PROBLEMS OF INVENTORY CONTROL IN FEED MILLS

by

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INTRODUCTION

Much of the investment currently being made in new facili-
ties, equipment and operational costs within the feed industry
is determined by intuition and judgment. Substantial improvement
in efficiency and higher profits can be achieved by applying
techniques presently available in Operations Research: inventory
control theory, linear programming, queuing theory, to mention
only some of them. These techniques are tools which can be used
to size some of the facilities required in the feed industry, or
any other industry, and to help management in the determination
of optimal operational policies.

This thesis describes and analyses the problem of finished
products inventory control in a feed mill with two main objectives,
depending upon the actual situation at a given time: first, for
the mill which is going to be built, to determine the size or
capacity of the storage facilities and the best way to handle
the products through them, and second, for the mill already pro-
ducing, to optimize actual operation.

In the feed business, customers do not place orders at
regular time intervals; like any other customers, they are not
always patient if their orders are delayed. Management should
establish some optimal or nearly optimal policy that combines
a proper production schedule with an adequate quantity of
stock, so the total operational cost will be a minimum and the
customer demands can be met at all times. Emergencies arising

from the possibility of being out-of-stock at a particular time
will disrupt the regular production schedule and reduce pro-
duction rate because of frequent formula changes.

Particular to feed mills is also the fact that they produce
a large number of different products with variations in formulas,
physical form (mash, pellets, crumbles), and variations in the
way they are sold, namely in bags or in bulk. Furthermore, the
products vary widely in demand. There are formulas for which
orders are received every day (poultry, swine or dairy feeds),
and others that are very seldom sold, such as rat or monkey
feed.

The present paper studies the products that, being demanded
in relatively high levels, are sold in bulk. In other words, it
is concerned with inventory control system for bulk feed and
optimal size of the bins that will hold such products in stock.

This is so only to simplify the application of the model,
but the theory itself can be extended without major difficulties
to the consideration of the whole system with all products (in
bags or in bulk) and facilities (bagged feed warehouse and bulk
feed bins).

The first part of the analysis is devoted to the determina-
tion of the demand distribution of a given feed per basic time
period. Considerations have been made for particular situations,
such as availability of historical data, market research or
typical pattern of demand within the feed industry, in the
construction of demand curves. Particular data from a typical

feed company is analyzed.

Once the demand distribution is set for a particular pro-
duct, the inventory control system is built upon the character-
istics of demand and costs as a regular application of the
theory available elsewhere. Some factors related to the size
of the facilities are included in the model so that when deriving
the parameters of the best inventory policy, the same model
produces the best size of facility to handle that inventory.

It is hoped that as a result of this work the feed industry
can start overcoming the old rules of thumb still being applied
in sizing warehouses and determining inventory policies. These
rules have been determined empirically for average situations
but are frequently inadequate in particular cases.

REVIEW OF LITERATURE

Among the techniques offered by operations research, linear programming has been the most widely used in the feed industry in what is known as "least-cost-formulation". Stafford and Snyder (13) have extended its utilization in building a model to evaluate courses of action in feed formulation, labor utilization, product mix and short run pricing policy.

Inventory control theory has been applied too in the feed industry as well as elsewhere. The same authors, Stafford and Snyder (14), published a paper in production planning and inventory control system in feed mills, in which they studied forecasting, inventory policy evaluation and allocation of productive resources.

Little has been published on sizing facilities within the feed industry using operations research tools. In the general field: Hancock and Kramer (6) investigated the influence of various parameters on warehouse sizing and developed a model in that respect. Homer (7) studied space-limited multiple item inventory with phased deliveries and derived formulas to determine warehouse size.

In the feed industry, only Pfost has published on sizing equipment (11). By applying inventory control models described by Morse in "Queues, Inventories and Maintenance," (10) Pfost has shown how to select and utilize selected feed mill facilities. He worked out the problem of deciding how large ingredient storage bins should be for a mill which is to be built; or given an

existing mill, which should be the best ordering policy for ingredients.

Most of the work in inventory control has been done assuming well known frequency distributions or density functions as demand distributions. Simulation has been used to some extent in those cases when demand has to be derived from combinations of two or more distributions (1). Waller (17) did some work in the analytical determination of this kind of distribution when studying methods of obtaining a distribution of the weight of an additive per portion of feed.

It is necessary that in general and particularly in the feed industry more realistic models be developed. Some of the major textbooks covering areas related to this thesis and additional references are listed at the end.

DETERMINATION OF THE DEMAND DISTRIBUTION

Definitions

It is known that the design of the inventory control policy
depends mainly on the demand distribution for the product being
studied. Of particular interest is the demand for feed in some
units per basic time period and/or per "lead time." As stated
in the Introduction, those feed formulas that have relatively
high demand levels and are produced to be sold in bulk will be
considered. This being the case we will choose the "ton" as
the unit to measure the demand, and the basic time period will be
measured in days. Thus, the demand per basic time period will be
given in tons/day.

To define the demand per lead time, we will first define
"lead time." Lead time "t", is the time required by the produc-
tion department of a feed mill to produce a given quantity of
feed needed for inventory. It is the time measured between the
instant an order is sent to the production department and the
instant that order, already filled, is sent back to inventory.

In general, the lead time is variable: in a feed mill,
that usually produces many different products, sometimes an
order for inventory can be filled almost instantaneously so
the lead time will be only the time needed to actually produce
the amount ordered; and sometimes the order has to wait in a
queue of several orders to be filled. We can take the average
of this variable lead time and consider it as the mean lead
time. Therefore, when we talk of lead time we will mean the

average lead time and consider it as a constant. (Notice that the lead time could be a fraction of a day.)

Lead time has to be measured in the same units chosen for the basic time period, likely in days.

Knowing the demand per day, the demand per lead time period is simply that demand times the lead time in days.

Definitions of demand per basic time period and per lead time period allow us to discuss distributions.

The demand distribution of a given feed formula in tons/day, and per lead time period have to be determined. The next sections are devoted to the actual determination of the first one. The demand distribution per lead time is easily determined by convoluting the demand distribution per day so many times as the lead time is a multiple of the basic time period (15).

Two Different Approaches

In determining the demand distribution in tons of feed per day one can consider two different approaches depending on the availability of historical data, market research studies or other sources of information.

The demand, tons/day, is the result or combination of the number of orders for the given feed, and the size of those orders.

If historical data exists from the mill being studied, or other similar mills, logically it would be easier to process that data in such way that analysis of the direct demand could be made.

If for any reason, the available data is insufficient to determine the demand directly, it might be possible to study separately the order size distribution and the distribution of the number of orders per day, and thus build up the distribution in tons/day.

If there is no historical data, a market research study is indicated. It is extremely difficult to obtain enough information from market research to build directly an estimated demand distribution. Therefore, the study of the market should be conducted to obtain information such as how many customers will the product have, how often will the customers order, and what size of order they would like. With this data, the final demand distribution can be built as before by combining the order size distribution and the distribution of the number of orders/day.

We shall investigate each of these two approaches separately. In order to do so, and as a method of showing how a practical situation can be analysed, data taken from a typical feed mill is used in the next section.

Analysis of Data

Data obtained from a feed mill of large production capacity is shown in the Appendix. The data represent the daily demand for a period of two months for nine high-demand formulas sold in bulk. The company owns the bulk trucks with which the feed is delivered. Each truck holds from 10 to 12 tons of feed (depending on the bulk weight of the formula) and is divided in four compartments. The price policy established by the

company is to charge the cost of transportation on a distance basis, independent of the size of the order placed by the customer. Salesmen discourage orders smaller than 3 tons.

The data has been analysed and some of the results are shown in Tables and Graphs in following pages. (Formulas are identified with the letter "F" and consecutive numbers.)

As shown in the sample distribution for order size (Table 1 and Figures 1 and 2), the company's price policy has had a marked influence on the way customers order. At least in the formulas with higher levels of demand (Figures 1A, 1B and 2A), the trend is to order a whole truck in order to minimize the cost of transportation per ton of feed. Nevertheless, in other formulas approximately all sizes have similar probabilities; in some others 1/2 truck is the most ordered size. It is noticed also that although salesmen discourage orders for less than 3 tons, there are still customers ordering as little as 1 ton. This might be due to the fact that those customers are relatively near to the feed mill and the price policy does not affect them much; or they are ordering special feeds used in small quantities.

For the order size distributions, there is no regular pattern in general, and at first glance it is difficult to fit them to any of the known distributions, with the exception of formulas F-8 and F-9 (Figs. 2C and 2D) which can be approximated by rectangular distributions. Later on it will be discussed how this data can be handled to fit functions which are general and easy to work with.

Table 1. Order size sample distribution of typical feeds sold in bulk.*

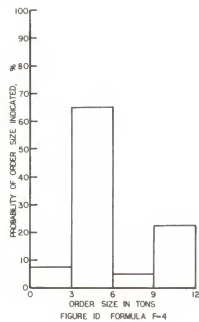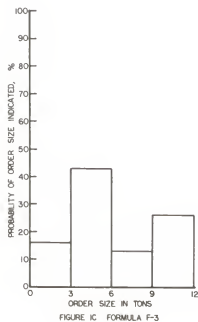| Size in tons	Frequency and (Probability, %)								
	F-1	F-2	F-3	F-4	F-5	F-6	F-7	F-8	F-9
0-2.9	9(5.5)	2(1.11)	9(17.0)	3(7.5)	6(9.0)	11(46.0)	3(4.5)	7(30.0)	7(26.0)
3-5.9	4(2.4)	3(1.60)	23(43.0)	26(65.0)	7(11.0)	6(25.0)	17(27.5)	5(22.0)	7(26.0)
6-8.9	11(6.7)	4(2.12)	7(13.0)	2(5.0)	10(15.0)	0(0.0)	8(13.0)	6(26.0)	7(26.0)
9-11.9	140(85.4)	180(95.25)	14(27.0)	9(22.5)	43(65.0)	7(29.0)	34(55.0)	5(22.0)	6(22.0)

*See Figs. 1 and 2.

FIGURE 1. ORDER SIZE SAMPLE DISTRIBUTIONS FOR FORMULAS F-1 TO F-4

FIGURE 2A. FORMULA F-5

FIGURE 2B. FORMULA F-7

FIGURE 2C. FORMULA F-8

FIGURE 2D. FORMULA F-9
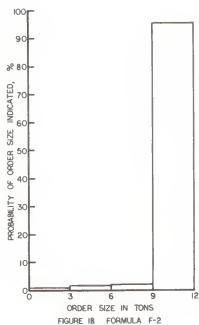
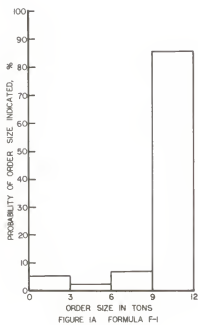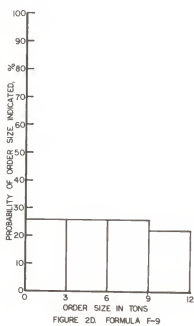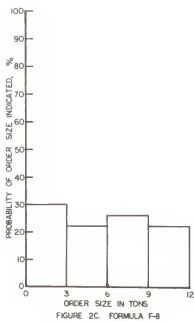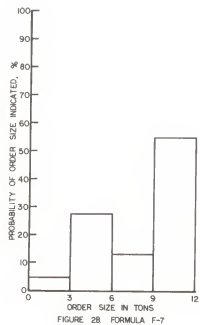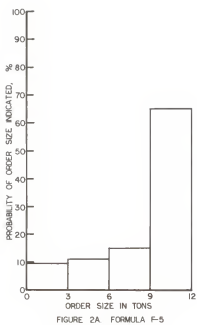FIGURE 2   ORDER SIZE SAMPLE DISTRIBUTIONS FOR FORMULAS F-5 TO F-9

By the shape of the curve and mainly by the condition that in almost all cases the mean and variance computed from the number of orders per day distribution are very close to each other (Table 3), it has been concluded that, in general, this distribution follows approximately a Poisson distribution. It will be assumed in this work that customers' orders per day follow a Poisson distribution.

Analysis made by Pfost and Thomas (12) on data gathered from another feed mill shows that customers' orders tend to accumulate near the weekend (Monday and Friday), building a sort of "seasonal" variation within a week period. It should be pointed out that although in the data being studied here there are some formulas for which customers' orders do seem to accumulate near the weekend, (formula F-7 for example), in general this trend has not been confirmed enough to get the same conclusion.

Another characteristic situation in the feed business is the actual existence of seasonal variations of demand throughout the year for particular formulas. This case will not be studied here. However, it is recognized that further research is needed in this area, not only in the feed industry but in the general application of the inventory control theory. The best method of handling the problem of seasonal variations seems to be simulation.

The graphs of the distribution of tons/day (Figures 4 and 5) show that for higher demand formulas, F-1 and F-2 (Figures 4A

Table 2.   Number of orders per day and frequency sample
           distribution of typical feeds sold in bulk.*

Number of orders per day	Frequency and (Probability, %)			
	F-1	F-2	F-3	F-5
0	0(0.0)	1(2.5)	9(22.5)	7(17.5)
1	1(2.5)	1(2.5)	14(35.0)	10(25.0)
2	5(12.5)	3(7.5)	12(30.0)	17(42.5)
3	8(20.0)	8(20.0)	5(12.5)	3(7.5)
4	10(25.0)	5(12.5)		2(5.0)
5	10(25.0)	10(25.0)		1(2.5)
6	3(7.5)	4(10.0)		
7	2(5.0)	2(5.0)		
8	1(2.5)	4(10.0)		
9		2(5.0)		

Note: The rest of the formulas have similar frequency
        distributions.   * See Fig. 3.

Table 3.   Sample mean and variance of number of orders per day.

	F-1	F-2	F-3	F-4	F-5	F-6	F-7	F-8	F-9
Mean	3.90	4.5	1.25	.95	1.57	.57	1.48	.55	.64
Variance	3.05	5.5	1.25	.68	1.42	.54	1.50	.50	.53
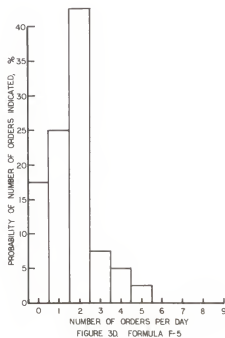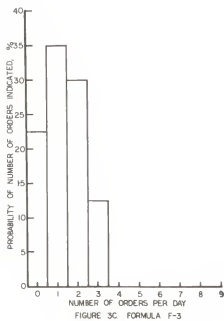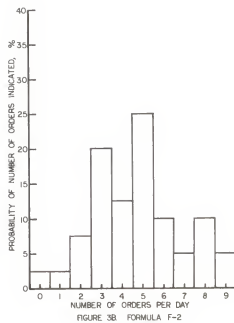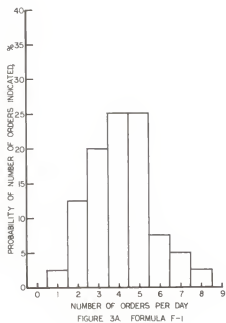
FIGURE 3   NUMBER OF ORDERS PER DAY SAMPLE DISTRIBUTIONS

and 4B), it would be possible to fit them to a Poisson or to a normal distribution. This first observation is partially confirmed by the fact that actually the demand in tons/day is the result of the order size and the number of orders/day distributions. As mentioned before, for these two formulas customers' orders are almost entirely in one size (the full truck), and if the number of orders follow the Poisson, the resulting demand in tons/day would be another Poisson distribution.

In general, for every formula, a general procedure is described in the next section to cover almost all possible shapes of demand distributions.

Table 4. Demand sample distributions in tons/day of typical
feeds sold in bulk.*

	Probability in %			Probability in %	
Tons/day	F-1	F-2	Tons/day	F-5	F-7
0-15	5.0	5.0	0-10	42.5	32.5
15-30	17.5	10.0	10-20	42.5	50.0
30-45	27.5	20.0	20-30	7.5	7.5
45-60	37.5	35.0	30-40	5.0	10.0
60-75	10.0	10.0	40-50	2.5	0.0
75-90	2.5	7.5			
90-105	0.0	12.5			
Mean	42.8 tons	53.2 tons	Mean	13.3 tons	12.2 tons

	Probability in %				
Tons/day	F-3	F-4	F-6	F-8	F-9
0-5	30.0	47.5	67.5	67.5	57.5
5-10	27.5	25.0	15.0	22.5	22.5
10-15	22.5	25.0	17.5	7.5	17.5
15-20	17.5	2.5	0.0	2.5	2.5
20-25	2.5	0.0	0.0	0.0	0.0
Mean	7.9 tons	5.7 tons	3.2 tons	3.1 tons	4 tons

*See Figs. 4 and 5

Note: As is shown, three different size of intervals have been
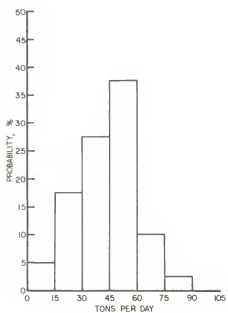taken according to the demand levels in each formula.
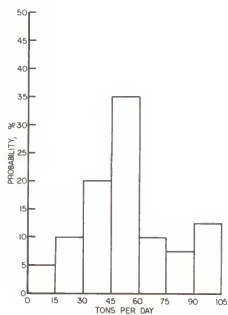
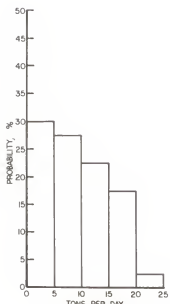FIGURE 4A. FORMULA F-1

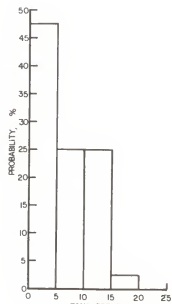FIGURE 4B. FORMULA F-2

FIGURE 4C. FORMULA F-3

FIGURE 4D. FORMULA F-4

FIGURE 4 DEMAND SAMPLE DISTRIBUTIONS IN TONS PER DAY FOR FORMULAS F-1 TO F-4
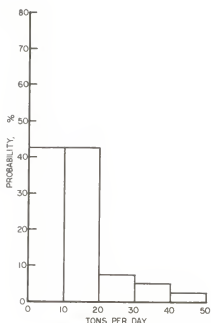
FIGURE 5A FORMULA F-5
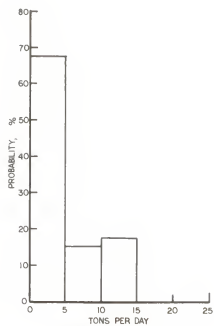
FIGURE 5B FORMULA F-6
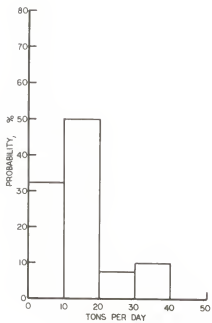
FIGURE 5C FORMULA F-7
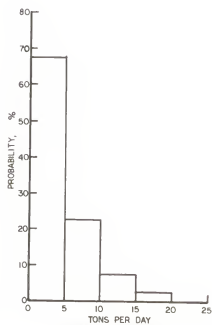
FIGURE 5D FORMULA F-8

FIGURE 5 DEMAND SAMPLE DISTRIBUTIONS IN TONS PER DAY FOR FORMULAS F-5 TO F-9

Direct Determination of Demand
from Historical Data

With the availability of historical data, the best method of determining the demand distribution is simply to analyse the data directly. The analysis that must be made in order to fit the sample distribution to known distributions or general mathematical functions easy to work with, is rather simple using known statistical methods.

The general procedure is to test whether the set of data may be looked upon as values assumed by a random variable having a given distribution. This can be done by means of the "chi-square test for goodness of fit," (5) and (16).

The first step could be to try distributions most likely to fit the set of data. For instance, for formulas F-1 and F-2, (Figs. 4A and 4B), evidently we will try to test goodness of fit to a Poisson distribution with parameter equal to the observed mean or to a normal if the mean is large enough. In other cases, the binomial distribution might be tried.

The Beta and Gamma distributions cover a great variety of distributions depending on the values of the parameters used in each case (16).

In the case of a Beta distribution, whose standard form applies for values of the variable between 0 and 1, a simple linear transformation of the set of data is indicated.

There exists still another method that could be applied when there is no possibility of fitting any of the best known

distributions, or when there is no hint about what distribution to use.

A polynomial distribution could be used to fit many general sample distributions. The idea is to use the polynomial function as a probability distribution that depending on the number of terms (powers) used, will fit a wide variety of shapes of curves within some fixed intervals.

Let "y" represent the demand of a given feed in tons/day, and f(y) its probability distribution.

Let

$$f(y) = A_n y^n + A_{n-1} y^{n-1} + \ldots + A_2 y^2 + A_1 y + A_0$$

be the general polynomial expression for the probability distribution of the demand, with the conditions:

$$f(y) \geq 0 \qquad \text{and} \qquad 0 \leq y \leq b$$

The lower limit of y is, logically, zero. The upper limit, "b", is the maximum demand that actually occurs or has occurred during the past. It could be some maximum quantity over which the probability of occurrence of higher demand in tons/day is likely to be zero or very close to zero. (At this point it should be recalled that this method like any other actually gives an approximation to the real probability distribution of the entire population). The upper limit "b" could be determined also on a past-experience basis but allowing an additional level of demand equivalent to the estimated increase for future periods.

The problem is to find the number of terms, and consequently the actual coefficients A_i in the polynomial function that

sufficiently fits the sample distribution.

In order to do so we can use the properties of the moments of distributions. Equating the moments of the population distribution with the corresponding moments of the sample we can get as many equations as are needed to solve for the unknown parameters (5).

The rth. sample moment of a set of observations y_1, $y_2, \ldots, y_n$ is defined as

$$m'_r = \frac{\sum_{i=1}^{n} y_i^r}{n}$$

and the rth. moment of a distribution is the expected value of the rth. power of 'y'.

$$u'_r = \int_y y^r f(y) \, dy$$

For $r = 1$, m'_1 is the sample mean and u'_1 the expected value of "y", or mean of the distribution.

The first equation can be that deduced from the property:

$$\int_y f(y) \, dy = 1$$

Thus the system of equations becomes:

$$\int_0^b f(y) \, dy = 1$$

$$\int_0^b y f(y) \, dy = \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\int_0^b y^2 f(y)\,dy = \frac{\sum_{i=1}^n y_i^2}{n}$$

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for as many equations as required.

The procedure will be by trial and error, starting with $f(y)$ having the minimum number of terms. Obviously when $f(y) = A_0$, only the first equation is needed; therefore,

$$A_0 = \frac{1}{b}$$

and $f(y) = \frac{1}{b}$, which is the rectangular distribution. It will be very easy to prove in this case if the sample corresponds actually to a rectangular distribution.

For irregular shapes two terms or more for $f(y)$ must be tried.

Each time a term is added to the polynomial function, a chi-square test for goodness of fit should be made comparing the observed values against those calculated from the $f(y)$, already derived up to that step. If the test is accepted, the $f(y)$ derived last is a sufficient fit to the sample distribution. If the test is rejected, further moments should be calculated.

It has been stated that by convoluting the distribution of tons of feed/day so many times as the number of days in the lead time, we are able to obtain the demand distribution per lead time.

Supposing we have already found the best fit to the set of data and this results in one of the distributions: Poisson, Binomial, Normal, Gamma or Beta. Convolutions of these well known distributions are the same distributions with mean and variance equal the original mean and variance, times the number of times the original distribution is being convoluted.

Procedures to derive convolutions of any distribution in general, and specifically for the polynomial, are discussed in the next section.

(Demand distribution per lead time will be designated with the notation $f_t(y)$).

Building the Demand Distribution by the Second Approach

This procedure may be used when, lacking sufficient data or having available only market research studies, it is necessary to build the demand distribution in tons/day by combining order size distribution with that of customers' orders/day.

Let "m" be the number of orders for a particular formula per day. The probability distribution of m, $f(m)$, is assumed to follow the Poisson.

If $f(y|m)$ is the conditional probability of demand in tons/day given that "m" orders are received, then, the marginal distribution $f(y)$ is simply the joint distribution of "y" and "m", $f(y,m)$, over all possible values of "m":

$$f(y) = \sum_m f(y,m) \; ; \; \text{when "m" is a discrete random variable,}$$

and

$$f(y) = \int_m f(y,m)\,dm \; ; \; \text{if "m" is a continuous random variable.}$$

But,

$$f(y,m) = f(y|m)\,f(m), \quad \text{hence}$$

$$f(y) = \sum_m f(y|m)f(m) \qquad \text{(discrete case)},$$

or

$$f(y) = \int_m f(y|m)f(m)\,dm \qquad \text{(continuous case)}.$$

Although we have defined $f(m)$ as a Poisson distribution, the continuous case is given along with the discrete because if the average number of orders/day is large enough, $f(m)$ could be approximated by a normal distribution.

From the last equation it is obvious that determining the expression for the demand given "m" orders, $f(y|m)$, $f(y)$ will be also determined.

Let x_i be the size in tons of the order i, where i can take values from 1 to "m". Then, the demand in tons/day is:

$$y = x_1 + x_2 + x_3 + \ldots + x_m,$$

or

$$y = \sum_{i=1}^{m} x_i \, .$$

The x_i's are independent random variables from the distribution of order size $f(x)$.

In order to derive a function $f(y|m)$ where "y" is the sum of "m" variables from the same order size distribution, we must determine first the $f(x)$.

Observation of the data (Figs. 1 and 2) shows that, in general, the order size distribution follows no standard pattern (with the exception of two of them (Figs. 2C and 2D) that behave almost as rectangular distributions).

We could repeat here what has been said in the last section about selecting one known distribution and try to fit the set of data to it. The polynomial approach could be applied as well in order to determine the order size distribution $f(x)$. As a matter of fact, we are going to derive here the $f(y|m)$ using the polynomial distribution as a general expression for $f(x)$. For, as we will see, $f(y|m)$ can be derived applying the theory of convolutions or sum of random variables, and these convolutions are well known in the case of the best known probability distributions.

Let

$$f(x) = A_n x^n + A_{n-1} x^{n-1} + \dots + A_2 x^2 + A_1 x + A_0$$

be the general polynomial expression for the order size probability distribution. The same procedure explained before has to be used here to determine the number of terms of this expression. Only a few comments about the limits of the order size could be made.

In general, order sizes are limited at the lower and upper end by some known and fixed levels established by

company policies. In actual practice, and this is confirmed
in the analysis of the data studied, the lower limit "a" is
usually the minimum order size equivalent to one of the compart-
ments of the bulk truck. The upper limit "b" is actually the
full size of the truck in tons.

In other words, assumptions might logically be made, that
the feed mill has set some standards for the order size by
which customers cannot order less than "a" (frequently a mixer
batch), or more than "b" tons per order. The lower limit in
some cases is relaxed to take the value of zero.

With $f(x)$ already derived we must go on in the determina-
tion of the conditional distribution of the demand tons/day
given that "m" orders are received, $f(y|m)$.

By the theory of sum of random variables, or convolutions,
we know that given

$$y = \sum_{i=1}^{m} x_i$$

x_i, independent random variables from the same distribu-
tion $f(x)$, where:

$$f(x) = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_2 x^2 + A_1 x + A_0; \text{ when } a<x<b$$

$$f(x) = 0 \quad ; \quad \text{elsewhere}$$

Then:

$$f(y|m) = \int_{x_m} \int_{x_{m-1}} \ldots \int_{x_2} f(y-x_2- \ldots - x_m) f(x_2) \ldots$$
$$f(x_m) dx_2 dx_3 \ldots dx_m$$

where, $f(y-x_2- \ldots - x_m) = f(x_1) = A_n(y-x_2- \ldots -x_m)^n + \ldots$
$$+ A_1(y-x_2- \ldots -x_m) + A_0$$

$$f(x_2) = A_n x_2^n + A_{n-1} x_2^{n-1} + \ldots + A_2 x_2^2 + A_1 x_2 + A_0$$

.
.
.
.

$$f(x_m) = A_n x_m^n + \ldots + A_1 x_m + A_0$$

Special Case

As an example, consider the case when $f(x)$ is a rectangular distribution between the limits "a" and "b".

Here, $A_0 = \dfrac{1}{b - a}$

Thus,

$$f(x) = \frac{1}{b - a} \qquad \text{when } a < x < b,$$
$$f(x) = 0 \qquad \text{elsewhere.}$$

<u>For $m = 2$; $y = x_1 + x_2$</u>

$$f(y|2) = \int_{x_2} f(y - x_2)\, f(x_2)\, dx_2 \; ,$$

if
$$\left. \begin{array}{l} a < x_1 < b \\ a < x_2 < b \end{array} \right\} \qquad \text{then } 2a < y < 2b$$

$$f(y|2) = \int_{a}^{y-a} \frac{1}{(b-a)^2} \, dx_2 \quad ; \quad \text{when } 2a < y < (a+b)$$

$$f(y|2) = \int_{y-b}^{b} \frac{1}{(b-a)^2} \, dx_2 \quad ; \quad \text{when } (a+b) < y < 2b \; .$$

Therefore,

$$f(y \mid 2) = \frac{y-2a}{(b-a)^2} \qquad \text{when } 2a < y < (a+b), \text{ and}$$

$$f(y \mid 2) = \frac{2b-y}{(b-a)^2} \qquad \text{when } (a+b) < y < 2b \; ;$$

this is the expression of a triangular distribution between the limits 2a and 2b, with mean (a+b).

For m = 3; $y = x_1 + x_2 + x_3$

$$f(y \mid 3) = \int_{a}^{y-2a} \int_{a}^{y-x_3-a} f(y-x_2-x_3) f(x_2) f(x_3) \, dx_2 \, dx_3$$
$$\text{when } 3a < y < 2a+b,$$

$$f(y \mid 3) = \int_{y-a-b}^{b} \int_{a}^{y-x_3-a} f(y-x_2-x_3) f(x_2) f(x_3) \, dx_2 \, dx_3 +$$

$$+ \int_{a}^{y-a-b} \int_{y-x_3-b}^{b} f(y-x_2-x_3) f(x_2) f(x_3) \, dx_2 \, dx_3$$
$$\text{when } 2a+b < y < a+2b,$$

and

$$f(y \mid 3) = \int_{y-2b}^{b} \int_{y-x_3-b}^{b} f(y-x_2-x_3) f(x_2) \, dx_2 \, dx_3 \; ; \text{ when}$$
$$a+2b < y < 3b$$

which yields:

$$f(y \mid 3) = \frac{(y-3a)^2}{2(b-a)^3} \qquad \text{when } 3a < y < 2a+b,$$

$$f(y \mid 3) = \frac{(y-3a)^2 - 3[y-(2a+b)^2]}{2(b-a)^3} \qquad \text{when } 2a+b < y < a+2b,$$

$$f(y|3) = \frac{(3b-y)^2}{2(b-a)^3} \qquad \text{when } a+2b < y < 3b.$$

We would like to get some general expression $f(y|m)$. In order to do so, we can take the particular case for $a=0$ and $b=1$ which has been worked out, (8) and (9), and make a transformation.

When $a=0$ and $b=1$:

$$g(w|m) = \frac{1}{(m-1)!} \sum_{r=0}^{n} (-1)^r \binom{m}{r} (w-r)^{m-1}$$

$$\text{when } n < w < n+1$$
$$\text{for } n = 0, 1, \ldots, m-1$$

Here, the total interval is $o < w < m$, and we want to change the variable and transform to: $ma < y < mb$

So, $y = w(b-a) + ma$

or $w = \frac{y-ma}{b-a}$

$$f(y|m) = g(w|m) \frac{dw}{dy}$$

$$f(y|m) = g(w|m) \frac{1}{b-a}$$

Then,

$$f(y|m) = \frac{1}{(b-a)^m(m-1)!} \sum_{r=0}^{n} (-1)^r \binom{m}{r} [y-ma-r(b-a)]^{m-1}$$

when $(m-n)a + nb < y < (m-n-1)a + (n+1)b$; for $n = 0, 1, \ldots, m-1$; and $m > 0$; which is the general formula for the demand in tons/day given "m" orders and, being the order size rectangularily distributed.

As "m" increases this function tends to approach the normal curve. In fact, it can be proved that when "m" is large enough this function approaches normality in virtue of the Central Limit theorem in general, and particularly by a theorem of Lindberg, 1922, (9) and (18).

The mean of the rectangular distribution is $\frac{a + b}{2}$ and the variance is $\frac{(b-a)^2}{12}$. Therefore, the mean and variance of $f(y|m)$, which is $f(x)$ convoluted "m" times, are:

$$\mu = \frac{m(a+b)}{2} \qquad \text{(mean)}$$

and $$s^2 = \frac{m(b-a)^2}{12} \qquad \text{(variance)}$$

Therefore if "m" is large enough, we can express $f(y|m)$ as a normal function:

$$f(y|m) = \frac{\sqrt{12}}{\sqrt{2\pi}\ (b-a)\ \sqrt{m}} \quad \exp \quad \left\{ - \frac{[y - \frac{m(a+b)}{2}]^2}{\frac{m(b-a)^2}{6}} \right\}$$

or,

$$f(y|m) = \frac{\sqrt{12}}{\sqrt{2\pi m}\ (b-a)} \quad \exp \quad \left\{ - \frac{3[2y-m(a+b)]^2}{m(b-a)^2} \right\}$$

General Case

Consider now the general case, when the polynomial function has more than one term.

For $m = 2$, $y = x_1 + x_2$; and for $m = 3$, $y = x_1 + x_2 + x_3$, $f(y|m)$ is derived exactly as for the rectangular distribution case, only that the proper $f(x)$ has to be used.

<u>For m > 3</u>; $(y = x_1 + x_2 + x_3 + \ldots + x_m)$.

Only for reasons of space, let:

$$f(y-x_2-x_3- \ldots -x_m)f(x_2)f(x_3) \ldots f(x_m) = g(X)$$

and $dx_2 dx_3 \ldots dx_m = dX$.

Then:

$$f(y|m) = \int_{a}^{y-(m-1)a} \int_{a}^{y-x_m-(m-2)a} \int_{a}^{y-x_m-x_{m-1}-(m-3)a} \ldots$$

$$\ldots \int_{a}^{y- \ldots -x_3-a} g(X)\,dX$$

when $ma < y < (m-1)a+b$,

$$f(y|m) = \int_{a}^{y-(m-2)a-b} \int_{a}^{y-x_m-(m-3)a-b} \ldots$$

$$\ldots \int_{a}^{y- \ldots -x_4-a-b} \int_{y-x_m-x_{m-1}- \ldots -x_3-b}^{b} g(X)\,dX$$

$$+ \int_{a}^{y-(m-2)a-b} \int_{a}^{y-x_m-(m-3)a-b} \ldots$$

$$\ldots \int_{a}^{y-x_m- \ldots -2a-b} \int_{y-x_m- \ldots -x_4-a-b}^{b} \int_{a}^{y-x_m- \ldots -x_3-a} g(X)\,dX$$

$$+ \int_a^{y-(m-2)a-b} \cdots \int_a^{y-x_m-\ldots-3a-b} \int_{y-x_m-\ldots-2a-b}^{b}$$

$$\int_a^{y-x_m-\ldots-x_4-2a} \int_a^{y-x_m-\ldots-x_3-a} g(X)\,dX$$

$$+$$
$$\vdots$$
$$+ \int_{y-(m-2)a-b}^{b} \int_a^{y-x_m-(m-2)a} \cdots \int_a^{y-x_m-\ldots-x_3-a} g(X)\,dX$$

$$\text{when } (m-1)a+b < y < (m-2)a+2b,$$

$$f(y\mid m) = \int_a^{y-(m-3)a-2b} \int_a^{y-x_m-(m-4)a-2b} \cdots$$

$$\cdots \int_a^{y-x_m-a-2b} \int_{y-x_m-\ldots-x_4-2b}^{b} \int_{y-x_m-\ldots-x_3-b}^{b} g(X)\,dX$$

$$+$$
$$\vdots$$
$$+$$
$$\vdots$$
$$+$$

$$+ \int_{y-(m-3)a-2b}^{b} \int_{y-x_m-(m-3)a-b}^{b} \cdots \int_{a}^{y-x_m- \ldots -2a} \int_{a}^{y-x_m- \ldots -x_3-a} g(X) dX$$

when $(m-2)a+2b < y < (m-3)a+3b$,

$f(y|m) = \ldots$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

and,

$$f(y|m) = \int_{y-(m-1)b}^{b} \int_{y-x_m-(m-2)b}^{b} \int_{y-x_m-x_{m-1}-(m-3)b}^{b} \cdots \int_{y- \ldots -x_3-b}^{b} g(X) dX$$

when $a+(m-1)b < y < mb$.

Here again, as in the case when $f(x)$ is a rectangular distribution, when "m" is large enough, $f(y|m)$ will approach the normal distribution, and even faster than the $f(y|m)$ for the rectangular case. Mean and variance, as before, equals "m" times the mean and variance of the original $f(x)$ being convoluted.

With the expression of $f(y|m)$, $f(y)$ can be derived.

With "m" having a Poisson distribution, and letting 'N' be the parameter of that distribution (N = mean = variance):

$$f(m) = \frac{N^m e^{-N}}{m!}$$

$$f(y) = \sum_m f(y|m) f(m)$$

$$f(y) = \sum_{m=o}^{\infty} f(y|m) \frac{N^m e^{-N}}{m!}$$

If the average number of orders/day, "N", is large enough, the Poisson distribution approaches a normal:

$$f(m) = \frac{1}{\sqrt{2\pi N}} \quad \exp\left[-\frac{(m-N)^2}{2N} \right]$$

and,

$$f(y) = \int_0^\infty f(y|m)\, f(m)\, dm$$

$$f(y) = \frac{1}{\sqrt{2\pi N}} \int_0^\infty f(y|m) \exp\left[-\frac{(m-N)^2}{2N} \right] dm$$

Again, when $f(y|m)$ is the convolution of a rectangular distribution $f(x)$:

$$f(y) = \sum_{m=0}^\infty \left\{ \frac{N^m e^{-N}}{(m-1)!\, m!\, (b-a)^m} \sum_{r=0}^m (-1)^r \binom{m}{r} [y-ma-r(b-a)]^{m-1} \right\} \; ;$$

this expression can be solved numerically as shown in the example at the end of this section.

If "m" is large, $f(y|m)$ approaches the normal distribution; and if "N" is large $f(m)$ approaches also the normal; then,

$$f(y) = \frac{\sqrt{12}}{2\pi\,(b-a)\,\sqrt{N}} \int_0^\infty m^{-1/2} \exp -\left\{ \frac{m^2}{2N} + \left[\frac{3(a+b)^2}{2(b-a)^2} -1\right] m \right.$$

$$\left. + \frac{6y^2}{m(b-a)^2} - \frac{6y(a+b)}{(b-a)^2} + \frac{N}{2} \right\} dm$$

which can be solved also numerically.

The derivation of $f_t(y)$, demand distribution per lead time, is simple now:

Convoluting 't' times $f(m)$, being 't' the number of days in lead time,

$$f_t(m) = \frac{(Nt)^m e^{-Nt}}{m!}$$

(The convolution of a Poisson distribution is another Poisson with parameter the original one times "t").

and,

$$f_t(y) = \sum_{m=0}^{\infty} f(y|m) \frac{(Nt)^m e^{-Nt}}{m!} \text{ for the discrete case.}$$

In the continuous case, substituting the parameter "Nt" for N in the expression of $f(y)$ already shown for that case, would be required.

EXAMPLE I

This is an example to show how to build the demand distribution $f(y)$ by considering separately the distributions of the order size, "x", and the number of orders per day, "m".

Consider the order size distribution of F-9 shown in Table 1 and Figure 2D. The number of orders follow a Poisson distribution with mean N=0.64 (Table 3). Assume that the lead time is one day.

Therefore,

$$f_t(y) = f(y)$$

$$F_t(y) = F(y)$$

and $$f(m) = \frac{(0.64)^m e^{-0.64}}{m!}$$

To determine the order size distribution the polynomial approach will be used.

Corresponding with the grouped data for the considered formula, the upper limit b = 12, and the lower limit a = 0. Following the method explained:

$$f(x) = A_o$$

$$\int_o^{12} f(x)\,dx = 1 \qquad ; \qquad \int_o^{12} A_o\,dx = 1$$

hence, $$A_o = \frac{1}{12}$$

and $$f(x) = \frac{1}{12} \quad \text{when } 0 < x < 12$$

$$f(x) = 0 \qquad \text{elsewhere}$$

Applying a Chi-square test of goodness of fit:
the null hypothesis is: the given data comes from a
population having a rectangular distribution.

Reject the null hypothesis if $\chi^2 \geq \chi^2_{\alpha, n-1}$

$$\chi^2 \geq \chi^2_{.05, 3}$$

$$\chi^2_{.05, 3} = 7.815$$

$$\chi^2 = \sum_{i=1}^{n} \frac{(P_i - e_i)^2}{e_i}$$

where

n = 4 (number of observations)

Order Size (tons)	Observed Frequency	$P_i(\%)$ Observed Probability	$e_i(\%)$ Calculated Probability
0-2.9	7	26	25
3-5.9	7	26	25
6-8.9	7	26	25
9-11.9	6	22	25

$$\chi^2 = \frac{1}{25} + \frac{1}{25} + \frac{1}{25} + \frac{9}{25} = 0.48$$

Therefore, the null hypothesis is accepted. (If the null
hypothesis had been rejected, one term would have been added
to the polynomial.)

Substituting the values of "a" and "b" in the general form
of $f(y|m)$ already derived for a rectangular order size
distribution:

$$f(y|m) = \frac{1}{(12)^m (m-1)!} \sum_{r=0}^{n} (-1)^r \binom{m}{r} (y-12r)^{m-1}$$

when $12n < y < 12(n+1)$

for $n = 0, 1, \ldots, m-1$

and $m > 0$.

For $m = 0$, $y = 0$ and $f(y)$ will be the probability of getting no orders.

For $m > 0$:

$$f(y) = \sum_{m=1}^{\infty} \left[\frac{e^{-0.64}(0.64)^m}{m!} \frac{1}{(12)^m(m-1)!} \sum_{r=0}^{n} (-1)^r \binom{m}{r}(y-12r)^{m-1} \right]$$

when $12n < y < (n+1)(12)$

for $n = 0, 1, \ldots, m-1$

Using Poisson probabilities:

For $m = 0$

$\quad f(y) = .5227$ when $y = 0$

For $m = 1$

$\quad f(y) = \dfrac{.3384}{12}$ when $0 < y < 12$

For $m = 2$

$\quad f(y) = \dfrac{.1102}{(12)^2} y$ when $0 < y < 12$

$\quad f(y) = \dfrac{.1102}{(12)^2} (24-y)$ when $12 < y < 24$

For $m = 3$

$\quad f(y) = \dfrac{.0241}{2(12)^3} y^2$ when $0 < y < 12$

$\quad f(y) = \dfrac{.0241}{2(12)^3} [y^2 - 3(y-12)^2]$ when $12 < y < 24$

$\quad f(y) = \dfrac{.0241}{2(12)^3} [y^2 - 3(y-12)^2 + 3(y-24)^2]$ when $24 < y < 36$

For all practical purposes for m > 3, f(y) = 0 because the
Poisson probabilities for this particular case are almost zero,
and they have to be divided in each case by the value $(12)^m$.

Therefore, the demand distribution is:

$f(y) = .5227$ when y = 0

$f(y) = \frac{.3384}{12} + \frac{.1102}{(12)^2} y + \frac{.0120}{(12)^3} y^2$ when 0 < y < 12

$f(y) = \frac{.1843}{12} - \frac{.0379}{(12)^2} y - \frac{.0241}{(12)^3} y^2$ when 12 < y < 24

$f(y) = \frac{.1084}{12} - \frac{.0723}{(12)^2} y + \frac{.0120}{(12)^3} y^2$ when 24 < y < 36

$f(y) = 0$ when y > 36

INVENTORY CONTROL MODEL AND OPTIMAL

SIZE OF BINS

Designing an inventory control system is finding the best possible policy to handle the inventory in order to meet future demand.

The best possible policy is simply that which, taking into account all cost and time factors involved, enables the system to satisfy customers' demands with a minimum total cost.

There are two major systems of inventory control as defined by Starr and Miller (15). The difference between them consists in the manner in which the problem of controlling the existing variables is handled.

These variables are related to the problem of how much to stock, how much of a given formula to produce at a given time, and how often should that formula be produced. Translated to the "inventory control terminology" this means, what level of inventory should be kept, what size of order should be sent to the production department in order to replenish the inventory, and when is replenishment needed.

Actually the variables are only two: the frequency of ordering and the amount ordered. (The term "order" refers here, and through the rest of this thesis, to orders sent by the inventory control personnel to the production department for a given quantity of feed to be produced for stock.)

From the point of view of average demand level, the two variables can be considered as only one since each of them

could be derived from the other one. But, when fluctuations above and below average demand are considered, those fluctuations can be covered by varying either the frequency of orders or the size of the orders.

These two possibilities determine the two major systems of inventory control.

The Q-system is characterized by having a fixed order size and a variable order period. Fluctuations in demand are covered by the frequency of ordering. The system works in the following fashion:

Whenever the stock on hand falls to a minimum level called the Reorder Point, (determined by considerations about the demand during the lead time), some fixed quantity of product is ordered to be produced for replenishing the inventory. The system is defined by determining the optimal reorder level and the order size.

The P-system is characterized by having a fixed order period and a variable order size. Fluctuations in demand are covered by different sizes of orders placed in successive periods. The system is defined by determining the optimal order period and some maximum level "M" on which basis the order size is calculated each time. The inventory is reviewed at the end of the order period, and the order size is deduced from the difference between the level "M" and the actual stock on hand at that instant.

Facts about which one of these two major systems should be chosen in a particular case, or which one is more economical

when actually applied, are fully discussed in the literature of inventory control theory, available elsewhere. The Q-system seems to be more generally applicable to feed mills and its model will be used here to derive the optimal size of the facilities. Nevertheless, the P-system could also be used, and the method to be described here could also be applied as both models are actually developed with similar assumptions and mathematical derivations.

Before going into the analysis for the model of the Q-system and its specific application in sizing the bins for feed in bulk, it is necessary to repeat some of the assumptions already made in the first part of this thesis, namely:

a. The demand distribution in tons of feed per day, $f(y)$, and per lead time period, $f_t(y)$, are known as a result of the analysis discussed earlier, or by other means.

b. $f(y)$ is stationary, the same from day to day. Demand is independent of the demand level in preceding periods.

c. The lead time, already defined, is constant. Cases when the lead time cannot be estimated as a constant (its average or maximum value) are discussed in detail by some authors. Such cases are analysed through the queueing theory, (10) and (15).

Let,

$\bar{y}$ = expected value of the demand of feed in tons/day.

D = average demand per year. If 5 days per week and 52 weeks per year are assumed, then $D = 260 \bar{y}$.

c = cost per ton of feed to be stored (ingredient and processing costs).

X = size of the order to be produced for replenishing the inventory, in tons.

R = average demand during the lead time period.

W = lead time period safety reserve. Reserve needed to cover fluctuations in demand during the lead time.

$R+W$ = Reorder point.

C_c = inventory carrying rate. Interest rate to apply to the amount of money invested in inventory. This rate will determine the estimated return on the amount of money tied up in inventory if it had been invested in some other place, inside or outside of the feed mill itself.

C_r = set-up cost. Cost of changing over the production process to produce some specific formula. It is mainly the cost of the time lost in changing formulas. At the end of the analysis of the model an entire section is devoted to discussing the determination of this cost in a feed mill.

K = fixed out-of-stock cost or cost of being out of stock. This is also discussed in detail later.

C_s = yearly storage cost. Cost of the space required to store the inventory of a given feed. Some authors have pointed out that this is a fixed cost with respect to the inventory system because in general there is no possibility of using the storage space

for any other activity. For that reason they have disregarded this cost in the inventory analysis. Although in some cases this argument could be true, the storage cost is included here to help in the determination of the optimal bin size as described next.

The inventory policy will be defined when the unknowns X and W are determined. The analysis must be directed to find some optimal values for X and W by balancing properly all the cost factors involved so that the total cost of the system is minimized.

The variables X and W are also two of the three parameters needed to determine the size of the bin that will hold the inventory. (The word "bin" is used as a general term. Actually a given feed formula could have one, two or more bins assigned.)

When the stock on hand falls to the Reorder Point (R+W), an order for X units is sent to the production department. During the lead time on the average, the demand will be R. Under average conditions when the order X is filled and sent back to inventory only W will be on hand. If the demand has exceeded R, the quantity W should have covered that excess, so at the arrival of X there would be on hand some level of inventory between zero and W, depending on the actual value of the excess. An extreme condition occurs when there has been no demand at all during the lead time. In this case, at the arrival of X the inventory level would be R+W.

The bin might be designed to cover this last extreme condition. In other words, the size of the bin should be that which would hold the largest possible inventory level.

Let "S" be the size of the bin to be determined. Then,

$$S = X + W + R$$

Consider "C_b" as the variable cost of the bin, or that part of the total cost that varies with the size of the bin. In other words, for a system of bins there will be some costs (equipment and others) that are fixed, and there will be some other costs that depend on the size of the bin. The latter are those whose sum will be called "C_b". This is expressed in dollars/ton of capacity and can be easily determined from experience or from data available elsewhere. The yearly storage cost, C_s, is then:

$$C_s = (X + W + R) C_b D_s$$

D_s is defined as depreciation rate in per cent per year. (This might also include interest, insurance, taxes, etc.)

Assuming that on the average, the reserve stock W is carried all year round; the average inventory carried is $\frac{X}{2}$; and knowing that the average number of orders per year is $\frac{D}{X}$, the yearly costs involved in the inventory system are:

Ordering cost = $\dfrac{DC_r}{X}$

Carrying cost = $(\frac{X}{2} + W) cC_c + (X + W + R) C_b D_s$

Out-of-stock-cost = $\dfrac{DK}{X} \displaystyle\int_{R+W}^{\infty} f(y)\,dy$

(The integral of the function between (R+W) and ∞ is the probability of being out of stock; or the probability that the demand exceeds (R+W).

Therefore, the total yearly cost:

$$T.C. = \frac{DC_r}{X} + (-\frac{x}{2} + W)cC_c + (X + W + R)C_bD_s + \frac{DK}{X} \int_{R+W}^{\infty} f(y)\,dy$$

The objective is to determine the unknowns X and W so as to get a minimum yearly cost. This is obtained by taking partial derivatives of the function T.C. with respect to each one of the variables and equating to zero (15). The two resulting equations are:

$$-\frac{DC_r}{X^2} + \frac{cC_c}{2} + C_bD_s - \frac{DK}{X^2} [1 - F_t(R+W)] = 0,$$

and $cC_c + C_bD_s - \frac{DK}{X} f_t(R+W) = 0$

Solving each equation for X and equating the results:

$$[f_t(R+W)]^2 = \frac{2(cC_c + C_bD_s)^2 \{C_r + K[1-F_t(R+W)]\}}{DK^2(cC_c + 2C_bD_s)}$$

To solve for W a trial and error procedure could be used: assume first $F_t(R+W) = 1$ and compute the corresponding value for $f_t(R+W)$. With this value go to the known function which gives the probability distribution of the demand per lead time, and deduce a value for W. With this value of W compute $F_t(R+W)$ and substitute it in the equation. Proceed as before as many

times as necessary to obtain two consecutive solutions close in value.

Once W is determined, X is calculated from the equation

$$X = \frac{DK}{cC_c + C_b D_s} \, f_t(R+W)$$

In this fashion the inventory control system and the bin size have been simultaneously derived.

There is still the possibility of reducing the bin size using a second approach which is more general than the previous one. It has been said that S must be equal to (X + W + R) to cover the extreme condition of no demand during the lead time, and the quantity X is arriving to be stocked.

Let T be some level of demand between zero and R, that can occur during the lead time.

Let the expression for the bin size be
$$S \overset{=}{+} X + W + (R-T) \; ; \; o < T < R$$

This means that management is willing to risk not having enough space to store all of the quantity X if the demand during the lead time happens to be less than T.

In order to investigate this case a new cost should be introduced into the system. This cost might be defined as some penalty cost for not having enough room to store what has been produced at a given time. To fully understand what this means, suppose that a feed mill has already built its bins and has sized them by some empirical methods. Later on an inventory control system is designed and in its application it is found

that the size of the bins is insufficient to handle the inventory which is produced under an "optimum" policy.

The cost of building additional storage space or the cost of having to store the feed originally produced to be sold in bulk in some other place would be considered the penalty cost referred to. Let "C_T" be this penalty cost. To visualize the case a graphical presentation of the problem has been developed (Fig. 6). There it has been assumed that the demand distribution during the lead time is normal with mean R.

The total yearly cost in this case is:

$$T.C. = \frac{DC_r}{X} + (\frac{X}{2} + W)cC_c + [X + W + (R-T)]C_b D_s$$
$$+ \frac{DC_T}{X} \int_0^T f(y)\,dy + \frac{DK}{X} \int_{R+W}^\infty f(y)\,dy$$

(The integral of the function between zero and T is the probability that the demand be T or less.)

There are now three unknowns: X, W, and T.
(Notice that when $T = 0$, the first case is reached.)

Taking partial derivatives with respect to these three variables and equating to zero, a system of three equations is obtained:

$$-\frac{DC_r}{X^2} + \frac{cC_c}{2} + C_b D_s - \frac{DK}{X^2}[1-F_t(R+W)] - \frac{DC_t}{X^2}F_t(T) = 0,$$

$$cC_c + C_b D_s - \frac{DK}{X}f_t(R+W) = 0,$$
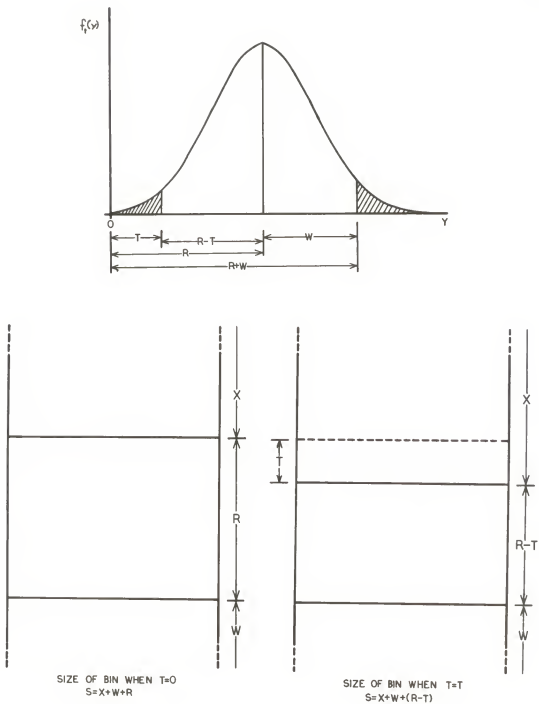
FIGURE 6. DEMAND DISTRIBUTION DURING LEAD TIME (ASSUMED NORMAL) AND SIZE OF BIN

and $\quad -C_b D_s + \dfrac{DC_t}{X} \, f_t(T) \; = 0$

This system can be solved for X, W and T, using a numerical procedure as the one previously used. Of course, the optimum values for X, W, and T depend on the real costs for each particular case. Therefore, the reduction of the size of the bin, by the quantity T, will depend on the cost structure. This structure determines whether X and W for the second case are going to be of a larger, equal or lesser size than those for the first case (T = 0).

Incomplete knowledge of the demand distribution does not mean that the inventory control model can not be developed. If only the mean and the standard deviation are known for instance, a model still can be worked out by applying Tchebycheff's inequality.

The results, of course, will not be the best policy, but that is the price of a lack of knowledge of the actual distribution. The more known about the distribution the nearer the results will be to the optimum.

Knowledge of the distribution moments, whether there exists immodality or not, etc., improve the resulting model. Those cases of partial information are fully discussed in the literature (15). Best results are logically obtained when the demand distribution is defined, as assumed in the model worked out.

Set-up Cost (C_r)

In general set-up cost is simply the cost of the lost time
during set-up. Within the feed mill it would be the time
lost in making the necessary changes to produce a new formula
when it has been producing a different one.

Whenever a change of formula occurs or even with the same
formula but changing the shape of the final product (mash, pellets,
crumbles) or changing from bulk to bags, some time is spent in
setting the equipment so the new formula can be produced at full
capacity. This operation is called "set-up", and the cost is
the set-up cost.

Deriving the set-up cost involves determining the period
of time spent between the time the scheduled production of the
current formula is finished and the time the mill is producing
the new formula at its regular capacity. Once this period of
time is determined and knowing the cost per hour of the mill
(labor, utilities, depreciation rates, etc.), the set-up·cost is
derived by multiplying the set-up time times the cost per hour.

Set-up cost for similarly processed formulas would be the
same. In other words, independent of the formula itself if
two formulas are produced in the same way (pellets in bulk or
mash in bags, for example) they will have, on the average, the
same set-up cost. By the same token, variation on these con-
ditions will vary the set-up cost.

There are several operations that have to be made in
changing formulas in a feed mill. The set-up time will be com-

puted on the "bottle-neck" operation; that which will take more
time in setting the mill up, since almost all of them can be
made simultaneously.

      In general time losses may occur at the following points.

a. Check if major ingredients are available in the
    ingredient bins. If not, order transfer from storage
    bins or warehouse of the quantity needed. Set grinders
    and grind if it is necessary. (Usually in normal
    operation this should have been done before the
    formula change.)

b. Check availability of minor ingredients, premix, etc.
    [same observation as in (a)].

c. Check if the flow from ingredient bins to final
    destination of the mix is clear. Set the flow, spouts,
    conveyors, etc. For bagged feed: Arrange for avail-
    ability of bags, labels, etc. For bulk feed: Set
    spouts and flow to the desired bin. For pelleted or
    crumblized feed: (1) change pellet mill die if it is
    necessary, (2) empty cooler, (3) set feed rate, steam
    rate, temperature to get optimal capacity in pellet
    mill and optimal pellet quality, and (4) fill cooler.

      Only careful time studies can determine the extent of
the time losses in an actual mill.

Cost of Being Out-of-Stock (K)

The out-of-stock cost is the cost of not carrying inventory. There are several situations to consider in regard to this cost. If an order for some amount of a particular feed arrives at the mill when this is out-of-stock:

a. If the customer is willing to wait the time required to fill the order, and the mill is not running 24 hours per day, the order could be filled by overtime production. The regular production schedule could be interrupted in order to make room for the particular order, and of course, overtime is needed to recuperate the normal production. Or the order could be run directly in overtime without interrupting the regular schedule. In any case the out-of-stock cost (K) would be: (Overtime production cost + set-up cost for interrupted normal schedule). The overtime production cost can be easily calculated, and usually amounts to the regular production cost plus the additional percentage of labor cost due to overtime.

Set-up cost was explained above.

b. If the customer is willing to wait some time to get his order filled but the mill is running 24 hours per day, the order could be lost (this case is discussed later), or the production schedule should be interrupted to make room for that particular order. In this latter case, recuperation of the lost production

could be done over the week-end with its consequent
costs (overtime cost plus set-up cost), or lost pro-
duction could be carried on indefinitely with risk
of going out-of-stock in other formulas and losing
additional orders.

c. Assuming that the mill is running 24 hours, seven
days a week, or if the customer is not willing to
wait for his order to be filled in a given time. This
depends on the emergency of the order itself from the
customer's point of view. (Animals could starve, for
instance.) In this case, the sale will be lost because
the customer will go to some other feed supplier to
get his order. The cost of out-of-stock would be
then, the profit lost due to not making a sale.

But usually this is not all because a customer who
has found one or more out-of-stocks in one given
formula, will become less likely to return and there-
fore, the customer will be lost. It could happen
that a particular customer usually buys not only the
formula we have been talking about, but several
others.

Thus, the cost of out-of-stock would be the profit
lost due to not making all possible future sales in
the different formulas the lost customer used to buy.
It is very difficult to measure this cost and perhaps
it could only be established by estimation. The way

to do this is by establishing a policy of permissible
percentage level of out-of-stocks, one out-of-stock
in a hundred times or one in a thousand times for
example. This kind of policy imputes automatically
a cost to the condition of being out-of-stock:

We have derived the following equation.

$$X = \frac{DK}{cC_c + C_b D_s} \, f_t \, (R + W)$$

in the analysis of the Q-system.

X can be approximate from the optimal lot-size formula
which is usually applied for inventory problems with fixed known
demand:

$$X = \sqrt{\frac{2 \, DC_r}{cC_c}}$$

therefore we could write:

$$f_t(R + W) \, \frac{DK}{cC_c + C_b D_s} = \sqrt{\frac{2 \, DC_r}{cC_c}}$$

or

$$K = \sqrt{\frac{2 \, cC_c C_r}{D}} \left[\frac{C_b D_s}{f_t(R + W)} \right] \, .$$

Here as before $f_t(R + W)$ is the lead time demand distribution.

If our policy is to permit one out-of-stock in a hundred
times:

$$1 - F_t(R + W) = .01$$

$$F_t(R + W) = .99$$

With this value of .99 we can evaluate our $F_t(y)$ and deduce the value of $(R + W)$, and therefore the value of $f_t(R + W)$. Substituting this in our formula for K we get an estimation of the cost of being out-of-stock for the particular policy we have set.

The approach of the optimal lot-size formula and the policy of permissible percentage level of being out-of-stock is an approximated method of analysis to derive the Q-system.

Considerations on the Design of
the System of Bins

Once the optimal theoretical size of the bin for each one
of the feed formulas under study has been determined, the
system of bins must be analysed as a whole.

Logically, because of the existing differences in the
demand and some other characteristics, each product will result
with a different bin size in theory. It is more economical,
of course, to have many or all bins of the same size. Besides,
there are some other known engineering and operational factors
that lead one to follow regular patterns in the construction
of the system of bins. For these reasons some combinations
should be made in order to achieve standard measures in the
height and sections of the bins. Following this idea, some
products could have two or more bins assigned, and some others
only one.

In the case of a feed formula having two or more bins
assigned, one of the bins could be used as the "reorder level";
in other words, the last bin of the set would be used as the
signal to place a new order. This method reduces considerably
the cost of the perpetual inventory that otherwise must be kept
in dealing with the Q-system of inventory control.

EXAMPLE II

To show how the model operates an example illustrating the first approach of determining the demand distribution $f(y)$, is shown next.

Consider formula F-1 which sample demand distribution in tons/day is shown in Table 4 and Figure 4A.

Sample mean and variance were calculated before the data was grouped being their values:

$$u = 42.8 \text{ tons} \qquad \text{(mean)}$$
$$s^2 = 280.11 \qquad \text{(variance)}$$
$$s = 16.75 \qquad \text{(standard deviation)}$$

Let us try to fit the data to a normal distribution by running a chi-square test of goodness of fit:

The null hypothesis is that data come from a population having a normal distribution. Reject the null hypothesis if

$$x^2 \geq x^2_{\alpha, \, n-3} \qquad \text{(n=number of observations; 2 parameters are used)}$$

$$x^2 \geq x^2_{.05, 3}$$

$$x^2 \geq 7.815$$

$$x^2 = \sum_{i=1}^{n} \frac{(P_i - e_i)^2}{e_i}$$

$$n = 6$$

Demand (tons/day)	P_i (%) Observed Probability	e_i (%) Expected Probability
0-15	5.0	4.33
15-30	17.5	17.51
30-45	27.5	32.81
45-60	37.5	29.72
60-75	10.0	12.47
75-	2.5	2.58

e_i has been calculated from a normal distribution with mean equal to 42.8 and standard deviation equal to 16.75.

$$\chi^2 = 3.47 < 7.815$$

Therefore the null hypothesis is accepted and $f(y)$ can be expressed as a normal distribution:

$$f(y) = \frac{1}{\sqrt{2\pi}\ (16.75)} \quad \exp\ [\ -\frac{1}{2}\ (\frac{y-42.8}{16.75})^2\ ]$$

To derive the best policy of inventory control and the size of bins needed for formula F-1, some estimation of costs is needed. (The cost data used here is estimated and it has been selected only as illustration).

Let:

Lead time = 1 day

R = u = 42.8 tons

   s = 16.75 tons

   D = (260)R = 11,128 tons/year

   c = $80/ton

   C_b = $30/ton

$$D_s = .10$$

$$C_c = .10$$

$$C_r = \$10$$

Two costs have not been given: the out-of-stock "K" cost, and the cost of running out of storage space.

Assume that the company has set a standard of 1% as permissible percentage of time of being out of stock. K can now be estimated by the formula

$$K = \sqrt{\frac{2 c C_c C_r}{D}} \left[\frac{C_b D_s}{f(R+W)} \right]$$

$F(R+W) = .99$ which yields $f(R+W) = .001$

(value found in normal tables for mean and standard deviation used).

Substituting correspondent values in the formula and solving:

$$K = \$114$$

The cost of running out of storage space can be given in a per ton basis. Let C_o be the notation for this per ton cost, and assume in this example, that:

$$C_o = \$5/\text{ton}$$

At this point a review of the model is indicated. Since the model derived included a fixed cost of running out of storage space, C_T.

When C_o is given instead of C_T the total yearly cost of the system is:

$$T.C. = \frac{DC_r}{X} + (\frac{X}{2} + W) cC_c + [X + W + (R-T)]C_b D_s + \frac{DK}{X} \int_{R+W}^{\infty} f(y)\,dy$$

$$+ \frac{D}{X} C_o \int_o^T (T-y) f(y)\,dy$$

The last term is the average yearly cost of running out of storage space, being $\frac{D}{X}$ the average number of orders per year and $\int_o^T (T-y) f(y)\,dy$ the expected value of the number of tons the demand "y" is less than the value "T".

Taking derivatives with respect to the three variables X, W, T, equating to zero, and assuming that the lead time is one day [which means that $f_t(y) = f(y)$; and $F_t(y) = F(y)$]; three equations are obtained:

$$(I) \qquad X^2 = \frac{2D\{C_r + K[1-F(R+W)] + C_o[T\,F(T) - \int_o^T y f(y)\,dy}{cC_c + 2 C_b D_s}$$

$$(II) \qquad f(R+W) = \frac{cC_c + C_b D_s}{DK} X$$

$$(III) \qquad F(T) = \frac{C_b D_s}{DC_o} X$$

This system can be solved numerically: assume a value for X, calculate F(T) out of equation (III) and f(R+W) out of equation (II). Deduce correspondent values for T and W from the known f(y). Substitute those values in equation (I) and compute the new value for X. Compare this value with that

previously assumed and proceed as before if they are not close to each other.

The only difficulty that can be found is in the term $\int_0^T yf(y)\,dy$. Let G be the notation for this term and see what happens when f(y) is normal as in the example being studied.

$$G = \int_0^T yf(y)\,dy = \int_0^T \frac{y}{\sqrt{2\pi}\,s}\ \exp\left[-\frac{1}{2}\left(\frac{y-u}{s}\right)^2\right]\,dy$$

Here the normal distribution is truncated at zero, however the Chi-square test assumes no negative values for the actual case.

Changing variables:

$$z = \frac{y-u}{s}$$

$$y = sz + u$$

$$dy = s\,dz$$

$$G = \frac{1}{\sqrt{2\pi}} \int_{-\frac{u}{s}}^{\frac{T-u}{s}} (zs + u)e^{-1/2\ z^2}\,dz$$

integrating by parts:

$$G = \frac{1}{\sqrt{2\pi}}\ \left\{\ e^{-\frac{1}{2}\left(\frac{T-u}{s}\right)^2}\left[-s + u\left(\frac{T-u}{s} + \frac{\left(\frac{T-u}{s}\right)^3}{3} + \frac{\left(\frac{T-u}{s}\right)^5}{15} + \frac{\left(\frac{T-u}{s}\right)^7}{105} + \ldots\right)\right] \right.$$

$$\left. + e^{-\frac{1}{2}\left(\frac{T-u}{s}\right)^2}\left[s + u\left(\frac{u}{s} + \frac{\left(\frac{u}{s}\right)^3}{3} + \frac{\left(\frac{u}{s}\right)^5}{15} + \ldots\right)\right]\right\}$$

Solving now the example:

a) Assume X = 167 (this value is calculated from the optimal lot size formula for X, given in the section devoted to discussing the out-of-stock cost K)

b) Substitute in (III) and solve:

$$F(T) = \frac{(30)(.10)}{5(11,128)} \ (167) = .009$$

$$T = 3.27 \text{ (from normal tables)}$$

c) Equation (II):

$$f(R+W) = \frac{(80)(.10)+(30)(.10)}{(11,128)(114)} \ (167) = .0014$$

Then, $(R+W) = (42.8 + 39.85) = 82.66$

d) Compute G with the value of T:

$$\frac{T-u}{s} = \frac{3.27 - 42.8}{16.75} = -2.37$$

$$\frac{u}{s} = \frac{42.8}{16.75} = 2.56$$

$$G = \frac{1}{\sqrt{2\pi}} \ \{e^{-2.80}[-16.75 + 42.8(-2.37-4.44-4.99-4.00-2.49-1.28-0.55)]$$

$$+e^{-3.28}[16.75+42.8(2.56+5.6+7.33+6.87+5+3+1.5+0.3)]\}$$

(The series converge to zero after the terms shown)

$$G = \frac{1}{\sqrt{2\pi}} \ (51.55 - 52.67)$$

$$G = -0.45$$

e) Compute [1 - F(R+W)] with the value of W:

[1 - F(R+W)] = .009 (from normal tables)

f) Compute X in equation (I)

$$X^2 = \frac{(2)(11,128)\{10 + 114(.009) + 5[3.27(.009) + 0.45]\}}{(80)(.10) + 2(30)(.10)}$$

$$X = 146$$

The whole procedure is repeated now with this new value of X. In this example the solution was found at the third trial:

$$X = 136 \qquad\qquad R+W = 84$$
$$W = 41$$
$$T = 3$$

The policy should be: whenever the stock on hand falls to 85 tons, order 116 tons.

The bin size:

$$S = X + W + (R-T)$$
$$S = 217 \text{ tons}$$

CONCLUSIONS

The main difficulties in developing models of inventory
control that lead to proper sizing of facilities are those
found when trying to get the demand distribution.

Management of existing feed mills should study directly
available data in tons of feed per day in order to build the
demand distribution. New mills have, perhaps, to study
separately order size and number of order distributions and
apply the second approach discussed here.

Data analysed for feed formulas sold in bulk, product of
particular conditions of the market, price policies and some
other characteristics for the feed mill studied, have shown
that:

a. Price policies have definite influence in order
   size distributions for feed formulas.

b. Customers' order frequency follow approximately a
   Poisson distribution.

c. In general no seasonal variation during a week
   period was observed.

Some of the methods that can be used to get general
functions with which future demand could be predicted within
reasonable risks have been discussed. Particularly, it has
been shown how to get the demand distribution as a marginal
probability function, when the order size is expressed by
means of a general polynomial function and the number of orders
per day follows a Poisson.

Some other techniques not discussed here, such as fitting experimental data to Pearson curves or simulation, can be equally applied.

Simulation is indicated for the design of inventory control systems where seasonal variations occur in weekly or yearly periods.

Inventory control systems covered by major textbooks in the field were discussed and several cost factors were analysed in applying those systems to the feed industry. The mathematical model of one of the systems was extended to derive parameters that allowed bulk feed bin sizing. A new concept of cost, the cost of running out of storage space, was introduced in the model to generalize more its application.

Set-up cost can only be determined by detailed time studies. Estimations of the out-of-stock cost and the penalty cost of no room for storage depend to a certain extent on management judgment.

Further research is needed in determining over-all policies of operation and facilities sizing in feed mills. Studies that consider all possible variables involved: production capacity and planning, ingredient procurement and storage, feed formulation, inventory for bagged and bulk feed, warehouses, etc.; will surely determine better policies than those resulting from partial analysis of each area separately.

ACKNOWLEDGMENTS

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—APPENDIX—

DATA FOR NINE FEED FORMULAS SOLD IN BULK
FROM TYPICAL LARGE CAPACITY FEED MILL. (2 months)

FORMULA F-1

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0	4(11.6)	1(11.6),1(4),1(10)	2(11.6),1(10.4)		3(11.6),1(10.4),1(4),1(3.5)
1	2(11.6),1(10.4),1(4)	2(11.6)	1(11.6),1(10.4),1(10),1(8.4)	3(11.6),1(4),1(8.4)	3(11.6),1(10),1(2.5)
2	6(11.6),1(10.4)	1(11.6),1(6)	3(11.6),1(2)	4(11.6)	1(11.6),1(10)
3	5(11.6),1(2)	2(11.6),1(8.4)	2(11.6),1(2)	1(11.6),1(11)	2(11.6),1(10.4),1(10)
4	3(11.6),1(11.2)	1(10.4),1(10)	3(11.6),1(10.4),1(2)	4(11.6),1(10.4)	2(11.6),1(9)
5	4(11.6),1(10.4),1(8.4),1(2.4),1(7.6)	1(11.6)	3(11.6),1(10.4),1(4)	3(11.6),1(10.4)	4(11.6),1(10)
6	3(11.6),1(11.2)	2(11.6),1(10.4),1(4)	3(11.6),1(10.4),1(7.6)	Holiday	Holiday
7	3(11.6),1(10),1(4),1(6),1(5.2)	2(11.6),1(8.4)	2(4),1(2.5)	4(11.6),1(9)	Holiday
8	4(11.6),1(10.4),1(11)	2(11.6),1(10.4),1(8.4),1(2.4)		3(11.6),1(10.4),1(10)	2(11.6),1(7.6)
9					

FORMULA F-2

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					
1	8(11.6)	4(11.6),1(7.6)	2(11.6),1(5.8)	3(11.6)	1(11.6)
2	6(11.6)1(7.6)	2(11.6)	3(11.6)	4(11.6)	3(11.6)
3	6(11.6)	4(11.6)	3(11.6)	5(11.6)	2(11.6)
4	5(11.6)	9(11.6)		3(11.6)	2(11.6)
5	8(11.6)	6(11.6)	4(11.6)	1(11.6),2(10.4),2(1.2)	3(11.6)
6	8(11.6),1(4)	5(11.6)	3(11.6)	Holiday	Holiday
7	3(11.6)	6(11.6)	7(11.6),1(4)	4(11.6)	Holiday
8	6(11.6)	5(11.6)	3(11.6),2(7.6)	4(11.6)	5(11.6)
9	7(11.6)	5(11.6)			

FORMULA F-3

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					1(9.6)
1	1(10.4)	—	1(5.4),1(4.8)	1(10.4),1(5.6)	1(4.8),1(2.4)
2	1(5.4),2(2.4)	1(9.6)	1(7.8)	2(2.4)	1(5.2)
3	1(2.4)	1(10.4),1(8.8)	1(11.2),1(5.4)	—	1(10.4),2(5.2)
4	1(10.4)	1(10.4),1(4.8)	—	¡	1(5.2),1(4.8)
5	3(5.2)	1(10.4)	—	—	1(7)
6	—	1(11.6),1(5.2),1(2.4)	1(7),1(5.2)	Holiday	Holiday
7	1(10.4)	1(11.6),1(7)	1(5.2)	1(5.2)	Holiday
8	—	1(10.4),1(6),1(3)	1(2.4)	—	1(5.6),1(5.2)
9	1(7)	1(4.8),1(2.4)			

FORMULA F-4

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					1(10.4)
1	–	–	1(4.8),1(1.3)	1(10.4)	–
2	1(5.4),1(4.8)	–	1(10.4),1(2.5)	1(4.8)	–
3	1(4.8),1(2.4)	–	1(5.4)	2(4.8)	1(4.8)
4	–	1(4.8)	1(10.4)	–	1(5.2),1(4.8)
5	1(5.2)	–	1(10.4),2(4.8)	–	1(3)
6	1(7.2)	1(6.2),1(5.2)	1(5.2),1(3)	Holiday	Holiday
7	1(9.6)	1(10.4),1(3)	1(5.2)	1(9.6),1(5.2)	Holiday
8	–	1(4)	1(4.8)	1(.0.4)	1(5.2)
9	1(3)	–			

FORMULA F-5
NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					1(9.6),1(8.8)
1	1(8.8)	2(9.6),1(4.4)	1(8.8),1(4.4)	1(9.6),1(2)	—
2	—	2(9.6),1(8.8)	1(8.8),1(2)	1(9.6),1(8.8)	1(9.6)
3	1(9.6),1(3)	—	2(9.6)	1(9.6)	1(2)
4	1(9.6)	1(9.6),1(8.8)	1(9.6),1(8.8)	1(9.6)	1(9.6)
5	1(9.6)	3(9.6),1(2)	—	1(9.6),1(8.8)	1(2)
6	—	5(9.6)	1(9.6),1(3)	Holiday	Holiday
7	—	3(9.6),1(3)	1(9.6),1(6)	2(9.6)	Holiday
8	2(9.6),1(2)	1(9.6)	2(9.6)	1(9.6),1(3.1)	—
9	1(9.6),1(3)	2(9.6)			

FORMULA F-6

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					−
1	−	1(10.4),1(2.6)	−	1(5.2),1(2.6)	−
2	−	−	1(10.4)	2(2.6)	−
3	−	1(10.4)	−	−	1(2.6)
4	1(10.4),1(2.8)	1(5.2)	−	1(5.2)	−
5	−	−	1(10.4)	−	−
6	−	1(3)	1(10.4)	Holiday	Holiday
7	1(2.6),1(2.4)	−	1(10.4)	1(3)	Holiday
8	−	1(5.2),1(2.6)	−	−	1(2.8)
9	−	1(2.4)			

FORMULA F-7

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					1(10.4),1(5.2)
1	1(10.4)	1(10.4),1(5.3)	1(,0.4)	—	1(10.4),1(6),1(5.2)
2	—	—	1(10.4),1(1.7)	1(7.8),1(5.2)	1(10.4),1(4.2)
3	—	—	1(10.4)	—	1(7.8)
4	1(10.4),1(6)	1(5.2),1(2.5)	1(10.4)	—	1(10.4)
5	1(10.4),1(5.6),1(5.2)	—	3(10.4)	—	2(10.4),1(10.6),1(5.2)
6	1(10.4)	1(10.4),1(5.2),1(3)	1(10.4)	Holiday	Holiday
7	2(10.4),1(7.8),1(5.2)	1(6),1(4.4)	1(10.4)	3(19.4),1(5.2),1(3)	Holiday
8	1(10.4)	1(7.8),1(5.2)	1(10.4),1(5.2)	—	1(7.8)
9	3(10.4)	1(10.4),1(4.8),1(1.7)			

FORMULA F-8

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					–
1	1(10.4)	–	–	–	1(6)
2	–	–	1(6)	–	1(2.8)
3	1(10)	–	–	–	–
4	1(5.2),1(3.2)	1(2.6)	1(10.4)	–	–
5	1(5.4),1(4.4)	–	–	–	1(2.6)
6	–	1(6),1(2.6)	–	Holiday	Holiday
7	1(2.6)	1(6)	–	1(6),1(2.6)	Holiday
8	1(5.2)	–	1(2.6)	–	1(10.4),1(9.2)
9	–	1(6)			

FORMULA F-9

NUMBERS OF ORDERS AND (ORDER SIZE IN TONS)

Week	Monday	Tuesday	Wednesday	Thursday	Friday
0					1(6),1(5.2)
1	—	1(5.2)	—	1(7.8)	—
2	1(6)	—	—	1(5.2)	1(7.8)
3	—	1(10.4)	—	—	—
4	—	1(5.2)	—	1(5.2)	1(6)
5	1(10.4),1(5.2)	—	1(10.4)	—	1(2.6)
6	—	1(10.4),1(2.6)	1(6)	Holiday	Holiday
7	1(2.6)	1(10.4)	—	1(6),1(2.6),1(3)	Holiday
8	—	—	1(2.6)	1(10.4)	1(2.6)
9	—	1(2.4)			

PROBLEMS OF INVENTORY CONTROL IN FEED MILLS

by

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1963

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AN ABSTRACT OF A MASTER'S THESIS

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requirements for the degree

MASTER OF SCIENCE

Department of Grain Science and Industry

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Manhattan, Kansas

1969

The feed industry can improve efficiency in general and achieve higher profits by applying Operations Research techniques in sizing facilities and determining operational policies. This thesis analyses the problem of finished product inventory control in feed mills with the objective of optimizing actual operation and determining the proper capacity of storage facilities. It studies in particular feed formulas produced to be sold in bulk and correspondent sizing of bulk feed bins. However, the analysis can be extended to the consideration of bagged feed and general warehouses.

The main difficulties in developing models of inventory control are those involved in determining the demand distribution for a given formula. The first part of the thesis is devoted to the determination of that probability distribution. Considerations have been made for practical situations, such as availability of historical data and market research studies, indicating procedures to follow while studying existing or new mills. Data from a particular feed mill with characteristic conditions of market and price policies is analysed. Results show that order size distributions for feeds are definitely influenced by price policies and customers' order frequency follow approximately a Poisson distribution.

Some of the methods that can be used to get general functions to predict future demand within reasonable risks are discussed. It is specifically shown how to get the demand distribution as a

marginal probability function, when the order size distribution is approximated by polynomial expressions and the number of orders follows a Poisson distribution.

The mathematical model of one of the major systems of inventory control is derived including factors related to the size of the bin that will hold the inventory. In this fashion the best inventory policy and the parameters needed to size the facility are simultaneously obtained. The model is generalized introducing a new cost: the cost of " running out of storage space ". Factors involved in inventory control are defined especially from the feed industry point of view and particular discussion is devoted to steps to be followed to determine set-up and out-of-stock costs in a feed mill.

This work shows the actual possibility of overcoming empirical methods still being used in the feed industry when sizing warehouses or setting inventory policies.