THE TORSION PROBLEM

by

SIRJANG L. TANDON

B. S., Howard University, Washington, D.C.

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F. C. Appl
Major Professor

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NOMENCLATURE

Symbols		Mechanical
x, yz	=	rectangular co-ordinates
K	=	a constant
G	=	shearing modulus of elasticity
M	=	torque
P	=	differential pressure
S	=	slope
T	=	tension
V	=	volume
τ	=	shearing stress
ϕ	=	stress function
Θ	=	angle of twist of the shaft per unit length
A	=	cross section area
J	=	polar second movement of circular section
r	=	radius of circle
Δ	=	$(\partial^2/\partial x^2) + (\partial^2/\partial y^2)$
		Electrical
C	=	capacity per square inch across insulation
E	=	instantaneous potential applied
E_m	=	maximum potential applied
E'	=	rms value of potential applied
i_x, i_y	=	current in paper in x and y directions per inch of width
n	=	frequency of a-c voltage

q	=	charge per unit area of paper
R	=	resistance across semiconducting layer, per square inch
t	=	time
ψ	=	potential at point in paper
ρ	=	resistance of paper per square

INTRODUCTION

The solution for the torsion problem for a circular section is very simple and is well known. The maximum shear stress τ_{cm} can be calculated by the simple formula [3]*

$$\tau_{cm} = \frac{2M}{\pi r^3} \quad .$$

and the torsional rigidity (M/θ) can be calculated by

$$\left(\frac{M}{\theta}\right)_c = \frac{MGr^4}{2} \quad .$$

The torsion analysis of bars of other cross sections is mathematically more difficult. Cross sections other than the circular have been solved by the well known "soap film" [3] and plastic film [4] which are based on the membrane analogy. This is an analogy which gives a valuable means to visualize the torsion properties of different cross sections.

The method described here was first suggested theoretically by Edamston [5] and then by Swannell [1]. This is an electrical analogy for the solution of Poission's equation which is both quick, cheap and easy to carry out experimentally.

THEORY OF TORSION

Saint-Venant in 1855 formulated the general torsion problem for a bar or a shaft of uniform cross-section [2]. He developed the following differential equation known as the equation of compatibility:

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* Refer to bibliography.

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = -2G\theta \qquad (1)$$

The function φ is a stress function. If φ satisfies the above differential equation along with the appropriate boundary conditions, then the shear stresses are given by the following equations.

$$\tau_{yz} = + \frac{\partial\Phi}{\partial x} \qquad (2)$$

$$\tau_{xz} = - \frac{\partial\Phi}{\partial y} \qquad (3)$$

Where τ_{yz} is the shearing stress in y-z plane and τ_{xz} is the shearing stress in xz-plane.

The stress function φ is constant on the boundary for zero surface stresses. Since this constant does not affect the stresses, it is set equal to zero. With zero φ on the boundary, it is found that the torque on the section is given by

$$M = 2\iint_A \varphi \, dA \qquad (4)$$

where M = torque applied to bar

A = cross section area of bar.

Equation (1) may be written as

$$\Delta^2\varphi = -2G\theta \qquad (5)$$

where $$\Delta^2\varphi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}$$

The right hand side of equation (5) is a constant. Once the integral values of φ are known, a surface "torsion hill" may be visualized in which Φ is the height of the torsion hill. Fig. 1 shows typical hills for simple sections. From these mathematical abstractions the torsion properties of a section can be derived. The necessary torque is twice the volume of the hill above the base. The shear due to this torque at any point in the cross section is equal to the maximum slope of the hill at that point and acts in the direction parallel to the contour line at that point. If only one value of $-G\theta$ is found by equation (5), all other cases may be obtained by simple proportions.

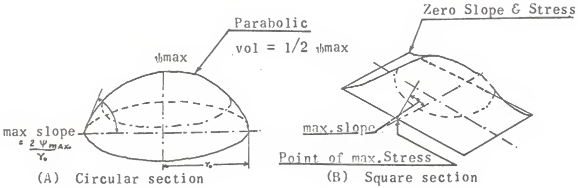

Fig. 1

PRANDTL'S MEMBRANE ANALOGY

Prandtl noted in 1903 that the equation for the deflection of a membrane subjected to a uniform pressure differential was of the same mathematical form as that for the stress function Φ, [4]. This makes it possible to solve torsion problems approximately, using the membrane analogy.

The deflection equation for a membrane is as follows:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = -\frac{P}{T} \qquad (6)$$

where z = deflection of the membrane above x-y plane

P = differential pressure on the membrane

T = tension in the membrane.

The above equation is obtained by considering the static deflection of a membrane due to a uniform differential pressure P. The deflection of the membrane is considered to be zero on the boundary of the region.

If a membrane of the same geometrical shape as the bar's cross-section is deflected so that $P/T \quad 2G\theta$ and the deflection at the boundary is zero then $z \quad \Phi$. The shearing stresses τ_{yz} and τ_{xy} may be found by substituting z for Φ in equations (2) and (3)

$$\tau_{yz} = + \frac{\partial z}{\partial x}$$
$$\tau_{xy} = - \frac{\partial z}{\partial y} \qquad (7)$$

The shear stress in any other direction is proportional to the slope of the membrane in a direction normal to the direction of the shear stress [2].* It is therefore seen that a membrane deflected through an opening in a flat plate by a differential pressure provides a convenient analogy of torsion.

When using the membrane analogy, it is inconvenient to determine P/T. This can be avoided, however, by having a circular opening in the plate together with the opening representing the cross section for which the stresses are to be determined. Since the same membrane is used for both openings, the same P/T value applies to both and the following equation 4 .

$$\frac{M}{\tau_c} = K \frac{V_c}{S_c} \qquad (8)$$

$$\frac{M}{\tau} = K \frac{V}{S} \qquad (9)$$

and

$$\frac{V_c}{V} = \frac{(M/\theta)_c}{(M/\theta)} \qquad (10)$$

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* Page 7 - 8

where M = torque applied to the base

τ = shear stress at any point

K = constant

V = volume enclosed by the membrane in the x-y plane

S = slope of the membrane at any point in the direction normal to the shear stress at that point

θ = angle twist of the shaft per unit length

M/θ = torsional rigidity

subscript c indicates values for the circular section.

The solution of the circular shaft is well known. The maximum shear stress τ_{cm} occurs at the boundary and is given by

$$\tau_{cm} = \frac{2M}{\pi R^3} \tag{11}$$

where R = outside radius of circular cross section

and torsional rigidity $(\frac{M}{\theta})_c$ = is found to be

$$(\frac{M}{\theta})_c = \frac{M\ G\ R^4}{2} \tag{12}$$

These equations for the circular section taken as reference, make it possible to determine the constant K. The maximum slope (at boundary) of the circular membrane and the volume under the membrane V_c are measured and substituted in equation (8). The results combined with equation (10) yield:

$$K = \frac{\pi R^3}{2} \quad \frac{S_{cm}}{V_c} \tag{13}$$

After measuring the membrane slope at any point of interest and the volume under the "hill" for any other section, the shear stress τ is obtained by substituting (13) into (9).

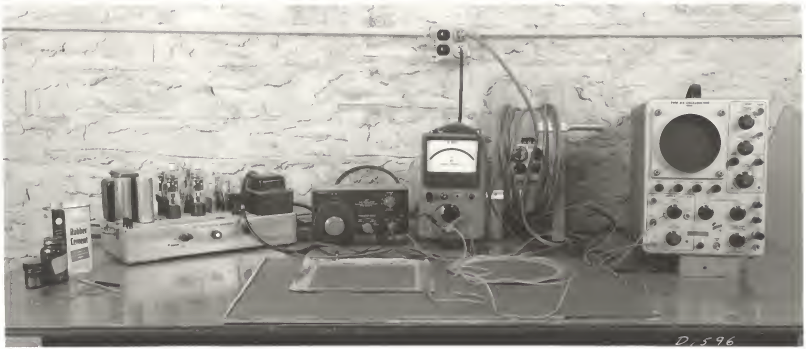


Fig. 2. A Typical Set Up

Thus $$\tau = \frac{1}{c}\left(\frac{S}{V}\right) M = \frac{2M}{\pi R^3}\left(\frac{V_c}{S_{cm}}\right)\left(\frac{S}{V}\right) \tag{14}$$

The torsional rigidity of the known section is obtained by substituting (12) into (10)

$$\left(\frac{M}{\theta}\right) = \frac{\pi G R^4}{2}\left(\frac{V}{V_c}\right) \tag{15}$$

ELECTRICAL ANALOGIES

D. C. Analogy

For simplicity an analogy based on d. c. potential will first be considered (Fig. 3). In this method, the layer between the resistance paper and the metal base is a "Semiconducting" layer which allows d. c. current to leak away from the paper into the metal base. This method is of limited practical use since such a semiconducting layer is very difficult to get and is very expensive.

The resistance properties of the paper are assumed uniform in all directions in its plane, and is taken as ρ ohms/square. See Fig. 4-A.

We find the current flowing in a given direction in terms of C and the potential gradient in that direction by considering a strip of a unit height and width (Fig. 4-C). The potential increase between AB and CD is $\partial\psi/\partial x \cdot \delta x$ and resistance $\rho\ \delta x$. Thus

$$i_x = \frac{\text{Potential drop}}{\text{resistance}} = \frac{-\partial\psi}{\partial x} \cdot \frac{\delta x}{\rho\delta x} = \frac{-\partial\psi}{\partial x}/\rho$$

Now a unit square (Fig. 5-D) is considered and net current flowing into the square across the four edges. Let the current across PQ be i_x, then the current flowing across RS is $i_x + \partial i_x/\partial x$. Hence the net inflow in the x-direction is $\partial i_x/\partial x$ which is equal to $(\partial^2\psi/\partial\psi^2)/\rho$ from the above.

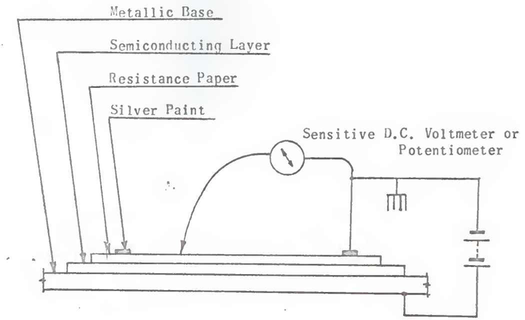


Fig. 3— D-C form of apparatus

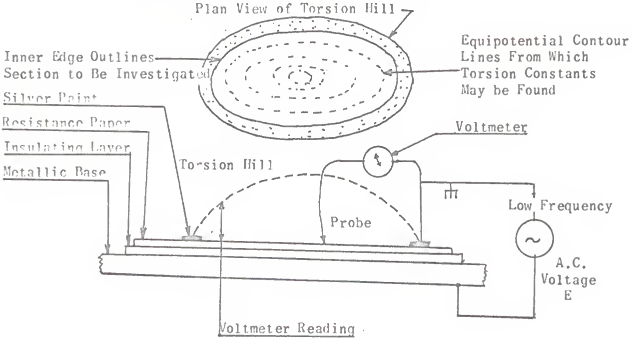
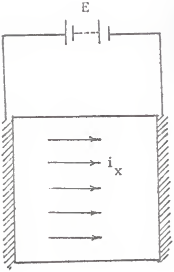
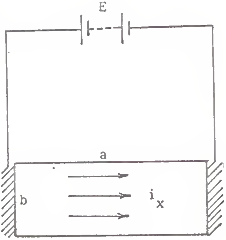


Fig. 2 — Layout of apparatus



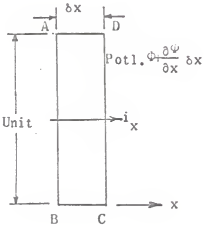
A. Resistance of Square

$i_x = \frac{E}{\rho}$

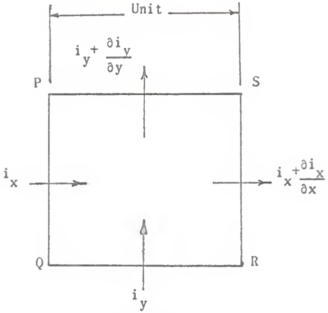


B. Resistance of Rectangle

$i_x = \frac{E}{\rho}\frac{b}{a}$



C. Current Flow Across Thin Strip



D. Current Flow Into Square

Fig. 4

Similarly the net inflow in the y direction is $(\partial^2\psi/\partial y^2)/\rho$ and the total inflow is obtained by adding the above two

$$\frac{1}{\rho}\left[\,\partial^2\psi/\partial x^2 + \partial^2\psi/\partial^2 y\,\right] \tag{16}$$

this can be written as $\Delta^2\psi/\rho$ where Δ is the operator "del." Equation (6) can be derived more elegantly by vector notation as follows.

Let a rectangular vol element dv with dimensions d x, d y and dz be located at a point where co-ordinates are x, y, z. Consider the two faces of area dxdy. The current entering the lower face. Vertical section i_z dxdy where i_z is the current density in z direction.

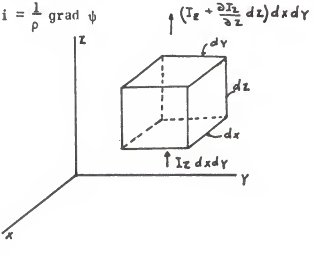
Current leaving the upper face is

$$(I_z + \frac{\partial I_z}{\partial Z}\, dZ)\ dxdy$$

Net current flow out of the given element through the faces parallel to x-y plane is

$$\frac{(I_z + \frac{\partial I_z}{\partial Z}\, dZ\ )\ dxdy - I_z\ dxdy}{dxdydZ} = \frac{\partial I_z}{\partial Z} \text{ in direction of Z}$$

We know by Flick's law that $i = \frac{1}{\rho}$ grad ψ



therefore

$$i_x = \frac{1}{\rho}\ \frac{\partial\psi}{\partial X}\ ,$$

$$i_z = \frac{1}{\rho}\ \frac{\partial\psi}{\partial Z}\ ,$$

and

$$i_y = \frac{\partial\psi}{\partial Y}$$

therefore

$$\frac{\partial I_z}{\partial Z} = \frac{1}{\rho}\ \frac{\partial^2\psi}{\partial Z^2}$$

Similarly from the y-z and x-z planes are

$$\frac{1}{\rho}\ \frac{\partial^2\psi}{\partial x^2} \text{ and } \frac{1}{\rho}\ \frac{\partial^2\psi}{\partial y^2}$$

Hence the net current out of a vol dv

$$= + \frac{1}{\rho}\left[\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \right]$$

$$= + \frac{1}{\rho} \; \Delta^2\psi$$

If there is no leakage from the top or the bottom of the paper then $\Delta^2\psi = 0$. This is Laplace's equation. Thus the measurement of potentials gives a solution of this equation.

On the other hand if there is a semiconducting layer between the paper and a base of uniform Potential E, then there is a Potential Drop of $(E - \psi)$ at any given point causing current leak across the layer. For the method to be accurate E must be high compared to ψ so that the leakage P.D. may be taken as uniform (and equal to E) all over the sheet.

If the resistance across unit area of the semiconducting sheet is R, the current leakage into the paper is E/R per unit area and thus

$$\frac{\Delta^2\psi}{\rho} + \frac{E}{R} = 0 \qquad (17)$$

A. C. Analogy

Now we can turn back to the actual method, where the a.c. potential is applied and the conducting layer is replaced by an insulator. The quantities i and ψ are functions of time and there will be a leakage current across the insulating layer, which will act as the dielectric of a capacitor formed by the resistance paper and the metal base. If q is the charge per unit area of this plate capacitor, then $q = C\,E$ and the current per unit area following into the paper from the base is

$$i = dq/dt = \frac{c\,dE}{dt} \qquad (18)$$

Here again the variations in the resistance paper potential are assumed to be small compared to E.

If the applied potential frequency is n we have

$$E = E_m \sin 2\pi\, nt$$

differentiating with respect to time

$$\frac{dE}{dt} = 2\pi n E_m \cos 2\pi n\, t$$

But $i = c\,\frac{dE}{dt}$

$$i = C 2\pi n E_m \cos 2\pi nt$$

and $\frac{E}{R} = C\, 2\pi n E_m \cos 2\pi nt$

Putting this value in equation (17) we get

$$\frac{\Delta^2\psi}{\rho} + C\, 2\pi n\, E_m \cos.\ 2\pi nt = 0$$

or

$$\Delta^2\psi + \rho\, C\, 2\pi n\, E_m \cos 2\pi nt = 0$$

$$\Delta^2\psi = -\rho\, C\, 2\pi n\, E_m \cos 2\pi nt \qquad (19)$$

The right hand side of equation (19) is uniform over the sheet but is a function of time [1]. Hence the value of ψ is also a function of time i.e. are sinusoidal but out of phase with the applied potential E. A voltmeter reading rms values therefore gives a solution

$$\Delta^2\psi = -2\pi\, n\rho\, CE' \qquad (20)$$

where E' is the rms voltage.

We have already shown that

$$\Delta^2\Phi = -2\ G\theta \quad \text{from equation (5)}$$

if $2G\theta = 2\pi n\, \rho\, C\, E'$ then there is an exact analogy between Φ and ψ [1].

All the values of the electrical constants are not required if a parallel experiment with a circular section is carried out on the same sheet using the same frequency.and potential. In this case the values of ψ found correspond to the same angle of twist for both sections, and the test section may be compared to the circular.

DESCRIPTION OF METHOD

Fig. 2 is a photograph of a typical lay out shown diagrammatically in Fig. 3. There are three layers, the bottom being metallic and of high electrical conductivity, the middle an insulating layer and top layer of resistance paper. The outline of the shape to be investigated is painted on the resistance paper with silver paint to form a conducting boundary of neglegible resistance. An A.C. voltage is applied between the bottom layer and the silver boundary and the potentials in the resistance paper are found at different points by means of probe and a sensitive vacuum tube voltmeter. These potentials contain all the information for the solution of the torsion problem if the electrical constants are known. Since these constants change with different set ups it is necessary to simultaneously determine measurements for a circular section, on the same sheet. The torsion properties of circular sections are known, so the electrical constants can be cancelled from the equations.

For accuracy the variation in potential in the paper should be small compared to the over all voltage. In other words the impedance of the capacitor formed by the resistance paper and the bottom sheet should be compared to the resistance paper.

RESULTS AND CALCULATIONS

Circular Section

Table 1 shows all the potential readings taken along the different sections. Fig. 6 shows the experimental points for the average of all points equidistante from the boundary. Table 1 shows that the experimental points lie very close to the theoretical curve, which can easily be shown to be a parabola. If ψ_m is the central value and R is the radius of the circle, then assuming the simple geometrical properties of a parabola

$$V_c = \frac{1}{2} \pi \gamma^2 \; x \psi_{max}{*}$$

$$= \frac{1}{2} \times \pi \times (4 \text{ in})^2 \times 7.2 \text{ (Unit)}^3$$

$$= 181 \text{ (Units)}^3$$

Maximum Shear Stress $\tau_{max} \equiv$ slope at the edge $= \dfrac{2\psi_{max}}{R} = \dfrac{2 \times 7.2}{4} = 3.6$

Square Section

Visualization of the shape of a soap film covering a square hole** and subjected to a slight pressure on one side shows that the maximum slope occurs at the center of each edge; readings were taken along the center line inter-secting the middle of an edge. Fig. 3 shows all the experimental points taken at the center line. In Table 2 all the points taken at different parts of the section

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* Taken from Fig. 6.

** See Fig. 1.

The value of the torsion "hill" was found* by dividing the whole section into small squares (.3"x.3") and taking the potential reading at the center of the small squares. Readings were required to take only 1/4 of the section. The slope was determined by the graph (Fig. 7).

Dimensionless expressions relating maximum stress, and torque may be determined using equation (14). Analytical results are available for these expressions, and are given in Table 3.

$$\frac{\tau_m D^3}{M} = \frac{2D^3 V_c}{R^3 S_{cm}} \frac{S_m}{V}$$

D = Length of the side of square

$$\frac{\tau_m D^3}{M} = \frac{2 \times (8.4)^3 \cdot 1 \times 4.6 \times 181}{(4)^3 \text{ (326 units) } 3.60}$$

$$= 4.18$$

Dimensionless expression containing torsional rigidity (M/θ) may be determined using equation (15)

$$\frac{M}{\theta G D^4} = \frac{\pi r^4 V}{2 D^4 V_c}$$

$$= \frac{\pi \times (4)^4 \times 326}{2 \times (814)^4 \times 181} = .144 \text{ dimensionless}$$

* See Table 2.

DISCUSSION AND CONCLUSIONS

The resistance paper can be obtained in rolls, and is commercially known as teledeltos paper.

The silver paint should be painted thickly. In order to get a smooth inner edge a crow pen was used for painting the silver paint. A thin aluminum foil was embedded in the paint (see photograph Fig. 2).

The base sheet used was one-quarter inch aluminum, 20 by 30 inches. Aluminum was used because it was easy to get and was very cheap. Considerable difficulty was experienced to get a good sandwich. First, domestic contact paper was used as an insulating layer but it was very difficult to obtain a flat surface. So in the final layout a thin cardboard was used. This was stuck to the plate and the resistance paper with rubber cement. Considerable care is needed to get a satisfactory assembly, as the whole experiment depends on the uniformity of the thin insulating layer.

The A.C. voltage and frequency may be varied within wide limits and it is not required to know them as long as they are constant. The values used were about 300 to 250 cps, provided by the oscillator and the voltages in the paper were found to be of the order of 1/100 of the supply voltage. The whole analogy is based upon the assumption that the variation in the potential in the paper be small compared to the applied voltage. The above values were more than ample to ensure the theoretical accuracy of the analogy. The silver paint was earthed and the potential readings were taken on a tube voltmeter. It was found necessary to use shielded wire to avoid stray voltage. A regular probe was used. Light contact only with the paper was found necessary.

Accuracy within a few percent was obtained. It may again be emphasized that the accuracy of both d.c. and a.c. methods depends on the potential variation across the plate being small compared to overall voltage.

ACKNOWLEDGMENT

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APPENDIX I

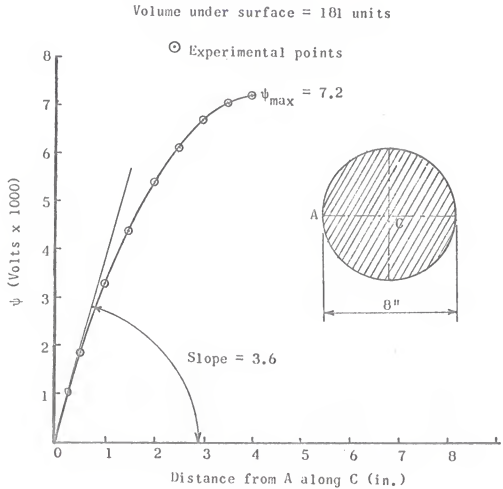


Fig. 6. Results for circular section.

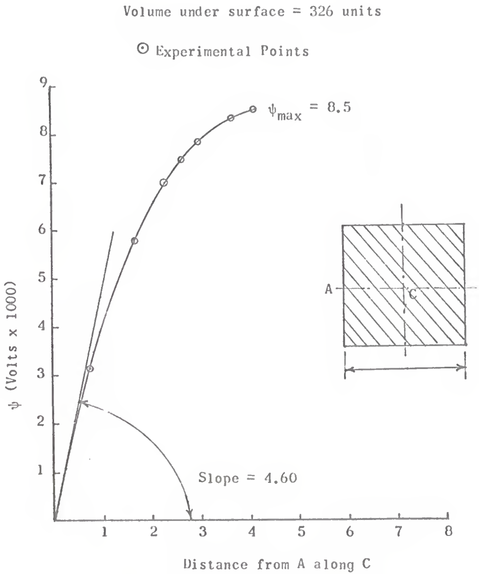


Fig. 7. Results for square section.

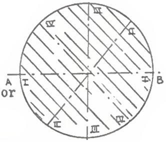
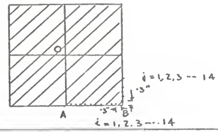


Table 1. ψ (volts x 1000) vs distance (in) for circular section.

Distance A-B (IN)	Experimental points				Average	Theoretical points
	I	II	III	IV		
0.00	0.00	0.00	0.00	0.00	0.00	---
0.25	1.05	1.05	1.05	1.05	1.05	---
0.50	1.90	1.90	1.90	1.95	1.91	1.81
1.00	3.25	3.25	3.25	3.30	3.26	3.2
1.50	4.35	4.40	4.35	4.40	4.375	4.38
2.00	5.25	5.40	5.25	5.40	5.37	5.40
2.50	6.00	6.10	6.10	6.10	6.10	6.10
3.00	6.60	6.60	6.60	6.70	6.60	6.60
3.50	7.00	7.00	7.1	7.1	7.05	7.04
4.00	7.20	7.20	7.2	7.2	7.20	7.20
4.50	7.00	7.00	7.1	7.0	7.05	7.04
5.00	6.662	6.62	6.664	6.6	6.66	6.60
5.50	6.11	6.11	6.10	6.00	6.10	6.10
6.00	5.45	5.35	5.40	5.35	5.38	5.40
6.50	4.50	4.50	4.5	4.35	4.50	4.38
7.00	3.25	3.25	3.25	3.225	3.25	3.20
7.50	1.95	2.00	2.00	1.95	1.98	1.81
7.75	1.06	1.06	1.06	1.06	1.06	---
8.00	0.00	0.00	0.00	0.00	0.00	---

Table 2. Volume under "hill" for square section. Readings of ψ at the center of (.3x.3) squares in order to calculate the value under hill.



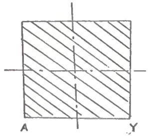
ψ values from A along B	ψ values from A along O: 1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1.35	2.40	3.6	4.6	5.25	6.18	6.8	7.38	7.8	8.2	8.5	8.8	8.75	8.8
2	1.31	2.40	3.46	4.40	5.20	6.18	6.6	7.28	7.65	8.2	8.4	8.7	8.7	8.75
3	1.28	2.45	3.40	4.4	5.20	5.6	6.6	7.18	7.6	8.0	8.3	8.6	8.6	8.6
4	1.35	2.42	3.40	4.3	5.18	5.78	6.4	6.98	7.4	8.0	8.18	8.38	8.4	8.5
5	1.25	2.30	3.32	4.20	4.8	5.58	6.2	6.0	7.2	7.6	7.95	8.0	8.18	8.15
6	1.36	2.28	3.32	4.0	4.7	5.2	5.62	6.14	6.8	7.25	7.5	7.7	7.82	7.8
7	1.25	2.18	3.18	3.80	4.45	5.0	5.70	6.1	6.4	6.8	7.2	7.2	7.38	7.38
8	1.25	2.10	3.1	3.60	4.25	4.70	5.3	5.25	6.0	6.9	6.6	6.95	6.8	6.8
9	1.25	2.05	2.8	3.40	3.85	4.40	4.9	5.21	5.42	5.78	6.0	6.0	6.2	6.18
10	1.35	1.91	2.58	3.20	3.60	3.80	4.38	4.70	4.8	5.18	5.3	5.2	5.3	5.21
11	1.38	1.88	2.2	3.0	3.18	3.40	3.85	4.0	4.2	4.30	4.5	4.6	4.72	4.6
12	1.40	1.75	2.00	2.35	2.20	2.85	3.15	3.3	3.4	3.48	3.6	3.6	3.72	3.70
13	1.38	1.54	1.70	1.60	1.95	2.20	2.40	2.40	2.4	2.5	2.6	2.6	2.65	2.60
14	1.25	1.25	1.28	1.28	1.25	1.35	1.38	1.38	1.35	1.35	1.3	1.35	1.45	1.35

Σ_{ij} = 1.35 + 1.31 + ... 2.60 + 1.35 = 908.01

Volume of "Hill" for $\frac{1}{4}$ of section = .30x .30x Σ_{ij} = .30 x .30 x 908.01

Volume of Hill for whole square section = 4 x .3 x .3 x 908.01 = 326.88 units

Table 3. Square Section ψ (volts x 100) Vs distance Along Y



	0.0	.70	1.70	2.70	3.70	Midway: 4.20	4.70	5.70	6.70	7.70	8.40
0.0	0.00										
.70		1.3	2.3	2.8	3.1	3.25	3.0	2.7	2.3	1.2	
1.70		2.15	4.2	5.2	5.8	5.8	5.75	5.2	3.75	2.0	
2.70		2.7	5.2	6.8	7.5	7.6	7.5	6.6	5.1	2.68	
3.70		8.9	5.75	7.6	8.3	8.4	8.3	7.4	5.6	2.9	
4.20		2.95	5.85	7.62	8.5	8.6	8.45	7.6	5.78	2.75	
4.70		2.9	5.75	7.58	8.35	8.4	8.35	7.4	5.70	2.9	
5.70		2.72	5.2	6.8	7.6	7.7	7.6	6.8	5.1	2.7	
6.70		2.3	4.25	5.35	5.95	6.0	5.98	5.4	4.2	3.0	
7.70		1.5	2.35	2.85	3.1	3.2	3.1	2.8	3.2	1.3	
8.40											

Table 4(a). Experimental and theoretical values for maximum shear stress.

	S_m	V	Experimental $\tau_m\ D^3/M$	Theoretical $\tau_m\ D^3/M$	% Error
Circle	3.6	181			
Square	4.6	326	4.18	4.80	12.8%

Table 4(b). Experimental and Theoretical values for torsional rigidity.

	S_m	V units	Experimental $M/\theta\ GD^4$	Theoretical $M/\theta\ GD^4$	% Error
Circle	3.6	181			
Square	4.6	326	1.44	1.41	+2.13

APPENDIX II

Extension to Hollow Sections

The electrical analogy method can easily be extended to sections containing one or more holes, by painting silver paint over the area occupied by the holes, thus forming areas of constant potential. The potentials of these areas automatically assume the correct value required by the analogy.

The current from any one of these areas flowing into the base sheet is $AC.\,2\pi\ n\ E_m \cos 2\pi\ nt$ where A is the area of the "hole."

The current in the paper flowing through the boundary is

$$\int i\,ds = \frac{\frac{\partial\psi}{\partial x}\,ds}{\rho}$$

where ds and n refer to directions along the normal to the boundary.

$$\int\frac{\frac{\partial\psi}{\partial x}\,ds}{\rho} + AC.\,2\pi\ n\ E_m \cos 2\pi\ nt = 0$$

$$\int\frac{\partial\psi}{\partial x}\,ds = -AC.\,2\pi\ \rho\ E' \text{ in rms values}$$

The required boundary value of ψ is that which satisfies the equation

$$\int\frac{\partial\psi}{\partial x}\,ds = -A\ 2G\theta$$

There is a direct analogy between x and p

$$-A2G\theta = -AC.\,2\pi\ n\ \rho\ E' \text{ rms value}$$

Hence the value of ψ at the boundary is automatically correct.

A TORSION PROBLEM

by

SIRJANG L. TANDON

B. S., Howard University, Washington, D. C.

AN ABSTRACT OF A MASTER'S REPORT

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This is an electrical analogy for the solution of torsion problems using a.c. current which is cheap, very quick and easy to operate experimentally. Accuracy within a few percent can be obtained. Values for a square section are calculated taking circular cross section as a reference and are compared to the theoretical values and the percentage error is determined. This method has advantages over the soap film method because the electrical quantities are much easier to read. There is a similar analogy using d.c. current but it is not as practical because semiconducting layer in sheet form is difficult to get and is very expensive.