CORRELATION STUDY BETWEEN
SAND EQUIVALENT AND PLASTICITY INDEX
TESTS

by

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By Leon W. Heidebrecht,[1] A. M. ASCE

SYNOPSIS

This paper considers the correlation between two parameters of aggregates used in bituminous mixes and bases, the sand equivalent and plasticity index. It shows the effectiveness of sand equivalent values in predicting plasticity index values.

The test values representing the parameters were obtained from aggregates used in bituminous, asphaltic concrete, shoulder and aggregate binder base construction. These values were determined for the above aggregates used in several counties in Kansas.

It is concluded that there appears to be no correlation between sand equivalent and plasticity index which could be applied to engineering practices.

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INTRODUCTION

The plasticity index test is presently being used in many specifications to control the quality of aggregates used in bituminous mixes and bases in highway construction.

Since production is ever increasing, it is desirable to speed up testing operations by avoiding the need for weighing the sample and drying it. Such a test for speeding quality control testing has been developed and is called the "Sand Equivalent Determination."[2]

The sand equivalent test has gained favor as a means of detecting excessive clay content in aggregate and is being used in many western states at the present time.

The State Highway Commission of Kansas presently specifies limits of plasticity index on base materials, road and plant mix aggregates for bituminous mixtures, asphaltic concrete aggregates, and other types of construction materials.[3] This paper will deal with the combined aggregates of bases, road and plant bituminous mixes and asphalt concrete mixes.

The State Highway Commission of Kansas has collected data for sand equivalent test values and corresponding plasticity index values on numerous samples representing aggregates used in bituminous mixes and bases. The values were analyzed by an electronic computer to

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[2] "Sand Equivalent Test for Control of Materials During Construction," by F. N. Hveem, Highway Research Board Proceedings, Vol. 32, 1953, p. 238.

[3] Standard Specifications for State Road and Bridge Construction, State Highway Commission of Kansas, 1960 Edition.

establish a regression line equation and confidence interval bands for each type of material. This was done using appropriate statistical procedures.[4]

For many years highway construction has been steadily increasing and this has made it increasingly important that the engineer have some ready and convenient test for detecting the presence of excessive amounts of detrimental clay or fine material in base or subbase materials.

Small amounts of clay may be detrimental to the performance of bituminous mixtures, especially when the clay exists as a coating on the surfaces of the larger grains of the combined aggregate. This coating prevents adhesion of the bitumen to the aggregate particles.

THE SAND EQUIVALENT TEST DETERMINATION

It might be well at this point to consider the work involved in the determination of the sand equivalent values. This test is intended to serve as a rapid field test to show the relative amounts of plastic fines in a graded aggregate or soil.

Sample Preparation for Sand Equivalent Determination

The sand equivalent test samples were prepared from the portion of the material passing the No. 4 sieve. It was essential that all fines were cleaned from the portion retained on the No. 4 sieve and

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[4] Statistical Methods, by George W. Snedecor, Iowa State University Press, Publisher, 1956.

included with that portion passing. The passing No. 4 portion was then split or quartered so enough material was obtained to fill a 88 ml. capacity tin measure to within 3/16" of the brim (see Figure A). Extreme care was taken when getting this smaller portion so a truly representative sample was obtained.

It has been found from experiments, that as the amount of material being reduced by splitting or quartering was decreased, the accuracy of providing representative portions was decreased. Since the sand equivalent test sample was already relatively very small, it was imperative that it be split or quartered evenly to the correct size. Adjustments were sometimes required to provide the desired test sample size. The adjustments were speeded up by determining the exact amount which was split down to an even tin measure full. This was done by dipping four full measures of the material of average specific gravity and weighing or by determining the volume in a cylinder before beginning the splitting operation. The four measures were then returned to the sample and the material was split or quartered until the predetermined weight or volume was obtained. When this weight or volume was obtained, the two successive quartering operations usually provided the amount of material required to fill the tin measure to within 3/16" of the brim.

Test Procedure

A solution of calcium chloride was prepared by diluting 88 cc of stock calcium solution to 1 gallon of distilled water. The test was normally performed without strict temperature control, however,

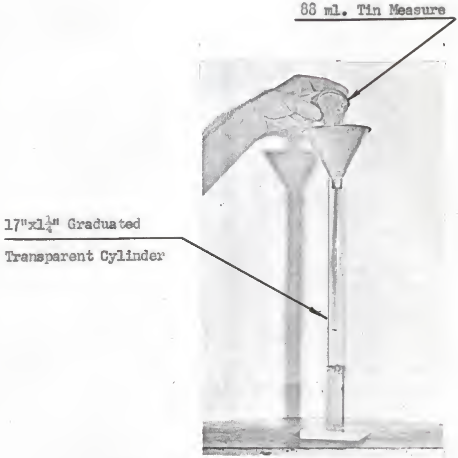
88 ml. Tin Measure
17"x1$\frac{1}{4}$" Graduated
Transparent Cylinder

Figure A

in the event of dispute, the material should be retested with the temperature of the working solution at 72° ± 5° F.

After preparing the calcium chloride solution, 4 inches of the solution was siphoned into a standard plastic cylinder (see Figure A). The prepared test sample was then poured into the cylinder from the measuring tin using a funnel to avoid spillage. The bottom of the cylinder was then tapped on the heel of the hand several times to release the air bubbles in the sand and promote thorough wetting of the sample. After allowing the wetted sample to stand undisturbed for 10 ± 1 minute, the cylinder was then corked with a rubber stopper and the material loosened by partially inverting the cylinder and shaking it simultaneously (see Figure B). After the material was loosened by this method, the cylinder was held in a horizontal position and shaken vigorously in a horizontal linear motion from end to end. This was done at the rate of three (3) cycles per second for thirty (30) seconds with a throw of 9 ± 1 inch. A cycle is defined as the complete back and forth motion.

The cylinder was then placed in an upright position and uncorked. The irrigator tube was then inverted into the cylinder and the material was rinsed from the cylinder walls as the irrigator was lowered (see Figure C). The irrigator was forced to the bottom of the cylinder by twisting and stabbing action during which time the calcium chloride solution was flowing from the irrigator tip. The irrigator tube was made of ¼ inch outside diameter brass tubing and was used for flushing the fine material into suspension above the coarser sand particles.

Figure B

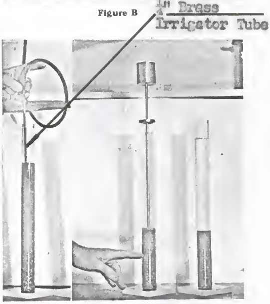

Figure C

This action was continued until the cylinder was filled to the 15 inch mark while the irrigator was being withdrawn. The flow was then regulated just before the irrigator was entirely withdrawn and the final level was adjusted to 15 inches.

The cylinder and contents were allowed to stand undisturbed for 20 minutes $\pm$ 15 seconds, the timing being started immediately after withdrawing the irrigator. At the end of the sedimentation period the level of the top of the clay suspension was read and recorded. This is referred to as the "clay reading." After getting the "clay reading" the weighted foot assembly was placed over the cylinder and lowered until it came to rest on the sand. When lowering the weighted foot it was necessary to take some precautions so that one of the centering screws could be seen at all times. This was necessary so that the level of the screw could be recorded as the "sand reading."

Both "sand" and "clay" readings were recorded to the next higher graduation when read. For example, a reading of 3.22 was recorded as 3.3.

The sand equivalent value was obtained by multiplying the "sand" reading by one hundred (100) and dividing this by the "clay" reading. This value was carried to tenths and rounded up to the next whole number.

If it were desired to average a series of sand equivalent values, the whole-number values were used and the average was rounded up to the next whole number.

Some precautions are necessary to give values which represent the sample being tested. For example, care must be exercised in providing a representative sample for the test. This involves using the prescribed quartering or splitting practices and in case of segregation or loss of fines it may be necessary to dampen the material.

The test should also be performed in a location, which is free from vibration, as this will cause suspended material to settle at a greater rate than normal.

It might be well at this point to discuss the operator's qualifications required to perform this test. An operator's test results are considered to be consistent if three tests performed by him, on a representative sample of a given material, do not vary more than 4 points from the average of these tests. If the operator's test results are not consistent, it is recommended that he not be allowed to continue until he has perfected the technique enough to obtain results that do not vary more than 4 points from the average of the three tests.

It is believed that a simple and relatively inexpensive shaker has been developed and, if so, would eliminate variation due to personal shaking techniques.

The values used for this study were obtained by hand shaking methods and the instructions above are those used by the State Highway Commission of Kansas.

It is the intent of this paper to study the correlation existing between the sand equivalent values and plasticity index values.

THE PLASTICITY INDEX DETERMINATION

All plasticity index values were obtained using the procedures outlined in the specifications used by Kansas State Highway Commission.[5]

DISCUSSION AND ANALYSIS OF DATA

Figures 1 through 4 represent data from combined aggregates of bases, road and plant bituminous mixes and asphalt concrete mixes. The data appears to be so scattered that it would be impossible to visually place any kind of a line which would represent the regression line. It would also be difficult to visually determine whether a linear or curvilinear regression line would best represent the data.

The analyses for the specific aggregates presented probably have no engineering applicability because of the variation shown in the plotted data. The large amount of scatter might be explained by between-operator variation and would make it economically unsound to reject material which might be acceptable according to the plasticity index specifications of the material.

The following analyses for the specific aggregates assume linear regression and are presented for information only.

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[5] Standard Specifications for State Road and Bridge Construction, State Highway Commission of Kansas, 1960 Edition.

CORRELATION OF PARAMETERS FOR SPECIFIC AGGREGATES

Road and Plant Mix Aggregates for Bituminous Construction

Figure 1 represents the values obtained for the aggregate used in bituminous bases, both road and plant mixes. The aggregates used by Kansas for this type of construction fall under the same specification limits and differ only in the method used to mix the oil, the road mix aggregates being mixed by use of a traveling pugmill and the plant mix aggregates mixed with oil at a stationary plant. The aggregates sampled as shown in Figure 1 were within specifications for BC-1[6] as required by State Highway Commission of Kansas Specifications. The gradation specifications are as follows for the BC-1 samples.

Sieve #	1"	3/8"	No. 8	No. 30	No. 200
Percent Retained	0%	5-30%	25-50%	80%	87-93%

Each sample obtained was split so both the sand equivalent test and plasticity index could be performed on each half of the representative sample. Figure 1 shows the two values plotted for four hundred and forty (440) samples. The independent variable X (sand equivalent) is plotted along the horizontal axis. Each measure of the dependent Y (plasticity index) is indicated by a point above the corresponding X. For the purpose of this study, it was assumed that no error existed in

6 Standard Specifications for State Road and Bridge Construction, State Highway Commission of Kansas, 1960 Edition.

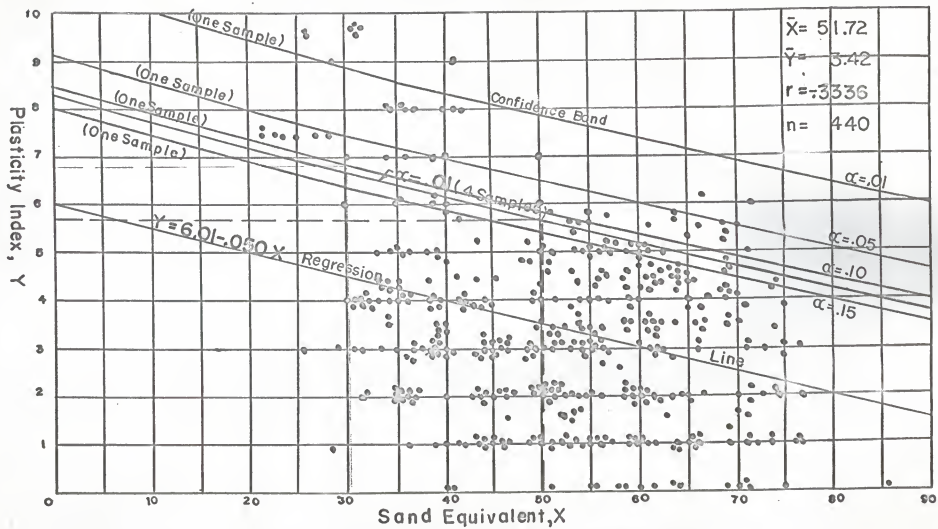

FIGURE I. ROAD AND PLANT MIX AGGREGATE

the sand equivalent values. Variation in the plasticity index values might be explained by between-operator variance, however, no data were available from which to determine the magnitude of this variation.

Figure 1 obviously indicates very scattered points, and since linear regression was assumed, the sample regression coefficient (b) of Y on X was calculated.[7] This regression coefficient was obtained from the formula

$$b = \frac{\Sigma(x-\bar{x})\ (y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

where $\Sigma(x-\bar{x})^2$ represents the summation of the squares of the deviations from the mean of all sand equivalent values and $\Sigma(x-\bar{x})\ (y-\bar{y})$ represents the summation of the products of the deviations from the means representing each parameter. The corresponding values obtained from an electronic computer were $\Sigma(x-\bar{x})^2 = 74{,}030$, $\Sigma(x-\bar{x})\ (y-\bar{y}) = -3{,}707$. This would give a computed regression coefficient b = -.0501. This would indicate that for each twenty units of increase in the sand equivalent, the plasticity index decreases by 20(-.0501) or -1.021 units.

A test of significance of b is given by $t = b/S_b$, degrees of freedom = n-2 when S_b is the sample standard deviation from the regression coefficient and $S_b = S_{y.x} / \sqrt{\Sigma(x-\bar{x})^2}$. $S_{y.x}$ represents the sample standard deviation from regression and is obtained by the following

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7 Statistical Methods, by George W. Snedecor, Iowa State University Press, 1956, p. 122.

formula

$$S_{y.x} = \sqrt{\frac{\Sigma\, d^2 y.x}{n-2}} = \sqrt{\frac{\Sigma (y-\bar{y})^2 - \frac{[\Sigma (x-\bar{x})\ (y-\bar{y})]^2}{\Sigma (x-\bar{x})^2}}{n-2}}$$

The following values were obtained using $\Sigma(x-\bar{x})^2 = 74{,}030$, $\Sigma(x-\bar{x})(y-\bar{y}) = -3{,}707$ and $\Sigma(y-\bar{y})^2 = 1668$; $S_{y.x} = \sqrt{3.385}$ and $S_b = \sqrt{\frac{3.385}{74{,}030}}$

and therefore $t = \frac{-.0501}{\sqrt{\frac{3.385}{74{,}030}}} = \frac{-.0501\ (\sqrt{74{,}030})}{\sqrt{3.385}} = 7.45^{**}$

**This indicates significance at .01 or less level when d.f. = n-2 = 438 and we therefore conclude that the regression coefficient is significantly different from zero. Now because the slope is significantly different from zero and it is known that the regression line passes through both $\bar{x}$ and $\bar{y}$, we are able to compute the regression line equation and plot it as shown in Figure 1.

It must be pointed out that the regression coefficient "b" calculated and the significant "t" only implied that the best fitting straight line had a slope different from zero and gave no information concerning the accuracy of the predicted relationship between variables. The correlation coefficient was computed to give the degree to which the two measurements werelinearly related. The correlation coefficient (r) was computed by the following formula.

$$r = \frac{\Sigma(x-\bar{x})\ (y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\ \sqrt{\Sigma(y-\bar{y})^2}}$$

This computed value was r = -.3336 and indicated that only $(-.3336)^2$ or 11% of the variation was explained, leaving 89% unexplained. For this reason, it was felt that test values would likely disagree with the predicted values from the equation.

The prediction ordinates for several confidence levels are shown in Figure 1 for information only.

Suppose that predictions are desirable for plasticity index from the sand equivalent of a material meeting the BC-1 specifications. As shown in Figure 1, the various confidence belts are plotted for different α levels. These confidence belt ordinates are computed using the sample standard deviation of Y which is found by the formula tS_y where

$$S_y = S_{y.x}\sqrt{\frac{1}{n.} + \frac{1}{n} + \frac{(x-\bar{x})^2}{\Sigma(x-\bar{x})^2}}$$

n. = Samples required for making prediction
n = Number of pairs of values used in establishing the regression equation

The confidence belt ordinate is then computed as tS_y or

$$tS_{y.x}\sqrt{\frac{1}{n.} + \frac{1}{n} + \frac{(x-\bar{x})^2}{\Sigma(x-x)^2}}$$

t can be obtained from any "Table of Percentage Points of the t- Distribution" using N-2 degrees of freedom and various α levels.

The confidence belt (one sided) or band is computed for various values of X (sand equivalent). When the belt ordinates are computed for predictions to be made using one sample, then n = 1 and the ordinate value is equal to

$$(t_{\alpha},\ d.f. = 438)\ \sqrt{3.385}\ \sqrt{1 + \frac{1}{440} + \frac{(X-51.72)^2}{74,030}}$$

The numerical values which have been substituted into the above formula are tabulated in Table I and are as follows: $n = 440$, $\bar{x} = 51.72$, $\Sigma(x-\bar{x})^2 = 74,030$ and $S_{y.x}{}^2 = 3.385$. Note that the ordinate from the regression line increases only slightly as X gets far removed from $\bar{x}$ for any one sided belt for α.

Now suppose one desires to make a prediction and wants to be 90% confident about his statement he would use the $\alpha = 0.10$ confidence interval. Note that the predictions could only be made from Figure 1 if one were willing to assume linear regression. As an example, suppose that a stockpile of BC-1 material were sampled and the sand equivalent value was determined as 30. The conclusion then could be made that the plasticity index would not be greater than 6.8 unless a 1-in-10 chance occurred in sampling.

Now suppose that the prediction becomes desirable using the average of four (4) values of sand equivalents and were to be made at the 99% confidence level. The confidence belt as plotted in Figure 1 for $\alpha = .01$ (4 samples) would be used. For example, if one were to sample a stockpile of material meeting BC-1 gradation specifications and obtain four (4) values of sand equivalent which averaged 50, it would be possible to predict from Figure 1 that the plasticity index of the material sampled would not be greater than 5.7 unless a one in one hundred chance that bad sampling occurred.

TABLE I

REGRESSION DATA FOR SAMPLE TEST VALUES

Group	n	$\bar{x}$	$\bar{y}$	$\Sigma(x-\bar{x})^2$	$\Sigma(x-\bar{x})(y-\bar{y})$	$\Sigma(y-\bar{y})^2$	b	t computed	t* tabled	$S^2_{y.x}$	y-intercept	r
Road and Plant Mix Aggregate for Bituminous Construction	440	(SE) 51.72	(PI) 3.42	74,030	-3,707	1668	-.0501	7.45**	2.35	3.385	6.01	-.3336
Hot Mix Aggregates for Asphaltic Concrete Construction	155	(SE) 53.91	(PI) 2.05	22,395	-1,194	256	-.0533	7.10**	2.36	1.259	4.93	-.4983
Aggregates Used for Shoulder Construction	179	(SE) 25.09	(PI) 4.19	10,548	-1,177	260	-.1116	13.50**	2.58	0.728	7.00	-.7104
Aggregate for Aggregate-Binder Base Construction	148	(SE) 32.11	(PI) 4.08	15,941	-1,259	822	-.0790	4.50**	2.38	4.947	6.62	-.3479

* $\alpha = .01$ ** Indicates significance for regression slope (b) at the $\alpha = .01$ level

It should also be pointed out that since the upper limit of the plasticity index is the critical value, all confidence interval bands are one sided. The regression line equation and the correlation coefficient are also indicated on the graph (Figure 1).

Hot Mix Aggregates for Asphaltic Concrete Construction

Another group of interest was the aggregate used for asphaltic concrete construction. The samples shown in Figure 2 were obtained from aggregates meeting the specification requirements set up for hot mix and represent aggregates used for asphaltic concrete pavement construction. The specifications used were again those used by the State Highway Commission of Kansas.

All sample values obtained are shown in Figure 2. The material sampled varied from HM-1, HM-2, HM-3, and HM-4. Therefore, the gradations shown below represent the upper and lower limits for all of the hot mix classes and the samples shown in Figure 2 were within these gradation ranges.

Sieve #	1"	3/4"	3/8"	#4	#8	#16	#30	#80	#200
% Ret.	0%	0-5%	15-40%	22-60%	45-72%	54-80%	63-92%	80-94%	87-99%

Each sample was split so both the sand equivalent test and the plasticity index (liquid limit less plastic limit) could be performed on each half of the representative sample. Figure 2 shows the values plotted for one hundred and fifty-five (155) samples. The independent variable X (sand equivalent) was plotted along the horizontal axis.

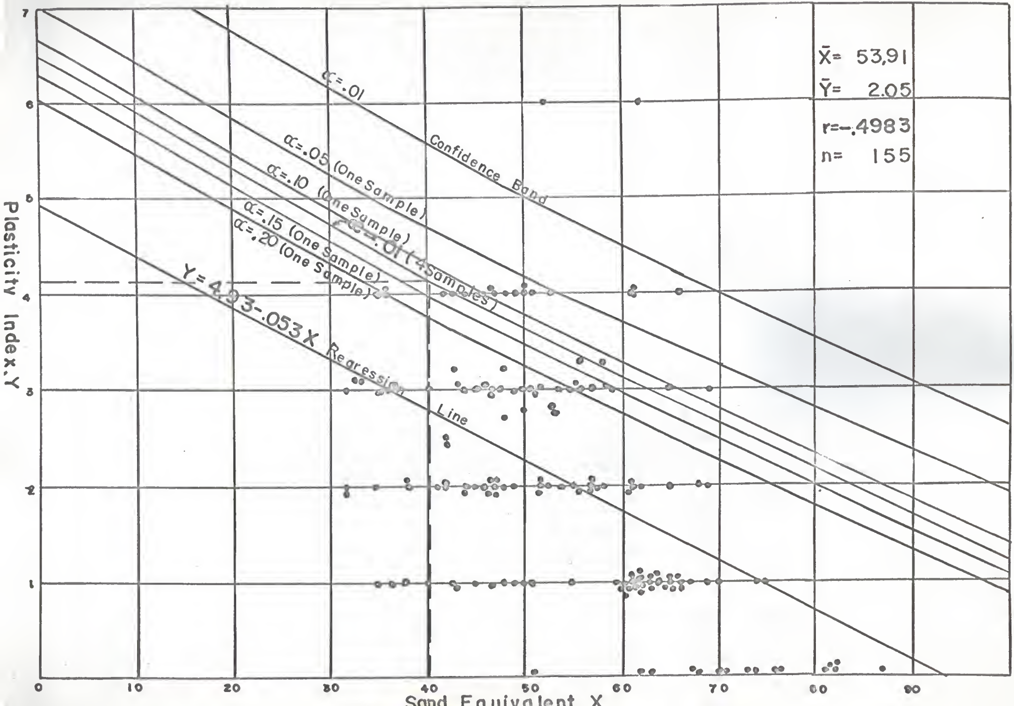

FIGURE 2. HOT MIX AGGREGATE FOR ASPHALTIC CONCRETE

Each measure of the dependent Y (plasticity index) was indicated by a point above the corresponding X.

The regression coefficient (b) of Y on X was calculated from the formula

$$b = \frac{\Sigma(x-\bar{x})\ (y-\bar{y})}{\Sigma(x-\bar{x})^2} \quad \text{where} \quad \Sigma(x-\bar{x})^2 = 22{,}395$$

and $\Sigma(x-\bar{x})\ (y-\bar{y}) = 1{,}194$. Substituting in these two values gave a computed regression coefficient of b = -.0533.

A test of significance indicated a computed value of t = 7.10.**

**This indicates significance at .01 or less level when d.f. = n-2 = 153 and it is therefore concluded that the slope (b) is significantly different from zero. The information needed to compute the regression coefficient is shown in tabulated form in Table I. In reduced form

$$t = \frac{b\sqrt{(x-\bar{x})^2}}{\sqrt{S_{y.x}{}^2}} = \frac{-.0533\sqrt{22395}}{\sqrt{1.259}} = 7.10^{**}$$

Since $\bar{x}$ = 53.91 and $\bar{y}$ = 2.05, where $\bar{x}$ is the mean of all x values and $\bar{y}$ is the mean of all y values, and the slope (b) is -.0533, the equation for the regression line can be written as Y = 4.93 - .053 X.

If predictions are desirable for material meeting hot mix specifications, confidence belts are provided in Figure 2. Again the confidence belt ordinates are computed using the formula tS_y, where tS_y equals the ordinate for various X values and

$$tS_y = tS_{y.x}\sqrt{\frac{1}{n.} + \frac{1}{n} + \frac{(x-\bar{x})^2}{\Sigma(x-\bar{x})^2}} \quad \text{and}$$

$n_{\circ}$ = Samples required for making prediction

n = Number of pairs of values used in establishing the regression equation

The ordinate for $\alpha = 0.10$ at $X = 55$ is computed substituting the following values,

$$\text{Ordinate} = tS_y = 1.28\ \sqrt{1.259}\ \sqrt{1 + \frac{1}{155} + \frac{(55.00-53.91)^2}{22,395}} = 1.48$$

The following t values are used for the different α levels, all of which are one sided and are obtained using one hundred and fifty-three (153) degrees of freedom: $t = 2.36$ for $\alpha = .01$, $t = 1.65$ for $\alpha = 0.05$, $t = 1.28$ for $\alpha = 0.10$, $t = 1.06$ for $\alpha = 0.15$, $t = 0.84$ for $\alpha = 0.20$. A simple meaning for the $\alpha = .05$ level with regard to t is that among samples of great size, drawn at random from a normal population, 5% of them are expected to be in a region where t is greater than 1.65.

Now suppose one desires to make a prediction and wants to be 90% confident about his statement, he would use the $\alpha = 0.10$ confidence belt shown in Figure 2. As an example, if a stockpile of material meeting HM specifications, as shown above, was sampled and the sand equivalent value was determined to be 50, the plasticity index should not be greater than 3.8 unless a 1-in-10 chance occurred in sampling.

It might also be desirable to use the average of four (4) values of sand equivalents to make a prediction concerning the plasticity index of a stockpile of HM material.

As an example, if one were to predict the plasticity index of a material meeting these gradation specifications after obtaining four (4) sand equivalent values which would average 40, it could be said that the plasticity index would probably be 4.1 or less unless a one in one hundred chance occurred that a bad sample was obtained. Again the regression line equation and the correlation coefficient are indicated in Figure 2.

One hundred and fifty-five (155) samples were used to establish this prediction equation and correlation coefficient.

Again the confidence interval bands plotted are one sided since the critical value is the upper limit of plasticity index permitted under HM specifications.

The correlation coefficient was computed and tested for significance. The formula

$$r = \frac{\Sigma(x-\bar{x})\,(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\;\sqrt{\Sigma(y-\bar{y})^2}}$$

where $\Sigma(x-\bar{x})\,(y-\bar{y}) = -1{,}194$, $\Sigma(x-\bar{x})^2 = 22{,}395$ and $\Sigma(y-\bar{y})^2 = 256$, represents the correlation coefficient and

$$r = \frac{-1194}{\sqrt{22395}\;\sqrt{256}} = -.4983$$

This computed r indicated that only $(-.4983)^2$ or about 25% of the variation was explained, leaving 75% unexplained. It was felt that such a low level of accuracy of prediction would lead to too many errors in accepting or rejecting materials.

Aggregates Used for Shoulder Construction

Figure 3 shows the plotted values obtained for aggregates used for shoulder construction. This material was of type AS-1 by specification designation. The gradation requirements for AS-1 are as follows:

Sieve #	2"	1½"	3/4"	#4	#10	#40	#200
Percent Retained	0%	0-5%	5-30%	35-60%	45-70%	60-84%*	80-92%*

*The ratio of percent passing the No. 200 to percent passing No. 40 shall be less than 3/4.

The samples which were split so both the sand equivalent and plasticity index could be determined numbered one hundred seventy-nine (179). As shown in Figure 3, the sand equivalent was plotted along the horizontal axis and each measure of the dependent plasticity index was indicated above the corresponding sand equivalent.

The regression coefficient b was computed from

$$b = \frac{\Sigma(x-\bar{x})\ (y-\bar{y})}{\Sigma(x-\bar{x})^2} \qquad \text{where}$$

$\Sigma(x-\bar{x})\ (y-\bar{y}) = -1{,}177$, $\Sigma(x-\bar{x})^2 = 10{,}548$ and $\Sigma(y-\bar{y})^2 = 260$. This gave a computed regression coefficient as $-.1116$.

A test of significance of b was computed as before by $t = b/S_b$ where S_b was the sample standard deviation from the regression coefficient and was obtained from

$$S_b = S_{y.x} / \sqrt{\Sigma(x-\bar{x})^2}$$

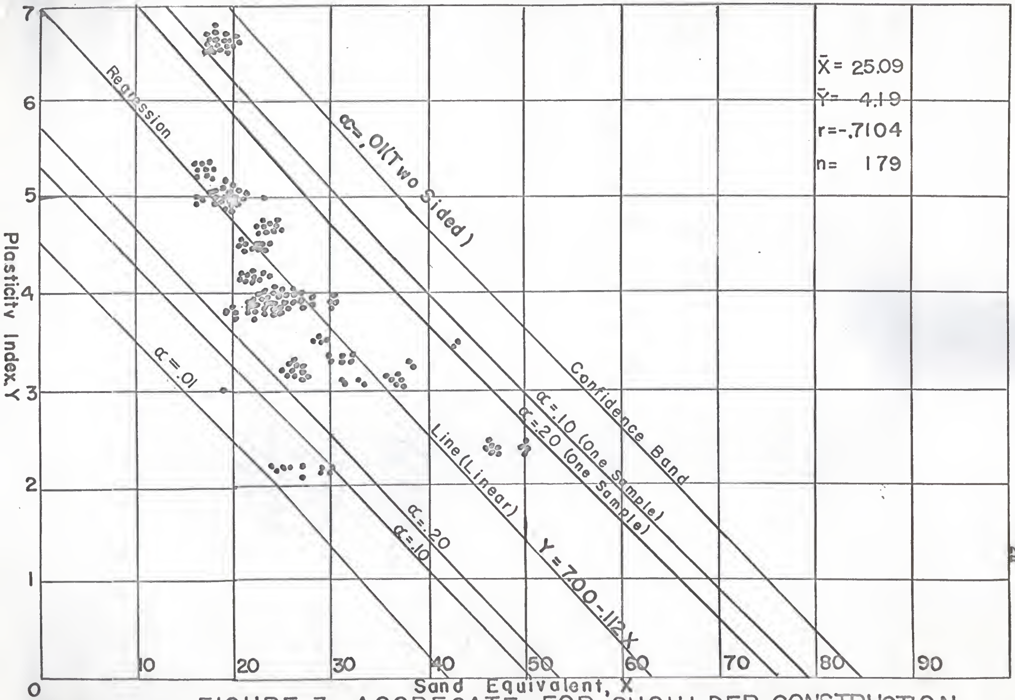

FIGURE 3. AGGREGATE FOR SHOULDER CONSTRUCTION

$S_{y.x}$ was the sample standard deviation from the regression and was obtained from the formula

$$S_{y.x} = \sqrt{\frac{\Sigma d^2 y.x}{n-2}} = \sqrt{\frac{\Sigma(y-\bar{y})^2 - \frac{[\Sigma(x-\bar{x})(y-\bar{y})]^2}{\Sigma(x-\bar{x})^2}}{n-2}}$$

$$S_{y.x} = \sqrt{\frac{260-(1177)^2 / 10548}{177}} = \sqrt{.728} = .855$$

$$\text{Therefore } t = \frac{b\sqrt{\Sigma(x-\bar{x})^2}}{S_{y.x}} = \frac{-.1116\sqrt{10,548}}{.855} = 13.5^{**}$$

**This indicates significance at .01 or less level when d.f. = 177 and the conclusion may be drawn that the slope of the regression line was significantly different from zero when linear regression was assumed. The means for both the sand equivalent and plasticity were obtained from $\bar{x} = \frac{X}{n}$ and $\bar{y} = \frac{Y}{n}$ where $\bar{X} = 25.09$ and $\bar{y} = 4.19$. Since the slope was known and it was realized that the regression line passed through both $\bar{x}$ and $\bar{y}$, it was possible to write the linear regression equation as $Y = 7.00 - .1116X$ and plot it as shown in Figure 3.

The above statements only indicated that the best fitting straight line had a slope different from zero but gave no indication as to the accuracy of making predictions using this equation. The correlation coefficient was computed as $-.7104$ and this indicated that about $(-.7104)^2$ or 50% of the variation was explained with the linear regression line.

This low correlation coefficient could give undesirable predictions when using the equation for engineering decisions, however, the confidence belts at various α levels are shown for information only in Figure 3.

As has been done for the other specific aggregates, the confidence belts are needed for various α levels if predictions are to be made for material meeting AS-1 specifications. However, since AS-1 specifications as allowed by Kansas Highway Commission 1960 Specifications indicate both an upper and lower limit on plasticity index, the confidence belts plotted are two sided bands for the various α levels. The confidence belt ordinates were computed by the following formula

$$\text{Ordinate} = tS_{y.x}\sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{(x-\bar{X})^2}}$$

This formula will no doubt look familiar as it is the same one used for computing one sided confidence band ordinates. However, the t values used in the computations are the t values representing the two sided t and are taken from any "t"-Distribution Table as plus and minus. Thus, the ordinates at any α level for any X value were plotted both above and below the regression line. The ordinate computed for X = 25 and α = .01 (two sided) is as follows when $S_{y.x}$ = .855

$$\text{Ordinate} = \pm 2.58\ (.855)\sqrt{1 + \frac{1}{179} + \frac{(25-25.09)^2}{10{,}548}} = \pm 2.21$$

Therefore, the ordinate was plotted +2.21 units above and -2.21 units below the regression line at X = 25.

Other t values taken from the Distribution Table for the various α levels are t = $\pm$1.645 for α = .10 and t = $\pm$1.28 for

α = .20 and these were used to compute the other belt ordinate for other α levels.

Now suppose one desires to make a prediction and wants to be 90% confident about the statement, one would use the α = .10 belt. As an example, if a stockpile of material meeting AS-1 gradation specifications was sampled and the sand equivalent value was determined as 40, it could be said with 90% confidence that the material would have a plasticity index between 1.0 and 4.0.

Aggregate for Aggregate-Binder Base Construction

All samples plotted for this group shown in Figure 4 were within specifications for AB-3 and were within the following gradation requirements:

Sieve #	2"	1½"	3/4"	No. 4	No. 10	No. 40	No. 200
% Retained	0%	0-5%	5-30%	35-60%	45-70%	60-84%*	80-92%*

*The fraction passing the No. 200 shall not exceed three-fourths (3/4) of the fraction passing the No. 40 sieve.

This combined material also must consist of eighty-five (85) percent or more produced by the mechanical crushing of limestone.

Figure 4 represents graphically points obtained from 148 samples meeting AB-3 gradation specifications, which were tested both for plasticity index and sand equivalent values.

The independent variable X (sand equivalent) was plotted along the horizontal axis and each measure of the dependent Y (plasticity index) was indicated by a point above the corresponding X.

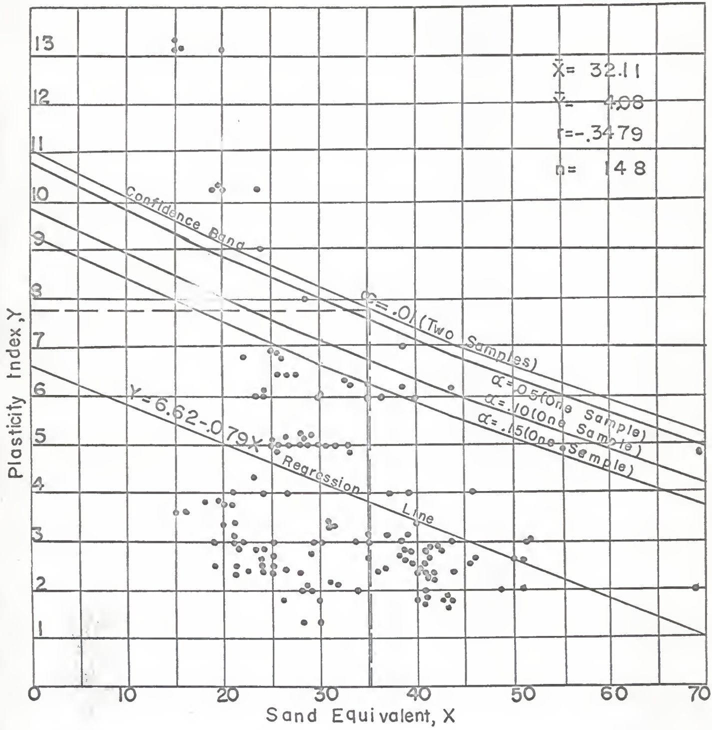

FIGURE 4. AGGREGATE FOR AGGREGATE-BINDER BASE CONSTRUCTION

The sample regression coefficient (b) of Y on X was calculated using the linear regression technique and represented by the formula

$$b = \frac{\Sigma(x-\bar{x})\ (y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

where $\Sigma(x-\bar{x})^2 = 15{,}941$, $\Sigma(x-\bar{x})\ (y-\bar{y}) = -1{,}259$ and $\Sigma(y-\bar{y})^2 = 822$. The regression coefficient was computed as

$$b = \frac{-1{,}259}{15{,}941} = -.0790$$

and was tested for significance by $t = b/S_b$ to be sure that it was larger than the tabled t for a specified α level for n-2 degrees of freedom. The sample standard deviation from the regression coefficient S_b was equal to $S_{y.x}/\sqrt{\Sigma(x-\bar{x})}$ where $S_{y.x}$ was the sample standard deviation from regression and was obtained from the following formula:

$$S_{y.x} = \sqrt{\frac{\Sigma d^2 y.x}{n-2}} = \sqrt{\frac{\Sigma(y-\bar{y})^2 - \frac{\left[\Sigma(x-\bar{x})\ (y-\bar{y})\right]^2}{\Sigma(x-\bar{x})^2}}{n-2}}$$

By substitution we see that $S_{y.x} = \sqrt{4.947}$ and since

$$S_b = \frac{S_{y.x}}{\sqrt{\Sigma(x-\bar{x})^2}} = \sqrt{\frac{4.947}{15{,}941}}$$

t is equal to

$$\frac{-.0790}{\sqrt{\frac{4{,}947}{15{,}941}}} = 4.50^{**}$$

**This indicates significance at .01 level or less when d.f. = n-2 = 146 and it may be concluded that b is significantly different from zero. The means of both sand equivalent values $\bar{x}$ and plasticity index $\bar{y}$ are 32.11 and 4.08 respectively. Since the linear regression line must pass through $\bar{x}$ and $\bar{y}$ and have a slope of -.0790, it is possible to show the regression line as indicated in Figure 4 and write the regression line equation as $Y = 6.62 - .0790X$.

If predictions are desirable for material meeting aggregate-binder base aggregate specifications, confidence belts are provided in Figure 4. The confidence belt ordinates are computed using the formula tS_y, where tS_y equals the following

$$tS_y = tS_{y.x}\sqrt{\frac{1}{n_.} + \frac{1}{n} + \frac{(x-\bar{x})^2}{\Sigma(x-\bar{x})^2}} \quad \text{and}$$

$n_.$ = Samples required for making prediction.

n = Number of pairs of values used in establishing the regression equation.

The ordinate for α = .10 at X = 35 is computed substituting the following values:

$$\text{Ordinate} = tS_y = 1.288\sqrt{4.947}\sqrt{1 + \frac{1}{148} + \frac{35-32.11}{15,941}} = 2.87$$

The following t values are used for the different α levels, all of which are one sided and are obtained using one hundred forty-six (146) degrees of freedom; t = 2.38 for α = .01, t = 1.66 for α = .05, t = 1.29 for α = .10, t = 1.07 for α = .15 and t = 0.84 for α =.20.

If a prediction were desirable using the $.10 = \alpha$ level, a stockpile of material could be sampled and inference made concerning the prediction of plasticity index from the sand equivalent. The material, of course, would need to meet the specifications as shown above before Figure 4 could be used for prediction. As an example, if one were to sample a stockpile and obtain a sand equivalent value of 30, the statement could be made that the plasticity index would not be greater than 7.2 due to chance alone more than one out of ten times.

It might also be desirable to use the average of two (2) values of sand equivalent to make a prediction concerning the plasticity index of a stockpile of this type of aggregate.

If two (2) samples were obtained from a stockpile of material meeting AB-3 specifications and the two values averaged 35, it could be said that the plasticity index of the stockpile would be 7.6 or less unless a one (1) in one hundred (100) chance occurred that a bad sample was obtained. This prediction is possible using a ninety-nine percent (99%) one sided confidence interval band shown in Figure 4 as $\alpha = .01$ (Two Samples).

The correlation coefficient was computed and tested for significance. The formula

$$r = \frac{\Sigma(x-\bar{x})\ (y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\ \sqrt{\Sigma(y-\bar{y})^2}}$$

where $\Sigma(x-\bar{x})\ (y-\bar{y}) = -1259$, $\Sigma(x-\bar{x})^2 = 15{,}941$ and $\Sigma(y-\bar{y})^2 = 822$, gave a correlation coefficient of $-.3479$. This indicated that only

$(-.3479)^2$ or 12% of the variation was explained and 88% was left unexplained when the linear regression equation was used.

DISCUSSION OF RESULTS

Variation in Plasticity Index Values

In looking at Figures 1 through 4, it is apparent that the values of Y plotted above and below the regression line tend to indicate much variability if the Y value is dependent on the X value. Since the Y values represent the plasticity index values and variation might exist, the causes of this variation are discussed below. It should be pointed out also that the variations to be discussed are variations which have been measured by analysis of variance in other experimental results. Since the values of plasticity index as shown in Figures 1 through 4 were determined by many different operators and in many different laboratories, the estimated variance due to different operators and laboratory procedure might be mentioned. It is not possible, however, to make a statement concerning the amount of the plasticity index variation as shown in Figures 1 through 4 which is due to different operators or any other cause. There is no way to substantiate a statement concerning this variability because an analysis of variance was not determined for any of the liquid limit values or the plastic limit values which were used to yield the plasticity index values as shown in Figures 1 through 4.

As a matter of interest, variance due to laboratory procedure has been estimated for other experimental work. G. E. H. Ballard and W. F. Weeks investigated the operator variance in the determination of the plastic limit of cohesive soils.[8] Five random samples were distributed to five zones of operators, where two operators of each group performed five replicate tests. It was their finding that the best estimator of the within operator variance (σ_ϵ^2) was 0.448, while the best estimator of the between-operators-within-zones variance (σ_δ^2) was 4.180. This clearly indicates that there is appreciably more variation between operators than replicate measurements by a given operator.

The variation due to laboratory procedure in the determination of the liquid limit has been estimated by analysis of variance of other experimental work.[9] The value for the estimate of the variation due to laboratory procedure was 5.71. In the conclusions, it was suggested one operator be used in a given series of tests to increase the accuracy of the variability study. This suggests that variation between operators does exist and may be of very undesirable magnitude.

For the matter of discussion, the following values of $S_{y.x}^2$ are given for the four aggregates observed in Figures 1 through 4.

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8 "The Human Factor in Determining the Plastic Limit of Cohesive Soils," by G. E. H. Ballard and W. F. Weeks, Materials Research and Standards, ASTM, Vol. 3 Number 9, p. 726.

9 "Variability of Engineering Properties of Brookston and Crosby Soils," by Delon Hampton, E. J. Yoder, and I. W. Burr, Highway Research Board Proceedings, Vol. 41, p. 621.

Aggregate	$S_{y.x}^2$
Road and Plant Mix Aggregate	3.385
Hot Mix Aggregate	1.259
Shoulder Aggregate	0.728
Aggregate Binder-Base Aggregate	4.947

It will be remembered that $S_{y.x}^2$ represents the mean square deviation from regression and furnishes an estimate of σ^2. As seen above, the variance (σ^2) due to operator or laboratory procedure variance was estimated to be 4.180 for the plastic limit and 5.71 for the liquid limit in other experimental results. As stated before, the magnitudes discussed above cannot be used to estimate the operator variance of the results shown in this paper. It is desirable, however, to point out the fact that the confidence band ordinates are computed using $S_{y.x}$ and if $S_{y.x}$ could be reduced by removing the effect of the among operator variance, it might improve the prediction ability.

It might also be pointed out that as the variance due to operator or laboratory occurs in both the liquid limit determination and the plastic limit determination, this may cause the variance to be even greater in the plasticity index since it is determined from subtracting the plastic limit from the liquid limit.

Other sources of variation which might contribute to variable plasticity index values could be termperature and pore water concentration. The values of plasticity index shown in Figure 1 through 4 were obtained using pore water suitable for drinking purposes, however, the concentration was not determined. The test values were also determined in many cases in field laboratories where strict temperature control could not be enforced.

Of course, the sand equivalent values are also subject to variation and may very well be responsible for more of the observed variability than the PI values.

Discussion of Power of Procedure

As has been pointed out in previous discussion, the correlation coefficients are significantly different from zero for each type of aggregate but they are numerically lower than desirable. Another way to state the difficulty would be that the power of the test is low and it may lead to rejecting too often if plasticity index specifications are used to set specification limits on sand equivalent values. For example, referring to Figure 1, if the upper limit of six (6) for plasticity index were used as a control and 90% confidence ($\alpha = .10$) was desirable, it would be possible to say that a minimum value of forty-five (45) would be required for sand equivalent. A minimum value of forty-five (45) for sand equivalent would enable one to predict that the plasticity index would not be greater than 6.1 unless a 1-in-10 chance error occurred in sampling. The difficulty seems to be, however, that when 45 was used as a minimum allowable value for sand equivalent, about 33% of the 440 samples were rejected. This was compared to 8% rejection of the 440 samples when using a maximum plasticity index allowable of 6. If the allowable minimum sand equivalent were determined for 85% confidence ($\alpha = .15$), the minimum allowable would be lowered to 35. A minimum of 35 for sand equivalent would reject about 10% of the 440 samples represented in Figure 1.

F. N. Hveem[10] found a relationship between sand equivalent value and resistance value and suggested the minimum values as shown in Figure 5. The resistance values plotted were the lateral pressure transmitted by the specimen while under load. The values were obtained from the stabilometer and reflected the internal friction or degree of lubrication.

Figure 2 indicates that about 24% of the 155 samples would be rejected if the maximum allowable value of plasticity index were 3.0. If the minimum sand equivalent allowed were 50, it would be possible to predict that the plasticity index would not be greater than 3.2 if 80% confidence (α = .20) were desired. The minimum sand equivalent of 50 would reject about 38% of the 155 samples. As shown in Figure 5, California Division of Highways specifies a minimum of 55 for Asphaltic Concrete, however, using this as a minimum would reject about 54% of the 155 samples shown in Figure 2.

Figure 3 indicates that if a minimum sand equivalent value of 20 were allowed for aggregate used for shoulder construction, one would be 80% confident that the plasticity index would be between 3.6 and 5.8. If the maximum plasticity index were 6, 10% of the 179 test samples would be rejected. The percent rejected using a minimum sand equivalent of 20 was 21% of the 179 tests. The data shown in Figure 3 suggests that curvilinear regression might be better than linear regression. It was felt that the summation of the deviations about

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10 Sand Equivalent Test for Control of Materials During Construction, by F. N. Hveem, Highway Research Board Proceedings, Vol. 32, 1953, p. 248.

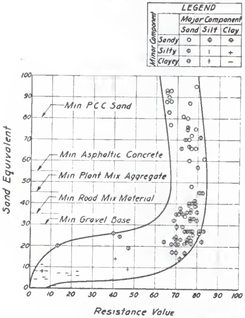
LEGEND
Major Component
Sand
Silt
Clay
Minor Component
Sandy
Silty
Clayey
Min PCC Sand
Min Asphaltic Concrete
Min Plant Mix Aggregate
Min Road Mix Material
Min Gravel Base
Sand Equivalent
Resistance Value
0
10
20
30
40
50
60
70
80
90
100

Figure 5

the regression would probably decrease and thereby decrease the confidence band ordinates about the curvilinear regression line. This might not benefit the power of the test, however, because the minimum limit of sand equivalent would probably not be lowered below 20 due to the rapidly decreasing resistance as the sand equivalent drops below 20 (see Figure 5).

Figure 4 indicates that if one wants to be 90% confident ($\alpha = .10$) that the plasticity index will be 8 or less, a minimum sand equivalent of 20 would be required. As can be seen in Figure 5, California requires a minimum sand equivalent of 30 for gravel base, but since eighty-five percent of the material shown in Figure 4 must be crushed limestone, by specifications, it is felt that an investigation to determine the resistance value at sand equivalent of 20 would be desirable. In using a maximum plasticity index of 8 as the control, about 6% of the 148 samples shown in Figure 4 would be rejected. The minimum value of 20 for sand equivalent at the $\alpha = .10$ level would reject about 7% of the 148 samples.

CONCLUSIONS

There appears to be no correlation between sand equivalent values and plasticity index values, which could be used for engineering decisions, from the data analyzed in this study. Since most of the correlation coefficients (r's) shown in Figures 1 through 4 are small this would indicate that most of the variation in plasticity index values (y-axis) is unexplained by sand equivalent values (x-axis).

It is felt that another investigation might be made using curvilinear regression or multiple regression techniques. Curvilinear regression might better fit the data as shown in Figures 1 through 4, but it is not known if exponential or polynomial curves would be the best fit. It is also felt that multiple regression techniques could be used to determine if other variables might be used to explain more of the variation than is presently explained. The percent passing the 200 sieve or the amount passing the 200 sieve in the material passing the number 4 sieve are two suggested variables which might explain some of the unexplained variation.

In studying the limited information available, it can be seen that correlation was not found, but this may be hidden by between-operator or laboratory procedure variance in the test data, for both plasticity index and sand equivalent values.

It should be pointed out that the use of the graphs for rejecting material would probably not be economically feasible because of the large amount of variation which has not been explained by the linear regression equations.

A C K N O W L E D G M E N T

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CORRELATION STUDY BETWEEN

SAND EQUIVALENT AND PLASTICITY INDEX

TESTS

by

Leon W. Heidebrecht

B. S., Kansas State University, 1957

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

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This paper considers the correlation between two parameters of aggregates used in bituminous mixes and bases, the sand equivalent and plasticity index. It shows that, from the data analyzed, there is no useful correlation between these values that would permit the prediction of plasticity indices from more rapidly obtained sand equivalent values.

The test values representing the parameters were obtained from aggregates used in bituminous, asphaltic concrete, shoulder and aggregate binder base construction. These values were determined for the above aggregates used in several counties in Kansas.