THEORY OF OSCILLATIONS IN A STRIATED DISCHARGE

by

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MASTER OF SCIENCE

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KANSAS STATE UNIVERSITY
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Approved by: [Signature]
Major Professor
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INTRODUCTION

Early experiments with glow discharges consisted of placing a probe or several probes into the discharge and observing what has been called a "plasma frequency" which has a characteristic frequency of about $10^8$ cycles per second (1, 2, 3). Many attempts have been made to interpret the observed phenomena mathematically.

The simplest treatment of the problem is to consider a region of uniform electron and ion density and then displace the electrons by a small amount (4). This treatment gives

\[ F = \frac{Ne^2}{M} \]  

where:

$N$ = electron number density,  
$M$ = electron mass,  
$e$ = electron charge  
$F$ = frequency of the oscillation.

This plasma oscillation frequency gives results of the right order of magnitude for the above experiments. Moreover, by letting $M$ equal the ion mass, the frequency is then an ion oscillation frequency (4).

Another way of obtaining the same result is the one applied by Tonks and Langmuir (5). They used two of Maxwell's equations to get the result

\[ F = \frac{Ne^2}{M + c^2/L^2} \]  

where:

$L$ = the wavelength of the plane waves in the discharge,  
$c$ = the velocity of light.

For very long wavelengths, this equation reduces to the plasma frequency (4).
Thomson (4), using another approach, also obtained a similar relation for ion oscillations.

\[
F = \frac{kT_i/L^2M + F_i^2}{(F_i^2L^2M/kT_e + 1)}
\]

where:

- \( Te \) = electron temperature,
- \( Ti \) = ion temperature,
- \( k \) = Boltzmann's constant,
- \( M \) = ion mass,
- \( F_i = Ne^2/ M \).

Other more complicated theories were also applied to these oscillations with indecisive results (6, 7, 8, 9).

In 1939, Merrill and Webb (10) published an experimental paper which dealt with probe measurements in plasmas. They found oscillations at frequencies \( 10^8 \) to \( 10^9 \) cycles per second as previous experimenters had found. More important were the observations that different potentials impressed on the probe produced different effects in the plasma and that certain violent oscillatory disturbances existed close to the cathode for some discharges.

In 1930, B. W. Fox (11) performed an experiment with a discharge tube in which he found oscillation frequencies of the order of \( 1.5 \times 10^4 \) to \( 2 \times 10^5 \) cycles per second, using the experimental apparatus of Webb and Pardue (2). Fox found that these oscillations were quite sensitive to pressure and current changes. The oscillations were independent of circuit resistances in series or any capacitance or inductance in parallel with the discharge. By using a magnet to "probe" the discharge from outside the tube, he discovered that the magnet had very little effect on the discharge except in the region of the Faraday dark
space.

Donahue and Dieke (12) performed an important experiment with discharge tubes in 1948. A photocell's output was connected to an oscilloscope which provided a measure of the oscillating portion of the light intensity of the discharge. Also, the oscilloscope was attached across a resistor in the circuit to measure the current oscillations developed by the discharge. Oscillations with frequencies from $10^3$ to $10^6$ cycles per second were discovered in all inert gases and mercury vapor type discharges. Two types of moving striations were discovered, a positive striation moving from the anode to the cathode and a negative striation moving from the cathode to the anode. The negative striation was found to move much faster than the positive striation. Both striations were observed in all parts of the discharge with the exception of Crookes dark space which did not give off enough light to enable this type of measurement to be made. When a positive striation and a negative striation met in the discharge, the motion of both striations ceased for a time until the striations again moved toward the cathode and anode respectively. It was this last experiment which led to the present series of experiments.

**EXPERIMENTAL PROCEDURES**

To verify Donahue and Dieke's results, a discharge system was arranged as pictured in figure 1. Two different cylindrical tubes with diameters of 2.3 centimeters were used. The system
EXPLANATION OF FIGURE 1

Schematic of discharge tube circuit.
FIGURE 1

A is a 1P23 photocell,
D is either a Heathkit Variable Voltage Regulated Power Supply model PS-3 or a General Radio type 673-A direct current power supply,
C is the discharge tube,
D is a Tektronics 545 Oscilloscope with plug-in units 53/54 D and 53/54 C.
EXPLANATION OF FIGURE 2

Schematic of signal generator circuit.
FIGURE 2
was first outgased to $10^{-6}$ millimeters of mercury and then deuterium gas was released into the system. The alternating voltage measurements were obtained by placing the probes from the oscilloscope across the ends of the discharge tube. Light intensity measurements consisted of connecting the photocell output to the oscilloscope and observing the alternating portion of the light intensity. The alternating current measurements were obtained by placing the probes of the oscilloscope across a 49 ohm resistor. The results are given in tables 1 and 2.

The oscillations depended on the applied voltage, filament current, current through the tube, gas pressure, position of the striations with respect to the cathode and plate, number of striations, and the type of glow at the plate. No direct relationship was observed between the variation of any one of these parameters and the appearance of oscillations in the tube. It proved impossible to make accurate determinations of the widths of the striations and the position of the striations with respect to the anode and cathode.

Time differences were found between the occurrences of current, voltage, and light intensity maxima. A dual trace plug-in unit was first used to measure the time difference between the voltage and light intensity maxima. A second method, which consisted of triggering the oscilloscope externally with the current oscillations, allowed a direct comparison of the time differences between the current, voltage, and light intensity maxima. Table 3 gives the results of these measurements.
EXPLANATION OF TABLE 1

This table gives some of the observed voltage oscillations along with their respective discharge parameters. The key for the symbols is found on table 2.
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EXPLANATION OF TABLE 2

This table gives a comparison of voltage and light intensity oscillations.
Comparison of voltage and light intensity oscillations

**TABLE 2**

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</table>

P. N. = Picture Number

PRES. = gas pressure in centimeters of oil. The oil has a specific gravity of 1.050.

FIL. = filament current in amperes.

TUBE = tube current in milliamperes.

FREQ. = oscillation frequency in cycles per second.

N. S. = number of striations.

M. D. = measuring device—photocell (PC) or voltage measurement across the tube (AT).
EXPLANATION OF TABLE 3

This table gives measurements of current, voltage, and light intensity oscillations.
Measurements of current, voltage, and light intensity oscillations.

### TABLE 3

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<th>S.W.</th>
<th>S.S.</th>
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P.N. = picture number.  
FIL. = filament current in amperes.  
TUBE = tube current in milliamperes.  
V. = tube voltage in volts.  
PRES. = gas pressure in centimeters of oil.  
N.S. = number of striations.  
S.W. = width of each striation in meters.  
S.S. = separation between the striations in meters.  
V/CM = millivolts per centimeter multiplying factor used in conjunction with the oscilloscope pictures.  
M. = type of measurement - current measurement (C), voltage measurement (V), or light intensity measurement (P).
A General Radio Company Type 1001-A signal generator was connected to the tube as shown in figure 2. Signals of desired frequency could then be impressed on the discharge. It was found that intermittent oscillations could be set up in the discharge by impressing a second frequency on top of the frequency at which the tube was oscillating. Further, by choosing a second frequency near the tube frequency and adjusting the magnitude of the signal properly, the striations in the tube could be made to oscillate visibly.

Light intensity measurements were made by looking at the positive column as a whole rather than by looking at separate parts of the discharge as was done by Donahue and Dieke. It was found that the scattered light intensity from the hot cathode was very much less than the light intensity from the positive column and could therefore be ignored during measurements.

DISCUSSION OF DIFFERENT THEORIES

Investigations were undertaken to determine if there was a mathematical development to explain the oscillatory phenomena observed in a striated discharge. The various treatments of Bohm and Gross (6) were studied but were found not to predict the discreet bands of oscillations found in the discharge.

The only case discussed by Bohm and Gross which may describe the phenomena is the section called Non-Linearity in Bunching or Plasma Shock Waves. Here Bohm and Gross assume a modulating potential and get a periodic charge density function for the
discharge along with a potential drop across regions of high charge density. Applying the experimental data to this theory does not give agreement with experimental results. This in part may be explained by the omission of the recombination process. The theory must account for the observed light intensity oscillations before it adequately describes the oscillation mechanism in the discharge.

Vlasov considered the interactions between the electrons in the electron beam, which are injected into the plasma, as the source of the oscillations. This treatment neglects ionization and recombination as important mechanisms in the discharge. Vlasov's treatment does not give a means of accounting for the experimentally observed light intensity oscillations.

The theory by Tonks (7) bears some investigation. Tonks assumed that a plasma could be considered as a series of capacitors and inductors in parallel combinations. By considering the number of collisions per electron per second to be too large to neglect, the expression

\[ F = \frac{S}{2} \frac{M}{(4Ne^2CL - M)} \]  

where:

- \( F \) = frequency,
- \( M \) = mass of the electron,
- \( N \) = electron concentration,
- \( e \) = charge on the electron,
- \( C \) = capacitance,
- \( L \) = inductance,
- \( S \) = the number of collisions per electron per second,

may be obtained. By redefining \( S \) to be the number of ionizing collisions per electron per second and redefining \( N \) as a function of position, this theory can give agreement with the present
experimental results. Tonks' theory does not, however, give an explanation for the information of striations in the discharge.

After examining these theories and others, it was decided that a new theory was in order to explain the appearance of oscillations in a striated discharge. The older theories were based on probe type measurements. These measurements yielded oscillations with frequencies of about $10^8$ cycles per second. No mention of any type of light oscillation was made and no phase measurements were taken. Therefore, the older theories did not have these results to work with and as a consequence fail to give a complete description of the discharge.

PURPOSED THEORY

To develop a satisfactory theory to explain oscillations of the magnitudes that are observed, consider three basic discharge characteristics. First a mechanism has to be developed to explain the stable oscillations found in the discharge. Then the process which causes intermittent oscillations as depicted by picture 57, see page 19, must be described. Finally, an adequate explanation must be given for the many discharge parameters for which no measurable oscillations are observed.

Consider a discharge tube with a hot cathode and plate. To initiate the discharge in the tube, a potential source is connected to the tube and at a certain definite voltage the discharge "strikes". The voltage then drops to a steady state value
EXPLANATION OF PLATE 1

Photographs are of the actual light intensity oscillations, current oscillations, and voltage oscillations. Oscilloscope data for these pictures are found in table 3.
Photographs are of the actual light intensity oscillations, current oscillations, and voltage oscillations. Oscilloscope data for these pictures are found in table 3.
which is great enough to maintain the discharge. To explain the theory of oscillations, we must also explain the difference between the steady state voltage and the initial voltage.

To explain this steady state voltage, consider that at the moment of ionization there are places of electron and ion densities which are more dense than at other points in the tube (13). Call these points "nodal points" and postulate that at these places the greatest amount of ionization occurs. In the case of a striated discharge, the striations coincide with the nodal points (13). Immediately after ionization, the electrons and ions start to move toward the anode and cathode respectively. This motion sets up positive and negative space charges in the region of the striation, see figure 3. This explains why a smaller steady state voltage is needed to maintain the discharge than the initial starting voltage.

Considering the initial conditions of the discharge, it is seen that as the applied potential is increased, the initial nodal point occurs at the plate, see figure 4. The positive space charge formed at the plate decreases the need for a high applied voltage since the space charge will contribute a part of this potential. Now since the field at this moment is higher than necessary to maintain the discharge, the nodal points will move toward the cathode until the steady state voltage is reached. The value of the steady state voltage depends on the pressure of the gas and the type of gas being used.

Consider the mechanics of the steady state discharge, see
EXPLANATION OF FIGURES 3, 4, AND 5

Figure 3 shows the steady state discharge.

Figure 4 shows the discharge just after the formation of the first nodal point at the plate. The electrons formed at the plate by the ionization process have been collected by the plate leaving a net positive space charge.

Figure 5 identifies the various space charges formed by the discharge which is operating in the steady state mode.
FIGURE 3

FIGURE 4

FIGURE 5
figure 5. The electrons are being continuously boiled off the cathode. As the electrons move along the tube, they gain energy until they have enough energy to produce ionization. Ionization takes place without altering the microscopic potential distribution since, at this point, the total charge is still constant, see figure 6. The ions move toward the cathode and the electrons move toward the plate causing a potential distribution function, as shown in figure 7, to form. The increased field caused by the excess of positive ions further disrupts the potential distribution attracting electrons toward the ions as seen in figure 8. The new electron space charge B caused further bunching of the electrons in the tube, see figure 9. The electron space charge B then neutralizes the positive ion space charge D by recombination, changing the potential distribution to that shown in figure 10. The negative space charge C then causes ionization at point D and the cycle repeats itself.

The calculations are based on sinusoidal density distribution functions for B and D, and considering that recombination occurs when B and D come together.

Before proceeding further, it is necessary to note that ionization takes place in all parts of the tube. The maximum amount, however, takes place at the striations. Ionization occurs in all parts of the tube because of the wide variation of energy which the electrons possess because of elastic scattering and cathode emission in all directions.
Figures 6 thru 10 show the formation of the various space charges along the discharge tube. The effects of the space charges on the potential function along the tube are shown by the graph of voltage vs distance from cathode. The figures are in a time sequence beginning with the ionization process.
DC = DISTANCE FROM CATHODE
The equation

\[ I = NeVA \]

where:

\( N \) = particle density,
\( V \) = electron velocity,
\( A \) = cross sectional area,
\( I \) = current,
\( e \) = charge on an electron,


gives the relationship of the current to the electron density and velocity. To determine what phase shifts should be observed experimentally, consider the discharge at the plate. A current minimum will occur either just before ionization when the density of charged particles is minimum or at the time of ionization since this is when the electrons have their lowest velocity. A light intensity maximum should then follow since the rate of recombination is increasing due to the increased charged particle density. The relation between light intensity and charged particle density is given by

\[ A = RN^+N^- \]

where:

\( A \) = light intensity,
\( R \) = recombination coefficient,
\( N^+ \) = positive ion concentration,
\( N^- \) = negative particle concentration.

A current maximum should be reached at this time since the electrons which caused the ionization plus the electrons formed from the ionization process will now be collected by the plate. As the recombination process continues, the voltage should approach a minimum due to the annihilation of the positive space charge. A light minimum will occur when practically all the positive ions
have recombined leaving low electron and ion densities.

To summarize the results, if the scope is triggered on the current maximum, almost simultaneous current and light intensity maxima should occur, the light intensity maximum occurring simultaneously or just slightly after the current maximum, and these maxima should be followed by a voltage minimum. These results may be seen in pictures 630 and 640, see page 21. The time relations may not be as simple for discharges with multiple frequencies such as seen in picture 620, see page 21.

Intermittent oscillations may be produced by finding an oscillation within the tube and then superimposing another oscillation of nearly equal amplitude upon it. Therefore, when intermittent oscillations occur in a discharge, it may be assumed that more than one frequency is present in the tube and that these frequencies interfere with each other producing the intermittent discharge.

This interference can be explained by oscillating striations and hence oscillating nodal points. If the striations oscillate at a low enough frequency, during the time when the striation is changing direction in its motion, it may remain in position long enough to set up some high frequency oscillations in the tube. A flickering of the striations was noticed in all cases where intermittent oscillations were observed.

This mechanism can also account for the discharges which have no stable oscillations. In this case, the striations may oscillate with a high enough frequency to prevent the formation
of any oscillations which the discharge might try to set up. High frequency striation oscillations have yet to be measured.

CALCULATIONS

The recombination coefficient is given by the equation

\[
\frac{dN^+}{dt} = \frac{dN^-}{dt} = -RN^+N^- \tag{7}
\]

where:

- \(N^+\) = the positive ion concentration,
- \(N^-\) = the negative particle concentration,
- \(R\) = the recombination coefficient (4).

For the most conditions, the recombination coefficient is a constant. Therefore, if one considers the time when the greatest density of ions occurs in the discharge - at the current minimum - and the time when the greatest amount of recombination takes place - at the light intensity maximum, a recombination coefficient can be calculated by using the above formula and trying different recombination coefficients until the calculated light intensity maximum occurs at the experimentally observed time.

To set up this problem for a numerical solution, consider that the number of positive ions recombining with electrons in any increment of time is given by

\[
N^+ = RN^+N^- t \tag{8}
\]

This is a good approximation as long as the time increment is small enough so that \(N^+\) and \(N^-\) may be regarded as constants within the time interval.

Now assume a distribution function at the position of a striation of the form
\( N_n^+ = A \sin(Wt(N) + \cdot \quad \text{and,} \quad N_n^- = B + A \sin(Wt(N) + \cdot \quad \text{where:} \)

\[ \theta \] is a phase angle such that \( N_n^+ \) is a maximum when \( Wt = 3/2 \),

\( B \) = the steady state electron density,

\( A \) = the oscillatory electron density.

The steady state electron density is responsible for a uniform light intensity along the tube. Superimposed on this steady state light intensity is the light intensity due to the oscillatory electron density at the striations. Since the light intensity at the striations is much greater than in the "dark" regions between the striations, the steady state electron density must be much smaller than the oscillatory electron density at the striation. This conclusion is due to the direct relation between charged particle density and light intensity cited earlier in this paper. Microwave measurements by Sodomsky (14) verify that the steady state electron density is much smaller than the oscillatory electron density occurring at the striations. Therefore, the steady state electron density may be ignored for recombination calculations dealing with striated discharges due to its small effect on the total light intensity.

'A' in equation (9) was evaluated by finding the maximum oscillatory current flowing through the 49 ohm resistor by obtaining the voltage fluctuation from the oscilloscope. The equation

\[ I = AeFXA \]

where:

\( F \) = frequency of the oscillation,

\( X \) = separation between striations which is defined as the wave
length of oscillation, is then used to evaluate $A$.

Since the mechanism within the striation is unknown, assume that the wave movement is interrupted within the striation as found by Donahue and Dieke (12). The amount of recombination in the electron and ion waves in any time increment $t_n$ may be given as follows assuming that recombination outside the striation is negligible.

\begin{align*}
Z(1) &= t_1^1 R N_1^+ N_1^- dt \\
Z(2) &= t_2^1 R (N_2^+ + N_1^- - Z(1))(N_2^- + N_1^- - Z(1)) dt \\
&
\vdots \\
Z(N) &= t_n^1 R (N_n^+ + N_{n-1}^+ + \ldots + N_1^+ - Z(1) - \ldots - Z(N - 1))(N_n^- + N_{n-1}^- + \ldots + N_1^- - Z(1) - \ldots - Z(N - 1)) dt
\end{align*}

where:

- $Z(N)$ = the amount of recombination between time $t_{n-1}$ and time $t_n$.
- The computer program used for the calculations is given on figure 11.

CONCLUSIONS

The calculated recombination coefficients for the data of pictures 620, 630, and 640 give recombination coefficients of $9 \times 10^{-14}$, and $5 \times 10^{-14}$ cubic meters per second, respectively. The theoretical recombination coefficients given by Francis (4) are about $10^{-19}$ cubic meters per second for ion-electron
recombinations and from $10^{-14}$ to $10^{-16}$ cubic meters per second for all processes.

The calculations for picture 630, see figure 13, show fair agreement between the experimental curve shape and the theoretical curve shape. The difference between the two is due to the choice of the distribution function. Part of the electrons involved in the recombination process may be contributed by the ionizing wave. This would make the electron density greater for the early part of the recombination calculations than has been assumed. This would account for the calculated light intensity peaks occurring after the experimentally observed light intensity peaks.

The calculations for picture 620, see figure 12, show the necessity of knowing the proper distribution function. The recombination coefficient was chosen so that the calculated light intensity maximum would coincide with the larger of the two experimentally observed light intensity maxima. The chosen distribution function ruled out the appearance of the second peak in the calculations.

The failure to observe frequencies in the range of $10^8$ to $10^9$ cycles per second in this investigation may be due to one of two reasons. From a comment made by Bohm and Gross (6) one may be led to believe that these frequencies are reflections of gas electrons from plasma sheaths. Our measurements were independent of any sheath interference and would therefore not measure such frequencies.
FIGURE 11

Fortran program for the IBM 1620 used to calculate recombination coefficients.
FIGURE 11

ENTER SOURCE PROGRAM, PUSH START

C0300 C RECOMBINATION CALCULATIONS
C0300 C BB=VMAX-VMIN OF VOLTAGE OSC.
C0300 C VA=APPLIED VOLTAGE
C0300 C R=RECOMBINATION COEF.
C0300 C SW=STRIATION WIDTH
C0300 C X=DIST. BETWEEN STRIATIONS
C0300 C RT=TIME BETWEEN CURRENT MIN AND LIGHT MAX
C0300 C F=MODULATION FREQUENCY
C0300 C K=PIE. NO.
C0300 C L=NO. OF STRIATIONS
C0300 C Q(N)=FRACTION OF IONS REMAINING
C0300 C TTT=TIME ELASPED
C0300 C MKS UNITS

C0400 C 51 FORMAT(E14.3,E14.8,E14.6)
C0400 C 50 FORMAT(E14.3,E14.8)
C0400 C 61 FORMAT(E14.3,E14.8,E14.0)
C0400 C 62 FORMAT(E9.3,2H9.3)
C0400 C 33 FORMAT(14,13)

C0464 4 READ1, BB, VA, SW
C0512 READ1, X, RT, F
C0560 READ1, RR
C0564 READ33, K, L
C0564 E0=.805E-11
C0564 TOSS=BB/49.C
C0564 P1=3.1415926
C0564 E=1.6038E-19
C0572 W=P1*F
C0574 TT=1.6E-02/F
C0600 A=TOSS/(E*F*F*P1*RR*RR)
C0636 3 DIMENSION T(101),SP(100),SN(100)
C0636 5 DO99=1,100
C0636 T(1)=0.0
C0632 T(N+1)=T(N)*TT
C0636 SP(N)=A*(SIN(W*T(N+1))+SIN(W*T(N)))/2.0
C0622 SN(N)=SP(N)
C0622 PUNCH56,SP(N),SN(N)
C0637 9 CONTINUE
C0642 PUNCH51,RT,TT,K
C0646 CONTINUE
C0650 15 STOP
C06520 END
Results of the recombination calculations for picture 620. The theoretical curve is dashed. The experimental curve is solid.
FIGURE 13

Results of the recombination calculations for picture 630. The theoretical curve is dashed. The experimental curve is solid.
FIGURE 13

PARTICLES/M$^3$

$10^{18}$

0  3  6  9  12  $10^{-5}$SEC.
The other reason for the failure to measure these frequencies may be that these oscillations have amplitudes which are smaller than the outside interference which was picked up by the measuring devices.

To use this theory for recombination coefficient measurements, the mechanics of recombination within the striation will have to be more thoroughly explored. Also a way of accurately determining the positive and negative charge distribution functions will have to be discovered.
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THEORY OF OSCILLATIONS IN A STRIA TED DISCHARGE

by

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AN ABSTRACT OF A MASTER'S THESIS

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1964
Oscillations of the light intensity, current, and voltage were found in striated deuterium discharges. The frequency of these oscillations varied from 34 to $10^6$ cycles per second. Phase measurements showed that there were time differences between the occurrence of the light intensity maximum, the current maximum, and the voltage maximum. The striations could be made to oscillate visibly by impressing a second frequency on the discharge in addition to the frequency with which the discharge was oscillating.

A theory was proposed to account for these phenomena. It was based on positive and negative space charges being set up in the regions of the striations. Ionization and recombination were the principle mechanisms for setting up the observed oscillations.

Calculations of the recombination coefficients based on this theory agreed with the accepted values. The predicted shape of the light intensity oscillation from the theory was close to the experimental shape of the light intensity oscillation as taken from the oscilloscope measurements.