ANALYSIS OF EXTERNAL GROWTH OF MAMMALIAN FETUSES

by

DALE LAMBERT PRESTON

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Approved by:

H T Gier

Major Professor

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General Introduction

Prenatal growth, which is the summation of the differentiation and maturation processes of development, can be quantified by recording the change in size of external organs and the increase in body weight. Correlation of these changes to the length of gestation can provide information as to those measurements which can be employed with a reasonable degree of accuracy. The segregation of those measurements which are expressive of the growth phenomenon is the objective of Study I.

Mathematical quantification of growth is followed in Study II where two statistical models are presented to correlate the growth process with time to determine theoretical estimates of length or weight. Based on these mathematical estimates, the rate of development of external organs and body weight are determined. In addition the correlation of the rate of growth of one organ with respect to another organ is examined to lend mathematical quantification to those representative measurements chosen in Study I.

Study I

Introduction

The prenatal development of animals provides a basis for determining factors which contribute to normal variation in development and thereby provide criteria for determining condition of abnormal development due to factors such as nutritional loss or genetic mutation.

The descriptive embryogeny has been elucidated, but attempts to correlate the developmental process as reflected by external measurements and subsequent body weights with respect to the length of development has remained an area of diversified investigation.

Winters, Green, and Comstock (1942) detailed bovine development from gametogenesis through the embryonic period and established the contour measurement as a criterion for aging. Maneely (1952) developed curves of crown-rump measurement using known ovulation ages of six breeds of dairy cattle. Hammond (1927) briefly described bovine specimens recovered at the end of each lunar month of pregnancy.

A descriptive outline of cow and dog embryogeny from the 20th day of gestation to 40 days prior to parturition in the cow and from the 15th to 56th day of gestation in the dog was presented by Henry (1958).

External organ development of the horse was the subject of studies by Zeitzschmann and Krolling (1955), Stoss (1944), and Roberts (1956), as reviewed by Bergin (1968). Bergin presented a summary of the horse embryogeny and graphs of external measurements for aging purposes.

The human was the subject of several reviews by Streeter (1920, 1926, 1940, 1941, 1945, 1948). His compilation of weight, sitting height, head size, and foot length provided the data used for the human analysis section of this study. The data reviewed consisted of 1200 observations on embryos and fetuses, mostly of known menstrual age.

Jones and Brewer (1941), Payne (1925), and Corner (1929), as reviewed by Henry (1958), have concentrated studies on human development up to the 10 somite stage.

Methods and Materials

The data assembled for growth studies and for the subsequent analysis represent four very different groups of mammals: cow, dog, horse, and human.

The cow embryos and fetuses were obtained from dairy cows from the Kansas State University dairy herd, supplemented with material from Armour Packing Co. and Rodeo Packing Co. in Kansas City, Missouri. These animals which came from the Kansas State dairy herds were of known age based on breeding records with the variation being ± 12 hours post-ovulation. The slaughter house material was aged on the basis of growth charts which gave a variation of ± 5 days.

Dog fetuses and embryos were derived from a colony of dogs which was maintained by this laboratory. It was a mixed colony and no one breed served as the dominant member of the population. Bitch size was kept within a 20-40 lb. interval. The aging of these animals was based on post-ovulation time with a variation of ± 1 day.

The horse fetuses and embryos were obtained from the Department of Veterinary Science, University of Kentucky; the Hill Packing Co., Topeka, Kansas; and the Veterinary Clinic, Kansas State University, School of Veterinary Medicine. The majority of these animals were from mares weighing 1000 ± 100 lbs. The age estimation is based on breeding records and growth charts with a variation of ± 5 days.

Data on human embryos or fetuses was compiled from the data assembled by Streeter (1920). These data were adjusted by subtracting two weeks from the menstrual age recorded by Streeter to coincide with the conceptional age. In addition the data were averaged for each time interval to obtain mean measurements. This was done because the human data was to be used only as a point of comparison to the cow, dog, and horse studies.

There was no uniformity in the methods of preservation of the embryos and fetuses in this study prior to the taking of measurements, introducing a source of error due to shrinkage.

Linear measurements taken on all four species were recorded in millimeters and the weight recorded in grams. The measurements on the cow, dog, and horse were taken as described by Henry (1958).

Contour - length from the tip of the snout over the forehead, along the mid-dorsal line, to the tip of the tail.

Crown-rump - the greatest length in a straight line from the tip of the forehead to the posterior surface of the thighs ventral to the tail.

Trunk - greatest length in a straight line from the point of the shoulder to the posterior surface of the thigh, ventral to the tail.

Head length - distance from the tip of the nose across the eye region to the top of the head.

Hindfoot - length from the back of the heel to the tip of the hoof or the longest toe, with the foot flattened on a ruler.

Forefoot - length from the dorsal surface of the distal end of the radius to the tip of the hoof or longest toe with the knee in flexion.

Tail - length from the base to the tip of the last caudal vertebra.

Ear - length from the base to the tip of the dorsal surface of the ear.

The human data assembled by Streeter include a head modulus measurement which represents the mean between the greatest horizontal circumference and the biauricular transverse arc of the head. This measurement is grouped in the analysis with head length. The crown-rump measurements used in this study were the flexed crown-rump measurements taken by Streeter.

The number of observations used in this study varied considerably within each group. The cow data range from 69 observations on the head width to 184 observations on the crown-rump. Dog observations range from 92 on the ear to 197 on the crown-rump. Horse observations range from 57 on the weight to 89 on the crown-rump. The human data is based on 98 mean values compiled from Streeter's original data set of approximately 1200 observations.

Results and Discussion

The use of linear external organ measurements and cubic weight in development of standards of aging must be employed with some limitation and selectiveness. The measurement selected should encompass the entire growth period under study and display a minimum of variation throughout the period of development. In addition the measurement used should not exhibit divergence resulting from imprecise measurement, inconsistent means of fixation, and breed heterogeneity.

Establishment of these criteria disposes of the tail and ear observations in the cow, dog, and horse because of lack of significant development prior to day 50 in the cow and day 24 in the dog. The horse tail and ear do not show significant development until after the 100th day, and age estimation prior to this is not precise. The cow, dog, and horse heterogeneity with respect to the tail and ear measurements is significant with the approach of parturition.

Head width is of little value as a criterion for age estimation because time and extent of head development and establishment of areas suitable for basing the measurement are not consistent among species or within breeds.

The trunk, head length, hindfoot, and crown-rump of the cow, dog, and horse are presented in Fig. 1-3 and the means of the human flexed crown-rump, foot, and head modulus in Fig. 4. Fig. 5 is a composite of the body weights of the cow, dog, and horse, and the mean body weight of the human.

Hindfoot and forefoot measurements exhibit parallel growth in the cow, dog, and horse. However, the hindfoot is the preferred measurement since its initial rate of development is somewhat greater than that of the forefoot, thus the possibility of error is reduced. With the exception of the human mean measurement the hindfoot shows a great amount of variation during the time of gestation, and significant divergence of measurements is indicated prior to parturition.

The trunk measurement in the horse as described by Bergin (1968) expresses little variation since it parallels the crown-rump and exhibits a similar deviation as the crown-rump with the approach of parturition. Contrarily, the trunk measurement of the dog displays an initial diversity which is maintained throughout gestation and is indicative of breed and litter variation. The cow trunk measurement shows a variation after the middle of gestation. This is attributable to the breed heterogeneity.

The head length of the cow displays a high rate of growth from the 20th day to the 60th day, making age estimation difficult. Heterogeneity within the cow, dog, and horse with respect to head length and the mean head modulus of the human eliminates these measurements as a criterion of age.

The linear measurements of contour and crown-rump can be accepted with equal frequency to the earliest stages of development. Winters, Green, and Comstock (1942) favor the contour measurement in the cow due to its existence prior to the development of significant crown-rump observations. Crown-rump measurements in the cow, dog, and horse display a marked correlation to the time interval of study as indicated by Figs. 1-4.

Divergence of the crown-rump measurement in the species examined was consistent and of an acceptable low level. Contour measurements while not presented graphically, show greater variation in the later stages of development which makes them not accurate for terminal age estimations. The crown-rump measurement of the dog as shown in Fig. 2 is representative of the lack of breed or litter variation exhibited through the crown-rump measurement as compared to the head length or hindfoot measurements.

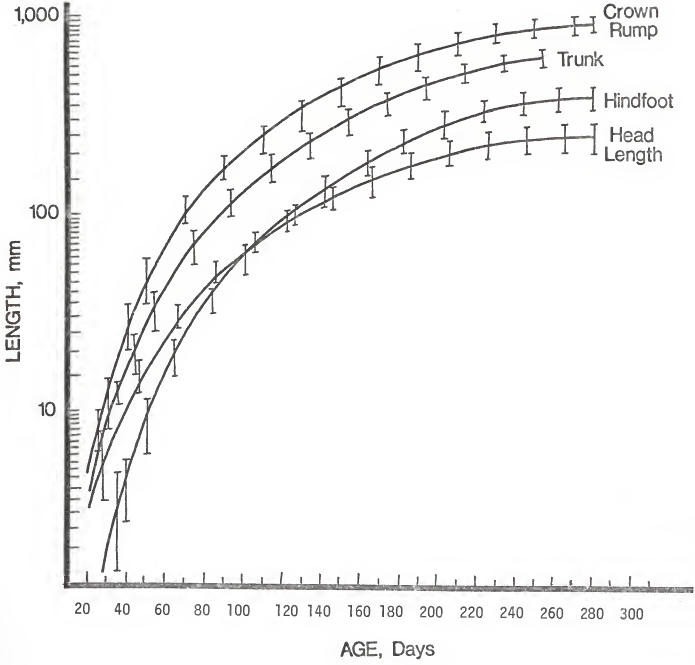
Body weights as presented in Fig. 5 indicate that this measurement can be used as an aging criterion if it is used in conjunction with an acceptable linear dimension such as crown-rump or contour to establish the age.

The estimate of age of any species studied is subject to error because the embryo and subsequent fetus are individuals and as such possess a genetic variation that, while within breed limitations, does contribute a heterogeneity which is unique.

EXPLANATION OF PLATE I

Figure 1. Cow linear measurements of the crown-rump, head length, hindfoot, and trunk. Measurements are in millimeters against time in days.

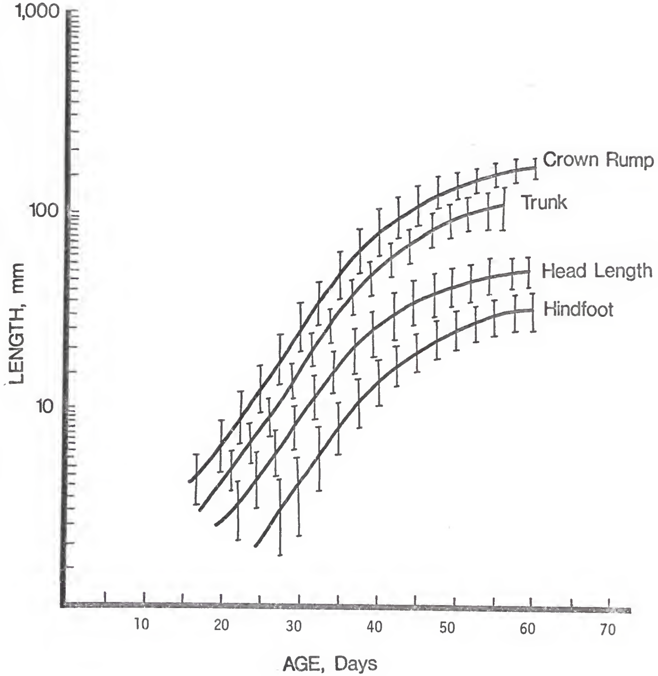
PLATE I


Crown
Rump
Trunk
Hindfoot
Head
Length
1,000
100
10
LENGTH, mm
20 40 60 80 100 120 140 160 180 200 220 240 260 280 300
AGE, Days

EXPLANATION OF PLATE II

Figure 2. Dog linear measurements of crown-rump, head length, hindfoot, and trunk. Measurements are in millimeters against time in days.

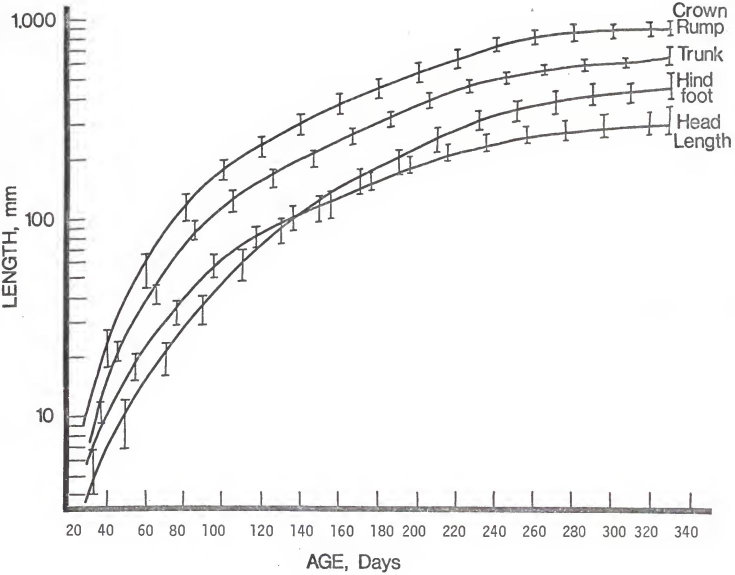
PLATE II


1,000
100
10
LENGTH, mm
Crown Rump
Trunk
Head Length
Hindfoot
10
20
30
40
50
60
70
AGE, Days

EXPLANATION OF PLATE III

Figure 3. Horse linear measurements of crown-rump, head length, hindfoot, and trunk. Measurements are in millimeters against time in days.

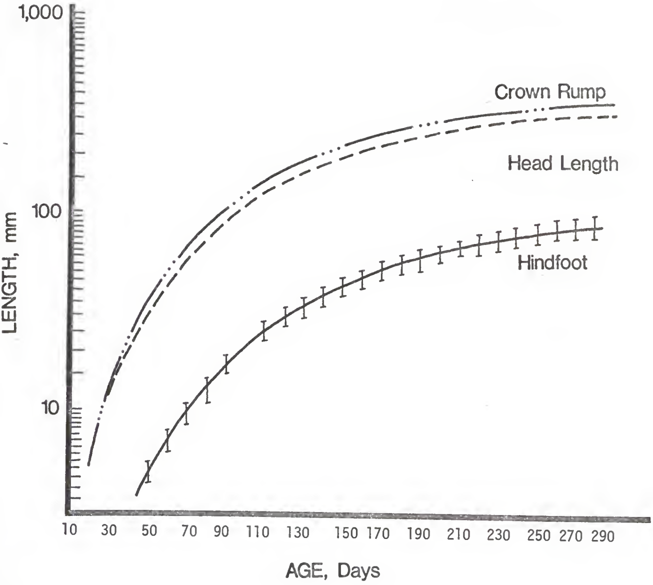
PLATE III


Crown
Rump
Trunk
Hind
foot
Head
Length
1,000
100
10
LENGTH, mm
20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340
AGE, Days

EXPLANATION OF PLATE IV

Figure 4. Human linear measurements of crown-rump, foot, and head modulus. Measurements are in millimeters against time in days.

PLATE IV

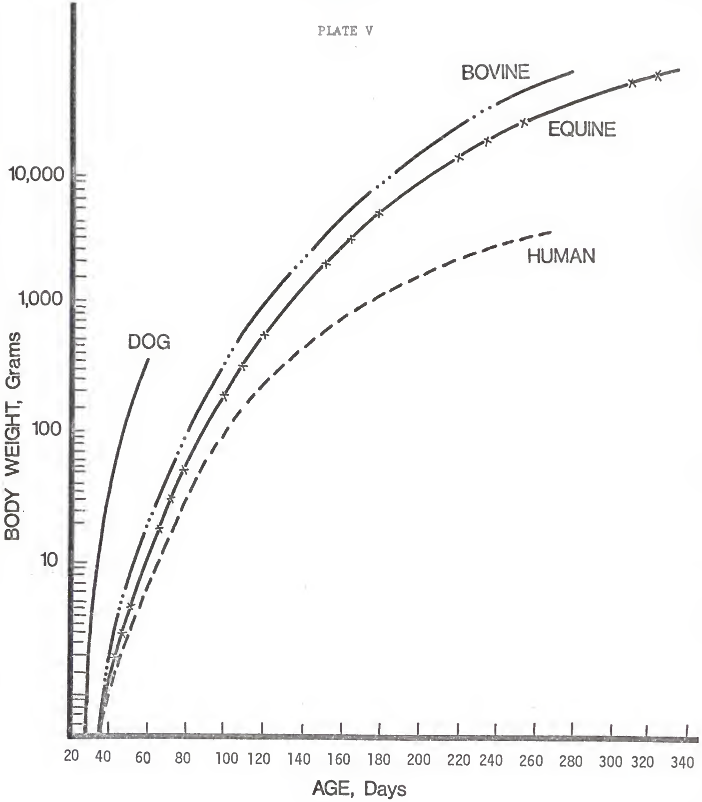


1,000
100
10
LENGTH, mm
Crown Rump
Head Length
Hindfoot
10 30 50 70 90 110 130 150 170 190 210 230 250 270 290
AGE, Days

EXPLANATION OF PLATE V

Figure 5. Body weights of cow, dog, horse, and human in grams against time in days. Human weights are mean values.

PLATE V


BOVINE
EQUINE
HUMAN
DOG
BODY WEIGHT, Grams
10,000
1,000
100
10
20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340
AGE, Days

STUDY II

Introduction

The growth process has been the subject of intensive investigation with attempts to formulate a specific, predictive model which will give numerical quantification to the complex growth process.

Pearl and Reed (1923) developed the logistic curve to estimate the United States Census. Though its widest applications have been actuarial, it is now a frequent tool of the biologist. Ricklefs (1967) gave a graphical solution for the logistic, Gompertz, and the von Bertalanffy equations.

The Gompertz equation as modified by Laird, Tyler, and Barton (1966) developed three parameters for explanation of the dynamics of growth. The most significant of these is the parameter estimating the rate of exponential decay in growth per unit of time. The Laird equation explains growth as the resultant between the initial exponential proliferation of the system as determined by the initial mass, and the rate at which this initial mass decays exponentially with respect to time. Winsor (1932) discussed the mathematical differences between the Gompertz and the logistic with respect to the inflection point and growth rate, as a function of time or as a function of size.

The von Bertalanffy model (1938) expressed the concept that the rate of growth is determined by the extent to which the anabolic rate of the animal exceeds the catabolic rate; further, that the anabolic rate is a function of the surface of the organism; and that the catabolic rate is a function of the volume. Fabens (1965) presented a computer method for fitting of the von Bertalanffy curve from recapture data. This equation is highly dependent upon the ability of the researcher to initially estimate the upper asymptote of growth of which observations will be expressed as a fraction.

Weiss and Kavanau (1957) presented a negative feedback hypothesis that defined growth as "the net balance of mass produced and reatined over mass destroyed and lost". Their presentation included the differential equations explaining the growth of chick embryos.

Huxley (1932, 1950) used the log-log transformation of both dependent and independent variables to approximate growth. This study developed the relative growth rates of organ to organ, or organ to body relationships of several invertebrates. Lumer (1937) stated that while the Huxley formulation makes a useful approximation to relative growth processes, it fails to take into consideration some of the features of determinate growth. This is due in part to the exponential growth of initial stages which tend to obscure the subtle but precise changes occurring at the end of the period. Brody (1945), using the Huxley approximation in consideration of linear growth to weight, found the curve to be insensitive to the growth process as the asymptotic levels were approached. Medawar (1950) differentiated the Huxley formulation to develop specific growth rates per unit of time. Bernadis and Skelton (1964) plotted the body size versus organ size to explain the differential growth rates of the organ are not equal to the body, and isogonic if the growth rates are equivalent.

Huggett and Widdas (1950) expressed the relationship that during a substantial period of pregnancy fetal development can be adequately represented by the cube root of the body weight. This was used to develop the growth rates for the pregnancy period.

Spencer and Coulombe (1964, 1965) applied the Hasse equation to man, rat, and monkey, expressing the relationship that the weight in terms of the cube of the dimension examined yields a modified cubic equation which is descriptive of the interuterine weight of the specimen. The growth process was then represented as a fractional portion of the gestation period in order to compare the growth rates of animals having varying gestation lengths.

In this investigation two statistical models were applied to the linear dimensions and body weights of the cow, dog, horse, and human. It is the objective of this study to show the applicability of the models to the growth phenomenon.

Methods and Materials

It has been the practice of this laboratory to use a semi-logarithmic plot of growth data to visually represent the growth process. The ordinate in most instances is the magnitude of the observed measurement and the abscissa is the time interval of study.

A polynomial of the form,

$$\log_{10} Y = \alpha + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \beta_4 X^4 \; .... + \beta_n X^n \quad (1)$$

was applied to the linear measurements and body weights used in this study. The independent variable X is the time of gestation and the dependent variable Y is the corresponding observed measurement. The power of the independent variable which gave the highest sums of squares attributable to regression and maintained significant beta coefficients at the .01 level was the degree of polynomial accepted as the growth model.

In order to develop a relation of rate of change of the Y variable with the X variable the relation,

$$dy/dx = rY$$

was proposed, where dy is the change in weight or linear length, and dx is the change in time, and Y is the observed measurement.

Solving for r

$$r = dy/dxY$$

and

$$r = d\log_e Y/dx.$$

In equation (1) this becomes

$$.434 \; d\log_e Y/dx = 0 + \beta_1 + 2\beta_2 X + 3\beta_3 X^2 + 4\beta_4 X^3. \quad (2)$$

Thus r, the rate of growth as approximated by the polynomial of equation (1) is

$$r = (\beta_1 + 2\beta_2 X + 3\beta_3 X^2 + 4\beta_4 X^3)/\; .434 \qquad (3)$$

if the Y measurement is a cubic dimension. If Y is a linear dimension the equation is multiplied by a factor of 3.

The data was also fit by the exponential model of the form,

$$Y = \alpha X^{\beta} \tag{4}$$

where Y is the observed measurement and X is the time of the gestation when the measurement was taken. This model was initially approximated by least-squares method after making logarithmic transformations of both variables. The initial estimates for the α and β paramaters provided from the least-squares fit were then used to approximate the starting estimates for an iterative solution.

A least-squares and iterative procedure were used to solve the exponential because the sums of squares for the residual in the least-squares model and the iterative model are not equivalent. Thus a better fit consistently on one method of solution would indicate that this method was making the correct assumptions for the data used.

The log-log transformation was also used to determine the relative growth rates of the linear measurements and body weights with respect to each other. If the growth rates are similar a least-squares regression will yield a straight line equation with a high correlation between variables.

The criterion for the acceptance of the polynomial model (1) as opposed to the exponential model (4) was the correlation coefficient and the statistic defined by the formula,

$$\Sigma(Y_{calc.} - Y_{obs.})^2/(N - K - 1)$$

where N is the number of observations and K is the number of coefficients.

The data analysis for this study was done on an IBM 360/50 digital computer. The computer programs used in the polynomial and exponential models were modifications of the IBM Scientific Subroutine Package. The iterative estimations of the exponential paramaters was performed by programs supplied by Jerrold Zar, Department of Biological Science, Northern Illinois University.

Results and Discussion

Comparative analysis of linear external organs and body weights by application of polynomial and exponential models reveals that the polynomial gives the best fit in the majority of cases. Table I shows the distribution of the data analyzed with the model which gave the best fit on the basis of the highest coefficient of correlation. Out of the 32 linear dimensions and body weights analyzed the polynomial gave the best fit in 25 cases, and the exponential accounted for the remaining seven. Of these seven exponential models, four were improved from their original least-squares estimations of log-log transformations by the iterative method.

The coefficients and statistics for the polynomial model are presented in Table I and II of the Appendix, the coefficients and statistics for the exponential least-squares estimates are presented in Table III and IV of the Appendix, and the improvements of these estimates by the iterative method are presented in Table V of the Appendix. Those iterative applications which did not close due to poor initial estimates, inability of the model to approximate the data, or insignificance of parameter estimates at the .01 level are not included in Table V.

The distribution of the organs analyzed with respect to the degree of polynomial which gave the best fit are presented in Table II. Dog linear dimensions and body weights with the exception of the ear were fit entirely by the second order model. The human data was best approximated by the third order polynomial, and the cow and horse were distributed through third and fourth order models.

The polynomial, while presenting a model which in most cases gives the best fit according to correlation and size of the deviation, does not

TABLE I. Distribution of Organs to Models Giving the Best Fit.

	Polynomial	Exponential	
		Least-squares	Iterative
WEIGHT	Bovine, Canine Equine, Human		
CROWN-RUMP	Bovine, Canine Equine, Human		
CONTOUR	Canine, Equine	Bovine	
TRUNK	Bovine, Canine Equine		
TAIL	Canine, Equine		Bovine
HEAD LENGTH	Bovine, Canine Equine		Human
HEAD WIDTH			Bovine, Equine
HINDFOOT	Bovine, Canine Equine, Human		
FOREFOOT	Bovine	Equine	
EAR	Bovine, Equine	Canine	

TABLE II. Distribution of Organs Analyzed by the Polynomial Model.

	Degree of Polynomial 1	2	3		4
BOVINE			Crown-rump Contour Head Length Head Width	Weight Tail Ear Forefoot	Hindfoot Trunk
CANINE	Ear	Contour, Tail Trunk, Weight Head Length Hindfoot Crown-rump			
EQUINE		Ear	Contour Forefoot Head Length Head Width	Tail	Hindfoot Weight Trunk Crown-rump
HUMAN			Crown-rump Foot Weight Head Modulus		

seem to be biologically valid. The fourth beta coefficient for the fourth order polynomial is negative, and application of this model gives a negative theoretical growth prior to parturition. Though the second coefficient is negative in the second order model, it does not have a magnitude which results in negative growth prior to birth. The third order model has a positive high order beta coefficient, and this shows positive growth throughout the gestation period.

The exponential model provided a good fit to the head length and head width of human, cow, and dog fetuses. It also provided a good fit to the cow tail, horse forefoot, and dog ear, and cow contour. All of these measurements show a relatively high variation of observed measurement throughout the gestation period and this is probably the reason for the acceptance of the exponential.

The exponential model gives a poor approximation in the majority of dimensions analyzed due to its insensitiveness to the change of inflection of the growth process. The exponent parameter of the model is the ratio of rate of change of the independent to the dependent variable and remains constant throughout gestation. Since the exponential model is attempting to fit the lower rate of growth and the terminus with the initial high exponential rate of growth, the entire slope is lowered, thus making the curve a poor approximation to the inflection and middle sections of the curve.

Figures 6 and 7 show the relation of the fits achieved by the polynomial and the exponential models to a linear measurement of crown-rump and a cubic dimension, body weight. These measurements on the cow fetuses are typical of the fits achieved on all dimensions and weights analyzed by the polynomial and exponential in the cow, horse, and human. The exponential, while not fitting the inflection portion of the curve, also gives a terminal estimation which is not reflective of the trend of the observed data. The exponential made its best fit to the dog data even though the polynomial gave the best approximation and highest correlation. Comparison of the raw data presented in Figure 2 of Study I shows that the dog fetuses have a marked linear trend throughout gestation and do not experience such a dramatic point of inflection as is present in other species. Figure 5 indicates that the dog body weight remains in a constant exponential phase during gestation paralleling the initial exponential growth of the cow, horse, and human.

Though the second beta coefficient for the canine polynomial model is negative, the independent variable is not of sufficient magnitude to make application of this model invalid as a result of negative growth with the approach of parturition.

Thus the polynomial is accepted in this study as the primary model for growth, based not only on the high correlations with time, but the small deviation from theoretical estimates. However, in the case of the fourth order models which are invalid, the third order should be accepted, and it should be noted that it does not differ significantly from the fourth order correlations.

The fitting of the polynomial is valid to the study of growth only if it gives an explanation of the physical laws which are determining growth. Application of the polynomial makes the inference that the growth process is not constant, but a state modified with the passage of time and the degree of development.

The Gompertz function which has been applied by Laird (1966) defines growth to be approximated by an exponential decay with respect to time.

Contrary to this the polynomial model presents 1-4 coefficients which are partial slopes of the entire curve. In addition the magnitude of these coefficients is of such an order that in the cow prior to day 100, the curve is approximated by a linear function. After this the powers of the independent variable reach such a magnitude that they exhibit sufficient influence on the curve with the highest beta coefficient increasing in this influence as the time interval expands.

Rates of growth estimated from the first derivative of the polynomial accepted as the growth model are presented in Figures 8-11. The rates of growth for body weight, crown-rump, and hindfoot are used because of their common acceptance as criterion for aging.

Dog growth in Figure 9 is a linear process decreasing in magnitude to the onset of parturition. In contrast to this the human growth rate in Figure 11 shows an initial linear decrease which then levels off and is followed by a slight rise due to the influence of the positive third beta coefficient. The cow and horse rates show initial linear decay followed by a levelling off which is then terminated by a rise if the model was of the third order or a decrease to a negative growth if the model was of the fourth order. This negative growth displayed by the fourth order model reflects the invalidity of this application. In all cases the linear dimension growth rates were multiplied by a factor of three to approximate the range of the body weight.

These growth rates suggest that there is a possible relation between the rate of growth, the length of gestation, and the percentage of gestation spent in development as opposed to the percent of gestation employed in maturation of already established organs. The linear decay of growth as evidenced during the entire period of gestation for the dog is probably due to the high percentage of the gestation period spent in development, whereas the cow, horse, and human have been established as a fetus by the end of the first third of the gestation when the decreasing linear trend levels to a constant growth rate.

Allometric relationships exist between organs or between body and organ when the logarithmic transformation of dimensions compared results in a linear relationship. Figures 12 and 13 are representative of the regression lines calculated in an allometric analysis of a weight versus a linear dimension and a linear dimension versus a linear dimension. Tables VI and VII of the Appendix show the coefficients of determination as calculated for all possible comparisons of the organs and weights analyzed. This was done in order to provide a mathematical basis for accepting one or more linear measurements as a criterion for aging.

As a premise is the fact that an animal is composed of many linear dimensions which are only a part of the whole and if summed and cubed are not reflective of the dimension of the whole body. All of the allometric relationships show a high degree of correlation with each other and to the body weight. The human data is presented only as a point of comparison and is not considered for analysis.

Since extremely high correlations exist and it is not feasible to separate these correlations into groups which fit better than others, those correlations which showed a consistency in and between species became the point of investigation.

A consistent high correlation does exist between crown-rump and body weight, contour and body weight, and trunk and body weight in the cow, dog, and horse. Furthermore the correlations between crown-rump and contour, crown-rump and trunk, and contour and trunk show a similar consistency in the cow and horse. Dog crown-rump and contour correlations are somewhat low in comparison to the levels of crown-rump and trunk, or contour and trunk relationships.

The hindfoot which is commonly used as a determinant of age does not differ significantly from the correlations achieved by the forefoot either with body weight or any of the other organs used in comparisons. Likewise the hindfoot and forefoot do not exhibit consistent correlations with crown-rump, contour, or trunk in the species examined.

On the basis of the high correlations expressed by the crown-rump, contour, and trunk as compared to the body weight, or compared among themselves, these measurements should be employed preferable in connection with each other to determine the age of the animal studied. Certainly no one linear dimension can consistently and unequivocally be the best determinant, but employment of these dimensions with the realization that variation is the hallmark of growth would probably give the best results in the majority of cases.

Summary

Comparison of the polynomial and exponential models indicates that the polynomial has the capacity to provide the best fit to the observed data in the majority of cases studied employing cow, dog, horse, and human linear dimensions and body weight.

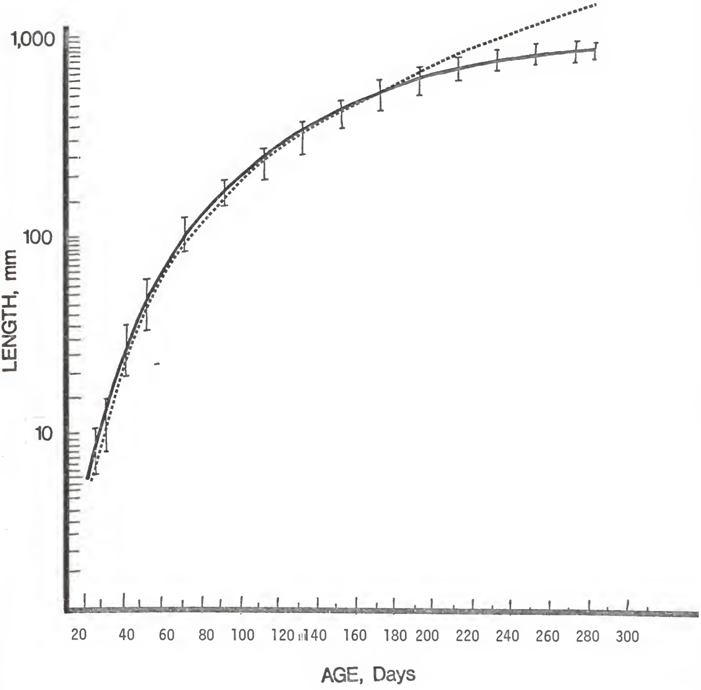
Rate of growth at any point of time as derived from the polynomial indicates that the growth process with the exception of the dog is not a constant process, but one that modifies itself with the passage of embryonic development and the establishment of fetal growth and maturation.

The allometric relationships provide information on those organs which show relative growth of the same magnitude and as a result are indicative of those linear dimensions which best approximate the geometric growth of the body.

EXPLANATION OF PLATE VI

Figure 6. Cow crown-rump measurement fit by the polynomial model (solid line) and the exponential model (dotted line).

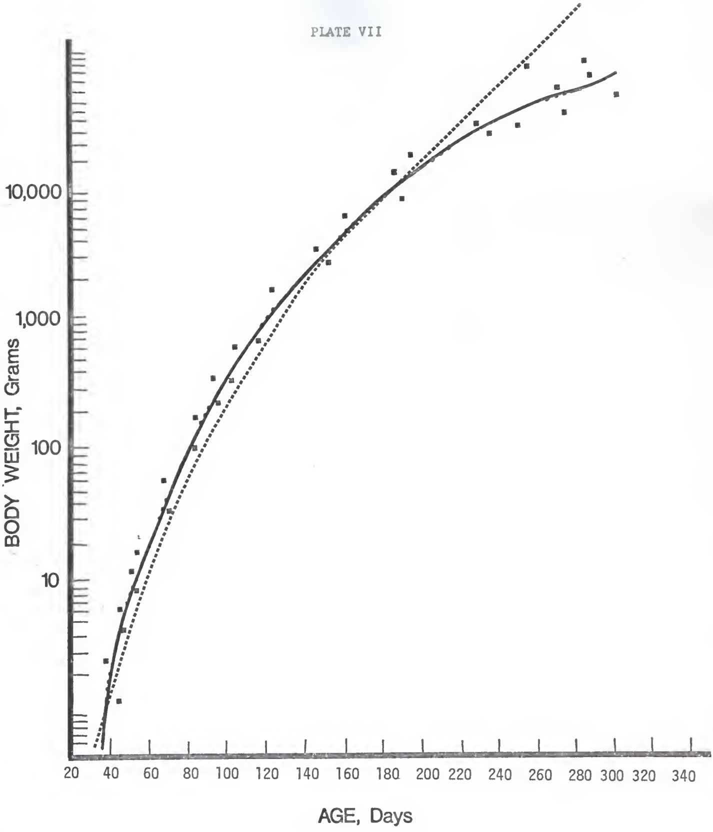
PLATE VI


1,000
100
10
LENGTH, mm
20 40 60 80 100 120 140 160 180 200 220 240 260 280 300
AGE, Days

EXPLANATION OF PLATE VII

Figure 7. Cow body weight as estimated by the polynomial model (solid line) and the exponential model (dotted line).

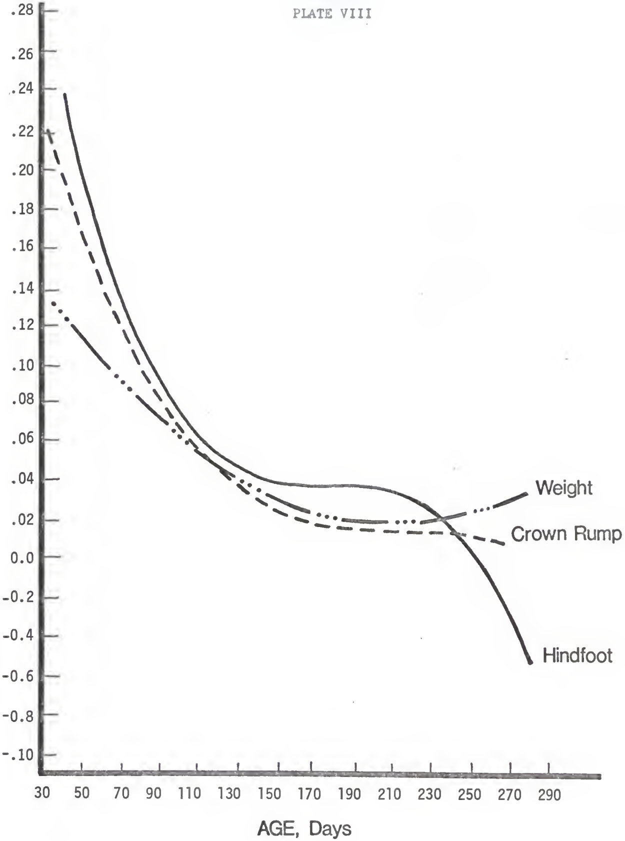
PLATE VII


BODY WEIGHT, Grams
10,000
1,000
100
10
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
AGE, Days

EXPLANATION OF PLATE VIII

Figure 8. Cow growth rates for the body weight, crown-rump, and hindfoot as determined by the first derivative of the accepted polynomial model.

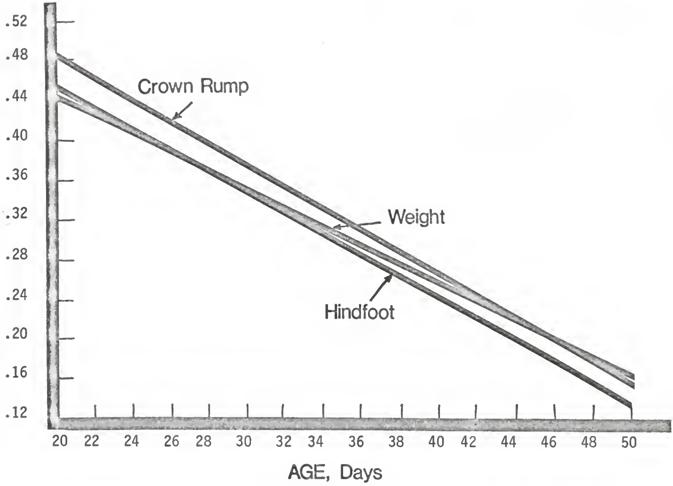
PLATE VIII


.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0.0
-0.2
-0.4
-0.6
-0.8
-.10
30
50
70
90
110
130
150
170
190
210
230
250
270
290
AGE, Days
Weight
Crown Rump
Hindfoot

EXPLANATION OF PLATE IX

Figure 9. Dog growth rates for the body weight, crown-rump, and hindfoot as determined by the first derivative of the accepted polynomial model.

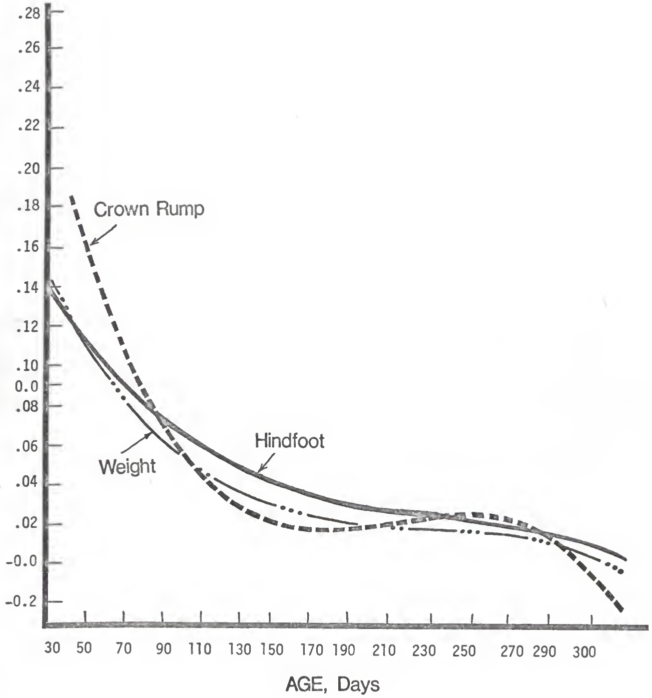
PLATE IX


.52
.48
.44
.40
.36
.32
.28
.24
.20
.16
.12
Crown Rump
Weight
Hindfoot
20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
AGE, Days

EXPLANATION OF PLATE X

Figure 10. Horse growth rates for the body weight, crown-rump, and hindfoot as determined by the first derivative of the accepted polynomial model.

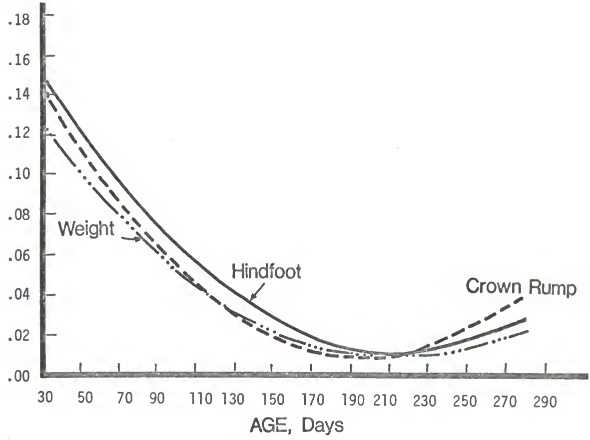
PLATE X


.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
0.0
.08
.06
.04
.02
-0.0
-0.2
Crown Rump
Hindfoot
Weight
30 50 70 90 110 130 150 170 190 210 230 250 270 290 300
AGE, Days

EXPLANATION OF PLATE XI

Figure 11. Human growth rates for the body weight, crown-rump and foot as determined by the first derivative of the accepted polynomial model.

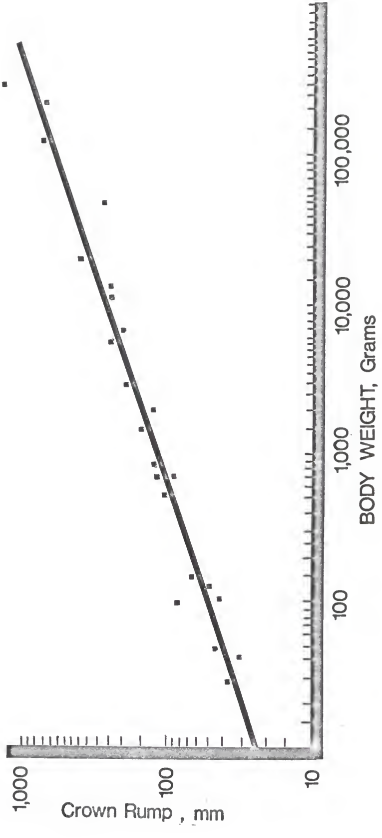
PLATE XI


.18
.16
.14
.12
.10
.08
.06
.04
.02
.00
Weight
Hindfoot
Crown Rump
30 50 70 90 110 130 150 170 190 210 230 250 270 290
AGE, Days

EXPLANATION OF PLATE XII

Figure 12. Allometric plot of horse crown-rump in millimeters against horse body weight in grams.

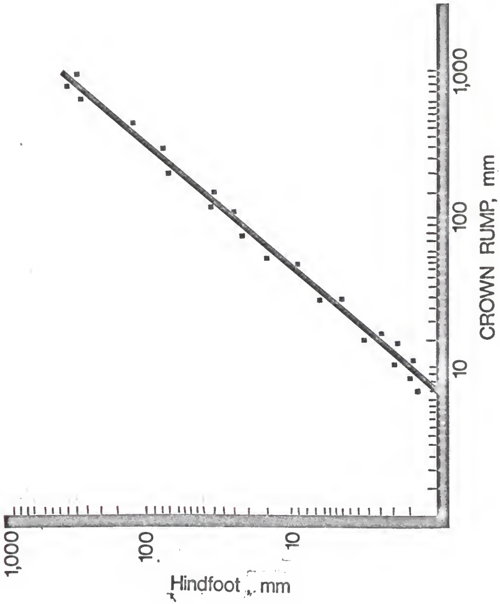
PLATE XII


1,000
100
10
Crown Rump , mm
100
1,000
10,000
100,000
BODY WEIGHT, Grams

EXPLANATION OF PLATE XIII

Figure 13. Allometric plot of cow hindfoot in millimeters against crown-rump in millimeters.

PLATE XIII



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APPENDIX

APPENDIX TABLE 1. Polynomial Coefficients for Body Weight ($\log Y = \alpha + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \beta_4 X^4$).

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	.3197	.191 ± .005			
	-1.2672	.482 ± .011	- 1.030 ± .037		
	-2.2881	.767 ± .019	- 3.260 ± .137	.510 ± .031	
	-3.063	1.073 ± .074	- 7.130 ± .090	2.431 ± .442	- .324 ± .074
CANINE	-2.323	.898 ± .028			
	-6.5240	2.918 ± .243	-23.420 ± 2.800		
	-6.3690	2.789 ± 1.985	-20.170 ± 47.708	-2.533 ± 37.471	
	-6.7610	3.042 ± 1.145	-27.740 ± 45.61	8.741 ± 71.841	-5.519 ± 37.917
EQUINE	.4032	.145 ± .007			
	-1.1139	.397 ± .015	- .686 ± .040		
	-2.0340	.639 ± .030	- 2.280 ± .189	.292 ± .034	
	-2.7650	.904 ± .081	- 5.190 ± .852	1.493 ± .344	- .163 ± .047
HUMAN	.0714	.152 ± .006			
	-1.3653	.414 ± .012	- .901 ± .040		
	-2.427	.741 ± .009	- 3.186 ± .073	.508 ± .016	
	-2.818	.873 ± .060	- 5.168 ± .714	1.475 ± .336	- .154 ± .054

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Ear.

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	.1160	.086 ± .003			
	-0.8000	.237 ± .008	- .506 ± .025		
	-1.2970	.358 ± .024	- 1.359 ± .167		
	-1.670	.490 ± .134	- 2.866 ± 1.481	.873 ± .674	- 0.111 ± .107
CANINE	- .880	.373 ± .016			
	-1.7295	.774 ± .192	- 4.590 ± 2.186		
	3.9680	-3.406 ± 2.402	+96.159 ± 57.870	-79.650 ± 45.586	
	.9534	- .989 ± .546	25.310 ± 27.760	7.500 ± 45.500	-35.700 ± 21.320
EQUINE	- .8528	.063 ± .003			
	-1.278	.132 ± .012	- .182 ± .031		
	- .9342	.044 ± .035	- .376 ± .214	- .010 ± .377	
	- .1752	- .231 ± .082	3.391 ± .840	- 1.331 ± .339	.170 ± .045

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Hindfoot.

	α	β_1 X 10^{-1}	β_2 X 15^{-4}	β_3 X 10^{-6}	β_4 X 10^{-8}
BOVINE	.6566	.090 ± .003			
	-0.1557	.240 ± .006	- .540 ± .022		
	-0.7595	.415 ± .012	- 1.918 ± .091		
	-1.3932	.674 ± .030	- 5.273 ± .373	1.992 ± .183	- 0.283 ± .031
CANINE	- .4047	.361 ± .011			
	-1.4880	.927 ± .084	- 7.000 ± 1.030		
	.8433	-.953 ± .628	+41.520 ± 16.080	-40.190 ± 13.290	
	- .2467	.084 ± .539	6.517 ± 23.110	9.620 ± 38.190	-25.230 ± 20.570
EQUINE	- .0510	.057 ± .002			
	- .5976	.143 ± .005	- .227 ± .012		
	- .8553	.208 ± .012	- .638 ± .073	.073 ± .013	
	-1.0524	.274 ± .010	- 1.341 ± .104	.357 ± .041	- .038 ± .005
HUMAN	.6376	.055 ± .002			
	.0087	.160 ± .005	- .342 ± .015		
	- .4794	.281 ± .008	- 1.200 ± .053	.180 ± .011	
	- .7053	.355 ± .034	- 2.020 ± .374	.547 ± .165	- .057 ± .025

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Trunk.

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	1.0890	.090 ± .004			
	.4494	.216 ± .006	- .510 ± .024		
	.0452	.248 ± .002	- 1.677 ± .099	.293 ± .025	
	- .4276	.561 ± .035	- 4.667 ± .476	1.912 ± .254	- .295 ± .046
CANINE	- .0670	.408 ± .012			
	-1.3110	1.108 ± .086	- 8.647 ± 1.090		
	.1152	- .098 ± .734	+22.840 ± 19.430	-26.880 ± 16.540	
	- .7576	.754 ± .506	- 7.250 ± 24.590	18.200 ± 43.75	-24.180 ± 25.100
EQUINE	.3328	.051 ± .002			
	- .2162	.142 ± .005	- .244 ± .013		
	.5920	.239 ± .010	- .874 ± .061	.114 ± .011	
	-1.089	.418 ± .021	- 2.832 ± .223	.914 ± .090	- .109 ± .012

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Head Length.

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	.9913	.070 ± .002			
	.3535	.190 ± .006	− .459 ± .020		
	− .1167	.333 ± .011	− 1.638 ± .082	.281 ± .019	
	− .6052	.547 ± .033	− 4.516 ± .418	1.767 ± .211	− .2583 ± .037
CANINE	.0756	.315 ± .009			
	−1.0202	.905 ± .048	− 7.462 ± .606		
	−1.4394	1.253 ± .277	−16.680 ± 7.270	7.851 ± 6.153	
	− .5543	.324 ± .376	18.540 ± 16.430	−49.480 ± 28.830	33.860 ± 17.470
EQUINE	.1317	.046 ± .002			
	− .3860	.131 ± .004	− .225 ± .011		
	− .6457	.197 ± .010	− .655 ± .063	.078 ± .011	
	− .8664	.277 ± .027	− 1.520 ± .277	.431 ± .111	− .048 ± .015
HUMAN	1.5920	.038 ± .002			
	1.1010	.114 ± .004	− .244 ± .012		
	.6850	.213 ± .006	− .919 ± .041	.139 ± .008	
	.350	.323 ± .029	− 2.130 ± .315	.675 ± .137	− .0821 ± .021

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Crown Rump.

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	1.3407	.076 ± .003			
	.6583	.211 ± .006	- 0.496 ± .020		
	.1484	.370 ± .009	- 1.818 ± .069	.315 ± .016	
	.3245	.584 ± .030	- 4.745 ± .389	1.835 ± .091	- .265 ± .034
CANINE	.0267	.433 ± .009			
	- .9600	1.019 ± .040	- 7.945 ± .539		
	.0676	.059 ± .180	19.865 ± 5.148	- 25.220 ± 4.645	
	-1.5340	2.025 ± .108	-66.670 ± 4.940	135.950 ± 9.35	-107.120 ± 6.269
EQUINE	.4773	.053 ± .003			
	- .1642	.161 ± .007	- .292 ± .019		
	.6600	.296 ± .013	- 1.200 ± .080	.167 ± .015	
	-1.1872	.502 ± .021	- 3.560 ± .222	1.160 ± .091	- .137 ± .013
HUMAN	1.3290	.056 ± .003			
	.7480	.167 ± .005	- .404 ± .019		
	.3260	.298 ± .006	- 1.500 ± .047	.266 ± .011	
	.1200	.382 ± .020	- 2.650 ± .260	.873 ± .130	- .108 ± .024

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Contour

	α	β_1 X 10^{-1}	β_2 X 15^{-4}	β_3 X 10^{-6}	β_4 X 10^{-8}
BOVINE	1.6520	.071 ± .002			
	1.1200	.176 ± .005	- .394 ± .015		
	.7870	.280 ± .007	- 1.276 ± .059	.213 ± .014	
	.462	.429 ± .027	- 3.313 ± .332	1.274 ± .169	- 0.185 ± .029
CANINE	.6798	.344 ± .011			
	- .4506	.977 ± .061	- .812 ± .078		
	-1.292	1.721 ± .340	-29.120 ± 9.372	18.580 ± 8.273	
	- .0998	.375 ± .414	-25.160 ± 19.76	-74.810 ± 36.960	58.150 ± 23.610
EQUINE	.7809	.048 ± .002			
	.2761	.134 ± .006	- .233 ± .016		
	- .0578	.224 ± .013	- .839 ± .083	.111 ± .015	
	- .4012	.364 ± .040	- 2.454 ± .433	.794 ± .179	- .094 ± .025

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Forefoot.

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	.6123	.085 ± .003			
	-0.1241	.224 ± .006	-0.501 ± .023		
	-0.6366	.375 ± .013	-1.714 ± .097	.283 ± .022	
	-1.094	.057 ± .038	-4.254 ± .477	1.572 ± .238	
EQUINE	- .1095	.057 ± .002			
	- .6366	.138 ± .005	- .213 ± .013		
	- .8421	.189 ± .014	- .537 ± .084	.057 ± .015	
	-1.023	.252 ± .031	- .121 ± .031	.329 ± .124	-.037 ± .017

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Tail.

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	.7772	.080 ± .003			
	.1648	.198 ± .006	− .427 ± .023		
	− .2253	.317 ± .014	− 1.400 ± .110	.229 ± .026	
	− .6621	.509 ± .039	− 4.019 ± .506	1.582 ± .257	− .234 ± .044
CANINE	− .3843	.408 ± .014			
	−1.3202	.911 ± .112	− 6.360 ± 1.400		
	3.0200	−2.665 ± .899	88.080 ± 23.630	−80.110 ± 19.980	
	.5256	− .257 ± .659	5.106 ± 29.840	40.499 ± 51.085	−62.170 ± 28.220
EQUINE	− .1509	.051 ± .002			
	− .6244	.126 ± .005	− .197 ± .014		
	− .8154	.173 ± .015	− .492 ± .092	.053 ± .016	
	− .7385	.146 ± .050	− .206 ± .508	− .064 ± .201	.016 ± .270

APPENDIX TABLE 1. (cont.) Polynomial Coefficients for Head Width.

	α	$\beta_1 \times 10^{-1}$	$\beta_2 \times 10^{-4}$	$\beta_3 \times 10^{-6}$	$\beta_4 \times 10^{-8}$
BOVINE	1.1465	.046 ± .002			
	.5744	.139 ± .008	- .327 ± .027		
	- .1053	.306 ± .021	-1.566 ± .150	.280 ± .037	
	-1.294	.709 ± .112	-6.254 ± 1.274	2.520 ± .603	-.376 ± .101
EQUINE	- .0708	.041 ± .002			
	- .5990	.127 ± .006	- .228 ± .015		
	- .9429	.215 ± .014	- .795 ± .085	.102 ± .015	
	-1.3659	.370 ± .058	-2.479 ± .6049	.786 ± .243	-.093 ± .033

APPENDIX TABLE 2. Polynomial Statistics for Body Weight.
($\log Y = \alpha + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \beta_4 X^4$)

	Order	R	R^2	Deviation	N
BOVINE	1	.9470	.8969	1988867000.	148
	2	.9910	.9830	17675060.	
	3	.9970	.9940	10826040.	
	4	.9940	.9890	5809748.	
CANINE	1	.9584	.9180	6374.	94
	2	.9766	.9538	716.	
	3	.9804	.9612	703.	
	4	1.0000	1.0000	1229.	
EQUINE	1	.9377	.8794	1823286000.	57
	2	.9902	.9806	55942370.	
	3	.9957	.9915	6348970.	
	4	.9960	.9920	4548442.	
HUMAN	1	.9350	.8743	9787651.	98
	2	.9899	.9800	290898.	
	3	.9991	.9983	33765.	
	4	.9961	.9920	41540.	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Crown Rump.

	Order	R	R^2	Deviation	N
BOVINE	1	.0934	.8343	77379.	284
	2	.9807	.9617	7431.	
	3	.9939	.9878	3151.	
	4	.9897	.9796	1480.	
CANINE	1	.9605	.9226	1283.	197
	2	.9815	.9634	105.	
	3	.9835	.9672	112.	
	4	.9992	.9985	160.	
EQUINE	1	.9066	.8219	767.	89
	2	.9757	.9525	150.	
	3	.9906	.9813	53.	
	4	.9960	.9920	22.	
HUMAN	1	.9183	.8432	10956.	95
	2	.9863	.9727	957	
	3	.9981	.9962	163.	
	4	1.0000	1.0000	33.	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Contour.

	Order	R	R^2	Deviation	N
BOVINE	1	.9364	.8969	112820.	180
	2	.9870	.9743	10073.	
	3	.9945	.9891	5949.	
	4	.9901	.9803	3305.	
CANINE	1	.9447	.8925	1192.	106
	2	.9731	.9470	272.	
	3	.9715	.9439	331.	
	4	.9691	.9392	345.	
EQUINE	1	.9276	.8604	1049.	89
	2	.9793	.9590	174.	
	3	.9873	.9747	64.	
	4	.9813	.9629	73.	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Trunk.

	Order	R	R^2	Deviation	N
BOVINE	1	.9243	.8543	44440.	94
	2	.9877	.9755	2040.	
	3	.9952	.9904	626.	
	4	.9958	.9917	387.	
CANINE	1	.9485	.8996	654.	130
	2	.9655	.9327	486.	
	3	.9623	.9261	581.	
	4	.9638	.9289	497.	
EQUINE	1	.9408	.8851	248.	84
	2	.9886	.9774	36.	
	3	.9950	.9902	11.	
	4	.9976	.9925	13.	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Head Width.

	Order	R	R^2	Deviation	N
BOVINE	1	.9169	.8407	217.3	69
	2	.9754	.9515	44.7	
	3	.9880	.9763	24.0	
	4	.9766	.9537	21.2	
EQUINE	1	.9151	.8372	8.6	85
	2	.9783	.9570	2.0	
	3	.9859	.9719	1.0	
	4	.9756	.9518	1.2	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Tail.

	Order	R	R^2	Deviation	N
BOVINE	1	.9337	.8717	8767.	151
	2	.9807	.9617	730.	
	3	.9876	.9753	423.	
	4	.9875	.9751	325.	
CANINE	1	.9387	.8811	71.6	115
	2	.9484	.8995	29.6	
	3	.9484	.8995	31.8	
	4	.9521	.9064	29.9	
EQUINE	1	.9573	.9164	26.7	76
	2	.9891	.9783	8.9	
	3	.9901	.9804	7.3	
	4	.9879	.9760	7.2	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Hind Foot.

	Order	R	R^2	Deviation	N
BOVINE	1	.9237	.8532	22447.	180
	2	.9831	.9665	1192.	
	3	.9928	.9856	644.7	
	4	.9940	.9881	207.2	
CANINE	1	.9416	.8867	41.2	127
	2	.9576	.9171	12.0	
	3	.9608	.9232	16.0	
	4	.9575	.9169	25.6	
EQUINE	1	.9584	.9186	104.	103
	2	.9914	.9829	14.3	
	3	.9930	.9861	9.3	
	4	.9993	.9986	10.2	
HUMAN	1	.9368	.8776	265.	89
	2	.9915	.9831	33.	
	3	.9978	.9957	8.	
	4	1.0000	1.0000	6.	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Front Foot.

	Order	R	R^2	Deviation	N
BOVINE	1	.9310	.8668	7521.	159
	2	.9835	.9674	657.	
	3	.9920	.9841	293.	
	4	.9919	.9830	136.	
EQUINE	1	.9595	.9207	73.4	99
	2	.9898	.9797	10.5	
	3	.9907	.9814	8.1	
	4	.9937	.9875	9.0	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Head Length.

	Order	R	R^2	Deviation	N
BOVINE	1	.9009	.8116	4317.	175
	2	.9760	.9527	397.	
	3	.9894	.9790	185.	
	4	.9857	.9717	90.	
CANINE	1	.9462	.8952	51.	162
	2	.9728	.9463	11.4	
	3	.9756	.9518	12.1	
	4	.9723	.9453	11.2	
EQUINE	1	.9394	.8825	47.	83
	2.	.9899	.9802	5.0	
	3	.9934	.9870	1.7	
	4	.9948	.9896	1.6	
HUMAN	1	.9302	.8453	1561.	80
	2	.9889	.9779	264.	
	3	.9975	.9951	72	
	4	.9953	.9907	50	

APPENDIX TABLE 2. (cont.) Polynomial Statistics for Ear.

	Order	R	R^2	Deviation	N
BOVINE	1	.9441	.8919	1117.	124
	2	.9875	.9751	40.9	
	3	.9895	.9791	24.6	
	4	.9731	.9469	39.4	
CANINE	1	.9304	.8656	71.	92
	2	.9338	.8720	89.	
	3	.9090	.8263	98.	
	4	.9732	.9471	93.	
EQUINE	1	.9422	.8878	13.1	78
	2	.9604	.9225	1.8	
	3	.9635	.9283	1.9	
	4	.9690	.9405	2.2	

APPENDIX TABLE 3. Exponential Least-Squares Coefficients.
($\log Y = \log \alpha + \beta_1 \log X$)

	$\log \alpha$	β_1
	BODY WEIGHT	
BOVINE	- 8.270	5.396 ± .044
CANINE	-12.759	8.796 ± .220
EQUINE	- 7.780	5.014 ± .077
HUMAN	- 6.949	4.408 ± .052
	CROWN RUMP	
BOVINE	- 1.837	2.042 ± .022
CANINE	- 3.696	3.439 ± .053
EQUINE	- 2.494	1.827 ± .013
HUMAN	- 1.075	1.533 ± .024
	CONTOUR	
BOVINE	- 1.170	1.827 ± .013
CANINE	- 2.616	2.942 ± .071
EQUINE	- 1.850	1.631 ± .025
	TRUNK	
BOVINE	- 2.183	2.132 ± .027
CANINE	- 4.092	3.565 ± .084
EQUINE	- 2.617	1.805 ± .024
	FOREFOOT	
BOVINE	- 3.084	2.345 ± .028
EQUINE	- 3.456	2.029 ± .028
	EAR	
BOVINE	- 4.344	2.703 ± .044
CANINE	- 5.258	3.677 ± .149
EQUINE	- 4.308	2.137 ± .085

APPENDIX TABLE 3. (cont.) Exponential Least-Squares Coefficients

	log α	β_1
	TAIL	
BOVINE	-2.573	2.148 ± .026
CANINE	-4.423	3.569 ± .115
EQUINE	-3.153	1.820 ± .031
	HEAD LENGTH	
BOVINE	-1.848	1.809 ± .026
CANINE	-3.078	2.779 ± .056
EQUINE	-2.556	1.638 ± .025
HUMAN	- .512	1.256 ± .025
	HEAD WIDTH	
BOVINE	-1.203	1.417 ± .042
EQUINE	-2.504	1.475 ± .033
	HINDFOOT	
BOVINE	-3.313	2.500 ± .029
CANINE	-4.128	3.240 ± .091
EQUINE	-3.347	2.007 ± .039
HUMAN	-2.252	1.754 ± .029

APPENDIX TABLE 4. Exponential Least-Squares Statistics.
($\log Y = \log \alpha + \beta_1 \log X$)

	R	R^2	Deviation	N
BODY WEIGHT				
BOVINE	.9950	.9901	36287124.	148
CANINE	.9710	.9400	2413.	94
EQUINE	.9901	.9869	92897302.	57
HUMAN	.9931	.9863	40805.	98
CROWN RUMP				
BOVINE	.9890	.9781	7419.	184
CANINE	.9771	.9548	255.	197
EQUINE	.9871	.9745	112	89
HUMAN	.9886	.9778	800.	98
CONTOUR				
BOVINE	.9950	.9901	37896974.	180
CANINE	.9711	.9434	424.	106
EQUINE	.9905	.9803	140	89
TRUNK				
BOVINE	.9925	.9851	256.	94
CANINE	.9626	.9264	138.	130
EQUINE	.9930	.9861	29.	84
TAIL				
BOVINE	.9892	.9786	499.	151
CANINE	.9460	.8949	35.	115
EQUINE	.9887	.9776	8.2	76

APPENDIX TABLE 4. (cont.) Exponential-Least Squares Statistics.

	R	R^2	Deviation	N
HEAD LENGTH				
BOVINE	.9823	.9649	84.	175
CANINE	.9687	.9342	32.	162
EQUINE	.9907	.9815	7.2	83
HUMAN	.9848	.9698	8.5	80
HEAD WIDTH				
BOVINE	.9711	.9431	61.	69
EQUINE	.9787	.9579	2.4	85
HINDFOOT				
BOVINE	.9886	.9773	1509.	180
CANINE	.9594	.9111	9.	127
EQUINE	.9816	.9635	12.9	104
HUMAN	.9876	.9753	70.6	89
FOREFOOT				
BOVINE	.9889	.9779	563.	159
EQUINE	.9909	.9819	10.7	99
EAR				
BOVINE	.9838	.9678	176.	124
CANINE	.9331	.8706	2.	92
EQUINE	.9445	.8922	7.9	78

APPENDIX TABLE 5. Iterative Coefficients and Statistics for Exponential Model $Y = \alpha X^{\beta}$.

	α	β	R	Deviation
BOVINE TAIL	.0104	1.881	.9926	308.
EQUINE HEAD LENGTH	.0146	1.374	.9953	3.5
HUMAN HEAD LENGTH	.8562	1.051	.9973	186.
EQUINE HEAD WIDTH	.0160	1.160	.9912	1.3

APPENDIX TABLE 6. Coefficients of Determination for Allometric Comparisons

BOVINE									
CROWN RUMP	.9917								
CONTOUR	.9975	.9880							
TRUNK	.9926	.9938	.9981						
TAIL	.9885	.9833	.9555	.9833					
HEAD LENGTH	.8926	.9255	.9684	.9561	.9577				
HEAD WIDTH	.9602	.9644	.9592	.9612	.9569	.9118			
HINDFOOT	.9642	.9745	.9821	.9954	.9845	.9646	.9706		
FOREFOOT	.9942	.9916	.9417	.9931	.9840	.9566	.9575	.9952	
EAR	.9826	.9727	.9254	.9581	.9722	.9264	.9684	.9778	.9741
	WEIGHT	CROWN R.	CONTOUR	TRUNK	TAIL	HEAD L.	HEAD W.	H.FT.	F.FT.

CANINE							
CROWN RUMP	.9730						
CONTOUR	.9910	.9517					
TRUNK	.9176	.9636	.9659				
TAIL	.9109	.9001	.9721	.9529			
HEAD LENGTH	.9560	.9434	.9798	.9796	.9423		
HINDFOOT	.9835	.9741	.9857	.9585	.9785	.9802	
EAR	.9550	.9588	.9546	.9219	.9042	.9610	.9406
	WEIGHT	CROWN R.	CONTOUR	TRUNK	TAIL	HEAD L.	HIND FT.

APPENDIX TABLE 7. Coefficients of Determination for Allometric Relationships

EQUINE									
CROWN RUMP	.9916								
CONTOUR	.9935	.9880							
TRUNK	.9951	.9965	.9931						
TAIL	.9708	.9611	.9799	.9684					
HEAD LENGTH	.9835	.9862	.9881	.9886	.9650				
HEAD WIDTH	.9750	.9742	.9676	.9737	.9410	.9776			
HINDFOOT	.9814	.9762	.9897	.9851	.9743	.9828	.9536		
FOREFOOT	.9747	.9706	.9868	.9814	.9733	.9794	.9606	.9936	
EAR	.9635	.9455	.9685	.9581	.9652	.9692	.9271	.9735	.9729
	WEIGHT	CROWN R.	CONTOUR	TRUNK	TAIL	HEAD L.	HEAD W.	H.FT.	F.FT.

HUMAN			
CROWN	.9965		
HEAD MODULUS	.9955	.9949	
FOOT	.9965	.9944	.9948
	WEIGHT	CROWN R.	HEAD MOD.

ANALYSIS OF EXTERNAL GROWTH OF MAMMALIAN FETUSES

by

DALE LAMBERT PRESTON

B.S., Kansas State University, 1966

AN ABSTRACT OF A MASTER'S THESIS

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ABSTRACT

Analysis of four types of mammalian embryos and fetuses of cow, dog, horse, and human was conducted by visual comparison of external measurements and body weight plotted on semi-logarithmic scales and application was made of two statistical models to determine theoretical estimates and correlations with time. The data analyzed consisted of a maximum of 184 observations on the cow, 197 observations on the dog, 89 observations on the horse, and 98 mean values compiled from approximately 1200 observations on the human recorded by G.L. Streeter.

External organ dimensions and body weight are often used as determinants of embryonic or fetal age. In the cow, dog, horse, and human there are linear dimensions which can yield close approximations of age. The crown-rump, contour, trunk, and hindfoot express little variation throughout gestation, are present through the majority of the gestation period, and do not reflect the breed heterogeneity with the approach of partuition.

Other body measurements such as tail, ear, head length, head width, and forefoot display too much variation during gestation and should not be used as a criterion of age.

Application of two statistical models, one a polynomial and the other an exponential, to the linear dimensions and the cubic weight measurements of the cow, dog, horse, and human determined the theoretical estimates and the correlation with time.

This comparison of the fit of the polynomial and the exponential models supported the previous acceptance of the crown-rump, contour, and trunk. Hindfoot, despite the common application of this measurement, did not reflect the high degree of correlation that was expressed by the

crown-rump, contour, and trunk.

From the polynomial model the rate of growth was determined by taking the first derivative of the equation. This revealed a significant difference in the rate of growth in the cow, dog, horse, and human. The dog which was fit by a second order polynomial had a constant linear decay of the growth rate. The human was fit entirely by third order equations and in the case of the cow and horse by third and fourth order equations. As a result initial linear decay of the growth rate in the cow, horse, and human was followed by a period of leveling off which was terminated by a slight rise if the model was of the third order or a decay if the model fit was of the fourth order.

This rate of growth as determined by the best fit polynomial equation indicates a possible relation of the rate of growth with the length of gestation and the percent of gestation spent in development with respect to the percent of gestation spent in maturation.

Allometric relationships between linear dimensions and body weight gave indication of similar rates of growth between crown-rump and body weight, contour and body weight, trunk and body weight, crown-rump and contour, and trunk and contour.

On the basis of the minimum variation during gestation, high correlation with the time of gestation, parallel growth as determined by polynomial derivatives and allometric relationships, the only linear dimensions which are consistent determinants of age are the crown-rump, contour, and trunk.