A NECESSARY CONDITION FOR UNIQUE SOLVABILITY OF INFINITE-TRANSMITTANCE ACTIVE NETWORK EQUATIONS

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I. INTRODUCTION

There has been much recent interest in formulating and solving the equations for general networks containing dependent sources. However, the conditions for unique solvability of the equations describing a mathematical model of a general active network are not well-understood[1].

Recently, Malik and Hale[2] have extended Seshu and Reed's solvability conditions for networks of passive elements and independent sources to networks containing dependent sources. The extended conditions state that the Laplace transformed equations describing a network N containing n finite-transmittance dependent sources have an unique solution if, and only if:

1) N contains no cut-set consisting only of elements from the set of independent and dependent current sources of N,

2) N contains no circuit consisting only of elements from the set of independent and dependent voltage sources of N,

3) det $[F - \Phi^{-1}] \neq 0$ identically in s,

where matrix Φ^{-1} is an n x n diagonal matrix with the reciprocals of the transmittances of the dependent sources as diagonal elements, and F is an n x n matrix such that element f_{ij} of F is a certain transfer function found from a passive network N_3 which is related to N.

The first two conditions are simply generalizations of the well-known facts that the voltages across a cut-set of independent current sources and the currents in a loop of independent voltage sources cannot be uniquely determined. The third condition, which is unique to the active network problem, requires interpretation and further study, which hopefully will lead to a better understanding of the structure and properties of active networks.

In general, the determinant of condition 3 may be expanded into a sum of 2^n determinants, the first term of which is the determinant of F. The determinant det $[F - \Phi^{-1}]$ can vanish in two ways: either all 2^n terms vanish individually, or the terms cancel one another and sum to zero. At this time there is no known way to describe how the terms can cancel one another and sum to zero. The possibility of individual terms vanishing, however, seems to be related to the network structure and lends itself more easily to further study. Every term in the expansion except the first term, det F, has the reciprocal of a transmittance for a coefficient. Thus, in the limit as all of the transmittances approach infinity, the third condition reduces to det $F \neq 0$. This suggests that studying networks containing infinite transmittances may provide valuable information about finite-transmittance networks. The idea of formulating equations for infinite-transmittance network is not new. Nathan[3], for example, has shown a matrix analysis method for networks containing "infinite gain" operational amplifiers.

The object of this paper is to state and prove a necessary condition for an uniquely solvable set of active network equations to remain uniquely solvable in the limit as all of the transmittances approach infinity. A specific statement of the necessary condition must be deferred until after the concept of a seg has been introduced.

II. NETWORK ANALYSIS TOOLS

A. Indefinite Admittance Matrix.

In formulating the equations for electrical networks there arise matrices with the property that the sum of the elements of every row and of every column equals zero. As a consequence, the matrices are singular and all the first cofactors associated with the determinants of such matrices are equal. This kind of matrix is called an "equicofactor matrix". The equicofactor matrix formed on node basis is called the "indefinite admittance matrix".

Shekel[4] first introduced the indefinite admittance matrix as a convenient means of completely specifying a linear n-terminal network "black box". Every practical linear electrical network may be analyzed by the matrix regardless of the number or type of its constituent elements or their manner of interconnection.

The indefinite admittance matrix is composed of the usual self- and mutual- admittances specified by the measurable terminal magnitudes of an n-terminal "black box", and can be constructed as follows.

Consider a linear n-terminal network as shown in Fig. 1. The assumptions are made that only the n terminals are physically accessible, the remaining structure is contained in an inaccessible "black box", and the terminals are the points at which external sources or loads may be connected. Let V_1, V_2, ... , V_n denote the potentials measured between terminals 1, 2, ... , n,

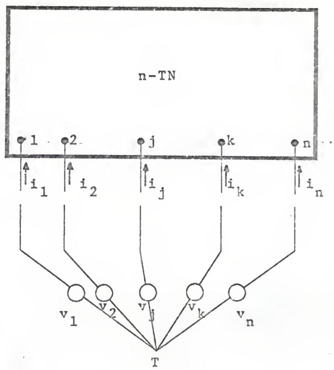

Fig. 1. An n-Terminal Network.

respectively, and some arbitrary but unspecified reference point r. Let $I_1, I_2, \ldots, I_n$ denote the currents entering 1, 2, ... , n, respectively, from outside the network. Choosing the reference point of zero potential arbitrarily outside the n-terminal "black box", and applying Kirchhoff's current law to each of these terminals, a set of n linear algebraic equations can be written matrically in

$$\underline{Y}\,\underline{V} = \underline{I} \quad , \qquad (2\text{-}1)$$

where

$$\underline{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix}, \qquad \underline{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}, \qquad \underline{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}.$$

In Eqn. (2-1), $\underline{Y}$ is an n x n matrix called the "indefinite admittance matrix". The elements of this matrix can be found from Eqn. (2-1) to be

$$y_{jk} = \left.\frac{I_j(s)}{V_k(s)}\right|_{V_i = 0, \text{ if } i \neq k} \qquad (2\text{-}2)$$

Thus, the self-admittance y_{kk} and the mutual-admittance y_{jk} can be found by connecting all the terminals except terminal k to the chosen reference point and exciting terminal k with a unit impulse; that is $V_k(s) = 1$. Then y_{kk} is the current $I_k(s)$ entering terminal k, and y_{jk} is the current $I_j(s)$ entering terminal j.

Some important properties of the indefinite admittance matrix are listed below:

Property 1. The sum of the elements in every column of the indefinite admittance matrix is zero.

$$\sum_{j=1}^{n} y_{jk} = 0 \text{ , for all } k \tag{2-3}$$

Property 2. The sum of the elements in every row of the indefinite admittance matrix is zero.

$$\sum_{k=1}^{n} y_{jk} = 0 \text{ , for all } j \tag{2-4}$$

The above properties can easily be established by applying Kirchhoff's current and voltage laws.

Property 3. For a passive, reciprocal n-terminal network, the indefinite admittance matrix is symmetrical.

$$y_{jk} = y_{kj} \tag{2-5}$$

Property 4. The indefinite admittance matrix is singular and all of its first cofactors are equal.

This is due essentially to the fact that the currents in the n terminals depend only on the differences of potential between these terminals rather than on the absolute potential of the terminals.

Property 5. The indefinite admittance matrices corresponding to networks connected in parallel are additive.

This can be seen from the fact that currents entering the corresponding nodes of parallel networks are additive. Every practical linear network can be split into elementary subnetworks in such a fashion that these subnetworks, no matter whether they are in series or in parallel in the original network, are relatively in parallel with respect to the reference node. This gives great advantages in analyzing networks of complicated structure. The following is a simple example in which the same indefinite admittance matrix will be obtained by means of different approaches.

Example. Find the indefinite admittance matrix of the network N as shown in Fig. 2(a). The zero potential reference node r is chosen arbitrarily outside of the network.

Solution:

A) Direct method.

Let V_1, V_2, V_3, V_4 denote the voltages between node 1, 2, 3, and 4, respectively, and the reference node r. Then, applying Kirchhoff's current law to each of these nodes, a set of linear algebraic equations is written as follows.

at node 1: $J = (y_1+y_2+y_5)V_1 - y_2V_2 - y_5V_3 - y_1V_4$.

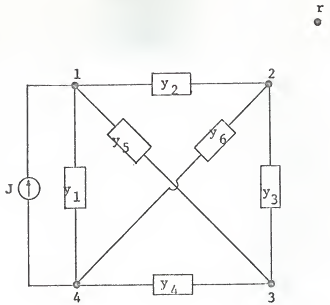

Fig. 2(a). Network N of the example.

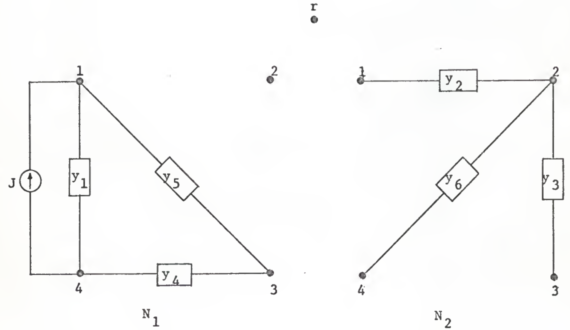

Fig. 2(b). Example by Additive Method, Subnetworks N_1 and N_2.

$$\text{at node 2:}\quad 0 = -y_2V_1 + (y_2+y_3+y_6)V_2 - y_3V_3 - y_6V_4 \;.$$

$$\text{at node 3:}\quad 0 = -y_5V_1 - y_3V_2 + (y_3+y_4+y_5)V_3 - y_4V_4 \;.$$

$$\text{at node 4:}\quad -J = -y_1V_1 - y_6V_2 - y_4V_3 + (y_1+y_4+y_6)V_4 \;.$$

Denote the coefficient matrix of the equations by $\underline{Y}$. Then, $\underline{Y}$ is a 4 x 4 indefinite admittance matrix of the network N. That is,

$$\underline{Y} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\ \begin{bmatrix} y_1+y_2+y_5 & -y_2 & -y_5 & -y_1 \\ -y_2 & y_2+y_3+y_6 & -y_3 & -y_6 \\ -y_5 & -y_3 & y_3+y_4+y_5 & -y_4 \\ -y_1 & -y_6 & -y_4 & y_1+y_4+y_6 \end{bmatrix} \end{array} .$$

B) Additive method.

Let N be split into two four-terminal subnetworks N_1 and N_2 with the elements of N divided between N_1 and N_2 as indicated in Fig. 2(b). Note that every node of the original network is considered to be a node of each subnetwork. It is seen that N may be formed by placing the two four-terminal networks N_1 and N_2 in parallel.

Denote the indefinite admittance matrices of N_1 and N_2 by $\underline{Y}_1$ and $\underline{Y}_2$, respectively. Then, as in part A),

$$\underline{Y}_1 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} y_1+y_5 & 0 & -y_5 & -y_1 \\ 0 & 0 & 0 & 0 \\ -y_5 & 0 & y_4+y_5 & -y_4 \\ -y_1 & 0 & -y_4 & y_1+y_4 \end{bmatrix} \end{array} ,$$

and

$$\underline{Y}_2 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} y_2 & -y_2 & 0 & 0 \\ -y_2 & y_2+y_3+y_6 & -y_3 & -y_6 \\ 0 & -y_3 & y_3 & 0 \\ 0 & -y_6 & 0 & y_6 \end{bmatrix} \end{array} .$$

The matrix sum of the above two indefinite admittance matrices of N_1 and N_2 gives

$$\underline{Y}_1 + \underline{Y}_2 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} y_1+y_2+y_5 & -y_2 & -y_5 & -y_1 \\ -y_2 & y_2+y_3+y_6 & -y_3 & -y_6 \\ -y_5 & -y_3 & y_3+y_4+y_5 & -y_4 \\ -y_1 & -y_6 & -y_4 & y_1+y_4+y_6 \end{bmatrix} \end{array}$$

$$= \underline{Y} \quad .$$

The resultant matrix is the same as was given in part A).

Since the indefinite admittance matrix is singular, its inverse does not exist, and an expression of the solution in the form $\underline{V} = \underline{Y}^{-1}\underline{I}$ is not obtainable. In order to find the solution of the network, the zero potential reference node is chosen to be one of the network terminals, say the kth. Then the remaining (n-1) voltages of the (n-1) terminals are measured relative to that of the kth terminal. The current in the kth terminal, by Kirchhoff's current law, is the negative of the sum of the remaining (n-1) terminal currents. Having V_k and I_k, a submatrix $\underline{Y}_d$ is formed by deleting the kth row and the kth column from the indefinite admittance matrix. The resulting (n-1) x (n-1) submatrix $\underline{Y}_d$ is called the "definite admittance matrix". It is non-singular and can be inverted to give a relation of the solution to the network in the form

$$\underline{V}_d = \underline{Y}_d^{-1} \underline{I}_d \quad , \qquad (2\text{-}6)$$

where the (n-1) vector $\underline{V}_d$ contains the voltages of the n-1 non-reference nodes relative to node k and where $\underline{I}_d$ is an (n-1) vector containing the currents at all of the nodes except the kth.

The foregoing study of the indefinite admittance matrix illustrated the principal advantage of this matrix as a general analysis tool; its additive property provides the flexibility in describing complex networks by terminal equations.

B. The Seg.

The usefulness of linear graph theory in electrical network analysis is in separating those properties of networks which arise from the physics of the network elements from those which are a consequence of the interconnection of the elements. The interconnection properties, the well-known laws of Kirchhoff, deal solely with the geometric structure of the network. Thus, networks are represented in accordance with their geometric structure as patterns of line segments called graphs. Each line segment of the graph represents a circuit element or a combination of circuit elements in the original network. Each vertex, or connection point of line segments, represents a node of the network.

There have been developed some useful classes of subgraphs such as trees, cotrees, paths, stars, cut sets, and circuits. Among these, the classes of stars and of cut sets have similar properties. The former is the set of line segments incident to a vertex. The latter is a minimal set of edges such that their removal from the graph leaves two unconnected subgraphs of the original graph. Both of them are useful in formulating Kirchhoff's current law. That is, the algebraic sum of the currents flowing through the elements of a star or of a cut set in a given reference direction is zero.

Reed[5] recently introduced a new class of subgraphs called segs. The segs are defined in terms of a segregation of the vertices of a graph into two all-inclusive, mutually-exclusive, non-empty sets, the X set and the NX set. This class of subgraphs includes both the stars and the cut-sets as special cases. The concept of a seg is particularly useful for two reasons. First, its definition is so simple and basic that it readily lends itself to mathematical treatment. Second and more important, the seg makes it possible to formulate Kirchhoff's current law in the most general known form. That is: "the algebraic sum of the currents flowing in a seg in a given reference direction is zero".

The definitions essential to establishing the concept of a seg are listed here.

Definition 1. Vertex Segregation: A vertex segregation is a classification of the vertices of a graph into two all-inclusive, mutually-exclusive, non-empty sets; the X set and the NX set.

Definition 2. Bridge: A bridge is an element of a graph with one vertex in the X set and one vertex in the NX set of a vertex segregation.

Definition 3. Seg: A seg is the set of all bridges corresponding to any vertex segregation of a graph. An orientation is assigned to each edge of a seg, directed from the X set vertex toward the NX set vertex.

Three segs are indicated by the heavy lines of the graphs in Fig. 3. A seg may or may not include all elements of a graph.

Also, it is possible to rearrange any graph such that the X set and the NX set of vertices are clustered in a convenient way for a given vertex segregation.

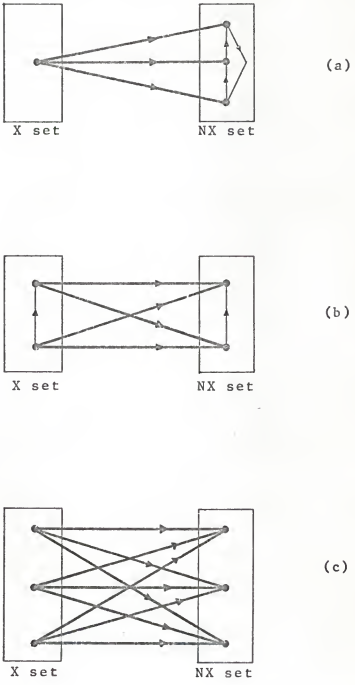

Fig. 3. Three Examples of Segs.

III. STATEMENT OF THE PROBLEM

The study is restricted to networks consisting of 2-terminal passive elements and dependent sources controlled by admittance currents and admittance voltages, called controlling variables. Imperfect transformers may be present in the network, subject to loose restrictions on their locations. Every controlling variable is associated with a passive, 2-terminal network element called a controlling element. A dependent source is simply a source whose strength (voltage or current) is proportional to another quantity (voltage or current), namely the controlling variable, in some other part of the network. The source need not have a terminal in common with its controlling element. The transmittance of a dependent source is the proportionality constant which relates the controlling variable and the source strength. All transmittances are assumed to be positive real numbers.

There are four possible combinations of the related quantities:

A type-A controlled source is a voltage-controlled current source in which the output current is proportional to the input voltage. An example of this controlled source is the idealized pentode wherein the plate current is proportional to the grid voltage. The proportionality constant g_m is commonly known as the transconductance.

A type-B controlled source is a current-controlled voltage source in which the output voltage is proportional to the input current. An example of this controlled source is the rotary d-c generator whose generated voltage is proportional to the field current.

A type-C controlled source is a voltage-controlled voltage source in which the output voltage is proportional to the input voltage. Many voltage amplifiers have properties approximating those of this type of controlled source. Among them, feedback amplifiers give a very good approximation. The proportionality constant μ is commonly known as the amplification factor.

A type-D controlled source is a current-controlled current source in which the output current is proportional to the input current. An example of this controlled source is the ideal transistor. The proportionality constant α is commonly known as the current-transfer ratio.

It is now possible to state the central problem of this report. A set of Laplace transformed equations is written in a certain manner for a connected network of arbitrary structure containing passive elements and n dependent sources of one of the types listed above. These equations contain the controlled and controlling variables among the unknowns; and are assumed to have a unique solution for finite transmittances. The object of this paper is to prove an assertion based upon intuitive physical reasoning. The assertion is that these equations have a unique solution in the limit as the transmittances approach infinity only if the controlling elements do not constitute a seg of the network.

IV. SOLUTION OF THE PROBLEM

The problem is approached by first formulating a set of equations for an active network in which the controlling elements are not constrained to constitute a seg. The coefficient matrix is then examined in the limit as the transmittances approach infinity. For this case, it is not possible to demonstrate that the coefficient matrix is either singular or nonsingular. Some additional information about the structure and/or element values is required in order to come to a definite conclusion.

A physical argument suggests that the coefficient matrix might vanish if the network contained a seg of controlling elements. This idea is explored by formulating the equations for a general connected active network in which a seg of controlling elements is imbedded. In order to simplify the approach, types of dependent sources in the network are considered individually. In the limiting case, the coefficient matrices of the equations for each type of dependent sources are clearly found to become singular. Furthermore, the singularity is shown to follow directly from the presence of the seg of controlling elements.

A. Active Network Without A Seg of Controlling Elements.

The general diagram of an active network N without a seg of controlling elements explicitly shown is given in Fig. 4. Network N is excited by an independent current source J. The admittances y_1, y_2, ... , y_k are the k controlling elements for the k

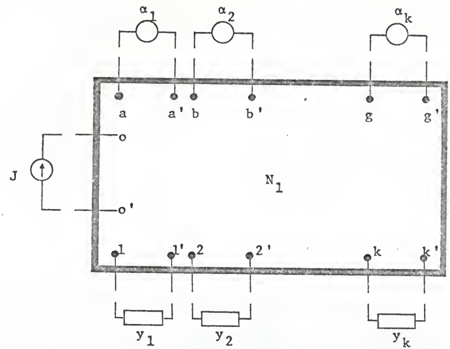

Fig. 4. Active Network N without A Seg of Controlling Elements.

dependent sources represented by circles. The α's are the source transmittances, with subscripts relating them to their respective controlling elements. The controlling elements and sources constrain a connected passive network N_1 having arbitrary structure, and possibly containing imperfect transformers. No magnetic coupling between N_1 and the controlling elements is permitted.

Using the indefinite admittance matrix, equations for N_1 are written in the form

$$\begin{bmatrix} I_{0'} \\ I_0 \\ I_A \\ I_{A'} \\ I_K \\ I_{K'} \end{bmatrix} = \begin{bmatrix} Y_{0'0'} & Y_{0'0} & Y_{0'A} & Y_{0'A'} & Y_{0'K} & Y_{0'K'} \\ Y_{00'} & Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0K'} \\ Y_{A0'} & Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AK'} \\ Y_{A'0'} & Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'K'} \\ Y_{K0'} & Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK} & Y_{KK'} \\ Y_{K'0'} & Y_{K'0} & Y_{K'A} & Y_{K'A'} & Y_{K'K} & Y_{K'K'} \end{bmatrix} \begin{bmatrix} V_{0'} \\ V_0 \\ V_A \\ V_{A'} \\ V_K \\ V_{K'} \end{bmatrix} . \qquad (4\text{-}1)$$

where

$$\begin{aligned} I_{0'} &= -J \\ I_0 &= J \end{aligned} \qquad (4\text{-}2)$$

$$I_A = \begin{bmatrix} i_a \\ i_b \\ \vdots \\ i_g \end{bmatrix}, \quad I_{A'} = \begin{bmatrix} i_{a'} \\ i_{b'} \\ \vdots \\ i_{g'} \end{bmatrix}, \quad I_K = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_K \end{bmatrix}, \quad I_{K'} = \begin{bmatrix} i_{1'} \\ i_{2'} \\ \vdots \\ i_{K'} \end{bmatrix}.$$

$$V_A = \begin{bmatrix} v_a \\ v_b \\ \vdots \\ v_g \end{bmatrix}, \quad V_{A'} = \begin{bmatrix} v_{a'} \\ v_{b'} \\ \vdots \\ v_{g'} \end{bmatrix}, \quad V_K = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_K \end{bmatrix}, \quad V_{K'} = \begin{bmatrix} v_{1'} \\ v_{2'} \\ \vdots \\ v_{K'} \end{bmatrix}.$$

Since each terminal current at a port must be the negative of the other terminal current at the port, the equations for the controlling elements are written in the form

$$\begin{bmatrix} -I_K \\ -I_{K'} \end{bmatrix} = \begin{bmatrix} P & -P \\ -P & P \end{bmatrix} \begin{bmatrix} V_K \\ V_{K'} \end{bmatrix} \qquad (4\text{-}3)$$

where P is k x k diagonal admittance matrix for the controlling elements. That is,

$$P = \text{diag.}\ (y_1, y_2, \ldots, y_k) \ . \qquad (4\text{-}4)$$

Again, using the fact that the terminal currents of a 2-terminal element are the negatives of each other,

$$I_A = -I_{A'} \quad , \qquad (4\text{-}5)$$

and

$$I_K = -I_{K'} \quad . \qquad (4\text{-}6)$$

Combining Eqns. (4-1), (4-2), (4-3), (4-5), and (4-6), and choosing 0' as zero potential reference node, the set of linearly independent equations is written for the whole passive part of the network in the form

$$\begin{bmatrix} J \\ I_A \\ -I_A \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0K'} \\ Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AK'} \\ Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'K'} \\ Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KK'}-P \\ Y_{K'0} & Y_{K'A} & Y_{K'A'} & Y_{K'K}-P & Y_{K'K'}+P \end{bmatrix} \begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K \\ V_{K'} \end{bmatrix} . \tag{4-7}$$

The coefficient matrix in Eqn. (4-7) is the nonsignular definite admittance matrix of the passive part of N.

Since the controlling variable $(V_K - V_{K'})$, the voltage vector across the controlling elements, is of particular interest inalater part of the problem, the following nonsingular transformation of variables is made:

$$\begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K \\ V_{K'} \end{bmatrix} = \begin{bmatrix} U & 0 & 0 & 0 & 0 \\ 0 & U & 0 & 0 & 0 \\ 0 & 0 & U & 0 & 0 \\ 0 & 0 & 0 & U & U \\ 0 & 0 & 0 & 0 & U \end{bmatrix} \begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K - V_{K'} \\ V_{K'} \end{bmatrix} . \tag{4-8}$$

where the U's denote identity matrices of appropriate orders.

Using Eqn. (4-8), Eqn. (4-7) is transformed into the following equivalent matrix equations:

$$\begin{bmatrix} J \\ I_A \\ -I_A \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0K}+Y_{0K'} \\ Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AK}+Y_{AK'} \\ Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'K}+Y_{A'K'} \\ Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KK}+Y_{KK'} \\ Y_{K'0} & Y_{K'A} & Y_{K'A'} & Y_{K'K}-P & Y_{K'K}+Y_{K'K'} \end{bmatrix} \begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K-V_{K'} \\ V_{K'} \end{bmatrix} . \qquad (4\text{-}9)$$

The dependent sources, which are constrained by the controlling variables, give an additional set of equations. For type-A dependent sources, these equations are written in matrix form as:

$$I_A = Q\ (V_K - V_{K'}) \quad . \qquad (4\text{-}10)$$

where Q is a k x k diagonal matrix of transmittances of dependent sources. That is,

$$Q = \text{diag.}\ (\alpha_1, \alpha_2, \ldots., \alpha_k) \quad . \qquad (4\text{-}11)$$

Equations (4-9) and (4-10) must be solved simultaneously. Combining these into one matrix equation gives

$$\begin{bmatrix} 0 \\ J \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -Q^{-1} & 0 & 0 & 0 & U & 0 \\ 0 & Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0K}+Y_{0K'} \\ -U & Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AK}+Y_{AK'} \\ U & Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'K}+Y_{A'K'} \\ 0 & Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KK}+Y_{KK'} \\ 0 & Y_{K'0} & Y_{K'A} & Y_{K'A'} & Y_{K'K}-P & Y_{K'K}+Y_{K'K'} \end{bmatrix} \begin{bmatrix} I_A \\ V_0 \\ V_A \\ V_{A'} \\ V_K-V_{K'} \\ V_{K'} \end{bmatrix} . \qquad (4\text{-}12)$$

The coefficient matrix of the above equation is assumed to be nonsingular for finite value of transmittance. Since the solvability of a set of equations is characterized by the singularity or nonsingularity of the coefficient matrix, attention is focused on the coefficient matrix in the limiting case.

Denote the coefficient matrix in Eqn. (4-12) by $\underline{C}$. Then, in the limit as Q approaches infinity, that is, all the transmittances approach infinity, $\underline{C}$ becomes

$$\lim_{Q\to\infty} \underline{C}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & U & 0 \\ 0 & Y_{OO} & Y_{OA} & Y_{OA'} & Y_{OK} & Y_{OK}+Y_{OK'} \\ -U & Y_{AO} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AK}+Y_{AK'} \\ U & Y_{A'O} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'K}+Y_{A'K'} \\ 0 & Y_{KO} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KK}+Y_{KK'} \\ 0 & Y_{KO} & Y_{KA} & Y_{KA'} & Y_{KK}-P & Y_{KK}+Y_{KK'} \end{bmatrix} . \qquad (4\text{-}13)$$

In Eqn. (4-13), the singularity of the coefficient matrix $\underline{C}$ in the limiting case is not obvious. It might be that for some particular network structure the coefficient matrix $\underline{C}$ may be simplified such that it will vanish in the limiting case. But in the absence of more detailed knowledge of N_1, no definite conclusion can be reached.

Physical insight gained from the study of high gain operational amplifiers suggests that as the transmittances approach infinity, one would expect the controlling variables to approach zero and the controlled variable to remain finite. If the controlling elements should constitute a seg of the network, in this limiting case, transmission across the seg would necessarily be zero. Thus part of the network would be electrically isolated from the remaining part. This line of reasoning suggests that a necessary condition for the unique solvability of the network equations might be that there must be no seg of controlling elements imbedded in the active network. This hypothesis is explored in the next section.

B. Active Network With A Seg of Controlling Elements.

Let N be the general active network containing n dependent sources shown in Fig. 5. The passive part of the network consists of two connected passive subnetworks, N_1 and N_2, bridged by a seg of controlling elements. Subnetworks N_1 and N_2 are of arbitrary structure and may contain imperfect transformers, however magnetic coupling must be confined within the black boxes labelled N_1 and N_2.

Using the indefinite admittance matrix, N_1 is described by the partitioned matrix equation

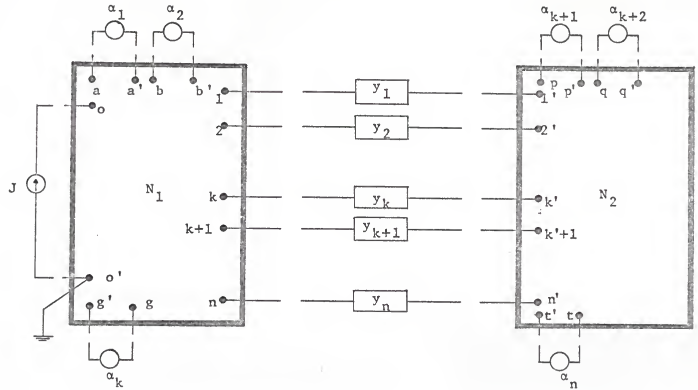

Fig. 5. Active Network N Containing A Seg of Controlling Elements.

$$\begin{bmatrix} I_{0'} \\ I_0 \\ I_A \\ I_{A'} \\ I_K \\ I_M \end{bmatrix} = \begin{bmatrix} Y_{0'0'} & Y_{0'0} & Y_{0'A} & Y_{0'A'} & Y_{0'K} & Y_{0'M} \\ Y_{00'} & Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0M} \\ Y_{A0'} & Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} \\ Y_{A'0'} & Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} \\ Y_{K0'} & Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK} & Y_{KM} \\ Y_{M0'} & Y_{M0} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM} \end{bmatrix} \cdot \begin{bmatrix} V_{0'} \\ V_0 \\ V_A \\ V_{A'} \\ V_K \\ V_M \end{bmatrix} \tag{4-14}$$

where

$$I_{0'} = -J$$

$$I_0 = J$$

$$I_A = \begin{bmatrix} i_a \\ i_b \\ \vdots \\ i_g \end{bmatrix}, \quad I_{A'} = \begin{bmatrix} i_{a'} \\ i_{b'} \\ \vdots \\ i_{g'} \end{bmatrix}, \quad I_K = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_k \end{bmatrix}, \quad I_M = \begin{bmatrix} i_{k+1} \\ i_{k+2} \\ \vdots \\ i_n \end{bmatrix}, \tag{4-15}$$

$$V_A = \begin{bmatrix} v_a \\ v_b \\ \vdots \\ v_g \end{bmatrix}, \quad V_{A'} = \begin{bmatrix} v_{a'} \\ v_{b'} \\ \vdots \\ v_{g'} \end{bmatrix}, \quad V_K = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix}, \quad V_M = \begin{bmatrix} v_{k+1} \\ v_{k+2} \\ \vdots \\ v_n \end{bmatrix}.$$

Similarly, the subnetwork N_2 is described by

$$\begin{bmatrix} I_{K'} \\ I_{M'} \\ I_W \\ I_{W'} \end{bmatrix} = \begin{bmatrix} Y_{K'K'} & Y_{K'M'} & Y_{K'W} & Y_{K'W'} \\ Y_{M'K'} & Y_{M'M'} & Y_{M'W} & Y_{M'W'} \\ Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\ Y_{W'K'} & Y_{W'M'} & Y_{W'W} & Y_{W'W'} \end{bmatrix} \begin{bmatrix} V_{K'} \\ V_{M'} \\ V_W \\ V_{W'} \end{bmatrix} \tag{4-16}$$

where

$$I_{K'} = \begin{bmatrix} i_{1'} \\ i_{2'} \\ \vdots \\ i_{k'} \end{bmatrix}, \quad I_{M'} = \begin{bmatrix} i_{k+1'} \\ i_{k+2'} \\ \vdots \\ i_{n'} \end{bmatrix}, \quad I_W = \begin{bmatrix} i_p \\ i_q \\ \vdots \\ i_t \end{bmatrix}, \quad I_{W'} = \begin{bmatrix} i_{p'} \\ i_{q'} \\ \vdots \\ i_{t'} \end{bmatrix},$$

$$V_{K'} = \begin{bmatrix} v_{1'} \\ v_{2'} \\ \vdots \\ v_{k'} \end{bmatrix}, \quad V_{M'} = \begin{bmatrix} v_{k+1'} \\ v_{k+2'} \\ \vdots \\ v_{n'} \end{bmatrix}, \quad V_W = \begin{bmatrix} v_p \\ v_q \\ \vdots \\ v_t \end{bmatrix}, \quad V_{W'} = \begin{bmatrix} v_{p'} \\ v_{q'} \\ \vdots \\ v_{t'} \end{bmatrix}. \tag{4-17}$$

It is particularly important to notice that the coefficient matrix in Eqn. (4-16) is the indefinite admittance matrix of the passive subnetwork N_2; and therefore the sum of the elements in every column (or row) is zero. The singularity of this matrix is a key part of the argument which follows.

The set of current equations for the seg of controlling elements is

$$\begin{bmatrix} -I_{K'} \\ -I_{K'} \\ -I_M \\ -I_{M'} \end{bmatrix} = \begin{bmatrix} P & -P & 0 & 0 \\ -P & P & 0 & 0 \\ 0 & 0 & R & -R \\ 0 & 0 & -R & R \end{bmatrix} \begin{bmatrix} V_K \\ V_{K'} \\ V_M \\ V_{M'} \end{bmatrix} \tag{4-18}$$

where P is a k x k diagonal matrix of controlling elements related to the dependent sources of N_1. That is,

$$P = \text{diag.}\ (y_1, y_2, \ldots, y_k) \quad . \tag{4-19}$$

Also, R is a (n-k) x (n-k) diagonal matrix of controlling elements related to the dependent sources of N_2. That is,

$$R = \text{diag.}\ (y_{k+1}, y_{k+2}, \ldots, y_n) \quad . \tag{4-20}$$

The port constraint relationships for the 2-terminal circuit elements are

$$\begin{aligned} I_A &= -I_{A'} \quad , \\ I_W &= -I_{W'} \quad , \\ I_K &= -I_{K'} \quad , \\ I_M &= -I_{M'} \quad . \end{aligned} \tag{4-21}$$

Combining Eqns. (4-14), (4-15), (4-16), (4-18), and (4-21), and choosing 0' as zero potential reference node, the set of linearly independent current equations which describes the passive part of the network N are formulated as

$$\begin{bmatrix} J \\ I_A \\ -I_A \\ 0 \\ 0 \\ 0 \\ 0 \\ I_W \\ -I_W \end{bmatrix} = \begin{bmatrix} Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0M} & 0 & 0 & 0 & 0 \\ Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} & 0 & 0 & 0 & 0 \\ Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} & 0 & 0 & 0 & 0 \\ Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KM} & -P & 0 & 0 & 0 \\ Y_{M0} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM}+R & 0 & -R & 0 & 0 \\ 0 & 0 & 0 & -P & 0 & Y_{K'K'}+P & Y_{K'M'} & Y_{K'W'} & Y_{K'W'} \\ 0 & 0 & 0 & 0 & -R & Y_{M'K'} & Y_{M'M'}+R & Y_{M'W} & Y_{M'W'} \\ 0 & 0 & 0 & 0 & 0 & Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\ 0 & 0 & 0 & 0 & 0 & Y_{W'K'} & Y_{W'M'} & Y_{W'W} & Y_{W'W'} \end{bmatrix} \begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K \\ V_M \\ V_{K'} \\ V_{M'} \\ V_W \\ V_{W'} \end{bmatrix} \tag{4-22}$$

In order to introduce the controlling variables into the equations, the following nonsingular transformation of variables is applied.

$$\begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K \\ V_M \\ V_{K'} \\ V_{M'} \\ V_W \\ V_{W'} \end{bmatrix} = \begin{bmatrix} U & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & U & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & U & 0 & U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & U & 0 & U & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & U & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & U & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & U \end{bmatrix} \begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K - V_{K'} \\ V_M - V_{M'} \\ V_{K'} \\ V_{M'} \\ V_W \\ V_{W'} \end{bmatrix} \tag{4-23}$$

Thus, Eqn. (4-22) is transformed into

$$\begin{bmatrix} J \\ I_A \\ -I_A \\ 0 \\ 0 \\ 0 \\ 0 \\ I_W \\ -I_W \end{bmatrix} = \begin{bmatrix} Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0M} & Y_{0K} & Y_{0M} & 0 & 0 \\ Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} & Y_{AK} & Y_{AM} & 0 & 0 \\ Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} & Y_{A'K} & Y_{A'M} & 0 & 0 \\ Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KM} & Y_{KK} & Y_{KM} & 0 & 0 \\ Y_{M0} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM}+R & Y_{MK} & Y_{MM} & 0 & 0 \\ 0 & 0 & 0 & -P & 0 & Y_{K'K'} & Y_{K'M'} & Y_{K'W} & Y_{K'W'} \\ 0 & 0 & 0 & 0 & -R & Y_{M'K'} & Y_{M'M'} & Y_{M'W} & Y_{M'W'} \\ 0 & 0 & 0 & 0 & 0 & Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\ 0 & 0 & 0 & 0 & 0 & Y_{W'K'} & Y_{W'M'} & Y_{W'W} & Y_{W'W'} \end{bmatrix} \begin{bmatrix} V_0 \\ V_A \\ V_{A'} \\ V_K - V_{K'} \\ V_M - V_{M'} \\ V_{K'} \\ V_{M'} \\ V_W \\ V_{W'} \end{bmatrix} . \tag{4-24}$$

which describes the passive part of N and includes all of the controlling variables as unknowns.

Now it is necessary to introduce the active network constraints into the equations. In accordance with the four types of dependent sources given previously, four cases are now considered.

Case 1. Type-A Controlled Sources.

A type-A controlled source is a voltage controlled current source in which the output current is proportional to the voltage across the controlling element. Thus, a set of constraint equations can be written in the form

$$I_A = Q\ (V_K - V_{K'}) \quad , \tag{4-25}$$

and

$$I_W = S\ (V_M - V_{M'}) \quad . \tag{4-26}$$

In Eq. (4-25), Q is a k x k diagonal matrix of transmittances associated with the dependent sources in N_1:

$$Q = \text{diag.}\ (\alpha_1, \alpha_2, \ldots., \alpha_k) \quad . \tag{4-27}$$

In Eq. (4-26), S is an (n-k) x (n-k) diagonal matrix of transmittances associated with the dependent sources in N_2:

$$S = \text{diag.}\ (\alpha_{k+1}, \alpha_{k+2}, \ldots., \alpha_n) \quad . \tag{4-28}$$

Using the procedure which was applied to the case where a seg was not explicitly shown, the equations for N are found to be

$$\begin{bmatrix} 0 \\ 0 \\ J \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -Q^{-1} & 0 & 0 & 0 & 0 & U & 0 & 0 & 0 & 0 & 0 \\ 0 & -S^{-1} & 0 & 0 & 0 & 0 & U & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{OO} & Y_{OA} & Y_{OA'} & Y_{OK} & Y_{OM} & Y_{OK} & Y_{OM} & 0 & 0 \\ -U & 0 & Y_{AO} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} & Y_{AK} & Y_{AM} & 0 & 0 \\ U & 0 & Y_{A'O} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} & Y_{A'K} & Y_{A'M} & 0 & 0 \\ 0 & 0 & Y_{KO} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KM} & Y_{KK} & Y_{KM} & 0 & 0 \\ 0 & 0 & Y_{MO} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM}+R & Y_{MK} & Y_{MM} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -P & 0 & Y_{K'K'} & Y_{K'M'} & Y_{K'W} & Y_{K'W'} \\ 0 & 0 & 0 & 0 & 0 & 0 & -R & Y_{M'K'} & Y_{M'M'} & Y_{M'W} & Y_{M'W'} \\ 0 & -U & 0 & 0 & 0 & 0 & 0 & Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\ 0 & U & 0 & 0 & 0 & 0 & 0 & Y_{W'K'} & Y_{W'M'} & Y_{W'W} & Y_{W'W'} \end{bmatrix} \begin{bmatrix} I_A \\ I_W \\ V_O \\ V_A \\ V_{A'} \\ V_K - V_{K'} \\ V_M - V_{M'} \\ V_{K'} \\ V_{M'} \\ V_W \\ V_{W'} \end{bmatrix} \tag{4-29}$$

Denote the coefficient matrix in Eqn. (4-29) by $\underline{C}_1$. It is assumed that $\underline{C}_1$ is nonsingular for finite value of transmittances. However, in the limiting case as Q and S approach infinity, $\underline{C}_1$ becomes

$$\lim_{\substack{Q\to\infty \\ S\to\infty}} \underline{C}_1$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & U & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & U & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{OO} & Y_{OA} & Y_{OA'} & Y_{OK} & Y_{OM} & Y_{OK} & Y_{OM} & 0 & 0 \\ -U & 0 & Y_{AO} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} & Y_{AK} & Y_{AM} & 0 & 0 \\ U & 0 & Y_{A'O} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} & Y_{A'K} & Y_{A'M} & 0 & 0 \\ 0 & 0 & Y_{KO} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KM} & Y_{KK} & Y_{KM} & 0 & 0 \\ 0 & 0 & Y_{MO} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM}+R & Y_{MK} & Y_{MM} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -P & 0 & Y_{K'K'} & Y_{K'M'} & Y_{K'W} & Y_{K'W'} \\ 0 & 0 & 0 & 0 & 0 & 0 & -R & Y_{M'K'} & Y_{M'M'} & Y_{M'W} & Y_{M'W'} \\ 0 & -U & 0 & 0 & 0 & 0 & 0 & Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\ 0 & U & 0 & 0 & 0 & 0 & 0 & Y_{W'K'} & Y_{W'M'} & Y_{WW} & Y_{W'W'} \end{bmatrix} . \tag{4-30}$$

The coefficient matrix $\underline{C}_1$ in the limiting case is seen to vanish by applying Laplace's Expansion according to minors formed from the first two partitioned rows. This gives

$$\det \left[\lim_{\substack{Q\to\infty \\ S\to\infty}} \underline{C}_1 \right]$$

$$= \begin{vmatrix} U & 0 \\ 0 & U \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & Y_{00} & Y_{0A} & Y_{0A'} & Y_{0K} & Y_{0M} & 0 & 0 \\ -U & 0 & Y_{A0} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} & 0 & 0 \\ U & 0 & Y_{A'0} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} & 0 & 0 \\ 0 & 0 & Y_{K0} & Y_{KA} & Y_{KA'} & Y_{KK} & Y_{KM} & 0 & 0 \\ 0 & 0 & Y_{M0} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_{K'K'} & Y_{K'M'} & Y_{K'W} & Y_{K'W'} \\ 0 & 0 & 0 & 0 & 0 & Y_{M'K'} & Y_{M'M'} & Y_{M'W} & Y_{M'W'} \\ 0 & -U & 0 & 0 & 0 & Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\ 0 & U & 0 & 0 & 0 & Y_{W'K'} & Y_{W'M'} & Y_{W'W} & Y_{W'W'} \end{vmatrix} . \tag{4-31}$$

In Eqn. (4-31), the 4 x 4 partitioned submatrix at the lower right is recognized as the coefficient matrix of Eqn. (4-16), hence its rows sum to zero. In summing the last four partitioned rows, the two identity matrices in these rows also sum to zero. This gives a zero row in the minor of the determinant. Therefore,

$$\det \left[\lim_{\substack{Q\to\infty \\ S\to\infty}} \underline{C}_1 \right] = 0 \tag{4-32}$$

In comparing Eqn. (4-13) and (4-31) and their corresponding networks, it becomes clear that the only essential difference in the two cases is that in the latter, a seg of controlling elements is explicitly included in the description. The conclusion is that not only is the coefficient matrix singular in the second case, but that it is singular as a direct result of the seg of controlling elements.

Network containing the other active source types are now examined.

Case 2. Type-B Controlled Sources.

A type-B controlled source is a current-controlled voltage source in which the output voltage is proportional to the current flowing through the controlling element. Thus, the set of constraint equations can be written in the form

$$V_A - V_{A'} = Q\ P\ (V_K - V_{K'}) \quad , \qquad (4\text{-}33)$$

and

$$V_W - V_{W'} = S\ R\ (V_M - V_{M'}) \quad , \qquad (4\text{-}34)$$

where P, R, Q, and S have been previously defined.

Combining Eqn. (4-24), the description of the passive part of the network, with these new constraint equations gives

$$
\begin{bmatrix} 0 \\ 0 \\ J \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & -P^{-1}Q^{-1} & P^{-1}Q^{-1} & U & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & U & 0 & 0 & -R^{-1}S^{-1} & R^{-1}S^{-1} \\
0 & 0 & Y_{OO} & Y_{OA} & Y_{OA'} & Y_{OK} & Y_{OM} & Y_{OK} & Y_{OM} & 0 & 0 \\
-U & 0 & Y_{AO} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} & Y_{AK} & Y_{AM} & 0 & 0 \\
U & 0 & Y_{A'O} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} & Y_{A'K} & Y_{A'M} & 0 & 0 \\
0 & 0 & Y_{KO} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KM} & Y_{KK} & Y_{KM} & 0 & 0 \\
0 & 0 & Y_{MO} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM}+R & Y_{MK} & Y_{MM} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -P & 0 & Y_{K'K'} & Y_{K'M'} & Y_{K'W} & Y_{K'W'} \\
0 & 0 & 0 & 0 & 0 & 0 & -R & Y_{M'K'} & Y_{M'M'} & Y_{M'W} & Y_{M'W'} \\
0 & -U & 0 & 0 & 0 & 0 & 0 & Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\
0 & U & 0 & 0 & 0 & 0 & 0 & Y_{W'K'} & Y_{W'M'} & Y_{W'W} & Y_{W'W'}
\end{bmatrix}
\begin{bmatrix} I_A \\ I_W \\ V_O \\ V_A \\ V_{A'} \\ V_K - V_{K'} \\ V_M - V_{M'} \\ V_{K'} \\ V_{M'} \\ V_W \\ V_{W'} \end{bmatrix}
\tag{4-35}
$$

Denote the coefficient matrix in Eqn. (4-35) by $\underline{C}_2$. In the limiting case as Q and S approach infinity, $\underline{C}_2$ becomes

$$\lim_{\substack{Q\to\infty \\ S\to\infty}} \underline{C}_2$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & U & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & U & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{OO} & Y_{OA} & Y_{OA'} & Y_{OK} & Y_{OM} & Y_{OK} & Y_{OM} & 0 & 0 \\ -U & 0 & Y_{AO} & Y_{AA} & Y_{AA'} & Y_{AK} & Y_{AM} & Y_{AK} & Y_{AM} & 0 & 0 \\ U & 0 & Y_{A'O} & Y_{A'A} & Y_{A'A'} & Y_{A'K} & Y_{A'M} & Y_{A'K} & Y_{A'M} & 0 & 0 \\ 0 & 0 & Y_{KO} & Y_{KA} & Y_{KA'} & Y_{KK}+P & Y_{KM} & Y_{KK} & Y_{KM} & 0 & 0 \\ 0 & 0 & Y_{MO} & Y_{MA} & Y_{MA'} & Y_{MK} & Y_{MM}+R & Y_{MK} & Y_{MM} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -P & 0 & Y_{K'K'} & Y_{K'M'} & Y_{K'W} & Y_{K'W'} \\ 0 & 0 & 0 & 0 & 0 & 0 & -R & Y_{M'K'} & Y_{M'M'} & Y_{M'W} & Y_{M'W'} \\ 0 & -U & 0 & 0 & 0 & 0 & 0 & Y_{WK'} & Y_{WM'} & Y_{WW} & Y_{WW'} \\ 0 & U & 0 & 0 & 0 & 0 & 0 & Y_{W'K'} & Y_{W'M'} & Y_{W'W} & Y_{W'W'} \end{bmatrix} \tag{4-36}$$

The resulting matrix is exactly the same as was given for type-A controlled sources in Eqn. (4-30). Therefore, the same conclusions are applicable to type-B controlled sources.

Case 3. Type-C Controlled Sources.

A type-C controlled source is a voltage-controlled voltage source in which the output voltage is proportional to the voltage variation across the controlling element. Thus, the set of constraint equations can be written in the form

$$V_A - V_{A'} = Q\ (V_K - V_{K'}) \quad , \tag{4-37}$$

and

$$V_W - V_{W'} = S\ (V_M - V_{M'}) \quad , \tag{4-38}$$

where Q and S are diagonal matrices of transmittances which were defined in Eqns. (4-27) and (4-28).

Evidently, this case is similar in every important respect to that of Case 2. The only difference is the absence of the scalar multiplication by the matrices of controlling elements, P and R in this case. This is apparent by comparing Eqns. (4-37) and (4-38) with Eqns. (4-33) and (4-34).

Case 4. Type-D Controlled Sources.

A type-D controlled source is a current-controlled current source in which the output current is proportional to the current flowing through the controlling element. The set of constraint equations for the network with this type of dependent sources are given below.

$$I_A = Q\ P\ (V_K - V_{K'}) \tag{4-39}$$

$$I_W = S\ R\ (V_M - V_{M'}) \tag{4-40}$$

Since these constraints differ from those of Case 1, Eqns. (4-25) and (4-26), only in scalar multipliers P and R, the result also holds for this case.

V. CONCLUSIONS

A set of equations describing a general active network were written and examined in the limiting case as all of the transmittances approached infinity. It was found that these equations had a unique solution in the limiting case only if the controlling elements did not constitute a seg of the network. Four types of dependent sources were considered individually, and the same result applied in each case regardless of the distribution of the sources in the network relative to the seg. These results make it possible to suggest a number of new problems and to make educated guesses concerning what the answers might be.

The first extension of this work should be to examine a network containing all four source types distributed arbitrarily relative to the seg. Since this would involve combining the equations used in this report in a straightforward manner, the same result is expected to apply to this more general case.

Another problem would involve examining the limiting case when only a subset of the controlling elements constitute a seg. The same physical reasoning which suggested the present investigation makes it reasonable to suppose the equations would be unsolvable also in this limiting case.

A very interesting problem which remains to be examined is suggested by the principle of duality. It is easy to show that there is a type of closure with respect to duality in the set of dependent source types considered. That is, the dual of any member of the set is itself a member of the set. The dual of the seg, called a cirk, has been well-established[9]. Just as the seg is a cut-set or disjoint union of cut-sets, the cirk is a circuit or disjoint union of circuits.

Suppose the equations of this report were changed by replacing every voltage variable by a current variable and conversely. Suppose also that the admittance matrices were interpreted as impedance matrices. Assuming that the dual network exists, one would expect the dual conclusion to hold. That is, the equations are not uniquely solvable in the limit if the network contains a cirk of controlling elements. The original result applies to planar and nonplanar networks. This dual argument would apply, of course, only to planar networks.

Another and more general approach would be to reformulate the equations with the dual condition in mind. An indefinite impedance matrix exists which provides an approach dual to the indefinite admittance matrix. Such a development might lead to a "dual" result for a nonplanar network. The difficulty of explicitly including the cirk elements in the equations might make this approach prohibitive. Since the dual argument has not been carefully explored, no definite conclusion on the dual condition can be made at this time.

Finally, these results should be examined to see what inferences are now possible in the general active network problem discussed in the introduction. The work of this report suggests that det F will vanish in the limiting case if the controlling elements constitute a seg. There is other evidence to support this conclusion[14]. However, the equations used in this report are not equivalent (related by a nonsingular transformation) to the equations resulting in the condition, det $[F - \Phi^{-1}] \neq 0$. The calculations in this report seem to indicate that the controlling variables must be included among the unknowns in order for the two approaches to give equivalent results. The reason for this remains obscure.

VI. REFERENCES

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A NECESSARY CONDITION FOR UNIQUE SOLVABILITY
OF INFINITE-TRANSMITTANCE ACTIVE NETWORK
EQUATIONS

by

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A necessary condition is presented to ensure that a uniquely solvable set of active network equations remains uniquely solvable in the limit as the transmittances approach infinity. This condition is that the set of controlling elements imbedded in the network does not constitute a seg of the network.

The problem is approached by first formulating a set of equations for an active network in which the controlling elements are not constrained to constitute a seg. In the limit as the transmittances approach infinity, it is not possible to demonstrate that the coefficient matrix is either singular or nonsingular. No definite conclusion can thus be reached.

Equations are then formulated for a connected active network in which a seg of controlling elements is explicitly shown. The coefficient matrix is assumed to be nonsingular for finite value of transmittances. In the limiting case, the coefficient matrices of the equations for each of the four possible types of dependent sources are clearly found to become singular. The singularity is shown to follow directly from the presence of the seg of controlling elements.