A STUDY OF THERMAL DETECTORS

by

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INTRODUCTION

Infrared Radiation

In 1800, Sir William Herschel discovered infrared radiation when the spectrum obtained by passing a ray of light through a prism was found to extend beyond the red end of the visible spectrum, using a sensitive mercury-in-glass thermometer (Kruse, 1962). This radiation in a wavelength range beyond the red region (0.75 micron) (micron = $10^{-6}$ meter) is called infrared radiation and has more heating power than the visible one. A large variety of sensitive detectors and measuring devices for infrared radiation have since been developed. The wavelength of visible light extends from about 0.4 to 0.75 microns, whereas that of the infrared radiation may lie between 0.75 and 1000 microns. The upper limit of 1000 microns is arbitrary and has been chosen so as not to extend into the radio wavelength. This band-width may further be divided into three parts:

- 0.75 - 1.5 microns  Near infrared
- 1.5 - 10 microns    Intermediate infrared
- 10 - 1000 microns   Far infrared

based on detection and measurement techniques.

Sources of Infrared Radiation

There are three basic types of infrared radiation sources, namely, thermally excited solids giving a continuous spectrum of radiation; excited gases giving a series of emission lines; and luminescent materials, which are in general optically excited
solids giving line or band spectra. Radiation from earth which cools the earth on a clear night is infrared radiation with a wavelength around 10 microns.

Uses of Infrared Radiation

Infrared radiation has found a large number of military, industrial, scientific, and domestic applications (Holter, 1962). For example, infrared radiation is used for communication devices. Its advantages are security, immunity to jamming, and compactness, and its disadvantage lies in its inability to penetrate clouds, dense fog, and heavy rain. Active sensory devices, generating their own infrared radiation and detecting it after reflection from objects, are used in Sniperscope, etc. Passive tracking and fire control systems make use of the infrared radiation of military targets, which are usually moving vehicle targets on the ground, in the sea, or airborne. Air-to-air homing missiles also use this technique. Passive surveillance sensors are relatively crude detectors which scan areas larger than those scanned by tracking devices, and are used primarily to determine areas of interest for the tracking device to probe more intensely. Airborne ground scanner is a device of this type.

The wavelength and intensity of the infrared radiation depends upon the temperature of the radiating device. An infrared detecting and measuring instrument can measure the temperature of hot bodies when it is not convenient to measure the temperature otherwise. The design of radiation pyrometers is also based on
this principle. The fast response of these detectors has made it possible to detect temperature changes of short duration, such as "hot-spots" produced by the friction between two surfaces. The heat transferred by a hot body to the ground underneath it is dissipated slowly by the ground after the removal of the hot body. So, if an infrared photograph of the ground is taken shortly after the removal of a hot body, the heated part of the ground will produce an image. Thus, photographs of recent past history of occupancy of ground can be taken. The design of control instruments in industrial automation is often based on infrared sensory devices, whereas artificial earth satellites carry radiometers to make meteorological observations of earth. The infrared spectroscopy is commonly used to study the molecular structures. Thermal detectors are also used for detection of microwave power, as the maximum frequency to which these can respond is lower than the normal modulating frequencies. Thermal detectors cannot demodulate the microwave frequencies and are used only for determining the average power. These can also be used in power measuring equipment in audio- and radio-frequency ranges. Here, the electrical power dissipated in a resistive element causes its temperature to rise and infrared radiation to be emitted, and this radiation is detected by a thermal detector. Infrared radiation is used to provide radiant heat in household heating systems. A considerable part of heat provided by an open fire and the bulk of that provided by the electrical heater is infrared. Room comfort requires a proper balance between radiative and convective heat. Infrared detective devices are also used in burglar alarm
Infrared Detection

The infrared radiation is invisible, therefore, special methods for detecting its presence and measuring its intensity are required. An infrared detector must convert electromagnetic radiation falling upon it into some other easily measurable form of energy, which in most cases is electrical. The principles of infrared detection can be classified into two categories, namely, quantum detection and thermal detection.

Quantum detectors make use of the property of the infrared radiation in freeing bound electrons from their atoms by absorption of individual quanta of radiation. The energy of a quantum of frequency $\nu$ or wavelength $\lambda$ is given by

$$\frac{hc}{\lambda}$$  \hspace{1cm} (1)

where $h = $ Planck's constant $= 6.625 \times 10^{-34}$ joule sec.,

and $c = $ velocity of light $= 2.998 \times 10^8$ metres/sec.

The response of a quantum detector is determined by the energy and, hence, the number of effective quanta incident on it. Only quanta having energy more than a certain energy $E_o$ are detected (Kruse, 1962), whereas others go undetected. $E_o$ depends upon the absorbing material of the detector and is called its work function. The value of $\lambda$ given by $hc/\lambda = E_o$ is called the long wave "limit" of the detector. Unfortunately this limit is very low, and therefore, a maximum of 9 microns wavelength has been detected by a lead selenide detector cooled by liquid hydrogen (Fig. 1).

In general, the quantum detectors have a much greater sensi-
Fig. 1. Comparison of idealized spectral responses of quantum- and thermal-detectors (Kruse, 1962).
tivity (about 100 times) and faster response (about 1000 times) than the thermal detectors. The limit on maximum detectable wavelength, and the dependence of number of electrons freed at the wavelength of the incident radiation, impose serious limitations on the use of the quantum detectors.

Thermal detectors are designed to take advantage of the heating property of the infrared radiation (DeWaard & Wormser, 1959). So, any temperature sensitive quantity or effect may be used as the design criterion of a thermal detector. The thermal detectors measure the rate of absorption of energy, regardless of the spectral content of the incident energy, and hence, no part of the radiation goes unmeasured. The thermal detectors, therefore, are useful, in spite of their low sensitivity and slow response.

PRINCIPLE OF THERMAL DETECTION

Many temperature-sensitive effects and materials have been used for designing thermal detectors and meters. Ultimate sensitivity in modern instruments is limited, not by their design peculiarities, but by fundamental random fluctuations inherent in the instruments (Kruse, 1962). The noise aspect will be considered later. Sensitivity of a detector is defined either as responsivity or ultimate sensitivity, where responsivity is the magnitude of the response to a given input radiation flux, and the ultimate sensitivity is defined as the minimum radiation flux observable by the detector under given conditions. A discussion of three commonly used thermal detectors follows.
Radiation Thermocouple

A thermocouple consists of two bimetallic junctions mounted in a blackened receiver. The metals must have different thermoelectric powers. The thermoelectric power $P_{ab}$ (volts per degree K) of a junction of two materials A and B is defined by

$$ E = P_{ab} \triangle T $$

where $\triangle T$ is the rise in temperature of the junction in °K, $E$ is the electromotive force (e.m.f.) (in volts) produced due to that temperature rise. Thermoelectric power of one material is measured relative to gold as a reference material. Usually antimony-bismuth and sometimes silver-bismuth junctions are used. One junction is called the "hot" junction, and it is exposed to radiation to be detected; the other, called the "cold" junction, is shielded from the radiation and kept at constant temperature by keeping it in contact with a "heat sink" in the form of a body of large thermal capacity (Fellgett, 1949). Sometimes a pair of couples is used. In that case they are made as identical as possible, and only one hot junction is exposed to the incident radiation. This way drifts in the ambient temperature do not affect the performance of the detector.

The difference in temperature between the hot and cold junction produces a thermoelectric e.m.f. which is proportional to the temperature difference and, hence, is an indication of the magnitude of the thermal energy of the incident radiation. The above-mentioned e.m.f. is open circuit e.m.f. across the output terminals. When the circuit is closed, the current passing
through the bimetallic junctions cools the hot junction and heats the cold junction. This effect is known as the Peltier effect. The cooling of the hot junction produces a thermoelectric e.m.f. in opposition to the original e.m.f. This effect is accounted for by the dynamic resistance, $R_d$ of the thermocouple.

$$R_d = P^2RT$$

where $P$ is the thermoelectric power (in volts per degree K) of the two materials, $R$ is the thermal impedance of the hot junction plus the receiver ($Z = \frac{\Delta T}{\Delta W}$), $T$ is the absolute temperature of the junction in degrees K.

With antimony-bismuth thermocouple and galvanometer the ultimate sensitivity is $10^{-10}$ watt, and it is governed by the galvanometer. The sensitivity could be improved by use of electronic amplifiers, but on the average, the time constant of the thermocouple (of the order of 4 sec.) is too large for efficient operation of amplifiers. To overcome this difficulty the radiation is chopped, and this provides alternating flux to the thermocouple and, hence, alternating e.m.f. as thermocouple output.

**Materials for Thermocouples**

Responsivity of a thermocouple is proportional to the combined thermoelectric powers of its materials (Hornig, 1947). Electrical resistance of the couple should be low, but, in general, materials with low resistance have a low thermoelectric power, and therefore, in order to obtain high responsivity, the thermal resistance should be high. The Wiedemann-Franz law states
\[
\frac{K}{(\sigma - T)} = L
\]

(4)

where \( K \) is the thermal conductivity in watt per meter °C, \( \sigma \) is the electrical conductivity in ohm\(^{-1}\) meter\(^{-1}\), \( T \) is the absolute temperature in degrees Kelvin and \( L \) is a constant of the material. \( L \) is called the Lorenz number. Another constant \( L_0 \) as obtained from theories of conductivities is

\[
L_0 = 2.45 \times 10^{-8} \text{ volt}^2/\text{°C}^2.
\]

(5)

Values of \( P \) and \( L/L_0 \) are given in Table 1 for various substances including metals, alloys, and tellerium. A value of \( L/L_0 \) greater than unity indicates that the substance has a very high electrical resistance. It should be noted that high values of \( P \) and \( L/L_0 \) usually occur together. Therefore, \( P^2/L \), a dimensionless quantity is a better index of merit than either of them taken separately. The final figure of merit, then, is \( 2P^2/(L_1+L_2) \) where \( L_1 \) and \( L_2 \) are the Lorenz numbers of the two materials.
Table 1. Properties of various thermoelectric materials (Hornig, 1947).

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermo-electric power $P$ ($\mu$V/°C)</th>
<th>Thermal conductivity $K$ (watt/m°C)</th>
<th>Electrical conductivity $\sigma \times 10^6$ (ohm⁻¹ m⁻¹)</th>
<th>Lorenz no. ratio $L/L_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>+2.9</td>
<td>4.24</td>
<td>61</td>
<td>0.95</td>
</tr>
<tr>
<td>Iron</td>
<td>+16</td>
<td>67</td>
<td>10</td>
<td>0.92</td>
</tr>
<tr>
<td>Nickel</td>
<td>-19</td>
<td>59</td>
<td>13</td>
<td>0.63</td>
</tr>
<tr>
<td>Antimony</td>
<td>+40</td>
<td>20</td>
<td>2.4</td>
<td>1.1</td>
</tr>
<tr>
<td>Bismuth</td>
<td>-60</td>
<td>8.3</td>
<td>0.83</td>
<td>1.35</td>
</tr>
<tr>
<td>Tellerium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) single</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>crystal</td>
<td>+436</td>
<td>1.8</td>
<td>0.0029</td>
<td>85</td>
</tr>
<tr>
<td>(b) polycrystal-line</td>
<td>+376</td>
<td>1.5</td>
<td>0.0033</td>
<td>62</td>
</tr>
<tr>
<td>(c) polycrystal-line</td>
<td>+372</td>
<td>1.0</td>
<td>0.0005</td>
<td>270</td>
</tr>
<tr>
<td>(d) Baker &amp; Co.</td>
<td>+119</td>
<td>2.0</td>
<td>0.032</td>
<td>8.4</td>
</tr>
<tr>
<td>Constantan</td>
<td>-38</td>
<td>21.2</td>
<td>2</td>
<td>1.35</td>
</tr>
<tr>
<td>Chromel---p</td>
<td>+30</td>
<td>20</td>
<td>1.2</td>
<td>2.3</td>
</tr>
<tr>
<td>95% Bi-5%Sn</td>
<td>+30</td>
<td>4.5</td>
<td>0.36</td>
<td>1.7</td>
</tr>
<tr>
<td>97% Bi-3%Sn</td>
<td>-75</td>
<td>7</td>
<td>0.58</td>
<td>1.65</td>
</tr>
<tr>
<td>90% Bi-10%Sn</td>
<td>-78</td>
<td>5.3</td>
<td>0.62</td>
<td>1.15</td>
</tr>
<tr>
<td>99.6% Te - 0.4%Bi</td>
<td>+191</td>
<td>2-3</td>
<td>0.035</td>
<td>approx. 10</td>
</tr>
<tr>
<td>99.1% Te - 0.9%Sb</td>
<td>+139</td>
<td>2-3</td>
<td>0.043</td>
<td>&quot;</td>
</tr>
<tr>
<td>98.5% Te - 1.5%Sb</td>
<td>+575</td>
<td>1.5-3</td>
<td>0.00029</td>
<td>&quot;</td>
</tr>
<tr>
<td>65%Sb - 35%Cd</td>
<td>+106</td>
<td>1.5-4</td>
<td>0.17</td>
<td>&quot;</td>
</tr>
<tr>
<td>75%Sb - 25%Cd</td>
<td>+112</td>
<td>1.5-4</td>
<td>0.14</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

Performance of some modern thermocouples is compared in Table 2. The first three are metal thermocouples. The fourth, Schwarz thermocouple, is made of semiconducting materials. Semiconductors are useful because of their very high thermoelectric power and low thermal conductivity (Brown, 1953). The difficulty with semiconductors is that their electrical resistivity is also high due to the same factors that contribute to the high thermoelectric power. Also, reproducibility of electrical and thermoelectric properties is difficult to achieve in mass production.
Table 2. Performance of some modern thermocouples.

<table>
<thead>
<tr>
<th>Name of the thermocouple</th>
<th>Materials used</th>
<th>Resistance in ohms</th>
<th>Minimum detectable power $W_m$ watts x $10^{-11}$ for 5c/s. with 1c/s. bandwidth</th>
<th>Time constant in milliseconds</th>
<th>Responsivity (volts per watt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jones</td>
<td>Bi &amp; 51.7% Sb 48.3% Cd</td>
<td>8</td>
<td>5</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>Hornig &amp; O'Keefe</td>
<td>97% Bi 3% Sb 95% Bi 5% Sn</td>
<td>5</td>
<td>5</td>
<td>30-40</td>
<td>6.5</td>
</tr>
<tr>
<td>Harris</td>
<td>Bi &amp; Sb or Bi &amp; Te</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Schwarz</td>
<td>See above</td>
<td>30-100</td>
<td>5</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>

(Jones, 1934; Hornig, 1947; Harris, 1934; Brown, 1953)

The Schwarz thermocouple consists of two alloys, one consisting of 27% Cu, 32% Ag, 33% Te, 7% Se, and 1% S; and the other 50% Ag$_2$S and 50% Ag$_2$Se. The former has positive thermoelectric power with respect to gold, and the latter has negative thermoelectric power with respect to gold. The above two materials in the form of cones or wedges are mounted on supporting pins of gold. The connections of these with the pins become cold junctions. A blackened gold foil acts as a receiver and is welded between the tips of the wedges. The connections between the gold foil and the thermoelectric materials constitute hot junctions. As the thermoelectric e.m.f.'s of the two materials are of opposite signs with respect to gold, they are additive.

The principle of construction is shown in Fig. 2. The whole
Fig. 2. Construction of Schwarz thermocouple. (Holter, 1962).

Fig. 3. Arrangement of bolometer used under d.c. conditions (Jones, 1953).

Fig. 4. Arrangement of bolometer for a.c. operation (Wormser, 1953).
assembly is mounted in an evacuated chamber with a window of fluorite (which has good infrared transmission properties). A compensating thermocouple of similar characteristics is sometimes mounted in the same envelope but its hot junction is not exposed to radiation. This thermocouple is connected in opposition to the main couple and serves to balance out drifts due to ambient temperature changes. Its advantage is that very thin supporting wires for receivers are not required, and, therefore, this type of thermocouple does not suffer from microphonic troubles.

Bolometer

A bolometer works on the principle that the resistance of conductors and semiconductors changes with the temperature (Shive, 1947, and Jones, 1953). The change of resistance for a given change in temperature is proportional to the temperature coefficient of resistance. This is approximately constant for most metals at ordinary temperatures. Therefore, many metals would be suitable for such an application; however, platinum is often used (and was used by Langley in the first bolometer in 1880), because it can be readily worked into very thin and strong strips.

A bolometer consists of two identical blackened platinum strips. One is exposed to radiation, and the other is shielded from it. Their resistances are then compared. When the radiation intensity is high, the resistance of the first strip decreases, while that of the second strip remains the same. When the ambient temperature changes, the resistances of both the
strips change equally. The difference in resistances due to the radiation is proportional to the radiation intensity, and it is either measured electronically or by the use of an accurate Wheatstone's bridge. Figure 3 shows one such Wheatstone's bridge circuit. R and R-AR are the resistances of the platinum strips; nR are the balancing resistances, one of which is variable. nR is several times larger than R, and Rg is galvanometer internal resistance.

For a.c. operation (chopped radiation) the circuit shown in Fig. 4 is used. Only one strip (R - ΔR) is used. The a.c. voltage across R₁ is amplified by an electronic amplifier and then indicated on a meter. The capacitor C is a coupling capacitor to block d.c. signal.

Theory of Bolometer

Let α, the temperature coefficient of resistance, be defined (Jones, 1953) as

\[ \alpha = \frac{1}{R} \frac{dR}{dT} \]  

where R is the resistance at temperature T. Consider the bolometer for a.c. operation. If resistance of the temperature-sensitive element changes from R to R-ΔR, voltage across R₁ changes by Δv, and if ΔR is much less than R, we get

\[ \Delta v = \frac{R₁ E \Delta R}{(R + R₁)^2} = \frac{R₁ i \Delta R}{R + R₁} = F i \Delta R \]  

where i is the direct current through the bolometer, and F = R₁/(R + R₁) is called the "bridge factor". For a given value
of \( i \) the maximum \( \Delta v \) is obtained when \( R_1 \) is much greater than \( R \), i.e., \( F = 1 \).

For Wheatstone's bridge \( F = \frac{1}{2} \), and galvanometer current is given by

\[
ig = \frac{i \Delta R}{2 R_g + (n + 1)R}
\]

(8)

Modern Metal Bolometer

When a platinum strip 0.1 micron thick, cooled on one side by radiation, is used, a time constant of 0.05 second is obtained because its thermal capacity is very small. This extra speed, however, is gained at the cost of sensitivity. Responsivity is found to vary as \( R^{\frac{1}{2}} \), but the time constant varies as \( R \). This makes it possible to gain a factor of 10 in speed with a loss of only about three times in responsivity. It is possible to design a bolometer to work at a particular frequency, and it is also possible to adjust the heat dissipation and thus achieve optimum response at that frequency. The heat loss may be adjusted by using a gaseous atmosphere to provide convection and conduction losses. The optimum condition is

\[
G = wC
\]

(9)

where \( G \) is the thermal conductance, \( w \) is the angular frequency of operation, \( C \) is the thermal capacity. Therefore, to obtain optimum response, the bolometer is made as thin as possible to have small thermal capacity \( C \), and losses are adjusted to get thermal conductance \( G \) to satisfy equation (9). Platinum and gold are the most popular metals for use in fast-responding bolometers,
because they can be made into thin strips. Nickel is sometimes used, because its temperature coefficient of resistance (0.006) is about the highest among metals.

**Semiconductor Bolometer**

As the temperature coefficient of resistance of semiconductors are much larger than those of metals (Kruse, 1963), semiconductors are very useful as bolometers. Also, due to their high values of resistivity, their responsivity may be 1000 times as large as that for metals. Hence, they can be coupled directly to an amplifier instead of using an intermediate transformer. Although $\alpha = 0.05^\circ C^{-1}$ at room temperature gives high responsivity, the ultimate limit of detection for a semiconductor bolometer is the same as that for a metal bolometer, because both are determined by noise inherent in the bolometer element.

**Thermistor Bolometer**

Thermistor is a new material developed by Bell Telephone Laboratories (Wormser, 1953, and Becker, 1946). The name stands for thermally sensitive resistor. Usually it consists of sintered oxides of manganese, cobalt and nickel. Thin flakes (10 microns thick) are mounted on "thermal sinks". Radiation falling on the flake warms it. If the radiation is removed, the flake returns to its original temperature with a decay time depending on the thermal conductance between the flake and the sink, and the radiation interchange with the surroundings. When the sink is solid the bolometer is said to be "solid backed",
and has fast response. If the flake is gas or vacuum backed, the response is slow. However, the latter has greater responsivity.

A thermistor is a nonlinear device as shown in Fig. 5. Its resistance is a function of the applied voltage. When operating in the negative resistance region, a ballast resistance must be used, otherwise the thermistor will burn out. Thermistor bolometers are used in the positive resistance region at a bias of about 0.6 of the peak of the curve (Fig. 5). Usually two identical flakes (one shielded from radiation) are used to compensate for the changes in the ambient temperature. A thermistor bolometer has a time constant of 2 to 8 msec., responsivity of 40 to 1100 V/Watt, and minimum detectable power of $2.5 \times 10^{-10}$ watt for 1 c/s. bandwidth.

Superconducting Bolometer (Andrews, 1946)

Certain materials, called superconductors, exhibit sharp drops of resistance to zero near absolute zero temperature. For pure metals this transition is abrupt, but for some alloys like columbium nitride (characteristics shown in Fig. 6), tantalum, niobium nitride and niobium stannide, the transition is relatively gradual. This region gives a very large value of $\alpha$ and, therefore, is a very useful phenomenon for thermal detection. Tantalum has transition near 4.4 °K, and it requires liquid helium as a coolant; while columbium nitride has the transition region near 14.3°K, so liquid hydrogen is used as a coolant. Triple point of hydrogen is just below this, so a coil heater is used to achieve this temperature. The temperature is very finely controlled by a
Fig. 5. Voltage current characteristics of a thermistor. (Kruse, 1962).

Fig. 6. Superconducting transition region in coulombium nitride (Andrews, 1946).

Fig. 7. Golay detector (Kruse, 1962).
servo, within ±0.00001 K. Fluctuations in temperature show up as noise in the bolometer.

Advantages of the superconducting bolometer are that the thermal noise is less at small temperatures and responsivity is very large, but the problems of low temperature maintenance and very precise temperature control are severe. Due to these two disadvantages, the superconducting bolometer is still undergoing research.

The superconducting bolometer has responsivity of 13.5 V/watt at 360 c/s., time constant 0.5 msec., and minimum detectable power 2.1 x 10^{-11} watt for 1 c/s. bandwidth.

Golay Cell (Golay, 1947; Zahl, 1946)

This is a pneumatic type of thermal detector based on the principle of the thermal expansion of gases. Its construction is shown in Fig. 7. A gas-filled chamber is connected by a passage to a flexible membrane having a reflecting film on the side opposite to the passage. An absorbing film of low thermal mass is fixed in the chamber. Infrared radiation from the left-hand side is absorbed by the absorbing film and gas. The gas expands and causes the flexible mirror to bulge out. This mirror displacement is detected by an optical arrangement shown in Fig. 7. A line grid is projected on the mirror from a visible light source, reflected by the flexible mirror and absorbed by a photocell. When the flexible mirror bulges, the amount of the light incident on the photocell changes. When the flexible mirror is in the initial unbulged position, the light from the source passes through
one-half of the line grid and after reflection from the flexible mirror reaches the other half. Initial adjustments are so made that no light passes out of the lower half of the grid. A slight bulge of the flexible mirror, however, causes a good amount of light to reach the photocell. The signal from the photocell is, then, electronically amplified. By this optical arrangement, 0.1° deflection of the flexible membrane can be detected.

An air-filled detector of this type has 3 msec. time constant and can detect \(1.4 \times 10^{-9}\) watt. A helium-filled detector has 0.6 msec. time constant, but its sensitivity is not as good as that of the air-filled one.

**MATHEMATICAL ANALYSIS OF THERMAL DETECTORS**

The idea of thermal impedance is analogous to electrical impedance in electrical circuitry (Holter, 1962), and therefore, thermal capacity and thermal conductance are also encountered.

Suppose a body at temperature \(T_1\) is connected to a source of heat at temperature \(T_2\). Then the thermal energy \(\Delta Q\) gained by the body when its temperature rise is \(\Delta T\) is given by

\[
\Delta Q = C \Delta T
\]

and in the limit, as \(\Delta T\) tends to zero,

\[
C = \frac{dQ}{dT}, \text{ where } C \text{ is the thermal capacity of the body in joules per degree K.}
\]

Thermal conductance describes the ease or difficulty with which heat flows through a body, when it is at a temperature different from that of a body it is in contact with. Heat flows from
a body at a higher temperature to that at a lower temperature; and the rate of flow of heat is proportional to the area A through which it passes and also to the temperature gradient in the direction of its flow $\frac{dT}{dx}$. These properties suggest that

$$\frac{dQ}{dt} = -K'A \frac{dT}{dx}$$

(12)

where $K'$ is the conductivity. When the temperature change $\Delta T^0K$ exists for a specific dimension 1 cm.,

$$\frac{dQ}{dt} = -K \Delta T,$$

(13)

where $K$ is the thermal conductance in joules/sec. $^0K$ and is given by

$$K = \frac{K'A}{l}$$

(14)

Table 3. Analogies between thermal and electrical parameters.

<table>
<thead>
<tr>
<th>Thermal parameter</th>
<th>:</th>
<th>Electrical parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature difference T</td>
<td>:</td>
<td>e.m.f. e</td>
</tr>
<tr>
<td>Rate of flow of heat $\frac{dQ}{dt}$</td>
<td>:</td>
<td>Current i</td>
</tr>
<tr>
<td>Thermal capacity C</td>
<td>:</td>
<td>Electrical capacity C</td>
</tr>
<tr>
<td>Thermal conductance K</td>
<td>:</td>
<td>Electrical conductance G</td>
</tr>
<tr>
<td>Thermal impedance Z</td>
<td>:</td>
<td>Electrical impedance Z</td>
</tr>
</tbody>
</table>

Now, drawing on the electrical analogy, it can be said that
time constant (in sec.) $\tau = C = RC$, where $R = \frac{l}{K}$ is the thermal resistance.
Equivalent Circuit of a Thermocouple

Consider a single couple made up of two junctions \( J_1, J_2 \) between metals A and B, as shown in Fig. 8 (Hornig, 1947). They are connected to a galvanometer through wires C. Let us assume that the meter, the junctions \( J_3 \) and \( J_4 \) are at uniform temperature, such that no thermoelectric effects take place anywhere except at \( J_1 \) and \( J_2 \).

Junction \( J_2 \) is assumed to be at constant temperature, due to very large thermal capacity. Junction \( J_1 \) is connected to a blackened receiver. Therefore, temperature of \( J_1 \) increases from \( T \) to \( T + \Delta T \). The temperature difference \( \Delta T \) causes thermoelectric e.m.f. \( E \) volts to be set up.

\[
E = P_{ab} \Delta T \tag{15}
\]

where \( P_{ab} \) is the thermoelectric power of metals A and B. Similarly, when a current \( i \) passes through junction \( J_1 \), the junction cools (Peltier effect), the amount of heating being given by

\[
\begin{align*}
W &= -\Pi_{ab} i, \\
\Pi_{ab} &= T P_{ab} \\
W &= -T P_{ab} i \tag{16}
\end{align*}
\]

where \( \Pi_{ab} \) in volts is called the Peltier coefficient and is related to \( P_{ab} \) as shown.

So, when a constant amount of radiation power \( W \) flows into the thermocouple, the equation for the balance of heat flow into the hot junction \( J_1 \) becomes

\[
\frac{\Delta T}{R} = W - P i T \tag{17}
\]
Fig. 8. Elementary thermocouple configuration (Holter, 1962).

Fig. 9. Electrical equivalent of a thermocouple (Fellgett, 1949).

Fig. 10. Variation of responsivity with operating frequency of an antimony-bismuth thermocouple (Holter, 1962).
where $P_i T$ accounts for the Peltier effect. Usually, chopped radiation is used for the thermocouple. So the radiation power becomes $W e^{j\omega t}$, where $\omega$ is the angular frequency of chopping (Holter, 1962). The thermal capacity $C$ also comes into play, and the equation (17) becomes

$$C \frac{d \Delta T}{dt} + \frac{\Delta T}{R} = W e^{j\omega t} - P_i T \tag{18}$$

Now, consider the case when an external e.m.f. $E e^{j\omega t}$ in series with a resistance $R_g$ is applied to the thermocouple having resistance $R$ and no inductance or capacitance. It is assumed that no heat flows to the thermocouple except that caused by its own changes in temperature with respect to its surroundings. Then the total e.m.f. is given by

$$E_t = E e^{j\omega t} + P \Delta T \tag{19}$$

where $\Delta T$ is the change in temperature due to the Peltier effect, $\Delta T$ is given by the equation (18),

$$C \frac{d \Delta T}{dt} + \frac{\Delta T}{R} = -i P T, \text{ where } i = I e^{j\omega t} \tag{20}$$

Combining equations (19) and (20),

$$E_t = E e^{j\omega t} - i P^2 R T/(1 + j\omega \tau) \tag{21}$$

where $\tau = CR$, and $i = E_t/(R + R_g)$

$$i(R + R_g) + iP^2 R T/(1 + j\omega \tau) = E e^{j\omega t} \tag{22}$$

$$i[R + R_g + P^2 R T/(1 + j\omega \tau)] = E e^{j\omega t} \tag{23}$$

$$i(R + R_g + Z_d) = E e^{j\omega t} \tag{24}$$

where $Z_d = P^2 R T/(1 + j\omega \tau) = R_d/(1 + j\omega \tau) \tag{25}$

$Z_d$ is called the dynamic impedance of the thermocouple, and the thermocouple becomes equivalent to the electrical circuit shown in Fig. 9.
In this circuit $R$ is the d.c. resistance of the thermocouple, $R_d = P^2RT$ is the dynamic resistance due to the Peltier effect, and $C_d$ is the dynamic capacitance of the thermocouple.

Now $\tau = R_d C_d = RC$ \hspace{1cm} (27)

but $R_d = P^2RT$ \hspace{1cm} (28)

$C_d = \frac{C}{P^2T}$ \hspace{1cm} (29)

Now consider the solution of the equation (18). For open circuit $i = 0$ and $E$ is given by

$$E = P \Delta T = PR W e^{jwt}/(1 + j\omega t)$$ \hspace{1cm} (30)

When the circuit is closed via galvanometer resistance $R_g$, current $i$ is given by

$$i = E/(R + R_g + Z_d), \text{ where } i = I e^{jwt} \hspace{1cm} (31)$$

$$= \frac{P R W}{(1+j\omega t)(R+R_g+Z_d)} \hspace{1cm} (32)$$

by combining equations (30) and (31)

Now, responsivity $r$ can be defined as the ratio of the amplitude of open circuit e.m.f. to the amplitude of the incident radiation power (Hornig, 1947).

$$r = \frac{|E|}{W} = \epsilon PR/(1 + w^2 \tau^2)^{\frac{1}{2}} \hspace{1cm} (33)$$

where $\epsilon$ is the emissivity (a pure number).

From this, it can be noticed that when $wRC$ is much greater than 1, the responsivity is inversely proportional to the frequency of operation (Fig. 10). Therefore, the thermocouples are operated at low frequencies, usually 5 c/s.
Minimum Detectable Power

The fluctuations in the incident radiation power, the detector and the measuring system finally limit the minimum power that the detector can detect. Here the first source, i.e., the fluctuations in radiation power, will be considered. Fluctuations in power flowing to and from a body in a radiation field set a limit to the amount of power that can be observed flowing to the body from an external source. Let $W_s$ be the signal power, i.e., power flowing from the external source. Then $W_s$ cannot be observed unless it is greater than the total fluctuation power \( (\Delta W_T)^2 \), that is, square root of time average of \( (\Delta W_T)^2 \). If the body is a thermal detector, there will also be Johnson noise \( (v_j^2) \), which is equivalent to power \( v_j^2 r^{-2} \) where \( r \) is the responsivity of the detector. Then the minimum detectable power $W_m$ watts is given by

\[
W_m^2 = \Delta W_T^2 + r^{-2} v_j^2
\]  

(34)

If \( r \) is sufficiently large,

\[
W_m^2 = \Delta W_T^2
\]  

(35)

\[
W_m^2 = 4kT^2 G \Delta f
\]  

(36)

So, $W_m$ is small when \( G \) is small, i.e., when the body is coupled to its surroundings by radiation alone.

Ultimate Sensitivity of the Thermal Detectors

The Thermocouple. Johnson noise and radiation fluctuation are the two sources of noise in the thermocouple detector. There-
fore, the total noise voltage $v_t$ is given by (Fellgett, 1949).

$\overline{v_t^2} = \overline{v_J^2} + \overline{v_T^2}$

$= 4kT \Delta f + r^2 \overline{\Delta W_T^2}$

$= 4kT \Delta f + r^2 (4kT^2 G \Delta f)$

$= 4kT \Delta f (R + r^2TG)$

But $r = \varepsilon PR/(1 + w^2C^2R^2)^{\frac{1}{2}}$ (33)

$\overline{v_t^2} = 4kT \Delta f \left( R + \frac{\varepsilon^2 P^2RT}{(1 + w^2C^2R^2)} \right)$

$= 4kT \Delta f \left( R + \frac{\varepsilon^2 P^2GT}{G^2 + w^2C^2} \right)$

$= 4kT \Delta f (R + \varepsilon^2 \text{ Real part of } Z_d), \ Z_d = P^2TZ$ (43)

where $Z_d$ is the dynamic impedance of the thermocouple, and consists of a resistance and a capacitance in parallel. $R_d = P^2RT$, and $C_d = C/P^2T$.

Thus, the fluctuations may be regarded as due only to Johnson noise in the total resistance of the circuit including the dynamic impedance. Current noise in the thermocouple need not be considered, because when the signal is near the ultimate sensitivity of the thermocouple, the current through it is so small that the current noise is negligible. Care must, however, be taken to see that there is no noise due to microphonics.

In equation (42), there are two sources of noise, the Johnson noise and the radiation fluctuations. Their ratio is

$\frac{\overline{v_J^2}}{\overline{v_T^2}} = \frac{R(G^2 + w^2C^2)}{\varepsilon^2 P^2GT}$ (44)
\[ = \frac{R}{R_d} \quad (45) \]

if \( G \) is much greater than \( wC \), because \( P^2T/G = \gamma G_d \). In practical thermocouples, described above, the dynamic resistance \( R_d \) is less than 15% of \( R \). Therefore, the radiation fluctuation noise is relatively small.

Now, let \( C = C_r + C_a + C_l \) \( (46) \)
and \( G = G_r + G_a + G_l \) \( (47) \)
thus breaking up \( C \) and \( G \) into their constituents associated with the receiver, the air, and the leads with subscripts \( r, a, l \), respectively. The electrical conductance is related to \( G_c \) only; the others are only thermal conductances. From equation (44), it should be noted that \( \sqrt{\gamma J^2} \) decreases if \( G_a \) and \( C_a \) are small. Therefore, it is always advantageous to evacuate the thermocouple.

Evacuation may increase the time constant, but that is not a very big disadvantage. In practice it is advantageous to keep \( G_l = G_r \).

The bolometer would seem to have many advantages over the thermocouple; but, actually, except for the superconducting bolometer, all the advantages of the bolometers have not been fully realized in practice (Kruse, 1962). In addition to the sources of noise present in the thermocouple, current or modulation or \( f \) noise occurs in the bolometer (Holter, 1962). This is the noise arising from the exciting current of the bolometer. This occurs in semiconductors and in very thin metallic films or wires.

Therefore, it appears in sensitive bolometers which depend on the change in the resistance of bolometer elements. First the current noise will be neglected, and only the Johnson noise and
the radiation fluctuation noise will be considered. Let the bolometer element be at a uniform temperature $T$. This temperature is usually above the temperature $T_o$ of its surroundings.

Then $W_m$ the minimum detectable power is given by the sum of the fluctuation power and the Johnson noise power.

$$W_m^2 = \frac{w_T^2 + r^2}{v_J^2}$$

$$= 4kT^2G \Delta f + 4kTR \Delta f r^{-2}$$

By an analysis similar to that of the thermocouple, it can be shown that responsivity $r$ of the bolometer is given by (Holter, 1962),

$$r = \frac{e_i R}{(G_e^2 + w^2C^2)^{1/2}}$$

Where $G_e = G - \alpha G_o(T - T_o) \frac{R_1 - R}{R_1 + R}$, $G_o = G/\beta$.

$\beta$ is a constant of the thermistor, $\alpha$ is the temperature coefficient of resistance, $i$ is the exciting current through the bolometer, $G_e$ is the effective thermal conductance of the bolometer element. Therefore, combining the equations (34) and (48), we get

$$W_m^2 = 4kT^2 \Delta f \left(G + \frac{(G_e^2 + w^2C^2)}{\varepsilon^2 i^2 \frac{2}{RT}}\right)$$

Here $i^2R$ is the electrical power dissipated in the element and converted into heat, which raises the element temperature to $T$, when the ambient temperature is $T_0$.

Then $i^2R = (T - T_0)G/\beta$

$$W_m^2 = 4kT^2 \Delta f G \left(1 + \frac{(G_e^2 + w^2C^2)\beta}{\varepsilon^2 i^2 \frac{2}{RT(T - T_o)G^2}}\right)$$

The first term corresponds to the radiation fluctuations, and the
second term corresponds to the Johnson noise.

\[
\frac{\overline{v_J^2}}{v_T^2} = \frac{(G_e^2 + \omega^2 C^2) \beta}{\varepsilon^2 \alpha^2 T(T-T_0)G^2}
\]  

(53)

From equations (52) and (53), it should be noted that \(\overline{v_J^2}/v_T^2\) decrease when \(wC\) is much smaller than \(G_e\). Therefore, the thermal capacity of the receiver should be as small as possible, and \(G_e\) should also be made as small as possible.

Now the effect of current noise on the minimum detectable power will be considered. For a semiconductor bolometer this noise will be at least comparable to the Johnson noise and radiation fluctuation noise (Holter, 1962). For metallic strips of 0.1 micron thickness the value of the current noise may be small or large, depending upon the crystalline structure of the strip material. The current noise is given by

\[
\overline{v_i^2} = B \alpha^2 \Delta f
\]

(54)

where \(B\) is a constant depending upon the material and the frequency of operation, and the constant \(x\) lies between 1.5 and 3.

This current noise is observed in all resistors except metallic wire resistors at low frequencies and at d.c. The amplitude of the noise power increases with increase in the current. The constant \(x\) is often equal to 2. Therefore, it is necessary to use wire-wound resistors in low frequency applications requiring high sensitivity. The behavior of a typical carbon resistor is shown in Fig. 12.

Now, adding the current noise to equation (48) for noise in
Fig. 11. Variation of minimum detectable power \( W_m \) with operating temperature \( T \) of the bolometer element. \( T_0 \) is the ambient temperature (Holter, 1962).

Fig. 12. Variation of r.m.s. noise voltage \( v_f \) with current \( i \) for a 1 MΩ carbon resistor, expressed in terms of Johnson noise voltage \( v_J \) (Holter, 1962).
the bolometer, \(W_m^2\) becomes (Holter, 1962)

\[
W_m^2 = 4kT^2 G f - 4kT R \Delta f r^{-2} + B i^x \Delta f r^{-2}
\]  

(55)

The third term can be written as

\[
W_{mi}^2 = \frac{B i^{x-2} f (G_e^2 + \omega^2 c^2)}{\epsilon^2 \omega^2 R^2}
\]  

(56)

using equation (49).

The value of \(W_{mi}^2\) usually depends upon the value of \(x\). If current noise predominates and \(x\) is less than 2, it is advantageous to operate the bolometer with large exciting current, within the limits of stability, at least till \(W_{mi}\) becomes comparable to the sum of the first two terms of \(W_m\) in equation (55). This will result in an optimum value of \(W_m\).

When \(x = 2\), \(W_{mi}\) is constant with respect to \(i\), and the value of \(i\) may be taken such that the first two terms are optimized. If \(W_{mi}\) predominates, \(W_m\) will hardly vary with \(i\) over a wide range. Actually, in practice, \(R\) varies substantially with \(T\) in semiconductors, and \(T\) depends upon \(i\). Therefore, there is a somewhat flat minimum in the value of \(W_{mi}\), and at this value \(W_{mi}\) is considerably greater than the other two terms of \(W_m\). It is almost 10 times as much. Due to this current noise, it is desirable to operate the bolometer at higher frequencies.

**Effect of Temperature on Minimum Detectable Power of the Bolometer**

In equation (55), if the temperature \(T\) is reduced, \(W_m\) should improve. This is particularly true for high frequency operation,
because then the current noise is negligible. However, if the temperature is further decreased to the liquid helium temperature range, it is found that $\xi$ decreases and the advantage is lost. The effect of low temperature on the $W_m$ is shown in Table 4.

Table 4. Variation of minimum detectable power $W_m$ and time constant with ambient temperature for a metal bolometer with $G = 10^{-4}$ watt/°C with conduction cooling predominating.

<table>
<thead>
<tr>
<th>$T_0$ (°K)</th>
<th>$W_m$ (watt/cycle)</th>
<th>$W_m$ (watt/cycle)</th>
<th>$\tau$ (milli sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($w \ll \tau^{-1}$)</td>
<td>($f = 300$ c/s.)</td>
<td></td>
</tr>
<tr>
<td>290</td>
<td>$5.1 \times 10^{-11}$</td>
<td>$6.0 \times 10^{-10}$</td>
<td>6</td>
</tr>
<tr>
<td>90</td>
<td>$1.6 \times 10^{-11}$</td>
<td>$1.3 \times 10^{-10}$</td>
<td>4</td>
</tr>
<tr>
<td>77</td>
<td>$1.3 \times 10^{-11}$</td>
<td>$1.0 \times 10^{-10}$</td>
<td>3.5</td>
</tr>
<tr>
<td>20</td>
<td>$3.5 \times 10^{-12}$</td>
<td>$5.0 \times 10^{-12}$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

(Wormser, 1953)

At lower temperatures, conduction cooling actually predominates.

FIGURES OF MERIT

The performance of the various detectors should be compared under standardized conditions of operation, because there are many variables. Secondly, there are a large number of characteristics which together describe the performance of a detector. Each of these characteristics can be modified to yield a figure of merit, which can be used for comparing the performance of rival detectors (Potter, 1959). The figures of merit can be grouped into four categories (Kruse, 1962):

1. Minimum detectable power, i.e., minimum intensity of
radiation power falling on the detector which will give rise to a signal voltage equal to the noise voltage from the detector.

2. Signal voltage per unit radiation power falling on the detector.

3. Variation of the signal with the wavelength of the incident radiation.


One of the most commonly used figures of merit is the Noise Equivalent Power or NEP (Potter, 1959). It is defined as the r.m.s. value of the sinusoidally varying radiation power falling upon a detector, which gives rise to an r.m.s. voltage equal to the r.m.s. noise voltage of the detector. The radiation source is standardized as blackbody at 500°K. The reference bandwidth is 1 c/s. or 5 c/s. The center temperature is room temperature (about 295°K) or the operating temperature if the detector is cooled. The reference area of the receiver is 1 cm². The intensity of radiation power and the field of view are not standardized, but should be specified.

$$\text{NEP} = \frac{\text{HA} \left( \frac{V_N}{V_S} \right)}{(\Delta f)^{\frac{1}{2}}} \text{ watt sec.}^{\frac{1}{2}}$$  \hspace{1cm} (57)

where H is the radiation power falling on the detector per unit area, A is the area of the detector, \( \frac{V_N}{V_S} \) is the N/S ratio of the voltages in bandwidth \( \Delta f \).

NEP (506°K, 900, 1) means it is NEP for 500°K source, 900 c/s. chopping frequency and 1 c/s. bandwidth. It should be noted
that the detecting capability of the detector increases as NEP decreases. NEP is measured in practice by measuring S/N ratio in a specified narrow electrical bandwidth, called the measurement bandwidth. Then the power required for S/N ratio of unity is calculated on linear basis.

Detectivity D (Kruse, 1962) is the reciprocal of NEP, under the same standard conditions of operation.

\[
D = \frac{1}{\text{NEP}}
\]  

(58)

Noise Equivalent Input (NEI) (Kruse, 1962) is another common figure of merit. It is the radiation power per unit area of detector required to give a signal to noise ratio of unity. The standardized conditions of operation are the same as those for NEP.

\[
\text{NEI} = \frac{\text{NEP}}{A} = \frac{1}{AD} \text{ watt cm}^{-2} \text{ sec}^{\frac{1}{2}}
\]

(59)

As many detector of infrared radiation exhibit NEP, which is proportional to square root of the area of the detector, a figure of merit \(D^*\) (called D-star) is defined as

\[
D^* = \frac{A^{\frac{1}{2}}}{\text{NEP}} = \frac{1}{(\text{NEI})A^{\frac{1}{2}}} = DA^{\frac{3}{2}} \text{ cm sec}^{-\frac{1}{2}} \text{ watt}^{-1} \quad \text{(Kruse, 1962)}
\]

(60)

The operating conditions are the same as for NEP, except that the reference bandwidth is always 1 c/s. \(D^*\) has become very popular and is often referred to as detectivity, even though \(D\) is the true detectivity.

A figure of merit which is very useful for a detector whose NEP is proportional to the square root of the area of the detector, and which is also current noise limited is Jones "S"
(Jones, 1953). Current noise voltage is inversely proportional to the square root of the measuring frequency. This frequency dependence is removed in "S".

\[ S = \frac{NEP}{A^2} f^{\frac{1}{2}} = \frac{f^{\frac{1}{2}}}{D^*} \text{ watt/cm.} \] (61)

Now, consider the problem of a figure of merit to characterize the signal voltage per unit radiation power. Noise is neglected. **Responsivity** \( r \) is defined as

\[ r = \frac{V_s}{R_A} = \frac{V_s}{W} \text{ volts/watt} \] (62)

where \( V_s \) is the signal voltage, \( W \) is the power of the radiation. The temperature of the blackbody radiator is kept at 500°K.

\[ r = \frac{V_N}{(NEP)(\Delta f)^{\frac{1}{2}}} = \frac{D^* V_N}{(A \Delta f)^{\frac{1}{2}}} \] (63)

Thirdly, we need a figure of merit for the spectral because of the detector (Kruse, 1962); in other words, the manner in which the detector output variation with change in the wavelength of monochromatic incident radiation. This spectral response is usually described by a graph. A very informative description is given by plot of \( \text{NEP}_\lambda \), \( \text{NEI}_\lambda \) or \( D^*_\lambda \) as a function of wavelength. \( \text{NEP}_\lambda \) refers to NEP of the detector at wavelength \( \lambda \). Long wave limit of detection is given by the wavelength at which the response falls to 1% or 50% of the maximum response.

Fourthly, consider the frequency response of a detector. One of the important characteristics of the detector is the dependence of the responsivity and the detectivity upon the frequency of chopping of the incident radiation. Ideally the char-
acteristic should be flat, but in practice the responsivity has characteristics similar to that of a low-pass filter. In general,

\[ r(f) = \frac{r_0}{(1 + w^2 \tau^2)^{1/2}} \quad (64) \]

where \( r(f) \) is the responsivity at frequency \( f \), and \( r_0 \) is the d.c. responsivity. When \( w\tau \) is much less than 1, \( r(f) \) is constant, and when \( w\tau \) is much greater than 1, \( r(f) \) is inversely proportional to the frequency of chopping \( \left( \frac{w}{2\pi} \right) \). Fig. 13 shows the dependence of the responsivity on the frequency of chopping.

Now consider the frequency dependence of the detectivity. From equation (63) we get

\[ D^* = \frac{rA^{1/2} (\Delta f)^{1/2}}{V_N} \quad (65) \]

which is free from the frequency factor except for \( r \). So, \( D^*(f) \) behaves the same way as \( r(f) \), but this is not true for current noise limited detectors. In this case noise voltage depends upon \( \frac{1}{f^{1/2}} \), unlike the Johnson noise and radiation fluctuation which do not depend upon the frequency of operation. So, for current noise limited detectors, we get

\[ D^*(f) = \frac{k f^{3/2}}{(1 + w^2 \tau^2)^{1/2}} \quad (66) \]

where \( k \) is a constant. This frequency dependence is shown in Fig. 14.

The value of \( D^*(f)_{\text{max.}} \) can be found thus
Fig. 13. Frequency dependence of responsivity (Kruse, 1962).

\[ r(f) = \frac{r_0}{(1+w^2\tau^2)^{1/2}} \]

\[ \frac{1}{2\pi \tau} \]

log frequency

Fig. 14. Dependence of \(D^*\) on chopping frequency for 1 noise limited detectors (Kruse, 1962).
\[
\frac{d}{df} D^*(f) = \frac{1}{2} k f^{-\frac{1}{2}} (1 + 4 \pi^2 f^2 \tau^2)^{\frac{1}{2}} - k f^2 \left( \frac{1}{2} (1 - 4 \pi^2 f^2 \tau^2) \right)^{-\frac{1}{2}}
\]
\[
x \cdot 8 \pi^2 f^2 \tau^2 = 0
\]
(1 + 4 \pi^2 f^2 \tau^2)^{\frac{1}{2}} = f (1 + 4 \pi^2 f^2 \tau^2)^{-\frac{1}{2}} \cdot 8 \pi^2 f \tau^2
\]
\[
1 + 4 \pi^2 f^2 \tau^2 = 8 \frac{2 f \tau^2}{2 \pi^2}
\]
\[
f = \frac{1}{2 \pi \tau}
\]

No figure of merit has been defined for frequency dependence of detectivity, but the current noise limited detector should be operated at frequency of chopping \( \frac{1}{2 \pi \tau} \).
Table 5. Performance of thermal detectors (Kruse, 1962).

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Detector</th>
<th>Operating temperature (°K)</th>
<th>(D^*(500^\circ K,f,l)) cm-cps(^{1/2}) watt(^{-1}) (Measuring freq. indicated)</th>
<th>Response time (m sec.)</th>
<th>Calculated optimum chopping freq. (cps)</th>
<th>Resistance per sq. cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thermo-couple</td>
<td>295</td>
<td>(1.4 \times 10^9)</td>
<td>36</td>
<td>(&lt;5)</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Thermistor Bolometer</td>
<td>295</td>
<td>(1.95 \times 10^8)</td>
<td>1.5</td>
<td>Freq. independent up to 30 cps.</td>
<td>2.4 M</td>
</tr>
<tr>
<td>3</td>
<td>NbN Bolometer</td>
<td>15</td>
<td>(4.8 \times 10^9)</td>
<td>0.5</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>Carbon Bolometer</td>
<td>2.1</td>
<td>(4.25 \times 10^9)</td>
<td>10</td>
<td>16</td>
<td>0.12 M</td>
</tr>
<tr>
<td>5</td>
<td>Golay cell</td>
<td>295</td>
<td>(1.67 \times 10^{10})</td>
<td>20</td>
<td>(&lt;5)</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 15. Frequency response of various detectors (Kruse, 1962).

1. Thermocouple (36 msec., 295°K)
2. Thermistor bolometer (1.5 msec., 295°K)
3. NbN bolometer (0.5 msec., 15°K)
4. Carbon bolometer (10 msec., 2.1°K)
5. Golay Cell (20 msec., 295°K)
CONCLUSION

Infrared radiation, besides being increasingly used in scientific, industrial and domestic spheres because of its compactness and inexpensiveness, has special military applications, due to the fact that many infrared systems are passive and jam-proof. Moreover, infrared radiation is invisible, and its detection by the enemy is difficult, and hence, countermeasures are difficult to design.

Two basic types of detectors, namely quantum detectors and thermal detectors, are frequently used. Quantum detectors detect individual quanta of the incident radiation, whose bandwidth is limited to a maximum of 9 microns. The response of the quantum detector also depends upon the wavelength of the incident radiation, because this detector essentially counts the number of quanta and not the total energy of the incident on it. If the radiation consists of more than one wavelength, the detector does not indicate the true intensity of the incident radiation. The thermal detector, however, detects the heat energy produced by the incident radiation and thus measures the energy content of the incident radiation irrespective of the wavelengths content. Therefore, thermal detectors can detect radiation of any wavelength.

A thermal detector converts infrared radiation energy into electrical energy, and thermocouple, bolometer, and Golay Cell are common types of thermal detectors. Thermocouple, although widely used in industry, has limited military applications
because of its fragility. The thermocouples have low thermal capacity and are designed for high responsivity and fast response. The sensitivity of a thermocouple is limited by that of the indicating galvanometers, but when chopped radiation and electronic amplifier are used, sensitivity can be considerably increased until it is limited by noise inherent in the system. Detectivity of up to $1.4 \times 10^9$ cm. cps$^{\frac{1}{2}}$ watt$^{-1}$ has been achieved. The noise in thermocouples is predominantly white noise.

The bolometer consists of a temperature-sensitive resistance. The thermistor bolometer is often used to detect infrared radiation from low-temperature objects, whereas metal bolometers are not used frequently. Radiometers and horizon scanners used for establishing vertical reference for artificial earth satellites employ thermistor bolometers. It has a very long-time constant, and therefore, its usefulness in missile seekers or trackers is limited. The detectivity $D^*$ of the thermistor bolometer is proportional to the square root of the response time, and a typical value of $D^*$ is $1.95 \times 10^8$ cm cps$^{\frac{1}{2}}$ watt$^{-1}$, which is lower than that of the quantum detectors. Therefore, the thermistor bolometer is used where the quantum detector cannot be used (beyond 9 microns). The superconducting bolometer is only a research tool, because it works at near absolute zero temperature and is critically temperature sensitive. Detectivity may be in the range of $4.8 \times 10^9$ cm cps$^{\frac{1}{2}}$ watt$^{-1}$.

The Golay Cell is a pneumatic type thermal detector and is very sensitive and is operated at room temperature. It is used in infrared spectroscopy, but it is too fragile to be useful in
military applications. Its $D^*_{\text{A}}$ is nearly $1.67 \times 10^9 \text{ cm cps}^{1/2} \text{ watt}^{-1}$.

The equivalent electrical impedance of a thermocouple can be represented by a resistance in series with the parallel combination of another resistance and a capacitance. The responsivity of the thermocouple, defined as the ratio of the amplitude of open circuit voltage of the thermocouple to the amplitude of the incident radiation power, is found to be inversely proportional to the frequency of operation of the thermocouple radiation chopping mechanism. Therefore, the thermocouple is operated at a low frequency.

The minimum detectable power of the thermal detectors is limited by the noise in the systems. For the thermocouples it is advantageous to keep the thermal conductance due to the thermocouple leads equal to that due to the receiver. The bolometers are known to introduce $1/f$ or modulation noise, in addition to the radiation fluctuation noise and Johnson noise also encountered in the thermocouple. Since $1/f$ noise is small at large frequencies, it is advantageous to operate the bolometer at high frequency. The noise in the thermal detectors is excessive at high temperatures, and therefore, it would seem desirable to operate these at low temperatures, but it is not practical to do so.

Many figures of merit have been devised to compare the performance of the thermal detectors; but none of them can adequately describe their performance, because there are many other important characteristics which need to be considered. For the
minimum detectable power, a figure of merit called \( D^* \) (D-Star) is popular. It is defined as

\[
D^* = \left( \frac{V_s}{V_n} \right)^\frac{3}{2} \frac{\Delta f}{W A^2} \text{ cm. sec.}^{-\frac{3}{2}} \text{ watt}^{-1}
\]  

(68)

where \( V_s/V_n \) is the signal to noise ratio in the bandwidth \( \Delta f \), \( W \) is the radiation power incident on the detector per unit area, and \( A \) is the area of the detector. Responsivity, radiation spectral response, and effect of chopping frequency are the other three common characteristics which are commonly compared.


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A STUDY OF THERMAL DETECTORS

by

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B. E., University of Poona, India, 1962

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963
In this report the principles of operation of the thermal detectors are described, and a mathematical analysis is made of their performance. Then the criteria for comparison of performance of the various thermal detectors are discussed.

The thermal detector converts infrared radiation energy into electrical energy, and the latter is then amplified and measured. Infrared radiation heats the detector element. This results in the use of temperature sensitive effects for thermal detectors. The design of radiation thermocouples is based on the thermoelectric effect occurring at a junction of two metals at different temperatures. Temperature sensitive resistors are used in bolometers. Golay Cell is a pneumatic device using the principle of thermal expansion of gases.

Modern applications of the thermal detectors require these instruments to detect very low intensity infrared radiation. The minimum detectable power is limited by the noise inherent in the instruments. Equivalent electrical circuit analysis is especially suitable for calculating and optimizing this minimum detectable power.

The several important characteristics of the thermal detectors cannot be expressed in one comprehensive figure of merit. and instead, for each desirable characteristic many figures of merit have been devised. These figures of merit are useful in finding the most suitable thermal detectors for various applications.