251

EFFECT OF NEUTRON IRRADIATION ON
TRANSISTOR CURRENT GAIN

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

A	Cross sectional area of conduction band
A_s	Effective surface recombination
C_{abs}	Absolute disintegration rate
C_i	Number of majority carriers in i region
C_T	Total count
D_E	Diffusion constant of majority carriers in the emitter region
D_n	Diffusion constant of electrons
D_p	Diffusion constant of holes
D_{Em}	Diffusion constant of minority carriers in the emitter region
D_{im}	Diffusion constant of minority carriers in the i region
E	Electric field strength
e	Electronic charge
E_c	Ionization energy
E_i	Energy of the incident particle
E_p	Energy transferred to the primary knock-on
$f(Z)$	Fall-off factor
f_T	Correction factor for the proportional counter
$f_{c\alpha}$	Transistor cutoff frequency
$g(Z)$	Field factor
$h(Z)$	Correction factor due to the electric field in the base region
I_B	Base current
I_{bR}	Current due to bulk recombination
I_C	Collector current
I_{co}	Leakage current

I_E	Emitter current
I_{eE}	Current due to injection of electrons from base to emitter region
I_{pE}	Current due to injection of holes into base region
I_R	Recombination current
I_{sR}	Surface recombination current
J_E	Total emitter junction current density
J_C	Total collector junction current density
J_n	Electron current density
J_{nC}	Electron current density from collector to base region
J_{nE}	Electron current density from emitter to base region
J_p	Hole current density
J_{pE}	Hole current density injected from collector into base region
J_{pC}	Hole current density injected from emitter into base region
K	Boltzmann's constant
K_J	Lifetime damage constant
L_B	Diffusion length of minority carriers in the base region
L_i	Diffusion length of minority carriers in the i region
L_n	Diffusion length of electrons
L_{nC}	Diffusion length of electrons in the collector region
L_{nE}	Diffusion length of electrons in the emitter region
L_p	Diffusion length of holes
M_1	Mass of incident particle
M_2	Mass of knock-on particle
M_e	Mass of electrons
M_i	Mass of impurity particle

N	Number of radioactive atoms
n	Electron density in N region
N_0	Avogadro's number
n_0	Original density of electrons in N region
N_T	Total number of atoms in foil
p	Hole density in P region
p_0	Original hole density in P region
p_E	Density of holes at the emitter junction
p_e	Number of holes leaving the region
p_r	Number of holes recombined
p_s	Density of holes present near the surface
r	Activity of the foil
s	Surface recombination velocity
t	Time of irradiation
t_2	Time counting started
t_3-t_2	Counting time
t'	Cooling time
V_E	Emitter voltage
V_C	Collector voltage
v	Velocity
W	Transistor base width
w	Weight of foil
α_J	Transistor damage constant
α_{cb}	Common base current gain
β	Common emitter current gain

β_a	Common emitter current gain after irradiation
β_b	Common emitter current gain before irradiation
γ	Fraction of current at emitter junction produced by emitter voltage and carried by minority carriers in base
ξ	Base transport factor
ξ_1	Empirical constant
λ	Decay constant of foil
μ_{BC}	Mobility of majority carriers in base region
μ_{iC}	Mobility of majority carriers in i region
μ_n	Electron mobility
μ_p	Hole mobility
σ_B	Base conductivity
σ_E	Emitter conductivity
σ_i	Conductivity of i region
τ	Minority carriers lifetime
τ_a	Minority carriers lifetime after irradiation
τ_b	Minority carriers lifetime before irradiation
ϕ	Integrated neutron flux

1.0 INTRODUCTION

1.1 General Discussion

One of the environments in which transistors must function properly is the nuclear radiation environment. Transistors used for nuclear reactor instrumentation and other electronic circuits around a reactor must operate reliably in a moderate flux of neutrons and gamma-rays for an extended period of time. Transistors which can operate reliably for long periods in Van Allen radiation are important to future communication satellites. Military equipment, using electronic components such as missiles, must operate properly during and after irradiation from a nuclear blast. Therefore it is important both to understand and be able to predict the extent of radiation damage to transistors.

In studying radiation effects on transistors, a parameter which is critical to the circuit performance and sensitive to radiation effects should be selected for study. For analysis and prediction of the radiation dependence of this parameter in a different radiation environment, the selected parameter should be explicitly related to the internal parameters of the device. For transistors, there are two related parameters which meet the above requirements, the common emitter forward current gain, β, and the common base forward current gain, α_{cb}. Evaluation of the change of either one of these parameters in a radiation field will provide information necessary for the design of radiation resistant transistors. A thorough study of the physical theory of semiconductors permits the development of relations for α_{cb} and β. Knowing the physical effect of radiation on semiconductor materials will then allow one to predict the change in α_{cb} and β in a radiation field.

1.2 Properties of Semiconductors

Semiconductors, the basic materials used in transistor fabrication, are often defined as electrical conductors with a conductivity between that of an insulator and of a metal. This definition, of course, does not completely describe a semiconductor.

Silicon and germanium, two typical semiconducting materials, each have four electrons in their outermost electron shells and have a tendency to form crystals in the pure state. The distribution of the four valence electrons in the germanium or silicon atom is such that one electron is shared with each of four neighboring atoms in the crystal. This implies that all the valence electrons are in a covalent bound state. In this case (only holds strictly at 0° K) the crystal has insulating properties, since there is no free electron for conduction. However, an electric field, a beam of light, thermal energy, or an energetic particle can supply enough energy to break the weak covalent bonds and liberate free electrons to serve as charge carriers. Thus, depending on the energy state of the crystal, either silicon or germanium may be an insulator or a conductor.

In semiconductors conduction is by means of free electrons and holes, which can be originated by either introducing energy into the crystal as noted above or by introducing donor and acceptor impurities into the crystal. At ordinary temperatures the crystal lattice is in continuous random agitation because of thermal energy. As a result, an individual electron of a covalent bond occasionally acquires enough energy at room temperature to break the bond and become a free electron. In the absence of an applied field, the free electron moves about the crystal in a random way. When an external electric field is applied, there is superimposed upon this random motion a steady drift toward the positive electrode that represents a flow of current carried by electrons. The empty place left in the crystal structure when an electron breaks away from the covalent bond is termed a hole. Once a hole is created, it moves about in the crystal in a random way, in the same manner as an electron only with a positive charge. Therefore, in the presence of an electric field, there is superimposed upon the random thermal motion a steady drift of the holes toward the negative electrode. This drift represents a current flow transported in the absence of, and is in addition to, the current carried by the electrons.

One of the distinctive characteristics of semiconductors is the extent to which their electrical properties depend upon impurity content and also the type and degree of the binding forces which exist between the atoms (8). A very small amount of certain types of impurities will tremendously alter the concentration of electron and hole current carriers, e.g., the introduction of a small number of phosphorus atoms into a germanium crystal. Since each phosphorus atom has five electrons in its outer orbit, one electron will be left from each atom of the impurity; that is, one electron does not enter into the covalent bonding. When freed from its parent impurity atom, this fifth valence electron moves at random through the crystal in the same manner as the free electrons present in an intrinsic semiconductor. When an electric field is applied, there is superimposed upon this random motion a steady drift toward the positive electrode.

The addition of an impurity with three electrons in its outer orbit, e.g., boron, has a different effect upon the lattice. In this case there are not enough electrons to satisfy all the covalent bonds; therefore, between each atom of the impurity and the surrounding atoms, there will be one electron void, a hole. This hole moves about in a random way due to thermal effects and when an electric field is applied, it tends to drift toward the negative electrode.

The study of imperfections in single crystal semiconductors shows that no such thing as a truly pure crystal exists. It is possible to list six general types of imperfections (35): holes and electrons, phonons, excitons, foreign atoms, lattice defects, and dislocations. Electrons and holes are considered to be imperfections of the crystal lattice regardless of how they are produced. A phonon, a quantized unit of an elastic wave, is used to describe the energy in a particular type of lattice vibration. Since the elastic waves are excited by thermal energy, the number of phonons increases with the temperature of the crystal (4). Phonons are effective in scattering electrons or holes which are moving through a crystal and must be considered in calculating the carrier mobilities. Excitons, mobile carriers like electrons and holes, are electrically neutral and do not contribute to electrical conduction; they are not important in the silicon and germanium semiconductors (35).

Any foreign atom, whether introduced purposely or not, constitutes an imperfection in the crystal. A lattice defect is created whenever the periodicity and order of the crystal lattice are disturbed by a misplaced atom. In general, there are two kinds of lattice defects: excess impurity atoms which squeeze into a lattice between the normal atoms, i.e., interstitial impurities, and vacancies which are simply the removal of atoms from their places in the lattice. It is customary to describe these lattice vacancies and interstitial atoms as Schottky and Frenkel defects (33). A Schottky defect is equivalent to a simple vacancy in the crystal lattice. The generation of Schottky defects proceeds by the migration of an ion to the surface of the crystal, leaving a vacancy behind. A Frenkel defect is produced when the atom is removed at one point leaving a vacancy, and appears at another point in the crystal as an interstitial atom.

The presence of lattice defects and dislocations change the electrical properties of a semiconductor crystal. These properties generally provide the most sensitive way of determining the presence of lattice defects, dislocations, and foreign atoms. Defects and dislocations affect the velocity at which charged carriers move through a semiconductor and consequently the electrical properties of diodes and transistors made from such semiconductor materials.

Impurity atoms that contribute holes are termed acceptors; they accept electrons from the germanium atoms. Those impurity atoms that contribute electrons are termed donors. By way of further definition P-type materials are semiconductors in which electrical conduction is primarily due to hole movement and N-type materials are those in which conduction is primarily due to electron movement. Electrons in N-type are called the majority carriers. Electrons in P-type material and holes in N-type material are known as the minority carriers.

1.3 Conduction in a Semiconductor

At absolute zero a semiconductor may contain a certain concentration of occupied electronic energy levels which lie in the normally "forbidden region" between the valence and conduction bands. Figure 1 shows the positions of the conduction band (E_c and above) and the valence band (E_v and below). These electrons are localized in the vicinity of the impurities and therefore do not contribute to the conductivity unless they are excited into the conduction band (above E_c) (5). Centers of this kind are called donor levels. In the energy level scheme, Figure 1, they are represented by a short bar to indicate that they are localized. An impurity semiconductor may also contain a certain density of holes which at absolute zero are trapped in levels lying in the forbidden gap, Figure 2. Such levels are called acceptor levels because they may become occupied by electrons excited from the filled band (below E_v). These excited electrons leave holes in the valence band, thus conduction becomes possible. (See Figure 2.)

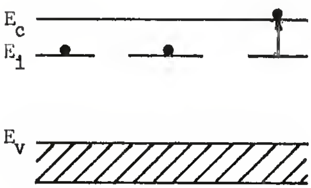

Figure 1. Donor levels

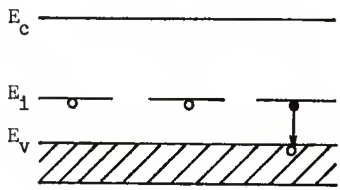

Figure 2. Acceptor levels

1.4 Radiation Damage to Semiconductors

The permanent changes in semiconductors produced by radiation result from the displacement of atoms. These displacements arise from the direct interaction of incident radiating particles with the semiconductor nuclei. The physical properties of semiconductors are extremely sensitive to small amounts of disorder introduced by such energetic radiating particles.

An energetic particle can lose kinetic energy by two major processes: electrostatic interaction and direct collision with the atoms of the semiconductor. The electrostatic interaction is the chief means of energy loss for charged particles; this lost energy causes electronic excitation and ionization of the traversed matter. If the energy of the incident particle is greater than the ionization energy E_c, then most of the particle's energy will be lost by this process. The ionization energy depends on the bond structure of the solid in question and is estimated by Seitz (30) for semiconductors as

$$E_c = \frac{M_1}{8} \frac{\Delta E}{M_e} \qquad (1)$$

where: M_1 is the mass of the impurity

M_e is the mass of the electron and

ΔE is the optical width of the forbidden gap.

In the direct collision process, an incident radiating particle may lose energy through elastic atomic or nuclear collisions or coulombic interaction. Any one of these processes may transfer sufficient energy to the struck semiconductor atom to displace it from its normal lattice site. The struck atom may in turn have sufficient energy to displace other atoms so that an avalanche of displacements may result from a primary collision. To estimate the amount of energy lost through a direct primary collision the following equation has been derived (see Appendix A)

$$E_p(\theta) = \frac{4\ M_1\ M_2}{(M_1 + M_2)^2} E_1 \sin^2(\theta/2). \tag{2}$$

Knowing E_p it should be possible in principle to calculate the number of atomic displacements.

Considerable theoretical effort has been applied to calculating the number of atomic displacements introduced by a given type of irradiation and in comparing these results with the observed changes in the physical properties of the irradiated materials (14). A major difficulty arises due to the uncertainty of the number of defects and the changes in the observed properties. The problem of the number of displacements introduced in a lattice must be broken down into two processes: First, the collision of the bombarding particle with an atom of the lattice produces a primary knock-on, which in turn may possess enough energy to create further displacements. Secondly, the primary knock-on creates secondary displacements. Calculating the number of these displacements can become quite involved and there is little agreement among investigators regarding the manner of calculation and calculated results.

The effects of irradiation on the electrical properties of semiconductors are usually interpreted in terms of the introduction of electronic states or energy levels lying within the forbidden energy gap (1). These levels are associated with defects in the crystal lattice which may tend to trap either holes or electrons. These trapping sites have profound effects on free electron and hole concentration and consequently on the electrical properties of the semiconductors. Figure 3 shows energy levels postulated for germanium bombarded with electrons, neutrons, deuterons, and gamma rays (17). The effects of gamma (3), and neutron bombardment (4) on N-type germanium are similar, the curve showing a decrease in electrical conductivity which after going through a minimum then increases with further bombardment. This indicates that the N-type material is converted to P-type.

Irradiation of P-type semiconductors reduces both the hole concentration and the charge carrier mobility, which in turn reduces the conductivity (13). In addition to changing majority carrier properties, nuclear irradiation also affects minority carrier properties. The lattice disorders act as a recombination center for minority carriers and hence reduces the number of charge carriers in the semiconductor, and also reduces the conductivity of the irradiated semiconductor.

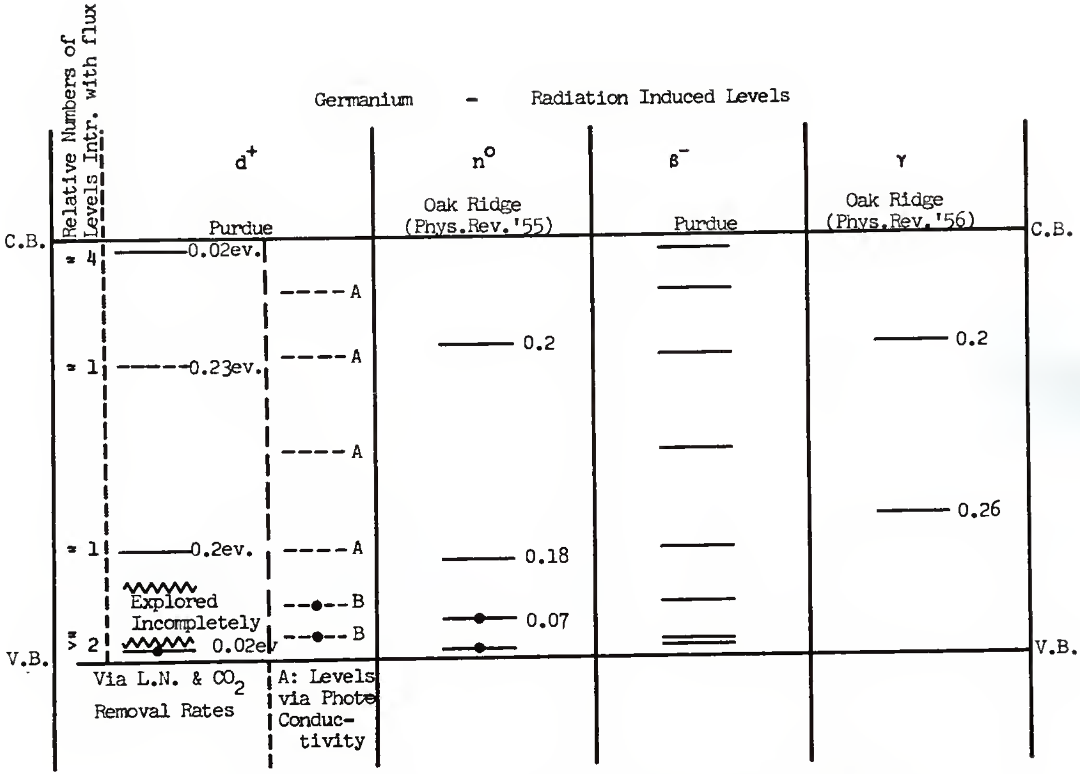

Figure 3. Energy-level scheme for deuteron, neutron, electron, and gamma radiation induced levels in germanium (2)

2.0 THEORETICAL DEVELOPMENT

2.1 Equations for Current Flow in Semiconductors

The fundamental equations in the analysis of transistor behavior are those which describe the motion of carriers in semiconductors under the combined influence of external fields and deviations from the thermal equilibrium densities of carriers. To find the total current in a semiconductor it is necessary to add the contributions from the drift current to the diffusion current of electrons and holes. The total current density, J_n, due to the electrons in a semiconductor is (37)

$$J_n = e\, n\, \mu_n\, E + e\, D_n \nabla n. \tag{3}$$

Similarly, the total current density due to holes, J_p,

$$J_p = e\, p\, \mu_p\, E - e\, D_p \nabla p \tag{4}$$

where: e is electronic charge

n is electron density

p is hole density

μ_n is the electron mobility

μ_p is the hole mobility

D_n is the diffusion constant for electrons

D_p is the diffusion constant for holes and

E is the electric field strength.

At low voltages, diffusion is the main cause of carrier flow and therefore the current due to the electric field, the drift current, is negligible in comparison to the diffusion current (23). Hence, for a transistor where the applied voltage and electric field produced is small, the hole and electron current densities take the following form

$$J_n = e\, D_n\, \nabla n \tag{5}$$

$$J_p = -\, e\, D_p\, \nabla p. \tag{6}$$

In the case of a PN junction (a sandwich of a P and an N material), when holes are injected from the P to the N region, the hole density in the N region rises from p_0 to p and as equilibrium is approached, the hole density decays back to p_0 due to recombination with electrons. This leaves the N region as a hole current. The rate of recombination of holes is proportional to the number of excess holes $(p - p_0)$ present at any given time (5).

$$\frac{\partial p_r}{\partial t} = -\, C\, (p - p_0) \tag{7}$$

where $\frac{\partial p_r}{\partial t}$ is the rate of change of holes in N-region due to recombination. The minority carrier lifetime in N region, τ_p, is defined so that

$$\frac{1}{C} \equiv \tau_p;$$

therefore,

$$\frac{\partial p_r}{\partial t} = -\, \frac{1}{\tau_p}\, (p - p_0). \tag{8}$$

The rate of decrease of hole density in the N-region due to holes leaving this region is

$$\frac{\partial p_e}{\partial t} = -\, \frac{\partial p_e}{\partial x} \cdot \frac{\partial x}{\partial t} = -\, \frac{\partial p_r}{\partial x} \cdot v. \tag{9}$$

The current density due to holes is given by the following equation:

$$J_p = p\ e\ v \tag{10}$$

where v is the hole speed.

The derivative of equation (10) with respect to x gives

$$\frac{\partial J_p}{\partial x} = \frac{\partial p}{\partial x} e\ v. \tag{11}$$

Combining equations (9) and (11) gives

$$\frac{\partial p_e}{\partial_t} = -\frac{1}{e}\frac{\partial J_p}{\partial x}. \tag{12}$$

Then adding equations (8) and (12) find the total rate of hole decrease at any part of the N region

$$\frac{\partial p}{\partial t} = -\frac{(p - p_0)}{\tau_p} - \frac{1}{e}\frac{\partial J_p}{\partial x}. \tag{13}$$

Thus equation (13) is the relation for hole density as a function of time and distance.

The relation between diffusion length and minority carrier lifetime is

$$L_p = (D_p\ \tau_p)^{1/2}. \tag{14}$$

Substituting equations (6) and (14) into equation (13) results in the diffusion equation for holes:

$$\frac{\partial p}{\partial t} = \frac{\partial^2 p}{\partial x^2} - \frac{p - p_0}{L^2_p}. \tag{15}$$

Similarly, the diffusion equation for electrons is

$$\frac{\partial n}{\partial t} = \frac{\partial^2 n}{\partial x^2} - \frac{n - n_o}{L^2_n} \tag{16}$$

where: n is the density of electrons in N-region

n_o is the original density of electrons in N-region and

L_n is the diffusion length of electrons.

In deriving equations (15) and (16), it is assumed that diffusion takes place only in the x-direction. For the general case of three dimensional diffusion, the applicable diffusion equations are the well known continuity equations

$$\frac{\partial p}{\partial t} = \nabla^2 p - \frac{p - p_0}{L^2_p} \tag{17}$$

$$\frac{\partial n}{\partial t} = \nabla^2 n - \frac{n - n_o}{L^2_n} . \tag{18}$$

For the steady state conditions $\frac{\partial p}{\partial t} = \frac{\partial n}{\partial t} = 0$ and therefore,

$$\nabla^2 p - \frac{p - p_0}{L^2_p} = 0 \tag{19}$$

$$\nabla^2 n - \frac{n - n_o}{L^2_n} = 0. \tag{20}$$

2.2 Equations for Common Base Current Gain of A Transistor

A transistor with a thin base region and small V_E, emitter voltage, and V_C, collector voltage, can be treated by extension of the PN junction theory developed above. In the case of a PNP transistor shown in Figure 4, the hole diffusion equation applicable to the base region is equation (19). Assuming that diffusion takes place only in one dimension, the general solution for equation (19) is

$$p - p_0 = A\, e^{-x/L_p} + B\, e^{x/L_p} \tag{21}$$

with the following boundary conditions

$$p = p_0\, e^{e V_E/K T} \quad \text{at } x = 0 \tag{22}$$

and

$$p = p_0\, e^{e V_C/K T} \quad \text{at } x = W \tag{23}$$

where: K is Boltzmann's constant

T is temperature and

W is the base width of transistor.

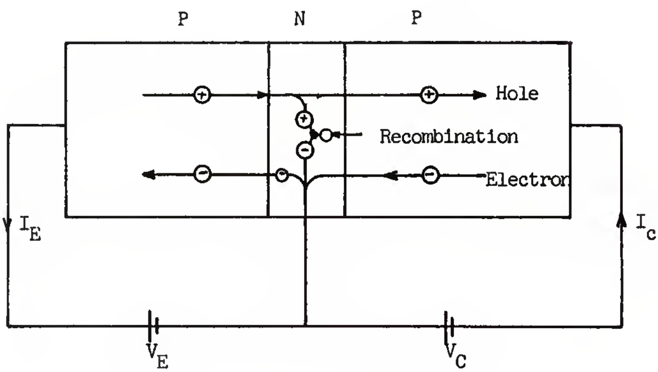

Figure 4. Various flow paths of electrons and holes in PNP transistor

Applying the boundary conditions to equation (21), the following relations are obtained:

$$p_0 \left(e^{e V_E/K T} - 1\right) = A + B \tag{24}$$

$$p_0 \left(e^{e V_C/K T} - 1\right) = A\, e^{-W/L_p} + B\, e^{W/L_p}. \tag{25}$$

Then, solving for A and B,

$$A = \frac{p_0 \left(e^{e V_E/K T} - 1\right) e^{W/L_p} - p_0 \left(e^{e V_C/K T} - 1\right)}{2 \sinh W/L_p} \tag{26}$$

and

$$B = \frac{p_0 \left(e^{e V_C/K T} - 1\right) - p_0 \left(e^{e V_E/ K T} - 1\right) e^{-W/L_p}}{2 \sinh W/L_p}. \tag{27}$$

Substituting for A and B in equation (21) gives

$$p - p_0 = \left[\frac{p_0 \left(e^{e V_E/ K T} - 1\right) e^{W/L_p} - p_0 \left(e^{e V_C/K T} - 1\right)}{2 \sinh W/L_p}\right] e^{-x/L_p}$$
$$+ \left[\frac{p_0 \left(e^{e V_C/K T} - 1\right) - p_0 \left(e^{e V_E/K T} - 1\right) e^{-W/L_p}}{2 \sinh W/L_p}\right] e^{x/L_p}. \tag{28}$$

This is the hole diffusion equation for the base region. The derivatives of this relationship at different values of x are used to describe the hole current density injected into the base region and the hole current density going from the base to the collector region.

The hole current density injected into the base region from the emitter region, J_{pE}, is

$$J_{pE} = - e\, D_p \frac{dp}{dx} \bigg|_{x = 0}. \tag{29}$$

Using $\frac{dp}{dx} \Big|_{x = 0}$ from equation (28) gives

$$J_{pE} = \frac{e\, D_p\, p_0}{L_p} \left[- (e^{e V_C/K T} - 1) \operatorname{csch} W/L_p + (e^{e V_E/K T} - 1) \coth W/L_p \right] . \tag{30}$$

The hole current density going from the base region to the collector region, J_{pC}, is

$$J_{pC} = - e\, D_p \frac{dp}{dx} \bigg|_{x = W}. \tag{31}$$

Substituting the derivative of p with respect to x from equation (28) gives

$$J_{pC} = \frac{e\, D_p\, p_0}{L_p} \left[(e^{e V_E/K T} - 1) \operatorname{csch} W/L_p - (e^{e V_C/K T} - 1) \coth W/L_p \right] . \tag{32}$$

Equations (30) and (32) give the current density through both the emitter and collector junctions due to holes. To calculate the total current density through these junctions, it is also necessary to calculate the contribution from electrons crossing the emitter and collector junction.

The electron current density in the emitter region corresponding to equation (20) is

$$n - n_o = D_n\, e^{x/L_{nE}}, \tag{33}$$

with the following boundary condition

$$n = n_o\, e^{e V_E/K T} \quad \text{at } x = 0, \tag{34}$$

where L_{nE} is the diffusion length of electrons in emitter.

The electron current density from emitter to base, J_{nE}, is

$$J_{nE} = - e D_n \left. \frac{dn}{dx} \right|_{x = 0} \cdot \tag{35}$$

Combining equations (33), (34) and (35) gives

$$J_{nE} = - \frac{e D_n n_o}{L_{nE}} (e^{e V_E/K T} - 1). \tag{36}$$

For the electron current density through collector, the solution corresponding to equation (20) is

$$n - n_o = D e^{x/L_{nC}}, \tag{37}$$

with the following boundary condition

$$n = n_o e^{e V_C/K T} \text{ at } x = W, \tag{38}$$

where L_{nC} is the diffusion length of electrons in the collector. The electron current density from collector to base, J_{nC}, is

$$J_{nC} = - e D_n \left. \frac{dn}{dx} \right|_{x = W} \cdot \tag{39}$$

Combining equations (37), (38) and (39) gives

$$J_{nC} = - \frac{e D_e n_o}{L_{nC}} (e^{e V_C/K T} - 1). \tag{40}$$

The electron and hole contributions to the junction's current density can be combined to find the total current density at the junction.

The total current density through the emitter and collector junctions are J_E and J_C respectively.

$$J_E = J_{nE} + J_{pE} \tag{41}$$

$$J_C = J_{nC} + J_{pC}, \tag{42}$$

or substituting equations (30) and (36) into equation (41) and substituting equations (32) and (40) into equation (42) gives

$$J_E = -\frac{e\,D_p\,p_0}{L_p}\,\mathrm{csch}\,W/L_p\,(e^{e\,V_C/K\,T} - 1)$$

$$+ \left[\frac{e\,D_n\,n_o}{L_{nE}} + \frac{e\,D_p\,p_0}{L_p}\,\coth W/L_p\right](e^{e\,V_E/K\,T} - 1) \tag{43}$$

and

$$J_C = \frac{e\,D_p\,p_0}{L_p}\,\mathrm{csch}\,W/L_p\,(e^{e\,V_E/K\,T} - 1)$$

$$- \left[\frac{e\,D_p\,p_0}{L_p}\,\coth W/L_p + \frac{e\,D_e\,n_o}{L_{nC}}\right](e^{e\,V_C/K\,T} - 1). \tag{44}$$

The derivative of these two equations will be used to develop relations for the transistor gain.

The current amplification factor, α_{cb}, is the variation of collector current, I_C, in response to a change in emitter current, I_E, with the collector voltage, V_C, held constant. Assuming that the collector and emitter junction areas are equal

$$\alpha_{cb} = \frac{\partial I_C}{\partial I_E}\bigg|\,V_C = \text{const} = \frac{\partial J_C}{\partial J_E}\bigg|\,V_C = \text{const} \tag{45}$$

or

$$\alpha_{cb} = \frac{\left.\dfrac{\partial J_C}{\partial V_E}\right| V_C = \text{const}}{\left.\dfrac{\partial J_E}{\partial V_E}\right| V_C = \text{const}}. \tag{46}$$

Substituting equations (43) and (44) into (46) gives

$$\alpha_{cb} = \frac{K\,T/e \cdot \dfrac{e\,D_p\,p_0}{L_p} \cdot \operatorname{csch} W/L_p \cdot e^{e\,V_E/K\,T}}{K\,T/e \left[\dfrac{e\,D_n\,n_o}{L_{nE}} + \dfrac{e\,D_p\,p_0}{L_p} \coth W/L_p\right] e^{e\,V_E/K\,T}},$$

or more simply,

$$\alpha_{cb} = \frac{\operatorname{sech} W/L_p}{\dfrac{D_n}{D_p} \cdot \dfrac{n_o}{p_0} \cdot \dfrac{L_p}{L_{nE}} \cdot \tanh W/L_p + 1}. \tag{47}$$

Since W/L_p is very small in most transistors, a first order approximation may be made for the hyperbolic functions such that (37)

$$\operatorname{sech} W/L_p \approx 1 - \frac{W^2}{2L^2_p} \tag{48}$$

and

$$\tanh W/L_p \approx W/L_p. \tag{49}$$

Substituting equations (48) and (49) into equation (47) gives

$$\alpha_{cb} = \frac{1 - W^2/2L^2_p}{1 + \dfrac{D_n}{D_p} \cdot \dfrac{n_o}{p_0} \cdot \dfrac{W}{L_{nE}}}. \tag{50}$$

From the definition of conductivity,

$$\sigma = e\,n\,\mu, \tag{51}$$

and assuming that (5)

$$\frac{D_p}{D_n} \approx \frac{\mu_p}{\mu_n} , \tag{52}$$

equation (50) can be rewritten in the following form:

$$\alpha_{cb} = \frac{1 - W^2/2L^2{}_p}{1 + \dfrac{\sigma_B}{\sigma_E} \dfrac{W}{L_{nE}}} \tag{53}$$

where: σ_B is the conductivity of base region and

σ_E is the conductivity of emitter region.

The base transport factor, ξ, is defined as the fraction of injected hole current from the emitter which reaches the collector. Therefore

$$\xi = \left. \frac{\partial J_{pC}}{\partial J_{pE}} \right|_{V_C = \text{const}} \tag{54}$$

or

$$\xi = \frac{\left. \dfrac{\partial J_{pC}}{\partial V_E} \right|_{V_C = \text{const}}}{\left. \dfrac{\partial J_{pE}}{\partial V_E} \right|_{V_C = \text{const}}} . \tag{55}$$

Substituting equations (30) and (32) into (55) gives

$$\xi = \text{sech } (W/L_p) \doteq 1 - W^2/2L^2{}_p \tag{56}$$

by the first order approximation used earlier.

Shockley (32) defines γ as that fraction of the current at the emitter junction produced by emitter voltage that is carried by minority carriers in the base region, or

$$\gamma = \frac{1}{1 + \frac{\sigma_B W}{\sigma_E L_{nE}}} . \tag{57}$$

Substituting equations (56) and (57) into (53) gives

$$\alpha_{cb} = \gamma \, \xi \tag{58}$$

where ξ is a function of transistor base width and the diffusion length of minority carriers in the base region and γ is a function of base and emitter conductivity, transistor base width and diffusion length of majority carriers in the emitter region. Equation (58) shows that the transistor amplification factor, α_{cb}, is a pure function of the transistor physical parameters. If any of these parameters change with irradiation, a change in α_{cb} would be expected. Hence by developing the relationships between the flux and the variation of these parameters, it is possible to predict the change in α_{cb} due to irradiation. Such equations are developed in the following sections.

2.3 Common Emitter Current Gain of a Transistor

The current amplification factor β is the grounded - emitter current gain, which is defined as the variation of collector current, I_C, in response to a change in base current, I_B, with the collector voltage, V_C, held constant.

$$\beta = \left. \frac{\partial I_C}{\partial I_B} \right|_{V_C} \tag{59}$$

β is a very sensitive parameter of transistors and critical to the circuit performance. This parameter is explicitly related to the transistor parameters, i.e., the material electrical properties, the geometry and the dimensions. When the emitter current, I_E, in a junction transistor is increased, β originally increased and after going through a maximum, it finally decreased steadily (37). Figure 5 shows this variation for a typical P-N-P alloy junction transistor. Operating temperature can also severely affect the transistor's current gain (29). This effect is particularly important for those transistors whose surface has not been stabilized to fix its recombination velocity (38). Absorbed water on the surface and other effects can change the performance of even inactive transistors. Therefore it is necessary that pre-irradiation transistor characteristic data be obtained shortly before irradiation, and that the temperature be fixed at a constant value during irradiation. This procedure was followed in this work.

Figure 6 shows the energy band structure of the P-N-P transistor and the conventional directions for both current flow and bias voltage. When operated as a transistor, the left junction, biased positively with respect to the base, acts as an emitter, and the right junction, biased negatively, acts as a collector.

The emitter current, I_E, results from the injection of holes into the base region, creating current I_{pE}, and the movement of electrons from the base to emitter region, creating current I_{eE}; thus

$$I_E = I_{pE} + I_{eE}. \tag{60}$$

In a good transistor, the current I_{eE} is very small and negligible compared with I_{pE} (37). Therefore the emitter current is

$$I_E \approx I_{pE}. \tag{61}$$

Some of the holes moving from the emitter to the base recombine with the

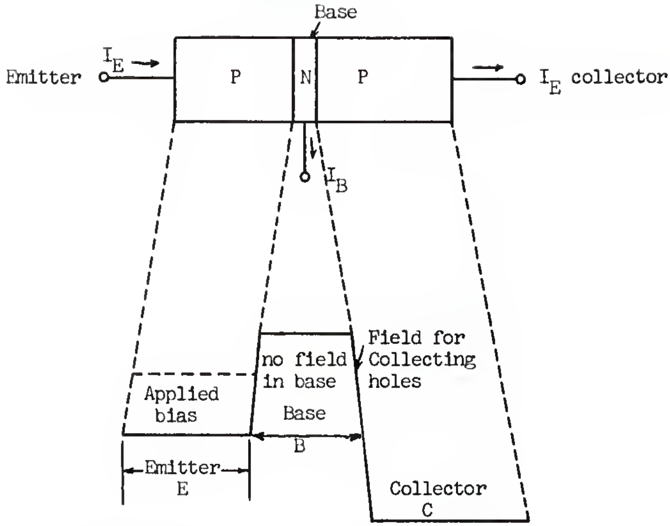

Figure 6. Schematic diagram for PNP junction transistor. The upper diagram shows the three parts of the transistor; the lower part shows the potential energy diagram for positive holes.

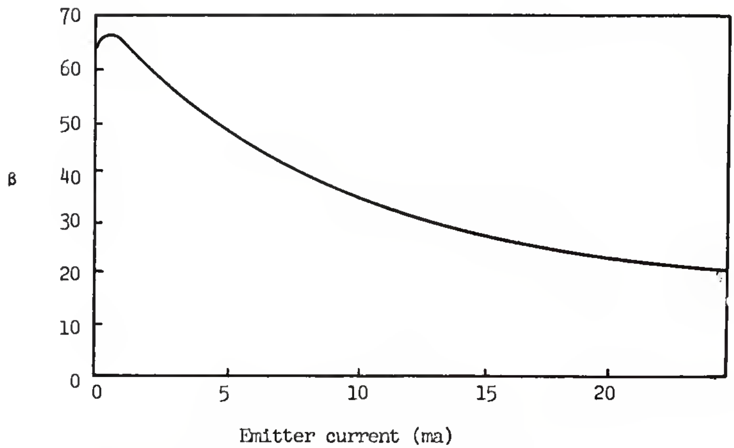

Figure 5. Variation of β with emitter current for a typical transistor

electrons generated in the base region. This recombination rate creates a current, I_R. The saturation current, I_{co}, which is composed of holes and electrons produced spontaneously by thermal energy in both the base and the collector region, and the current due to the holes which did not recombine, constitute the collector current, Figure 7. Therefore the base current I_B, is

$$I_B = I_R + I_{eE} - I_{co}. \tag{62}$$

From equation (62), the following relationship can be obtained.

$$\frac{\partial I_B}{\partial I_E} = \frac{\partial I_R}{\partial I_E} + \frac{\partial I_{eE}}{\partial I_E} - \frac{\partial I_{co}}{\partial I_E} \tag{63}$$

The recombination current, I_R, is composed of two parts; one is due to the surface recombination and the other results from the bulk recombination. Therefore

$$I_R = I_{sR} + I_{bR} \tag{64}$$

where: I_{sR} is current due to the surface recombination and

I_{bR} is current due to the bulk recombination.

Therefore from equations (63) and (64),

$$\frac{\partial I_B}{\partial I_E} = \frac{\partial I_{sR}}{\partial I_E} + \frac{\partial I_{bR}}{\partial I_E} + \frac{\partial I_{eE}}{\partial I_E} - \frac{\partial I_{co}}{\partial I_E}. \tag{65}$$

Since, as noted earlier, $I_{pE} >> I_{eE}$; therefore $I_E \approx I_{pE}$ and

$$\frac{\partial I_B}{\partial I_E} = \frac{\partial I_{sR}}{\partial I_{pE}} + \frac{\partial I_{bR}}{\partial I_E} + \frac{\partial I_{eE}}{\partial I_{pE}} - \frac{\partial I_{co}}{\partial I_{pE}}. \tag{66}$$

I_{co} and I_{pE} are produced in different regions of the transistor and they are independent; therefore

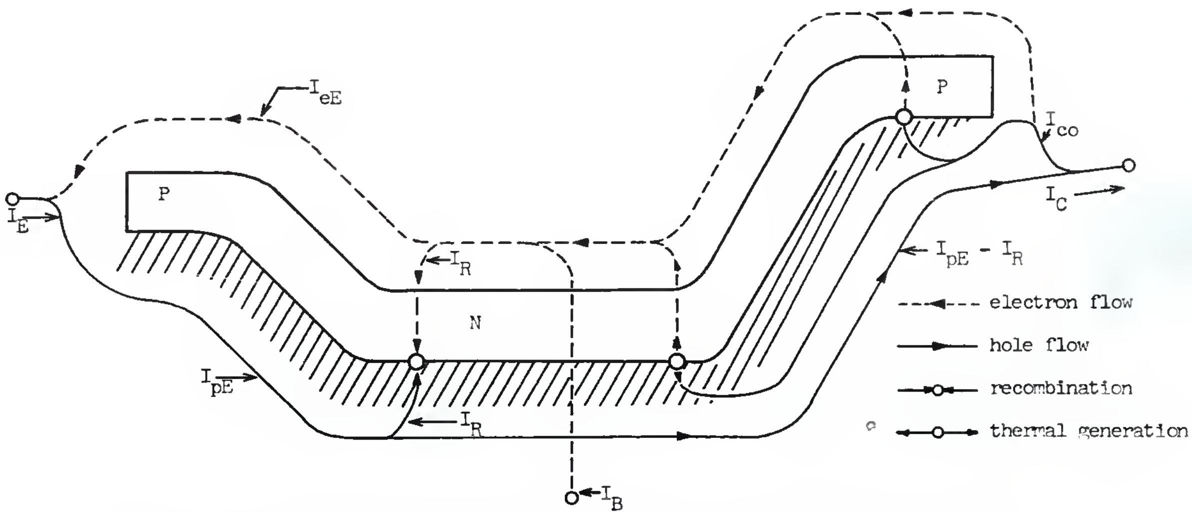

Figure 7. Potential distribution through a PNP transistor showing, schematically, the paths followed by holes and electrons

$$\frac{\partial I_{co}}{\partial I_{pE}} = 0,$$

and thus

$$\frac{\partial I_B}{\partial I_E} = \frac{\partial I_{sR}}{\partial I_{pE}} + \frac{\partial I_{bR}}{\partial I_{pE}} + \frac{\partial I_{eE}}{\partial I_{pE}}. \tag{67}$$

For equation (67) to be useful, it is necessary to express the partial differentials in terms of physical parameters of the transistor. To derive such relations, the following assumptions which are used by Shockley, Sparks, and Teal (32) are adopted. These assumptions are:

1. The donors and acceptors are fully ionized.
2. The density of minority carriers is much smaller than the density of majority carriers in each region.
3. The net rate of recombination in any region is linear in the deviation of the minority carrier density from its thermal equilibrium value.
4. The electric field in the base region is negligible.
5. The change in base region conductivity due to injected charge is trivial.

Assumptions (2) and (3) permit the use of linear equations for the currents arising from carrier injection and therefore

$$\frac{\partial I_{bR}}{\partial I_{pE}} = \frac{I_{bR}}{I_{pE}} = C_1, \tag{68}$$

$$\frac{\partial I_{eE}}{\partial I_{pE}} = \frac{I_{eE}}{I_{pE}} = C_2 \text{ and} \tag{69}$$

$$\frac{\partial I_{sR}}{\partial I_{pE}} = \frac{I_{sR}}{I_{pE}} = C_3. \tag{70}$$

To find C_1, C_2 and C_3 Ohm's law,

$$I_{pC} = I_{pE} - I_{bR}, \tag{71}$$

was applied.

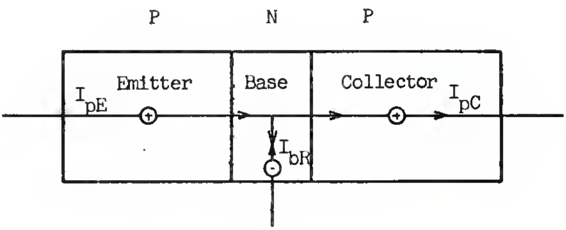

Figure 8. Hole and recombination current in a transistor

Assuming that the collector and emitter area are equal, it follows from equation (54) that

$$\xi = \frac{\partial I_{pC}}{\partial I_{pE}} = \frac{I_{pC}}{I_{pE}} . \tag{72}$$

Substituting equation (71) into (72) gives

$$\xi = 1 - \frac{I_{bR}}{I_{pE}}. \tag{73}$$

Comparing the terms in equations (56) and (73) gives

$$\frac{I_{bR}}{I_{pE}} = W^2/2L^2{}_p = C_1. \tag{74}$$

Equation (74) is limited to a PNP transistor. For the general case, equation (74) takes the following form:

$$\frac{I_{bR}}{I_{pE}} = 1/2\ (W/L_B)^2 \tag{75}$$

where L_B is the diffusion length of minority carriers in the base region.

An expression for C_2 can be derived by substituting equations (30) and (36) into the following equation:

$$C_2 = \frac{\left.\dfrac{\partial I_{eE}}{\partial V_E}\right|_{V_C}}{\left.\dfrac{\partial I_{pE}}{\partial V_E}\right|_{V_C}}.$$

The result of this substitution is

$$C_2 = \frac{\sigma_B\, W}{\sigma_E\, L_E} \tag{76}$$

where: σ_B and σ_E are conductivity of base and emitter regions respectively and L_E is the diffusion length of the minority carrier in the emitter region.

To find C_3 it should be noted that nearly all the surface recombination in an alloy junction transistor occurs in an area which is ring-shaped and surrounds the emitter junction (37), figure 7. The current due to surface recombination of holes is given by equation (77):

$$I_{sR} = e\ s\ A_s\ P_s \tag{77}$$

where: s is recombination velocity

A_s is the effective surface area for recombination and

P_s is the density of holes present near the surface.

However since the area where major surface recombination occurs is very near to the emitter, it can be assumed that

$$P_s = P_E \tag{78}$$

where P_E is the density of holes at the emitter junction.

From the definition of current density given by equation (29)

$$J_p = -\ e\ D_E \text{ grad } P_E = I_{pE}/A \tag{79}$$

where: A is the cross sectional area of conduction path

D_E is the diffusion constant of majority carrier in emitter region and

J_p is the hole current density.

For plane parallel geometry, where recombination may be neglected as far as its effect on minority carrier density is concerned, equation (79) can be integrated. The result of integration is

$$e\ A\ D_E\ P_E = I_{pE}\ W$$

or

$$P_E = \frac{I_{pE}\ W}{e\ A\ D_E}\ . \tag{80}$$

Substituting equations (78) and (80) into equation (77) gives

$$\frac{I_{sR}}{I_{pE}} = \frac{s\ A_s\ W}{A\ D_E}. \tag{81}$$

From equations (81) and (70) the following relation is obtained:

$$C_3 = \frac{s\ A_s\ W}{A\ D_E}. \tag{82}$$

Equations (74), (76) and (77) show that C_1, C_2 and C_3 are functions of the transistor physical parameters. Since it is possible to express β as a function of C_1, C_2 and C_3, β is also a function of the transistor physical parameters.

To develop the relationship between β and the physical parameters Kirchhoff's law,

$$I_E = I_B + I_C, \tag{83}$$

is employed. Differentiating with respect to I_E gives

$$1 = \frac{\partial I_B}{\partial I_E} + \frac{\partial I_C}{\partial I_E} \tag{84}$$

and also

$$\frac{\partial I_C}{\partial I_B} = \frac{\partial I_C}{\partial I_E} \cdot \frac{\partial I_E}{\partial I_B}. \tag{85}$$

Since $\frac{\partial I_C}{\partial I_E} = \alpha_{cb}$ and $\frac{\partial I_C}{\partial I_B} = \beta$, equations (84) and (85) can be employed to obtain the following relation:

$$\frac{\partial I_C}{\partial I_B} = \beta = \frac{\alpha_{cb}}{1 - \alpha_{cb}}. \tag{86}$$

α_{cb}, the common base configuration current gain, is the variation of collector current in response to a change in emitter current with the collector voltage held constant. This parameter was derived and discussed in the previous section.

In a properly operating transistor, α_{cb} is very close to unity (31); therefore equation (86) and (84) can be combined to give

$$\beta = \frac{1}{1 - \alpha_{cb}} = \frac{\partial I_E}{\partial I_B}. \tag{87}$$

Substituting equations (76), (82) and (87) into equation (75) gives

$$1/\beta = \frac{s\ A_s\ W}{D_E\ A} + \frac{\sigma_B\ W}{\sigma_E\ L_E} + 1/2\ (W/L_B)^2. \tag{88}$$

This equation shows the relation between the common emitter current gain and the physical properties and parameters of a transistor.

2.4 Theory of Radiation Damage in Transistors

Bombardment of a semiconductor material by energetic particles changes the conductivity (27) and the minority carrier lifetime (7) of the device. The relations between these parameters and the integrated flux have been formulated empirically by several investigators for fast neutrons (3) and for other energetic particles (19).

For the initial part of the irradiation σ was found to be a linear function of flux. Thus,

$$\sigma_i = \sigma_{io} \pm \xi_i\ e\ \mu_i\ \phi \tag{89}$$

where the subscript i, n or p is used depending on the initial resistivity type. The choice of sign depends on σ_{io}; for N-type materials it is negative and independent of the value of σ_{io}. For P-type materials, if σ_{po} is greater than $20\Omega^{-1}cm^{-1}$, the sign is negative. If σ_{po} is less than $20\Omega^{-1}cm^{-1}$, the sign is positive (21).

Lofferski (21) states that since transistor failure from changes in τ occurs before failure from changes in σ, it is sufficiently accurate to use equation (88) in the analysis of transistors. Although Lofferski's results indicate that changes in surface recombination velocity do occur during the early part of irradiation, these changes are transient, (24) they are not great enough to limit the operation of a transistor, and they are less extensive and/or important than those changes in minority carrier lifetime. However the bulk changes (changes in τ) continue indefinitely under irradiation and control the useful life of a transistor.

Equation (88) implies that $1/\beta$ is the sum of three independent terms which can be referred to as the surface recombination term, emitter efficiency term and bulk recombination term following the sequence in the equation. For the reasons mentioned before, it is assumed that the surface recombination term, which depends primarily on surface recombination velocity, does not change during irradiation.

It has been shown by Rappaport (27) that over many orders of magnitude, there is a reciprocal relation between τ and flux. Thus,

$$\frac{1}{\tau_{ai}} = \frac{1}{\tau_{bi}} + \frac{\phi}{K_j} \qquad (90)$$

where: τ_{bi} is the value of the minority carrier lifetime in the i region before irradiation

τ_{ai} is the value of the minority carrier lifetime in i region after irradiation

ϕ is the irradiation flux and

K_j is the lifetime damage constant, dependent upon the type of j material used in the base region.

The relation between minority carrier lifetime and diffusion length of carriers is commonly given by equation (91); similarly the relation between mobility and conductivity is expressed by equation (92).

$$L^2_i = D_{im} \tau_{im} \tag{91}$$

$$\sigma_i = C_i \, e \, \mu_{ic} \tag{92}$$

where: L_i is the diffusion length of minority carriers in the i region

D_{im} is the diffusion constant of minority carriers in the i region

σ_i is the conductivity of base or emitter

C_i is the number of majority carriers in the i region

e is the electronic charge and

μ_{ic} is the mobility of the majority carriers in the i region.

Substituting equation (90) into (91) gives the following relation:

$$\frac{1}{L_i} = \left[\frac{1}{D_{im}}\left(\frac{1}{\tau_{bi}} + \frac{\phi}{K_j}\right)\right]^{1/2}. \tag{93}$$

Substituting equation (93) into the emitter efficiency term gives

$$\frac{\sigma_E W}{\sigma_E L_E} = \frac{\sigma_B W}{\sigma_E} \cdot \left(\frac{1}{D_{Em}}\right)^{1/2} \cdot \left(\frac{1}{\tau_{bi}} + \frac{\phi}{K_j}\right)^{1/2}. \tag{94}$$

Considering the change in conductivity due to irradiation, the emitter efficiency term takes the following form:

$$\frac{\sigma_B W}{\sigma_E L_E} = H \left(\frac{1}{\tau_{bi}} + \frac{\phi}{K_j} \right)^{1/2} \tag{95}$$

where

$$H = \frac{\sigma_{Bio} \pm \xi_{iB} e \mu_{iB} \phi}{\sigma_{Eio} - \xi_i e \mu_{iE} \phi} \cdot \frac{W}{(D_{im})^{1/2}} .$$

Substituting equation (91) into the bulk recombination term of equation (88) gives

$$1/2 \frac{W^2}{L^2_B} = \frac{W^2}{2D_{im}} \left(\frac{1}{\tau_{bB}} + \frac{\phi}{K_j} \right) . \tag{96}$$

The surface recombination, emitter efficiency and bulk recombination terms have been calculated as a function of flux, and typical results are shown in Figure 9. These results show that the bulk recombination term only changes appreciably during the irradiation time, and therefore equation (96) predicts that $1/\beta$ is directly proportional to the integrated flux or time of irradiation if the flux is constant.

In the preceding derivation, the electric field in the base region was neglected. However Webster (37) shows that if the electric field in the base region is considered, equation (88) would have the following form:

$$1/\beta = \frac{s A_s W}{D_E A} g(Z) + \left[\frac{\sigma_B W}{\sigma_E L_E} + 1/2 (W/L_B)^2 \right] f(Z) \tag{97}$$

where: $g(Z)$ is called the field factor and is discussed in Appendix B

$f(Z)$ is called the fall-off factor and is discussed in Appendix B

I_E is the emitter current

μ_{BC} is the mobility of majority carriers in the base region

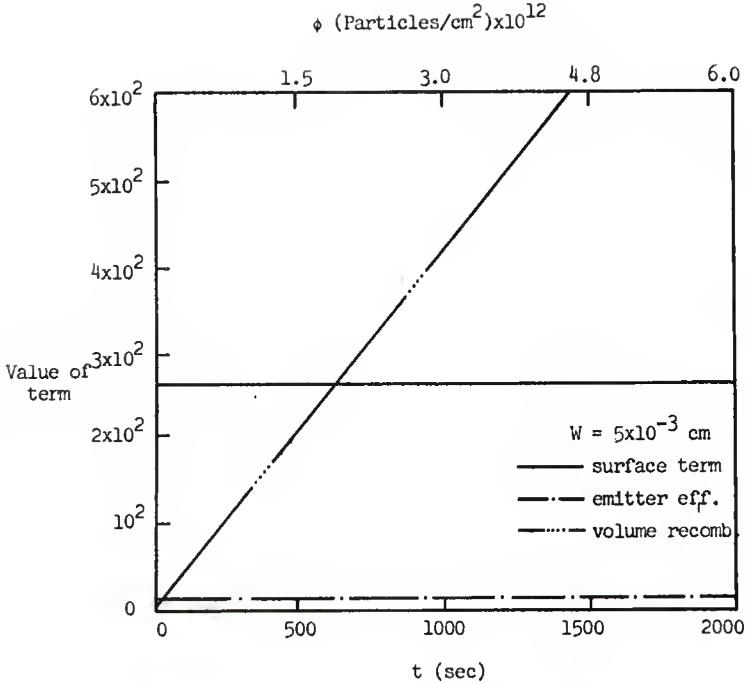

Figure 9. The flux behavior of three components of β (21)

D_{Em} is the diffusion length of minority carriers in emitter region and

$$Z = \frac{W \mu_{BC} I_E}{D_{Em} A \sigma_B} .$$

After irradiation, the equation for $1/\beta$ can be written in the following form:

$$1/\beta_a = 1/\beta_b + \Delta 1/\beta \qquad (98)$$

where $\Delta 1/\beta$ is proportional to flux.

From equations (98), (97), (96), (95) and considering the previous discussion and assumptions in regard to the change in conductivity during irradiation, the following conclusion is obtained.

$$\frac{Ws\ A}{D_E\ A} g(Z) \frac{\partial s}{\partial \phi} << f(Z)\ W/L_E \frac{\partial(\frac{\sigma_B}{\sigma_E})}{\partial \phi} << f(Z) \frac{W^2}{2D_{im}} \frac{\partial(\frac{1}{\tau})}{\partial \phi}.$$

Therefore,

$$\frac{\partial(1/\beta)}{\partial \phi} = \frac{W}{2D_{im}} \frac{\partial(\frac{1}{\tau})}{\partial \phi} h(Z) \qquad (99)$$

where: $h(Z)$ is a function of (1) emitter current, (2) mobility of majority carriers in the base region and (3) diffusion length of minority carriers in the emitter region. Messenger and Spratt (24) give the following equation for $h(Z)$:

$$h(Z) = \frac{(1 + Z)}{(1 + 2Z)} \left\{ \frac{c}{2a} - (\frac{c}{a} - 1) \left[\frac{\ln (1 + \frac{a\ Z}{c})}{(\frac{a\ Z}{c})^2} - \frac{1}{\frac{a\ Z}{c} (1 + \frac{a\ Z}{c})} \right] \right\} \qquad (100)$$

where: a and c are constants of the recombination process.

Substituting equation (90) into (99) gives

$$\frac{\partial(1/\beta)}{\partial\phi} = \frac{W^2}{2D_{im}} \cdot \frac{1}{K_j} \cdot h(Z). \tag{101}$$

The solution of this differential equation is

$$1/\beta_a = 1/\beta_b + \frac{W^2}{2D_{im}} \cdot \frac{\phi}{K_j} \cdot h(Z). \tag{102}$$

At zero emitter current h(Z) is equal to unity (33); therefore

$$1/\beta_a = 1/\beta_b + \frac{W^2}{2D_{im}} \cdot \frac{1}{K_j} \cdot \phi. \tag{103}$$

Since W, D_{im}, and K_j are constants

$$1/\beta_a = 1/\beta_b + \alpha_j \ \phi \tag{104}$$

where:

$$\alpha_j = 1/2 \ \frac{W^2}{D_{im}} \cdot \frac{1}{K_j} . \tag{105}$$

The relation between the transistor cutoff frequency and its base width is commonly given as

$$W^2 = \frac{1.22 \ D_{im}}{\pi \ f_{c\alpha}} . \tag{106}$$

Substituting equation (105) into (106) gives

$$\alpha_j = \frac{1.22}{2\pi} \cdot \frac{1}{f_{c\alpha}} \cdot \frac{1}{K_j} . \tag{107}$$

At values of emitter current different from zero, the transistor damage constant is

$$\alpha_j = \frac{1.22}{2\pi} \cdot \frac{1}{f_{c\alpha}} \cdot \frac{1}{K_j} \cdot h(Z). \tag{108}$$

K_j is the lifetime damage constant which depends only on the type of materials used in the base region. Therefore, if K_j is known for a type of material used in a transistor's base region, the transistor damage constant could be evaluated from equation (108). To calculate K_j, equation (101) can be used. In applying this equation, $\frac{\partial(1/\beta)}{\partial\phi}$ is first determined experimentally from the collector characteristic curves; the necessary procedure will be described fully in a later section. (See Section 5.2).

3.0 NEUTRON FLUX MEASUREMENT

3.1 General Discussion

The absolute number of disintegrations per unit time of an irradiated foil is a function of flux, time of irradiation, foil weight, decay constant and activation cross section of the foil. Therefore, if all these variables except the flux is known accurately, the fast neutron flux can be determined. However in most cases the activation cross section is not known accurately; hence it is necessary to choose foils that have the best known activation cross section for the particular neutron energy being used.

3.2 Theory

The activation rate of a foil in a neutron flux is

$$dr = N(x,\ y,\ z)\sigma_{act}(E,\ x,\ y,\ z)\ dE\ dv$$

where: $N(x,y,z)$ is the foil's density of atoms at x,y,z

$\sigma_{act}(E)$ is the microscopic activation cross section and

$\phi(E,x,y,z)$ is the neutron flux at x,y,z.

Assuming the neutron flux is constant within the thin foil, then

$$r = N_T \int_0^\infty \sigma(E)\ \phi(E)\ dE \tag{109}$$

where: $N_T = \frac{N_o\ w}{A}$ is the total number of atoms in the foil

N_o is Avagadro's number

A is the foil atomic weight and

w is the foil weight in gms.

The foil activity at any time, t, during irradiation will obey the following equality:

Rate of change of radioactive atoms = Rate of production - Rate of decay. If N is the number of radioactive atoms at time t, and if λ is the decay constant for N, then

$$\frac{dN}{dt} = r - \lambda N. \qquad (110)$$

Solution of this differential equation with the B.C., N = 0 at t = 0 is

$$N = r/\lambda\ (1 - e^{-\lambda\ t}). \qquad (111)$$

The activity of the foil at time t during irradiation is

$$N\ \lambda = r(1 - e^{-\lambda\ t}). \qquad (112)$$

After the irradiation time t, the number of radioactive atoms and the activity of these atoms will decrease according to the decay law.

$$N_1\ \lambda = N_o\ \lambda\ e^{-\lambda\ t'} \qquad (113)$$

where: N_o is the number of radioactive atoms at the end of irradiation (t'=0)

t' is time after irradiation and

λ is the foil decay constant.

A measurement of this activity between time t_2' and t_3' gives the following total number of counts

$$f_T\ C_T = \int_{t_2}^{t_3} N_o\ e^{-\lambda\ t'}\ dt'$$

or

$$f_T\ C_T = N_o\ (e^{-\lambda\ t_2'} - e^{-t_3'})$$

or

$$f_T\ C_T = N_o\ e^{-\lambda\ t_2'} \left[1 - e^{-\lambda\ (t_3' - t_2')}\right] \qquad (114)$$

where f_T is a correction factor which is discussed in the following pages.

From equation (114), λN_o is

$$\lambda N_o = \frac{f_T\, C_T\, \lambda}{e^{-\lambda\, t_2'}\,(1 - e^{\lambda\,(t_3' - t_2')})} . \tag{115}$$

Substituting equation (115) into (112) gives

$$r = \frac{f_T\, C_T\, \lambda}{e^{-\lambda\, t_2'}\,(1 - e^{\lambda\,(t_3' - t_2')})(1 - e^{-\lambda\, t})} . \tag{116}$$

From equations (109) and (116), the relation between flux and count rate is

$$N_T \int_0^\infty \sigma(E)\ \phi(E)\ dE = \frac{f_T\, C_T\, \lambda}{e^{-\lambda\, t_2'}\,(1 - e^{\lambda\,(t_3' - t_2')})(1 - e^{-\lambda\, t})} . \tag{117}$$

In the case where the neutrons are monoenergetic, equation (117) takes the following form:

$$\phi = \frac{f_T\, C_T\, \lambda}{e^{-\lambda\, t_2'}\,(1 - e^{\lambda\,(t_3' - t_2')})(1 - e^{-\lambda\, t})\ N_T\, \sigma_{act}} . \tag{118}$$

If $t_3' - t_2' << t_1/2$ and $(t_3' - t_2') << .639$, then $1 - e^{(t_3' - t_2')} = \lambda\,(t_3' - t_2')$. Therefore equation (118) may be simplified into the following form:

$$\phi = \frac{f_T\, C_T\, \lambda}{e^{-\lambda\, t_2'}\,(1 - e^{-\lambda\, t})(t_3' - t_2')\ N_T\, \sigma_{act}} , \tag{119}$$

where f_T is a correction factor which satisfies the following relation:

$$C_{abs} = C_T\, f_T$$

with C_{abs} as the absolute disintegration rate and

C_T as the measured count rate.

In the case of beta counting, assuming each integration in the foil produces one beta, the correction factor is

$$\frac{1}{f_T} = f_g \; f_w \; f_e \; f_\gamma \; f_{bs} \; f_s$$

where: f_g is the counter geometry correction factor

f_w is the counter window absorption correction factor

f_e is the counter efficiency for β particles

f_γ is the gamma background correction factor

f_s is the foil self-absorption and self scatter correction factor and

f_{bs} is the correction factor for beta particle back scattering from the support material.

Since equation (119) is used in calculating the flux, it is important to choose a foil for which the activation cross section is well known. Cu^{63}, when irradiated by 14.1 Mev neutrons, shows the following reaction:

$$Cu^{63} + {}_0n^1 \longrightarrow Cu^{62} + 2\,{}_0n^1.$$

Cu^{62} decays by $\beta +$ (2.91 Mev). Counting the positrons, it is possible to calculate the absolute activity of the sample; however, since the activation cross section of Cu^{63} (n,2n) Cu^{62} is not known accurately, (σ_{act} = .5b ± 10%), it is not possible to measure the absolute flux with less than 10% error. Al^{27} foils have been used extensively by the staff of Argonne National Laboratories, and it is believed (9) to be the best foil for measuring the 14.1 Mev neutron flux resulting from the reaction

$$H^3 \; (d,n)He^4.$$

Irradiation of Al^{27} foils with fast neutrons results in the following reaction:

$$Al^{27} + n^1 \longrightarrow Na^{24m} + \alpha .$$

Na^{24m} is metastable and has a half life of 0.02 sec. It will decay to Na^{24} which in turn has a half life of 15 hr. Decay of Na^{24} results in production of two gammas with energies of 2.75 Mev and 1.37 Mev and a β with energy of 1.394 Mev. An energy decay scheme for Na^{24m} is shown in Figure 10. Other reactions resulting from fast neutron irradiation of Al are:

$$Al^{27}\ (n,p)\ Mg^{27}$$

and

$$Al^{27}\ (n,\gamma)\ Al^{28}.$$

The decay scheme of Mg^{27} and Al^{28} are shown in Figure 11. Some of the characteristics of these three reactions are tabulated in Table I. It is obvious that the activation cross section for the Al^{27} (n,p) Mg^{27} and Al^{27} (n,γ) Al^{28} reactions are very small. In addition the half lives of the products are less than ten minutes which is small in comparison to the 15 hour Na^{24} half life. If after irradiation, ten or fifteen minutes are allowed to elapse before counting Al^{27} foils, the counts observed will be due only to the decay of Na^{24}. By measuring the β (1.394 Mev) count rate of Na^{24} the absolute activity of the foil and the absolute flux can be calculated using equation (119).

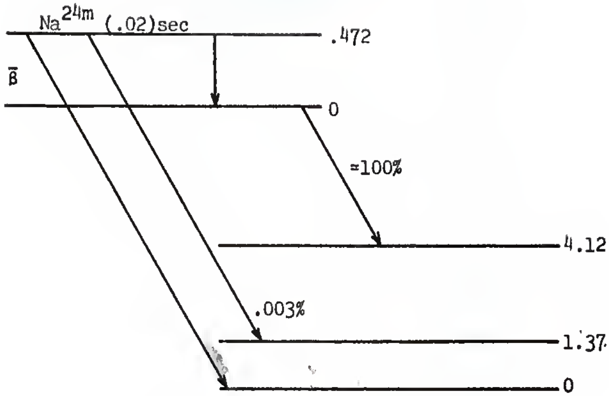

Figure 10. Decay scheme of Na^{24m}

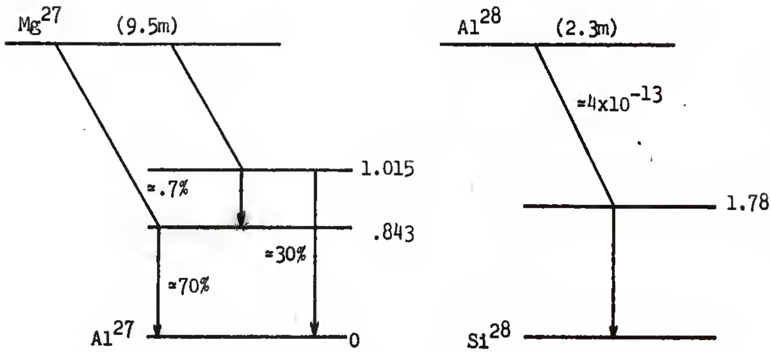

Figure 11. Decay schemes of Mg^{27} and Al^{28}

Table I. Activation cross sections of Al^{27} and Cu^{63} for 14.1 Mev neutrons

	T1/2	σ_{act}(mb)	gamma radiation	other
$Al^{27}(n,\alpha)\ Na^{24}$	15h	121 ± % 3	1.37(100%).2.75(100%)	β^- 1.394(100%)
$Al^{27}(n,p)\ Mg^{27}$	9.45m	70	.843(70%).1.015(30%)	β^- 1.75(58%)1.59(42%)
$Al^{27}(n,\gamma)\ Al^{28}$	2.27m	.53 ± .15	1.78(100%)	β^- 2.87(100%)
$Cu^{63}(n,2n)\ Cu^{62}$	9.8m	500 ± %10	several gammas	β^+ 2.91

4.0 EXPERIMENTAL

4.1 Apparatus

The Kansas State University Neutron Generator was used for transistor irradiation. This Texas Nuclear Model 9504 Neutron Generator, designed primarily for the production of intense neutron fluxes, consists of a deuteron ion source and focusing and accelerating electrodes to direct the deuteron beam on a tritium target. The following reaction between the deuteron beam and the tritium target produces nominal 14.1 Mev neutrons.

$$H^2 + H^3 \longrightarrow H_e{}^4 + {}_on^1 + 17.6 \text{ Mev}$$

A Tektronic type 575 Transistor Curve Tracer was used to display the transistor dynamic characteristic curves. These curves, produced on the screen of a cathode-ray tube, were photographed with a Hewlett Packard Model 196A Oscilloscope C.mera.

Table II lists the various electronic components used for neutron dosimetry. Two types of aluminum foil were used for absolute counting; the first group of foils was produced by Radiation Equipment and Accessories Corporation. These foils were 0.50 inches in diameter, 0.03 inches in thickness, and weighed approximately 0.2215 grams per foil. The second group produced at K.S.U. had a 0.50 inch diameter and weighed 0.001 grams.

The beta particle counting system consisted of a B.J. Electronics Model DD7 continuous flow proportional counter, a Baird Atomic Model 225 Proportional counter preamplifier, a Baird Atomic Model 132 scaler, and a Baird Atomic Model 322 timer. The K.S.U. Nuclear Engineering Inventory No. for the scaler was 144;

Table II. A list of the electronic equipment used

Component	Type and Model	Nuclear Engr. Inventory No.
Transistor-curve tracer	Tektronix Model 575	No. 1074
Oscilloscope camera	Hewlett Packard Model 196A	
Scaler	Baird Atomic Model 132	No. 145
Timer	Baird Atomic Model 960	No. 146
Preamplifier	Baird Atomic Model 255	No. 208
BF3 probe	Radiation Counting Lab. RCL-10504	No. 320
Scaler	Baird Atomic Model 132	No. 144
Timer	Baird Atomic Model 960	No. 147
Proportional counter	B.J. Electronics Model DD7	No. 370
Preamplifier	Baird Atomic Model 255	No. 209

that of the proportional counter was 370. The local identification of the preamplifier was K.S.U. Nuclear Engineering Laboratory Inventory No. 209. The proportional counter had a tungsten collector wire with a one-half inch loop. The counting gas was Olin-Matheson P-10, a mixture of 10 percent Methane and 90 percent argon.

The flux monitor system in the neutron generator room consisted of a Baird Atomic model 132 scaler, a Baird Atomic model 255 proportional counter preamplifier, a Baird Atomic model 322 timer, and an RCL model 10504 BF3 probe. The BF3 probe's active volume length of 12 inches and diameter of one inch, contained $B^{10}F3$ gas at a pressure of 12cm of Hg at an enrichment of 96 percent. The BF3 probe was placed in a cadmium foil covered box surrounded by paraffin blocks.

4.2 Procedure

Two series of experiments were performed. The first determined the neutron flux of the generator and standardized the absolute counting system. The second series observed the variation in transistor collector characteristic curves during neutron irradiation.

The calibrated gas flow proportional counter was operated at 1950 volts, with a pulse height sensitivity setting of 0.8. This counting system was checked for stability before and after each trial with a standard Ra-D-E source, Nuclear Chicago Serial No. 2354. All foil data were taken with the counting system reading 12,000 ± 100 cpm from the standard Ra-D-E source. Background was measured for a 20 minute period before and after counting each foil. These experiments are described in detail in Appendix D.

For the second series of experiments, a transistor socket was mounted on an Alden Chassis Card and then connected to the curve tracer with 30 feet of low capacitance cable. To reduce noise interference, both ends of the cable were grounded. Foil holders, one-half inch in diameter, were also mounted on the Alden Chassis Card so that the transistor was sandwiched between them. The chassis card was placed one inch from the neutron generator target cooling jacket and aligned so the beam would pass first through an aluminum foil, then the transistor and finally through the second aluminum foil.

Following the initial measurement of the collector characteristic curve at zero flux, the transistor was placed in the transistor socket and the aluminum foils were placed in the foil holders. During irradiation, the variation of collector characteristics curves was observed on the curve tracer and pictures of the curves were taken at intervals. The corresponding neutron flux for each curve was obtained from a calibrated BF3 counter (see Appendix D) which had been calibrated against the proportional counter. To check the BF3 counter, at the end of each experiment, the aluminum foils placed on the Alden Chassis Card were counted by the proportional counter and their absolute activity checked against the BF3 results.

4.3 Transistors Analyzed

As mentioned earlier, there are basically four different types of transistors: PNP Ge, NPN Ge, PNP Si, and NPN Si. Many different techniques are used by manufacturers to design and produce transistors; however, these techniques have in common the fundamental problems of (1) growing suitable crystals,

(2) forming junctions in them, (3) attaching leads to the structure and (4) encapsulating the resulting transistor. The ultimate aim of all transistor fabrication techniques is to construct two parallel PN junctions with both controlled spacing between them (base width) and controlled impurity content in the emitter and collector regions.

An attempt was made to purchase different types of transistors with known physical parameters (desired known parameters include width, diffusion length of electrons and holes, minority carrier lifetime, etc.). However after a long period of communication with different manufacturers, it was determined that all but one of the manufacturers either did not know the exact value of these parameters or were not willing to give out the information.

A set of 2N94A, NPN Ge, transistors was presented, however to the Nuclear Engineering Department at Kansas State University by the Sylvania Company who also supplied some of the desired physical parameters (see Table IV). The 2N94A transistors were irradiated with the Kansas State University neutron generator and characteristic curves determined.

In addition to the transistor neutron irradiation data collected at Kansas State University, irradiation data were available in the literature for several general purpose transistors. The literature data, in the form of collector characteristic curves, were treated by the theory developed in Section 2.4. A comparison of neutron damage constants determined for the Kansas State University and literature irradiated specimens are presented in the Discussion of Results.

Table III. Transistors analyzed

Transistor	Type	Material	Manufacturer	Nominal $f_{c\alpha}$
4JD1E17	PNP	Ge	G.E.	1 MC
2N139	PNP	Oe	R.C.A.	7.5 MC
T1166	PNP	Ge	Philco	40 MC
T1041	PNP	Ge	Philco	1 MC
L5405	PNP	Ge	Philco	.75 MC
2N176	PNP	Ge	Philco	225 KN
T1257	PNP	Si	Philco	35 KC
2N1155	NPN	Si	Texas Instr.	7.5 MC
2N94A	NPN	Oe	Sylvania	455 KC

Table IV. Sylvania specifications for 2N94A transistor

β	W	σ_E	σ_B
20 to 80 at 8 volts	$= 2.5\times10^{-3}$ cm	$= 6.5\times10^{2}(\text{ohm-cm})^{-1}$	$= 6.3\times10^{-1}(\text{ohm-cm})^{-1}$

Emitter area
$= 1.2\times10^{-3}\text{cm}^2$

5.0 DATA PRESENTATION AND ANALYSIS

5.1 Presentation of Data

Collector characteristic curves of 2N94A transistors were obtained by the experimental procedure described earlier. Figure 12 shows the variation in a collector characteristic curve for a typical 2N94A transistor during neutron irradiation. The standard flux deviation shown for each curve is due to error present in the aluminum cross section measurement and errors inherent in calculating the absolute activity.

Since it was not possible to purchase other types of transistors with a known base width, the raw collector characteristic data for eight other types of transistors were obtained from Inland Testing Laboratories (36); these data are shown in Figures 13 through 20. Flux values corresponding to these curves are accurate to within ±9% (36). The flux deviation shown in Figures 12 through 20 is the standard deviation of each value calculated from the well known equation

$$\sigma = \pm\ 1.4830\ P$$

where: P is the probable error and

σ is the standard deviation.

5.2 Analysis of Data

From equation (101), section 2.4, it is obvious that the variation of $1/\beta$ with integrated neutron flux is a function of transistor base width, the base region diffusion constant for minority carriers, emitter current and lifetime damage constant K_j. The transistor cutoff frequency is a function of the

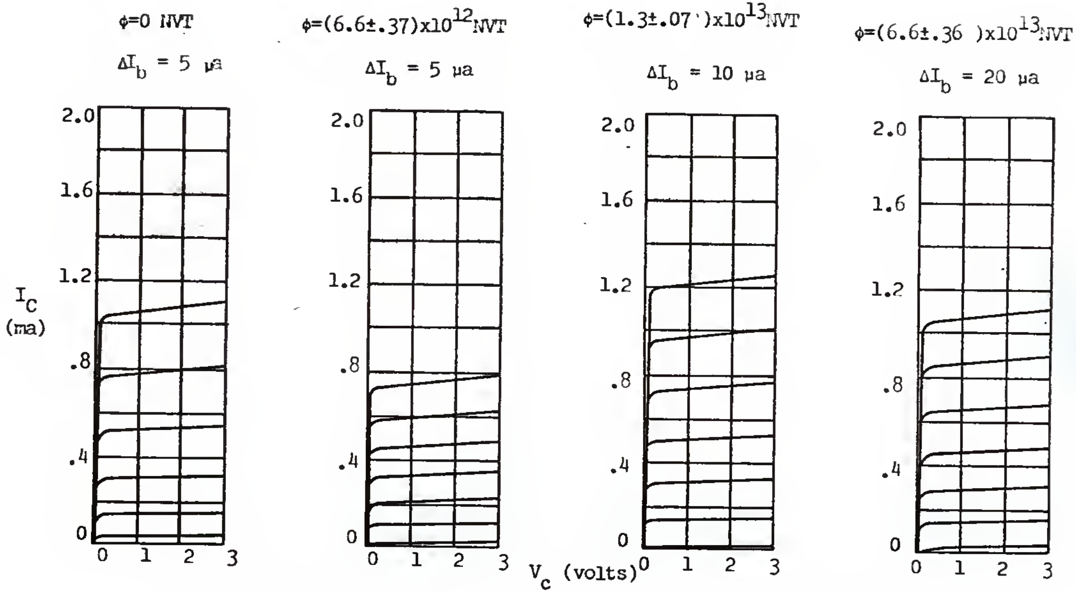

Figure 12. Typical collector characteristics of 2N94A NPN Ge Transistor

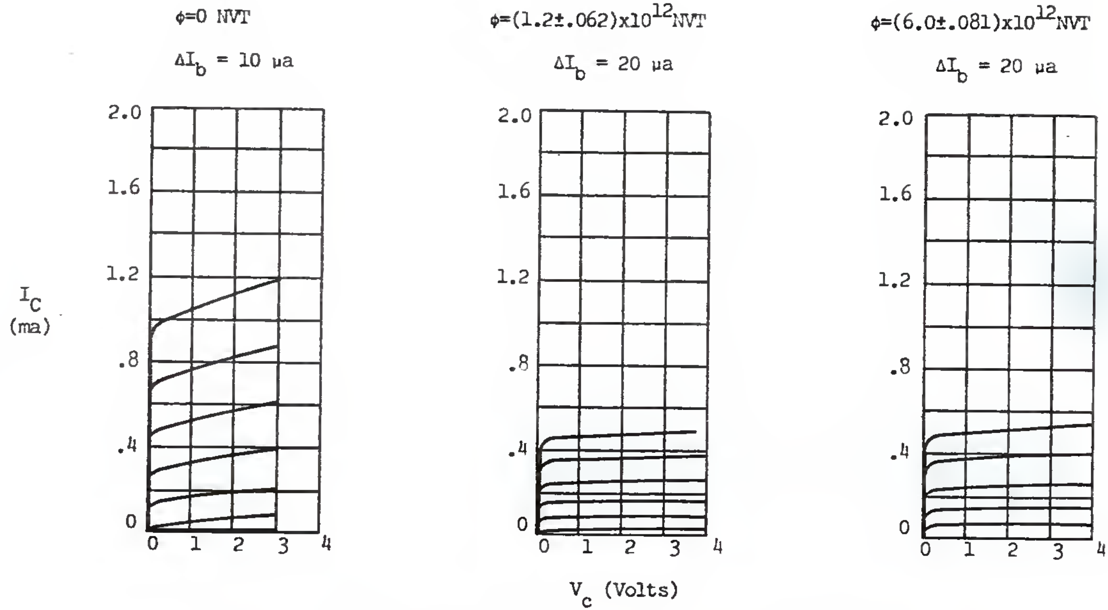

Figure 13. Typical collector characteristics of 2N1155 NPN Si Transistor (36)

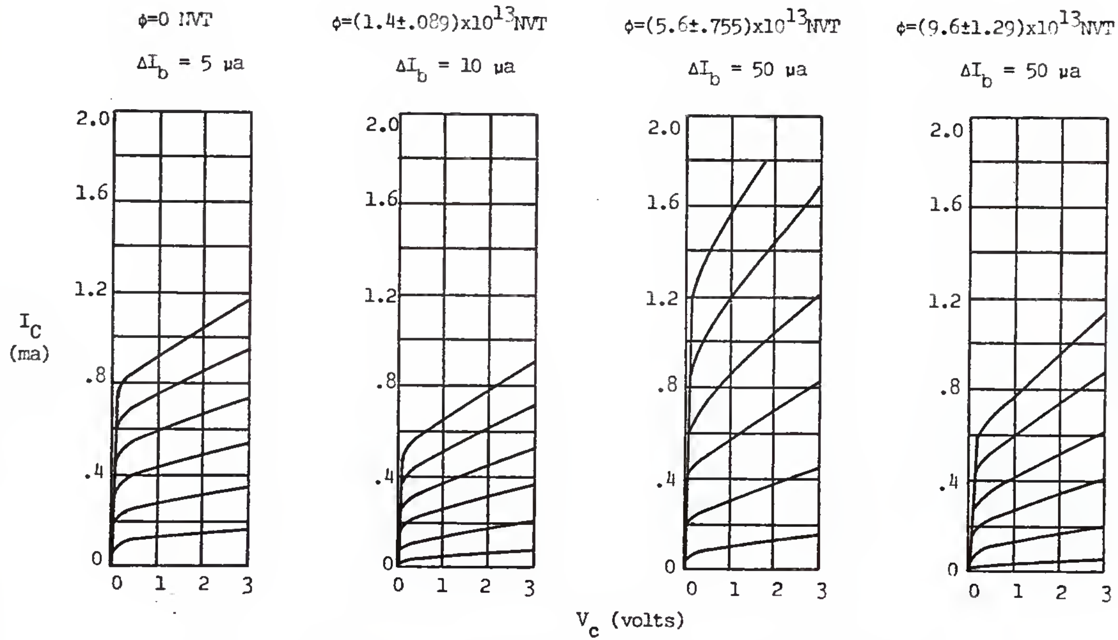

Figure 14. Typical collector characteristics of T-1257 PNP S_1 Transistor (36)

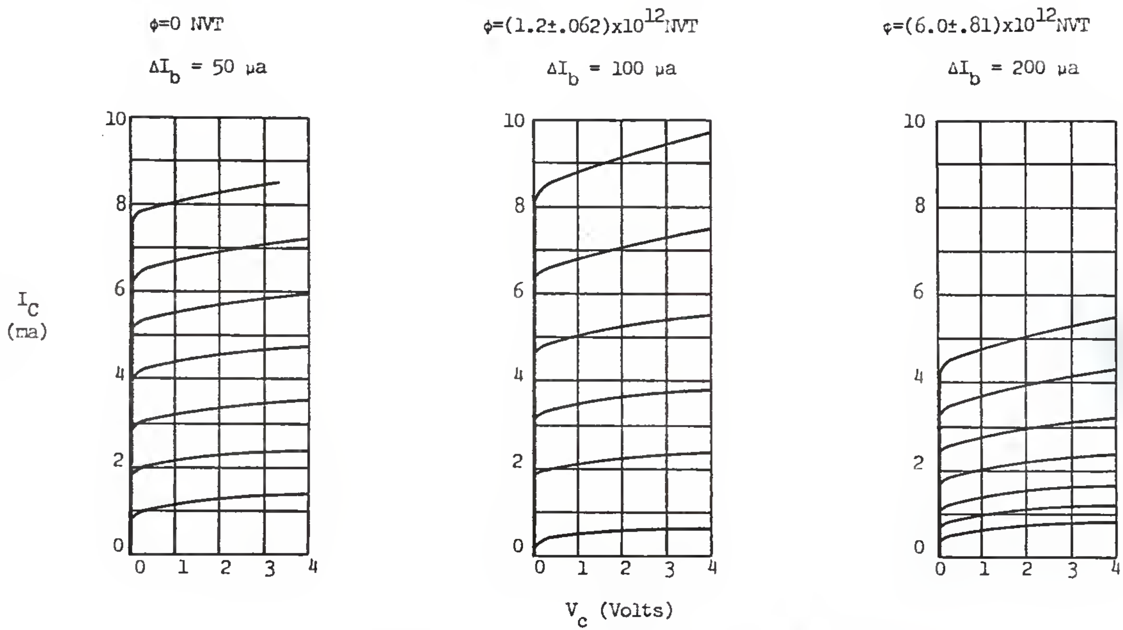

Figure 15. Typical collector characteristics of 2N176 PNP Ge Transistor (36)

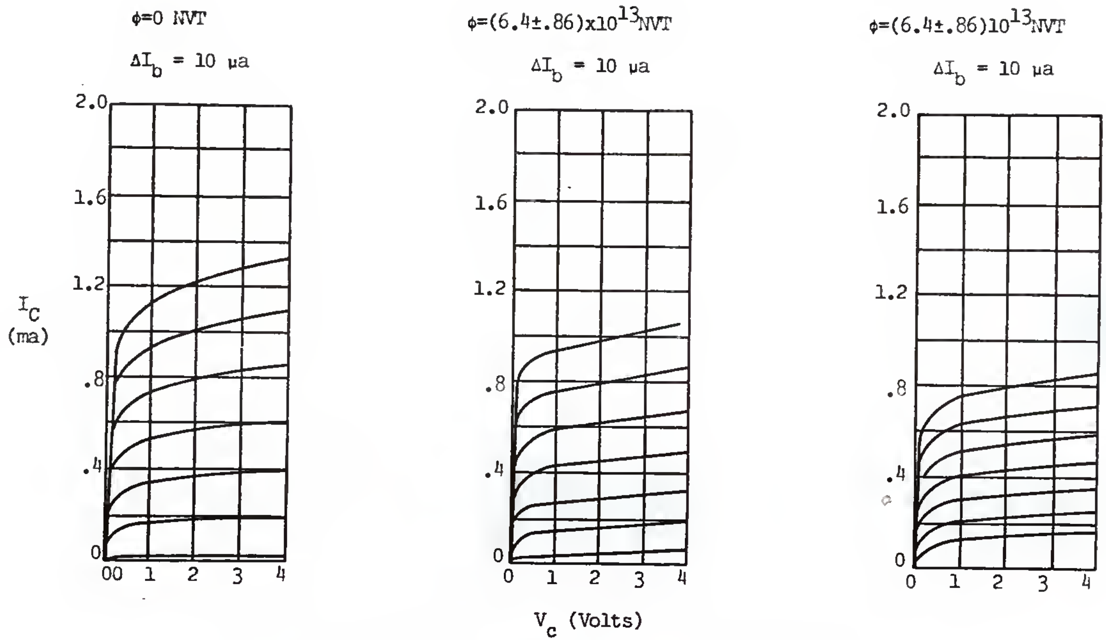

Figure 16. Typical collector characteristics of L5405 PNP Ge Transistor (36)

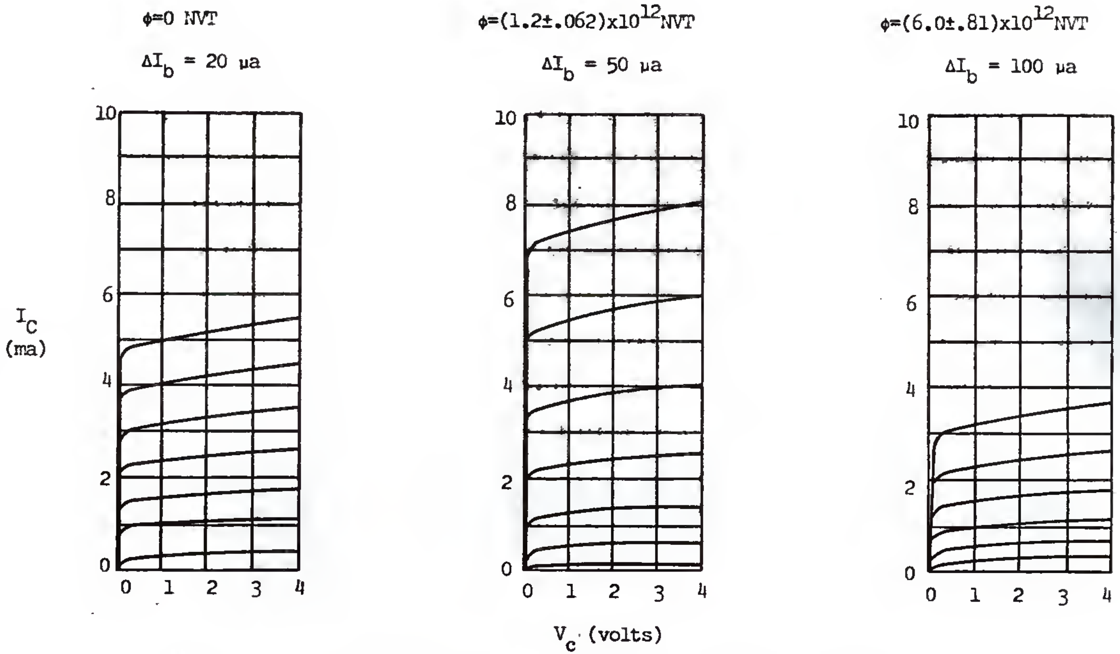

Figure 17. Typical collector characteristics of T1041 PNP Ge Transistor (36)

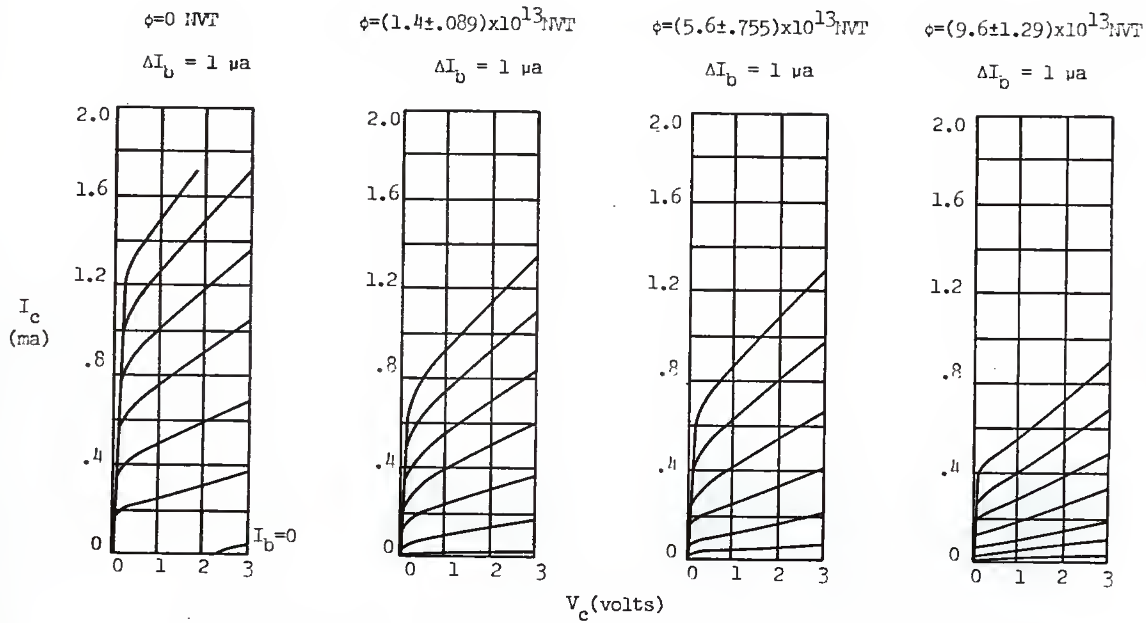

Figure 18. Typical collector characteristics of T-1166 PNP Ge Transistor (36)

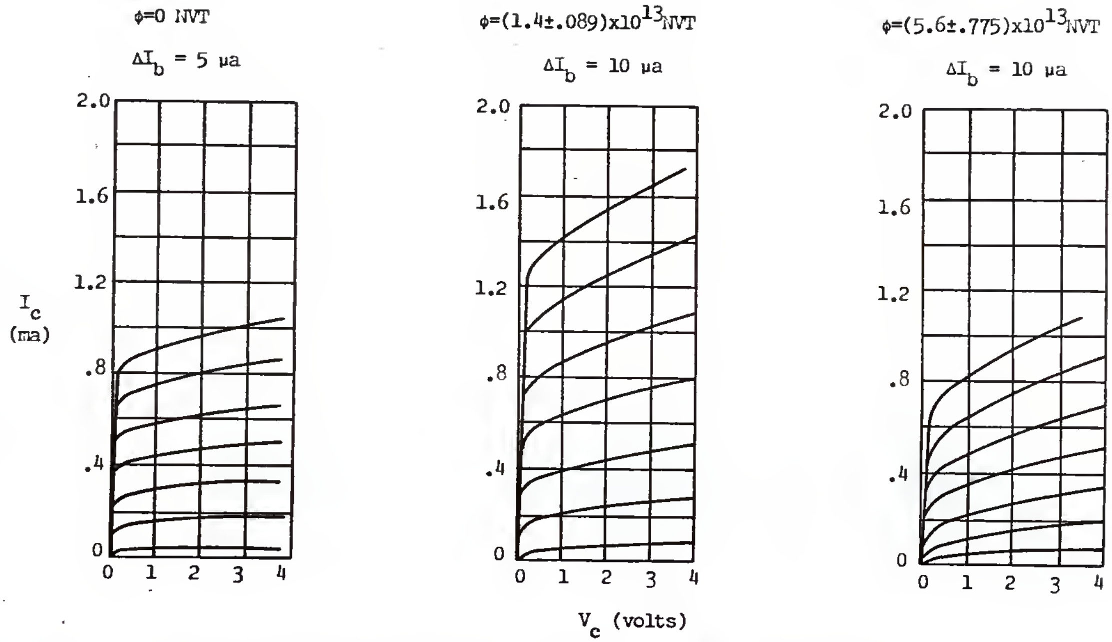

Figure 19. Typical collector characteristics of 2N139 PNP Ge Transistor (36)

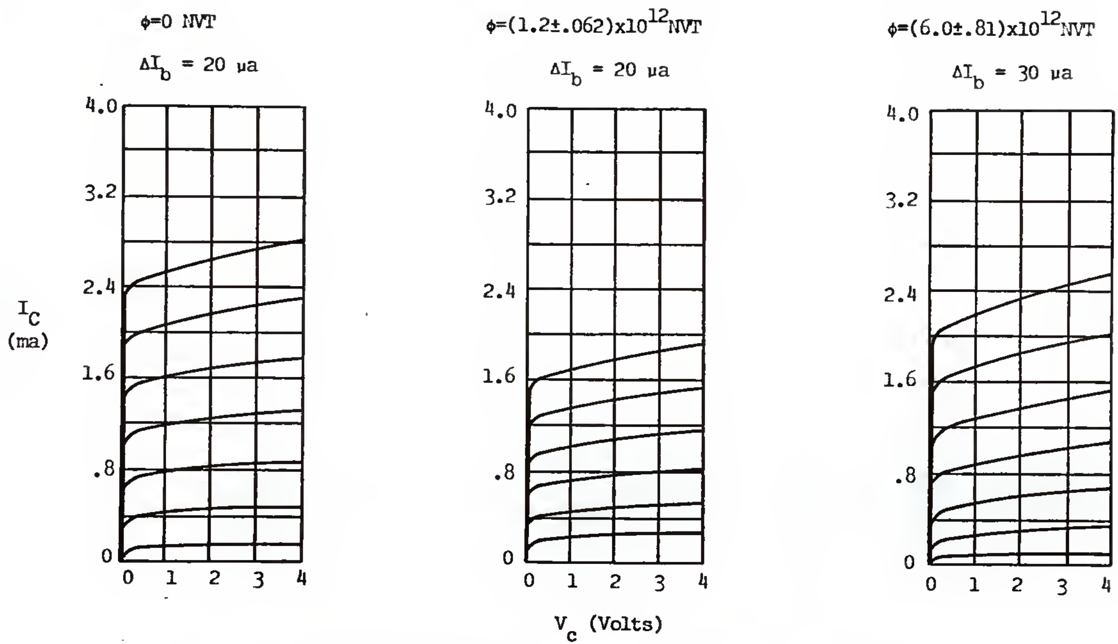

Figure 20. Typical collector characteristics of 4JD1E17 PNP Ge Transistor(36)

transistor base width as shown in equation (106). Substituting equation (106) into (101) gives

$$\frac{\partial(1/\beta)}{\partial\phi} = \frac{1.22}{2\pi} \cdot \frac{1}{f_{c\alpha}} \cdot \frac{1}{K_j} \cdot h(Z). \tag{121}$$

At zero emitter current, Z is equal to zero and h(Z) is equal to unity (see Section 2.4); therefore

$$\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E=0} = \frac{1.22}{2\pi} \cdot \frac{1}{f_{c\alpha}} \cdot \frac{1}{K_j} \tag{122}$$

or

$$K_j = \frac{1.22}{2\pi} \cdot \frac{1}{f_{c\alpha}} \cdot \frac{1}{\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E=0}} . \tag{123}$$

The value of $\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E=0}$ can be calculated from the collector characteristic curves shown in Figures 12 through 20, and since $f_{c\alpha}$ is a known parameter, K_j can be calculated from equation (123).

Equation (121) can be rewritten in the following form:

$$h(Z) = \frac{2\pi}{1.22} \cdot f_{c\alpha} \cdot K_j \cdot \frac{\partial(1/\beta)}{\partial\phi}. \tag{124}$$

Knowing the value of constant K_j (assuming K_j is independent of emitter current) and calculating $\frac{\partial(1/\beta)}{\partial\phi}$ for different values of I_E, it is possible to calculate h(Z) as a function of the emitter current from equation (124). The transistor damage constant can then be calculated from equation (108) since all the parameters in this equation are now known.

The IBM-1410 Computer was programmed to determine K_j, h(Z), α_j, and ($\alpha_j \cdot h(Z)$). The necessary data for calculating these parameters are obtained from the collector characteristic curves, corresponding to different values of the base current at V_C equal to one volt and known flux. An example listing of the necessary computer input data is shown in Appendix C. After introducing the necessary data, the computer program passes through the following steps to calculate the required parameters (the theory used in the computer programs is ~~are~~ discussed in detail in Appendix C):

1. After fitting a polynomial through I_B and I_C, $1/\beta$ and I_E are calculated using the equations

$$I_E = I_B + I_C \tag{83}$$

   and

$$1/\beta = \left.\frac{\partial I_B}{\partial I_C}\right|_{V_C = 1 \text{ volt}} .$$

   This step is repeated for different sets of data corresponding to different values of the flux.

2. The inverse of the common emitter current gain is plotted vs emitter current for different values of flux. Direct reproductions of the computer plots are shown in Figures 21 through 29.

3. For different values of emitter current, the computer calculates corresponding pairs of $1/\beta$ and ϕ data.

4. The computer program then fits a straight line through the $1/\beta$ vs ϕ data and calculates the line slope, $\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E}$. The standard deviation of the $1/\beta$ vs ϕ slope is calculated using equation (125);

$$\sigma(b) = \pm \left[\sum_{i=1}^{n} \left[f(X_i) = Y_i \right]^2 \Big/ (n-2) \sum_{i=1}^{n} X_i^{\,2} \right]^{1/2} \tag{125}$$

   This equation was used assuming the data would fit a linear plot and that the error in calculation of $1/\beta$ is very small compared with the

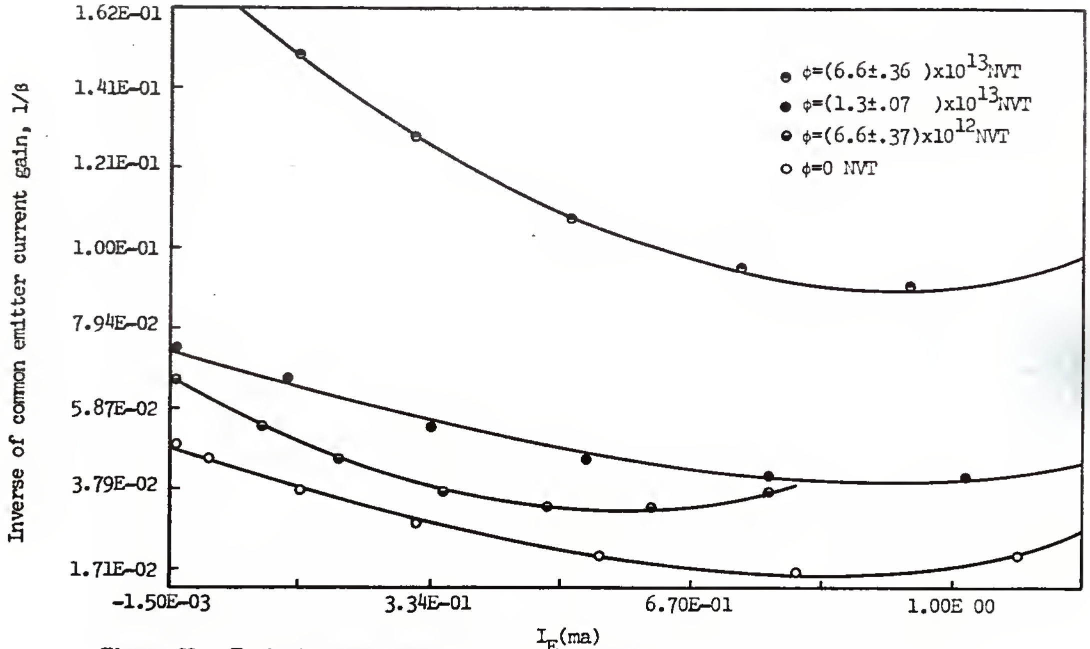

Figure 21. Typical variation of 1/β as a function of flux and emitter current for a 2N94A NPN Ge Transistor with collector voltage at 1 volt

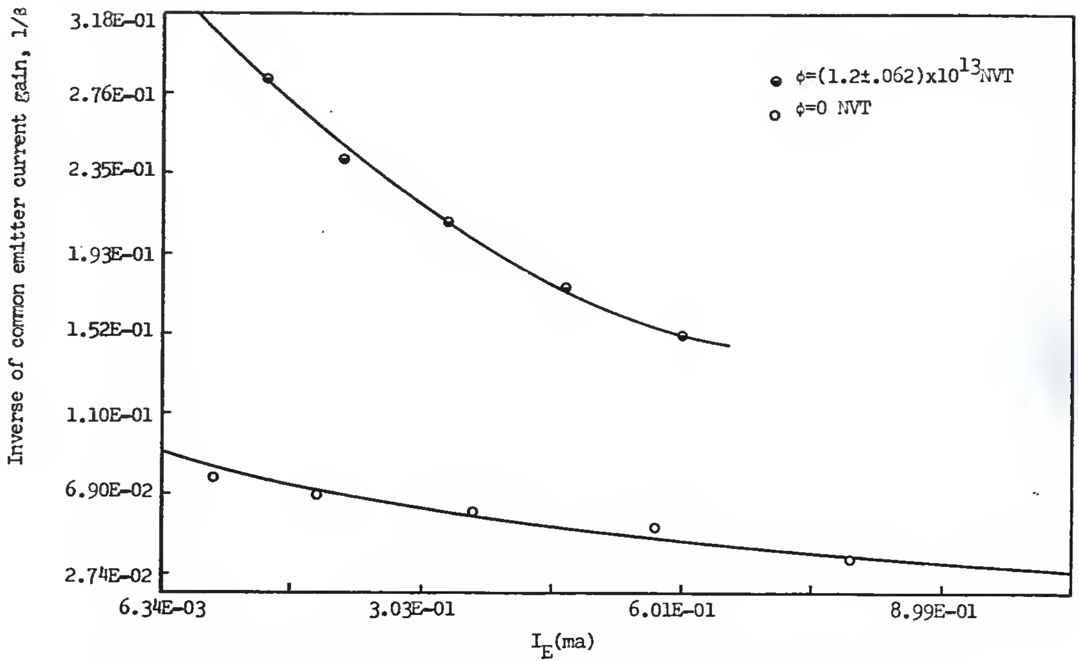

Figure 22. Typical variation of $1/\beta$ as a function of flux and emitter current for a 2N1155 NPN Si Transistor with collector voltage at 1 volt

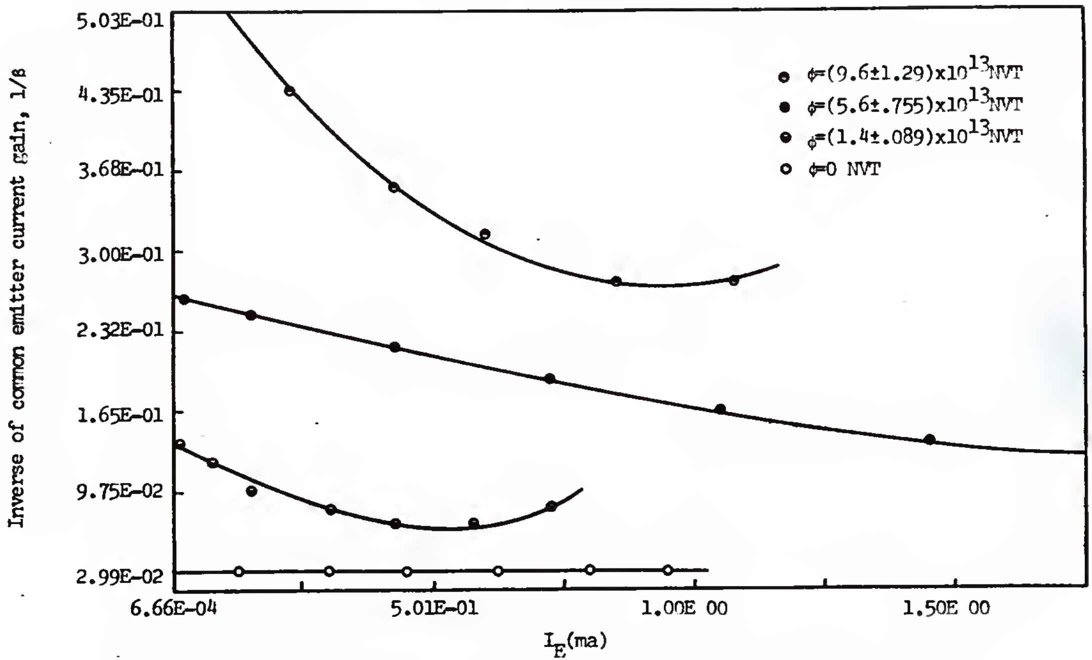

Figure 23. Typical variation of $1/\beta$ as a function of flux and emitter current for a T1257 PNP Si Transistor with collector voltage at 1 volt

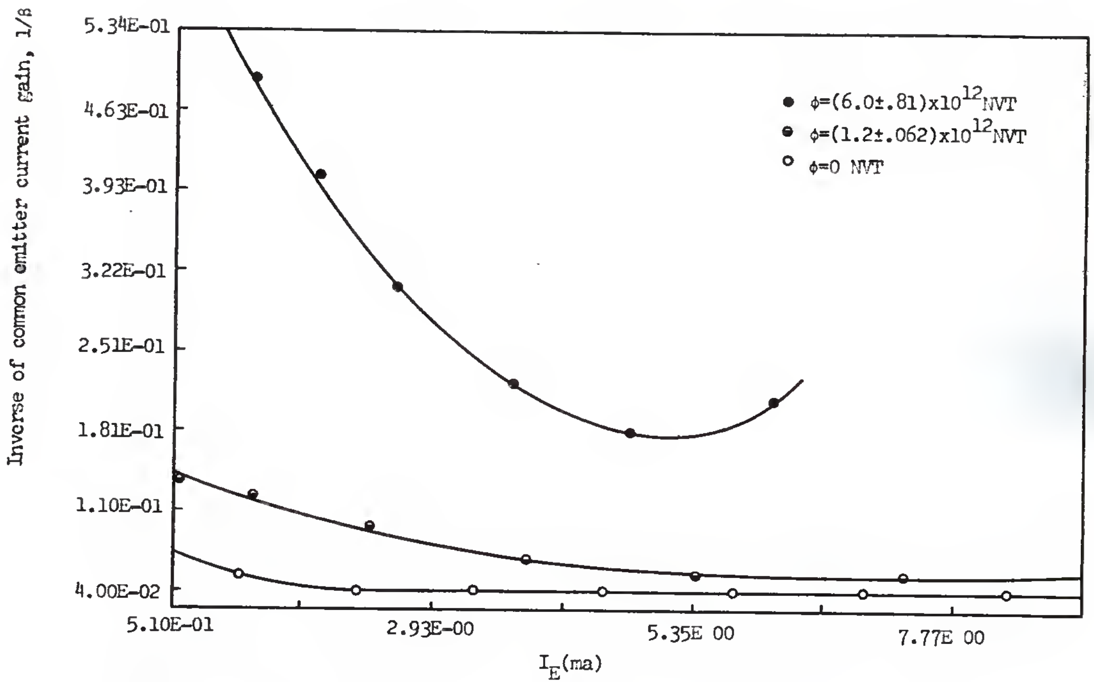

Figure 24. Typical variation of $1/\beta$ as a function of flux and emitter current for a 2N176 PNP Ge Transistor with the collector voltage at 1 volt

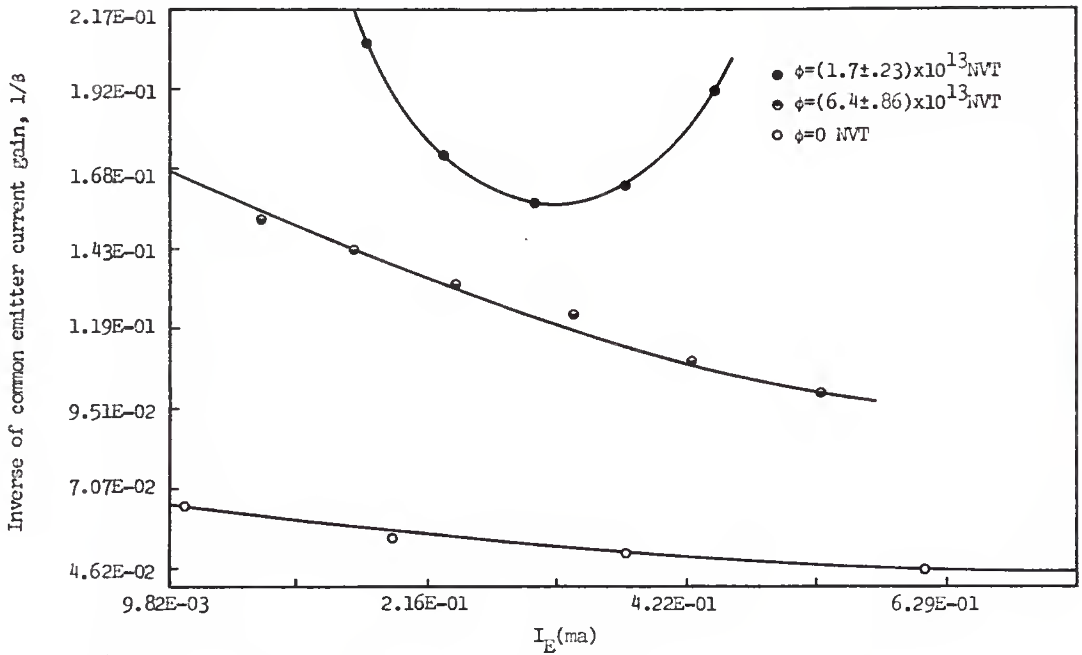

Figure 25. Typical variation of $1/\beta$ as a function of flux and emitter current for a L5405 PNP Ge Transistor with the collector voltage at 1 volt

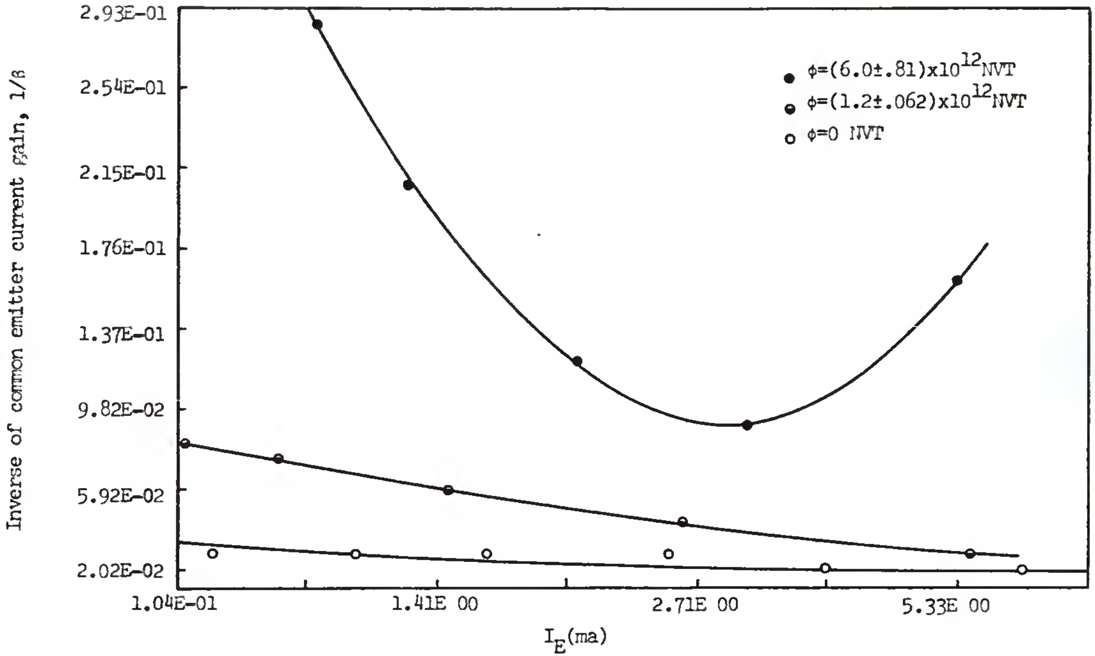

Figure 26. Typical variation of 1/β as a function of flux and emitter current for a T1041 PNP Ge Transistor with the collector voltage at 1 volt

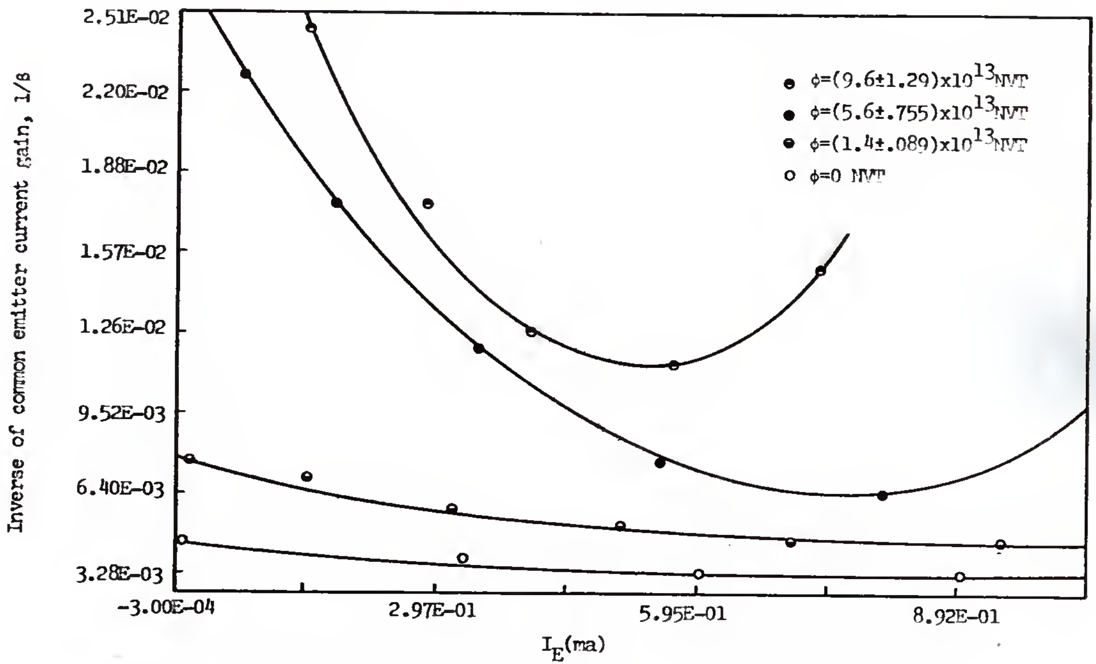

Figure 27. Typical variation of 1/β as a function of flux and emitter current for a T1166 PNP Ge Transistor with the collector voltage at 1 volt

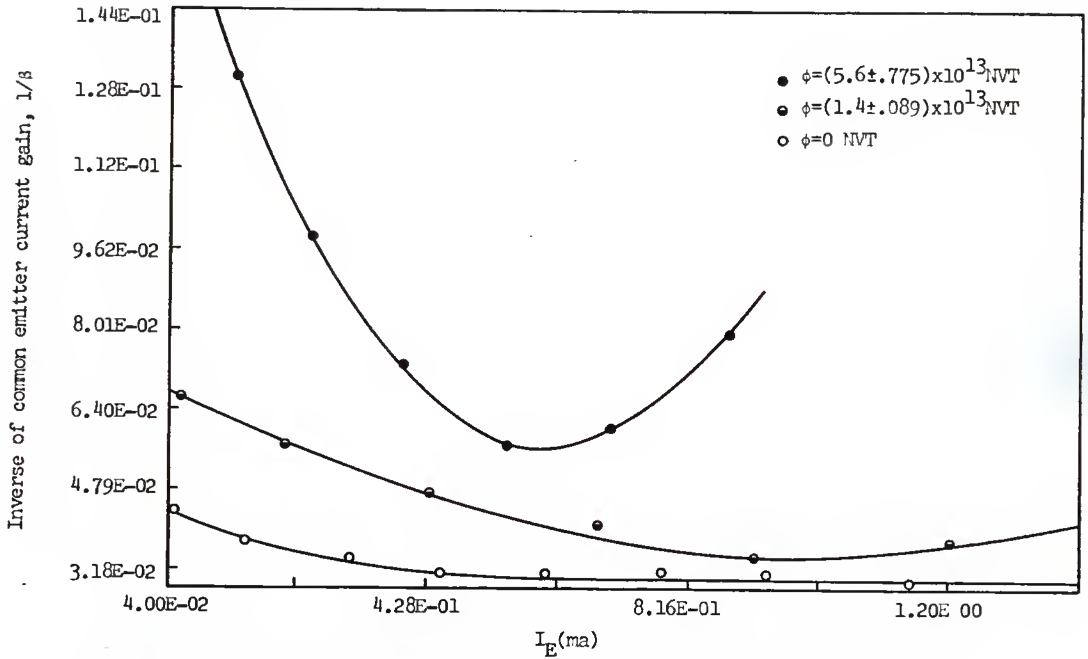

Figure 28. Typical variation of $1/\beta$ as a function of flux and emitter current for a 2N139 PNP Ge Transistor with the collector voltage at 1 volt

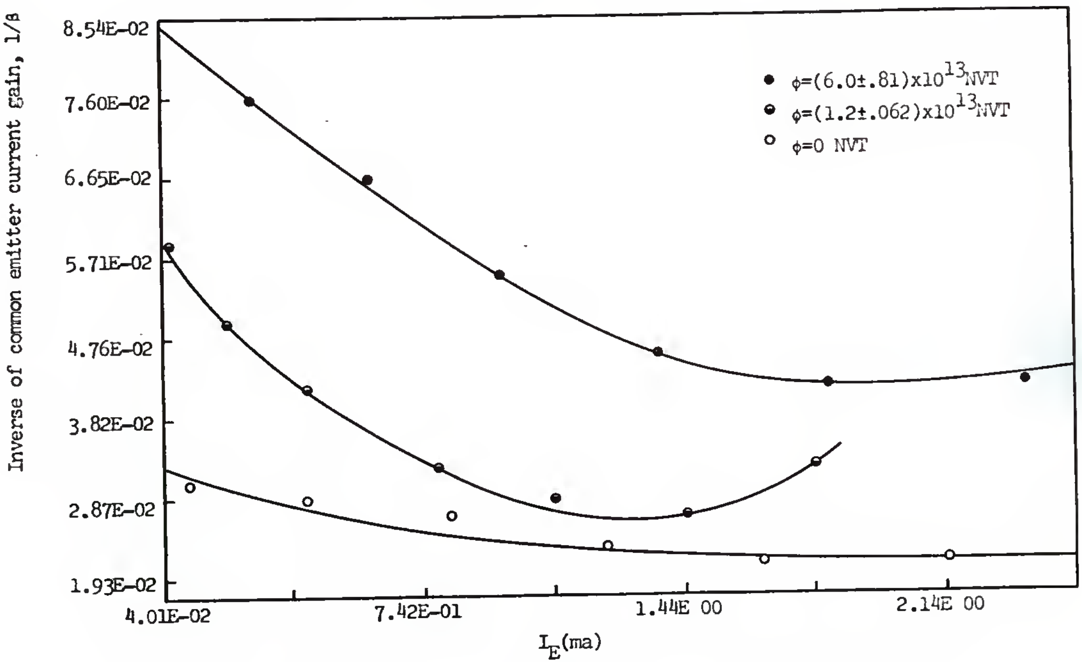

Figure 29. Typical variation of 1/β as a function of flux and emitter current for a 4JD1E17 PNP Ge Transistor with the collector voltage at 1 volt

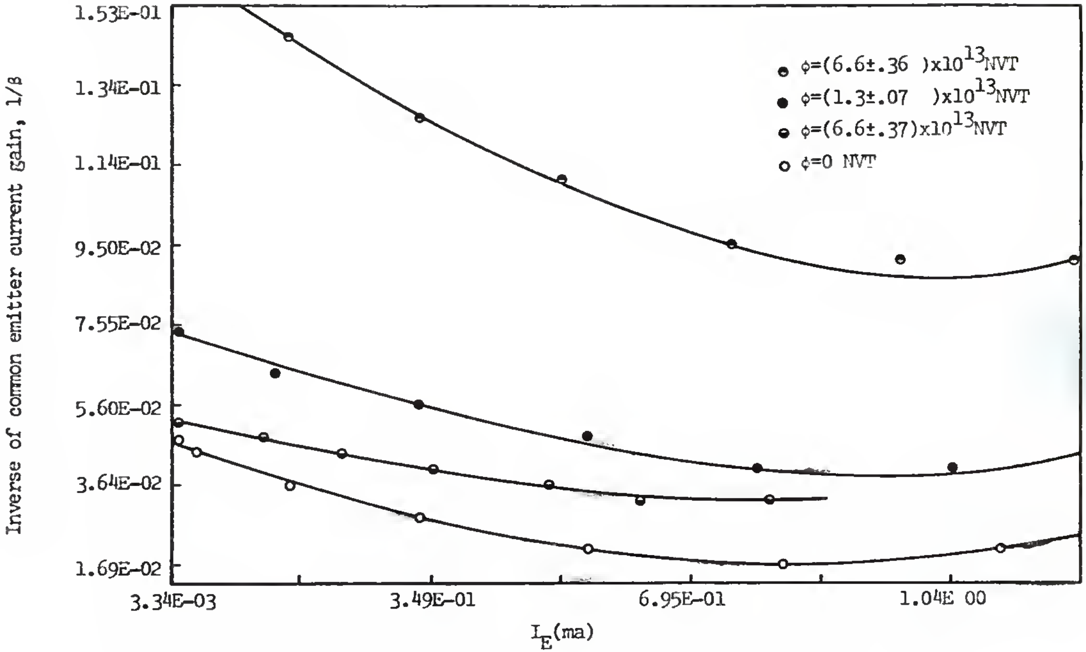

Figure 30. Typical variation of $1/\beta$ as a function of flux and emitter current for a 2N94A NPN Ge Transistor with collector voltage at 3 volts

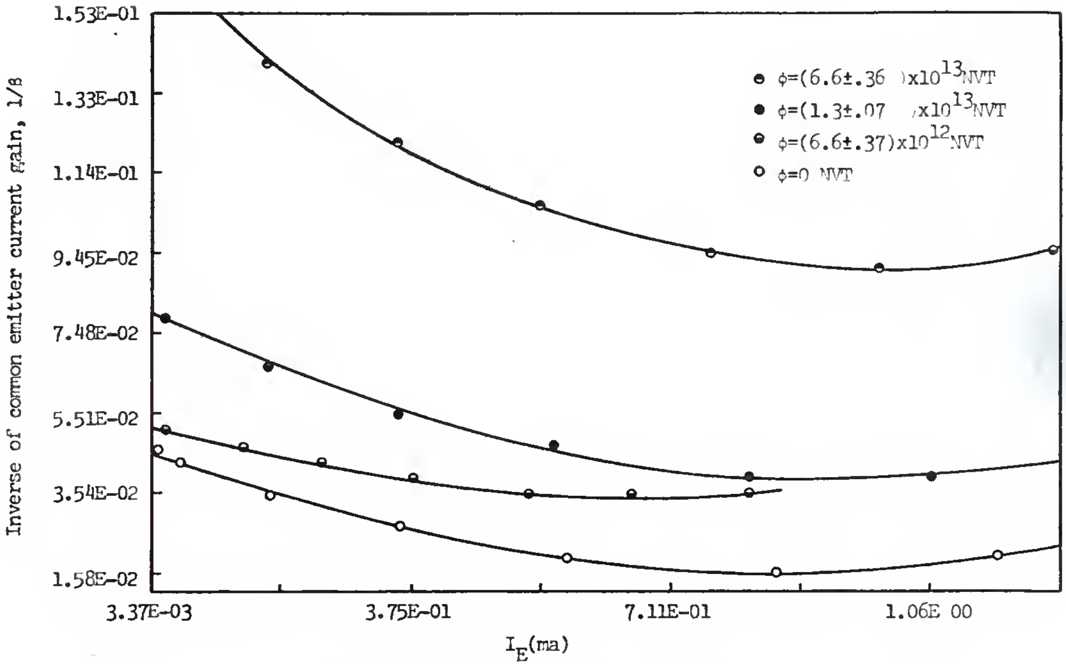

Figure 31. Typical variation of 1/β as a function of flux and emitter current for a 2N94A NPN Ge Transistor with collector voltage at 2 volts

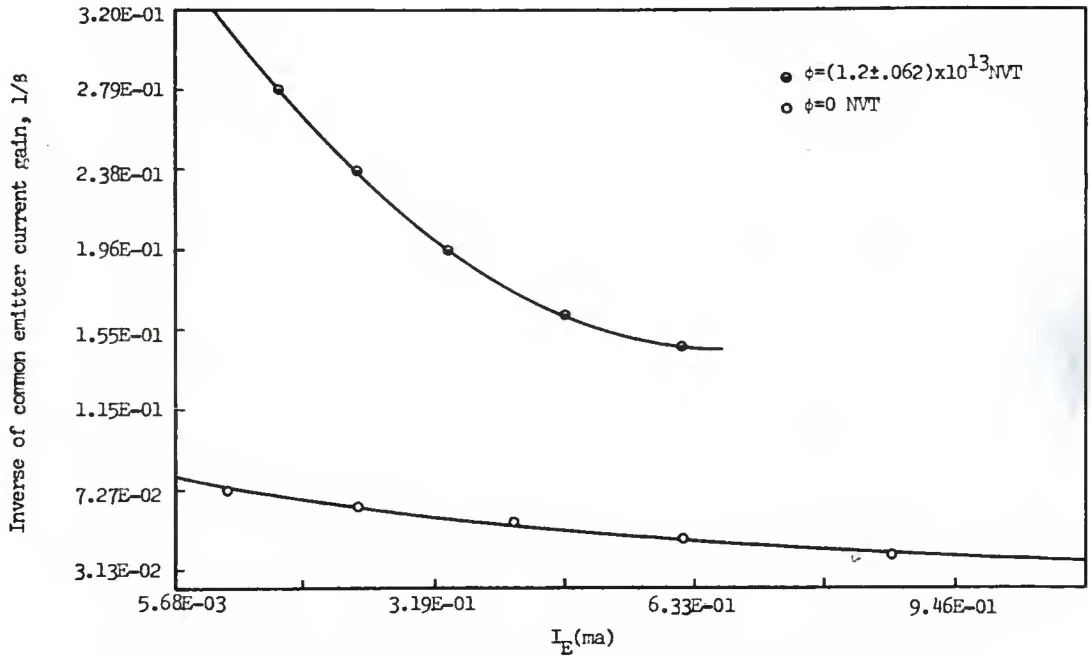

Figure 32. Typical variation of 1/β as a function of flux and emitter current for a 2N1155 NPN Si Transistor with collector voltage at 2 volts

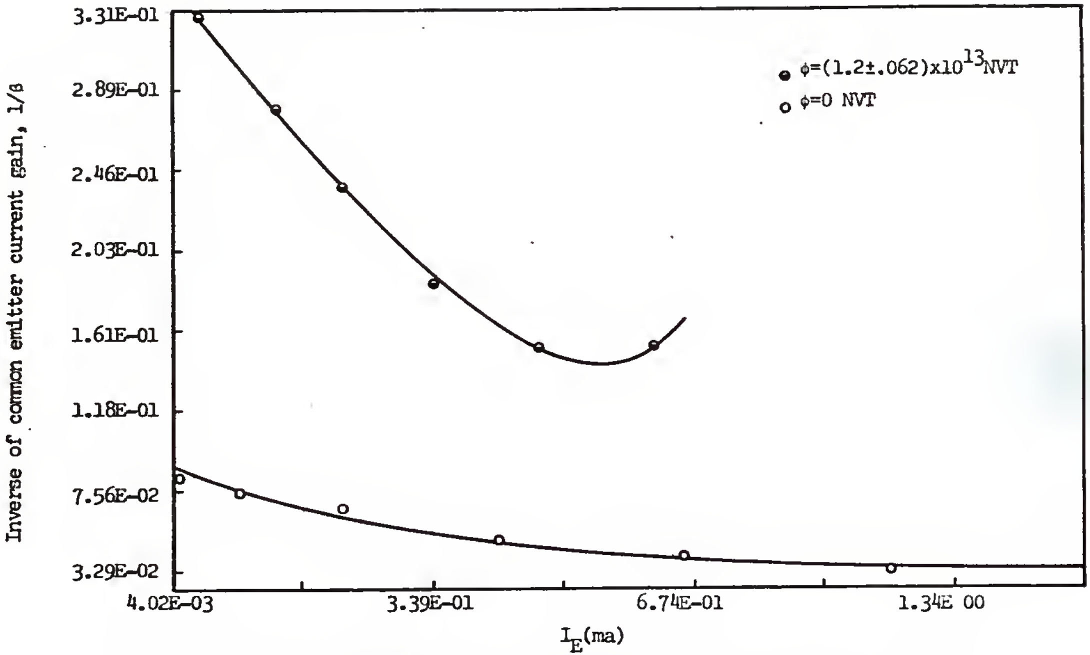

Figure 33. Typical variation of $1/\beta$ as a function of flux and emitter current for a 2N1155 NPN Si Transistor with collector voltage at 3 volts

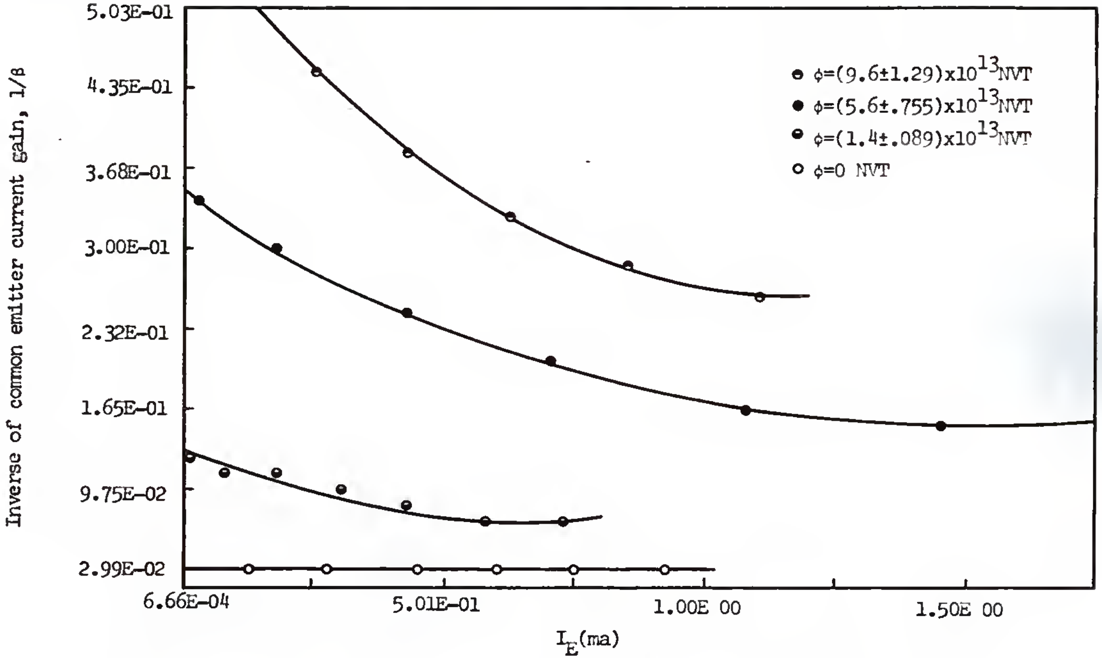

Figure 34. Typical variation of $1/\beta$ as a function of flux and emitter current for a T1257 PNP Si Transistor with collector voltage at 2 volts

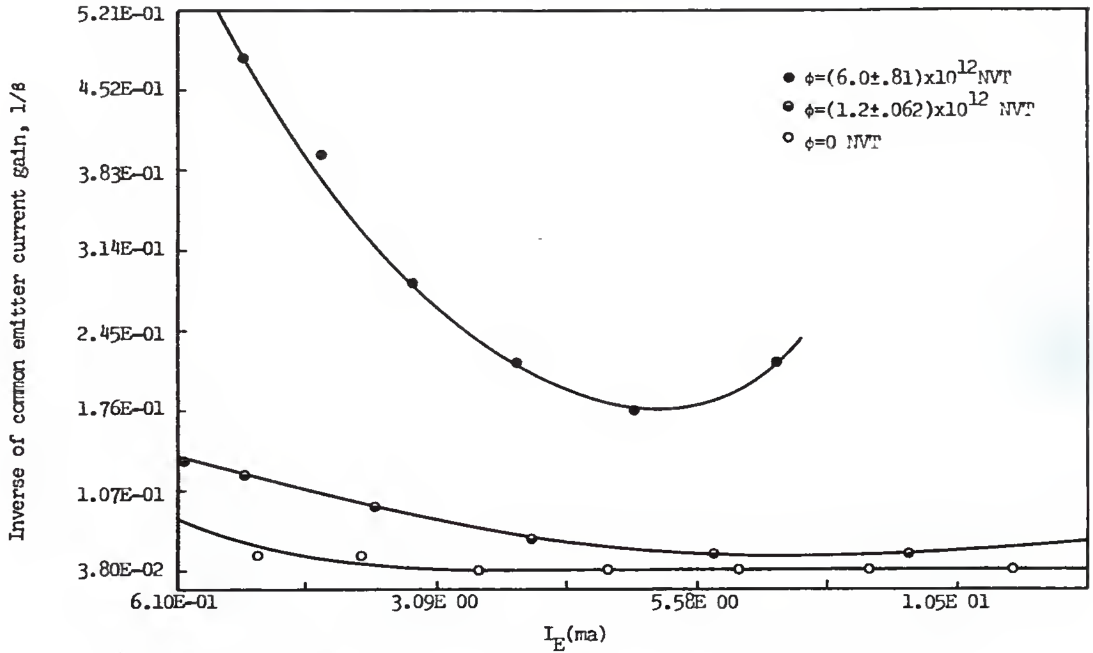

Figure 35. Typical variation of $1/\beta$ as a function of flux and emitter current for a 2N176 PNP Ge Transistor with collector voltage at 2 volts

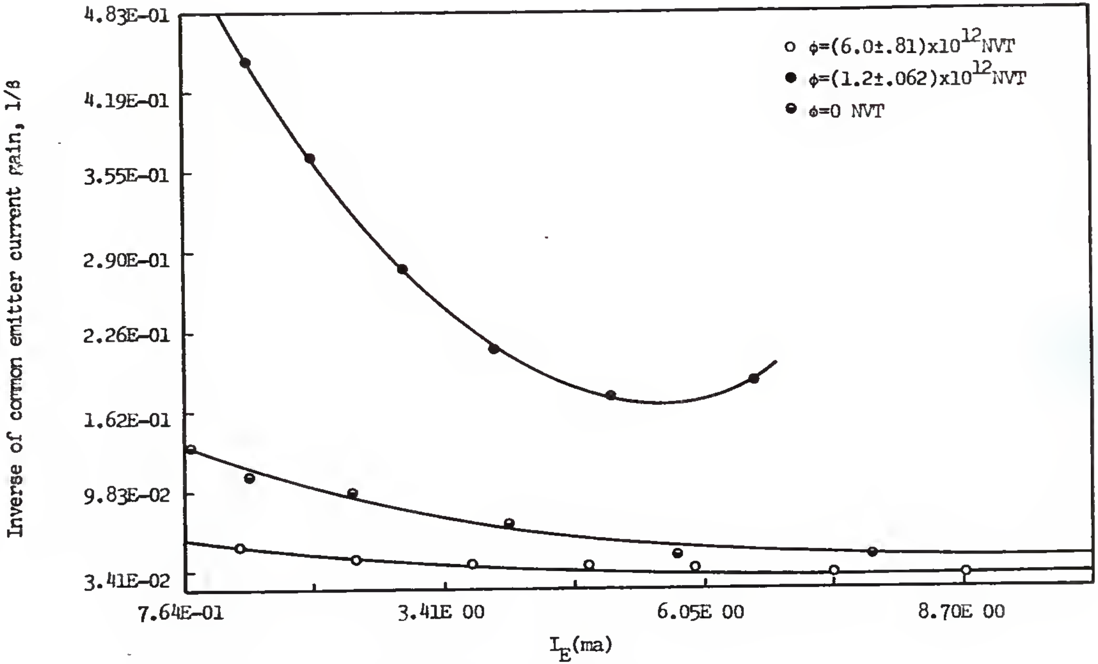

Figure 36. Typical variation of 1/β as a function of flux and emitter current for a 2N176 PNP Ge Transistor with collector voltage at 3 volts

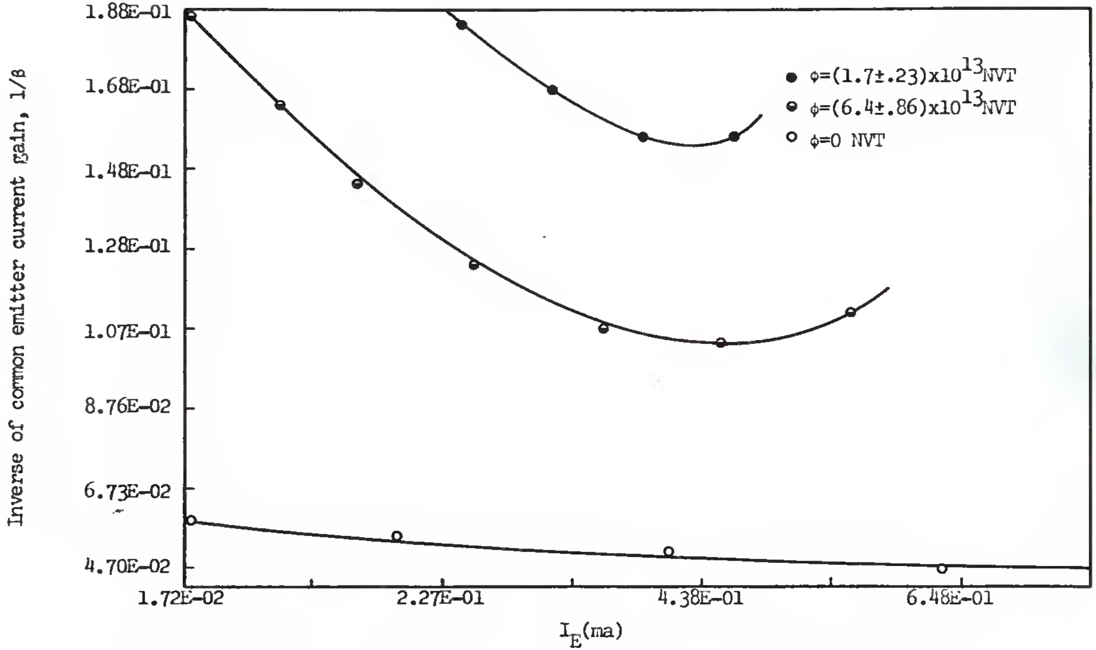

Figure 37. Typical variation of $1/\beta$ as a function of flux and emitter current for a L5405 PNP Ge Transistor with the collector voltage at 2 volts

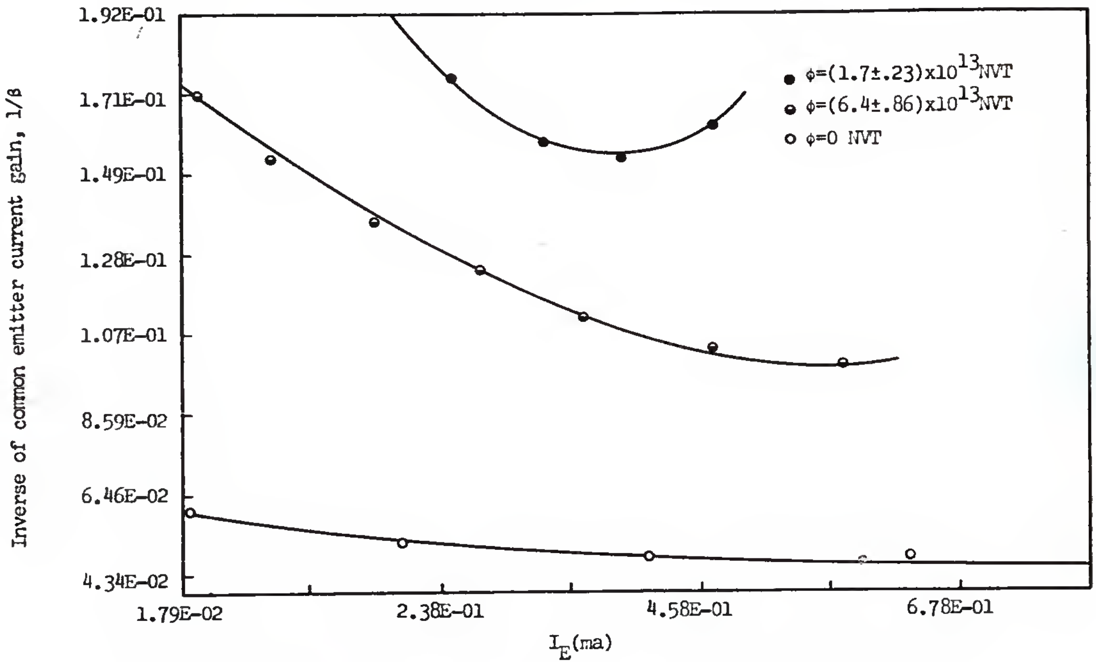

Figure 38. Typical variation of 1/β as a function of flux and emitter current for a L5405 PNP Ge Transistor with the collector voltage at 3 volts

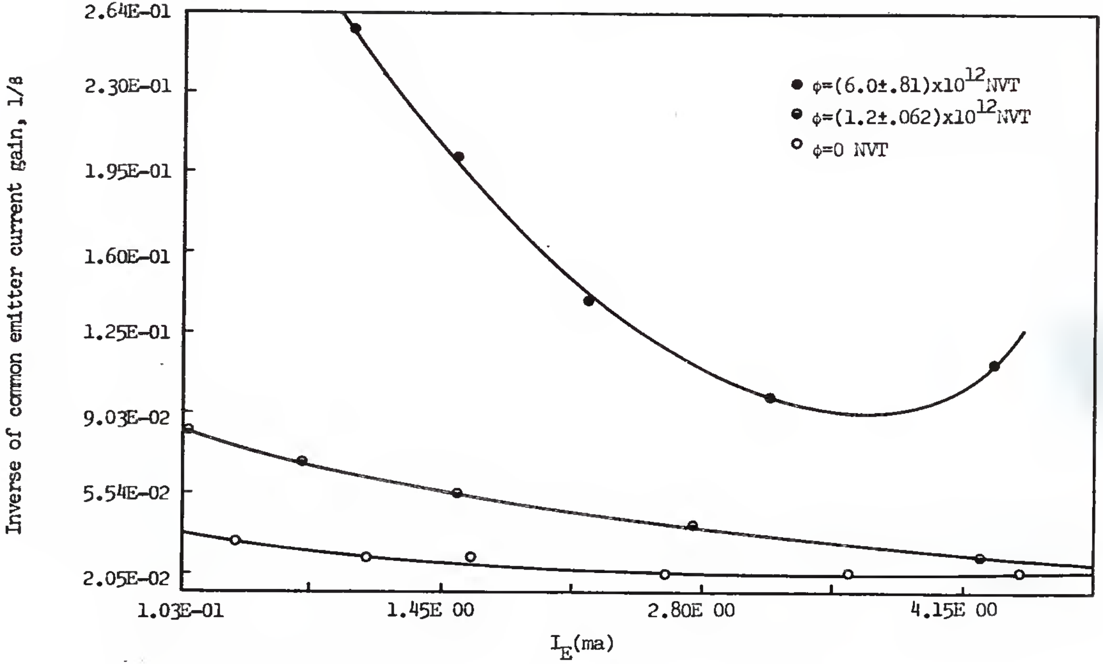

Figure 39. Typical variation of $1/\beta$ as a function of flux and emitter current for a T1041 PNP Ge Transistor with the collector voltage at 2 volts

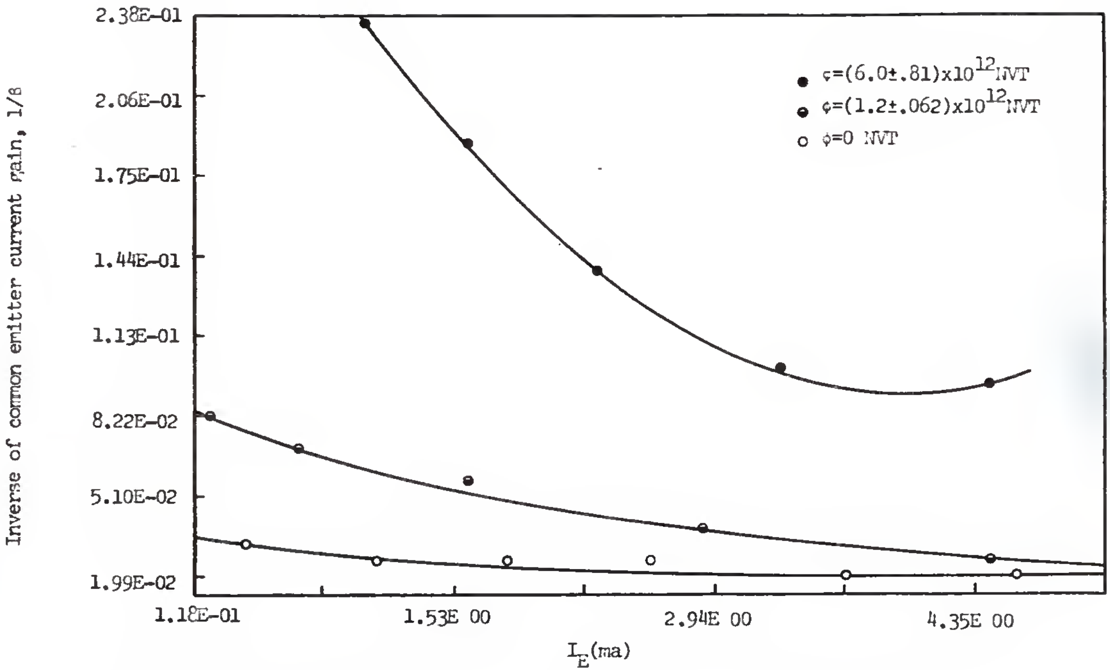

Figure 40. Typical variation of 1/β as a function of flux and emitter current for a T1041 PNP Ge Transistor with the collector voltage at 3 volts

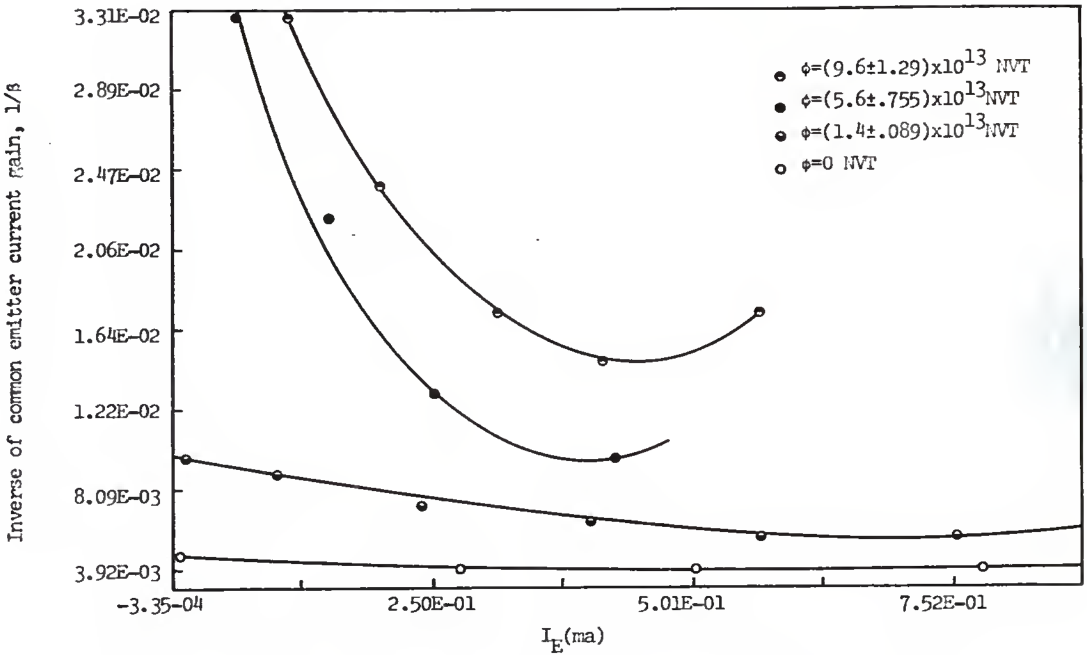

Figure 41. Typical variation of $1/\beta$ as a function of flux and emitter current for a T1166 PNP Ge Transistor with the collector voltage at 2 volts

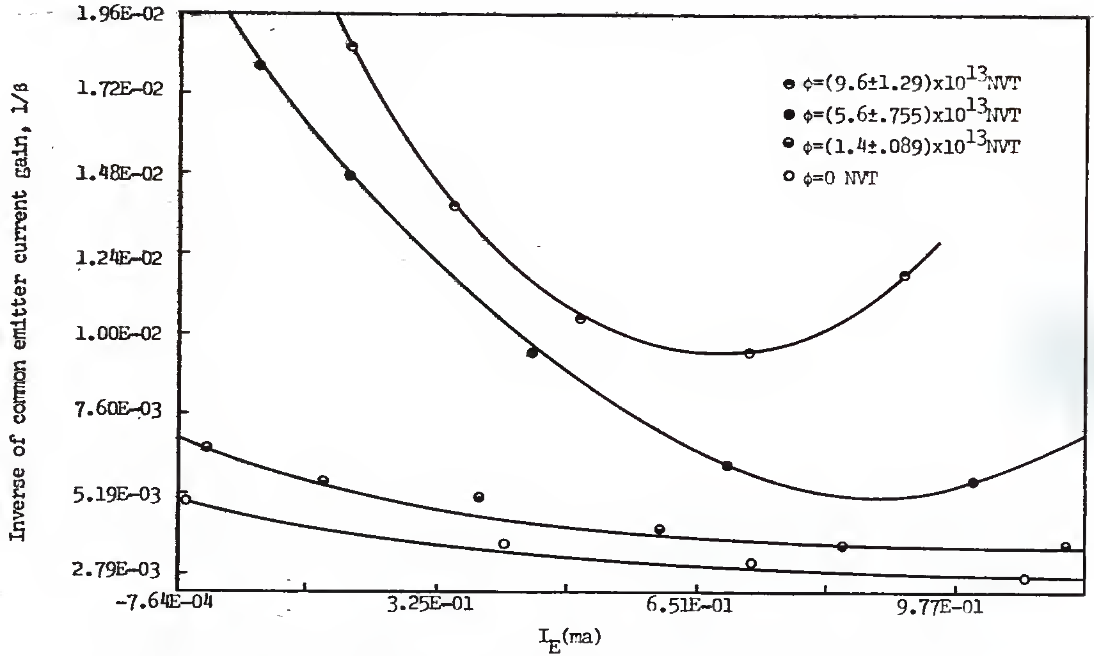

Figure 42. Typical variation of $1/\beta$ as a function of flux and emitter current for a T1166 PNP Ge Transistor with the collector voltage at 3 volts

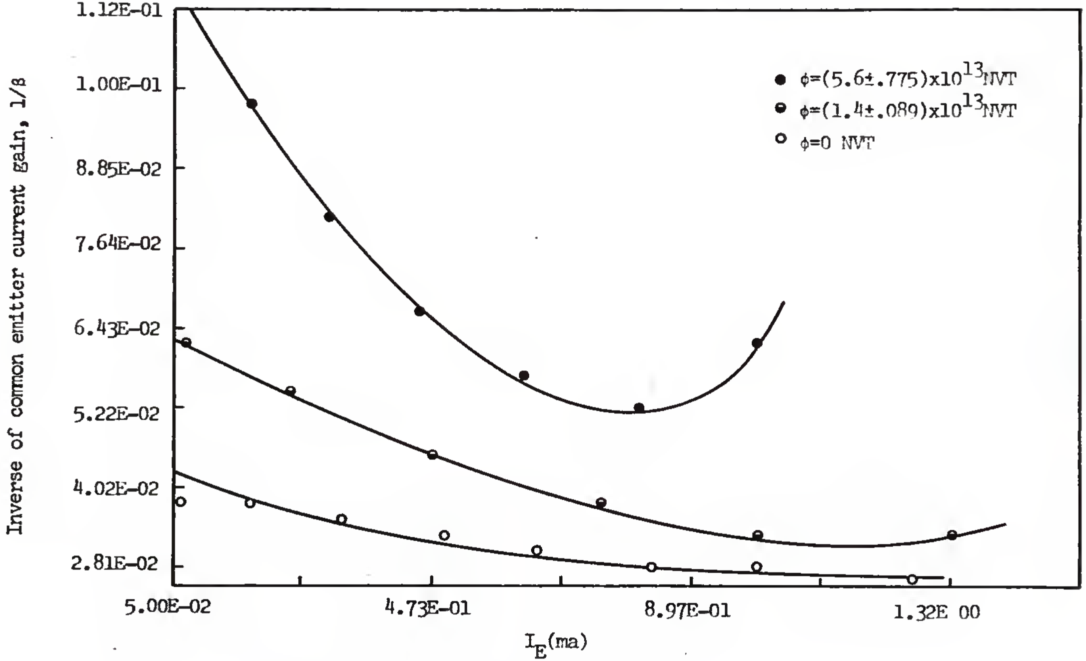

Figure 43. Typical variation of 1/β as a function of flux and emitter current for a 2N139 PNP Ge Transistor with the collector voltage at 2 volts

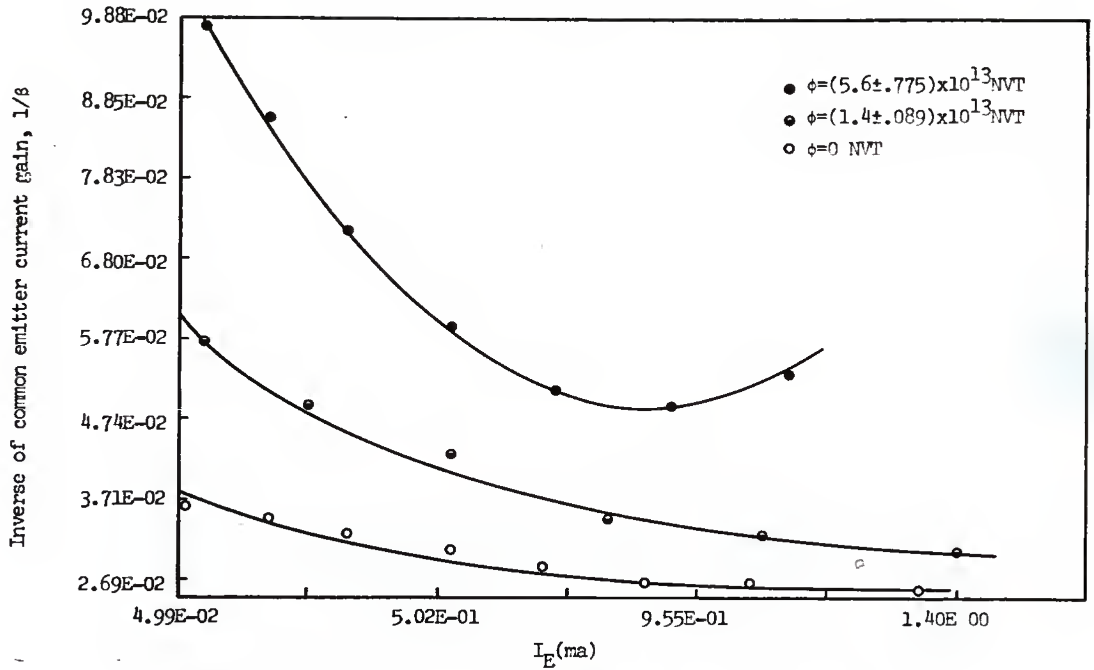

Figure 44. Typical variation of 1/β as a function of flux and emitter current for a 2N139 PNP Ge Transistor with the collector voltage at 3 volts

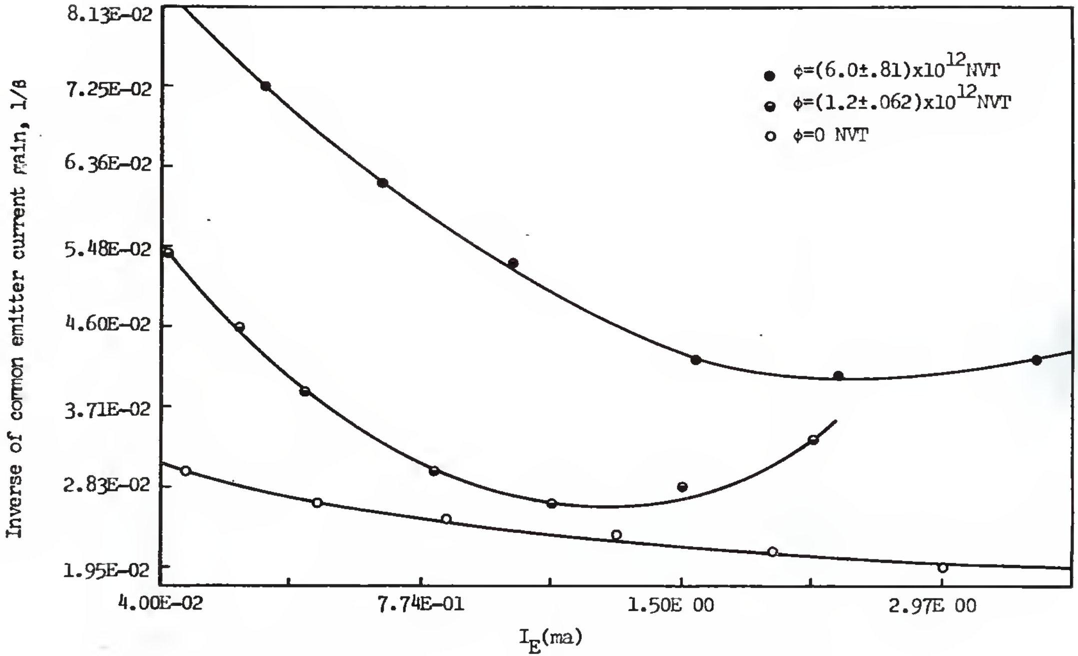

Figure 45. Typical variation of $1/\beta$ as a function of flux and emitter current for a 4JD1E17 PNP Ge Transistor with the collector voltage at 2 volts

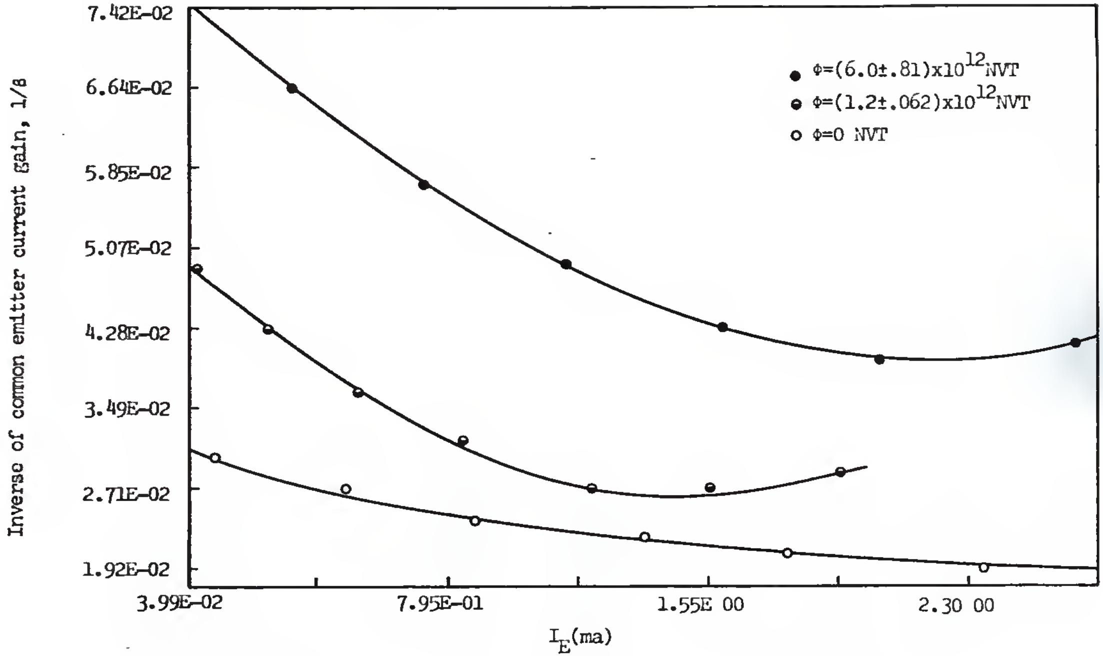

Figure 46. Typical variation of $1/\beta$ as a function of flux and emitter current for a 4JD1E17 PNP Ge Transistor with the collector voltage at 3 volts

error in ϕ.

5. Once $\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E}$ is known for different values of I_E, equations (123), (124) and (108) can be used to compute K_j, $h(Z)$ and α_j.
6. Finally the computer plots values of $h(Z)$ and $(\alpha_j \cdot h(Z))$ as a function of I_E; see Figures 47 through 55.

The above six steps are also repeated for collector voltages of two and three volts.

5.3 Results

For each transistor analyzed the inverse of common emitter current gain, $1/\beta$, as a function of I_E was determined at several neutron dose levels; in each case the graphs for the most representative transistor of a given type are shown. For a collector voltage of one volt, the results are presented in Figures 21-29 and for collector voltages of two and three volts they are shown in Figures 30-46. These graphs show that for a constant value of emitter current, $1/\beta$ increases with the accumulated neutron dose i.e., β decreases with integrated flux.

The values of lifetime damage constant, K_j, for the 2N94A transistors irradiated at K.S.U. are shown in Table V (It should be noted that the larger the value of K_j, the smaller the radiation damage susceptibility). The measured pre-irradiation collector characteristic curves for the four 2N94A transistors were not exactly the same. This is usually true for general purpose transistors since the manufacturers do not have complete control over the transistor physical parameters which determine the collector characteris-

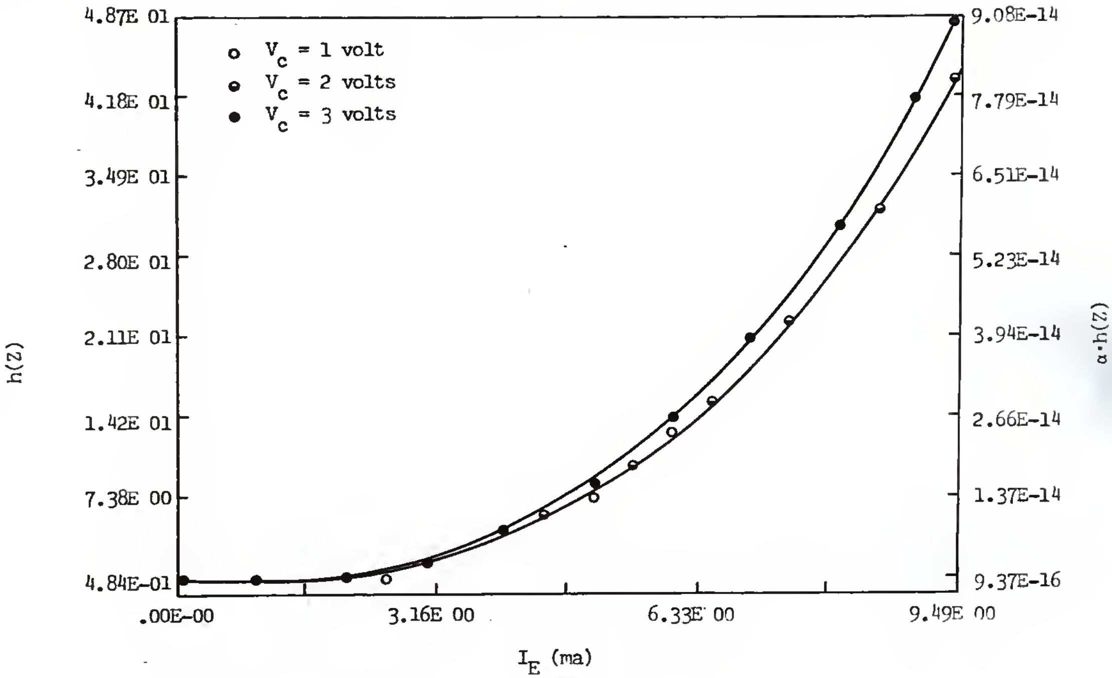

Figure 47. h(Z) and α·h(Z) as a function of emitter current for a typical 2N94A NPN Ge Transistor for collector voltages of 1, 2 and 3 volts

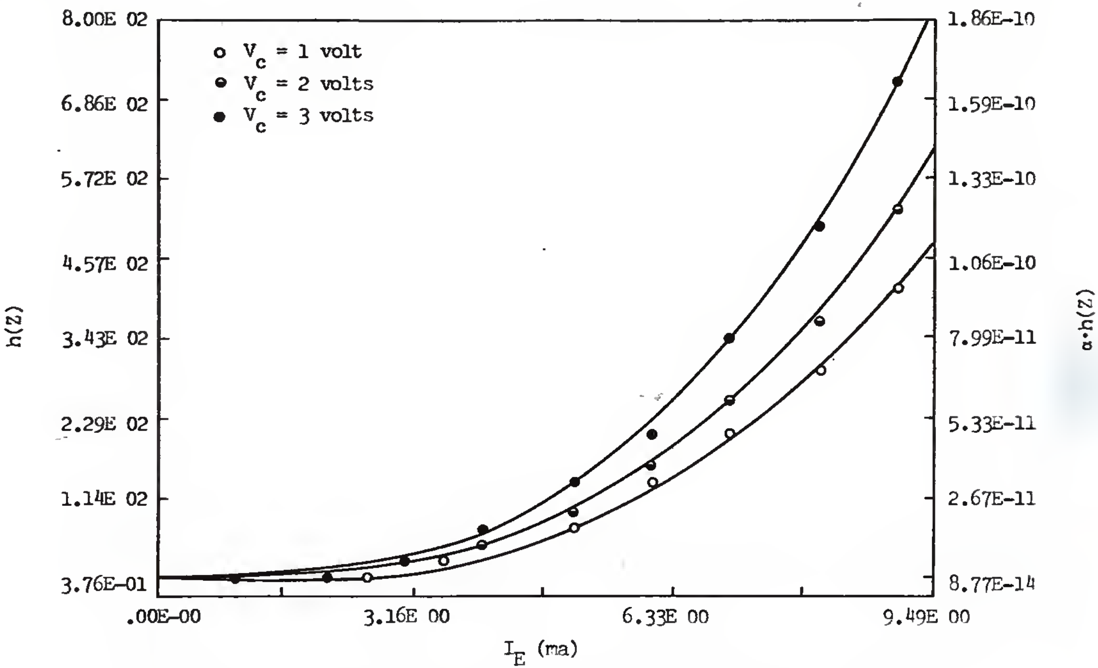

Figure 48. h(Z) and α·h(Z) as a function of emitter current for a typical 2N1155 NPN Si Transistor for collector voltages of 1, 2 and 3 volts

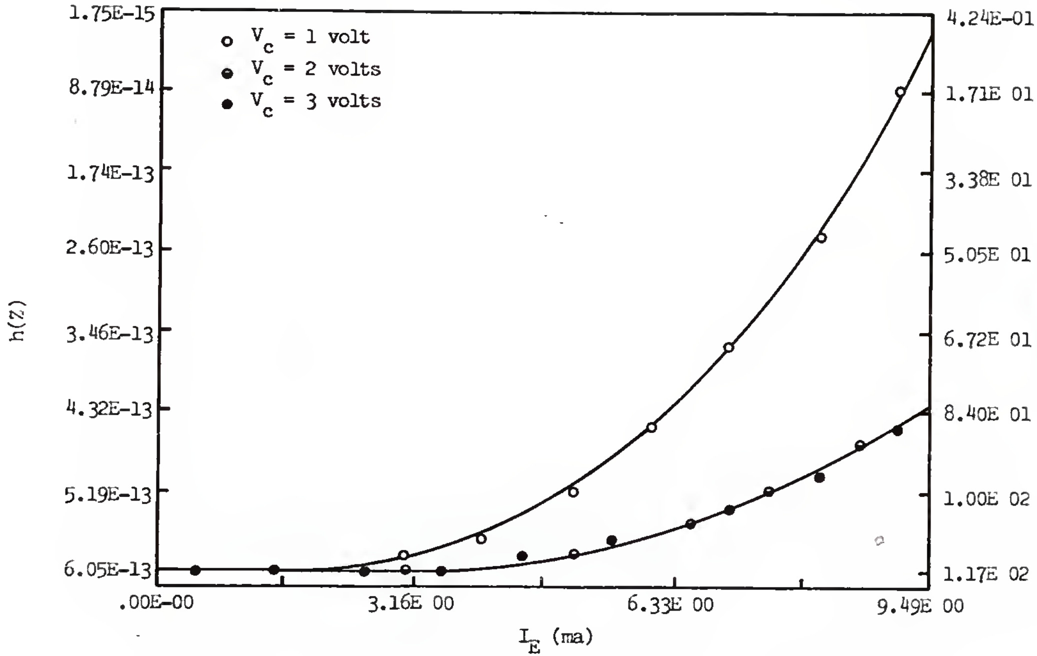

Figure 49. h(Z) and $\alpha \cdot h(Z)$ as a function of emitter current for a typical T1257 PNP Si Transistor for collector voltages of 1, 2 and 3 volts

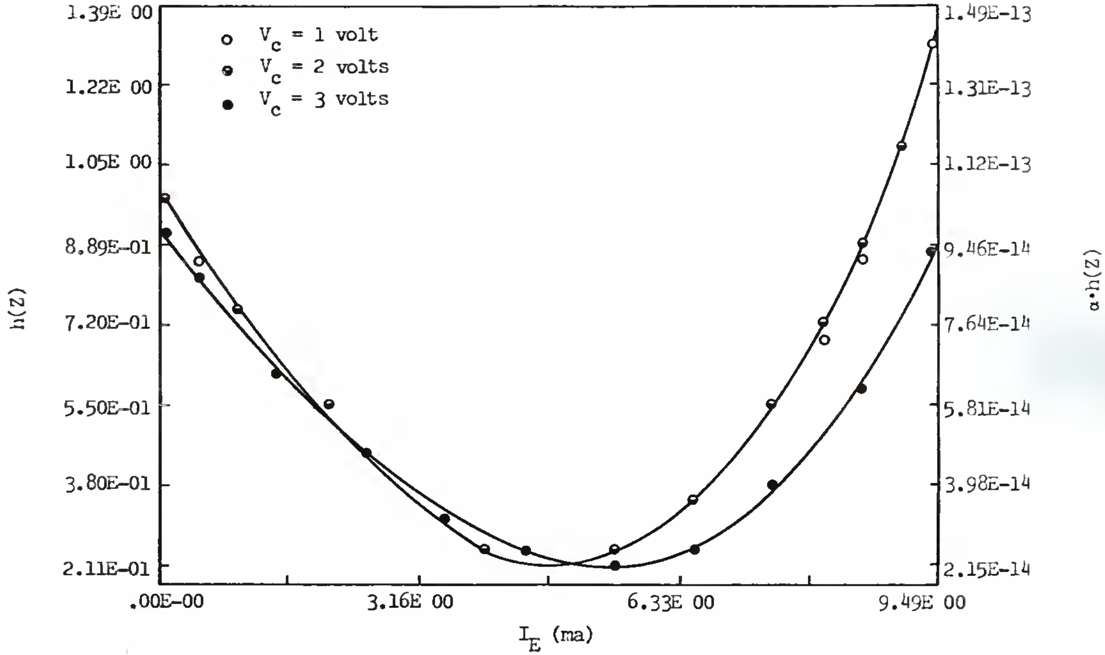

Figure 50. $h(Z)$ and $\alpha \cdot h(Z)$ as a function of emitter current for a typical 2N176 PNP Ge Transistor for collector voltages of 1, 2 and 3 volts

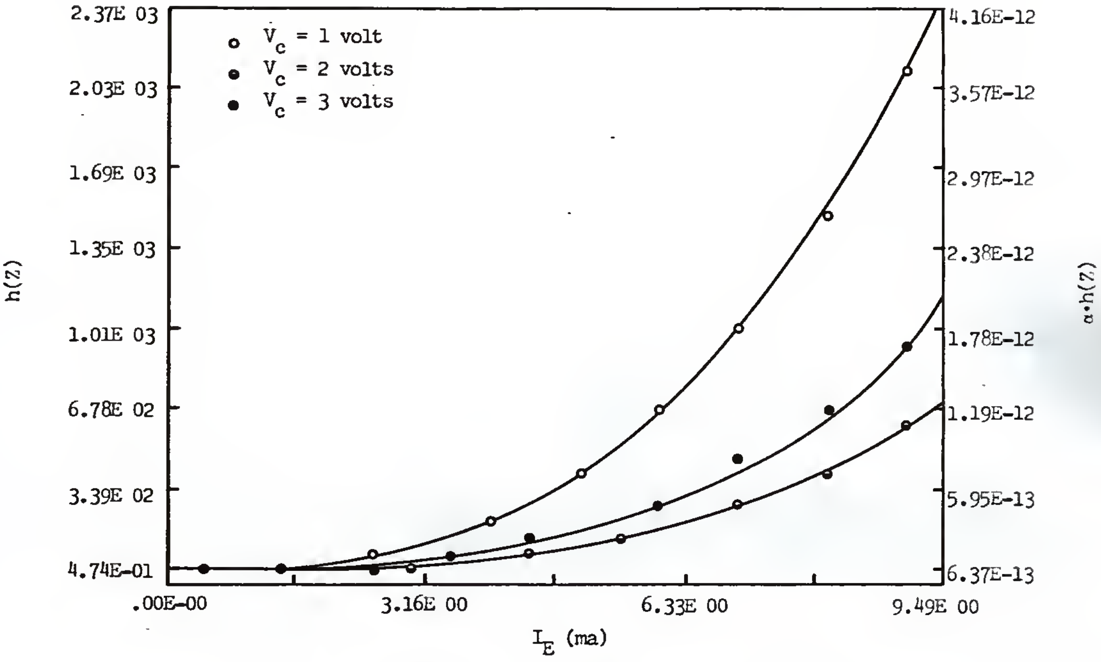

Figure 51. h(Z) and α·h(Z) as a function of emitter current for a typical L5405 PNP Ge Transistor for collector voltages of 1, 2 and 3 volts

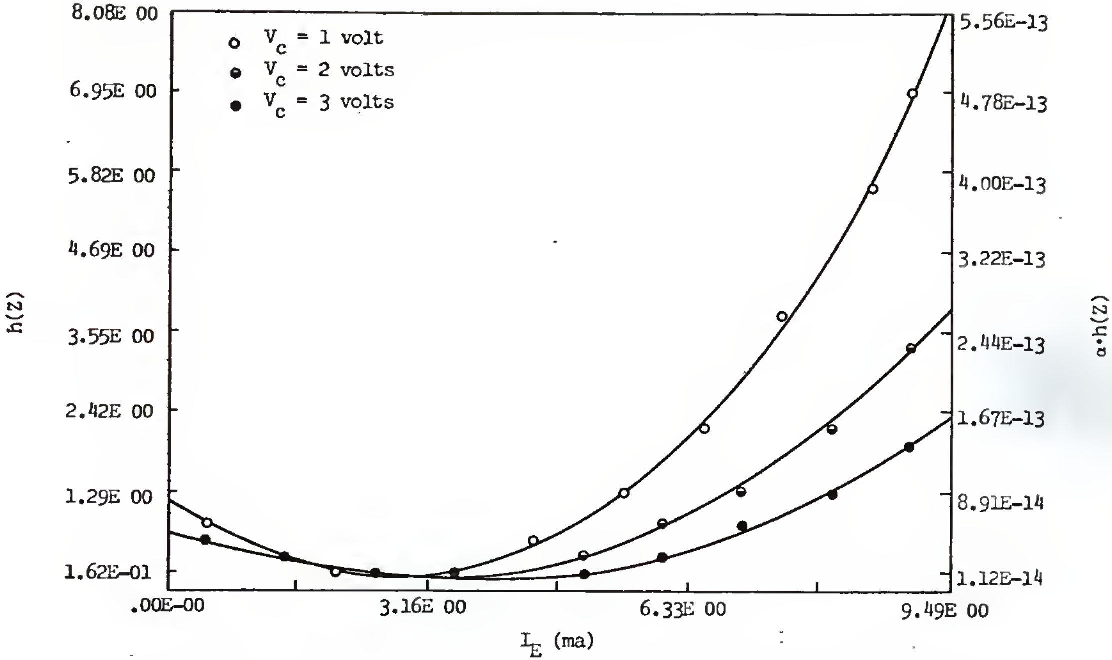

Figure 52. h(Z) and α·h(Z) as a function of emitter current for a typical T1041 PNP Ge Transistor for collector voltages of 1, 2 and 3 volts

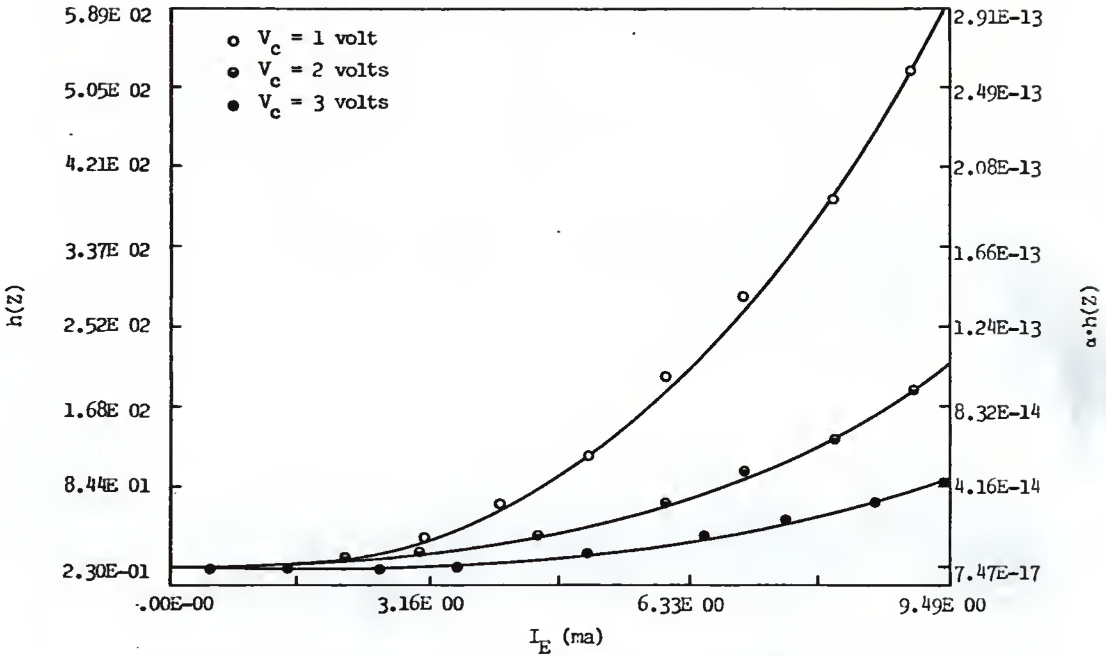

Figure 53. h(Z) and α·h(Z) as a function of emitter current for a typical T1166 PNP Ge Transistor for collector voltages of 1, 2 and 3 volts

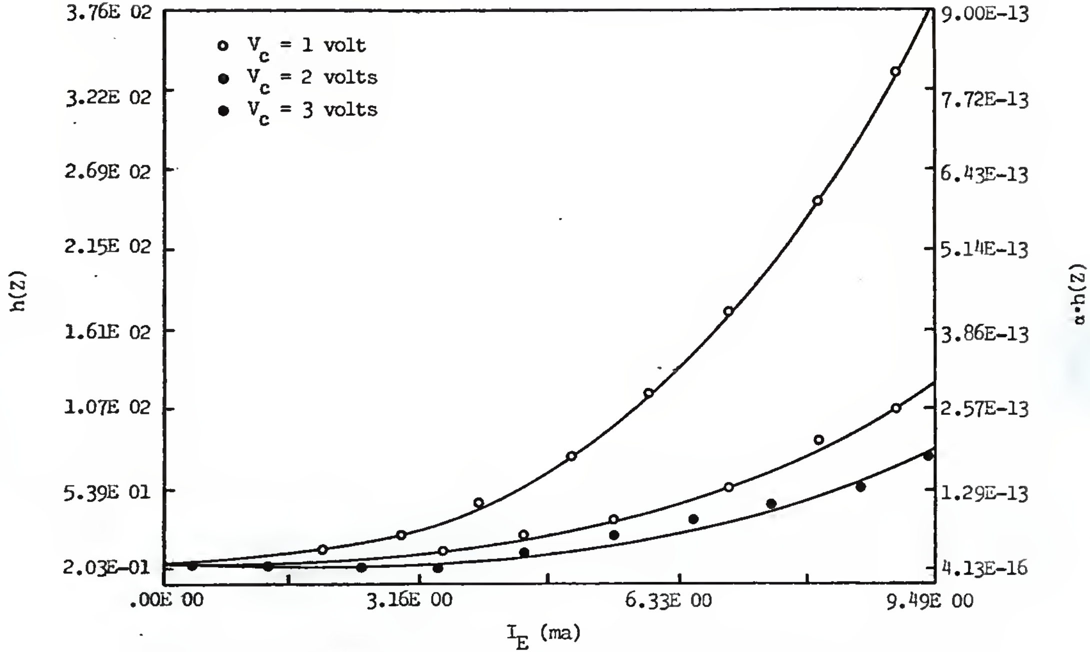

Figure 54. h(Z) and α·h(Z) as a function of emitter current for a typical 2N139 PNP Ge Transistor for collector voltages of 1, 2 and 3 volts

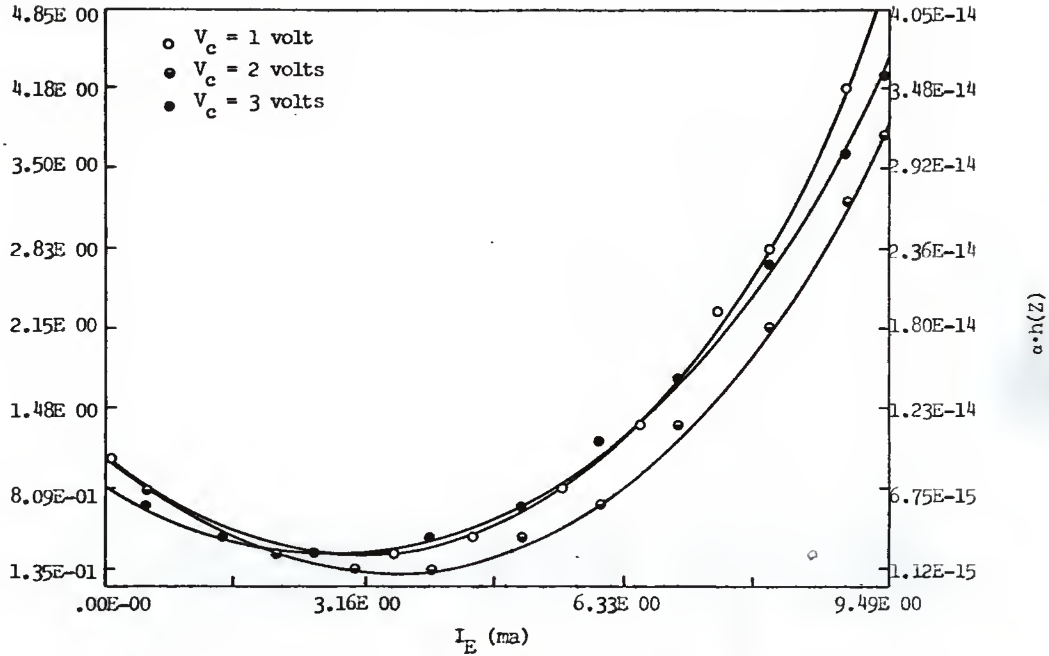

Figure 55. h(Z) and α·h(Z) as a function of emitter current for a typical 4JD1E17 PNP Ge Transistor for collector voltages of 1, 2 and 3 volts

Table V. Lifetime damage constants for 2N94A transistors

Transistor No.	Lifetime damage constant, K_j (NVT-μsec)		
	V_C = 1 volt	V_C = 2 volts	V_C = 3 volts
1	$(1.39\pm.10)\times10^{13}$	$(1.24\pm.09)\times10^{13}$	$(1.29\pm.10)\times10^{13}$
2	$(2.09\pm.15)\times10^{13}$	$(2.06\pm.15)\times10^{13}$	$(2.22\pm.17)\times10^{13}$
3	$(2.11\pm.16)\times10^{13}$	$(2.16\pm.16)\times10^{13}$	$(2.26\pm.17)\times10^{13}$
4	$(2.00\pm.15)\times10^{13}$	$(2.07\pm.24)\times10^{13}$	$(1.37\pm.34)\times10^{13}$

Table VI. Transistor damage constant for 2N94A transistors

Transistor No.	Transistor damage constant, α_j*		
	V_C = 1 volt	V_C = 2 volts	V_C = 3 volts
1	$(2.79\pm.21)\times10^{-15}$	$(3.13\pm.24)\times10^{-15}$	$(3.00\pm.23)\times10^{-15}$
2	$(1.85\pm.14)\times10^{-15}$	$(1.88\pm.14)\times10^{-15}$	$(1.74\pm.14)\times10^{-15}$
3	$(1.83\pm.14)\times10^{-15}$	$(1.79\pm.14)\times10^{-15}$	$(1.71\pm.15)\times10^{-15}$
4	$(1.93\pm.15)\times10^{-15}$	$(1.86\pm.15)\times10^{-15}$	$(2.81\pm.31)\times10^{-15}$

* The α_j values shown in Table VI can be used to calculate β_a for an irradiated transistor. For example using equation (104), $1/\beta_a = 1/\beta_b + \alpha_j\phi$, the change in β for a 2N94A transistor at an integrated flux, NVT, of 6.6×10^{12} produces a 31% difference in the value of β i.e., values of β will drop from 24.00 at zero flux to 16.53±0.05. Considering the standard deviation in α_j and its effect in calculated values of β, it will be noted that this effect will be most significant at high values of NVT. At the highest flux considered in this work, 6.6×10^{13}, the mean value of β and its standard deviation for a 2N94A transistor after irradiation are 5.78±0.03 (this is a 75.92 mean percent change in β_a as compared with the zero flux value).

tic curves. Since these parameters are not exactly the same the lifetime damage constant and the corresponding transistor damage constant, α_j, varies slightly from one 2N94A transistor to another. These variations can be seen in Tables V and VI.

Transistor lifetime damage constants for transistors with various base materials are shown in Table VII. This table indicates that these constants are a function of materials used in the transistor base region. The average value of lifetime damage constants for the four types of transistors presented in Table VII are tabulated in Table VIII. This table shows that (1) germanium transistors are more radiation resistant than similar silicon transistors, (2) that N-type silicon base are more resistant than P-type silicon base and (3) that P-type germanium base are more resistant than N-type germanium base transistors.

Merrill and Bilinski (23), using a somewhat different analysis than used here, have calculated lifetime damage constants (see Table XI). However their analysis neglects the electric field effect in the base region and does not consider the dependency of K_j on collector voltage. Their results concur with ours that germanium transistors are more radiation resistant than similar silicon transistors and that N-type silicon base are more resistant than P-type silicon base transistors. The results in Table IX also show that N-type germanium base are more radiation resistant than P-type germanium base; this result is not consistant with that of the writer.

Tables V, VII, and VIII show that K_j varies with the collector operating voltage. However this variation is anomalous and as yet no reasonable theoretical explanation exists. From Tables V, VII, and VIII and graphs of $1/\beta$

Table VII. Transistor lifetime damage constants* at different collector voltages

Transistor Analyzer		Lifetime damage constant, K_J (NVT-μsec) V_C = 1 volt	V_C = 2 volts	V_C = 3 volts
4JD1E17	PNP Ge	$(2.27\pm.31)\times10^{13}$	$(2.33\pm.32)\times10^{13}$	$(2.56\pm.36)\times10^{13}$
2N139	PNP Ge	$(1.08\pm.14)\times10^{13}$	$(1.64\pm.22)\times10^{13}$	$(1.94\pm.27)\times10^{13}$
T1166	PNP Ge	$(9.83\pm1.42)\times10^{12}$	$(1.38\pm.20)\times10^{13}$	$(1.86\pm.27)\times10^{13}$
T1041	PNP Ge	$(2.82\pm.40)\times10^{12}$	$(3.25\pm.46)\times10^{12}$	$(3.69\pm.52)\times10^{12}$
L5405	PNP Ge	$(1.47\pm.18)\times10^{12}$	$(1.99\pm.23)\times10^{12}$	$(1.77\pm.20)\times10^{12}$
2N176	PNP Ge	$(8.25\pm1.36)\times10^{12}$	$(8.04\pm1.30)\times10^{12}$	$(9.92\pm1.48)\times10^{12}$
T1257	PNP Si	$(1.07\pm.14)\times10^{12}$	$(1.38\pm.19)\times10^{12}$	$(2.19\pm.31)\times10^{12}$
2N115	NPN Si	$(1.17\pm.16)\times10^{11}$	$(1.13\pm.16)\times10^{11}$	$(1.11\pm.15)\times10^{11}$
2N94A†	NPN Ge	$(1.90\pm.21)\times10^{13}$	$(1.88\pm.20)\times10^{13}$	$(1.79\pm.36)\times10^{13}$

* Collector characteristic data obtained from literature

† Average of values in Table V

Table VIII. Lifetime damage constants for fast neutrons *

Transistor Base Material	K_j (NVT-μsec) V_C = 1 volt	V_C = 2 volts	V_C = 3 volts
N-type Ge	$(9.32\pm1.8)\times10^{12}$	$(1.11\pm.24)\times10^{13}$	$(1.29\pm.33)\times10^{13}$
P-type Ge	$(1.90\pm.21)\times10^{13}$	$(1.88\pm.20)\times10^{13}$	$(1.79\pm.36)\times10^{13}$
N-type Si	$(1.07\pm.14)\times10^{12}$	$(1.38\pm.19)\times10^{12}$	$(2.19\pm.31)\times10^{12}$
P-type Si	$(1.17\pm.16)\times10^{11}$	$(1.13\pm.16)\times10^{11}$	$(1.11\pm.15)\times10^{11}$

* This is the summary of results from Table VII.

Table IX. Lifetime damage constants for fast neutrons (23)

Material	Transistor Type	K_j (NVT-μsec)
N-type Ge	PNP Ge	$(5.0\pm2.0)\times10^{13}$
P-type Ge	NPN Ge	$(2.4\pm0.4)\times10^{13}$
N-type Si	PNP Si	$(2.8\pm0.8)\times10^{12}$
P-type Si	NPN Si	$(3.2\pm1.1)\times10^{12}$

vs I_E it is clear that any one transistor operated at a particular current and voltage experiences minimum damage. It is not possible as yet to predict a priori what this current and voltage is.

Values of α_j calculated for the four 2N94A transistors are tabulated in Table VI for three values of the collector voltages. Curves of h(Z) and $(\alpha_j \cdot h(Z))$ as a function of emitter current are shown for several transistors in Figures 47-55. These graphs indicate that each transistor experiences the least damage at certain values of the emitter current and collector voltage. In addition these operating conditions are not the same for different types of transistors. However it is also clear from Table VI that for a particular type of transistor (e.g. the 2N94A) these operating conditions are very similar. Therefore, although it is not possible to predict theoretically the values of emitter current and collector voltage that will produce minimum damage, this information can be obtained empirically using a transistor of the same type.

It is generally noted that an increase in $f_{c\alpha}$ will decrease the transistor damage constant; this can be noted by comparing the results for the 4JD1E17, 2N139, T1166, and 2N176 transistors. Comparison of α_j for these transistors, Table X, implies that as $f_{c\alpha}$ increases, the transistor damage constant decreases. Therefore it can be said that transistors with high cutoff frequency (small base width) are more radiation resistant than those with low cutoff frequency (large base width). However there are exceptions to this observation, for example the T1041 transistor. This transistor has the same cutoff frequency as the 4JD1E17 but its α_j is higher (see Table X); a result possibly due to a difference in the manufacturing process since they

Table X. Transistor damage constants at different collector voltages

Transistor Analyzed	Transistor damage constant, α_J		
	V_C = 1 volt	V_C = 2 volts	V_C = 3 volts
4JD1E17	$(8.53\pm1.17)\times10^{-15}$	$(8.32\pm1.14)\times10^{-15}$	$(7.30\pm1.02)\times10^{-15}$
2N139	$(2.39\pm.33)\times10^{-15}$	$(1.57\pm.21)\times10^{-15}$	$(1.33\pm.18)\times10^{-15}$
T1166	$(4.93\pm.71)\times10^{-16}$	$(3.50\pm.50)\times10^{-16}$	$(2.60\pm.37)\times10^{-16}$
T1041	$(6.88\pm.97)\times10^{-14}$	$(5.96\pm.83)\times10^{-14}$	$(5.25\pm.74)\times10^{-14}$
L5405	$(1.75\pm.20)\times10^{-15}$	$(1.29\pm.15)\times10^{-15}$	$(1.45\pm.17)\times10^{-15}$
2N176	$(1.04\pm.01)\times10^{-13}$	$(1.07\pm.01)\times10^{-13}$	$(9.77\pm0.56)\times10^{-14}$
T1257	$(5.15\pm.65)\times10^{-15}$	$(3.99\pm.55)\times10^{-15}$	$(2.53\pm.34)\times10^{-15}$
2N1155	$(2.20\pm.31)\times10^{-13}$	$(2.27\pm.31)\times10^{-13}$	$(1.11\pm.15)\times10^{-13}$

are produced by two different companies.

The value of flux used in plotting each collector characteristic curve is the flux incident on the front face of a transistor. If we assume the flux from the neutron generator is similar to that from an isotropic point source, then the neutron flux incident on any particular part of the transistor can be calculated according to a $1/R^2$ dependence. To check the validity of the point source assumption, a series of foils was placed in front of the neutron generator in the pattern shown in Figure 56. After irradiation, the foils were counted and the corrected activity was plotted against the distance from the neutron generator cooling jacket, Figure 57. The slope of this line did not show $1/R^2$ dependence; however if two centimeters were added to each distance then the slope of the new line did show this dependence. It is interesting to note that the tritium target is about two centimeters from the cooling jacket end of the neutron generator.

5.4 Conclusions

Based upon (1) the experimental results, (2) analysis of the literature data, and (3) the discussion presented in earlier sections, the following conclusions are drawn:

1. Transistor current gain decreases with the integrated neutron flux incident on the transistor. The extent of gain loss is a function of transistor base material, collector voltage and emitter current.
2. For a given transistor type, the transistor with wider base width experiences more damage.
3. The lifetime damage constant is a function of semiconductor base

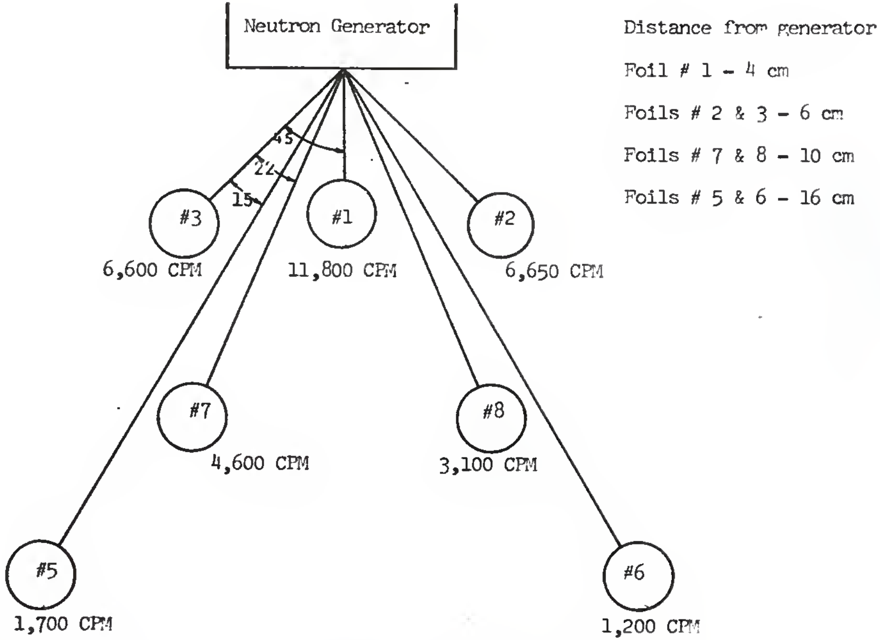

Figure 56. Position of foils with respect to neutron generator and corresponding CPM for each foil

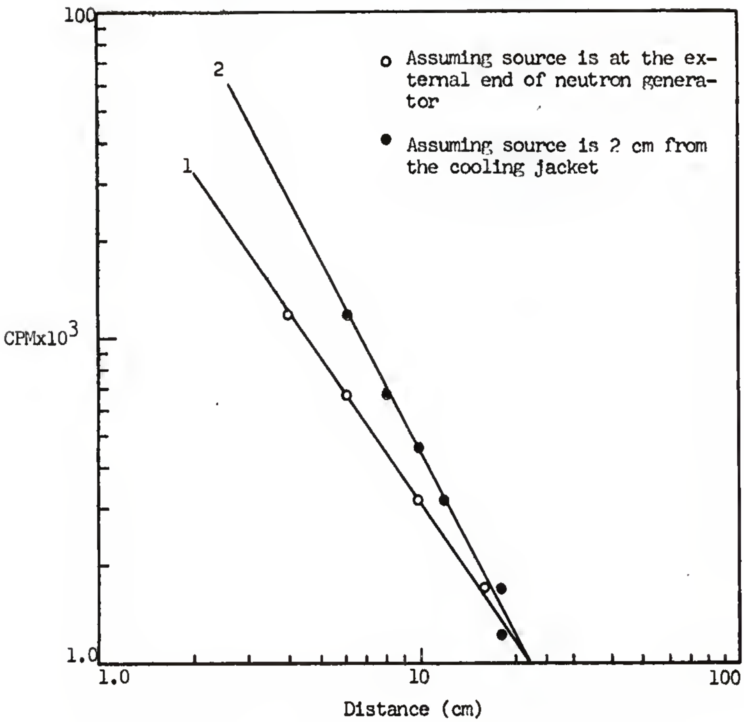

Figure 57. Decay of activity of neutron generator as a function of distance from the source

material. Germanium transistors are more radiation resistant than similar silicon transistors.

4. The lifetime damage constant for P-type germanium base transistors is higher than for N-type; the lifetime damage is higher for N-type silicon base transistors than for P-type.
5. The transistor damage constant for a particular transistor can be decreased by operating at a certain collector voltage and emitter current.
6. α_j and K_j were found to be convient means of reporting the transistor and lifetime damage respectively; the writer suggests that other writers use these same parameters in reporting their work.

6.0 ACKNOWLEDGMENT

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7.0 LITERATURE CITED

1. Aukerman, L. K.
   The Effects of Nuclear Radiation on Semiconductor Materials, REIC Report No. 1, Battele Memorial Institute, March, 1959.

2. Aukerman, L. K. and Reid, F. J.
   Energy Levels Produced in Semiconductors by High Energy Radiation, REIC Technical Memorandum No. 4, Battele Memorial Institute, July, 1958.

3. Cleland, J. W., Crawford, J. H., and Pigg, J. C.
   Fast Neutron Bombardment of N-type Germanium, Phys. Rev., Vol. 98, pp. 1742, 1955.

4. Cleland, J. W., Crawford, J. H., and Holmes, D. K.
   Fast Neutron Bombardment of P-type Germanium, Phys. Rev., Vol. 99, pp. 1170, 1955.

5. Dekker, A. J.
   Solid State Physics, Englwood Cleffs, New Jersey, 1962.

6. Dunlap, C. W.
   An Introduction to Semiconductors, Wiley, New York, 1957.

7. Easely, J. W. and Dooley, J. A.
   On the Neutron Bombardment Reduction of Transistor Current Gain, Journal of Applied Physics, Vol. 31, pp. 1024, 1960.

8. Fitchen, C. F.
   Transistor Circuit Analysis and Design, Van Nostrand Company, Princeton, New Jersey, 1960.

9. Golchert, N. W.
   Measurement of Cross Sections Produced by 14.1 Mev Neutrons on Technetium-99, Master Thesis, Illinois Institute of Technology, Chicago, Illinois, 1965.

10. Gordon, Frederick
    Nontransient Nuclear Reactor Experiments on Power Transistors, U. S. Army Signal Research and Development Laboratory, Fort Manmouth, New Jersey, 1959.

11. Greiner, R. A.
    Semiconductor Devices and Applications, McGraw-Hill, New York, 1961.

12. Hannay, N. B.
    Semiconductors, Reinhold Publishing Corporation, New York, February, 1959.

13. Harrison, W. A. and Seitz, F.
On the Theory of Radiation Damage, Phys. Rev., Vol. 98, pp. 1530A, 1955.

14. Harwood, J. J., Hausner, H. H., and Morse, J. G.
The Effects of Radiation on Materials, Reinhold Publishing Corporation, New York, 1958.

15. Hicks, D. A., Keller, J. B., Robinson, J. B., Durkee, R. K., Orr, J.R., and Clarke, B. M.
Radiation Damage to Transistors, AD244806, Boeing Airplane Company, 1958.

16. Hood, J. A.
Degradation of NPN Silicon Planar Transistors with Bombardment by High-Energy Neutrons, Sandia Corporation, Albuquerque, New Mexico, July 1965.

17. Kleitman, D. and Fan, H. Y.
Purdue University Report to Signal Corporation, RF1258, March 1957.

18. Kinchin, G. H. and Rease, R. S.
Report Progress in Physics, Vol. 18, pp. 16, 1955.

19. Klontz, W.
Neutron Irradiation of Silicon Semiconductors, Purdue University Special Report, 1957.

20. Lofferski, J. J.
Analysis of the Effect of Nuclear Radiation on Transistors, Journal of Applied Physics, Vol. 29, pp. 35, 1958.

21. Lofferski, J. J. and Rappaport, P.
Electron Voltaic Study of Electron Bombardment Damage and Its Thresholds in Ge and Si, The Physical Review, Vol. 98, pp. 1861, 1955.

22. Longo, T. A.
Neutron Irradiation of Silicon Semiconductors, Purdue University Special RUport to Signal Corp., August 1957.

23. Merril, R. and Bilinski, J. R.
Selecting Transistors for Radiation Environments, Electronic, December 1959.

24. Messenger, G. C. and Spratt, J. P.
The Effects of Nuclear Irradiation on Germanium and Silicon, Proceeding of the IRE, Vol. 46, pp. 1038, 1958.

25. Mitchell, J. C. and Perkins, J. R.
Nuclear Radiation Tolerant Circuitry Design and Test, AD261975, Wright Air Development Division, 1961.

26. Poblenz, F. W.
Analysis of Transistor Failure in a Nuclear Environment, IEEE Transactions on Nuclear Science, January 1963.

27. Rapport, P.
Minority Carrier Lifetime in Semiconductors as a Sensitive Indicator of Radiation Damage, Phys. Rev., Vol. 94, pp. 1409A, 1954.

28. Reid, F. J.
The Effects of Nuclear Radiation on Semiconductor Materials, REIC Report No. 1, Battele Memorial Institute, December 1957.

29. Sah, C. T.
IRE Transactions on Electron Devices, ED-9#1, pp. 94, January 1962.

30. Seitz, F.
Semiconductor Theory, Discussions of the Faraday Society, Vol. 5, pp. 271, 1949.

31. Shelton, R. D.
The Effects of Nuclear Radiation on Electronic Components, AD91458, Scientific Report No. 3, Admiral Corporation, 1956.

32. Shockly, W., Sparks, M., and Teal, C. K.
The P-N Junction Transistors, Phys. Rev., Vol. 83, pp. 151, July 1951.

33. Shive, J. H.
The Properties, Physics, and Design of Semiconductor Devices, Van Nostrand Company Inc., pp. 287, 1959.

34. Snyder, W. S., and Neufeld, J.
Phys. Rev., Vol. 97, pp. 1636, 1955.

35. Vides, L. B.
The Physical Theory of Transistors, McGraw Hill, New York, 1963.

36. Walston, J. A. and Miller, J. R.
Transistor Circuit Design, Texas Instruments Incorporated, McGraw Hill, New York, 1963.

37. Webster, W. M.
On the Variation of Junction Transistor Current Amplification Factor With Emitter Current, Proceedings of IRE, Vol. 42, pp. 914, 1954.

38. Weider, H. H.
Performance of Solid State Materials and Devices Subject to a Nuclear Radiation Flux, AD143467, U. S. Naval Ordnance Laboratory, 1957.

39. Xavier, M. A.
Research to Determine the Effects of Nuclear Radiation on Semiconductors Electronic Components, AD125271, Scientific Rbport No. 2, U. S. Air Force General Publications, 1957.

40. Xavier, M. A.
Research to Determine the Effects of Nuclear Radiation on Semiconductors Electronic Components, AD157984, Scientific Report No. 3, Inland Testing Laboratories, 1958.

8.0 APPENDICES

8.1 APPENDIX A

Energy transferred to primary knock-on by fast neutrons

Elastic scattering of a neutron with a nucleus results in a discrete loss of neutron energy and subsequent transfer of energy to the recoil nucleus. The incident neutron with mass M_1, and velocity v_1 collides with the stationary nucleus with mass M_2. Conservation of kinetic energy and momentum in the L-system requires that

$$1/2\ M_1\ v_1^2 = 1/2\ M_1\ V_1^2 + 1/2\ M_2\ V_2^2 \tag{A-1}$$

$$M_1\ v_1 = M_1\ V_1 + M_2\ V_2. \tag{A-2}$$

Conservation of momentum in center of mass system gives

$$(v_1 - v_m)M_1 = M_2\ v_m. \tag{A-3}$$

Simplifying equation (A-3) gives

$$v_m = \frac{v_1\ M_1}{M_1+M_2} \tag{A-4}$$

where v_m is the speed of center of mass in L-system.

Speed of the neutron before collision in the C-system is

$$v_1 - v_m = v_1 - \frac{v_1\ M}{M_1 + M_2} = \frac{M_2\ v_1}{M_1 + M_2}\ . \tag{A-5}$$

Speeds of neutron and nucleus before collision, in the C-system are given by equations (A-4) and (A-5), respectively. The principle of the conservation of momentum requires that

$$\left[\frac{M_2\ v_1}{M_1 + M_2}\right] M_1 - \left[\frac{M_1\ v_1}{M_1 + M_2}\right] M_2 = M_1\ v_a - M_2\ v_b = 0 \tag{A-6}$$

or

$$v_a = \frac{M_2}{M_1}\ v_b \tag{A-7}$$

where v_a and v_b are speeds of neutron and nucleus after collision, respectively, Figure A-1. Applying the law of conservation of energy in C-system gives

$$1/2\left[\frac{M_2\ V_1}{M_1 + M_2}\right]^2 M_1 + 1/2\left[\frac{M_1\ v_1}{M_1 + M_2}\right]^2 M_2 = 1/2\ M_1\ v_a + 1/2\ M_2\ v_b\ . \tag{A-8}$$

Simplifying equation (A-8) and using equation (A-7) gives

$$v_a = \frac{M_1}{(M_1 + M_2)}\ v_1 \tag{A-9}$$

$$v_b = \frac{M_2}{(M_1 + M_2)}\ v_1. \tag{A-10}$$

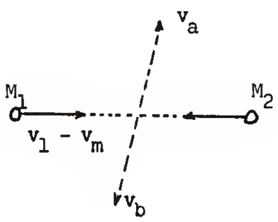


Figure A-1. Velocities in C.M. system

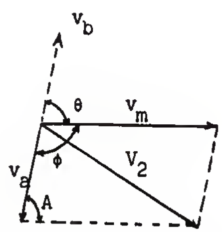


Figure A-2. Velocity of neutron in L system

The energy of the incident neutron is given as

$$E_1 = 1/2\ M_1\ v_1{}^2. \tag{A-11}$$

If V_2 is the velocity of the neutron after collision in the L-system, then by law of cosines

$$V_2{}^2 = v_b{}^2 + v_m{}^2 - 2\ v_b\ v_m\ \cos A$$

or

$$V_2{}^2 = v_b{}^2 + v_m{}^2 + 2\ v_b\ v_m\ \cos\theta. \tag{A-12}$$

Substituting equations (A-4) and (A-9) into equation (A-12), the result is

$$V_2{}^2 = \frac{v_1{}^2\ M_1{}^2 + v_1{}^2\ M_1{}^2}{(M_1 + M_2)^2} + 2\ \frac{v_1{}^2\ M_1{}^2}{(M_1 + M_2)^2}\cos\Phi$$

or

$$V_2{}^2 = \frac{2\ M_1{}^2\ (1 + \cos\Phi)}{(M_1 + M_2)^2}\ v_1{}^2. \tag{A-13}$$

The energy transferred to the knock-on is

$$E_p = 1/2\ V_2{}^2\ M_2. \tag{A-14}$$

Substituting for $V_2{}^2$ from equation (A-13) gives

$$E_p = 2\ E_1\ M_1\ M_2\ \frac{(1 + \cos\Phi)}{(M_1 + M_2)^2}\ . \tag{A-15}$$

The tigonometric laws predict that

$$\cos\Phi = -\cos(\pi - \Phi) = -\cos\theta.$$

Substituting equation (A-13) into (A-14) and using trigonometric relation $1 - \cos\theta = 2\ \sin^2(1/2X)$ results in the following equation:

$$E_p(\theta) = E_1 \frac{4\ M_1\ M_2}{(M_1 + M_2)^2} \sin^2(\theta/2). \tag{A-16}$$

The maximum energy is transferred to the recoil when $\sin^2(\theta/2) = 1$ or $\theta = 180^\circ$.

Therefore maximum recoil energy is

$$E_{p_{max}} = \frac{4\ M_1\ M_2}{(M_1 + M_2)^2} E_1. \tag{A-17}$$

8.3 APPENDIX B

Consideration of electric field in the base region

In the early section, where a relation for transistor amplification factor is developed, it was assumed that the electric field in the base region is neglibigle; however, it is possible to consider this electric field and develop some correction factors for variation of current amplification with change of electric field in the base region. Webster (32) gives a detailed analysis of this variation. A summary of his analysis is shown below.

Including the effects of the electric field in the base region, the current densities for conduction electrons and holes are

$$J_n = n\, e\, \mu_n\, E + e\, D_n\, \nabla n \qquad \text{(B-1)}$$

$$J_p = p\, e\, \mu_p\, E - e\, D_p\, \nabla p \qquad \text{(B-2)}$$

where: J_n is the current density of electrons in the base

J_p is the current density of holes in the base

n and p are electron and hole densities

μ_n and μ_p are electron and hole mobilities

D_n and D_p are electron and hole diffusion coefficients and

$E = -\nabla V$, where V is the electric potential.

Since the net charge density in the base region is essentially zero,

$$n + N_a = p + N_d \qquad \text{(B-3)}$$

where N_a and N_d are acceptor and donor ion densities. In N-type material (base region of PNP transistors), N_a is very small and it may be set equal to zero and further it is assumed that N_d is constant in the base region. This implies that

$$n \approx p + N_d \tag{B-4}$$

and

$$\nabla n = \nabla p. \tag{B-5}$$

If a transistor is to be useful, J_n must be very small compared to J_p. Hence, it can be assumed that $J_n = 0$. This assumption and equations (B-4), (B-5) and (B-1) give the following equation:

$$E = - \frac{D_n}{\mu_n} \frac{1}{N_d + p} \nabla p. \tag{B-6}$$

Substituting equation (B-6) into (B-1) and considering that $\mu_p D_n = \mu_n D_p$ gives,

$$J_p = (- e D_p \nabla p) (1 + \frac{p}{N_d + p}). \tag{B-7}$$

If $p \ll N_d$, then equation (B-7) reduces to equation (B-8).

$$J_p = - e D_p \nabla p \tag{B-8}$$

This is the same equation that was used in the text with the assumption that the electric field in the base region is negligible. Therefore in general if $p \ll N_d$, this assumption is good. This assumption is not correct for a transistor which is operating at a current density in excess of about 0.1 ampere per square centimeter. A reasonably good manner of taking into account the electric field in the base region is to simply multiply the base conductivity by the ratio $\frac{p + N_d}{N_d}$ where it appears in the equation for current amplification factor. $\frac{p + N_d}{N_d}$ is the ratio of the electron density in the base with the injected holes present to the density of electrons in the base in the absence of holes.

The ratio p/N_d is given by Webster to be equal to Z/2, where

$$Z = \frac{W\ \mu_n\ I_E}{D_p\ A\ \sigma_B} \quad \text{for PNP transistors}$$

$$Z = \frac{W\ \mu_p\ I_E}{D_n\ A\ \sigma_B} \quad \text{for NPN transistors.}$$

The theory developed in Section 2.3 predicts that

$$\frac{\partial I_{eE}}{\partial I_{pE}} = \frac{I_{eE}}{I_{pE}} = \frac{\sigma_B\ W}{\sigma_E\ L_E}\ . \tag{B-9}$$

Considering the electric field in the base region, equation (B-9) takes the following form:

$$\frac{I_{eE}}{I_{pE}} = \frac{\sigma_B\ W}{\sigma_E\ L_E}\ (1 + Z/2). \tag{B-10}$$

Taking the derivative of equation (B-10) gives

$$\frac{\partial I_{eE}}{\partial I_{pE}} = \frac{\sigma_B\ W}{\sigma_E\ L_E}\ (1 + Z/2 + \frac{I_{pE}}{2}\ \frac{\partial Z}{\partial I_{pE}}). \tag{B-11}$$

Since Z and I_{pE} are linearly related (see equations above), therefore $I_{pE}\ \frac{\partial Z}{\partial I_{pE}}$ is equal to Z. Thus the effect of differentiation is to double the Z term.

$$\frac{\partial I_{eE}}{\partial I_{pE}} = \frac{\sigma_B\ W}{\sigma_E\ L_E}\ (1 + Z) \tag{B-12}$$

Webster (32) uses a similar type of analysis to derive correction factors for the volume recombination and surface efficiency terms. The results of his analysis are shown in equations (B-13) and (B-14).

$$\frac{\partial I_{bR}}{\partial I_{pE}} = 1/2\ (\frac{W}{L_B})^2\ (1 + Z) \tag{B-13}$$

$$\frac{\partial I_{sR}}{\partial I_{pE}} = \frac{W_s\ A_s}{D_p\ A}\ g(Z) \tag{B-14}$$

where: $A_s = \pi\ d\ \Delta d$, d is the diameter of emitter and Δd is the width of the absorbing ring which is equal to W and

g(Z) and (1 + Z) are correction factors due to electric field in the base region.

Therefore the approximate equation for a current amplification factor, considering the electric field in the base region, for a PNP transistor is

$$1/\beta = \frac{s\ W\ A_s}{D_p\ A}\ g(Z) + \left[\frac{\sigma_B\ W}{\sigma_E\ L_E} + 1/2\ \left(\frac{W}{L_B}\right)^2\right]\ (1 + Z) \tag{B-15}$$

and for NPN transistor is

$$1/\beta = \frac{s\ W\ A_s}{D_n\ A}\ g(Z) + \left[\frac{\sigma_B\ W}{\sigma_E\ L_E} + 1/2\ \left(\frac{W}{L_B}\right)^2\right]\ (1 + Z). \tag{B-16}$$

The correction factors g(Z) and (1 + Z) are plotted as a function of Z in Figures (B-1) and (B-2).

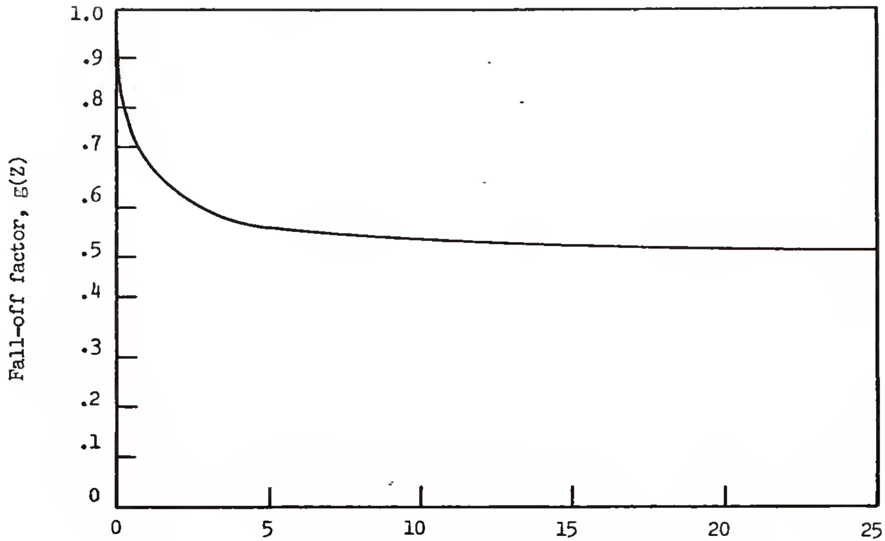


Figure B-1. The field factor as a function of Z(32)

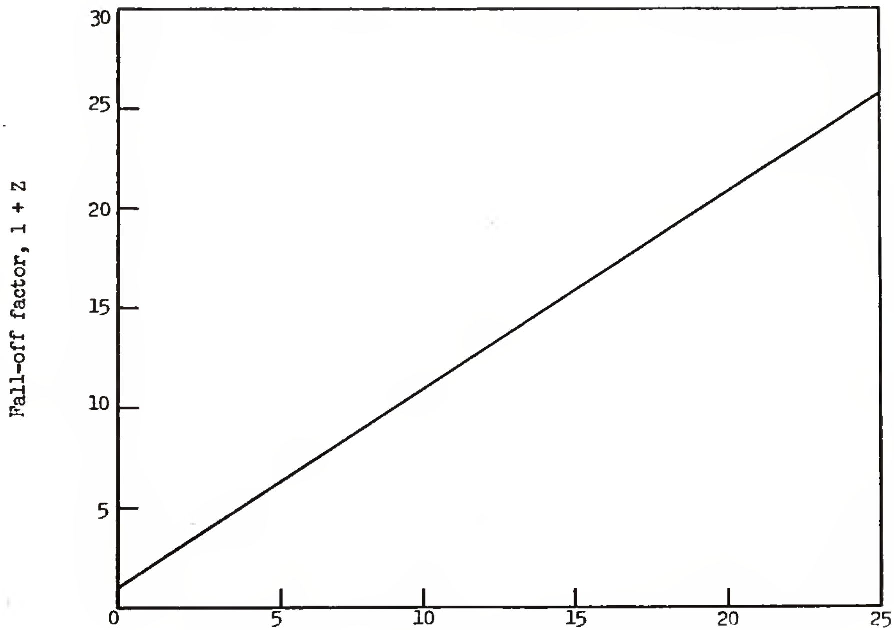


Figure B-2. The fall-off factor as a function of Z(32)

8.3 APPENDIX C

8.3.1 Description and explanation of computer programs

The group of programs discussed in this section are written to compute the damage coefficients and other pertinent information for an irradiated transistor. The programs are written for the IBM-1410 computer in the FORTRAN II language. Since the size of IBM-1410 core storage is only 40,000 characters, it is necessary to divide the program into six phases. Each phase consists of a main program and several subprograms. The following list shows the order of the main program and subprograms in each particular phase:

Phase 1; LSTSQARE, CRAM, SOLVE, RESIDU

Phase 2; LEASTSQ, SOLN

Phase 3; MAPDATTA, PLOT

Phase 4; SLOP, LSTSQ

Phase 5; DAMAGE, FINAL, CRAM, SOLVE, RESIDU

Phase 6; GRAPH, PLOT

Each one of these programs and subprograms are considered in more detail, later in this Appendix. Table C-I lists the variables utilized in these programs and Table C-II shows the block diagram for the programs. The source program is listed, logic diagram shown and the computer's input data for one transistor is tabulated in this Appendix.

Three work tapes, 6, 7 and 8, are used in these programs. The tapes are used for storing information from one phase and feeding them as input data to another phase. Approximately 35 minutes is required to compile these sets of programs and about 23 minutes to compute the results for each transistor.

Table C-I. Symbols used in program for transistor analysis

Symbol	Used in phase	Meaning
X(I)	1	Collector current
Y(I)	1	Base current
CY(I)	1	Calculated base current by polynomial fit
DY(I)	1	Inverse of transistor's common emitter current gain
CZ(I)	1	X(I) + CY(I)
B(K)	1	Coefficients of the polynomial fit to collector and base current
NDP	1,2,3,5,6	Number of data points
NCOEF	1,2,5	Minimum number of coefficients in the polynomial fit
NCOEFL	1,2,5	Maximum number of coefficients in the polynomial fit
NCM1	1,2,5	Degree of analysis
SMU	1,2,4,5	Standard deviation
RELERR	1,2,5	Relative error
X(I)	2,3	Emitter current
Y(I)	2,3	Inverse of transistor's common emitter current gain
B(K)	2	Coefficients of the polynomial fit to emitter current and $1/\beta$
SL	4	$1/\beta \; \partial I_E$
CONS1	4,5,6	h(Z)
CONS2	4,5,6	$\alpha \cdot h(Z)$

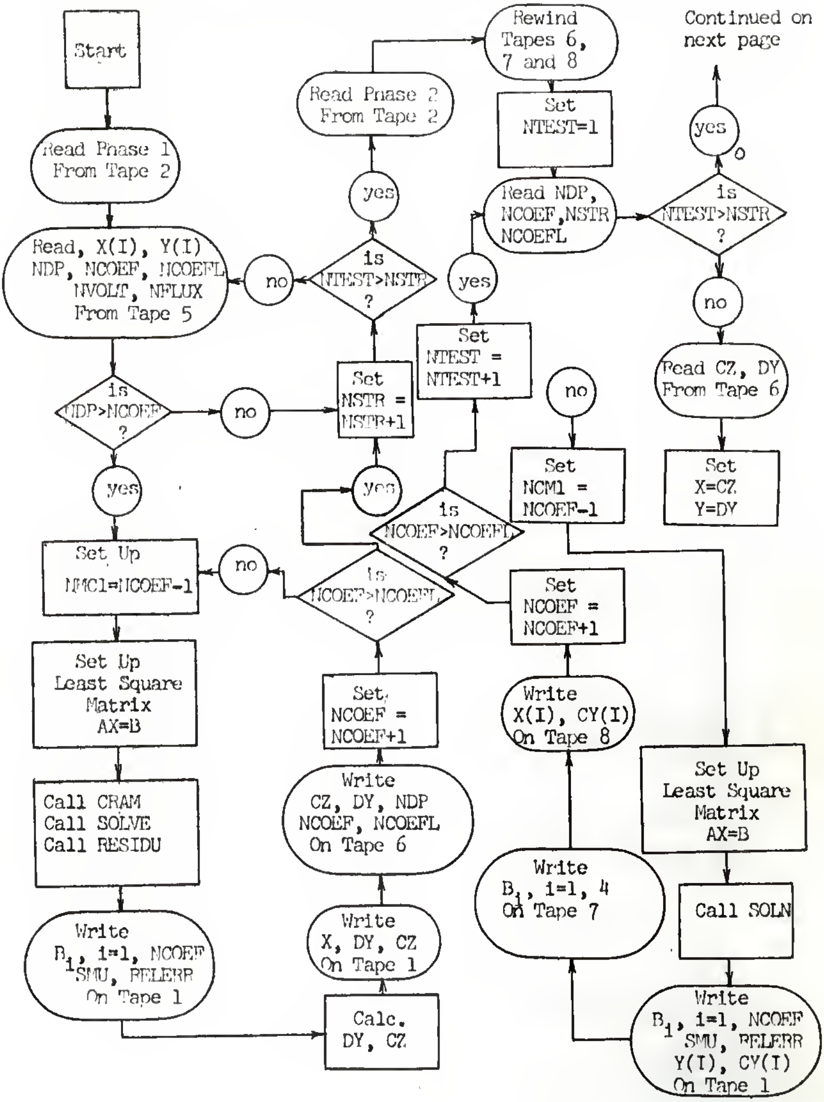
Table C-I. Continued

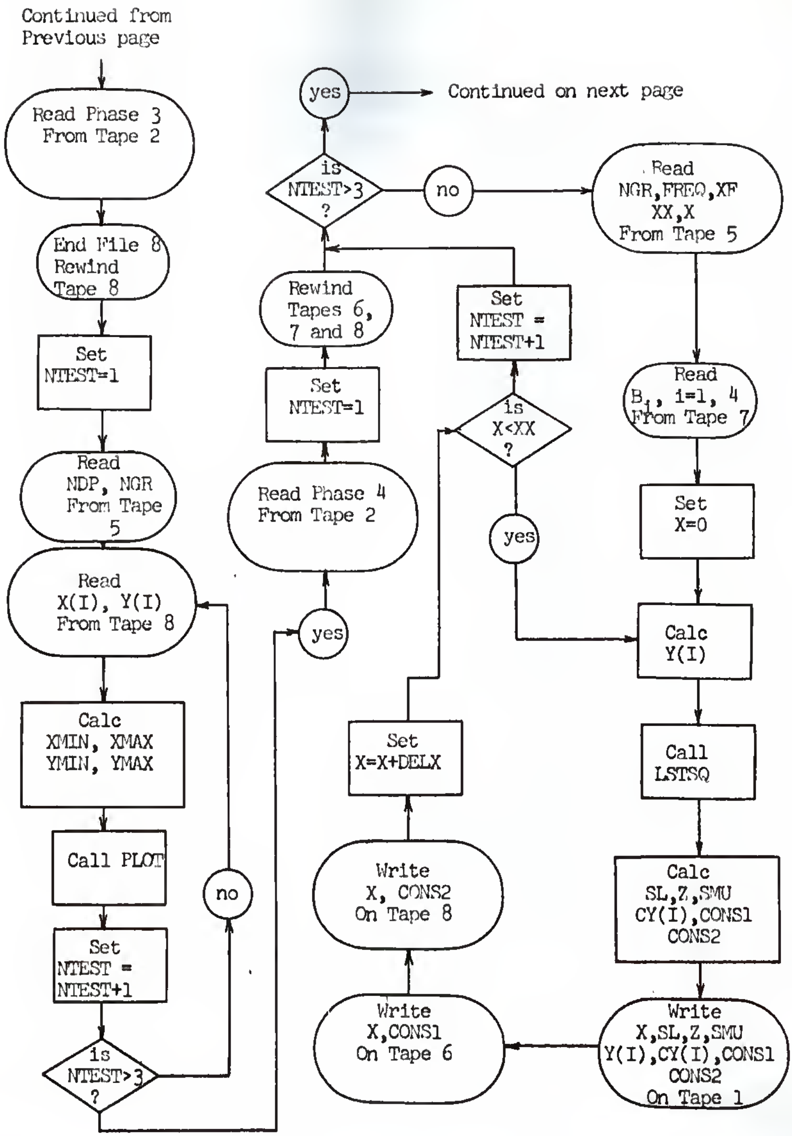
Symbol	Used in phase	Meaning
CONSK	4	Lifetime damage constant
CONSA	4	Transistor's damage constant
Z	4	Intercept of straight line through emitter current and flux points
Y(I)	4	$1/\beta$
CY(I)	4	Calculated $1/\beta$ by polynomial fit
X(I)	4,5,6	Emitter current
N	4	NGR
B(I)	5	Coefficients of polynomial fit to h(Z) and emitter current
CO1MA	6	Maximum h(Z)
CO2MA	6	Maximum h(Z)
CO1MI	6	Minimum $\alpha \cdot h(Z)$
CO2MI	6	Minimum $\alpha \cdot h(Z)$
NVOLT	1	Number of collector voltages at which the analysis is performed
NFLUX	1	Number of flux conditions
NTEST	1,2,3,4,5,6	A dummy used for testing
CY(I)	2	Calculated $1/\beta$ by polynomial fit
N	1,2	NCOEF
XMIN	3,6	Minimum emitter current
XMAX	3,6	Maximum emitter current
YMIN	3	Minimum $1/\beta$

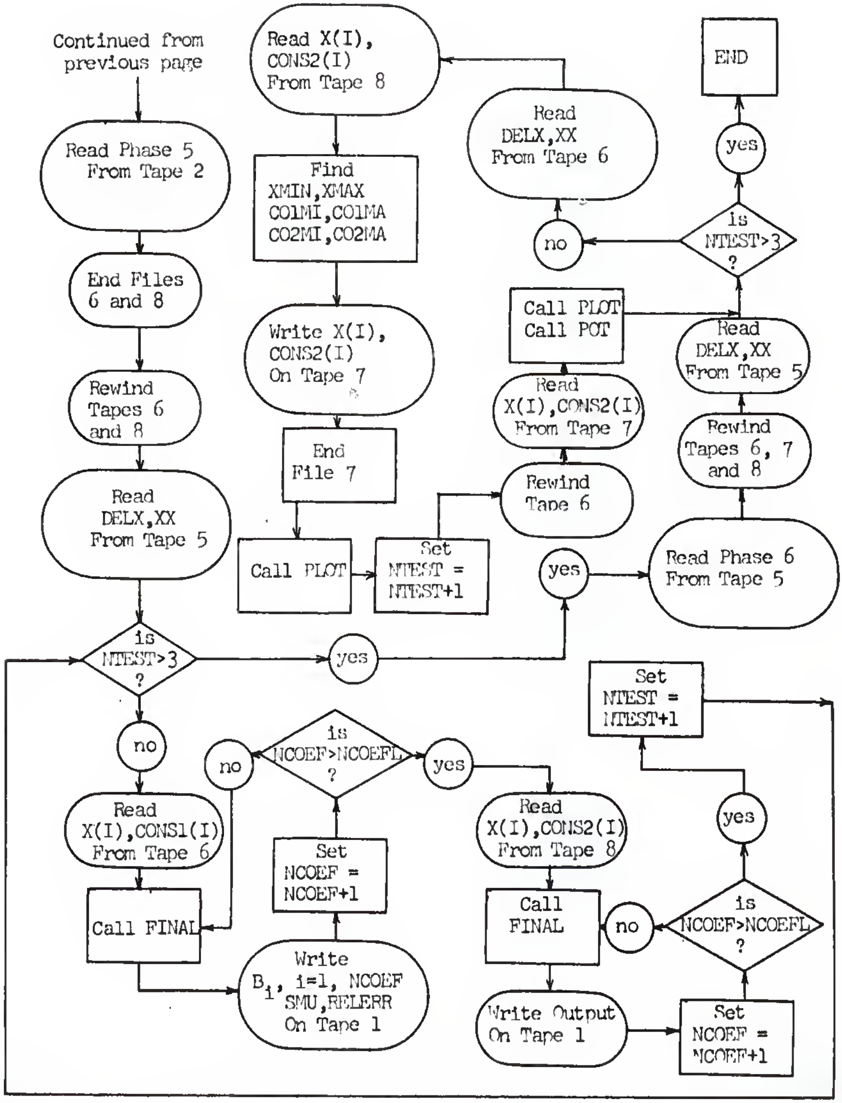
Table C-I. Continued

Symbol	Used in phase	Meaning
YMAX	3	Maximum $1/\beta$
NGR	3	Number of sets of data corresponding to different flux values for a transistor
NP	3	Number of points that must be plotted on one graph
NPTS	3,6	NP
NPLOTS	3	NGR
NCARDS	3,6	Number of information cards to be read by subprogram PLOT
XF	4	Flux
Freq	4	Cutoff frequency of transistor
DELX	4,5,6	ΔI_E
A,B,C,D	4	Coefficients of third degree polynomial fit to I_E and ϕ

Table C-II. Computer program logic diagram for Appendix C




Continued from Previous page
Read Phase 3 From Tape 2
End File 8 Rewind Tape 8
Set NTEST=1
Read NDP, NGR From Tape 5
Read X(I), Y(I) From Tape 8
Calc XMIN, XMAX YMIN, YMAX
Call PLOT
Set NTEST = NTEST+1
is NTEST>3 ?
no
yes
Read Phase 4 From Tape 2
Set NTEST=1
Rewind Tapes 6, 7 and 8
is NTEST>3 ?
yes
Continued on next page
no
Read NGR,FREQ,XF XX,X From Tape 5
Read B_i, i=1, 4 From Tape 7
Set X=0
Calc Y(I)
Call LSTSQ
Calc SL,Z,SMU CY(I),CONS1 CONS2
Write X,SL,Z,SMU Y(I),CY(I),CONS1 CONS2 On Tape 1
Write X,CONS1 On Tape 6
Write X, CONS2 On Tape 8
Set X=X+DELX
is X<XX ?
yes
Set NTEST = NTEST+1



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Read Phase 5 From Tape 2
End Files 6 and 8
Rewind Tapes 6 and 8
Read DELX,XX From Tape 5
is NTEST>3 ?
no
Read X(I),CONS1(I) From Tape 6
Call FINAL
Write B_i, i=1, NCOEF SMU,RELERR On Tape 1
Set NCOEF = NCOEF+1
is NCOEF>NCOEFL ?
no
yes
Read X(I),CONS2(I) From Tape 8
Call FINAL
no
Write Output On Tape 1
Set NCOEF = NCOEF+1
is NCOEF>NCOEFL ?
yes
Set NTEST = NTEST+1
yes
Read Phase 6 From Tape 5
Rewind Tapes 6, 7 and 8
Read DELX,XX From Tape 5
is NTEST>3 ?
yes
END
no
Read DELX,XX From Tape 6
Read X(I), CONS2(I) From Tape 8
Find XMIN,XMAX CO1MI,CO1MA CO2MI,CO2MA
Write X(I), CONS2(I) On Tape 7
End File 7
Call PLOT
Set NTEST = NTEST+1
Rewind Tape 6
Read X(I),CONS2(I) From Tape 7
Call PLOT Call POT

8.3.2 Program LEASQARE

In analyzing the experimental data it was necessary to fit polynomial least squares curve to the data and then use the resulting coefficients in further computations. LEASQARE is a program which uses the method of least squares to fit m sets of data points to a polynomial with real coefficients of the form

$$f(X) = b_0 + b_1 X + b_2 X^2 + \ldots. + b_n X^n \qquad \text{(C-1)}$$

where b_0, b_1, ... b_n are determined such that E is minimum.

$$E\ (b_0,\ b_1,\ \ldots.\ b_n) = \sum_{i=1}^{m} \left[Y_i - f(X_i)\right]^2 \qquad \text{(C-2)}$$

For E (b_0, b_1, ... b_n) to be minimum, the first partial derivatives of E (b_0, b_1, ... b_n) with respect to b_0, b_1, ... b_n must vanish and these coefficients must satisfy the equation (C-3).

$$\frac{\partial E(b_0,\ b_1,\ \ldots.\ b_n)}{\partial b_i} = 0 \qquad i = 0,\ 1,\ \ldots.\ n \qquad \text{(C-3)}$$

Equation (C-2) may be rewritten in the following form

$$E(b_0,\ b_1,\ \ldots.\ b_n) = \sum_{i=1}^{m} \left[\ Y_i - b_0 - b_1 X_i - \ \ldots.\ -b_n X_i^n\ \right]. \qquad \text{(C-4)}$$

Differentiating equation (C-4) with respect to b_0, b_1, ... and b_n gives,

$$\frac{\partial E}{\partial b_0} = \sum_{i=1}^{m} 2(-1) \left[Y_i - b_0 - b_1 X_i - b_2 X_i{}^2 - \ \ldots.\ -b_n X_i^n\right] = 0$$

$$\frac{\partial E}{\partial b_1} = \sum_{i=1}^{m} 2(-X_i)\left[\ Y_i - b_0 - b_1 X_i - b_2 X_i{}^2 - \ \ldots.\ -b_n X_i^n\right] = 0 \qquad \text{(C-5)}$$

$$\vdots$$

$$\frac{\partial E}{\partial b_n} = \sum_{i=1}^{m} 2(-X_i^n)\left[\ Y_i - b_0 - b_1 X_i - b_2 X_i{}^2 - \ \ldots.\ -b_n X_i^n\right] = 0$$

These equations may be rewritten in the following form by collecting the coefficients of b_0, b_1, ... and b_n.

$$\begin{aligned}
\sum_{i=1}^{m} Y_i &= b_0 n + b_1 n + b_1 \sum_{i=1}^{m} X_i + \dots + b_n \sum_{i=1}^{m} X_i^n \\
\sum_{i=1}^{m} X_i Y_i &= b_0 \sum_{i=1}^{m} X_i + b_1 \sum_{i=1}^{m} X_i^2 + \dots + b_n \sum_{i=1}^{m} X_i^{n+1} \\
&\vdots \\
\sum_{i=1}^{m} X_i^n Y_i &= b_0 \sum_{i=1}^{m} X_i^n + b_1 \sum_{i=1}^{m} X_i^{n+1} + \dots + b_n \sum_{i=1}^{m} X_i^{2n}
\end{aligned} \tag{C-6}$$

This is a system of n+1 linear equations in n+1 unknowns, b_i(i=0, 1, ... n). This system of equations may be written in the matrix form.

$$\begin{vmatrix} n & \sum_{i=1}^{m} X_i & \dots & \sum_{i=1}^{m} X_i^n \\ \sum_{i=1}^{m} X_i & \sum_{i=1}^{m} X_i^2 & \dots & \sum_{i=1}^{m} X_i^{n+1} \\ \vdots & \vdots & & \vdots \\ \sum_{i=1}^{m} X_i^n & \sum_{i=1}^{m} X_i^{n+1} & \dots & \sum_{i=1}^{m} X_i^{n+n} \end{vmatrix} \begin{vmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{vmatrix} = \begin{vmatrix} Y_i \\ X_i Y_i \\ \vdots \\ X_i^n Y_i^n \end{vmatrix} \tag{C-7}$$

Program LEASQARE produces the above matrix equation from the input data points and then this matrix equation is solved by subprograms CRAM, SOLVE and RESIDU for the unknown coefficients b_0, b_1, ... and b_n. LEASQARE also calculates the slope of the fitted polynomial by using equation (C-8).

$$f'(X_i) = \sum_{i=1}^{m} A_i X_j^{i-1}. \tag{C-8}$$

Equations (C-9) and (C-10) are used in LEASQARE to calculate the standard deviation and the relative error.

$$\sigma = \pm\left[\frac{1}{m} \sum_{i=1}^{m} (Y_i - f(X_i))^2 \right]^{1/2} \tag{C-9}$$

and

$$r = \frac{100}{m} \sum_{i=1}^{m} \frac{f(X_i) - Y_i}{Y_i} . \tag{C-10}$$

This program is the control program for the first phase. In this phase the input data, which consists of collector current, base current, number of data points and the minimum and maximum degree of polynomial least square fit is read and the preliminary output is produced. Emitter current and the inverse of common emitter current gain are evaluated using equations (C-11) and (C-12) as follows:

$$I_E = I_B + I_C \tag{C-11}$$

$$1/\beta = \left. \frac{\partial I_B}{\partial I_C} \right|_{V_C = \text{const}} . \tag{C-12}$$

The output from this program is stored on tape 6 and is used as input data for phase 2. This procedure is repeated for different sets of input data which in turn corresponds to different values of the flux and collector voltage.

8.3.3 Subprogram CRAM

This program reduces a given matrix to an upper right triangular matrix which indeed is the first step in the Crout reduction method for solving a matrix equation of the form

$$(A)\ \{X\} = \{B\} .$$

To solve the coefficient matrix {X} by Crout reduction, matrix M is chosen such that

$$(M)\ (T)\ \{X\} = (M)\ \{C\} \qquad \text{(C-13)}$$

where:

$$(M)\ (T) = (A)$$

$$(M)\ \{C\} = \{B\}$$

and (M) is a lower triangular matrix and (T) is an upper right triangular matrix.

Matrix equation (C-13) may be rewritten in the following form:

$$(M)\ (TX - C) = 0. \qquad \text{(C-14)}$$

Since (M) is not a zero matrix

$$(TX - C) = 0. \qquad \text{(C-15)}$$

Equation (C-15) can be rewritten in the following form:

$$\begin{vmatrix} t_{11} & t_{12} & t_{13} & \cdots & t_{1n} \\ 0 & t_{22} & t_{23} & \cdots & t_{2n} \\ 0 & 0 & t_{23} & \cdots & t_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & t_{nn} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{vmatrix} = \begin{vmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \\ \gamma_n \end{vmatrix} \qquad \text{(C-16)}$$

In general it is not necessary to choose an M matrix to obtain equation (C-16) from AX = B. It is possible to change matrix equation A to upper right hand matrix by using elementary row or column operation. This method which is known as the modified Crout reduction, is used in this subprogram to reduce matrix A to an upper right triangular matrix; the basic equations used are:

$$a'_{ij} = a_{ij} - \sum_{k=1}^{j-1} a'_{ik}\, a'_{kj} \qquad (i \geq j) \qquad \text{(C-17)}$$

$$a'_{ij} = \frac{1}{a_{ii}}\, a_{ij} - \sum_{k=1}^{i-1} a'_{ik}\, a'_{kj} \qquad (i < j). \qquad \text{(C-18)}$$

where i and j range from 1 to n, and the a_{ij} are elements of the matrix A. Prime denotes that this element has been operated on. A detailed description of this subprogram is on file at the K.S.U. Computing Center.

8.3.4 Subprogram SOLVE

This subprogram solves the matrix equation AX = B for values of X, after CRAM has reduced the matrix A to T, where T is an upper right triangular matrix. The column matrix on the right hand side of equation (C-16) is produced by using equation (C-19).

$$b'_i = \frac{1}{a'_{ii}} \left[b_i - \sum_{k=1}^{i-1} a'_{ik} \, b'_k \right] \tag{C-19}$$

where b'_i corresponds to γ_i of equation (C-16), a'_{ii} corresponds to diagonal elements of matrix T, and b_i corresponds to elements of the column matrix B.

After matrix equation (C-16) is set up, equation (C-20), the coefficient matrix, is obtained.

$$X_i = b'_i - \sum_{k=i+1}^{n} a'_{ik} \, X_k \tag{C-20}$$

Equations (C-19) and (C-20) are the basic equations used in subprogram SOLVE. A detailed description of this subprogram is on file in the K.S.U. Computing Center.

8.3.5 Subprogram RESIDU

This subprogram minimizes the error matrix or residuals obtained when using the Crout reduction method in solving for the coefficient matrix. After the coefficient matrix is calculated by CRAM and SOLVE, RESIDU calculates {e} from equation (C-21).

$$A\,\{X\} - \{B\} = \{e\} \tag{C-21}$$

Then, the matrix equation $A\,\{X\} = \{e\}$ is solved for a coefficient matrix $\{X\}$ by calling CRAM and SOLVE. This new coefficient matrix is added to the previous one and the result is substituted in equation (C-21) for a new $\{e\}$. This iteration process is continued until the matrix determined after the n^{th} iteration is negligible with respect to the sum of the previous (n-1) coefficient matrices. A detailed description of this subprogram is on file in the K.S.U. Computing Center.

8.3.6 Subprogram LEASTSQ

LEASTSQ is the control program for the second phase. In this phase the input data are read from tape 6. Using a least squares analysis, a fourth degree polynomial is fitted to the input data, I_E and $1/\beta$. The coefficients of the polynomial are stored on tape 7 and the resulting values of $1/\beta$ and I_E are stored on tape 8. This program is a modified version of LEASQARE.

8.3.7 Subprogram SOLN

After program LEASTSQ performs the least squares analysis and produces the matrix equation:

$$AX = B$$

then, subprogram SOLN is called to solve for the values of coefficient matrix $\{X\}$. This subprogram uses elementary row operations to reduce equation (C-22) to the form of equation (C-23).

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix} \qquad \text{(C-22)}$$

$$\begin{vmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{vmatrix} = \begin{vmatrix} b'_1 \\ b'_2 \\ \vdots \\ b'_n \end{vmatrix} \qquad \text{(C-23)}$$

In the process of elementary row operation to reduce matrix A to an identity matrix, matrix B changes to matrix B'. To change the matrix equation (C-22) to (C-23), the program divides each row of A and B matrices by its corresponding a_{ii} to obtain,

$$\begin{vmatrix} 1 & \frac{a_{12}}{a_{11}} & \cdots & \frac{a_{1n}}{a_{11}} \\ \frac{a_{21}}{a_{22}} & 1 & \cdots & \frac{a_{2n}}{a_{22}} \\ \vdots & & & \vdots \\ \frac{a_{n1}}{a_{nn}} & \frac{a_{n2}}{a_{nn}} & \cdots & 1 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{vmatrix} = \begin{vmatrix} \frac{b_1}{a_{11}} \\ \frac{b_2}{a_{11}} \\ \vdots \\ \frac{b_n}{a_{nn}} \end{vmatrix} ; \qquad \text{(C-24)}$$

then, the elements a_{ij}, $i \neq j$ are replaced by zero by adding appropriate multiples of each row to the other rows. This operation forms matrix equation (C-23). Since after this operation matrix A is changed to an identity matrix it follows that

$$\begin{vmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{vmatrix} = \begin{vmatrix} b'_1 \\ b'_2 \\ \vdots \\ b'_2 \end{vmatrix} \tag{C-25}$$

A detailed description of this program is on file in the K.S.U. Computing Center.

8.3.8 Subprogram MAPDATA

This program is the control program for the third phase. The third phase reads $1/\beta$ and I_E for different values of flux from tape 8 and after calculating the minimum and maximum values of $1/\beta$ and I_E, it calls subroutine PLOT to plot the set of input data. This process is repeated for different values of the collector voltage.

8.3.9 Subprogram PLOT

This subprogram plots a graph of some of the computed results. To print titles and axes on the graphs, a set of control cards are first read in from tape 5. A list of the control cards used in this program are shown in Table C-III. A complete description of this subprogram is on file in the K.S.U. Computing Center.

8.3.10 Subprogram SLOP

This program is the control program for the fourth phase. Equations for $1/\beta$ as a function of flux are read in from tape 7, then for a constant value of emitter current, this program fits a straight line through the points of the ϕ vs $1/\beta$ data. Using the slope of this line and equations (C-26) and (C-27), the transistor damage constant and the lifetime damage constant for a transistor operating at a constant collector voltage are evaluated.

$$\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E=0} = \alpha_j \tag{C-26}$$

$$K_j = \frac{1.22}{2\pi} \cdot \frac{1}{f_{c\alpha}} \cdot \frac{1}{\alpha_j} \tag{C-27}$$

To find the dependancy of h(Z) on emitter current, at different values of the emitter current, the value of $\left.\frac{(\partial 1/\beta)}{\partial\phi}\right|_{I_E=\text{const}}$ is evaluated and the results are applied in equation (C-28),

$$\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E} = \alpha_j\ h(Z)\ . \tag{C-28}$$

Since α_j is a constant and also not a function of emitter current,

$$h(Z) = \frac{\left.\frac{\partial(1/\beta)}{\partial\phi}\right|_{I_E}}{\alpha_j} \tag{C-29}$$

This process is repeated for several values of the collector voltage. The resulting values of α_j h(Z) and h(Z) are stored on tapes 8 and 6 respectively and they are used as input to the fifth and sixth phase.

8.3.11 Subprogram LSTSQ

This program determines by least square analysis the slope, intercept and standard deviation of the best straight line through the data.

Fitting a straight line equation,

$$Y = a + b\,X, \tag{C-30}$$

to n points $(X_1, Y_1) \cdot (X_2, Y_2) \ldots\ldots (X_n\, Y_n)$, one obtains equations

$$e_1 = a + bX_1 - Y_1$$

$$e_2 = a + bX_2 - Y_2$$

$$\vdots$$

$$e_n = a + bX_n - Y_n$$

These equations are not necessarily equal to zero since the straight line does not always pass exactly through the points. The principle of least squares requires that the best representation of the data is that which makes the sum of the squares of residuals a minimum. Therefore, to find the best values of a and b, the first partial derivatives of E is set equal to zero,

$$\frac{\partial E}{\partial a} = \frac{\partial E}{\partial b} = 0 \tag{C-32}$$

where

$$E = \sum_{i=1}^{n} e_1 \;= (a + b\,X_1 - Y_1)^2 + \ldots\ldots + (a + b\,X_n - Y_n)^2\,. \tag{C-33}$$

Substituting equation (C-33) into (C-32) gives

$$\frac{\partial E}{\partial a} = 2(a + b\,X_1 - Y_1) + 2(a + b\,X_2 - Y_2) + \ldots\ldots + 2(a + b\,X_n - Y_n) = 0 \tag{C-34}$$

$$\frac{\partial E}{\partial b} = 2X_1(a + b\,X_1 - Y_1) + 2X_2\,(a + bX_2 - Y_2) + \ldots\ldots$$

$$+ 2X_n\,(a + bX_n - Y_n) = 0. \tag{C-35}$$

Collecting the terms of the unknown coefficients a and b gives,

$$\sum_{i=1}^{n} Y_i = n\,a + b\sum_{i=1}^{n} X_i{}^2 \tag{C-36}$$

$$\sum_{i=1}^{n} X_i\ Y_i = \sum_{i=1}^{n} X_i + b \sum_{i=1}^{n} X_i^2 \qquad \text{(C-37)}$$

Equations (C-36) and (C-37) can now be solved simultaneously for a and b. Multiplying equations (C-36) by $\Sigma\ X_i$ and (C-37) by n and summing the results gives,

$$b \left[n \sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2 \right] = n \sum_{i=1}^{n} X_i\ Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i$$

or

$$b = \frac{n \sum_{i=1}^{n} X_i\ Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n \sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}. \qquad \text{(C-38)}$$

Substituting equation (C-38) into (C-37) and simplifying the terms gives the following equation for a:

$$a = \frac{\sum_{i=1}^{n} X_i^2 \sum_{i=1}^{n} Y_i^2 - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i\ Y_i}{n \sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}. \qquad \text{(C-39)}$$

This subprogram uses equations (C-38) and (C-39) to calculate the best straight line through a set of n data points. The standard deviation for b is calculated using the equation

$$\sigma(b) = \pm \left[\sum_{i=1}^{n} \left[f(X_i) - Y_i\right]^2 \Big/ (n-2) \sum_{i=1}^{n} X_i^2 \right]^{1/2} \qquad \text{(C-40)}$$

8.3.12 Subprogram DAMAGE

This is the control program for the fifth phase. The fourth phase reads the stored information on tapes 6 and 8 and then by calling subprograms FINAL CRAM, VOLVE AND RESIDU it fits different degrees of polynomial to α_i h(Z) vs I_E and h(Z) vs I_E.

8.3.13 Subprogram FINAL

This subprogram is a modified version of program LEASQARE. It is written in the subprograms and can be used with any program which reads the input data. The theory used in this subroutine is discussed under LEASQARE. After performing the least squares analysis, it calls CRAM, SOLVE and RESIDU to solve the coefficient matrix.

8.3.14 Subprogram GRAPH

GRAPH is the control program for the sixth phase. It reads the input data from tapes 6 and 8. After finding the limits for the graphs, it calls subprogram PLOT and it plots graphs of the stored results in the printed output. Again, to print graph titles and axes a set of control cards used are shown in Table C-IV.

Table C-III. LISTING OF COMPUTER PROGRAM

```
MON$$     JOB TRANSISTOR ANALYSIS                              BOROOKHIM
MON$$     COMT 10 MINUTES,25 PAGES,BOROOKHIM,NUCLEAR ENGG
MON$$     ASGN MJB,12
MON$$     ASGN MGO,16
MON$$     MODE GO,TEST
MON$$     EXEQ FORTRAN,,,,,,,LSTSQARE
      DIMENSION X(40),Y(40)
      DIMENSION CY(40),DY(40)
      DIMENSION CZ(40)
      COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
C     X STAND FOR COLLECTOR CURRENT, Y FOR BASE CURRENT, CZ FOR EMITTER
C     CURRENT AND DY FOR INVERSE OF COMMON EMITTER CURRENT GAIN
C     A IS MATRIX OF X SUMMATION COEFFECIENTS,IP AND V ARE INTERNAL
C       VECTORS USED IN CRAM SUBROUTINE,B IS VECTOR OF XY SUMMATION
C       RESIDUALS,MATRIX(AH)=MATRIX(A) INITIALLY,VECTOR(BH)=VECTOR(B)
C       INITIALLY
C     THE POLYNOMIAL COEFFECIENTS ARE STORED IN VECTOR(B) AT
C       COMPLETION OF RESIDU SUBROUTINE
C     THIS IS THE CONTROL PROGRAM FOR PHASE ONE
C     THIS PROGRAM ANALYZES THE RADIATION DAMAGE TO TRANSISTOR GAIN
C     THE INPUT DATA TO THIS PART OF PROGRAM CONSIST OF EMITTER CURRENT
C     AND BASE CURRENT, DEGRRE OF ANALYSIS,NUMBER OF POINTS .
C     THE IMFORMATION FROM THIS PHASE ARE STORED ON TAPE AND ARE USED
C     IN NEXT PHASE
C     THIS PART OF PROGRAM WAS ORIGINALLY WRITTEN BY TOM HILL BUT IT IS
C     MODIFIED BY MANOUCHEHR BOROOKHIM FOR USE IN THIS PROGRAM
    1 FORMAT(3I3)
    2 FORMAT(20H/ANALYSIS OF DEGREE ,I3)
    3 FORMAT(2F12.6)
    4 FORMAT(3H X=,F12.6,4H  Y=,F12.6)
   24 FORMAT(3H X=,F8.6,4H  Y=,F8.6)
    5 FORMAT(3H B(,I2,2H)=,E16.8)
    6 FORMAT(20HTSTANDARD DEVIATION=,F16.8)
    7 FORMAT(35HTINSUFFECIENT NUMBER OF DATA POINTS)
    8 FORMAT(3H X=,F12.6,14H CALCULATED Y=,F16.8,16H EXPERIMENTAL Y=,F16
     1.8)
   28 FORMAT(3H X=,F10.8,14H CALCULATED Y=,F10.8,16H EXPERIMENTAL Y=,F10
     1.8)
    9 FORMAT(15HTRELATIVE ERROR=,F16.8)
   10 FORMAT(F12.6,5X,F12.6,5X,F12.6)
   11 FORMAT(8H     ATX,10X,15HINVERSE    GAIN,5X,15HEMITTER CURRENT)
   12 FORMAT(2I3)
      REWIND6
      NTEST=1
      READ(1,12)NVOLT,NFLUX
```

```
      NSTR=NVOLT*NFLUX
      WRITE(6)NSTR
 1000 READ(1,1)NDP,NCOEF,NCOEFL
      WRITE(6)NDP,NCOEF,NCOEFL
C     NCOEFL IS LAST COEFFECIENT CALCULATED--20 OR LESS
C     NDP IS NUMBER OF DATA POINTS--20 OR LESS
C     NCOEF IS NUMBER OF COEFFECIENTS TO BE CALCULATED--20 OR LESS
C       THEREFORE,NCOEF-1 IS DEGREE OF LEAST SQUARE FIT
      IF(NDP.EQ.0)CALL EXIT
      READ (1,3)(X(I),Y(I),I=1,NDP)
      WRITE(3,4)(X(I),Y(I),I=1,NDP)
C     TEST TO DETERMINE IF SUFFECIENT NUMBER OF DATA POINTS
 1002 IF(NDP.GE.NCOEF) GO TO 1001
      WRITE(3,7)
      GO TO 1000
 1001 NCM1=NCOEF-1
      WRITE(3,2)NCM1
      DO 101 K=1,NCOEF
      DO 102 J=1,NCOEF
C     SET MATRIX(A) OF X SUMMATION COEFFECIENTS TO ZERO
  102 A(K,J)=0.
C     SET VECTOR(B) OF X,Y SUMMATION RESIDUALS TO ZERO
  101 B(K)=0.
      DO 107 J=1,NCOEF
      DO 107 I=1,NDP
      IF(J.GT.1) GO TO 105
      DO 103 K=1,NCOEF
C     GENERATE FIRST ROW OF MATRIX(A)
  1(3 A(1,K)=A(1,K)+X(I)**(K-1)
      GO TO 107
C     GENERATE LAST COLUMN OF MATRIX(A)
  105 A(J,NCOEF)=A(J,NCOEF)+X(I)**(J+NCOEF-2)
C     GENERATE VECTOR(B)
  1(7 B(J)=B(J)+Y(I)*X(I)**(J-1)
C     THE FIRST ROW AND LAST COLUMN OF MATRIX(A) HAS BEEN FORMED
C       VECTOR(B) HAS ALSO BEEN FORMED
      DO 109 J=2,NCOEF
      DO 109 K=1,NCM1
C     GENERATE REMAINDER OF MATRIX(A) BY SHIFTING ELEMENTS OF TOP ROW
C       AND LAST COLUMN DOWNWARD AND TO THE LEFT
  109 A(J,K)=/(J-1,K+1)
      DO 120 J=1,NCOEF
      DO 121 K=1,NCOEF
C     SET MATRIX(AH) EQUAL TO MATRIX(A)
  121 AH(J,K)=A(J,K)
C     SET VECTOR(BH) EQUAL TO VECTOR(B)
  120 BH(J)=B(J)
      CALL CRAM(NCOEF,1)
      CALL SOLVE(NCOEF)
      CALL RESIDU(NCOEF)
      WRITE(3,5)(I,B(I),I=1,NCOEF)
C     PROCEED WITH CALCULATION OF STANDARD DEVIATION
```

```
      DEVESQ=0.
      RELDEV=0.
      DO 200 I=1,NDP
C     CALCULATE Y VALUE,CY,UTILIZING LEAST SQUARE POLYNOMIAL
      CY(I)=B(1)
      DO 201 J=2,NCOEF
  201 CY(I)=B(J)*X(I)**(J-1)+CY(I)
      RELDEV=RELDEV+(CY(I)-Y(I))/Y(I)
C     CALCULATE DEVIATION SQUARED BETWEEN EXPERIMENTAL AND LEAST SQUARE
C       POINT
  200 DEVESQ=(Y(I)-CY(I))**2+DEVESQ
      WRITE(3,8)(X(I),CY(I),Y(I),I=1,NDP)
      SMU=SQRT(DEVESQ/FLOAT(NDP))
      WRITE(3,6)SMU
      RELERR=RELDEV/FLOAT(NDP)*100.
      WRITE(3,9) RELERR
      WRITE(2,9)RELERR
C     GENERATE FIRST DERIVATIVE FOR POINTS OF INPUT
C     FIRST DERIVATIVE CORRESPOND TO INVERSE OF COMMON EMITTER GAIN
      WRITE(3,11)
      DO111I=1,NDP
      DY(I)=B(2)
      IF(NCOEF.EQ.2)GO TO114
      DO112J=3,NCOEF
      DY(I)=DY(I)+FLOAT(J-1)*X(I)**(J-2)*B(J)
  112 CONTINUE
  114 CZ(I)=X(I)+CY(I)
  111 WRITE(3,10)X(I),DY(I),CZ(I)
      WRITE(6)(CZ(I),DY(I),I=1,NDP)
C     TEST FOR DEGREE OF ANALYSIS
      NCOEF=NCOEF+1
      IF(NCOEF.GT.NCOEFL)GOTO300
      GOTO1002
C     TEST  TO SEE IF THERE IS ANYMORE INPUT DATA
  300 NTEST=NTEST+1
      IF(NTEST.GT.NSTR)GO TO 400
      GOTO1000
  400 CONTINUE
      STOP
      END
     MON$$      EXEQ FORTRAN,,,,,,,CRAM
      SUBROUTINE CRAM(N,I)
C     CROUT REDUCTION OF AUGMENTED MATRICES
C     THIS PROGRAM PERFORMS A CROUT REDUCTION ON A MATRIX A.
C     WITH I=1, THE CROUT REDUCTION IS PERFORMED WITH ROW INTERCHANGES.
C     WITH I=2, THE CROUT REDUCTION IS PERFORMED WITHOUT ROW CHANGES.
      COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
 2240 FORMAT(1HK,5HP1VOT,I3,19HIS LESS THAN 1.E-12)
 2241 FORMAT(1HK,5HP1VOT,I3,7HIS ZERO)
      GO TO (2200,2201),I
 2200 IDMV=1
      GOTO2202
```

```
 2201 IDMV=2
C     REDUCTION OF MATRIX
 2202 DO 2204 IDK=1,N
      V(IDK)=ABS(A(IDK,1))
      DO2204 IDI=2,N
      IF(V(IDK)-ABS(A(IDK,IDI)))2203,2204,2204
 2203 V(IDK)=ABS(A(IDK,IDI))
 2204 CONTINUE
      DO 2222 IDK=1,N
      DETR=-1.
      IDK1=IDK-1
      DO2214 IDI=IDK,N
      DETPR=0.0
      IF(IDK-1)2208,2208,2206
 2206 DO2207 IDJ=1,IDK1
 2207 DETPR=DETPR+A(IDI,IDJ)*A(IDJ,IDK)
 2208 A(IDI,IDK)=A(IDI,IDK)-DETPR
      GO TO(2212,2225),IDMV
 2212 DETS=ABS(A(IDI,IDK))/V(IDI)
      IF(DETS-DETR)2214,2214,2213
 2213 DETR=DETS
      IP(IDK)=IDK-IDI
      GO TO 2214
 2225 IP(IDK)=0
 2214 CONTINUE
      IDK2=IDK-IP(IDK)
      DETR=A(IDK2,IDK)/V(IDK2)
      IF(ABS(DETR)-1.E-12)2230,2230,2232
 2230 WRITE(3,2240)IDK
      IF(A(IDK2,IDK)) 2232,2231,2232
 2231 WRITE(3,2241)IDK
      CALL EXIT
 2232 V(IDK2)=V(IDK)
      V(IDK)=DETR
      DO2222 IDJ=1,N
      DETR=A(IDK,IDJ)
      IF(IDJ-IDK)2215,2215,2216
 2215 A(IDK,IDJ)=A(IDK2,IDJ)
      GO TO 2220
 2216 DETPR=0.0
      IF(IDK-1)2219,2219,2217
 2217 DO2218 IDI=1,IDK1
 2218 DETPR=DETPR+A(IDK,IDI)*A(IDI,IDJ)
 2219 A(IDK,IDJ)=(A(IDK2,IDJ)-DETPR)/A(IDK,IDK)
 2220 IF(IP(IDK))2221,2222,2222
 2221 A(IDK2,IDJ)=DETR
 2222 CONTINUE
      RETURN
      END
     MON$$      EXEQ FORTRAN,,,,,,,SOLVE
      SUBROUTINE SOLVE (N)
C     AFTER CALLING CRAM THIS SUBROUTINE WILL COMPUTE THE SOLUTION
```

```
C          VECTOR OF THE MATRIX EQUATION AX=B. BEFORE RETURNING TO THE
C          MAIN PROGRAM THE SOLUTION VECTOR IS STORED IN B.
      COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
      DO 2256 IDK = 1,N
      IDK1 = IDK - 1
      IDK2 = IDK-IP(IDK)
      DETR = B(IDK)
      DETPR = 0.0
      IF(IDK-1) 2253, 2253, 2257
 2257 DO 2252 IDI = 1, IDK1
 2252 DETPR = DETPR+A(IDK,IDI)* B(IDI)
 2253 B(IDK) = (B(IDK2) - DETPR) / A(IDK,IDK)
      IF (IP(IDK)) 2254, 2256, 2256
 2254 B(IDK2) = DETR
 2256 CONTINUE
      DO 2263 IDI2 = 1,N
      IDI = N + 1 - IDI2
      DETPR = 0.0
      IDI1 = IDI + 1
      IF (N - IDI) 2263, 2263, 2261
 2261 DO 2262 IDJ = IDI1,N
 2262 DETPR = DETPR + A(IDI,IDJ)* B(IDJ)
 2263 B(IDI) = B(IDI) - DETPR
      RETURN
      END
     MON$$      EXEQ FORTRAN,,,,,,,RESIDU
      SUBROUTINE RESIDU(N)
C     AFTER THE CRAM AND SOLVE SUBROUTINES HAVE BEEN CALLED THIS
C       SUBROUTINE WILL COMPUTE THE RESIDUALS IN THE COEFFICIENT
C       VECTOR T AND ITERATE ON THE ANSWER VECTOR B UNTIL THERE IS
C       NO CHANGE IN B FROM ONE ITERATION TO THE NEXT. THE SUBROUTINE
C       ASSUMES THAT THE ORIGINAL MATRIX IS IN S AND THAT THE ORIGINAL
C       COEFFICIENT VECTOR IS IN T.
      COMMON A(20,20),IP(20),V(20),B(20),S(20,20),T(20)
      DO 1 I=1,N
      V(I)=B(I)
      DO 1 J=1,N
    1 T(I)=T(I)-S(I,J)*B(J)
    2 DO 3 I=1,N
    3 B(I)=T(I)
      CALL SOLVE (N)
      DO 10 I=1,N
      DO 10 J=1,N
   10 T(I)=T(I)-S(I,J)*B(J)
      J=0
      DO 5 I=1,N
      B(I)=B(I)+V(I)
      IF(B(I)-V(I))6,5,6
    6 J=1
    5 V(I)=B(I)
      IF(J) 7,7,2
    7 RETURN
```

```
      END
     MON$$     EXEQ FORTRAN,,,,,,,LEASTSQ
      DIMENSION X(40),Y(40)
      DIMENSION CY(40)
      COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
C     THIS PROGRAM FITS A FOURTH DEGREE POLYNOMIAL TO INVERSE OF
C     COMMON EMITTER GAIN AND EMITTER CURRENT VALUES
C     X STAND FOR EMITTER CURRENT AND Y FOR INVERSE OF GAIN
    1 FORMAT(3I3)
    2 FORMAT(20H/ANALYSIS OF DEGREE ,I3)
    3 FORMAT(2F12.6)
    4 FORMAT(3H X=,F12.6,4H  Y=,F12.6)
    5 FORMAT(3H B(,I2,2H)=,E16.8)
    6 FORMAT(20HTSTANDARD DEVIATION=,F16.8)
    7 FORMAT(35HTINSUFFECIENT NUMBER OF DATA POINTS)
    8 FORMAT(3H X=,F12.6,14H CALCULATED Y=,F16.8,16H EXPERIMENTAL Y=,F16
     1.8)
   28 FORMAT(3H X=,F10.8,14H CALCULATED Y=,F10.8,16H EXPERIMENTAL Y=,F10
     1.8)
    9 FORMAT(16HTRELATIVE ERROR=,F16.8)
   11 FORMAT(8H      ATX,10X,15HFIRSTDERIVATIVE,5X,15HEMITTER CURRENT)
      END FILE 6
      REWIND 6
      REWIND7
      REWIND8
      NTEST=1
      READ(6)NSTR
 1000 READ(6)NDP,NCOEF,NCOEFL
      IF(NDP.EQ.0)CALL EXIT
      READ(6)(X(I),Y(I),I=1,NDP)
      WRITE(3,4)(X(I),Y(I),I=1,NDP)
 1002 IF(NDP.GE.NCOEF) GO TO 1001
      WRITE(3,7)
      GO TO 1000
 1001 NCM1=NCOEF-1
      WRITE(3,2)NCM1
      DO 101 K=1,NCOEF
      DO 102 J=1,NCOEF
  102 A(K,J)=0.
  101 B(K)=0.
      DO 107 J=1,NCOEF
      DO 107 I=1,NDP
      IF(J.GT.1) GO TO 105
      DO 103 K=1,NCOEF
  103 A(1,K)=A(1,K)+X(I)**(K-1)
      GO TO 107
  105 A(J,NCOEF)=A(J,NCOEF)+X(I)**(J+NCOEF-2)
  107 B(J)=B(J)+Y(I)*X(I)**(J-1)
      DO 109 J=2,NCOEF
      DO 109 K=1,NCM1
  109 A(J,K)=A(J-1,K+1)
      DO 120 J=1,NCOEF
```

```
      DO 121 K=1,NCOEF
  121 AH(J,K)=A(J,K)
  120 BH(J)=B(J)
      CALL SOLN(A,B,NCOEF)
      WRITE(3,5)(I,B(I),I=1,NCOEF)
      WRITE(7) B(1),B(2),B(3),B(4)
      DEVESQ=0.
      RELDEV=0.
      DO 200 I=1,NDP
      CY(I)=B(1)
      DO 201 J=2,NCOEF
  201 CY(I)=B(J)*X(I)**(J-1)+CY(I)
      RELDEV=RELDEV+(CY(I)-Y(I))/Y(I)
  200 DEVESQ=(Y(I)-CY(I))**2+DEVESQ
      WRITE(3,8)(X(I),CY(I),Y(I),I=1,NDP)
      SMU=SQRT(DEVESQ/FLOAT(NDP))
      WRITE(3,6)SMU
      RELERR=RELDEV/FLOAT(NDP)*100.
      WRITE(3,9) RELERR
      WRITE(8)(X(I),CY(I),I=1,NDP)
C     TEST FOR DEGREE OF ANALYSIS
      NCOEF=NCOEF+1
      IF(NCOEF.GT.NCOEFL)GOTO300
      GOTO1002
C     TEST FOR INPUT DATA
  300 NTEST=NTEST+1
      IF(NTEST.GT.NSTR)GO TO 400
      GOTO1000
  400 CONTINUE
      STOP
      END
     MON$$      EXEQ FORTRAN,,,,,,,SOLN
      SUBROUTINE SOLN(A,B,N)
      DIMENSION A(20,20),B(20)
   25 FORMAT(1HL,18HMATRIX IS SINGULAR)
      DO 3 KK=2,N
      K=KK-1
C*****TEST FOR ZERO DIAGONAL, INTERCHANGE ROWS.
      IF(A(K,K).NE.0.) GO TO 6
      DO5I=KK,N
      IF(A(I,K).EQ.0.)GOTO5
      F=B(K)
      B(K)=B(I)
      B(I)=F
      DO 7 J=K,N
      F=A(K,J)
      A(K,J)=A(I,J)
    7 A(I,J)=F
      GO TO 6
    5 CONTINUE
      WRITE(3,25)
      STOP
```

```
C*****REDUCE A TO UPPER TRIANGULAR
    6 DO1J=KK,N
    1 A(K,J)=A(K,J)/A(K,K)
      B(K)=B(K)/A(K,K)
      A(K,K)=1.
      DO3I=KK,N
      F=A(I,K)
      DO2J=K,N
    2 A(I,J)=A(I,J)-F*A(K,J)
    3 B(I)=B(I)-F*B(K)
      IF(A(N,N).EQ.0.)WRITE(3,25)
      IF(A(N,N).EQ.0.)STOP
      F=A(N,N)
      A(N,N)=1.
      B(N)=B(N)/F
C*****REDUCE A TO IDENTITY
      DO10II=2,N
      I=II-1
      DO10K=II,N
      F=A(I,K)
      B(I)=B(I)-F*B(K)
      DO10J=K,N
   10 A(I,J)=A(I,J)-F*A(K,J)
      RETURN
      END
     MON$$     EXEQ FORTRAN,,,,,,MAPDATA
      DIMENSION X(40),Y(40)
C     THIS PROGRAM PRINTS THE GRAPHS OF EMITTER CURRENT VS INVERSE OF
C     COMMON EMITTER GAIN FOR DIFFERENT VALUES OF FLUX
    1 FORMAT(2I3)
    2 FORMAT(16H EMITTER CURRENT,6X,12HINVERSE BETA//)
    3 FORMAT(F12.6,8X,F12.6)
    4 FORMAT(6H XMAX=,F12.6,2X,5HXMIN=,F12.6,2X,5HYMAX=,F12.6,5X,5HYMIN=
     1,F12.6//)
      END FILE 8
      REWIND 8
      NTEST=1
   10 READ(1,1)NDP,NGR
      WRITE(3,2)
      NP=NDP*NGR
      DO20 L=1,NGR
      J=NDP*(L-1)+1
      K=J+NDP-1
      READ(8)(X(I),Y(I),I=J,K)
   20 WRITE(3,3)(X(I),Y(I),I=J,K)
      XMIN=X(1)
      XMAX=X(1)
      YMIN=Y(1)
      YMAX=Y(1)
C     FIND THE LIMITS FOR THE GRAPHS
      DO30I=2,NP
      IF(XMIN.GT.X(I))XMIN=X(I)
```

```
      IF(XMAX.LT.X(I))XMAX=X(I)
      IF(YMIN.GT.Y(I))YMIN=Y(I)
   30 IF(YMAX.LT.Y(I))YMAX=Y(I)
      WRITE(3,4)XMAX,XMIN,YMAX,YMIN
      XMAX=1.6*XMAX
      YMAX=1.3*YMAX
      CALL PLOT(X,XMIN,XMAX,0,Y,YMIN,YMAX,0,0.,0.,0.,0,NP,NGR,1,3,2)
C     TEST FOR INPUT DATA
      NTEST=NTEST+1
      IF(NTEST.LE.1)GOTO10
      STOP
      END
     MON$$      EXEQ FORTRAN,,,,,,,PLOT
      SUBROUTINEPLOT(X,XMIN,XMAX,LX,Y,YMIN,YMAX,LY,Z,ZMIN,ZMAX,LZ,NPTS,
     1NPLOTS,NCOPYS,NCARDS,NDIM)
C     PROGRAMMED BY ED KOBETICH, KSU DEPT. OF PHYSICS, JUNE 1964.
      DIMENSIONX(1),Y(1),Z(1),SX(7),TITLE(8),L(134),NCH(41),MOP(18)
    1 FORMAT(8A10)
    2 FORMAT(58A1,2A9,4A1)
    3 FORMAT(1H1,26X,8A10)
    4 FORMAT(1H ,A1,1PE9.2,122A1)
    5 FORMAT(133A1)
    6 FORMAT(1PE17.2,5E20.2,F16.2)
    7 FORMAT(1PE17.2,100X,E16.2)
    8 FORMAT(1PE17.2,E60.2,E56.2)
    9 FORMAT(1PE17.2,2E40.2,E36.2)
   10 FORMAT(1PE17.2,3E30.2,E26.2)
   11 FORMAT(1PE17.2,4E24.2,E20.2)
   12 FORMAT(1HA,62X,2A9)
      SLOG(F)=ALOG(F)/2.302585
      LLX=LX+1
      NDD=NCARDS+1
      GOTO(15,13,14,13),NDD
   13 READ(1,1)(TITLE(I),I=1,8)
   14 IF(NDD.GE.3)READ(1,2)(MOP(I),I=1,18),(NCH(I),I=1,40),TAB1,TAB2,ND,
     1NP,NM,NB
   15 NCH(41)=NB
      NPN=NPTS/NPLOTS
      IF(LX.GT.0)GOTO17
      CX=120./(XMAX-XMIN)
      SX(1)=XMIN
      SX(7)=XMAX
      U=XMIN
      DO16K=2,6
      U=(XMAX-XMIN)/6.+U
   16 SX(K)=U
      GOTO19
   17 KX=120/LX
      CX=KX
      SX(1)=XMIN
      DO18K=2,LLX
   18 SX(K)=10.*SX(K-1)
```

```
19 CALLPCT(X,XMIN,LX,NPTS,0,120.,CX)
   IF(LY.GT.0)GOTO20
   CY=50./(YMAX-YMIN)
   GOTO21
20 KY=50/LY
   CY=KY
   NY=SLOG(YMIN)
21 CALLPCT(Y,YMIN,LY,NPTS,1,50.,CY)
   IF(NDIM.LT.3)GOTO24
   IF(LZ.GT.0)GOTO22
   CZ=40./(ZMAX-ZMIN)
   GOTO23
22 ZLZ=LZ
   CZ=40./ZLZ
23 CALLPCT(Z,ZMIN,LZ,NPTS,0,40.,CZ)
24 DO50NN=1,NCOPYS
   M1=1
   T1=33.
   LYY=LY
   TT=50.
   WRITE(3,3)(TITLE(I),I=1,8)
   DO43KK=1,51
   N=1
   NNN=NPN
   JED=1
   T=51-KK
   DO25J=1,133
25 L(J)=NB
   L(133)=ND
   IF(LY.GT.0)GOTO26
   L(13)=NP
   IF(T.GT.TT)GOTO30
   SCALE=T/CY+YMIN
   L(133)=NP
   N=0
   TT=TT-5.
   IF(T.LE.0.)SCALE=YMIN
   GOTO30
26 GOTO(27,27,28,28,27,28),LY
27 SS=KY*LYY
   GOTO29
28 SS=KY*LYY+1
29 L(13)=ND
   IF(T.GT.SS)GOTO30
   SCALE=10.**(NY+LYY)
   N=0
   LYY=LYY-1
   L(13)=NP
   L(133)=NP
30 IF(50..EQ.T)GOTO31
   IF(0..NE.T)GOTO37
31 DO32J=14,133
```

```
   32 L(J)=NM
      IF(LX.GT.0)GOTO34
      DO33J=13,133,10
   33 L(J)=NP
      GOTO36
   34 DO35J=13,133,KX
   35 L(J)=NP
   36 IF(50..EQ.T)L(133)=ND
   37 DO40LM=1,NPLOTS
      DO39I=JED,NNN
      IF(Y(I).NE.T)GOTO39
      J=X(I)
      IF(NDIM.NE.3)GOTO38
      IZ=Z(I)
      L(J+13)=NCH(IZ+1)
      GOTO39
   38 L(J+13)=NCH(LM)
   39 CONTINUE
      JED=NNN+1
      NNN=NNN+NPN
   40 CONTINUE
      IF(T1.NE.T)GOTO41
      IF(15..GE.T)GOTO41
      L(2)=MOP(M1)
      M1=M1+1
      T1=T1-1.
   41 IF(N.EQ.1)GOTO42
      WRITE(3,4)L(2),SCALE,(L(J),J=12,133)
      GOTO43
   42 WRITE(3,5)(L(J),J=1,133)
   43 CONTINUE
      GOTO(44,45,46,47,48,49,44),LLX
   44 WRITE(3,6)(SX(K),K=1,7)
      GOTO50
   45 WRITE(3,7)(SX(K),K=1,LLX)
      GOTO50
   46 WRITE(3,8)(SX(K),K=1,LLX)
      GOTO50
   47 WRITE(3,9)(SX(K),K=1,LLX)
      GOTO50
   48 WRITE(3,10)(SX(K),K=1,LLX)
      GOTO50
   49 WRITE(3,11)(SX(K),K=1,LLX)
   50 WRITE(3,12)TAB1,TAB2
      RETURN
      END
     MON$$      EXEQ FORTRAN
      SUBROUTINEPOT(V,VMIN,LV,NP,J,VC,C)
C     PROGRAMMED BY ED KOBETICH, KSU DEPT. OF PHYSICS, JUNE 1964.
C     THIS SUBPROGRAM SCALES THE COORDINATES OF THE POINTS
C     SO THEY CAN BE PLOTTED.
      DIMENSIONV(1)
```

```
      IF(LV.GT.0)GOTO2
      DO1I=1,NP
    1 V(I)=FLOAT(IFIX(C*(V(I)-VMIN)+.5))
       GOTO4
    2 DO3I=1,NP
    3 V(I)=FLOAT(IFIX(C*(ALOG(V(I)/VMIN)/2.302585)+.5))
    4 IF(J.GT.0)GOTO7
      DO6I=1,NP
      IF(V(I).LT.0.)GOTO5
      IF(V(I).LE.VC)GOTO6
    5 V(I)=VC+1.
    6 CONTINUE
    7 RETURN
      END
     MON$$     EXEQ FORTRAN,,,,,,,SLOP
      DIMENSION XF(20),A(20),B(20),C(20),D(20),Y(40),CY(40)
C     THIS PROGRAM COMPUTES DAMAGE CONSTANTS K AND ALPHA
C     H(Z) IS COMPUTED FOR DIFFERENT VALUES OF EMITTER CURRENT
    1 FORMAT(I3)
    2 FORMAT(E14.8)
    3 FORMAT(6H FLUX=,E16.8,5X,6HFLUX2=,E16.8,5X,6HFLUX3=,E16.8//)
   13 FORMAT(6HFLUX1=,E9.4,3X,6HFLUX2=,E9.4,3X,6HFLUX3,E9.4)
    4 FORMAT(1HL,13H EMITTER CUR.,3X,10H     SLOPE,8X,9HINTERCEPT,2X,11H
     1STAND. DIV.,1X,21HCAL.1/BETA FIT.1/BETA,5X,8HK CONST.,12X,12HALPHA
     2 CONST./)
    5 FORMAT(1HK,F12.6,3X,E16.8,5X,F8.6,3X,F8.6,3X,F8.6,3X,F8.6,E16.8,5X
     1,F16.8)
    6 FORMAT(59X,F8.6,3X,F8.6)
    7 FORMAT(1HL,2X,2HA=,E16.8,2X,2HB=,E16.8,2X,2HC=,E16.8,2X,2HD=,E16.8
     1)
   17 FORMAT(2HA=,E12.6,2X,2HB=,E12.6,2X,2HC=,E12.6,2X,2HD=,E12.6)
    8 FORMAT(2F10.5)
    9 FORMAT(32H CUTOFF FREQUENCY OF TRANSISTOR=,E14.8)
   25 FORMAT(25HLIFETIME DAMAGE CONSTANT=,E16.8)
   26 FORMAT(28HTRANSISTOR DAMAGE  CONSTANT=,E16.8)
      END FILE 7
      REWIND 7
      REWIND 6
      REWIND 8
   40 READ(1,1)NGR
      READ(1,2)(XF(I),I=1,NGR)
      READ(1,2)FREQ
      READ(1,8)DELX,XX
      NTEST=1
   30 CONTINUE
      DO10I=1,NGR
   10 READ(7)A(I),B(I),C(I),D(I)
      WRITE(3,7)(A(I),B(I),C(I),D(I),I=1,NGR)
      X=0.
      WRITE(3,4)
  100 DO20I=1,NGR
   20 Y(I)=A(I)+B(I)*X+C(I)*X*X+D(I)*X*X*X
```

```
      CALL LSTSQ(XF,Y,NGR,SL,Z,SMU,CY)
      CONSK=.19416/(FREQ*SL)
      CONSA=SL
      WRITE(3,5)X,SL,Z,SMU,Y(1),CY(1),CONSK,CONSA
      WRITE(3,6)(Y(I),CY(I),I=2,NGR)
      CONS1=1.
      CONS2=SL
      WRITE(6)X,CONS1
      WRITE(8)X,CONS2
      X=.0+DELX
  130 CONTINUE
 1100 DO120I=1,NGR
  120 Y(I)=A(I)+B(I)*X+C(I)*X*X+D(I)*X*X*X
      CALL LSTSQ(XF,Y,NGR,SL,Z,SMU,CY)
      CONS1=SL/CONSA
      CONS2=SL
      WRITE(6)X,CONS1
      WRITE(8)X,CONS2
      WRITE(3,5)X,SL,Z,SMU,Y(1),CY(1),CONS1,CONS2
      WRITE(3,6)(Y(I),CY(I),I=2,NGR)
      X=X+DELX
      IF(X.LT.XX)GO TO 130
C     TEST FOR INPUT DATA
      NTEST=NTEST+1
      IF(NTEST.LE.1)GOTO30
      STOP
      END
     MON$$     EXEQ FORTRAN,,,,,,,LSTSQ
      SUBROUTINE LSTSQ(X,Y,N,SL,Z,SMU,CY)
      DIMENSION X(40),Y(40),CY(40)
C      THIS SUBPROGRAM FITS A STRAIGHT LINE THROUGH A SET  OF POINTS
C     SL IS THE SLOP OF THE LINE
C     Z IS THE INTERCEPT,N IS NUMBER OF DATA POINT
      SX=0.
      SY=0.
      SXZ=0.
      SXY=0.
      DO2I=1,N
      SX=SX+X(I)
      SY=SY+Y(I)
      SXY=SXY+X(I)*Y(I)
    2 SXZ=SXZ+X(I)**2
      Q=N
      DEN=Q*SXZ-SX**2
      SL=(Q*SXY-SX*SY)/DEN
      Z=(SXZ*SY-SX*SXY)/DEN
C     CALCULATION OF STANDARD DIVIATION
      DEVESQ=0.
      DO4 I=1,N
      CY(I)=Z+SL*X(I)
    4 DEVESQ=(Y(I)-CY(I))**2+DEVESQ
      DER=(Q-2.0)*SXY
```

```
      SMU=SQRT(DEVESQ/DER)
      RETURN
      END
     MON$$      EXEQ FORTRAN,,,,,,,DAMAGE
      DIMENSION X(30),CONS1(30),CONS2(30)
      COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
C     THIS PROGRAM FITS A SERIES OF POLYNOMIAS TO H(Z) AND CURRENT
    1 FORMAT(1HL,53X,37HCOEF. FOR  H(Z) VS CURRENT ANALYSIS  /)
    2 FORMAT(1HL,50X,38HCOEF. FOR ALPHA*H(Z) VS CUR. ANALYSIS /)
    3 FORMAT(2F10.5)
      REWIND 6
      REWIND 8
      READ(1,3)DELX,XX
      NDP=XX/DELX
      NTEST=1
    9 CONTINUE
      NCOEF=2
      NCOEFL=4
      WRITE(3,1)
      DO10 I=1,NDP
   10 READ(6)X(I),CONS1(I)
      CALL FINAL(X,CONS1,NDP,NCOEF,NCOEFL)
      WRITE(3,2)
      NCOEF=2
      NCOEFL=4
      DO20 I=1,NDP
   20 READ(8)X(I),CONS2(I)
      CALL FINAL(X,CONS2,NDP,NCOEF,NCOEFL)
C     TEST FOR INPUT DATA
      NTEST=NTEST+1
      IF(NTEST.LE.1)GOTO9
      STOP
      END
     MON$$      EXEQ FORTRAN,,,,,,,FINAL
      SUBROUTINE FINAL(X,Y,NDP,NCOEF,NCOEFL)
      DIMENSION X(20),Y(20)
      DIMENSION CY(20),DY(20)
      COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
    2 FORMAT(20H/ANALYSIS OF DEGREE ,I3)
    5 FORMAT(3H B(,I2,2H)=,E16.8)
    6 FORMAT(20HTSTANDARD DEVIATION=,E16.8)
    9 FORMAT(16HTRELATIVE ERROR=,E16.8)
 1001 NCM1=NCOEF-1
      WRITE(3,2)NCM1
      DO 101 K=1,NCOEF
      DO 102 J=1,NCOEF
  102 A(K,J)=0.
  101 B(K)=0.
      DO 107 J=1,NCOEF
      DO 107 I=1,NDP
      IF(J.GT.1) GO TO 105
      DO 103 K=1,NCOEF
  103 A(1,K)=A(1,K)+X(I)**(K-1)
```

```
      GO TO 107
  105 A(J,NCOEF)=A(J,NCOEF)+X(I)**(J+NCOEF-2)
  107 B(J)=B(J)+Y(I)*X(I)**(J-1)
      DO 109 J=2,NCOEF
      DO 109 K=1,NCM1
  109 A(J,K)=A(J-1,K+1)
      DO 120 J=1,NCOEF
      DO 121 K=1,NCOEF
  121 AH(J,K)=A(J,K)
  120 BH(J)=B(J)
      CALL CRAM(NCOEF,1)
      CALL SOLVE(NCOEF)
      CALL RESIDU(NCOEF)
      WRITE(3,5)(I,B(I),I=1,NCOEF)
      DEVESQ=0.
      RELDEV=0.
      DO200I=1,NDP
      CY(I)=B(I)
      DO201J=2,NCOEF
  201 CY(I)=B(J)*X(I)**(J-1)+CY(I)
      RELDEV=RELDEV+(CY(I)-Y(I))/Y(I)
  200 DEVESQ=(Y(I)-CY(I))**2+DEVESQ
      SMU=SQRT(DEVESQ/FLOAT(NDP))
      WRITE(3,6)SMU
      RELERR=RELDEV/FLOAT(NDP)*100.
      WRITE(3,9) RELERR
      NCOEF=NCOEF+1
      IF(NCOEF.LE.NCOEFL)GOTO1001
      RETURN
      END
     MON$$      EXEQ FORTRAN,,,,,,,CRAM
      SUBROUTINE CRAM(N,I)
      COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
 2240 FORMAT(1HK,5HP1VOT,I3,19HIS LESS THAN 1.E-12)
 2241 FORMAT(1HK,5HP1VOT,I3,7HIS ZERO)
      GO TO (2200,2201),I
 2200 IDMV=1
      GOTO2202
 2201 IDMV=2
 2202 DO 2204 IDK=1,N
      V(IDK)=ABS(A(IDK,1))
      DO2204 IDI=2,N
      IF(V(IDK)-ABS(A(IDK,IDI)))2203,2204,2204
 2203 V(IDK)=ABS(A(IDK,IDI))
 2204 CONTINUE
      DO 2222 IDK=1,N
      DETR=-1.
      IDK1=IDK-1
      DO2214 IDI=IDK,N
      DETPR=0.0
      IF(IDK-1)2208,2208,2206
 2206 DO2207 IDJ=1,IDK1
```

```
2207 DETPR=DETPR+A(IDI,IDJ)*A(IDJ,IDK)
2208 A(IDI,IDK)=A(IDI,IDK)-DETPR
     GO TO(2212,2225),IDMV
2212 DETS=ABS(A(IDI,IDK))/V(IDI)
     IF(DETS-DETR)2214,2214,2213
2213 DETR=DETS
     IP(IDK)=IDK-IDI
     GO TO 2214
2225 IP(IDK)=0
2214 CONTINUE
     IDK2=IDK-IP(IDK)
     DETR=A(IDK2,IDK)/V(IDK2)
     IF(ABS(DETR)-1.E-12)2230,2230,2232
2230 WRITE(3,2240)IDK
     IF(A(IDK2,IDK)) 2232,2231,2232
2231 WRITE(3,2241)IDK
     CALL EXIT
2232 V(IDK2)=V(IDK)
     V(IDK)=DETR
     DO2222 IDJ=1,N
     DETR=A(IDK,IDJ)
     IF(IDJ-IDK)2215,2215,2216
2215 A(IDK,IDJ)=A(IDK2,IDJ)
     GO TO 2220
2216 DETPR=0.0
     IF(IDK-1)2219,2219,2217
2217 DO2218 IDI=1,IDK1
2218 DETPR=DETPR+A(IDK,IDI)*A(IDI,IDJ)
2219 A(IDK,IDJ)=(A(IDK2,IDJ)-DETPR)/A(IDK,IDK)
2220 IF(IP(IDK))2221,2222,2222
2221 A(IDK2,IDJ)=DETR
2222 CONTINUE
     RETURN
     END
    MON$$     EXEQ FORTRAN,,,,,,,SOLVE
     SUBROUTINE SOLVE (N)
     COMMON A(20,20),IP(20),V(20),B(20),AH(20,20),BH(20)
     DO 2256 IDK = 1,N
     IDK1 = IDK - 1
     IDK2 = IDK-IP(IDK)
     DETR = B(IDK)
     DETPR = 0.0
     IF(IDK-1) 2253, 2253, 2257
2257 DO 2252 IDI = 1, IDK1
2252 DETPR = DETPR+A(IDK,IDI)* B(IDI)
2253 B(IDK) = (B(IDK2) - DETPR) / A(IDK,IDK)
     IF (IP(IDK)) 2254, 2256, 2256
2254 B(IDK2) = DETR
2256 CONTINUE
     DO 2263 IDI2 = 1,N
     IDI = N + 1 - IDI2
     DETPR = 0.0
```

```
      IDI1 = IDI + 1
      IF (N - IDI) 2263, 2263, 2261
 2261 DO 2262 IDJ = IDI1,N
 2262 DETPR = DETPR + A(IDI,IDJ)* B(IDJ)
 2263 B(IDI) = B(IDI) - DETPR
      RETURN
      END
     MON$$      EXEQ FORTRAN,,,,,,,RESIDU
      SUBROUTINE RESIDU(N)
      COMMON A(20,20),IP(20),V(20),B(20),S(20,20),T(20)
      DO 1 I=1,N
      V(I)=B(I)
      DO 1 J=1,N
    1 T(I)=T(I)-S(I,J)*B(J)
    2 DO 3 I=1,N
    3 B(I)=T(I)
      CALL SOLVE (N)
      DO 10 I=1,N
      DO 10 J=1,N
   10 T(I)=T(I)-S(I,J)*B(J)
      J=0
      DO 5 I=1,N
      B(I)=B(I)+V(I)
      IF(B(I)-V(I))6,5,6
    6 J=1
    5 V(I)=B(I)
      IF(J) 7,7,2
    7 RETURN
      END
     MON$$      EXEQ FORTRAN,,,,,,,GRAPH
      DIMENSION X(60),CONS1(60),CONS2(60)
C     THIS PROGRAM PLOTS THE OUTPUT FROM TAPES 6 AND 8
    2 FORMAT(6H XMAX=,E16.8,5X,5HXMIN=,E16.8/)
    3 FORMAT(7H CO1MA=,E16.8,5X,6HCO1MI=,E16.8/)
    4 FORMAT(7H CO2MA=,E16.8,5X,6HCO1MI=,E16.8/)
    5 FORMAT(2F10.5)
    6 FORMAT(3X,3E16.8)
      END FILE6
      END FILE8
      REWIND 6
      REWIND8
      REWIND 7
      READ(1,5)DELX,XX
      NDP=XX/DELX
      NTEST=1
      NBA=1*NDP
   10 CONTINUE
      DO15 K=1,NBA
   15 READ(6)X(K),CONS1(K)
      DO20 J=1,NBA
   20 READ(8)X(J),CONS2(J)
      XMIN=X(1)
```

```
      XMAX=X(1)
      CO1MI=CONS1(1)
      CO1MA=CONS1(1)
      CO2MI=CONS2(1)
      CO2MA=CONS2(1)
C     FIND THE LIMITS OF GRAPHS
      DO 30 I=2,NBA
      IF(XMIN.GT.X(I))XMIN=X(I)
      IF(XMAX.LT.X(I))XMAX=X(I)
      IF(CO1MI.GT.CONS1(I))CO1MI=CONS1(I)
      IF(CO1MA.LT.CONS1(I))CO1MA=CONS1(I)
      IF(CO2MI.GT.CONS2(I))CO2MI=CONS2(I)
   30 IF(CO2MA.LT.CONS2(I))CO2MA=CONS2(I)
      WRITE(3,2)XMAX,XMIN
      WRITE(3,3)CO1MA,CO1MI
      WRITE(3,4)CO2MA,CO2MI
      WRITE(3,6)(X(J),CONS1(J),CONS2(J),J=1,NBA)
      XMAX=2.0*XMAX
      CO1MA=1.4*CO1MA
      CO2MA=1.4*CO2MA
      WRITE(7)(X(I),I=1,NBA)
      END FILE 7
      CALL PLOT(X,XMIN,XMAX,0,CONS1,CO1MI,CO1MA,0,0.,0.,0.,0,NBA,1,1,3,2
     1)
      REWIND 7
      READ(7)(X(I),I=1,NBA)
      CALL PLOT(X,XMIN,XMAX,0,CONS2,CO2MI,CO2MA,0,0.,0.,0.,0,NBA,1,1,3,2
     1)
      NTEST=NTEST+1
      IF(NTEST.LE.1)GOTO10
      STOP
      END
     MON$$      EXEQ FORTRAN,,,,,,PLOT
      SUBROUTINEPLOT(X,XMIN,XMAX,LX,Y,YMIN,YMAX,LY,Z,ZMIN,ZMAX,LZ,NPTS,
     1NPLOTS,NCOPYS,NCARDS,NDIM)
C     PROGRAMMED BY ED KOBETICH, KSU DEPT. OF PHYSICS, JUNE 1964.
      DIMENSIONX(1),Y(1),Z(1),SX(7),TITLE(8),L(134),NCH(41),MOP(18)
    1 FORMAT(8A10)
    2 FORMAT(58A1,2A9,4A1)
    3 FORMAT(1H1,26X,8A10)
    4 FORMAT(1H ,A1,1PE9.2,122A1)
    5 FORMAT(133A1)
    6 FORMAT(1PE17.2,5E20.2,E16.2)
    7 FORMAT(1PE17.2,100X,E16.2)
    8 FORMAT(1PE17.2,E60.2,E56.2)
    9 FORMAT(1PE17.2,2E40.2,E36.2)
   10 FORMAT(1PE17.2,3E30.2,E26.2)
   11 FORMAT(1PE17.2,4E24.2,E20.2)
   12 FORMAT(1HA,62X,2A9)
      SLOG(F)=ALOG(F)/2.302585
      LLX=LX+1
      NDD=NCARDS+1
```

```
      GOTO(15,13,14,13),NDD
   13 READ(1,1)(TITLE(I),I=1,8)
   14 IF(NDD.GE.3)READ(1,2)(MOP(I),I=1,18),(NCH(I),I=1,40),TAB1,TAB2,ND,
     1NP,NM,NB
   15 NCH(41)=NB
      NPN=NPTS/NPLOTS
      IF(LX.GT.0)GOTO17
      CX=120./(XMAX-XMIN)
      SX(1)=XMIN
      SX(7)=XMAX
      U=XMIN
      DO16K=2,6
      U=(XMAX-XMIN)/6.+U
   16 SX(K)=U
      GOTO19
   17 KX=120/LX
      CX=KX
      SX(1)=XMIN
      DO18K=2,LLX
   18 SX(K)=10.*SX(K-1)
   19 CALLPOT(X,XMIN,LX,NPTS,0,120.,CX)
      IF(LY.GT.0)GOTO20
      CY=50./(YMAX-YMIN)
      GOTO21
   20 KY=50/LY
      CY=KY
      NY=SLOG(YMIN)
   21 CALLPOT(Y,YMIN,LY,NPTS,1,50.,CY)
      IF(NDIM.LT.3)GOTO24
      IF(LZ.GT.0)GOTO22
      CZ=40./(ZMAX-ZMIN)
      GOTO23
   22 ZLZ=LZ
      CZ=40./ZLZ
   23 CALLPOT(Z,ZMIN,LZ,NPTS,0,40.,CZ)
   24 DO50NN=1,NCOPYS
      M1=1
      T1=33.
      LYY=LY
      TT=50.
      WRITE(3,3)(TITLE(I),I=1,8)
      DO43KK=1,51
      N=1
      NNN=NPN
      JED=1
      T=51-KK
      DO25J=1,133
   25 L(J)=NB
      L(133)=ND
      IF(LY.GT.0)GOTO26
      L(13)=NP
      IF(T.GT.TT)GOTO30
```

```
      SCALE=T/CY+YMIN
      L(133)=NP
      N=0
      TT=TT-5.
      IF(T.LE.0.)SCALE=YMIN
      GOTO30
   26 GOTO(27,27,28,28,27,28),LY
   27 SS=KY*LYY
      GOTO29
   28 SS=KY*LYY+1
   29 L(13)=ND
      IF(T.GT.SS)GOTO30
      SCALE=10.**(NY+LYY)
      N=0
      LYY=LYY-1
      L(13)=NP
      L(133)=NP
   30 IF(50..EQ.T)GOTO31
      IF(0..NE.T)GOTO37
   31 DO32J=14,133
   32 L(J)=NM
      IF(LX.GT.0)GOTO34
      DO33J=13,133,10
   33 L(J)=NP
      GOTO36
   34 DO35J=13,133,KX
   35 L(J)=NP
   36 IF(50..EQ.T)L(133)=ND
   37 DO40LM=1,NPLOTS
      DO39I=JED,NNN
      IF(Y(I).NE.T)GOTO39
      J=X(I)
      IF(NDIM.NE.3)GOTO38
      IZ=Z(I)
      L(J+13)=NCH(IZ+1)
      GOTO39
   38 L(J+13)=NCH(LM)
   39 CONTINUE
      JED=NNN+1
      NNN=NNN+NPN
   40 CONTINUE
      IF(T1.NE.T)GOTO41
      IF(15..GE.T)GOTO41
      L(2)=MOP(M1)
      M1=M1+1
      T1=T1-1.
   41 IF(N.EQ.1)GOTO42
      WRITE(3,4)L(2),SCALE,(L(J),J=12,133)
      GOTO43
   42 WRITE(3,5)(L(J),J=1,133)
   43 CONTINUE
      GOTO(44,45,46,47,48,49,44),LLX
```

```
44 WRITE(3,6)(SX(K),K=1,7)
   GOTO50
45 WRITE(3,7)(SX(K),K=1,LLX)
   GOTO50
46 WRITE(3,8)(SX(K),K=1,LLX)
   GOTO50
47 WRITE(3,9)(SX(K),K=1,LLX)
   GOTO50
48 WRITE(3,10)(SX(K),K=1,LLX)
   GOTO50
49 WRITE(3,11)(SX(K),K=1,LLX)
50 WRITE(3,12)TAB1,TAB2
   RETURN
   END
  MON$$      EXEQ FORTRAN
   SUBROUTINEPOT(V,VMIN,LV,NP,J,VC,C)
C     PROGRAMMED BY ED KOBETICH, KSU DEPT. OF PHYSICS, JUNE 1964.
C     THIS SUBPROGRAM SCALES THE COORDINATES OF THE POINTS
C     SO THEY CAN BE PLOTTED.
   DIMENSIONV(1)
   IF(LV.GT.0)GOTO2
   DO1I=1,NP
 1 V(I)=FLOAT(IFIX(C*(V(I)-VMIN)+.5))
    GOTO4
 2 DO3I=1,NP
 3 V(I)=FLOAT(IFIX(C*(ALOG(V(I)/VMIN)/2.302585)+.5))
 4 IF(J.GT.0)GOTO7
   DO6I=1,NP
   IF(V(I).LT.0.)GOTO5
   IF(V(I).LE.VC)GOTO6
 5 V(I)=VC+1.
 6 CONTINUE
 7 RETURN
   END
  MON$$      EXEQ LINKLOAD
             PHASELSTSQARE
             CALL LSTSQARE
             PHASELEASTSQ
             CALL LEASTSQ
             PHASEMAPDATA
             CALL MAPDATA
             PHASESLOP
             CALL SLOP
             PHASEDAMAGE
             CALL DAMAGE
             PHASEGRAPH
             CALL GRAPH
```

Table C–IV EXAMPLE OF INPUT DATA FOR A TRANSISTOR

```
    MCN$$     JOB  TRANSISTOR NO. K-3  ANALYSIS
    MCN$$     ASGN MJB,12
    MCN$$     EXEQ LSTSQARE,MJB
 1  4
 7  4  4
 .000        0.000
 .042        0.005
 .260        0.010
 .330        0.015
 .560        0.020
 .840        0.025
1.160        0.030
 7  4  4
 .020        0.000
 .120        0.005
 .240        0.010
 .360        0.015
 .520        0.020
 .660        0.025
 .800        0.030
 7  4  4
 .000        0.000
 .160        0.010
 .340        0.020
 .540        0.030
 .780        0.040
1.040        0.050
1.280        0.060
 7  4  4
 .040        0.000
```

```
 .160        0.020
 .240        0.040
 .480        0.060
 .700        0.080
 .960        0.100
1.160        0.120
    MCN$$      EXEQ LEASTSQ,MJB
    MCN$$      EXEQ MAPDATA,MJB
  7  4
INVERSE BETA VS EMITTER CURRENT-TRANS. K-3
INVERSE GAIN       1234                                  EMITTER CURRENT   .+-
    MCN$$      EXEQ SLOP,MJB
  4
.00000000E 00
.66000000E 13
.13000000E 14
.65000000E 14
.50000000E 01
.5       10.
    MCN$$      EXEQ DAMAGE,MJB
.5       10.
    MCN$$      EXEQ GRAPH,MJB
.5       10.
H(Z) VS EMITTER CURRENT - TRANS. K-3
H(Z)               1                                     EMITTER CURRENT   .+-
ALPHA*H(Z) VS EMITTER CURRENT - TRANS. K-3
ALPHA*H(Z)         4                                     EMITTER CURRENT   .+-
```

APPENDIX D

Neutron Dosimetry and Standarization of Counting Systems

To accurately measure the integrated neutron flux it was suggested by the Argonne National Laboratory staff (9) that aluminum activation foils be used. The aluminum activation cross section for 14.1 Mev neutrons is better known than that of other materials (see Table I). In utilizing the aluminum foils it was necessary to establish a stable accurate counting system of known efficiency. A gas flow beta proportional counter was selected as most suitable for this purpose; such a counter was set up and standardized against a similar counter at Argonne National Laboratory.

The Argonne National Laboratory gas flow proportional counter serves as a calibration reference within the Argonne National Laboratories and it is believed (9) to be the most accurate system available there for measurement of absolute activity. The count per second (CPS) obtained from this system are corrected to disintegration per second (DPS) using the following relation:

$$DPS = \frac{CPS}{(0.5)(1.47)e^{-.693t/\mu}} \qquad (D-1)$$

Where 0.5 is the system geometry correction factor, 1.47 the stainless steel backscatter correction factor and $e^{-.693t/\mu}$ is the foil thickness, t, correction factor.

To calibrate the Kansas State University beta proportional system, four foils were irradiated with the neutron generator and then counted with both the Kansas State University and Argonne National Laboratory systems. The resulting CPS values were corrected for background and using equation (D-1) the Argonne National Laboratory results were converted to DPS. The DPS obtained with the Argonne National Laboratory system and CPM from the Kansas State University system are plotted as a function of cooling time in Figure D-1 and D-2. Since the DPS obtained with the Argonne National Laboratory system are the same as C_{abs} and C_T is the same as CPS for the Kansas State University system (see Equation (120), Section 3.1) therefore

$$f_T = \frac{C_{abs}}{C_T} = \frac{DPS_{A.N.L.}}{CPS_{K.S.U.}} \; . \qquad \text{(D-2)}$$

f_T for the four foils and the average accepted value for the Kansas State University system are shown in Table D-I. Knowing the f_T value, the integrated neutron flux incident upon the surface of the foil is calculated from equation (119), Section 3.1.

Table D-I Correction factor for K.S.U. proportional counter

Foil No.	1	2	3	4	Ave.
f_T	1.550	1.552	1.541	1.516	1.539±.023
Expected half life	15 hrs	15 hrs	15 hrs	15 hrs	-
Measured half life	14.9	15.0	15.1	15.0	-

To monitor the transistor incident flux during the irradiation, it was necessary to set up a BF3 proportional counter (see Section 4.1). To calibrate this system, the transistor was removed from the socket and an aluminum foil

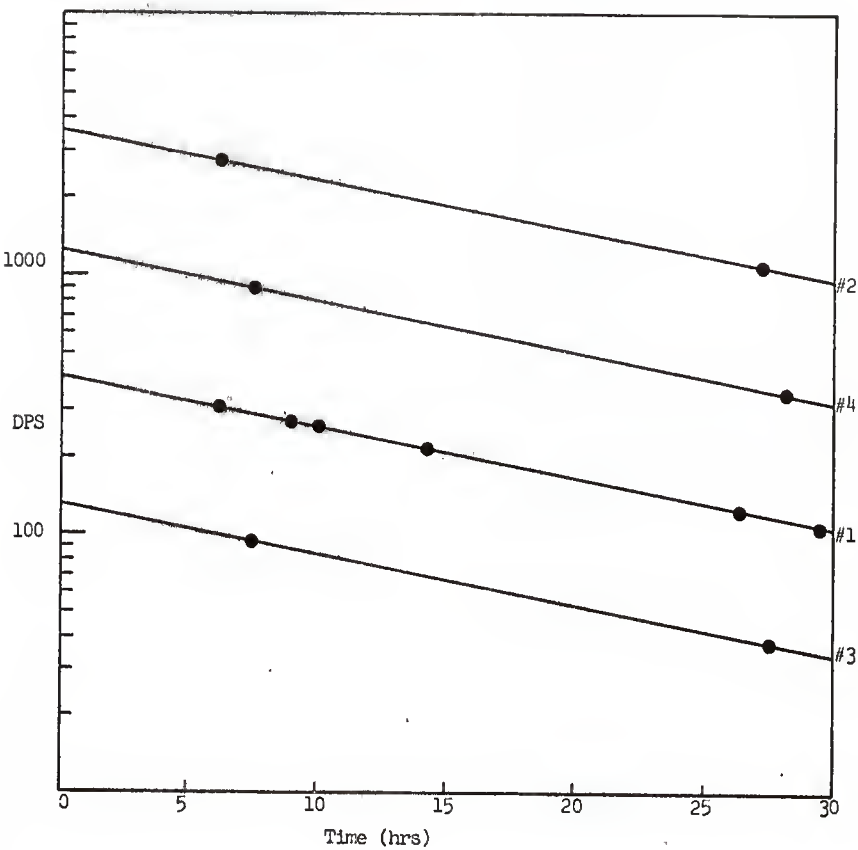

Figure D-1. Absolute activity (DPS) obtained from A.N.L. as a function of cooling time

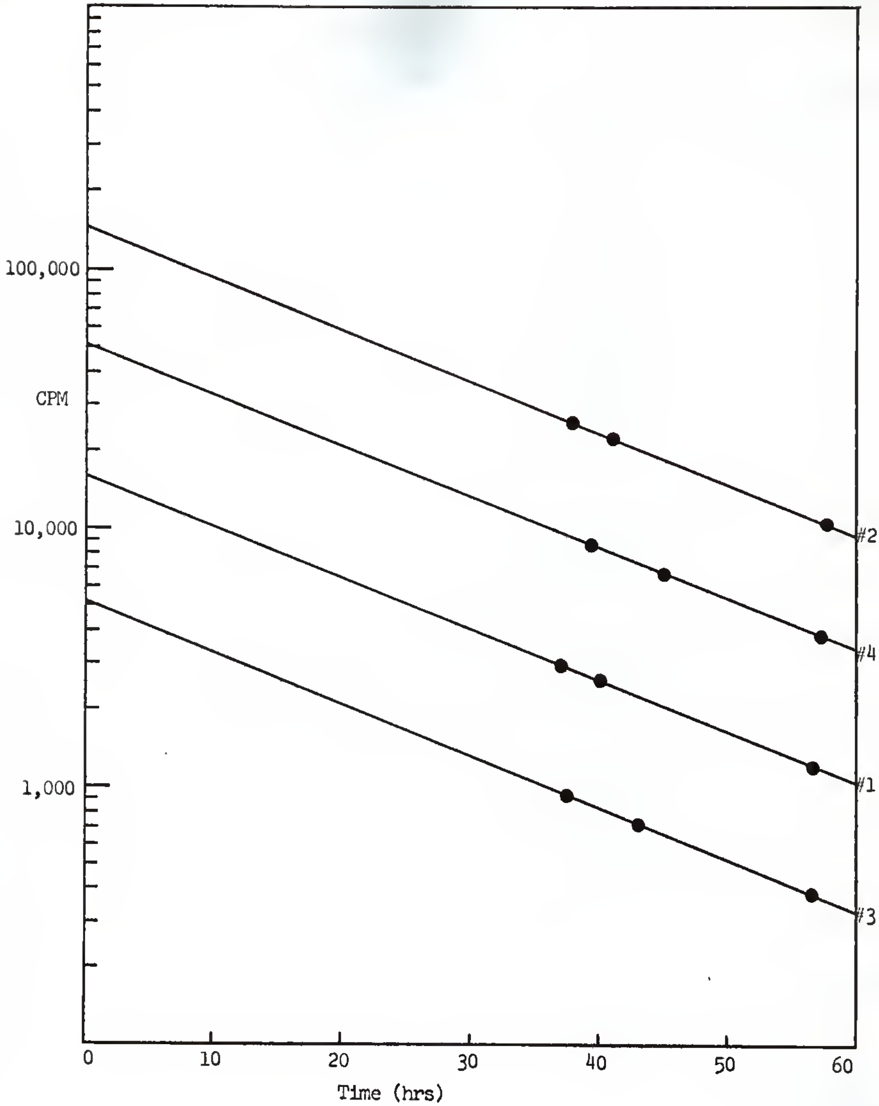

Figure D-2. Activity (CPM) obtained with K.S.U. system as a function of cooling time was placed in its place so that the front face of this foil was exactly the same distance from the neutron generator target as the front face of a transistor when it is placed in the socket. Two aluminum foils were placed in the foil holders around the transistor socket. After irradiation the count obtained from BF3 counter was calibrated against the absolute activity of each foil. For irradiation of transistors the middle foil was removed and two new foils were placed in the foil holders. The two outside foils were used for checking the stability of the BF3 counter.

EFFECT OF NEUTRON IRRADIATION ON TRANSISTOR CURRENT GAIN

by

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ABSTRACT

The purpose of this work was to study the effects of neutron irradiation on semiconductor materials and in particular to investigate the effect of 14.1 Mev neutrons on the performance of several commonly used transistors. Since transistor current gain is both a critical parameter in transistor performance and a function of the transistor-semiconductor physical parameters, current gain was selected as a means of measuring the neutron radiation damage.

The known effects of neutron irradiation on the minority carriers and conductivity of the semiconductor materials were used to derive a theoretical expression for the variation of transistor current gain with neutron irradiation. To check the validity of this relation, a set of transistors was irradiated with fast neutrons and collector characteristic curves obtained. A computer program was developed to analyze these data and calculate a lifetime damage constant. The value of this constant depends on the transistor semiconductor material. The computer also calculates a transistor damage constant; this constant is a function of transistor base width, lifetime damage constant for the base region, and operating conditions including the emitter current and collector voltage.

Damage constant results are compared with recent literature results. The literature results in the form of lifetime damage constants indicate that the order of radiation resistivity for germanium and silicon semiconductors is N-type Ge>P-type Ge>N-type Si>P-type Si. The results obtained in this work follow this order except that P-type Ge is found to be more resistant to neutron irradiation than N-type Ge.

In summary, the general results obtained are:

1. Transistor current gain decreases with the integrated neutron flux incident on the transistor. The extent of gain decrease is a function of transistor base width, semiconducting material used, and collector voltage and emitter current.
2. For a given transistor type, the transistor with wider base width showed more damage.
3. The lifetime damage constants determined were a function of the base region semiconductor materials.
4. Germanium transistors were found to be more radiation resistant than similar silicon transistors. Lifetime damage constants were higher for P-type germanium than for N-type; N-type silicon lifetime constants, however, were higher than those for P-type.