SOLAR FURNACE DESIGN

by

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NOMENCLATURE

C - Concentration ratio

d - Diameter of the sun's image (circular image in the focal plane), inches

D - Diameter of the paraboloid at the edge, inches

E - Overall efficiency factor

f - Focal length of paraboloid, inches

H - Height of mirror, feet

N - Number of mirror segments

P - Heat flux received per unit area within the sun image at the focus

P_a - Actual flux per unit area received at furnace site = \( \eta_a P_o \), cal/cm²/min

P_o - Solar constant, measured above the atmosphere, 2 cal/cm²/min

P - Total power of flux passing through the sun's image, cal/min

R - Distance between a point on concentrator and focus, inches

T - Absolute temperature, °K

T_m - Maximum absolute temperature attained, °K

T_s - Absolute temperature of the sun - 5800 K

W - Width of mirror, feet

\( \beta \) - Ratio of surface area of paraboloid to projected area of the mirror opening

\( \gamma \) - Index of geometrical perfection

\( \epsilon \) - Emmissivity coefficient

\( \eta_a \) - Efficiency taking into account losses through the atmosphere

\( \eta_r \) - Reflectivity efficiency factor
Nomenclature (Continued)

\( \eta_s \) - Reflectivity efficiency factor taking into account losses due to shadowing and non-reflecting portions of the optical parts of the furnace.

\( \theta \) - Angular aperture or rim of angle of a parabolic concentrator

\( \sigma \) - Stefan-Boltzmann constant
INTRODUCTION

Daylight now falling on the earth escaped the surface of the sun eight minutes ago, but its energy was created deep within the Solar Furnace before the birth of civilization. Each second, four million tons of hydrogen transforms itself to radiant energy which eventually floods into space. The sun delivers to us in just three days as much heat and light as would be produced by burning the earth's entire oil and coal reserves and all the wood of its forests. Yet the earth receives only about two-billionth of the sun's radiant energy.

In 1925 Sir Arthur Eddington, a British astronomer, proposed the theory now accepted as correct. It is atomic or nuclear energy that makes the stars bright. This energy, the same as that of the hydrogen bomb, comes from the process we call nuclear fusion, in which the nuclei or cores, of hydrogen atoms collide, uniting to form helium nuclei and giving off burst of energy. Within the sun, each second, 564 million tons of hydrogen are converted to 560 million tons of helium. The remaining four million tons each second radiate away as heat and light. Hydrogen nuclei, impelled by tremendous heat, collide with such violence that thermonuclear fusion could occur and the nuclear energy thus released maintains the sun's temperature.

As we look at the sun from earth, we can "see" only three layers directly--through visible light, infrared, and radio, observed by ground observations; and through ultraviolet and X-rays detected by instruments in rockets and satellites, from 93,000,000 miles away, the sun showers earth with cosmic rays and electromagnetic radiations. Fortunately for man, our atmosphere blocks the most dangerous high energy rays, but lets through light and part of the heat.

*Numbers in brackets designate references at end of report.
Radiant energy from the sun travels directly by waves moving with the speed of light, as one feels the heat of fire at a distance. Tremendous power is released by a brilliant burst of light and all other electromagnetic wavelengths, from X-rays and ultraviolet to infrared and radio waves; by protons and electrons accelerated to more than half the speed of the light; and clouds of ionized, or electrified, gases that sweep through space at hundreds of miles per second.

Civilization requires energy; not a wheel of commerce or industry would turn without it. Today, a very large fraction of all energy comes from coal, oil, and other fossil fuels which were produced over the centuries by the decay of animal and vegetable matter whose growth depended upon sunlight. These resources are now being consumed at an enormous and constantly increasing rate. Once used, they are gone forever. The level of energy consumption per person in the United States is nearly eight times greater than it is for the world in a whole. For us, the ability to command a vast reservoir of energy means continuing improvement of our standard of living. However, nearly one billion people from Casablanca to Calcutta are in dire need of even a small increase of energy-energy which they must have at their command if they are to improve the nature of their existence.

The amount of solar energy reaching the earth each year is over 30,000 times that presently used in the same time period. In view of the rapidly diminishing supplies of fossil fuels, it is clear that we must soon devise means for converting solar energy more economically into useful power.

The launching of Sputnik I opened up a completely new area for application of solar energy power-conversion systems. Solar radiation has been the major source of energy for earth satellites and space probes up to the present time and its value as an important energy source is likely to con-
tinue for spacecraft of the future. This is in contrast to the utilization of solar energy on earth where, owing to the present relative abundance of other forms of energy, solar energy has not been utilized to any significant extent. The possibility should not be overlooked, however, that the development of truly reliable and low-cost-solar-energy system as a result of the present space exploration efforts may also find important terrestrial applications.

Solar energy available on earth is one kw on about 15 square feet of land. The roof of a house having an area of 100 square meters (33 by 33 ft) receives in eight hours on a bright day about 500,000 kilocalories, or two million BTU, of solar heat. This is equivalent to burning of about 150 pounds of coal or 15 gallons of gasoline. This heat would give 58 kilowatt-hours of electricity if produced by a boiler, engine, and dynamo system having an efficiency of 10 per cent.

The sun's radiation as ordinarily received is not hot enough to be of much technical use. It must be concentrated by focusing from a large area onto a small heating surface, or it must be retained in a heat trap so that the heated object does not lose its heat too rapidly.
Although the chief difficulty in producing power from the sun's energy is the limitation of low temperatures, it is true also that solar energy is used for achieving the highest possible temperatures for laboratory research. A large parabolic mirror focusing radiation on a small area can give temperatures of about 3,000 to 5,000 °C. France has done important work in this field.

**Utilization of Solar Energy for the Attainment of High Temperatures**

The attainment of high temperature is a necessity for an industrial society. Metallurgy, chemical synthesis, fabrication, heating, lighting, etc., are dependent on the ability to attain high temperatures. In addition, much of present-day research is in the field of high temperatures.

The common methods used to achieve the required temperatures are flames, electric resistance heating, induction heating, and electric arc. Short-time heating at high temperatures can be obtained by "exploding" wires, detonations, and shock waves.

In each case there exist some drawbacks as to possible applications. One is limited by the shortness of time, the products produced, electric and magnetic fields, the necessity of special materials that can contaminate, and limitations of furnace materials as to temperatures attainable.

It has run across the minds of many that it should be possible, in some manner, to approach this extremely high temperature on earth. It is towards this end that the concentration of the sun's radiation has been achieved by means of parabolic mirrors and lenses. It has been possible by these means to achieve relatively high temperatures in a short period of time and for
durations up to eight hours, depending on the weather conditions, time of year, and location. It is possible, by suitable enclosure of the samples to be heated, to obtain any desired transparent atmosphere free of electric and magnetic fields, contamination, and reaction with sample holders or furnace elements.

**Historical**

The earliest use of solar radiation to attain elevated temperatures is generally attributed to Archimedes. Archimedes is supposed to have used a large hexagonal mirror to burn the Roman fleet which besieged Syracuse. The first evidence of the attainment of high temperatures by means of solar radiation is the use by Averani and Targioni in 1695, at Florence, of large burning glass to make a diamond, previously considered unalterable, disappear. Since that time, lenses and reflectors have been used intermittently for various purposes, but without any appreciable contribution to the development of these instruments.

**Present**

Due to the development for military purposes of large searchlight mirrors, which meet the needs of solar furnaces perfectly, a large number of these mirrors have been recently used for this purpose. By using these searchlight mirrors of a high aperture parabolic shape as a collector of solar radiation, one can readily attain high temperatures. For reflectors, the parabola is as near an ideal shape as can be obtained because any light parallel to its optical axis will be reflected at its surface so that the light will pass through its focal point. The higher the relative aperture (diameter/focal length of mirror), the greater will be the concentration of light at the focal area. The diameter of the image obtained is the product of the distance of the mirror from
the focal point and the angle subtended by the source (for the sun the angle is 32' or 0.00931 radians, on the average, \(d=2f \tan 16'\)). In the case of small aperture mirrors as used in the telescopes, use of the product of the focal length and angle subtended by the image is precise enough for determining the diameter of the focal area. For large-aperture mirrors, the diameter of the image varies as one goes from the optical axis to the outer edge of the mirror. This property of parabolas, i.e., the increase of distance from the mirror to the focal point with increasing aperture, causes a doubling of the image diameter at 90° aperture (as measured at the focus from the optical axis to the edge of the mirror). The collecting area increases as the aperture increases; but this, to some extent, is negated by distribution of the collected radiation over a larger area. To obtain very high temperatures, it is necessary to concentrate the energy in as small an area as possible.

To attain high temperatures one needs a target for the intense radiation. This target introduces additional problems with large-aperture reflectors. At large angles, a flat target will reflect most of the light coming to it and, at 90° aperture, all the light, according to Lambert's Law. These large angles also spread the image over a large area of a mirror of 90° aperture is completely ineffective. An analysis shows, taking into account reflection at the target and mirror and change of image size with angle, that the maximum input to a flat collector is at 35° and that beyond 70° no appreciable contribution is obtained to the total flux density. The situation is somewhat altered for the case of a hemispherical target, in which case the radiation is received even at 90°. However, since the area of the hemisphere is twice that of the flat target, it is found that the flux per unit area is slightly lower than for the flat target. The maximum effective area on a
mirror for the hemispherical target is found to be at 40° aperture.

The other method used to concentrate light is by means of lenses. The only known large installation of this type in use is the one used by Professor Duwez (4) at the California Institute of Technology (Fig. 1). Here the light is refracted by lenses onto mirrors which reflect it through an almost hemispherical nest of smaller lenses, which further concentrate the light. The distances traveled by the light are almost constant. However, the problems of spherical and chromatic aberration becomes important. In addition, there is still the reflection problem at high angles at the target.

Reports of the utilization of searchlight reflectors of large aperture as solar furnaces are being made continually by various institutions. In 1921, Straubel used silvered-glass searchlight mirrors. The mirrors were two meter in diameter and had a focal length of 86 cm. After the last war, the French Government obtained a number of German search-light mirrors. The mirrors obtained were two meter in diameter, 83 cm focal length, and had a silvered-glass surface. Conn William (consulting Physicist, Kansas City, Missouri) used an aluminum mirror three meters in diameter and 86 cm focal length, having an image diameter of 0.8 cm.

The apparent movement of the sun through the sky and its change of declination daily necessitates a mounting of the solar furnaces, or the auxiliary flat mirrors, that will allow for easy movement. The equatorial mounting seems to be most popular. The declination is set each day and the motion through the sky is followed by photocells, bimetallic strips, astronomical clocks (synchronous motors or falling arm with governor), or by manual operation. The intensity at the target has been varied by means of a diaphragm over the entire mirror, by a reflecting cylinder near the target,
and by defocusing.

Recently the solar furnaces are used in the field of nitrogen fixation and melting of iron. It is Trombe’s belief that it is waste of high temperature energy to vaporize steam when the same amount of energy can melt iron. The past uses of solar furnace have been primarily confined to the study of ceramic materials and their products. This instrument easily provides a high temperature, controllable atmosphere heat source. In the way of research the solar furnace is also adaptable to the study of the thermodynamic properties of refractory materials. Since it is a source of high energy density, it can be used to evaluate thermal shock resistivity. Use of solar furnace for research work includes the high temperature chemistry and physics of silicates, high melting oxides, borides, carbides, silicides and nitrides; the thermal properties of materials in solid or gaseous form at high temperatures; the determination of liquid curves. Other fields of high-temperature research are: the determination of the heat flux at the focus of the concave mirror up to the highest temperature attainable; the rate of transfer of heat for various mirror combinations; and high temperature spectrographic work. Use of solar furnace for semi-industrial includes the production of glasses with high refractive indices; the growing of single crystals; and the development of heat-resistant alloys.

It has been already pointed out that solar furnaces for attaining high temperatures may be divided in two groups: (1) experimental furnaces for pure research work, and (2) semi-industrial furnaces. The first group includes furnaces for attaining high temperatures by concentrating solar energy in a small area into which the sample to be heated is brought. Paraboloidal mirrors are used, as a rule, similar in design and definition to
searchlight mirrors. The greater the focal length, the greater will be the diameter of the image of the sun; and the larger the aperture, the greater will be the amount of solar energy collected. Experimental solar furnaces may be operated with fixed values of aperture and focal length, or with variable effective aperture, or with variable focal length.

(1) CONSTANT APERTURE AND CONSTANT FOCAL LENGTH

(Solar Furnace Used for Experimental Work)

The incident energy from the sun is collected by means of a paraboloidal mirror in a small area. In some cases the paraboloidal mirror remains stationary and a plane mirror is used as a heliostat which follows the apparent movement of the sun. In other cases, a plane or convex mirror is inserted between the main mirror and the sample in order to shift the image of the sun to a desired location or to change its size.

The sample to be tested is brought in to the image of the sun where it quickly melts if sufficient energy is available. The products of melting are collected in a crucible made from or lined with the same material that is being studied. Melting can be carried out in oxidizing atmosphere or the sample and its holder can be enclosed in a transparent container for heating in a vacuum or at elevated pressure, with or without protecting atmosphere.

(2) VARIABLE APPARENT APERTURE

In this case a shutter in the form of a cylinder which moves along the axis of the paraboloidal is used. It serves for control of the amount of incident radiation by varying the effective aperture of the mirror. Since the position of the control cylinder can be closely adjusted, an accurate control of the temperature of the sample is possible, the temperature equilibrium
attained being a function of the amount of incident and absorbed radiation, emitted radiation, and losses by convection and conduction.

The temperature of a sample heated below its melting point can be observed on the side opposite to the incident radiation if the sample is small enough to attain uniform temperature. In most cases, however, a small sample will melt quickly. It becomes necessary to measure the temperature of the sample or target from the same side from which it receives radiant energy. The result is that the incident radiation from the sun (reflected by a concave mirror) and the temperature radiation of the sample are superimposed. They are separated by means of two sectors which intermittently block out the incident radiation from the sample while temperature readings are taken. The sector rotates at 2,500 rpm.

(3) VARIABLE FOCAL LENGTH

The mirror made from glass or metal is replaced by a surface of liquid mercury; the mercury rotating with and in a container at constant angular velocity. The surface of liquid forms a paraboloid of revolution, the shape of which depends upon the characteristics of the liquid and its angular velocity. A change in the rate of revolution causes a change of focal length. At the same time, the diameter of sun's image changes, and also the amount of energy received per unit area of the sample, causing an increase or decrease of the temperature of the target. Since the rate of revolution of the liquid can be easily adjusted, close control of the temperature of the sample is attained. (Harold Heywood, Department of Mechanical Engineering, Imperial College of Science & Tech., University of London)
The theoretical and practical factors which affect the specific performance characteristics and costs of solar furnaces are reviewed in detail. The specific items which affect the performance characteristics and costs of solar furnace systems can be classified in six general categories. These are: the type of furnace, methods of mounting the concentrator and tracking the sun, optics, material and accuracy of construction, site, and types of target or use. There are also two other factors which affect the over-all cost of an installation, namely, the facilities and staff and operation costs.

(1) Type of Furnace

Included in the category, the type of furnace, is the method of concentrating the sun's ray. There are three principal forms of concentrators: the lens, oriented flat plates, and the parabolic surface. The factors which should be evaluated to determine the relative costs of various types of concentrators are the number, size, and cost of the reflecting segments (or lens), the cost of segment support brackets, the cost of adjustment of the segments, and the proportional size and cost of the components of the concentrator (concentrator primary frame, supporting structure, and foundation).

In comparing the different types of reflecting concentrators, there are factors which indicate that the cost of a segmented parabolic concentrator would be lower than that of flat-plate type. Trombe stated, in reference to the 35 ft. solar furnace at Montlouis, that an energy concentration at the focal point obtained by approximately 20,000 flat mirrors could be obtained by using 3,500 curved mirror segments. Thus, a preliminary evaluation of these factors indicates that the cost of an oriented flat plate concentrator would
Fig. 1  SCHEMATIC DIAGRAM OF LENS-TYPE FURNACE

CONCENTRATOR WITH HORIZONTAL AXIS

CONCENTRATOR WITH VERTICAL AXIS

Fig. 2
DIRECT CONCENTRATOR

CONCENTRATOR WITH VERTICAL AXIS
WITH ONE AUXILIARY MIRROR

Fig. 3
be about twice that of a segmented parabolic concentrator having equal performance characteristics. This is based on the cost of the reflecting surfaces alone, assuming the mirror to be of the same material. If consideration is given to the number of mirror-supporting brackets, relative size of the supporting structures and foundation, and other factors, the cost differential would probably be greater.

In comparing the relative cost of the lens-type concentrator (Fig. 1) and the parabolic concentrator, there are several factors which indicate that the cost of a large furnace employing a lens type concentrator would be higher. The other factor which indicates that the cost of a lens-type concentrator would be higher than a parabolic type is the optical losses.

All these factors indicate that for the Research (Experimental) Solar furnace the parabolic type is most suitable.

(2) Mounting (The Concentrator) and Tracking (The Sun)

There are at least four methods of mounting the reflector-type of concentrator, each of which has certain advantages depending upon the size of the solar furnace and the intended use. In one, the individual flat-plate mirror segments each track the sun and horizontally reflect the sun's ray which are superimposed on the target. A second type is the unit in which the entire concentrator follows either the entire, or a limited, portion of the sun's path. A third type is composed of fixed concentrator with a horizontal axis and movable flat mirror (heliostat) which is used to track the sun and direct rays into the concentrator. In this case the rays are reflected and concentrated to the target, on the horizontal axis. In the last type, the concentrator and one flat mirror are fixed, with concentrator faced toward the ground. In this case a second flat mirror or heliostat is required, which tracks the sun.
The last two cases are illustrated in Fig. 2. The fixed mirror can be dispensed with the vertical arrangement if the concentrator is mounted high enough to prevent shadowing of the movable heliostat under it, but this is normally impractical. To avoid a tall tower and for experimental purposes a fourth type is suitable. Now we have to see whether assumed mounting is economical or not.

In determining the relative costs of mountings which employ auxiliary mirrors, the effect of additional reflecting surfaces as well as effect of the types of mounting and tracking equipment have to be considered. This is because no surface is 100 per cent efficient as a reflector, but varies in performance depending on the material. A system employing more reflecting surfaces would therefore require a larger concentrator and larger auxiliary mirrors to give some performance characteristic. For furnace system employing parabolic concentrators and having the same performance characteristics but different number of auxiliary mirrors, the following relationship holds:

\[
\left( \eta_r \right)_1^{1/2} \sin \theta_1 = \left( \eta_r \right)_2^{1/3} \sin \theta_2
\]

\[
\frac{D_2}{D_1} = \frac{\sin \theta_2 (1 + \cos \theta_1)}{\sin \theta_1 (1 + \cos \theta_2)}
\]

where \( \eta_r \) is the individual reflectivity efficiency factor taking into account losses by reflectivity or absorptivity of the optical parts of the furnace and \( \theta \) is the rim angle of a parabolic concentrator. Subscript 1 and 2 refer to the system employing one and two auxiliary mirrors, respectively. \( D \) is the diameter of paraboloid.

Assuming the furnace system which employs one auxiliary mirror to have a concentrator with a diameter of 100 ft., a rim angle of 50°, and all reflecting surfaces constructed of the same material with \( \eta_r = 0.85 \), then
required diameter of concentrator using two auxiliary mirrors is obtained from above equations.

\[(0.85)^2 \cdot \sin^2 50^\circ = 0.85^3 \cdot \sin^2 \theta_2\]

\[\theta_2 = 64.5^\circ\]

\[\therefore D_2 = \frac{100 \cdot \sin 64.5^\circ (1 + \cos 50)}{\sin 50^\circ (1 + \cos 64.5)}\]

\[= 135 \text{ ft.}\]

which shows that a concentrator of 2 is more costly. But in case "I" we have to put the concentrator at higher height (fig. 3), and for wind load the tall tower should be strong, which makes system I more costly. While use of a single auxiliary mirror may give somewhat higher values of possible concentration ratios, solar furnaces as they are envisioned today are considered as research tools, and it is the over-all usefulness of these tools that must be considered, not merely the efficiency of some abstract object. If we see the lists of furnaces in the United States (given on page 55), we will find that the majority of research furnaces are of the type having two auxiliary mirrors. So we select the second type.

Two methods can be used in constructing the target platform: (1) by using a cantilever type construction and having the target platform at one end, or (2) by having a movable target tower which is designed to follow the path of the reflected image. For the first method, access to the target area would be poor, and the second case the mechanism required to move the tower a cantilever to support the target platform, the shutter design would be difficult because of the interference of the cantilever. Thus on technical basis, the system employing a fixed concentrator and movable heliostat appears to be more desirable.
(3) Optics

The next design category to be considered is the optics or the optical geometry of the concentrator. In this group there are five important variables: the aperture, ratio of aperture to focal length, image or target diameter, average target flux, and uniformity of the flux distribution at the target.

The first factor in this category which will be evaluated from both a technical and an economic standpoint is the rim angle, or the ratio of aperture to focal length of the concentrator, since this affects the relation between mirror and image size and determines the flux. To determine the optimum rim angle \( \theta \), the effect of increasing rim angle on the flux delivered by the concentrator will be determined.

For the case of a parabolic-type concentrator, the optimum rim angle depends upon the basis on which it is chosen. For example, the maximum change in the concentration ratio with respect to change in \( \theta \), and thus in the flux of the image, occurs at the inflection point of the curve, as seen in Fig. 4, where concentration ratio \( C \) is plotted versus rim angle \( B \). This inflection point occurs at a rim angle of \( 45^\circ \). In this case, as shown in Fig. 4, any dollar invested to increase the rim angle by one degree returns the most flux at \( 45^\circ \).

From a cost viewpoint, there are three bases on which the optimum rim angle may be selected. These are: (1) the amount of flux per unit cost assuming that the useful value of flux increases as the total quantity of flux delivered by the concentrator increases (so that value/cost equals flux/cost multiplied by flux); and (3) the rate of change of flux per unit
Variation of concentration ratio with rim angle.

Fig. 4

Variation of flux and cost of a solar furnace employing a parabolic concentration with rim angle.

Fig. 5
cost. An illustration of the variation of the optimum rim angle using these three bases is given in Fig. 5. For this example, two assumptions were made: first, that the cost of a solar furnace system consisting of a fixed parabolic concentrator and a movable heliostat varies with the diameter of the concentrator to the 1.5 power (the results of design study indicates that the cost varies approximately with the diameter to the 1.5 power), and second, that the focal length of the concentrator is constant, so that the size of the sun image at the focal plane would be constant. If the purpose of the solar furnace in a particular application is to replace a conventional type of furnace, i.e. electric or gas fired, then from an engineering viewpoint the flux per unit cost is one of the most important factors. For this case, as can be seen from the flux/cost versus rim angle curve in Fig. 5 (curve B) the maximum ratio occurs at a rim angle of approximately 45°.

When the maximum temperature obtainable (i.e. maximum flux) is taken to be the prime factor, then the unit value of the flux can be taken to be related to the total quantity of flux delivered by the furnace. Specially, the optimum rim angle was determined by assuming the value of the flux to be equivalent to the total flux delivered. For this case, from the flux²/cost curve in Fig. 5 (C) the optimum value of the rim angle is approximately 70°.

In the last case, shown by curve A in Fig. 5, the maximum differential change in flux with cost occurs at a rim angle of 30°. In this case, therefore, a dollar invested for a degree increase in rim angle greater than 30° would not supply as much flux as a degree increase at 30°. Since the first two cases are the practical limits of interest, it appears that optimum value of the rim angle of a parabolic concentrator is between 50° to 70°.
Although the rim angle is an important factor with respect to furnace design, it may be necessary for certain applications to provide other heat flux levels and image diameters than the design values. In a system employing parabolic concentrators there are four methods which can be used to achieve different flux levels and/or image diameters. These are: (1) the use of control shutter placed between the focal plane and the concentrator; (2) shading of a portion of a concentrator and/or the heliostat, and (3) position adjustment and deformation of the individual mirror segments of the concentrator to obtain a different focal length; and (4) movement of the target off-focus.

The first two methods, the use of a control shutter and the shading of the concentrator and/or heliostat, can be used to reduce the heat flux level without loss of uniformity in the flux at the focal plane or without a change in the image size. A control shutter can also be designed to reduce image size. The third method, adjustment and deformation of the individual mirror segments, can be used to achieve either higher or lower flux levels and larger or smaller images by increasing or decreasing the focal length of the paraboloid. For a given aperture, a smaller image diameter and higher flux are obtained by decreasing the focal length, whereas increasing the focal length will result in a larger image size and lower flux. The last method, moving the target off-focus, will result in a larger or smaller image size, but this is accompanied with some loss of uniformity of heat flux within the image.

(4) Construction Factors

The next design category which will be considered is the material of construction and the method of construction. Selection of structural
materials will depend on the strength-weight relations and the corrosion resistance required. The choice of reflector materials will be governed by reflectivity and corrosion resistance as well as by the optics and the spectrum desired. Of the three types of reflecting materials, metal, front silvered-glass, and back-silvered glass, the latter has been determined to be the most favorable.

The disadvantage of metal as a reflecting surface are poor corrosion resistance and inability of any resulting oxide coating to reflect the entire spectrum. Although the reflecting qualities of front-silvered glass are good, corrosion is again a problem. The reflecting surface of back-silvered glass can be protected from corrosion by various means. One factor which is a disadvantage for back-silvered glass is the addition of an absorption loss by the glass. This loss can be minimized, however, by the use of a good grade of optical glass.

The accuracy of construction of the reflecting surfaces, i.e. how costly they approach the perfect geometrical shape, will affect the performance characteristics of the furnace system. Therefore, consideration will be given to variation of the cost with estimated degrees of accuracy of the reflecting surfaces. For this purpose an efficiency factor $E$, and an index of geometrical perfection $\gamma$, were determined for number of furnaces employing parabolic concentrators. The efficiency term includes all of the sources of inefficiency in furnace construction and is defined by

$$E = \eta_r \eta_s \eta_a \gamma.$$  

where $\gamma$ is index of geometric perfection, $\eta_r$ is the over-all reflectivity efficiency of the reflecting surface, $\eta_s$ is the shadowing efficiency factor.
which accounts for the non-reflecting surfaces, and \( \eta_a \) is the fraction of the solar constant which is available at the site (Manhattan).

Another definition of the efficiency factor can be obtained from

\[
E = \frac{p_{\text{actual}}}{46.1 \times 10^3 p \cdot \sin^2 \theta} = \frac{p_{\text{actual}}}{C_{\text{theoretical}} p_0}
\]

where \( p_{\text{actual}} \) is the heat flux actually received per unit area within the sun image at the focus, and \( p_0 \) is the solar constant, measured above atmosphere. In Table I the efficiency factor and index of geometrical perfection are given for seven furnaces.

The effect of shadowing of the reflecting surfaces and the reduction in performance caused by non-reflecting portions of the furnace also should be included in the category dealing with the method of construction. Shadowing losses are caused by structures and other portions of the furnace, i.e. mirror adjustment brackets and shutters, which block the sun's ray from the reflectors and target. The percent of the voids (spaces between the reflecting segments of the concentrator and auxiliary mirrors) should be considered in the furnace design. Both of these factors tend to reduce the performance characteristics and should be kept to minimum.

(5) Location

The fifth category which should be considered is the location or site of the furnace. Selection of a site should be considered from two viewpoints, the solar climate and the overall cost of the installation. The solar climate should be investigated to determine the average solar radiation and the fraction that is direct radiation. The portion of the sun's energy which is received as a direct radiation and can be concentrated is affected by many
factors. A study of the relative importance of these factors has indicated that water vapor and atmospheric pollutants have more effect on the energy available for solar furnace work than has the altitude.

(6) **Type of Target Use.**

The last category is the type of target and intended applications of the solar furnace. In designing a furnace, consideration must be given to the types of target that will be used. The emissivity or absorptivity of the target and the heat losses by conversion and conduction effect the maximum attainable temperature. Also, the intended use of the furnace, i.e. as high temperature source or production unit, must be considered to determine the type of mounting, the required work space at the focus, and other related factors.

**CONCLUSIONS**

In summary, the technical and economical considerations indicate that the best design for the experimental (research) solar furnace should consist of

1) a parabolic concentrator.
2) a fixed concentrator with its axis vertical, using two auxiliary mirrors and using a heliostat to follow the sun.
3) a rim angle between 50° to 70°, depending upon the economic value of flux increments.
4) a reflecting surface of back-silvered glass.
Derivation of Formulae Used: (29)

In designing a solar furnace for a specified heat flux at the target, there are two classes of factors (theoretical and operational) which should be considered. Both affect the maximum flux (or temperature) a solar furnace can achieve.

The first class is related to the optical geometry of the concentrator and includes six important design variables: the aperture, ratio of aperture to focal length, image diameter, average heat flux at the image, uniformity of flux distribution at the image, and the efficiency at which energy received by the concentrator is concentrated.

When a parallel beam of light is received on a paraboloid of revolution in a direction parallel to its axis, the reflected light rays converge into a point which is the focus of the paraboloid. The rays coming from the sun, however, are not parallel rays, since the sun has a finite size. From the earth, the sun appears as a circular disc subtending an angle of about 32\(\text{'}\) of arc. As first approximation, it may be considered that the intensity of the radiation is uniform over the surface of the sun. Actually, the black-body temperature of the sun is greater at the center and progressively decreases toward the edges.

When the axis of the parabolic mirror is pointed towards the sun, the axis of the incident cone of light is parallel to the axis of paraboloid, and as shown in Fig. 6, the axis of the reflected cone passes through the focus of the paraboloid. The reflected cone coming from a point such as B in Fig. 6 will intersect the focal plane and form an ellipse, and hence the image of
Fig. 6  PARABOLIC MIRROR WITH REFLECTION OF INCIDENT RAYS INTO FOCAL PLANE
the sun coming from point B is badly distorted. In fact, only the part of the paraboloid located near the apex will reflect the sun rays into a circular image in the focal plane. The term "sun image" will refer to that circular image formed by the portion of the parabola near the apex. The diameter of the sun image depends only on the focal length of the paraboloid, and is given by the simple formula

\[ d = 2 f \tan 16' = \frac{f}{107.3} \]

in which \( f \) is the focal length and 16 minutes angle is half of the angle subtended by the sun, seen from the earth. When a very precise image of the sun is wanted, as in a telescope, the diameter of the paraboloid must be relatively small in comparison with the focal length. For a solar furnace, the maximum amount of energy must be collected and focused into the solar image; hence, the diameter of the paraboloid should be as large as possible compared with the focal length. Instead of paraboloid size, it is more convenient to consider the angle between the axis of the paraboloid and a line joining the focal point to the edge of paraboloid. The optimum conditions will be obtained when this angle, which will be called the rim angle of the furnace is equal to 90°. Various factors involved in the construction of parabolic solar furnaces restrict the value of the rim angle to 50° to 70°, as shown already on pp. 19. The other reason is given as follows:

Problem to consider is the variation of the heat flux within the solar image with the rim angle, assuming that the paraboloid is geometrically perfect. The intersection of the reflected cone and the focal plane is an ellipse, and the major axis of the ellipse increases with the angle \( \theta \) and also with distance FB. This means that the outer portion of the paraboloid
reflects longer and wider ellipse around the theoretical sun image on the focal plane and, as a consequence, the amount of radiation focused into the sun image by an element of the paraboloid decreases as the angle increases. The problem is to compute the total amount of radiation energy falling into the sun image as a function of the rim angle of the mirror (see Fig. 4).

The energy per unit time which is concentrated within the sun image is the power. This power can be obtained by integrating the contributions of all of the elements of the mirror over its surface. Considering an element of the mirror around point B (Fig. 6) at a distance \( R \) from the focal point and at an angle \( \theta \), the power contributed by this element is the ratio between the area of the sun image, namely, \( \pi/4d^2 = \pi (f/1214.6)^2 \), and the area of the ellipse resulting from this intersection of the reflected cone and the focal plane, namely \( \pi(R/214.6)^2 (1/\cos \theta) \). Considering a ring at azimuthal angle \( \theta \), its contribution within the sun image is \( (f/R)^2 \cos \theta \). The power reflected from a ring element of area \( \Delta S \) is equal to the product of the solar constant \( p_o \) and the reduced area of the ring element corresponding to normal incidence \( \Delta S \cos (\theta/2) \), or \( p_o \Delta S \cos (\theta/2) \).

The solar constant \( p_o \) is defined as the total solar radiation received at normal incidence outside the atmosphere and is about 2 Cal/min cm\(^2\). The power contributed by the ring element to the solar image is therefore

\[
\Delta p = \left(\frac{f^2}{R^2}\right) (\cos \theta) (p_o \Delta S \cos \theta/2)
\]

or

\[
\Delta p = p_o \left(\frac{f^2}{R^2}\right) \cos \theta \cdot \cos (\theta/2) \Delta S
\]

The area of a concentric ring of the paraboloid is given by

\[
\Delta S = 2\pi R \sin \theta \left[ (\Delta R)^2 + R^2 (\Delta \theta)^2 \right]^{1/2}
\]

From the geometry of the parabola,

\[
R = \frac{2f}{(1 + \cos \theta)}
\]
and hence,

\[ \frac{1}{N} \frac{\Delta R}{\Delta \theta} = \tan \theta/2 \]

as a consequence

\[ \Delta S = 2 \pi R^2 \sin \theta \cdot \text{Sec} (\theta/2) \Delta \theta. \quad (3) \]

Combining Eqs. (2) and (3), and changing from differences to differentials,

\[ dP = 2 \pi p_o^2 \sin \theta \cdot \cos \theta \cdot d\theta. \quad (4) \]

This expression is the power contributed to the sun-image area by a differential ring of azimuthal angle \( \theta \). By integrating Eq. (4), the total power focused by the mirror will be obtained. This total power is

\[ P = \pi p_o^2 \sin^2 \theta \quad (5) \]

in which \( \theta \) is the rim angle of the mirror. When an object is placed at the focus of the furnace, it casts a shadow on the mirror. If \( \theta \) is the azimuthal angle corresponding to this shadow, Eq. (5) must be replaced by

\[ P = \pi p_o^2 (\sin^2 \theta - \sin^2 \theta_1) \quad (6) \]

which takes into account the power lost due to the presence of an object within the solar image.

So far it has been assumed that the total amount of solar radiation reached the paraboloid and reflected without losses. (The latitude, elevation, and weather of the site influence the amount of the sun's energy available for concentration.) This is, of course, practically impossible, since part of the radiation is absorbed by the atmosphere and part of it is lost because of imperfect reflection by the mirror surface. Two coefficients will be used to take these losses into consideration. The first coefficient \( \eta_a \) is the fraction of the solar constant which is available at the furnace site as a result of atmospheric absorption. The second coefficient \( \eta_r \) is the fraction
of radiation which is actually reflected by the mirror. With these two factors, Eqs. (5) and (6) may be written

\[ P = \eta_a \eta_r p_o f^2 \sin^2 \theta \]  
(7)

and

\[ P = \eta_a \eta_r p_o f^2 (\sin^2 \theta - \sin^2 \theta_1) \]  
(8)

Instead of considering the total power passing through the sun image, it is more convenient to consider the heat flux per unit area of the sun image, since this quantity is independent of the size of the image and hence independent of the size of furnace. The heat flux, or power per unit area, designated by \( p \), is readily obtained by dividing the value of \( P \) as given by Eq. (7) by the area of the sun image, which is \( \pi (f/214.6)^2 \). The heat flux is therefore given by

\[ p = \eta_a \eta_r p_o (214.6)^2 \sin^2 \theta \]

\[ = 46.1 \times 10^3 \eta_a \eta_r \sin^2 \theta \]  
(9)

The last operational factor involves the inefficiencies that the degree of accuracy of construction of the concentrator surface introduces. In the equations above it was assumed that the parabolic reflecting surface was geometrically perfectly paraboloid. This is of course impossible to obtain, and the performance of any solar furnace will be less than that predicted by Eq. (9). The index of geometrical perfection, \( \gamma \), or a concentrator may be defined as the quotient of the actual concentration ratio to the theoretical concentration ratio after correction for atmospheric, reflective, and shadowing losses. Therefore, the actual heat flux available at the target is given by

\[ p = 46.1 \times 10^3 \eta_a \eta_r \gamma p_o \sin^2 \theta \]  
(9a)

\[ = 46.1 \times 10^3 \eta_a \eta_r \gamma_s \eta_s p_o \sin^2 \theta \]  
(9b)

considering shadowing efficiency \( \eta_s \).
Concentration Ratio and Concentration Efficiency

The concentrator ratio of a solar furnace is defined as the ratio of the heat flux within the image to the actual flux received on the earth at normal incidence, assuming that this flux would have been reflected at normal incidence by as many reflecting surfaces as there are in the furnace. The concentration ratio is thus the number by which the flux at the furnace site, $p_a$, corrected for reflectivity losses must be multiplied in order to obtain the flux within the sun image. This ratio called $C$ is a function of the mirror (or mirrors) reflectivity. By definition, it is given by

$$C = \frac{p}{(\eta_a \eta_r p_o)}$$

$$= 46.1 \times 10^3 \sin^2 \theta$$

Obviously, the concentration ratio increases with rim angle $\theta$ of the furnace and maximum value for $\theta = 90^\circ$ is 46,100.

Another parameter in evaluating the performance of a solar furnace is the concentration efficiency. This efficiency is defined as the ratio of the power received within the sun image to the total power reflected by the parabolic mirror. The concentration efficiency $\eta_c$ is thus

$$\eta_c = \frac{p}{\eta_r \eta_a r \pi (D^2/4)}$$

using Eq. (7) and $\eta_a p_o = p_a$ we will get

$$\eta_c = \frac{p}{\eta_a \eta_r r p_o \pi (D^2/4)} \frac{2 \sin^2 \theta}{(1 + \cos \theta)^2} = \left( \frac{1 \pm \cos \theta}{2} \right)^2$$

The variation of the concentration ratio $C$ and the concentration efficiency $\eta_c$ with rim angle $\theta$ as shown in Fig. 5. Instead of rim angle, the aperture
ratio of the mirror, namely, the ratio of the diameter $D$ of the mirror to its focal length $f$, may be used. The relationship between focal-length, diameter of mirror and rim angle is

$$\frac{D}{f} = \frac{4 \sin \theta}{1 + \cos \theta} = 4 \tan \theta/2 \quad (13)$$

**Maximum Attainable Temperature**

Eq. (5) represents what may be described as the potentially available power passing through the sun image in the focal plane. When a solid body is placed at or near the solar image, the shape and the absorptivity of this solid will determine the fraction of the potentially available energy which is actually absorbed by the solid. In order to speak about the maximum temperature than can be obtained at the sun image, it is necessary to describe the condition under which this temperature is being created by the presence of a solid body absorbing the radiation.

In the case of a cavity having an opening equal to the area of the sun image, and assuming that this cavity is perfectly insulated from the surroundings, all of the flux received will be used to raise the temperature of the cavity, which will act as a perfect blackbody. In this case, the absorptivity coefficient may be taken as unity and independent of the angle $\theta$. Eqs. (5) and (6) therefore give the total power absorbed into the cavity. Flux is given by Eq. (9a). It is now possible to compute the maximum attainable temperature by equating the heat flux received at the sun image to that radiated back under equalibrium conditions.

$$P = \sigma \left( T_m^4 - T_o^4 \right) \quad (14)$$

in which $T_m$ is the cavity temperature, $T_o$ is ambient temperature, and $\sigma$ is the Stefan-Boltzmann constant. Since the surrounding temperature $T_o$ is very small with respect to the temperature $T_m$, the term $T_o^4$ can be neglected.
and Equ. (14) becomes

$$p = \sigma T_m^4$$

(15)

It is useful to express the ratio $p/\sigma$ in terms of temperature of the sun. The power radiated by the sun which has radius $R_s$ and surface temperature $T_s$ is $4\pi R_s^2 \sigma T_s^4$. The solar power flowing through a spherical surface $4\pi R e^2$ concentric with sun is $4\pi R e^2 P_o$, $R e$ being the distance between the earth and sun. Equating these two solar powers, we obtain

$$4\pi R_s^2 \sigma T_s^4 = 4\pi R e^2 P_o$$

$$\therefore \frac{P_o}{\sigma} = T_s^4 \frac{R_s^2}{R e^2}$$

$$\frac{R e}{R s} = 215$$

Equating (15) and (16)

$$T_m = \frac{T_s}{14.65} \left( \eta \eta_r \eta_s r c \right)^{\frac{1}{2}}$$

$$= T_s \left( \eta \eta_r \eta_s r \right)^{\frac{1}{2}} \left( \sin\theta \right)^{\frac{1}{2}}$$

(17)
DESIGN

Data: - Manhattan

39.3° Latitude
96.6° Meridian

Average Temp 55°F Altitude 1010 ft.

Solar radiation incident upon horizontal surface (14),(6),(1)

March 350 Langleys/day or gm. cal./cm²/day
June 600 Langleys/day
September 400-450 Langleys/day
December 150 Langleys/day

\[ \eta_r = 0.85 \text{ for each reflecting surface} \]

\[ \eta_s = \text{is the fraction of the sun's rays which is available} \]
\[ \text{for concentration, accounting for losses due to shadowing} \]
\[ \text{effect of the target and furnace structure.} \]

\[ \eta_s = 0.9 \]

\[ \gamma = 0.6 \]

If we see survey of old furnaces in U.S.A. \( \gamma \)
ranges between 0.4 to 0.5 but due to recent development of manufacturing paraboloidal mirror we can get \( \gamma = 0.6 \)

\[ \eta_{a'} = \text{values given as above.} \]

**Parabolic Concentrator Design**

Specifications:

The solar furnace must be able to attain a thermal flux density of approximately 200 to 250 cal/sec/cm² over an area of uniform irradiance \( \frac{1}{4} \) in diameter. The solar furnace should consist of a fixed parabolic concentrator with vertical axis, heliostat mirror to track the sun, and a target tower.
Using (9b)

\[ \sin^2 \theta = \frac{p}{46.1 \times 10^3 \times (\eta_r)^3 \eta_s \eta_a \eta_o} \]

The individual reflectivity coefficient in this equation is \((\eta_r)^3\) to account for three reflecting surfaces

\[ \eta_a \eta_o = \frac{600}{8 \times 60} = 1.25 \text{ cal/min/cm}^2 \]

taking eight effective hours daily.

\[ \therefore \sin^2 \theta = \frac{225}{46.1 \times 10^3 \times 0.85^3 \times 0.9 \times 0.6 \times 1.25/60} = 0.706 \]

\[ \sin \theta = 0.84 \]

\[ \theta = 57^\circ 8' 30'' \]

\[ f = 107.3 \text{ d} \quad d = \frac{1}{4}'' \]

\[ = 107.3 \times 0.25 \]
\[ = 26.825'' \]

\[ D = f \times 4 \times \tan \frac{\theta}{2} \]
\[ = 26.825 \times 4 \times \tan \left(28^\circ 34' 15''\right) \]
\[ = 107.3 \times 0.5446 \]
\[ = 58.5'' \]

The objective is to design a furnace for research (High Temperature) work only. So it is better to use paraboloid mirror which is readily available. General Electric is supplying a paraboloid mirror of 5' diameter, or we can get 5' diameter paraboloid mirrors from used military searchlights. So it is suggested to use 60'' diameter mirror instead of 58.5''.
\[ D = f \times 4 \times \tan \frac{\theta}{2} \]
\[ 60 = 107.3 \times \tan \theta/2 \]

\[ \therefore \frac{\theta}{2} = 29^\circ 12' \]
\[ \theta = 58^\circ 24' \]

\[ \therefore \; r = 46.1 \times 10^3 \times (\eta_f)^3 \eta_g \eta_a p_0 \sin^2 \theta \]
\[ = 46.1 \times 10^3 \times 0.85^3 \times 0.9 \times 0.6 \times \frac{1.25 \sin^2 (58^\circ 24')}{50} \]
\[ = 231.9882 \; \text{cal/sec/cm}^2 \]

The surface area of concentrator is obtained by the following formula for the surface area of a parabolic surface

\[ \text{Area} = \frac{\pi}{2^2} \left[ \left(4f^2 + \frac{D^2}{4}\right)^{3/2} - 8f^3 \right] \]
\[ = \frac{\pi}{3 \times 26.825} \left[ \left(4 \times 26.8^2 + \frac{60^2}{4}\right)^{3/2} - 8 \times 26.8^3 \right] \]
\[ = 3060 \; \text{in}^2 \]
\[ = 21.3 \; \text{ft}^2 \]

This surface area can also be expressed in terms of the projected area of the concentrator as

\[ \text{Area} = \beta \pi \left( \frac{D}{2} \right) \]

where \( \beta \) is function only of \( D/f \) (or \( \theta \))

\[ = \frac{32}{3 (D/f)^2} \left\{ \left[ 1 + \left(\frac{D^2}{150} \right) \right]^{3/2} - 1 \right\} \]

or

\[ = \frac{2}{3} \cot^2 \frac{\theta}{2} \left( \sec^3 \frac{\theta}{2} - 1 \right) \]
substituting value of $\theta$

$$f_2 = \frac{2}{3} \cot^2 (29^\circ 12') \left[ \sec^2 (29^\circ 12') - 1 \right]$$

$$= 1.082$$

'.'. Area = $1.082 \times \frac{\pi}{4} \times 5^2$

$$= 21.3 \, \text{ft}^2$$

**Maximum Attainable Temperature**

$$T_m = T_s \left( \gamma_c \gamma_a \gamma_r \gamma_s \right)^{\frac{1}{6}} (\sin \theta)^{\frac{1}{3}} \quad (17)$$

$T_s$ - temp. of sun = 5800 K.

$$T_m = 5800 \left( 0.625 \times 0.85^3 \times 0.9 \times 0.6 \right)^{\frac{1}{6}} \times \sin (58^\circ \cdot 24')^{\frac{1}{3}}$$

$$= 5800 \times (0.207)^{\frac{1}{6}} (0.8517)^{\frac{1}{3}}$$

$$= 5800 \times 0.679 \cdot 0.9227$$

$$= 3615 \cdot 345 \, \text{K}$$

$$= 3342 \cdot 345 \, \text{C}$$

This temperature will attain in the target of the type as shown in Fig. 7.

(14)

**Heliostat Mirror Design**

Specifications:

Use two auxiliary mirrors. The size and tracking capabilities of the heliostat must be such as to keep the concentrator filled with solar radiation under the following conditions: (1) the heliostat must be able to track the sun during the day over an angle of $120^\circ$; (2) the elevation must be sufficient to cover seasonal variations in the sun's elevation; (3) the
Fig. 7
heliostat must be able to assume angles with the horizontal ranging from 50° to 80°. The heliostat reflecting surface should be constructed of a flat, back-silvered glass having minimum reflectivity coefficient of 0.85.

All structures erected between the heliostat and concentrator must offer minimum obstruction to the transmission of the sun's ray. The total allowable shadowing losses, including the loss due to nonreflecting surfaces, must be less than 10 per cent.

Procedure

An analysis is made of the size and shape required for heliostat mirror as a function of: latitude, hours of operation, angular diameter of the sun, the distance between paraboloid concentrator and heliostat, and the arrangement of the axes.

Elevation of the sun \( = 90 - \phi + \delta \)

where \( \phi = \) Latitude of site. \( 39.3° \)
\( \delta = \) declination of the sun. Maximum +23.5° in summer and minimum -23.5°

\( \therefore \) minimum elevation = 90 - 39.3 - 23.5 = 21.2°
maximum elevation = 90 - 39.3 + 23.5 = 74.2°

This elevation range is sufficient to cover seasonal variations in the sun's elevation.

Angle of the heliostat with horizontal \( = 90 - (90 - \phi + \delta)/2 \)

Minimum angle of the heliostat with horizontal = 90 - 74.2/2 = 52.9°
Maximum angle of the heliostat with horizontal = 90 - 21.2/2
\( = 79.4° \)
FIG. 3 DIAGRAM OF SOLAR FURNACE INSTALLATION
Fig. 9  FIXED MIRROR 7.07' x 5'

Fig. 10  DEPENDENCE OF OVER-ALL HEIGHT OF HELIOSTAT MIRROR ON THE LATITUDE (φ) AND MAXIMUM DECLINATION OF THE SUN (δ) APPROXIMATELY OVER-ALL HEIGHT $H = 2R \sec \left(90 - \phi + \delta\right)$.
FIG. 11 FOR 120° SUN MOVEMENT Heliocast turns through 60°

FIG. 12 Shape of conventional heliocast
— 6 hours
— 8 hours
— 10 hours
— 12 hours
Fixed mirror

\[ \text{Width} = 5' \]
\[ \text{Height} = (5/\cos 45^\circ) = 7.0' \]

Heliostat Mirror

\[ H = 2R \sec \left(90 - \frac{\theta + \delta}{2}\right) \quad \text{where} \quad 2R = 5' \]
\[ H = 5 \sec \left(90 - 39.3 + 23-5\right)/2.0 \]
\[ = 6.266' \]

To track the sun during the day over an angle of 120° the heliostat mirror must move 60°

\[ \therefore \text{width} = 5/\cos 30^\circ \]
\[ = 5.77' \]

Shape of conventional heliostat is shown in Fig. 12.

A Guidance System For Solar Furnace

There must be some system by which we can automatically track the sun about any two orthogonal axes. When the solar furnace is used for high temperature research, it is essential that the position of the hot zone on the sample be very stable. This can be achieved by keeping the heliostat mirror focused on the sun. Manual focusing results in jerky motion and is therefore unsatisfactory. Mechanical rotation has to be guided so as to follow precisely the apparent motion of the sun.

There are several methods to achieve this. Arc of an electric light uses principal that resistance changes with temperature. On our furnace this system cannot be used since it has a considerable time lag and low sensitivity. Another method uses single photosensitive detector in conjugation
with rotating shutter to generate an a-c error signal.

A method more nearly satisfying our requirements was developed by the Army Medical Corps. A photosensitive light receptor should be constructed, consisting of four gas phototubes mounted in quadrature at the end of a collimating tube (Fig. 13). The phototubes are optically isolated in individual chambers, and those in opposite quadrants are paired. One pair is aligned parallel to the horizontal axis of rotation of the mirror. The other pair automatically becomes parallel to vertical axis. The collimating tube is capped by aperture plate with round hole centered over the four phototubes. The diameter of this opening is selected to give a spot size just fitting inside one chamber. A diffusion disc is placed in front of the phototube mounting assembly to minimize the effect of variation on cathode surface sensitivity. A fitter is mounted in front of the aperture to control the intensity of light reaching the phototubes. The entire assembly is mounted (vertically at the periphery) of the paraboloidal mirror. When the mirror is focused on the sun, an image of the aperture falls on the center of the diffusing disc in such a way as to provide equal illumination to all four phototubes, and hence equal voltage output from the tubes. Any deviation from the focus moves the spot off center. This causes unequal amount of light to fall on the phototubes and results in unequal voltage outputs. The voltage outputs of the phototubes contain the vector magnitude information which can be used to determine both the direction and magnitude of the spot shift. This information is used in an error-signal amplifier to generate a voltage varying both in polarity and amplitude. The amplified error-signal voltage is added algebraically to control voltage which determines the basic drive current. The addition
Fig. 13 SENSING - DEVICE PHOTOTUBE ASSEMBLY

Fig. 14 BLOCK DIAGRAM OF DRIVING AND GUIDING SYSTEM
of the error-signal voltage corrects any inaccuracies in the tracking speed. If the phase of the error signal is selected so as to provide a faster drive when the sensing head is lagging behind and a slower drive when it is leading, then the system becomes self-balancing. This results in correct tracking. A block diagram of the entire driving and guiding system is shown in Fig. 14.

The electronic system for the azimuthal guiding is illustrated in Fig. 15. Two phototubes $V_1$ and $V_2$ are mounted parallel to the vertical axis of the parabola. Equal amounts of light falling of $V_1$ and $V_2$ produce equal grid voltages on the tubes $V_3$ and $V_4$. When the primary side of the transformer $T_2$ goes positive, $V_3$ and $V_4$ conduct according to their grid voltages, and therefore equal plate voltages will be obtained. The voltage across the two halves of the primary coil of the transformer $T_2$ are opposed to each other, and, since in this case they are numerically equal, the result is zero voltage on the secondary coil of $T_2$. If, however, unequal amounts of light fall on $V_1$ and $V_2$, the voltages across the two halves of the primary coil of $T_2$ are no longer equal. They are proportional to the amount of the grid bias developed by $V_1$ and $V_2$. A secondary voltage is then produced on $T_2$ which is proportional in amplitude to the difference between the plate voltages of $V_3$ and $V_4$. The polarity of this secondary voltage shifts depending on which of the two phototubes receives more light.

The basic drive current is supplied to the armature of the drive-motor a full wave rectifier circuit using two beam power amplifier tubes, $V_5$ and $V_6$. Grid bias is obtained for these tubes from the balance potentiometer which is adjusted to give a maximum voltage slightly beyond the cut-
off value for \( V_5 \) and \( V_6 \). The secondary voltage from \( T_2 \), i.e., the error signal, is superimposed on the grid bias of \( V_5 \) and \( V_6 \), thus increasing or decreasing the current to the armature of the drive-motor. Balancing control for the error signal amplifier tubes are not necessary. Any error signal that will be produced due to the different sensitivity of the phototubes is constant and can be compensated for by the proper setting of the rate potentiometer. On the other hand, the rate at which the system corrects itself may be affected by any unbalance in these tubes. Proper operation, however, has been achieved with phototubes mismatched by as much as 50 per cent.

The electronic system for the elevation guiding is basically the same. Some modification is necessary, however, caused by differences in the nature of rotation. Azimuth tracking is always in the same direction and at the same speed. Elevation tracking requires reversal of direction at the zenith position of the sun and continuous change in angular velocity. The reversal of direction is accomplished by the installation of two relays into the elevation-signal amplifier. The first relay reverses the direction of current from the elevation-control phototube pair. The second relay reverses the polarity of the motor-armature current. The switch of both relays is actuated by an arm mounted on the vertical shaft of the mirror. When the mirror reaches the 12 o'clock noon position, the arm throws the switch which in turn energizes the relays, thus reversing the elevational rotation of the mirror. The rotation of the vertical shaft is used also for the change in angular velocity of the elevational rotation. A center tapped rate potentiometer is mounted in the stationary base of the mirror. The rotation of the vertical shaft is transmitted by gears to the moving contact of the potentiometer.
As a vertical shaft rotates towards the 12 o'clock noon position, the rate potentiometer decreases the basic drive current of the elevation motor. At 12 o'clock noon (astronomical time) the angular velocity of the motor is zero. Accordingly the output of the rate potentiometer is also zero. After 11 o'clock the basic drive current increases, and since the polarity of the armature current was changed by activation of the relay, the mirror rotates towards the horizon. Superimposed on this changing basic drive current is the amplified error signal correcting any deviations from the true tracking speed as described in connection with the azimuthal system.

The constant field current to both motors is supplied by a selenium full wave rectifier. Photocells are placed in collimating tubes mounted vertically at the periphery of the parabolic mirror.

Advantages:

High sensitivity and complete absence of "hunting," that is, overcorrection. In addition it was found that the passing of a cloud in front of the sun does not upset the tracking. Since in this case no error signal is furnished by the photocubes, the driving motors receive only the basic drive current. Accordingly, the mirror will follow approximately the apparent rotation of the sun in spite of the screening effect of the cloud. When the cloud passes, the mirror may be somewhat of the true position due to the absence of correction. This deviation, however, will be immediately corrected when the phototubes receive direct sunlight and the precise tracking is resumed.
Principal advantages and disadvantages of a solar furnace in comparison to other means of obtaining very high temperatures may be summarized as follows: Heating samples by solar energy presents a means of carrying out experiments under very pure conditions in an atmosphere which remains oxidizing up to the highest temperatures. No interference occurs from electric or magnetic fields, from products of combustion or from heated furnace walls made from materials different from the sample being tested. Specimens may be heated or heat-treated in predetermined cycles. Physical changes occurring in a sample may be directly observed up to the highest temperatures while the material is being heated or cooled. On the other hand furnaces heated by gas, oil, coal, or electric energy can be heated for twenty four hours, while solar furnaces can only be used during daylight hours. Solar furnaces depend on favorable atmospheric conditions. The area of heating is limited in size depending on the focal length of the main mirror and the definition of the sun's image.

Hence advantages to be gained in high temperature research from the use of a solar furnace are: (1) an entirely "pure" and uncontaminated source of heat; (2) a heat source of high quantity and quality; (3) an accurately controllable heat flux, and (4) a comparative facility of manipulation of the work.
SUMMARY (OR THE RESULTS)

Vertical paraboloid solar furnace having two auxiliary mirrors having phototube cell control to heliostat.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture</td>
<td>60&quot;</td>
</tr>
<tr>
<td>Focal length</td>
<td>26.825&quot;</td>
</tr>
<tr>
<td>Focal length/aperture</td>
<td>0.448</td>
</tr>
<tr>
<td>Effective conc. surface</td>
<td>21.3 ft²</td>
</tr>
<tr>
<td>Theoretical image diameter</td>
<td>0.25&quot;</td>
</tr>
<tr>
<td>Rim angle</td>
<td>58° 24'</td>
</tr>
<tr>
<td>Theoretical Max. Temperature</td>
<td>3342 °C</td>
</tr>
<tr>
<td>Fixed mirror</td>
<td>5' x 7.07'</td>
</tr>
<tr>
<td>Heliostat mirror</td>
<td>5.77' x 6.27'</td>
</tr>
</tbody>
</table>

The cost of unit exclusive of research time has been approximately estimated as $5,500, including $3,000 for materials and $2,500 for labor.
ACKNOWLEDGMENT

The author would like to express his deep sense of gratitude to Dr. Hugh Walker who has given patient guidance and encouragement throughout the work on this report. The author would also like to thank Dr. R. G. Nevins for kind help and supplying material for this report.
REFERENCES


APPENDIX A

A SURVEY OF SOLAR FURNACE INSTALLATIONS IN

THE UNITED STATES

(1) A. D. Little, Inc., Cambridge, Mass.
(2) Arizona State College, Tempe, Ariz.
(5) California Institute of Technology, Pasadena, Calif.
(6) Convair, Fort Worth, Tex.
(7) Convair, San Diego, Calif.
(8) E. J. dePont de Nemours & Co., Wilmington, Del.
(9) Fordham University, New York, N.Y.
(11) General Electric Co., Cleveland, Ohio
(12) Kennecott Copper Corp., Salt Lake City, Utah
(14) National Bureau of Standards, Washington, D.C.
(15) Quartermaster Research and Development Command, Natick, Mass.
(16) Stanford Research Institute, Menlo Park, Calif.
(17) Sandia Corp., Albuquerque, N.M.
(18) University of Minnesota, Minneapolis, Minn.
(19) University of Utah, Salt Lake City, Utah
(20) W.M.C. Precision Works, Kansas City, Mo.

APPENDIX B.

<table>
<thead>
<tr>
<th>Furnace</th>
<th>Diameter (in)</th>
<th>Reflecting Material</th>
<th>( \Phi ) (cfs/ft²)</th>
<th>Temp. (°F)</th>
<th>Rand (cfs/sec)</th>
<th>( \gamma )</th>
<th>( \gamma' )</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muntz 17</td>
<td>35</td>
<td>Glass</td>
<td>41</td>
<td>3270</td>
<td>...</td>
<td>0.85</td>
<td>(0.82)*</td>
<td>24.0</td>
<td>41.5</td>
<td>30.6</td>
<td>22.5</td>
<td>9</td>
</tr>
<tr>
<td>Algiers</td>
<td>27.6</td>
<td>Aluminum</td>
<td>36.5</td>
<td>3270</td>
<td>...</td>
<td>0.65</td>
<td>0.60</td>
<td>30.3</td>
<td>43.2</td>
<td>36.7</td>
<td>26.0</td>
<td>10</td>
</tr>
<tr>
<td>Aluminium</td>
<td>5</td>
<td>Glass</td>
<td>60</td>
<td>3670</td>
<td>...</td>
<td>0.76</td>
<td>0.50</td>
<td>21.5</td>
<td>35.7</td>
<td>27.6</td>
<td>42.1</td>
<td>11</td>
</tr>
<tr>
<td>Japan</td>
<td>6.5</td>
<td>Aluminum</td>
<td>75</td>
<td>2370</td>
<td>...</td>
<td>0.76</td>
<td>0.80</td>
<td>5.3</td>
<td>5.4</td>
<td>4.0</td>
<td>6.6</td>
<td>12</td>
</tr>
<tr>
<td>Convex</td>
<td>5</td>
<td>Glass</td>
<td>60</td>
<td>...</td>
<td>665*</td>
<td>1.0</td>
<td>0.80</td>
<td>57.6</td>
<td>71.5</td>
<td>...</td>
<td>...</td>
<td>13</td>
</tr>
<tr>
<td>Convex</td>
<td>10</td>
<td>Aluminum</td>
<td>85</td>
<td>...</td>
<td>96*</td>
<td>1.0</td>
<td>0.60</td>
<td>13.6</td>
<td>13.5</td>
<td>...</td>
<td>...</td>
<td>13</td>
</tr>
<tr>
<td>Nat. Inst. S.</td>
<td>5</td>
<td>Rodium</td>
<td>60</td>
<td>3570</td>
<td>...</td>
<td>0.76</td>
<td>(0.69)*</td>
<td>23.5</td>
<td>42.6</td>
<td>29.6</td>
<td>33.5</td>
<td>14</td>
</tr>
</tbody>
</table>

* Value of \( \gamma' \) corrected from value given in reference indicated because flux meter used to obtain data had an opening larger than the sun image. The origin to \( \gamma' \) has not been corrected to \( \gamma' \) of unity.
Appendix C

C C TO FIND FLUX DENSITY AND TEMPERATURE AT VARIOUS SEASONS (MONTHS),
READ, CN, ES, GYA, THET
THET=THET*.01745
DO 2 =1,4
READ, P1
P2=P1*.85.
FA=P2/2.
TM=5600.**((TE*ER*ER*ES*GYA)**C.25)*((SIN(THET))**0.5)
P=(.81*V.850)*((ER***3.)*ES*GYA*P2*((SIN(THET))**2.)
PUNCH 1, P1, P, TM
1 FORMAT (F3E.1, F12.3, F12.3)
2 CONTINUE
END

0.85 0.90 6.60 58.6
1.00
3.25
6.05
C C TO FIND FLUX DENSITY AND TEMPERATURE AT VARIOUS SEASONS (MONTHS),
MARCH 350.0 159.326 3159.677
JUNE 60.0 231.963 3615.343
SEPTMBER 425.0 164.329 3316.722
DECEMBER 150.0 57.997 2958.433
SOLAR FURNACE DESIGN

by

MAHENDRAKUMAR KESHAVLAL PUNATAR

B. E. (Mechanical) University of Poona, Poona, India 1965

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1967
The theoretical and practical factors which affect the specific performance characteristics and costs of solar furnaces are reviewed in detail. Technical and economic considerations indicate that the best design for a solar furnace should incorporate the following features:

1. A parabolic concentrator and black body cavity target.
2. A stationary concentrator with its axis vertical, using two auxiliary mirrors, one fixed and another movable mirror to follow the sun.
3. An angle between the image area and the rim of the concentrator of between 50° to 70°.
4. A reflecting surface consisting of back-silvered glass.

The actual heat flux available at the target is expressed as

\[ p = 46.1 \times 10^3 \ E \ p_o \sin^2 \theta. \]

where \( E \) is the overall efficiency factor of the solar furnace, \( p_o \) is the solar constant, and \( \theta \) is the rim angle. \( E \) is defined as

\[ E = \eta_a \eta_r \eta_s \gamma, \]

where \( \eta_a \) accounts for loss of radiant energy due to the atmosphere, \( \eta_r \) is the combined reflectivity coefficient of the reflecting surfaces of the concentrator and auxiliary mirrors, \( \eta_s \) accounts for the nonreflecting portion of the concentrator because of shadowing and open areas between mirror segments, and \( \gamma \) is an index which represents the degree of geometrical perfection of the parabolic concentrator.

Maximum attainable temperature is expressed as

\[ T_m = T_s \ E \frac{1}{\gamma} \cdot \sin^{\frac{1}{2}} \]

where \( T_s \) is the temperature of the sun.

The design study, and sample design for 5' parabolic concentrator is
given. A tabulation of calculated E and r values, and survey of number of actual solar furnaces is given in Appendixes A and B.