COORDINATED ELECTRIC VEHICLE CHARGING WITH RENEWABLE ENERGY SOURCES

by

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Abstract

Electric vehicles (EVs) are becoming increasingly popular because of their low operating costs and environmentally friendly operation. However, the anticipated increase of EV usage and increased use of renewable energy sources and smart storage devices for EV charging presents opportunities as well as challenges. Time-varying electricity pricing and day-ahead power commitment adds another dimension to this problem. This thesis, describes development of coordinated EV charging strategies for renewable energy-powered charging stations at homes and parking lots. We develop an optimal control theory-based charging strategy that minimizes power drawn from the electricity grid while utilizing maximum energy from renewable energy sources. Specifically, we derive a centralized iterative control approach in which charging rates of EVs are optimized one at a time. We also propose an algorithm that maximizes profits for parking lot operators by advantageously utilizing time-varying electricity pricing while satisfying system constraints. We propose a linear programming-based strategy for EV charging, and we specifically derive a centralized linear program that minimizes charging costs for parking lot operators while satisfying customer demand in available time. Then we model EV charging behavior of Active Consumers. We develop a real-time pricing scheme that results in favorable load profile for electric utility by influencing EV charging behavior of Active Consumers. We develop this pricing scheme as a game between electric utility and Active Consumers, in which the electric utilities decide optimal electricity prices that minimize peak-to-average load ratio and Active Consumers decide optimal charging strategy that minimizes EV charging costs for Active Consumers.
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Chapter 1

Introduction

Growing concerns regarding global warming, air pollution, and oil shortages have motivated research and development of energy efficient, clean vehicles known as Electric Vehicles (EVs). Thanks to their environmental, social, and economic benefits, EVs are expected to become a major component of the power grid. Compared to conventional gasoline vehicles, distinct advantages of EVs, such as low pollution, low per-mile cost, ease of maintenance, opportunity to use renewable energy (RE) sources, and fast charging, cause EVs to be the potential future of transportation. New generations of EVs and Plug-in Hybrid EVs (PHEVs) have extended per-charge mileage, thereby increasing their convenience for use in everyday life. For example, Chevy Volt can travel up to 40 miles on electricity and has a 375-mile range when using an internal combustion engine electric generator [1]. Moreover, EVs are more energy efficient than gasoline vehicles, making EVs less expensive to operate.

1.1 Background

Over past few years, interest of the U.S. auto industry in EVs has increased significantly as most car companies have PHEVs in market. The U.S. Department of Energy has set a goal of lowering the purchasing cost of EVs by 2025, thereby increasing the number of EVs on
the road. According to [2], a total of 17,080 PHEVs were sold in 2011, increasing to 52,581 in 2012, 97,507 in 2013, and 123,049 in 2014. As of April 2015, 32,433 PHEVs had already been sold [2]. Sales of the EV Nissan Leaf have already passed the 100,000 milestone. By the year 2025, 3.3 million EVs are expected to be in use [3]. Figure 1.1 obtained from [4] shows the increase of EV sales since December 2010. According to Figure 1.1, we can say that the number of EVs in use is expected to increase significantly in the near future.

![Visualizing Electric Vehicle Sales](chart.png)

**Figure 1.1: Electric Vehicle Sales [4]**

Anticipated increase in the number of EVs presents some challenges associated with charging of EVs. Existing EV infrastructure is not sufficient to support anticipated future EV penetration. Therefore, increase in number of EVs will create new demands for charging facilities at homes, places of employment, and parking stations. If not properly managed, the anticipated use of EVs could adversely impact power system operation and stability.
Studies in [5] illustrated the inability of the current distribution system to accommodate a large number of EVs. Therefore, development of effective strategies based on coordination among EVs, non-EV loads, and renewable energy sources is critical for the reliability of future smart distribution systems.

Use of renewable energy sources for EV charging makes EV charging green in true sense; however, coordinating EV charging with randomness of renewable generation is challenging. Environmental benefits of EVs have been well documented in [6]. However, without the use of renewable energy sources such as solar and wind power to charge EVs, the carbon footprint simply shifts from tail pipes of vehicles to power generating stations. Therefore, renewable energy sources for EV charging must be considered in order to maintain cost-effective, pollution-free, environmentally friendly charging. Increased utilization of renewable energy sources will reduce greenhouse emissions and reduce charging costs for EVs. However, the random nature of solar and wind generation creates challenges for charging station operators to optimally utilize renewable energy. In addition, the random nature of renewable generation, especially wind generation, can result in transients in load profile. Therefore, more effective charging strategies that account for randomness of renewable generation are required.

In addition to technical challenges, we need to account for multiple stakeholders (e.g., electric utility, end user), each with individual economic objectives that are involved in the operation and use of EV charging stations. EV consumers want to charge their EVs at minimum charging costs and charging station operators want to maximize their profits from EV charging. However, once a substantial number of EVs are connected to the grid, the electric power industry would face unprecedented problems, such as power surge and voltage fluctuations, especially when more EV consumers require rapid or fast charging. Therefore, financial benefits must be provided in order to encourage consumers to charge their EVs during off-peak hours. In addition, electric utilities want to flatten the load profile in order to reduce the cost of electricity generation, which can be achieved by offering financial benefits.
to EV consumers when they charge their EVs at night. This thesis proposes charging strategies that satisfy all the above interests.

1.2 Related Work

There have been significant efforts to design charging strategies for EVs. These can be categorized into efforts related to 1) residential EV charging and impact on power system stability and operation, 2) vehicle-to-grid discharge, and 3) use of time varying electricity pricing to control multiple EV charging.

The primary emphasis of prior work has been on residential EV charging has been on levelling the load profile [7]. [7, 8] and [9] focus on minimizing generation costs and system losses using the valley filling optimization approach. Another realm of research, classified as “scenario analysis” [10, 11, 12, 13], focuses on EVs’ impact on peak power demand and low voltage problems. However, a majority of past research assume a deterministic model of vehicle behavior in order to optimize charging strategies [14, 15]. The final dimension of prior EV research involves charging schemes typically based on heuristic multi-agent distributed algorithms [16, 17, 18]. A majority of prior efforts focus on home-level EV charging, but very little work has been done to address EV charging in a commercial setting. For example, no systematic approach addresses coordinated EV charging in a commercial parking lot that has access to renewable energy sources.

Another realm of EV research is related to vehicle-to-grid (V2G) discharge. The idea is to make the plug-in EV bidirectional so that homes or utilities can draw power back from the plug-in EV’s batteries. A practical demonstration of V2G scenarios is reported in [19], which provides real-time frequency regulation from an electric car. The results show that V2G could be a prominent application in the global transition to emerging green and sustainable energy [19].

EV charging can be controlled or uncontrolled. Based on studies in [20, 21], people
often charge their EVs as soon as they reach home, which may cause a daily charging peak. In order to mitigate the impacts on the grid, certain strategies have been implemented. In literature, time-of-use (TOU) pricing and demand response (DR) have been proposed to interpret demand during high-demand hours [22]. A number of efforts have focused on controlling EV charging using TOU pricing [23, 24, 25, 26, 27, 28, 29, 30]; most of this prior literature assume fixed pricing structures. However, research related to use of real-time electricity to influence multiple EV charging is limited. A multi-tiered pricing scheme is proposed in [31], but it lacks deep insight into distribution level constraints and does not consider renewable generation. A regret minimization-based algorithm is proposed in [32], including design of a charging policy that ensures that an aggregated charging profile results in a valley-filling profile. A mathematical model of active consumer is developed in [33].

1.3 Contributions

In this work, we propose coordinated EV charging strategies that exploit renewable energy sources while considering technical and economic objectives within a smart distribution framework.

1.3.1 Coordinated EV Charging Strategies in a Commercial Parking Lot

Chapter 2 proposes charging strategies for multiple EVs in a commercial parking lot. The goal of this research is to coordinate multiple EV charging with renewable generation as well as varying electricity prices. The commercial parking lot has access to solar and wind energy to charge EVs. In order to satisfy customers’ charging demands, charging stations typically allow variable charging rates. The parking lot is also assumed to have a storage battery to store additional energy in order to increase profits for the parking lot operator.

• An optimal control theory-based charging strategy is developed in section 2.2 for
multiple EVs in a commercial parking lot, maximizing use of renewable energy.

- A linear programing-based charging strategy is developed in section 2.3 that considers economic aspects of multiple EV charging in addition to technical objectives. The goal of this research is to maximize the profit for parking lot operators by coordinating multiple EV charging with real-time electricity pricing and renewable generation.

- The assumption is made that parking lot owners typically make day-ahead power purchase commitments with the electric power utility in order to obtain reduced electricity prices for a certain amount of energy. Additional energy required to charge EVs can be purchased in real-time at increased price. When demand is less than the scheduled power commitment or total renewable generation is higher than demand, the parking lot can sell back additional energy at a lower price.

- Within this framework, optimal charging rates for EVs are sought that will meet customer demands and maximize profit for the parking lot operator. Charging rates of each EV at all time slots are optimization variables in this scheduling problem. Constraints of this optimization are (1) time available for charging each EV, (2) demand of charge for each vehicle in the charging station, (3) amount of renewable energy available from distributed generation (DG), such as solar and wind generation, and (4) maximum power that can be drawn from a distribution transformer. A provably optimal linear programing-based charging solution is proposed. With minor modifications, this framework could also be applied to residential scenarios. Details of this work are presented in Chapter 2 and in the following publications:


[35]: K. Jhala, B. Natarajan, A. Pahwa, and L. Erickson, “Coordinated electric vehicle charging for commercial parking lot with renewable energy sources,” in *Smart Grid, IEEE Trans-
1.3.2 Active Consumer-Based Real-Time Pricing Scheme

In this thesis, a real-time pricing scheme is proposed that helps utility achieve favorable load profile at the transformer. The key idea in this research is use of a spatial and temporal real-time pricing scheme in order to influence charging behavior of multiple EVs in a demographic area.

- A system structure is considered in which a transformer is connected to feeders that are connected to an arbitrary number of homes with possible renewable energy sources. The presence of Active Consumers of EVs who react to changes in electricity pricing in real-time is assumed.

- Behavior of Active Consumers is modeled. Based on our model of Active Consumer we develop an optimal pricing scheme that minimizes peak-to-average load ratio at the transformer.

- An optimal EV charging algorithm for EV consumers is developed that minimizes EV charging costs using current and future system parameters. We develop game-theory-based approach where electric utility determines electricity prices based on consumers’ reaction to changes in electricity prices and Active Consumers adjust their EV load based on real-time electricity prices.

- From simulation, the proposed approach is shown to result in a favorable load profile at the transformer. This work is presented in Chapter 3 and in the following publication: [36]: K. Jhala, B. Natarajan, A. Pahwa, and L. Erickson, “Active consumer-based real-time electricity pricing scheme,” in Smart Grid, IEEE Transactions on (to be submitted)
1.4 Organization of Thesis

The rest of thesis is organized into specific chapters. Chapter 2 develops a system model for multiple EV charging in a commercial parking lot with renewable generation, including proposal of two novel charging strategies that maximize use of renewable generation and maximize profits for parking lot operators. Chapter 3, develops an Active Consumer-based optimal pricing scheme that uses real-time electricity pricing and influences charging behavior of Active Consumers, resulting in a favorable load profile for electric utilities. Finally, Chapter 4 summarizes major contributions of this research and presents ideas for future work.
Chapter 2

Optimal Electric Vehicle Charging Strategies

In this chapter, we develop optimal charging strategies for multiple EVs parked in charging stations at a commercial parking lot. Two charging strategies are developed for charging multiple electric vehicles (EVs): 1) Maximization of the use of renewable energy 2) Maximization of profit for parking lot operator by coordinating multiple EV charging with real-time electricity pricing.

2.1 Commercial Electric Vehicle Charging Setup

In this work, we consider a parking lot with $M$ charging stations. In addition to energy from the power grid, the parking lot has solar and wind energy available to charge EVs. The system predicts the amount of future solar and wind generation and determines current EV charging strategy based on that prediction, which are random function of time. Solar power can be modeled as random function of time [37],

$$ P_{solar}(t) = S(t) + n_s(t) $$
where, $S(t)$ is the predicted value of solar generation with some prediction method and $n_s \sim \mathcal{N}(0, \sigma_s^2)$ is the prediction error modeled as zero mean Gaussian random variable with some variance $\sigma_s^2$. Available wind energy from a wind generator is also modeled as a random function of time [38],

$$P_{\text{wind}}(t) = W(t) + n_w(t)$$

where, $W(t)$ is the predicted value of wind generation with some prediction method, and $n_w \sim \mathcal{N}(0, \sigma_w^2)$ is a prediction error modeled as zero mean Gaussian random variable with some variance $\sigma_w^2$. Therefore, the total available power from renewable energy sources corresponds to,

$$P_{\text{re}}(t) = P_{\text{solar}}(t) + P_{\text{wind}}(t).$$

For the analysis purpose, we use predicted value of renewable generation to calculate optimal charging strategy.

For every vehicle arriving to the parking lot, the system can sense the state of charge of the battery for each vehicle. Based on initial state of charge and final desired charge level, the system calculates charge demand for that particular EV. $DOC_i$ indicates demand of charge of the $i^{th}$ EV, which is modeled as a beta random variable with parameter $\alpha$ and $\beta$, as suggested in [39]. $DOC_i \sim C_b \text{Beta}[\alpha_D, \beta_D]$, where $C_b$ indicates maximum battery capacity. $DOC_i$ can also be sensed as soon as customers plug in their EVs. Therefore, $DOC_i$ is treated as a known quantity for the analysis purpose.

We divide the charging period into $T$ number of time slots of equal length $\Delta t$. When a user enters the parking lot, the system asks for departure time. Based on arrival and departure time, the system calculates the number of time slots available for charging each EV. $t_{a_i}$ is number of slots available for charging and $t_{a_i}\Delta t$ is total time available for charging the $i^{th}$ EV. In this study, $t_{a_i}$ is modeled as a gamma random variable with parameter $\alpha$ and $\beta$ [39], $t_{a_i} \sim \Gamma(\alpha, \beta)$. When a new customer arrives in the parking lot he can indicate his approximate parking duration which can be treated as a known quantity for the purpose of
All charging stations in a parking lot can support multiple charging rates. We assume that the system can charge an EV at five distinct charging rates. At any time $t$, the charging rate of $i^{th}$ EV (i.e., $P_{ci}(t)$) can take any value from set $P_c$.

$$P_c \in \{0, r_1, r_2, r_3, r_{max}\}$$

where, $0 < r_1 < r_2 < r_3 < r_4 = r_{max}$. $r_{max}$ indicates maximum allowable charging rate and $P_{ci}(t) = 0$ indicates that the $i^{th}$ EV is not charging at time $t$. Due to the random nature of $t_{ai}$ and $DOC_i$, some EVs can create conditions such that the system cannot satisfy the EV charge demand, even by charging EV at the highest charging rate. In order to avoid this unwanted scenario, we isolated EVs that can cause this condition. That is, for EVs with higher demand of charge but not sufficient time available for charging, charging rate selection is left to the customers. Any EV with $t_{ai} < k_i \frac{DOC_i}{r_{max}}$ is treated as a special case because it is not in the parking lot for a sufficient amount of time. $k \geq 1$ is a relaxation factor, and the value of $k$ can be decided based on various methods discussed later. Additionally, some customers may choose to charge their EVs partially instead of charging fully. For these EVs, the customer has a choice to select his or her own charging rate based on desired level of charge at the completion of the charging process. The parking lot allows customers to select various charging rates at different costs. For example a customer can pay more in order to select a high charging rate. Based on the customer’s selected charging rate, the system calculates a new demand of charge as $P_{ci}t_{ai}$ if $t_{ai} < k_i \frac{DOC_i}{r_{max}}$. For others, the demand of charge is unchanged.

The value of relaxation factor $k$ can be a treated as a fixed constant value for all cases for simplicity. This value can be decided based on past experiences. A better approach is to select value of $k$ dynamically for every upcoming EV as $k_i$ based on present scenario of
EV load and renewable generation. For example, 

\[ k_i = K \frac{1}{T} \sum_{t=1}^{T} \left[ \frac{\sum_{j=1, j \neq i}^{M} P_{c_i}(t)}{P_{re}(t)} \right], \tag{2.1} \]

where, \( K \) is constant. Here, right hand side of equation (2.1) corresponds to the scaled average of the total load to the total generation ratio.

### 2.2 Approach to Maximize Use of Renewable Energy

In this section, we develop optimal control theory-based optimal EV charging strategy that maximizes the use of renewable generation by coordinating multiple EV charging with renewable generation. In a scenario, in which charging demand of an EV must be satisfied in given amount of time, the primary research question pertains to EV charging should be scheduled and then deciding the charging rate at different time slots in order to maximizes renewable energy usage.

#### 2.2.1 Problem Formulation

The energy used to charge EVs comes from both the power grid and distributed generations (DGs). The goal is to minimize total energy drawn from grid which maximizes the share of renewable energy sources while satisfying EV demand in given time. So we can write the general optimization problem as,

\[
\min_{P_{c_i}(t)} J = \sum_{t=0}^{T} \left[ \sum_{i=1}^{M} P_{c_i}(t) \Delta t - P_{solar}(t) \Delta t - P_{wind}(t) \Delta t \right]^2 \tag{2.2}
\]

subject to

\[
\sum_{t=0}^{t_{a_i}} P_{c_i}(t) \Delta t = DOC_i \tag{2.3}
\]
\[-P_{c_i}(t) \leq 0, \forall t\]  \hspace{1cm} (2.4)

\[P_{c_i}(t) - r_{\text{max}} \leq 0, \forall t\]  \hspace{1cm} (2.5)

\[
\left[\sum_{i=1}^{M} P_{c_i}(t) \Delta t\right] - P_{\text{solar}}(t) \Delta t - P_{\text{wind}}(t) \Delta t \leq P_{\text{max}}
\]  \hspace{1cm} (2.6)

where, \(T\) is number of distinct charging time slots.

The quadratic cost function in equation (2.2) is related to the total energy drawn from power grid which is the objective that needs to be minimized and \(P_{c_i}(t)\) is optimization variable. Constraint (2.3) ensures that the sum of energy delivered in available time slots is equal to the initial demand. Constraint (2.4) and (2.5) ensures the lower and upper bounds on charging rates respectively. The upper bound on power drawn from grid is given by constraint (2.6).

Although, \(DOC_i\) and \(t_{a_i}\) are modeled as random quantities, for perspective of optimization problem they are treated as known parameters as \(DOC_i\) can be sensed as soon as vehicle is parked and \(t_{a_i}\) can be provided by user. Optimization problem assumes average deterministic values of \(P_{\text{solar}}(t)\) and \(P_{\text{wind}}(t)\) as predicted value of solar and wind energy.

Since our goal is to minimize the power drawn from grid at all times, constraint (2.6) is assumed to be inactive. In the following discussion, we will drop this constraint. In the next section, we develop an optimal control theoretic formulation, which is less computationally complex, and provide an analytical solution.

### 2.2.2 Optimal Control Theoretic Solution

Solving the general optimization problem (2.2)-(2.6) and finding the charging profile of all EVs simultaneously is computationally complex. So we use optimal control theoretic approach and find the charging profile of one EV at a time. The optimal control problem
for one EV can be formulated by fixing the charging rate for all other charging EVs as constants. We define our state variable $x_i(t)$ as $DOC$ for $i^{th}$ vehicle ($DOC_i$), and the control variable of problem $u_i(t)$ is the charging rate of $i^{th}$ EV at time $t$ (i.e. $P_{ci}(t)$). We define $C(t)$ as available renewable energy at time $t$, which is sum of available solar and wind energy at time $t$.

$$C(t) = P_{solar}(t) + P_{wind}(t)$$

Charging rates of all other EVs are assumed to be constant for each iteration. $U_{-i}(t)$ is sum of charging rate of all other EVs except $i$.

$$U_{-i}(t) = \sum_{j \neq i} u_j(t)$$  \hspace{1cm} (2.7)

The optimization problem in (2.2)-(2.5) can now be formulated as the following control problem for each individual EV:

$$J^*_i = \min_{u_i(t)} \sum_{t=0}^{T} [(u_i(t) + U_{-i}(t))\Delta t - C(t)\Delta t]^2$$  \hspace{1cm} (2.8)

subject to

$$x_i(t + 1) = x_i(t) - u_i(t)\Delta t$$  \hspace{1cm} (2.9)

$$\sum_{t=0}^{T} u_i(t)\Delta t = x_i(t_0)$$  \hspace{1cm} (2.10)

$$-u_i(t) \leq 0, \forall t$$  \hspace{1cm} (2.11)

$$u_i(t) - r_{max} \leq 0, \forall t$$  \hspace{1cm} (2.12)

The objective (2.8) is to find the optimal charging rate for $i^{th}$ EV that minimizes the energy drawn from grid and maximizes the renewable energy share in EV charging while
making sure that all EV’s charging demand is satisfied in given time that is taken care by equation (2.10). Equation (2.9) is state equation of optimal control problem.

Therefore, the Lagrangian can be written as

\[
\mathcal{L} = \sum_{t=0}^{T} [(u_i(t) + U_{-i}(t)) \Delta t - C(t) \Delta t]^2 \\
+ \lambda_1(t + 1) [x_i(t + 1) - x_i(t) + u_i(t) \Delta t]
\]

\[+ \lambda_2 \left[ \sum_{t=0}^{T} u_i(t) \Delta t - x_i(t_0) \right] - \mu_1(t) u_i(t) + \mu_2(t) [u_i(t) - r_{max}] \quad (2.13)\]

Hamiltonian can be written as:

\[
\mathcal{H} = [(u_i(t) + U_{-i}(t)) \Delta t - C(t) \Delta t]^2
\]

\[- \lambda_1(t + 1) [x_i(t) - u_i(t) \Delta t] + \lambda_2 [u_i(t) \Delta t - x_i(t_0)]
\]

\[- \mu_1(t) u_i(t) + \mu_2(t) [u_i(t) - r_{max}] \quad (2.14)\]

where, \( \lambda_1 \in \mathbb{R} \) is the co-state variable and \( \mu_1(t) \) and \( \mu_2(t) \) are Lagrangian multipliers.

The solution to the problem is obtained by considering the following adjoint system equations:

\[
\lambda_1(t) = \frac{\partial \mathcal{H}}{\partial x_i(t)} = -\lambda_1(t + 1) \quad (2.15)
\]

from condition \( \lambda_1(T) = 0 \)

\[
\lambda_1(t) = \lambda_1(t + 1) = 0, \forall t \quad (2.16)
\]

In the discrete-time case, stationarity conditions are defined as

\[
\frac{\partial \mathcal{H}}{\partial u_i(t)} = 2 \Delta t [u_i(t) + U_{-i}(t) - C(t)] + \lambda_1(t + 1) \Delta t
\]
\[ u_i(t) \Delta t = C(t) \Delta t - U_{-i}(t) \Delta t - \frac{\lambda_1(t + 1)}{2} - \frac{\lambda_2}{2} + \frac{\mu_1(t)}{2 \Delta t} - \frac{\mu_2(t)}{2 \Delta t} \]

But \( \lambda_1(t) = 0, \forall t \)

\[ u_i(t) \Delta t = C(t) \Delta t - U_{-i}(t) \Delta t - \frac{\lambda_2}{2} + \frac{\mu_1(t)}{2 \Delta t} - \frac{\mu_2(t)}{2 \Delta t} \]  \hfill (2.18)

from Equations (2.18) and (2.10)

\[ \lambda_2 = \frac{1}{T} \left[ \sum_{t=1}^{T} (C(t) - U_{-i}(t)) \Delta t + \frac{\mu_1(t) - \mu_2(t)}{2 \Delta t} \right] - \frac{2 x_i(t_0)}{T} \]  \hfill (2.19)

We can solve above problem from Equations (2.10), (2.18), and (2.19) by considering the following cases and selecting the corresponding minimum.

**Case - I** \( \mu_1 = 0, \mu_2 = 0 \)

\[ u_i(t) \Delta t = C(t) \Delta t - U_{-i}(t) \Delta t - \frac{\lambda_2}{2} \]  \hfill (2.20)

**Case - II** \( \mu_1(t) > 0, \mu_2(t) = 0 \)

From complimentary slackness condition \( \mu_1 u_i(t) = 0 \), we can write

\[ u_i(t) = 0 \]

and,

\[ \mu_1(t) = \frac{1}{2 \Delta t} \left[ u_i(t) \Delta t - C(t) \Delta t + U_{-i}(t) \Delta t + \frac{\lambda_2}{2} \right] \]

**Case - III** \( \mu_1(t) = 0, \mu_2(t) > 0 \)
From complimentary slackness condition $\mu_2(u_i(t) - r_{max}) = 0$, we can write

$$u_i(t) = r_{max}$$

and,

$$\mu_2(t) = 2\Delta t \left[ C(t)\Delta t - U_{-i}(t)\Delta t - \frac{\lambda_2}{2} - u_i(t)\Delta t \right]$$

After computing optimal charging rates for one EV, optimal values of $u_i(t)$ are quantized to available charging rate options. This process is repeated for all $i$’s and known values of $u_j(t)$ as $j \neq i$ are used to set $U_{-i}(t)$. Since new EVs may arrive and others may leave, this process is repeated whenever such event occurs.

**Lemma 1.** Let $J^{*\text{opt}}$ be the minimum cost function corresponding to (2.2) and $J^{*\text{opt}}_i$ be the minimum objective function value corresponding to (2.8), then $J^{*\text{opt}}_i \rightarrow J^{*\text{opt}} \rightarrow J^{\text{opt}}$

**Proof:** Convergence of $J^{*\text{opt}}_i$ to $J^{\text{opt}}$ follows from approach similar to iterative optimization approach similar to [40].

$$J^{*\text{opt}}_1 \geq J^{*\text{opt}}_2 \geq \cdots \geq J^{*\text{opt}}_M \geq J^{*\text{opt}} = J^{\text{opt}}$$

Optimizing $J^{*\text{opt}}_i$ can never increase over all grid power consumption $J^{*\text{opt}}$. Therefore rather than simultaneously optimizing the charging rate of all vehicles, we can optimize the charging rate of each vehicle at a time without loss of generality. Consequently, results converge into global optimum. By this we can say that, minimizing the charging rate of one EV at a time as in equation (2.8) and finding optimal charging strategy for each EV sequentially provides global optimal charging strategy of all EVs.
2.2.3 Results

To test the optimal control theoretic framework, we set up a simulation with 10 vehicles in a parking lot for 30 times slots. Each time slot is 15 minutes long, totaling 7.5 hours. During this period, we use real data for solar generation and wind generation to set our $S(t)$ and $W(t)$ and added random noise. The maximum allowable generation for solar and wind generation was set to 30kW each. The sum of both these sources is referred to as total renewable generation as shown in Figure 2.1.

![Figure 2.1: Total Power Used to Charge EVs](image)

Parameters for a vehicle's state, the $DOC$, was taken as a random variable with Beta distribution having parameters $\alpha_D = 5, \beta_D = 2$ and $C_b = 70kWh$. Time available for
charging was a gamma random variable with $\alpha = 10$ and $\beta = 4$. To identify the vehicles that are not in the system for sufficient amount of time value of $k = 1.2$. For EVs that are not in the system for sufficient amount of time in the system, new $DOC_i$ is calculated by multiplying the $t_{ai}$ with chosen charging rate from $P_c$.

The proposed optimal control theory based algorithm was applied to above model. In this approach we find the optimal charging rate of each vehicle sequentially assuming the charging rates of all other vehicles as constant. We use only one iteration of optimizing the charging rates of each EVs sequentially. Simulation result for one realization is shown in Figure 2.1. As expected, results indicate that the sum of charging rates follow the trend of renewable generation. Because of a lack of vehicles in the parking lot, system could not fully utilize available renewable energy after the 23rd time slot. As shown in Figure 2.1, all of energy required to charge EVs comes from renewable energy sources.

In practice, the amount of renewable energy share for charging EVs depends on $DOC$ and available time to charge, which are random quantities. We find the percentage renewable energy share in EV charging and plotted histogram for thousand monte carlo repetitions of our scenario. Figure 2.2 shows the histogram percentage share of renewable energy to charge EVs when the prediction error of solar and wind energy is set to zero. From Figure 2.2, we can see that in more than 50% of the cases, percent DG share in EV charging is 95% or higher demonstrating the potential of our proposed approach.

Efficiency of this algorithm depends on the accuracy of prediction of renewable generation. To check the robustness of our algorithms to prediction errors, we consider a range of prediction variances and evaluate the performance. With zero prediction noise, renewable energy share for charging EVs is 93% on average. We simulated the algorithm thousand times for different prediction noise values and found average renewable energy share to charge EVs. From Figure 2.3 it can be seen that the average performance of optimal control method deteriorates with increase in prediction noise. While this result is not surprising, it is important to note that with reasonable prediction error variance of 6 or below, the
proposed approach assures 90\% or more share of renewable energy.

We also compare the result of control theoretic solution with global optimum. From Figure 2.2 and Figure 2.3 it can be seen that proposed algorithm provides results very close to the optimal solution with comparatively lower complexity. In thousand Monte Carlo repetitions of our scenario, we monitored the execution time of one iteration of global optimization and our proposed optimal control theoretic solution on same machine. In case of global optimization method, the average simulation time for one iteration is 41.70 seconds. The average simulation time for optimal control theoretic method is 1.78 seconds only. It shows that our proposed algorithm is 23.5 times faster than general global optimization method.

Figure 2.2: Histogram of Percentage RE Share in EV Charging
Figure 2.3: Performance of Algorithm with Prediction Noise

2.3 Techno-Economic Approach to Minimize Charging Cost

In this section, we develop linear programing-based optimal EV charging strategy that maximizes profit by coordinating multiple EV charging with renewable generation and real-time electricity prices.

2.3.1 Economic Aspects of EV Charging Setup

In addition to renewable generation, additional energy required to charge EVs is drawn from the utility grid. The parking lot operator makes a day-ahead power purchase $P_{da}(t)$ with the utility in order to achieve a reduced electricity price $c_{da}$. The operator can purchase
additional electricity at higher electricity pricing rate \( (c_r(t)) \) if real-time consumption \( P_{rt}(t) \) is higher than what was predicted. However, if real time electricity demand is lower than the day-ahead prediction, operator must pay penalty. The operator can sell this under-utilized energy back to the utility at lower price \( (c_p(t)) \).

In order to minimize mismatch between real-time demand and day-ahead commitment, the parking lot has a storage battery. When EV demand is low or electricity prices are low, the system is expected to store excess energy in the storage battery and use this energy when electricity prices are high. Let \( B_{max} \) indicate maximum battery capacity. At the beginning of the time period of interest, we model the initial state of charge \( B_0 \) as a uniform random variable between 0 and \( B_{max} \), i.e.,

\[
B_0 \sim \text{uniform}[0, B_{max}].
\]

However, in practice the state of charge \( B_0 \) can be monitored by the parking lot operator and can be treated as a known quantity for the analysis purpose. Let \( v(t) \) indicate charging and discharging rate of battery. When \( v(t) > 0 \), the system is charging the storage battery (Corresponding to the case of excess energy), and \( v(t) < 0 \) indicates that the system is using energy from the storage device in order to charge EVs. We assume that the battery cannot store or deliver energy at a rate higher then \( v_{max} \) because of the power rating of the battery.

In this scenario, we have to satisfy charging demand of each customer in available time; the main research question is how can we coordinate multiple EV charging and decide optimal charging rate for all EVs in each time slot in a manner that maximizes profit for the parking lot operator?

### 2.3.2 Problem Formulation and Solution

The goal of the optimization problem is to determine optimal charging strategy for EVs in order to maximize profits for parking lot owners, which can be written as summation
of income subtracted by charging cost at each time slot \( t \). So we can write the general optimization formulation as,

Maximize

\[
J = \sum_{t=1}^{T} \text{Income}(t) - \text{Cost}(t) \quad (2.21)
\]

subject to

\[
\sum_{t=0}^{t_{a_i}} P_{c_i}(t) \Delta t = \text{DOC}_i, \forall i \quad (2.22)
\]

\[
\left[ \sum_{i=0}^{M} P_{c_i}(t) + v(t) \right] - P_{re}(t) \leq P_{max}, \forall i \quad (2.23)
\]

\[
0 \leq P_{c_i}(t) \leq r_{max}, \forall t \leq t_{a_i} \quad (2.24)
\]

\[
B_{min} \leq B_0 + \sum_{t=1}^{t' \leq T} v(t) \leq B_{max}, \forall t' \leq T \quad (2.25)
\]

Income in (2.21) is money earned by selling electricity to customers and cost is defined as the amount the owner must pay to the utility. In addition to minimizing charging costs, the system must satisfy charging demand of each customer as imposed by constraint in (2.22). Because the transformer has limited power-delivering capacity, the system cannot draw power more than \( P_{max} \) from grid, especially when electricity prices are low. Equation (2.23) is the constraint on maximum allowable power that can be drawn from the grid. The system cannot discharge any EV or charge at a rate higher than \( r_{max} \). Charging rates of EVs is limited by constraint (2.24).

Money earned from a customer is a fixed amount from optimization standpoint, so maximizing profit is equivalent to minimizing the cost (amount of money the parking lot operator pays the utility). Therefore, we can minimize the amount of money the parking
lot owner pays toward the utility grid. Objective function (2.21) can be written as

\[ J = \sum_{t=1}^{T} \left[ \sum_{i=1}^{M} c_{si} P_{ci}(t) \Delta t \right] \]

\[ - \left[ (c_{da}(t) - c_{p}(t)) P_{da}(t) \right] \Delta t - (c_{p}(t) P_{rt}(t)) \Delta t \]

\[ - (c_{r}(t) - c_{p}(t)) Ra \left[ P_{rt}(t) - P_{da}(t) \right] \Delta t \] (2.26)

The first term in Equation (2.26) can be written as

\[ \sum_{t=1}^{T} \left[ \sum_{i=1}^{M} c_{si} P_{ci}(t) \Delta t \right] \]

\[ = \sum_{i=1}^{M} c_{si} \left[ \sum_{t=1}^{T} P_{ci}(t) \Delta t \right] = \sum_{i=1}^{M} c_{si} DOC_{i}, \]

which is a fixed number because \( DOC_{i} \) and \( c_{si} \) are fixed for each EV. In order to maximize profit, we minimize the cost only as the income from charging EVs is a constant.

Therefore, maximizing \( J \), or profit, is identical to minimizing \( J' \), or the total cost of charging.

\[ J' = \sum_{t=1}^{T} \left[ (c_{da}(t) - c_{p}(t)) P_{da}(t) \right] \Delta t + (c_{p}(t) P_{rt}(t)) \Delta t \]

\[ + (c_{r}(t) - c_{p}(t)) Ra \left[ P_{rt}(t) - P_{da}(t) \right] \Delta t \] (2.27)

Solving the optimization problem (2.21) can be computationally complex because the objective function is not differentiable at the optimal point. However, (2.27) suggests that the alternative objective function is piecewise linear as shown in Figure 2.4. By using this property, we can derive a linear programing-based formulation that is easier to solve and less computationally complex.

For given time varying electricity price \( (c_{da}(t), c_{p}(t) \text{ and } c_{r}(t)) \), and day-ahead power
commitment $P_{da}(t)$, cost $J'$ is a piecewise linear function of real-time power consumption $P_{rt}(t)$ as shown in Figure 2.4. Equation (2.27) can be rewritten as,

$$J' = \max(l_1, l_2)$$  \hfill (2.28)

where,

$$l_1 = c_p(t) \left[ \sum_{i=1}^{M} P_{ci}(t) + v(t) \right] \Delta t + (c_{da}(t) - c_p(t))P_{da}(t)\Delta t - c_p(t) \cdot P_{re}(t)\Delta t,$$

and
\[ l_2 = c_r(t) \left[ \sum_{i=1}^{M} P_{c_i}(t) + v(t) \right] \Delta t \]
\[ + (c_{da}(t) - c_r(t)) P_{da}(t) \Delta t - c_r(t) \cdot P_{re}(t) \Delta t. \]

For a given objective function, the optimization problem can be written as a piecewise linear minimization.

\[
\min \sum_{t=1}^{T} y(t)
\]
subject to

\[ l_1 \leq y(t) \]
\[ l_2 \leq y(t) \]

\[ \sum_{t=1}^{T} P_{c_i}(t) \Delta t - DOC_i = 0, \forall i \]

\[ \left[ \sum_{i=0}^{M} P_{c_i}(t) + v(t) \right] - P_{re}(t) \leq P_{max}, \forall i \]
\[ 0 \leq P_{c_i}(t) \leq r_{max} \]

\[ B_{min} - B_0 \leq \sum_{t=1}^{t'} v(t) \leq B_{max} - B_0, \forall t' \leq T \]

where, \(y(t)\) is a dummy variable.

In order to write the above problem formulation in vector form, we introduce some vector notation as

\[ \mathbf{P}_c^T(t) = [P_{c_1}(t), P_{c_2}(t), \cdots, P_{c_M}(t), v(t)] \]
\[
\begin{align*}
\text{doc}^T &= [\text{DOC}_1, \text{DOC}_2, \ldots, \text{DOC}_M] \\
\mathbf{r}_{\text{min}}^T &= [0, 0, \ldots, 0, v_{\text{min}}] \\
\mathbf{r}_{\text{max}}^T &= [r_{\text{max}}, r_{\text{max}}, \ldots, r_{\text{max}}, v_{\text{max}}] \\
\mathbf{e}_1^T &= [0, 0, \ldots, 0, 1]
\end{align*}
\]

The above optimization can be written as a linear program (LP) with six inequality and one equality constraint. That is

\[
\min \sum_{t=1}^{T} y(t) \tag{2.29}
\]

subject to

\[
c_p(t) \left[ \mathbf{P}_c^T(t) \cdot 1 - P_{\text{re}}(t) \right] + (c_{\text{da}}(t) - c_p(t)) \left( P_{\text{da}}(t) \frac{y(t)}{\Delta t} \right) \leq 0
\]

\[
c_r(t) \left[ \mathbf{P}_c^T(t) \cdot 1 - P_{\text{re}}(t) \right] + (c_{\text{da}}(t) - c_r(t)) \left( P_{\text{da}}(t) \frac{y(t)}{\Delta t} \right) \leq 0
\]

\[
\mathbf{P}_c^T(t) \cdot 1_{M+1} \leq P_{\text{max}} + P_{\text{re}}(t), \forall t
\]

\[
\sum_{t=0}^{T} \mathbf{P}_c^T(t) \cdot \Delta t - \text{doc} = 0, \forall i
\]

\[
\mathbf{r}_{\text{min}} \leq \mathbf{P}_c(t) \leq \mathbf{r}_{\text{max}}
\]

\[
\sum_{t=1}^{t'} \mathbf{e}_1^T \cdot \mathbf{P}_c(t) \leq B_{\text{max}} - B_0, \forall t' \leq T
\]

\[
- \sum_{t=1}^{t'} \mathbf{e}_1^T \cdot \mathbf{P}_c(t) \leq B_0 - B_{\text{min}}, \forall t' \leq T
\]

The equivalent LP in matrix notation can be written as

\[
\min \mathbf{c}^T \mathbf{x} \tag{2.30}
\]
subject to

\[ Ax \leq b \]
\[ D^T x = \text{doc} \]
\[ r_{\text{min}} \leq x(t) \leq r_{\text{max}} \]

where,

\[ c = [0, 1, 0, 1, \cdots, 0, 1]^T, \]
\[ x = [P_c(1), y(1), P_c(2), y(2), \cdots, P_c(T), y(T)]^T, \]
\[ A = [A_1, A_2, A_3, A_4]^T, \]
\[ b = [-b_1(t = 1), -b_2(t = 1), -b_1(t = 2), -b_2(t = 2), \cdots, -b_1(t = T), -b_2(t = T), P_{\text{max}}, B_0 - B_{\text{min}}, B_{\text{max}} - B_0]^T, \]
\[ D = [I_{M \times M}, 0^T, I_{M \times M}, 0^T, \cdots, I_{M \times M}, 0^T]^T \]

Here, \( A_1 = \text{diag}(\tilde{A}(t = 1), \tilde{A}(t = 2), \cdots, \tilde{A}(t = M)) \)

where,

\[ \tilde{A}(t) = \begin{bmatrix} c_p(t) \Delta t \cdot 1_{M+1}^T & -1 \\ c_r(t) \Delta t \cdot 1_{M+1}^T & -1 \end{bmatrix}; \]
\[
A_2 = \begin{bmatrix}
1_{M+1}^T & 0 & 0^T & \cdots & 0^T & 0 \\
0^T & 0 & 1_{M+1}^T & \cdots & 0^T & 0 \\
& & & \ddots & & \\
0^T & 0 & 0^T & \cdots & 1_{M+1}^T & 0
\end{bmatrix};
\]

\[
A_3 = \begin{bmatrix}
e_2^T & 0^T & 0^T & \cdots & 0^T \\
e_2^T & e_2^T & 0^T & \cdots & 0^T \\
e_2^T & e_2^T & e_2^T & \cdots & 0^T \\
& & & \ddots & \\
e_2^T & e_2^T & e_2^T & \cdots & e_2^T
\end{bmatrix},
\]

where,

\[
e_2^T = [0, 0, \cdots, 0, 1, 0];
\]
\[
A_4 = \begin{bmatrix}
-e_2^T & 0^T & 0^T & \cdots & 0^T \\
-e_2^T & e_2^T & 0^T & \cdots & 0^T \\
-e_2^T & -e_2^T & e_2^T & \cdots & 0^T \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-e_2^T & -e_2^T & -e_2^T & \cdots & -e_2^T
\end{bmatrix} .
\]

Additionally,

\[
b_1(t) = c_p(t)P_{re}(t)\Delta t - (c_{da}(t) - c_p(t))P_{da}(t)\Delta t
\]

\[
b_2(t) = c_r(t)P_{re}(t)\Delta t - (c_{da}(t) - c_r(t))P_{da}(t)\Delta t
\]

Solving this linear-program helps find the optimal charging strategy \( P_{ci}^*(t) \) for all EVs. After every time interval \( \Delta t \) we update the values of system parameters (renewable generation and time varying electricity prices) and find the optimal charging strategy.

### 2.3.3 Results

To test our proposed linear-programing framework, we setup a parking lot simulation with 10 charging stations. We consider each time slot to be 15 minutes duration, and the simulation was conducted for 30 time slots. For solar and wind generation, we used a scaled version of real data [34]. Total maximum generation capacity for solar and wind generation was set to 35kW.

A vehicle’s \( DOC \) was taken as a random variable with Beta distribution with parameters \( \alpha_D = 5, \beta_D = 2, \) and \( C_b = 70kWh \) [34]. In order to identify vehicles that are not in the system for a sufficient amount of time, value of \( k \) was set as 1.2. For the EVs that were
not in the system for a sufficient amount of time, a new $DOC_i$ is calculated by multiplying $t_{a_i}$ with the chosen charging rate from $P_c$. We use a time of use (TOU) electricity pricing scheme, with different pricing scheme for different time slots. However, we can use proposed approach also for real time pricing scenario. Parking lot makes day-ahead power consumption commitment at discounted rate as shown in Figure 2(a); however, parking lot operator must pay a higher price for purchasing additional energy in real-time in case of higher demand. If a parking lot operator cannot use day-ahead scheduled energy, he or she can sell

**Figure 2.5: Simulation Results**
that energy back to the grid at a reduced price. Electricity prices for all three scenarios are shown in Figure 2(a).

Figure 2.5 shows results for one particular scenario with figure 2.5(b) showing the number of EVs available for charging in the parking lot. Arrival of EVs is modeled as exponential random variable with mean equal to 12 time slots; and time available for charging \((t_a)\) is a gamma random variable with \(\alpha = 10\) and \(\beta = 4\) [34]. Based on arrival time and time available for charging number of EVs in parking station during each time-slot can be calculated. Total renewable generation available at each time \(t\) is shown in Figure 2.5(c). For renewable generation we used scaled value of real data for solar and wind generation. Sum of solar and wind generation at each time is used as total renewable generation.

The charging rate of all EVs increases when electricity prices are lower and decreases when electricity prices are higher. Figure 2.5(c) and 2.5(d) shows that overall power consumption of the system is higher when the price of electricity is lower (time slot 11 to 20). During this period, system increases the charging rates for most of EVs present in the system. By doing this parking lot is buying more energy when electricity prices are low in order to maximize profit for parking lot operator. Figure 2.5(d) shows that power consumption of parking lot is much higher than day ahead power commitment when electricity prices are low. When electricity prices are high again (time slot 21 to 30) system is not using energy more than day-ahead commitment in order to avoid buying energy at higher cost.

The initial state-of-charge of the battery is modeled as a uniform random variable between minimum and maximum state-of-charge (i.e., 0 and \(B_{max} = 50\)). Parking lot charges the storage battery when electricity prices are low as shown in Figure 2.5(e). This stored energy can be used in future when necessary. After time slot 21, when electricity prices return to a high state, the system begins discharging the storage battery and uses that energy to charge EVs, which can be seen in Figure 2.5(e).

Next, we run a Monte Carlo simulation with 1000 runs reflecting the randomness in renewable generations, \(DOC\), \(t_a\) and storage battery state. The cumulative distribution
Figure 2.6: Total Charging Cost

function (CDF) of charging cost for all EVs in the system is shown in Figure 2.6, which demonstrates that the average cost of charging for smart price aware charging proposed in this work is significantly lower than dumb charging. In a dumb charging system, time varying electricity pricing is not considered while deciding charging strategies and all EVs are charged at fixed rates.

Results of our simulation show that the proposed linear programming based approach can significantly reduce the electricity cost of EV charging while efficiently utilizing renewable generation and battery storage thereby increasing profits for parking lot operators while satisfying customer demand. This approach elegantly incorporates time varying pricing. The results suggest that utilities can control charging behavior/net power consumption of
charging stations by controlling the real time electricity prices.

2.4 Summary

In this research, we model an EV charging scenario in a parking lot with renewable energy sources, storage battery, and random arrivals and departures of EVs. We propose two strategies to charge multiple EVs in a commercial parking lot. In Case 1, we formulate an optimization problem that maximizes the use of available renewable energy for EV charging. Then we propose a control theory-based iterative solution through which we obtain optimal charging rate for all vehicles. We demonstrate the proposed approach shows a near-optimal solution and is robust to prediction errors. In Case 2, we formulate an optimization problem that maximizes profits for parking lot operators by coordinating EV charging with renewable generation, battery storage, and time-varying electricity prices. A LP-based solution for the formulated problem is proposed and the proposed algorithm is shown to reduce electricity costs associated with EV charging.
Chapter 3

Active Consumer-based Real-Time Electricity Pricing Scheme

This chapter proposes a game theory-based spatially varying optimal residential electricity pricing scheme that can be used to influence charging behavior of multiple electric vehicle (EV) consumers of a demographic area. The goal of this research is to achieve favorable load profile at the transformer that results into favorable load profile at higher levels of the power system.

3.1 System Model

We consider a scenario where $N$ feeders are connected to a transformer and each feeder is connected to $M_n$ number of homes. Figure 3.1 shows a transformer connected to $N$ feeders that are further connected to $M_n$ number of homes. We treat each feeder as a node. At $t^{th}$ time slot, $n^{th}$ node has some residential load ($L_n(t)$) and load due to EV charging, the sum of which is total load at node $n$. Here it must be noted that EV load is not considered as part of residential load ($L_n(t)$). Total residential load at $n^{th}$ node is sum of residential load
<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>denotes maximum allowable price change in one time-slot</td>
</tr>
<tr>
<td>$\theta_j(t)$</td>
<td>denotes EV charging status at time $t$</td>
</tr>
<tr>
<td>$\lambda_1, \lambda_2(t)$</td>
<td>denotes Lagrange multipliers</td>
</tr>
<tr>
<td>$C_n(t)$</td>
<td>denotes load correction factor at node $n$ at time $t$</td>
</tr>
<tr>
<td><strong>dom</strong></td>
<td>denotes domain of a function</td>
</tr>
<tr>
<td>$DG_n(t)$</td>
<td>denotes total distributed generation at node $n$ at time $t$</td>
</tr>
<tr>
<td>$DG_j(t)$</td>
<td>denotes total distributed generation at $j^{th}$ home at time $t$</td>
</tr>
<tr>
<td>$j$</td>
<td>denotes index for home in $n^{th}$ node</td>
</tr>
<tr>
<td>$l_j(t)$</td>
<td>denotes residential load of $j^{th}$ home in $n^{th}$ node at time $t$</td>
</tr>
<tr>
<td>$L_n(t)$</td>
<td>denotes total residential load of $n^{th}$ node at time $t$</td>
</tr>
<tr>
<td>$l_{EV_j}$</td>
<td>denotes maximum available charging rate for EV at $j^{th}$ home in $n^{th}$ node</td>
</tr>
<tr>
<td>$L_{EV_n}(t)$</td>
<td>denotes total EV load of $n^{th}$ node at time $t$</td>
</tr>
<tr>
<td>$L_{max_n}$</td>
<td>denotes maximum allowable load at node $n$</td>
</tr>
<tr>
<td>$M_n$</td>
<td>denotes number of homes connected to $n^{th}$ node</td>
</tr>
<tr>
<td>$n$</td>
<td>denotes index for feeder or node</td>
</tr>
<tr>
<td>$N$</td>
<td>denotes number of nodes connected to transformer</td>
</tr>
<tr>
<td>$\hat{p}_n(t)$</td>
<td>denotes real-time electricity price at time $t$</td>
</tr>
<tr>
<td>$p_n(t)$</td>
<td>denotes normalized value of real-time electricity price at time $t$</td>
</tr>
<tr>
<td>$p_{flat}$</td>
<td>denotes normalized value of flat rate electricity price</td>
</tr>
<tr>
<td>$t$</td>
<td>denotes index of time slot</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>denotes duration of each time slot</td>
</tr>
<tr>
<td>$T$</td>
<td>denotes total number of time slots</td>
</tr>
</tbody>
</table>

**Table 3.1: Notations**

36
of all $M_n$ homes ($l_n(t)$) connected to that node as shown in equation 3.1.

$$L_n(t) = \sum_{j=1}^{M_n} l_j(t), \forall t \tag{3.1}$$

We assume certain percentage of EV penetration in each node, but that percentage may differ per node. Using smart metering techniques it is possible to know which homes in the grid have EV charging station installed. Therefore, the exact number of EV charging stations for each node is assumed to be known. For this research, a fixed charging rate is assumed to be available to charge EVs (i.e., Level 1 charging). Therefore, at time $t$, each EV can be in one of the following states: charging at Level 1 charging rate or not charging. Total EV load at $n^{th}$ node ($L_{EV_n}(t)$) is the sum of all $M_n$ homes’ EV loads connected to that node:

$$L_{EV_n}(t) = \sum_{j=1}^{M_n} \theta_j(t) l_{EV_j}; \forall t, \tag{3.2}$$

where $\theta_j(t) \in \{0, 1\}$ and $L_{EV_n}(t)$ is total load due to charging of multiple EVs at node.
$n$ at time $t$, and $\theta_j(t) l_{EV_j}$ is EV load at each home. For homes without EV, $l_{EV_j}$ is set to zero, indicating no EV load. For homes with EV, $l_{EV_j}$ is maximum available charging rate and $\theta_j(t)$ indicates whether or not EV is charging, where $\theta_j(t) = 0$ indicates that EV is not charging and $\theta_j(t) = 1$ indicates that EV is charging. The key idea is when electricity prices are changed in real time, a subset of customers can respond to changes in electricity pricing by changing their charging behavior in real time. These customers are termed as “Active Consumers”.

3.2 Classification of Consumers

It is not necessary that all the customers with EVs will respond to changes in electricity prices in real time. Based on consumers’ response to real-time pricing, we classify consumers as Active Consumers or Traditional Consumers.

![Classification of Consumers](image)

**Figure 3.2: Classification of Consumers**

EV charging of Active Consumers is controlled by an automated device that optimally schedules EV charging at each home in order to minimize EV charging costs while ensuring that EVs are charged within available time frame. Therefore, throughout the entire charging period, the automated controller can turn charging on and off based on electricity pricing, allowing the Active Consumers to respond instantaneously to real-time electricity price changes by having their automated device turned on.

Traditional Consumers do not have an automated device installed in their home; instead
they manually manage their charging activity. Some Traditional Consumers schedule EV charging based on electricity price trends, but they cannot be as responsive to real-time price variations because they do not have an automated device; this type of "Traditional Consumers" are referred to as Price Aware Consumers. Other Traditional Consumers do not care about electricity prices and so they charge their EVs according to their preference. However, real-time pricing can be used to control the charging behavior of Active Consumers and to influence charging behavior of Price Aware Consumers.

### 3.3 Real-Time Electricity Prices

The primary objective of this research is to use real-time electricity pricing to influence EV charging behavior of consumers in order to allow utilities to reduce peak power demand and achieve favorable load profile at the transformer. Based on the price of electricity ($\hat{p}_n(t)$) at node $n$ during $t^{th}$ time slot, EV customers connected to node $n$ adjust their charging behavior. The electric utility is assumed to be able to have different electricity prices for each node (i.e., feeder). Based on present and predicted values of future residential loads, renewable generation and the number of EVs connected to a particular node utility can determine real-time electricity price. However, in practice, the cost of electricity is affected by many factors including generation cost, distribution cost, and demographic area. Determination of actual electricity value by accounting for all factors is out of the scope of this research. Therefore, normalized value of the price of electricity is used for analysis.

$$p_n(t) = \frac{\hat{p}_n(t) - p_{min}}{p_{max} - p_{min}},$$  \hspace{1cm} (3.3)

where $p_{max}$ and $p_{min}$ are maximum and minimum allowable values of electricity price in considered time window. Thus, when $\hat{p}_n(t)$ is at its minimum value, $p_n(t)$ goes to zero; when $\hat{p}_n(t)$ is at its maximum value, $p_n(t)$ goes to 1.

Active Consumers base their charging decisions on real-time pricing, energy demands
of vehicles, and time available for charging. When \( p_n(t) = 0 \) (i.e., electricity price is at its minimum value), most Active Consumers charge their EVs with probability 1; however, when \( p_n(t) = 1 \) (i.e., electricity price is at its maximum value), most Active Consumers do not charge their EVs. Therefore, in this research, normalized electricity pricing is treated as the probability of not charging for EV. Actual charging decision also depends on vehicles energy demands, time available for charging, residential load and Distributed Generation, all of which are unknown to the electric utility, in addition to real-time electricity price. In this study, EV charging decision is modeled as weighted Bernoulli random variable:

\[
\theta_j(t) \sim \text{Bernoulli}(1 - p_n(t))
\]

(3.4)

where \( p_n(t) \) is real-time electricity price during \( t^{th} \) time slot for homes connected to \( n^{th} \) node.

### 3.4 Algorithm and Flow Chart

In the real-time electricity price-based load management system proposed in this study, the electric utility decides optimal electricity prices at time \( t \) based on current and predicted value of future load and distributed generations at each node. Electric utility publishes real-time electricity values so that Active Consumers can make charging decision at time \( t \) based on predicted values of future electricity prices and future residential load, and distributed generation values. After Active Consumers make their charging decision based on electricity price at time \( t \), the electric utility calculates the difference between predicted value of load at time \( t \) and actual value of load at time \( t \) and uses this value as a correction factor \( (C_n(t)) \) for the next time slot. Based on predicted value of future load, future DG, and value of correction factor at last time slot, the electric utility decides optimal real-time price for the next time slot and publishes that price, allowing Active Consumers to make their charging decision. The process is repeated for each time slot. In summary, the charging behavior of Active Consumers and real-time variation in expected load result in changes in future
real-time electricity prices; consequently, changes in real-time electricity prices influence the charging behavior of Active Consumers, as shown in Figure 3.3.

3.5 System Constraints and Problem Formulation

In this study, the goal of the proposed problem formulation is to determine an optimal electricity pricing scheme that results in a favorable load profile at the transformer when the EV charging behavior of Active Consumers is influenced. However, the system constraint
and fairness factors have to be considered throughout EV charging, and the pricing signal must ensure that the load at any node does not exceed feeder capacity, as ensured by the following constraint:

\[
\left[ \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right] + L_n(t) - DG_n(t) \leq L_{max_n}, \forall n, t
\]  

(3.5)

where \( L_{max_n} \) is capacity of \( n^{th} \) feeder.

In order to be fair to consumers while implementing real-time pricing this study has to ensure that utilities on average are charging the same prices or less per kWh. The following constraint ensures fairness of pricing:

\[
\frac{\sum_t p_n(t) \left[ \left( \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right) + L_n(t) - DG_n(t) \right]}{\sum_t \left[ \left( \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right) + L_n(t) - DG_n(t) \right]} \leq p_{flat},
\]  

(3.6)

where \( p_{flat} \) is a reference electricity pricing that is a fixed flat rate. In order to be fair with Traditional Consumers who do not respond to electricity price in real-time, the price of electricity is prevented from changing abruptly within small period of time.

\[
|p_n(t + 1) - p_n(t)| \leq \alpha, \forall t, n
\]  

(3.7)

where \( \alpha \) is maximum allowable price change in one time slot. After modeling Active Consumers’ behavior, the system solves the following optimization problem in order to determine an optimal set of pricing signals:

Objective functions at the substation level is to minimize peak-to-average load ratio and achieve flatter load profile at the transformer.
minimize\(_{p_n(t)}\)

\[
\max_t \left[ \sum_{n=1}^{N} \left[ \left( \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right) + L_n(t) - DG_n(t) + C_n(t-1) \right] \right]
\]

\[
\text{avg}_t \left[ \sum_{n=1}^{N} \left[ \left( \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right) + L_n(t) - DG_n(t) + C_n(t-1) \right] \right]
\]

subject to

\[
\left[ \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right] + L_n(t) - DG_n(t) \leq L_{max_n}, \quad \forall n, t
\]

\[
\sum_t p_n(t) \left[ \left( \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right) + L_n(t) - DG_n(t) \right] \leq p_{flat}, \quad \forall n
\]

\[
(p_n(t) - p_n(t-1))^2 \leq \alpha^2, \quad \forall n, t
\]

At every time \(t\), an updated prediction of future load and correction factor utility decides the optimal pricing scheme based on current load and implement only current electricity prices. The utility is able to choose to publish the expected future electricity price trend along with current pricing in order to help EV consumers more accurately determine their charging strategy better.

3.5.1 Convexity Analysis:

This section proves convexity of the objective function. First we defined some theorems that will help us prove convexity of objective function.

**Theorem 1.** Non-negative weighted sum of convex function is convex \([41]\).

**Theorem 2.** If \(f_1\) and \(f_2\) are convex functions, then their point-wise maximum \(f\), defined by

\[
f(x) = \max \left\{ f_1(x), f_2(x) \right\},
\]

with \(\text{dom} f = \text{dom} f_1 \cap \text{dom} f_2\), is also convex \([41]\).
Therefore, \( \left( \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right) + L_n(t) - DG_n(t) + C_n(t - 1) \) is a linear function of \( p_n(t) \) for given \( n \) and \( t \). According to Theorem 1, \( \sum_{n=1}^{N} \left( \sum_{j=1}^{M_n} (1 - p_n(t)) l_{EV_j}(t) \right) + L_n(t) - DG_n(t) + C_n(t - 1) \) is also a convex function of \( p_n(t) \) at given time \( t \). Use of Theorem 2 demonstrates that the numerator of Equation 3.8 is also convex. Similarly, the denominator is Equation 3.8 is also convex because it is average of convex function over time. Therefore the objective function 3.8 is a convex function.

### 3.6 Active Consumer Behavior

To test effectiveness of proposed pricing strategy we check effect of the proposed pricing strategy on load profile. An optimal charging strategy for Active Consumers is developed in order to observe changes in load profile when customers charge EVs using the strategy based on given a pricing scheme. All Active Consumers make charging decision based on current electricity price and future price prediction in order to minimize the cost of EV charging. We use convex optimization as a tool to obtain the optimal charging strategy for Active Consumers.

The residential EV charging scenario for this study contains random arrival and departure time of EV, thereby creating a random time window for charging EV. When the consumer reaches home, the time of next departure is entered into the smart controller by consumer. Based on given arrival \( t_a \) and departure time \( t_b \), the smart controller calculates the total number of time slots available for charging. Therefore, arrival and departure times are considered to be known quantities for the purpose of analysis; however, for simulation purposes, arrival time and departure time are generated as random values. Each EV has unique demand of charge \( (DOC_j) \) that depends on the user’s driving and charging patterns. When an EV is charging, the smart controller is able to sense actual demand of charge; therefore, demand of charge is treated as a known quantity for analysis purposes; however, demand of charge \( (DOC_j) \) is modeled as a random variable for simulation purposes. Each
controller solves the following optimization problem below and decides the optimal charging strategy:

minimize $l_{EV_j}(t)$

$$\sum_{t=t_a}^{t_b} (p_n(t)(l_{EV_j}(t) + l_j(t) - DG_j(t))) \Delta t^2$$

subject to

$$\sum_{t=t_a}^{t_b} l_{EV_j}(t) \Delta t = DOC_j$$

$$l_{EV_j}(t)(l_{EV_j}(t) - r) = 0, \forall t$$

where $p_n(t)$ is real-time electricity price at node $n$ at time $t$, $DOC_j$ is demand of charge of EV that must be satisfied within available time frame from $t_a$ to $t_b$, and $DG_j(t)$ is available distributed generation at time $t$. Only one charging level $r$ kW, which is 120 V (3.3 kW) Level 1 charging, is assumed. The smart controller at home solves this problem at each time slot with length of $\Delta t$ duration, using predicted value of future distributed generation $DG_j(t)$ and electricity price $p_n(t)$.

The Lagrangian for the system can be written as

$$\mathcal{L} = \sum_{t=t_a}^{t_b} [p_n(t)(l_{EV_j}(t) + l_j(t) - DG_j(t))]^2$$

$$+ \lambda_1 \left( \sum_{t=t_a}^{t_b} l_{EV_j}(t) \Delta t - DOC_j \right) + \lambda_2 (l_{EV_j}(t)(l_{EV_j}(t) - r))$$

$$\frac{\partial \mathcal{L}}{\partial l_{EV_j}(t)} = (l_{EV_j}(t) + l_j(t) - DG_j(t))p_n^2(t) \Delta t^2$$

$$+ \lambda_1 \Delta t + 2\lambda_2 l_{EV_j}(t) - \lambda_2 r = 0$$
$$l_{EV_j}(t) = \frac{\lambda_2(t)r + (DG_j(t) - l_j(t))p_n^2(t)\Delta t^2 - \lambda_1\Delta t}{p_n^2(t)\Delta t^2 + 2\lambda_2(t)} \quad (3.12)$$

From constraint (2):

$$\lambda_2(t) = \frac{\lambda_1\Delta t - (DG_j(t) - l_j(t))p_n^2(t)\Delta t^2}{r} \quad (3.13)$$

or

$$\lambda_2(t) = \frac{\lambda_1\Delta t - (DG_j(t) - l_j(t))p_n^2(t)\Delta t^2 + rp_n^2(t)\Delta t^2}{3r} \quad (3.14)$$

From constraint (1):

$$\lambda_1 = DOC_j - \sum_t \frac{\lambda_2(t)r + (DG_j(t) - l_j(t))p_n^2(t)\Delta t^2}{p_n^2(t)\Delta t^22\lambda_2(t)} \frac{1}{\sum_t \frac{\Delta t^2}{p_n^2(t)\Delta t^22\lambda_2(t)}} \quad (3.15)$$

$$\lambda_1 = \frac{\Delta tDOC_j}{\Delta t + 1} \quad (3.15)$$

From this we can estimate how Active Consumers will react to proposed real-time electricity pricing can be made. However, it must be noted that although the charging strategy is deterministic at home level, the charging strategy at the transformer level is not deterministic due to lack of knowledge of EV’s available time frame for charging and demand of charge.

### 3.7 Simulation and Results

In order to test the proposed optimal real-time pricing scheme, a scenario of a residential area is created in which four feeders are connected with a transformer; each feeder is connected to 1000 homes. Random distribution of EVs in homes is adopted with different percentage EV penetration for each feeder. In this case, random distribution of EVs is considered in a geographical area; therefore, the number of EVs connected to a particular feeder is different,
which is known to utility. For this simulation, the numbers of EVs per feeder are 21, 43, 21, and 33. The scenario depicted a residential area for 8 hours with fluctuating electricity prices every 5 minutes (\(\Delta t = 5\) minutes). Simulation is conducted for 96 time slots.

The residential load of each home \(l_j(t)\) is modeled as a random variable with a trend component and some random noise above it [42].

\[
l_j(t) = \hat{l}_j(t) + n_j(t)
\]

(3.16)

where \(\hat{l}_j\) is a deterministic trend and \(n(t) \sim N(0, \sigma^2)\) is Gaussian noise. The trend component of residential load is \(\hat{l}_j(t) = 2.2 + 0.8\sin(1 + \frac{4\pi t}{3T})\) with a maximum value of 3 kW at peak load hours and a minimum value of 1.4 kW at night. The value of \(\sigma^2\) is set equal to 1, and the residential load of a particular home is never less than a specific value (1 kW).

In this game theory-based approach, the electric utility decides optimal pricing scheme based on current load at each feeder and predicted value of future load and then publishes these prices for customers. However, electricity prices may differ for customers connected to different feeders. Customers decide whether or not to charge based on current electricity prices and predicted future electricity prices. Customers’ reaction to the current electricity price utility determines optimal electricity price for the next time slot. In this case, real-time electricity prices influence the charging behavior of Active Consumers and Active Consumers’ reaction to real-time pricing influences the real-time electricity pricing.

Figure 3.4 shows simulation for one scenario in which the utility does not consider value of past correction factor while making the current decision. Therefore, the assumption was made that the utility has perfect knowledge of future residential load and customers have perfect knowledge of future electricity prices. Figure 3.5 shows simulation for a scenario in which the utility considers the value of the past correction factor while making a current decision. As shown in Figure 3.5, it is clear that performance is improved due to real-time feedback.

The proposed approach is then tested to determine if it would work with future load
prediction errors while real-time pricing is being decided. Therefore, independent identically distributed (i.i.d.) zero mean Gaussian noise is added to the value of future residential load and customers’ reactions to generated real-time electricity prices is observed. Figure 3.6 compares three cases: 1) when prediction is perfect (zero noise), 2) when noise variance is 30 kW, and 3) when noise variance is 50 kW. As shown in Figure 3.6, performance of the proposed approach degrades with prediction error. However, no new peaks are generated even when prediction noise variance is high.
Figure 3.5: Real-time electricity load and pricing considering correction factor $C_n(t)$

### 3.8 Summary

In this chapter, we model a geographically distributed residential area with renewable generation, and random distribution of Active Consumers in this region. A game theory-based optimal pricing scheme is proposed that can help utilities to achieve favorable load profile by influencing EV charging behavior of Active Consumers. The game theory-based approach is used for utility that measure electric load in real time and decides optimal electricity
price based on current load and predicted value of future load and generation values. Active Consumers utilizes a smart controlling device, manage their charging in response to real-time electricity pricing, and based on reaction from Active Consumers utility decides the optimal electricity price for the next time slot. Performance of the proposed real-time pricing strategy is verified via simulation, demonstrating that it works satisfactorily with noise in future prediction.

**Figure 3.6: Real-time electricity load with prediction error**
Chapter 4

Conclusion and Future Work

In this thesis, we develop coordinated charging strategies for multiple electric vehicles (EVs). The objective of this research is to coordinate multiple EV charging with renewable generation and real-time electricity pricing as well as to achieve favorable load profile for electric utility. In this chapter, we summarize the contributions of this thesis and discuss future research directions.

4.1 Summary of Key Contributions

This thesis develops optimal charging strategies for multiple EVs that have access to renewable energy source in addition to power from utility grid. In chapter 2, we model EV charging in a parking lot with solar and wind energy sources. First, we formulate an optimization problem that maximizes the use of available renewable energy for EV charging. Then we propose a control theory based iterative through which we obtain optimal charging rate for all vehicles. We demonstrate the proposed approach shows near optimal solution and is robust to prediction errors. Then, we extend this case and include economic aspects in it. We consider time varying electricity pricing and a storage battery in addition to previous formulation. We formulate an optimization problem that maximizes profits for parking lot
operators by coordinating EV charging with renewable generation, battery storage, and time varying electricity prices. We propose a linear programing-based solution for the formulated problem and show that the proposed algorithm reduces electricity cost associated with EV charging.

In chapter 3, we develop game theory based optimal pricing scheme where utility decides optimal electricity pricing in real time based on current load on power grid and Active Consumers make their EV charging decisions in real time based in electricity prices. We develop a system structure of demographic residential area where electric utilities can have spatial and temporal electricity pricing. The main objective of electric utility is to obtain a favorable load profile by minimizing peak-to-average ratio and to minimize EV charging cost for Active Consumers. This allows electric utilities to influence multiple EV charging by changing real-time electricity pricing.

4.2 Future Work

In this section, we present possible future research directions for optimal charging strategies for commercial parking lot and for active consumer based real-time pricing scheme.

- Simulations in section 2.3 considers a fixed battery storage. Our simulation results can be extended to check effects of changes in battery size on charging cost and finding optimal size of battery.

- Similarly, our simulation can be extended to check the performance of the algorithm with error in predicted values of future electricity prices and renewable generation.

- In our analysis chapter 3, we used convex optimization methods to find optimal pricing scheme for electric utility and optimal charging strategy for active consumers. Our analysis can be extended to reduce computational complexity by using model predictive control method.
Next, we present some new area of investigation in demand response (DR) in active consumers. The first dimension of work relates to residential demand response, which typically leverages price and generation forecasts to either shift or reduce load consumption in order to maximize some utility function (e.g., energy costs) under some constraints (levels of comfort/convenience).

- One approach is to implement direct load control where a utility or aggregator can remotely control certain loads in a household based on an a priori agreement. User privacy is the primary barrier for the large scale implementation of such direct load control methods.

- Alternately, smart pricing (e.g., critical peak pricing (CPP), TOU pricing and real time pricing (RTP)) can be used to encourage consumers to individually manage their loads.

- A plethora of deterministic centralized and distributed optimization, model predictive control, reinforcement learning as well as game theoretic methods have been proposed to attack these problems.

- Recently, there have been some efforts to systematically model the uncertainties in this framework and implement stochastic versions of optimization, dynamic programming, model predictive control methods.
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