OPTIMUM DESIGNS FOR POLYPHASE SQUIRREL-CAGE INDUCTION MOTORS BY DIGITAL COMPUTER

by

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INTRODUCTION

This thesis presents a program set up in an IBM 650 digital computer system for optimizing design of a polyphase squirrel-cage induction motor with a single winding wire-wound stator and a die-cast single-cage rotor to meet a set of customer specifications and manufacturing specifications, while working toward a minimum amount of copper and iron.

The need for such a program is very great when the motor manufacturing industry makes changes in its standards or when efforts are being made to establish new standards. The standard ratings and the standard dimensions for these ratings are arrived at by the National Electrical Manufacturers Association (NEMA). As better materials become available to the industry and as the industry makes better usage of these materials through more knowledge about motor performance and application and as better manufacturing processes become available, the industry standards are changed to take advantage of these factors. The amount of material permitted for a given motor rating is continually being decreased. This program will help indicate what new standards should be and will also help in the determination of the details for the various ratings once the new standards are arrived at. This program will also help in the development of special motor ratings not covered by industry standards.

Figure 2 is a flow chart which outlines the overall logic of the program. Symbols used in the flow chart are described in Fig. 1.
Letter symbols used in this thesis are selected from standards proposed by the A.I.E.E. when applicable. Appendix 1 is a list of letter symbols for the equivalent circuit (Fig. 8) and Appendix 2 is a list of symbols for dimensional values.

INPUT

Input data must provide the computer with a complete description of the motor required by the customer and also with any restriction on the materials, dimensions, or design proportions found necessary or desirable from the manufacturer's viewpoint.

Three parts or groups of input data are shown in Appendix 3. These are customer specifications, manufacturing specifications, and optional specifications.

Among customer specifications, items 1 through 10 specify the ratings and the working conditions of the motor, while items 11 through 18 specify its performance characteristics. It is desirable to enter all of these specifications, but items 14 through 17 may be omitted (by putting in as zeros) if the values involved are not limited. For those items not specified in the input, the computer will not make comparisons or scoring for them during the optimizing process. At the present time the cooling capability of the motor is not included in this program, so the temperature rise and time listed in item 8 are for reference only. The insulation class in item 10 is limited to Class A, B, or H, as defined by the NEMA standard.
In manufacturing specifications a suitable combination of the number of stator slots and the number of rotor slots should be so selected and put in to secure good performance and to avoid troubles, such as dead points, noise and synchronous cusps, which are not considered in this program. The types of iron and the types of wire are limited to those used in Westinghouse Buffalo Works, as listed in Appendix 5 and Appendix 6. The conductivity of wire, the weight factor of wire, and the conductivity of the rotor alloy are expressed in terms of copper by assuming the values for copper to be one. All items in manufacturing specifications have to be put in with proper values, except that the conductivity of the rotor alloy can be put in as zero and the computer will select a suitable value. The computer has the freedom to use 50 per cent or 30 per cent conductivity alloy.

All items in Part C of the input are optional and they are independent one from the other. For those not specified in the input, the computer will deal with them as independent variables and suitable values will be selected for them during the optimizing process.

Input data are punched on two I.B.M. cards and are then read into the computer. Figure 3 shows the arrangement of the input cards.

OUTPUT

As soon as input data have been read into the computer, they are arranged and printed out. The print out of input data not
only provides a reference of specifications for the problem that the computer is working on, but also can be used for visually checking to see if any errors were made in the specifications or during the card punching process.

From each set of optional specifications either specified in the input or selected by the computer, a set of magnetic parts is designed. For them a set of windings is designed to meet the input specifications, and its performance is calculated. The calculated performance characteristics are then compared against the specified values and scores are made. Should there be any improvement or adjustment needed, another set of windings is designed by using a combination of the same magnetic parts. This procedure continues on until there are no more improvements or adjustments needed, or until all possible combinations for the particular set of magnetic parts have been tried. Then from among those designs, the one with the best performance characteristics is selected and printed out in the output. Output data contain a part of the input data, the constants defining the magnetic parts, the winding data, and the performance expected. For those parts failing to give any reasonable design, a code indicating the cause of the failure substitutes for the performance in the print out.

Output items are listed in Appendix 4 and the arrangement of the print out is shown in Fig. 4.
CHECKS

Checks on input data, as listed in Appendix 7, are made by the computer to see that the input specifications are reasonable, compatible, and/or available. When any one of the input data fails to pass the checks, a stop code will be printed out to indicate the cause of the failure and the computer proceeds to the next problem.

Other checks are made continuously during the designing, calculating, and optimizing process to see if factors involved are within the electrical, magnetic, mechanical, or manufacturing limitations. Whenever there is any failure to meet the limitations, corrections are made automatically by the computer and calculations continue on.

INITIALIZATION

In this program both magnetic parts and windings for the motor are basically designed from input specifications. For those not specified in the input, first approximations are initiated by the computer calculating from other related input values available by using formulas listed in Appendix 8. These items are then dealt with as independent variables throughout the whole program and they are adjusted by proper increments within certain limits whenever it is desirable. Formulas for first approximations, limitations, and increments, which are listed in Appendix 8, were derived from optimum values based on wide studies.
and statistical analyses from a large number of motors built in the past with the great help of other programs set up in the same computer system. Other initializations used in designing parts and in calculating performance characteristics will be discussed in their appropriate sections.

**STATOR PUNCHING DESIGN**

A round punching with parallel sided teeth and with round-bottom slots, which is commonly used in wire-wound motors, is selected for stator cores in this program. Figure 5 shows the shape and the dimensional symbols for such a stator punching. The following factors define the dimensions of a stator slot.

1. The outside diameter of the stator punching - D.
2. The punched diameter of the stator punching - Dp.
3. The ratio of the effective stator tooth magnetic section per pole to the effective stator yoke magnetic section per pole - \( \frac{A_{t1}}{A_{y1}} \).
4. The ratio of the stator tooth flux density to the air-gap flux density - \( \frac{B_{t1}}{B_g} \).
5. The number of poles - P.

Appendix 9 is a list of formulas used in calculating dimensions for a stator punching. The value \( g_1 \) gives the allowance used for finishing the stator bore. The dimensions \( W_{10} \) is the smallest value selected from values \( (W_{10})_1, (W_{10})_2, \) and \( (W_{10})_3 \). Among them, \( (W_{10})_1 \) and \( (W_{10})_2 \) are derived from practical values commonly used in the past and \( (W_{10})_3 \) is so fixed as to secure a
value for $W_{11}$ which is at least .06 inch greater than $W_{10}$, enabling the use of a top wedge to hold coils in the slot. The value of $W_{10}$ is so fixed as to secure a value which is equal to or greater than 0.09 inch so that through this opening at least a coil using the smallest size wire available in the manufacturing can be put into the slot. The width of the tooth, $tw_1$, is so proportioned as to meet the specified ratio, $B_{t1}/B_g$. The depth of the straight portion of the slot, $d_{13}$, is limited by the ratio $A_{t1}/A_{y1}$ and the number of poles, $P$. From these values calculated above, other dimensions are derived and a stator punching is completed. However, certain limitations are set, based either on the mechanical aspects or on the manufacturing aspects and checks are made by the computer during the calculating process to insure that all dimensions are within proper limits. If any one of the calculated dimensions fails to meet its limits, necessary corrections or adjustments (see Appendix 12) are made either in the slot dimension or in those independent specifications which define a stator punching and following the same procedure a new stator punching will be designed. Immediately following the completion of a stator punching, design constants which are related to it and which will be used later in this program are calculated and stored by the computer.

**ROTOR PUNCHING DESIGN**

Two types of rotor punchings are used in this program. A "Type I" rotor punching, as shown in Fig. 6A, has slots with
round tops and round bottoms and parallel sided teeth. A "Type II" rotor punching, as shown in Fig. 6B, has slots with round tops and round bottoms and with parallel sides. Both are single cage. A rotor punching with double cage slots is not included in this program at the present time. The following factors define the dimensions of a rotor slot.

1. The punched diameter of the rotor punching - $D_p$, which is the same as the punched diameter of the stator punching.

2. The bore diameter of the rotor punching - $D_b$.

3. The ratio of the effective stator tooth magnetic section per pole to the effective stator yoke magnetic section per pole - $A_{t1}/A_{y1}$.

4. The ratio of the stator tooth flux density to the air-gap flux density - $B_{t1}/B_g$.

5. The number of poles - $P$.

Appendix 10 is a list of formulas used in calculating the dimensions for a rotor punching. The value $g_2$, added to $g_1$, gives the proper air gap $g$, which is a function of the finished diameter of the stator, $D_1$, and the number of poles, $P$, as shown in Appendix 8. The opening, $W_{20}$, is the smaller value selected between $(W_{20})_1$ and $(W_{20})_2$, both calculated from formulas derived from practical values used in the past. A slot without an opening (i.e., $W_{20} = 0$) is used when the calculated value of $W_{20}$ is smaller than 0.02 inch due to manufacturing reason. The depth of the opening, $d_{20}$, is a function of the opening, $W_{20}$, and the
number of poles, \( P \), in order to give a proper upper slot reactance. The tooth width, \( t_w \), is so calculated as to maintain the tooth flux density of the rotor the same as that of the stator.

In "Type I" rotor, the value \( r_{22} \) is calculated immediately following the determination of \( t_w \), while in "Type II" rotor, \( r_{22} \) is determined after \( d_{23} \) has been found. In determining \( (d_{23})_{\text{max}} \), \( (d_{23})_1 \) is limited by the minimum value, 0.03 inch, for the \( r_{23} \) value which is permissible in the manufacturing process, \( (d_{23})_2 \) is so limited to make the ratio of the effective tooth section per pole to the effective yoke magnetic section in the rotor not greater than that in the stator, and \( (d_{23})_3 \) limits the depth of the rotor slot to less than 1.5 inches because of both electrical and magnetic reasons. The value \( (d_{23})_{\text{max}} \) is set equal to the smallest value among \( (d_{23})_1 \), \( (d_{23})_2 \), and \( (d_{23})_3 \). It is apparent that \( (d_{23})_{\text{max}} \) must be a positive value. Should \( (d_{23})_{\text{max}} \) be a negative value, corrections and adjustments are made as shown in Appendix 12 and a new stator punching as well as new rotor punchings have to be designed following the same procedure. For each value of \( (d_{23})_{\text{max}} \), five rotor punchings are designed by using different values of \( d_{23} \) with a difference of 20 per cent of \( (d_{23})_{\text{max}} \) from one to the other. Other dimensions of the rotor slot are calculated following the determination of \( d_{23} \), and from its dimensions constants which are related to it and which will be used later in this program, are computed and stored. Thus from one set of independent variables five rotor punchings for each type of rotor are designed to match a stator punching.
Actual dimensions for an end ring are not calculated in this program. However, for a specified punched diameter, \( D_p \), the maximum and minimum dimensions in manufacturing a ring to match the stator and rotor set and the constants related to them are calculated by using formulas listed in Appendix II. An end ring with an area within these limits is then chosen and used in the rotor winding and its constants are then calculated proportionally from the maximum and minimum values previously calculated. The end ring used is also checked against its loss dissipation capability which will be discussed in a later section.

**MAGNETIC PARTS**

The combination of one stator punching and five rotor punchings for each rotor type provides essential magnetic parts for designing a motor. In this program stator records are stored in the drum of the computer, while rotor records are written and stored in the tape with Type I rotor punchings preceding Type II and arranged and numbered according to the depth, \( d_{23} \), of the slot in a descending sequence.

It is apparent that each set of magnetic parts is basically defined by two constants, \( P \) and \( D_b \), and four independent variables, \( B_{t1}/B_g \), \( A_{t1}/A_{y1} \), \( D \), and \( D_p/D \) (this in turn gives \( D_p \)). By changing any one of the four independent variables, a new set of magnetic parts is designed. They are the factors which play a
great role in the optimization. On the other hand, a stator punching can be definitely determined when those constants and variables are defined. The rotor punching is defined by these same constants and variables plus rotor punching type and number.

It is for this reason that these values, instead of actual dimensions of parts, are given in the output. These constants and variables can be used as input to a companion program which will give the exact dimensions of the parts desired.

WINDINGS

From a set of magnetic parts designed from the same variables, various combinations can be selected to form a motor by putting in the proper windings. Combinations start by using the same stator punching and reading a rotor punching from the tape each time according to the arranged sequence. With this set of punchings, the minimum core length is selected and all constants related to them are calculated. A first iteration for the turns per coil, based on a series delta connection, is then calculated to meet the specified pull-out torque. If the conductivity of the alloy used in casting the rotor is not specified, the computer will try an alloy with 50 per cent conductivity first. An alloy with 30 per cent conductivity can be chosen if it is necessary.

In order to meet the minimum and maximum full-load slips specified in the input, the secondary resistance must lie within certain limits. A first approximation for the secondary
resistance is a value lying between those limits. The first approximation for the secondary resistance, its minimum value, its maximum value, and its increment are calculated by formulas listed in Appendix 8. With the total secondary resistance selected and with the rotor bar resistance calculated, a ring with a desirable area is then selected. During the preceding process, checks are made by the computer to see if those parts selected are reasonable and are within proper limits. If they are, calculations are made to find the primary resistance and pounds of copper based on the turns per coil selected and to get a more accurate value of the secondary and leakage reactance. All constants which are related to turns per coil are then modified.

EQUIVALENT CIRCUIT

A per unit system which has the rated output watts of the motor to be calculated as its base is used in the performance calculation. Formulas for calculating constants for the per unit system are listed in Appendix 13. Figure 8 shows the equivalent circuit used in calculating performance characteristics. The letter symbols used in the circuit are described in Appendix 1.

In this equivalent circuit \( R_M \) represents the core loss and is connected directly across the line instead of being in parallel with the magnetizing reactance, \( X_M \), as is the case in the exact equivalent circuit. This not only simplifies the
calculation of the circuit, but also gives satisfactory results for motors involved in this program. A resistance $K_{SR} R_2/\alpha$ is inserted in series with the secondary circuit to represent the stray-load loss based on the assumption that the stray-load loss is a function of the power transferred across the air gap.

**NO-LOAD PERFORMANCE**

In calculating the no-load performance, a first approximation for the secondary emf is calculated as follows.

$$E_2 = (E_1)(X_M')/(X_1 + X_M')$$

With this approximation, flux densities in the air gap, the stator yoke, the stator teeth, the rotor yoke, and the rotor teeth are calculated.

Saturation curves for different paths for different types of iron are listed and stored in the computer as tables. By checking the type of iron specified, the computer looks up its proper table and then by comparing and interpolating flux density in each path, ampere-turns required are calculated. The actual secondary emf is then determined. If the difference between the calculated value and the assumed one is greater than five-tenths of one per cent of the assumed value, the calculation has to be repeated by using the calculated value of $E_2$ as a new approximation. The same process continues on until the limit is met and the calculation of iron losses then follows.

Formulas for calculating no-load performance are listed in Appendix 14.
OTHER PERFORMANCE

Other performance calculations are made by applying formulas in Appendix 15 to the equivalent circuit shown in Fig. 8 and by considering the slip of the motor as an independent variable. Both deep bar effect and saturation effect are included in this program. They are calculated by formulas given in Appendix 17 and Appendix 18, respectively. In calculating the saturation effect, first the primary current is calculated by using unsaturated circuit constants and then with the calculated current corrections are made on the circuit constants. If the difference between the constants used in calculating the current and the constants just being corrected is greater than one-tenth of one per cent of their average value, a new calculation of the current is made by using the corrected constants and a comparison follows. The same procedure repeats until the proper limit has been met. This provides an accuracy which can seldom be done in long-hand calculations.

As stated before, the design of a stator winding in this program is first aimed to meet the specified pull-out torque and since the slip is the only element considered as an independent variable in the equivalent circuit after the winding has been defined, a slip where the pull-out torque is expected to occur is first calculated and the performance at this slip is then computed. If the output torque at this breakdown point is not equal to the specified value, a new turns per coil is assumed and calculations are repeated.
When a design has its pull-out torque equal to the specified value, calculations for locked rotor performance and for full-load performance follow. In calculating full-load performance, a slip equal to 125 per cent of the per unit secondary resistance is used as its first approximation. It is then modified until a slip where the horse power output is within five-tenths of one per cent of the rated value has been found. In these performance calculations only those values which are to be printed out are calculated and stored.

LIMITATIONS

In selecting parts and in calculating performance, various tests are made to check whether the selected parts and the calculated performance are within reasonable limits. Whenever any part or any performance fails to meet limits, corrections or adjustments are made immediately by changing the parts or by varying the factors involved and calculations continue on. In the case when all available parts and all variable limits have been exhausted or when there is no way to improve the situation by using the available parts, decisions are made to change the independent variables and a new set of parts has to be designed.

Some limits set for this purpose are as follows. In selecting the end ring when an end ring with a maximum permissible area is used, if the average current density in the ring is greater than that in the rotor bar, or if the full-load slip is higher than that specified, the specified full-load slip is first
increased and then, if it fails, the specified pull-out torque is increased and new windings to meet this pull-out torque are designed following the same procedures previously described. When an end ring with a minimum permissible area is used, if the average current density in the end ring is less than a certain per cent \((K_r)\) of that in the rotor bars, or if the full-load slip is lower than that specified, the specified full-load slip is first decreased and then, if it fails, alloy with a lower conductivity or a rotor punching with a smaller slot area, if available, is used.

In calculating the no-load performance, if the flux density in some iron path is over 140 kilolines per square inch for specified over voltage and if there is no design achieved by using this particular set of magnetic parts, a longer core is used.

In calculating the full-load performance, if the motor designed fails to deliver the specified horsepower output, a new winding with a higher specified pull-out torque is designed.

**SCORING**

Having all performance characteristics calculated, scores are made by comparing these calculated values against those specified in the input. For items not specified in the input, no score will be given to them and their output values are assumed to be acceptable.

Appendix 16 shows the scoring system used in this program.
Penalty scores are given to items when their actual performance cannot meet the specified values and plus scores are given to items when their actual performance are better than those specified. Different scoring points are used for different items. These scoring points are listed in Appendix 16 and are weighed according to their effects on the overall acceptability of general purpose motors. Penalty scores or plus scores for each item are then added separately to give the total penalty score and the total plus score. The total penalty score and the total plus scores are independent from one another, and they cannot be used to compensate one another.

The scoring system not only gives a comparative score for the overall performance of a motor but also plays an important role in the whole program by guiding the optimizing processes.

OPTIMIZATION (1)

The optimization in this program starts by changing the stator winding but keeping all other parts the same to see if a better overall design can be made. It is known that the stator winding in this program is originally designed to meet the minimum pull-out torque specified and it is also considered that a design with a higher than specified pull-out torque is not undesirable. Thus in the first step of optimization, the specified pull-out torque is increased to see if a stator winding can be designed to give better overall performance. However, when this consideration is being made, factors such as the starting current,
the full-load power factor, and the full-load slip (comparing to the specified minimum full-load slip) are checked to see if any one of them alone is outside of a reasonable value by enough to cause rejection of the parts. In this program, parts are considered rejectable when the total penalty score for the motor designed from them is greater than 10 points. It is apparent that a stator winding giving a higher pull-out torque will tend to make the situation worse when any one of the factors has a penalty score greater than 10 points already. The full-load efficiency is also checked to see if it is at the maximum value for the parts. Whenever there is an indication that an increase in the specified pull-out torque will result in a better stator winding, the computer will continue to increase the specified pull-out torque designing new windings and calculating and scoring the performance. This optimizing process continues on until it reaches a point where either it fails to meet limitations as stated before, or there are indications that no better windings can be further achieved. Of all designs using the same parts but with different windings, only the "best one", that is, the one with the highest total plus score among those with the lowest total penalty score has its windings and performance characteristics stored.

If the total penalty score of the "best one" is zero, the computer will proceed to the second step of the optimization. If it is not zero, searches are made to find out the factor which causes the most penalty score and necessary improvements are made by changing the combination of parts. For example,
when the starting torque is too low or the starting current is too high, improvements are made first by raising the specified full-load slip and then by extending the core length. When the power factor or the efficiency is too low, a longer core may be tried. In making improvements, checks are made to see if the factor being changed has exceeded its limits or if it has been changed previously in an opposite direction. When either one of those conditions happens, the computer will proceed to the second step of the optimization. Otherwise, it tends to continue on the first step of the optimization until a design with a "zero" total penalty score has been found.

OPTIMIZATION (2)

In its first step of the optimization, the computer has optimized through a set of parts designed from the same independent variables by making different possible combinations, and from them it picks a "best one". Thus it leaves to the second step of the optimization for the computer the decision as to whether it is possible to make further improvements in designs by changing independent variables and designing new parts. Some sets of parts fail to make any combination to form a proper design. In this case searches are made to find the cause of the failure, and from the cause necessary steps to be followed are decided upon. For example, if the failure is due to a too high or a too low full-load slip, the ratio, $B_b/B_g$, has to be increased. If it fails to give a sufficient horsepower output, a stator core
with a bigger outside diameter, D, is then considered to be desirable.

If there are one or more designs being made from the parts, it checks to see if the permissible locked time of the "best one" among them is equal to or greater than the minimum value specified. If it is not, the ratio $B_{tl}/B_g$ has to be increased by a certain amount depending on how far it is off from the specified locked time. If its locked time meets the requirement and if its total penalty score is within a reasonable limit (10 points in this program), checks are made to see if the flux densities in all iron paths are less than 140 kilolines per square inch when the motor works at the specified over voltage. If it fails, efforts are made to see if the core length can possibly be extended to help the situation. If not, an increase in the outside diameter, D, of the stator core has to be taken. If the "best one" performs within the flux density limit at the specified over voltage, it further checks to see if its locked time is much more than that required. In this program when the locked time is twice or more as much as the specified minimum value, the computer will try to reduce the area of the stator slots by decreasing the ratio $B_{tl}/B_g$ to see if a design using less copper is possible. If the locked time is within its upper and lower limits, a stator core with a smaller outside diameter, D, is tried to see if any saving in the iron is possible.

If the total penalty score of the "best one" is not within the 10-point limit, decisions are made to determine what necessary steps have to be taken in order to improve the factor which
causes the most penalty score. For example, an increase in the ratio, \( B_{t1}/B_g \), is first tried if the main failure is due to a low power factor or a low efficiency and a bigger outside diameter, \( D \), of the stator core has to be used if the main failure is due to a low starting torque or a high starting current.

In changing independent variables, the computer follows the proceedings:

1. The ratio \( B_{t1}/B_g \) (can be decreased or increased).
2. The outside diameter, \( D \), of the stator core (can be decreased or increased).
3. The ratio \( A_{t1}/A_{y1} \) (can be decreased only).
4. The ratio \( D_p/D \) (can be changed around the initial value as a center).

During the changing process, if it happens that the factor being changed has already reached its limit, or that there is an indication that the direction of the change will not help the optimization, the computer will proceed to change the next factor and the process continues on until a proper change is achieved or until there are no more changes possible. Provided a proper change exists, factors preceding the one being changed in the procedure will return to their initial values, and factors following will maintain their same values. Using this new set of variables, the computer will continue on its designing and optimizing process until no more changes are possible. At this time the optimizing process for this problem is complete and the program proceeds to the next problem.
SUMMARY

This program has been used successfully in arriving at the details of a new line of motors using a new type of magnetic sheet steel and a new winding and insulation system. The designs were arrived at in a matter of days as compared to a matter of years in the case of completely manual procedures.

The optimization in this program is based primarily on the performance characteristics of the designs and on the scoring system derived for general purpose motors. For special motors the scoring system must be modified in order to approach "the optimum design". Although there is an approach toward minimum cost in this program, real cost calculations are not at present a part of this program. Until real costs and cooling capability calculations are made a part of this program, the engineer must do these optimizing functions.

This program has not covered all of the considerations that must be made in arriving at "the optimum design", but it has taken much of the burdensome calculations and routine decision making away from the design engineer. With this burden removed, the design engineer now has the time to consider factors which were previously ignored. The effects of these new factors when formulated can be added to this program or to other programs and will result in a closer approach to "the optimum design".

The trend in engineering design groups has been toward less and less engineering and toward more and more routine calculations. It is felt that the use of large digital computers by engineering design groups will reverse this trend.
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### LETTER SYMBOLS FOR EQUIVALENT CIRCUIT

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_g$</td>
<td>Flux density in air gap</td>
</tr>
<tr>
<td>$B_L$</td>
<td>Fictitious value of air-gap flux density used to find saturation effect</td>
</tr>
<tr>
<td>$B_{t1}$</td>
<td>Flux density in stator teeth</td>
</tr>
<tr>
<td>$B_{t2}$</td>
<td>Flux density in rotor teeth</td>
</tr>
<tr>
<td>$B_{y1}$</td>
<td>Flux density in stator magnetic yoke</td>
</tr>
<tr>
<td>$B_{y2}$</td>
<td>Flux density in rotor magnetic yoke</td>
</tr>
<tr>
<td>$\text{COND}_1$</td>
<td>Conductivity of wire (based on copper as 1)</td>
</tr>
<tr>
<td>$\text{COND}_2$</td>
<td>Conductivity of rotor alloy (based on copper as 1)</td>
</tr>
<tr>
<td>$E$</td>
<td>Unit voltage</td>
</tr>
<tr>
<td>$E_1$</td>
<td>Voltage per phase across stator winding</td>
</tr>
<tr>
<td>$E_2$</td>
<td>Voltage per phase induced in rotor winding referred to stator</td>
</tr>
<tr>
<td>$\text{EFF}$</td>
<td>Efficiency</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency, in cycles per second.</td>
</tr>
<tr>
<td>$F &amp; W$</td>
<td>Friction and windage loss.</td>
</tr>
<tr>
<td>$\text{GF}_1$</td>
<td>Stator gap factor</td>
</tr>
<tr>
<td>$\text{GF}_2$</td>
<td>Rotor gap factor</td>
</tr>
<tr>
<td>$\text{HP}$</td>
<td>Horsepower output</td>
</tr>
<tr>
<td>$I$</td>
<td>Current per phase</td>
</tr>
<tr>
<td>$I$</td>
<td>Unit current</td>
</tr>
<tr>
<td>$I_L$</td>
<td>Line current</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Starting line current</td>
</tr>
</tbody>
</table>
Appendix 1 (Cont.)

K\textsubscript{d}  Distribution factor
K\textsubscript{db}  Deep bar factor
K\textsubscript{LM}  Deep bar multiplier for rotor lower slot reactance
K\textsubscript{m}  Leakage pitch factor
K\textsubscript{L1}  Stator lower slot constant
K\textsubscript{L2}  Rotor lower slot constant
K\textsubscript{LF}  Chording leakage factor
K\textsubscript{p}  Pitch factor
K\textsubscript{r}  Minimum ratio of average current density in end ring to average current density in rotor bar
K\textsubscript{rb}  Deep bar multiplier for rotor bar resistance
K\textsubscript{s}  Stack factor
K\textsubscript{SL}  Saturation multiplier for stator upper slot reactance
K\textsubscript{S2}  Saturation multiplier for rotor lower slot reactance
K\textsubscript{SL}  Stray-load loss factor
K\textsubscript{t}  Coil pitch
K\textsubscript{u1}  Stator upper slot constant
K\textsubscript{u2}  Rotor upper slot constant
K\textsubscript{Z}  Saturation multiplier for zig-zag reactance
N  Actual rotational speed
\bar{N}  Unit rotational speed
N\textsubscript{s}  Synchronous rotational speed
P  Number of poles
\bar{P}  Unit power
PAR  Paths in parallel
PF  Power factor
Appendix 1 (Cont.)

q  Number of phases

R_1  Primary resistance per phase

R_2  Secondary resistance per phase referred to primary

R_M  Resistance simulating iron loss

s  Slip of motor, expressed as a friction of synchronous speed

T  Torque output

T_s  Unit torque

TPC  Number of turns per coil

T_s  Starting torque

V  Line voltage

X_1  Primary leakage reactance per phase

X_{1E}  Primary end winding leakage reactance per phase

X_{1SL}  Primary lower slot leakage reactance per phase

X_{1SU}  Primary upper slot leakage reactance per phase

X_{1ZZ}  Primary zig-zag leakage reactance per phase

X_2  Secondary leakage reactance, referred to primary

X_{2E}  Secondary end winding leakage reactance, referred to primary

X_{2SL}  Secondary lower slot leakage reactance, referred to primary

X_{2SU}  Secondary upper slot leakage reactance, referred to primary

X_{2ZZ}  Secondary zig-zag leakage reactance, referred to primary

X_M  Magnetizing reactance
Appendix 1 (Concl.)

$X_M'$ Magnetizing reactance due to air-gap flux alone

$X_{SK}$ Secondary skew leakage reactance, referred to primary

$Z$ Unit impedance

$\phi$ Space fundamental component of mutual air-gap flux per pole per phase

ADDITIONAL SUBSCRIPTS

• Per unit value

FL Full-load quantities

$\mathbf{i}$ Initialized quantities

max Maximum quantities

min Minimum quantities

NL No-load quantities

PO Pull-out torque quantities

$F(\ )$ Function of

* Letter symbols, other than those described above, are defined by formulas
APPENDIX 2

LETTER SYMBOLS FOR DIMENSIONAL VALUES*

\[ \begin{align*}
A_b & \quad \text{Bar area} \\
A_g & \quad \text{Air-gap area per pole} \\
A_r & \quad \text{End ring area} \\
A_{s1} & \quad \text{Available area of one stator slot for the winding} \\
A_{t1} & \quad \text{Effective stator tooth section per pole} \\
A_{t2} & \quad \text{Effective rotor tooth section per pole} \\
A_{y1} & \quad \text{Effective stator yoke magnetic section area} \\
A_{y2} & \quad \text{Effective rotor yoke magnetic section area} \\
D & \quad \text{Outside diameter of stator core} \\
D_1 & \quad \text{Finished stator bore diameter} \\
D_2 & \quad \text{Finished diameter of rotor core} \\
D_p & \quad \text{Rotor bore diameter} \\
D_p & \quad \text{Punched inside diameter of stator punching and punched} \\
& \quad \text{outside diameter of rotor punching} \\
D_r & \quad \text{Outside diameter of end ring at the core} \\
d_{1c} & \quad \text{Effective stator tooth length} \\
d_{2c} & \quad \text{Effective rotor tooth length} \\
g & \quad \text{Length of single air gap} \\
g_e & \quad \text{Effective length of single air gap} \\
L & \quad \text{Measured core length} \\
L_{eff} & \quad \text{Effective core length}
\end{align*} \]

*See Figs. 5, 6, and 7, and Appendices 9, 10, and 11 for other letter symbols for dimensional values.
Appendix 2 (Concl.)

$L_r$  Gross core length

$L_y1$  Effective length of stator yoke magnetic section

$L_y2$  Effective length of rotor yoke magnetic section

$S_1$  Number of stator slots

$S_2$  Number of rotor slots

$T_k$  Thickness of iron

$V_t1$  Effective volume of stator tooth section per pole

$V_y1$  Effective volume of stator yoke magnetic section
APPENDIX 3

INPUT

A. CUSTOMER SPECIFICATIONS
   1. Rated horsepower
   2. Poles
   3. Frequency
   4. Phase
   5. High voltage
   6. Number of voltages
   7. Percentage of overvoltage
   8. Temperature rise and time
   9. Enclosure
  10. Insulation class
  11. Minimum (pull-out torque/full-load torque)
  12. Minimum full-load slip
  13. Maximum full-load slip
  14. Minimum full-load efficiency
  15. Minimum full-load power factor
  16. Maximum starting line current
  17. Minimum (starting torque/full-load torque)
  18. Minimum locked time

B. MANUFACTURING SPECIFICATIONS
   1. Number of stator slots
   2. Number of rotor slots
   3. Type of iron
Appendix 3 (Concl.)

4. Thickness of iron
5. Type of wire
6. Conductivity of wire
7. Weight factor of wire
8. Maximum fill factor of wire in slot
9. Conductivity of alloy in rotor

C. OPTIONAL SPECIFICATIONS

1. Outside diameter of stator core
2. Punched diameter of punchings (or ratio of punched diameter to outside diameter)
3. Rotor bore diameter
4. \( \frac{\text{Effective stator tooth section per pole}}{\text{effective stator yoke magnetic section per pole}} \)
5. \( \frac{\text{Stator tooth flux density}}{\text{air-gap flux density}} \)
6. Rotor type
APPENDIX 4

OUTPUT

A. INPUT

B. PARTS

1. Outside diameter of stator core
2. Punched diameter of punchings
3. Rotor bore diameter
4. Ratio of At1 to Ay1
5. Ratio of Bt1 to Bg
6. Rotor type and number
7. Ring area
8. Conductivity of alloy in rotor
9. Turns per coil
10. Core length
11. Pounds of copper
12. Pounds of iron

C. PERFORMANCE

1. (Pull-out torque)/(full-load torque)
2. Slip at full load
3. Efficiency at full load
4. Power factor at full load
5. Starting line current in amperes
6. (Starting torque)/(full-load torque)
7. Air-gap flux density
8. Maximum flux density
Appendix 4 (Concl.)

9. Maximum locked time in seconds
10. Plus score
11. Penalty score

D. STOP CODE

APPENDIX 5

TYPES OF IRON

1. Type 1625 - A.I.S.I. M-36 (~ 1% silicon)
2. Type 9080 - A.I.S.I. M-19 (~ 3% silicon)
3. Type 9116 - A.I.S.I. M-27 (~ 2-3/4% silicon)

APPENDIX 6

TYPES OF WIRE

1. Type 12310-2 Special Bondar enamel covered
2. Type 12400-3 Heavy silicone enamel covered
3. Type 1880-6 Single Bondar enamel covered
4. Type 1880-7 Heavy Bondar enamel covered
5. Type 1880-10 Heavy Formvar enamel covered
6. Type 7854-3 Double glass covered
APPENDIX 7

CHECKS ON INPUT

1. Poles $P$
   (1) $P \neq 0$
   (2) $P$ is divisible by 2

2. Phase $q$
   $q = 2$ or $3$

3. Number of stator slots $S$
   $S_1$ is divisible by $[(q)\text{(number of voltages)}]$

4. Outside diameter of stator core $D$
   (1) $D > D_p$
   (2) $D > D_b$

5. Punched diameter of punchings $D_p$
   (1) $D_p < D$
   (2) $D_p > D_b$

6. Rotor bore diameter $D_b$
   (1) $D_b < D$
   (2) $D_b < D_p$

7. Type of iron (available Appendix 5)

8. Type of wire (available Appendix 6)

9. Insulation class (A, B, or H)

10. Maximum full-load slip $s_{max}$
    (1) $s_{max} \leq 0.2$
    (2) $s_{max} \geq s_{min}$
Appendix 7 (Concl.)

11. Minimum full-load efficiency $\text{EFF}_{\text{min}}$

$$\text{EFF}_{\text{min}} \leq 100 - (3.5 + \% s_{\text{min}}) \%$$

12. Minimum (pull-out torque/full-load torque)

$$\left( \frac{T_p}{T_{\text{FL}}} \right)_{\text{min}} \leq (K)(I_s)$$

where $K = 0.42$ if $s_{\text{max}} \leq 0.036$

$$K = 2.15 (s_{\text{max}})^{1/2}$$ if $s_{\text{max}} > 0.036$

13. Minimum (starting torque/full-load torque)

$$\left( \frac{T_s}{T_{\text{FL}}} \right)_{\text{min}} \leq (K)(I_s)$$

where $K = 2.03 (s_{\text{max}})^{1/2}$
APPENDIX 8

VARIABLES AND LIMITATIONS
(Length dimensions are expressed in inches)

<table>
<thead>
<tr>
<th>Variable</th>
<th>First approximation</th>
<th>Minimum value</th>
<th>Maximum value</th>
<th>Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_b )</td>
<td>[ 0.750 (TFL)^{0.286}, ] ( \text{if } N_s \leq 1800 ) [ 0.765 (TFL)^{0.256}, ] ( \text{if } N_s &gt; 1800 )</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( D_p/D )</td>
<td>[ 0.955 - \frac{0.570}{(P)^{0.500}} ]</td>
<td>0.45</td>
<td>0.90</td>
<td>0.05 - Centered around first approximation</td>
</tr>
</tbody>
</table>

Where

\[ R = 0.333 \ln \left( \frac{T_p}{P}\right) \]

\[ D_p = e^R K = \frac{1.350}{(P)^{1.570}}, \] \( \text{if } P \leq 6 \)
\[ K = \frac{0.131}{(P)^{0.285}}, \] \( \text{if } P > 6 \)

\[ D = \frac{(D_p)_1}{(D_p/D)_1} = 1.30 (D)_1 \]
\[ A_{tl}/A_{yl} = 1.570 \]
\[ B_{tl}/B_{lg} = 1.700 \]
### Appendix 8 (Concl.)

<table>
<thead>
<tr>
<th>Variable</th>
<th>First approximation</th>
<th>Minimum value</th>
<th>Maximum value</th>
<th>Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>$L_{\text{min}}$</td>
<td>$(0.25 + 0.0625P)(\frac{\pi D_p}{P})$</td>
<td>$(0.75 + 1.04P)(\frac{\pi D_p}{P})$</td>
<td>$0.2 \ (L_{\text{max}} - L_{\text{min}})$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>if $P \leq 12$</td>
<td>if $P \leq 12$</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>$K D_1$</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td></td>
<td>if $P = 2$</td>
<td>if $P = 4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K = 0.00255$</td>
<td>$K = 0.00255$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>if $P = 6$</td>
<td>if $P = 6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K = 0.00240$</td>
<td>if $P \geq 8$</td>
<td></td>
</tr>
<tr>
<td>$\dot{R}_2$</td>
<td>$\dot{R}_{2\text{min}} + 2(\text{Increment } \dot{R}_2)$</td>
<td>$0.72 \ (s_{\text{min}})$</td>
<td>$0.88 \ (s_{\text{max}})$</td>
<td>$0.2 \ (\dot{R}<em>{2\text{max}} - \dot{R}</em>{2\text{min}})$</td>
</tr>
</tbody>
</table>
APPENDIX 9

FORMULAS FOR STATOR PUNCHING
(Length dimensions are expressed in inches)

1. $g_1 = 0.003$

2. $D_1 = D_p + 2g_1$

3. $tp_1 = \pi D_p / S_1$

4. $w_{10}$ = the smallest value of $(w_{10})_1$, $(w_{10})_2$, and $(w_{10})_3$.
   a. $(w_{10})_1 = tp_1 / 3$
   b. $(w_{10})_2 = 0.0016(D_p) + 0.120$
   c. $(w_{10})_3 = \frac{(\pi)(D_p)(1 - B_g/B_{t1}) - (0.06)(S_1 - \pi \tan 30^\circ)}{(S_1 - \pi \tan 30^\circ)}$

**LIMITATION:** $w_{10} \geq 0.090$

5. $d_{10} = \frac{1}{3}(w_{10}) \tan 30^\circ$

**LIMITATION:** $d_{10} \geq 0.030$

6. $tw_1 = \frac{\pi D_p}{(S_1)\left(\frac{B_{t1}}{B_g}\right)}$

7. $w_{1x} = \frac{1}{2}(w_{11} - w_{10}) = \frac{\pi(D_p - 2d_{10}) - S_1(tw_1 + w_{10})}{2(S_1 - \pi \tan 30^\circ)}$

**LIMITATION:** $w_{1x} \geq 0.030$

8. $d_{11} = (w_{1x}) \tan 30^\circ$

9. $w_{11} = w_{10} + 2w_{1x}$

**LIMITATION:** $1.5 w_{10} \leq w_{11} \leq 2tw_1$
Appendix 9 (Cont.)

\[
(D - D_p - 2d_{10} - 2d_{11}) - \left(\frac{4}{3} \frac{w_{11}}{2} - \frac{(S_1)(tw_1)}{(P)(A_{tl}/A_{yl})}\right)
\]

10. \(d_{13} = \frac{\pi}{2 + \left(\frac{4}{3} \frac{\pi}{S_1}\right)}\)

11. \(r_{13} = \left(\frac{\pi}{S_1}\right)d_{13} + \frac{2}{3} w_{11}\)

12. \(d_1 = d_{10} + d_{11} + d_{13} + r_{13}\)

LIMITATION: \(D - D_p - 2d_1 \geq 0.06 D\)

13. \(d_{lc} = d_1 - \left(\frac{r_{13}}{3}\right)\)

LIMITATION: \(d_{lc} \leq 10 tw_1\)

14. \(K_s = 0.93, \text{ if } TK < 0.020\)

\(K_s = 0.96, \text{ if } TK \geq 0.020\)

15. \(h_{31} = \frac{1}{3} d_1\)

16. \(A_{S1} = \left(\frac{w_{11} + 2r_{13}}{2}\right) - a(d_1 - r_{13} - d_{10} - b) + \left(\frac{\pi}{2}\right)(r_{13} - c)^2\)

<table>
<thead>
<tr>
<th>: Class &quot;A&quot; :</th>
<th>: Class &quot;B&quot; :</th>
<th>: Class &quot;H&quot; :</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>b</td>
<td>0.175</td>
<td>0.175</td>
</tr>
<tr>
<td>c</td>
<td>0.025</td>
<td>0.025</td>
</tr>
</tbody>
</table>

17. \(L_{eff} = (K_s)(L)\)

18. \(A_{tl} = (S_1)(tw_1)(L_{eff})/(P)\)

19. \(A_{yl} = (D - D_p - 2d_{lc})(L_{eff})\)

20. \(L_{yl} = \pi(D + D_p + 2d_{lc})/4(P)\)

21. \(V_{tl} = (A_{tl})(d_{lc})(P)\)

22. \(V_{yl} = (A_{yl})(L_{yl})(P)\)
Appendix 9 (Concl.)

23. \( GF_1 = \frac{(tp_1)(4.40 \ g + 0.75 \ w_{10})}{(tp_1)(4.40 \ g + 0.75 \ w_{10}) - (w_{10})^2} \)

24. \( K_{ul} = \frac{(d_{10} - s_1)}{w_{10}} + \frac{4(d_{11})}{3(w_{10}) + (w_{11})} \)

25. \( K_{L1} = K \left( \frac{A_1}{A_1 + A_2} \right)^2 + \frac{0.25(X^4 - Y^4) - B(X^2 - Y^2) + (B)^2 \ Ln(X/Y)}{2(B - Y^2)^2 \ tan \alpha} \)

where

\[
K = \frac{(r_{13})^4}{2(A_1)^2} \left[ \theta + \frac{\theta^3}{8} - \frac{\sin 2\theta}{3} + \frac{\theta \cos 2\theta}{4} - \frac{\sin 4\theta}{2} \right]
\]

\( A_1 = (r_{13})^2 (\theta - \frac{1}{8} \sin 2\theta) \)

\( A_2 = d_{13}(r_{13} + \frac{1}{8} \ w_{11}) \)

\( B = (X)^2 + (A_1/tan\alpha) \)

\( X = (r_{13} - r_{13} \ sin^2 \alpha)/(sin\alpha) \)

\( Y = \frac{1}{8}(w_{11}) \ cos \alpha \)

\( \theta = \frac{\pi}{2} + \alpha \)

\( \alpha = \cos^{-1} \left[ \frac{(d_{13}) \left[ (d_{13})^2 + (\frac{1}{8} \ w_{11})^2 - (r_{13})^2 \right]^{\frac{1}{2}} + \frac{1}{8}(w_{11})(r_{13})}{(d_{13})^2 + (\frac{1}{8} \ w_{11})^2} \right] \)
APPENDIX 10

FORMULAS FOR ROTOR PUNCHING
(Length dimensions are expressed in inches)

1. \( g_2 = g - g_1 \)
2. \( D_2 = D_p - 2 \, g_2 \)
3. \( t p_2 = \pi \, \frac{D_p}{S_2} \)
4. \( w_{20} = \) the smallest value of \((w_{20})_1\) and \((w_{20})_2\)
   a. \((w_{20})_1 = 0.0015 \, (D_p) + 0.040\)
   b. \((w_{20})_2 = tp_2/3\)
   \textbf{LIMITATION:} \( w_{20} \geq 0.020 \)
5. \( w_{20}' = w_{20}, \ \text{if} \ w_{20} \neq 0 \)
   \( w_{20}' = 0.020, \ \text{if} \ w_{20} = 0 \)
6. \( d_{20} = K \, (w_{20}') \)
   where \( K = 1.140, \ \text{if} \ P = 2 \)
   \( K = 1.000, \ \text{if} \ P \neq 2 \)
   \textbf{LIMITATION:} \( d_{20} \geq g_2 + 0.005 \)
7. \( tw_2 = \frac{\pi \, D_p}{B_{t1}} \left( \frac{S_2}{B_g} \right) \)
   \textbf{LIMITATION:} \( tw_2 \geq 0.050 \)
8. \( (d_{23})_{\text{max}} = \) the smallest value of \((d_{23})_1, \ (d_{23})_2, \ \text{and} \ (d_{23})_3\)
   \textbf{TYPE I ROTOR}
   a. \((d_{23})_1 = \frac{1}{2} \left[ (D_p - 2\, d_{20} - 2\, r_{22}) - \left( \frac{S_2}{\pi} \right)(0.060 + tw_2) \right] \)
Appendix 10 (Cont.)

\[
(D_p - \frac{1}{3}D_b - 2d_{20} - 2r_{22}) - \frac{4}{3}(r_{22}) - \frac{(S_2)(t_{w2})}{(P)(A_{t1}/A_{y1})}
\]

b. \((d_{23})_2 = \frac{2}{2 - (4/3)(\pi/S_2)}

c. \((d_{23})_3 = 1.500 - 2\ r_{22}

**TYPE II ROTOR**

d. \((d_{23})_1 = \frac{\pi(D_p - 2\ d_{20}) - (S_2)(t_{w2}) - 0.060(\pi + S_2)}{(4/3)\pi}

e. \((d_{23})_2 = (K_2 - r_{22x})/K_1

f. \((d_{23})_3 = (1.500 - 2\ K_2)/(1 - 2\ K_1)

where \(K_1 = \frac{2\ \pi}{3(\pi + S_2)}

\[K_2 = \frac{\pi(D_p - 2\ d_{20}) - S_2(t_{w2})}{2(\pi + S_2)}\]

\[K_3 = (D_p - \frac{1}{3}D_b - 2d_{20})(\frac{A_{t1}}{A_{y1}}) - \frac{(S_2)(t_{w2})}{P}\]

\[r_{22x} = \frac{(K_1)(K_3) - 2\ K_2(A_{t1}/A_{y1})}{[(10/3)K_1 - 2](A_{t1}/A_{y1})}\]

**LIMITATION:** \((d_{23})_{max} \geq 0

9. \(d_{23} = 1.00\ (d_{23})_{max} \text{ for rotor } #1
\)
\(d_{23} = 0.80\ (d_{23})_{max} \text{ for rotor } #2
\)
\(d_{23} = 0.60\ (d_{23})_{max} \text{ for rotor } #3
\)
\(d_{23} = 0.40\ (d_{23})_{max} \text{ for rotor } #4
\)
\(d_{23} = 0.20\ (d_{23})_{max} \text{ for rotor } #5

10. **TYPE I ROTOR**

\[
r_{22} = \frac{\pi(D_p - 2\ d_{20}) - (S_2)(t_{w2})}{2(\pi + S_2)}
\]
Appendix 10 (Cont.)

**TYPE II ROTOR**

\[
\begin{align*}
\mathbf{r}_{22} &= \frac{\pi(D_p - 2d_{20}) - (S_2)(tw_2) - (4/3)(\pi)(d_{23})}{2(\pi + S_2)} \\
\text{LIMITATION:} \quad \mathbf{r}_{22} &\geq 0.030
\end{align*}
\]

11. **TYPE I ROTOR**

\[
\mathbf{r}_{23} = \mathbf{r}_{22} - \left(\frac{\pi}{S_2}\right)(d_{23})
\]

**TYPE II ROTOR**

\[
\mathbf{r}_{23} = \mathbf{r}_{22}
\]

12. \(d_2 = d_{20} + \mathbf{r}_{22} + d_{23} + \mathbf{r}_{23}\)

13. \(d_{2c} = d_2 - (\mathbf{r}_{23}/3)\)

14. \(A_b = \frac{1}{2} \pi \left[ (\mathbf{r}_{22})^2 + (\mathbf{r}_{23})^2 \right] + \left[ (\mathbf{r}_{22} + \mathbf{r}_{23}) \right] (d_{23})\)

15. \(A_{t2} = (S_2)(tw_2)(L_{eff})/(P)\)

16. \(A_{y2} = \left[ D_p - 2d_{2c} - (2/3)D_b \right](L_{eff})\)

17. \(L_{y2} = \pi \left[ D_p - 2d_{2c} + (2/3)D_b \right]/4(P)\)

18. \(G_{F2} = \frac{tp_2(4.40g + 0.75w_{20}')}{(tp_2)(4.40g + 0.75w_{20}') - (w_{20}')^2}\)

19. \(K_{u2} = (d_{20} - \mathbf{g}_2)/w_{20}'\)

20. \(K_{L2} = K_1 \left( \frac{A_1}{A_1 + A_2 + A_3} \right)^2 + K_2 \left( \frac{A_1 + A_2}{A_1 + A_2 + A_3} \right) + K_3\)

**TYPE I ROTOR**

\[
\begin{align*}
K_1 &= \frac{(\mathbf{r}_{23})^4}{2(A_1)^2} \left( -\frac{\theta^3}{3} - \frac{\sin 2\theta}{4} + \frac{\theta \cos 2\theta}{2} - \frac{\sin 4\theta}{32} \right) \\
K_2 &= \frac{0.25(Y^4 - X^4) + B(Y^2 - X^2) + (B)^2 \ln (Y/X)}{2(B - Y^2) \tan \alpha}
\end{align*}
\]
Appendix 10 (Cont.)

\[
K_3 = \frac{1}{2 \left[ \frac{1}{\pi} + \frac{A}{(r_{22})^2} \right]^2} \left[ \frac{\pi^3 - \varrho^3}{3} + \frac{A^2}{(r_{22})^2} + \frac{1}{8} \right] (\pi - \theta) \\
+ \frac{A}{(r_{22})^2} (\pi^2 - \varrho^2) + \frac{\sin 2\theta}{4} + \frac{A}{2(r_{22})^2} + \frac{\pi - \theta \cos 2\theta}{2} \\
- \frac{A}{(r_{22})^2} \left( \frac{\cos 2\theta}{2} + \frac{\sin 4\theta}{32} \right)
\]

A_1 = (r_{23})^2 (\theta - \frac{1}{3} \sin 2\theta)

A_2 = (Y^2 - X^2) \tan \alpha

A_3 = (r_{22})^2 \left[ (\pi - \theta) + \frac{1}{3} \sin 2\theta \right]

A_4 = (r_{22})^2 (\theta - \frac{1}{3} \sin 2\theta)

A = A_1 + A_2 - A_4

X = r_{23} \cos \alpha / \tan \alpha

Y = r_{22} \cos \alpha / \tan \alpha

B = (A_1 / \tan \alpha) - X^2

\theta = \frac{1}{3} \pi - \alpha

\alpha = \sin^{-1} \left( \frac{r_{22} - r_{23}}{d_{23}} \right)

**TYPE II ROTOR**

A_1 = \frac{1}{3} \pi (r_{23})^2

A_2 = 2(r_{22})(d_{23})

A_3 = A_1

K_1 = 0.1424
Appendix 10 (Concl.)

\[ K_2 = \frac{\frac{d_{23}}{r_{22}}}{3} \frac{(A_1)^2 + (A_1)(A_2) + (A_2)^2/3}{(A_1 + A_2)^2} \]

\[ K_3 = \frac{1}{2 \left[ \pi + \frac{A_2}{(r_{22})^2} \right]^2} \left[ 11.596 + 8.402 \frac{A_2}{(r_{22})^2} + 1.571 \frac{(A_2)^2}{(r_{22})^4} \right] \]
APPENDIX 11

FORMULAS FOR END RING

(Length dimensions are expressed in inches)

1. \((D_r)_1 = (1 - 0.022) \, D_p\)
2. \((d_3)_{\text{max}} = \) the smaller value of \((d_3)_1\) and \((d_3)_2\)
   a. \((d_3)_1 = 0.207 \, (D_p) + 0.100\)
   b. \((d_3)_2 = \frac{1}{2} \left[(D_r)_1 - D_b - 0.25\right]\)
3. \((h_3)_{\text{max}} = (d_3)_{\text{max}}\)
4. \((A_r)_{\text{max}} = (h_3)_{\text{max}} \left[ (d_3)_{\text{max}} - 0.0874 \right]\)
5. \((D_r)_{\text{min}} = (D_r)_1 - (d_3)_{\text{max}}\)
6. \((d_3)_{\text{min}} = 0.0692 \, (D_p) + 0.1300\)
7. \((h_3)_{\text{min}} = \) the larger value of \((h_3)_1\) and \((h_3)_2\)
   a. \((h_3)_1 = 0.2 \, (d_3)_{\text{min}}\)
   b. \((h_3)_2 = 0.100 + K\)

   where \(K = 0, \) if \(D_p \leq 6.000\)
   \[K = 0.032 \, (D_p - 6.000), \) if \(D_p > 6.000\)
8. \((A_r)_{\text{min}} = (h_3)_{\text{min}} \left[ (d_3)_{\text{min}} - 0.0874 \right]\)
9. \((D_r)_{\text{max}} = (D_r)_1 - (d_3)_{\text{min}}\)
10. \(h_{32} = \frac{1}{8} \, (D_p - D_r) + 1/4 \, d_3\)
11. \(K_r = 1/ \left[ 1.25 \, (P)^{2/3} \right]\)
### APPENDIX 12

**ADJUSTMENTS FOR STATOR PUNCHING AND ROTOR PUNCHING DESIGN**

<table>
<thead>
<tr>
<th>Cause of failure</th>
<th>Proceeding of adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>((W_{10})_1 &lt; 0.09)</td>
<td>Increase (D)</td>
</tr>
<tr>
<td>((W_{10})_3 &lt; 0.09)</td>
<td>(1) Increase (\frac{B_{tl}}{B_g})</td>
</tr>
<tr>
<td>(W_{1x} &lt; 0.03)</td>
<td>(2) Increase (D)</td>
</tr>
<tr>
<td>(d_{13} &lt; 0)</td>
<td>(d_{10} = 0.03)</td>
</tr>
<tr>
<td>(d_{10} &lt; 0.03)</td>
<td>(W_{10} = W_{11}/1.5)</td>
</tr>
<tr>
<td>(W_{11} &lt; 1.5 (W_{10}))</td>
<td>(W_{10} = W_{11}/1.5)</td>
</tr>
<tr>
<td>(d_{1c} &gt; 10 (tw_1))</td>
<td>Decrease (\frac{B_{tl}}{B_g})</td>
</tr>
<tr>
<td>(D - D_p - 2 d_1 &lt; 0.06 D)</td>
<td>(1) Decrease (\frac{B_{tl}}{B_g})</td>
</tr>
<tr>
<td>(W_{11} &gt; 2 (tw_1))</td>
<td>(2) Decrease (\frac{A_{tl}}{A_y1})</td>
</tr>
</tbody>
</table>

**STATOR**

**ROTOR**

\(W_{20} < 0.02\)

\(d_{20} < g_2 + 0.005\)

\(W_{20} = 0\)

\(d_{20} = g_2 + 0.005\)
Appendix 12 (Concl.)

<table>
<thead>
<tr>
<th>Cause of failure</th>
<th>Proceeding of adjustment</th>
</tr>
</thead>
</table>
| $\text{tw}_2 \leq 0.05$ | (1) Decrease $\frac{B_{tl}}{B_g}$  
                                                                  (2) Increase $D$ |
| $(d_{23})_1 < 0$ | (1) Increase $\frac{B_{tl}}{B_g}$  |
| $(d_{23})_2 < 0$ | (2) Increase $D$ |
| $r_{22} < 0.03$ | (1) Decrease $\frac{B_{tl}}{B_g}$  
                                                                  (2) Increase $D$ |
| $(d_{23})_3 < 0$ |                           |
APPENDIX 13

FORMULAS FOR CONSTANTS

A. UNIT VALUES

1. \( P = 746 \) (HP)

2. \( E = V \)

3. \( I = \frac{P}{qE} \)

4. \( Z = \frac{E}{I} \)

5. \( T = 7.04 \frac{P}{N} \)

6. \( N = N_s = 120 \frac{f}{P} \)

B. WINDING CONSTANTS

1. \( (\text{Throw}) = \left( \frac{S_1}{P} \right) + 0.99 \) *

   where \( K = \frac{5}{6}, \) if \( P \neq 2 \)

   \( K = \frac{2}{3}, \) if \( P = 2 \)

   * rounded to the nearest integer.

2. \( (\text{Span}) = (\text{Throw}) - 1 \)

3. \( K_t = (\text{Span})P/(S_1) \)

4. \( K_p = \sin \left[ \frac{\pi}{2} (K_t) \right] \)

   \( (a) \sin \left[ \frac{\pi}{2} (\frac{1}{q}) \right] \)

5. \( K_d = \frac{(S_1) \sin \left[ \frac{\pi}{2} (\frac{a}{q}) \right]}{2 (S_1)(q)} \)

   where \( a = \) the highest common factor of \( S_1 \) and \( (P)(q) \).

   * 60° phase belt is used for 3-phase motors in this program.
Appendix 13 (Cont.)

6. \( K_m = K_t \), \( if \ q = 2 \)
   \( K_m = 0.75 K_t \), \( if \ q = 3 \ and \ K_t \leq \frac{1}{3} \)
   \( K_m = 1.50 K_t - 0.25 \), \( if \ q = 3 \ and \ \frac{1}{3} < K_t \leq \frac{2}{3} \)
   \( K_m = 0.75 K_t + 0.25 \), \( if \ q = 3 \ and \ \frac{2}{3} < K_t \leq 1 \)

7. \( K_{LF} = \frac{K_m(3K_{L1} + 4K_{u1}) + K_{L1}}{4(K_{L1} + K_{u1})} \)

C. CIRCUIT ELEMENTS

1. \( \dot{R}_1 = 8.27 \times 10^{-7} \left[ \frac{2(S_1)(TPC)(MCL)}{(q)(PAR)^2(COND_1)(COND\_AREA_1)} \right] \left( \frac{1}{Z} \right) \)

2. \( \dot{R}_2 = 8.27 \times 10^{-7}(A)(q) \left[ \frac{(K_{rb})(L)}{(S_2)(A_r)(COND_2)} \right] \left( \frac{1}{Z} \right) \)
   \( + \frac{2(D_T)}{(\pi)(P)^2(A_r)(COND_2)} \left( \frac{1}{Z} \right) \)

3. \( \dot{X}_m' = 6.38 \times 10^{-8}(A)(f)(L_f)(q) \left[ \frac{(D_2 + g)}{(P)^2(g_e)} \right] \left( \frac{1}{Z} \right) \)

4. \( \dot{X}_{1SL} = 20.04 \times 10^{-8}(A)(f)(L_f)(q)(K_{LF}) \left[ \frac{(K_u1)}{(S_1)(K_d)^2(K_p)^2} \right] \left( \frac{1}{Z} \right) \)

5. \( \dot{X}_{1SU} = 20.04 \times 10^{-8}(A)(f)(L_f)(q)(K_{LF}) \left[ \frac{(K_u1)}{(S_1)(K_d)^2(K_p)^2} \right] \left( \frac{1}{Z} \right) \)

6. \( \dot{X}_{2SL} = 20.04 \times 10^{-8}(A)(f)(L_f)(q) \left[ \frac{(K_{L2})}{(S_2)} \right] \left( \frac{1}{Z} \right) \)

7. \( \dot{X}_{2SU} = 20.04 \times 10^{-8}(A)(f)(L_f)(q) \left[ \frac{(K_u2)}{(S_2)} \right] \left( \frac{1}{Z} \right) \)

8. \( \dot{X}_{1E} = 8.2 \times 10^{-8}(A)(f)(q) \left[ \frac{(h_1 + h_{11})}{2(P)} \right] \left( \frac{1}{Z} \right) \)
Appendix 13 (Concl.)

9. \( \dot{X}_{2E} = 18 \times 10^{-8} \frac{(A)(f)(q)(D_2 + g)}{(P)^2} \frac{(h_{31} + h_{32})}{1.7\ h_3 + 1.2\ d_3 + 1.45(h_{31} + h_{32})} \frac{1}{(Z)} \)

10. \( \dot{X}_{1ZZ} = 21 \times 10^{-8} \frac{(A)(f)(L_f)(q)(K_{LF})}{(D_2 + g)} \frac{(D_2 + g)}{(g_e)(S_1 + S_2)(S_1)(K_d)^2(K_p)^2} \frac{S_1}{(S_1 + S_2)} \frac{1}{(Z)} \)

11. \( \dot{X}_{2ZZ} = 21 \times 10^{-8} \frac{(A)(f)(L_f)(q)(K_{LF})}{(D_2 + g)} \frac{(D_2 + g)}{(g_e)(S_1 + S_2)(S_1)(K_d)^2(K_p)^2} \frac{S_2}{(S_1 + S_2)} \frac{1}{(Z)} \)

12. \( \dot{X}_{SK} = (Q)(\dot{X}_M) \)

where \( Q = 1 - \left( \frac{\sin(\theta_{SK})}{\theta_{SK}} \right)^2 \), and \( \theta_{SK} = \frac{(P)(SKEW)}{2(D_2)} \)

13. \( \dot{X}_1 = \dot{X}_{1SL} + (K_{s1})\dot{X}_{1SU} + \dot{X}_{1E} + (K_z)\dot{X}_{1ZZ} \)

14. \( \dot{X}_2 = (K_{LM})\dot{X}_{2SL} + (K_{s2})\dot{X}_{2SU} + \dot{X}_{2E} + (K_z)\dot{X}_{2ZZ} + (s)\dot{X}_{SK} \)

\* \( g_e = g(GF_1)(GF_2) \)

\( L_f = L + 2\ g_e \)

\( A = \left[ \frac{2(S_1)(TPC)(K_d)(K_p)}{(PAR)(q)} \right]^2 \)
APPENDIX 14

FORMULAS FOR NO-LOAD PERFORMANCE

1. \( E_1 = V \)

2. \( E_{21} = \left( \frac{X_M'}{X_1 + X_M'} \right) E_1 \)

3. \( \phi = \frac{E_{21}}{10^8} \times \left[ \frac{2(S_1)(K_d)(K_p)(TPC)}{(PAR) (q)} \right] \)

4. \( \phi' = (\pi/2) \phi \)

5. \( B_g = \phi' / A_g \)

   where \( A_g = \pi \left( D_2 + g \right) \left( L_f \right)/(P) \)

6. \( B_{t1} = \phi' / A_{t1} \)

7. \( B_{t2} = \phi' / A_{t2} \)

8. \( B_{y1} = \phi / A_{y1} \)

9. \( B_{y2} = \phi / A_{y2} \)

10. \( (NI)_g = 0.313 \left( B_g \right) (E_e) \)

11. \( (NI)_{t1} = \left[ F_t \left( B_{t1} \right) \right] \left( d_{1c} \right) \)

12. \( (NI)_{t2} = \left[ F_t \left( B_{t2} \right) \right] \left( d_{2c} \right) \)

13. \( (NI)_{y1} = \left[ F_y \left( B_{y1} \right) \right] \left( L_{y1} \right) \)

14. \( (NI)_{y2} = \left[ F_y \left( B_{y2} \right) \right] \left( L_{y2} \right) \)

15. \( (NI)_T = (NI)_g + (NI)_{t1} + (NI)_{t2} + (NI)_{y1} + (NI)_{y2} \)

16. \( \dot{X}_M = X_M' \left( NI \right)_g / (NI)_T \)

17. \( E_2 = \left( \frac{\dot{X}_M}{X_1 + X_M} \right) E_1 \)

18. \(|(E_{21} - E_2)| \leq 0.005 E_{21}\)
Appendix 14 (Concl.)

19. \( F_w(B) = 1.732 \times 10^{-10} \ (B)^2 \), if \( B \leq 105 \times 10^3 \)

\[
F_w(B) = 1.581 \times 10^{-20} \ (B)^4, \ \text{if} \ B > 105 \times 10^3
\]

20. \( W_1 = \left[ F_w(B_{bl}) \right] V_{tl} + \left[ F_w(B_{y1}) \right] V_{yl} \)

21. \( K_w = (f/60)^{1+1/4} \)

22. \( K_y = 800 \ (TK)^2 + 0.500 \)

23. Core loss = \( (K)(K_w)(K_y)(W_1) \)

where \( K = 1.250 \), for iron Type 1625

\( K = 0.938 \), for iron Type 9080 and 9116

24. \( \dot{E}_1 = \frac{E_{1}}{E} \)

25. \( \dot{R}_m = \frac{(\dot{E}_1)^2}{\text{(Core loss)}/F} \)

<table>
<thead>
<tr>
<th>B (X10^3)</th>
<th>( F_t(B) )</th>
<th>( F_y(B) )</th>
<th>( F_t(B) )</th>
<th>( F_y(B) )</th>
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<td>179.0</td>
<td>247.0</td>
<td>182.0</td>
</tr>
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</table>
APPENDIX 15

FORMULAS FOR OTHER PERFORMANCE

1. \( (s)P_0 = \frac{\dot{R}_2}{[(\dot{R}_1)^2 + (\dot{x}_1 + \dot{x}_2)^2]^{\frac{1}{2}}} \)

2. \( a = (1 + K_{SL})(\dot{R}_2/s) \)
   where \( K_{SL} = 0.030, \) if \( P = 2 \)
   \( K_{SL} = 0.015, \) if \( P \neq 2 \)

3. \( b = (\dot{x}_2 + \dot{x}_M)/a \)

4. \( c = b (\dot{x}_2) + a \)

5. \( d = b (\dot{x}_M) + c \)

6. \( \dot{x}_{2f} = c (\dot{x}_M)/d \)

7. \( \dot{R}_{2f} = (\dot{x}_M)^2/d \)

8. \( \dot{x}_T = \dot{x}_1 + \dot{x}_{2f} \)

9. \( \dot{R}_T = \dot{R}_1 + \dot{R}_{2f} \)

10. \( Z_T = [(\dot{R}_T)^2 + (\dot{x}_T)^2]^{\frac{1}{2}} \)

11. \( \dot{I}_w = \frac{E_1}{Z_T} (\dot{R}_T) + \frac{E_1}{ \dot{R}_M} Z_T \)

12. \( I_x = \frac{E_1}{Z_T} (\dot{x}_T) \)

13. \( \dot{i} = \left[ (\dot{I}_w)^2 + (\dot{I}_w)^2 \right]^{\frac{1}{2}} \)

14. \( I = (\dot{i})(\dot{I}) \)

15. \( T = \frac{(\dot{i})^2 \dot{R}_{2f}}{1 + K_{SL}} - (F & W)_{NL} (1 - s)^2 \)

16. \( \dot{H}P = (1 - s) \dot{T} \)

17. INPUT = \( \dot{I}_w \)
Appendix 15 (Concl.)

18. $I_L = 1.732 I$, for 3-phase, $\Delta$-connection

$I_L = I$, for 2-phase, and for 3-phase, $Y$-connection

19. $PF = \frac{i_w}{I}$

20. $EFF = \frac{(HP)}{(INPUT)}$

21. Stator $I_{2R}^2 = (I^2R_1)(P)$

22. Locked time (in seconds) = $\frac{30,000}{(Stator I_{2R}^2)_s}$

23. $(s_{FL})_1 = 1.25 R_2$
### Appendix 16

### Scoring System

<table>
<thead>
<tr>
<th>Factor</th>
<th>Critical Range</th>
<th>Penalty score</th>
<th>Plus score</th>
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<tbody>
<tr>
<td>Full-load slip</td>
<td>Range</td>
<td>1% = 10 Pts</td>
<td>--</td>
</tr>
<tr>
<td>Full-load efficiency</td>
<td>Minimum</td>
<td>1% = 3 Pts</td>
<td>1% = 3 Pts</td>
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<tr>
<td>Full-load power factor</td>
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<td>1% = 1 Pt</td>
<td>1% = 1 Pt</td>
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<tr>
<td>Starting torque</td>
<td>Minimum</td>
<td>1% = 1 Pt</td>
<td>1% = 1 Pt</td>
</tr>
<tr>
<td>Starting current</td>
<td>Maximum</td>
<td>1% = 1 Pt</td>
<td>1% = 1 Pt</td>
</tr>
</tbody>
</table>
APPENDIX 17

FORMULAS FOR DEEP BAR EFFECT

1. \[ K_{db} = \left[ 0.1607 \left( COND_2 \right)^{1/3} + 0.1964 \left( COND_2 \right) \left[ 2 - \left( COND_2 \right) \right] \right] \left( sf \right)^{3/8} \left( d_2 e \right) \]

2. Multiplier for bar resistance
   \[ K_{rb} = 1.00, \quad \text{if } K_{db} \leq 1.00 \]
   \[ K_{rb} = 0.88 \left( 0.17 + K_{db} \right), \quad \text{if } 1.00 < K_{db} \leq 2.30 \]
   \[ K_{rb} = K_{db}, \quad \text{if } 2.30 < K_{db} \]

3. Multiplier for secondary lower slot reactance
   \[ K_{LM} = 1.00, \quad \text{if } K_{db} \leq 0.50 \]
   \[ K_{LM} = 0.114 \left( 6.5 - K_{db} \right), \quad \text{if } 0.50 < K_{db} \leq 1.40 \]
   \[ K_{LM} = 0.246 \left( 5.04 - K_{db} \right), \quad \text{if } 1.40 < K_{db} \leq 3.00 \]
   \[ K_{LM} = \left( 1.30/K_{db} \right)^{5/6}, \quad \text{if } 3.00 < K_{db} \]
FORMULAS FOR SATURATION EFFECT

1. \( B_L = \frac{a}{0.628(b)(g)} \)

   where \( a = I \left[ K_{LF} + (K_d) \frac{S_1}{S_2} \right] \frac{(TPC)}{(PAR)} \frac{2E_2}{E_1} \)

   \( b = 2.5 \frac{g}{t_p1 + t_p2} + 0.64 \)

2. Multiplier for zig-zag reactance

   \( K_z = 1.00, \) if \( B_L \leq 0.71 \times 10^5 \)

   \( K_z = 0.2 \left( 5.7 - B_L \times 10^{-5} \right), \) if \( 0.71 \times 10^5 < B_L \leq 1.35 \times 10^5 \)

   \( K_z = \left[ 1.1/(B_L \times 10^{-5}) \right]^{2/3}, \) if \( 1.35 \times 10^5 < B_L \)

3. Multiplier for primary upper slot reactance

   \( K_{s1} = 1 - \left( \frac{c_1}{c_1 + w_{10'}} \right) \frac{K_{u1'}}{K_{u1}} \)

   where \( c_1 = (tp_1 - w_{10}) (1 - K_z) \)

   \( w_{10'} = 1.5 w_{10} \)

   \( K_{u1'} = \frac{[d_{10} - g_1] + 0.58 d_{11}}{w_{10}} \)

4. Multiplier for secondary upper slot reactance

   \( K_{s2} = 1 - \frac{c_2}{c_2 + w_{20'}} \)

   where \( c_2 = (tp_2 - w_{20'}) (1 - K_z) \)
APPENDIX 19

TYPES OF ENCLOSURE

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRPR</td>
<td>Drip proof</td>
</tr>
<tr>
<td>FCST</td>
<td>Fan-cooled standard</td>
</tr>
<tr>
<td>FCXP</td>
<td>Fan-cooled explosion proof</td>
</tr>
<tr>
<td>NVSS</td>
<td>Non-vent smooth standard</td>
</tr>
<tr>
<td>NVRS</td>
<td>Ribbed frame without hood and blower</td>
</tr>
<tr>
<td>NVRX</td>
<td>Ribbed frame without hood and blower</td>
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<tr>
<td></td>
<td>Explosion proof</td>
</tr>
<tr>
<td>SPPR</td>
<td>Splash proof</td>
</tr>
</tbody>
</table>
APPENDIX 20

FORMULAS FOR FRICTION AND WINDAGE LOSS

For \( D_b \leq 2.571 \),

\[
F & W = 4.73 (D_b)^2 \frac{N}{1000} + .002 (DBL)^4 \frac{N}{1000}^3
\]

For \( D_b > 2.571 \),

\[
F & W = 4.23 (D_b)^{2.5} \left(\frac{N}{1000}\right) + .002 (DBL)^4 \frac{N}{1000}^3
\]

LIMITATION: \((F \& W)_{N L} \leq .05\)

where \( F \& W \) = friction and windage loss at speed \( N \) in watts

\( D_b \) = rotor bore diameter in inches

\( DBL \) = blower diameter in inches

\( N \) = motor speed in rpm

FORMULA FOR BLOWER DIAMETER

<table>
<thead>
<tr>
<th>Encl.</th>
<th>Poles</th>
<th>Blower diameter ( DBL )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCST</td>
<td>( P = 2 )</td>
<td>( DBL = D - .025 (D)^2 )</td>
</tr>
<tr>
<td>&amp; FCXP</td>
<td>( P \neq 2 )</td>
<td>( DBL = D - (34/D^2) )</td>
</tr>
<tr>
<td>Others</td>
<td>Any</td>
<td>( DBL = D_p )</td>
</tr>
</tbody>
</table>

Remarks: \( D = \) outside diameter of core

\( D_p = \) punched diameter of core

LIMITATIONS: \( D_p \leq DBL \leq D \)
Fig. 1. Symbols for flow charts.
Fig. 2. Over-all flow chart.
### CARD 1

<table>
<thead>
<tr>
<th>IDENT</th>
<th>HP</th>
<th>P</th>
<th>f</th>
<th>q</th>
<th>V</th>
<th>#F</th>
<th>MIN. M</th>
<th>MIN. M</th>
<th>MAX. M</th>
<th>MIN. M</th>
<th>MIN. M</th>
<th>MIN. M</th>
<th>Locked</th>
<th>TIME</th>
<th>OVE</th>
<th>V</th>
<th>RISE</th>
<th>°C</th>
<th>TIME</th>
<th>ENCL.</th>
<th>CLASS</th>
<th>CARD</th>
</tr>
</thead>
<tbody>
<tr>
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<td>XXX.XX</td>
<td>XX</td>
<td>XXX</td>
<td>X</td>
<td>XXXX</td>
<td>X.XXXX</td>
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<td>XXXX</td>
<td>AAAA</td>
<td>A</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1 to 5</td>
<td>6 to 10</td>
<td>11 to 12</td>
<td>13 to 15</td>
<td>17 to 20</td>
<td>21</td>
<td>22 to 24</td>
<td>25 to 27</td>
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<td>30</td>
<td>31</td>
<td>36</td>
<td>39</td>
<td>h5</td>
<td>48</td>
<td>51</td>
<td>5h</td>
<td>53</td>
<td>57</td>
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</tbody>
</table>

### CARD 2

| IDENT | TYPE | TK | IRON | S1 | S2 | TYPE | WIRE | WIRE | MAX. | COND | D | Dp | OR | DP/D | Dy | Ay1 | Bt1 | Bg | TYPE | CARD |
|-------|------|----|------|----|----|------|------|------|------|      |     |    |    |      |    |     |     |    |      |     |
| AAAAA | XXXX | .XXX | XXX | XXX | XXXXX | X.XXX | X.XX | X.XX | XX.XX | XX.XX | XX.XX | XX.XX | X.XX | XXXX | XXXX | X.XX | X |     |   |
| 1 to 5 | 6 to 9 | 10 to 12 | 13 to 15 | 18 | 21 | 24 | 28 | 32 | 35 | 39 | h4 | 45 | 50 | 55 | 59 | 63 | 80 |   |   |

**Fig. 3.** Input card form.
Fig. 5. Stator punching.
A. Type I

B. Type II

Fig. 6. Rotor punching.
Fig. 7. End winding.
Equivalent circuit.
OPTIMUM DESIGNS FOR POLYPHASE SQUIRREL-CAGE INDUCTION MOTORS BY DIGITAL COMPUTER

by

HSI MAO CHEN

B. S., National Amoy University, Amoy, China, 1949

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY OF AGRICULTURE AND APPLIED SCIENCE

1960
This thesis presents a program set up in an IBM 650 digital computer system for optimizing design of a polyphase squirrel-cage induction motor with a single winding, wire-wound stator and a die-cast single-cage rotor to meet a set of customer specifications and manufacturing specifications, while working toward a minimum amount of copper and iron.

The need for such a program is very great when the motor manufacturing industry makes changes in its standards or when efforts are being made to establish new standards.

Input data consist of three parts: customer specifications, manufacturing specifications, and optional specifications, which provide the computer with a complete description of the motor required by the customer and also with any restriction on the materials, dimensions, or design proportions found necessary or desirable from the manufacturer's viewpoint.

In these kinds of designs, hundreds of variable parameters are involved; however, in this program, they are narrowed down to five independent variables, namely:

1. The outside diameter of the stator punching.
2. The inside diameter of the stator punching.
3. The shaft diameter through the rotor punching.
4. The ratio of the effective stator tooth magnetic section per pole to the effective stator yoke magnetic section per pole.
5. The ratio of the stator tooth flux density to the air-gap flux density.
These independent variables are either specified in the input or selected by the computer from optimum values based on wide studies and statistical analyses from a large number of motors built in the past. From each set of independent variables, a set of magnetic parts is designed. From these magnetic parts a set of windings is designed to meet the input specifications, and its performance is calculated. The calculated performance characteristics are then compared with the specified values and scores are made according to their effects on the overall acceptability of general purpose motors.

The optimization in this program starts by changing windings but keeping all other parts the same to see if a better overall design can be made. Of all designs using the same parts but with different windings, only the "best one", that is, the one with the highest total plus score among those with the lowest total penalty score has its windings, constants defining the magnetic parts, and performance characteristics printed out in the output.

In the second step of the optimization, the computer makes decisions as to whether it is possible to make further improvements in designs by changing independent variables and designing new parts. Provided a proper change exists, using this new set of variables, the computer will continue on its designing and optimizing process until no more changes are possible. At this time the optimizing process for this problem is complete.

The optimization in this program is based primarily on the performance characteristics of the designs and on the scoring
system derived for general purpose motors. It has been used successfully in arriving at the details of a new line of motors using a new type of magnetic sheet steel and a new winding and insulation system. The designs were arrived at in the matter of days as compared to a matter of years in the case of completely manual procedures.