A STUDY OF UNSTEADY DISCHARGE AND INFLOW EFFECTS ON THE STATES INSIDE A VESSEL WITH AND WITHOUT HEAT TRANSFER FOR PERFECT GASES

by

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CHAPTER I

INTRODUCTION

The discharge of a compressible fluid from its pressurized container into an environment, such as the atmosphere, and the inflow of the fluid from the outside to a vessel, are frequently encountered in engineering applications. The discharge of combustion products from their combustion chamber; the discharge of fuel from its pressurized tank; and flow processes in refrigerating and heating are typical ones.

The state of the fluid inside a vessel may change as a result of heat flow and/or mass flow between the fluid and the surroundings. Heat may be exchanged between the vessel itself and the fluid it contains. For mass transfer, the flow rate to or from a vessel is ordinarily dependent on the pressure difference in the duct and the configuration of the duct that carries the fluid into or out of the vessel. A converging nozzle or a pipe is usually used for the discharge purpose and the kinetic energy is often neglected in the analyses of inflow problems. The type of the inflow duct is, therefore, immaterial.

The procedures as demonstrated herein for the discharge process provide a way to predict the conditions of a perfect gas inside a vessel from the measurable elapsed time. Also, the conditions can be predicted from the total amount of mass discharged if the outflow gas can be weighed without any difficulties. As to the inflow process, the way of prediction of states inside a vessel, based on the total amount of heat transferred and/or inflow gas, is demonstrated.

CHAPTER II

THE PROBLEM

Nomenclature

English alphabets:

A	Cross sectional area, ft^2
$\mathcal{C}$	Integral constant
C	Polytropic constant defined by $Pv^n = C$
c_p	Specific heat at constant pressure, Btu/lbm ^oF
c_v	Specific heat at constant volume, Btu/lbm ^oF
F	Frictional work per unit mass, Btu/lbm
g	Gravitational acceleration, 32.174 ft/sec^2
g_c	Dimensional constant, 32.174 lbm-ft/ lbf/sec^2
h	Enthalpy per unit mass, Btu/lbm
J	Joule's unit converging factor, 778 ft-lbf/Btu
k	c_p/c_v
l	Height of a vessel, ft
M	$(n + k)/2nk - 1$, dimensionless
m, m_c	Total mass inside a vessel, lbm
$\dot{m}$	mass flow rate, lbm/sec
N	$(k - 1)/k$, dimensionless
n	Polytropic exponent
P	Pressure, psia.
Q	Heat, Btu

q	Same as Q, except for a unit mass, Btu/lbm
R	Gas constant, ft-lbm/lbm ^{o}R
S	Exponent in binomial integral
S_t	Allowable tensile stress, lbf/in^2
T	Temperature, ^{o}R
t_I, t_c	Thickness of a vessel, inches
U	Internal energy of a system, Btu
u	Same as U, except for a unit mass, Btu/lbm
V	Velocity, ft/sec Capacity of a vessel, ft^3 Volume of the material of a vessel, ft^3
$\mathcal{V}$	Specific volume, ft^3/lbm
$W_{s,out}$	Rotary shaft work out per unit mass, Btu/lbm
X	$(1 - P^{-N})^{\frac{1}{2}}$
Y	$(P^N - \beta)^{\frac{1}{2}}$
Z	Elevation, ft

Greek letters:

α	$\left[\dfrac{2 g_c k}{C^{\frac{1}{n}} (k-1)} Pe^{\frac{2}{k}} \right]^{\frac{1}{2}}$
β	$Pe^{\frac{k-1}{k}}$
γ	$V/(Ae \alpha n C^{\frac{1}{n}})$
ε	$(m_i c_v)/(m_t c_{pt})$
θ	Elapsed time, sec

$$\psi \quad \left[g_c \; k \; (\frac{2}{k+1})^{\frac{k+1}{k-1}} \right]^{\frac{1}{2}}$$

Subscripts:

Numerical subscript: representing that particular node

e : Representing a gas condition at exits

b : Representing an ambient condition

* : Representing a critical condition

i : Representing an initial condition inside a vessel

in : Representing an inflow gas condition

t : Representing a condition of the material wall of a vessel

Statement of the Problem

The Discharge Process. A vessel of fixed capacity with a converging nozzle is initially filled with a perfect gas. Suddenly the gas is allowed to escape through the nozzle into the atmosphere. As a result, the state within the vessel changes continuously as long as the inside gas is being discharged. The effect of heat exchange between the gas inside the vessel and the surroundings is taken into account.

The relation between the pressure change and the elapsed time, due to mass flowing out, is first to be determined, and a numerical example is to be shown.

The Inflow Process. A theoretical analysis on the inflow problem, in which a heat flow-mass relation and a heat flow-pressure relation are considered, is demonstrated. Also a numerical example for this case is presented.

The equations thus derived enable one to predict the instantaneous state of a perfect gas inside a vessel.

CHAPTER III

ILLUSTRATIVE PROBLEMS

1. Fundamental Governing Equations for Fluid Flow.

The basic equations of compressible fluid flow, known as the energy equation and the equation of continuity, can be derived by considering two arbitrary coordinates normal to the flow stream, where the conservations of both mass and energy have to be satisfied.

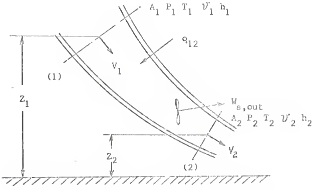

Fig. 1. Control surfaces for mass and energy balances.

Starting with the assumption of the flow being steady and one dimensional, one can make the mass balance between the sections (1) and (2) of Fig. 1 from the condition that the instantaneous mass flow rate across the sections must be identical. Thus

$$\frac{A_1 V_1}{\mathcal{V}_1} = \frac{A_2 V_2}{\mathcal{V}_2} = \ldots\ldots\ldots\ldots = \frac{A\ V}{\mathcal{V}} = \dot{m} = \text{Constant} \quad (1)$$

or in its differential form

$$\frac{dA}{A} + \frac{dV}{V} - \frac{d\mathcal{V}}{\mathcal{V}} = 0 \qquad (1a)$$

The energy balance for fluids flowing in steady motion, as shown in Fig. 1, is

$$q_{12} = (u_2 - u_1) + (V_2^2 - V_1^2)/2g_c J + (Z_2 - Z_1)\, g/g_c J + (P_2 \mathcal{V}_2 - P_1 \mathcal{V}_1)/J + Ws{,}out/J \qquad (2)$$

or by the definition $h = u + P\mathcal{V}/J$,

$$q_{12} = (h_2 - h_1) + (V_2^2 - V_1^2)/2g_c J + (Z_2 - Z_1)\, g/g_c J + Ws{,}out/J \qquad (2a)$$

or in its differential form

$$dq = dh + VdV/g_c J + dZ \cdot g/g_c J + dWs{,}out/J \qquad (2b)$$

Equation (2b) applies both to frictional and frictionless flow. For flow with friction

$$q_{12} = (u_2 - u_1) + \int_1^2 P\,d\mathcal{V}/J - F/J \qquad (3)$$

where

$$\int_1^2 P\,d\mathcal{V}/J = (P_2 \mathcal{V}_2 - P_1 \mathcal{V}_1)/J - \int_1^2 \mathcal{V}\,dP/J \qquad (4)$$

From (3) (4), and (2), one gets

$$(V_2^2 - V_1^2)/2g_c \cdot J + (Z_2 - Z_1)g/g_c \cdot J + W_{s,out}/J = -\int_1^2 \mathcal{V}\, dP/J - F/J \qquad (5)$$

or

$$V dV/g_c \cdot J + dZ\, g/g_c \cdot J + d\, W_{s,out}/J + dF/J + \mathcal{V}\, dP/J = 0 \qquad (5a)$$

The equations thus derived are to be applied to solve the discharge and the inflow problems herein. Also, the equation of state, $PV = mRT$, is introduced in solving the problems related to a perfect gas.

2. Unsteady Discharge of a Perfect Gas from a Vessel Through a Converging Nozzle.

(i) Reversible Adiabatic Flow of a Perfect Gas Through a Converging Nozzle.

Equation (2b) can be applied to this case and is simplified as

$$V dV / g_c J = - dh \tag{6}$$

Substituting $dh = c_p dT = \frac{kR}{J(k-1)} dT$ into equation (6) and integrating between the two sections, one obtains

$$V_e = \left[\frac{2 g_c kRT}{k-1} \left(1 - \frac{T_e}{T}\right) + V^2 \right]^{\frac{1}{2}} \tag{7}$$

By the relations $\frac{T_e}{T} = \left(\frac{P_e}{P}\right)^{\frac{k-1}{k}}$ and $P \mathcal{V} = RT$, and neglecting V, one gets

$$V_e = \left\{ \frac{2 g_c kP\mathcal{V}}{k-1} \left[1 - \left(\frac{P_e}{P}\right)^{\frac{k-1}{k}} \right] \right\}^{\frac{1}{2}} \tag{7a}$$

The mass flow rate $\dot{m}$, from equation (1), is

$$\dot{m} = \frac{A_e V_e}{\mathcal{V}_e} = A_e \left\{ \frac{2 g_c kP\mathcal{V}}{(k-1)\,\mathcal{V}_e^2} \left[1 - \left(\frac{P_e}{P}\right)^{\frac{k-1}{k}} \right] \right\}^{\frac{1}{2}} \tag{8}$$

or

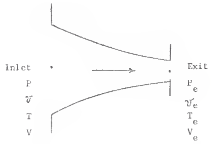

Fig. 2. Discharge of fluid through a converging nozzle.

$$\dot{m} = A_e \left\{ \frac{2g_c k}{k-1} \frac{P}{\mathcal{V}} \left[\left(\frac{P_e}{P}\right)^{\frac{2}{k}} - \left(\frac{P_e}{P}\right)^{\frac{k+1}{k}} \right] \right\}^{\frac{1}{2}} \tag{8a}$$

When the flow at the exit is in the critical condition the pressure ratio, P_e/P, becomes P^*/P. Here P^*/P is called the critical pressure ratio and is defined as the ratio of the pressure at a section of maximum flow rate in the nozzle to the pressure at the nozzle inlet. As a result, P^*/P is obtained by the condition $d\dot{m}/d(P_e/P) = 0$, thus

$$\frac{P_e}{P} = \frac{P^*}{P} = \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}} \tag{9}$$

and

$$\frac{T^*}{T} = \left(\frac{P^*}{P}\right)^{\frac{k-1}{k}} = \frac{2}{k+1} \tag{10}$$

$$\frac{\mathcal{V}^*}{\mathcal{V}} = \left(\frac{P}{P^*}\right)^{\frac{1}{k}} = \left[\frac{k+1}{2}\right]^{\frac{1}{k-1}} \tag{11}$$

Substituting (9) and (11) into (7a), one obtains

$$V^* = \left[\frac{2g_c k}{k+1} P\mathcal{V}\right]^{\frac{1}{2}} = \left[\frac{2g_c k}{k+1} RT\right]^{\frac{1}{2}} = \left[g_c k R T^*\right]^{\frac{1}{2}} \tag{12}$$

The mass flow rate, $\dot{m}$, is therefore

$$\dot{m} = \frac{A^* V^*}{\mathcal{V}^*} = A^* \psi \sqrt{\frac{P}{\mathcal{V}}} \tag{13}$$

where

$$\psi = \left(\frac{2g_c k}{k+1}\right)^{\frac{1}{2}} \Bigg/ \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} = \left[g_c k \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]^{\frac{1}{2}} \tag{14}$$

From an inspection of equations (8a) and (13) one realizes that the flow rate is a function of the inlet condition and the pressure ratio when k is specified, and is only a function of the inlet condition if the outlet of the nozzle is at a critical condition.

(ii) Reversible Diabatic Process Within a Vessel with Choking at the Nozzle Exit ($P_b \leq P_e = P^*$).

It is permissible to assume that the state at the nozzle inlet is identical with that inside the vessel at any point of time. In addition, if the process within the vessel follows $P\mathcal{V}^n = C$, where n and C are constants, then one can substitute this relation into equation (13). Therefore

$$\dot{m} = \frac{A^* \Psi}{C^{\frac{1}{2n}}} P^{\frac{n+1}{2n}} \tag{15}$$

or

$$dm = \frac{A^* \Psi}{C^{\frac{1}{2n}}} P^{\frac{n+1}{2n}} \, d\theta \tag{16}$$

But

$$m_c = \frac{V}{\mathcal{V}} \quad \text{and} \quad \mathcal{V} = (C/P)^{\frac{1}{n}}$$

Differentiating on both sides,

$$dm_c = V d\left(\frac{1}{\mathcal{V}}\right) = \frac{V}{nC^{\frac{1}{n}}} P^{\frac{1-n}{n}} \, dP \tag{17}$$

Since $\quad dm = -dm_c$

therefore

$$d\theta = -\frac{V}{C^{\frac{1}{2n}} n A^{*} \psi} P^{\frac{1-3n}{2n}} dP \tag{18}$$

or

$$\theta = -\frac{2V}{C^{\frac{1}{2n}} A^{*} \psi (1-n)} P^{\frac{1-n}{2n}} + \mathcal{C} \tag{18a}$$

The integration constant is obtained by using the boundary condition $\theta = 0$ at $P = P_i$. Thus

$$\theta = \frac{2V}{(n-1) A^{*} \psi \sqrt{RT_i}} \left[\left(\frac{P_i}{P}\right)^{\frac{n-1}{2n}} - 1 \right] \quad \text{for} \quad n \neq 1 \tag{19}$$

and

$$\theta = \frac{V}{A^{*} \psi \sqrt{RT_i}} \ln \frac{P_i}{P} \qquad \text{for} \quad n = 1 \tag{20}$$

The mass in the vessel at any state can be expressed in terms of the initial mass, m_i, and the pressure ratio in the vessel.

$$m = \frac{V}{\mathcal{V}} = \frac{V}{\left(\frac{P_i \mathcal{V}_i^n}{P}\right)^{\frac{1}{n}}} = m_i \left(\frac{P}{P_i}\right)^{\frac{1}{n}} \tag{21}$$

The net mass discharged is

$$\Delta m = m_i - m = m_i \left[1 - \left(\frac{P}{P_i}\right)^{\frac{1}{n}} \right] \tag{21a}$$

Similarly,

$$\mathcal{V} = \mathcal{V}_i \left(\frac{P_i}{P}\right)^{\frac{1}{n}} \tag{22}$$

$$T = T_i \left(\frac{P_i}{P}\right)^{\frac{1-n}{n}} \tag{23}$$

Heat added to the substance inside a vessel, from the first law of thermodynamics, is equal to the sum of the increase in internal energy of the substance and the energy flow out accompanying the discharged mass. Mathematically,

$$Q_{in} = (U - U_i) - \int_{m_i}^{m} h\,dm_c$$

$$= (c_v mT - c_v m_i T_i) - \int_{m_i}^{m} c_p T\,dm_c \tag{24}$$

Substituting equations (17) (18) and (23) into (24), one obtains

$$Q_{in} = \frac{c_v V}{R}\,[P - P_i] - \frac{c_p V}{nR}\,[P - P_i]$$

$$= \frac{(P_i - P)V}{R}\,\left[\frac{c_p}{n} - c_v\right]$$

$$= \frac{(P_i - P)V}{J\ (k - 1)}\,\left[\frac{k}{n} - 1\right] \tag{25}$$

or

$$Q_{in} = \frac{(P_i - P)V}{J} \qquad \text{for} \quad n = 1 \tag{25a}$$

$$Q_{in} = 0 \qquad \text{for} \quad n = k \tag{25b}$$

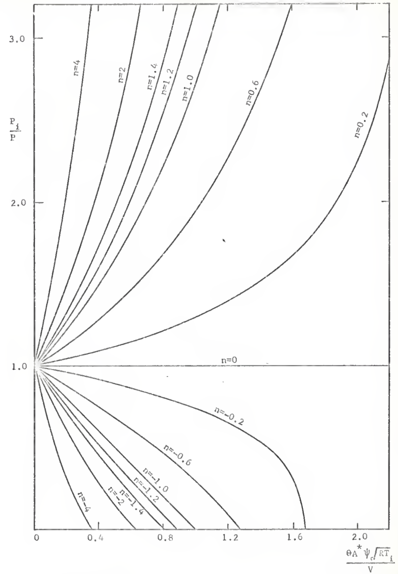

Fig. 3. Time parameter versus pressure ratio for sonic discharge of a perfect gas.

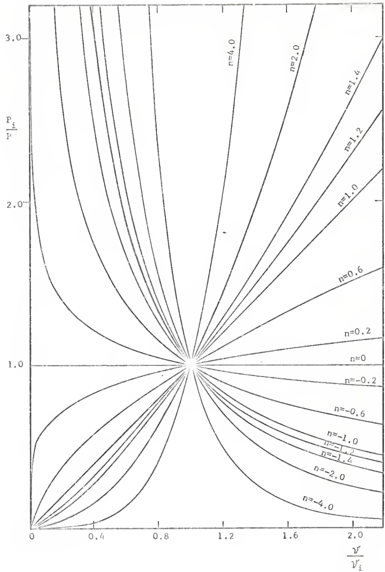

Fig. 4. Specific volume ratio versus pressure ratio for sonic discharge of a perfect gas.

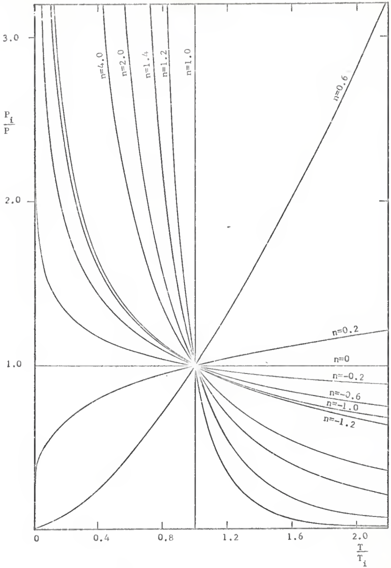

Fig. 5. Temperature ratio versus pressure ratio for sonic discharge of a perfect gas.

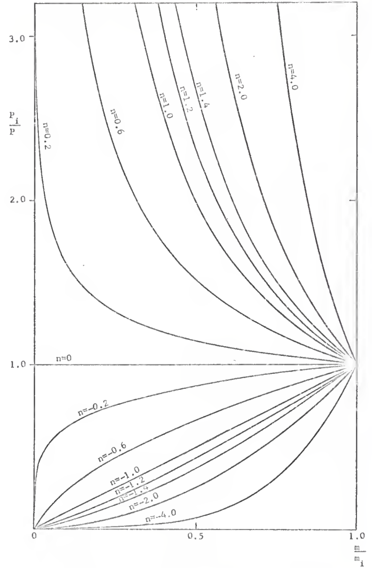

Fig. 6. Mass ratio versus pressure ratio for sonic discharge of a perfect gas.

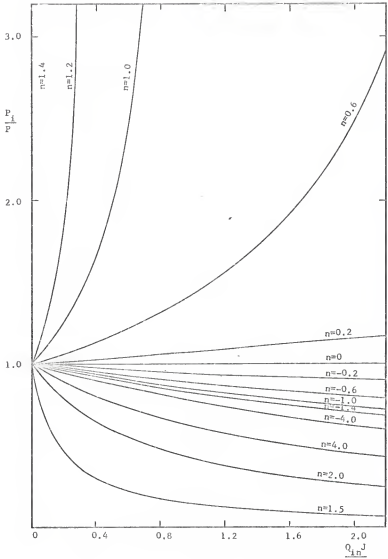

Fig. 7-A. Heat-in parameter versus pressure ratio for sonic discharge of a perfect gas.

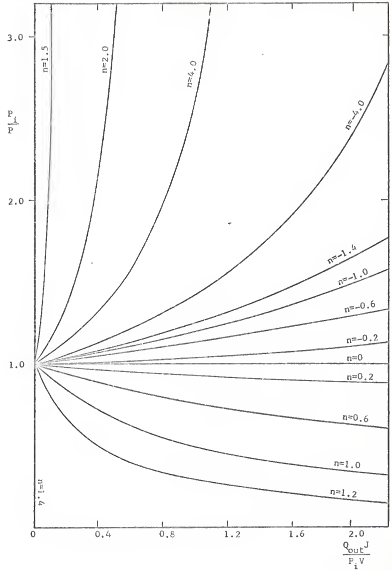

Fig. 7-B. Heat-out parameter versus pressure ratio for sonic discharge of a perfect gas.

Also, equation (25) gives the following information:

When

$P_i > P$

heat is added if $n < k$

heat is out if $n > k$

When

$P_i < P$

heat is added if $n > k$

heat is out if $n < k$.

The equations thus derived are summarized in Fig. 3 to Fig. 7-B.

(iii) Reversible Diabatic Process Within a Vessel with Subsonic Flow at the Nozzle Exit ($P_b = P_e > P^*$).

Assume the gas within the vessel follows $P\mathcal{V}^n = C$. Equation (8a) therefore becomes

$$\dot{m} = A_e \left\{ \frac{2g_c\, k}{C^{\frac{1}{n}}\,(k-1)} P_e^{\frac{2}{k}} P^{\frac{k-n}{nk}} \left[P^{\frac{k-1}{k}} - P_e^{\frac{k-1}{k}} \right] \right\}^{\frac{1}{2}} \tag{26}$$

or

$$\dot{m} = A_e \,\alpha\, P^{\frac{k-n}{2nk}} \left[P^{\frac{k-1}{k}} - \beta \right]^{\frac{1}{2}} \tag{27}$$

where

$$\alpha = \left[\frac{2g_c\, k}{C^{\frac{1}{n}}\,(k-1)} P_e^{\frac{2}{k}} \right]^{\frac{1}{2}} \text{, and } \beta = P_e^{\frac{k-1}{k}}$$

The mass flowing out, dm, during time $d\theta$ is then

$$dm = A_e \alpha \, P^{\frac{k-n}{2nk}} \left[P^{\frac{k-1}{k}} - \beta \right]^{\frac{1}{2}} d\theta \qquad (28)$$

But

$$m_c = \frac{V}{\mathcal{V}}, \quad \text{and} \quad \mathcal{V} = \left(\frac{C}{P}\right)^{\frac{1}{n}}$$

or

$$dm_c = \frac{V}{n\,C^{\frac{1}{n}}} P^{\frac{1-n}{n}} dP \qquad (17)$$

$$dm = -\,dm_c \qquad (18)$$

Equating (28) and (17) by the relation given in (18), one gets

$$d\theta = -\frac{V}{A_e \alpha \, n C^{\frac{1}{n}}} P^{\frac{n+k-2nk}{2nk}} \left[P^{\frac{k-1}{k}} - \beta \right]^{-\frac{1}{2}} dP \qquad (29)$$

or

$$\theta = -\gamma \int P^{\frac{n+k-2nk}{2nk}} \left[P^{\frac{k-1}{k}} - \beta \right]^{-\frac{1}{2}} dP + \mathcal{C} \qquad (30)$$

where

$$\gamma = \frac{V}{A_e \alpha \, nC^{\frac{1}{n}}} = \frac{m_i}{A_e \alpha \, nP_i^{\frac{1}{n}}}$$

Let $M = \frac{n + k - 2nk}{2nk}$, $N = \frac{k - 1}{k}$ and $S = -\frac{1}{2}$, then equation (30) becomes

$$\theta = -\gamma \int P^M (P^N - \beta)^S \, dP + C \qquad (31)$$

The general solution of the binomial integral in equation (31) is not available and even a particular solution, corresponding to the given constants k and n, is usually very involved. For approximation, the Simpson method may be applied in solving the integral. However, an exact solution can be obtained if and only if the condition that $\frac{M+1}{N} + S$ or $\frac{M+1}{N}$ is an integer is satisfied. For instance, if k = 1.4, then the condition becomes

$$\frac{M+1}{N} = \frac{7}{4\,n} + 1.25 = \text{Integer} \qquad (32)$$

or

$$\frac{M+1}{N} + S = \frac{7}{4\,n} + 0.75 = \text{Integer} \qquad (32a)$$

A number of values of n which will satisfy (32) or (32a) are listed in Tables 1 and 2. The Tables inform the possible region of the exact solutions. Many solutions can be obtained in the region $1 \geq n \geq -1$, but no exact solution can be obtained if $7 < n < -7$. Some of the exact solutions corresponding to the given constants n and k (=1.4) are listed in Table 3.

(iv) Reversible Adiabatic Process Within a Vessel Containing Air with Subsonic Flow at the Nozzle Exit (n=k=1.4).

A reversible diabatic process, following $Pv^n = C$, becomes a reversible adiabatic process if the exponent n is replaced by k. Therefore, the pressure-time relation of a reversible adiabatic process inside a vessel can be obtained from the equation in Table 3, where the value of n is 1.4.

TABLE 1

VALUES OF n FOR EXACT SOLUTIONS OF EQUATION (31), TYPE 1

k=1.4

(N+1)/N	n	(N+1)/N	n
	-1.4000	0	-1.4000
1	-7.0000	-1	-0.7778
2	2.3333	-2	-0.5385
3	1.0000	-3	-0.4118
4	0.6364	-4	-0.3333
5	0.4667	-5	-0.2800
6	0.3684	-6	-0.2414
7	0.3043	-7	-0.2121
8	0.2593	-8	-0.1892
9	0.2258	-9	-0.1707
10	0.2000	-10	-0.1556
11	0.1795	-11	-0.1429
12	0.1628	-12	-0.1321
13	0.1498	-13	-0.1228
14	0.1373	-14	-0.1148
15	0.1273	-15	-0.1077
16	0.1186	-16	-0.1015
17	0.1111	-17	-0.0959
18	0.1045	-18	-0.0909
19	0.0986	-19	-0.0864
20	0.0933	-20	-0.0824
30	0.0609	-30	-0.0424
40	0.0452	-40	-0.0424
50	0.0359	-50	-0.0341
60	0.0298	-60	-0.0286
70	0.0255	-70	-0.0246
80	0.0222	-80	-0.0215
90	0.0197	-90	-0.0192
100	0.0177	-100	-0.0173
110	0.0161	-110	-0.0157
120	0.0147	-120	-0.0144
130	0.0136	-130	-0.0133
140	0.0126	-140	-0.0124
150	0.0118	-150	-0.0116

TABLE 2

VALUES OF n FOR EXACT SOLUTIONS OF EQUATION (31), TYPE 2

k=1.4

(M+1)/N+S	n	(M+1)/N+S	n
	-2.3333	0	-2.3333
1	7.0000	-1	-1.0000
2	1.4000	-2	-0.6364
3	0.7778	-3	-0.4667
4	0.5385	-4	-0.3684
5	0.4118	-5	-0.3043
6	0.3333	-6	-0.2593
7	0.2800	-7	-0.2258
8	0.2414	-8	-0.2000
9	0.2121	-9	-0.1795
10	0.1892	-10	-0.1628
11	0.1707	-11	-0.1489
12	0.1556	-12	-0.1373
13	0.1429	-13	-0.1273
14	0.1321	-14	-0.1186
15	0.1228	-15	-0.1111
16	0.1148	-16	-0.1045
17	0.1077	-17	-0.0986
18	0.1015	-18	-0.0933
19	0.0959	-19	-0.0886
20	0.0909	-20	-0.0843
30	0.0598	-30	-0.0569
40	0.0446	-40	-0.0429
50	0.0355	-50	-0.0345
60	0.0295	-60	-0.0288
70	0.0253	-70	-0.0247
80	0.0221	-80	-0.0217
90	0.0196	-90	-0.0193
100	0.0176	-100	-0.0174
110	0.0160	-110	-0.0158
120	0.0147	-120	-0.0145
130	0.0135	-130	-0.0134
140	0.0126	-140	-0.0124
150	0.0117	-150	-0.0116

TABLE 3

PRESSURE-TIME RELATIONS FOR SUBSONIC DISCHARGE k=1.4

n=7	$\theta = \frac{\gamma}{N}\left[\frac{X_i}{P_i^{-N}} - \frac{X}{P^{-N}}\right] + \frac{\beta\gamma}{2N}\left[\ln\left\lvert\frac{1+X_i}{1-X_i}\right\rvert - \ln\left\lvert\frac{1+X}{1-X}\right\rvert\right]$
n=2.3333	$\theta = \frac{2\gamma}{3N} Y_i^3 \left[1 - \left(\frac{Y}{Y_i}\right)^3\right] + \frac{2\beta\gamma}{N} Y_i \left[1 - \frac{Y}{Y_i}\right]$
n=1.4	$\theta = \frac{\gamma}{2N}\left[\frac{X_i}{P_i^{-2N}} - \frac{X}{P^{-2N}}\right] + \frac{3\beta\gamma}{4N}\left[\frac{X_i}{P_i^{-N}} - \frac{X}{P^{-N}}\right] + \frac{3\beta^2\gamma}{8N}\left[\ln\left\lvert\frac{1+X_i}{1-X_i}\right\rvert - \ln\left\lvert\frac{1+X}{1-X}\right\rvert\right]$
n=1	$\theta = \frac{2\gamma}{5N} Y_i^5 \left[1 - \left(\frac{Y}{Y_i}\right)^5\right] + \frac{4\beta\gamma}{3N} Y_i^3 \left[1 - \left(\frac{Y}{Y_i}\right)^3\right] + \frac{2\beta^2\gamma}{N} Y_i \left[1 - \frac{Y}{Y_i}\right]$
n=0.2	$\theta = \frac{\gamma}{N}\Big\{2\beta^9 [Y_i - Y] + 6\beta^8 [Y_i - Y]^3 + \frac{72}{5}\beta^7 [Y_i - Y]^5 + \frac{168}{7}\beta^6 [Y_i - Y]^7$ $+ \frac{252}{9}\beta^5 [Y_i - Y]^9 + \frac{252}{11}\beta^4 [Y_i - Y]^{11} + \frac{168}{13}\beta^3 [Y_i - Y]^{13} + \frac{72}{15}\beta^2 [Y_i - Y]^{15}$ $+ \frac{18}{17}\beta [Y_i - Y]^{17} + \frac{2}{19} [Y_i - Y]^{19}\Big\}$

TABLE 3 (Continued)

n=-0.2	$\theta = \frac{\gamma}{N\beta^8}\left\{2\,[X_i - X] - \frac{14}{3}\,[X_i^3 - X^3] + \frac{42}{5}\,[X_i^5 - X^5] - 10\,[X_i^7 - X^7] + \frac{70}{9}\,[X_i^9 - X^9] - \frac{42}{11}\,[X_i^{11} - X^{11}] + \frac{14}{13}\,[X_i^{13} - X^{13}] - \frac{2}{15}\,[X_i^{15} - X^{15}]\right\}$
n=-1	$\theta = \frac{2\gamma}{N\beta}\,X_i\,[1 - \frac{X}{X_i}]$
n=-1.4	$\theta = \frac{2\gamma}{N\beta^{\frac{1}{2}}}\,[\tan^{-1}\sqrt{\frac{P_i^N}{\beta} - 1} - \tan^{-1}\sqrt{\frac{P^N}{\beta} - 1}]$
n=-2.3333	$\theta = \frac{\gamma}{N}\,[\ln\left\|\frac{1 + X_i}{1 - X_i}\right\| - \ln\left\|\frac{1 + X}{1 - X}\right\|]$
n=-7	$\theta = \frac{2\gamma}{N}\,Y_i\,[1 - \frac{Y}{Y_i}]$
where	$X = [1 - \beta P^{-N}]^{\frac{1}{2}}$, $X_i = [1 - \beta P_i^{-N}]^{\frac{1}{2}}$
	$Y = [P^N - \beta]^{\frac{1}{2}}$, $Y_i = [P_i^N - \beta]^{\frac{1}{2}}$

$$\theta = \frac{\gamma}{2N}\left[\frac{X_i}{P_i^{-2N}} - \frac{X}{P^{-2N}}\right] + \frac{3\beta\gamma}{4N}\left[\frac{X_i}{P_i^{-N}} - \frac{X}{P^{-N}}\right]$$

$$+ \frac{3\beta^2\gamma}{8N}\left[\ln\left|\frac{1+X_i}{1-X_i}\right| - \ln\left|\frac{1+X}{1-X}\right|\right] \tag{33}$$

Let

$$\theta = \theta_1 + \theta_2 + \theta_3$$

where

$$\theta_1 = \frac{\gamma}{2N}\left[\frac{X_i}{P_i^{-2N}} - \frac{X}{P^{-2N}}\right]$$

$$\theta_2 = \frac{3\beta\gamma}{4N}\left[\frac{X_i}{P_i^{-N}} - \frac{X}{P^{-N}}\right]$$

$$\theta_3 = \frac{3\beta^2\gamma}{8N}\left[\ln\left|\frac{1+X_i}{1-X_i}\right| - \ln\left|\frac{1+X}{1-X}\right|\right]$$

Substituting $\beta = P_e^{\frac{k-1}{k}} = P_e^N$ and $X = [1 - \beta P^{-N}]^{\frac{1}{2}}$ into the above three equations, one obtains

$$\theta(\theta_1) = \left[\left(\frac{P_e}{P}\right)^{-4N} - \left(\frac{P_e}{P}\right)^{-3N}\right]^{\frac{1}{2}} = \left[\left(\frac{P_i}{P_e}\right)^{4N} - \left(\frac{P_i}{P_e}\right)^{3N}\right]^{\frac{1}{2}} - \frac{2N\theta_1}{\beta^2\gamma} \tag{34}$$

$$\theta(\theta_2) = \left[\left(\frac{P_e}{P}\right)^{-2N} - \left(\frac{P_e}{P}\right)^{-N}\right]^{\frac{1}{2}} = \left[\left(\frac{P_i}{P_e}\right)^{2N} - \left(\frac{P_i}{P_e}\right)^{N}\right]^{\frac{1}{2}} - \frac{4N\theta_2}{3\beta^2\gamma} \tag{35}$$

$$\theta(\theta_3) = \ln \frac{1 + [1 - (\frac{P_e}{P})^N]^{\frac{1}{2}}}{1 - [1 - (\frac{P_e}{P})^N]^{\frac{1}{2}}} = \ln \frac{1 + [1 - (\frac{P_i}{P_e})^{-N}]^{\frac{1}{2}}}{1 - [1 - (\frac{P_i}{P_e})^{-N}]^{\frac{1}{2}}} - \frac{8N\theta_3}{3\,\beta^2\gamma} \qquad (36)$$

Equations (34), (35) and (36) are plotted in Fig. 8. Once the pressure ratio P_e/P is specified, the corresponding values of the functions θ_1, θ_2 and θ_3 can be found immediately.

(v) Reversible Isothermal Process Within a Vessel Containing Air with Subsonic Flow at the Nozzle Exit (n=1).

Following the same procedures in the case (iv), one gets

$$\theta(\theta_1) = [(\frac{P_e}{P})^{-N} - 1]^{\frac{5}{2}} = [(\frac{P_i}{P_e})^{N} - 1]^{\frac{5}{2}} - \frac{5N\theta_1}{2\,\gamma\,\beta^{5/2}} \qquad (37)$$

$$\theta(\theta_2) = [(\frac{P_e}{P})^{-N} - 1]^{\frac{3}{2}} = [(\frac{P_i}{P_e})^{N} - 1]^{\frac{3}{2}} - \frac{3N\theta_2}{4\,\gamma\,\beta^{5/2}} \qquad (38)$$

$$\theta(\theta_3) = [(\frac{P_e}{P})^{-N} - 1]^{\frac{1}{2}} = [(\frac{P_i}{P_e})^{N} - 1]^{\frac{1}{2}} - \frac{N\theta_3}{2\,\gamma\,\beta^{5/2}} \qquad (39)$$

Equations (37), (38) and (39) are plotted in Fig. 9. The values of θ_1, θ_2 and θ_3 corresponding to a pressure ratio P_e/P can be found providing P_i, P_e, A_e, V_i and $\mathcal{V}_i$ are given. The total time elapsed, θ, is simply the sum of θ_1, θ_2 and θ_3.

Equations (37) and (38) can be neglected for an approximate solution. From the relation $\theta = \theta_1 + \theta_2 + \theta_3$, one gets

$$\theta \cong \theta_3 = \frac{2\gamma\beta^{5/2}}{N}\left[\left\{\left(\frac{P_i}{P_e}\right)^N - 1\right\}^{\frac{1}{2}} - \left\{\left(\frac{P_e}{P}\right)^{-N} - 1\right\}^{\frac{1}{2}}\right] \tag{39a}$$

(vi) Calculations on Heat Transfer.

Assume the tank is perfectly insulated, and the gas and the walls of the tank are at the same temperature at any instant. As the temperature of the material of the tank decreases from Tt_i to T_t, or T_i to T, heat is transfered from the tank walls into the gas within the tank as a result of the decrease in the internal energy of the walls. Mathematically,

$$d\,Q_{in} = -d\,U_t = -c_{pt}\,m_t\,dT_t = -c_{pt}\,m_t\,d\,T \tag{40}$$

But

$$\begin{aligned} d\,Q_{in} &= -h\,dm + d(mu) \\ &= -c_p\,T\,dm + m\,du + u\,dm \\ &= -c_p\,T\,dm + m\,c_v\,dT + c_v\,T\,dm \end{aligned} \tag{41}$$

Equating (40) and (41) yields

$$c_{pt}\,m_t\,dT = (c_p - c_v)T\,dm - m\,c_v\,dT \tag{42}$$

By separating variables, equation (42) can be integrated. The integral constant is determined subject to the condition $T = T_i$ at $m = m_i$. The final result is

$$\frac{T}{T_i} = \left(\frac{1 + \dfrac{m\,c_v}{m_t\,c_{pt}}}{1 + \dfrac{m_i\,c_v}{m_t\,c_{pt}}}\right)^{k-1} \tag{43}$$

Let $\mathcal{E} = \dfrac{m_i\,c_v}{m_t\,c_{pt}}$, then

$$\frac{T}{T_i} = \left(\frac{1 + (\frac{m}{m_i})\varepsilon}{1 + \varepsilon} \right)^{k-1} \qquad (43a)$$

For a perfect gas

$$\frac{T}{T_i} = \frac{P}{P_i} \; \frac{m_i}{m} \qquad (44)$$

Rearranging yields

$$\frac{P}{P_i} = \frac{m}{m_i} \left(\frac{1 + (\frac{m}{m_i})\varepsilon}{1 + \varepsilon} \right)^{k-1} \qquad (45)$$

Combining (44), (45) and (40), one gets

$$Q_{in} = - \Delta U_t = c_{pt} \, m_t \, T_i \left[1 - \left(\frac{1 + (\frac{m}{m_i})\varepsilon}{1 + \varepsilon} \right)^{k-1} \right] \qquad (46)$$

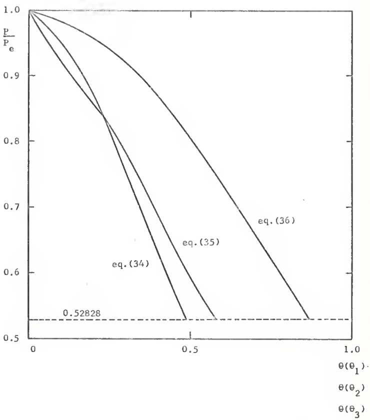

Fig. 8. Functions of time parameter versus pressure ratio for a subsonic discharge of air, $Q_{in}=0$ and $k=n=1.4$.

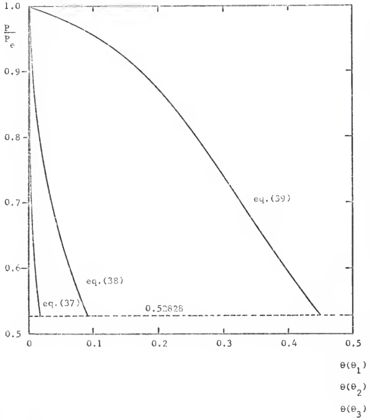

Fig. 9. Functions of time parameter versus pressure ratio for a subsonic discharge of air, T=constant and n=1.

3. Unsteady Inflow of a Perfect Gas to a Vessel with Constant Inflow Pressure, Temperature and Enthalpy.

(i) The Adiabatic Process and the Diabatic Process within a Vessel at an Initial Condition P_i and T_i.

The processes are irreversible because of the mixing of gases at different temperatures. Considering the vessel as a control volume and taking an energy balance, one comes to the result

$$
\begin{aligned}
Q_{in} &= (U - U_i) - (m - m_i)\, h_{in} \\
&= c_v\, m_i\, T_i \left[\frac{mT}{m_i T_i} - 1\right] - c_p\, m_i\, T_{in} \left[\frac{m}{m_i} - 1\right]
\end{aligned}
\tag{47}
$$

Substituting the perfect gas relation, PV = mRT, together with $c_v = R/J(k - 1)$ and $c_p = kR/J(k - 1)$ into equation (47), one gets

$$
Q_{in} = \frac{P_i V}{J(k - 1)} \left[\left(\frac{P}{P_i} - 1\right) - \frac{k\, T_{in}}{T_i} \left(\frac{P\, T_i}{P_i\, T} - 1\right)\right] \tag{48}
$$

If the process follows $P v^n = C$, equation (48) becomes

$$
Q_{in} = \frac{P_i V}{J(k - 1)} \left\{\left(\frac{P}{P_i} - 1\right) - \frac{k\, T_{in}}{T_i} \left[\left(\frac{P}{P_i}\right)^{\frac{1}{n}} - 1\right]\right\} \tag{49}
$$

For an adiabatic process within a vessel, $Q_{in} = 0$, the general equation (48) becomes

$$
\frac{dP}{P} = \frac{1}{1 - \frac{T}{k\, T_{in}}} \frac{dT}{T} \qquad (Q_{in} = 0) \tag{50}
$$

If the process still follows $P v^n = C$, the relation $\frac{P}{P_i} = \left(\frac{T}{T_i}\right)^{\frac{n}{n-1}}$ can be expressed in its differential form as

$$\frac{dP}{P} = \frac{n}{n-1} \frac{dT}{T} \tag{51}$$

Equating (50) and (51), one gets

$$\frac{n}{n-1} = \frac{1}{1 - \frac{T}{k\,T_{in}}} \tag{52}$$

Rearranging yields

$$n = \frac{k\,T_{in}}{T} \tag{52a}$$

The equation (52a) thus derived is subject to two conditions, namely, $Q_{in} = 0$ and $P\,\upsilon^n$ = a constant. When the process is isothermal or $n = 1$, the temperature inside the vessel remains constant at a value equal to $k \cdot T_{in}$. It also gives an information that T can not be equal to T_{in} because it will result in $n = k$, an irreversible, diabatic, isentropic process, which is inconsistent with the case as discussed here. As shown in the next section, $n = 1$ and $T/T_{in} = k$ applies for the case of a perfect gas flowing to an initially evacuated vessel in which there is no heat flow.

(ii) <u>Adiabatic Process Within a Vessel at an Initial Condition of Evacuation</u>.

Since $Q_{in} = 0$, $U_i = 0$ ($m_i = 0$), the energy equation (47) can be simplified as

$$mc_p\,T_{in} = m\,c_v\,T \tag{53}$$

or

$$\frac{T}{T_{in}} = \frac{c_p}{c_v} = k \tag{53a}$$

This gives an interesting result that for constant inlet temperature the states in the vessel are isothermal. Furthermore, one can find the relation between the mass flowing in and the pressure within the vessel from the perfect gas relation. Thus

$$\frac{V}{RT} = \frac{m}{P} = \text{Constant} \tag{54}$$

(iii) Diabatic Process Within a Vessel at an Initial Condition of Evacuation.

Since $U_i = 0$ ($m_i = 0$), the energy equation (47) is simplified as

$$Q_{in} = U - mh_{in} \tag{55}$$

or

$$Q_{in} = \frac{P\ V}{J(k-1)}\left[1 - k\,\frac{T_{in}}{T}\right] \tag{55a}$$

If the process follows $P\,v^n = C$, one can substitute the relation $T = T_i\,(\frac{P}{P_i})^{\frac{n-1}{n}}$ into equation (55a), thus

$$Q_{in} = \frac{P\ V}{J(k-1)}\left[1 - k\,\frac{T_{in}}{T_i(\frac{P}{P_i})^{\frac{n-1}{n}}}\right] \tag{56}$$

Equation (56) is applicable to the adiabatic process also. Substituting $Q_{in} = 0$ into equation (55a), one finds the condition $k = T/T_{in}$ has to be satisfied. Moreover, by comparing equation (55a) with equation (56), one finds $T = T_i$, if $n = 1$. The tank temperature remains constant in an adiabatic process if n is unity, and if the tank is initially evacuated.

4. Numerical Examples for the Discharge Problem.

Statement. A cylindrical tank having a volume of 100 ft^3 is initially filled with air at 100 psia and $600^{\circ}R$. Suddenly the air is allowed to escape to the atmosphere (14.7 psia) through a frictionless converging nozzle of one inch diameter.

(i) Tank Specifications

The size of the tank is determined on the condition of minimum wall material to be used for the fixed capacity. Since the configuration of the tank is assumed to be cylindrical, the size of the tank can be evaluated.

$$V = \pi r^2 \ell = 100$$

or $$\ell = 100/\pi r^2$$

$$A = 2\pi r^2 + 2\pi r \ell$$

$$\frac{dA}{dr} = 2\pi (2r - 100/\pi r^2) = 0$$

Solving for r, one gets

$$r = (50/\pi)^{\frac{1}{3}} = 2.515 \text{ ft.}$$

and $$\ell = 100/\pi r^2 = 5.031 \text{ ft.}$$

If the allowable tensile stress of the material being used is 18,000 psi, the thickness of the material

$$t_{\ell} = \frac{Pr}{2S_t} = \frac{100 \times 2.515 \times 12}{2 \times 18{,}000} \cong \frac{1''}{12}$$

and $$t_c = \frac{Pr}{S_t} = \frac{100 \times 2.515 \times 12}{18{,}000} \cong \frac{1''}{6}$$

One may select a steel plate having a thickness of ¼ inch.

(ii) Reversible Adiabatic Process Within the Tank ($n = k = 1.4$).

It is assumed that the geometry of the converging nozzle is fixed. The characteristic of this kind of nozzle indicates that the state at the nozzle exit is in the critical condition ($P_e = P^*$) whenever $P_b \leq P^*$. On the other hand, the state at the nozzle exit becomes subsonic with $P_e = P_b$ when $P_b > P^*$. Both critical flow and subsonic flow will exist in this problem as the tank pressure is reducing from 100 psia to 14.7 psia.

When $P_e = P_b = P^* = 14.7$ psia, the corresponding pressure in the tank is

$$P = \frac{14.7}{\left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}} = \frac{14.7}{0.52828} = 27.8261 \text{ psia}$$

Therefore, the regions are classified as

$100 \geq P \geq 27.8261$ Critical condition at exit

$27.8261 > P \geq 14.7$ Subsonic condition at exit

For the region $100 \geq P \geq 27.8261$, the following equations apply.

$$\theta = \frac{2V}{(k-1)\, A^* \psi \sqrt{RT_i}} \left[\left(\frac{P_i}{P}\right)^{\frac{k-1}{2k}} - 1\right] \tag{19}$$

$$\frac{P^*}{P} = \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}} \tag{9}$$

$$\dot{m} = \frac{A^* \psi}{C^{\frac{1}{2n}}} P^{\frac{k+1}{2k}} \tag{15}$$

$$\frac{T}{T_i} = \left(\frac{P}{P_i}\right)^{\frac{k-1}{k}} \quad (22) \qquad , \qquad \frac{\upsilon}{\upsilon_i} = \left(\frac{P_i}{P}\right)^{\frac{1}{k}} \quad (23)$$

For the region $27.8261 > P \geq 14.7$, the following equations apply.

$$\theta = \frac{\gamma}{2N}\left[\frac{(1-\beta P_i^{-N})^{\frac{1}{2}}}{P_i^{-2N}} - \frac{(1-\beta P^{-N})^{\frac{1}{2}}}{P^{-2N}}\right]$$

$$+ \frac{3\beta\gamma}{4N}\left[\frac{(1-\beta P_i^{-N})^{\frac{1}{2}}}{P_i^{-N}} - \frac{(1-\beta P^{-N})^{\frac{1}{2}}}{P^{-N}}\right] \tag{33}$$

$$\frac{T}{T_i} = \left(\frac{P}{P_i}\right)^{\frac{k-1}{k}} \tag{22}$$

$$\frac{\mathcal{V}}{\mathcal{V}_i} = \left(\frac{P_i}{P}\right)^{\frac{1}{k}} \tag{23}$$

$$\dot{m} = A_e \alpha \left[P^{\frac{k-1}{k}} - \beta\right]^{\frac{1}{2}} \tag{27}$$

The properties P, m, $\mathcal{V}$, and T of the air within the tank, and the elapsed time corresponding to the pressure reduction are evaluated by the above equations, and are expressed in Fig. 10 and Table 4. Also, by using the charts ranging from Fig. 3 to Fig. 8, the same answers can be expected.

TABLE 4

ANSWERS TO THE SELECTED DISCHARGE PROBLEM, $Q_{in} = 0$

P	P^*	P_e	θ	$\mathcal{V}$	T	$\dot{m}$
100	52.8281	52.8281	0.0000	2.2208	600.000	1.7058
95	50.1867	50.1867	0.9707	2.3037	591.271	1.6324
90	47.5453	47.5453	2.0016	2.3944	582.207	1.5585
85	44.9039	44.9039	3.1002	2.4942	572.777	1.4840
80	42.2625	42.2625	4.2752	2.6046	562.941	1.4088
75	39.6211	39.6211	5.5373	2.7275	552.655	1.3330
70	36.9797	36.9797	6.8995	2.8652	541.868	1.2565
65	34.3383	34.3383	8.3777	3.0210	530.515	1.1791
60	31.6969	31.6969	9.9919	3.1987	518.520	1.1009
55	29.0555	29.0555	11.7677	3.4039	505.789	1.0218
50	26.4141	26.4141	13.7385	3.6437	492.201	0.9412
45	23.7727	23.7727	15.9484	3.9285	477.605	0.8604
40	21.1312	21.1312	18.4587	4.2733	461.800	0.7777
35	18.4899	18.4899	21.3561	4.7010	444.514	0.6939
30	15.8484	15.8484	24.7704	5.2481	425.361	0.6078
27.8261	14.7	14.7	26.4641	5.5378	416.316	0.5698
26	-	14.7	+1.5454	5.8129	408.320	0.5360
24	-	14.7	+3.4010	6.1550	399.088	0.4941
22	-	14.7	+5.4849	6.5496	389.289	0.4453
20	-	14.7	+7.9003	7.0110	378.831	0.3864
18	-	14.7	+10.8633	7.5590	367.597	0.3111
16	-	14.7	+15.0377	8.2225	355.432	0.1996
14.7	-	14.7	+22.1562	8.7355	346.930	0.0000

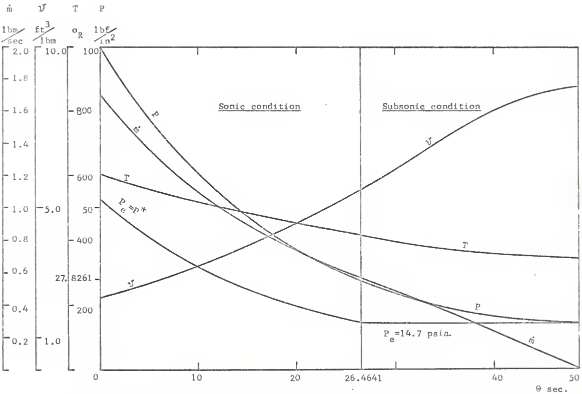

Fig. 10. The example of the discharge of air through a converging nozzle, $Q_{in} = 0$.

(iii) Reversible Isothermal Process Within the Tank (n = 1).

The two pressure regions still exist in this problem as the pressure is reduced from P_i to P_e.

For the region $100 \geq P \geq 27.8261$, the following equations apply.

$$\theta_{in} = \frac{(P_i - P)V}{J} \tag{25a}$$

$$\theta = \frac{V}{A^* \psi \sqrt{RT_i}} \ln \frac{P_i}{P} \tag{20}$$

$$\dot{m} = \frac{A^* \psi}{\sqrt{RT_i}} \cdot P \tag{15}$$

Also, equations (22), (23) and (9) apply.

For the region $27.8261 > P \geq 14.7$, the following equations apply.

From Table 3 (n = 1)

$$\theta = \frac{2\Gamma}{5N} [(P_i^N - \beta)^{\frac{5}{2}} - (P^N - \beta)^{\frac{5}{2}}]$$

$$+ \frac{4\beta\Gamma}{3N} [(P_i^N - \beta)^{\frac{3}{2}} - (P^N - \beta)^{\frac{3}{2}}]$$

$$+ \frac{2\beta^2\Gamma}{3N} [(P_i^N - \beta)^{\frac{1}{2}} - (P^N - \beta)^{\frac{1}{2}}]$$

$$\dot{m} = A_e \alpha P^{\frac{k-n}{2nk}} [P^N - \beta]^{\frac{1}{2}} \tag{27}$$

Also, equations $P\mathcal{V} = C$, and (25a) apply.

The results are summarized in Table 5, and are expressed in Fig. 11. The charts ranging from Fig. 3 to Fig. 9 are available for the calculation also.

TABLE 5

ANSWERS TO THE SELECTED DISCHARGE PROBLEM

$T = 600^{\circ}R = \text{Constant}$

P	P^*	P_e	θ	ϑ	Q_{in}	$\dot{m}$
100	52.8281	52.8281	0.0000	2.2208	0	1.7058
95	50.1867	50.1867	1.3540	2.3377	92.545	1.6205
90	47.5453	47.5453	2.7812	2.4678	185.090	1.5352
85	44.9039	44.9039	4.2901	2.6127	277.635	1.4499
80	42.2625	42.2625	5.8904	2.7760	370.180	1.3646
75	39.6211	39.6211	7.5941	2.9611	462.725	1.2793
70	36.9797	36.9797	9.4153	3.1726	555.270	1.1940
65	34.3383	34.3383	11.3715	3.4167	647.815	1.1088
60	31.6969	31.6969	13.4845	3.7014	740.360	1.0235
55	29.0555	29.0555	15.7814	4.0379	832.905	0.9382
50	26.4141	26.4141	18.2973	4.4417	925.450	0.8529
45	23.7727	23.7727	21.0786	4.9352	1017.995	0.7676
40	21.1312	21.1312	24.1877	5.5521	1110.540	0.6823
35	18.4899	18.4899	27.7126	6.3452	1203.085	0.5970
30	15.8484	15.8484	31.7818	7.4028	1295.630	0.5117
27.8261	14.7	14.7	33.7675	7.9811	1335.868	0.4747
26	–	14.7	+1.7934	8.5417	1368.666	0.4422
24	–	14.7	+3.9242	9.2535	1406.684	0.4030
22	–	14.7	+6.2886	10.0947	1443.702	0.3587
20	–	14.7	+8.9935	11.1042	1480.720	0.3071
18	–	14.7	+12.2638	12.3380	1517.738	0.2435
16	–	14.7	+16.7960	13.8802	1554.756	0.1536
14.7	–	14.7	+24.1367	15.1077	1578.818	0.0000

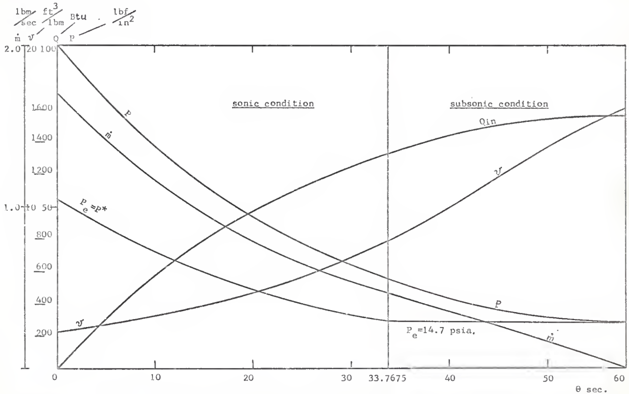

Fig. 11. The example of the discharge of air through a converging nozzle, T=constant.

(iv) Reversible Diabatic, Non-Isothermal Process Within the Tank.

Assume the tank is perfectly insulated, and the air and tank temperatures are the same at any instant.

From case (i) of this example, the data of the tank are 5.031 ft. high, 2.515 ft. radius, and ¼ inch thickness. On these bases the volume of the material (steel) of the tank, V, is 2.484 cu. ft., and 1213.8 lb. (the density of steel is assumed to be 7.83 x 62.4 = 488.6 lb/ft).

For the steel walls of the tank, assume

$$c_{pt} = c_{vt} = 0.12 \quad \text{Btu/lbm}^{o}\text{F}$$

$$\varepsilon = \frac{m_i\, c_v}{m_t\, c_{pt}} = \frac{\frac{100 \times 144 \times 100}{53.3 \times 600} \times 0.1714}{1213.8 \times 0.12} = 0.052987$$

Substituting $\varepsilon = 0.053$, and $P/P_i = 14.7/100$ into equation (45), one can solve for m/m_i by trial and error. Thus

$$\frac{m}{m_i} = 0.1496$$

From equation (44)

$$\frac{T}{T_i} = \frac{P\, m_i}{P_i\, m} = \frac{14.7}{100} \cdot \frac{1}{0.1496} = 0.98262$$

Therefore

$$\begin{aligned} Q_{in} &= -\Delta U_t \\ &= c_{pt}\, m_t\, [T_i - T] = c_{pt}\, m_t\, T_i\, [1 - \frac{T}{T_i}] \\ &= 0.12 \times 1213.8 \times 600\, [1 - 0.98262] \\ &= 1518.7 \text{ Btu.} \end{aligned}$$

The answer is checked as follows.

$$U_i = c_v m_i T_i = 0.1714 \times \frac{100 \times 144 \times 100}{53.3 \times 600} \times 600 = 4630.69 \quad \text{Btu.}$$

$$U = c_v mT = 0.1714 \times \left[0.1496 \times \frac{100 \times 144 \times 100}{53.3 \times 600}\right] \times \left[600 \times 0.98262\right]$$

$$= 680.7 \quad \text{Btu}$$

$$h_{out} = c_p T dm = c_p \bar{T}(m_i - m)$$

$$= 0.24 \times \left[\frac{600 + 600 \times 0.98262}{2}\right] \times \left[\frac{100 \times 144 \times 100}{53.3 \times 600} - 0.1496\right.$$

$$\left. \times \frac{100 \times 144 \times 100}{53.3 \times 600}\right] = \frac{0.24 \times 14400 \times 0.8504 \times 594.78}{319.8}$$

$$= 5468.07$$

$$Q_{in} = (U - U_i) + h_{out} = (680.7 - 4630.69) + 5468.07 = 1518.1 \quad \text{Btu.}$$

5. Numerical Examples for Inflow Problems.

Statement. A cylindrical tank having a volume of 100 ft^3 is initially filled with air at 20 psia and 500°R. Air at 100 psia and 700°R is allowed to flow into the tank. The inflow kinetic energy is neglected.

(i) Adiabatic Process.

Equation (48) applies. By the condition $Q_{in} = 0$, one gets

$$\frac{P}{P_i} - 1 = \frac{k\,T_{in}}{T_i}\left(\frac{P\,T_i}{P_i\,T} - 1\right)$$

and from the equation of state, PV = mRT, one gets

$$\frac{T_i}{T} = \frac{P_i\,m}{P\,m_i} \quad (V = \text{Constant})$$

Combining the above two equations yields

$$\frac{P}{P_i} = \frac{k\,T_{in}}{T_i}\left(\frac{m}{m_i} - 1\right) + 1$$

$$= \frac{1.4 \times 700}{500}\left(\frac{m}{m_i} - 1\right) + 1 = 1.96\left(\frac{m}{m_i}\right) - 0.96$$

Rearranging yields

$$m = \frac{m_i\left(\frac{P}{P_i} + 0.96\right)}{1.96} = \frac{\frac{20 \times 144 \times 100}{53.3 \times 500}\left(\frac{P}{20} + 0.96\right)}{1.96}$$

and

$$T = \frac{PV}{mR} = \frac{P \times 100 \times 144}{m \times 53.3} = \frac{1.96 \times 25\,P}{\frac{P}{20} + 0.96}$$

The results are shown in Fig. 12 and are summarized in Table 6.

TABLE 6

ANSWERS TO THE SELECTED INFLOW PROBLEM

$Q_{in} = 0$, $m_i = 10.8068$ lbm.

P	T	m	v	Δm
20	500	10.8068	9.2535	0
25	554.298	12.1852	8.2067	1.3784
30	597.561	13.5639	7.3727	2.7571
35	632.841	14.9420	6.6925	4.1352
40	662.162	16.3204	6.1273	5.5136
45	686.915	17.6988	5.6501	6.8920
50	708.092	19.0772	5.2419	8.2704
55	726.415	20.4556	4.8886	10.6488
60	742.424	21.8341	4.5800	11.0273
65	756.532	23.2125	4.3080	12.4057
70	769.058	24.5909	4.0665	13.7841
75	780.255	25.9692	3.8507	15.1624
80	790.322	27.3477	3.6566	16.5409
85	799.424	28.7261	3.4812	17.9193
90	807.692	30.1045	3.3218	19.2977
95	815.236	31.4829	3.1763	20.6761
100	822.148	32.8614	3.0431	22.0546

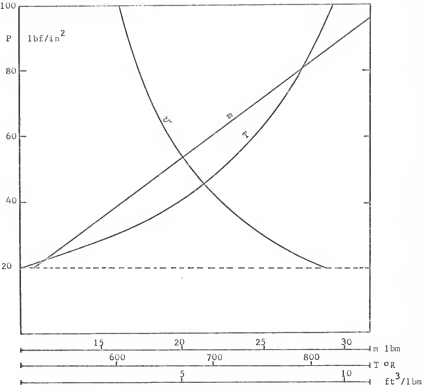

Fig. 12. The example of the inflow of air, $Q_{in} = 0$.

(ii) Diabatic Process (n = constant).

Assume n = 1.25 = constant. Therefore equation (49) applies.

$$Q_{in} = \frac{P_i V}{J(k-1)} \left\{ \left(\frac{P}{P_i} - 1\right) - \frac{k\, T_{in}}{T_i} \left[\left(\frac{P}{P_i}\right)^{\frac{1}{n}} - 1\right] \right\}$$

$$= \frac{20 \times 144 \times 100}{778\ (1.4 - 1)} \left\{ \left(\frac{P}{20} - 1\right) - \frac{1.4 \times 700}{500} \left[\left(\frac{P}{20}\right)^{\frac{1}{1.25}} - 1\right] \right\}$$

From the general equation (47)

$$Q_{in} = m\, c_v\, T - m_i\, c_v\, T_i - m\, c_p\, T_{in} + m_i\, c_p\, T_{in}$$

$$= m\, (c_v\, T - c_p\, T_{in}) - m_i\, (c_v\, T_i - c_p\, T_{in})$$

Rearranging yields

$$m = \frac{Q_{in} + m_i\, (c_v\, T_i - c_p\, T_{in})}{c_v\, T - c_p\, T_{in}}$$

$$= \frac{Q_{in} + m_i\, (c_v\, T_i - c_p\, T_{in})}{c_v\, T_i \left(\frac{P}{P_i}\right)^{\frac{n-1}{n}} - c_p\, T_{in}}$$

$$= \frac{\frac{20 \times 144 \times 100}{778\ (1.4 - 1)} \left\{ \left(\frac{P}{20} - 1\right) - \frac{1.4 \times 700}{500} \left[\left(\frac{P}{20}\right)^{\frac{1}{1.25}} - 1\right] \right\} + \frac{20 \times 144 \times 100}{53.3 \times 500} \times (0.1714 \times 500 - 0.24 \times 700)}{0.1714 \times 500 \times \left(\frac{P}{20}\right)^{\frac{0.25}{1.25}} - 0.24 \times 700}$$

The results are shown in Fig. 13 and is summarized in Table 7.

TABLE 7

ANSWERS TO THE SELECTED INFLOW PROBLEM

n = 1.25 = constant

P	Q_{in}	m**	Q_{in}/lbm
20	0	10.8068	0
25	*-123.144	12.9169	-9.54
30	-232.284	14.9437	-15.57
35	-330.207	16.9035	-19.53
40	-418.821	18.8080	-22.25
45	-499.509	20.6652	-24.16
50	-573.325	22.4814	-25.50
55	-641.087	24.2616	-26.41
60	-703.452	26.0096	-27.07
65	-760.957	27.7286	-27.46
70	-814.046	29.4213	-27.62
75	-863.097	31.0900	-27.85
80	-908.430	32.7364	-27.80
85	-950.323	34.3624	-27.75
90	-989.018	35.9693	-27.50
95	-1024.727	37.5583	-27.32
100	-1057.640	39.1307	-27.00

*Negative sign indicates heat-out

**Total mass inside the tank

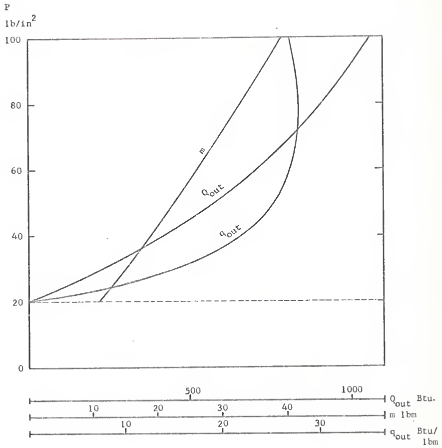

Fig. 13. The example of the inflow of air, n = 1.25 = constant.

(iii) Diabatic Process (q_{in} = Constant).

Assume 50 Btu is added per pound of mass inside the tank. Equation (47) applies.

$$Q_{in} = m\,(c_v\, T - c_p\, T_{in}) - m_i\,(c_v\, T_i - c_p\, T_{in})$$

$$q_{in} = (c_v\, T - c_p\, T_{in}) - \frac{m_i}{m}\,(c_v\, T_i - c_p\, T_{in})$$

$$= (c_v\, T - c_p\, T_{in}) - \frac{m_i RT}{PV}\,(c_v\, T_i - c_p\, T_{in})$$

Rearranging yields

$$T = \frac{q_{in} + c_p\, T_{in}}{c_v - \frac{R\, m_i}{PV}\,(c_v\, T_i - c_p\, T_{in})}$$

$$= \frac{50 + 0.24 \times 700}{0.1714 - \frac{20}{500 \times P}\,(0.1714 \times 500 - 0.24 \times 700)}$$

$$= \frac{218}{0.1714 + \frac{82.3}{25\, P}}$$

$$m = \frac{PV}{RT} = \frac{P \times 144 \times 100}{53.3 \cdot \frac{218}{0.1714 + \frac{82.3}{25\, P}}}$$

A mean value of n can be evaluated by using the relation

$$\frac{T}{T_i} = \left(\frac{P}{P_i}\right)^{\frac{n-1}{n}}$$

Rearranging yields

$$n = \frac{1}{1 - \dfrac{\ln\left(\dfrac{T}{T_i}\right)}{\ln\left(\dfrac{P}{P_i}\right)}} = \frac{1}{1 - \dfrac{\ln\left[\dfrac{218}{\left(0.1714 + \dfrac{82.3}{25\ P}\right) \times 500}\right]}{\ln\left[\dfrac{P}{20}\right]}}$$

The results are shown in Fig. 14 and are listed in Table 8.

TABLE 8

ANSWERS TO THE SELECTED INFLOW PROBLEM

q_{in} = 50 Btu/lbm

P	T	m	n
20	648.809	8.3281	0
25	719.282	9.3902	-1.588
30	775.433	10.4523	-12.159
35	821.225	11.5144	8.823
40	852.283	12.5765	4.570
45	891.413	13.6386	3.484
50	918.961	14.7007	2.978
55	942.584	15.7627	2.180
60	963.465	16.8248	2.482
65	981.778	17.8870	2.339
70	998.038	18.9490	2.231
75	1018.572	20.0111	2.105
80	1025.641	21.0732	2.076
85	1037.456	22.1352	2.018
90	1048.189	23.1973	1.969
95	1057.982	24.2594	1.927
100	1066.954	25.3215	1.890

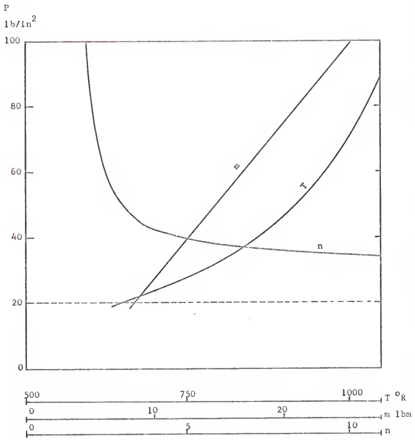

Fig. 14. The example of the inflow of air, q_{in} = 50 Btu/lbm = constant.

CHAPTER IV

CONCLUSION AND DISCUSSION

The change of states of a perfect gas, due to mass flow and/or heat flow, follows the basic equations including $PV = mRT$ and the equations derived from the first law of thermodynamics.

In the discharge process all the properties such as mass, temperature and specific volume, and the elapsed time can be expressed as a function of both the pressure ratio, P_i/P, and the initial conditions. An instantaneous state inside a vessel can, therefore, be predicted from the quantity of the outgoing mass by utilizing equation (21a), or can be predicted from the elapsed time by using equations (19) and (20) for a sonic discharge condition at the nozzle exit.

(i) When a perfect gas is discharged to an environment through a converging nozzle providing the flow is in a critical condition, the figures ranging from Fig. 3 to Fig. 7-B apply. When it is discharged by means of a Laval nozzle, the figures are available also, if the flow at the throat is still in a critical state.

(ii) No general solution in pressure-time relation is available when the exit is in a subsonic discharge condition. However, an exact solution corresponding to a set of n and k is possible if the condition $(M + 1)/N + S =$ an integer or $(M + 1)/N =$ an integer is satisfied. Some of the solutions, assuming $k = 1.4$, are shown in Table 3.

In the inflow process, the quantity of mass inflow is assumed to be measurable. Therefore, the prediction of the states inside the tank can be accomplished from the known amount of mass flowing in.

(iii) If a tank is initially filled with gas having an initial temperature equal to $k \cdot T_{in}$, the temperature inside the tank will remain at a constant value in an adiabatic process, when the inflow gas temperature, T_{in}, remains constant.

(iv) If a tank is initially at a condition of evacuation, the temperature inside the tank will remain a constant value equal to $k \cdot T_{in}$ in an adiabatic process. This is consistent with the result described in iii.

The technique of predicting a state within a gas vessel has been shown. Charts relating the properties and the elapsed time for various kinds of gases at subsonic discharge conditions can be plotted by the equations demonstrated herein, and it is recommended that the construction of the graphs would be desirable.

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A STUDY OF UNSTEADY DISCHARGE AND INFLOW EFFECTS ON THE STATES INSIDE A VESSEL WITH AND WITHOUT HEAT TRANSFER FOR PERFECT CASES

by

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A technique, based on mass flow and/or heat flow, for the prediction of a state of a perfect gas inside a vessel has been formulated by utilizing a series of fundamental equations including the equation of state, and those for mass and energy conservations.

A vessel with a converging nozzle of fixed geometry has been selected for the study of the discharge problem. In the analyses of inflow problems, the inlet stagnation temperature was assumed to be constant.

In a discharge process a general solution to the equation, representing the relation between the instantaneous vessel pressure and the elapsed time, is available for the case of a sonic discharge at the outlet. Only a particular solution, corresponding to a specified gas and heat flow process, is available when the discharge velocity is subsonic. The analyses, both on sonic and subsonic discharge, lead to a result which enables one to predict the instantaneous state of the perfect gas inside a vessel from the elapsed time.

In an inflow process the total amount of mass flowing to a vessel is assumed to be measurable. The instantaneous properties are, therefore, expressed as a function of mass inflow. As a result, a prediction of the states inside the gas vessel is possible from the known quantity of mass flowing in.

Two numerical examples on the discharge and inflow of air are presented.