.0D0
            IF (A.GE.B) RETURN
            BA=0.5D0*(B-A)
            AB=0.5D0*(A+B)
            DO 10 I=1,NHALF
         10 GLQUAD=GLQUAD+WEIGHT(I)*(FN(AB+BA*ROOT(I))+FN(AB-BA*ROOT(I)))
            GLQUAD=BA*GLQUAD
            RETURN
            END
```

```
      C
            FUNCTION FI(V)
      C*** SUBROUTINE COMPUTES VALUES OF EITHER (V/VRATED)**3*BETA OR JUST BETA
      C     DISTRIBUTION
            IMPLICIT REAL*8(A-H,O-Z)
            COMMON/LINK2/AIJ,BIJ,VMAXIJ,VRATED,PRATED,FRACT3,FRACT4
            FI=(V/VMAXIJ)**(AIJ-1.D0)*(1.D0 -V/VMAXIJ)**(BIJ-1.D0 )
            RETURN
            ENTRY V3FI(V)
            V3FI=(V/VRATED)**3*(V/VMAXIJ)**(AIJ-1.D0)*(1.D0-V/VMAXIJ)**(BIJ-
           1 1.D0)
            RETURN
            END
```

```
              C
              C           TO FIND THE UNCONSTRAINED MINIMUM OF A FUNCTION OF MANY
              C           VARIABLES BY SIMPLEX PATTERN SEARCH METHOD STARTING FROM
              C           AN ARBITRARY POINT ENTERED.
              C
              C
              C        DESCRIPTION OF PARAMETERS
              C           FX      - WITH (N) DIMENSIONS, THE ENTERING POINT WHEN
              C                     CALLING AND THE MINIMUM POINT WHEN RETURNING.
              C           FY      - FUNCTION VALUE AT RETURNING POINT IN RETURN.
              C           N       - NUMBER OF VARIABLES OF THE PROBLEM.
              C           D       - WITH (N) DIMENSIONS, THE STEP-SIZES FOR EACH
              C                     DIMENSION FOR INITIAL SIMPLEX SET-UP.
              C           ITOUT   - FREQUENCY OF WITHIN-SEARCH INTERMEDIATE PRINT-OUT
              C                     DESIRED. WHEN PUT ITOUT=THE NUMBER IN  ITMAX,
              C                     NO INTERMEDIATE PRINT-OUT WILL BE OUTPUT.
              C           ITMAX   - MAXIMUM NUMBER SEARCH ITERATION ASSIGNED. WHEN
              C                     EXCEEDED, THE SEARCH WILL BE TERMINATED AND RETURN
              C                     THE LAST MINIMUM DATA SEARCHED.
              C           EPSI    - STOPPING CRITERION. WHEN EPSI .GE. SY, STANDARD
              C                     DEVIATION OF FUNCTION VALUES EVALUATED AT CURRENT
              C                     SIMPLEX VERTICES,RETURN THE MINIMUM DATA .
              C           ALPHA   - REFLECTION COEFFICIENT, SUGGESTED VALUE IS 1.0 .
              C           BETA    - CONTRACTION COEFFICIENT, SUGGESTED VALUE IS 0.5 .
              C           GAMMA   - EXPANSION COEFFICIENT, SUGGESTED VALUE IS 2.0 .
              C
              C        SUBROUTINE NEEDED
              C           SUBROUTINE OBJN(X,Y,N)    - FOR COMPUTE FUNCTION VALUE Y
              C                                       AT X(I), WHERE:   I=1,2,....,N .
              C
              C
              C
0001                 SUBROUTINE SIMPX(FX,FY,N,D,ITOUT,ITMAX,EPSI,ALPHA,BETA,GAMMA)
              C
0002                 IMPLICIT REAL*8(A-H,O-Z)
0003                 REAL*8 X(9,8),Y(9),FX(N),D(N)
0004            1003 FORMAT(3X,65(1H*))
0005            1011 FORMAT(5X,5HOY = ,E11.5,9H  ITER = ,I4,10H  NOPT. = ,I4,10H  NOCVN
                    1 = ,I4)
0006            1012 FORMAT(7X,8HNORFT = ,I4,4X,8HNOEXP = ,I4,10H  NOCNT = ,I4,10H  NOC
                    1UT = ,I4)
0007            1013 FORMAT(7X,24HCURRENT SEARCHED DATA ../10X,3HY= ,E11.5,1H.)
0008            1014 FORMAT(10X,2HX(,I3,4H) = ,E11.5,1H,,5X,3HDX(,I3,4H) = ,E11.5,1H,)
0009            1015 FORMAT(7X,8HYMEAN = ,E15.8,9H ,  SY = ,E15.8,2H .)
0010            1016 FORMAT(5X,24H**CUT STEP-SIZES TIMES  ,I3,2H .)
0011            1023 FORMAT(5X,26H** ITERATION NO. EXCEEDED ,I5,2H .)
0012                 MULT=1
0013                 NOPT=0
0014                 NOCUT=0
0015                 NOCVN=0
0016                 ITER=0
0017                 NORFT=0
0018                 NOEXP=0
0019                 NOCNT=0
0020                 FN=N
0021                 NM=N+1
0022                 CALL OBJN(FX,YF,N)
0023                 LOCAT=1
0024                 IWAY=1
```

```
      C   SET UP INITIAL SIMPLEX .
        2 DO 6 J=1,N
          DO 3 I=1,J
        3 X(I,J)=  FX(J)-D(J)
          FJ=J
          X(J+1,J)=  FX(J)+FJ*D(J)
          IF(J-N)4,6,6
        4 JM=J+2
          DO 5 I=JM,NN
        5 X(I,J)=  FX(J)
        6 CONTINUE
          DO 8 I=1,NN
          DO 7 J=1,N
        7 FX(J)=X(I,J)
          CALL OBJN(FX,YF,N)
        8 Y(I)=YF
          INI=1
      C   REARRANGE ORDER (OVERALL) .
        9 I=1
          NS=N+1
       10 IF(Y(I)-Y(NS))13,11,11
       11 YTEM=Y(NS)
          Y(NS)=Y(I)
          Y(I)=YTEM
          DO 12 J=1,N
          FX(J)=X(NS,J)
          X(NS,J)=X(I,J)
       12 X(I,J)=  FX(J)
       13 IF(NS-I-1)15,15,14
       14 NS=NS-1
          GO TO 10
       15 I=I+1
          IF(I-N-1)16,17,17
       16 NS=N+1
          GO TO 10
       17 IF(INI) 65,65,501
      501 LOCAT=2
          IWAY=2
          GO TO 120
      C   COMPUTE THE CENTROID .
       18 DO 20 J=1,N
          PXT=X(1,J)
          DO 19 I=2,N
       19 PXT=PXT+X(I,J)
       20 X(N+2,J)=PXT/FN
      C   **MAKE REFLECTION MOVE .
          DO 21 J=1,N
          X(N+3,J)=X(N+2,J)+ALPHA*(X(N+2,J)-X(N+1,J))
       21 FX(J)=X(N+3,J)
          CALL OBJN(FX,YF,N)
          Y(N+3)=YF
          NOPT=NOPT+1
          LOCAT=3
          IWAY=3
          GO TO 500
       22 IF(Y(N+3)-Y(1))29,23,23
       23 IF(Y(N+3)-Y(N))24,26,26
      244 IWAY=7
```

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FORTRAN IV G LEVEL  21                SIMPX             DATE = 78135          23/29/01

0079         24 DO 25 I=1,N
0080         25 X(N+1,I)=X(N+3,I)
0081            Y(N+1)=Y(N+3)
0082            ITER=ITER+1
0083            NORFT=NORFT+1
0084            GO TO 100
0085         26 IF(Y(N+3)-Y(N+1))27,49,49
0086         27 DO 28 I=1,N
0087         28 X(N+1,I)=X(N+3,I)
0088            Y(N+1)=Y(N+3)
0089            ITER=ITER+1
0090            NORFT=NORFT+1
0091            GO TO 49
           C   **MAKE EXPANSION MOVE .
0092         29 DO 30 J=1,N
0093            X(N+4,J)=X(N+2,J)+GAMMA*(X(N+3,J)-X(N+2,J))
0094         30 FX(J)=X(N+4,J)
0095            CALL OBJN(FX,YF,N)
0096            Y(N+4)=YF
0097            NOPT=NOPT+1
0098            LOCAT=4
0099            IWAY=4
0100            GO TO 500
0101         31 IF(Y(N+4)-Y(1))32,244,244
0102         32 DO 33 I=1,N
0103         33 X(N+1,I)=X(N+4,I)
0104            Y(N+1)=Y(N+4)
0105            ITER=ITER+1
0106            NOEXP=NOEXP+1
0107            GO TO 100
           C   **MAKE CONTRACTION MOVE .
0108         49 DO 50 J=1,N
0109            X(N+5,J)=X(N+2,J)+BETA*(X(N+1,J)-X(N+2,J))
0110         50 FX(J)=X(N+5,J)
0111            CALL OBJN(FX,YF,N)
0112            Y(N+5)=YF
0113            NOPT=NOPT+1
0114            LOCAT=5
0115            IWAY=5
0116            GO TO 500
0117         51 IF(Y(N+5)-Y(N+1))52,60,60
0118         52 DO 53 I=1,N
0119         53 X(N+1,I)=X(N+5,I)
0120            Y(N+1)=Y(N+5)
0121            ITER=ITER+1
0122            NOCNT=NOCNT+1
0123            NOCVN=NOCVN+1
0124            GO TO 110
           C   **CUT DOWN STEP-SIZES .
0125         60 DO 62 I=2,NM
0126            DO 61 J=1,N
0127            X(I,J)=(X(I,J)+X(1,J))/2.000
0128         61 FX(J)=X(I,J)
0129            CALL OBJN(FX,YF,N)
0130         62 Y(I)=YF
           C   **REARRANGE ORDER (OVERALL ) .
0131            INI=0
0132            GO TO 9
```

```
   65 NOCUT=NOCUT+1
      NOPT=NOPT+N
      NCCVN=NOCVN+1
      LOCAT=6
      IWAY=6
      GO TO 120
  100 NOCVN=0
C   **REARRANGE CROER ( SHOT-DOWN ).
  110 IOR=N
  111 IF(Y(IOR+1)-Y(IOR))112,120,120
  112 YTEM=Y(ICR+1)
      Y(IOR+1)=Y(IOR)
      Y(IOR)=YTEN
      DO 113 J=1,N
      FX(J)=X(IOR+1,J)
      X(IOR+1,J)=X(IOR,J)
  113 X(IOR,J)=FX(J)
      IF(IOR-1)120,120,114
  114 ICR=ICR-1
      GO TO 111
C   **TEST FOR OPTIMALITY .
  120 FNM=NM
      YM=Y(1)
      DO 121 I=2,NN
  121 YM=YM+Y(I)
      YM=YM/FNM
      SY=(Y(1)-YN)**2
      DO 122 I=2,NN
  122 SY=SY+(Y(I)-YM)**2
      SY=(SY/FN)**0.500
      IF(LOCAT-6) 123,500,123
  123 IF(LOCAT-2) 500,500,124
  124 IF(SY-EPS1) 125,125,18
  125 LOCAT=8
  500 IF(NCPT-ITMAX) 505,505,560
  505 GO TO (2,18,530,530,530,540,560,560),LOCAT
  530 IF(NOPT-ITOUT*MULT) 533,531,531
  531 MULT=MULT+1
      IF(NOPT-ITOUT*MULT) 532,531,531
  532 WRITE(6,1011)Y(1),ITER,NOPT,NCCVN
      WRITE(6,1012)NCRFT,NOEXP,NCCNT,NOCUT
      WRITE(6,1015)YM,SY
      WRITE(6,1013)YF
      DO 534 IN=1,N
  534 WRITE(6,1014)IN,FX(IN),IN,X(1,IN)
      WRITE(6,1003)
  533 IWAY=IWAY-2
      GO TO (22,31,51,123,18),IWAY
  540 WRITE(6,1016)NOCUT
      GO TO 123
  560 IF(LCCAT-8) 561,562,562
  561 WRITE(6,1023)ITNAX
  562 DO 564 I=1,N
  564 FX(I)=X(1,I)
      FY=Y(1)
      RETURN
      END
```

WINTER WHEAT-SORGHUM FARM   WICHITA, KANSAS OBSERVED WIND SPEED DATA

AVERAGE VELOCITY FOR ENTERPRISE DURING EACH 3.00 HOUR INTERVAL BEGINNING AT MIDNIGHT (IN KNOTS):

SEASON 1:	8.740	8.723	9.240	11.41	12.31	11.73	8.880	9.017
SEASON 2:	10.01	9.739	9.899	11.28	12.17	11.60	9.654	10.03
SEASON 3:	10.89	10.87	11.57	14.02	14.59	14.38	11.53	10.68
SEASON 4:	8.584	7.734	8.670	10.24	10.62	10.90	10.15	8.921

ELECTRICAL POWER DEMAND FOR ENTERPRISE DURING EACH 3.00 HOUR INTERVAL BEGINNING AT MIDNIGHT (IN KW):

SEASON 1:	10.46	8.265	20.09	12.98	10.97	12.03	20.40	17.10
SEASON 2:	24.21	24.52	36.34	26.73	24.72	25.53	41.40	33.10
SEASON 3:	11.46	12.02	21.09	11.23	9.220	4.780	15.90	12.60
SEASON 4:	7.910	5.715	19.89	12.78	17.82	16.13	24.50	16.85

AVERAGE POWER REQUIREMENT= 17.836 KW.   AVERAGE WIND SPEED= 10.587 KNOTS
EXTREMA OF POWER REQUIREMENTS ARE 41.400000 4.7800000 KW.

ASSUMED COST OF ELECTRICITY = 0.8000-01 ($/KWH)
VALUE OF ELECTRICITY SOLD = 0.0 ($/KWH)

GEN. RATING (KW)	VRATED (KTS)	VCUTIN (KTS)	ANN. WIND COST ($)	GEN. ELECT. (KWH)	PURCH. ELECT. (KWH)	SOLD/WASTED (KWH)	ANN. NET SAVINGS ($)
18.440	19.200	8.9119	2037.24	37750.2	118490.	5570.26	462.458
10.160	16.425	7.6238	1868.46	31220.4	125020.	964.164	151.955
26.720	16.425	7.6238	3600.65	62379.1	93860.9	22263.6	470.050
18.440	24.750	11.488	1226.01	20742.5	135498.	2072.79	120.262
18.440	8.1000	3.7597	11446.5	96384.8	59855.2	22932.0	-6659.27
18.440	20.588	9.5559	1771.89	32626.8	123613.	4446.60	385.657
35.000	20.587	9.5559	2776.18	45016.3	107224.	21350.7	436.062
43.280	16.425	7.6238	5084.96	75698.0	80542.1	61403.0	-327.858
24.650	19.547	9.0729	2402.59	44537.8	111702.	11203.4	546.754
16.370	15.384	7.1408	2925.91	56643.7	105596.	7091.20	378.291
30.342	19.287	8.9521	2856.06	51310.8	104929.	19297.7	518.349
28.272	22.409	10.401	2013.02	36211.8	120028.	9464.11	369.781
27.108	17.921	8.3181	3055.48	54892.9	101347.	18226.9	555.560
21.416	18.181	8.4389	2518.31	46430.2	109810.	9745.84	552.514
23.874	16.555	7.6842	3275.43	58051.8	98188.2	16517.7	532.149
24.456	18.799	8.7257	2583.27	47675.8	108564.	12342.1	571.011
30.148	18.539	8.6049	3077.58	54773.1	101467.	21325.1	517.732
23.579	18.270	8.4804	2667.62	49020.2	107212.	12283.1	573.310
20.947	19.149	8.8880	2235.78	41565.0	114675.	7922.12	518.388
25.568	18.228	8.4606	2834.53	51689.1	104551.	15042.8	576.645
24.711	17.699	8.2153	2935.39	53286.5	102954.	14966.6	577.011
24.838	17.150	7.9601	3137.88	56253.7	99986.3	16513.6	560.588
26.680	17.657	8.1955	3112.51	55785.7	100454.	18243.4	555.383
24.369	18.117	8.4092	2774.48	50788.6	105451.	13575.3	579.550
23.512	17.589	8.1639	2871.08	52205.8	104034.	13511.0	572.054
25.054	18.068	8.3864	2844.14	51870.7	104369.	14650.8	579.098
24.712	18.486	8.5803	2691.10	49453.8	106786.	13280.6	577.683
24.712	18.289	8.4890	2749.23	50391.8	105848.	13682.7	579.945
24.027	18.338	8.5118	2681.39	49279.9	106960.	12691.6	576.158

24.797	18.136	8.4178	2802.73	51236.0	105004.	14128.6	580.316
24.454	17.963	8.3379	2829.02	51648.4	104592.	14013.0	580.306
24.882	17.582	8.3465	2857.66	52089.6	104150.	14588.9	579.640
24.497	18.083	8.3935	2795.09	51119.8	105120.	13817.9	580.609
24.840	18.255	8.4734	2769.42	50706.4	105534.	13535.0	579.759
24.551	18.036	8.3718	2813.90	51415.8	104824.	13991.1	580.667
24.251	17.984	8.3475	2806.10	51279.5	104961.	13691.8	579.565
24.661	18.098	8.4002	2803.62	51254.4	104986.	14014.8	580.671

OPTIMUM GENERATOR SIZE= 24.7 KW RATED SPEED= 18.1 (KTS) CUT-IN SPEED= 8.40 (KTS)
MAXIMUM SAVINGS ($)= 580.67137

WIND MODELS AND OPTIMUM SELECTION OF WIND TURBINE SYSTEMS

by

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ABSTRACT

To obtain accurate estimates of extractable energy from the wind at a given location, it is first necessary to obtain an accurate description of the expected distribution of wind speeds. For such analyses, the use of analytical representations for the wind speed distributions often simplifies the calculation as well as smooths out statistical fluctuations in the observed wind speed data. In the first phase of this study, three techniques for fitting a Weibull distribution to observed wind speed data are examined. In addition, the beta distribution is introduced as an alternative wind speed distribution model and a matching-moments scheme is presented to obtain the beta distribution's parameters. Two goodness of fit tests are performed on each analytical distribution to test the appropriateness of each model in describing 544 observed wind speed distributions. It was found that least squares fitting techniques produce Weibull distributions which poorly represent wind data, but that both Weibull and beta distributions give excellent fits to the data when the parameters are obtained with a matching-moments technique.

In the second phase of this work, the analytically fit wind speed distributions are used in a methodology to select the optimally sized wind turbine generator system for given demand power requirements. Such an optimally sized system will yield the maximum net economic savings for an enterprise which uses the system. In particular, the sensitivity of the optimal wind system to various problem parameters such as electricity cost and wind and load characteristics are investigated. It was found

that to accurately estimate the capability of a particular wind turbine system to supply the energy needs of a specified demand load, daily and seasonal wind speeds and demand loads as well as diurnal variations in these characteristics must be known.