POLARIZATION DEPENDENT RADAR RETURN FROM ROUGH SURFACES

BY

KUMAR KRISHEN

B.A.,JAMMU & KASHMIR UNIVERSITY, 1959.
B.TECH., UNIVERSITY OF CALCUTTA, 1962.
M.TECH., UNIVERSITY OF CALCUTTA, 1963.

A MASTER'S THESIS

Submitted in partial fulfillment of the

requirements of the degree

MASTER OF SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

Approved by

W. W. Koepsel
Major Professor

TABLE OF CONTENTS

INTRODUCTION

The well known laws of reflection permit (in case of a plane interface) to write the reflected field in terms of the incident field, the wavelength of the incident field, the angle of incidence and the electrical properties of the reflecting interface. This could also be interpreted in a different way, in that, the electrical properties of the medium can be determined by measuring the reflected wave. However, most radar targets are not plane but rough.

In the case of rough surfaces the reflected energy is scattered in various directions. The scattering of electromagnetic waves after reflection from a randomly rough surface makes it difficult to predict exactly the reflected field in a particular direction. This makes it difficult to know the electrical properties of the medium of reflection from the scattered field because the latter is not precisely known. The problem of electromagnetic wave scattering has thus assumed importance.

The physicists and engineers are confronted with the problem of electromagnetic wave scattering at many places, some of which are mentioned below.

In the long range atmospheric propagation of short waves beyond the limits of radio horizon, the rough layers of troposphere scatter the radio waves. In the short wave propagation by bouncing the waves from ionosphere the irregularities in the ionospheric layers scatter the short waves.

In random media (in which index of refraction varies from point to point) the energy of electromagnetic waves is scattered

The reflected signals, received from the Moon, show considerable deformation of the shape of incident pulse. This deformation is associated with the scattering properties of the surface of the Moon.

In the case of radio communication between two stations on earth, the rough surface of earth causes the wave propagated by the reflection from the earth to scatter. This makes it difficult to compute the received field strength theoretically.

In radar, the changes in the structure of the target, a terrain or sea surface make the received reflections fluctuate rapidly. Thus in order to predict the target nature, the scattering properties of the target must be known.

In case the reflector is not a perfectly conducting surface the complications are increased. So far there is no reliable scattering theory which takes account of both a partially conducting and a rough interface simultaneously.

One of the important properties associated with the scatter of electromagnetic waves is that the polarization plane rotates after reflection, depending on the slope of the scattering element. The scattered field thus is specified by the polarization apart from the amplitude and phase. There are however serious complications involved in the calculation of the polarization of the scattered waves.

Depolarization of electromagnetic waves has in many instances proved useful. At low frequencies, for short wave transmission, it is often convenient to use polarization as a method for discriminating against noise. It has been seen that very considerable gains in signal to noise ratio in daytime are achieved by using antennas insensitive to vertical polarization on the frequencies where most of the interference is propagated via the ionosphere. In television channels polarization discrimination is useful. Similarly in circularly polarized radar or microwave relays and other devices polarization has been used effectively to achieve the necessary purpose. In analyzing sea echoes polarization dependent radar return has proved useful in understanding the mechanism of scattering from sea surface resulting in a "sea clutter". Recently polarization dependence of scattering cross section of sea has been reported. (Long, 38)

More recently the problem of depolarization has aroused a tremendous interest among the engineers working on the problem of scattering of electromagnetic waves from rough surfaces. This is in part due to the use of radar for the purpose of exploration of celestial bodies. One of the frequently used targets is the Moon. The problem might also yield useful results for military purposes. However, in specifying the polarization of scattered waves the solution of the scattered waves should be sufficiently exact. For this reason polarization of the scattered wave has been so far calculated in case of particular models of rough surfaces for example Twersky model (Twersky; 44) and surfaces which are gently undulating and can be approximated by a single plane (Beckmann; 5). The general solution for a two dimensionally rough surface is thus by far unknown. The progress seems less considerable in part because of the mathematical complexity and in part because of the complexity in the experimental setups needed for the study of such a phenomena.

In general, for calculating the depolarization the same assumptions are made as for calculating the scattered field (e,g):

(a) The radius of curvature of the scattering elements is taken much larger than the wavelengths of the incident radiation.

(b) Shadowing effects are neglected.

(c) Multiple scattering is not taken into account.

(d) The dimensions of the scattering elements of the rough surface are taken large compared to a wavelength.

(e) Only far fields are included in calculations.

The concept of the change of polarization in plane layers is well known, however, in reality plane layers are not more often encountered. This makes the problem of depolarization important in the geophysical exploration of the electrical properties of rough layers.

In theoretical part of this thesis an attempt has been made to include, both the roughness parameters and electrical properties of the layer of reflection, in the returned polarization. It will be shown that the polarization factor of the reflected signal depends both on the roughness parameters and the dielectric properties of the rough layer. The concept of most probable polarization has been brought into picture because the received polarization factor obeys a certain distribution depending on the distribution of the slopes of the surface. The average cross polarization factor <D> is obtained as a complicated function. Except for particular cases the theoretical expressions are unwieldy, and it is difficult to recognize the parts played by:

(a) The properties of the layer

(b) The statistical properties of the surface of the layer

In order to investigate a correlation between the electrical and roughness properties of a statistically rough surface (of a dielectric or partially conducting layer) and the cross polarization distribution caused by the surface, an experiment was performed. Major stress has been placed on this experimental approach and this forms the second part of this thesis. The target model chosen consisted of a perfectly conducting plane covered by a dielectric layer of random thickness. The following experimental results were obtained.

(a) Graphs of <D> versus θ_2 for a number of targets having different dielectric constants ε but otherwise identical statistics.

(b) Graphs of <D> versus ε, with layer statistics as a parameter.

(c) Dependence of <D> on angle of incidence and range.

The experimental results show that cross polarization measurements can be used to draw information about the dielectric properties of the target for a known target roughness statistics. The results of the experimental investigation make it possible to believe $\epsilon = 2$ a plausible value for the lunar surface (Hagfors; 25).

HISTORICAL BACKGROUND.

In a famous series of experiments Heinrich Hertz in 1886 established the fact that radio waves were reflected from solid objects. Hulsmeyer in 1904 obtained a patent on a proposed way of using this property in an obstacle detector. It was in 1922 that Marconi proposed the use of short waves for radio detection. In the latter part of 1930's successful radar systems were developed. Ever since radar has been gaining dimensions in its applications. It has considerably helped in understanding the basic phenomena of electromagnetic wave propagation and reflection. Radar is an indispensable tool in research in these fields nowadays.

The locus of $\vec{E}$ describes the polarization of the electromagnetic waves. Stratton (1941) gave a mathematical formulation through polarization ellipse of the three types of polarization e.g. linear, circular and elliptic. The reflection properties of the two components (horizontal and vertical) of a linearly polarized wave was found to differ considerably (Pfannenberg 1926), experimentally. The theoretical expressions for these coefficients (R^+, $\bar{R}$) were derived by Stratton (1941) by satisfying boundary conditions at the surface of reflection. This, among other things, clearly pointed out the importance of 'polarization' in plane wave reflection from a plane medium.

However, when the surface of reflection is irregular with small or large undulations, the reflected wave is not in general precisely known. The reflected wave is scattered in various directions depending on the type of roughness.

The solution to the problem of scattering has been attempted by engineers and physicists for the last sixty years but an exact solution is still not found. Realizing dim prospects of an exact solution, experimental investigations were stepped up during early 1940's and are still on the same pace. Modern scattering theories apply Kirchhoff's approximation which is required to evaluate Helmholtz integral. In general, returned power is calculated in terms of roughness parameters of the surface (Beckmann 1961). This has improved the formulation of scattering theory to some extent.

Associated with the problem of scattering was the question as to what happens to the polarization of the incident wave when reflected from a rough surface. It was shown that scattering from rough objects (say a cylinder) changes the polarization of the incident wave (Kerr 1947). In order to measure the polarization of a reflected wave, Stratton (1941) introduced a factor called "polarization factor". This factor could measure the polarization of the reflected wave when a linearly polarized wave was illuminating a plane medium boundary.

In case of rough surfaces, the knowledge of return power (average) does not seem sufficient for formulating a general theory of depolarization. Beckmann (1961) has however derived an expression for a gently rough surface which seems to be a reasonable start. Meanwhile, sufficient experimental data are also not available. In 1960, Copeland showed a method of classification and identification of radar targets by the measurement of polarization properties.

More recently, depolarization from rough medium has aroused the interest of engineers and physicists. Depolarization thus might give some relationship to the roughness and dielectric properties of a medium. Hagfor's (1965) has applied depolarization measurements in predicting the lunar surface.

SCATTERING FROM ROUGH SURFACES, GENERAL KIRCHHOFF SOLUTION

I. Surface Rough in One Dimension.

The rough surface is described by the function

$$\zeta = \zeta(x,y).$$

The mean level of the surface is given by

$$Z = o.$$

$\vec{X}_o$, $\vec{Y}_o$ and $\vec{Z}_o$ denote unit vectors along x, y, z directions (Fig. 1) respectively.

The incident wave is assumed plane and of unit amplitude; given by

$$E_1 = \exp.\ (i\ \vec{K}_1 \cdot \vec{r} - i\ \omega t) \tag{1}$$

where

$$|\vec{K}_1| = \frac{2\pi}{\lambda}$$

and

$$\vec{r} = x\vec{X}_o + y \cdot \vec{Y}_o + z \cdot \vec{Z}_o \quad .$$

for points on the surface;

$$\vec{r} = x\ \vec{X}_o + y\ \vec{Y}_o + \zeta(x,y) \cdot \vec{Z}_o \ . \tag{2}$$

The scattering angle is the angle included by $\vec{Z}_o$ and $\vec{K}_2$ and the angle of incidence is the angle between $\vec{K}_1$ and $\vec{Z}_o$, (Fig. 1) where

$$|\vec{K}_2| = |\vec{K}_1| = \frac{2\pi}{\lambda} \quad .$$

The polarization of $\vec{E}_1$ is vertical if $\vec{E}_1$ lies in the plane of incidence $\vec{K}_1$, $\vec{Z}_o$ and horizontal if $\vec{E}_1$ is perpendicular to the plane $\vec{K}_1$, $\vec{Z}_o$. The same convention is applied to $\vec{E}_2$ with respect to the scattering plane $\vec{K}_2$, $\vec{Z}_2$. The quantities

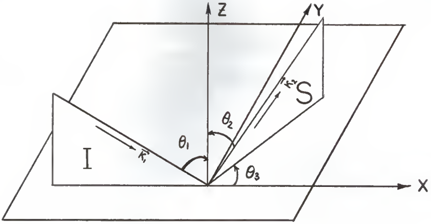


FIG.1 THE SCATTERING GEOMETRY: I, THE PLANE OF INCIDENCE & S THE SCATTERING PLANE.

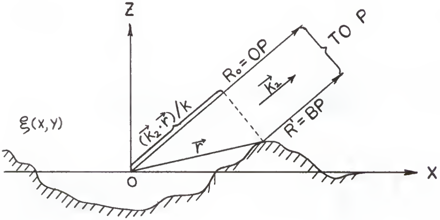


FIG. 2 DERIVATION OF EQUATION (4).

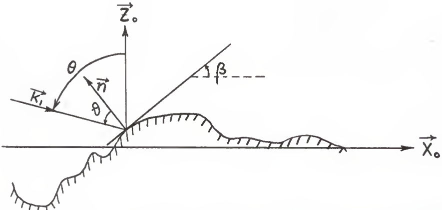


FIG.3 THE LOCAL SCATTERING GEOMETRY. ϑ, THE LOCAL ANGLE OF INCIDENCE WITH RESPECT TO NORMAL. θ, ANGLE OF INCIDENCE DEFINED WITH RESPECT TO Z_o.

associated with the vertical polarization are denoted by subscript '+' and those associated with horizontal polarization by '-'.

For the calculation of field E_2 at the point of observation P (Fig. 2), Helmholtz integral is applied. The field E_2 is given by

$$E_2(P) = \frac{1}{4\pi} \iint_S \left(E \frac{\partial \Psi}{\partial n} - \Psi \frac{\partial E}{\partial n}\right) ds \tag{3}$$

where
$$\Psi = \frac{e^{iK_2R'}}{R} ,$$

and R' is the distance from the point of observation P to any point $\left(x, y, \zeta(x,y)\right)$ on the surface.

Consider the point P in the far field region (Fig. 2) then

$$K_2R' = K_2R_o - \vec{K}_2 \cdot \vec{r} \tag{4}$$

where R_o is the distance of P from origin 0.

The radius of curvature of the irregularities on the surface is assumed large compared with the wavelength of the incident wave. With this assumption and application of Kirchhoff approximations the field and its normal derivative on the surface S are given by

$$E_S = (1+R)E_1 \tag{5}$$

and

$$\left(\frac{\partial E}{\partial n}\right)_S = \nabla(E_S) \cdot \vec{n} \tag{5a}$$

In the following derivation for $\left(\frac{\partial E}{\partial n}\right)_S$ the factor $e^{-i\omega t}$ is suppressed. Thus the incident wave is

$$E_1 = e^{i\vec{K}_1 \cdot \vec{r}}$$
$$= e^{i(K_{1x} \cdot x + K_{1z} \cdot Z)} .$$

Since, $\vec{K}_1$ is perpendicular to $\vec{Y}_o$.

Hence $\nabla E_1 = \left(\vec{X}_o \frac{\partial}{\partial x} + \vec{Y}_o \frac{\partial}{\partial y} + \vec{Z}_o \frac{\partial}{\partial z}\right) e^{i\ (K_{1x}\cdot x + K_{1z}\cdot Z)}$

$$= i\ (K_{1x}\cdot\vec{X}_o + K_{1z}\cdot\vec{Z}_o)\cdot E_1$$

$$= i(\vec{K}_1\cdot\vec{n})E_1,$$

where $\vec{n}$ is unit vector normal to the surface.

The reflected wave is

$$E_R = +Re^{i(\vec{K}_2\cdot\vec{r})}$$

$$= +Re^{i(K_{2x}\cdot X + K_{2z}\cdot Z)}$$

$$\nabla E_R = +iR(\vec{K}_2\cdot\vec{n})e^{i(\vec{K}_2\cdot\vec{r})} \quad .$$

Field on the surface is $E_1 + E_R$, since $|K_2| = |K_1|$,

$$\frac{\partial E}{\partial n} = (E_1 + E_R)_S\cdot n = i(1-R)\vec{K}_1\cdot\vec{n}\cdot E_1 \quad : \qquad (5b)$$

For one-dimensionally rough surfaces

$$\zeta(x,y) = \zeta(x),$$

$$\Psi = \frac{e^{i(K_2R_o - \vec{K}_2\cdot\vec{r})}}{\bar{R}_o}$$

$$\frac{\partial\Psi}{\partial n} = \nabla\Psi\cdot\vec{n} = -i\cdot\frac{e^{i(K_2R_o-\vec{K}_2\cdot\vec{r})}}{R}\cdot\vec{K}_2\cdot\vec{n}$$

$$= -i(\vec{K}_2\cdot\vec{n})\cdot\Psi \ .$$

Use of the above equations in (3) gives

$$E_2 = \frac{1}{4\pi}\iint -i\quad \Psi\left[(1+R)\vec{K}_2 + (1-R)\ \vec{K}_1\right]\cdot\vec{n}\ ds\cdot E_1$$

$$= \frac{1}{4\pi}\iint E_1\left[R(\vec{K}_1-\vec{K}_2)-(\vec{K}_1+\vec{K}_2)\right]\cdot\vec{n}\cdot\Psi\cdot ds \quad . \qquad (5c)$$

If the vectors

$$\vec{V} = \vec{K}_1 - \vec{K}_2$$

$$\vec{P} = \vec{K}_1 + \vec{K}_2$$

are introduced in (5c) then

$$E_2 = \frac{i\ e^{iKR_o}}{4\pi R_o}\int_S (R\vec{V}-\vec{P})\cdot\vec{n}\ e^{i\vec{V}\cdot\vec{r}}ds$$

Where $K = |\vec{K}_1|$,

$$\vec{V} = K(\sin\theta_1 - \sin\theta_2)\vec{X}_o - K(\cos\theta_1 + \cos\theta_2)\vec{Z}_o \ ,$$

$$\vec{P} = K\ (\sin\theta_1 + \sin\theta_2)\vec{X}_o + (\cos\theta_2 - \cos\theta_1)\vec{Z}_o \ ,$$

$$\vec{n} = -\vec{X}_o \sin\beta + \vec{Z}_o \cos\beta \ ,$$

$$\vec{r} = x\ \vec{X}_o + \zeta(x)\vec{Z}_o, \text{ and } \tan\beta = \zeta'(x) \ .$$

If the surface extends from L to -L then

$$E_2 = \frac{ike^{ikR_o}}{4\pi R_o} \int_{-L}^{L} (a\zeta' - b)e^{i(v_x \cdot x + v_z \zeta)} .dx. \tag{6}$$

Where $a = (1-R)\sin\theta_1 + (1+R)\sin\theta_2$

$b = (1+R)\cos\theta_2 - (1-R)\cos\theta_1$.

Define the scattering coefficient as

$$\rho = \frac{E_2}{E_{20}}$$

where E_{20} is the field reflected in the specular direction ($\theta_2 = \theta_1$) by a smooth, perfectly conducting plane of the same dimensions under the same angle of incidence at the same distance, when the incident wave is horizontally polarized.

For specular reflection

$V_x = 0$ and for a smooth surface $\zeta = \zeta' = 0$

Hence from (6)

$$E_{20} = \frac{ike^{ikR_o} L \cos\theta_1}{\pi R_o} \ . \tag{7}$$

from (6) and (7),

$$\rho = \frac{1}{4L\cos\theta_1} \int_{-L}^{L} (a\zeta' - b) \cdot e^{iv_x \cdot x + iv_z \zeta} .dx. \tag{8}$$

Evaluation of this integral gives,

$$\rho = \frac{1}{4L}\,\overline{\text{Cos}\theta_1}\left[\int_{-L}^{L} a\zeta'\, e^{i\vec{v}\cdot\vec{r}}\,dx - \int b\cdot e^{i\vec{v}\cdot\vec{r}}\cdot dx\right] . \qquad (9)$$

$$\text{Now, } a\int_{-L}^{L} \zeta'\; e^{i\vec{v}\cdot\vec{r}}.dx = \frac{a}{iv_z}\int_{-L}^{L} \zeta'\; v_z\cdot e^{i\zeta\cdot v_z}.i)\cdot\;(e)^{iv_x\cdot x}\cdot dx$$

$$= \frac{a}{iv_z}\left[e^{iv_x\cdot x+i\zeta\cdot v_z}\Big|_{-L}^{L} - iv_x\int_{-L}^{L} e^{i\vec{v}\cdot\vec{r}}\cdot dx\right]$$

$$= \frac{-av_x}{v_z}\int_{-L}^{L} e^{i\vec{v}\cdot\vec{r}}dx - \frac{ia}{v_z}\left[e^{i\vec{v}\cdot\vec{r}}\right]\Big|_{-L}^{L} . \qquad (10)$$

From (9) and (10)

$$\rho = \frac{1}{4L\text{Cos}\theta_1}\left[-\left(b+\frac{av_x}{v_z}\right)\int_{-L}^{L} e^{i\vec{v}\cdot\vec{r}}.dx - \frac{ia}{v_z}\; e^{i\vec{v}\cdot\vec{r}}\Big|_{-L}^{L}\right] .$$

In case of perfectly conducting surface

$$R^{+} = 1 \quad , \quad R^{-} = -1 \quad .$$

Hence for vertically and horizontally polarized waves

$$\rho^{\pm}\;(\theta_1,\theta_2) = \pm\;\sec\;\theta_1\;\frac{1+\text{Cos}\,(\theta_1+\theta_2)}{\text{Cos}\theta_1+\text{Cos}\theta_2}\quad\frac{1}{2L}\int_{-L}^{L} e^{i\vec{v}\cdot\vec{r}}.dx+\frac{e^{\pm}(L)}{2L} \;.(11)$$

where

$$e^{\pm}(L) = \frac{i\sec\theta_1\;\sin\theta^{\pm}}{K\,(\text{Cos}\theta_1+\text{Cos}\theta_2)}\;e^{i\vec{v}\cdot\vec{r}(x)}\Big|_{-L}^{L}$$

$$\theta^{+}=\theta_2 \qquad \theta^{-}=\theta_1$$

Equation (11) gives the general solution for a perfectly conducting, one dimensionally rough surface, with

$$\vec{v}\cdot\vec{r} = \frac{2\pi}{\lambda}\left[(\sin\theta_1-\sin\theta_2)x-(\cos\theta_1+\cos\theta_2)\zeta(x)\right]$$

II. Surface Rough in Two Dimensions.

If the procedure followed in the case of one dimensionally rough surface is followed in the case of a two dimensionally rough surface, then

$$\rho = \frac{1}{4xy\cos\theta_1}\int_{-x}^{x}\int_{-y}^{y}(a\zeta_x' + c\zeta_y' - b).e^{i\vec{v}\cdot\vec{r}}\cdot dx\cdot dy \qquad (12)$$

where

$$\vec{v} = K\left[(\sin\theta_1-\sin\theta_2\sin\theta_3)\vec{x}_o-\sin\theta_2\sin\theta_3\cdot\vec{y}_o-(\cos\theta_1-\cos\theta_2\ \vec{z}_o\right]$$

$$a = (1-R)\sin\theta_1+(1+R)\sin\theta_2\cos\theta_3,$$

$$b = (1+R)\cos\theta_2-(1-R)\cos\theta_1,$$

$$c = (1+R)\sin\theta_2\cdot\sin\theta_3.$$

In this derivation R is not in general equal to the Fresnel coefficient.

Integration of R.H.S. of (12) by parts gives

$$\rho = \frac{1}{4xy\cos\theta_1}\left[-\left(b+\frac{av_x+cv_y}{v_z}\right)\int_{-x}^{x}\int_{-y}^{y} e^{iv\cdot r}.dx.dy- -\frac{ic}{v_z}\int_{-x}^{x} e^{i\vec{v}\cdot\vec{r}}\Big|_{-y}^{y} dx -\frac{ia}{v_z}\int_{-y}^{y} e^{i\vec{v}\cdot\vec{r}}\Big|_{-x}^{x}.dy\right] \qquad (13)$$

Substitution of the values of a,b,c and $R^{\pm}=1$, $R^{-}=-1$ in (13) gives,

$$\rho^{\pm}(\theta_1,\theta_2,\theta_3)=\pm\frac{1+\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2\cos\theta_3}{\cos\theta_1(\cos\theta_1+\cos\theta_2)}\cdot\frac{1}{A}\int_{-x}^{x}\int_{-y}^{y} e^{i\vec{v}\cdot\vec{r}}\,dxdy+$$

$$+\frac{e^{\pm}(x,y)}{A} \qquad (14)$$

where $A = 4xy$.

Where $A>>\lambda^2$; the second term can be neglected compared to first, hence

$$\rho^{\pm} = \pm\frac{F_3}{A}\int_A e^{i\vec{v}\cdot\vec{r}}\,dxdy\ , \qquad (15)$$

where $$F_3(\theta_1,\theta_2,\theta_3) = \frac{1+\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2\cos\theta_3}{\cos\theta_1(\cos\theta_1+\cos\theta_2)}\ .$$

DEPOLARIZATION OF ELECTROMAGNETIC WAVES SCATTERED FROM A ROUGH SURFACE

I. Definitions.

The incident wave shall be denoted by $\hat{\vec{E}}_1$ where the accent $(\hat{\ })$ denotes a complex scalar. Similarly the laterally scattered wave reflected from the point 0 (Fig. 4) is denoted by $\hat{\vec{E}}_2$. If $\hat{\vec{E}}_1$ is polarized in any arbitrary direction, then resolution of $\hat{\vec{E}}_1$ along horizontal and vertical directions gives

$$\hat{\vec{E}}_1 = \hat{E}_1^+ \cdot \vec{e}_1^{\,+} + \hat{E}_1^- \cdot \vec{e}_1^{\,-} \tag{1}$$

Where $\vec{e}_1^{\,+}$ and $\vec{e}_1^{\,-}$ are the unit vectors along vertical and horizontal directions respectively. The polarization of the incident wave is defined by the ratio

$$\hat{p} = \frac{\hat{E}^+}{\hat{E}^-} \quad . \tag{2}$$

From equation (2) it follows that if

Imaginary $\hat{p} = o$ $\hat{\vec{E}}_1$ is linearly polarized,

Imaginary $\hat{p} > o$ implies right handed rotational polarization,

Imaginary $\hat{p} < o$ implies left handed rotational polarization,

$\hat{p} = o$ implies horizontal polarization,

$\hat{p} = \infty$ implies vertical polarization,

$\hat{p} = i$, $\hat{\vec{E}}_1$ is right handed circularly polarized,

$\hat{p} = -i$ $\hat{\vec{E}}_1$ is left handed circularly polarized.

Hence, p_1 (complex) uniquely defines the polarization of the incident wave.

From $\hat{p}$ other quantities relating to polarization, such as the parameters of polarization ellipse, can be obtained.

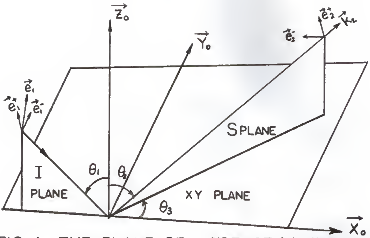


FIG. 4 THE PLANE OF INCIDENCE (I), THE SCATTERING PLANE (S) HORIZONTAL (–) AND VERTICAL (+) POLARIZATION.

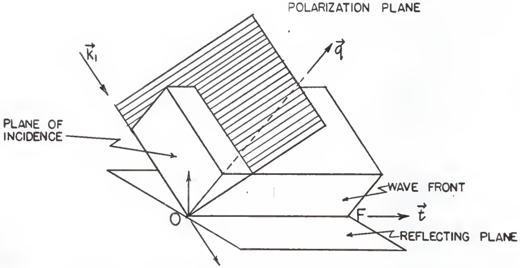


FIG. 5 ORIENTATION OF DIRECTION OF PROPGATION & POLARIZATION PLANE WITH RESPECT TO THE REFLECTING PLANE.

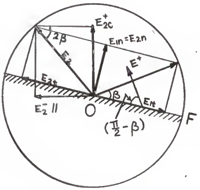


FIG. 6 DECOMPOSITION OF E_1 & E_2, $\overline{OF}$ CORRESPONDS TO $\overline{OF}$ IN FIG. 5.

The 'depolarization factor' is defined by the ratio

$$q = p_2/p_1 \tag{3}$$

where p_2 denotes the polarization of the scattered wave. Substitution of the values of p_2 and p_1 in (3) gives

$$q = \frac{E_2^+}{E_1^+} \cdot \frac{E_1^-}{E_2^-} = \frac{\bar{R}^+}{\bar{R}^-} \tag{4}$$

where $\bar{R}^+$ and $\bar{R}^-$ reduce to R^+ and R^- only when the scatterer is a properly oriented smooth plane.

The media of propagation are assumed linear, consequently, R^+ is independent of E_1^+ and R^- is independent of E_1^-. When the incident wave is depolarized after reflection from a rough surface, a horizontally polarized wave ($\bar{E}_1$) gives rise to two components $\bar{E}_{2||}$, a component parallel to the incident wave, and E_{2c}^+, a component perpendicular to the incident polarization (Fig. 6). Similarly for vertically polarized component of the incident wave, the reflected wave consists of two orthogonal components, perpendicular and parallel respectively to the incident wave.

Thus for the reflected wave

$$E_2^+ = E_{2||}^+ + E_{2c}^- \qquad E_2^- = E_{2||}^- + E_{2c}^+ \tag{5}$$

from (4) and (5),

$$\bar{R}^+ = \bar{R}_{||}^+ + \frac{\bar{R}_c}{p_1} \qquad \bar{R}^- = \bar{R}_{||}^- + p_1 \cdot \bar{R}_c^- \tag{6}$$

where

$$\bar{R}_{||}^{\pm} = \frac{E_{2||}^{\pm}}{E_1^{\pm}} \quad . \tag{7}$$

from (4) and (6)

$$q = \frac{p_1 R^+_{11} + R^+_c}{p_1 R^-_{11} + p_1{}^2 R^-_c} \tag{8}$$

In the case longitudinal scattering (scattering in the plane of incidence),

$$R^+_c = R^-_c = o \quad ,$$

and

$$q = \frac{R^+_{11}}{R^-_{11}} = \frac{E^+_{211}}{E^-_{211}} = \frac{\rho^+}{\rho^-} \quad . \tag{9}$$

In case the wave is laterally scattered from (8) it follows that

$$p_2 = \frac{p_1 R^+_{11} + R^+_c}{R^-_{11} + p_1 R^-_c} \quad . \tag{10}$$

Another important quantity associated with depolarization is the cross-polarization defined as $D = \left| \frac{E_2^{\perp}}{E_2^{11}} \right|$ (11)

where $E_2^{\perp}$ and E_2^{11} are the scalar values of the components parallel and perpendicular respectively to the incident field $\vec{E}_1$. Cross-polarization is useful in case of linear polarization only. D is related to q by the relation

$$D = \left(\frac{q-1}{\cot x_1 + q \tan x_1} \right) \tag{12}$$

In case cross polarization is random a statistical distribution of D is required and this is what is determined in the experimental approach to the problem of depolarization in this thesis.

II. Case of Smooth Plane

In the case of a smooth plane the scattering is only longitudinal, hence

$$q = \frac{\rho^+}{\rho^-} = \frac{(1+R^+)\cos\theta_2-(1-R^+)\cos\theta_1}{(1+R^-)\cos\theta_2+(R^--1)\cos\theta_1} \qquad (13)$$

for perfectly conducting case

$$R^+ = 1 \quad , \quad R^- = -1, \text{ thus from} \qquad (13)$$

$$q = -\frac{\cos\theta_2}{\cos\theta_1}$$

for specular direction this yields

$$q = -1 \quad .$$

Since $q \neq 1$, it follows that the smooth plane will in general depolarize the incident wave except the horizontal and vertical polarization.

III. Depolarization by a Rough Surface in the Plane of Incidence.

From the results already derived it follows that for depolarization in the plane of incidence

$$q = \frac{\rho^+}{\rho^-} \quad .$$

When $\theta_3 = o$ in the general solution for two dimensional case

$$q = -\frac{F_2\int_{-L}^{L} e^{i\vec{v}\cdot\vec{r}}\,dx + e^+(L)}{F_2\int_{-L}^{L} e^{i\vec{v}\cdot\vec{r}}\,dx - \bar{e}(L)} \quad . \qquad (14)$$

In case the 'edge effect' is small $q = -1$ which expresses the fact that $R^+ = -R^-$. This is true for irregularities large compared to a wavelength.

However, when the shape of the rough surface is such that the radii of curvature are large compared to a wavelength, only those solutions are considered which yield

$$q = \frac{E_2^+}{E_2^-}$$

in the plane of incidence, are considered. For example, in the case of Twersky model (Twersky, 44)

$$|q| = \frac{1+\cos 2\theta_1}{2-\cos 2\theta_1} \tag{15}$$

In the case of a random surface both ρ^+ and ρ^- are random but in the plane of incidence these are correlated by the relation

$$\rho^+ = -\rho^-, \text{ thus}$$

$$q = -1 .$$

For finite conductivity these quantities are not related by a deterministic relationship and 'q' turns out to be a ratio of two random variables. The distribution of q can be found (Marsaglia, 39) from the distribution of ρ^+ and ρ^-; and an expected depolarization can be calculated.

In case the dimensions of irregularities are large compared to a wavelength, it is possible to approximate the rough surface by randomly oriented planes.

In such cases the depolarization caused by a number of sheets of equal electrical properties and orientation but different areas and positions is

$$P_2 = \frac{E_2}{E_2^-} = \frac{\Sigma_j A_j E_{20}^+}{\Sigma_j A_j E_{20}^-} = \left|\frac{E_{20}^+}{E_{20}^-}\right| \frac{e^{i\phi_o^+}\ \Sigma_j e^{i(\phi_j^+ - \phi_o^+)}}{e^{i\phi_o^+}\Sigma_j e^{i(\phi_j^- - \phi_o^-)}} \tag{16}$$

Where $E_{20} = |E_{20}| \exp(i\phi_o)$ is the field reflected by any one arbitrarily selected sheet of area A_o' and $A_j = A_j'/A_o'$.

Since phase shift $\phi_j - \phi_o$ depends only on the position of the sheet S_j with respect to S_o,

$$\phi_j^+ - \phi_o^+ = \phi_j^- - \phi_o^-$$

After cancelling sums in (16),

$$p_2 = |p_{20}|\ e^{(i\ \arg.\ p_{20})} = p_{20} \qquad (17)$$

Equation (17) indicates that the field scattered by a rough surface into the direction (θ_2, θ_3) is depolarized in the same way as the field reflected by a plane of same electrical properties inclined in such a way that it will reflect the incident field into the direction (θ_2, θ_3).

IV. Depolarization of Laterally Scattered Waves.

The local plane of incidence is defined by the plane $\vec{K}_1$ and $\vec{n}$ where $\vec{n}$ is the normal to the scattering element (Fig. 4). According to Snell's law of reflection $\vec{K}_1$, $\vec{K}_2$ and $\vec{n}$ lie in the same plane. In case $\vec{n}$ does not lie in the plane of incidence it is laterally scattered and consequently depolarized.

It can be shown in the case of a perfectly conducting plane that an element of surface with slope unity will change the polarization of the oncoming vertically polarized wave (near grazing incidence) to horizontally polarized wave. Similarly, a horizontally polarized wave will be depolarized and will be reflected as vertically polarized wave from such a plane.

The scattering geometry is defined (Fig. 5) by two angles the local grazing angle and β, the polarization angle. Assume the reflecting plane is tilted by an angle β (Fig. 6) with respect to horizontal. If the reflecting plane is assumed perfectly conducting and a horizontally polarized wave is incident on this plane, such that,

$$\vec{E}_1 = \vec{E}_1^-$$

then

$$E_{1n} = E_{2n} \quad , \quad E_{1t} = -E_{2t} \quad .$$

Resolution of $\vec{E}_2$ into two components, $\vec{E}_{2ll}$, $\vec{E}_{2c}$; parallel and perpendicular respectively to $\vec{E}_1^-$; gives

$$E_{2ll} = -\,E_1 \cos 2\beta, \; E_{2c} = E_1 \sin 2\beta \quad .$$

From these it follows from definition,

$$\begin{aligned} \mathcal{R}_{ll}^- &= -\cos 2\beta \\ \mathcal{R}_c^- &= \sin 2\beta \quad . \end{aligned} \tag{18}$$

The polarization angle of $\vec{E}^+$ with respect to same reflecting plane is

$$\beta^+ = \pi/2 - \beta \quad .$$

Hence,

$$\mathcal{R}_{ll}^+ = \cos 2\beta \; , \; \mathcal{R}_c^+ = \sin 2\beta \quad . \tag{19}$$

For any β according to law of conservation of energy,

$$(\mathcal{R}_{ll}^{\pm})^2 + (\mathcal{R}_c^{\pm})^2 = 1 \quad . \tag{20}$$

From equations (6), (18), and (19), it follows that

$$\begin{aligned} \mathcal{R}^+ &= \cos 2\beta + \frac{1}{p_1} \sin 2\beta \\ \mathcal{R}^- &= -\cos 2\beta + p_1 \sin 2\beta \end{aligned} \tag{21}$$

Hence for the perfectly conducting plane the depolarization equation obtained from (21) is

$$p_2 = \frac{\tan 2\beta + p_1}{p_1 \tan 2\beta - 1} \quad . \tag{22}$$

In case $\beta_2 \neq \beta$,

$$p_2 = \frac{p_1(1-\tan\beta\tan\beta_2)+\tan\beta+\tan\beta_2}{p_1(\tan\beta+\tan\beta_2)+\tan\beta\tan\beta_2-1} \ . \tag{23}$$

DEPOLARIZATION DUE TO ONE DIMENSIONALLY; GENTLY UNDULATING ROUGH SURFACE.

It is possible to obtain a relation between the expected polarization and the slope distribution of a rough conducting surface by defining polarization of the reflected wave as the ratio

$$P_2 = \frac{|E_2^+|^2 - |E_2^-|^2}{|E_2^+|^2 + |E_2^-|^2} \quad . \qquad (1)$$

Some of the results derived by Beckmann (6) based on Kirchhoff's approximations have been used.

For the one dimensionally rough surface the scattering coefficients are defined by

$$\begin{aligned} \rho^+ &= E_2^+/E_{20} \\ \rho^- &= E_2^-/E_{20} \end{aligned} \qquad (2)$$

where E_{20} is the field reflected in the direction of specular reflection ($\theta_2=\theta_1$) by a smooth perfectly conducting plane of the same dimensions under the same angle of incidence at the same distance, when the incident wave is horizontally polarized. Only those cases are considered where the reflected wave is linearly polarized.

From (1) and (2),

$$P_2 = \frac{|\rho^+|^2 - |\rho^-|^2}{|\rho^+|^2 \quad |\rho^-|^2} \qquad (3)$$

The scattering coefficients are given by (Beckmann 6),

$$\rho^\pm = \pm\sec\theta_1 \frac{1+\cos(\theta_1+\theta_2)}{\cos\theta_1+\cos\theta_2} \cdot \frac{1}{2L} \cdot \int_{-L}^{L} e^{i\vec{v}.\vec{r}} \cdot dx + \frac{e^\pm(L)}{2L} \qquad (4)$$

Where $\quad e^{\pm}(L) = \dfrac{i\sec\theta_1\sin\theta^{\pm},}{K(\cos\theta_1+\cos\theta_2)}$

$$\theta^{+} = \theta_2 \;, \quad \theta^{-} = \theta_1$$

and $\quad \vec{v}\cdot\vec{r} = \frac{2\pi}{\lambda}\left[(\sin\theta_1-\sin\theta_2)x-(\cos\theta_1+\cos\theta_2)\zeta(x)\right]$

Substitution of (4) in (3), yields to the first degree of approximation

$$p_2 = \frac{|e^{+}(L)|^2 - |e^{-}(L)|^2}{|e^{+}(L)|^2 + |e^{-}(L)|^2} \tag{5}$$

Thus the edge effect seems to play an important role in depolarization of the incident wave, (Fung, et al... and Hagfors....).

Substitution of $e^{+}(L)$ and $e^{-}(L)$ in (5) yields,

$$p_2 = \frac{\sin^2\theta_1 - \sin^2\theta_1}{\sin^2\theta_2 + \sin^2\theta_1} \;. \tag{6}$$

The scattered pattern of the reflected wave will depend on the angle of incidence θ_1 (Fig. 7), the degree of roughness apart from λ of the incident wave. The reflecting surface is assumed perfectly conducting. In the event the angle θ_1 and λ are constant, the reflected pattern will depend fully on the type of roughness. When the surface roughness is assumed Gaussian then this pattern will be determined by the limits of the slope variation of the surface.

With these considerations now θ_2 can be linked to the slope of the scattering element in the following way

$$\vec{k}_1 = \sin\theta_1\cdot\vec{a}_{x_o}+\cos\theta_1\cdot\vec{a}_{zo} \;. \tag{7a}$$

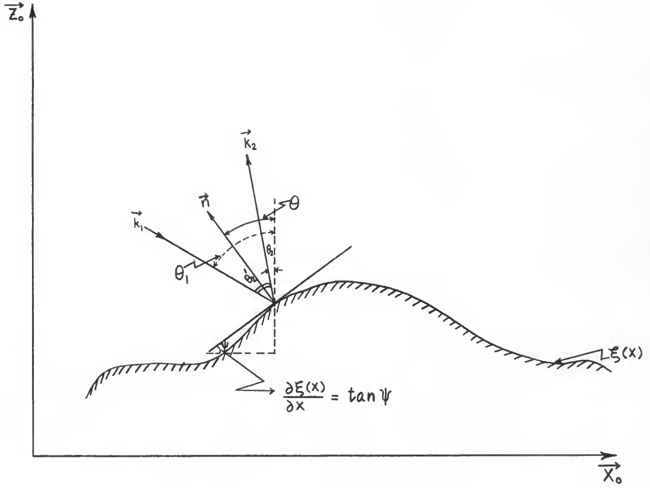


FIG. 7 DERIVATION OF EQUATION (10).

Unit vector normal to the surface element is given by,

$$\vec{n} = \frac{\zeta_x \cdot \vec{a}x_o + 1 \cdot \vec{a}z_o}{\sqrt{1+\zeta_x^2}} . \tag{7b}$$

Now, $\cos\theta'_2 = \vec{k}_1 \cdot \vec{n}$

$$= \frac{\zeta_x \cdot \sin\theta_1 + \cos\theta_1}{\sqrt{1+\zeta_x^2}} \tag{7c}$$

If θ be the angle between $\vec{n}$ and $\vec{z}$, then

$$\cos\theta = \vec{z}_o \cdot \vec{n} = \frac{1}{\sqrt{1+\zeta_x^2}} \tag{7d}$$

$$\text{Hence, } P_2 = \frac{\sin^2(\theta'_2-\theta) - \sin^2\theta_1}{\sin^2(\theta'_2-\theta) + \sin^2\theta_1} \tag{8}$$

Where $\quad 0 \le |\theta'_2-\theta| \le \pi/2$.

With, $\quad \sin(\theta'_2-\theta) = \sin\theta'_2\cos\theta - \cos\theta'_2\sin\theta$

$\sin^2(\theta'_2-\theta) = \sin^2\theta'_2\cos^2\theta + \cos^2\theta'_2\sin^2\theta - \dfrac{\sin^2\theta'_2 \sin^2\theta}{2}$

and using above relations

$$\sin^2(\theta'_2-\theta) = (-)\frac{(\zeta_x^2-1)\sin\theta_1 + 2\zeta_x\cos\theta_1}{(1+\zeta_x^2)} . \tag{9}$$

Substitution of (9) in (8) gives,

$$P_2 = \frac{\left[(\zeta_x^2-1)\sin\theta_1 + 2\zeta_x\cos\theta_1\right]^2 - (1+\zeta_x^2)^2\sin^2\theta_1}{\left[(\zeta_x^2-1)\sin\theta_1 + 2\zeta_x\cos\theta_1\right]^2 + (1+\zeta_x^2)^2\sin^2\theta_1} \tag{9a}$$

or

$$P_2 = \frac{4\zeta_x^2\cos2\theta_1 + 2\zeta_x(\zeta_x^2-1)\sin2\theta_1}{\left[(\zeta_x^2+1)^2 - \cos2\theta_1(\zeta_x^2-1)^2 + 2\zeta_x(\zeta_x^2-1)\sin2\theta_1\right]} \tag{9b}$$

For gently undulating surface, hence assuming ζ_x is normally distributed with mean value 1; the average value of p_2 is given by,

$$\langle p_2\rangle = \frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{1-\varepsilon}^{1+\varepsilon} \frac{4\zeta_x^2\cos 2\theta_2+2\zeta_x(\zeta_x^2-1)\sin(\theta_1 2)}{\left[(\zeta_x^2+1)^2-\cos 2\theta_1(\zeta_x^2-1)^2+2\zeta_x(\zeta_x^2-1)\sin 2\theta_1\right]} \times$$

$$\times \exp -\frac{1}{2}\left(\frac{\zeta_x-1}{\sigma}\right)^2 d\zeta_x. \tag{10}$$

This is the final result, assuming the incident wave is linearly polarized with $p_1=o$.

DEPOLARIZATION DUE TO A LAYER OF RANDOM THICKNESS

The method of approach is to assume, first a horizontally polarized component and calculate the horizontally polarized component in the layer. Second component, (i,e) the vertical component of the incident wave, will give rise to a vertically polarized component in the layer. The ratio of the vertical to the horizontal component of the field in the layer gives the polarization in the layer. This wave is then incident on the rough surface (by which the layer is terminated) which is perfectly conducting and the returned polarization is calculated. The returned polarization is averaged over the slopes of the rough bottom surface. Proper consideration is given to the transmission of the reflected wave ($\vec{k}_3$) back into medium number one.

Assume $\vec{E}_o^-$ is normal to the plane (Fig. 8) of incidence; $\vec{E}_1^-$ (the transmitted component into the layer) normal to the incident plane is given by

$$\vec{E}_1^- = \frac{(2\mu_1 k_o \cos\theta_o)}{\mu_1 k_o \cos\theta_o + \mu_o k_1 \cos\theta_1} \vec{E}_o^+ \tag{1}$$

$$k_1 \cos\theta_1 = \sqrt{k_1^2 - k_o^2 \sin^2\theta_o} \tag{2}$$

The layer is assumed to be a dielectric. If, now $\vec{E}_o^+$ (in the plane of incidence) is incident on the layer. The field due to this in the layer is

$$\vec{E}_1^+ = \frac{2\mu_1 k_o}{\mu_1 k_o + \mu_o k_1} \vec{E}_o^+ \tag{3}$$

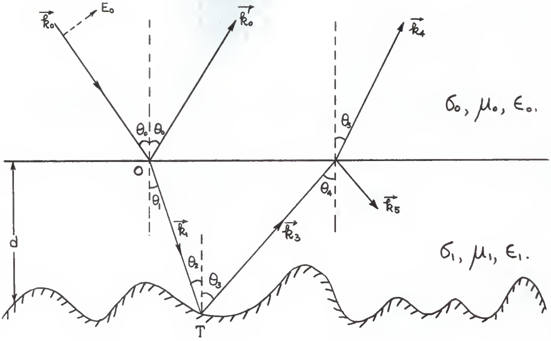


FIG. 8 SCATTERING GEOMETRY FOR A LAYER. DERIVATION OF EQUATION (22).

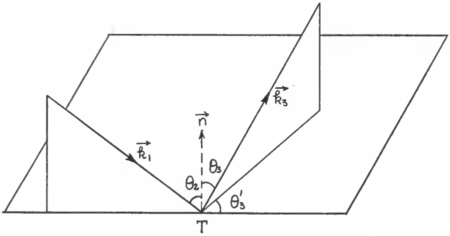


FIG. 9 SCATTERING GEOMETRY FOR A LAYER.

The ratio of (1) and (3) in the layer gives the polarization of the wave in the layer as

$$\frac{\vec{E}_1^+}{\vec{E}_1^-} = \left(\frac{2\mu_1 k_o}{\mu_1 k_o+\mu_o k_1}\right)\left(\frac{2\mu_1 k_o \cos\theta_o}{\mu_1 k_o \cos\theta_o+\mu_o k_1 \cos\theta_1}\right) \frac{\vec{E}_o^+}{\vec{E}_1^-} \quad . \tag{4}$$

Denote $p_1 = \frac{\vec{E}_1^+}{\vec{E}_1^-}$ and $p_o = \frac{\vec{E}_o^+}{\vec{E}_1^-}$.

Now an electromagnetic wave with polarization p_1 given by (4) is incident on the rough termination and is scattered in direction θ_3, θ_3' as shown in Fig. 9.

If 'β' be the polarization angle (i,e) the angle concluded by wave with polarization factor p_1 with the line of intersection of the incident wave front and the reflecting plane.

Then the polarization of the reflected plane wave is given by

$$p_3 = \frac{\tan 2\beta+p_1}{p_1\tan 2\beta-1} \tag{5}$$

The two components of the reflected wave from the bottom of the layer (horizontal and vertical) are $\vec{E}_3^-$ and $\vec{E}_3^+$.

In figure 8,

$$\theta_4 = \theta_3 \quad . \tag{6}$$

The transmitted wave due to $\vec{E}_3^-$ into medium 1 is given by

$$\vec{E}_4^- = \frac{2\mu_o k_5 \cos\theta_4}{\mu_o k_5 \cos\theta_4+\mu_1 k_4 \cos\theta_5} \tag{7}$$

where

$$k_4 \cos\theta_5 = \sqrt{k_4^2-k_5^2\sin^2\theta_4} \quad . \tag{8}$$

Similarly the transmitted wave due to $\vec{E}_3^+$ is given by

$$\vec{E}_4^+ = \frac{2\mu_o k_5}{\mu_o k_5 + \mu_1 k_4} \vec{E}_3^- \tag{9}$$

$$k_3 = k_5 \tag{10}$$

Hence polarization of the finally emerging wave in medium 1 is given by the ratio of

$$p_4 = \frac{\vec{E}_4^+}{\vec{E}_5^-} = \left(\frac{2\mu_o k_5 \cos\theta_4}{\mu_o k_5 \cos\theta_4 + \mu_1 k_4 \cos\theta_5}\right)\left(\frac{2\mu_o k_5}{\mu o k_5 + \mu_1 k_4}\right) \cdot \frac{\vec{E}_3^+}{\vec{E}_3^-} \tag{11}$$

After recognizing $p_3 = \frac{\vec{E}_3^+}{\vec{E}_3^-}$, equation (11) gives,

$$p_4 = \left(\frac{2\mu_o k_5}{\mu_o k_5 + \mu_1 k_4}\right)\left(\frac{2\mu_o k_5 \cos\theta_4}{\mu_o k_5 \cos\theta_4 + \mu_1 k_4 \cos\theta_5}\right)\left(\frac{\tan 2\beta + p_1}{p_1 \tan 2\beta - 1}\right) p_3 \,. \tag{12}$$

Denote,

$$A = \frac{2\mu_1 k_o}{\mu_1 k_o + \mu_o k_1} \,,$$

$$B = \frac{2\mu_1 k_o \cos\theta_o}{\mu_1 k_o \cos\theta_o + \mu_o k_1 \cos\theta_1} \,,$$

$$C = \frac{2\mu_o k_5}{\mu_o k_5 + \mu_1 k_4} \,,$$

and $$D = \frac{2\mu_o k_5 \cos\theta_4}{\mu_o k_5 \cos\theta_4 + \mu_1 k_4 \cos\theta_5} \,,$$ then

$$p_4 = C \cdot D \, \frac{\tan 2\beta + A \cdot B \cdot p_o}{A \cdot B \tan 2\beta - 1} \,. \tag{13}$$

Now, β will be related to the roughness of the bottom of the layer.

The equation for the perpendicular to a surface

$$\phi = f(x,y) \quad , \quad Z = o \text{ (mean value)}$$

is given by

$$\vec{n} = \frac{\nabla[f(x,y)\bar{z}=o]}{|\nabla[f(x,y)\bar{z}=o]|} \quad ; \tag{14}$$

or

$$\vec{n} = \frac{(-\zeta_x)\cdot\vec{a}_x+(-\zeta_y)\cdot\vec{a}_y+1\cdot\vec{a}_z}{\sqrt{\zeta_x^2+\zeta_y^2+1}} \tag{15}$$

where $\qquad \zeta_x = \frac{\partial f}{\partial x}$,

and $\qquad \zeta_y = \frac{\partial f}{\partial y}$.

Unit vector normal to the plane of $\vec{k}_1$ and $\vec{n}$ is

$$0\vec{F} = \frac{\vec{n}x\vec{k}_1}{|\vec{n}x\vec{k}_1|} \quad . \tag{16}$$

The angle between $0\vec{F}$ and 0Y-axis is β, hence,

$$\cos\beta = \frac{\sin\theta_2-\zeta_x\cos\theta_2}{\sqrt{\zeta_y^2+(\sin\theta_2-\zeta_x\cos\theta_2)^2}} \tag{17}$$

or

$$\tan\beta = \frac{\zeta_y}{\sin\theta_2-\zeta_x\cos\theta_2} \quad . \tag{18}$$

In order not to complicate matters, assume narrow bandwidth antennas and hence, take average values for θ_4, θ_5 and θ_2.

Hence,

$$p_4 = C\ D\ \frac{2\tan\beta+A.B.p_o(1-\tan^2\beta)}{A.B.p_o\cdot 2\ \tan\beta-(1-\tan^2\beta)} \tag{19}$$

where

$$\tan\beta = \frac{\zeta y}{\sin\theta_2-\zeta_x\cos\theta_2} \quad . \tag{19a}$$

After combining (19) and (19a),

$$p_4 = C.D. \frac{2\zeta y(\sin\theta 2-\zeta x\cos\theta 2)+A.B.p_o(\sin^2\theta_2+\zeta_x^2\cos^2\theta_2-\zeta_x\sin 2\theta_2-\zeta_y^2}{A.B.p_o\cdot 2\zeta_y(\sin\theta_2-\zeta_x\cos\theta_2)-(\sin^2\theta_2+\zeta_x^2\cos^2\theta_2-2\zeta_x\sin\theta_2\cos\theta_2-\zeta_y^2)} \quad (20)$$

Treat p_4 as a function of ζ_x and ζ_y and assume the probability density functions for these slopes as Gaussian and given by

$$P(\zeta x) = \frac{1}{\sqrt{2\pi\sigma^2_{\zeta_x}}} \exp\left(\frac{-\zeta x^2}{2\sigma_{\zeta_x}{}^2}\right)$$

$$P(\zeta_y) = \frac{1}{\sqrt{2\pi\sigma^2_{\zeta_y}}} \exp\left(\frac{-\zeta_y^2}{2\sigma_{\zeta_y}{}^2}\right) \quad . \quad (21)$$

If now $P(\zeta_x)$ and $P(\zeta_y)$ are independent the average value of p_4 can be calculated as,

$$\langle P_4\rangle_{avg} = \int_{\zeta_x}\int_{\zeta_y} p_4(\zeta_x,\zeta_y)d\zeta_x\cdot d\zeta_y\cdot P(\zeta_x)\cdot P(\zeta_y). \quad (22)$$

CROSS-POLARIZATION MEASUREMENTS FOR THE DETERMINATION OF TARGET SURFACE PROPERTIES

I. Introduction.

A survey of the literature available on the polarization measurements indicate that no substantial results have thus far been obtained. This is in part due to the fact that extensive experimental investigations have not been conducted. The investigations conducted so far were of the following nature.

Two orthogonal polarizations were transmitted either successively or simultaneously and the two reflected components received simultaneously. From the results thus obtained, some conclusions have been presented in regard to general efficiency of transmission, the fading due to rough surfaces and noise figures; in case of the two transmitted polarizations (for example, Gent, etal, 22). Another class of experiments reported present the dependence of radar cross section area of the transmitted polarization. This type of investigation has in most of the cases, been reported with respect to sea echoes, (for example, Long, 38). Some experiments were, of course, conducted to measure depolarization from certain class of terrains (for example, Kessler, etal, 1943), but due to insufficient data, no general conclusions could be drawn (Kerr (ed),36). However, recently (Hagfor's, 25) depolarization measurements have been applied to signals reflected from the Moon.

In order to use cross-polarization measurements for predicting surface properties of rough layers, a series of experiments was performed. Measurements of E_c and E_d (subscripts c, and d, stand for cross and direct sense of

BLOCK DIAGRAM OF THE EXPERIMENTAL SET UP

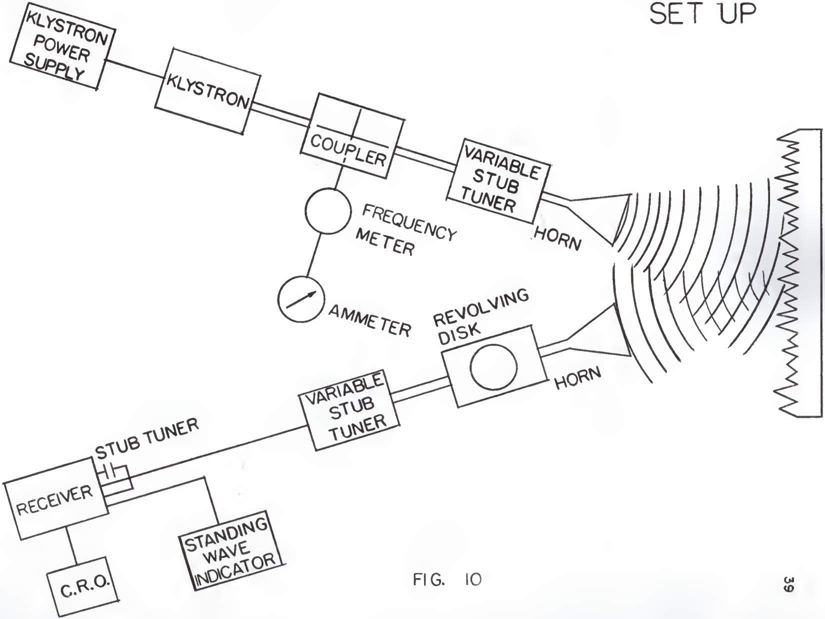


FIG. 10

FIG. 11. PHOTO OF THE EXPERIMENTAL SET UP.

FIG. 12. PHOTO OF TARGET

polarization as referred to the incident wave polarization). The incident wave was a vertically polarized linear wave. Three target models were chosen for investigation. The target models were layers of random thickness with a Gaussian distribution of the surface. Measurements of the cross-polarization factor were obtained for various directions of the incident wave and for various scattering angles and ranges.

The cross-polarization measurements indicate that for a particular type of roughness it is possible to predict the dielectric constant of the surface of reflection for a particular type of material of the layer. The dependence of average cross-polarization factor <D> on the standard deviation of the surface slope distribution is also indicated. This type of investigation is expected to aid in the radar exploration of rough surfaces.

II. Equipment.

The equipment used to obtain the average cross-polarization is shown in Figure 10. Two antennas were used and thus the isolation between the transmitter and receiver was increased. The antennas used are DBG-520*, 20 db gain, 15° beam width horns. The receiver was a R-111/APR-5A microwave superheterodyne receiver. The input to the receiver was tuned by a stub tuner to provide proper match. Both horns were also tuned using DBE 919 variable stub tuners to reduce reflections. A Varian Associates 6312 Klystron was used as the signal source.

In order to reduce signal variations the system stability was thoroughly checked. The possible sources of signal fluctuations are

a. fluctuation in the transmitted signal,

b. variation in transmitter and local ocsillator frequency,

c. variation in gain of receiver and video amplifier,

d. motion of transmitter, receiver, target due to winds,

e. receiption from some other sources.

However, none of these proved to be significant in this case. The source 'd' was almost eliminated by fixing the target rigidly and taking readings during the periods of minimum wind speed.

The amplitude stability was quite good during repeated tests which were performed on the equipment. From experimental observations it was seen that the system losses do not exceed more than 2 db.

Experimental Parameters

Operating frequency	10.02 KMC/S.
Antenna gain	20.0 db.
Polarization transmitted	Vertical
Polarization received	Vertical and horizontal
Pulse repetition frequency	1000.0 C/S.
Standing wave indicator bandwidth	40 C/S.

*DeMornay-Bonardi.

The receiving antenna was mounted in such a way as to receive either horizontal or vertical polarization. The whole assembly could be moved with relative ease to measure the field at any point (Figure 11). The target chosen was circular 4 feet diameter disc which could be revolved through 360 degrees. On the disc were blocks of wood whose angles are specified as a Gaussian distribution (1) having 45^{o} mean (Fig. 18). The apex angle of the pyramid varies from 20^{o} to 70^{o} in 5^{o} steps. The second target was also of wood but the distribution of the slope was changed (Fig.18, (2)and Fig. 12).

The third target was made with styrofoam and was identical to that of wood of distribution as in (1) Fig. 18. The layer thickness was minimum 10 cms. and maximum about 20 cms.

The experiment was performed on the roof of Seaton Hall in order to avoid multiple reflections. A Hewlett-Packard standing wave indicator was used to observe the received electric field strength. The difference between the horizontal and vertical field strengths (in dbs) gave a measure of cross-polarization (in dbs) directly. This method of measurement reduced the noise level considerably. The standing wave indicator is tuned to 1000.0 C/S.

III. Measurement Procedure.

In order to get various slope distributions with respect to the incident wave, the target was rotated in 10 degree steps. Thus, at a particular point in the scattering region thirty-six readings of E_c and E_d were recorded corresponding to the orientations of the target. The transmitter was aligned in such a fashion as to illuminate the target at the center point. The transmitter distance D_T was chosen as 10 feet so that the target was in far field region. The minimum receiver distance D_R was 10 feet. At any particular point say D_T = 10 feet, D_R = 10 feet, $\theta_1 = \theta_2 = 30^{\circ}$, the average cross-polarization factor <D> was calculated from the group of thirty-six readings. The average value of cross-polarization factor was plotted (Figs 13,14); with θ_2 or D_R as a parameter; the length of the line gives the standard deviation of the group.

The measurement procedure was repeated for obtaining the data in the following manner.

(a) The transmitter was kept at a fixed angle $\theta_1 = 30^{\circ}$ and $D_T = 10'$ and the distance D_R was variad for Wood I (with statistics shown in Fig. 18) and resulting <D> versus θ_2. was obtained.

(b) For $D_T = D_R = 10'$; graphs of <D> versus θ_2 were obtained for $\theta_1 = 0^{\circ}$, 30°, and 45° with the target as Wood I, Wood II, and styrofoam.

(c) In order to text consistancy the experiment was repeated for the case where $D_R = D_T = 10'$ and $\theta_1 = 30^{\circ}$. The target chosen for this was Wood I, (Fig.14).

(d) In order to obtain <D> versus distance of the receiver D_R, the experiment was performed with $D_T = 10'$, $\theta_1 = 0^o$ and $\theta_2 = 5^o$, (Fig. 14b).

In all the above cases, the height of the transmitter and receiver were the same and thus, θ_3 was zero in all these cases.

AVERAGE CROSS-POLARIZATION FACTOR FOR WOOD I
$D_T = D_R = 10'$, $\theta_3 = 0$
JUNE 1965.

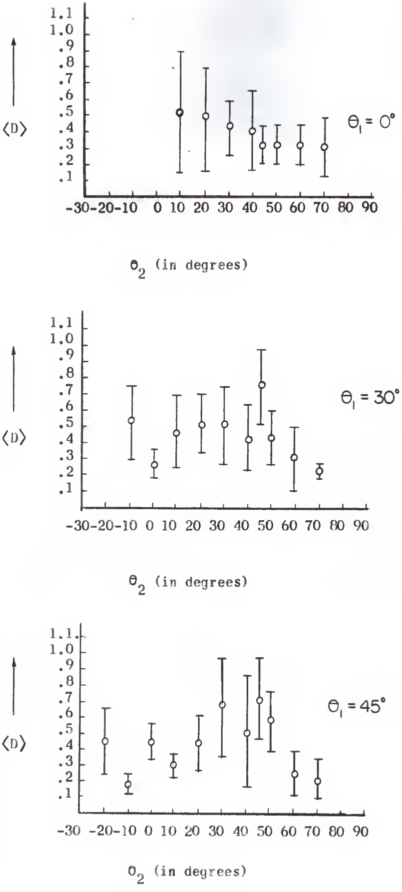


FIG. 13

AVERAGE CROSS-POLARIZATION FACTOR FOR WOOD I
$D_T = 10'$, $\theta_3 = 0°$, $\theta_1 = 30°$
JUNE, 1965

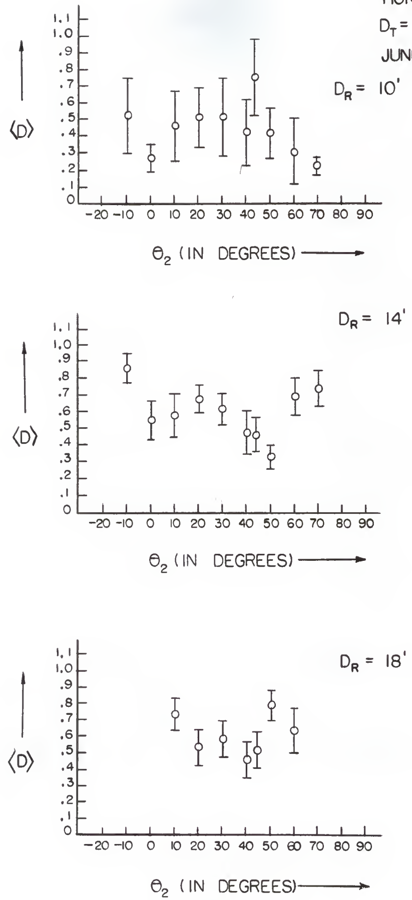
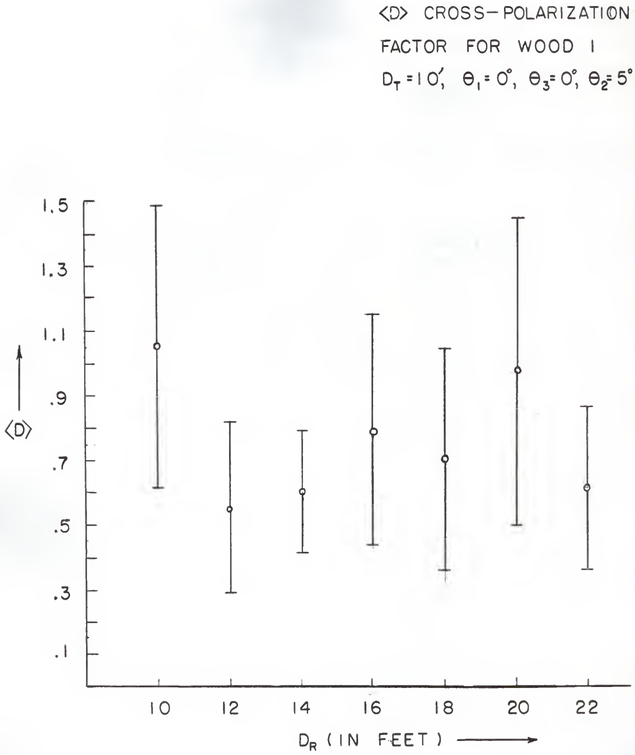


FIG. 14



⟨D⟩ CROSS-POLARIZATION FACTOR FOR WOOD I
$D_T = 10'$, $\theta_1 = 0°$, $\theta_3 = 0°$, $\theta_2 = 5°$
1.5
1.3
1.1
.9
.7
.5
.3
.1
⟨D⟩
10
12
14
16
18
20
22
D_R (IN FEET)

FIG. 14 b.

AVERAGE CROSS-POLARIZATION FACTOR FOR WOOD II
$D_T = D_R = 10'$, $\theta_3 = 0$
AUGUST 1965

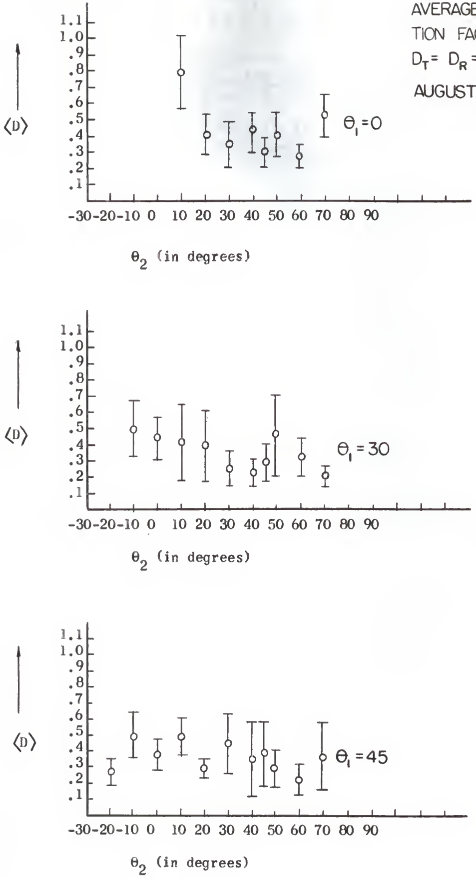


FIG. 15

AVERAGE CROSS-POLARIZATION FACTOR FOR STYROFOAM $D_T = D_R = 10'$, $\theta_3 = 0$ OCTOBER 1965.

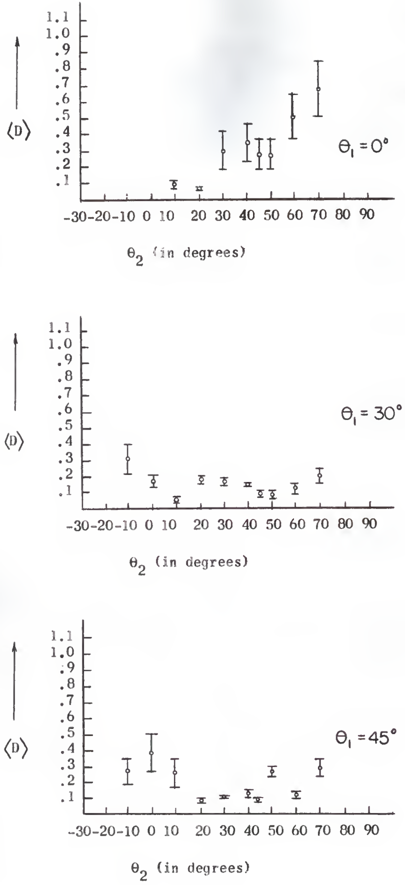


FIG. 16

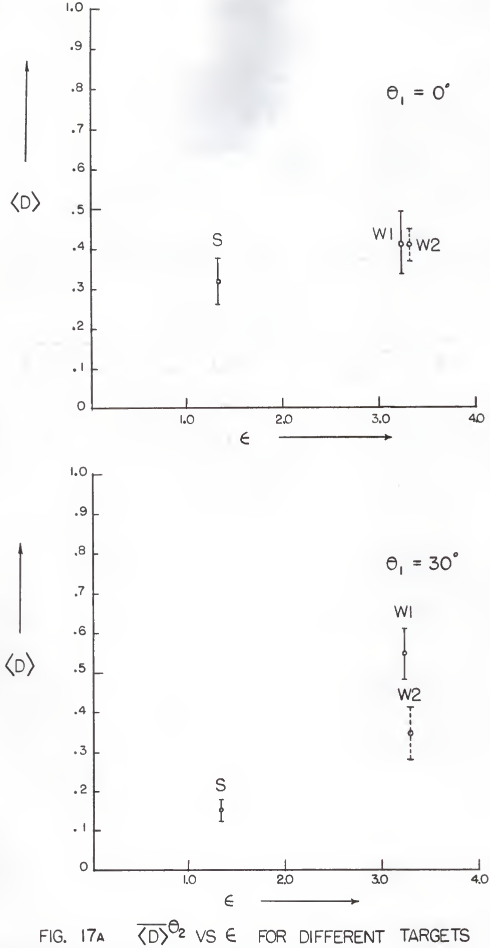


FIG. 17A $\overline{\langle D \rangle}^{\theta_2}$ VS ϵ FOR DIFFERENT TARGETS

$D_T = D_R = 10'$, $\theta_3 = 0°$.

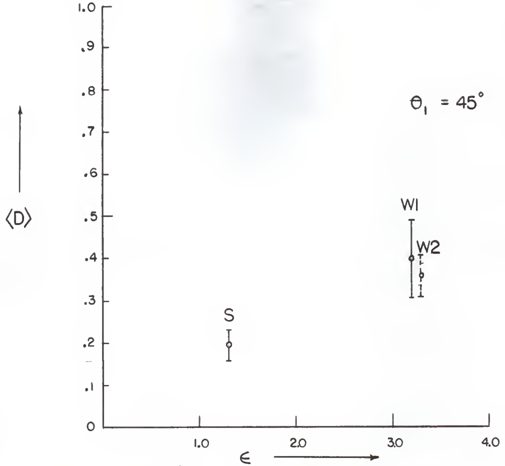


FIG. 17B $\overline{\langle D\rangle}^{\theta_2}$ VS ϵ FOR DIFFERENT TARGETS

$D_T = D_R = 10'$, $\theta_3 = 0°$.

NOTE: FOR STYROFOAM (S) THE <D> WILL BE AT $\epsilon = 1.03$ INSTEAD OF $\epsilon = 1.3$.

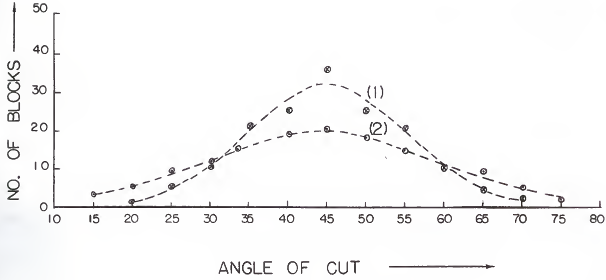


FIG. 18 SLOPE DISTRIBUTION FOR

1. WOOD & STYROFOAM
2. WOOD (CHANGED DISTRIBUTION)

IV. Discussion of the Results of Cross-Polarization Measurements.

In the case of wood I (with statistics shown in Fig.18) the <D> (averaged over different target orientations) vs θ_2 (Fig.13) for $\theta_1 = 0^\circ$ shows that values of <D> are scattered around 0.45. The same graph drawn for $\theta_1 = 30^\circ, 45^\circ$ shows that <D> vs θ_2 fits a parabolic variation. The maximum value of <D> is seen to scatter around $\theta_2 = 45$. The average value of <D> scatters around 0.45 in these cases. The graphs indicate that when θ_1 is increased <D> is increased in a particular direction. For θ_2 more than 75° the power scattered was considerably less and hence measurements could not be conducted.

For the case of Wood II (statistics shown in Fig. 18) <D> vs θ_2 curves (Fig.15) show that the value of <D> is in general decreased compared with Wood I. Wood I and Wood II targets are made of the same wood but with difference in surface statistics. The decrease in <D> in this case is expected because the number of blocks with smaller angle of cut has been increased. From the graphs it is also seen that for the case of Wood II the value of <D> does not increase as much as in the case of Wood I when θ_1 is increased.

The value of <D> does not increase considerably when the range <D> is increased. The value of <D> still remains in the vicinity of 0.5 for $\theta_1 = 30^\circ$, (Fig. 14).

In the case of styrofoam the (Fig.16) statistics was the same as in the case of Wood I but the value of <D> is seen to scatter around 0.18. The variation of <D> with θ_1 is also pronounced.

From the nature of $\langle D\rangle$ vs θ_2 graphs for Wood I and styrofoam it seems reasonable to have a graph of $\overline{\langle D\rangle}^{\theta_2}$ vs ε, the dielectric constant of the target at 3 cm wavelength, $\overline{\langle D\rangle}^{\theta_2}$ denotes here the value of $\langle D\rangle$ averaged over θ_2 (Fig.17a,b). This graph indicates that for the same statistics Wood I ($\varepsilon \doteq 3.2$) has greater $\overline{\langle D\rangle}^{\theta_2}$ compared to styrofoam ($\varepsilon \doteq 1.03$). Based on this result, possibly a distinction can be made between different targets on the basis of $\overline{\langle D\rangle}^{\theta_2}$.

For Wood II, the graphs (Fig.17a,b) are shown slightly shifted only to avoid overlapping of the graphs for Wood I and Wood II. From the graph it is seen that the value of $\overline{\langle D\rangle}^{\theta_2}$, for Wood II is less than for Wood I. This is expected because of the distribution of the surface roughness. This shows that for the same target the surface properties can be distinguished on the basis of $\overline{\langle D\rangle}^{\theta_2}$ measurements.

CONCLUSION.

The depolarized return from a statistically rough surface is dependent on the roughness parameters of the surface. In case of a perfectly conducting rough surface the average cross-polarization factor is believed to contain information about the roughness parameters of the surface. In the case of a layer terminated by a rough conducting interface, which has been considered for theoretical calculation, it is seen that the average cross-polarization factor can be calculated if the layer electric properties and probability density of the slopes of the rough interface are known.

From the theoretical expressions derived for the case of a layer it is difficult to recognize the part played by the different parameters involved in the average cross-polarization factor $\langle D \rangle$. Thus, an experimental approach was envisaged.

For the target model chosen (Wood I) the average cross-polarization factor was about 0.46. This value of $\langle D \rangle$ is justified for the following reasons.

(a) An unrealistic model with mean slopes as 45°. This was so chosen in order that $\langle D \rangle$ would be large and hence, insure ease of measurement.

(b) The use of wood insured a larger degree of depolarization (Kerr, 1964). After the change of statistics the value of $\langle D \rangle$ is reduced, this is in consistency with theoretical calculations because the target chosen

(Wood II) for this case had lesser slopes of 45°. This makes it possible to conclude that <D> can give indication as to the roughness of the layer, keeping other parameters constant.

The target made of styrofoam had identical statistics as Wood I. The <D> for this case was 0.18. Thus, the measurements of <D> indicate that <D> is dependent upon the dielectric properties of the target. <D>, for the same statistics of the surface roughness, is lesser for the target with less ε.

Evans and Pettengill (19) reported the percentage polarization of Moon echoes, this when averaged over the pulse gives approximately 50% of depolarization caused by ths surface of the Moon. Recently, Hagfor's (24) reported a depolarization of about 60% for the Moon echoes. Further, Hagfor's concludes that Moon's surface will have the dielectric constant of about 2.6. These investigations when compared with the experimental observations reported herein show a marked agreement.

When this investigation will be conducted on various target models, it is believed that the results will aid in the better prediction of the dielectric properties of the surface of the Moon, and Mars. This type of investigation seems to be attempted for the first time.

RECOMMENDATIONS FOR FUTURE STUDIES

The problem of depolarization of electromagnetic waves when reflected by rough surfaces has not been attacked seriously thus far. After using the Kirchoff approximations the computed results are quite elaborate. This hinders from forming definite conclusions as to the effect of various parameters on the depolarized return. However, by using a statistical approach it is possible to reach at some compact results. Furthermore, these results can be processed on a high speed digital computer. It might thus be possible to approach the problem theoretically.

Meanwhile, various models can be prepared and experimental investigations carried on. The models selected should be mostly simulations of the targets encountered in actual radar explorations. The materials of the targets may be varied and cross-polarization measurements made with variable frequency signals. The target statistics can be made variable and average cross-polarization factor $\langle D \rangle$ can be recorded for various angles of incidence. A complete set of such experiments when performed with care, most probably, will yield a workable 'Depolarization Criterion' enabling one to determine the surface properties (both statistical and electrical) of the surface of reflection from the knowledge of the distribution of $\langle D \rangle$ in a particular direction.

The present measurements on depolarization are confined to either power measurements of same sense polarized and cross polarized components of the reflected electric field or the ratio of these two components. However, it seems the individual pulse shapes in these two components might also have a close dependence on the electric properties of a layered medium. Thus for the multilayer problem the pulse deformation in these two components of electric field might yield some information about the layers if the data is carefully interpreted. Thus, an intensive experimental approach to the problem of depolarization seems to be worthwhile.

ACKNOWLEDGMENT

The author wishes to express his deep appreciation to Dr. Wellington W. Koepsel, Major Professor and Head of the Department of Electrical Engineering, for his guidance, encouragement, and patience.

The author wishes to thank Dr. Petr Beckmann* for his encouraging letter and fruitful suggestions.

The author also wishes to thank Dr. S. H. Durrani and Mr. D. E. Kaufman of the Department of Electrical Engineering from whom helpful advice has been obtained through stimulating discussions.

*Professor, University of Colorado, Boulder, Colorado.

REFERENCES

1. Ament, W. S.
Toward a theory of reflection by a rough surface. Proc. I.R.E. 41, 142-146 (1953)

2. Ament, W. S.
Reciprocity and scattering by certain rough surfaces. Trans. I.R.E. AP-8, 167-174 (1960).

3. Beckmann, P.
The reflection of electromagnetic waves and synthesis of media. Acta Technica CSAV 2, 311-355 (1957).

4. Beckmann, P.
The scattering of waves from a periodic surface. Czechosl. J. Phys. (1961a).

5. Beckmann, P.
The depolarization of electromagnetic waves scattered from rough surfaces. Acta Techn. CSAV 6, (1961b)

6. Beckmann, P. and Spizzichino, A.
The scattering of electromagnetic waves from rough surfaces. Macmillan & Company, N.Y. 1963.

7. Booker, H. G.
Introduction to techniques for handling elliptically polarized waves with special reference to antennas. Proc. I.R.E., vol. 39, 533-534 (1951).

8. Brekhovskikh, L. M.
Waves in layered media. Academic Press, New York and London (1960).

9. Briggs, B. H.
Roughness of the moon as a radar reflector. Nature 187, 490 (1960).

10. Burrows, C. R. and Atwood, S. S.
Radio wave propagation. Academic Press (1949).

11. Copeland, J. R.
Radar target classification by polarization properties. Proc. I.R.E., Vol. 48, 1290-1296,(1960).

12. Cowan, E. W.
X-Band sea return measurements, R. L. Report No. 870, 19 Jan. (1946).

13. Daniels, F. B.
Radar determination of scattering properties of moon. Nature, Vol. 187, No. 4735, 399 (1960).

14. Daniels, F. B. and Bauer, S. J.
The ionospheric Faraday effect and its applications Jour. Franklin Inst. Vol 267, No. 3, (1959).

15. Davies, H.
The reflection of electromagnetic waves from a rough surface. Proc. I.E.E. Pt. III 101, 209-214, (1954).

16. Deryugin, L. N.
The reflection of a longitudinally polarized plane wave from a surface of rectangular corrugations (in Russian), Radiotekhn. 15, No. 5, 9-16, (1960b).

17. Dyce, R. B.
Investigations of moon echoes SRI upper atmosphere clutter Research Final Report, Part IX, Jan.(1960).

18. Eckart, C.
The scattering of sound from sea surface. J. Acoust, Soc. Am. 25 566-570, (1953).

19. Evans, J. V. and Pettengill, G. H.
The scattering behaviour of the Moon at wave lengths of 3.6, 68, and 784 centimeters. Jour. Geo. Res. Vol. 68, No. 2, Jan 15 (1963).

20. Evans, J. V. and Pettengill, G. H.
The scattering properties of the lunar surface of radio wave lengths. The solar system vol. IV. The University of Chicago Press, 129-161 (1963).

21. Fung, A. K., Moore, R. K. and Parkins, B. E.
Notes on backscattering and depolarization by gently undulating surfaces. Jour. Geo. Res. Vol. 70, No. 6 1559-1561, Mar. 15.(1965).

22. Gent, H. (et al)
Polarization of radar echoes, including aircraft, precipitation and terrain. Proc. I.E.E. Vol. 110, 2139-2148 (1963).

23. Graves, C. D.
Radar polarization power scattering matrix. Proc. I.R.E. Vol. 44. 248-252, (1956).

24. Hagfors, T. (et al)
Evidence of a tenuous surface layer on the Moon as derived from radar observations, presented to 'science', M.I.T. July, (1965).

25. Hagfors, T.
Comments on letter by A. K. Fung, R. K. Moore, and B. E. Parkins, 'Notes on Backscattering and Depolarization by Gently Undulating Surfaces' Jour. Geo. Res. Vol. 70, No. 6 1563. Mar. 15, 1965.

26. Hayre, H. S.
Depolarization and surface roughness. U. S. National Committee, Fall (1963).

27. Hunter, I. M.
Polarization of radar echoes from meteorological precipitation. Nature, 173, 195, (1954).

28. Isakovich, M. A.
The scattering of waves from a statistically rough surface (in Russian) Zhurn. Eksp. Teor. Fiz. 23, 305-314 (1952).

29. Jones, D. S.
The theory of electromagnetism. The Macmillan Company New York (1965).

30. Katz, I. and Spetner, L. M.
Polarization and depression angle dependence of radar terrain return. J.N.B.S. Vol. 64D No. 5, 483-486(1960).

31. Katz, I.
Radar reflectivity of the earth's surface. A.P.L. Tech. Digest 11-17, Feb. (1963).

32. Katzin, M.
On the mechanism of sea clutter. Proc. I.R.E. 45, 44-54 (1957).

33. Katzin, M. (et al)
E.M.R. Corp. Rept. CRC-5198-4, Mar (1960).

34. Kaufman, D. E. and Hayre, H. S.
Electromagnetic wave scattering from very rough surfaces composed of correlated large and small scale roughness. Proc. IEEE. 1157-58-August. (1965).

35. Kennaugh, E. M.
Polarization properties of radar reflections. Ant. Lab. The Ohio State University Rept. No. 389-12 (1952).

36. Kerr, D. E. (Ed)
Propagation of short waves, McGraw-Hill (1964).

37. Krishen, K., Koepsel, W. W. and Durrani, S. H.
Radar cross-polarization measurements for the determination of target surface properties (communicated to IEEE International Conference 1966, March).

38. Long, M. W.
On the polarization and the wavelength dependence of sea echo Trans. IEEE, AP Vol. AP-13 No. 5, 749-754 (1965).

39. Marsaglia, G.
Ratios of normal variables and ratios of sums of uniform variables. Journal Am. Statistical Assoc. Vol. 60, No. 309, 193-204 (1965).

40. Peake, W. H.
I.R.E. Nat. Conv. Rec. Pt. I, 38-41, Mar.(1959).

41. Stratton, J. A.
Electromagnetic theory. McGraw-Hill Company, N.Y. and London (1941).

42. Taylor, R. C.
Terrain return measurements at X, K_u , and K_2 band. Presented at Nat. IRE Meeting in New York (1959), Ant. Lab. The Ohio State University.

43. Twersky, V.
Multiple scattering of radiation by an arbitrary planar configuration of parallel cylinders and by two parallel cylinders. J. Appl. Phys. 23, 407-414 (1952).

44. Twersky, V.
On the scattering and reflection of electromagnetic waves by rough surfaces. Trans. I.R.E. AP-5 81-90 (1957a).

45. Twersky, V.
On scattering and reflection of sound by rough surfaces. Jour. Acoust. Sco. Am. 29, 209-225 (1957b).

46. Westfold, K. C.
New analysis of polarization of radiation and faraday effect in terms of complex vectors. Jour. Opt. Soc. Am. Jv. 49, No. 7, 717-723 (1959).

POLARIZATION DEPENDENT RADAR RETURN
FROM
ROUGH SURFACES

By

KUMAR KRISHEN

B.A.,JAMMU & KASHMIR UNIVERSITY, 1959.
B.TECH., UNIVERSITY OF CALCUTTA, 1962.
M.TECH., UNIVERSITY OF CALCUTTA, 1963.

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

ABSTRACT

For the radar exploration of the surface properties of a target, such as the Moon, cross-polarization technique is developed. Assuming a Gaussian distribution for the surface slopes, an expression is derived in case of a layer with random thickness which shows that the depolarization factor depends on the dielectric properties of the medium, the slope structure of the rough interface, the wave length and the angle of incidence. Due to the complexity of this expression emphasis was placed on the experimental observations.

The experimental investigations were conducted on a target model with Gaussian surface structure. The following results were obtained:

(a) graphs of $\langle D \rangle$ versus θ_2 for targets with identical statistics but different dielectric constants

(b) graphs of $\langle D \rangle$ versus ε, with layer statistics as a parameter; these were drawn after averaging $\langle D \rangle$ over θ_2

(c) the dependence of $\langle D \rangle$ on θ_1 and range

The experimental results indicate that for a particular type of the target statistics cross-polarization factor can be used to identify the dielectric constant of the surface of reflection.