HIGHER-LEVEL LEARNING IN AN ELECTRICAL ENGINEERING LINEAR SYSTEMS COURSE

By

CHEN JIA

B.S., Beihang University, 2007

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Electrical & Computer Engineering

College of Engineering

KANSAS STATE UNIVERISTY

Manhattan, Kansas

2015

ABSTRACT

Linear Systems (a.k.a., Signals and Systems) is an important class in an Electrical Engineering curriculum. A clear understanding of the topics in this course relies on a well-developed notion of lower-level mathematical constructs and procedures, including the roles these procedures play in system analysis. Students with an inadequate math foundation regularly struggle in this class, as they are typically able to perform sequences of the underlying calculations but cannot piece together the higher-level, conceptual relationships that drive these procedures.

This dissertation describes an investigation to assess and improve students' higher-level understanding of Linear Systems concepts. The focus is on the topics of (a) time-domain, linear time-invariant (LTI) system response visualization and (b) Fourier series conceptual understanding, including trigonometric Fourier series (TFS), compact trigonometric Fourier series (CTFS), and exponential Fourier series (EFS). Support data, including exam and online homework data, were collected since 2004 from students enrolled in *ECE 512 - Linear Systems* at Kansas State University. To assist with LTI response visualization, two online homework modules, Zero Input Response and Unit Impulse Response, were updated with enhanced plots of signal responses and placed in use starting with the Fall 2009 semester. To identify students' conceptual weaknesses related to Fourier series and to help them achieve a better understanding of Fourier series concepts, teaching-learning interviews were applied between Spring 2010 and Fall 2012. A new concept-based online homework module was also introduced in Spring 2011. Selected final exam problems from 2007 to 2012 were analyzed, and these data were supplemented with detailed mid-term and final exam data from 77 students enrolled in the Spring 2010 and Spring 2011 semesters. In order to address these conceptual learning issues, two frameworks were applied: Bloom's Taxonomy and APOS theory.

The teaching-learning interviews and online module updates appeared to be effective treatments in terms of increasing students' higher-level understanding. Scores on both conceptual exam questions and more traditional Fourier series exam questions were improved relative to scores received by students that did not receive those treatments.

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CHAPTER 1: INTRODUCTION

A. Project Overview

One primary goal of the projects that supported this work (NSF grants DRL–0816207 and DUE–0206943) was to create a knowledge base related to the evolution of students' problem solving skills. Over the course of these projects, the broader team has investigated the development and transfer of problem solving skills in undergraduate mathematics, physics, and electrical engineering courses at Kansas State University (KSU). The target course in the KSU Electrical and Computer Engineering (ECE) department was Linear Systems – a required course taken by all electrical engineering and computer engineering students that relies on the understanding of mathematical concepts learned in earlier courses, up to and including differential equations.

The problem solving skills of Science, Technology, Engineering, and Mathematics (STEM) students clearly develop and change during their education. However, this educational process is not necessarily a well-coordinated effort in which the complexity and type of problems change in an orderly fashion. In the limited scope of these short-term NSF projects, the team cannot investigate students' problem-solving characteristics and changes through these students' entire academic careers. However, it is sensible to piece together parts of this knowledge base by looking at the development of these skills in several related academic courses within the mathematics, physics and electrical engineering programs.

For example, in the KSU ECE curricula, the senior-level Linear Systems course is a late course that relies on fundamental math and physics knowledge. This course, described in the next section, was the host course for this work. A combination of qualitative and quantitative methods were applied in the research to (a) better understand where students struggle, (b) identify tools and methods that can be used to help students learn, and (c) increase the understanding level of signals concepts in this higher level class.

B. Linear Systems

Linear Systems is a required course in most electrical and computer engineering curricula that addresses subjects such as convolution, Fourier series, and continuous/discrete Fourier transforms. Linear Systems (a.k.a., "Signals and Systems") is often seen as a "weed out" class for electrical engineering programs and is dreaded by some students as such [1-3]. This course is widely perceived as useful but difficult, as the subjects tend to be higher-level concepts that rely on a well-developed understanding of lower-level mathematical constructs and procedures. The study of these topics requires a certain level of mathematical sophistication. Students with an inadequate mathematical foundation and a poor sense of the underlying systems theory regularly struggle with such subjects, as they are typically able to perform sequences of the underlying calculations but cannot piece together the higher conceptual relationships that drive these procedures. As a result, many students are unable to address exam questions and analysis problems that deviate from a solution recipe described in the textbook, and they often cannot explain how slight changes in mathematical renderings will affect system or signal behavior.

To improve students' learning experiences in Linear System courses, substantial research has addressed course content rearrangement, exam redesign and development, virtual laboratory environments, employment of computer software, etc. For example, since 1999, active learning techniques (e.g., concept tests) have supported student learning in the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology, and some of these techniques have been used in the Signals and Systems module. In Fall 2002, oral problems were introduced as part of the requirements in Signals and Systems. "Even though a wide range of active learning techniques were found to be effective in revealing some student misconceptions, the results of this study indicate that these techniques had limited power to expose the breadth and depth of misconceptions [4]" (excerpt shortened – see original source). Interviews revealed a more detailed set of student misconceptions and helped to uncover their origins. Such information can help teachers adjust instruction techniques and develop improved active learning methods, increasing students' conceptual knowledge [4].

At the University of California, Berkeley, Signals and Systems courses have been recently redesigned and have involved efforts to teach early parts of these courses completely in the digital domain. This is driven by the idea that some concepts are more sensible to introduce in a discrete-time setting [5]. Their approach to teaching Fourier analysis starts with rudimentary Cartesian vectors and builds up to complex Euclidean vector spaces. It identifies a geometric structure that brings together Fourier decompositions, with a benefit of reducing the need for lengthy derivations and algebra [6].

At Kansas State University, researchers began implementing more active and 'intentional' education techniques for Linear Systems in 2002, such as detailed score recording/analysis and online homework module development and implementation [7, 8]. Each semester, final scores consisted of points earned from homework, projects, three mid-term exams, and a final exam, where homework contained handwritten and online portions. Homework scores have been tallied per problem (versus per assignment), projects have been graded with detailed rubrics, and exam scores have been recorded question by question, yielding a substantial database of performance metrics to use for cross correlations and other analyses. This was an important first step towards analyzing students' overall performance and the specific areas where they students.

In Spring 2010, teaching-learning interviews were added to this KSU course, focusing on parameter variations in Fourier series [9]**.** Such interviews are a powerful method to capture a person's knowledge and the fluidity of his or her thinking [10]. Fourier series are important in a Linear Systems course, which connects the time domain to the frequency domain, and an understanding of the roles of Fourier series parameters, as well as the relationships between them, relates directly to a student's knowledge of these time-frequency concepts. Trigonometric Fourier series and compact trigonometric Fourier series were used as the starting point for this work, where a knowledge of parameter variations as they relate to changes in signal behavior are assumed to indicate higher-level understanding. Such parameter questions had existed in mid-term and final exams for a while, but in Spring 2010, these relationships were better formalized into exam problems that would allow students' understanding of these ideas to be more readily tracked. Finally, in the Spring 2011 semester, a new online homework module was developed and offered to students that focused on parameter relationships without the requirement for significant student calculations. The details of this work and the associated results are presented later in this dissertation.

CHAPTER 2: BACKGROUND

A. Fourier Series

A.1. Overview of Fourier Series Theory

Any periodic signal, $f(t)$, can be decomposed into a sum of sinusoids, each with a different amplitude, phase, and frequency. A **trigonometric Fourier series (TFS)**, $f_{TFS}(t)$, can be expressed as [11]

$$f_{TFS}(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)$$

Here,

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)dt$$

is the 'DC,' or average, value of the signal over a time interval of duration $T_0 = 1/f_0$ seconds, where $f_0 = \omega_0/2\pi$ is referred to as the "fundamental" frequency. The coefficients a_n and b_n represent the amplitudes of the cosines (even functions) and sines (odd functions), respectively, that constitute the signal. These coefficients are determined using the expressions

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\cos(n\omega_0 t)dt,\ n = 1,2,3,\ldots$$

and

$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\sin(n\omega_0 t)dt,\ n = 1,2,3,\ldots,$$

where n is an integer that represents the number of harmonics used to reconstruct the signal. The coefficients a_n and b_n are positive or negative real numbers. Also, note that if the original signal,

$f(t)$, is not periodic, the Fourier series approximation assumes periodicity outside of the original time range (i.e., for $t < t_1$ and $t > t + T_0$).

The information in a trigonometric Fourier series can be encapsulated in a set of coefficients, C_n and θ_n, that represent the magnitudes and phases of these sinusoidal components. The affiliated series is known as a **compact trigonometric Fourier series (CTFS)**, where the signal $f(t)$ is expressed as [11]

$$f_{CTFS}(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(2\pi n f_0 t + \theta_n)\,.$$

These compact coefficients are related to the original Fourier series coefficients through the following relationships:

$$C_0 = a_0,\; C_n = \sqrt{a_n{}^2 + b_n{}^2}\,,\ \text{and}\ \ \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right).$$

An **exponential Fourier series**, which uses exponential basis functions $e^{jn\omega_0 t}$ as the components for the signal representation, is described by the expression

$$f_{EFS}(t) \;= D_0 + \sum_{n=1}^{\infty} D_n e^{jn\omega_0 t} + D_{-n} e^{-jn\omega_0 t} \;=\; \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}, \quad t_1 \le t \le t_1 + T_0$$

where

$$D_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t) e^{-jn\omega_0 t}\, dt$$

The D_n coefficients for the exponential Fourier series are complex numbers: $D_n = |D_n|$ at $\angle D_n$ and $D_{-n} = |D_{-n}|$ at $\angle D_{-n}$. Further,

$$D_0 = a_0 = C_0\,,$$

$$|D_n| = |D_{-n}| = \frac{1}{2} C_n \quad (n \neq 0)\,,$$

$\angle D_n = \theta_n$ and $\angle D_{-n} = -\theta_n$, and

$$D_n = \frac{1}{2}\left(a_n - jb_n\right) \quad (n \neq 0)\,.$$

The index n for an exponential Fourier series is valid over the interval $-\infty \leq n \leq +\infty$: negative frequencies are present in the exponential Fourier series.

A.2. Fourier Series Parameter Concepts

The TFS parameters play clear roles in the signal shape/behavior, so when the signal shape changes in a certain way, the parameters will change in a commensurate manner. These changes can be identified by a student that understands sine/cosine properties and has a good understanding of Fourier series. For example, a_0 and C_0 represent the signal baseline, and if the signal maintains its shape but shifts up/down, only a_0 and C_0 will change. In this case, there is no need to recalculate these parameters using their definitions. Additional types of changes in signal behavior have straightforward effects on these coefficients. Such behavioral changes include vertical inversion, time scaling, amplitude changes, time inversion, and time shifts – see Table 1. These relationships were covered in lecture but were not always practiced with in-class examples. Since students need solid trigonometric and function knowledge, coupled with a good understanding of Fourier series, to explain these relationships, this topic space was considered a good arena within which to assess their higher level understanding.

Table 1. Signal behavior versus Fourier series parameter variations (N/C means 'no change').

Signal Behavior Change	Fourier Series Parameter Variations							
	a_0 & C_0	a_n	b_n	ω_0	C_n	θ_n	$\lvert D_n \rvert$	$\angle D_n$
Baseline shift	Shift	N/C	N/C	N/C	N/C	N/C	N/C	N/C
Vertical inversion	*(-1)	*(-1)	*(-1)	N/C	N/C	$\pm\pi$	N/C	$\pm\pi$
Time scaling	N/C	N/C	N/C	Scaling	N/C	N/C	N/C	N/C
Amplitude Scaling	Scaling	Scaling	Scaling	N/C	Scaling	N/C	Scaling	N/C
Time inversion	N/C	N/C	*(-1)	N/C	N/C	*(-1)	N/C	*(-1)
Time shift	N/C			N/C	N/C	±Scaling	N/C	±Scaling

Coefficient changes that result from changes in signal behavior are illustrated in the upcoming figures and the mathematical expressions that follow each figure:

- **Figure 1** – vertical inversion,
- **Figure 2** – time scaling,
- **Figure 3** – amplitude scaling,
- **Figure 4** – time inversion, and
- **Figure 5** – time shift.

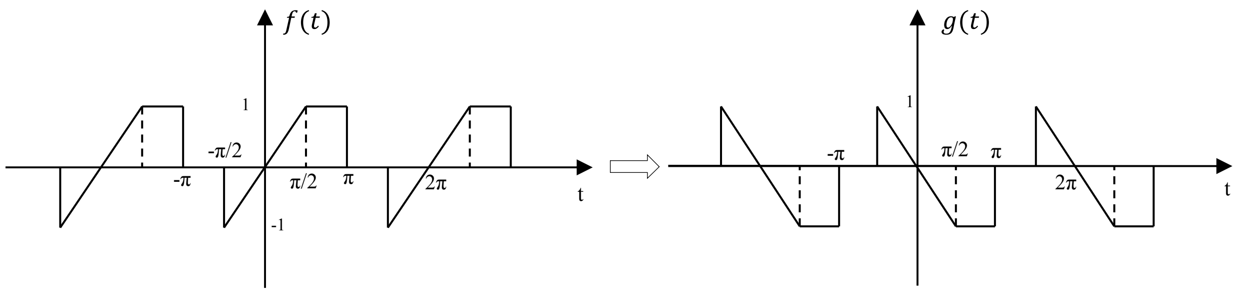

Figure 1. Vertical inversion example.

$g(t) = -f(t)$;

$g_{TFS}(t) = -f_{TFS}(t) = -(a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)))$; $2\pi f_0 = \omega_0$

$a_0' + \sum_{n=1}^{\infty}(a_n' \cos(n\omega_0' t) + b_n \sin(n\omega_0' t)) = -(a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)))$;

So, $\underline{a_0' = -a_0;\ a_n' = -a_n;\ b_n' = -b_n;\ \omega_0' = \omega_0}$

$g_{CTFS}(t) = -f_{CTFS}(t) = -(C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n))$;

$\sum_{n=1}^{\infty} C_n' \cos(\omega_0' nt + \theta_n') = -\sum_{n=1}^{\infty} C_n \cos(\omega_0 nt + \theta_n)$;

C_n is always positive or zero, so the sign change needs to be made on θ_n.

So, $\underline{C_0' = -C_0;\ C_n' = C_n;\ \theta_n' = \theta_n \pm \pi;\ \omega_0' = \omega_0}$

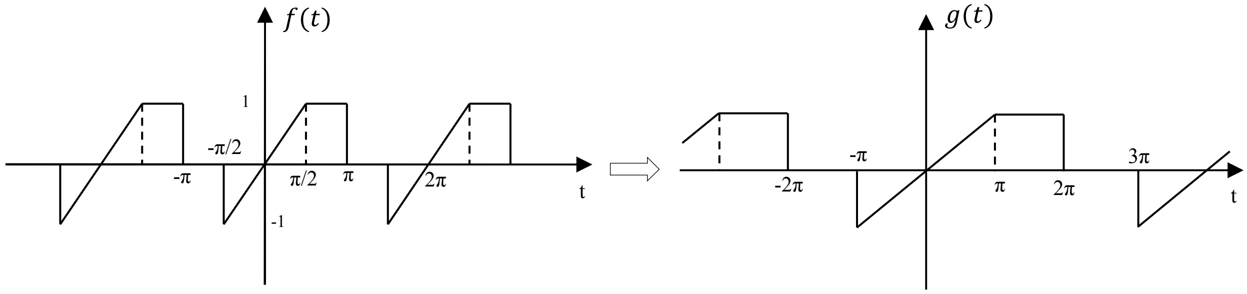

Figure 2. Time scaling example.

$$g(t) = f(t/2);\ g_{TFS}(t) = f_{TFS}(t/2) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega_0 t/2) + b_n \sin(n\omega_0 t/2));$$

$$g_{CTFS}(t) = f_{CTFS}(t/2) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t/2 + \theta_n);\ n\omega_0 t/2 = n(\omega_0/2)t;$$

So, $\underline{\omega_0' = \omega_0/2}$ is enough to specify the change.

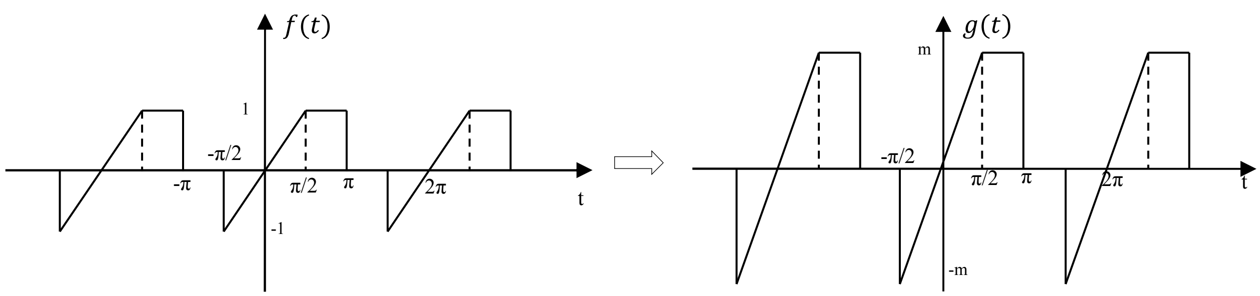

Figure 3. Amplitude scaling example.

$$g(t) = mf(t);\ g_{TFS}(t) = mf_{TFS}(t) = m \times (a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)));$$

$$a_0' + \sum_{n=1}^{\infty}(a_n' \cos(n\omega_0' t) + b_n \sin(n\omega_0' t)) = m \times (a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)));$$

So, $\underline{a_0' = ma_0;\ a_n' = ma_n;\ b_n' = mb_n;\ \omega_0' = \omega_0}$

$$g_{CTFS}(t) = mf_{CTFS}(t) = m \times (C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n));$$

So, $\underline{C_0' = mC_0;\ C_n' = mC_n;\ \omega_0' = \omega_0;\ \theta_n' = \theta_n}$

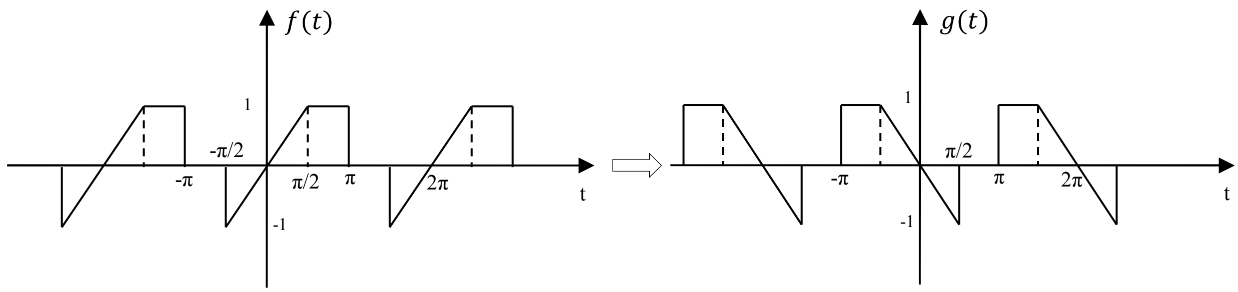

Figure 4. Time inversion example.

$g(t) = f(-t)$;

$$g_{TFS}(t) = f_{TFS}(-t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega_0(-t)) + b_n \sin(2\pi n f_0(-t)));$$

$$a_0' + \sum_{n=1}^{\infty}(a_n' \cos(n\omega_0' t) + b_n \sin(n\omega_0' t)) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega_0(-t)) + b_n \sin(n\omega_0(-t)));$$

So, $\underline{a_0' = a_0;\ a_n' = a_n;\ b_n' = -b_n;\ \omega_0' = \omega_0}$

$$g_{CTFS}(t) = f_{CTFS}(-t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0(-t) + \theta_n));$$

$$\sum_{n=1}^{\infty} C_n' \cos(\omega_0' nt + \theta_0') = \sum_{n=1}^{\infty} C_n \cos(-\omega_0 nt + \theta_n) = \sum_{n=1}^{\infty} C_n \cos(-(\omega_0 nt - \theta_0));$$

So, $\underline{C_0' = C_0;\ C_n' = C_n;\ \theta_n' = -\theta_n;\ \omega_0' = \omega_0}$

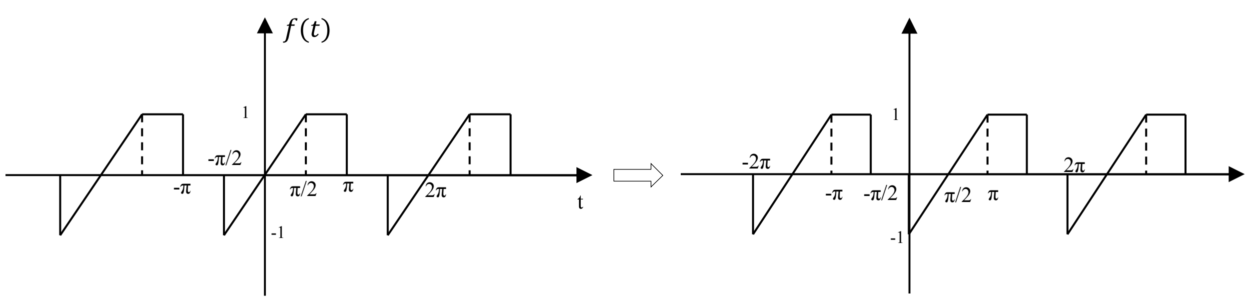

Figure 5. Time shift example.

$g(t) = f(t - \pi / 2)$;

In this case, it is difficult to achieve the change in signal behavior by making simple changes (by inspection) to the TFS parameters.

For the CTFS representation:

$$g_{CTFS}(t) = f_{CTFS}(t-\pi/2) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0(t-\pi/2)+\theta_n));$$

$$\sum_{n=1}^{\infty} C_n' \cos(\omega_0' nt + \theta_0') = \sum_{n=1}^{\infty} C_n \cos(\omega_0 n(t-\pi/2)+\theta_n) = \sum_{n=1}^{\infty} C_n \cos(\omega_0 nt + \theta_0 - \omega_0 n\pi/2);$$

So, $\underline{C_0' = C_0;\ C_n' = C_n;\ \theta_n' = \theta_n - \omega_0 n\pi/2;\ \omega_0' = \omega_0}$

B. Time-Domain Analysis of LTI Systems

Linear time-invariant (LTI) systems relate to applied mathematics and have direct applications in spectroscopy, circuits, signal processing, control theory, and other technical areas. A good example of an LTI system is an electrical circuit made up of resistors, capacitors, and inductors. Two methods of LTI system analysis are often applied: the time-domain method and the frequency-domain method [11]. In the time-domain method, the response of a linear system can be expressed as the sum of two components – the zero-input component and the zero-state component:

Total Response = Zero-Input Response + Zero-State Response.

The zero-input response is the system response when the input is zero, or $f(t) = 0$, which means the system output is the result of the initial system conditions alone. On the other hand, the zero-state response is the system response that results from the non-zero external input, $f(t)$, when the system is in a 'zero' state, meaning the internal energy storage is zero.

A linear differential equation consistent with the total response of such a system is introduced for the purpose of analysis. The input signal, $f(t)$, and the output signal, $y(t)$, are related through this expression:

$$(D^n + a_{n-1}D^{n-1} + \ldots + a_1D + a_0)\, y(t) = (b_mD^m + b_{m-1}D^{m-1} + \ldots + b_1D + b_0)\, f(t)$$

where D represents d/dt, yielding

$$Q(D)\, y(t) = P(D)\, f(t)$$

where

$$Q(D) = D^n + a_{n-1}D^{n-1} + \ldots + a_1 D + a_0$$

and

$$P(D) = b_m D^m + b_{m-1}D^{m-1} + \ldots + b_1 D + b_0.$$

Zero-Input Response

When the input signal, $f(t)$, equals zero, then

$$Q(D)\, y_0(t) = 0,$$

where the notation for the output signal, $y_0(t)$, is chosen to represent the system output due to initial conditions only. The notion that $y_0(t)$ should have the same form for all of its n successive derivatives yields

$$y_0(t) = ce^{\lambda t},$$

which is a solution to

$$(D^n + a_{n-1}D^{n-1} + \ldots + a_1 D + a_0)\, y_0(t) = 0.$$

Therefore,

$$Dy_0(t) = dy_0(t)/dt = c\lambda e^{\lambda t}$$
$$D^2 y_0(t) = d^2 y_0(t)/dt = c\lambda^2 e^{\lambda t}$$
$$\ldots\ldots$$
$$D^n y_0(t) = d^n\, y_0(t)/dt = c\lambda^n e^{\lambda t}$$

so

$$c\,(\lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0)\, e^{\lambda t} = 0$$

and

$$\lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0 = 0.$$

The left side of this equation has the same form as the polynomial $Q(D)$, yielding

$$Q(\lambda) = (\lambda - \lambda_1)\,(\lambda - \lambda_2)\,\ldots\,(\lambda - \lambda_n) = 0$$

The equation has n solutions (characteristic roots, or eigenvalues): $\lambda = \lambda_1,\ \lambda_2,\ \ldots,\ \lambda_n$, so $Q(D)\, y_0(t) = 0$ also has n possible solutions: $c_1 e^{\lambda_1 t}$, $c_2 e^{\lambda_2 t}$, … , $c_n e^{\lambda_n t}$, where c_1, c_2, … , c_n are arbitrary constants. A general solution is given by the sum of these n solutions as

$$y_0(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \ldots + c_n e^{\lambda_n t}$$

Repeated Roots

The solution above assumes that the n characteristic roots are distinct. If repeated roots occur, the solution form needs to be modified slightly as

$$y_0(t) = (c_1 + c_1 t + \cdots + c_1 t^{k-1})e^{\lambda_1 t} \; + \; c_{\tau+1}e^{\lambda_{\tau+1} t} \; + \; \dots + c_n e^{\lambda_n t}$$

with the characteristic polynomial

$$Q(\lambda) = (\lambda - \lambda_1)^k \, (\lambda - \lambda_2) \, \dots \, (\lambda - \lambda_n)$$

Complex Roots

The solution for complex roots is similar to that for real roots. For a real system, when the coefficients of $Q(\lambda)$ are real, the complex roots must occur in conjugate pairs. The zero-input response given a pair of complex conjugate roots can be presented as

$$y_0(t) = c_1 e^{(\alpha+j\beta)t} + c_2 e^{(\alpha-j\beta)t}.$$

For a real system, the response $y_0(t)$ must also be real, which requires c_1 and c_2 to be conjugates as well:

$$c_1 = \frac{c}{2} e^{j\theta} \text{ and } c_2 = \frac{c}{2} e^{-j\theta}$$

so that

$$y_0(t) = \frac{c}{2} e^{j\theta} e^{(\alpha+j\beta)t} + \frac{c}{2} e^{-j\theta} e^{(\alpha-j\beta)t},$$

or

$$y_0(t) = ce^{\alpha t} \cos(\beta t + \theta).$$

Unit Impulse Response

The unit impulse function, $\delta(t)$, is utilized to determine the response of a linear system to an arbitrary input, $f(t)$. The entire system response to an input can be seen as the sum of its responses to a collection of unit impulse functions that comprise the input, so if the system response to a unit impulse input is found, the system response to an arbitrary input can be determined. The following is a method to determine the unit impulse response, $h(t)$, of an LTI system described by the n^{th}-order differential equation

$$Q(D)\, y(t) = P(D)\, f(t)$$

where $Q(D)$ and $P(D)$ are the same polynomials as used above in the zero input response analysis [11]. The unit impulse response, $h(t)$, is given by

$$h(t) = b_n\delta(t) + [P(D)\, y_n(t)]\, u(t),$$

where b_n is the coefficient of the n^{th}-order term in $P(D)$, and $y_n(t)$ is a linear combination of the characteristic modes of the system subject to the following initial conditions:

$$y_n^{n-1}(0) = 1, \text{ and } y_n(0) = y_n'(0) = y_n''(0) = \cdots = y_n^{n-2}(0) = 0$$

This implies the following:

$n = 1$: $y_n(0) = 1$

$n = 2$: $y_n(0) = 0$, $y_n'(0) = 1$

$n = 3$: $y_n(0) = 0$, $y_n'(0) = 0$, and $y_n''(0) = 1$

…

If the order of $P(D)$ is less than the order of $Q(D)$, $b_n = 0$, and the impulse term in $h(t)$ is zero.

Given the unit impulse response, $h(t)$, the system's response to a delayed impulse, $\delta(t - n\Delta\tau)$, will be $h(t - n\Delta\tau)$, and the system's response to $[f(n\Delta\tau)\,\Delta\tau]\,\delta(t - n\Delta\tau)$ will be $[f(n\Delta\tau)\,\Delta\tau]\,h(t - n\Delta\tau)$. As a result, the zero-state response, $y(t)$, to the input, $f(t)$, is given by

$$y(t) = \int_{-\infty}^{\infty} f(\tau) * \mathrm{h}(t-\tau)d\tau$$

otherwise known as the convolution integral.

C. Online Homework Modules

Online education tools are more and more popular, since they offer flexible access to learning resources and help to maintain student engagement. Some of these resources, offered through universities and publishers, provide alternatives to traditional homework or are used in tandem with it [12-14]. Automatic grading is always implemented as well, which offers benefits for both students and instructors: the grading cycle is shortened (meaning students can get immediate feedback and guidance from the system), and instructors are released from grading, which is always time consuming. Database support offers researchers the opportunity to track learning elements that are not often recorded directly but may offer insight into learning [7, 15, 16].

In the KSU *ECE 512 – Linear Systems* course, online homework modules have been applied since the Spring 2004 semester [7, 8, 17]. The online system was developed with the thought that it would improve the homework experience and yield data sets useful to assess mathematical knowledge retention over multiple semesters. Prior to that work, and continuing to the present, the KSU Department of Mathematics has also developed and utilized online homework tools for trigonometry, calculus, and differential equations courses [15, 16]. To facilitate the assessment of knowledge retention between these early mathematics courses and follow-on linear systems courses in Electrical and Computer Engineering, the architecture of the software system for the mathematics modules was mirrored in the online homework modules for Linear Systems. Ideally, this architectural mapping could be applied to learning modules for numerous types of courses that rely on results in mathematical format, such as statistics, accounting, and so on.

The software architecture is shown in Figure 6. The main webpage is written primarily in PHP[18] embedded into HTML [19] scripts, which displays problem sets, receives student responses, and provides help [20]. In the back end, the grading parser is programmed in Java [21], and the database is constructed with PostgreSQL [22]. Some JavaScript [23] code, called as Java parser, is used for the main page to check expression syntax. The relationships between these languages are also depicted in Figure 6, as well as the software capabilities of the online system. On the left, the client tier represents the system users and requires an Internet browser, such as Internet Explorer (IE), Firefox, and Google Chrome. The Java runtime environment must be installed on the client computer. The middle (application) tier contains an Apache server [24], which fulfills basic web server requirements and supports PHP. The resource tier hosts the PostgreSQL database. The functionality of the online homework system consists of the problem generator, the expression parser, and the database. The problem generator creates problem parameters, prepares problem statements, determines solutions, grades answers, and creates help listings. The expression parser checks student input syntax and 'reads' the input functions. The PostgreSQL database stores student information, records student/system interactions, and saves problem set scores [7, 8, 17].

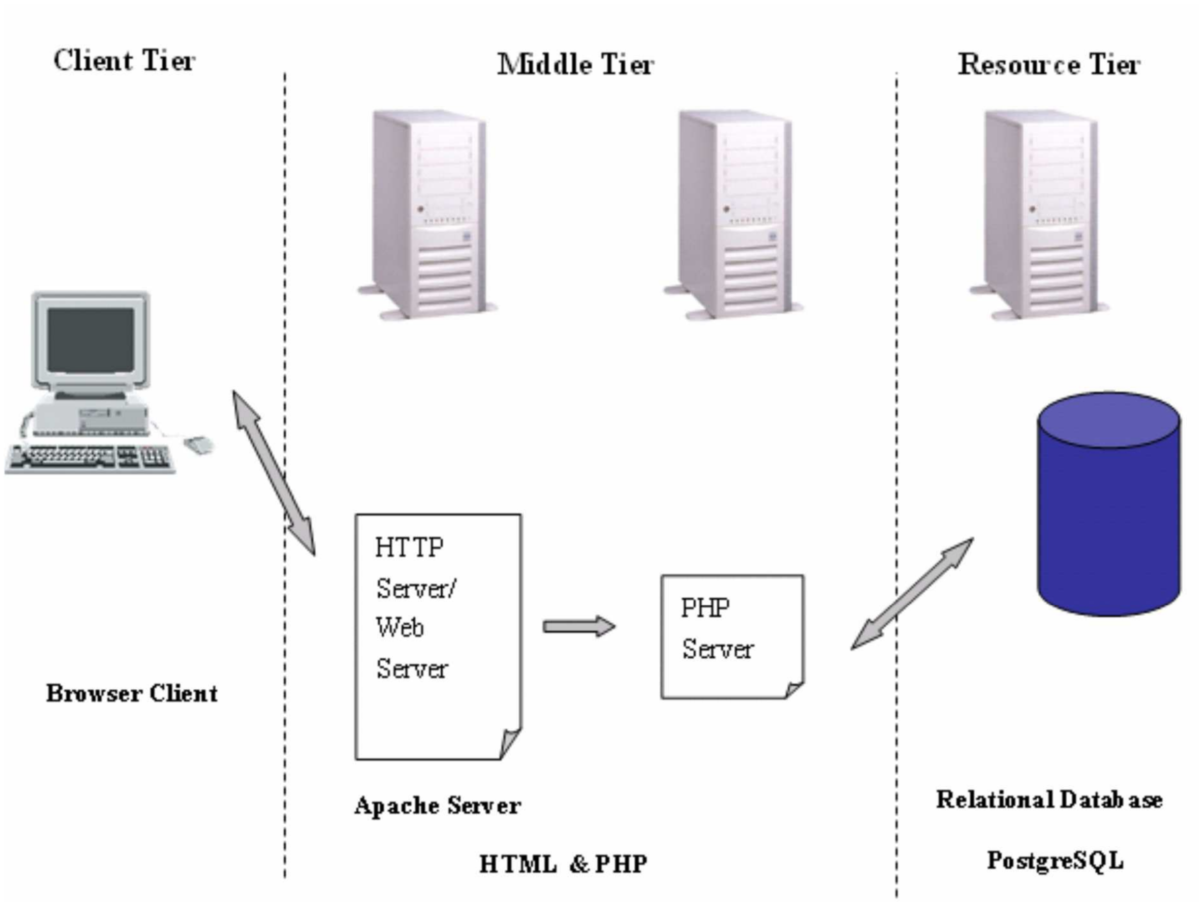

Figure 6. Online homework software configuration [20].

The module problems are similar to hand-written exercises employed in class. However, for each assignment, the online system generates a random set of problems that are unique to each student. Additionally, prior to the due date, a student can work as many homework sets as they desire (each one is new) until they obtain the desired score. Ten modules have been utilized to date [20]:

1. **Complex Arithmetic:** The first complex number module addresses multiplication, division, and complex number magnitudes in Cartesian coordinates.
2. **Complex Conversions:** The second module starts with Cartesian-polar conversions then finishes with a problem to merge sine/cosine functions at the same frequency into one cosine with a magnitude and a phase.

3. **Signal Graphing:** Each of three multiple choice problems gives a mathematical expression that is a combination of impulse, rectangle, exponential, sinusoidal, and unit step functions. The student must choose one graph of four that matches the expression.
4. **Zero Input Response:** This transient response module seeks the output expression for an unforced 2nd-order, time-invariant system. The differential equation and initial conditions are specified, and the student must enter the time-domain expression. Three problems address three types of eigenvalue (root) pairs: distinct real roots, repeated real roots, and complex roots.
5. **Unit Impulse Response:** The second transient response module seeks a system's unit impulse response given its differential equation. It addresses 2nd-order systems with distinct, repeated, and complex roots.
6. **Trigonometric Fourier Series:** Given three problems, the student must calculate the Fourier coefficients for the sine and cosine basis sets used to rebuild each function. Time-domain signals consist of even/odd functions built from pulse, saw tooth, parabola, triangle, trapezoid, ramp, and exponent functions. Coefficients are entered as expressions of *n*, the harmonic index.
7. **Compact Trigonometric Fourier Series:** This module assumes a basis set of cosines with different magnitudes and phases. The coefficients can be complicated expressions of *n*, so the students find and enter numerical coefficients for $n = 0$ to 3.
8. **Exponential Fourier Series:** This module uses reconstructions that employ complex exponential basis sets.
9. **Fourier Series Concepts:** This experience emphasizes changes in Fourier coefficients that occur as a result of changes in signal behavior.
10. **Discrete Fourier Transforms:** Given analytical signals, the student chooses sample rates and signal durations that retain important signal information.

As part of the recent research described in this dissertation, signal plots were added to the Zero Input Response and Unit Impulse Response modules to aid with visualization [8]. A new module focusing on Fourier series conceptual understanding (module 9 in the listing above) was inserted between the Compact Trigonometric Fourier Series module and the Exponential Fourier Series module. Details of this work will be introduced in Chapter 4.

D. Learning Theory/Framework Application

Learning frameworks for education have been discussed for over a century and should be part of any attempt to understand how material is learned and how changes in teaching methods can affect learning and retention. As noted in Dubinsky and McDonald [25], "models and theories in mathematics education can

- support prediction,
- have explanatory power,
- be applicable to a broad range of phenomena,
- help organize one's thinking about complex, interrelated phenomena,
- serve as a tool for analyzing data, and
- provide a language for communication of ideas about learning that go beyond superficial descriptions."

The Linear Systems course that supports this research is based on mathematics, and these six features map well to the facets of this work. To better understand how to address the important issues in this research, Bloom's Taxonomy [26] and APOS Theory [25] were applied. Both frameworks are briefly introduced in the next two sections, and elements of these frameworks will be applied to this research in Chapter 5.

D.1 Bloom's Taxonomy

Bloom's taxonomy presents a scheme to classify the various levels of cognition associated with learning and expertise [26]. It organizes cognitive ability and behavior into six levels of increasing abstractness or complexity (see Figure 7). Subject areas within a linear systems course often address multiple levels within Bloom's taxonomy simultaneously, which is in contrast to some courses leading up to linear systems which focus primarily on procedural calculations and plotting (Bloom's levels 1 through 3/4). Even in Linear Systems, traditional homework and exam questions can lean toward the lower levels of this taxonomy, and higher-level questions intentionally inserted into exams (to separate students that truly understand from those that do not) can draw complaints. In summary, students often struggle with, e.g., higher-level Fourier series concepts that are more consistent with levels 4 through 6, where they are asked to construct signals out of more rudimentary building blocks, assess changes in signal behavior due to changes in the associated coefficients, describe signal characteristics based solely on a depiction of the Fourier series coefficients versus frequency, etc.

1. **Knowledge**: arrange, define, duplicate, label, list, memorize, name, order, recognize, relate, recall, repeat, reproduce, state (***Remembering:*** *Can the student recall or remember the information?*)
2. **Comprehension**: classify, describe, discuss, explain, express, identify, indicate, locate, recognize, report, restate, review, select, translate (***Understanding:*** *Can the student explain ideas or concepts?*)
3. **Application**: apply, choose, demonstrate, dramatize, employ, illustrate, interpret, operate, practice, schedule, sketch, solve, use, write (***Applying:*** *Can the student use the information in a new way?*)
4. **Analysis**: analyze, appraise, calculate, categorize, compare, contrast, criticize, differentiate, discriminate, distinguish, examine, experiment, question, test (***Analyzing:*** *Can the student distinguish between the different parts?*)
5. **Synthesis**: arrange, assemble, collect, compose, construct, create, design, develop, formulate, manage, organize, plan, prepare, propose, set up, write (***Evaluating:*** *Can the student justify a stand or decision?*)
6. **Evaluation**: appraise, argue, assess, attach, choose, compare, defend, estimate, judge, predict, rate, core, select, support, value, evaluate (***Creating:*** *Can the student create a new product or point of view?*)

Figure 7. Classification levels in Bloom's taxonomy [9].

D.2 APOS Theory

APOS theory "begins with the hypothesis that mathematical knowledge consists in an individual's tendency to deal with perceived mathematical problem situations by constructing mental *actions*, *processes*, and *objects* and organizing them in *schemas* to make sense of the situations and solve the problems [25]." This hierarchy of mental constructions is therefore referred to as *APOS Theory*.

An *action* is a "transformation of objects perceived by the individual as essentially … step-by-step instructions on how to perform the operation [25]." That is to say, the individual can only do the calculation to get the correct result with the specific algorithms they have been introduced, but without any further thinking of the meaning of the steps and the ideas lying behind. For example, when the student is facing a question which seems familiar but doesn't know how to do it, he or she will go back to the catalog of procedures and find one that matches [27].

A *process* represents the level at “which the individual can think of as performing the same kind of action, but no longer with the need of external stimuli [25].” An individual at the process level of understanding can reflect on the process, describe the steps, and recognize the reason for steps [27]. As an example, to solve a problem, the student doesn’t need to check the notes or standard algorithms, but they can perform the correct procedures and calculations.

“An *object* is constructed from a process when the individual becomes aware of the process as a totality and realizes that transformations can act on it [25].” The individual can reflect on a set of similar processes and construct transformation on the concept [28]. All the steps can be rearranged or reversed, and the whole process can be viewed as an “input-output” process

Finally, “a *schema* for a certain mathematical concept is an individual’s collection of actions, processes, objects, and other schemas which are linked by some general principles to form a framework in the individual’s mind that may be brought to bear upon a problem situation involving that concept [25].” All of the related objects and concepts are interconnected in the individuals’ mind to form the schema; schemas can also perform as objects to be parts of higher schemas [28, 29].

APOS Theory is a description of the cognitive process and “can be used directly in the analysis of data by a researcher [25].” In this linear systems research, some students’ cognitive development seemed to be located between two of these levels depending on the topic. As a result, intermediate levels are introduced as well:

Action to Process: An individual moves back and forth between Action and Process [30]. This implies the individual still needs external guidance, such as a prescribed set of steps; however, s/he also has some understanding of the steps and can recognize mismatches between the example algorithms and the question of the moment, perhaps through mistakes that are made. However, the student cannot fully correct the mistakes without assistance.

Process to Object: The individual has a good understanding of all the steps and is able to rearrange the steps when necessary without checking their notes. S/he has the ability to make decisions at every step and may be able to construct concept transformations on some of the topics. However, the concept ideas are still not fully clear, and a bit of guidance is therefore needed to form some of these concept transformations.

Conceptual understanding was the focus of this linear systems research, and overall it appears that the object level is a suitable level to target for this type of course. To target the schema level, more analysis from later courses is required: courses for which Linear Systems is a prerequisite.

CHAPTER 3: TEACHING-LEARNING INTERVIEWS

Interactive teaching-learning interviews were conducted with Linear Systems students to gauge their levels of conceptual understanding with regard to the linear systems material, specifically in the area of Fourier series. This approach has the potential to identify student thought processes and areas of misunderstanding that may be difficult to observe from typical course examinations. This chapter addresses the interview method, the questions placed in front of the students, and the thematic areas where students appear to struggle when reasoning their way through Fourier series exercises.

A. Method

A.1 Overall Approach

Student Population and Interview Timing. The author interviewed one hundred and forty students in the Spring 2010, Spring 2011, Fall 2011, Spring 2012, and Fall 2012 sections of *ECE 512 – Linear Systems*, a course offered by the Kansas State University (KSU) Department of Electrical & Computer Engineering. These students were predominantly undergraduate seniors in Electrical Engineering or Computer Engineering. The interviews were conducted at the point in the semester (a) after the students had submitted Fourier series handwritten assignments, used the online linear systems modules, and taken exams on these same subjects but (b) before the final exam, implying that the students had absorbed the material to a level of understanding that would be typical at the end of a semester. All interviews were conducted over a period of two weeks just prior to the final exam for the course.

Interview Protocol. Each interview was conducted as a one on one, teaching-learning interview and was videotaped for follow-on analyses, where the camera was directed over the shoulder of the student so that it recorded video of only the work surface in front of them. Prior to each interview, the student signed a consent form (KSU IRB protocol #4691) stating their willingness to participate in this research exercise within the context of the overarching course experience. Four separate Fourier series problems were provided to each student (see the next section), where the student was asked to *describe their work out loud* as they progressed through each problem. Note that these interview components were not provided in interactive "question form" (questions followed by responses) but rather in "exercise form," as if the student was thinking out loud as they sat down to work a sequence of homework problems. When a student reached a point where their response was incorrect or they could not continue, the interviewer provided help/prompts in comment/question format. On average, the interview process took about an hour per student. Areas of conceptual misunderstanding were recorded both during each interview and during follow-on analyses of the video recordings.

Motivation for the Focus on Coefficient Roles. These interview problems were chosen to specifically address the roles of the Fourier series coefficients with respect to the shapes or behaviors of the reconstructed Fourier series. An understanding of these roles is an indication that a student has learned Fourier series concepts at a higher conceptual level, and past experience with exams that address Fourier series has taught the authors that describing these coefficient roles is a task where students begin to falter, even if they are adept at performing the calculations to determine the coefficients.

Interview Protocol and Updates. The interview protocol was updated twice. The interviews in the first two semesters (Spring 2010 and Spring 2011) shared the exact same protocol – protocol #1. The Fall 2011 and Spring 2012 protocols were the same (protocol #2), and then the interview questions were updated again for the Fall 2012 semester (protocol #3). The main part of the protocol was almost the same in all three cases, focusing on TFS and CTFS parameter variations and coefficient roles with regard to signal behavior (refer to the next section and the Appendices for specific problem descriptions). In the first update, one EFS coefficient question was added to the protocol, which had already incorporated TFS and CTFS coefficient exercises in prior semesters. In the second update, the relationships between the TFS, CTFS, and EFS coefficients were addressed. The first update was based on the nature of learning Fourier series, where students typically learn TFS and CTFS representations before moving to EFS problems. The second update to protocol #3 was driven by students' areas of doubt that were identified when working with protocol #2. In short, both updates were focused on the addition of EFS concepts, where the aim was to enhance each student's understanding of EFS coefficients roles. To control the interview time, the TFS and CTFS questions were scaled back, where the dropped questions covered areas where students struggled the least.

A.2 Interview Questions and Answers

Three versions of interview questions existed because of the two protocol updates. The questions listed in the following sections represent the entire question set and the associated answers; questions that were moved around are marked.

Problem #1: A trigonometric Fourier series is used to describe the signal $f(t) = t^2 - 2$ over the time range of t = [-2, 2] seconds (see Figure 8). Determine the trigonometric Fourier series, $f_{TFS}(t)$, for this signal. (Answers are listed in Table 2.)

- Before you start to solve the problem, estimate the sign of a_0.
- Can you describe how you solved the problem? (Use of even/odd symmetry or neither; the overall process; other details)
- Given $f_{TFS}(t)$, what is the value of $f(t)$ at $t = 0$?
- Assume the signal is represented as an Exponential Fourier Series (EFS), where the D_n coefficients contain both real and imaginary parts. Explain these coefficients in detail (added in Fall 2011).

Table 2. Answers for interview problem #1.

• Sign of a_0: negative
• $a_0 = -2/3$; $a_n = \frac{16}{n^2\pi^2}\cos(n\pi)$; $b_n = 0$.
• $f(0) = -2$. Note: $f(0) = -\frac{2}{3} + \sum_{n=1}^{\infty} a_n \cos(0)$ does not provide a direct result, so the student must understand the need to consult the plot rather than the Fourier series.
• $D_n = 0.5*a_n$ contains only real parts, because $b_n = 0$.

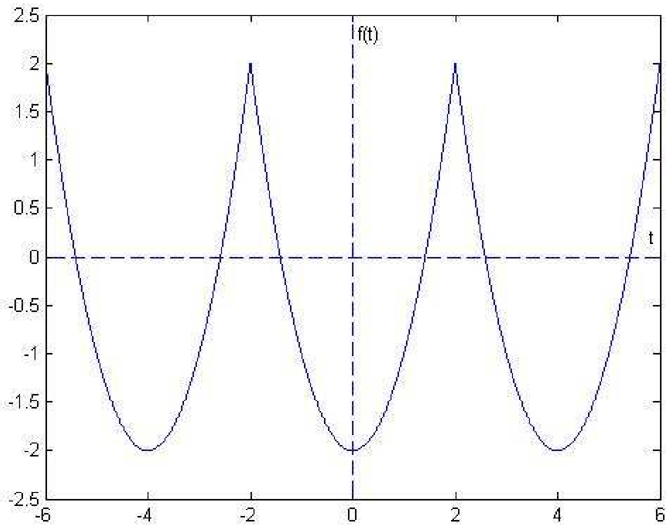

Figure 8. The parabolic function, $f(t) = t^2 - 2$, used in interview problem #1.

Problem #2 (Spring 2010 and Spring 2011): The parameters for $f_{TFS}(t)$ (a_0, a_n, and b_n) are known for the original signal in Figure 9a. Identify how the parameters ω_0, a_0, a_n, and b_n change if the original signal changes to each of the signals in Figure 9a, Figure 9b, and Figure 9c. (Answers are listed in Table 3.)

Figure 9b was deleted starting with the Fall 2011 interviews to accommodate the EFS additions to the protocol. Figure 9c was moved to Problem #4 in Fall 2012.

Table 3. Answers for interview problem #2.

Figure 9**b**	Figure 9**c**	Figure 9**d**
• $\omega_0' = \omega_0/2$	• $\omega_0' = \omega_0$	• $\omega_0' = \omega_0/2$
• $a_0' = a_0$	• $a_0' = a_0$	• $a_0' = a_0 + 1$
• $a_n' = a_n$	• $a_n' = a_n$	• $a_n' = a_n$
• $b_n' = b_n$	• $b_n' = -b_n$	• $b_n' = b_n$

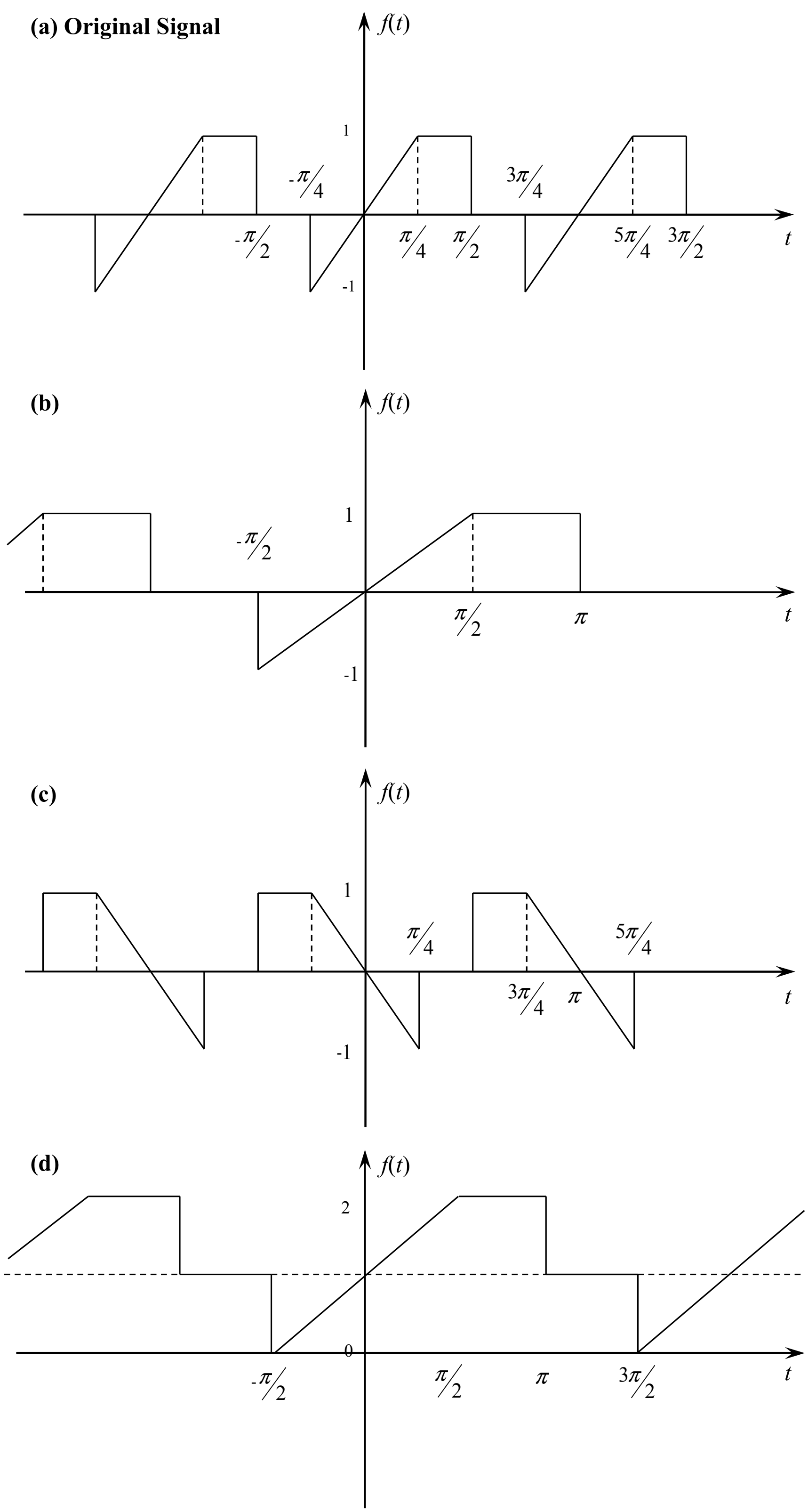

Figure 9. Signals for interview problem #2.

Problem #3 (Spring 2010, Spring 2011, Fall 2011 and Spring 2012 Version): The parameters for $f_{CTFS}(t)$ (C_0, C_n, and θ_n) are known for the original signal in Figure 10a. Identify how the parameters ω_0, C_0, C_n, and θ_n change given the signals in Figure 10b and Figure 10c. (Answers are listed in Table 4.)

Figure 10c was moved to Problem #4 in the Fall 2012 semester.

Table 4. Answers for interview problem #3.

Figure 10**b**	Figure 10**c**
• $\omega_0' = \omega_0$ • $C_0' = C_0$ • $C_n' = C_n$ • $\theta_n' = \theta_n - \frac{\pi}{4} n\omega_0$	• $\omega_0' = \omega_0$ • $C_0' = -C_0$ • $C_n' = C_n$ • $\theta_n' = \theta_n + \pi$

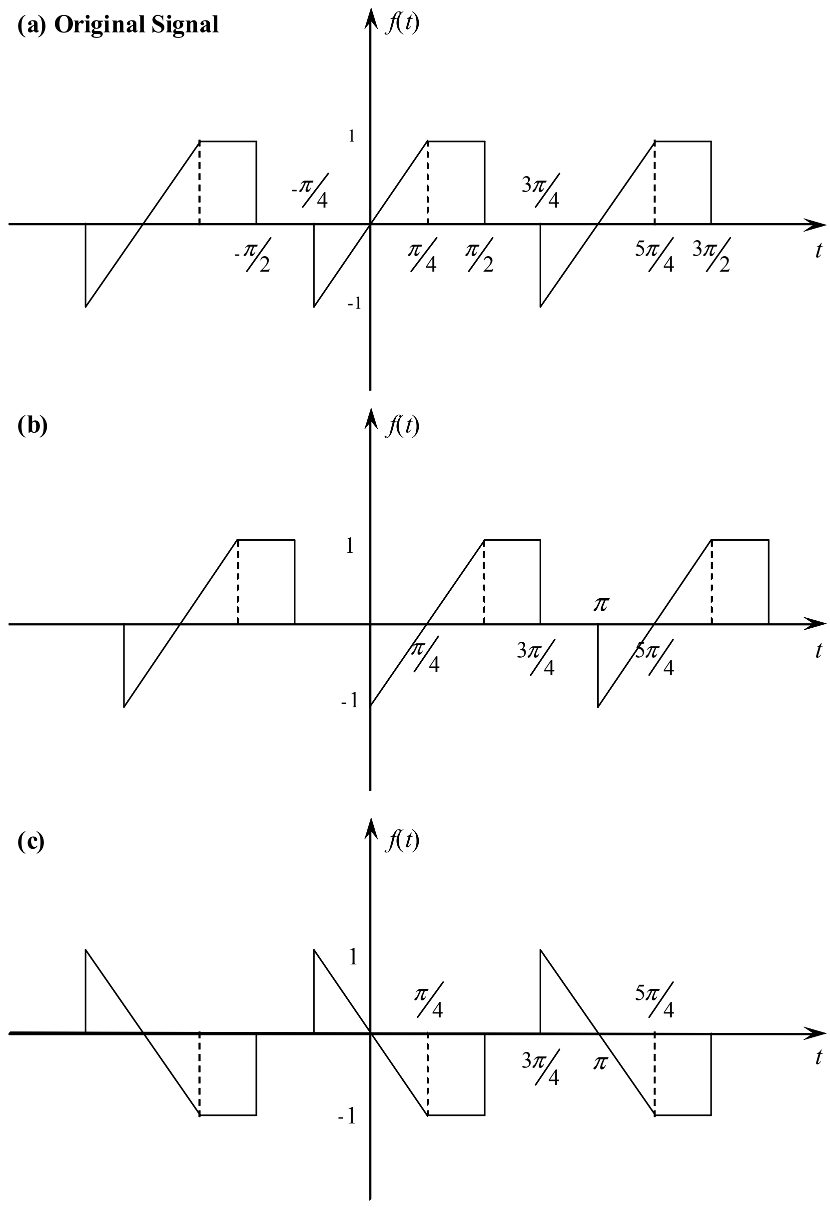

Figure 10. Signals for interview problem #3.

Problem #4 (Added in Fall 2012): In this problem, the original parameters are all known for the original signal in Figure 11a. Identify how all of the parameters in Table 5 change given the signals in Figure 11b and Figure 11c.

Table 5. Question format and answers for interview problem #4.

	Figure 11b	Figure 11c
ω_0'	$= \omega_0$ (N/C)	$= \omega_0$ (N/C)
a_0'	$= a_0$ (N/C)	$= -a_0$
a_n'	$= a_n$ (N/C)	$= -a_n$
b_n'	$= -b_n$	$= -b_n$
C_n'	$= C_n'$ (N/C)	$= C_n'$ (N/C)
θ_n'	$= -\theta_n$	$= \theta_n \pm \pi$
$\lvert D_n' \rvert$	$= \lvert D_n \rvert$	$= \lvert D_n \rvert$
$\angle D_n'$	$= -\angle D_n$	$= \angle D_n \pm \pi$
D_n'	$= D_n^*$ (Complex conjugate of D_n)	$= -D_n$

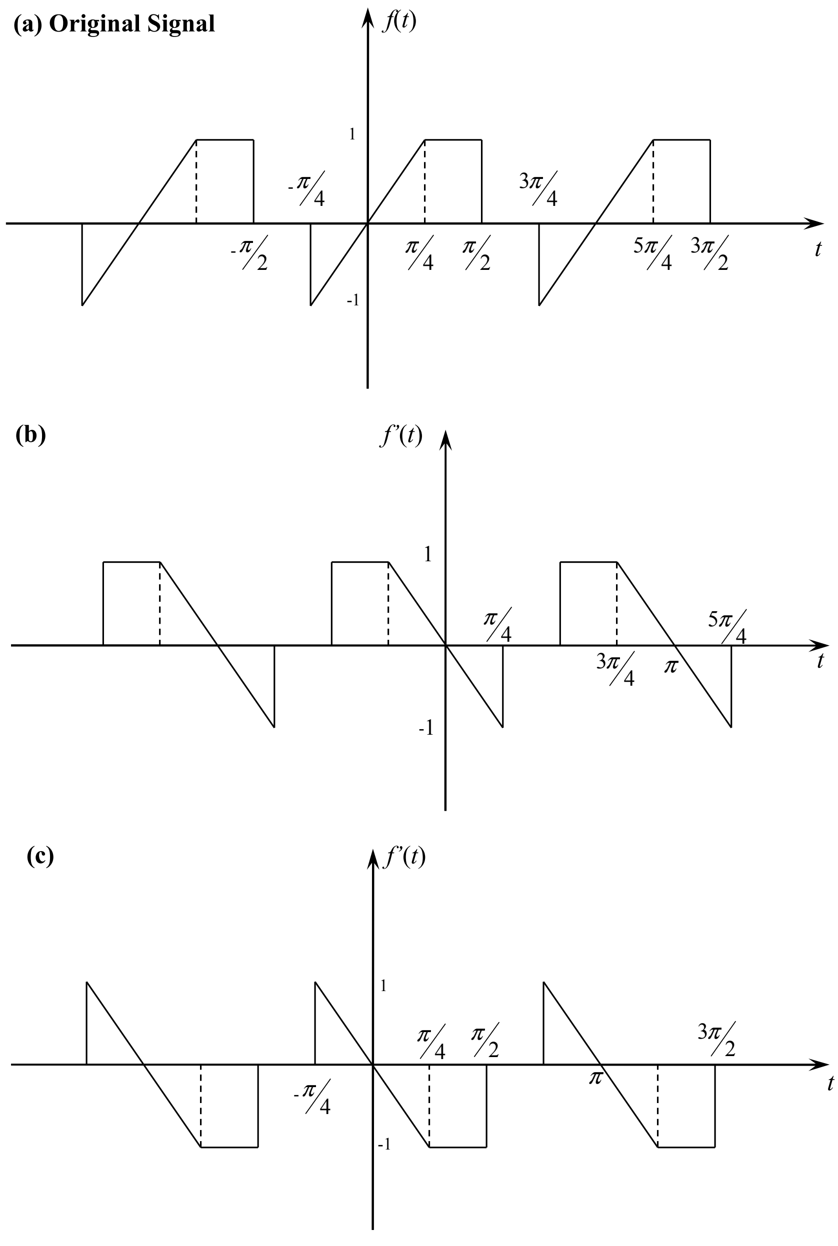

Figure 11. Signals for interview problem #4.

Problem #5 (Added in Fall 2012): A periodic signal is represented with a CTFS, and the plots of C_n and θ_n are given in Figure 12. Draw the plots of $\angle D_n$ and $|D_n|$. (Answers are noted in Figure 13.)

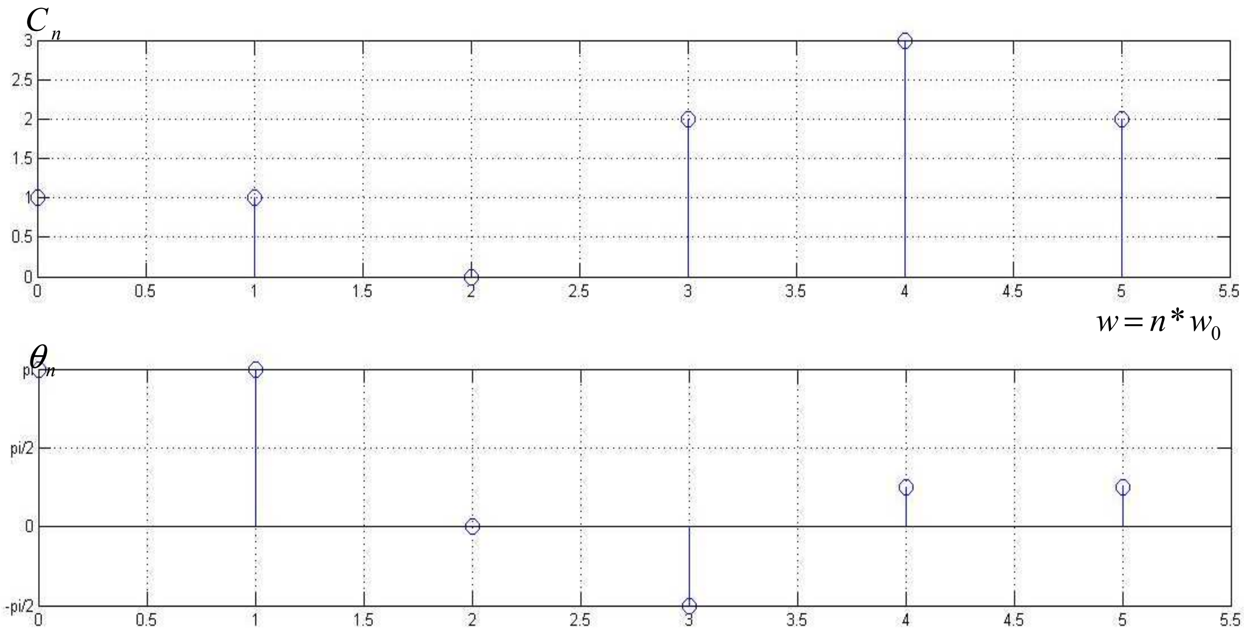

Figure 12. Coefficients for interview problem #5.

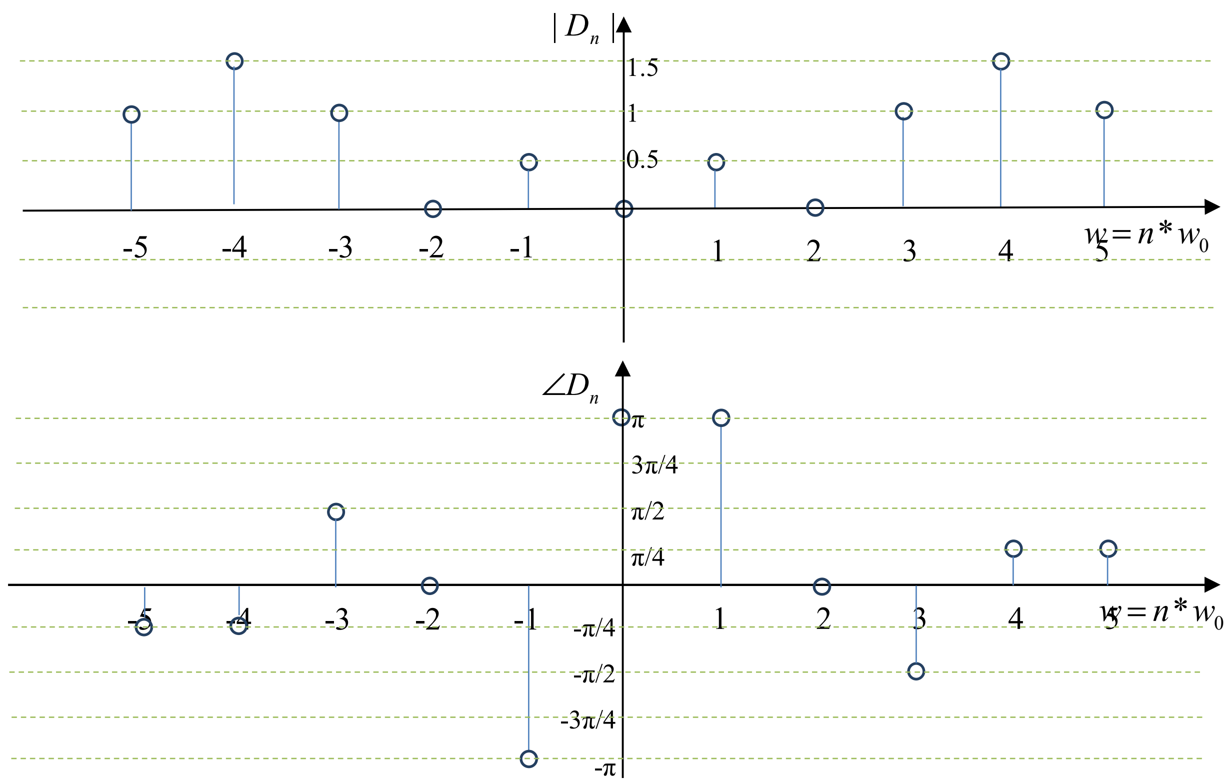

Figure 13. Coefficient solutions for interview problem #5.

Problem #6 (Problem #4 in Spring 2010 and Spring 2011): The parameters for $f_{CTFS}(t)$ (C_0, C_n, and θ_n) are known for the three original signals in Figure 14. If we wish to use these signals as building blocks to construct the signals in Figure 15, which signal(s) should we use? What changes in the respective $f_{CTFS}(t)$ parameters would be needed to make that happen?

One Acceptable Answer for Figure 15a. We choose the signal in Figure 14b to generate the signal in Figure 15a. In this case, the signal in Figure 14b will be used twice. First, an instance of the signal, f_1, can be flipped about the *t* axis, yielding $C'_{n1} = -C_{n1}$ (here $C_{01} = 0$). Then, the result will be delayed by half of the period (π/2 seconds in this case), which means a new phase $\theta'_{n1} = \theta_{n1} - \frac{\pi}{2} n\omega_0$. Another instance of the signal in Figure 14b, called f_2, can be added to f_1 to obtain signal f_3. Further, the amplitude of f_3 will be multiplied by ½, which means $C'_{n3} = \frac{1}{2} C_{n3}$. The final step is to raise the entire signal by ½, which means $C'_{03} = C_{03} + 1/2$.

One Acceptable Answer for Figure 15b. The signals in Figure 14a and Figure 14c can be used to generate the signal in Figure 15b given their period and duty cycle. The dashed lines in Figure 15b are drawn to assist the reader. The procedure is similar to that used for Figure 15a, only a bit more complex.

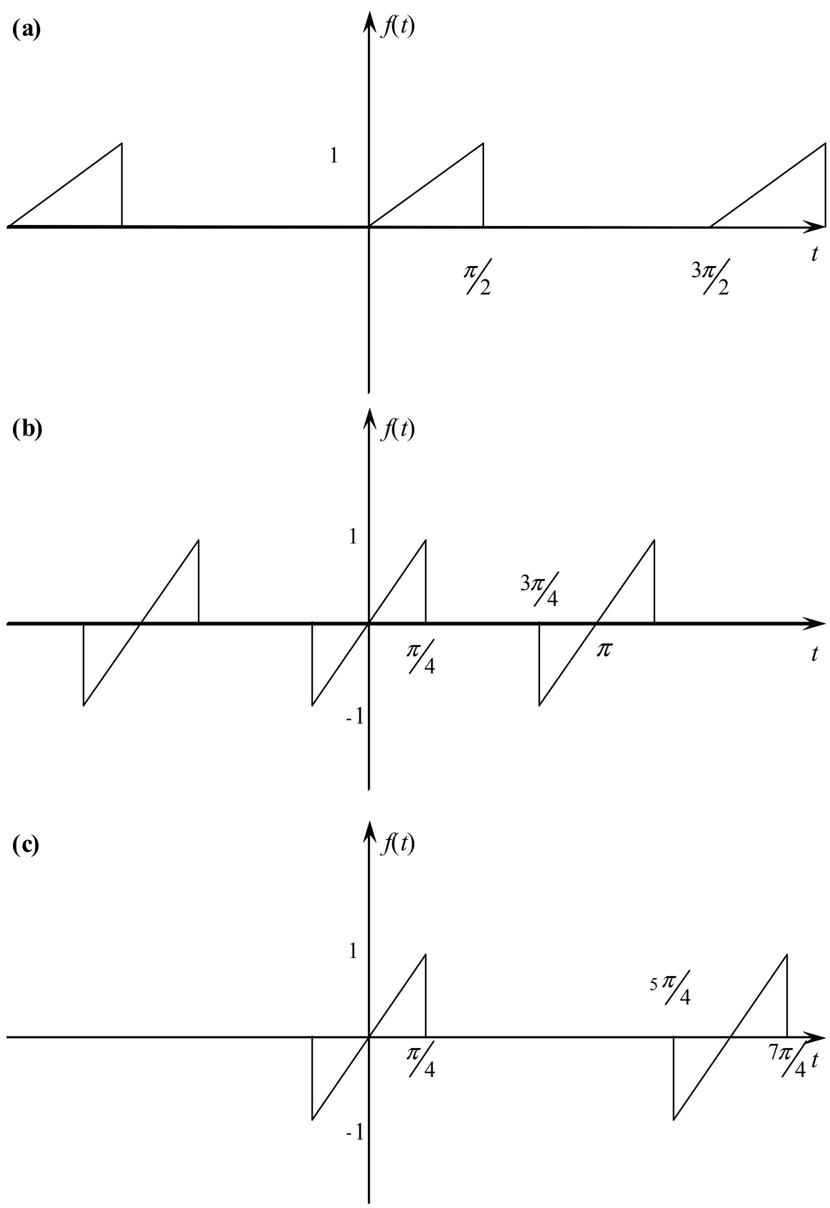

Figure 14. Building block signals for interview problem #4.

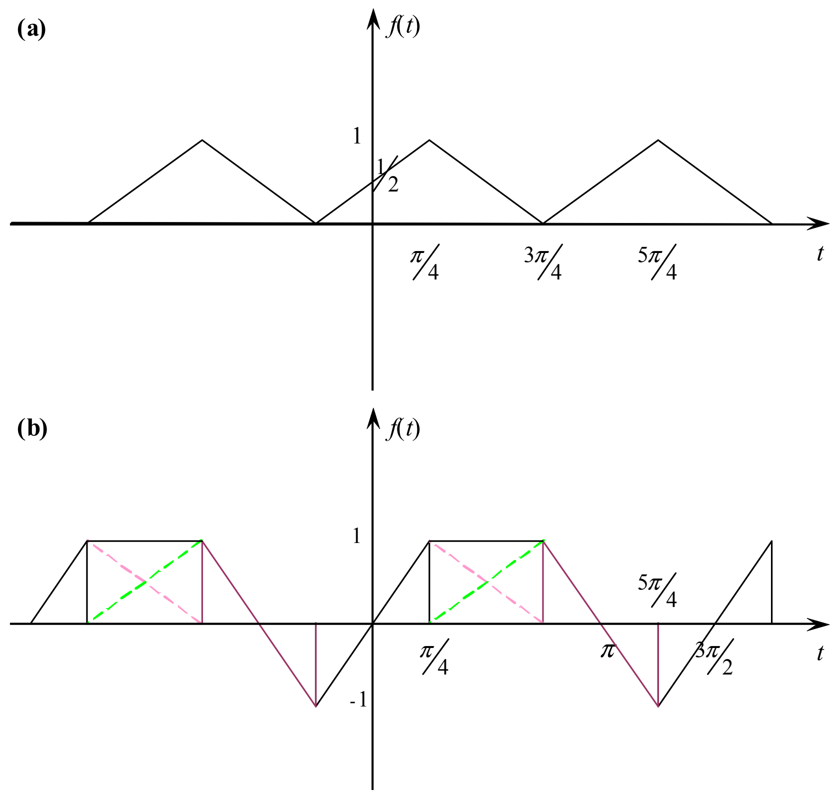

Figure 15. Target signals for interview problem #4.

B. Hints and Example Student Answers

Since these were teaching-learning interviews, hints and other necessary help were given to the students when they really struggled and were stuck in the interview sessions. In most cases, the answers and correct procedures were not given to the students directly, but related hints were provided. The main purpose of a hint would be to help a student recall the related domain rules or facts that the student may have misunderstood or forgotten. When a student gave unexpected answers, hints would help them find their mistake(s) and correct those answers. Hints could be general questions to help the students think or specific questions that more directly related to the areas of difficulty.

General hint examples:

- How do we get this answer?
- What equation should we use to derive the result?
- Can you explain the fomulas you are using?
- Can you think about this question from another perspective?
- Are you sure about the answer? If not, which part do you think might have some problems?

Specific hints could relate to specific questions. For example, referring to Problem #3 earlier in this chapter (original version - Figure 10), the correct answer should be

$$\omega_0' = \omega_0;\ C_0' = -C_0;\ C_n' = C_n;\ \theta_n' = \theta_n + \pi$$

One procedure could be the following:

- $g(t) = -f(t)$
- $g_{CTFS}(t) = -f_{CTFS}(t)$
- Same period, so $\omega_0' = \omega_0$
- $C_0' + \sum_{n=1}^{\infty} C_n' \cos(\omega_0' nt + \theta_n') = -(C_0 + \sum_{n=1}^{\infty} C_n \cos(\omega_0 nt + \theta_n))$
- $C_0' = -C_0$
- $\sum_{n=1}^{\infty} C_n' \cos(\omega_0' nt + \theta_n') = -\sum_{n=1}^{\infty} C_n \cos(\omega_0 nt + \theta_n)$

Because of the definition of C_n, where $C_n = \sqrt{a_n^{\ 2} + b_n^{\ 2}}$, C_n is positive or zero, so

$C_n' = C_n$, and $\cos(\omega_0' nt + \theta_n') = -\cos(\omega_0 nt + \theta_n)$

- To make the cosine function negative, we can apply $\theta_n' = \theta_n + \pi$.

The mistakes students often made when addressing problem #3 included the following:

1. When interpreting the figure, some students were not sure how the new signal related to the original signal: whether $g(t) = -f(t)$ or $g(t) = f(-t)$;
2. The baseline looked like it had a zero offset, which it did not, and some students would give the wrong answer of $C_0' = 0$;
3. To make the whole summation negative, some students would assign $C_n' = -C_n$, which was mathematically more straight forward. But, as explained above, C_n is the positive square root of $a_n^{\ 2} + b_n^{\ 2}$; we do not allow it to be negative. The way to make the summation negative is to induce a phase change in the cosine function.
4. Some students were unsure how to make the cosine function negative. For example, they did not think about the possibility of a phase change, or they would apply the wrong phase change, such as $+\pi/2$.

Hints provided to help the students address their mistakes:

1. For a general function, how does $f(t)$ behave compared to $-f(t)$? In our problem, could you find a method to double check your answer?
2. Back to the first problem, we talked about the physical meaning of an integral as "the area under the curve." Here, could you apply that knowledge and place a shadow inside the area we should focus on?
3. What is the defination of C_n? If C_n cannot be negative, what else we can do to make the summation negative?
4. In general, how do you negate a cosine function? If we draw a random cosine function and then draw the negative version of that signal, how much of a phase shift needs to be applied to accomplish this change?

Other frequently-used hints for the various questions follow.

Problem 1

- To estimate the sign of a_0 using the defination of a_0, we just need to find the sign of the integral. How can we find the sign of the definite integral?
- The definite integral of a function can be considered the area under the curve. Could you darken the area we should seek?
- What is the definition of even/odd functions?
- When we integrate an even function, can we apply techniques to simplify the calculation?
- How does D_n relate to a_n and b_n? (Added in the first update.)

Problem 2

- How has the new signal changed in comparison to the original signal?
- Mathematically, how should the function change to achieve this change in appearance?
- From the plot, the frequency of the signal has obviously changed: the signal looks stretched out. How does the frequency then change? Can we calculate the new frequency? Which parameter relates directly to the periodicity of the signal?
- The whole signal is shifted up by 1. How will the baseline, or a_0 in the TFS, change here?
- The original signal is represented in the TFS expression. How could the new signal relate to the original parameters? For example …

  $g_{TFS}(t) = -f_{TFS}(t)$;

  $f_{TFS}(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t))$;

  $g_{TFS}(t) = a_0' + \sum_{n=1}^{\infty}(a_n' \cos(n\omega_0' t) + b_n' \sin(n\omega_0' t))$;

  What is the relationship between the parameters of $g_{TFS}(t)$ and $f_{TFS}(t)$?

Problem 3

- There is a delay in the signal. What delay is it?
- How can we achieve this 'time delay?'
- We can change the phase θ_n to achieve the time delay. But, is the phase shift equal to the time delay?

- To shift the signal to the right, or delay it, how should the function change? $g(t) = f(t+a)$ or $g(t) = f(t-a)$ (assuming a is positive)?
- Is C_0 positive, negative, or zero? Could you draw a shadow on the area we should count as the 'area under the curve?' How then should C_0' change?
- What is the definition of C_n? If C_n cannot be a negative number, what can we do to make the whole result negative?
- How do we make a cosine function the negative of itself, meaning $-\cos(x) = \cos(??)$?
- Compare the two phase shifts. What are the differences between them?

Problem 4 (Some overlapping hints are not listed)

- How does D_n relate to θ_n and C_n? Then, how do C_n and θ_n relate to $|D_n|$ and $\angle D_n$?
- If two complex numbers have the same real part and opposite imaginary parts, what are these complex numbers are called?
- If two complex numbers have the same real part and opposite imaginary parts in Cartesian form, what are the relationships between their absolute values and angles?
- Could you draw a random complex number, such as $z = a + jb$, in both Cartesian form and Polar form?
- If $|D_n|$ remains the same and $\angle D_n$ shifts by π, how will D_n change?

Problem 5

- What is the relationship between D_0 and C_0? How about C_n and $|D_n|$?
- How do D_n and D_{-n} relate to each other?
- Does D_0 equal $|D_0|$? What if D_0 is negative? If we have a real number with a π phase shift, what does that mean? Is it still real? Could you draw the number on a polar coordinate system?
- Is $|D_n|$ even or odd? Is $\angle D_n$ even or odd? What if D_0 is negative?

Problem 6

- To build a new periodic signal based on the original one(s), the frequency is important. In our case, should the frequency be equal to the original one(s), or should be even or odd times of the original?
- An auxiliary line is provided to help the student – see Figure 16:

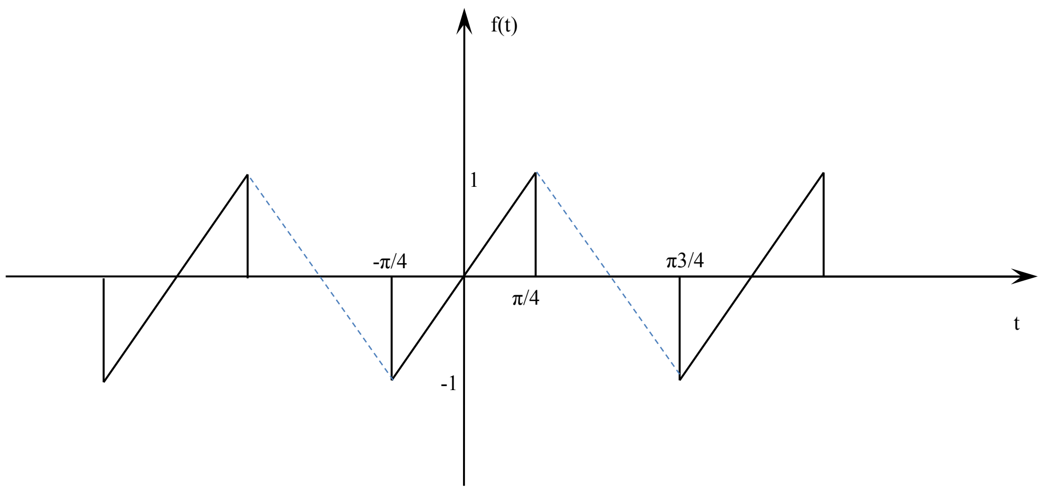

Figure 16. Auxiliary line to help the student solve Problem 6.

How can we get the auxiliary line? How can we get our desired signal from this?

- What can we do to change the frequency?
- How can we make a signal all positive? We cannot use the absolute value of the signal because the absolute value of an addition is not the addition of the absolute values ($|a+b| \neq |a|+|b|$).
- What are the final parameters of the new signal? Can you present these parameters in terms of the original parameters?

C. Interview Results and Discussion

C.1 Overall Themes: Concepts that Cause Students to Struggle

Notes taken during the individual interviews were analyzed to summarize the types of concepts that caused students to struggle and the types of hints that were often necessary to help students make progress on certain types of problems. Videos acquired during each of these sessions were also analyzed to corroborate and supplement these findings. This section provides an annotated listing of the types of concepts that were problematic for the students and the types of hints that were supplied to help them move forward. The number of students that struggled with each of the issue below are tallied as a function of semester in Table 6.

1. Physical meaning of the term 'integral.' For a one-dimensional function, the meaning of the term 'integral' is often defined functionally as accumulation but visually as 'the area under the curve', meaning the area between the curve and the independent axis. This is one concept that all students in an upper-level linear systems course should understand, as they have experienced it multiple times in various contexts. The first question in the first problem sought to assess this understanding when the interviewer asked students to estimate the sign of a_0 by looking at the $f(t)$ curve. Some of the students (about 1/5) had forgotten the meaning of 'integral' altogether, and another group of students (about 1/5) understood the concept but either did not know how to apply it or applied it in the wrong way for this problem, such as visualizing the area between the curve and minus infinity as a literal interpretation of 'area under the curve.'

2. Properties of even and odd functions. For a function with even symmetry, $f_e(t) = f_e(-t)$, whereas a function with odd symmetry has the property $f_o(t) = -f_o(-t)$. In a trigonometric Fourier series formulation, cosine (even) and sine (odd) functions specify the building blocks of the series and are paired with the coefficients a_n and b_n with the understanding that these coefficients specify the amplitudes of these basis functions. Students are instructed that if a function, $f(t)$, is even, then its Fourier series will only require a_n coefficients; if it is odd, only b_n coefficients are required. Even so, these interviews indicated that about 40% students still had trouble understanding the even or odd character of cosine and sine functions. For example, if t is changed to $-t$, these students had difficulty understanding the commensurate change in

$\sin(n\pi\omega_0 t)$ and $\cos(n\pi\omega_0 t)$ behavior and therefore the related changes to a_n and b_n. Regarding integration of an even function, as in problem #1, the definite integral from $-t$ to t should be twice the integral from 0 to t, while the definite integral of an odd function from $-t$ to t yields 0. When addressing integrals in these interviews, about 40% students did not use this concept to save time or used it in the wrong way, such as choosing wrong integration limits.

3. Properties of the sine and cosine functions. Other sine and cosine properties were also used in the interview problems. For example, when working with a compact trigonometric Fourier series representation, the C_n coefficient is defined as a magnitude (positive number), so when a term such as $C_n \times \cos(n\pi\omega_0)t$ is negated, the minus sign must be addressed through the angle of the cosine by changing $\cos(n\pi\omega_0)t$ to $\cos(n\pi\omega_0 t \pm \pi)$. 35% of the students struggled with this idea.

4. Inverse frequency/period relationships. One of the problems required each student to find the period and then the fundamental frequency (problem #1, question 2). Others addressed changes in period or frequency, such as problem #2 question 1 (see Figure 9b), which asked how the frequency would change if the signal was stretched to be doubly wide. The equations for the relationship between frequency and period, such as $\omega_0 = 2\pi f_0$ and $T_0 = \frac{2\pi}{\omega_0}$, were given on a formula sheet, yet 15% students still had trouble; some students were inclined to say the frequency also doubled.

5. Math-to-plot versus plot-to-math disconnect. Students seem uncomfortable establishing relationships between mathematical equations and plots and explaining changes in one given changes in the other. In problem #2, question 2 and problem #3, question 2, the plots were flipped about the vertical axis and horizontal axis, respectively. In the first case, most of the students could reason that the function changed from $f(t)$ to $f(-t)$, whereas a few students (about 10%) misunderstood. In the second case, some of the students described the result as $f(-t)$ rather than the correct response, $-f(t)$. If they needed a hint, students were asked to compare the previous $f(-t)$ plot with the plot in front of them and consider the differences. Eventually, most of the students came up with the correct answer, but about 80% students struggled with this concept at some level.

6. The mistaken equivalence between phase shift and time shift. In problem #3, question 1, the plot was moved π/4 seconds to the right in the time domain, so the CTFS function would become

$$f_{CTFS}(t-\pi/4) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0(t-\pi/4)+\theta_n) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n - \pi n\omega_0/4),$$

where the phase representation has changed to $\theta_n - \pi n\omega_0/4$. Most students specified a phase of $\theta_n - \pi/4$ as the answer, which implies their willingness to accept 'phase shift' as equal to 'time shift,' which is incorrect. This issue was arguably the most common mistake in the interview, as over 90% students did not get the right answer without a hint, and 5% of the students made an initial mistake but soon corrected themselves. Only 5 students out of over 100 interviewees answered this question completely correct.

7. Incorrect use of lookup aids. A few students (less than 10%) have trouble using lookup aids such as integral tables and Fourier series conversion tables. Both types of tables were made available during these interviews, yet two students stumbled by using the wrong integral table or wrong Fourier series equations, such as the use of $\int sinx\ dx$ instead of $\int xsinx\ dx$.

8. Inability to start in the middle. When addressing CTFS problems, some of the students feel the need to start back at the TFS representation and then move those coefficients into the CTFS and EFS (after the first update) domain, which inevitably adds calculations and therefore time. This usually leads to the correct answers but also implies a reliance on calculations and recipes rather than an understanding of the concepts of magnitude and phase.

9. Poor understanding of D_n's absolute value, $|D_n|$, and argument, $\angle$Dn (after the first update). About 1/3 of the students are not confident when using an absolute value and angle to present D_n, whereas most the students are able to use the Cartesian form: $D_n = ½(a_n - jb_n)$. These students also have doubts when using C_n and θ_n to represent D_n, and they have trouble building a direct connection between C_n and $|D_n|$ and between θ_n and $\angle D_n$. This is the main reason why the second update was implemented: to address their knowledge of the connections between parameters in different types of Fourier series.

10. Lack of clarity about the relationship between D_0 and C_0 (after the first update). Some students (about 10%) mistakenly apply the relationship between $|D_n|$ and C_n ($|D_n| = 0.5C_n$ for $n \neq 0$) to the relationship between D_0 and C_0 when drawing D_0 (Problem #5 in A.2). Most of these students could correct their mistakes with some hints, but the number of the students who have trouble with this is considerable.

Table 6. Concepts with which students struggled as a function of semester.

Semester (Population)	S10 (24)	S11 (24)	F11 (21)	S12 (14)	F12 (20)	Total (103)
Issue 1	10	9	9	5	7	40
Issue 2	11	9	8	6	7	41
Issue 3	6	12	8	5	5	36
Issue 4	5	3	2	1	4	15
Issue 5	20	15	16	10	17	78
Issue 6	24	22	20	13	19	98
Issue 7	2	2	1	2	1	8
Issue 8	2	1	3	2	3	11
Issue 9	N/A	N/A	N/A	6	6	12 (/34)
Issue 10	N/A	N/A	N/A	3	5	8 (/34)

C.2 Additional Notes

Unforeseen Benefits of Tutoring Sessions. One unforeseen benefit of this interview process was that, in some cases, the interview as planned turned into more of a personal tutoring session. This led to unsolicited feedback from many of the participants that indicated the hour-long interview was worth their time from that viewpoint alone, irrespective of the fact that they received course credit for participating in the interviews. More specifically, some students mentioned that the pace and feel of the interview were different from in-class learning (which would be expected), since they could work with the instructor individually and spend their interaction time on issues directly related to their areas of misunderstanding.

Findings Regarding Interviewer Help/Prompts. From these interviews, it is clear that most of the students have reached a satisfactory level of capability with regard to the types of mathematical calculations that one must perform in order to calculate Fourier series representations of signals. Based on students' responses to the help, most of their issues with learning Linear Systems were not problems associated with basic mathematics knowledge, but rather with the Linear Systems course material only.

Generalizations of Areas Where Students Struggle. Section C.1 noted specific areas where students struggled within the context of the Fourier series problems presented in the interviews. The following listing seeks to generalize and expand upon these areas of struggle with the thought that more overarching changes in pedagogy might be applied to address them.

- Students often have difficulty 'seeing' the relationship between (a) mathematical representations and signal features and (b) changes in mathematical representations as they relate to changes in the visual appearance of a signal.
- The general issue of performing the mathematical operations versus understanding their impact or purpose is an important discussion point. For signals constructed from basic functions, students have trouble getting past the details of a mathematical process that employs basis functions so that they can visualize the way in which these signals are constructed from those fundamental building blocks. In the case of Fourier series, these basis functions are cosines and sines, but other basis sets exist (e.g., t^n for polynomial functions).
- Visually pulling a signal apart can be a struggle for some students. For example, in a Fourier series context, it is hard for some students to visually remove the baseline (even component) of a signal and realize that the remaining signal may actually have odd symmetry on its own.
- Presenting a student with a mathematical shortcut does not ensure that they will understand when its use is or is not justified. This is demonstrated in Fourier series calculations by the use of symmetry to shorten the coefficient calculation process.
- The inverse relationship between time and frequency is always an issue. This issue not only relates to the misperception that a wider sinusoid means a higher frequency, but it includes

misperceptions such as (a) making a signal longer increases its bandwidth as represented by its TFS coefficients, or (b) (in the discrete domain) sampling a signal more quickly improves the resolution of the coefficients in the frequency domain.

- The mistaken equivalence between time shift and phase shift speaks to students' fundamental misunderstandings about Fourier series. If a waveform is shifted, then all of the sinusoids that comprise that waveform must also be shifted. These sinusoids are at different frequencies, yet they must retain alignment relative to one another in order to retain the overall signal shape, so each sinusoid (building block) experiences a different phase shift, even though the time shifts are all equal.
- Students find the absolute value and angle form (i.e., the polar form) of a complex number to be uncomfortable. They prefer to use the Cartesian form, $z = a + jb$, and calculate the absolute value and argument from this. When plotting a complex number on a set of coordinate axes, the polar form is seldom used, even if these numbers are originally given in polar form. For example, when asked to draw a stem plot of D_n based on C_n and θ_n, some students need to convert C_n and θ_n back to a_n and b_n first instead of using $|D_n|$ and $\angle D_n$, which come from C_n and θ_n more directly.
- When students do not know quite how to proceed, they fall back on process and recipe rather than think about the problem at a high level. For example, to describe the change in phase due to a time shift, most would be more comfortable recalculating the Fourier C_n and θ_n values from scratch rather than reason through the change in coefficient values.

CHAPTER 4: ONLINE HOMEWORK MODULE ENHANCEMENT

A. Signal Visualization in the ZIR and UIR Modules

The former instantiations of the zero-input response (ZIR) and unit-impulse response (UIR) modules required homework-like solutions, where mathematical answers were entered into text fields. Updates to these modules include plotting facilities in the help files that graphically depict the solution for the system response, note the initial conditions on the curve, and mark features such as bounding curves and zero crossings.

An example zero-input response for an underdamped system is plotted in Figure 17. Note that the solid red line representing the response is a dashed line for $t < 0$ to indicate that the signal does not exist prior to time zero. Bounding exponentials are marked as gray dashed lines. Zero crossings are marked with red circles, and the initial condition and slope at time zero are represented by the dashed green line. Colors of the expressions in the text above the plot are matched to the colors of the corresponding curves/marks.

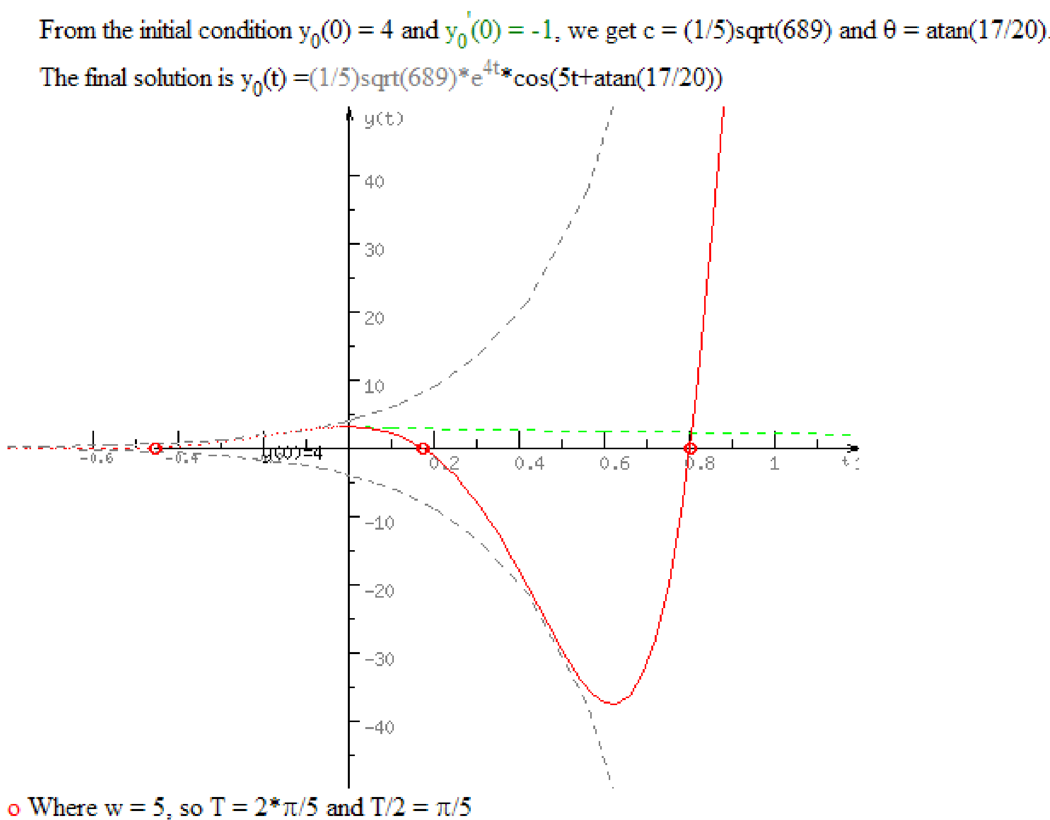

Figure 17: **Output signal visualization for a zero-input response problem.**

These visualizations are intended to offer the students a better understanding of the ZIR and UIR signal features relative to the mathematical expressions that describe them. These include the signal envelopes, relative time delays, zero-crossing points, bounding curves, signal trajectories, and the roles that the pieces of the mathematical expressions play in the shapes of the curves.

B. New Homework Module for Fourier Series Conceptual Understanding

B.1 Protocol and Timing

As mentioned above, during the Spring 2010 semester, 24 students enrolled in Linear Systems were interviewed, and the interview results indicated that the students' understanding of Fourier series concepts needed to improve. Based on the problems the students faced during the interviews, a new module was added to the online homework set after the TFS/CTFS modules. In this module, the students are offered an experience related to TFS/CTFS ideas that does not require computations but rather focuses on concepts. Each question asks the student to identify the differences between an original signal and a signal generated from it, then specify changes to the main parameters (based on the original parameters) that are required to realize the new signal. The goal is to help students understand each parameter's contribution to the entire signal as well as the relationship between the graphical representation and the functional representation, ideally improving their conceptual understanding and moving them to, e.g., a higher conceptual level within to Bloom's taxonomy.

B.2 Questions and Solutions

The new module contains three questions that cover basic changes in signal appearance and the resulting parameter variations. These questions are similar to the major part of the interview questions about the parameter variations, but the shapes of the signals are generated randomly (neither even nor odd), and the variations are randomly generated and combined. In this case, it is hard to address these parameter-variation questions just by memorizing answers to previously generated problems. Care was taken to avoid combining too many parameter changes in one question, which could cause misunderstanding and possibly frustration and guessing. To make each variation obvious and clear to the students, every question involves only one or two shape changes.

Example questions are illustrated in Figure 18, Figure 19, Figure 20, and Figure 21. All of these signals are periodic: the first three are depicted in TFS form, and the last is depicted in CTFS form. Students need to fill in the blanks as shown. The solution procedures are similar to the examples utilized in the interview session in Chapter 3.

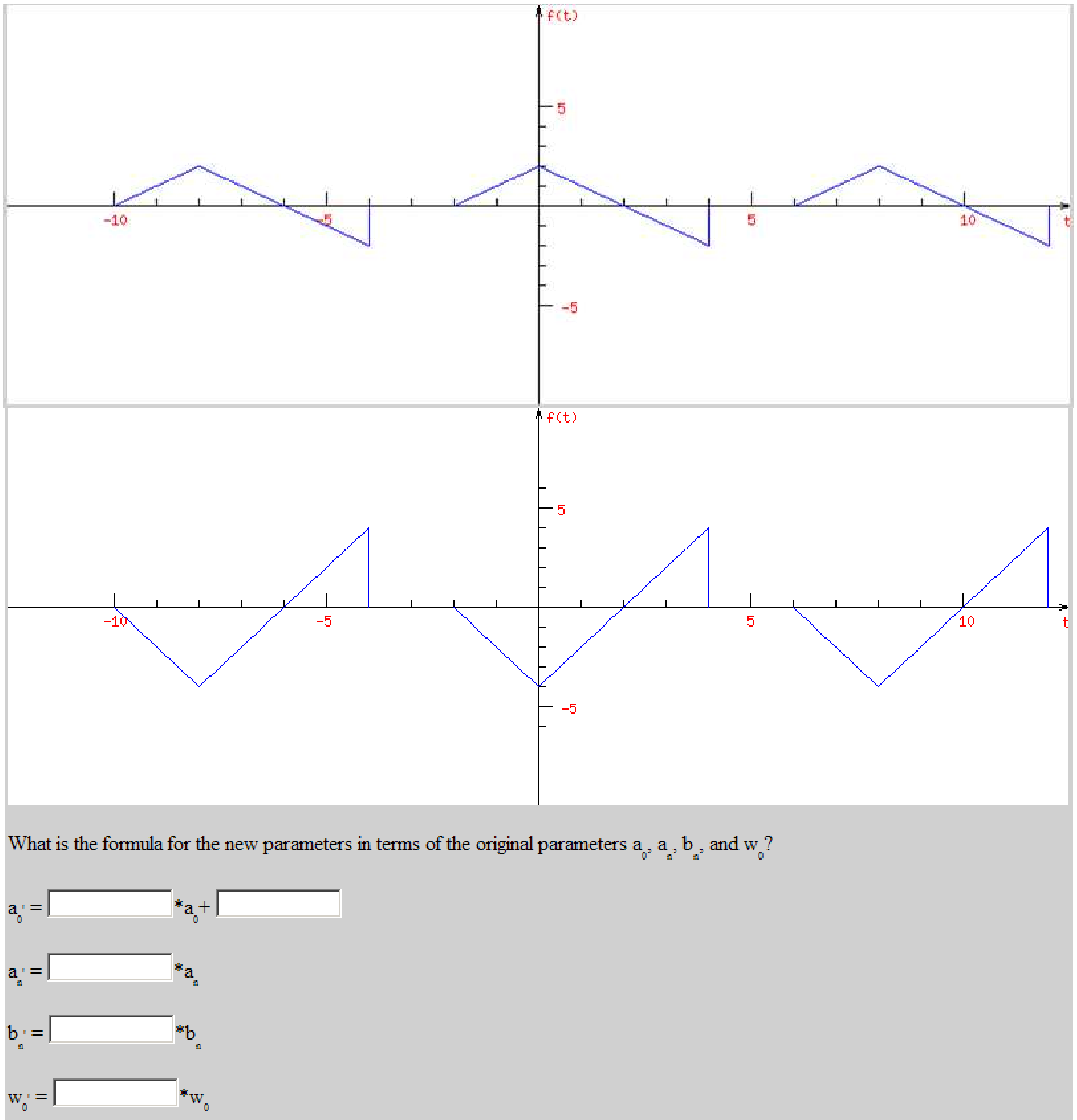

Figure 18. Example comparison signals and answer entry fields (TFS).

In Figure 18, the amplitude of the signal is inverted and doubled. The answer should be

- $a_0^{'} = -2 * a_0 + 0$
- $a_n^{'} = -2 * a_n$
- $b_n^{'} = -2 * b_n$
- $\omega_0^{'} = 1 * \omega_0$

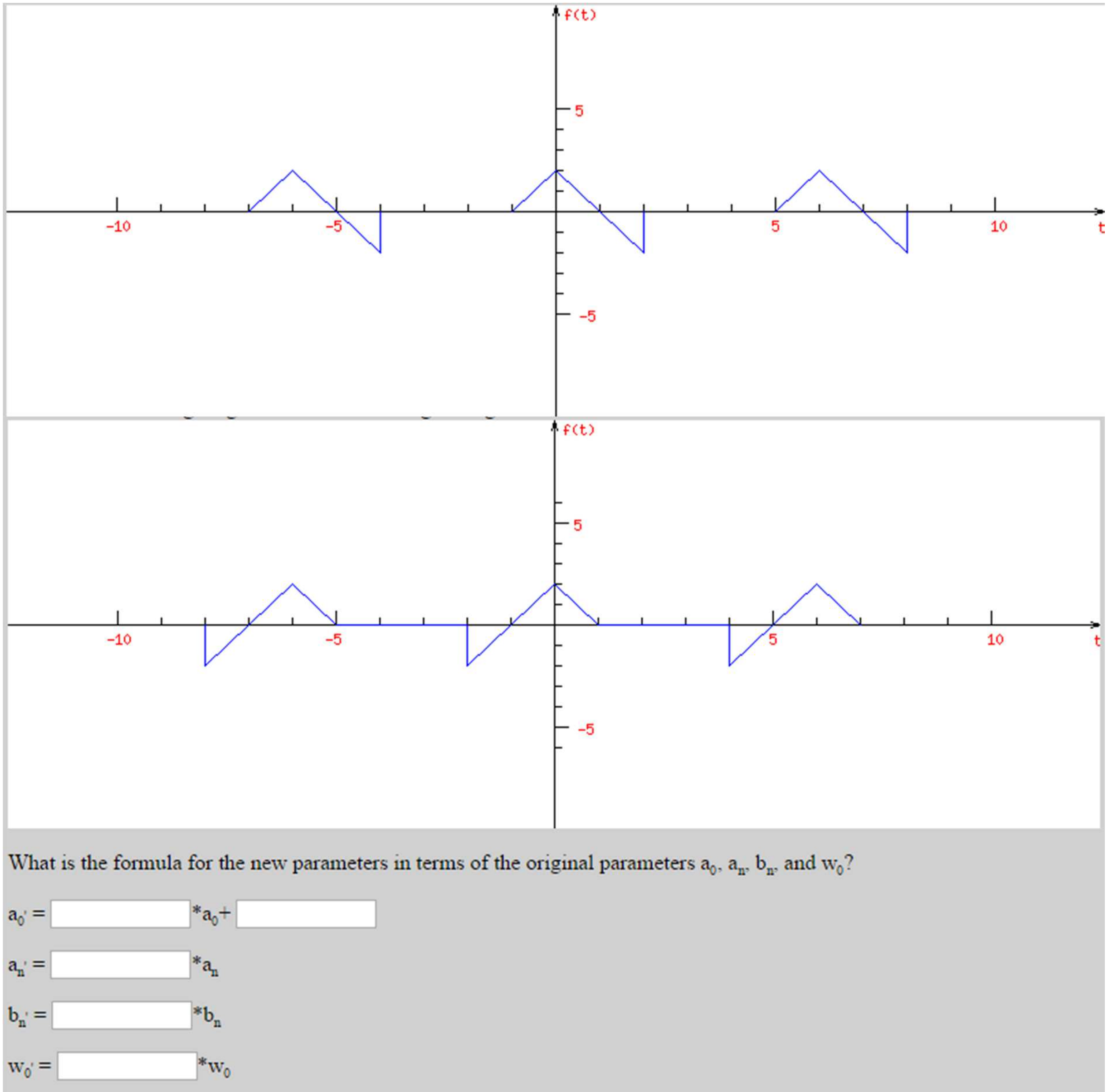

Figure 19. Example comparison signals and answer entry fields (TFS).

In Figure 19, the signal is time inverted relative to the original signal (flipped over the *y* axis).The answer should be

- $a_0' = 1 * a_0 + 0$
- $a_n' = 1 * a_n$
- $b_n^{\prime} = -1 * b_n$
- $\omega_0' = 1 * \omega_0$

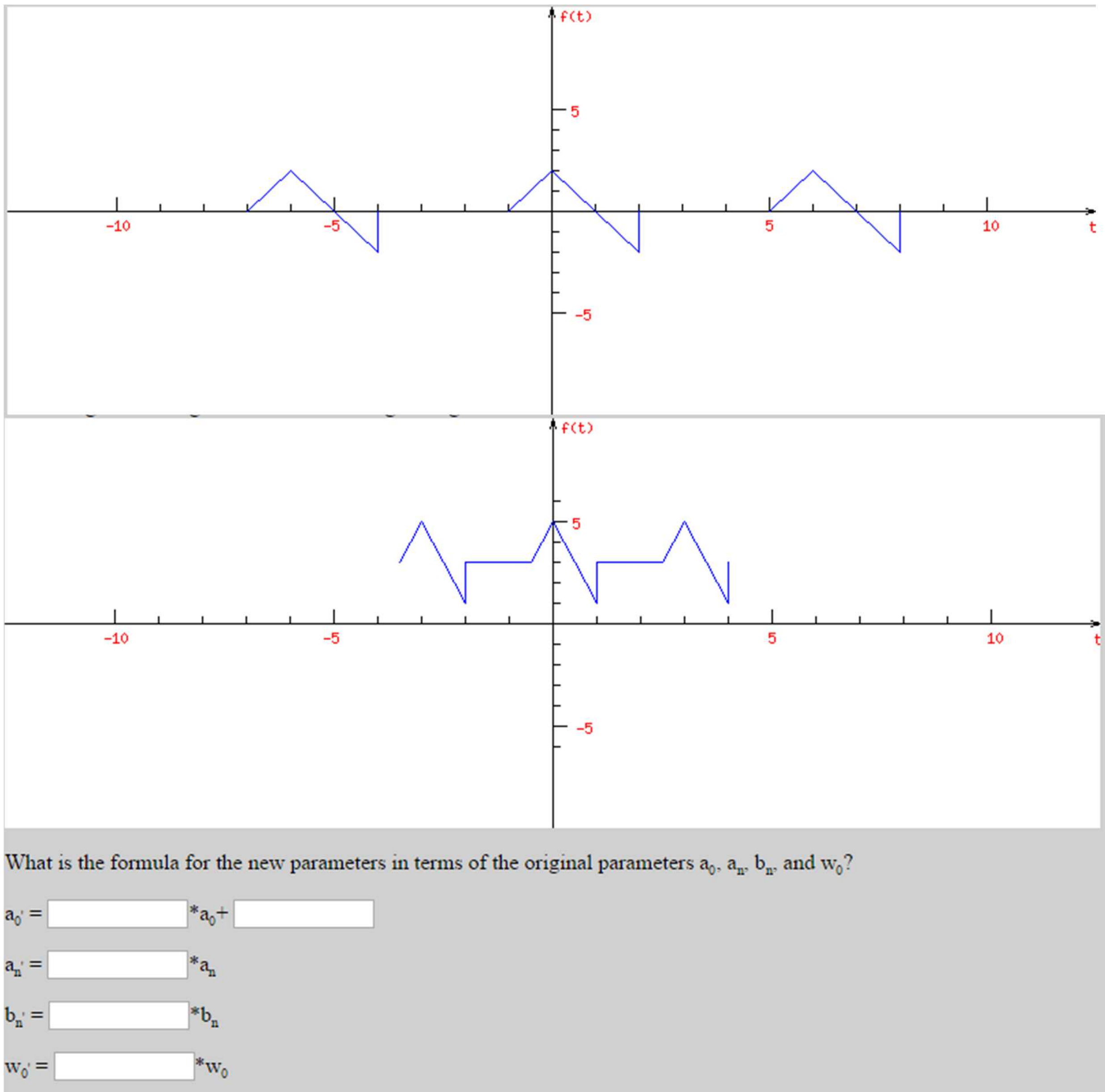

Figure 20. Example comparison signals and answer entry fields (TFS).

In Figure 20, the frequency of the signal is doubled, and the baseline of the signal is shifted up by 3. The answer should be

- $a_0' = 1 * a_0 + 3$
- $a_n' = 1 * a_n$
- $b_n' = 1 * b_n$
- $\omega_0' = 2 * \omega_0$

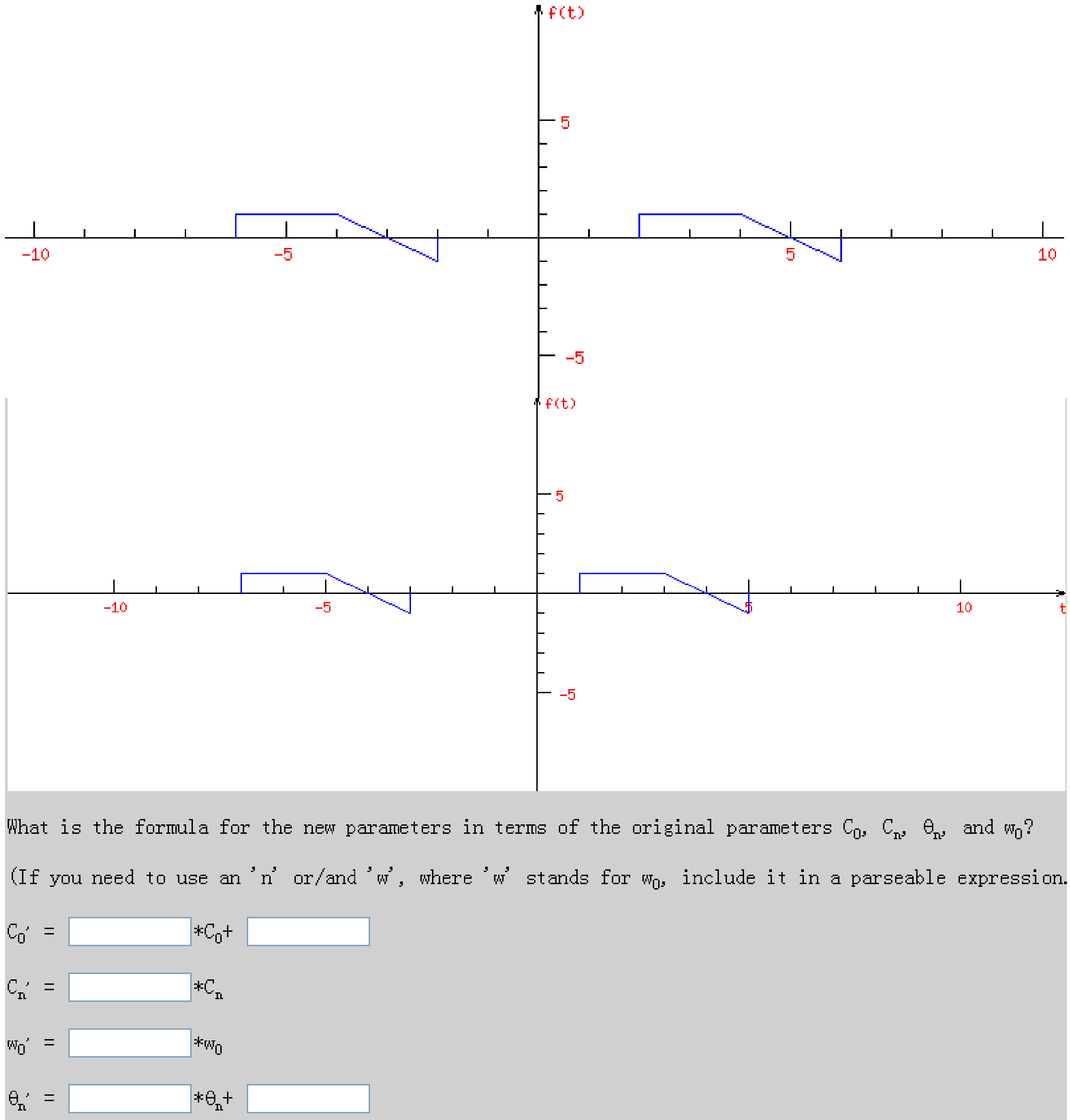

Figure 21. Example comparison signals and answer entry fields (CTFS).

In Figure 21, the signal is time advanced by 1 second. The answer should be

- $\omega_0^{'} = \omega_0$
- $C_0^{'} = C_0$
- $C_n^{'} = C_n$
- $\theta_n^{'} = \theta_n + n\omega_0$

CHAPTER 5: FRAMEWORK APPLICATION

In this engineering education research, two frameworks were applied to address the issues associated with student conceptual understanding: Bloom's Taxonomy and APOS Theory. This chapter presents an analysis of this research in the context of these frameworks.

A. Bloom's Taxonomy

Bloom's Taxonomy was applied in this research first. It divides educational objectives into three "domains": cognitive, affective, and psychomotor (sometimes loosely described as "knowing/head", "feeling/heart" and "doing/hands" respectively [26]). Within the domains, learning at the higher levels is dependent on having attained prerequisite knowledge and skills at lower levels. The work presented here focuses solely on the cognitive domain with reference to the levels of cognition noted in see Figure 7.

A.1 Previous Exam Questions Analysis

The idea of "higher level learning" or "conceptual learning" in Linear Systems has been implemented in practice for some time. For example, the signals in **Error! Reference source ot found.** and Figure 23, along with the accompanying questions listed below, have been used regularly on the final exam since 2007. The question addresses the students' understanding of Fourier series, and the sub-questions have been mapped to Bloom's taxonomy level(s).

Typical higher-level exam questions related to the signals in **Error! Reference source not found.** and Figure 23:

*A periodic signal f(t) is depicted below (***Error! Reference source not found.***):*

a) Is f(t) even, odd, or neither? (circle one)

even odd neither

Is F(ω) purely real, purely imaginary, or neither? (circle one)

real imaginary neither

b) If you represent f(t) with a trigonometric Fourier series, $f_{TFS}(t)$, which coefficients of $f_{TFS}(t)$, if any, can be determined by inspection, and what is their value?

c) What is $f_{TFS}(0.5)$?

d) *Find the trigonometric Fourier series, $f_{TFS}(t)$, that represents the signal. Take advantage of symmetry if possible. Simplify the expression(s) for the Fourier coefficients so that they* ***do not*** *contain any trigonometric functions.*

e) *What 3 ways would the Fourier series from part (d) need to be altered before it could represent the signal g(t) below (Figure 23)? (You do not need to know the answer to part (d) to address this question.)*

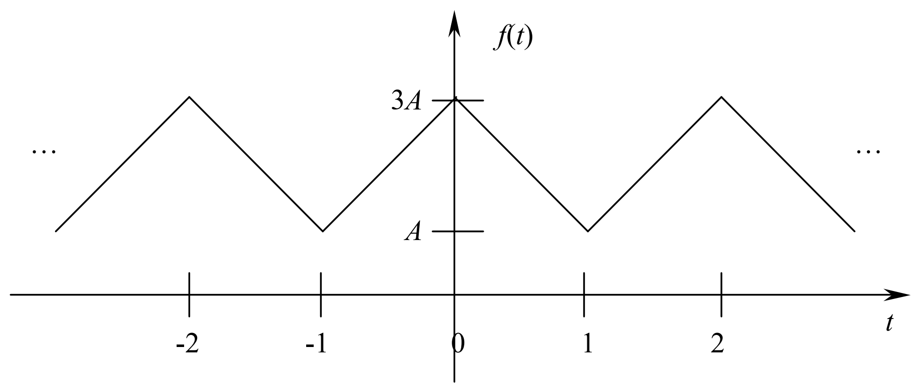

Figure 22. Example signal for a final exam question related to TFS.

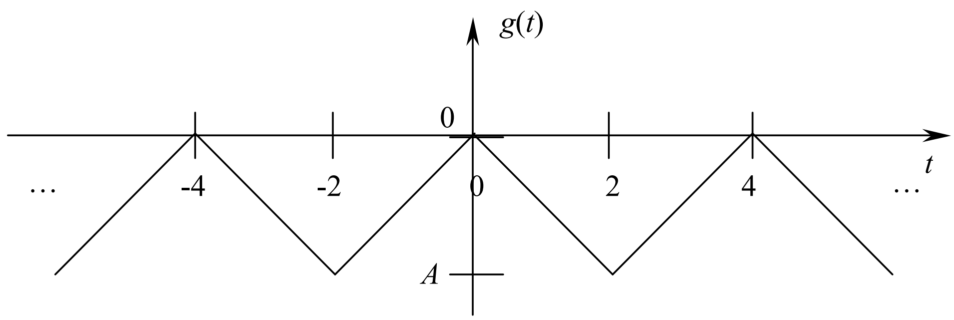

Figure 23. Altered signal with parameter changes.

It is not always possible to create a clear one-to-one mapping between a problem and a Bloom's level. However, this set of levels is offered as a generally sensible match to FS concepts. In the example above, sub questions a, b, c and d are traditional and are addressed in the Knowledge, Comprehension, Application, and Analysis stages. Items a and b could be addressed by memorizing/classifying, so they could be mapped to Knowledge/Comprehension. For item c, a basic understanding of Fourier series is required, and the students need to apply Fourier series knowledge and basic function knowledge to get the answer, which implies the Application level. Item d is a basic and classic TFS question – find the TFS that represents the signal. To address this item correctly, a student needs to memorize the procedures, select the parameters/equations to use, apply the existing equations, and calculate the result, addressing Bloom's levels 1→4.

Item e introduces the idea of parameter variations versus signal appearance in Fourier series. It requires an understanding of the role each parameter plays and the effects that parameter changes induce in signal plots. The knowledge was discussed in the class, but the relationships were not given to the students directly for memorizing. So, the students need to form their own understanding to interpret the question and respond. This short question therefore reflects conceptual understanding as related to Bloom's levels 4→6 to some degree. This question was used for several consecutive years on the final exam, and student ability to better address the question as teaching tools were introduced will be discussed in Chapter 6.

A.2 Interview and New Online Homework Module Analyses

The questions in the interviews and in the new online homework module are similar in nature, addressing comparable elements of Fourier series conceptual understanding. Student performance in both situations is therefore analyzed together in the context of the Bloom framework.

In comparison to the existing questions above that have been used in the final exam, the interview and module questions have more variation and involve more parameters. The hope was that the increased variation in these questions would lead to a more detailed understanding of the students' conceptual Fourier series knowledge relative to the Bloom's taxonomy levels. These questions do not ask for significant calculations, meaning the solution procedures were not required directly as with traditional questions. Rather, in order to give acceptable answers (especially in the interviews, where the interviewee needed to explain their answers to the interviewer), the students needed to comprehend the concepts well. These concepts included not only Fourier series ideas, but also the definition of a function, the properties of even/odd functions, the properties of trigonometric functions, the notion of period/frequency in periodic functions, and the purpose of integrals. These interview and module questions can be matched with Bloom's taxonomy levels, perhaps not one to one directly, but generally paired with the higher levels: 4 to 6. The following paragraphs analyze the details of the interview questions from a more cognitive perspective. Given their similarity between the interview questions and the new online module questions, this kind of analysis could be applied to the module as well.

Refer back to Chapter 3 where the interview questions are listed. The first question – the 'warm-up' question of the interview, is more like a traditional question. The student can just follow the procedures they learned in class or in the book to solve the major part: the question can be mainly addressed in Bloom's Taxonomy levels 1 to 3, or Knowledge, Comprehension, and Application.

The main content of the interview – signal appearance changes with parameter variations – starts in the second question and continues to the next-to-the-last question in all the three versions of the interview. This whole part of the interview maps to Bloom's levels 3 to 6 (Application, Analysis, Synthesis, and Evaluation in the framework), similar to sub-question e in the final exam problem set above. In this part of the interview, all parameter roles need to be understood well, and the properties of a function need to be utilized sensibly. The general procedures to address these questions, as noted in Chapter 3, involve representing the function in the Fourier series expanded form, then utilizing the properties of the trigonometric or exponential basis functions to identify how the parameters will change when the associated signal changes. Each student needs to identify the differences between the new and original signals, then determine how these changes could be represented mathematically in function form (step ① - refer to the example affiliated with Figure 24, which is discussed below). Next, the new and original signals can both be expanded as Fourier series (step ②), and the student needs to rearrange and reconstruct the expression to match the standard Fourier series form (step ③). Finally, the student needs to assess the restrictions of the new parameters (e.g., definitions or properties of these parameters) to achieve the correct result (step ④).

Back to the example analysis in Chapter 3, these four procedures can be applied:

- *The parameters for $f_{CTFS}(t)$ (C_0, C_n, and θ_n) are known for the original signal in Figure 23 below. Identify how the parameters ω_0, C_0, C_n, and θ_n change given the signal in Figure 23c.*

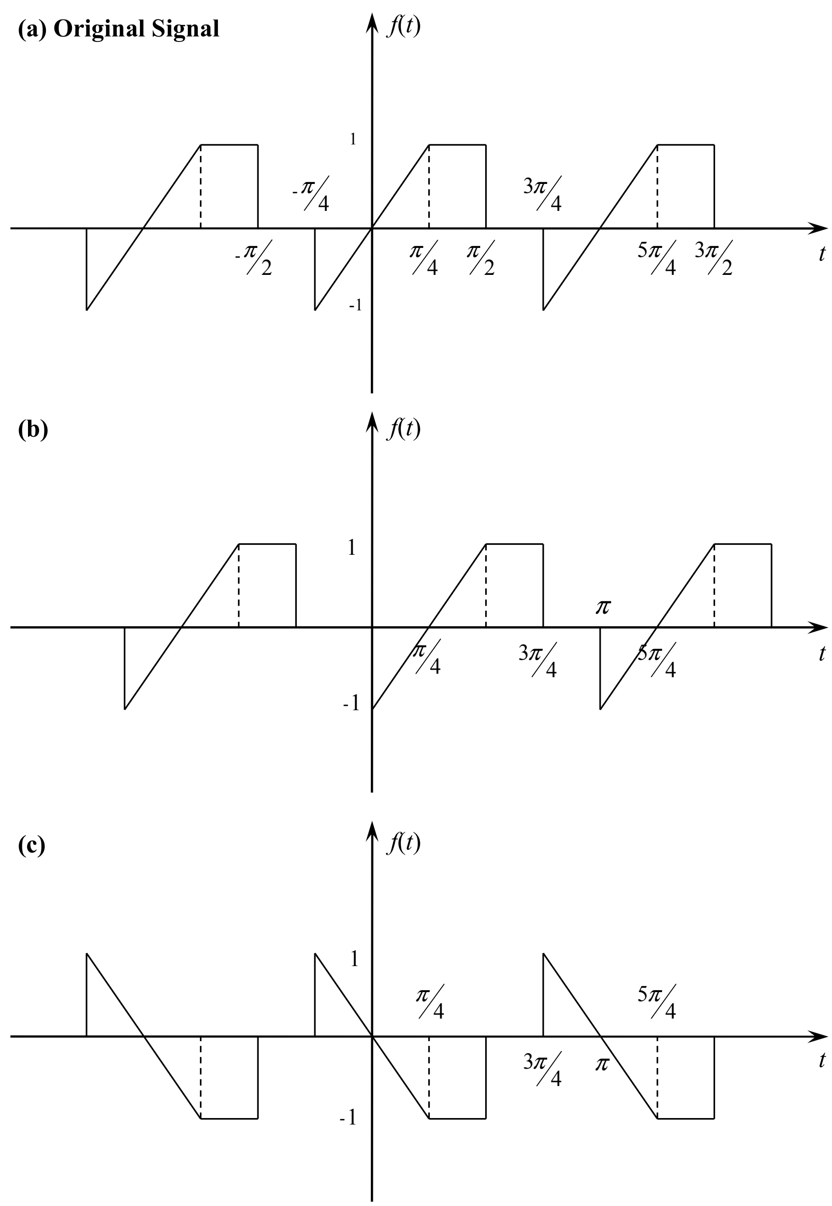

Figure 24. Example signals for matching Bloom's taxonomy levels to interview analyses.

- $g(t) = -f(t)$; ------------- ①
- $g_{CTFS}(t) = -f_{CTFS}(t)$;------------- ②
- Same period, so $\omega_0' = \omega_0$; ------------- ②
- $C_0' + \sum_{n=1}^{\infty} C_n' \cos(\omega_0' nt + \theta_n') = -(C_0 + \sum_{n=1}^{\infty} C_n \cos(\omega_0 nt + \theta_n))$;------------- ②

- $C_0' = -C_0$;-------------③
- $\sum_{n=1}^{\infty} C_n'\cos(\omega_0'nt + \theta_n') = -\sum_{n=1}^{\infty} C_n \cos(\omega_0 nt + \theta_n)$ -------------③

Because of the definition of C_n as $C_n = \sqrt{a_n^{\ 2} + b_n^{\ 2}}$, C_n is positive or zero, so

$C_n' = C_n$, and $\cos(\omega_0'nt + \theta_n') = -\cos(\omega_0 nt + \theta_n)$; -------------④

- To make the cos function negative, we can apply $\theta_n' = \theta_n + \pi$; -------------④

From the analysis above, the process could be mapped to levels 4 to 6 in Bloom's taxonomy, consistent with key words such as "compare," "analyze," "arrange," and "construct."

In the final interview question, the student is asked to build a new signal using the original signal(s). The student needs to be well-versed in parameter roles and function properties, since they need to select the signal(s) to use, arrange the signal(s), and construct the new signal. They need to explain their idea and defend their design through mathematical derivation. It is a creative procedure, and the product will be the new signal, so this question relates to levels 5 and 6 in Bloom's taxonomy.

B. APOS Theory Application and Results

As introduced in Chapter 2 – Background, APOS Theory is most widely used in mathematics education. Since Linear Systems is math intensive, APOS theory functions well as a framework, as it is easy to apply lessons learned from the literature to the analyses performed here.

Using detailed APOS theory analyses, the researcher can compare the success or failure of a student on a mathematical task with the specific mental constructions they may or may not have made. For example, if two students appear to agree in performance up to a specific mathematical point and then one student can take a further step while the other cannot, the researcher can try to explain the difference by pointing to the mental constructions of actions, processes, objects and/or schemas that the former student appears to have made but the other has not. The mapping between student understanding and the APOS levels seems natural.

Each semester, the interviews were offered about four or five weeks after the students initially focused on Fourier series in class, so students were not expected to remember all Fourier series details. All of the useful equations were provided, as well as an integral table. In the interview, the first traditional question maps to the action and process levels. Most of the students can calculate the parameters for the TFS expression readily, either with help from the given formulas or by themselves. Fewer than 20% of the students needed hints about which expanded form to use or how to properly apply the integral tables. About 40% of the students needed hints to apply the “area under the curve” concept. From an educational perspective, it appeared that about 80% of the students had reached the Process level, while the rest were still stuck in the Action level.

In the rest of the interview, the interviewee needed to think at the process and object levels to complete most of the questions. In the general procedure, as introduced above in this chapter – section A. 2 – an interviewee needed to understand parameter roles and the relationships between the parameters and the plots/functions to get the correct results. As a simple example, if the student understands the baseline, a_0 or C_0, as an object and can identify the sign of this parameter by looking at the plot, then they can describe changes in this parameter as the visual character of the signal changes without performing any calculations. In the example in section A.2, the baseline of the signal was sometimes (about 40% of the time) assumed to be equal to zero because of mistakes or carelessness. So, even in this simple case, an individual needs an object-level understanding to finish the question correctly without hints. Other signal-shape changes are considered harder than baseline changes, so most follow-on questions need at least an object-, if not schema-level, understanding. For example, in Chapter 3, section A.2, Problem #3b (the time shift question), over two thirds of the students were aware that the time shift related to a phase shift, however less than 5% of the students got the correct result without any hints. To obtain the correct answer to this question, an object-level understanding of Compact Trigonometric Fourier Series (CTFS) is required, specifically with regard to the role of θ_n in accomplishing the time delay while maintaining the relative alignment of the sinusoids that make up the Fourier series. For the student to understand CTFS thoroughly at the object-level, a schema-level understanding can be needed with regard to function behavior, signal appearance as it relates to a mathematical function, n-term summations, and other background mathematics knowledge.

Based on the analysis of the students' performances in these interviews, four groups of students can be identified:

1. Action/Action-to-Process Level: About 10% of the students performed at a level lower than the process level. Since no examples and notes were provided in the major part of each interview, these students could not finish the interview easily, even with lots of hints. For this group, each interview session was more like a tutoring class. The interviewer needed to provide the results and explain all of the questions in detail.

In the interview, the basic TFS equations were given as were listed in Chapter 2, section A.1. When facing the first question in the interview (see Chapter 3, section A.2, Problem #1), the students in the Action/Action-to-Process range had a sense of which equations they needed and followed the equations on the note sheet to find all of the parameters one by one. They could not deviate from the notes and did not rely on shortcuts, such as using the even nature of a function to simplify an integral calculation. When asked about the sign of a_0, they would get confused, and the did not appear to know or understand the meaning of "area under the curve." For the major part of the interview, which addressed parameter variations without any notes, they did not have the understanding of the concepts to finish the questions. They needed hints or explanations in most of the problem steps. For example, in Problems #2 to #5, almost all of the hints listed in Chapter 3, section B needed to be provided. For some questions, the full solutions needed to be provided with detailed explanations, which led to the interview being an extra tutoring session for the students in this level. These students might even have trouble understanding the meaning of certain parameters. For example, when a signal was time inverted (refer to Chapter 2, section A.2, *Time Inversion*), some students in this group wanted to apply negative time, $-t$, or negative frequency, $-\omega$, directly to achieve the change. The hint of "the definition of frequency" would be given, and they would need further help to get the correct answer.

2. Process Level: More than 50% of the students were located in this zone. They have a solid understanding of Fourier series and have some sense of the changes in signals brought about by changes in coefficients. They can answer most questions with hints. The correction ratio for their answers in the major part of the interview was not very high, and in some cases they could not prove their results mathematically. These students could understand the procedures the interviewer showed to them and apply these procedures to later questions with hints or by themselves.

The students in the process level and higher had no trouble working on the first question in the interview (refer to Chapter 3, section A.2, Problem #1). The students in the process group did not rely on the notes as much as the first group, but they still did not use any shortcuts to simplify the integral calculations. They have knowledge of the "area under the curve" but might need hints to get correct results. For the parameter variation question, once they got started, they had the basic idea of the relationship between the parameters and the signal appearance, but their understanding of the parameters was limited. They could realize how the signal changed, and they tried to build connections between the changes and the parameters. However, since either they could not prove the relationships mathematically, or their answers were not calculated directly from the math equations, their correction ratio was not high, and they were not confident in their answers. In this case, hints and explanations were provided, and the idea of how to get the relationship between the parameters and the signal appearance through math equations was introduced to these students. As the interview progressed, these students would have a better understanding of the role of the mathematical equations and would use them more often, but they still needed hints to use all of the equations correctly in most cases. For example, when they were asked to address amplitude inversion (refer to Chapter 2, section A.2, *Amplitude Inversion*), some of the students just applied the negative sign to the summation $\sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$, but they forgot about C_0. Most of the students in this group wanted to use a negative C_n to solve the problem. Here, hints regarding "the sign of C_0" and "the definition of C_n" would be provided. For the time-delay question (refer to Chapter 2, section A.2, *Time Delayed*), students invariably had trouble relating the time delay to the phase shift. Even with hints, they made mistakes, such as assuming that the time shift was equal to the (constant) phase shift – ignoring the *n* term in the phase shift. A full explanation would then be needed. Most students in this group did not reach the last question (signal reconstruction – refer to Chapter 3, section A.2, Problem #6) because of time limitations.

3. Process-to-Object Level: About 35% of the students were categorized in this level. They could understand most questions and the Fourier series expressions, and they had the basic understanding to identify changes in signal appearance and which parameters would change as a result. The students in this group always made some mistakes, but they were able to correct their mistakes with hints in most cases. They could explain their thoughts, and some mistakes were corrected during their self-explanations.

During the interview, the students in this group had the ability to finish the first question in a short time. They had a good understanding of the basic Fourier series expressions, and they were able to apply simplifications (such as even function properties in the context of integrals) to accelerate their calculation procedures. Most of them could use "area under the curve" knowledge to estimate a_0 and its sign. For the parameter variation questions, they were able to start the questions without any hints. When they made mistakes, they were asked to explain their answers, which allowed some mistakes to be found and corrected. If they still could not get the correct answer, hints would be provided, and in most cases, the difficulties could be overcome. One exception was still the time delay problem. Most of the students in this group tried to build the connection between the time delay and a phase change. However, they could not perform the mathematical procedures correctly. The most common mistake was the mistaken equivalency between the time shift and a constant phase shift, the same area of struggle for the students in the Process group, but their method to deal with the mistake was different. In this group, with the hint "does the time shift always equal the phase shift?," the interviewee always reacted with "no" and then tried to find the mathematical solution for the question. With the hint of, "As with the amplitude change, what is the expression of the new function in terms of the original function?," they were able to find the correct expression and then derive the phase change in terms of the original phase, θ_n, and the index n. At this point, they were able to get the answer, but they still had difficulty explaining it clearly, and the explanation would be given to help them understand. Most students in this group could reach the last question, and most of them needed the reference line to perform the reconstruction of the signal, as was introduced in Chapter 3, section A.2, Problem #6 and Figure 16. With the guide (auxiliary) line and other hints, such as, "How do you move the baseline of a signal?," they were able to get the correct answer by themselves.

4. Object Level: Only about 4% of the students (5 out of 140) were considered to perform at the full object level. They did not appear to feel challenged by the interview, and they finished most of the questions correctly without any hints. While they might make some slight mistakes, they were not critical. Hints provided to this group did not need to be specific; they can troubleshoot by themselves. All five of these students finished their interviews. They understood Fourier series very well and could link back to the related mathematics concepts of functions, summations, and signal plotting.

These four students had solid mathematics backgrounds in functions, integrals, and trigonometric functions, which helped them to get results faster than the other students, allowing them to finish the whole interview question set. For the parameter variation questions, they were able to solve the problems by deriving the math equations from the definition of the Fourier series, and they could also explain the changes from the signal graphing perspective, using the properties of sine/cosine functions and even/odd functions. For the time delay question, they could apply similar strategies in mathematic form to derive the result, and three of these four students could explain the results clearly. For the last question, two students needed the guide lines to get the correct results, but they could explain the result thoroughly. Based on their performance, the students in this group had a schema level understanding of the basic mathematics concepts, and they could utilize their knowledge fluently and build connections easily between Fourier series expressions and the visual representations of signals.

C. Reasons to Switch from Bloom's Taxonomy to APOS Theory

During the previous research, Bloom's taxonomy was applied as a framework to help quantify the cognitive levels of students in terms of their abilities to reason their way through Fourier series problems. Some value was generated from this framework – particularly the need to push the students toward exercises that required critical thinking and higher-level understanding. However, the primary limitation of this framework is that it is very broad and can therefore be difficult to apply within a technical learning environment. Each type/category of mathematical reasoning can arguably fall within several Bloom's levels depending on the type of problem, making it difficult to quantify the conceptual understanding state of a student.

APOS theory, which has broader use in mathematics education, is more 'object-like.' It deals in a more focused way with the state of a student and the characteristics that place the student in a certain conceptual level. The theory focuses on the cognitive process of mathematics concepts and the psychology of the learning process. As noted in the previous section, APOS theory appears to be a good match for education research in the area of linear systems; likely because of the mathematical nature of the topic. The matching of APOS theory to the students' behavior in this course is very natural.

Within the context of the NSF REESE program that funded this effort, the KSU Mathematics Department also employed APOS theory, but in the context of differential equations [31]. Applying APOS theory to Linear Systems also therefore helps to maintain some local consistency in terms of educational research frameworks. Bloom's Taxonomy is more general, as mentioned, not only with regard to the cognitive process, but also with regard to affective and psychomotor studies; we just looked at one part of the framework.

CHAPTER 6: EXAM MODIFICATION AND RESULT ANALYSIS

A. Exam Data Analysis - Short Term

Exam data from two semesters, Spring 2010 and Spring 2011, were analyzed to evaluate how the interviews and the new conceptual module were working. Each semester prior to that time, students would learn Trigonometric Fourier Series (TFS) and Compact Trigonometric Fourier Series (CTFS) before the second mid-term exam, and these subjects would be covered on that second exam as well as on the final exam. In Spring 2010 and Spring 2011, some conceptual understanding questions based on TFS and CTFS parameters were added to the second midterm exam. Conceptual questions had actually been consistently used on the final exam since Spring 2007. In summary, four sets of exam scores are of note here and relate to mid-term exam 2 and the final exam for the two semesters.

Four groups of students are of interest for this two-semester, short-term analysis. In Spring 2010, the Fourier series conceptual module was not ready, so none of the students used the module. However, 24 students participated in the interviews, while 18 others did not. In Spring 2011, all of the students used the conceptual module and 24 students participated in the interviews, while 11 did not. The four groups are noted in Table 7 below. The analysis in this section is based on student scores for the four exams: two in each semester, where each group had two exam scores.

	2010		**2011**	
Treatment Group	**Group 1**	**Group 2**	**Group 3**	**Group 4**
Interview	Yes (24)	No (18)	Yes (24)	No (11)
New Module	No	No	Yes (35)	Yes (35)

Table 7. Four groups of students assessed for the short-term analysis.

1. *Interviewed Versus Non-Interviewed Students*

As mentioned before, four groups of students are of interest (two groups per semester): interviewed students and non-interviewed students. To analyze the influence of the interviews, students from the same semester were compared against each other.

From the Spring 2010 exam data (scores on the conceptual Fourier series questions only), as shown in Figure 25, the average scores of the two groups were less than 10% apart after the second mid-term exam, which was taken before the interviews. The interviewed students received (on average) about 68%, while the non-interviewed received about 60%, a difference of about 8%. For the final exam, the scores of the interviewed students improved by 9 percent overall, from 68% to 77%; whereas the non-interviewed students' average score regressed to 56%. The difference was therefore about 20% between two groups.

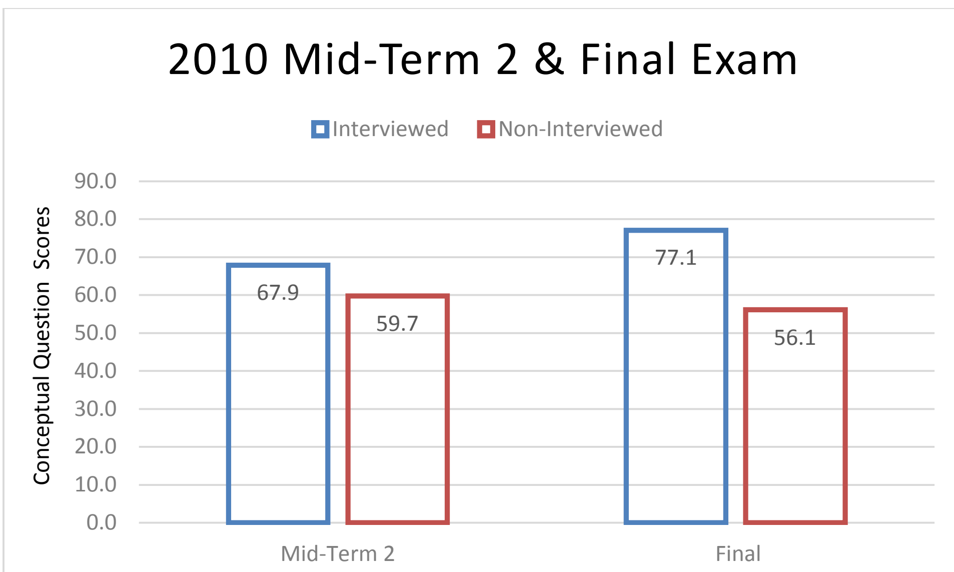

Figure 25. Spring 2010 exam scores (mid-term 2 and final exams) for interviewed and non-interviewed students

In Spring 2011, the result was a bit different. For mid-term exam 2, the average scores on the conceptual problems for the two groups were close: 75% compared to 73%. For the final exam, the average scores on the conceptual questions were lower for all the students, no matter whether they were interviewed or not. The average scores of the interviewed students dropped by 6%

from 75% to 69%, while the average scores of the non-interviewed students dropped by 30%, from 73% to 43%. This disparity is substantial.

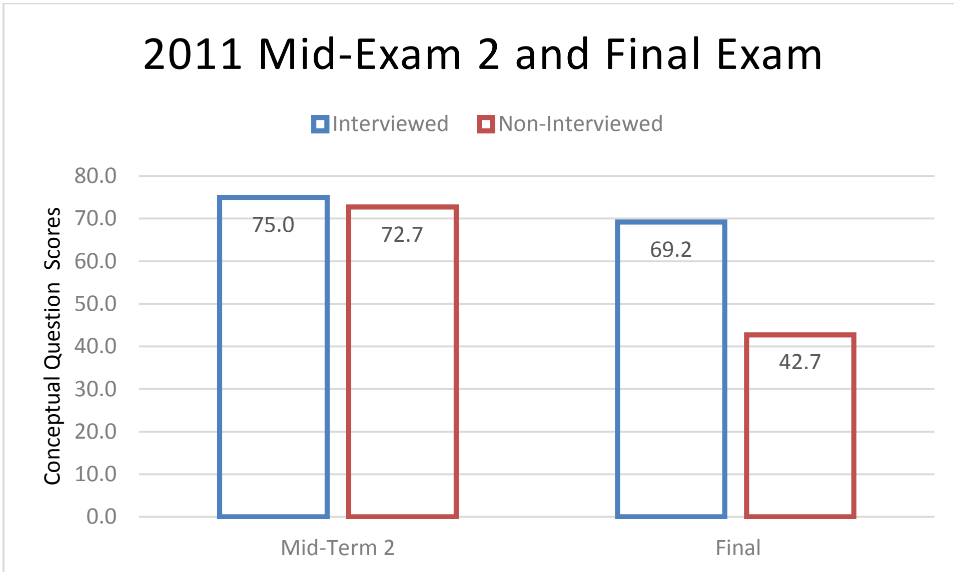

Figure 26. Spring 2011 exam scores (mid-term 2 and final exams) for interviewed and non-interviewed students.

From the two figures above, the interviews appear to have increased the disparity in student scores on the conceptual questions related to parameter variations – for both semesters. If the interviewed and non-interviewed students are placed in two big groups, the difference in scores was increased from about 5 percent on mid-term exam 2 to over 20 percent on the final exam.

Although in Spring 2011 neither of the two groups improved (the mid-term and final exam conceptual questions were not the same), the difference in performance between the interviewed students and the non-interviewed students was substantial. Note that the performance for both groups could have been influenced by the overall difficulty of the exam and the setting of the questions. In Spring 2010, more filter questions were included on the final exam at the expense of the traditional Fourier series questions, which were on most of the final exams since 2007, including Spring 2011. (Different filter topics were covered at the end of Spring 2010 and therefore also included on the final exam.) However, it is clear in both cases that students performed better if they participated in the interviews. Based on this exam performance and the students' responses to the interviews (refer to Chapter 3, section C.2), the interviews appear to have a positive influence with regard to student understanding of parameter variation questions, or conceptual understanding in TFS and CTFS.

2. *Student Performance With/ Without the Enhanced Online Homework Module*

To analyze the effect of the homework module on conceptual understanding, students were grouped by semester: the Spring 2011 students used the new conceptual module, which was not available in Spring 2010. In Spring 2011, the conceptual module was used before the second mid-term exam. As shown in Figure 25 and Figure 26, the average scores on the conceptual questions for the second mid-term exam increased significantly, from 64% to 74%. However, in Spring 2011, the,students did not perform well on the same types of questions for the final exam. As noted above in section A.1, the reasons may be varied. To get a better understanding of the students' overall performance on both the individual exams versus the entire semester, the grade point averages (GPAs), based on 4.0 scale, of each exam and the two overall semesters were calculated, as listed in Table 8. The reason we are using GPA other than average test score is that GPA can reflect the students' performance on the entire course (homework and projects included) with little effect from the difficulty levels of the test questions.

Table 8. Student exam performance in the Spring 2010 and Spring 2011 semesters.

	2010		**2011**	
Exam GPA	Mid-Term 2	Final	Mid-Term 2	Final
	2.25	2.38	2.30	1.76
Overall Class GPA of the Course	2.14		2.03	

From Table 8, the comparison data yield the following:

- Mid-Term 2: the two semester GPAs were close, with a difference of 0.05
- Final exam: the Spring 2010 GPA was higher by 0.6, which was a significant difference
- Overall: the Spring 2010 GPA was higher by 0.1, which was considerable

The following thoughts are therefore offered:

1. The Mid-Term 2 exams in both semesters had similar difficulty levels.
2. The students in Spring 2010 performed better in the course in general.
3. The students in Spring 2011 faced more difficulty on the final exam.

This information suggests that the decrease in average score on the conceptual questions in the final exam might have been caused by other effects (in the exam or the students) that had little relation to the conceptual questions themselves. Analyzing the data from Figure 25 and Figure 26 in this light, one can surmise that the new online homework module has a positive effect on the conceptual understanding of Fourier series.

Also, if the conceptual questions on the mid-term 2 and final exams are combined together for the four groups, then Group 3, which benefited from both the interviews and the new module, had the best performance, which suggested the two treatments in aggregate were successful with regard to helping the students with Fourier series conceptual understanding.

Because these analyses were performed using data from such a short time frame, longer term data involving 8 semesters and over 200 students were addressed, as discussed next.

B. Exam Data Analysis - Long Term

As mentioned before, in the final exam, conceptual questions have been included since Spring 2007. The data were traced back to that semester, focusing on student grades on (a) the TFS/CTFS conceptual questions and (b) the traditional TFS questions. The results are depicted in Table 9 and Figure 27. Before Spring 2010, there were no practice problems available that were related to parameter variations, but since then, both interviews and online modules have become available. Similar conceptual problems from final exams were analyzed semester by semester and then compared in two big groups: those that received the treatments (interviews and online modules) and those that did not. The questions were the same and comparable, resembling the example question in the Bloom's taxonomy portion of the Background. Some final exams did not have traditional TFS questions where students solved for parameters using integrals, so they were not counted. One example is the Spring 2010 semester – although that final exam contained conceptual questions, only small additional basic TFS questions were included, so that final exam was included in the analysis from last section but is not included here.

Table 9. Long-term student performance on final exams (basic TFS questions and coefficient variation questions)

	S07	S08	F08	S09	F09	S11	F11	S12
No. of Students	33	29	38	30	21	35	23	22
Basic TFS	Totally 17 Points							
Average	11.50	11.95	11.50	12.98	10.04	13.12	13.48	13.93
Average%	67.65	70.32	67.65	76.38	59.06	77.16	79.29	81.95
Average(Two Groups)	11.68					13.47		
Change Percentage %	10.52							
TFS Coeff Shift	Totally 5 Points							
Average	2.88	2.28	3.03	2.07	2.64	3.52	3.80	3.68
Standard Deviation	1.31	1.15	1.42	1.25	1.35	1.50	1.39	1.45
Average%	57.59	45.56	60.53	41.33	52.86	70.33	75.91	73.64
STDEV%	26.28	23.09	28.37	25.01	27.04	30.00	27.89	29.04
Average(Two Groups)	2.61					3.65		
Change Percentage %	20.87							

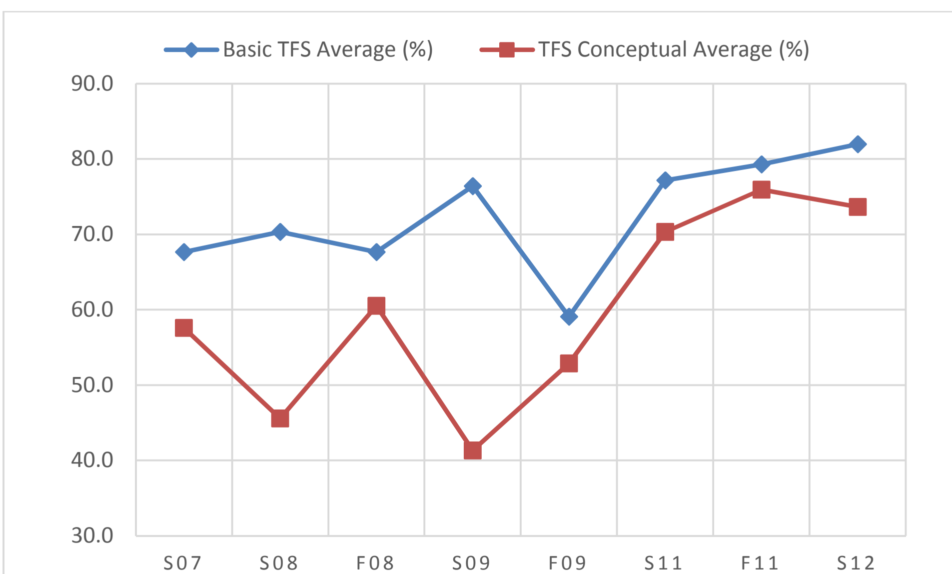

Figure 27. Long-term student performance on final exam conceptual questions.

From Table 9 and Figure 27, the grades received for TFS questions clearly increased with the help of the interviews and the online homework modules. The average scores on the TFS conceptual questions increased from 2.61 to 3.65, or 20%. Likewise, the scores for the traditional TFS questions increased from 11.68 to 13.47, or 10.5%. Attempts were made to keep the whole final exam and the individual questions in the same difficult level – indeed, with the exception of just a couple of exams, all of these final exams were the same and were not returned to students. The conceptual questions were graded to the same standard for every semester. The treatments were aimed at the conceptual questions, so a better understanding of this type of problem was expected. However, from the data, the students also improved their scores on the traditional TFS questions. Even the highest average score of the previous semesters (76.38% in S09) was still lower than the S11 semester, whereas the lowest average score was 77.16% after the treatments were applied.

CHAPTER 7: FUTURE WORK

This section addresses future efforts that can continue the work described in this dissertation. The research presented here focused on time-domain LTI systems and Fourier series, where the research team was able to identify areas of struggle and implement treatments, including teaching-learning interviews and a conceptual online homework module. These enhancements have made a positive impact on student learning, and Bloom's taxonomy and APOS theory have offered theoretical support and guidance to this work. Future work may continue with similar research applied to a broader area.

A. Application to Additional Linear Systems Topics and Method Enhancement

In Linear Systems, a wide range of topics are covered in both the time domain and frequency domain, as noted in Chapter 1, section B. This research addressed only a subset of those topics. Other topics, such as convolution, Fourier transforms, Laplace transforms, and discrete systems can prove difficult for students, based on their previous performance, but these topics are important to future studies in electrical engineering. Further research on these topics using methods similar to those employed here could help to improve student learning and help instructors to better understanding student difficulties in these topic areas.

The future research design could roughly imitate this current method using four main steps:

1. find areas where students struggle given exam scores and survey results,
2. create interviews, which can be the initial treatment, to gather in-depth data and more detailed information,
3. generate treatments in addition to the interviews, such as new online homework modules or innovative projects, and
4. apply detailed analyses on these data to evaluate the effectiveness of the treatments and enhance these methods.

The APOS learning framework has the potential to support these efforts. As discussed earlier, Fourier series form a connection between the time and frequency domain, so further studies related to frequency domain knowledge (e.g., Fourier Transforms, modulation, windowing, filters, etc.), naturally reflect students' conceptual understanding of Fourier series and earlier time domain topics. If applying APOS theory, this may allow a schema-level assessment of that earlier knowledge.

B. Applying This Research Method in Other Engineering Classes

As introduced in Chapter 1, Linear Systems is an important fundamental class in an Electrical Engineering curriculum. Several high level classes, such as Communication Systems, Digital Filtering, Electro-acoustics, and other courses related to signal analysis and processing, are based on the knowledge from Linear Systems. The current research can be expanded into those courses, as described above in section A, allowing the whole curriculum to gain similar benefits as were experienced in this course. In other engineering majors, such as mechanical engineering, this kind of research method can be applied as well. A learning framework such as APOS theory can help to guide and organize these research methods and directions, where the ultimate goal is to improve mathematics- and physics-based learning experiences for all engineering students.

CHAPTER 8: CONCLUSION

In this research, we addressed conceptual learning of junior and senior students enrolled in a Linear Systems class offered by the KSU Electrical and Computer Engineering Department. The areas of interest included Fourier series and LTI system response visualization. Fourier series provide an important connection between knowledge in the time and frequency domains, while LTI systems are important for signal processing and areas of electrical engineering study. Teaching-learning interviews and online homework modules were offered as treatments to improve student learning experiences and increase their conceptual understanding of these topics.

Students considered the teaching-learning interviews to be helpful and even considered them to be tutoring sessions in some cases. Enhancements of the online homework modules were useful as well. Exam data were analyzed to assess the effectiveness of these treatments for Fourier series learning, and the results were positive and matched expectations in most cases. Average scores on conceptual TFS problems, both on mid-term and final exams, were increased, and the better conceptual understanding also led to performance improvements on traditional TFS questions.

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