USING EMPIRICAL HAZARD FUNCTION IN
DISTINGUISHING FAILURE MODELS

by

SAYED AMIR FARTAJ

B.S., Kansas State University, 1980
M.S., Kansas State University, 1983

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A MASTER'S REPORT

Submitted  in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Statistics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1987

Approved by:

_Doris  Grosh_
Major Professor

A11207 301602

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CHAPTER I

INTRODUCTION

Suppose X is a random variable representing the service life to failure
of a specified material, structure, or device. Such a variable represents
an attempt to describe statistically the length of life of the material.
On the basis of actual observations of times to failure, it is difficult
to distinguish among various probability distributions. In order to discri-
minate among probability functions that can not be easily distinguished from
each other, it is reasonable to appeal to a concept that permits differen-
tiation among distributions, namely the hazard function.

The hazard function is known by a variety of names in various applica-
tions. It is used by actuaries under the name of "force of mortality" to
compute mortality tables. Its reciprocal for the normal distribution is
known as "Mill's ratio". Papers in extreme value distribution theory call
it the intensity function, while in reliability theory the name hazard
function and instantaneous failure function are used.

The hazard rate is dependent on the number of failures in a given time
period and the number of components exposed to failure. In order to evaluate
the hazard rate, the number of failures per unit of time must be related to
the number of components that are exposed to failure, giving the following
definition of h(t).

$$h(t) = \frac{\text{number of failure per unit time}}{\text{number of components exposed to failure}}$$

The central focus in the present study was to determine the usefulness of the hazard function in distinguishing between various failure models such as the exponential, gamma and Weibull distributions. In order to achieve this goal, sets of failure data were simulated from three assumed failure rate distribution functions with known parameters.

CHAPTER II

DEVELOPEMENT OF THE HAZARD FUNCTION

Let $F(x)$ be the distribution function of the random variable X, and let $f(x)$ be its probability density function. Then the hazard rate, $h(x)$ is defined as

$$h(x) = \frac{f(x)}{1-F(x)} \qquad (2.1)$$

where $1-F(x)$ is called the reliability at time x. The hazard rate as a function of time has a probabilistic interpretation; namely, $h(x)d(x)$ represents the failure probability per unit time at time x, given that failure has not yet occurred at time x. Differences in hazard rates are significant elements in the comparison of densities. In this paper the use of the empirical hazard function in distinguishing three distributions (exponential, gamma and Weibull) was studied.

## 2.1 The Negative Exponential Distribution

In reliability studies, the exponential distribution plays a role of importance analogous to that of the normal distribution in other areas of statistics. Although many life random variables can not be adequately described by the exponential distribution, an understanding of the theory

in the exponential case facilitates the treatment of more general situations
The utility of the exponential distribution is due to its simplicity and
thus it is widely used as a model for the life of many products.

The probability density function (p.d.f.) of the exponential distribution
can be written as

$$f(x) = \lambda \exp(-\lambda x) \qquad (2.2)$$

where the parameter $\lambda$ is called the failure rate.

The cumulative distribution function (CDF) of the exponential distribu-
tion is given by

$$F(x) = 1 - \exp(-\lambda x) \qquad (2.3)$$

By substituting equations (2.2) and (2.3) into (2.1), the hazard function
for the exponential distribution can be expressed as

$$h(x) = \lambda \qquad (2.4)$$

This instantaneous failure rate is constant over time, a key characteristic
of the exponential distribution. A constant hazard rate means that the
probability of failure is independent of age, so an old equipment which is
still operating is just as good as a new one.

## 2.2 The Gamma Distribution

The gamma distribution is a natural extention of the exponential distribution and has sometimes been considered as a model in life test problems. The probability density function (p.d.f.) of the Gamma distribution can be written as

$$f(x) = \frac{x^{\beta-1}}{\alpha^{\beta}\Gamma(\beta)} \exp^{-x/\alpha} \qquad (2.5)$$

where $\Gamma(\beta)$ is the gamma function evaluated at $\beta$. The parameter, $\beta$, is called the shape parameter while $\alpha$ is called the scale parameter. One of the main reasons the gamma distribution was chosen as a distribution is because of the large number of shapes the distribution can assume. By varying the value of $\beta$ the shape of the distribution is changed.

The cumulative distribution function (C.D.F.) of the gamma distribution is defined as

$$F(x) = [\frac{1}{\Gamma(\beta) \ \alpha^{\beta}}] \ \int x^{\beta-1} \ \exp(-x/\alpha) \ dx$$
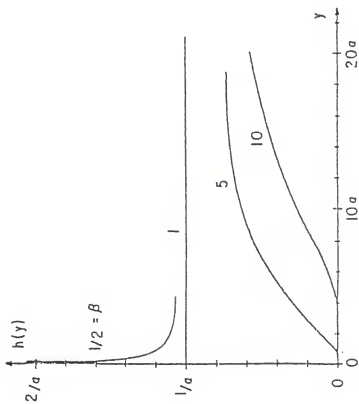
$$= \Gamma(x/\alpha,\beta) \qquad (2.6)$$

where

$$\Gamma(u,\beta) = [1 \ / \ \Gamma(\beta)] \ \int u^{\beta-1} \exp(-u) \ du$$

is the incomplete gamma function. Therefore the hazard function of the gamma distribution can be expressed as
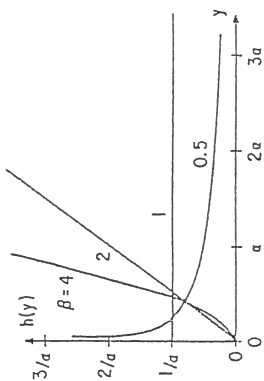
$$h(x) = \frac{[1 \, / \, \Gamma(\beta) \, \alpha^\beta \,] \;\; x^{\beta-1} \; \exp(-x/\alpha)}{[1 \, - \, \Gamma(x/\alpha, \; \beta)]} \qquad (2.7)$$

Figure 2.1 shows gamma hazard functions. For $\beta < 1$, h(x) decreases to a constant value $1/\alpha$. For $\beta > 1$, h(x) starts at zero and increases to a constant value $1/\alpha$. For $\beta = 1$ (exponential distribution), h(x) has the constant value $1/\alpha$.

Gamma Hazard Function

FIGURE 2.1



Weibull Hazard Function

FIGURE 2.2

## 2.3   The Weibull Distribution

The Weibull distribution has similar properties to those of Gamma distribution, that is, it is a two parameter distribution having a shape parameter $\beta$ and a scale parameter $\alpha$, and by varying these parameters a large number of shapes can be generated. This makes it extremely flexible in fitting data, and it empirically fits many kinds of data.

The probability density function (p.d.f.) of the Weibull distribution can be written as

$$f(x) = (\beta/\alpha^\beta) \, x^{\beta-1} \exp[-(x/\alpha)^\beta] \qquad (2.8)$$

The cumulative distribution function (C.D.F.) of the Weibull distribution is given by

$$F(x) = 1 - \exp[-(x/\alpha)^\beta] \qquad (2.9)$$

Therefore the hazard function of the Weibull distribution can be expressed as

$$h(x) = (\beta/\alpha)(x/\alpha)^{\beta-1}. \qquad (2.10)$$

Figure 2.2 shows Weibull hazard functions. A power function of time, $h(x)$ increases with time for $\beta > 1$ and decreases with time for $\beta < 1$. For $\beta = 1$ (the exponential distribution), the failure rate is constant. The ability to describe increasing or decreasing failure rates contributed to making the Weibull distribution popular for life data analysis.

CHAPTER III

EMPIRICAL HAZARD FUNCTION

The hazard rate is in some sense a measure of the rate at which failures occur. However, it is not simply the number of failures that occur in a given period of time because it is dependent upon the size of the sample being considered. For example, the number of failures in a 100-component sample is less than that in a 1000-component sample if the same time period is considered and the components are identical, yet the hazard rate should be the same. Similarly, if the number of failures in a given time period is the same for 100-component sample and a 1000-component sample, where the operating conditions are different, the components of the first samples would appear to be more failure prone than those of the second sample and the hazard rate should be greater.

This discussion indicates that the hazard rate is dependent on the number of failures in a given time period and the number of components exposed to failure.

## 3.1 Evaluation of the Hazard Rate

In a simple life testing experiment a number of items are subjected to tests, and the data consist of the recorded lives of all or some of the items. Since the items are likely to fail at any time, it is quite customery

to assume that the life of the item is a random variable with a distribution function which is the probability that the item fails before time t.

The following equations and mathematical relationships between the various functions do not assume any specific form of the functions and are equally applicable to all probability distributions used in the hazard evaluation.

Consider the case in which a fixed number N of identical components is tested. Let n(t) be the number of survivors (the number of items still alive or functioning adequately) at time t. The failure count during the interval $\Delta t$ is given by:

$$n(t) - n(t+\Delta t)$$

It is of more interest to obtain the failure count per unit time, Which is

$$\frac{n(t) - n(t+\Delta t)}{\Delta t} \; .$$

Now, to derive the empirical hazard function, the above expresion is standardized by the initial count n(t) at the beginning of the interval of interest:

$$h^*(t) = \frac{n(t) - n(t+\Delta t)}{\Delta t \cdot n(t)}$$

This is the empirical analogue of the theoretical hazard function h(t) defined in expression 2.1 .

Two cases of collecting data can be considered.

## Case 1

This case can be called continuous monitoring, where the number of units tested is small and the exact time of each failure is recorded.

## Case 2

This case occurs where the system is observed from time to time, at time points $t_i$ for each of which the number of survivors $n(t_i)$ is noted.

### 3.2  The Simulation Procedure

In order to simulate samples of different failure models, namely, the exponential, gamma, and Weibull distributions, a Fortran language program was written using an IMSL (International Mathematical and Statistical Library) random variable generator. This program created and then sorted the samples of the chosen size. The empirical hazard function was computed using the expression given in section 3.2, as given below

$$h^*(t) = \frac{n(t) - n(t+\Delta)}{\Delta t \cdot n(t)}$$

The Fortran data was then passed by appropriate job control language into the SAS program which has a simple and inexpensive plotting routine.

In the evaluation of the hazard function two cases were considered. In the Case 1, the size of grouped failure count are fixed, in other words $n(t)-n(t+\Delta)$ was maintained at a constant value. In the Case 2, the interval $\Delta t$ was kept constant. This interval was determined by dividing the largest value of the random variable by the desired number of groups.

Two typical programs that were used in this investigation are given in Appendix A. The first program is for the exponential distribution of Case 1 with group size of five. The second program is for Case 2 of the Weibull distribution with $\Delta t$ of about 0.5, shape parameter of four and scale parameter of one.

CHAPTER IV

PRESENTATION OF RESULTS

The purpose of this chapter is to present the results of the simulated failure data. Three different types of failure models were simulated (exponential, gamma and Weibull). These failure data that are presented graphically in the following sections, indicate the use of the empirical hazard function in distinguishing between the above failure models. For each set of failure data two plots were depicted. Plot (a) described the failure data set alone, as compared to plot (b) in which the SPLINE was constructed over the simulated failure data (SPLINE is a curve fitting technique used in the SAS program).

## 4.1 The Exponential Distribution

As mentioned in section 2.1, the exponential distribution has a constant hazard rate. Thus, on the plot of hazard function versus time to failure, the hazard function must look like a straight curve with zero slope.

In Figure 4.1 the empirical hazard function for Case 1 with sample size of 150 and group size of five is shown. It is clear that the hazard rate is not constant over time. Figures 4.2 and 4.3 show Case 2, the first plot corresponds to sample size of 150 items, while the second plot has a sample size of 300 items. Note that the sample in each case was subdivided into 30

intervals (groups).

It is evident that in both plots the hazard function fails to look anything like a straight line. Morecver, it is observed that by increasing the size of grouped data, the curve becomes more smooth, but still not recognizable as a straight line.

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HAZARD FUNCTION FOR
EXPONENTIAL DISTRIBUTION
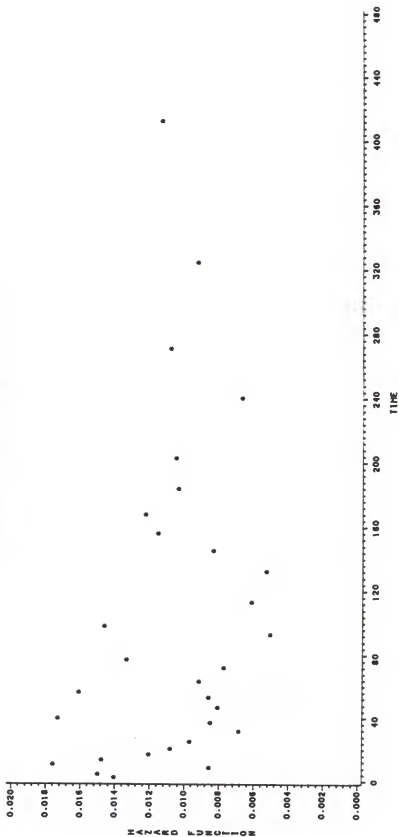(FIXED GROUP SIZE, NR=150)

FIGURE 4.1 a

HAZARD FUNCTION FOR
EXPONENTIAL DISTRIBUTION
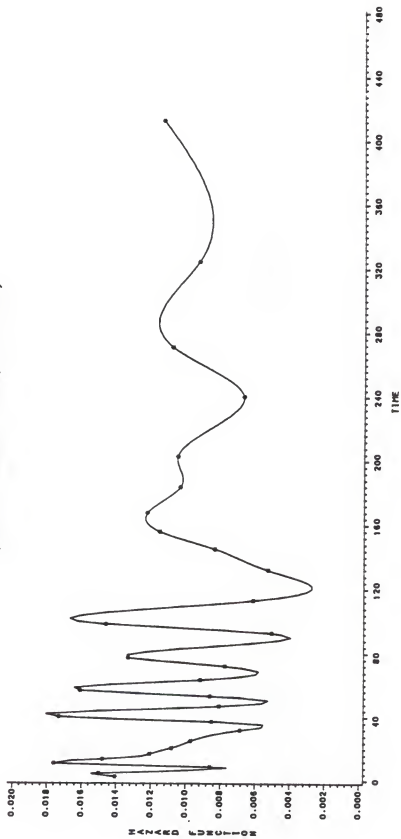(FIXED GROUP SIZE, NR=150)

FIGURE 4.1 b

HAZARD FUNCTION FOR
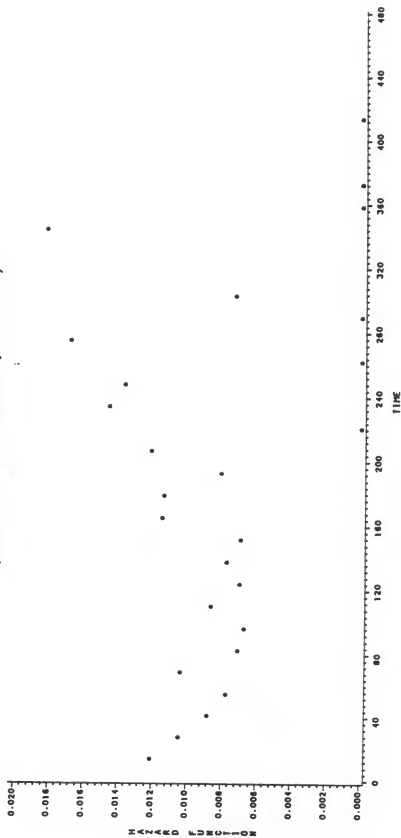EXPONENTIAL DISTRIBUTION
(FIXED TIME INTERVAL, NR=150)

FIGURE 4.2 a

HAZARD FUNCTION FOR
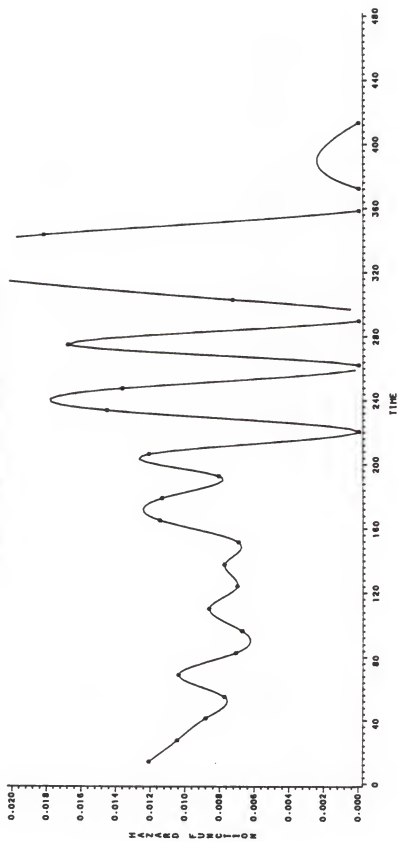EXPONENTIAL DISTRIBUTION
(FIXED TIME INTERVAL, NR=150)

FIGURE 4.2 b

HAZARD FUNCTION FOR
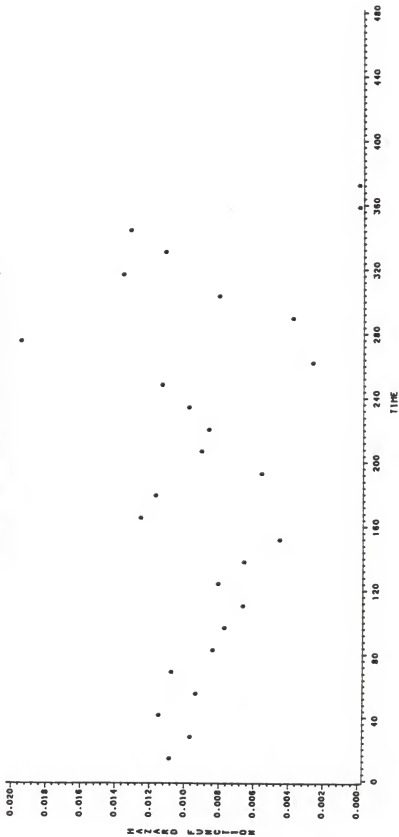EXPONENTIAL DISTRIBUTION
(FIXED TIME INTERVAL, NR=300)

FIGURE 4.3 a

HAZARD FUNCTION FOR
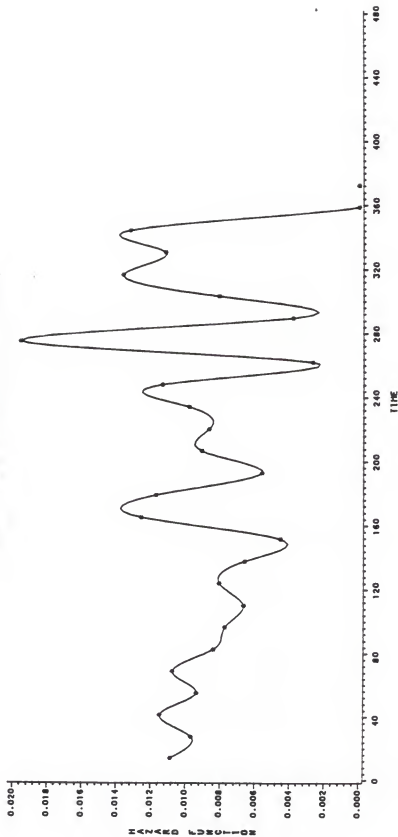EXPONENTIAL DISTRIBUTION
(FIXED TIME INTERVAL, NR=300)

FIGURE 4.3 b

## 4.2   The Gamma Distribution

In the study of the hazard function for the gamma distribution, three figures are constructed. These figures are similar to those of the previous section. The gamma distribution which is considered here has shape parameter of five and scale parameter of one.

In Figure 4.4 (Case 1), it is clear that the resulting graph is inconsistent with the prediction from the theory (Figure 2.1). The similar results are obtained by looking at Figures 4.5 and 4.6 (Case 2). The resultant curves are also inconsistent with their theoretical curves.

HAZARD FUNCTION FOR
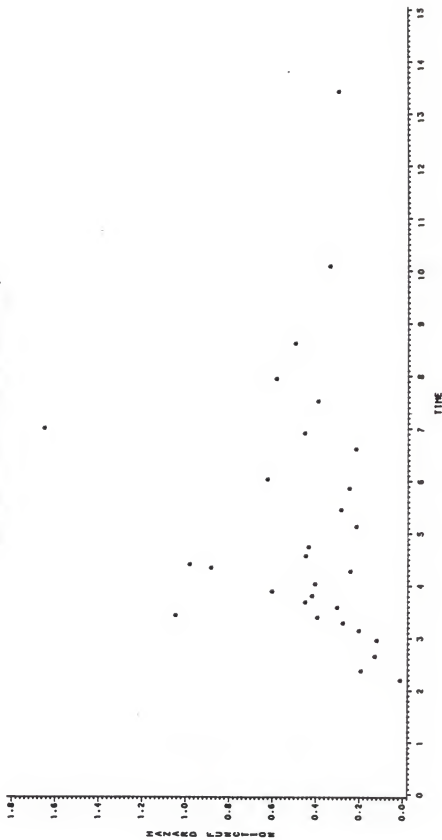GAMMA DISTRIBUTION
(FIXED GROUP SIZE, NR=150)

FIGURE 4.4 a

HAZARD FUNCTION FOR
GAMMA DISTRIBUTION
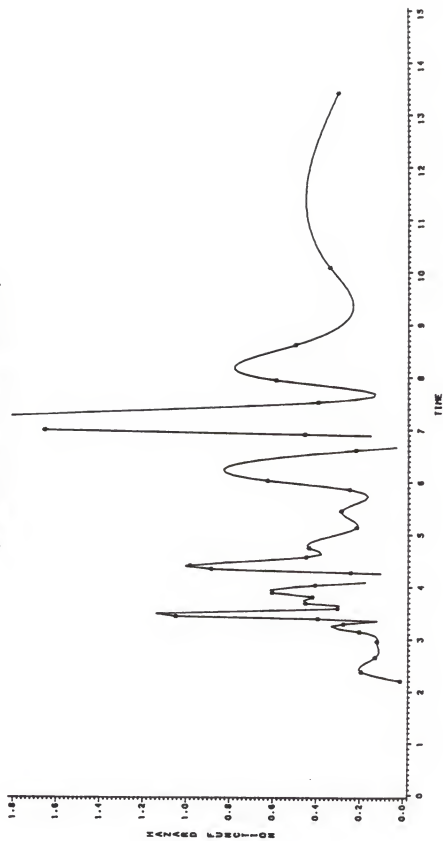(FIXED GROUP SIZE, NR=150)

FIGURE 4.4 b

HAZARD FUNCTION FOR
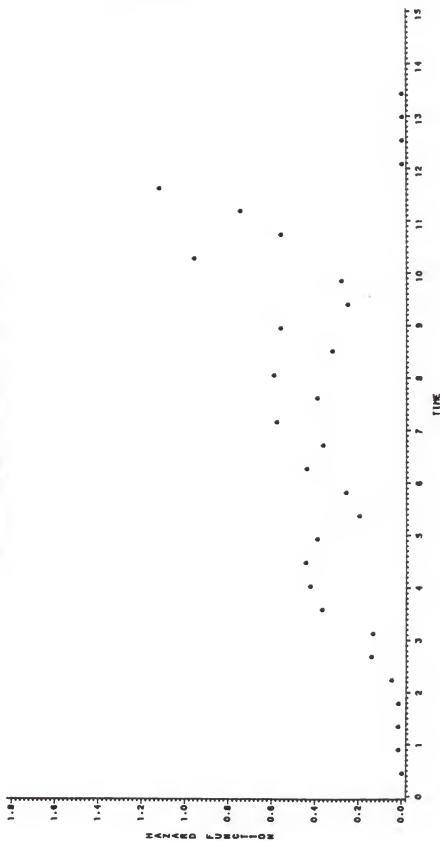GAMMA DISTRIBUTION
(FIXED TIME INTERVAL, NR=150)

FIGURE 4.5 a

HAZARD FUNCTION FOR
GAMMA DISTRIBUTION
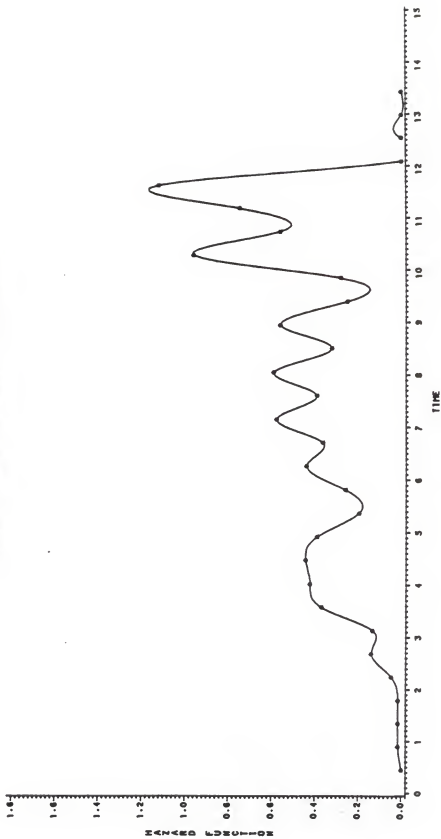(FIXED TIME INTERVAL, NR=150)

FIGURE 4.5 b

HAZARD FUNCTION FOR
GAMMA DISTRIBUTION
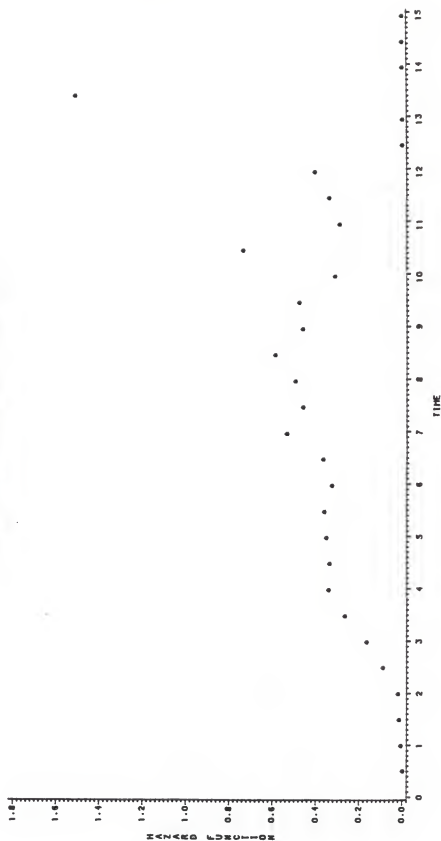(FIXED TIME INTERVAL, NR=300)

FIGURE 4.6 a

HAZARD FUNCTION FOR
GAMMA DISTRIBUTION
(FIXED TIME INTERVAL, NR=300)
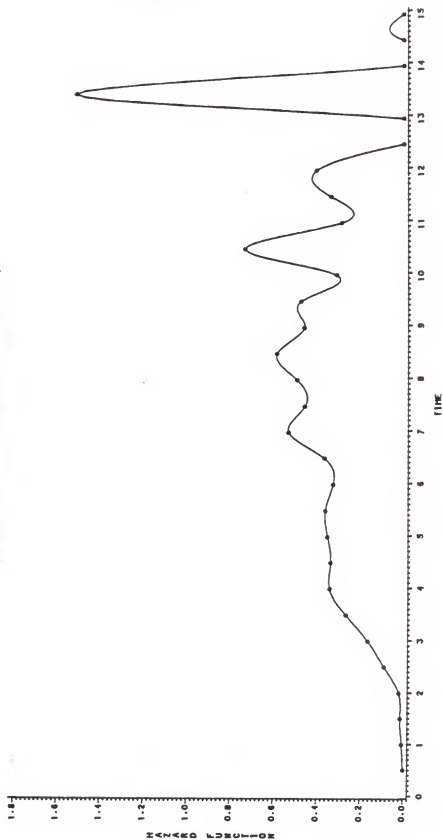
FIGURE 4.6 b

4.3 <u>The Weibull Distribution</u>

In order to show the hazard function for the Weibull distribution, three similar figures to those which were used previously are depicted. The Weibull distribution under study has shape parameter of four and scale parameter of one.

In Figure 4.7 (Case 1), it is observed that the response curve is in agreement with the prediction of the theory (Figure 2.2). Similarly, in Figures 4.8 and 4.9 (Case 2), the hazard function plot appears to look like its theoretical curve.

HAZARD FUNCTION FOR
WEIBULL DISTRIBUTION
(FIXED GROUP SIZE, NR=150)

FIGURE 4.7 a)

HAZARD FUNCTION FOR
WEIBULL DISTRIBUTION
(FIXED GROUP SIZE, NR=150)

FIGURE 4.7 b)

HAZARD FUNCTION FOR
WEIBULL DISTRIBUTION
(FIXED TIME INTERVAL, NR=150)

FIGURE 4.8 a

HAZARD FUNCTION FOR
WEIBULL DISTRIBUTION
(FIXED TIME INTERVAL, NR=150)

FIGURE 4.8 b

HAZARD FUNCTION FOR
WEIBULL DISTRIBUTION
(FIXED TIME INTERVAL, NR=300)

FIGURE 4.9 a

HAZARD FUNCTION FOR
WEIBULL DISTRIBUTION
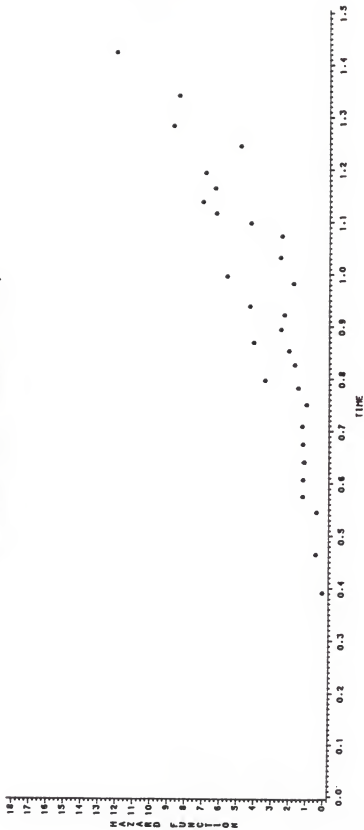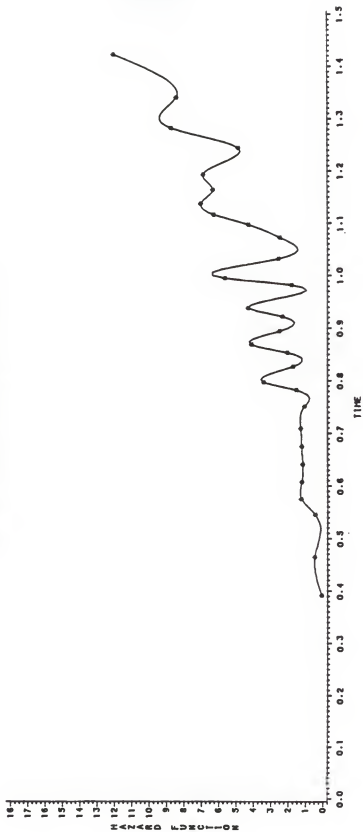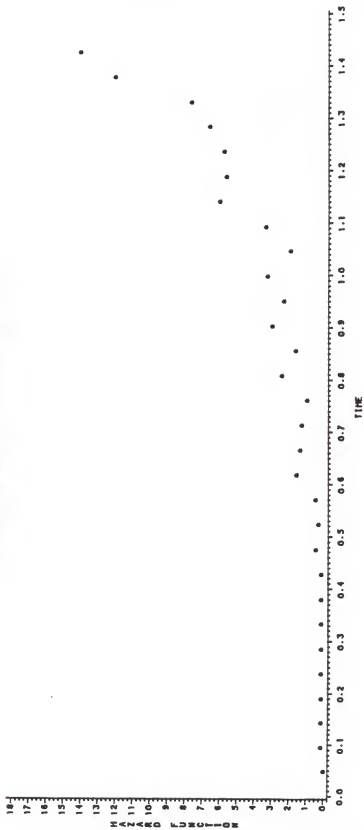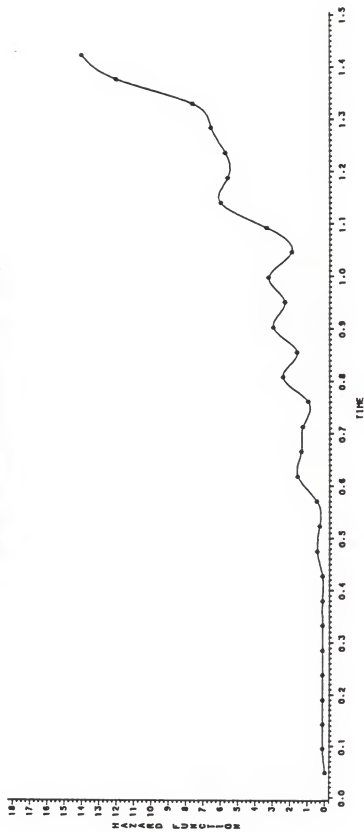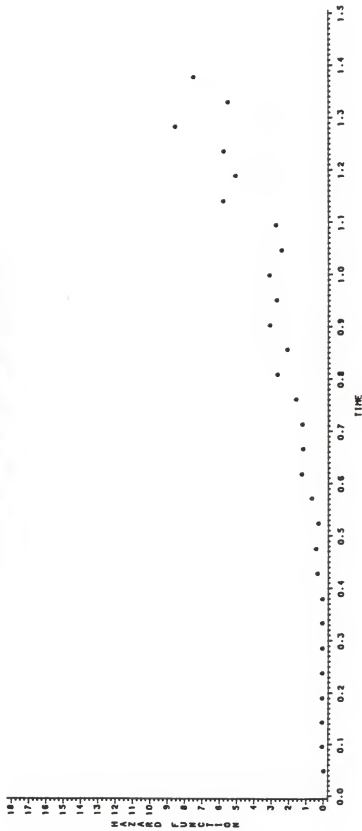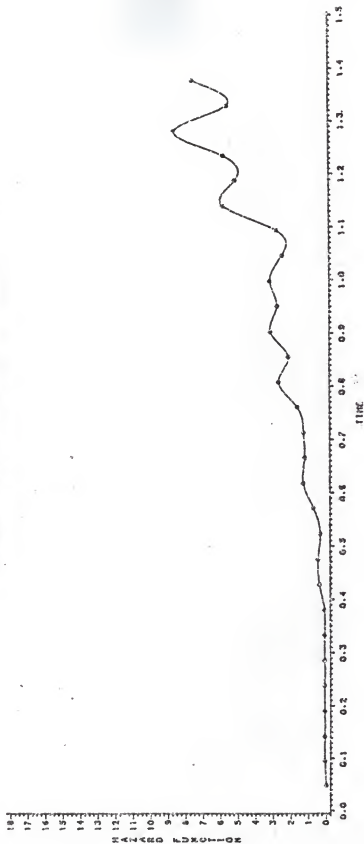(FIXED TIME INTERVAL, NP=300)

FIGURE 4.9 b

## V  SUMMARY AND CONCLUSIONS

This report attempted to investigate the use of the hazard function in distinguishing between various failure models, namely the exponential, gamma and Weibull distributions. In order to achieve this goal, a Fortran language program was written using an IMSL random variable generator. This program computed the empirical hazard function for an assumed failure rate distribution function with known parameters (see Appendix A). The computed hazard function was then plotted for grouped failure times by using a SAS program.

In determining the hazard function, two cases of grouping were considered. In the first case the size of grouped failure times were fixed, as compared to the second case which the interval Δt was kept at constant.

The figures presented in chapter IV (empirical hazard function) and in chapter II (theoretical hazard function) were compared. Only in the case of the Weibull distribution, the empirical hazard function plot appeared to closely follow its theoritical curve. Otherwise, it was concluded that the resulting hazard function plots were relatively useless for deciding which model to select. It was also observed that the increase in the size of grouped failure times had little effect upon reproducing the hazard function accurately.

The above results of this study show that the empirical hazard function in general is not a useful technique to distinguish between various failure models.

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APPENDIX A

A Sample Program For The Exponential Distribution (Case 1)

```fortran
//FORTRAN  JOB (  SSN   , ACOUNT# ), AMIR,TIME=(0,15)
/*REGION      500K
//  EXEC      FORTVCLG,PARM='NOMAP'
    INTEGER NR, NS, N
C         NR is the sample size.
C         NS is the number of samples (plots) to be generated.
C         N  is the group size.

    REAL XM, R(300), H(300), T(300)
C         XM is the mean of the exponential distribution and is the
C            reciprocal of the failure rate .
C         R(.) is the output vector of random exponential values from
C            the IMSL generator. After sorting, it is also the vector
C            of ordered failure times.
C         H(.) is the vector of Hazard function values corresponding
C            to the times given by R(.).

    DOUBLE PRECISION DSEED
    DSEED = 123457.D0
    XM = 100.
    NR = 100
    NS = 1
    N  = 5
    DO 200 M =1, NS
        CALL GGEXN (DSEED, XM, NR, R)
C           This is the invocation of the random number generator.

        CALL VSRTA (R,NR)
C           This is the IMSL sort procedure which orders the failure C
    times.

  H(1) = N / (R(N) * NR)                                          C
      Now the hazard function values are calculated.
C           the value for H(1) must be done separately because
C           Fortran does not like zero as an index. All the other
C           H values are calculated inside the following DO-LOOP.

        T(1) = R(N)
        MM   = N + 1
        J    = 2
        DO 100 I= MM, NR, N
          H(J) = N / ((N(R)-I+1) * (R(I+N-1)-R(I-1)))
          T(J) = R(I+N-1)
          J    = J + 1
100     CONTINUE
```

```
        NR2 = NR / N
        DO 120 J= 1, NR2
C           Now the data for a sample is written to disk (unit 10)
            WRITE (10,99)  J, T(J), H(J), NR2
 99         FORMAT  (5X, I5, 5X, F10.3, 5X, F10.5, 5X, I5)
120      CONTINUE
200   CONTINUE
      STOP
      END


C           THE NEXT FEW JCL CARDS ARE THE JCL NEEDED TO READ THE
C           FORTRAN GENERATED DATA SETS OFF DISK AND ALLOW THEM TO
C           BE USED AS DATA IN THE SAS PROGRAM WHICH FOLLOWS.

/*
//LKED.SYSLIB    DD
//               DD
//               DD
//        DD DSN=SYS1.IMSL.SPFLIB,DISP=SHR
//GO.FT10F001 DD DSAN=&&TEMP,DISP=(NEW,PASS),
//    UNIT=SYSDA,SPACE=(100,(80,10)),
//    DCB=(RECFM=FB,LRECL=80,BLKSIZE=6160)
//GO.SYSIN       DD *
/*
// EXEC     SAS
//DATASET     DD  DSN=&&TEMP,DISP=(OLD,DELETE)

DATA BIGLIST;
INFILE DATASET;
INPUT I 6-10    T 16-25   H 31-40   NR 46-50;
DATA SAMPLE1;
SET BIGLIST;
IF _N_ <= NR THEN OUTPUT SAMPLE1;
PROC PLOT DATA=SAMPLE1;
TITLE HAZARD FUNCTION FOR EXPONENTIAL DISTRIBUTION.;
PLOT H*T='*';
/*
```

A Sample Program For The Weibull Distribution (Case 2)

```
      INTEGER NR, NS, N
C          NR is the sample size.
C          NS is the number of samples (plots) to be generated.
C          N  is the group size.

      REAL ALPHA, BETA, R(300), H(300)
C          ALPHA is the scale factor.
C          BETA  is the shape parameter.
C          R(.) is the output vector of random exponential values from
C             the IMSL generator. After sorting, it is also the vector
C             of ordered failure times.
C          H(.) is the vector of Hazard function values corresponding
C             to the times given by R(.).

      DOUBLE PRECISION DSEED
      DSEED = 123457.D0
      ALPHA = 1.0
      BETA  = 4.0
      NR = 300
      NS = 1
      N  = NR
      DO 200 M =1, NS
         CALL GGWIB (DSEED, BETA, NR, R)
C             This is the invocation of the random number generator.

         CONST = -1.0/ BETA
         COEF = ALPHA ** CONST
         DO 3 I=1, NR
           R(I) = COEF * R(I)
3        CONTINUE
      CALL VSRTA (R,NR)
C          This is the IMSL sort procedure which orders the failure
C          times.

         DELTA = R(NR)/ 30.
         T = DELTA
         C = 0.0
         J = 0.0
         DO 300 I=1, NR
1000        IF (R(I) .LE. T) THEN
                C = C+1
                GO TO 300
             ELSE
                J = J+1
                H(J) = C/ (DELTA * N)
                R(J) = T
                T = T + DELTA
```

```
              N = N-C
              C = 0.0
              GO TO 1000
          ENDIF
300       CONTINUE
          NR2 = J
          DO 120 J= 1, NR2
C             Now the data for a sample is written to disk (unit 10)
              WRITE (10,99)  J, R(J), H(J), NR2
 99           FORMAT  (5X, I5, 5X, F10.3, 5X, F10.5, 5X, I5)
120       CONTINUE
200   CONTINUE
      STOP
      END

C         THE NEXT FEW JCL CARDS ARE THE JCL NEEDED TO READ THE
C         FORTRAN GENERATED DATA SETS OFF DISK AND ALLOW THEM TO
C         BE USED AS DATA IN THE SAS PROGRAM WHICH FOLLOWS.

/*
//LKED.SYSLIB   DD
//             DD
//             DD
//     DD DSN=SYS1.IMSL.SPFLIB,DISP=SHR
//GO.FT10F001 DD DSAN=&&TEMP,DISP=(NEW,PASS),
//    UNIT=SYSDA,SPACE=(100,(80,10)),
//    DCB=(RECFM=FB,LRECL=80,BLKSIZE=6160)
//GO.SYSIN      DD *
/*
// EXEC     SASGRAPH
//DATASET     DD  DSN=&&TEMP,DISP=(OLD,DELETE)

DATA BIGLIST;
INFILE DATASET;
INPUT I 6-10   T 16-25   H 31-40   NR 46-50;
DATA SAMPLE1;
SET BIGLIST;
IF _N_ <= NR THEN OUTPUT SAMPLE1;
GOPTIONS VSIZE=7 HSIZE=10;
C       This statement gives SAS/GRAPH information about the graphics
C       devise its using; what default colors, type fonts, and type
C       sizes to use; and other details about the pictures you want
C       to produce.

PROC GPLOT DATA=SAMPLE1;
C       The GPLOT procedure is the SAS/GRAPH version of the PLOT
C       procedure.

LABEL T='TIME';
LABEL H='HAZARD FUNCTION';
```

```
TITLE1 .F=TRIPLEX .H=3 HAZARD FUNCTION FOR;
TITLE2 .F=TRIPLEX .H=3 WEIBULL DISTRIBUTION;
TITLE3 .F=TRIPLEX .H=2 (FIXED TIME INTERVAL, NR=300);
FOOTNOTE .H=2 .F=TRIPLEX FIGURE 4.9 b;
PLOT H*T / HAXIS=0 TO 1.5 BY .1
           VAXIS=0 TO 18 BY 1;
SYMBOL1 V=STAR I=SPLINE;
/*
```

## ACKNOWLEDGMENT

USING EMPIRICAL HAZARD FUNCTION IN
DISTINGUISHING FAILURE MODELS


by


SAYED AMIR FARTAJ


B.S., Kansas State University, 1980
M.S., Kansas State University, 1983


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AN ABSTRACT  OF A REPORT


Submitted  in partial fulfillment of the
requirements for the degree


MASTER OF SCIENCE


Department of Statistics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1987

ABSTRACT

This research project specifically addressed the usefulness of the hazard function in distinguishing between various failure models such as the exponential, gamma and Weibull distributions. In order to acheive this goal, a Fortran language program was written to simulate sets of failure data from three assumed failure rate distribution functions. The computed hazard function from Fortran program was then plotted for grouped failure times by using a S.A.S program. Two cases of grouping were considered. In the first case the size of grouped failure times were fixed, as compare to the second case which the interval $\Delta t$ was kept at constant.

The failure data  are presented graphically, indicate the use of the empirical hazard function  in distinguishing between the three failure models.