DRYING OF FEED PELLETS

by

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NOMENCLATURE

A - Cross-section area of dryer

c - Specific heat of medium

D - Diffusion coefficient

d - Pellet diameter

$h_g$ - Coefficient of heat transfer through air film

$h_m$ - Thermal conductivity of solid

$h_s$ - Humidity of saturated air in the grain bin--lb. water per lb. of dry air

$h_1$ - Humidity of air entering dryer, lb. water per lb. of dry air

K - Thermal diffusivity

$K_g$ - Coefficient of water vapor diffusion

$K_m$ - Coefficient of water vapor diffusion through solid

L - Depth of bed

$L'$ - Distance from surface to limit of zone of vaporization

$M_a$ - Average moisture content

$M_c$ - Critical moisture content

$M_e$ - Equilibrium moisture content of bed

$M_i$ - Moisture content at given time

$M_s$ - Dynamic equilibrium moisture content

$M_x$ - Average moisture content of bed at end of maximum drying rate

$M_0$ - Initial moisture content

$M_2$ - Final moisture content

m - Rate of drying constant for deep bed

n - Integer
P - Vapor pressure

$P_z$ - Vapor pressure in zone of vaporization

Q - Air blowing through the bed (C.F.M.)

q - Water concentration

r - Radius of the sphere (grain)

S - Surface area of solid

$T_a$ - Temperature of air to bed

$T_d$ - Dry bulb air temperature

$T_g$ - Mean grain temperature

$T_{gl}$ - Initial grain temperature

$T_{pl}$ - Mean pellet temperature

$T_{po}$ - Initial pellet temperature

$T_s$ - Temperature at the solid surface

$T_w$ - Wet bulb air temperature

$T_Z$ - Temperature of the limit of vaporization zone

V - Volume of solid

$W_a$ - Mass rate flow of air, lb. of dry air per hr. sq. ft.

$W_g$ - Weight of grain in bin, lbs. dry matter

$W_w$ - Weight of water removed

W - Weight of water removed at given time

$\theta$ - Total time

$\theta_I$ - Time of drying maximum rate period

$\theta_{II}$ - Time of drying decreasing rate period

$\lambda$ - Latent heat of moisture

$\omega_f$ - Rate of free water transfer
INTRODUCTION

One of the most important processes and most problematic in the modern feed mill is the production of hard pellets and range cubes.

Drying is necessary when the pellet mash has been conditioned by steaming and the excess moisture has to be removed for safe storage of the finished pellet for relatively long periods.

Young and Pfost (29) found that pelleting efficiency and durability were directly related to the amount of steam added. Smith (25) and Young and Pfost (29) found that fines can be reduced by adding steam which heats the mash feed and causes a higher gelatization.

Also, during the pressing process friction adds heat to the pellet. The pellet must therefore be cooled to the ambient temperature.

The following two problems are basic in the pellet drying and cooling process:

1. The removal of the additional moisture, added by steam, to dry the pellet to a proper moisture content, 10 to 12 per cent (wet basis), depending on the relative humidity in the area.

2. The cooling of the pellet to the ambient temperature, i.e., from 160°F - 200°F. to 70°F - 90°F.

Usually pellets have been cooled and dried by large amounts of natural air.
Three-quarter-inch dairy feed range cubes with bloat guard were produced in the Kansas State University pilot feed mill during the summer of 1965 and it was found that even after 45 minutes cooling of 1,000 lbs. by a 1110-CFM conventional "California" pellet cooler, the range cubes became moldy after less than two weeks, which limited the desired storage period. The problem obviously is to transfer the moisture from the center of the range cube in a relatively short drying period.

The relative humidity of the air used for drying and cooling has a direct effect on the quantity of air required; where the relative humidity level is 90 to 95 per cent; pellets cannot be dried properly.

When cold air passes through hot pellets, the temperature of the air increases. With an increase in air temperature, the relative humidity decreases; thus the capacity of the air to pick up moisture is increased and at the same time heat is removed from the hot pellets. As the pellets cool, the air ceases to be heated and becomes saturated with moisture.

In high humidity areas some feed mills use coolers of higher air capacity; consequently, the air velocity is increased so that the air blowing through the bed carries out some of the fines. This residue together with some condensed water accumulates in the conveyor pipes and cyclone.

Using heated air can reduce greatly the relative humidity and evaporative cooling will prevent too much increase in the temperature of the pellets. Consequently it is possible to dry high moisture content pellets in high relative humidity areas.
By using heated air, the air requirements are reduced and the problem of fine in the air system is reduced.

Pellets dried with hot air must be cooled to surrounding temperature with natural air. In high humidity areas, some moisture can be regained during this process. It is dependent on the difference between the hot, dry pellet equilibrium moisture vapor pressure and the cold air vapor pressure. But in any case, moisture content cannot be changed too much because usually the vapor pressure of the colder air is lower than that of the hotter pellets.

The drying theory for pellets is based on a combination of heat transfer and mass transfer that interact simultaneously. The solution involves a complicated mathematical process that, until now, has not been satisfactorily applied. Most of the investigators have neglected the mass transfer factor. By using experimental data and applying dimensional analysis methods, this research was planned to develop an equation which would describe the pellet drying process.

Recent rapid increases in production of larger pellets, such as range cubes, have made pellet drying a more critical problem.
REVIEW OF LITERATURE

Stroup (27) considered removal of almost all of the moisture added to the mash feed in the form of steam before pelleting by passing 500 cubic feet per minute per ton of ambient air through a column of 1/4-inch diameter pellets for a period of 6 to 7 minutes. The length of time that a pellet should be exposed to the air stream, he concluded, is related to the diameter of the pellet; approximately 2 1/2 to 3 times longer retention time is required for each 1/4 inch increase in diameter.

Experience has shown that about 500 CFM of air is required for cooling one ton of 1/4-inch diameter pellets. The amount of air should be larger when larger pellets are processed.

Stroup also reported that:

The relative humidity of air used for cooling and drying has a direct effect on the quantity of air required. Pellets can be cooled by air that is at relative humidity of 90-95%. When this air has passed over the hot pellets, the temperature of the air naturally has increased, the relative humidity has therefore decreased; thus the capacity of the air to pick up moisture has increased and this phenomenon causes two very desirable effects. 1) The moisture of the pellets is carried away. 2) The heat is removed from the hot pellet.

This is the only published work on drying and cooling of feed pellets. Therefore the literature on drying of other hygroscopic material, especially cereal grain, has been reviewed with especial attention to results and conclusions which might apply to pellet drying.
Fundamental Concepts of Moisture Movement

Before the concepts of movement of moisture in hygroscopic material can be discussed, a number of terms must be defined.

**Free Water.** This is the water that exerts the same vapor pressure and which has the same latent heat of vaporization as does pure water at the same temperature; it is largely held in the interstitial spaces between particles.

**Bound Water.** This is the water up to the lowest concentration that is equilibrium with saturated air. It exerts a vapor pressure less than that of liquid water at the same temperature. (22) "Liquid may become bound by retention in small capillaries, solution in cells of fiber walls, homogeneous solution throughout the solid, and chemical or physical adsorption on solid surfaces."

**Adsorption.** The phenomenon of adsorption is distinct from that of primary chemical bonding; the water is held more loosely than by chemical bonding and forces that hold it are essentially those known as "hydrogen bonding".

Once adsorption has occurred, the molecules are restrained to two-dimensional free movement over the surface of the solid. This loss of one degree of freedom is accompanied by a decrease in entropy; the decrease in heat content is termed the "heat of adsorption" and accounts for the generally observed exothermic nature of all adsorption processes.

**Equilibrium Moisture Content.** A very wet sample of material loses moisture to the air and continues the process until an equilibrium state is reached. The moisture content attained at
equilibrium is termed the "equilibrium moisture content."

It is usually considered to be dependent upon the temperature and humidity of the ambient air as well as upon the particular material considered.

**Dynamic Equilibrium Moisture Content.** Hall (15) suggested that the moisture content at dynamic equilibrium might be determined by a method in which the "atmosphere surrounding the product or products is mechanically moved".

**Adsorption-Desorption Curves.** The relationship between the vapor pressure of water over solid material and the moisture content of the solid material is usually determined by experiment and is given in the form of a curve relating the moisture content of the solid material to the relative humidity of air in equilibrium with it. The moisture content corresponding to a relative humidity of 100 per cent is that which divides the "free" and "bound" water states. If the solid is wetter than this, it contains some free water and a maximum amount of bound water. If the solid is drier than this, it contains only bound water.

Since the curve of moisture content versus relative humidity (Fig. 1) is an equilibrium curve, theoretically the same curve should represent the end points of both water adsorption and water desorption processes applied to the material. When both adsorption and desorption curves for the porous hygroscopic materials have been obtained experimentally. However, the curves do not always coincide, but the desorption curves give higher moisture content for a given relative humidity than
FIGURE 1  HYSTERESIS IN MOISTURE SORPTION IN WHEAT
do the adsorption curves (5, 8, 9).

**Constant Rate Drying Period.** During a period of constant rate drying, the solid is so wet that water can be transferred from the interior of the solid to its drying surface as fast as it can be evaporated. The rate of evaporation is controlled by the transfer rates of heat and water vapor through the air film around the solid. The constant rate period is affected by the external conditions of air velocity, humidity, and temperature as well as by the area of the drying surface.

**Falling Rate Period.** When the solid continues to dry after the constant rate period has ended, the solid can no longer supply water to the surface fast enough to keep up with the rate of evaporation. As a result, the rate of drying decreases. It is assumed that internal moisture transfer completely controls the rate of drying.

The duration of the falling rate period usually establishes the size of dryer required (27).

Since more experimental and theoretical attention has been given to the constant rate period and to methods of predicting the rate in this period, it appears to be well understood. On the other hand, the falling rate period is not well understood and theoretical models have not been well established to describe adequately the mechanism of moisture movement. This is especially true for the cereal grains because of the complex nature of this structure, the great variety of grains, and the difficulties in describing them.
Mechanisms of Moisture Movement

**Liquid Diffusion.** Diffusion of liquid moisture may result because of concentration gradients between the interior of the solid where the concentration is high and the surface where it is low. These gradients are set up during drying from the surface. Diffusion is characteristic of slow-drying materials. Resistance to the mass transfer of water vapor to the air is usually negligible and the diffusion in the solid controls the overall drying rate. This type of moisture transport probably occurs in the removal of water from solids such as clays, paper, wood, and grains.

The liquid diffusion mechanism can be explained as the rate of drying or a function of the average moisture content diffusivity \( (3, 4, 21, 22) \). It has been shown that diffusivity usually decreases rapidly with decreasing moisture content. Published data are average values over the range of moisture content considered.

**Capillary Movement.** Free (unbound) moisture in granular and porous solids, such as clays, sand, and paint pigments, moves through the capillaries and interstices of the solids by a mechanism involving surface tension. At first, moisture moves by capillarity to the surface rapidly enough to maintain a uniformly wetted surface and the rate of drying is constant. The water is replaced by air entering the solid through relatively few openings and cracks. Eventually, as the surface moisture is drawn to spaces between the granules of the surface, the wetted area at the surface decreases. When the subsurface
reservoirs dry up, the liquid surface recedes into the capillaries, and evaporation occurs below the surface in a zone or plane which gradually recedes deeper into the solid. During this period, diffusion of vapor within the solid will occur from the place of vaporization to the surface.

Barrer (7) indicated that the heat of adsorption would be doubled if a molecule were adsorbed in a crack so that it touched both parallel walls.

Ceaglake (10) and Hougen (16) have suggested that for the solids which do not dry in accordance with the diffusion theory, capillary flow is often important. It should be noted that capillary movement becomes more important for solids that are extremely wet and which have very porous structures. In grain drying, the capillary movement may play little or no effect in the mechanism since the range of moisture content may not be high and the germ and endosperm of the grain seed are surrounded by a pericarp fruit coat and a seed coat which does not contain any pores or capillaries connecting the interior with the atmosphere (22).

Theory of Drying Various Materials

**Grain Drying.** Babbit (3) studied adsorption and desorption of water vapor by wheat under different conditions. He was the first to apply a theoretical equation to fit the drying rates of cereal grain. With assumptions that a kernel of wheat is a sphere of homogeneous material and that under the driving force of the concentration gradient the moisture diffuses through
solids according to Fick's law, the following equation was obtained:

\[ M_1 = M_0 - (M_0 - M_2) \left[ 1 - \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n^2D\pi^2}{r^2}\right) \right] \] (1)

The derivation of equation (1) can be obtained from various sources (1, 4).

Equation (1) cannot be rigidly applied unless it is known that the diffusion coefficient is independent of moisture content and the value of the diffusion coefficient is known. Because the final moisture content was assumed, the end point was not zero and a better method must be found to evaluate the final moisture content. The equation must be solved by an uncertain process of trial and error.

Simmonds et al (24) studied the rate of drying wheat grain by drying deep beds in air of constant inlet temperature and humidity. It was found that two drying rate periods may be considered. The maximum rate drying period occurs until the drying front reaches the top of the bed and is represented by the equation:

\[ \frac{M_0 - M_x}{\theta_I} = \frac{dM_s}{d\theta} = \frac{A W_A(h_s - h_1)}{W_g} \] (2A)

whence

\[ \theta_I = \frac{W_g(M_0 - M_x)}{A W_A(h_s - h_1)} \] (2B)

As soon as the drying front reaches the top of the bed, the rate of drying starts to decrease and is designated as the
decreasing rate of layer drying, as given by the equations:

\[
\frac{dM_a}{d\theta} = 2.303 \, m(M_a - M_e) \tag{3A}
\]

\[
\theta_{II} = -\frac{1}{m} \log_{10} \frac{(M_x - M_e)}{(M_2 - M_e)} \tag{3B}
\]

\[
\theta = \theta_I + \theta_{II} \tag{3C}
\]

The air velocities used were sufficient to nullify the effects of external resistance to mass transfer but they were not high enough to provide reasonable control over the grain temperature which increased gradually with drying time. Simmonds et al. found an equation relating the grain temperature to the moisture content at a given stage in the drying process:

\[
T_g = T_a - (T_a - T_w) \frac{(M_i - M_e)}{(M_c - M_e)} \tag{4}
\]

They stated that the most likely mechanism would appear to be that of diffusion of liquid water from the interior followed by evaporation at the surface. Since the drying state is independent of air velocity, this indicates that capillary effects play little part in the mechanism. McEwen (19) tested the concept that if there is no resistance to mass transfer in the air film surrounding the kernel, it is assumed that the kernel surface immediately obtains the final moisture content in equilibrium with the surrounding air. He used equation (3) and found that the value of \(M_e\) which gave the best fit of his
experimental data was the dynamic equilibrium moisture content and not the final equilibrium moisture content.

Becker and Sallams (4) tested the experimental data for the diffusion mechanism of wheat at moisture content 14.5 to 25 per cent (wet basis) with a diffusion equation practically identical with that used by Babbitt (3).

\[
\frac{M_1 - M_s}{M_0 - M_s} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n^2 \pi^2 D \Theta}{r^2}\right)
\]  

(5)

This equation must be limited to the interval of moisture content from 20.8 to 12 per cent (wet basis). Baker (6) developed mathematical methods for the general solution of problems on unsteady state diffusion in solids of arbitrary shape.

The solution was obtained from Fick's law of diffusion by assuming that the mechanism of internal liquid flow which controls the drying rate is a diffusion mechanism.

When the time approaches zero,

\[
\frac{M_1 - M_s}{M_0 - M_s} = 1 - \frac{2}{\sqrt{\pi}} \frac{s}{r} \sqrt{D \Theta} + \frac{f''(0)}{2} \frac{s^2}{V} D \Theta
\]  

(6)

For the case when the time approaches infinity:

\[
\frac{M - M_s}{M_0 - M_s} = \frac{s}{\beta^2} \exp\left\{-\beta^2 \left(\frac{s}{V}\right)^2 D \Theta\right\}
\]  

(7)

Baker derived from equation (7) the equation for the diffusion coefficient:
Wang and Hall (30) have studied the moisture movement in hygroscopic material. Based on Babbitt's work (3) that vapor pressure is the main drying force for the moisture movement during drying, a mechanism and mathematical equation for hygroscopic materials were proposed for moisture movement inside a kernel of grain. By assuming that the medium is isotropic to both heat and vapor diffusion, simultaneous partial differential equations which govern the internal movement of moisture inside a spherical material were proposed.

a. The vapor diffusion equation:

\[
\frac{dM}{d\theta} = D \frac{d^2P}{dV^2}
\]  

b. The heat diffusion equation:

\[
\frac{dT}{d\theta} = K \frac{d^2T}{dV^2} - \frac{\lambda}{C} \frac{dM}{d\theta}
\]

The same investigators tried to confirm this partial differential equation, but were unable to obtain a practical working equation.

Allen (2) used dimensional analysis and found for deep bed drying of corn:
This equation was developed from data obtained from eight runs. The kernel size was not taken into account; the average size of a corn kernel was used.

Because this equation was developed on the basis of experimental data, it takes into account all of the variables except size.

**Paper Drying.** McCready (19) studied the adiabatic air drying of hygroscopic solids. The experiments were carried out on slabs of paper pulp and asbestos. The mechanism of drying hygroscopic solids during the falling rate period has been suggested. Under regular drying conditions only the water diffuses through the solid as liquid water and when bound water is transferred, it is transferred as vapor. During this period water is vaporizing in a zone of vaporization beneath the surface; then the stream of water vapor diffuses toward the surface. A partial pressure difference over the vaporization zone provides a driving force for the vapor diffusion process.

Two partial differential equations apply to the mechanism. One of the equations applies to the diffusion of water vapor and the other to the flow of heat. Both equations apply only where there is no free water present in the zone of vaporization.

a. Diffusion of water vapor:

\[
\log_{10} \frac{h W_a}{(M_0 - M_s)W_w} = 1.75 \frac{T_g - T_l}{T_a - T_l} - \frac{0.19 + \log_{10} \left( \frac{M_0 - M_1}{M_0 - M_s} \right)}{1.21} \tag{11}
\]
\[ e \frac{dq}{d\theta} = \frac{d}{dL'} (K_m \frac{dP}{dL'}) \]  

(12)

b. The corresponding equation for heat flow:

\[ e \left[ C(1 + M) \frac{dT}{d\theta} - \lambda \frac{dq}{d\theta} \right] = \frac{d}{dL'} (h_m \frac{dT}{dL'}) \]  

(13)

Any attempt to integrate equations (12) and (13) for the general case is hopeless.

Consider the slow drying of a nonporous solid under such conditions that these assumptions can be made:

1. The vapor diffusion resistance is so high in comparison with the heat flow resistance that the slab remains at nearly constant temperature. Then \( T \) and \( P_z \) are constant.

2. The diffusion coefficient \( K_m \) is constant. This assumes negligible shrinkage.

3. The relationship between \( q \) and \( P/P_z \) is linear, so

\[ q = \frac{\lambda}{P_z} (P + b) \]  

(14)

and

\[ \frac{dq}{d\theta} = \frac{\lambda}{P} \frac{dP}{d\theta} \]  

(15)

An approximate equation can be derived for the case where the solid possesses negligible bound water and when it is assumed that thermal conductivity and vapor diffusivity are constant through the zone of vaporization, and that thermal
effects other than latent heat of water are small.

\[(P_s - P_a)K_g = \left(\frac{P_z - P_g}{L'}\right)K_m = \omega_f = \left(\frac{T_a - T_g}{\lambda}\right)h_g = \left(\frac{T_s - T_z}{L'}\right)h_m\]  \hspace{1cm} (16)

**Drying of Clays.** Many investigators have studied the drying of simple geometrical shapes of clay models. Several attempts have been made to devise mathematical theories to account for the experimental data. Most of these theoretical attempts were based on the diffusion process.

Sherwood (22) considered that moisture movement was essentially a diffusion process, the driving force of which was the moisture content gradient.

\[\frac{dM}{d\theta} = D \frac{d^2M}{dX^2}\] \hspace{1cm} (17)

Macey (17), however, considered that by analogy with heat flow, the driving force should be the hydrostatic pressure gradient \(\frac{dP}{dX}\), and that allowance should be made for the variation of aqueous conductivity \(C\), with moisture content. This gave:

\[J = C \frac{dP}{dX}\] \hspace{1cm} (18)

If \(P = \alpha e^{\beta M}\) , \(C = \theta \theta e^{\phi M}\) where \(\alpha, \beta, \theta, \phi\) are constant, and it was found that \(\beta\) and \(\phi\) were approximately equal for the clays studied. Equation (18) can be written as the heat-flow equation:
\[
\frac{dM}{d\theta} = D \frac{d^2M}{dx^2}
\] (19)

That is to say, the increased conductivity at higher moisture contents approximately compensated for the exponential relation between pressure and moisture content; thus the system behaves as if it were a diffusion process independent of moisture content.
MATERIAL AND METHODS

One poultry formula (Table 1) was pelleted at three different conditioning temperatures and three different pellet diameters. Three air temperatures, two air velocities, and three depths of bed in the dryer were used in this study.

The feed was ground, mixed, and pelleted at the pilot feed mill of Kansas State University.

It was assumed that (1) granulation was held constant by using the same hammermill and the same screen size; (2) the material was uniform after the mixing operation; (3) the pellet structure was kept uniform by using the same die and the same setting of the rolls and the same clearance between each roll and die.

Factors which may affect drying rates are:

1. The initial temperature of the pellet after pelleting.
2. The air temperature used to dry the pellet.
3. Temperature in the approximate center of the bed which is a combination of pellet surface temperature and air temperature.
4. Total weight of the air blown through the bed.
5. Air velocity.
6. Relative humidity of the air prior to entering the bed.
7. Initial moisture content of the pellet.
8. The weight of the water evaporated from the pellets.
9. The dynamic equilibrium moisture content.
10. The effect of the distance between the surface and the
Table 1. Feed formula used for pellets.  
Code No. P-17, Chick grower.

<table>
<thead>
<tr>
<th>Ingredients</th>
<th>Per cent</th>
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<tr>
<td>Soybean oil meal</td>
<td>15</td>
</tr>
<tr>
<td>Ground yellow meal</td>
<td>25</td>
</tr>
<tr>
<td>Ground milo</td>
<td>25</td>
</tr>
<tr>
<td>Ground oats</td>
<td>10</td>
</tr>
<tr>
<td>Wheat middlings</td>
<td>10</td>
</tr>
<tr>
<td>Dehydrated alfalfa meal</td>
<td>5</td>
</tr>
<tr>
<td>Premix A</td>
<td></td>
</tr>
<tr>
<td>Meat and bone meal</td>
<td>5</td>
</tr>
<tr>
<td>Fish meal</td>
<td>2.5</td>
</tr>
<tr>
<td>Limestone</td>
<td>1</td>
</tr>
<tr>
<td>Dicalcium phosphate</td>
<td>1</td>
</tr>
<tr>
<td>Salt</td>
<td>0.5</td>
</tr>
<tr>
<td>+ Premix B</td>
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center of the pellet, given as a function of pellet diameter.

The air velocity has a direct effect on the moisture movement in the pellet.

By multiplying the weight of the air forced through the bed in a fixed time interval by the potential of a unit of air to carry water, one obtains the maximum amount of water that potentially can be removed. The water holding potential of air can be found by using the psychometric chart (Fig. 2).

To use the psychometric chart, consider the point on the 100 per cent R.H. curve which corresponds to the wet bulb temperature of the air entering the fan. Now proceed along the enthalpy line which intersects the 100 per cent R.H. curve until one intersects the vertical line corresponding to the dry bulb temperature of the air entering the fan (A in Fig. 2). Point A is at the R.H. of the air entering the fan. Proceeding from point A on a line of constant weight of water, we eventually intersect the temperature line of the air entering the bed. Call this point of intersection B (Fig. 2).

Now, through point B there is a particular R.H. curve which shows the R.H. corresponding to the temperature of the air entering the bed for the given starting conditions. Also, through point B we have an enthalpy line which intersects the 100 per cent R.H. curve. Call this point of intersection C. But point C corresponds to a particular potential of air to carry water after it leaves the bed. Thus the difference in weight of water per pound of dry air between point C and point A is the
potential of the air to carry water before entering and after leaving the bed (i.e., the change in the potential of the air to carry water as a result of its passing through the bed).

The difference between the dynamic equilibrium moisture content and the initial moisture content of the pellet is the potential moisture transfer.

Dynamic equilibrium moisture was determined by placing a small sample of mash in a perforated container in the bottom of the bed and allowing it to dry for the entire drying period. The drying area was held constant (one square foot) in order to eliminate the number of dimensions.

The time gradients were held constant in order to get uniform tests.

Moisture of pellets and mash was determined by using the standard AACC (18) methods 44-15 and 44-17.

**Experimental Dryer Design.** The experimental dryer was designed according to the following specifications:

1. Air capacity - 0 to 100 CFM
2. Air temperature - ambient to 200° F
3. Relative humidity - ambient to 95 per cent R.H.

Processes were needed:

1. To weigh the water which has been removed from the pellets according to the time gradients.
2. To adjust the air temperature and to hold it constant ±5° F.
3. To adjust and to measure the rate of air flow.
4. To add relative humidity to the air.
5. To measure the relative humidity of the air.
6. To measure the temperature in the approximate center of the bed in each of the possible bed heights (6", 12", 24").
7. To measure loss of air pressure through the bed.
8. To dump the wet pellets into the dryer and the dry pellets out.

The conclusions were to mount the drying chamber (1)* on a scale (2)* and to feed this chamber from the top; the bottom having been constructed from fixed, heavy screen with 1/8-inch holes (3)*, to invert the chamber in order to dump the pellets out, to mount the chamber on a stand (4)* with two bearings, to insulate the drying chamber (5)*, thus keeping heat inside the bed. It can then be assumed that the drying will be an adiabatic process.

In order to adjust the air temperature and to control it at different air capacities, duct type, electrical air heating elements were used. Each heater was controlled by a separate switch. These elements were mounted in an insulated air oven (6)* through which the air to be heated is forced over the heating element and heated by the iron sheath plus the metal elements.

Four small electrical air heating elements (7)* were mounted close to the bottom of the bed and were controlled by a thermostat (8)* and a three-position switch (9)*. These small

*See Fig. 3B.
FIGURE 3A EXPERIMENTAL VERTICAL DRYER

FIGURE 3B SCHEMATIC DRAWING OF EXPERIMENTAL VERTICAL DRYER
heaters kept the incoming air temperature constant to within ±5° F. The air is supplied by a 1/4-hp air handling fan (10)* with 100-CFM capacity and 6-inch water pressure.

To measure the air flow, perforated sheet metal orifices were used. Due to the low rate of flow, a commercial perforated plate (11)* with 3/16-inch diameter holes, 18 holes per square inch, and 3-inch diameter exposed area was used. To measure the air pressure drop through the orifice an inclined tube manometer (12)* was installed. The orifice and manometer were calibrated at the Kansas State University Department of Mechanical Engineering and the results were plotted on log log paper (Fig. 4) and used during the tests. A bypass (13)* was needed to divert the air around the heaters (6)* to run the cooling tests. A flexible adapter (14)* was used between the fixed unit and the drying chamber which is mounted on a scale. Dry and wet bulb thermometers were mounted after the fan (15)* to measure the relative humidity of the incoming air. Steam jets (16)* were used to add moisture to the incoming air in order to increase the relative humidity. A thermometer (17)* was mounted between the small heating element (7)* and the bottom of the bed. This measured the temperature of the air forced into the bed. Three 3/4-inch nipples were mounted on the wall of the chamber in order to mount the thermometer which measures the temperature in the center of the bed.

*See Fig. 3B.
Figure 4 Calibration of perforated sheet metal 3/16" holes 18 per in.

G C.F.M

0.01

0.02

0.03

0.04

0.05

0.06

0.07

0.08

0.09

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

1.0

1.1

1.2

1.3

1.4

1.5

1.6

1.7

1.8

1.9

2.0
Experimental Design

Table 2 may be considered to explain the variables studied in this research.

Table 2. Explanation of variables studied.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Symbol</th>
<th>Range of variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initial pellet temperature</td>
<td>$T_p$0</td>
<td>a. 120-140° F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. 160-170° F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. 190-200° F.</td>
</tr>
<tr>
<td>2. Air temperature entering bed</td>
<td>$T_a$</td>
<td>a. $100^\circ$ F. ±5°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. $200^\circ$ F. ±5°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. $100^\circ$ F. ±5°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d. $160^\circ$ F. ±5°</td>
</tr>
<tr>
<td>3. Rate of air flow through bed</td>
<td>$Q$</td>
<td>a. 50 CFM</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. 100-115 CFM</td>
</tr>
<tr>
<td>4. Elapsed time in drying</td>
<td>$\theta$</td>
<td>Two-minute intervals to 16 minutes and after that 20, 25, 30, and 35 min.</td>
</tr>
<tr>
<td>5. Diameter of pellet</td>
<td>$d$</td>
<td>a. 3/16 inch</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. 3/8 inch</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. 3/4 inch</td>
</tr>
<tr>
<td>6. Depth of bed</td>
<td>$L$</td>
<td>a. 6 inches</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. 12 inches</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. 24 inches</td>
</tr>
<tr>
<td>7. Dry bulb temperature of air leaving the fan</td>
<td>$T_d$</td>
<td></td>
</tr>
<tr>
<td>8. Wet bulb temperature of air leaving the fan</td>
<td>$T_w$</td>
<td></td>
</tr>
</tbody>
</table>
Table 2 (concl.).

<table>
<thead>
<tr>
<th>B. Dependent variables</th>
<th>Variable name</th>
<th>Symbol:Units:Variable dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. Temperature in approximate center of the bed</td>
<td>$T_p$ °F.</td>
<td>$T_a$, $T_{p0}$, and $Q$</td>
</tr>
<tr>
<td>10. Weight of pellet in the bed before drying</td>
<td>$W$ lbs.</td>
<td>$L$</td>
</tr>
<tr>
<td>11. Change in weight of dried pellet</td>
<td>$-W$ lbs.</td>
<td>$T_a$, $T_{p0}$, $Q$, $h$, $W$, and $\theta$</td>
</tr>
<tr>
<td>12. Initial pellet moisture content (wet basis)</td>
<td>$M_0$ %</td>
<td>$T_{p0}$</td>
</tr>
<tr>
<td>13. Final pellet moisture content (wet basis)</td>
<td>$M_2$ %</td>
<td>$T_a$, $T_{p0}$, $Q$, $h$, $W$, and $\theta$</td>
</tr>
<tr>
<td>14. Dynamic equilibrium moisture content (wet basis)</td>
<td>$M_s$ %</td>
<td>$T_a$, $h$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Computed variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15. Density of air entering the bed</td>
<td>$\rho$ lb./cu. ft.</td>
</tr>
<tr>
<td>16. Weight of air entering the bed</td>
<td>$W_a$ lbs.</td>
</tr>
<tr>
<td>17. Velocity of air through the bed</td>
<td>$V$ ft./min.</td>
</tr>
</tbody>
</table>

Since the method of deriving the drying equation in this experiment is that of dimensionless groups, it is necessary to combine $d$ and $L$ to one dimensionless $d/L$.

It is essential that this group be held constant for each phase of the experiment. This group was allowed to assume only
the values given in Table 3. The reason for this will be apparent after a detailed discussion of the drying equation.

Table 3.

<table>
<thead>
<tr>
<th>d</th>
<th>L</th>
<th>d/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/16&quot;</td>
<td>6&quot;</td>
<td>1/32</td>
</tr>
<tr>
<td>3/8&quot;</td>
<td>12&quot;</td>
<td>1/32</td>
</tr>
<tr>
<td>3/4&quot;</td>
<td>24&quot;</td>
<td>1/32</td>
</tr>
<tr>
<td>3/16&quot;</td>
<td>12&quot;</td>
<td>1/64</td>
</tr>
<tr>
<td>3/8&quot;</td>
<td>24&quot;</td>
<td>1/64</td>
</tr>
<tr>
<td>3/4&quot;</td>
<td>12&quot;</td>
<td>1/16</td>
</tr>
</tbody>
</table>

Since the experiment was designed with the (d), (L) combination limitation given in Table 3, the total number of experiments was determined by all possible combinations of values of the given independent variables (Table 2).

Variable  | $T_p$ | $T_a$ | $Q$ | $h$ | $d/L$  
-----------|-------|-------|-----|-----|--------
No. of times | 3 x 2 x 2 x 2 x 6 = 144

with 13 time gradient readings.

However, when $d = 3/4"$ it is not possible to use $T_p = 120 - 140 ^\circ F$; thus the number of experiments is diminished by 48. Each experiment was performed twice, so that the revised number of experiments was $2 \times (144 - 48) = 192$.

Another 40 tests were run to test the cooling after drying. In this test not all the combinations were tested.
Dimensional Analysis

As indicated in the review of literature, most of the investigators found that drying hygroscopic material is a combination of heat and mass transfer. This interaction is very difficult to solve by the use of simultaneous differential equations with boundary values. Most of the investigators prefer to neglect the mass transfer factor. One way to combine all the pertinent factors into one drying equation is to use the experimental data to develop an equation. This method is known as Dimensional Analysis. One such method of development commonly used is the Buckingham pi groups method.

The Buckingham pi groups method is based on the fact that the number of dimensionless and independent quantities that can be used to express a relationship between the variables in any phenomenon, is equal to the difference between the total number of quantities involved and the number of dimensions in which those quantities may be measured. This equation may be written as:

\[ S = n - b \]

where \( S \) = the number of pi terms

\( n \) = the total number of quantities involved

\( b \) = the number of basic dimensions involved.

The pi terms must be dimensionless and independent.

The variables which were investigated for possible relevance to the pellet drying process are:

1. \((M_0 - M_1)\), the amount of moisture removed from the pellet over a time interval.
2. \((M_0 - M_g)\), the potential moisture removed from the pellet.

3. \(T_{p0}\), the initial temperature of the pellet.

4. \((T_{p0} - T_s)\), the temperature difference between the pellet initially and the air which causes the evaporation.

5. \((T_{p0} - T_{p1})\), the difference in temperatures between the initial temperature and the combination of the pellet surface temperature and the air in the center of the bed that can represent the vapor pressure at a certain time.

6. \(h = (h_s - h_l)\), available humidifying potential (absolute humidity of drying air when saturated at constant enthalpy in the bed minus the absolute humidity of drying air at entry into the bed).

7. \(W_a\), the weight of the air that was forced through the bed during a given interval of time.

8. \(\Delta W\), the weight of the water removed from the pellet during a given interval of time.

9. \(\theta\), time.

10. \(V\), the velocity of air in the bed.

11. \(d\), the pellet diameter.

12. \(L\), the depth of the bed.

The following techniques described by Murphy (14), a general relationship indicating the dependence of \((M_0 - M_1)\) to other variables, may be written as:
\((M_0 - M_1) = \prod \left[ (M_0 - M_s), (T_p_0), (T_p_0 - T_a), T_p_0 - T_p, (h), (W_a), (W), (\theta), (V), (d), (L) \right] \quad (20)\)

Equation (20) may be written as:

\[
C_1(M_0 - M_1)^{C_1}(M_0 - M_s)^{C_2}(T_p_0)^{C_3}(T_p_0 - T_a)^{C_4}(T_p_0 - T_p)^{C_5} \\
(h)^{C_6}(W_a)^{C_7}(W)^{C_8}(\theta)^{C_9}(V)^{C_{10}}(d)^{C_{11}}(L)^{C_{12}} = 1 \quad (21)
\]

To form \(pi\) groups, \(L\) was used for length, \(W\) for weight, \(T\) for temperature, and \(\theta\) for time.

It should be noted that since moisture and relative humidity are dimensionless, they could form three \(pi\) groups themselves. These \(pi\) groups containing moisture will be designated as \(M\).

The corresponding dimensional equation is found to be:

\[
(M)^{C_1}(M)^{C_2}(T)^{C_3}(T)^{C_4}(M)^{C_5}(W)^{C_6}(W)^{C_7}(\theta)^{C_8}(L^{L-1})^{C_{10}} \\
(L)^{C_{11}}(L)^{C_{12}} = 0 \quad (22)
\]

from which five auxiliary equations may be written:

\[
L: \quad C_{12} \quad C_{11} \quad C_{12} = 0 \quad (23)
\]

\[
W: \quad C_7 \quad C_8 = 0 \quad (24)
\]

\[
T: \quad C_3 \quad C_4 \quad C_5 = 0 \quad (25)
\]

\[
\theta: \quad C_9 - C_{10} = 0 \quad (26)
\]

and separately,

\[
M: \quad C_1 \quad C_2 \quad C_6 = 0.
\]

The number of \(pi\) groups \(S\) is then obtained from the equation \(S = n - b\).
Since \( n = 9 \) (i.e., 12 minus the 3 terms related to \( M \)) and \( b = 4 \),

\[
S = 9 - 4 = 5 \text{ pi groups.}
\]

Since four simultaneous linear equations are available for solving the nine unknowns, arbitrary values must be assigned to five of the unknowns.

Let \( C_3, C_5, C_7, C_{10}, \) and \( C_{12} = 0 \).

The determinant found by the coefficients of the remaining terms is:

\[
\begin{array}{cccc}
C_4 & C_8 & C_9 & C_{11} \\
L : & 0 & 0 & 0 & 1 \\
W : & 0 & 1 & 0 & 0 \\
T : & 1 & 0 & 0 & 0 \\
\theta : & 0 & 0 & 1 & 0 \\
\end{array}
\]

Since the value of this determinant is nonzero, the resulting equations are independent and therefore the prior selection is valid.

When \( C_3 \) is set as \(-1\) and \( C_5, C_7, C_{10}, \) and \( C_{12} = 0 \), then \( C_4 + C_5 = 1 \), which implies \( C_4 = 1 \). Therefore

\[
\pi_2 = \frac{T_{p0} - T_a}{T_{p0}}.
\]

When \( C_5 \) is set as \( 1 \) and \( C_3, C_7, C_{10}, \) and \( C_{12} = 0 \), then \( C_3 + C_4 = -1 \), which implies \( C_4 = -1 \). Therefore
\[ \pi_3 = \frac{T_{p0} - T_{p1}}{T_{p0} - T_{p2}} \]

When \( C_7 \) is set as 1 and \( C_3, C_5, C_{10}, \) and \( C_{12} = 0 \), then \( C_8 = -1 \). Therefore

\[ \pi_4 = \frac{W_a}{W} \]

When \( C_{10} \) is set as -1 and \( C_3, C_5, C_{10}, \) and \( C_{12} = 0 \), then \( C_9 = 1 \) and \( C_{11} + C_{12} = 1 \), which implies \( C_{11} = 1 \). Therefore

\[ \pi_5 = \frac{d}{V \theta} \]

When \( C_{12} \) is set as -1 and \( C_3, C_5, C_7, \) and \( C_{10} = 0 \), then \( C_{10} + C_{11} = 1 \), which implies that \( C_{11} = 1 \). Therefore

\[ \pi_6 = \frac{d}{L} \]

The two pi groups determined by the moisture dimensionless groups are:

\[ \pi_1 = \frac{M_0 - M_1}{M_0 - M_s} \]

and

\[ \pi_7 = \frac{h}{M_0 - M_s} \]

Some pi groups can be eliminated by combining two pi groups with the same physical aspects since their constants and
exponents have been determined from the experimental data. Therefore $\pi_4$ and $\pi_7$ would combine and

$$\pi_8 = \frac{h \times \Delta W}{(M_0 - M_s) \Delta W}$$

The terms involved in the above pi groups can be physically explained as follows:

a. $\frac{M_0 - M_1}{M_0 - M_s} = \text{the dryness ratio. This is a measure of the amount of moisture removed with respect to the potential moisture to be removed.}$

b. $\frac{\Delta T_p - \Delta T_a}{\Delta T_p} = \text{the ratio of the temperature ranges of the pellet and the air with respect to the initial pellet temperature.}$

c. $\frac{\Delta T_p - \Delta T_1}{\Delta T_p} = \text{the ratio of the temperature ranges of pellet and air.}$

d. $\frac{h \times \Delta W}{(M_0 - M_s) \Delta W} = \text{the ratio of the drying potential of the air to the task of drying.}$

e. $d/L = \text{the effect of the surface areas of the pellet to the height of the bed through which the saturated air must pass.}$

f. $d/\Theta = \text{the ratio between the distance of the surface from the center of the pellet or the surface area and}$
the velocity of the air passing the pellet. These two dimensions have the major effect on the capillary force.

The general relationship can be written as:

\[
\frac{M_0 - M_1}{M_0 - M_s} = F \left[ \left( \frac{T_{p0} - T_a}{T_{p0}} \right), \left( \frac{T_{p0} - T_{p1}}{T_{p0} - T_a} \right), \left( \frac{h \times W_a}{(M_0 - M_s) \Delta W} \right), \left( \frac{d}{v \theta} \right), \left( \frac{d}{L} \right) \right]
\]  

(27)

The equation derived from this relationship can be:

\[
\frac{M_0 - M_1}{M_0 - M_s} = K \left( \frac{T_{p0} - T_a}{T_{p0}} \right)^{g_1} \left( \frac{T_{p0} - T_{p1}}{T_{p0} - T_a} \right)^{g_2} \left( \frac{h \times W_a}{(M_0 - M_s) \Delta W} \right)^{g_3} \left( \frac{d}{L} \right)^{g_4} \left( \frac{d}{v \theta} \right)^{g_5}
\]  

(28)

To determine the equation, the constant \( K \) and the exponents \( g_1 \ldots g_5 \) must be found. That can be done by:

a. Holding all independent groups except one constant; this group can vary with the test and would establish a relationship between it and the dependent group \( \pi_1 \). This procedure is repeated for each of the individual groups. By plotting this relationship on log log paper, a linear regression curve can be found. The slope of the line would give the exponent. This plotting technique cannot be used easily when so many variables are involved.
b. The relationship between the dependent pi groups and the independent pi groups can be found by fitting a multiple linear regression hyperplane. The general form of the equation for the estimated regression hyperplane is given by the equation:

\[ \hat{Y} = a + b_1(X_i - x_i) + b_2(X_2i - x_2) + \ldots + b_n(x_ni - x_n) \]

(29)

Let

a. \( \log_{10} \left( \frac{M_0 - M_1}{M_0 - M_s} \right) = A \)

b. \( \log_{10} \left( \frac{T_{P0} - T_a}{T_{P0}} \right) = B \)

c. \( \log_{10} \left( \frac{T_{P0} - T_{P1}}{T_{P0} - T_a} \right) = C \)

d. \( \log_{10} \left( \frac{h \times W_a}{(M_0 - M_s) \Delta W} \right) = D \)

e. \( \log_{10} \left( \frac{d}{L} \right) = E \)

f. \( \log_{10} \left( \frac{d}{V_0} \right) = F \)

g. \( \log_{10}(K) = a \)

and \( g_i = b_i \) for \( i = 1, 2, \ldots, 5. \)
Equation (29) can be written as:

\[ A = a + b_1(B) + b_2(C) + b_3(D) + b_4(E) + b_5(F) \]  

(30)

This equation (30) is in the same form as equation (29).

Therefore by using a multiple linear regression computer program, equation (30) can be solved and \( b_i \) for \( i = 1, 2, \ldots, 5 \) and \( K \) can be found.

The standard program used supplied additional information that helped to test:

1. Each individual \( b_i \) for significance by t-test.
2. The multiple R square, \( R^2 \); as described by Fryer (12)
   "... about \( R^2 \) (in per cent) of the observed variability among the \( X \)'s, as measured by \( (Y)^2 \) is assignable to multiple linear regression on \( \pi_1, \pi_2, \ldots, \pi_n. \)"

The time value of \( Y \), or as in equation (30) \( A \), can be determined as recommended by Davis (11) and Snedecor (26).

\[ Y = \hat{Y} + t(1-\alpha/2)(n-1-p) \cdot S_Y^* \]  

(31)

Where \( S_Y^* \) = variance of \( Y \) regression, equation (31) can be expressed as:

\[ \hat{Y} - t(1-\alpha/2)(n-1-p) \cdot S_Y^* < Y < \hat{Y} + t(1-\alpha/2)(n-1-p)S_Y^* \]  

(32)

Since there were more than 120 observations and \( p = 4 \), \( t_1 - \alpha/2 \) can be found from t table always with infinite observation and only \( \alpha \) will change the value. Therefore, \( 1 - \alpha/2 \) will represent the power of \( Y \) or \( A \). Note: The value of \( t_1 - \alpha/2 (\infty) \cdot S_Y^* \) added or subtracted from \( A \) and not from
Pelleting Process

The pellet mill used throughout the tests was a California Master Model pellet press with a standard size conditioning chamber driven by a 25-horsepower motor. Steam at 90 PSIG was injected into the conditioning chamber and the moisture from it penetrated the mash feed. The mash moves in a spiral motion due to rotary action. A thermometer was mounted on the conditioning chamber outlet to give the temperature reading. The moist, hot mash feed flows into the vertical die where it is pressed by the two rollers through the die holes. The mash moves toward the space between the die and the rollers by centrifugal force. The pressing process adds heat to the pellet in two ways--by friction, and from the heat which is transferred to or from the machine through the die.

As the conditioning temperature is increased, the difference between it and the final temperature is decreased. When the conditioning temperature reaches 190° F. or above, it will be higher than the final temperature (Fig. 5). This is explained by the following two facts.

a. Moisture is introduced to the mash by the steam heating process. The dampened mash reduces friction which cuts down on heat produced by friction.

b. The mass of the machine is a specific temperature because of the mechanical friction of the gears,
FIG. 5. THE RELATION BETWEEN MASH TEMP. AND PELLET TEMP.
bearings, etc. The mass of the machine is much larger than the die which gets direct heat from the mash feed; therefore when the heat of the die is greater than the heat of the machine, the additional heat is absorbed by machine and transferred to all the surrounding mass, making the pellet cooler than the mash feed.

In order to get the proper temperature for the tests, it was necessary to follow Fig. 5. It was cross checked at intervals.

After the pelleting process, the pellets were collected in a container marked to measure the volume of the bulk pellets. Samples were taken to measure temperature and moisture. The temperature test was made by placing the pellets in a thermos container with a thermometer for one-half hour.

Drying Process

The perforated can containing the dynamic equilibrium moisture content sample was placed on the screen (Fig. 3, part 3) at the bottom of the dryer. Then the container with the hot wet pellets was weighed and dumped into the dryer. The scale was balanced and the fan turned on. The air flow, heat, and relative humidity were adjusted. Readings were taken according to the time intervals. If the same weight reading appeared four times in succession, the testing was stopped. After completion of the test, the pellets were dumped by inverting the dryer chamber (Fig. 3, part 1). Samples were taken for moisture test and pellet durability. The perforated can with the
dynamic equilibrium sample was emptied into a bottle for moisture testing.

Cooling Process

The cooling process was tested in order to determine cooling data about heat-dried pellets.

The pellets were dried to approximately 10 per cent moisture content at various temperatures, based on information from previous tests.

Samples were taken to measure the exact moisture content and to measure the temperature by the thermos container and thermometer method used previously.

The scale was balanced and the bypass (Fig. 3, part 13) was set for cooling and the fan turned on. Readings were taken on the weight changes and the temperature in the center of the bed according to the time gradients. If the same temperature reading appeared four times in succession, the testing was stopped and samples were taken for moisture and final temperature tests. These data could show the effect reduction of temperature has on the ability of the pellet to gain moisture and the duration of time of the cooling process.
RESULT AND DISCUSSION

Dynamic Equilibrium Moisture Content

The dynamic equilibrium moisture content data from 144 tests were statistically tested to study the effect of (1) air temperature or the relative humidity, (2) the rate of air flowing through the bed, and (3) the interaction between (1) and (2). The effect of (1) and (2) and the interaction between the two were tested to find if there is relation with moisture content.

Complete randomized block design test was used. The method is described by H. C. Fryer (13).

Table (4) was computed from 144 data tests with four cells and 36 replications in each cell.

Table 4. Air temperature—rate of air flow. Sum, sum of square, and mean of dynamic moisture content.

<table>
<thead>
<tr>
<th>Ta 100°F</th>
<th>Q1 = 100 CFM</th>
<th>Q2 = 50 CFM</th>
<th>Total about air temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum</td>
<td>340.09</td>
<td>372.595</td>
<td>712.685</td>
</tr>
<tr>
<td>Sum X²</td>
<td>3219.40</td>
<td>4137.83</td>
<td>7081.23</td>
</tr>
<tr>
<td>Mean X</td>
<td>9.5%</td>
<td>10.64%</td>
<td>9.94%</td>
</tr>
<tr>
<td>Ta 200°F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td>260.555</td>
<td>243.80</td>
<td>504.355</td>
</tr>
<tr>
<td>Sum X²</td>
<td>1894.256</td>
<td>1647.264</td>
<td>3561.52</td>
</tr>
<tr>
<td>Mean X</td>
<td>7.25%</td>
<td>6.88%</td>
<td>7%</td>
</tr>
<tr>
<td>Total about Q</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td>600.645</td>
<td>616.395</td>
<td>1217.04</td>
</tr>
<tr>
<td>Sum X²</td>
<td>5113.656</td>
<td>5785.094</td>
<td>11788.74</td>
</tr>
<tr>
<td>Mean X</td>
<td>8.46%</td>
<td>8.56%</td>
<td>8.48%</td>
</tr>
<tr>
<td>Total X² = 10646.242</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In order to use the Analysis of Variance Method, the following determination was used.

1. \[ CF = \frac{(1217.04)^2}{144} = 10286.0 \]

2. \[ SS_{T_a} = \frac{(712.685)^2 + (504.355)^2}{72} - 10286 = 301.4 \]

3. \[ SS_Q = \frac{(600.645)^2 + (616.385)^2}{72} - 10286 = 1.73 \]

4. \[ SS_{T \times Q} = \frac{(340.09)^2 + (372.595)^2 + (260.55)^2 + (243.8)^2}{36} \]

\[ - 10286 - 1.73 - 301.4 = 16.96 \]

5. Total SS = 10646.242 - 10286 = 360.242

6. Error SS = 360.242 - 301.4 - 1.73 - 16.96 = 40.152

The following hypotheses were tested in Table 5.

1. \( H_0 \ T(\beta_1 \text{ and } \beta_2 = 0) \) alternative \( H_a \ T(\beta_1 \text{ and } \beta_2 \neq 0) \), where 1 and 2 correspond to \( T_{a1} = 100^\circ \text{ F.} \) and \( T_{a2} = 200^\circ \text{ F.} \)

2. \( H_0 \ Q(\tau_1 \text{ and } \tau_2 = 0) \) alternative \( H_a \ T(\tau_1 \text{ and } \tau_2 \neq 0) \), where 1 and 2 are corresponding to \( Q_1 = 100 \text{ CFM} \) and \( Q_2 = 50 \text{ CFM} \).
Table 5. Analysis of variance of data from Table 4.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>Computed F</th>
<th>( F ) .01, (1, 140)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of air flow Q</td>
<td>1</td>
<td>1.73</td>
<td>1.73</td>
<td>( \frac{1.73}{.2826} = 6.03 )</td>
<td>6.81</td>
</tr>
<tr>
<td>Air temperature ( T_a )</td>
<td>1</td>
<td>301.4</td>
<td>301.4</td>
<td>( \frac{301.4}{.2826} = 1050.9 )</td>
<td>6.81</td>
</tr>
<tr>
<td>( Q \times T_a )</td>
<td>1</td>
<td>16.96</td>
<td>16.96</td>
<td>( \frac{16.96}{.2826} = 59.24 )</td>
<td>6.81</td>
</tr>
<tr>
<td>Error</td>
<td>140</td>
<td>40.152</td>
<td>.2826</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total about X</td>
<td>143</td>
<td>360.242</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion.

1. There is a significant difference between the interaction. This proves that there is an effect on the dynamic equilibrium moisture content due to the interaction of air temperature and rate of air flow.

2. There is no significant difference between the mean dynamic equilibrium moisture content by varying the rate of air flow. Therefore the rate of air flow does not affect the dynamic equilibrium moisture content variances.

3. There is a significant difference between the mean dynamic equilibrium moisture content by varying the air temperature.
As a result, statistically, the only factor that can greatly vary the equilibrium moisture content is the air temperature and related relative humidity.

The interval of the standard deviation $\sigma$ between all the dynamic equilibrium moisture content population within a certain air temperature and the dynamic equilibrium moisture content would be interesting to estimate and test (Table 6). The method used is from H. C. Fryer (13).

**Pellet Drying Mechanism**

**Effect of Time.** In order to study the mechanism of pellet drying, the methods used by other investigators were adopted.

1. The drying curve (Figs. 6-11) will show the behavior of the drying process in time. As a result the pellet feed drying test shows:
   a. Relatively short drying period, 12-35 minutes.
   b. The final moisture content lies between the interval of 10% - 12.5 moisture content (w.b.) dependent upon air temperature and relative humidity.
   c. The initial moisture content lies between 13% - 15.5% dependent upon initial pellet temperature and the pressure of the steam.

Generally, the 3/8-inch and the 3/16-inch pellet diameter has the same shape drying curve except that the initial moisture content of the 3/8-inch diameter is always higher than that of the 3/16-inch diameter. The 3/4-inch pellet diameter curve
Table 6. The confidence intervals of the standard deviation $\sigma$ and the mean $\mu$, about the dynamic equilibrium moisture content population.
When $T_a = 100^\circ F.$ and $200^\circ F.$, $M_g$(mean) = 9.94% and 7.0%, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$T_a = 100^\circ F.$</th>
<th>$T_a = 200^\circ F.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum($x^2$)</td>
<td>$7081.23 - \frac{712.682}{72} = 30.53$</td>
<td>$3561.52 - \frac{(504.35)^2}{72} = 28.82$</td>
</tr>
<tr>
<td>$S_x = \sqrt{\frac{\Sigma(x^2)}{n(n-1)}}$</td>
<td>$\sqrt{\frac{30.53}{5100}} = 0.0775$</td>
<td>$\frac{28.82}{5100} = 0.075$</td>
</tr>
<tr>
<td>$\mu$ *</td>
<td>CI$_{95}$: 9.94 - 0.0775$x^2$ $\leq \mu$</td>
<td>CI$_{95}$: 7 - 0.075$x^2$ $\leq \mu$</td>
</tr>
<tr>
<td></td>
<td>$\leq 9.94 + 0.0775 \times 2$</td>
<td>$\leq 7 + 0.075 \times 2$</td>
</tr>
<tr>
<td></td>
<td>CI$_{95}$: 9.885 $\leq \mu$ $\leq 10.095$</td>
<td>CI$_{95}$: 6.95 $\leq \mu$ $\leq 7.15$</td>
</tr>
<tr>
<td>$\sigma^2$ **</td>
<td>CI$_{95}$: $\frac{30.53}{95.05} \leq \sigma^2$</td>
<td>CI$_{95}$: $\frac{28.82}{95.05} \leq \sigma^2$</td>
</tr>
<tr>
<td></td>
<td>$\leq \frac{30.53}{48.76}$</td>
<td>$\leq \frac{28.82}{48.76}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>CI$_{95}$: $\sqrt{0.322} \leq \sigma \leq \sqrt{1.627}$</td>
<td>CI$_{95}$: $\sqrt{0.304} \leq \sigma \leq \sqrt{0.59}$</td>
</tr>
<tr>
<td></td>
<td>CI$_{95}$: 0.57 $\leq \sigma$ $\leq 0.79$</td>
<td>CI$_{95}$: 0.55 $\leq \sigma$ $\leq 0.71$</td>
</tr>
</tbody>
</table>

* t value from (13) $t$ .05 = 2.

** $\chi^2$ values from (26), $\chi^2$.975 = 48.76, $\chi^2$.025 = 95.05.
behaves differently. They dry faster in the first 4 to 6 minutes and then the drying changed to a linear drying curve except for Figs. 10 and 11, where it starts fast for a minute and then changes slowly and nonlinearly, showing a lower rate of drying than the 3/16-inch and 3/8-inch pellet diameter.

2. The drying rate versus moisture content shows the behavior of the drying rate. It can be plotted by differentiating the drying versus time curve, and then $dM/d\theta$ would plot against $M$.

   a. By increasing the temperature or rate of flow of air more drying energy is used and the pellet dries faster.

   b. By increasing the drying energy the rate of drying becomes more constant, as shown in Figs. 10 and 11. The drying rate curve in Figs. 8 and 9 starts differently but quickly becomes constant.

   c. In the lowest drying energy, Figs. 6 and 7, the constant rate of drying is short and a fast change occurred in the rate of drying.

   d. In all the 3/4-inch tests there is a fast change in drying. Then it becomes constant except in Fig. 10 where it is hard to compare behavior with the rest. This can be due to experimental error.

3. By using Fig. 9 as the average of all these curves, by using the average drying conditions, and analyzing them, it appears that:

   a. The 3/8-inch pellet diameter has a more or less
constant drying rate. Therefore according to previous theories, free water is removed during this period.

b. The 3/4-inch pellet diameter has a fast change in drying rate within a small moisture interval and then the drying rate becomes constant.

Since feed pellet drying mechanism is being recognized for the first time and until now no investigator has found the behavior of the drying rate curve, this first great change in the rate of drying can be named the "accelerated rate of drying" and might be explained as follows.

1. Free water lies on the surface of the particles of the pellet. These particles are pressed together to a uniform shape but still have spaces between themselves. The larger the pellet diameter, the larger the spaces between the particles. Therefore the resistance of water to move is less and the water moves easily to the surface of the pellet where it evaporates.

2. Free water and water can move easily to the exposed areas of pellets and is removed by the drying air within a short time. This can be considered as a capillary movement. The remainder of the water, if pellets were dried further, will be removed by the difference between the vapor pressure of the water and the material.

Conclusion. The moisture removed from the pellet in this research after conditioning and pressing process is free or
Fig. 7: Pellet Drying Characteristics.

\[ T_a = 100^\circ F \]
\[ T_{po} = 160^\circ F \]
\[ Q = 50 \text{ C.F.M.} \]
\[ d = 3/16'' \]
\[ d = 3/8'' \]
FIG. 8. PELLET DRYING CHARACTERISTICS.
FIG. 9. PELLET DRYING CHARACTERISTICS.
FIG. 10. PELLET DRYING CHARACTERISTICS.

Ta = 200°F
Tpo = 160°F
Q = 100 C.F.M.
\[ T_a = 200^\circ F \quad T_{po} = 200^\circ F \quad Q = 100 \text{ G.F.M.} \]

**FIG. 11. PELLET DRYING CHARACTERISTICS.**
unbound water.

Effect of Relative Humidity

The change in pellet moisture versus the weight of the drying air at three different absolute humidities, \( h \), can be studied from Fig. 12.

1. When \( h = 0.0255 \) lb. of water per pound of dry air and the temperature of the air is 200° F., it is equal to relative humidity 1-3 per cent. When the curve reached 10 per cent moisture content the tests were stopped, although there was a potential for further drying.

2. When \( h = 0.0074 \) and temperature of the air is 100° F., it would be equal to 25-30 per cent relative humidity. The moisture was removed more or less the same as in (1) but after 70 lbs. of air per square foot of drying area the drying changed and became almost negligible (about 0.2 per cent of moisture for 130 lbs. of air).

3. When \( h = 0.0045 \) lb. of water per pound of dry air and air temperature of 100° F., the relative humidity of this air is 55-60 per cent. In this case when the moisture went down to 12.5 per cent, there was no change of moisture as drying continued.

In case 1 and 2 the initial pellet temperatures were 200° F. Probably that was the source of heat that affected the moisture movement but after a short period when the pellet was cooled by the air no moisture could be removed.
FIG. 12. EFFECT OF WEIGHT OF THE AIR ON MOISTURE REMOVAL FROM THE PELLETS.
Because these tests were run with a minimum of 100°F air temperature and not the regular room temperature, the highest relative humidity was 60 per cent and not 90 per cent relative humidity. If relative humidity stands at 90 per cent and 80°F, \( h = 0.004 \) lb. of water to a pound of dry air. Then the minimum moisture of the pellet would be about 13 per cent.

Figures 13-15 show the amount of moisture removed versus the potential of water that could be removed \( W_a \times h \). Each figure represents different pellet diameters.

All three show more or less the same relationship between \( (M_0 - M_1) \) and \( W_a \times h \) when \( h = 0.025 \) and \( h = 0.0075 \). Figure 15 with 3/4-inch pellet diameter and \( h = 0.004 \), shows a small continuous rate of drying throughout the range investigated as contrasted with Figs. 13 and 14 which show that after a small value of \( W_a \times h \) the amount of drying becomes zero.

The continuous drying of 3/4-inch pellets in Fig. 15 can be explained by:

1. Longer period of heat transfer that is caused by the distance between the surface and center of the pellet.
2. The capillary movement of the free water as was explained in the previous section.

An average linear regression can be plotted but this line will have large variances.

**Conclusion.**

1. There is no way to remove moisture from the pellet under certain humidity conditions by using ambient air.
2. There is a positive relationship between the amount of
FIGURE 14 EFFECT OF POTENTIAL AIR MOISTURE CARRYING CAPACITY ON MOISTURE REMOVAL FROM 3/8 IN. PELLET
water removed from pellet to the potential amount of water that could be removed. This relationship approaches zero. This means that the air is almost saturated.

**Determination of Pi Groups and Exponents**

**Graphic Method.** By holding all the independent pi groups except one constant, the relationship between the one varied and the dependent pi group was found. The linear regression line was plotted.

Points become too spread out (without pattern); the linear regression line cannot be plotted and the relationship between the independent and dependent pi groups seems to be zero.

These relationships were plotted on log log paper and the relationship represented the exponents of the pi group.

The independent pi groups that were held constant were

\[
\frac{T_{p_0} - T_a}{T_{p_0}} \quad \text{and} \quad \frac{d}{L}
\]

and the rest were held constant in different time which show three different linear regression in each figure.

The test was not designed to hold all the independent pi groups constant; therefore it was necessary to find three different combinations by holding these groups as a constant.

\[
\frac{T_{p_0} - T_a}{T_{p_0}}
\]

is a group that has the same relation to the dependent group \[
\frac{M_0 - M_i}{M_0 - M_s}
\]

because the terms do not vary within particular
tests; therefore it is always constant.

Figure 16 shows no relationship between the independent group \( \frac{T_{p0} - T_{pa}}{T_{p0} - T_{p1}} \) and the dependent group. The exponent can be assumed as zero. Therefore this pi group can be excluded in this equation.

Figure 17 shows the relationship between the independent pi group \( \frac{h W_a}{(M_0 - M_s)\Delta W} \) and the dependent group with three variations of \( d/\Theta \) as follows: .0075, .00045, .00035, and \( \frac{T_{p0} - T_a}{T_{p0}} = .25 \) and \( \frac{d}{L} = .03125 \).

Three negative linear regression lines were plotted with the result that all lines were approximately parallel. Therefore there was a definite relationship between this group and the dependent group. The negative slope represents the negative exponent for this group.

Figure 18 shows the relationship between the independent pi group \( d/\Theta \) and the dependent pi group. With three variations of the independent group \( \frac{h W_a}{(M_0 - M_s)\Delta W} = .001, .003, .0007, \) and \( \frac{T_{p0} - T_a}{T_{p0}} = .25 \) and \( \frac{d}{L} = .03125 \).

Three linear regression lines were plotted where two of them were parallel, the third changes slope. Therefore average slope of the three can be recorded as the exponent.

All three lines show a definite relationship with negative
FIG. 16. EFFECT OF TEMPERATURE RELATION ON MOISTURE REMOVAL.

\[ \frac{T_{po} - T_{pi}}{T_{po} - T_{a}} \]

\[ \frac{s_{W} - \theta_{W}}{\theta_{W} - \theta_{W}} \]
FIG. 18. EFFECT OF P15 ON MOISTURE REMOVAL.
slope; therefore a negative exponent can be recorded.

Figure 19 uses the same groups as Fig. 18 except

\[
\frac{T_{p0} - T_a}{T_{p0}} = .55.
\]

Three linear regression lines can be plotted with negative slope. Although all three lines are not parallel, the average slope has more or less the same angle as the average slope in Fig. 18.

In Fig. 20 the independent group d/L was plotted against the dependent group with three variations of \( \frac{h W_a}{(M_0 - M_s)\Delta W} \) as follows: .001 - .0012, .003 - .005, and .002 - .0026, and

\[
\frac{T_{p0} - T_a}{T_{p0}} = .25 \quad \text{and} \quad \frac{d}{V\theta} = .0082.
\]

It was difficult to find enough data for each of these variations and as a result unequal behavior of the different variations is shown. The relationship between d/L and the dependent group is still not zero. Therefore only by using the computer program can this relationship be found.

**Multiple Linear Regression (Computer Program).** The exponents and constants were determined by using a Multiple Linear Regression Computer Method.

As mentioned in the previous section (Materials and Methods, page 38), equation (30) can be computed by using a Multiple Linear Regression Analysis. Equation (30) can be written as follows:
FIGURE 19 EFFECT OF PI 5 ON MOISTURE REMOVAL FROM PELLET
\[
\log\left(\frac{M_0 - M_1}{M_0 - M_3}\right) = \log K + g_2 \log\left(\frac{T_{P_0} - T_a}{T_{P_0}}\right) + g_4 \log\left(\frac{h}{(M_0 - M_3)\Delta w}\right)
\]

\[
+ g_6 \log\left(\frac{d}{L}\right) + g_5 \log\left(\frac{d}{V_d}\right)
\]

As mentioned in the Graphic Method section, as a result of Fig. 16 the independent group \(\frac{T_{P_0} - T_{P_a}}{T_{P_0} - T_{P_1}}\) is excluded. As a result of Fig. 20 different combinations of \(d\) and \(L\) were tried to find the best relationship between them and the dependent group. Because \(d\) represents the group of \(d/V_{d}\) three different combinations of \(L\) were used, where \(L\) is equal to 6 inches, 12 inches, and 24 inches.

By using the t-test for testing the exponent, the most significant exponent can be found. The multiple \(R^2\) will show the amount of variation of the independent in relation to the dependent variable. The number of independent groups is important. All the combinations except one with \(d/L\) will have three independent groups while in the combination of \(d/L\) group there are four independent groups. Table 7 shows the results of the analysis.
Table 7. The exponents $K$, t-test, and multiple $R^2$ as they were computed from ten different pi group combinations by using the multiple linear regression computer program.

1. $L = 6''$

a. $\pi_1 = K \times (\pi_2)^{g_2} \times (\pi_4)^{g_4} \times (\pi_5)^{g_5}$

\[
g_1 = .05 \quad - .175 \quad - .5
\]

Computed $t = 3.56 \quad - 4.73 \quad - 19.9$

Conclusion: probably reject $H_0$ probably reject $H_0$ definitely reject $H_0$

Multiple $R^2 = .68$

b. $\pi_1 = K \times (\pi_2)^{g_2} \times (\pi_7)^{g_7} \times (\pi_5)^{g_5}$

\[
g_1 = .017 \quad - .28 \quad - .365
\]

Computed $t = 1.113 \quad - 7.02 \quad - 19.0$

Conclusion: accept $H_0$ reject $H_0$ definitely reject $H_0$

Multiple $R^2 = .7$

2. $L = 12''$

a. $\pi_1 = K \times (\pi_2)^{g_2} \times (\pi_4)^{g_4} \times (\pi_5)^{g_5}$

\[
g_1 = .07 \quad - .24 \quad - .38
\]

Computed $t = 7.3 \quad - 10. \quad - 22.6$

Conclusion: reject $H_0$ reject $H_0$ definitely reject $H_0$

Multiple $R^2 = .5$

b. $\pi_1 = K \times (\pi_2)^{g_2} \times (\pi_7)^{g_7} \times (\pi_5)^{g_5}$

\[
g_1 = .065 \quad - .44 \quad - .257
\]

Computed $t = .64 \quad - 17.6 \quad - 21.4$

Conclusion: definitely accept $H_0$ definitely reject $H_0$ definitely reject $H_0$

Multiple $R^2 = .61$
Table 7 (cont.).

3. L = 24"

| a. $\pi_1 = K \times (\pi_2)^{\xi_2} \quad (\pi_4)^{\xi_4} \quad (\pi_5)^{\xi_5}$ |
|---|---|---|
| $\xi_i = \quad .077$ | - | .16 | - | .412 |
| Computed t = | 9.14 | - | 8.35 | - | 27.76 |
| Conclusion: definitely reject $H_0$ | definitely reject $H_0$ |
| Multiple $R^2 = .74$ |

| b. $\pi_1 = K \times (\pi_2)^{\xi_2} \quad (\pi_7)^{\xi_7} \quad (\pi_5)^{\xi_5}$ |
|---|---|---|
| $\xi_i = \quad .024$ | - | .27 | - | .306 |
| Computed t = | 2.735 | - | 11.88 | - | 28.8 |
| Conclusion: probably reject $H_0$ | definitely reject $H_0$ | definitely reject $H_0$ |
| Multiple $R^2 = .78$ |

4. L = 6", 12", and 24"

| a. $\pi_1 = K \times (\pi_2)^{\xi_2} \quad (\pi_4)^{\xi_4} \quad (\pi_5)^{\xi_5}$ |
|---|---|---|
| $\xi_i = \quad .0655$ | - | .246 | - | .412 |
| Computed t = | 10.263 | - | 18.44 | 37.7 |
| Conclusion: definitely reject $H_0$ | definitely reject $H_0$ | definitely reject $H_0$ |
| Multiple $R^2 = .57$ |

| b. $\pi_1 = K \times (\pi_2)^{\xi_2} \quad (\pi_4)^{\xi_4} \quad (\pi_6)^{\xi_6} \quad (\pi_5)^{\xi_5}$ |
|---|---|---|---|
| $\xi_i = \quad .064$ | - | .325 | .216 | - | .48 |
| Computed t = | 10.2 | -21.86 | 10.5 | - | 39.16 |
| Conclusion: All definitely reject $H_0$ |
| Multiple $R^2 = .606$ |
Table 7 (Concl.).

|  |  |  |  |  |  |
|---|---|---|---|---|
| c. $\pi_1 = K \times (\pi_2)^{g_2} (\pi_7)^{g_7} (\pi_6)^{g_6} (\pi_5)^{g_5}$ |  |  |  |  |
| $g_1$ = | .002 | .4 | .0052 | .253 |
| Computed $t$ = | .32 | 23.66 | .23 | 31.455 |
| Conclusion: definitely accept $H_0$ definitely reject $H_0$ definitely accept $H_0$ definitely reject $H_0$ |
| Multiple $R^2 = .62$ |
| d. $\pi_1 = K \times (\pi_2)^{g_2} (\pi_7)^{g_7} (\pi_5)^{g_5}$ |  |  |  |  |
| $g_1$ = | .0023 | .4 | .254 |
| Computed $t$ = | .33 | 23.67 | 32.326 |
| Conclusion: definitely accept $H_0$ definitely reject $H_0$ definitely reject $H_0$ |
| Multiple $R^2 = .62$ |

Discussion.

1. Since 4(a) and 4(b) are the only combinations of the $\pi$ groups all the hypotheses that $H_0(\beta_1 = 0)$ alternative $H_2(\beta_1 \neq 0)$ are definitely rejected. Therefore the most significant for each individual $g_1$ in the equation are 4(a) and 4(c) combinations.

2. About 57 per cent of the observed variability among the $\pi_1$ is assignable to multiple linear regression on $\pi_2$, $\pi_4$, and $\pi_5$, because $R^2 = .57$ in 4(a). 60.6 per cent of the observed variability among the $\pi_1$ is assignable to multiple linear regression on $\pi_2$, $\pi_4$, $\pi_6$, and $\pi_5$, because $R^2 = .606$. Only three independent pi groups are
involved in \(4(a)\) combinations compared to four in \(4(b)\). Therefore 60 per cent with four groups represents a more powerful correlation than 57 per cent with three groups.

3. Only \(4(b)\) combination based on much more information, i.e., factors that can affect in drying process. \(4(b)\) combination will be preferable over all other combinations that were computed.

It should be noted that the same equation was derived in the previous section (Materials and Methods, page 38) by using the dimensional analysis method. For one exception, it was found by the graphic method Fig. 16 that \(g_3\) equals zero.

Log \(K = -2.3\); therefore \(K = .02\).

The pellet drying equation can be written as follows:

\[
\frac{M_0 - M_1}{M_0 - M_s} = .02 \left(\frac{T_a - T_p}{T_p_0}\right)^{.06} \left(\frac{h \times W_a}{(M_0 - M_s)\Delta W}\right)^{-33} \left(\frac{d}{L}\right)^{.2} \times \left(\frac{d}{V \Theta}\right)^{.5} \times CF
\]

or

\[
\frac{M_0 - M_1}{M_0 - M_s} = .02 \left(\frac{T_a - T_p}{T_p_0}\right)^{.06} \left(\frac{(M_0 - M_s)\Delta W}{h \times W_a}\right)^{33} \left(\frac{d}{L}\right)^{.2} \times \left(\frac{d}{V \Theta}\right)^{.5} \times CF
\]

The correction factor \(CF\) can be calculated from the equation:
\[ CF = t_{1-\alpha/2}(\infty) \times S_Y \]  

(36)

Only the positive value can be affected; therefore a one-tail t-test is used with \( t = 2.256 \), with \( \beta = 99 \) per cent, and infinite DF; \( S_Y^2 = 0.0535 \) as was computed from the computer program,

\[ \log CF = \sqrt{0.0535} \times 2.3 = 0.53 \quad CF = 3.4. \]  

(37)

The rate of drying interval lies between the value with correction factor and without correction factor.

**Discussion About the Equation.**

1. The pellet diameter can be calculated from \( \pi_5 \) and \( \pi_6 \). It becomes:

\[ (d)^2 \times \left( \frac{1}{d} \right)^5 = \left( \frac{1}{d} \right)^3 \]

Therefore, when \( d \) increases, the rate of drying will decrease or the velocity or time will increase.

2. By increasing the velocity, the rate of drying will increase.

3. The rate of drying can be increased by increasing the drying time. All the rest of the groups are directly related. By changing one value all the other groups will change also. The only factor that affects the change of value or the correlation between these groups is the exponent of the group.

4. By increasing the air temperature \( h \) increases and \( M_s \) decreases; therefore the denominator of \( \pi \) will decrease.
$\pi_2$ will increase and also $\pi_3$. The difference in their exponents is $g_1 = 1$, $g_2 = .06$, $g_3 = .33$. It would seem that $h$ has to be the numerator but because of the difference in their exponents within the drying interval of 17 - 10 per cent moisture content, the change in value of the denominator at the rate of drying is larger than the change of $h$. Therefore $h$ will appear in the denominator.

It should be considered that in Figs. 13-15, pages 60-62, a positive relationship between $M_0 - M_1$ was found and $W_s \times h$ was found. This shows that adding $M_0 - M_s$ in the denominator a negative relationship occurred.

An example of the use of the derived equation is shown in the appendix.

**Cooling**

After completing drying, using hot air, it was necessary to cool the pellet to the ambient temperature. Therefore cooling tests were run. The two main problems tested were:

1. The cooling period required to cool pellets to the ambient temperature.

2. Whether cooling may add moisture to the pellets.

Figure 21 shows the effect of cooling temperature versus time. Generally, with 25 per cent relative humidity, 100°F air temperature and 100 CFM of air flow, 8-10 minutes will be needed to cool the pellet. The cooling time depends on the initial pellet temperature. With 60 per cent relative humidity,
Fig. 21. Effect of cooling time on pellet temp.

- Ta = 100° F
- Q = 100 CFM
- L = 12" R.H.
- H = 25% R.H.
- H = 50% R.H.
100° F. air temperature and 100 CFM of air flow, the time was 16-18 minutes to cool the pellet.

When using air of 60 per cent relative humidity, one-half to one per cent moisture was added to the pellet; whereas with air of 25 per cent relative humidity, one to two per cent moisture was removed.

Problem 1 was tested statistically to find the significant difference in cooling time between different pellet diameters plus relative humidity and difference in the initial pellet temperatures.

The three-way classification, complete randomized block design (13), was used.
Table 8. Three-way classification, statistical analysis of effect of R.H., pellet diameter, and initial pellet temperature as affecting pellet cooling time.

<table>
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<td>8</td>
<td>10</td>
<td>10</td>
<td>16</td>
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<td>10</td>
<td>10</td>
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<td>16</td>
<td>16</td>
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<td>26</td>
<td>28</td>
<td>28</td>
<td>48</td>
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<td>227</td>
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<tr>
<td>RH</td>
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<td>82</td>
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<td>145</td>
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<td>10</td>
<td>8</td>
<td>12</td>
<td>18</td>
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<td>10</td>
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<td>8</td>
<td>12</td>
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<td>18</td>
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<td>28</td>
<td>30</td>
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<td>52</td>
<td>54</td>
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<tr>
<td>248</td>
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<td>RH</td>
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<td>54</td>
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<td>103</td>
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<td>88</td>
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<td>160</td>
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<td>Total</td>
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<tr>
<td>170</td>
<td></td>
<td></td>
<td></td>
<td>305</td>
</tr>
<tr>
<td>Total SS = 6833</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub class = [\frac{20373}{3}] - 6267.36 = 523.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ CF = \frac{225625}{36} = 6267.36 \]

\[ (\text{R.H.}) \text{ SS} = \frac{(170)^2 + (305)^2}{18} - 6267.36 = 506.25 \]

\[ (d) \text{ SS} = \frac{(54 + 100)^2 + (58 + 102)^2 + (58 + 103)^2}{12} - 6267.36 = 2.39 \]

\[ (T_p^0) \text{ SS} = \frac{(227)^2 + (248)^2}{18} - 6267.36 = 12.25 \]

\[ (\text{R.H.} \times d) \text{ SS} = \frac{(54)^2 + (58)^2 + (58)^2 + (100)^2 + (102)^2 + (103)^2}{6} - 6267.36 - 506.25 - 2.39 = .166 \]

\[ (d \times T_p^0) \text{ SS} = \frac{(26+48)^2 + (28+48)^2 + (24+49)^2 + (28+52)^2 + (30+54)^2}{6} - 6267.36 - 2.39 - 12.25 = .166 \]

\[ (\text{R.H.} \times T_p^0) \text{ SS} = \frac{(82)^2 + (145)^2 + (88)^2 + (160)^2}{9} - 6267.36 - 506.2 - 12.25 = 2.19 \]

\[ (\text{R.H.} \times d \times T_p^0) = 6791 - 6267.36 - 2.39 - 12.25 - 506.25 - .166 - .166 - 2.19 = .228 \]

Sample SS = 6833 - 523.64 = 6309.36
Table 9. Analysis of variance for data from Table 8.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>DF</th>
<th>SS</th>
<th>Ms</th>
<th>Computed F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative humidity of the air, R.H.</td>
<td>1</td>
<td>506.25</td>
<td>506.25</td>
<td>$\frac{506.25}{262.85} \geq 1.92$</td>
</tr>
<tr>
<td>Pellet diameter, d</td>
<td>2</td>
<td>2.39</td>
<td>1.195</td>
<td>$\frac{1.195}{262.85} &lt; 1.9$</td>
</tr>
<tr>
<td>Initial pellet temperature, $T_{p_0}$</td>
<td>1</td>
<td>12.25</td>
<td>12.25</td>
<td>$\frac{12.25}{262.85} &lt; 1.92$</td>
</tr>
<tr>
<td>R.H. x d</td>
<td>2</td>
<td>.166</td>
<td>.083</td>
<td>$\frac{.083}{262.85} &lt; 1.9$</td>
</tr>
<tr>
<td>d x $T_{p_0}$</td>
<td>1</td>
<td>2.19</td>
<td>2.19</td>
<td>$\frac{2.19}{262.85} &lt; 1.92$</td>
</tr>
<tr>
<td>R.H. x $T_{p_0}$</td>
<td>2</td>
<td>.166</td>
<td>.083</td>
<td>$\frac{.083}{262.85} &lt; 1.9$</td>
</tr>
<tr>
<td>R.H. x $T_{p_0}$ x d</td>
<td>2</td>
<td>.228</td>
<td>.114</td>
<td>$\frac{.114}{262.85} &lt; 1.9$</td>
</tr>
<tr>
<td>Sample</td>
<td>24</td>
<td>6309.36</td>
<td>262.89</td>
<td></td>
</tr>
</tbody>
</table>

**Conclusion.** There is a significant difference between the two relative humidities. That would mean that only a difference of relative humidity can change the cooling time and the pellet diameter or the initial pellet temperature does not affect cooling time at all.

The interval of the standard deviation $\sigma$ between all the cooling time population within certain relative humidity and
the confidence interval about the mean \( \mu \) of the cooling time would be interesting to estimate and test.

The method used is from Fryer (13).

Table 10. The standard deviation \( \sigma \) interval and the confidence interval and the confidence interval about the mean of the cooling time.

<table>
<thead>
<tr>
<th></th>
<th>25% R.H., ( T_p = 130-140^\circ F )</th>
<th>50% R.H., ( T_p = 140^\circ F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>9.444</td>
<td>17.8</td>
</tr>
<tr>
<td>Sum ( (x^2) )</td>
<td>( 1636 - \frac{(170)^2}{18} = 30.45 )</td>
<td></td>
</tr>
<tr>
<td>( \bar{x} = \sqrt{\frac{\Sigma(x^2)}{n(n-1)}} )</td>
<td>( \sqrt{\frac{30.45}{18(17)}} = .315 )</td>
<td></td>
</tr>
<tr>
<td>( \mu^* )</td>
<td>CI95: 9.444-.315x2.1 ( \leq \mu ) ( \leq 9.444+.315 \times 2.1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CI95: 8.78 ( \leq \mu \leq 10.1 )</td>
<td></td>
</tr>
<tr>
<td>( \sigma^2 )**</td>
<td>CI95: ( \frac{30.45}{30.19} \leq \sigma^2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CI95: ( \frac{11.6}{17.5} \leq \sigma^2 )</td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>CI95: 1.02 ( \leq \sigma \leq 2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CI95: .78 ( \leq \sigma \leq 2.3 )</td>
<td></td>
</tr>
</tbody>
</table>

*\( t \) value from (13), \( t_{0.05} = 2.1 \)

**\( \chi^2 \) value from (26), \( \chi_{0.975}^2 = 7.56, \chi_{0.025}^2 = 30.19 \).
CONCLUSIONS

1. Relative humidity of the air and pellet diameter are the two major factors affecting pellet drying.

2. In high relative humidity areas it is almost impossible to dry feed pellets of more than 1/4-inch diameter to satisfactory storage moisture content by using the ambient air.

3. In high relative humidity areas feed pellets of more than 1/4-inch diameter can be dried to the same storing moisture content by using hot air.

4. Equation (35) can be used in designing a dryer or to determine the time and capacity required of a dryer.

5. Cooling after drying using hot air will be affected only by the relative humidity of the air.

6. Because of the increasing production of range cubes, it is highly recommended to install hot air dryers between the pellet mill and cooler.
ACKNOWLEDGMENTS

The writer wishes to express sincere appreciation to Dr. H. B. Pfoest, his major instructor, for guidance and suggestions during the research and preparation of the manuscript.

Thanks are due to Dr. W. J. Hoover, Head of the Department of Grain Science and Industry, and Dr. J. A. Shellenberger for providing physical facilities used in the work.

Smith-Kline and French Laboratories are thanked for financial support of the research work.

The timely help and patience of Mr. Ray Fortner during the pilot plant experiments is gratefully acknowledged.

Finally, it is a pleasure to thank the members of the Department of Grain Science and Industry and especially Drs. M. McMasters and V. Headly, and faculty members from many departments on the campus who most willingly gave assistance and inspiration during the course of this work.


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29. Young, L. R., and H. B. Pfost.  

30. Wang, J. K., and C. W. Hall.  
APPENDIX
APPENDIX

Example for solving a pellet drying problem by using the drying equation developed using poultry ration No. P-17. Whole pellet diameter: \( d = 3/8 \) inch.

\[
\begin{align*}
T_p^0 &= 160^\circ F. \\
T_s &= 200^\circ F. \\
T_b &= 80^\circ F. \\
H &= 60\% \\
M_s &= 7\% \\
Q &= 2000 \text{ CFM} \\
\rho_{\text{air}} &= 0.06 \text{ lb./ft.}^3 \\
\rho_{\text{pellet}} &= 42 \text{ lbs./ft.}^3
\end{align*}
\]

A horizontal dryer available with drying area \( A = 20 \text{ ft.}^2 \) and the depth of the layer being dried is one foot. The safe moisture content for storage \( M_2 = 12 \) per cent.

1. How much time, 6 minutes, will it take to dry this pellet?

2. At what speed will the conveyor in this dryer have to move?

3. What is the maximum capacity that can be dried?

1. Step a. By using the psychrometric chart, \( h \) can be found as \( h = h_s - h_1 \).

\[
\begin{align*}
h_1 &= 0.0135 \text{ lb. water/lb. of dry air} \\
h_s &= 0.038 \text{ lb. water/lb. of dry air} \\
h &= 0.038 - 0.0135 = 0.0245 \text{ lb. water/lb. of dry air.}
\end{align*}
\]

Step b. \( V = Q/A = 2000/20 = 100 \text{ ft./min.} \)

Step c. \( \Delta W = (M_0 - M_2) \times W_p \times L \times A \times \rho_p \)

\[
= 1 \times 20 \times 42 = 840 \text{ lbs.}
\]
\[ \Delta W = 0.4 \times 840 = 33.6 \text{ lbs. of water.} \]

Step d. The drying equation can be written as:

\[
\frac{M_0 - M_1}{M_0 - M_s} = 0.02 \times \left( \frac{T_a - T_{p0}}{T_{p0}} \right)^{0.06} \times \left( \frac{(M_0 - M_s)\Delta W}{h \times W} \right)^{33} \\
\times \left( \frac{d}{L} \right)^2 \times \left( \frac{V}{d} \right)^5 \tag{1}
\]

Since \( W_a = Q \times f_{\text{air}} \times \theta \), therefore:

\[
\frac{M_0 - M_1}{M_0 - M_s} = 0.02 \times \left( \frac{T_a - T_{p0}}{T_{p0}} \right)^{0.06} \times \left( \frac{(M_0 - M_s)\Delta W}{h \times Q \times x} \right)^{33} \times \left( \frac{d}{L} \right)^2 \\
\times \left( \frac{V}{d} \right)^5 \times (\theta)^5 \times \left( \frac{1}{\theta} \right)^{33} \tag{2}
\]

and

\[
\log \left( \frac{M_0 - M_1}{M_0 - M_s} \right) = 0.23 + 0.06 \log \left( \frac{T_a - T_{p0}}{T_{p0}} \right) + 0.33 \log \left( \frac{(M_0 - M_s)\Delta W}{h \times Q \times x} \right) \\
+ 0.2 \log \left( \frac{d}{L} \right) + 0.5 \log (\theta) - 0.33 (\log(\theta)) \pm (Cf)
\]

\[-0.17 \log \theta = -2.3 \pm (Cf) + 0.06 \log \left( \frac{200 - 160}{160} \right) \]

\[+ 0.33 \log \left( \frac{0.082 \times 33.6}{0.0245 \times 2000 \times 0.06} \right) + 0.2 \log \left( \frac{3/8}{12} \right) \]

\[+ 0.5 \log \left( \frac{160 \times 12}{3/8} \right) - \log \left( \frac{0.032}{0.082} \right) \]
Step e. \( \log C_f = .53 \).

\[
-2.3 + .53 + .06 \times (-.07) + .33(-.03)
+ .2 \times (-1.5) + .5 \times (3.7) - .41
\]

Step f. \( \log \theta = \frac{.128}{.17} = .76 \)

\( \theta = 5.76 \), or about 6 minutes.

2. The length of the dryer is 10 feet; therefore the speed of the conveyor will be \( \frac{10}{6} = 1.66 \text{ ft./min.} \)

3. 840 lbs. in 6 minutes; therefore the capacities

\[
\frac{840 \times 60}{6} = 8400 \text{ lb./hr.}, \text{ or } \frac{8400}{2000} = 4.2 \text{ ton/hr.}
\]
DRYING OF FEED PELLETS

by

YOVAL SHULMAN

Technion-Israel Institute of Technology, 1959

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Grain Science and Industry

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1967
The purpose of this research was to determine the factors which influence the rate of drying and cooling feed pellets. The method used to relate these factors was dimensional analysis.

An experimental bed-type dryer was constructed which provided for variations in bed depth, air flow rate, air temperature, and relative humidity of the drying air. Various temperatures and sizes of pellets were obtained by adjusting the pellet mill. The feed formula was held constant and was not an investigated variable.

It was found that the drying rate increased significantly by increasing the air temperature or the diameter of the pellet and increasing the velocity of the air flow through the bed. The cooling rate is significantly increased by decreasing the relative humidity of the air, using cooler air and using higher air flow rates.

The variables were related by deriving a set of equations using dimensional analysis. The exponents and constants for the equations were determined by applying multiple linear regression techniques to the experimental data.

The equation which best describes the phenomenon for pellet drying is:

\[
\left( \frac{M_0 - M_1}{M_0 - M_s} \right) = 0.02 \times \left( \frac{T_0 - T_a}{T_0} \right)^{0.06} \times \left( \frac{h x W_a}{(M_0 - M_s)\Delta W} \right)^{-0.325} \\
\times \left( \frac{d}{L} \right)^{2.16} \times \left( \frac{d}{V_0} \right)^{-0.48}
\]
This research should provide information to enable engineers to design dryers for typical complete feed rations. Constants for formulas which do not contain large quantities of cereal grains need to be investigated. Estimates for cooling feed pellets, after heated air has been used for drying, are possible. Only relative humidity can affect the cooling estimates.