A STUDY OF THE DISAPPOINTMENT MODEL IN DECISION MAKING
UNDER UNCERTAINTY

by

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INTRODUCTION

1.1 PURPOSE OF THIS STUDY

The techniques that the decision maker uses to reach a
decision are intuitive judgement and mathematic computation.
To make decisions based on a human being's intuitive
judgement is the most practiced decision process,
especially in the highest management level. However, making
decisions based on intuitive judgement can be dangerous and
unreliable since real world problems are usually out of any
individual's comprehension.

We may observe that decision makers often agree with
the logical procedure of decision analysis but still feel
uncomfortable at an intuitive level with its implications.
Because human beings fear making the wrong decision,
therefore, some people find decision making under uncertain-
ty difficult. Disappointment has important implications for
the study of decision making under uncertainty. Although
the axioms of Von Neumann and Morgenstern (19) are the
cornerstones of decision analysis, they can not be expected
to hold if preference has not been calculated over all
attributes of interest to the decision maker. A disappoint-
ment model built by David E. Bell (4) can explain the

implications of psychological reaction — disappointment — in decision-making situations, and can provide a preference model for decision makers to reach a rational decision.

1.2 LITERATURE SURVEY

Some general references to decision theory at a level roughly comparable to this report are: Drucker, P. (1956, Section 2.3); Greenwood, W. T. (1965 and 1969, Section 2.2); Hamburg, Morris (1977, Sections 2.4 and 2.5); Jedamus, P. and Frame (1969, Section 2.3); Wasserman, P. and Silander (1964, Section 2.2); and White, J. A., Agee and Case (1984, Sections 2.4 and 2.5). Other general references of interest include Fishburn, Peter C. (1970); Keeney, Ralph L.(1982); Lindley, D. V. (1973); Morris, William T. (1968); and White, D. J. (1970).

With regard to utility theory, some key historical references are Fishburn, Peter C. (1970, Section 2.6); and von Neumann and Morgenstern (1953, Section 2.6.1). The history of utility theory and advanced applications are discussed in Arrow, K.J. (1951); Kahneman, D. and A. Tversky (1979); and Savage, L. J. (1954). These references deal at least in part with the axiomatic development and application of utility.

Some other aspects of utility theory include risk aversion, which is discussed in Arrow, K. J. (1971); Crouch,

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Edmund A. C. and Wilson (1982); Dyer, J. S. and Sarin (1982); Kahneman, D. and A. Tversky (1979); Pratt, John W. (1964); Ross, Stephen A. (1981); and Stone, B. K. (1973).

For the study of this report — the disappointment reaction in decision making under uncertainty — the primary references used are Allais, Maurice (1953); Bell, David E. (1985); Kahneman, D. and A. Tversky (1979); and von Neumann and Morgenstern (1953)

1.3 REPORT ORGANIZATION

A disappointment model built by David Bell is introduced in this study. It is not well known and deserves wider readership. Chapter 1 gives the purpose of this study and the references used in this study. Chapter 2 briefly explains the definition and history of decision making, and reviews the methodology and procedure of decision analysis. Chapter 3 represents this disappointment model and investigates the implications of psychological reactions — disappointment and elation — in decision situations. Finally the conclusions are given in chapter 4.

CHAPTER II

DECISION MAKING UNDER UNCERTAINTY

2.1 DEFINITION OF DECISION MAKING

The decision making process involves getting the facts
about a problem, weighing them against specified criteria,
and then deciding which of several alternatives to select.

Decisions play an important role in our everyday lives,
thus life is a constant sequence of decision-making
situations. Every action we take, with the exception of a
few involuntary physiological actions, such as breathing,
can be thought of as a decision. Of course, most of these
decisions are quite minor because the consequences involved
are not very important. However, some involve millions of
dollars or even life and death. Indeed, decision making may
constitute one of the highest forms of human activities.

2.2 HISTORY OF DECISION MAKING

Decision making theories and methods have dominated the
management literature in the past decade. An investigation
by Greenwood (9) mentioned that before 1950, decision making
was not used in management literature and was not given much
importance. Management was more inclined towards human
relations, organization theory and economic analysis, than

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towards decision theory. Later, more emphasis was laid on
business decision making. Greenwood added that decision
making and methods have been developed in attempt to resolve
particular management problems and from the perspective of
particular academic disciplines, especially psychology,
sociology, mathematics, statistics, and logic. That is why
the literature on decision making is scattered and as yet
not properly gathered or integrated.

Between 1945 and 1948, an exhaustive survey was made on
the literature of decision making by Paul Wasserman and Fred
S. Silander (20). The findings were published in a summa-
rized form by Cornell University in 1958 under a McKinsey
Foundation grant entitled Decision Making — An Annotated
Bibliography . The findings revealed that decision making
was used in small groups concerning psychological studies of
individual, group and leadership factors. The idea of
management decision making was originated by psychologists,
mathematicians, and statisticians; its methods being
derived from the fields of mathematics and statistics.

2.3 HOW TO MAKE A DECISION

Peter Drucker (7) said that decisions will always have
to be based on judgement. They will always remain decisions
for a future which will continue to be unpredictable. They
will always entail risks. Nevertheless, a decision maker by

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following fairly simple steps can greatly improve his
performance. There are basically four steps involved in
decision making and they may be enumerated as follows:

1. Defining the problem: What kind of problem have
   we to solve? What is its critical factor? When do
   we have to solve it? What is the cost involved in
   its solution?

2. Defining expectations: What do we want to gain by
   solving it?

3. Developing alternative solutions: Which of several
   plans offers the surest way to avoid unexpected
   outcomes.

4. Knowing what to do with the decision after it is
   reached, i.e. implementation of the decision.

Attention to these rules will help the decision maker
avoid the three most common pitfalls in the making of
decisions. These are:

1. Finding the right answer for the wrong problem-few
   things are as useless.

2. Making the decision at the wrong time.

3. Making decisions that do not result in action.

Paul Jedamus and Robert Frame (12) explained that if
the procedure discussed above is followed step by step, the
decision made will be the best, not with certainty but with
higher probability (confidence).

2.4 STRUCTURE OF THE DECISION-MAKING PROBLEM

The decision problem under study may be represented by a model in terms of the following elements:

1. *The decision maker.* The agent charged with the responsibility for making the decision. The decision maker is viewed as an entity and may be a single individual, a corporation, a government agency, etc.

2. *Alternative courses of action.* The decision involves a selection among two or more alternative courses of action, referred to simply as "acts". The problem is to choose the best of these alternative acts. Sometimes the decision maker's problem is to choose the best of alternative "strategies", where each strategy is a decision rule indicating which act should be taken in response to a specific type of experimental or sample information.

3. *Events.* Occurrences that affect the achievement of the objectives. These are viewed as lying outside the control of the decision maker, who does not know for certain which event will occur. The events constitute a mutually exclusive and complete set of outcomes; hence, one and only one of them can occur. Events are also referred to as "states of nature", or "states of the world".

4. *Payoffs.* A measure of net benefit to be received by the decision maker under particular circumstances. These payoffs are summarized in a payoff table or payoff matrix, which displays the consequences of each action selected and each event that occurs.

5. *Uncertainty.* The indefiniteness concerning which events or states of nature will occur. This uncertainty is indicated in terms of probabilities assigned to events. A matrix decision model with general symbolism, adapted from Morris Hamburg (11), is given in Table 2.1.

The symbolism employed is defined as follows:

A_j = an *alternative* or *strategy* under the decision maker's control, where $j = 1, 2, \ldots, n$.

S_k = a *state of nature* that can occur after alternative A_j is chosen, where $k = 1, 2, \ldots, m$.

θ_{jk} = the *outcome* of choosing alternative A_j and having state S_k occur.

$V(\theta_{jk})$ = the *value* of outcome θ_{jk}, which may be in terms of dollars, time, distance, or utility.

P_k = the *probability* that state S_k will occur. It is assumed that the probability of a particular state occurring does not depend on the alternative chosen by the decision maker.

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	P_1	P_2	$\cdots$	$\cdots$	P_k	$\cdots$	$\cdots$	P_m
$\lambda_j \diagdown^{S_k}$	S_1	S_2	$\cdots$	$\cdots$	S_k	$\cdots$	$\cdots$	S_m
λ_1	$V(\theta_{11})$	$V(\theta_{12})$	$\cdots$	$\cdots$	$V(\theta_{1k})$	$\cdots$	$\cdots$	$V(\theta_{1m})$
λ_2	$V(\theta_{21})$	$V(\theta_{22})$	$\cdots$	$\cdots$	$V(\theta_{2k})$	$\cdots$	$\cdots$	$V(\theta_{2m})$
$\vdots$	$\vdots$	$\vdots$			$\vdots$			$\vdots$
$\vdots$	$\vdots$	$\vdots$			$\vdots$			$\vdots$
λ_j	$V(\theta_{j1})$	$V(\theta_{j2})$	$\cdots$	$\cdots$	$V(\theta_{jk})$	$\cdots$	$\cdots$	$V(\theta_{jm})$
$\vdots$	$\vdots$	$\vdots$			$\vdots$			$\vdots$
$\vdots$	$\vdots$	$\vdots$			$\vdots$			$\vdots$
λ_n	$V(\theta_{n1})$	$V(\theta_{n2})$	$\cdots$	$\cdots$	$V(\theta_{nk})$	$\cdots$	$\cdots$	$V(\theta_{nm})$

Table 2.1. A matrix decision model with general symbolism.

2.5 DIFFERENT SITUATIONS UNDER WHICH ONE HAS TO DECIDE

There are three situations under which one has to decide, as explained by Archer (2):

1. Decision under certainty

2. Decision under risk

3. Decision under uncertainty

2.5.1 Decision Under Certainty

It is reasonable to assume in many decision situations that only one state of nature is relevant. Then, the decision maker assumes this single state will occur with certainty, i.e. with probability = 1.0. This kind of case is termed a decision under assumed certainty.

In terms of the matrix decision model, a decision under assumed certainty would appear in Table 2.2.

A_j $\backslash$ S_k	1.0 S
A_1	$V(\theta_1)$
A_2	$V(\theta_2)$
$\vdots$	$\vdots$
A_j	$V(\theta_j)$
$\vdots$	$\vdots$
A_n	$V(\theta_n)$

Table 2.2. The matrix model for a decision under certainty

In this situation, the payoffs resulting from the selection of a particular strategy is known. It is assumed

that the payoffs resulting from the decision can be precisely measured; in other words, only one state of nature is assumed to exist. Prediction is involved, based on assumed outcomes. The assumption of certainty simplifies the decision but ignores variations in condition which often exist, leading to improper decisions.

Example :

A man wants to invest one thousand dollars for three years. From the present trend of market interest rate, he can choose either of two alternatives:

1. Invest $ 1000 at 5% compounded annually for three years, or

2. Invest $ 1000 at 5.5% compounded annually for two years and for the third year at 4% compounded annually.

The criterion for selection of a particular alternative is to maximize the interest earned. The solution to the above problem according to this criterion is as follows:

<u>Alternative 1 :</u>

F = Future value of the deposit after n interest periods
$$= P(1+i)^n$$

where,

P = present amount

i = interest rate per period

n = number of interest period

F = a future sum of money

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The future value of the deposit after three years using this relation is:

$$F_1 = 1000(1+0.05)^3$$
$$= 1000(1.05)^3$$
$$= 1000(1.168)$$
$$= \$ 1168$$

Alternative 2 :

The future value of the deposit after two years is:

$$F_2 = 1000(1+0.055)^2$$
$$= 1000(1.055)^2$$
$$= 1000(1.113)$$
$$= \$ 1113$$

For the third year, he has:

$$P = \$ 1113$$
$$i = 4\%$$
$$n = 1$$

The future value of deposit after the third year is:

$$F_3 = 1113(1+0.04)^1$$
$$= 1113(1.04)$$
$$= \$ 1157.52$$

From these calculations the first alternative will be selected, since the future value of deposit after three years is greater than from the second alternative.

2.5.2 Decision Under Risk

A decision situation is called a decision under risk
when the decision maker elects to consider several states
and the probabilities of their occurrence are explicitly
stated. In some decision problems, the probability values
may be objectively known from historical records or objec-
tively determined from analytical calculations.

In this case, the decision maker must review the payoff
matrix resulting from the various states of nature, along
with their probabilities of occurrence. In order to arrive
at a decision, the payoff is weighed by the associated prob-
ability. The expected value of a strategy is the sum of the
payoffs, each multiplied by (i.e. weighted by) its respec-
tive probability of occurrence. The appropriate decision is
to select the strategy with optimum expected value (largest,
for maximization of the payoff unit). The matrix model for
a decision under risk is as same as Table 2.1.

Example :

The payoffs mentioned below are profits. The criterion
for selection of a strategy is to maximize the profit.
There are three states of nature which occur with probabili-
ties (0.25, 0.5, 0.25) as shown in Table 2.3.
The strategies represent different inventory levels, i.e.

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      Strategies:     Inventory levels
   A₁ ----- A₂ ----- A₃ ----- A₄ ----- A₅
  (200)   (250)   (300)   (350)   (400)
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A_j \ S_k	0.25	0.5	0.25	Expected
	S_1	S_2	S_3	Values
A_1	100	100	50	87.5
A_2	90	120	100	107.5
A_3	70	120	140	105.0
A_4	40	90	190	102.5
A_5	0	50	160	65.0

Table 2.3. Payoff (profit) matrix

For a particular strategy, the profit is different for the several states of nature as shown above. As mentioned in the beginning, the profit is to be maximized, therefore, a strategy with maximum average profit will be chosen.

The optimal strategy is therefore the stocking of 250 units, for the expected value (the average profit from such a decision in the long run) is higher, 107.5, than for any other stategy, as summarized in the right portion of Table 2.3.

2.5.3 Decision Under Uncertainty

A decision situation where several states are possible and sufficient information is not available to assign proba-

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bility values to their occurrence is termed a decision under uncertainty. In this case, the possible criteria for selecting the optimum strategy are:

1. *The Laplace Criterion:* If one can not assign probabilities to the states, the states should be considered as equally probable.

Example :

Applying this criterion to the previous example of Table 2.3, the probability value assigned to each of the three states is 1/3. Then,

$$E(A_1) = 100(1/3) + 100(1/3) + 50(1/3)$$
$$= 83.33$$
$$E(A_2) = 90(1/3) + 120(1/3) + 100(1/3)$$
$$= 103.33$$
$$E(A_3) = 70(1/3) + 120(1/3) + 140(1/3)$$
$$= 110.00$$
$$E(A_4) = 40(1/3) + 90(1/3) + 190(1/3)$$
$$= 106.67$$
$$E(A_5) = 0(1/3) + 50(1/3) + 160(1/3)$$
$$= 70.00$$

and A_3 would be chosen, that is, the stocking of 300 units.

2. *The Maximin Criterion:* The matrix model is expressed in terms of profit. The decision maker regards nature as an antagonist and expects the worst possible

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outcome (the smallest profit). He therefore selects the strategy that will yield the greatest minimum profit.

Example :

Applying this criterion to the previous example of Table 2.3, thus, alternative A_2 with 90 maximin value is selected as the alternative that will maximize the minimum present worth value that could occur.

3. *The Minimax Criterion:* The matrix model is expressed in terms of loss. The decision maker expects the worst possible outcome (the greatest loss), and selects the strategy that will yield the smallest loss. Both criteria 2 and 3 are the most conservative (pessimistic) decision rules.

Example :

Applying this criterion to the previous example of Table 2.3, thus, alternative A_1 with 100 minimax value is selected as the alternative that will minimize the maximum present worth value that could occur.

4. *The Maximax Criterion:* The matrix model is usually expressed in terms of profit. In this case, the decision maker therefore selects the strategy with the highest possible payoff.

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Example :

Applying this criterion to the previous example of Table 2.3, thus, alternative A_4 with 190 maximax value is selected as the alternative that will maximize the maximum present worth value that could occur.

5. *The Minimin Criterion:* The matrix model is expressed in terms of loss. In this case, an optimistic philosophy of choice is to select the strategy that affords the opportunity to obtain the smallest loss. Both criteria 4 and 5 are optimistic decision rules.

Example :

Applying this criterion to the previous example of Table 2.3, thus, alternative A_5 with 0 minimin value is selected as the alternative that will minimize the minimum present worth value that could occur.

6. *Hurwicz Criterion:* In this case, the decision maker uses the weighted average of the minimum and the maximum payoffs to select the best strategy. Weights are designed to reflect the decision maker's subjective degree of pessimism. The weight given to minimum payoff is chosen arbitrarily by decision maker.

Example :

Suppose the decision maker concerned with the example

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problem of Table 2.3 was a middle-of-the-road type of person and assigns $\alpha = 0.5$. Then,

H_1 for A_1 is $100(0.5) + 50(0.5) = 75$

H_2 for A_2 is $120(0.5) + 90(0.5) = 105$

H_3 for A_3 is $140(0.5) + 70(0.5) = 105$

H_4 for A_4 is $190(0.5) + 40(0.5) = 115$

H_5 for A_5 is $160(0.5) + 0(0.5) = 80$

Choosing the maximum of these values is to select alternative A_4 with Hurwicz value 115.

7. *Savage Criterion (Minimax Regret):* This criterion, proposed by L. J. Savage, introduces and defines a quantity termed regret. A matrix consisting of regret values is first developed. Then the maximum regret value for each alternative A_j is determined, and the alternative associated with the minimum regret value is chosen from the set of maximum regret values.

Example :

Applying the Savage criterion to the example of Table 2.3, the regret matrix given in Table 2.4 is obtained. The maximum regret values are 140, 90, 50, 60 and 100 for alternatives A_1, A_2, A_3, A_4 and A_5, respectively. Thus, the minimum of these values is 50, and alternative A_3 would be preferred.

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A_j \ S_k	S_1	S_2	S_3	Maximum regret value
A_1	0	20	140	140
A_2	10	0	90	90
A_3	30	0	50	50
A_4	60	30	0	60
A_5	100	70	30	100

Table 2.4. Regret matrix for the minimax regret example

2.6 DECISION MAKING BASED ON EXPECTED UTILITY

In the decision analysis discussed up to this point, the criteria of choice were the maximization of expected monetary value. This criterion can be interpreted as a test of preferredness that selects as the optimal act the one that yields the greatest long-run average profit. That is, in a decision problem, the optimal act is the one that would result in the largest long-run average profit if the same decision had to be made repeatedly under identical environmental conditions. In general, in such decision-making situations, as the number of repetitions becomes large, the observed average payoff approaches the theoretical expected

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payoff. However, many of the most important personal and business decisions are made under unique sets of conditions, and in some of these occasions it may not be realistic to think in terms of many repetitions of the same decision situation.

In all of the foregoing discussion the payoffs and losses have been expressed in monetary terms. This is not always the case; it is easy to think of examples in which the consequences of a decision are nonmonetary. The consequences may involve quantities of a good or a service. If the good or the service has a known monetary value, then the payoffs can be expressed in dollars, pounds, francs, or whatever. Otherwise, the decision maker has to build another criterion for nonmonetary decision situations to express and to select the best alternative.

It is reasonable to depart sometimes from the criterion of maximizing expected monetary values in making decisions. One's decisions will clearly depend upon one's attitude toward risk, which in turn will depend on a combination of factors such as one's level of assets, liking or distaste for gambling, and psycho-emotional constitution.

To recapitulate, Morris Hamburg (11) summarized the problem concerning decision making in problems involving payoffs that depend upon risky outcomes. Monetary payoffs are sometimes inappropriate as a measuring device, and it

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appears appropriate to substitute some other set of values or "numeraire", which reflects the decision maker's attitude toward risk. A clever approach to this problem — the maximization of expected utility value — has been furnished by Von Neumann and Morgenstern, who developed the so-called Von Neumann and Morgenstern utility measure. The concept of this approach is a focus for this report.

Essentially, the theory of utility makes it possible to measure the relative value to a decision maker of the payoffs, or consequences, in a decision problem. In a general sense, the payoff represents the consequence to the decision maker of taking a particular action and having a particular state of nature occur. This includes all aspects of the consequences, monetary or otherwise.

2.6.1 Axioms Of Utility

Basically, what is needed is an objective function which aggregates all the individual objectives and an attitude toward risk. In decision analysis, such an objective function is referred to as a utility function. Using the axioms of utility, it is possible for an individual to assess a utility function.

Formally, a utility function U can be interpreted in terms of a preference relationship; thus $u(X)$, the utility of the consequence X, indicates the desirability of X

relative to all other consequences. The four basic axioms
of utility are as follows, assuming X is measured in dollars
or other tangible goods:

1. If payoff X_1 is preferred to payoff X_2, then
   $U(X_1) > U(X_2)$;
   if X_2 is preferred to X_2, then
   $U(X_2) < U(X_2)$;
   and if neither is preferred to the other, then
   $U(X_1) = U(X_2)$.

2. If you are indifferent between (a) receiving payoff
   X_1 for certain and (b) taking a bet or lottery in
   which you receive payoff X_2 with probability p and
   payoff X_3 with probability $1-p$, then
   $U(X_1) = pU(X_2) + (1-p)U(X_3)$

3. If an individual prefers X_1 to X_2 and X_2 to X_3, he
   will also prefers X_1 to X_3. This is referred to as
   the principle of transitivity. It extends also to
   indifference relationships.

4. If a payoff or consequence of an act is replaced by
   another, and one is indifferent between the former
   and new consequences, then one should also be
   indifferent between the old and new acts. This is

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often referred to as the principle of substitution.

5. The utility function is bounded. This means that utility can not increase or decrease without limit. As a practical matter, this simply means that the range of possible monetary values is limited. For example, at the lower end the range may be limited by a bankruptcy condition.

It is important to note that a utility function is not unique, even for a specific individual, and in any event a person's utility function will not necessarily remain the same over time. If, for a particular person, a function U satisfies the above axioms, then the function $W = c + dU$ also satisfies the axioms, where c and d are constants with d greater than 0. In words, it is said that a utility function is only unique up to a positive linear transformation.

Following directly from the foregoing axioms of utility function, alternatives with higher expected utilities should be preferred to those with lower expected utilities.

It should be emphasized that the development and discussion of utility theory presented in this report are only a brief, rough development and discussion of the most important points of the theory of utility, although it will suffice for the purpose of this report.

2.6.2 Risk In Decision-Making Problems

The importance of risk to decision making is attested by its position in decision theory (Allais 1953; Arrow 1965), by its standing in managerial ideology (Peters and Waterman 1982), and by the burgeoning interest in risk assessment and management (Crouch and Wilson 1982).

Studies of utility commonly make hypotheses about properties of the utility function that should be hold for "most people". These studies generally assume that people are risk averse in monetary gambles and that the extent of their risk aversion (15) decreases as they become wealthier (16).

Early treatments by Pratt (1964), Arrow (1965) and others, as well as more recent work (Ross 1981), assumed that individual human decision makers are risk averse, that is, that when faced with one alternative having a given outcome with certainty, and a second alternative which is a gamble but has the same expected value as the first, an individual will choose the certain outcome rather than the gamble.

Suppose you are offered the following choice: you receive three oranges for sure or receive a lottery in which you get eight oranges with a 0.5 chance and zero oranges with a 0.5 chance. Futher suppose you prefer more oranges to fewer oranges in the range of zero to eight oranges. If

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you are indifferent between these two options, (three
oranges for sure versus the lottery) then you would be
classified as a risk averse individual according to the
Pratt-Arrow definition of risk aversion. Thus, the economic
explanation of risk aversion is that each additional dollar
is worth slightly less due to satiation (decreasing marginal
value). Imagine two individuals with equal wealth and
identical tastes for consumables. One is timid, nervous,
and full of self-doubt; the other is outgoing,
self-confident, and with a sense of purpose. We might
suppose that the latter will be less risk averse than the
former. Indeed, his relative risk attitude, in the sense of
Dyer and Sarin (1982)(8), may even be risk prone.

2.6.3 Characteristics And Types Of Utility Function

Several types, or classes, of utility functions can be
distinguished, although there are utility functions not
falling into any of the classes to be described. The
utility functions depicted in Figures 2.1, 2.2 and 2.3 rise
consistently from the lower left to the upper right side of
the chart. That is, the utility curves have positive slopes
throughout their extent. This is a general characteristic
of utility functions; it simply implies that people
ordinarily attach greater utility to a larger sum of money
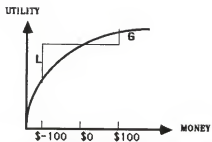than to a smaller sum. Economists have noted this

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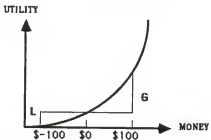Figure 2.1. Utility curve for a "Risk-Avoider"
(Concave Function)

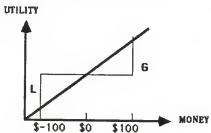Figure 2.2. Utility curve for a "Risk-Taker"
(Convex Function)

Figure 2.3. Utility curve for an individual who is
neutral to risk (Linear Function)

psychological trait in traditional demand theory and have referred to it as a "positive marginal utility for money".

The curve shown in Figure 2.1 illustrates the utility curve of an individual who has a diminishing marginal utility for money, although the marginal utility is always positive. Mathematically, this type of utility function is called a concave function and is characteristic of a "risk-avoider". A person characterized by such a utility curve would prefer a small but certain monetary gain to a gamble whose expected monetary value is greater but may involve a large but unlikely gain, or a large and not unlikely loss.

In Figure 2.2 is shown the utility curve for a "risk-taker". This type of person willingly accepts gambles that have a smaller expected monetary value than an alternative payoff received with certainty. For such an individual, the attractiveness of a possible the gamble tends to outweigh the fact that the probability of such a payoff may indeed by very small. Mathematically, this type of function is called convex function.

The linear function shown in Figure 2.3 depicts the behavior of a person who is "neutral" or "indifferent" to risk. For such a person every increase of, say, $1,000 has an associated constant increase in utility and thus he is neither a risk-avoider nor a risk-taker. This type of

individual would use the criterion of maximizing expected monetary value in decision-making problems, because this would also maximize expected utility.

To see why these terms, "risk-avoider", "risk-taker" and "risk neutral", aptly describe the curves, consider the following bet:
you win $100 with probability 0.5 and you lose $100 with probability 0.5. This can be thought of as a bet of $100 on the toss of a fair coin. In terms of expected payoff, you should be indifferent about the bet since it has an expected payoff of zero. In terms of expected utility, however, the decision as to whether or not to take the bet depends on the shape of your utility function.

In Figure 2.1 through 2.3, the gain denoted by G in utility if the bet is won is

$$G = U(100) - U(0)$$

and the loss denoted by L in utility if the bet is lost is

$$L = U(0) - U(-100)$$

The expected utility denoted by EU of the bet is

$$EU(\text{bet}) = 0.5U(100) + 0.5U(-100)$$

The alternative action is not to bet, and the expected utility of this is just

$$EU(\text{not bet}) = U(0)$$

Under what circumstances would you take the bet ? Using the expected utility rules, you would take the bet if

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$$EU(bet) > EU(\text{not bet});$$

that is, if

$$EU(\text{bet}) - EU(\text{not bet}) > 0.$$

But from the above equations,

$EU(\text{bet}) - EU(\text{not bet})$

$= 0.5U(100) + 0.5U(-100) - U(0)$

The right-hand side of this equation can be written as

$$0.5U(100) + 0.5U(-100) - 0.5U(0) - 0.5U(0)$$

which is equal to

$[0.5U(100) - 0.5U(0)] - [0.5U(0) - 0.5U(-100)]$

$= 0.5G - 0.5L$

$= 0.5(G-L).$

Therefore, the decision rule is as follows:

Take the bet if $0.5(G-L) > 0$.

Do not take the bet if $0.5(G-L) < 0$.

In order to make the decision in this example, you need only look at the sign of $G-L$.

For the curve in Figure 2.1, G is smaller than L, so that $(G-L)$ is negative, and you should not take the bet. Since you will not take a bet with an expected monetary payoff of zero, you are called a "risk-avoider". In fact, with this curve it is possible to find some bets with expected monetary payoffs greater than zero that you would consider unfavorable in terms of expected utility.

In Figure 2.2, G is greater than L, and as a result you

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should take the bet. Furthermore, there are some bets with negative expected monetary payoffs that you would consider to be favorable bets. As a result, this curve represents the utility function of a "risk-taker".

Finally, $G = L$ in Figure 2.3. In this case you are indifferent between taking the bet and not taking it, and thus you are neither a risk-avoider nor a risk-taker. For a person with a linear utility function (that is, the curve is a straight line), maximizing expected utility is equivalent to maximizing expected monetary payoff.

CHAPTER III

THE DISAPPOINTMENT MODEL

3.1 INTRODUCTION

Most financial decision analyses presume that if two
consequences have the same dollar outcomes they will be
equally preferred, implying that the requisite of
decision analyses is that two identically attractive
consequences have to be the same utility values and vice
versa. For most people it is apparent that they will feel
much happier when they win the top prize of $10,000 in a
lottery than when they receive the lowest prize of $10,000
in a lottery. There exists a psychological reaction — dis-
appointment — in such lottery. The satisfaction you feel
with the prize you win in a lottery will directly depend
upon your expectations.

3.2 DISAPPOINTMENT IN DECISION PROBLEM

In order to reward your outstanding performance over
the past year, your boss decides to give you a $5,000 bonus.
If you never expected a bonus, you will be excited to get
it. However, if you expected a $10,000 bonus, you will
naturally be disappointed. The disappointment is a psycho-
logical reaction caused by comparing the actual outcome of a

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lottery to your prior expectations. The higher your expectations, the greater will be your disappointment.

Of course, there are many other "reference effect" situations. Although a $5,000 bonus perhaps exceeds your expectations, it still causes the disappointment to learn that your colleague got a $10,000 bonus. Consequently, it is apparent that the most influential reference point is the status quo of the decision maker.

We recognize that a decision maker will tend to make economic tradeoffs to remove the possibility of disappointment in a transaction. People who are particularly hostile to disappointment may adopt a pessimistic outlook on the future. If you are given a 50-50 lottery between $1,000 and $0, you have a 0.5 chance that you will feel disappointed when the lottery is resolved. Consequently, you may prefer to exchange the lottery for a sure $400; decreasing marginal values can remove the possibility of disappointment. The amount that you are willing to pay to avoid having to take the bet, $100, is a risk premium.

Of course, people who feel that the "thrill of winning" is worth potential enjoyment may adopt the opposite action. Generally speaking, disappointment is a psychological reaction to an outcome that does not match up to the prior expectations. The greater the disparity, the greater the disappointment.

3.3 PARADOXES OF THE SUBSTITUTION PRINCIPLE OF UTILITY THEORY

The substitution principle is key to the derivation of expected utility theory. This principle is used to investigate inconsistency in preference orderings. We will use the examples of Kahneman and Tversky (13)(1979) both to illustrate the violations of the substitution principle of utility theory and to show that a disappointment model provides an explanation for them. These examples are based on Maurice Allais (1). The number of respondents who answered the problems is denoted by N, and is abstracted from the original examples of Kahneman and Tversky. The percentage of subjects who choose each option is shown in brackets. The symbol (x, p) stands for a lottery where the player wins x dollars with probability p and wins nothing with probability $1-p$.

Problem 1 : Choose between

    A : ($4,000, 0.8) and B : ($3,000, 1)   N = 95

      (20% chose this)   (80% chose this)

Problem 2 : Choose between

    C : ($4,000, 0.2) and D : ($3,000, 0.25) N = 95

      (65% chose this)   (35% chose this)

Figure 3.1 and 3.2 show the representation of problem 1 and 2 as decision trees. The symbol, *, in the figures shows the preferred choice of a majority of subjects. The symbol

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square ($\square$) represents a node where the decision maker must decide which branch to choose depending upon the criterion for selection. The symbol circle ($\bigcirc$) represents a chance node, where each branch coming out from the node has an ascertainable probability of occurance.

To summarize:

$\square$ represents an action (of the decision maker)

$\bigcirc$ represents an event (of the state of the nature)

Note that $C = (\$4,000, 0.2)$ can be represented as $(A, 0.2)$ and $D = (\$3,000, 0.25)$ as $(B, 0.25)$. Over half the respondents didn't obey the expected utility theory. In order to explain that the preferences in problem 1 and 2 are not compatible with the theory, we assume that $u(0) = 0$, and the option of B implies

$$u(\$3,000) > (0.8)\,u(\$4,000),$$

whereas the option of C implies

$$(0.2)\,u(\$4,000) > (0.2)\,u(\$3,000)$$

which is the reverse inequality of the option B. The substitution principle of utility theory asserts that if B is preferred to A, then any probability mixture (B, p) must be preferred to the probability mixture (A, p). Evidently, the subjects violate this principle. It is apparent that reducing the probability of winning from 1.0 to 0.25 has a greater influence than reducing it from 0.8 to 0.2.
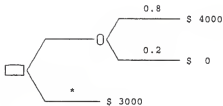
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Figure 3.1.   The representation of problem 1 as a decision
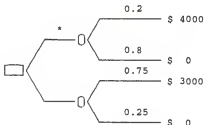   tree

Figure 3.2.   The representation of problem 2 as a decision
              tree

Problem 3 : Consider the following 2-stage game. In the
first stage, the player has a choice between a 0.75 chance
to end the game without winning anything and a 0.25 chance

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to get into the second stage. If the player reaches the
second stage, the options faced are:

E : ($4,000, 0.8)   and   F : ($3,000, 1)   N = 141
        (22% chose this)   (78% chose this)

This is illustrated in Figure 3.3. The symbol, *, in the
figure shows the preference pattern of subjects.

In this problem, there is a probability of

        (0.25)(0.80) = 0.20 to win $4,000

and a probability of

        (0.25)(1.0) = 0.25 to win $3,000.

Therefore, the final outcomes and probabilities are

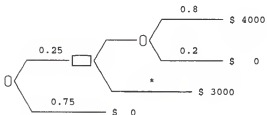   ($4,000, 0.2) and ($3,000, 0.25),

as in problem 2.

Figure 3.3.   The representation of problem 3 as a decision
        tree

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The result should be the same as that in problem 2. The essential difference between problems 3 and 2 is whether the uncertainty is resolved in two stages or one. Explicitly, people will always agree with the step-by-step logic of the above analysis but they still feel uncomfortable with the final conclusion. However, the result in problem 3 is apparently contrary to the pattern of preference in problem 2. There is an important hypothesis in this report — that psychological reactions of disappointment play a role in the informal analysis of decision making but always are ignored in the rational economic evaluation.

In problem 1, if you accept the gamble and got nothing, there exists the disappointment that you would feel because the higher expectation — 0.8 chance to win $4,000 — is abruptly frustrated. This phenomenon will warn people to select $3,000 for sure on the grounds of the basic security. In problem 2, one does not have much chance to win in either lottery, so losing is almost to be expected. There is no great disparity in the disappointment that one would feel at losing either lottery C and D. Consequently, people may prefer to choose the $4,000 gamble because of higher expected monetary value and implications of disappointment are similar to each lottery. In problem 3, if the player passes the first stage successfully, then he is likely to become extremely afraid to lose what he has obtained and his

expectations rise dramatically.

3.4 PRELIMINARY ASSUMPTIONS

Some authors differentiate between what they call
"decision making under risk" (decision making when the
states of the nature are not known but probabilities for the
various possible states are known) and "decision making
under uncertainty" (decision making when the states of the
nature are not known and probabilities for the various
possible states are not known). Under the subjective
interpretation of probability, it is always possible to
assign probabilities for the possible events, or states of
the nature (22). Hence, the "risk versus uncertainty"
dichotomy is artificial (in fact, it is nonexistent
according to the subjective interpretation of probability),
and in this report any decision-making problem in which the
states of nature are not known for certain is called
decision making under uncertainty.

The word elation is used to describe euphoria — the
opposite of disappointment — associated with an outcome
that exceeds one's prior expectation. These feelings, dis-
appointment and elation, may make decision makers reflect
when considering uncertain alternatives. In order to avoid
unnecessary complication, this report will only consider the
effects of disappointment and elation in decision-making

problems under uncertainty. It is presumed that decision makers have constant marginal values for money, they never suffer from regret, from envy, from other visible or invisible influences.

It is worth mentioning that reference points such as status quo, regret, and an assumption of nonconstant marginal value for money are excluded from the following analysis, only because their presence would complicate both the analysis and our understanding of the effect disappointment has on decision making, not because these factors are unimportant.

We will denote by $L(x, p, y)$ an offered lottery having a probability p of yielding payoff $x and a probability $(1-p)$ of yielding payoff $y. The expected monetary value (EMV) of such lottery is $px + (1-p)y$. It should be emphasized that x is at least as preferred as y (i.e. $x \geq y$) and p is the probability of winning.

Assumption: (Constant marginal value for payoffs)

It is reasonable to suppose that the prior expectations for a lottery $L_1(2x, p, 2y)$ will be exactly twice those for the lottery $L(x, p, y)$. Similarly, the prior expectations for a lottery $L_2(x+k, p, y+k)$ would be an amount k higher than those for the lottery $L(x, p, y)$. Figure 3.4 shows these lotteries.
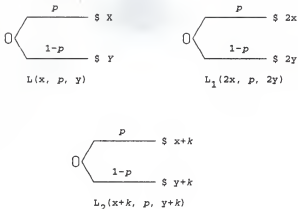
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Figure 3.4. The graph for the considered lotteries

Proof: The EMV of $L_1(2x, p, 2y)$ is:

$$2xp + 2y(1-p)$$

$$= 2[xp + y(1-p)]$$

$$= 2[\text{EMV of } L(x, p, y)].$$

The EMV of $L_2(x+k, p, y+k)$ is $(x+k)p + (y+k)(1-p)$.

The difference between $L(x, p, y)$ and $L_2(x+k, p, y+k)$ is:

$$[(x+k)p + (y+k)(1-p)] - [xp + y(1-p)]$$

$$= px + pk + y - py + k - pk - px - y + py$$

$$= k$$

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3.5 THE DISAPPOINTMENT MODEL

The preliminary assumption — constant marginal value for money — indicates that the decision maker would be risk neutral if it were not for the effects of psychological reactions, disappointment and elation. However, the purpose of this report is to explore the implications of disappointment and elation in decision-making situations. A disappointment model built by David E. Bell is represented as follows.

If someone is offered an unresolved lottery $L(x, p, y)$, the expected monetary value of $L(x, p, y)$ is $px + (1-p)y$. This is shown in Figure 3.5.
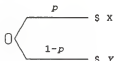
Figure 3.5. The graph for an unresolved offered lottery

Case 1: When y occurs, it means that one loses the lottery. The disappointment of a decision maker might be in direct proportion to the differences between what he got and what the expected monetary value is. The disappointment

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denoted by D can be quantified as follows. Letting d be a
constant reflecting the degree to which a unit of money
affects the decision maker psychologically ($d \geq 0$), we can
write:

$$D = d(\text{ EMV } - y)$$
$$= d\{[px + (1-p)y] - y\}$$
$$= d(px + y - py - y)$$
$$= dp(x-y) \tag{1}$$

Case 2: when x occurs, it means that one wins the
lottery. The sense of elation of a decision maker is
presumed to be in direct proportion to the difference
between what he got and the expected monetary value. The
elation denoted by E can be quantified as follows. Letting
e be a constant reflecting the degree to which a unit money
won affects the decision maker psychologically ($e \geq 0$), we
can write:

$$E = e(x - \text{EMV})$$
$$= e\{x - [px + (1-p)y]\}$$
$$= e(x - px - y + py)$$
$$= e(1-p)(x-y) \tag{2}$$

Thus disappointment and elation have been defined as
positive quantities describing reverse psychological
reactions. Hence, equations (1) and (2) are actually

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identical in structure and constants e and d have been allowed to be different. Following Bell's treatment, the utility value of the decision maker's multiattribute preferences over dollars and disappointment (or elation) should be positive linear and additive transformation:

Total utility = Economic Payoff + Psychological Reaction
$$= \text{EMV} + [pE + (1-p)(-D)]$$
$$= \text{EMV} + [pE - (1-p)D] \qquad (3)$$

where psychological reaction is positive for elation and negative for disappointment.

Consider the following three special cases:

Case 1: If $p = 0$ then
$$D = d \cdot 0 \cdot (x-y)$$
$$= 0 \qquad \text{and}$$
$$E = e(1-0)(x-Y)$$
$$= e(x-y)$$

i.e, there is no chance to win, so losing the lottery is to be expected and the degree of disappointment is zero from equation (1).

Case 2: If $p = 1$ then
$$D = d \cdot 1 \cdot (x-y)$$
$$= d(x-y) \qquad \text{and}$$

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$$E = e(1-1)(x-y)$$
$$= 0$$

i.e, the probability of winning is 1, so winning the lottery is a certain event and the degree of elation is zero from equation (2).

Case 3: If $d = e$ then the psychological reaction in this lottery will be:

$$pE + (1-p)(-D)$$
$$= p[e(1-p)(x-y)] + (1-p)[-dp(x-y)]$$
$$= pe(1-p)(x-y) - dp(1-p)(x-y)$$
$$= (1-p)(x-y)p(e-d)$$
$$= (1-p)(x-y)p \cdot 0$$
$$= 0$$

i.e, the psychological reaction, disappointment and elation, are identically compelling in such lottery, and they cancel each other when taking expectations. The disappointment and elation do not affect in decision making in this case but they still play an important role in inducing the desirability of individual outcomes.

3.6 ILLUSTRATION OF THE DISAPPOINTMENT MODEL

One of the purposes of this report is to investigate the implications of disappointment and elation in

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decision-making problems. In an unresolved lottery, if a decision maker experiences disappointment more than elation, i.e, $d > e$, then according to equation (3) the certainty equivalent (CE) of this lottery should be:

$$CE = EMV + \text{Psychological Reaction}$$
$$= [px + (1-p)y] + [pE + (1-p)(-D)]$$
$$= px + (1-p)y + (e-d)p(1-p)(x-y) \qquad (4)$$

Consider a special "unit" lottery $L(1, p, 0)$ for which

$$CE = 1p + 0(1-p) + (e-d)p(1-p)(1-0)$$
$$= p + (e-d)p(1-p)$$

where $e - d < 0$. The surprising result is that the decision maker has decreasing marginal value for money. Figure 3.6 shows the marginal values for money against the probability of winning. Although the preliminary assumption of the disappointment model is constant marginal value for money, it leads to risk-averse behavior by the decision maker when a relative aversion to disappointment over elation (say $d > e$) exists. According to the principles of utility theory, the utility function of this lottery can be deduced as follows:

For the implicit definition: By the linearity of money for lottery $L(1, p, 0)$, and assuming that

$$u(0) = 0, \ u(1) = 1,$$

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let $u(L)$ be the utility value of such a lottery. Then

$$u(L) = pu(x) + (1-p)u(y)$$

$$= pu(1) + (1-p)u(0)$$

$$= pu(1)$$

$$u(L) = p$$

Then the certainty equivalent CE is defined by

$$u(L) = u(CE), \text{ so that}$$

$$u(CE) = u[p + (e-d)p(1-p)]$$

$$= p$$

For the explicit definition:

Let $CE = p + (e-d)p(1-p) = X$ (say), and what we have
to do is to solve for the value of $u(X)$

Extend the equation and let $d-e = K$. Then

$$p + (e-d)p(1-p) = X$$

$$p - Kp(1-p) - X = 0$$

$$p - Kp + Kp^2 - X = 0$$

which leads to

$$p = \frac{(K-1) + \sqrt{(1-K)^2 + 4KX}}{2K}$$

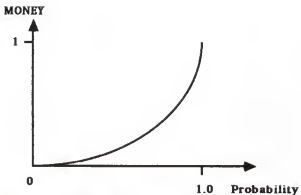$$u(X) = p = \frac{(K-1) + \sqrt{(1-K)^2 + 4KX}}{2K}$$

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Figure 3.6. Marginal Values for money against the probability of winning
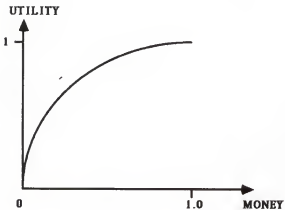

Figure 3.7. Utility function for money

Figure 3.7 graphs the utility function of this lottery. According to this utility function, we can conclude that

it is increasing for $0 \leq X \leq 1$ ($d-e < 1$),

it is a concave function ($d > e$),

it displays decreasing risk aversion and the risk primium of this function is always greater than 0 (22).

A premise we may obtain is that if a decision maker can not model the formulation of expectations explicitly, then, at least implicitly, the expectations are linear in the payoffs and for a given probability of winning, p, disappointment and elation are proportional to the difference between outcomes and expectations. Otherwise, the marginal values for money should be the type of Figure 3.7, i.e, showing decreasing marginal values for money. The preliminary assumption, constant marginal value for payoffs, discussed in Section 3.4 is important here.

Consider the previous discussion for the paradoxes of substitution principle of utility theory in Section 3.3, since the decision makers do not always follow the principle based on expected monetary values alone, it is reasonable to inject the ideas of disappointment and elation.

For the four alternatives used in problems 1, 2 and 3, the certainty equivalents for each alternative can be calculated by using equation (4) and they are shown in Table 3.1.

lottery L(x, p, y)	CE for each lottery
L_1(3,000, 1, 0)	3000
L_2(4,000, 0.8, 0)	$3200 + 640 \cdot (e-d)$
L_3(3,000, 0.25, 0)	$750 + 563 \cdot (e-d)$
L_4(4,000, 0.2, 0)	$800 + 640 \cdot (e-d)$

Table 3.1. Certainty equivalents for the considered
alternatives

1. For L_1(3,000, 1, 0), we have

   $CE = px + (1-p)y + (e-d)p(1-p)(x-y)$

   $= 1 \cdot 3000 + (1-1)0 + (e-d)1(1-1)(3000-0)$

   $= 3000$

2. For L_2(4,000, 0.8, 0), we have

   $CE = (0.8)4000 + (1-0.8)0 + (e-d)(0.8)(1-0.8)(4000-0)$

   $= 3200 + 640(e-d)$

3. For L_3(3,000, 0.25, 0), we have

   $CE = (0.25)3000 + (1-0.25)0 + (e-d)(0.25)(1-0.25)3000$

   $= 750 + 563(e-d)$

4. For L_4(4,000, 0.2, 0), we have

   $CE = (0.2)4000 + (1-0.2)0 + (e-d)(0.2)(1-0.2)4000$

   $= 800 + 640(e-d)$

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According to the preference patterns of the empirical obser-vation, $L_1(3,000, 1, 0)$ is preferred to $L_2(4,000, 0.8, 0)$ and $L_4(4,000, 0.2, 0)$ is preferred to $L_3(3,000, 0.25, 0)$, leading to the following two inequalities,

$$3000 > 3200 + 640 \cdot (e-d)$$

and

$$800 + 640 \cdot (e-d) > 750 + 563 \cdot (e-d)$$

Solving these two inequalities we got a range for the values of d-e, i.e,

$$0.3125 < d-e < 0.6494$$

which means that d is greater than e. Quantities inside this range will also support the empirically observed rank orders.

A common ratio effect for the behavioral rule derived by Kahneman and Tversky (13) is shown below.

For $x > y > 0$ and any $0 < r < 1$, if a lottery of $L(x, q, 0)$ is equally preferred (denoted by «») with a lottery of $L(y, p, 0)$, then the lottery of $L(x, qr, 0)$ is preferred to the lottery of $L(y, pr, 0)$, i.e, $L(x, qr, 0) \gg L(y, pr, 0)$.

Proof: If $L(x, q, 0)$ «» $L(y, p, 0)$
using the equations (3) and (4), we have
$$qx + (e-d)q(1-q)x = py + (e-d)p(1-p)y \qquad (5)$$

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In order to prove that $L(x, qr, 0) \gg L(y, pr, 0)$, then we must have an inequality of the form:

$$xqr + (e-d)qr(1-qr)x > ypr + (e-d)pr(1-pr)y \quad (6)$$

Multiply (5) by r^2 and get:

$$qr^2x + (e-d)q(1-q)xr^2 = pr^2y + (e-d)p(1-p)yr^2$$

Substract this equality from (6) then

LHS $= qxr(r-1)(1+e-d)$

and

RHS $= pyr(r-1)(1+e-d)$.

The inequality obtained is LHS >RHS. i,e,

$$qxr(r-1)(1+e-d) > pyr(r-1)(1+e-d) \quad (7)$$

If it is true then $qx > py$, which implies that $L(x, qr, 0) \gg L(y, pr, 0)$.

The equality (5) also indicates that the inequality (7) will be true if

$$1 + (e-d)(1-q) < 1 + (e-d)(1-p)$$

and if

$$(e-d)(p-q) < 0.$$

Proof: From (5), $qx + (e-d)q(1-q)x = py + (e-d)p(1-p)y$

$\Rightarrow \quad qx[1 + (e-d)(1-q)] = py[1 + (e-d)(1-p)]$

If $qx > py$ is true then

$$1 + (e-d)(1-q) < 1 + (e-d)(1-p)$$

Sequentially extend the above inequality:

$$(e-d)(1-q) < (e-d)(1-p)$$

$$(e-d)[(1-q) - (1-p)] < 0$$

$$(e-d)(p-q) < 0$$

Because $d > e$ and $e-d < 0$, it implies that $p-q > 0$,
i.e, $p > q$.

Example: Two lotteries shown in Figure 3.8 are

L(4,000, 0.8, 0) and L(3,000, 1, 0). We have seen
that they are equally preferred between each
other. For $r = 0.25$, $x = 4000 > y = 3000$, and
$qx = 4000(0.8) = 3200$ is greater than $py = 3000 \cdot 1$
= 3000, and $p = 1 > q = 0.8$, a new lottery
L(4,000, 0.2, 0), where $0.2 = (0.8)(0.25)$, will be
preferred to another new lottery L(3,000, 0.25, 0).
The combined lottery is shown in Figure 3.9, where
the symbol, *, indicates the empirically observed
preference. Thus the common ratio effect supports
the empirical observation and explains why people
selected a lottery which violated the substitution
principle of utility theory.

A conclusion from the above example can be obtained:

If L(x, q, 0) «» L(y, p, 0) then as long as

1. $qx > py$ and

2. $p > q$

then L(x, qr, 0) » L(y, pr, 0).

Figure 3.8.   The representations of lotteries
L(4,000, 0.8, 0) and L(3,000, 1, 0)
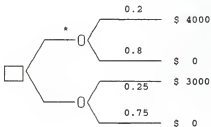

Figure 3.9.   The combination lottery of L(4,000, 0.8, 0) and
L(3,000, 1, 0)

Note that no matter what d and e are, using equation
(4) the quantity $(d-e)p(1-p)(x-y)$ which are psychological
reactions, disappointment and elation, can be treated as a

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measure of the risk involved in the lottery. It is ulti-
mately the reduction part in equation (4) caused by the
existence of uncertainty. This measure reflects the effects
of decreasing marginal values for money. There are many
well-known literature sources, such as Stone's parameters'
family (18), the variance measure of risk by Pratt (15)
which discuss this matter.

3.7 VERIFYING THE DISAPPOINTMENT MODEL

There are four lotteries shown in Figure 3.10 to
inspect the disappointment model represented in the fore-
going sections. The expected monetary values in these
lotteries are all same, $1,000. Using equation (1), the
disappointment on losing the lottery, i.e, on receiving $0,
will be the same quantity, $1000d$, in all four lotteries.
For example, the disappointment in lottery L_1 should be:

$$dp(x-y)$$
$$= d(0.1)(10000-0)$$
$$= 1000d.$$

However, according to the actual observation, the
probability of winning in lottery L_4, 0.999, is large enough
that one can almost be convinced that he will win the
lottery. Therefore, people may observe that the disappoint-
ment in the fourth lottery, $L_4(1,001, 0.999, 0)$, should be

the largest one among all four cases, and the order of their
disappoint- ment should be L_4, L_3, L_2 and L_1. The most
important thing that we need to know for this disappointment
model is whether the levels of disappointment in these
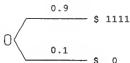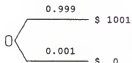lotteries are the same or not.
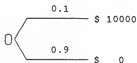
$L_1(10,000, 0.1, 0)$
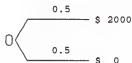
$L_2(2,000, 0.5, 0)$

$L_3(1,111, 0.9, 0)$

$L_4(1,001, 0.999, 0)$

Figure 3.10. Four lotteries are used to inspect the
disappointment model

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Simultaneously, consider another test for the elation model. It is represented in Figure 3.11 which contains the same top prize of $10,000 and the expected monetary values are all same, $9,000. Using equation (2), the elation on winning the lottery, i.e, on receiving $10,000, should be the same quantity, 1000e, in all four lotteries. For example, the elation in lottery L_1 should be:
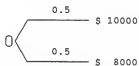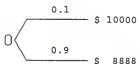
$$e(1-p)(x-y)$$
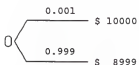$$= e(1 - 0.9)(10000-0)$$
$$= 1000e$$
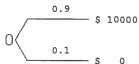
$L_1(10,000, 0.9, 0)$

$L_2(10,000, 0.5, 8,000)$

$L_3(10,000, 0.1, 8,888)$

$L_4(10,000, 0.001, 8,999)$

Figure 3.11. Four lotteries are used to inspect the elation model

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Similarly, the probability of losing in lottery L_4, 0.999, is so large, that is to be expected to lose. Therefore, the elation in the fourth lottery, $L_4(10,000, 0.001, 8,999)$, should be the largest one among these four cases and the order of their elation should be L_4, L_3, L_2 and L_1.

The above discussion shows that psychological reactions, disappointment and elation, may depend not only upon a level of prior expectations, but also in a direct way upon the probability with which the outcome will occur.

Since there is no formal model that can be constructed accurately for the psychological reactions, disappointment and elation, in all various circumstances, it should be emphasized that the simple model expressed by equations (1), (2), (3) and (4) just interprets that the psychological reactions, disappointment and elation, can be considered and be modeled systematically in decision situations. The expression of expectations is not unique, even for a specific individual. It may differ from person to person. A pessimist may have lower expectations, but an optimist may expect more. A mathematician may expect the probabilities of occurance for every outcome, but a business man maybe expects greater payoffs for each outcome. Different individuals have different expectations. It is understandable that in circular decision situations decision makers may spend much time to determine their decisions.

3.8 PREFERENCE MODEL INVOLVING DISAPPOINTMENT (AND ELATION)

The discussion in the previous sections indicates that disappointment and elation depend not only upon the formed expected payoffs, but also directly upon the probabilities involved. There is a preference model originated by Kahneman and Tversky in their prospect theory (13) and developed by David Bell which captures the above concepts.

We will denote an offered unresolved lottery by $L_0(x, p, y)$, an outcome x resulting from $L_0(x, p, y)$ by $L_1(x, p, y)$ and an outcome y resulting from $L_0(x, p, y)$ by $L_2(x, p, y)$. Let $C_i(x, p, y)$ be the certainty equivalents of the situations $L_i(x, p, y)$ (i = 0, 1, 2). It means that $C_0(x, p, y)$ represents the certainty equivalent of traditional utility theory for the unresolved lottery $L_0(x, p, y)$, where $C_1(x, p, y)$ and $C_2(x, p, y)$ are the cash equivalents for the outcomes.

If there is no considerations of psychological reactions, — disappointment and elation — $C_0(x, p, y)$ should be the following expression and the relationship of $C_i(x, p, y)$ is shown in Figure 3.12.

$$C_0(x, p, y) = pC_1(x, p, y) + (1-p)C_2(x, p, y)$$

The foregoing expression derives from our preliminary assumption of constant marginal values for money. This situation can be called "Risk Neutrality in the Absence of Disappointment and Elation".
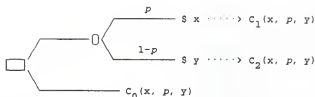
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Figure 3.12. The representation of cash equivalents for
$C_i(x, p, y)$ (i = 0, 1 and 2)

By using the concept of constant marginal value for payoffs, a theorem derived by Kahneman and Tversky (13) and used by David Bell (4) is shown below:

Theorem : For i = 0, 1 and 2, the situations $L_i(x, p, y)$ have certainty equivalents of $y + (x-y)\pi_i(p)$ for some functions $\pi_i(p)$.

Proof : $C_i(x, p, y) = y + C_i(x-y, p, y-y)$
$$= y + C_i(x-y, p, 0)$$
$$= y + (x-y)C_i(1, p, 0)$$
$$= y + (x-y)\pi_i(p)$$

$\pi_i(p)$ takes the place of $C_i(1, p, 0)$ and stands for a behavioral subjective probability,

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To summarize:

> $y + (x-y)\pi_0(p)$ is the certainty equivalent of the unresolved situation.
>
> $y + (x-y)\pi_1(p)$ is the certainty equivalent of the winning situation.
>
> $y + (x-y)\pi_2(p)$ is the certainty equivalent of the losing situation.

It is worth mentioning that $\pi_i(p)$ is not a conventional probability but can be applied to the traditional utility model, called $u(x)$, as a behavioral value.

Generally speaking, a lottery $L(x, p, y)$ is divided into two components:

(1) the riskless component, i.e. the minimum gain or loss which has more chance to be obtained or paid.

(2) the risky component, i.e. the additional gain or loss which is actually at stake.

The value of such lottery equals the value of the riskless component, i.e. y, plus the value difference between the outcomes, i.e. $x-y$, multiplied by the weight associated with the more extreme outcome.

For example: the value of a lottery $L(400, 0.25, 100)$ is:

$$100 + \pi(0.25)(400-100) = 100 + 300\pi(0.25)$$

We will substitute $\pi(p)$ for $\pi_0(p)$, $w(p)$ for $\pi_1(p)$ and $l(p)$ for $\pi_2(p)$. The function $w(p)$ represents the value of

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psychological gains, elation, that comes with winning in the lottery L(1, p, 0). The function $l(p)$ represents the value of psychological losses, disappointment, that comes with losing in the lottery L(1, p, 0). A flexible model represented in Figure 3.13 can be expressed as follows:

$$C_0(x, p, y) = y + (x-y)\pi(p)$$
$$C_1(x, p, y) = y + (x-y)w(p)$$
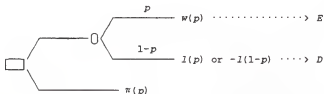$$C_2(x, p, y) = y + (x-y)l(1-p)$$

Figure 3.13. The representation of the lottery L(1, p, 0) involving functions $\pi(p)$, $w(p)$ and $l(p)$.

There are three properties for the functions π, w and l.

(1) Because of sure-thing indifference, for $i = 0$ and 1, $L_i(x, p, x)$ is equally preferred to $L_i(x, 1, y)$ and for $i = 0$ and 2, $L_i(y, p, y)$ is equally preferred to $L_i(x, 0, y)$. To summarize, we have

$$\pi(0) = w(1) = l(1) = 0$$
$$\pi(1) = w(0) = l(0) = 1$$

(2) For a quantity q, if $p > q$, then

$L_0(x, p, y)$ is preferred to $L_0(x, q, y)$

and

$L_i(x, q, y)$ is preferred to $L_i(x, p, y)$.

It means that π is a increasing function of p, w and l are decreasing functions of p. It is shown in Figure 3.14.

(3) $\quad C_0(x, p, y) = pC_1(x, p, y) + (1-p)C_2(x, p, y)$

$y + (x-y)\pi(p) = p[y + (x-y)w(p)] +$

$\qquad\qquad\qquad\qquad (1-p)[y + (x-y)l(1-p)]$

$y + (x-y)\pi(p) = py + p(x-y)w(p) + y - py -$

$\qquad\qquad\qquad\qquad (x-y)l(1-p) + p(x-y)l(1-p)$

$\qquad \pi(p) = p + pw(p) - (1-p)l(1-p)$

It means that the certainty equivalent of an un-resolved lottery $L(x, p, y)$ can be represented by the above equation.
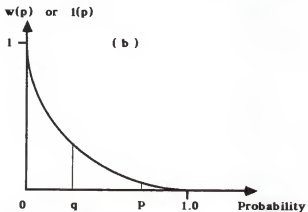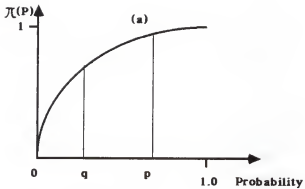
Figure 3.14. (a) Increasing function $\pi(p)$ and

(b) decreasing function $w(p)$ and $l(p)$

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People may find that the disappointment model introduced in Section 3.5 is a special case for using the foregoing model. Therefore, this preference model is very flexible for purposes of our study — an investigation of the effects of psychological reactions.

Using equation (4),

$$CE = EMV + Psychological\ Reaction$$

$$\pi(p) = [1p + 0(1-p)] + [p \cdot E + (1-p) \cdot (-D)]$$

$$\pi(p) = p + (e-d) \cdot p \cdot (1-p)(1-0)$$

$$\pi(p) = p + p(e-d) - p^2(e-d)$$

$$\pi(p) = p + pe - p^2 e - pd + p^2 d$$

$$\pi(p) = p + p[e(1-p)] - p[d(1-p)]$$

$$\pi(p) = p + pw(p) - pl(p)$$

$$\pi(p) = p + pw(p) - (1-p)l(1-p)$$

We can show that $w(p) = (1-p)e$,

$$l(p) = (1-p)d\ \text{and}$$

$$\pi(p) = p[1 + (e-d)(1-p)].$$

Therefore, the disappointment model is really fit for the preference model.

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CHAPTER IV

CONCLUSION

Utility theory is often criticized because it fails to predict actual behavior for some quite straightforward comparisons between alternatives with uncertain consequences (5). It is understandable why decision makers may be skeptical of expected utility analysis as a prescriptive tool when it apparently fails even for some simple comparisons.

We may observe that decision makers often agree with the logical procedure of decision analysis but feel uncomfortable at an intuitive level with its implications. Far from encouraging departure from traditional economic analysis, all the discussions of this report may explain that what is currently omitted from expected utility analysis deserves to be omitted and that what psychological reactions should be concerned in forcing economically inefficient decisions.

The concept of psychological reaction — disappointment — is integrated into utility theory in a prescriptive model (4). This studied model is perfectly adaptable to the case of nonconstant marginal values for money, by making an appropriate transformation of the attribute scale.

For many decision analyses, it is easy to understand that any quantitative analysis must explain the various conflicting objectives of the decision maker. Psychological reaction is an appropriate objective that should be included in any decision analysis if the decision maker considers it as a criterion for decision. In particular, a consumer may wish to spend some dollars in avoiding disappointment, an aspect of risk aversion that doesn't seem to be reflected by a utility function over dollar assets alone.

By using normative analyses, it merely indicates that the psychological behavior is the logical result of such an objective. The psychological impacts of a decision are generated by the same thought process used in making a decision, namely that the value of an outcome is judged relative to various reference points such as status quo, foregone assets, and prior expectations (4).

If we are interested only in the effects of disappointment on decision making, then only the function π need be assessed, which may be done by the obvious mechanism of asking directly for certainty equivalents for the gambles $(1, p, 0)$.

However, it would be important, in any prescriptive analysis that incorporates disappointment, for the assessment procedure to require explicit tradeoffs between psychology and economy. Assessment of the functions w and 1

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requires the decision maker to compare outcomes (and the
psychological consequences that go with them) instead of
alternatives.

Explicitly, the disappointment model captures the idea
that people's reaction to decision outcomes is a function
not only of the absolute value of their payoff, but also to
the change in their expectations and to the likelihood of
such outcomes. Thus, we have looked at the implications of
disappointment for decision making in standard situations
including violations of the substitution principle.

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A STUDY OF THE DISAPPOINTMENT MODEL IN DECISION MAKING
UNDER UNCERTAINTY

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ABSTRACT

This report investigates the implications of using
disappointment as a criterion in a decision-making problem.
This procedure was developed after the concept of regret in
decision analysis. Utility theory is often criticized
because it fails to predict actual behavior for some quite
straightforward comparisons between alternatives with
uncertainty consequences.

A simple model of David E. Bell incorporating
disappointment is introduced, which offers an explanation
for systematic violations of the substitution principle of
utility theory and investigates the behavioral implications
for a decision maker. Under the basic assumptions for this
report, the resulting preference model is shown. This model
permits a straightforward assessment task on the part of the
decision makers.

This report is a study of descriptive behavior to force
recognition of the importance of psychological impacts to
the decision maker.