NOISE IN SEMICONDUCTOR DEVICES

by

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INTRODUCTION

Semiconductors became popular after the invention of the transistor in 1948, at the Bell Telephone Laboratories. The scientific importance of this invention by J. Bardeen, W. H. Brattain, and W. Shockley was recognized by the award of the 1956 Nobel prize in Physics. Although silicon and germanium are the most widely used semiconductors, intensive present research is being carried out on other compound materials such as germanium silicon alloys, gallium arsenide, indium phosphide, and silicon carbide.

Transistors, switching diodes, tunnel diodes, and varactors are some of the useful semiconductor devices used invariably in place of vacuum tubes. Their advantages over vacuum tubes are less power consumption, low voltage requirement, considerable reduction in the size of equipment, considerably longer life, and the expected future low manufacturing cost as compared to the vacuum tubes.

Like all other electron devices, semiconductors introduce noise in the signal being processed. A good deal of research on the noise properties of semiconductor devices has resulted in better manufacturing and optimization techniques.

MATHEMATICAL BACKGROUND

The basic principles of the theory of random noise used to obtain the statistical properties of noise are given in this section.

Probability Density Functions, Averages, Variances, and Correlation

If R(u) is a continuous random variable and may take on any value between $-\infty$ to $+\infty$, then the distribution function F(x) associated with R(u) is defined as the probability of R(u) being less than or equal to x, which is written symbolically as (Bendat, 1958)

$$F(x) = [\text{Prob } R(u)] \leq x \qquad (1)$$

Its probability density function is defined as

$$f(x) = \frac{d}{dx} F(x) \qquad (2)$$

and the probability of R(u) being in the range x and x + Δ x is

$$P[x < R(U) \leq x + \Delta x] = \int_{x}^{x+\Delta x} f(x)dx = f(x)dx \qquad (3)$$

where F(x) has been assumed to be differentiable.

The expected value or the mean value $\overline{R}$, mean square value $\overline{R^2}$, and variance of R, σ^2 can be written as

$$\overline{R} = E(R) = \sum_{\substack{i=1 \\ k\to\infty}}^{k} x_i \text{ Prob}[R = x_i] = \int_{-\infty}^{\infty} xf(x)dx \qquad (4)$$

$$\overline{R^2} = E(R^2) = \sum_{\substack{i=1 \\ k\to\infty}}^{k} x_i^2 \text{ Prob}[R = x_i] = \int_{-\infty}^{\infty} x^2 f(x)dx \qquad (5)$$

$$\sigma^2 = E\left[(R - \bar{R})^2\right] = \sum_{i=1}^{k} (x_i - \bar{R})^2 \left[\text{Prob}(R = x_i)\right]$$

$$\underset{k \to \infty}{=} \int_{-\infty}^{\infty} (x - \bar{R})^2 f(x)dx \qquad (6)$$

Some of the common discrete and continuous probability distributions are given below.

Binomial Distribution. If p is the probability of occurrence of a certain event, A, and (1 - p) is the probability of the failure of the event, then the probability of x occurrences of the event in n trials is

$$f(x) = \binom{n}{x} p^x (1 - p)^{n-x} \qquad (7)$$

The mean value and variance of R are

$$\bar{R} = np$$

$$\text{Variance} = \overline{(R - \bar{R})^2} = \bar{R^2} - (\bar{R})^2 = np(1 - p) \qquad (9)$$

Poisson Distribution. When n becomes very large and p is close to zero, binomial distribution in the limit approaches Poisson distribution, which is given as

$$f(x) = \frac{m^x}{x!} \exp\left[-m\right] \qquad (10)$$

with mean and variance both equal to m.

Normal Distribution. When n is very large and p is neither close to zero nor unity, binomial distribution in the limit approaches normal distribution, for which

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp -(x - m)^2/2\sigma^2 \qquad (11)$$

where m and σ^2 are the mean and the variance, respectively.

One-dimensional distribution discussed above can be easily extended to a two-dimensional one. If R(u) and S(v) are continuous random variables, then

$$\text{Prob}\left[R(u) \le x \text{ and } S(v) \le y\right] = F(x, y) \tag{12}$$

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y} \tag{13}$$

$$E(R^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x, y)dydx \tag{14}$$

$$E(S^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 f(x, y)dydx \tag{15}$$

$$E(RS) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\ f(x, y)dydx \tag{16}$$

E(RS) is the cross-correlation function of R and S. The correlation coefficient, C, is given as

$$C = \frac{E(RS)}{\sqrt{E(R^2)\ E(S^2)}} \tag{17}$$

Power Spectral Density Function and Its Relationship with Autocorrelation Function

If x(t) is a randomly varying signal for $-T < t < T$, then the average signal power across a unit resistor can be expressed as

$$P_{av} = \frac{1}{2T}\int_{-T}^{T} x^2(t)dt \tag{18}$$

By Fourier analysis it can be shown that (Bendat, 1958)

$$P_{av} = \int_{0}^{\infty} g(f)df \tag{19}$$

where g(f) is called the power spectral density function of x(t).

Autocorrelation in the case of a stationary random process, defined as a random process with statistical properties which remain unchanged with translation in time, is given as

$$\Gamma_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)\,dt \tag{20}$$

and the autocorrelation coefficient $c(\tau)$ is

$$c(\tau) = \frac{\Gamma_{xx}(\tau)}{\overline{x^2(t)}} \tag{21}$$

Autocorrelation function and power spectral density function of a stationary process are related.

$$g_{xx}(f) = 4 \int_0^\infty \Gamma_{xx}(\tau) \cos 2\pi f\tau \, d\tau \tag{22}$$

Substitution of Eq. (21) yields

$$g_{xx}(f) = 4\,\overline{x^2(t)} \int_0^\infty C(\tau) \cos 2\pi f\tau \, d\tau \tag{23}$$

If a signal x(t) with spectral density function $g_{xx}(f)$ is passed through a filter with frequency response Y(f), then the output, y(t), with spectral density function $g_{yy}(f)$ is given as

$$g_{yy}(f) = g_{xx}(f)\, |Y(f)|^2 \tag{24}$$

If $\overline{x^2(t)}$ and $C(\tau)$ are known, spectral density can be calculated from Eq. (23). $\overline{x^2(t)}$ is usually known from statistical considerations, and $C(\tau)$ can be calculated by the use of the Langevin differential equation (Van Der Ziel, 1958).

The Langevin differential equation relates the fluctuating response u(t) of the system to a random noise signal H(t). For an LCR circuit this noise signal is the noise emf of the circuit. It is assumed that enough information is known about H(t) from statistical consideration of the process. The Langevin equation is written as

$$L(p)u(t) = H(t) \tag{25}$$

where L(p) is a polynomial in operator p defined as p = d/dt.

The following example of spontaneous temperature fluctuations illustrates the use of the Langevin differential equation. A small body with a heat capacity 'C' is considered. If H(t) is the fluctuating heat exchange with the surroundings, then the Langevin equation is written as

$$c\,\frac{d\theta}{dt} + \beta\theta = H(t) \tag{26}$$

where β is the coefficient of heat transfer to the surroundings and θ is the fluctuations in temperature of the body around its equilibrium temperature T.

If $\theta = \theta_0$ at $t = 0$, taking average values for $t > 0$ for this value of θ_0 and denoting the average by $\overline{}^{\theta_0}$ above the quantity, gives

$$\overline{H(t)}^{\theta_0} = 0 \tag{27}$$

since H(t) is assumed to be independent random source, thereby making H(t) independent of θ_0 for $t > 0$. Equation (26) reduces to

$$C\,\frac{\overline{d\theta}^{\theta_0}}{d\tau} + \beta\,\overline{\theta}^{\theta_0} = 0 \tag{28}$$

Therefore

$$\overline{\theta}^{\theta_0} = \theta_0\, e^{-\beta(\tau/c)} \tag{29}$$

Autocorrelation $\overline{\theta\theta_0}$ is calculated by first taking the average for a fixed value of θ_0 and then averaging over θ_0.

$$\overline{\theta_0\theta} = \overline{\overline{\theta_0\theta}^{\theta_0}} = \overline{\theta_0\overline{\theta}^{\theta_0}} = \overline{\theta}_0^{\,2}\ e^{-\beta(\tau/c)} \tag{30}$$

$$\text{Autocorrelation coefficient } c(\tau) = \frac{\overline{\theta_0\theta}}{\overline{\theta_0}^{\,2}} = e^{-\beta(\tau/c)} \tag{31}$$

SOURCES OF NOISE

The most important and understood sources of noise in semiconductor devices are thermal noise, shot noise, and flicker noise. Thermal noise occurs in any conductor and is caused by random motion of its current carriers, e.g., holes and electrons in semiconductors. This type of noise is usually represented by

$$\sqrt{\overline{e}^2} = \sqrt{4\ KTR_n \Delta f} \tag{32}$$

as equivalent noise voltage source, or

$$\sqrt{\overline{i}^2} = \sqrt{4\ KTG_n \Delta f} \tag{33}$$

as equivalent noise current source, and is appropriately inserted in series or in parallel with the noisy resistor. In the above equations, K is the Boltzman's constant $K = 1.38\text{x}10^{-23}$ joule/°K , T is the absolute temperature in °K, and R_n or G_n is the equivalent noise resistance or conductance, respectively.

Shot noise has characteristics of white noise over all frequencies of practical range, namely, it has a uniform spectral density. It is caused by the random changes in the number of charge carriers, each of which has a finite charge, +e for a hole and -e for an electron. For frequencies which are comparably less than the inverse of transit time of the charge carriers, power spectral density function for shot noise is given as

$$g(f) = 2\, e\, \bar{I} \tag{34}$$

where $\bar{I}$ = the average direct current

e = the charge of an electron in magnitude

$g(f)$ = the power spectral density function of current fluctuations

The phenomenon of flicker noise is not yet completely understood. However, several theories have been developed which discuss the origin and behavior of flicker noise. Flicker noise occurs at low frequencies and has a spectrum of the form $f^{-\alpha}$ (Fonger, 1959) with α close to unity. There are two components of flicker noise, both with a low-frequency spectrum--surface noise and leakage noise.

Surface noise is caused by the modulation of the flow of carriers by charge centers at potential barriers. There are two types of energy levels at the surface of a semiconductor, namely, "slow" states and "fast" states. The first act mainly as traps for the majority carriers and the latter as recombination centers for minority carriers. The fluctuating property of the slow states modulates the conductivity of the junction, which is the cause of flicker noise in bulk material. In addition, it modulates the capture cross section of the recombination centers, which is the cause of surface noise in diodes and transistors.

The source of leakage noise is the fluctuations in the leakage current caused by a thin conducting film bypassing the junction. Leakage occurs at the perimeter of the junction and increases with an increase in back bias. Therefore leakage noise predominates in back-biased junctions.

Both flicker noise and surface noise are very sensitive to ambient atmosphere, but are completely independent of temperature.

SHOT NOISE

The noise due to charge carriers moving under a drift is discussed first and it is followed by the discussion of fluctuations of charge carriers moving due to a concentration gradient, or, in other words, due to diffusion property.

Shot Noise Due to Drifting Carriers

If an electric field $\bar{E}$ is applied across a semiconductor material of length L, then drift velocity, v_d, can be written as

$$v_d = \mu \bar{E} \tag{35}$$

where μ $\left(\frac{m^2}{\text{volt-sec}}\right)$ is the mobility of the charge carriers.

Average drift time τ_d of the charge carriers from one electrode to the other is

$$\tau_d = L/\mu \bar{E} \tag{36}$$

If the total number of charge carriers in the sample is equal to N, then the average current is

$$I = \frac{e\,N}{\tau_d} = \frac{eN\mu\bar{E}}{L} \tag{37}$$

where e is the charge of a carrier, +e for a hole, and -e for an electron. Fluctuating current is

$$i(t) = \frac{e(\delta N)\mu\bar{E}}{L} \tag{38}$$

Autocorrelation function of i(t), using Eq. (21), can be written as

$$\Gamma_{ii} = \overline{i(t)\, i(t+\tau)} = \overline{i^2(t)}\; c(\tau) \tag{39}$$

Power spectral density of i(t), using Eq. (23), is

$$g_{ii}(f) = 4\, \overline{i^2(t)} \int_0^\infty c(\tau) \cos \omega\tau \, d\tau \tag{40}$$

Substitution of Eq. (38) in Eq. (40) yields

$$g_{ii}(f) = 4\left(\frac{e\mu\bar{E}}{L}\right)^2 (\overline{\delta N})^2 \int_0^\infty c(\tau) \cos \omega\tau \, d\tau \tag{41}$$

It is necessary to know $\overline{\delta N^2}$ and $c(\tau)$ to obtain $g_{ii}(f)$. The problem of computing $c(\tau)$ can be considered in two steps. In the first step, average free time, τ_0, of the charge carriers is assumed to be small as compared with the drift time. $c(\tau)$ can then be written as (Van Der Ziel, 1958)

$$c(\tau) = \exp(-\tau/\tau_0) \qquad (\tau_0 < \tau_d) \tag{42}$$

In the second step, τ_0 is considered to be large as compared to the drift time. In other words, if fluctuation δN_0 is present at time t = 0, then $(1 - \tau/\tau_d)$ of δN_0 will, on the average, be present at a time $t = \tau$. Therefore

$$c(\tau) = \begin{cases} 1 - \dfrac{\tau}{\tau_d} & (\tau < \tau_d) \\ 0 & \text{(otherwise)} \end{cases} \tag{43}$$

Substituting Eqs. (42) and (43) in Eq. (41) gives

$$g_{ii}(f) = \begin{cases} 4\left(\frac{e\mu\bar{E}}{L}\right)^2 \overline{\delta N^2} \frac{\tau_0}{1 + \omega^2\tau_0^2} & \tau_0 < \tau_d \\ 2\left(\frac{e\mu\bar{E}}{L}\right)^2 \overline{\delta N^2} \,_d \left(\frac{\sin \frac{1}{2}\omega\tau_d}{\frac{1}{2}\omega\tau_d}\right)^2 & \tau_0 > \tau_d \end{cases} \tag{44}$$

$\overline{\delta N^2}$ is calculated from the statistical considerations of the charge carriers. It has different solutions under different conditions of temperature and impurity level of the semiconductor (Van Der Ziel, 1958).

Shot Noise Due to Diffusing Carriers

In most of the semiconductor devices charge carriers move under diffusion and experience recombinations on their way. Both diffusion and recombination occur in random and thus produce shot noise. The problem of shot noise due to diffusing carriers has been solved by two different approaches--the corpuscular approach, and the collective approach. In the corpuscular approach shot noise is considered to be due to a series of random and independent crossing of the junction by the individual charge carrier. On the other hand, in the collective approach process of random diffusion, recombination and generation of charge carriers are analyzed.

The following sections describe shot noise in junction diodes, transistors, and tunnel diodes employing corpuscular approach.

Junction Diodes

In a junction diode the average current and forward admittance are given as (Joyce and Clarke, 1962)

$$I = I_0 \left[\exp\left(\frac{ev}{KT}\right) - 1 \right] \tag{45}$$

$$Y = G_0 = \frac{e}{KT} \left[I + I_0 \right] \tag{46}$$

where V = the forward bias voltage of the junction

T = the absolute temperature of the diode in K°

K = the Boltzmann constant

I_0 = the back-biased current of the diode under saturation

In the following analysis holes are considered to be the only charge carriers. Current, I, can be considered to be made up of two components, $(I + I_0)$ and $(-I_0)$. Therefore the charge carriers can be separated into the following three groups.

Group 1. Holes flowing from the p region into the n region and recombining there, thus giving independent current pulses which carry a current $(I + I_0)$.

Group 2. Holes flowing from the p region into the n region and returning to the p region before having combined, thus giving independent and random double current pulses, each consisting of two single short current pulses of opposite polarity. The second pulse is delayed by random delay time with respect to the first one. This group of holes determines the high-frequency noise characteristics of the diode.

Group 3. Holes generated in the n region and diffusing into the p region, thereby contributing a current $(-I_0)$. This current is independent of the applied voltage, and therefore does not contribute to the diode admittance Y.

At low frequencies the effect of group 2 is negligible. Therefore the noise current generator $\overline{i^2}$ in parallel with the junction admittance is

$$\overline{i^2} = 2e(I + I_0)\Delta f + 2\, eI_0\Delta f \tag{47}$$

At higher frequencies the effect of charge carrier of group 2 cannot be ignored. The diode admittance, Y, now consists of two parts, G_0 due to group one, and an unknown part Y_2 due to group 2. Junction capacitance of the diode also affects the admittance. A modified form of the admittance is

$$Y = G_0 + Y_2 + j\omega c_T = G + jB \tag{48}$$

where c_T is the junction capacitance of the diode and is drawn as a parallel capacitance across its terminals in its equivalent circuit.

Charge carriers of group 2 return to the p region by diffusion caused by the thermal energy of the charge carriers. These carriers thus produce thermal noise, which is given as $4\,KT(G - G_0)\Delta f$. Therefore noise current defined in Eq. (47) at higher frequencies is

$$\overline{i^2} = 2e(I + I_0)\Delta f + 2eI_0\Delta f + 4\,KT(G - G_0)\Delta f \tag{49}$$

$$= 4\,KTG\Delta f - 2\,eI\Delta f \tag{50}$$

An equivalent shot noise model for a diode is shown in Fig. 1(a) (Van Der Ziel, 1958).

Junction Transistors

In junction transistors the emitter current, I_e, can be considered to be made up of two independent components $(I_e + I_{ee})$ and $(-I_{ee})$, where I_{ee} is the back-bias current of the emitter base diode under saturation. Therefore charge carriers (in a p-n-p transistor) can be divided into the following five groups.

<u>Group 1</u>. Emitter holes injected into the base region and collected by the collector.

<u>Group 2</u>. Emitter holes injected into the base region and recombining in that region with free electrons.

<u>Group 3</u>. Emitter holes injected into the base region and returning to the emitter.

<u>Group 4</u>. Holes generated in the base region and collected by the emitter.

<u>Group 5</u>. Holes generated in the base region and collected by the collector.

At low frequencies the effect of group 3 is negligible. Therefore an equivalent noise current generator in parallel with the emitter junction is

$$\overline{I_1}^2 = 2e(I_e + I_{ee})\Delta f + 2e\, I_{ee}\Delta f \tag{51}$$

Fluctuations in collector current, I_c, also produce shot noise, and its equivalent noise current generator in parallel with the collector junction is

$$\overline{I_2}^2 = 2e\, I_c\Delta f \tag{52}$$

The collector current is given as

$$I_c = \beta_0(I_e + I_{ee}) + I_{cc} = \beta_0 I_e + (\beta_0 I_{ee} + I_{cc}) \tag{53}$$

where β_0 is the transportation factor of the base defined as the ratio of the holes injected into the base to the holes collected by the collector, and I_{cc} is the collector current when the emitter current is zero.

Equation (53) can be written as

$$I_c = \beta_0 I_{\underset{e}{\cancel{ee}}} + I_{c0} \qquad (54)$$

where $$I_{c0} = \beta_0 I_{ee} + I_{cc}$$

The current $\beta_0(I_e + I_{ee})$ flows from emitter to collector. Therefore correlation $i_1{*}i_2$ of the noise current generators i_1 and i_2 is

$$\overline{i_1{*}i_2} = 2e\,\beta_0(I_e + I_{ee})\Delta f \qquad (55)$$

where the asterisk mark denotes the conjugate.

The transconductance, Y_{ce}, of the transistor is defined as

$$Y_{ce} = \left.\frac{\partial I_e}{\partial V_e}\right|_{i_e=0} = G_{ce0} = \frac{\beta_0 e(I_e + I_{ee})}{KT} \qquad (56)$$

The analysis given above can be generalized by considering both the holes and electrons as the charge carriers. Besides holes, now there are electrons flowing from emitter to base, base to emitter, and collector to base. Emitter current is partly caused by holes injected from the emitter into the base and partly by electrons injected from the base into the emitter. Equations (55) and (56) are now written as

$$i_1{*}i_2 = 2e\alpha_0(I_e + I_{ee})\Delta f \qquad (57)$$

$$Y_{ce} = G_{ce0} = \frac{\alpha_0 e(I_e + I_{ee})}{KT} \qquad (58)$$

where $\alpha_0 = Y_0\beta_0$ is the d-c amplification factor of the transistor, and Y is the emitter efficiency defined as the proportion of the emitter current carried by holes.

Equations (57) and (58) yield good results up to frequencies of the order of 10 to 15 megacycles. For higher frequencies, noise current generators in Eqs. (51) and (52) can be modified by a reasoning similar to that for the diode and the resulting expressions are

$$\overline{i_1}^2 = 4\ KTG_e \Delta f - 2e\ I_e \Delta f \tag{59}$$

$$\overline{i_2}^2 = 2e\ I_c \Delta f \tag{60}$$

where G_e is similar to G in Eq. (50).

At low frequencies collector current follows the emitter voltage almost instantaneously, while at high frequencies finite diffusion time of the charge carriers, holes in this case, should be taken into account. Therefore transfer admittance at high frequencies can be written as

$$Y_{ce} = G_{ce0} \exp\left[-j\omega\mathcal{T}\right] \tag{61}$$

where $\mathcal{T}$ is the diffusion time of the holes considered to be the same for all the holes.

If diffusion time $\mathcal{T}$ is assumed to have a distribution $g(\mathcal{T})d\mathcal{T}$ $(0 \leq \mathcal{T} < \infty)$, Eq. (61) can be written as

$$Y_{ce} = \int_0^\infty G_{ce0} \exp\left[-j\omega\mathcal{T}\right] g(\mathcal{T})d\mathcal{T} \tag{62}$$

Cross correlation in the modified form is

$$\overline{i_1{}^* i_2} = 2e\alpha_0(I_e + I_{ee})\Delta f \exp\left[-j\omega\mathcal{T}\right] \tag{63}$$

Substitution of Eq. (58) in Eq. (63) yields

$$\overline{i_1 i_2{}^*} = 2\ KTG_{ce0}\Delta f \exp\left[-j\omega\mathcal{T}\right] \tag{64}$$

Taking distribution of $\mathcal{T}$ into account,

$$\overline{i_1^* i_2} = \int_0^\infty 2KT\ G_{ce0}\,\Delta f\ \exp\left[-j\omega\tau\right]\ g(\tau)d\tau = 2\ KT\ Y_{ce}\,\Delta f \qquad (65)$$

An equivalent noise model for a transistor is given in Fig. 1(b) (Van Der Ziel, 1958) and is redrawn in Fig. 1(c) (Van Der Ziel, 1958), where total effect of shot noise is represented by a noise current generator $\sqrt{\overline{i}^2}$ across the collector and a noise emf generator $\sqrt{\overline{e}_e{}^2}$ in series with the emitter. Expressions for $\overline{e}_e{}^2$ and $\overline{i}^2$ are

$$\overline{i}^2 = \overline{(i_2^* - \alpha^* i_1^*)(i_2 - \alpha\, i_1)}$$

$$= i_2{}^2 - \alpha\overline{{}^*i_1{}^* i_2} - \alpha\,\overline{i_1 i_2{}^*} + \left|\alpha\right|^2 i_1{}^2 \qquad (66)$$

Substitution for $\overline{i_1}^2$, $\overline{i_2}^2$, $\overline{i_1{}^* i_2}$, and $\overline{i_1 i_2{}^*}$ from Eqs. (51), (52), and (57) yields

$$\overline{i}^2 = 2e\,\Delta f\left[I_c - \left|\alpha\right|^2 I_e\right] \qquad (67)$$

Replacing I_c by its equivalent $\alpha_0 I_e + I_{c0}$ gives

$$\overline{i}^2 = 2e\Delta f\left[\alpha_0 I_e - \left|\alpha\right|^2 I_e + I_{c0}\right] \qquad (68)$$

and

$$\overline{e}_e{}^2 = \overline{i}_1{}^2\left|Z_e\right|^2$$

$$= \left[4\ KTG_e\Delta f - 2\ eI_e\Delta f\right]\left|Z_e\right|^2$$

$$= \left[4\ KTG_e\Delta f - 2\ e\,\Delta f(I_e + I_{ee} - I_{ee})\right]\left|Z_e\right|^2$$

$$= 2KT\Delta f\left[2\ G_e - G_{e0} + G_{e00}\right]\left|Z_e\right|^2 \qquad (69)$$

where $G_{e0} = \dfrac{e}{KT}(I_e + I_{ee})$

$$G_{e00} = \frac{eI_{ee}}{KT}$$

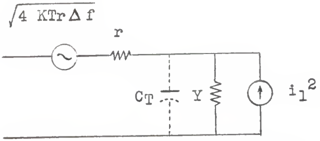

Fig. 1(a). Equivalent circuit for diode shot noise.

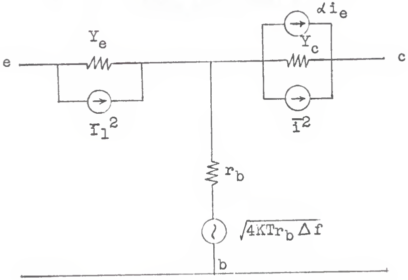

Fig. 1(b). Equivalent circuit for transistor shot noise.

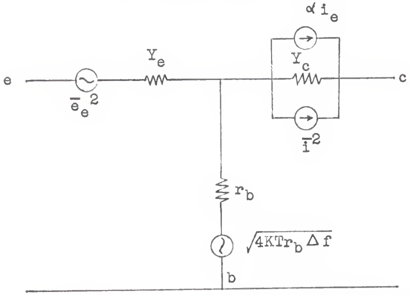

Fig. 1(c). Equivalent circuit for transistor shot noise with Thevenin generator on emitter side.

$$\overline{e_e{*}i} = |Z_e| \; \overline{i_1{*}(i_2 - \alpha_0 i_1)}$$
$$= |Z_e| \left[\overline{i_1{*}i_2} - \alpha_0 \overline{i_1^2}\right]$$
$$= |Z_e| \left[2\ KTY_{ce}\Delta f - \alpha_0(4KTG_{e0}\Delta f - 2eI_{ee}\Delta f)\right] \quad (70)$$

If $I_e \gg I_{ee}$, Eq. (70) can be written as

$$\overline{e_e{*}i} = 2\ KT|Z_e|\Delta f\left[Y_{ce} - G_{e0}\right] \quad (71)$$

At low frequencies Y_{ce} is nearly equal to G_{e0}; therefore the correlation $\overline{e_e{*}i}$ is negligible. At higher frequencies Y_{ce} may be considerably greater than G_{e0}, and the correlation may have a significant value.

Tunnel Diodes

The tunnel diode was discovered by Esaki in 1957 during his experimental study of heavily doped p-n junctions. Distinct characteristics of the tunnel diode are its negative conductance property with high frequency, and high-speed performance. The tunnel diode can be used as an amplifier, an oscillator, a switch, frequency converter, harmonic generator, or mixer.

The current-voltage characteristic of a typical tunnel diode is illustrated in Fig. 2 (Paucel, 1963). The region of main interest is the negative conductance region bounded by peak and valley voltages V_p and V_v. Without going into the details of the characteristics, a small signal equivalent circuit is presented in Fig. 3 (Paucel, 1963). Series resistance, R_s, represents ohmic losses of the semiconductor bulk material external to the

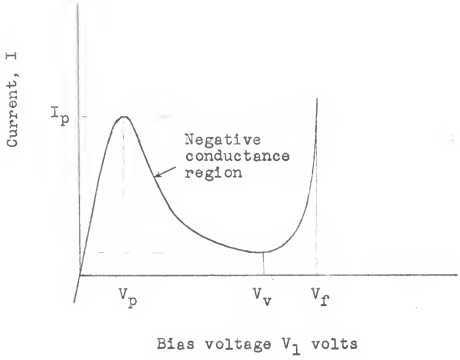

Fig. 2. Current-voltage characteristics of a typical tunnel diode.

junction. L_s represents inductance associated with the package and semiconductor, and C is the junction capacitance. L_s and C limit its high-frequency operation.

There are two principal sources of noise in a tunnel diode: shot noise, which is produced by the random flow of the charge carriers across the p-n junction, and thermal noise due to the bulk resistance of the semiconductor. The tunnel diode displays very little or no flicker noise when operated in the region of negative conductance (Yajima and Esaki, 1957).

It was found (Esaki, 1958) that in a tunnel diode there are two current streams flowing across the junction in opposite directions. The current flowing in the reverse direction is the familiar Zener current. Esaki explained the origin of these currents with the help of the energy diagram of Fig. 4 (Esaki, 1958), drawn for an unbiased junction.

If the current flowing from the valence band to the conduction band and that from the conduction band to the valence band are denoted as I_Z and I_E, respectively, then expressions for I_Z and I_E can be written as (Esaki, 1958)

$$I_Z = A\int_{E_c}^{E_v} f_v(E)\,\rho_v(E) Z_{v\to c}\left[1 - f_c(E)\right]\rho_c(E)\; dE \qquad (72)$$

$$I_E = A \quad {}_{E_c}^{E_v} \quad f_c(E)\,\rho_c(E) Z_{c\to v}\left[1 - f_v(E)\right]\rho_v(E)\; dE \qquad (73)$$

where $Z_{v\to c}$ and $Z_{c\to v}$ are the probabilities of penetrating the band gap and may be assumed to be approximately equal, $f_c(E)$ and $f_v(E)$ are the probability functions denoting that a state E in the conduction and valence bands is occupied, respectively, $\rho_c(E)$

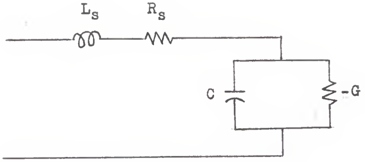

Fig. 3. Equivalent circuit of tunnel diode.

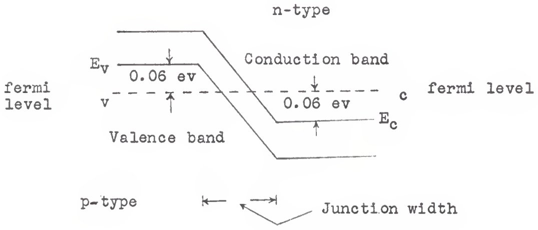

Fig. 4. Energy diagram of the p-n junction at 300° K and no bias voltage.

and $\rho_v(E)$ are the energy level densities in the conduction and valence bands, respectively, A is a constant, and E is the energy level of the conduction or valence band. Integration ranges over the overlap region of the two bands.

Under biased conditions net current flowing across the junction can be written as (assuming $Z_{c \rightarrow v} = Z_{v \rightarrow c}$ = a constant),

$$i = I_E - I_Z = A' \int_{E_c}^{E_v} \left[f_c(E) - f_v(E) \right] \rho_c(E) \rho_v(E) dE \qquad (74)$$

where A' is a constant.

Both current streams explained above are assumed to be independent of each other; thus both produce shot noise. Therefore equivalent saturated diode current for a tunnel diode is (Tiemann, 1960)

$$\begin{aligned} i_{eq} &= I_Z + I_E \\ &= A' \int_{E_c}^{E_v} \left[f_c(E) + f_v(E) - 2f_c(E) f_v(E) \right] \rho_c(E) \rho_v(E) dE \qquad (75) \end{aligned}$$

It was shown by Tiemann (1960) that this expression reduces to exact value for Johnson noise in the region of zero bias. This indicates full agreement of the model with the requirements of thermodynamics, which are that any electron device under thermal equilibrium should show thermal noise. Figure 5 (Tiemann, 1960) describes the behavior of the currents discussed above. Moreover the currents i and i_{eq} are shown to approach each other when more than a few times KT/e of bias is applied.

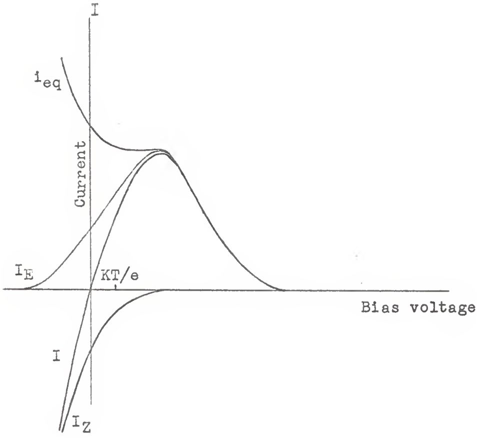

Fig. 5. Theoretical tunnel diode characteristics.

FLICKER NOISE

Flicker Noise in a Semiconductor Material

In semiconductors a large amount of noise over and above thermal and shot noise is found at low frequencies. Johnson discovered this phenomenon in 1925 in saturated tubes. The noise has a spectrum of the form $(f)^{-\alpha}$ with α close to unity. The frequency range over which flicker noise extends is from 2.5×10^{-4} cycles to a few kilocycles. Flicker noise is very much dependent upon surface conditions of the semiconductor. It is independent of temperature variations.

The phenomenon of flicker noise is due to the trapping of charge carriers at the surface of the material. The trapping process has a time constant τ_0. This time constant is not uniform over the entire surface, but has been shown to have a probability density function $p(\tau_0)$ (Van Der Ziel, 1958).

$$p(\tau_0) = \begin{cases} \dfrac{1}{\tau_0 \log \tau_2/\tau_1} & \text{for } \tau_1 < \tau_0 < \tau_2 \\ 0 & \text{otherwise} \end{cases} \tag{76}$$

In an N-type semiconductor, if $\overline{N}$ is the average of N, which is the number of charge carriers, then the spectral density $g_N(f, \tau_0)$, using Eq. (22), is

$$g_{\delta N}(f, \tau_0) = 4\int_0^{\infty} \Gamma_{NN}(\tau) \cos 2\pi f \tau \, d\tau \tag{77}$$

$$\Gamma_{nn}(\tau) = \overline{(N - \overline{N})_t (N - \overline{N})_{t+\tau}} = (\delta N)^2 \exp\left[-\tau/\tau_0\right] \tag{78}$$

where $\exp\left[-\tau/\tau_0\right]$ is the autocorrelation coefficient.

Substitution of $\Gamma_{nn}(\tau)$ in Eq. (77) gives

$$g_{\delta N}(f, \tau_0) = 4(\delta N)^2 \frac{\tau_0}{1 + \omega^2 \tau_0^2} \tag{79}$$

Also

$$g_{\delta N}(f) = \int_0^\infty g_{\delta N}(f, \tau_0) p(\tau_0) d\tau_0 \tag{80}$$

or

$$g_{\delta N}(f) = \frac{4(\delta N)^2}{\omega \log_e(\tau_2/\tau_1)} \left[\tan^{-1} \omega\tau_2 - \tan^{-1} \omega\tau_1\right] \tag{81}$$

The spectral density $g_{\delta N}(f)$ varies as $1/\omega$ over a wide range provided that τ_2 and τ_1 are far apart.

Flicker Noise in Diodes and Transistors

Origin of flicker noise in semiconductor devices, as previously explained, is caused by the random changes in the recombination rate of the charge carriers near the surface of the semiconductor by the charged centers. This causes fluctuations in the current flowing through the device. Resistance of a forward biased junction is strongly dependent on current. Therefore fluctuations of the current vary the resistance of the junction randomly, which is known as the modulation of the junction resistance.

Consider a p-n junction (Van Der Ziel, 1958) in which all the current is carried by holes. If the recombination rate is written as s, and assumed to be the same for all the surface elements, then the current density, J, disappearing at the surface can be written as

$$J = e\ p\ s \text{ amp/m}^2 \tag{82}$$

where p is the hole density and e is the charge of an electron.

Junction current, I, is given as

$$I = \int J\ d\ A \quad \text{amperes} \tag{83}$$

the integration being taken over all surface elements at which recombination takes place. The fluctuation δJ in the current density due to fluctuation δs in recombination rate is

$$J = ep\ \delta s = J/s\,[\delta s] \tag{84}$$

Spectral densities of δJ and δs, from Eq. (84), are

$$g_J(f) = \frac{J^2}{s^2}\, g_s(f) \tag{85}$$

Fluctuating current $\bar{I}_s^{\,2}$ can therefore be written as

$$\bar{I}_s^{\,2} = \int \frac{J^2}{s^2}\, g_s(f) dA\ df \tag{86}$$

where the integration is taken over all the surface recombination elements. As surface noise is proportional to 1/f and spectral density is a function of frequency, f, the current density, J, and the surface recombination velocity, s, it may be assumed (Van, Der, Ziel, 1958) that

$$g_s(f) = \frac{cJ^{\alpha}\, S^{\beta}}{f^{Y}} \tag{87}$$

where α and β are unknown constants and Y is assumed to be close to unity. Substituting Eqs. (86) and (87),

$$\bar{I}_s^{\,2} = \frac{cS^{\beta - 2}}{f^{Y}}\, \Delta f \int J^{2+\alpha}\ dA \tag{88}$$

if $\alpha = -1$. Then

$$\overline{i_s}^2 = \frac{cS^{\beta-2}}{f^{\gamma}} I \Delta f \tag{89}$$

Gianola (1956) found that experimental results agree very closely with Eq. (89). Fonger (1957) found that lower values of s result in lesser noise for constant current, which shows that β, in Eq. (89), is larger than two.

As the junction resistance depends upon the junction current, therefore the fluctuating current, $\overline{i_s}^2$, modulates the junction resistance randomly, and the alternating-current impedance of the junction differs from that of the direct current. At high frequencies alternating-current impedance may have a considerable amount of reactive component, but at low frequencies at which flicker noise is important the reactive component is negligible. Therefore at frequencies of the order of a few kilocycles, junction impedance can be approximated by pure resistance.

Flicker noise models for a diode and a transistor are shown in Fig. 6(a) and Fig. 6(b), respectively (Van Der Ziel, 1958). In Fig. 6(a), i_s represents noise current due to surface noise and i_L represents noise current due to leakage noise. Effect of modulation of series bulk resistance, R, is taken into account by connecting e_b in series with it, and i_s and i_L across the junction resistance, R_0, and the modulation resistance, R_{mb}, of the diode. Modulation resistance is given as

$$R_{mb} = I \frac{\partial R}{\partial I} \tag{90}$$

where I is the diode current.

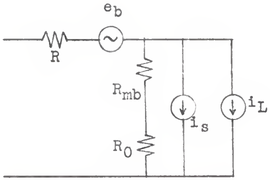

Fig. 6(a). Equivalent circuit for diode flicker noise.

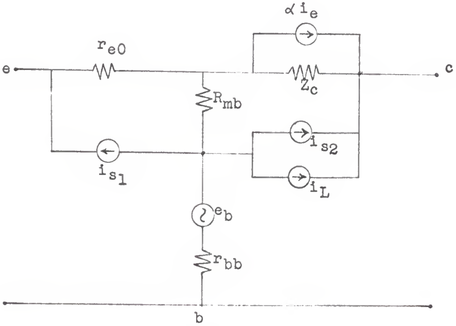

Fig. 6(b). Equivalent circuit for transistor flicker noise.

The way in which the modulation effect has been accounted for is somewhat arbitrary (Van Der Ziel, 1958). Most of the effect of modulation is taken into account by connecting i_s across R and R_{mb}. The part of modulation effect that is not taken into account by i_s is incorporated into the equivalent thermal noise emf, e_b.

The noise model for a junction transistor shown in Fig. 6(b) is a direct extension of Fig. 6(a) (Van Der Ziel, 1958).

Flicker noise is not a limiting factor in the operation of semiconductor devices because it can be reduced to a practicable limit by improving manufacturing techniques pertaining to the surface and junction perimeter of the device, and by using low voltages for back biasing of the junctions so as to keep the leakage to a minimum.

NOISE FIGURE

Noise figure is a measure of the noisiness of a device. It is a standard specification used to compare the noisiness of amplifiers. Noise figure is generally given in decibels and is defined as

$$F = \frac{\text{Total mean square noise voltage at the output}}{\begin{array}{l}\text{Mean square noise voltage at the output due to}\\ \text{thermal noise of the source resistance}\end{array}} \tag{91}$$

Transistor Amplifiers

The noise figure of a transistor amplifier can be calculated from the noise models established previously. The model of Fig. 1(c) is simplified for calculations of the noise figure (Nielsen, 1957) for frequencies below alpha cut-off frequency. Alpha cut-off frequency is defined as the frequency at which the grounded base current gain α of the transistor is reduced by root two. The following simplifications were suggested by Nielsen (1957).

1. Frequency dependence of the emitter noise generator is neglected.

2. Equivalent noise generators may be considered to be independent.

3. Space charge and junction capacitance effects are negligible.

Equations (68) and (69) can now be reduced to

$$\overline{i}^2 = 2e\Delta f\left[I_c - |\alpha|^2 I_e\right] = 2e\ I_c\left[1 - \frac{|\alpha|^2}{\alpha_0}\right]\Delta f \qquad (92)$$

and $$\overline{e}_e^{\,2} = 2\ e\ I_e \Delta\ f \qquad (93)$$

In Eqs. (92) and (93), I_{ee} was neglected in comparison with I_e.

Amplification factor α at higher frequencies is (Joyce and Clarke, 1962)

$$\alpha = \frac{\alpha_0}{1 + J(f/f_\alpha)} = \frac{\alpha_0}{\left[1 + (f/f_\alpha)^2\right]^{\frac{1}{2}}} \underline{/-\tan^{-1} f/f_\alpha} \qquad (94)$$

where f_α is the alpha cut-off frequency.

From Eqs. (92) and (94),

$$\overline{i}^2 = 2\ e\ I_c\left[1 - \frac{\alpha_0}{1 + (f/f_\alpha)^2}\right]\Delta f$$

$$= 2e\ I_c(1 - \alpha_0) \frac{\left[1 + \left(\frac{f}{\sqrt{1 - \alpha_0}\ f_\alpha}\right)^2\right]}{\left[1 + (f/f_\alpha)^2\right]} \Delta f \qquad (95)$$

The noise generator $\overline{i}^2$ may now be replaced by a noise emf generator, $\overline{e}_c^{\,2}$, in series with the collector impedance Z_c, when

$$\overline{e}_c^{\,2} = 2e\ I_c(1 - \alpha_0)\ Z_c^2 \frac{\left[1 + \left(\frac{f}{\sqrt{1 - \alpha_0}\ f_\alpha}\right)^2\right]}{\left[1 + (f/f_\alpha)^2\right]} \Delta f \qquad (96)$$

Substituting $\frac{KT}{r_e e} = I_e = \alpha_0 I_c$ in Eqs. (96) and (93),

$$\overline{e}_c^{\,2} = 2KT\alpha_0(1 - \alpha_0)\ Z_c^2 \frac{\left[1 + \left(\frac{f}{\sqrt{1 - \alpha_0}\ f_\alpha}\right)^2\right]}{\left[1 + (f/f_\alpha)^2\right]} \Delta f \qquad (97)$$

and $\overline{e}_e^{\,2} = 2\ KT\ r_e\ \Delta f$ (98)

where r_e is the emitter resistance.

A simplified noise model is shown in Fig. 7 (Nielsen, 1957) where

$$\overline{e}_s^{\,2} = 4\ KTR_s \Delta f \qquad (99)$$

is the thermal noise generator of the source resistance, R_s.

An arbitrary polarity of noise generators was chosen to make the calculations simple. Now e_0 and I_e are

$$\begin{aligned} e_0 &= e_c - e_b + I_e r_b + \alpha I_e Z_c \\ &= e_c - e_b + I_e(r_b + \alpha Z_c) \qquad (100) \end{aligned}$$

$$I_e = \frac{e_s + e_e + e_b}{R_s + r_b + r_e} \qquad (101)$$

From Eqs. (100) and (101),

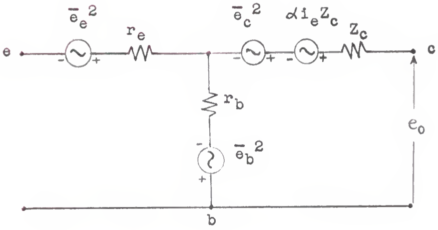

Fig. 7. Simplified noise model of a transistor.

$$e_0 = \frac{e_s(r_b + \alpha Z_c)}{R_s + r_b + r_e} + \frac{e_b(\alpha Z_c - R_s - r_e)}{R_s + r_b + r_e} + \frac{e_e(r_b + \alpha Z_c)}{R_s + r_b + r_e} + e_c \quad (102)$$

Assuming $r_b << \alpha Z_c$ and $R_s + r_e << \alpha Z_c$,

$$e_0 = \frac{e_s(\alpha Z_c)}{R_s + r_b + r_e} + \frac{e_b(\alpha Z_c)}{R_s + r_b + r_e} + \frac{e_e(\alpha Z_c)}{R_s + r_b + r_e} + e_c \quad (103)$$

Taking the mean square value of both sides and recognizing that all the noise generators are independent and therefore the mean of cross-product terms is zero,

$$\bar{e}_0^2 = \frac{\left[\bar{e}_s^2 + \bar{e}_b^2 + \bar{e}_c^2\right]\left[\alpha Z_c\right]^2}{(R_s + r_b + r_e)^2} + \bar{e}_c^2 \quad (104)$$

Noise figure, F_b, for this circuit can be calculated from Eq. (91) as

$$F_b = \frac{\bar{e}_0^2}{\bar{e}_s^2 \dfrac{|\alpha Z_c|^2}{(R_s + r_b + r_e)^2}} = \frac{\bar{e}_0^2 \, (R_s + r_b + r_e)^2}{\bar{e}_s^2 \, |Z_c|^2} \quad (105)$$

which can be simplified using Eq. (104) as

$$F_b = 1 + \frac{\bar{e}_b^2}{\bar{e}_s^2} + \frac{\bar{e}_e^2}{\bar{e}_s^2} + \frac{\bar{e}_c^2 \, (R_s + r_b + r_e)^2}{\bar{e}_s^2 \quad |Z_c|^2} \quad (106)$$

Substitution of Eqs. (97) through (99) in (106) yields

$$F_b = 1 + \frac{r_b}{R_s} + \frac{r_e}{2R_s} + \frac{\alpha_0(1-\alpha_0)(R_s+r_b+r_e)^2\left[1 + \left(\frac{f}{1-\alpha_0 \, f_\alpha}\right)^2\right]}{2 \, R_s r_e \alpha^2\left[1 + (f/f_\alpha)^2\right]} \quad (107)$$

From Eqs. (94) and (107),

$$F_b = 1 + \frac{r_b}{R_s} + \frac{r_e}{2R_s} + \frac{(1-\alpha_0)(R_s+r_b+r_e)^2 \left[1 + \left(\frac{f}{\sqrt{1-\alpha_0}\; f_\alpha}\right)^2\right]}{2\, R_s\, r_e\, \alpha_0} \tag{108}$$

The noise figure for common collector and common emitter configurations can similarly be calculated to be

$$F_c = 1 + \frac{r_b}{R_s} + \frac{r_e}{2R_s} + \frac{(1-\alpha_0)\left[1 + \left(\frac{f}{\sqrt{1-\alpha_0}\; f_\alpha}\right)^2\right](R_s+r_b)^2}{2\, r_e R_s \left[1 + (f/f_\alpha)^2\right]} \tag{109}$$

and

$$F_e = 1 + \frac{r_b}{R_s} + \frac{r_e}{2R_s} + \frac{(1-\alpha_0)(R_s+r_b+r_e)^2 \left[1 + \left(\frac{f}{\sqrt{1-\alpha_0}\; f_\alpha}\right)^2\right]}{2\, \alpha_0\, r_e\, R_s} \tag{110}$$

for $\alpha_0 \approx 1$; $r_e << R_s + r_b + r_e$, which is usually the case, and $f < f_\alpha$

$$F_b \approx F_c \approx F_e$$

However, at frequencies above f_α , F_c tends to be constant due to the frequency dependent term in the denominator, whereas F_e and F_b continue to increase.

Figure 8 (Nielsen, 1957) describes the behavior of the noise figure in the common base and common emitter transistor amplifier. The noise figure is high at low frequencies but decreases as the inverse of the frequency. This is caused by flicker noise and can be reduced to a practicable limit by proper surface treatment and other manufacturing techniques. In the midband region the noise figure stays at a constant value of two to six decibels. This is caused by shot noise and thermal noise. Noise figure increases at higher frequencies with a slope of six

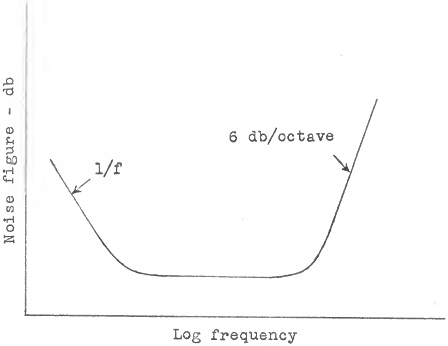

Fig. 8. Behavior of noise figure as a function of frequency.

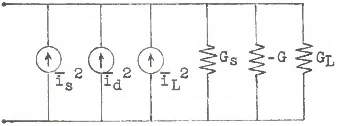

Fig. 9. Equivalent circuit of small signal tunnel diode amplifier.

decibels per octave. This increase is due to the frequency dependence of the amplification factor, α , as given by Eq. (94). Increase in noise figure at higher frequencies is also attributed to increase in correlation between $\overline{e_e}^2$ and $\overline{i}^2$. Noise figure is also affected by the source resistance, R_s. It was found (Nielsen, 1957) that the noise figure reaches a minimum for a particular value of source resistance. The effect of source resistance on noise figure is considered in detail in the next section.

Tunnel Diode Amplifiers

Noise figure of a tunnel diode amplifier is computed from its small signal equivalent circuit shown in Fig. 9 (Tiemann, 1960), where $\overline{i_s}^2$ and $\overline{i_L}^2$ are the Johnson noise generators of the source and the load, respectively, and $\overline{i_d}^2$ is the noise current generator of the tunnel diode. Expressions for the noise generators are

$$\overline{i_s}^2 = 4\ KT_sG_s \Delta f \tag{111}$$

$$\overline{i_d}^2 = 2\ e\ i_{eq} \Delta f \tag{112}$$

$$\overline{i_L}^2 = 4\ KT_LG_L \Delta f \tag{113}$$

where T_s and T_L are the absolute temperatures of the source and the load, respectively.

The noise figure for this circuit, using Eq. (91), is

$$F = \frac{\overline{i_s}^2 + \overline{i_L}^2 + \overline{i_d}^2}{\overline{i_s}^2} = 1 + \frac{T_LG_L}{T_sG_s} + \frac{e\ i_{eq}}{2KT_sG_s} \tag{114}$$

Usually $i_{eq} \approx i$, the diode current; therefore Eq. (114) can be written as

$$F \simeq 1 + \frac{T_L G_L}{T_s G_s} + \frac{ei}{2\ KT_s G_s} \tag{115}$$

Also the power gain of the amplifier is given as

$$K = \frac{\text{Output power at the load}}{\text{Available power from the source}}$$

Therefore $$K = \frac{e^2\ G_L}{i^2/4\ G_s} = \frac{4\ G_L G_s e^2}{i^2} \tag{116}$$

Also $$e^2 = \frac{i^2}{(G_s + G_L - G)^2} \tag{117}$$

Substituting Eq. (117) in (116),

$$K = \frac{4\ G_L G_s}{(G_s + G_L - G)^2} \tag{118}$$

Equations (115) and (118) show that for low noise figure G_s should be as large as possible, and for high gain G should be nearly equal to $(G_s + G_L)$. Therefore for low noise figure and high gain

$$G_s \gtrsim G$$

and $$G_L \approx 0$$

For this case Eq. (115) becomes

$$F = 1 + \frac{e}{2\ KT_s} \left(\frac{i}{G}\right) \tag{119}$$

The quantity (i/G) appearing in Eq. (119) is the horizontal distance along the voltage axis between the points at which tangent at the operating point crosses it, and the point vertically below the operating point, as is shown in Fig. 10 (Tiemann, 1960).

Therefore the most favorable operating point, as well as an

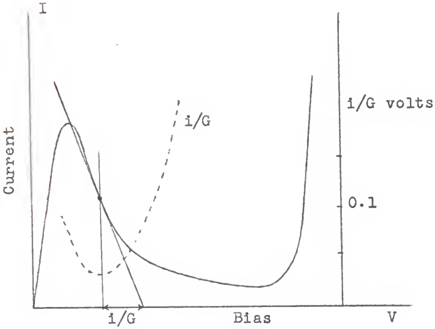

Fig. 10. Effect of operating point on the magnitude of (i/G).

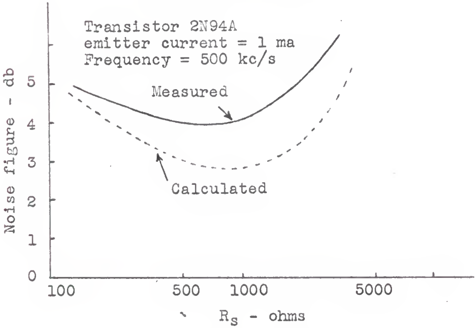

Fig. 11. Plot of noise figure versus source resistance.

absolute figure of merit for the shot noise of the diode, can be obtained from the current-voltage characteristics. It was found experimentally (Esaki, 1958) that diodes made from large band gap semiconductors have larger values of (i/G) at the optimum operating point than diodes made from semiconductors with smaller band gaps. Therefore for best noise performance small band gap materials are preferred, although they result in low temperature stability.

Optimization

Minimum noise figure is the major consideration when dealing with low level and very high-frequency signals. The effect of source resistance on the noise figure of a common emitter transistor amplifier is shown in Fig. 11 for a type 2N94A transistor (Nielsen, 1957). Mathematically, optimum source resistance may be computed from Eq. (110) which is rewritten as

$$F_e = 1 + \frac{K_1}{R_s} + \frac{(K_2 + R_s)^2 K_3}{R_s} \qquad (120)$$

where $K_1 = r_b + r_e/2$

$K_2 = r_b + r_e$

and $K_3 = (1 - \alpha_0)\left[1 + \left(\frac{f}{\sqrt{1 - \alpha_0}\, f_\alpha}\right)^2\right]\frac{1}{2\alpha_0 r_e}$

To find the minima of Eq. (120), it is written as

$$F_e = 1 + \frac{K_1}{R_s} + \frac{K_2^2 K_3}{R_s} + 2\,K_2 K_3 + K_3 R_s \qquad (121)$$

$$\frac{\partial F_e}{\partial R_s} = -\frac{K_1}{R_s^2} - \frac{K_2^2 K_3}{R_s^2} + K_3$$

Setting $\frac{\partial F_e}{\partial R}$ equal to zero gives,

$$R_{s(opt)} = \sqrt{\frac{K_1}{K_3} + K_2^2} \tag{122}$$

$R_{s\text{-optimum}}$ depends upon frequency because K_3 is frequency dependent. At frequencies below the alpha cut-off frequency, $R_{s(opt)}$ is the same for all three configurations, i.e., common base, common emitter, or common collector. Nielsen (1957) found in his experiments that for common emitter connection, source resistance for maximum gain and minimum noise figure comes out to be the same.

Substitution of Eq. (122) in (121) gives

$$F_{min} = 1 + \frac{K_1}{R_{s(opt)}} + \frac{K_2^2 K_3}{R_{s(opt)}} + 2\,K_2K_3 + K_3R_{s(opt)} \tag{123}$$

Degradation of noise figure resulting from the use of source resistance other than $R_{s(opt)}$ is seen from the expression

$$\frac{F_e}{F_{min}} = \frac{1 + \frac{K_1}{R_s} + \frac{K_2^2 K_3}{R_s} + 2\,K_2K_3 + K_3R_s}{1 + \frac{K_1}{R_{s(opt)}} + \frac{K_2^2K_3}{R_{s(opt)}} + 2\,K_2K_3 + K_3R_{s(opt)}} \tag{124}$$

which is rewritten as

$$\frac{F_e}{F_{e\,min}} = \frac{\frac{1}{K_3} + \frac{1}{R_s}\left(\frac{K_1}{K_3} + K_2^2\right) + 2\,K_2 + R_s}{\frac{1}{K_3} + \frac{1}{R_{s(opt)}}\left(\frac{K_1}{K_3} + K_2^2\right) + 2\,K_2 + R_{s(opt)}} \tag{125}$$

Using Eq. (122) in (125) yields

$$\frac{F_e}{F_{e\,min}} = \frac{\frac{1}{K_3} + 2\,K_2 + \frac{R^2_{s(opt)}}{R_s} + R_s}{\frac{1}{K_3} + 2\,R_{s(opt)} + 2\,K_2} \;. \tag{126}$$

Dividing numerator and denominator by 2 $R_{s(opt)}$ gives

$$\frac{F_e}{F_{e\,min}} = \frac{\frac{1}{R_{s(opt)}}\left[\frac{1}{2K_3} + K_2\right] + \frac{1}{2}\left[\frac{R_{s(opt)}}{R_s} + \frac{R_s}{R_{s(opt)}}\right]}{\left[\frac{1}{R_{s(opt)}}\right]\left[\frac{1}{2K_3} + K_2\right] + 1}$$

$$= 1 + \frac{\frac{1}{2}\left[\frac{R_{s(opt)}}{R_s} + \frac{R_s}{R_{s(opt)}}\right] - 1}{\frac{1}{R_{s(opt)}}\left[\frac{1}{2K_3} + K_2\right] + 1} \;. \tag{127}$$

The plot of $\frac{F_e}{F_{e\,min}}$ versus R_s is symmetrical and has a value unity for $R_s = R_{s(opt)}$ as is also evident from Eq. (127), which may be rewritten as

$$\frac{F_e}{F_{min}} = 1 + \frac{\frac{1}{2}\left[\frac{R_{s(opt)}}{R_s} + \frac{R_s}{R_{s(opt)}}\right] - 1}{K + 1} \tag{128}$$

$$\text{where} \quad K = \frac{1}{R_{s(opt)}}\left[\frac{1}{2K_3} + K_2\right] \tag{129}$$

Figure 12 shows a plot of $\frac{F_e}{F_{min}}$ as a function of $\frac{R_s}{R_{s(opt)}}$ or $\frac{R_{s(opt)}}{R_s}$ on semi-log scale for various values of K.

From the foregoing discussion it is inferred that the noise figure of transistor amplifiers can be minimized by: Reducing the base resistance of the transistor, increasing the alpha cut-off frequency of the transistor, choosing a source resistance closer to its optimum value, increasing the emitter resistance, r_e, which in turn implies decreasing the emitter current because the former is inversely proportional to the latter, and by using common emitter amplifiers.

The noise figure of a tunnel diode amplifier can be calculated from its current-voltage characteristic. In order to reduce the noise figure, the tunnel diodes are manufactured from small band gap semiconductor materials.

CONCLUSION

Shot noise in transistors is essentially constant below the alpha cut-off frequency. However, at frequencies higher than this, shot noise increases due to frequency dependence of the amplification factor α, and the increase in cross correlation between the equivalent noise generators of the emitter and that of the collector. Flicker noise occurs at low frequencies and

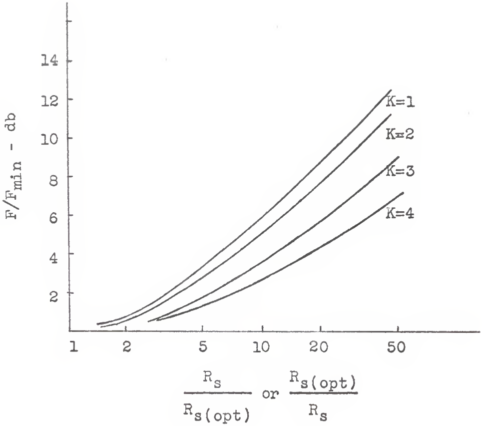

Fig. 12. Degradation of noise figure due to source resistance other than the optimum one.

is not a limiting factor in the operation of the semiconductor devices. It can be reduced by improving manufacturing techniques pertaining to the surface and the junction perimeter of the devices and using low voltage for back biasing of the junction so as to keep the leakage across the junction to a minimum. In order to keep the noise figure of transistor amplifiers to a minimum, transistors with low base resistance, large alpha cut-off frequency, small collector saturation current, and alpha close to unity are used. Source resistance is kept close to a value necessary for optimum noise figure.

In tunnel diodes, the equivalent noise current is found to be approximately equal to the forward diode current for a bias greater than KT/e volts. The noise figure of a tunnel diode amplifier can be calculated from its current voltage characteristics. In order to reduce the noise figure, tunnel diodes with small operating current and made from small band-gap semiconductor materials are preferred.

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NOISE IN SEMICONDUCTOR DEVICES

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This report is a study of the noise in semiconductor devices and the methods for reducing their noise figure. First the origin of noise in semiconductors is discussed. Then the discussion is extended to cover diodes, transistors, and tunnel diodes. Three types of noise--thermal noise, shot noise, and flicker noise--are studied, and their mathematical expressions derived. Both low- and high-frequency effects are discussed. The equivalent noise circuits for diode, transistor, and tunnel diode amplifiers are included. Using the equivalent circuits, the noise figures are calculated. The optimization of the noise figure and the effect of source resistance and operating point on it are also included.

It is concluded that in order to minimize noise figure in transistor amplifiers, transistors should have the grounded base amplification factor close to unity, a large alpha cut-off frequency, and a small collector saturation current. Grounded emitter circuits are found to possess low noise figure. In tunnel diodes, shot noise is found to be dominant, and in order to obtain low noise figures, tunnel diodes must be made from small band-gap semiconductor materials.