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On Monitoring of Multiple Non-linear Profiles

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On Monitoring of Multiple Non-linear Profiles

Most state-of-the-art profile monitoring methods involve studies of one profile. However, a process may contain several sensors or probes that generate multiple profiles over time. Quality characteristics presented in multiple profiles may be related multiple aspects of product or process quality. Existing charting methods for simultaneous monitoring of each multiple profile may result in higher false alarm rates. Or worse, they cannot correctly detect potential profile relationship changes. In this study, we propose two approaches to detect process shifts in multiple nonlinear profiles. A simulation study was conducted to evaluate the performance of the proposed approaches in terms of average run length under different process shift scenarios. Pros and cons of the proposed methods are discussed. A guideline for choosing the proposed methods is introduced. In addition, a hybrid method combining the salient points of both approaches is explored. Finally, a real-world data set from a vulcanization process is used to demonstrate the implementation of the proposed methods.

Keywords: Big Data, Profile Analysis, Multiple Profiles Analysis, Multivariate SPC, Multivariate EWMA.

Introduction

Quality characteristics represented as profiles have been studied in recent quality monitoring literature in recent years. For examples, Jin and Shi (1999) examined the stamping tonnage data over time within a cycle. Kang and Albin (2000) introduced a calibration issue during the etch step of a semiconductor manufacturing process. Walker and Wright (2002) studied the density of wood board over a section. Chang and Gan (2006) showed the monitoring stability of a calibration process in order to assure its accuracy. Paynabar and Jin (2011) presented pressing force profile signals in a valve seat assembly operation. Chang et al. (2012) investigated the temperature profile from a curing process for high-pressure hose products. These studies only consider one profile type in their respective applications.

Taking the advantage of information technology, engineers no longer measure the quality characteristics by hands but through automatic data sensors. This paradigm shift has resulted in a tremendous amount of data. For example, Jin and Shi (1999) reported that one tonnage sensor within a sampling interval could collect 1500 data points for each part, and the database would store 2.88×10^6 data points for 30 presses in 16 hours of production. Although modern database management systems can handle and store those huge datasets, it is very difficult for conventional multivariate statistical process control techniques, such as, Multivariate Hotelling's T^2 control chart, to deal with big data and multiple types of profiles simultaneously.

Profile analysis can be simply characterized into two categories, linear and nonlinear profiles according to the shape complexity of a profile of interest. With respect to linear profile applications, model parameters are the subjects of monitoring because linear profiles are easy to be presented by, for example, a simple linear regression model. Many studies monitored either intercept or slope parameter of the calculated simple linear regression model or monitoring both. For example, Kang and Albin (2000) proposed two approaches, the first one monitored slope and intercept with the Hotelling's T^2 control chart, while the second one monitored average residuals between sample profiles and reference profile followed by exponentially weighted moving average (EWMA) chart and *R* chart. Kim *et al.* (2003) showed their method of three univariate EWMA charts monitoring slope, intercept, and the variance of deviation between samples and regression line performs better than EWMA/*R* chart in terms of average run length (ARL).

Studies in nonlinear profile analysis can be categorized into four types—applying multiple and polynomial regression (Zou et al., 2007; Kazemzadeh et al. 2008; Mahmoud 2008), applying nonlinear regression models (Ding et al., 2006; Williams et al., 2007; Shiau et al., 2009; Chang and Yadama 2010; Chen and Nembhard 2011;), use of mixed models (Jensen et al., 2008; Jensen and Birch, 2009; Qiu et al., 2010; Paynabar and Jin, 2011), and use of wavelets (Reis and Saraiva, 2006; Zhou et al., 2007; Chicken et al., 2009). For more detail of those methods to monitor the process stability can be found in Woodall (2007) and Noorossana *et al.* (2011).

The studies mentioned above only consider one profile type for process monitoring. However, data points that collected in a process or system may be characterized by two or more profiles. Noorossana *et al.* (2010) investigated a calibration application between desired force and the real force produced by 1600-ton hydraulic press machine. The machine consists of a set of cylinders, pistons and hydraulic pipe controlled by a programmable logic controller (PLC) for input and output factors adjustment. The input variable known as the desired force or nominal force is given by a motor placed on the top of machine so that four real forces or the response variables collected from four cylinders of the press can be measured by a PLC. Since four response variables can be considered as correlated linear profiles, Noorossana *et al.* (2010) proposed a multivariate simple linear profile method to deal with this problem. Specifically, all linear profiles are of the same type but for each press cylinder. Their method cannot monitor multiple correlated nonlinear profiles of different types that measure different process characteristics.

An example of multiple correlated nonlinear profiles can be found in a curing process of high-pressure hose products. According to Chang et al.'s (2012) study, the high-pressure hose products are covered layers of rubber and metal wires, which are loaded and cured in a heated chamber called an autoclave or vulcanizer, equipped with several sensors in different locations for monitoring air temperature, condensation water temperature, and chamber pressure. Although the key factor of curing process is the air temperature, the other profiles, such as, chamber pressure profiles, monitored simultaneously also play important roles in the curing process. Note that, high chamber pressure will increase the speed of reaching target air temperature. Also, a sealed chamber helps air temperature climb quickly and stably to the setup temperature point. On the other hand, a leaking vulcanizer requires more energy consumption to maintain the same temperature during the curing stage. Therefore, it is easier for quality engineers to monitor the pressure profile using statistical process control (SPC) tool for saving energy.

Chang *et al.* (2012) only investigated the air temperature profile of the curing process. When this process is out of control, it is very difficult for quality engineers to pinpoint the root cause. It is possible that the chamber is not airtight but both temperature and pressure maintain their target values. However, the relationship between the temperature and pressure profiles may have changed. In this study, both air temperature and pressure profiles are considered simultaneously. This example provides an illustration of how the proposed framework addresses multiple profile process monitoring in general. This paradigm motivates us to develop a novel approach for simultaneous monitoring of multiple correlated nonlinear profiles. A two-profile simulation study is conducted to evaluate the performance of the proposed charting methods.

Figure 1 shows overall air temperature and pressure profiles that generated from a typical curing process. It may be possible to construct an underlining physics equation between the air temperature and pressure so that the quality characteristics can then be transformed from profiles to parameters as variables used in multivariate control chart. However, this underlining equation is not easy to be formulated, and it cannot be generalized for all applications. In other words, this equation (if it can be formulated) can only be used in this curing process application. In addition, according to Figure 1, it is obvious that the temperature and pressure profile are correlated, and yet, it is hard to define such a correlation between profiles in general. Although monitoring those profile types using multiple multivariate control charts independently provides a solution, high false alarm rate and low detecting power is the major concern given that profiles are correlated to each other. This study provides a general SPC framework for multiple correlated linear or nonlinear profiles.

The focus of this study is the development of a proper process monitoring strategy for monitoring multiple correlated nonlinear profiles. This study examines two alternative solutions, in which profiles are first fitted by B-splines according to Chang and Yadama's method (2010). Then the deviations of the observed profile from the fitted profile are recorded to generate a vector of plotting statistics. A multivariate EWMA (MEWMA) control chart is then used for process monitoring. The first proposed method converts absolute deviations at each profile into a summary statistic. The second proposed method contains several numbers for each profile because a profile is segmented into p sections where $p>1$, where p is a constant to be determined from engineering knowledge or the complexity of a profile. Each section is represented by a summary absolute deviation statistic. A two-profile simulation study is conducted to characterize property of the proposed approaches in terms of ARL. We will also discuss the pros and cons of both approaches, and how we combine the salient features of both methods into a hybrid approach. The proposed hybrid method combines method I and method II, which monitors each section or segment by a MEWMA control chart. Multiple (p) MEWMA charts need to be maintained for the proposed hybrid method. A real-world data set from a curing process is used to demonstrate the implementation of the proposed methods.

The organization of this study is the following. First, the modified Chang and Yadama's (2010) method is briefly summarized, as well as the MEWMA procedure, followed by the proposed methods. Second, the experimental design of the simulation is introduced for testing the robustness of the proposed methods. Third, the ARL property for all proposed methods is introduced. Finally, the discussion and conclusion will be drawn in this section.

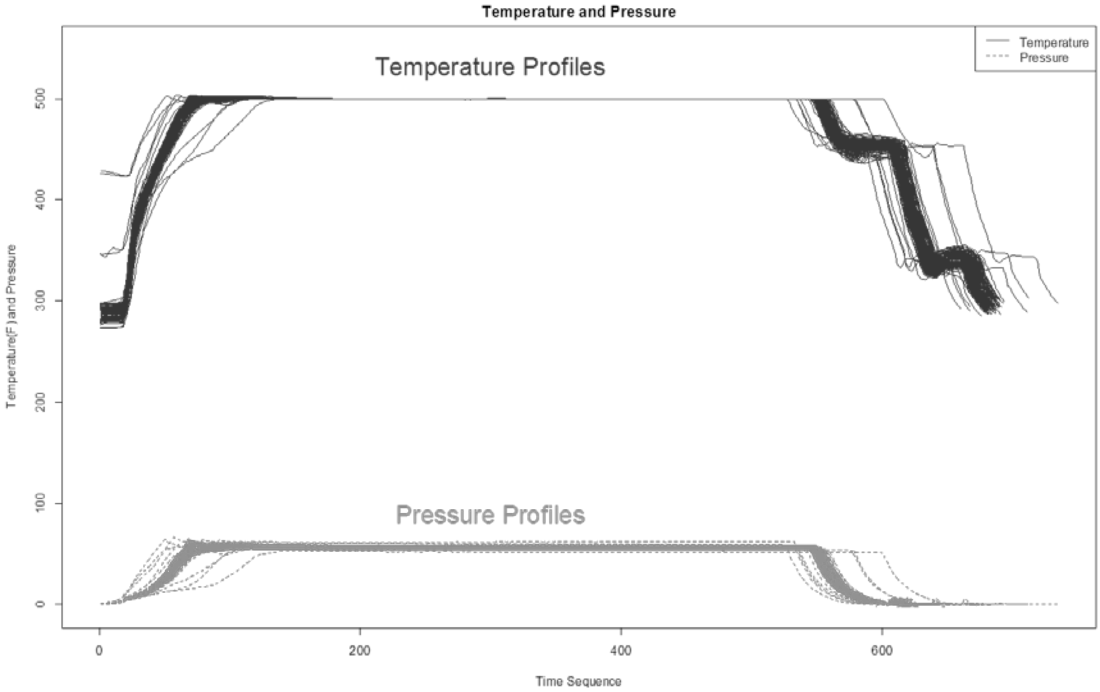


Figure 1 Overall air temperature and pressure profiles from the curing process of high-pressure hose products.

The Proposed Methods

In this study, two methods, method I and method II, are proposed to monitor the stability of the process whose quality characteristics are multiple profiles. Both methods consist of one multivariate control chart for all profiles and share the common modelling treatment, i.e., the modified Chang and Yadama's method (see Chang and Yadama's, 2010). Therefore, this section summarizes the original Chang and Yadama's method followed by the modified version. Then, the MEWMA procedure is presented. Finally, the proposed method I and II are shown in the last part of this section.

Modified Chang and Yadama's Method

Chang and Yadama (2010) proposed a control charting framework to monitor non-linear profiles in detecting shape changes. They proposed single segment and multiple segments approach for monitoring one profile. We will only introduce their multiple segments approach in this study because this approach provides more details of fault location for diagnosis purposes. The procedure of their proposed multiple segments approach is shown in Figure 2. Note that D_{ij} in Step 5 can be presented as equation (1) where $\overline{x}_{jk}$ is the mean profile of segment k that can be calculated by using B-spline fitting. Moreover, the MTY decomposition method shown in Step 7 can be found in Mason, Tracy, and Young's (2001) study for interpreting T^2 control chart signals.

$$D_{ij} = \frac{\sum_{k=1}^{c} \left| x_{ijk} - \overline{x}_{jk} \right|}{c}, \qquad i = 1,2,\ldots,n; \;\; j = 1,2,\ldots,p, \tag{1}$$

where $|\cdot|$ represents absolute value; n is the number of profiles and i is the index of profile; p is the number of segments of a profile and j is index of the segment; c is the number of control points within each segment so that a B-spline can be fit to each segment, and k is the index of control point. All numbers of n, p and c are known or well defined according to the manufacturing process and the complexity of a profile. The default c is equal to 16 according to Chang and Yadama's study (2010).

Procedure of Chang and Yadama's Method

Step 1: Apply Discrete Wavelet Transformation (DWT) to the given profile to the desired level, so that the DWT coefficients can be obtained.

Step 2: Reconstruct the DWT coefficients to original domain in two sets, signal of mean and signal of variance.

Step 3: Partition the mean signal into p segments.

Step 4: Apply B-spline with c control points to each segment, so that a control point matrix of order $n \times c \cdot p$ is constructed, where n is the profile replicates.

Step 5: The mean distance difference vector $Y_i = [D_{i1}, D_{i2}, \ldots, D_{ip}]$ is calculated, in which is associated with the control point matrix.

Step 6: To monitor the profile stability, the Hotelling's T^2 statistics on Y_i are calculated using the mean distance difference vectors.

Step 7: If process is in-control, go to Step 5; otherwise the MTY decomposition method is used for identifying the responsible T^2 components.

Figure 2 Procedure of Chang and Yadama's Method.

For the curing example in this study, since the curing process is control by the PLC, the within profile variance is very small. In other words, noises in each profile are very small. Therefore, we do not consider the use of the discrete wavelet transformation (or DWT) method for modelling the variance profiles as originally proposed in steps 1 and 2 in Chang and Yadama (2010). Moreover, according to Chang and Chou's (2009) study, the B-spline fitting is sufficient without applying the DWT to the Chang and Yadama's method when the monitoring of the profile shape change is the only consideration. Therefore, the B-spline fitting technique is applied to construct the proposed control charts. In addition to these changes, a MEWMA control chart is selected to be the charting tool due to its sensitivity and flexibility. In addition, the MTY decomposition method can be replaced by Chang and Chou's (2010) marginal cumulative sum (or CUSUM) glyphs because of its benefits of visualization and capability of dealing with high dimensional dataset of up to 20. The procedure of modified Chang and Yadama's method is shown in Figure 3. The MEWMA control chart is introduced in the next section.

Procedure of the Modified Chang and Yadama's Method
Step 1: Partition the mean signal into p segments.
Step 2: Apply B-spline with c control points to each segment, so that a control point matrix of order $n \times c \cdot p$ is constructed, where n is the number of profiles.
Step 3: The mean distance difference vector $Y_i = [D_{i1}, D_{i2}, \dots, D_{ip}]$ is calculated, in which is associated with the control point matrix.
Step 4 To monitor the profile stability, the MEWMA statistics are calculated using the mean distance difference vectors.
Step 5: If process is in-control, go to Step 3; otherwise marginal CUSUM glyphs method is used for identifying the responsible T^2 components.

Figure 3 Procedure of Modified Chang and Yadama's Method

Multivariate EWMA Control Chart

The charting technique based on MEWMA is introduced in this section. The MEWMA was first developed by Lowry et al. (1992). It is the extension version of the EWMA for solving multivariate quality control problem. The procedure of MEWMA is presented as follows. The MEWMA statistics T^2 of the i^{th} observation is shown in equation (2), where Z_i is the extension form of univariate EWMA as shown in equation (3). Note that $0 \leq \lambda \leq 1$ and Z_0=0. The selection of chart parameters, λ and H, can be found in Prabhu and Runger's (1997). The variance-covariance matrix of Z's, Σ_{Z_i}, can be calculated using equation (4), where Σ is a variance-covariance matrix, which is either known or can be estimated from a phase I control charting procedure with m individual observations according to equations (5), (6), and (7). Moreover, Σ, shown in the equation (5) performs better than that of using the conventional approach if there was no trend, cycle, etc., in the process. If the process was totally random, the variance-covariance structure determined by equation (5) and the conventional approach would have no difference (Holmes and Mergen, 1993).

$$T_i^2 = Z_i^{'} \Sigma_{Z_i}^{-1} Z_i \tag{2}$$

$$Z_i=\lambda(x_i)+(1-\lambda)Z_{i-1} \tag{3}$$

$$\Sigma_{Z_i} = \frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]\Sigma \tag{4}$$

$$\Sigma = \frac{V'V}{2(m-1)} \tag{5}$$

$$V = [v_1' \; v \; \cdots \; v_{m-1}']' \tag{6}$$

$$v_i = x_{i+1} - x_i \tag{7}$$

<u>Method I: One Chart for All Profiles and One Segment per Profile</u>

The first method proposed in this study is straightforward. We first apply the Step 1 to Step 5 of the modified Chang and Yadama's method in Figure 3 with one segment to each profile so that multiple profiles become a vector with each element representing one profile. As shown in equation (8), given a set of profiles, X_i is the i^{th} observation and m is the number of types of profile and D_j can be calculated by using equation (1) with the number of segment of 1. For example, if the number of multiple profiles is two types with 512 data points in a profile, such as, temperature and pressure profiles of curing process of high-pressure hose products in this study, the number of types of profile m is equal to two. Therefore, the X_i is transformed from a matrix of size 2×512 into a 2×1 matrix. After Step 1 to Step 5 in Figure 3 is applied, the input variables expressed as equation (8) will be input into the MEWMA. Then a process will be stopped if any out-of-control signal takes place.

$$X_i = [D_1\ D_2 \dots D_j]_i',$$
where i is the index of observations and j=1,2,…,m (8)

Method II: One Chart for All Profiles and Multiple Segments per Profile

The second approach in this study is similar to the method I except that the number of segments of each type of profile p is greater than 1. The number of segments p is defined by users' pre-knowledge about the process of interest. In our real world case, the curing process of high-pressure hose product, the p is equal to three because the quality engineer in the PH cooperate specified the process consists of three stages. Users can also segment the profile based on the section of interests. The choice of p is a balance of diagnostic need and computational resources. Once the number of the segments is determined by quality engineers, the procedure of the method II and method I are identical. The input variables are specified in equation (9), where X_i represents the i^{th} observation, k is the index of segment, and j is the index of profile. The advantage of using segmentation is that the quality engineers can gain more details of fault locations when a diagnosis is needed. In other words, the segmentation method is sensitive to partial profile shape changes. The main drawback of method II is that the type I error of the MEWMA will increase when the number of segments increases. For example, if the process consists of 5 profiles with 4 segments each, the vector X_i will contain 20 elements while the one segment approach only has 5 elements in the vector.

$$X_i = [D_{11}, D_{12}, \dots, D_{kj}]_i'$$
where k=1,2,…,p; j=1,2,…,m; i is the index of observations. (9)

A Simulation Study

In order to study the performance of the proposed methods, all charts are established with in-control ARL approximately 200, denoted by ARL_0=200. Moreover, the out-of-control ARL is denoted by ARL_1 in the simulation study. ARL_1 is used to evaluate the charting performance of the proposed approaches at the same false alarm rate. The simulation study conducts two correlated four-parameter logistic curves. The curve equation is adapted from Jensen and Birch (2009). They used this curve in their simulation study for generating ARL property and then applied this property to the real world case. Note that, their study only consider one type of profile. In this study, the curve equation will be extended to multiple profiles case. Specifically, in each sample, two correlated profiles are studied simultaneously. Equation (10) shows the operated function in this simulation study. Note that the coefficients in the equation (10), *A*=5, *B*=8, *C*=0.6, and *D*=0, are as same as the setup in Jensen and Birch's (2009) study. Further, to simulate the correlated multiple non-linear profiles, it is assumed that the parameters between profiles are generated from independently and identically multivariate normal distribution from one observation to another. If the parameters *a* and *b* are said to be generated from independently and identically multivariate normal distribution with mean vector μ and variance-covariance matrix Σ, it is denoted as $(a,b)' \sim NM(\mu, \Sigma)'$. The equation of simulated multiple correlated non-linear profiles is shown in equation (11) of which coefficients follow the following distribution: $(a_{0i}, b_{0i})' \sim MN(\mu_0, \Sigma_0)'$, $(a_{1i}, b_{1i})' \sim MN(\mu_1, \Sigma_1)'$, *and* $(e_{1i}, e_{1i})' \sim MN(\mu_e, \Sigma_e)'$. Specific setup for the parameters of multivariate normal distribution is given in equation (12). Note that equation (10) determines the shape of two multiple correlated profiles in the simulation study. Following the profile property of the high-pressure hose products in the curing example, in which the noises within each profile are very small, and the variance-covariance matrix of error terms in equation (11) is denoted as Σ_e specified by equation (12). An example of 25-pair in-control multiple correlated nonlinear profiles generated by equation (11) is shown in Figure 4.

$$y_{ij} = A + \frac{D-A}{1+\left(\frac{x_{ij}}{C_i}\right)^{B_i}} + \epsilon_{ij} \text{ for } i = 1,2,3,\dots,m; j = 1,2,3,\dots,n_i \tag{10}$$

$$\begin{cases} f_{ij} = a_{0i} + a_{1i}y_{ij} + e_{1i} \\ g_{ij} = b_{0i} + b_{1i}y_{ij} + e_{2i} \end{cases} \tag{11}$$

$$\mu_0 = (10,-5)', \mu_1 = (10,3)', \Sigma_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \Sigma_1 = \begin{pmatrix} 0.1^2 & 0.01\rho \\ 0.01\rho & 0.1^2 \end{pmatrix}, \text{ and } \mu_e = (0,0)', \Sigma_e = \begin{pmatrix} 0.1^2 & 0.01\rho \\ 0.01\rho & 0.1^2 \end{pmatrix} \text{ where} \tag{12}$$

ρ is correlation parameter.

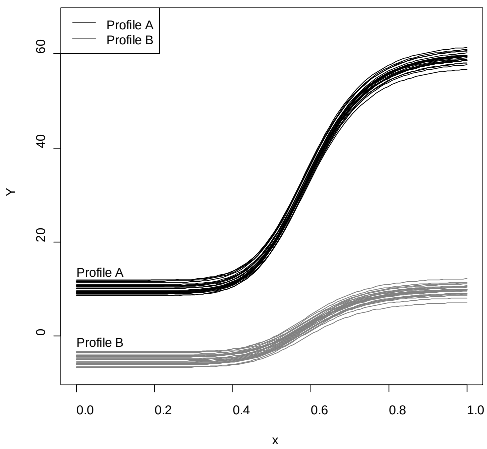


Figure 4 An example of 25-pair in-control profiles.

Experimental Design

To test the robustness of the proposed methods, profiles shifted in shape is considered in this simulation study with various testing factors. Figure 5 shows the experimental design of the simulation study. In this research, only shape changes are considered due to the shape change could be caused by a number of factor combinations. Given two types of profiles, Profile A and Profile B, possible factor combinations include two categories and five scenarios for the profile shapes changes are given according the scheme in Figure 5 for the simulation study.

In addition to shifted types, correlations between profiles and shift sizes are also considered in this simulation study. For the factor of correlation between profiles, there are three attributes to be tested, i.e., low (ρ=0.3), moderate (ρ=0.5), and high correlated (ρ=0.9). As for shift sizes, three magnitudes: small, medium, and large shift, are considered in this simulation study. The in-control ARL is fixed at approximately 200 for method I and method II. Therefore, there are total 45 cases to be tested for the proposed methods. Detailed parameter settings for the simulation study are discussed as follows:

Methods:

- Method I and II.

Shifted Types:

- Only one type of profiles changed:
    - Entire profile changed: only profile A changed (Scenario 1).
    - Partial profile changed: only profile B changed (Scenario 2).
- Both type of profiles changed:
    - Entire profile changed: both profile A and profile B changed (Scenario 3).
    - Partial profile changed: both profile A and profile B changed (Scenario 4).
    - Mixture changed: entire profile changed on profile A and partial profile changed on profile B (Scenario 5).

Correlation between profiles:

- ρ=0.3, 0.5, and 0.9

Shift size:

- Small, Medium, and Large shift

Performance:

- Average Run Length with in-control ARL of approximately 200.

Figure 5 Design of experiment of simulation study.

<u>Only the shape of either profile A or B is shifted</u>

Scenario 1: The shape of profile A shifts entirely and the shape of profile B unchanged

In this scenario, the shape of profile A is vertically shifted away from the reference profile. With respect to the real world case, the curing process for high-pressure hose products, it is possible that the temperature profiles are shifted and go above the reference profile if the PLC or a thermocouple malfunctions, but the chamber is still airtight so that the pressure profiles are in control. This scenario may result in defective hose products due to overheating in the chamber. In this simulation study, we consider whether the shape of profile A is shifted entirely and go above the reference profile or not. Note that coefficient a_0 in equation (11) controls the vertical magnitudes change of profile A. Small shift is defined as the magnitude of a_0 from 10 to 12; medium shift is magnitude of a_0 from 10 to 14; and the large shift scenario is magnitude of a_0 from 10 to 16. Figure 6 (a) shows the profiles in scenario 1 where the shape of profile A exhibits a large shift. Twenty-five in-control profiles and another 25 shifted profiles with correlation ρ=0.5 are superimposed on top of each other.

Scenario 2: The shape of profile B shifts partially and the shape of profile A unchanged

In this scenario, we alter b_1 in equation (11) to change the partial shape of profile B. Scenario 2 examines whether the proposed method is capable of detecting a leaking chamber during the operation process. As described in scenario 1, scenario 2 also considers three different correlations between profiles. Figure 6 (b) shows the graphics of scenario 2 profiles with large shift magnitudes along with ρ=0.5. From Figure 6 (b), profile B changes at the middle of the profile. The magnitude of small shift in the simulation model is b_1 changed from 3 to 3.5. For medium shift, b_1 is changed from 3 to 4, while b_1 changed from 3 to 4.5 represents large shift in this simulation study.

Shapes of both profiles are shifted

Scenario 3: Shapes of both profiles A and B are shifted entirely

This scenario simulates the case both shapes of profiles A and B are changed entirely. Since the parameters a_0 and b_0 control vertical shifts in equation (11), a_0 is altered to change the shape of profile A higher than the reference profile, and b_0 is manipulated to make the entire profile B shift below the reference profile. The setting of correlations for scenario 3 is the same as that in the scenario 1 and 2, but the magnitudes of shift sizes are different from the former settings and given as follows: (1) small shift: a_0 shifted from 10 to 12 and b_0 shifted from -5 to -3; (2) medium shift: a_0 shifted from 10 to 14 and b_0 shifted from -5 to -1; and (3) large shift: a_0 shifted from 10 to 16 and b_0 shifted from -5 to 1. The superimposed profiles A and B with large shifted for scenario 3 is shown in Figure 6 (c). Scenario 3 intends to simulate the case where both temperature and pressure profiles are changed, especially when malfunctions take place in both thermocouples and pressure sensors or the door of vulcanizer is not sealed.

Scenario 4: Shapes of both profiles A and B are partially changed

Scenario 4 examines the case when shapes of both profiles are partially changed. In this case, profiles A and B are all in control before the middle point of the process, but both profile A and profile B are shifted after the middle point. Figure 6 (d) shows the overall profiles of A and B including shifted profiles with large shifted magnitude.

The shift size in this scenario is as the follows: (1) small magnitude size of shift is simulated by altering a_1 from 10 to 10.5 and b_1 from 3 to 3.5; (2) medium shift is generated by changing a_1 from 10 to 11 and b_1 from 3 to 4; (3) large shift is created by changing the parameters a_1 and b_1 from 10 to 11.5 and 3 to 4.5, respectively. The correlations setting for each profiles in this scenario is the same as scenario 1 as well.

Scenario 5: Shape of profile A is shifted entirely and the shape of profile B is shifted partially

The parameter settings for the profiles in scenario 5 are given as following: (1) small shift: a_0 shifted from 10 to 12 and b_1 shifted from 3 to 3.5; (2) medium shift: a_0 shifted from 10 to 14 and b_1 shifted from 3 to 4; (3) large shift: a_0 shifted from 10 to 16 and b_1 shifted from 3 to 4.5. Specifically, we examine if the proposed method is capable of detecting changes when the shape of temperature profile A is shifted entirely but the pressure profile B is changed after half of the process. Figure 6 (e) shows the superimposed profile A and profile B with large shift magnitude.

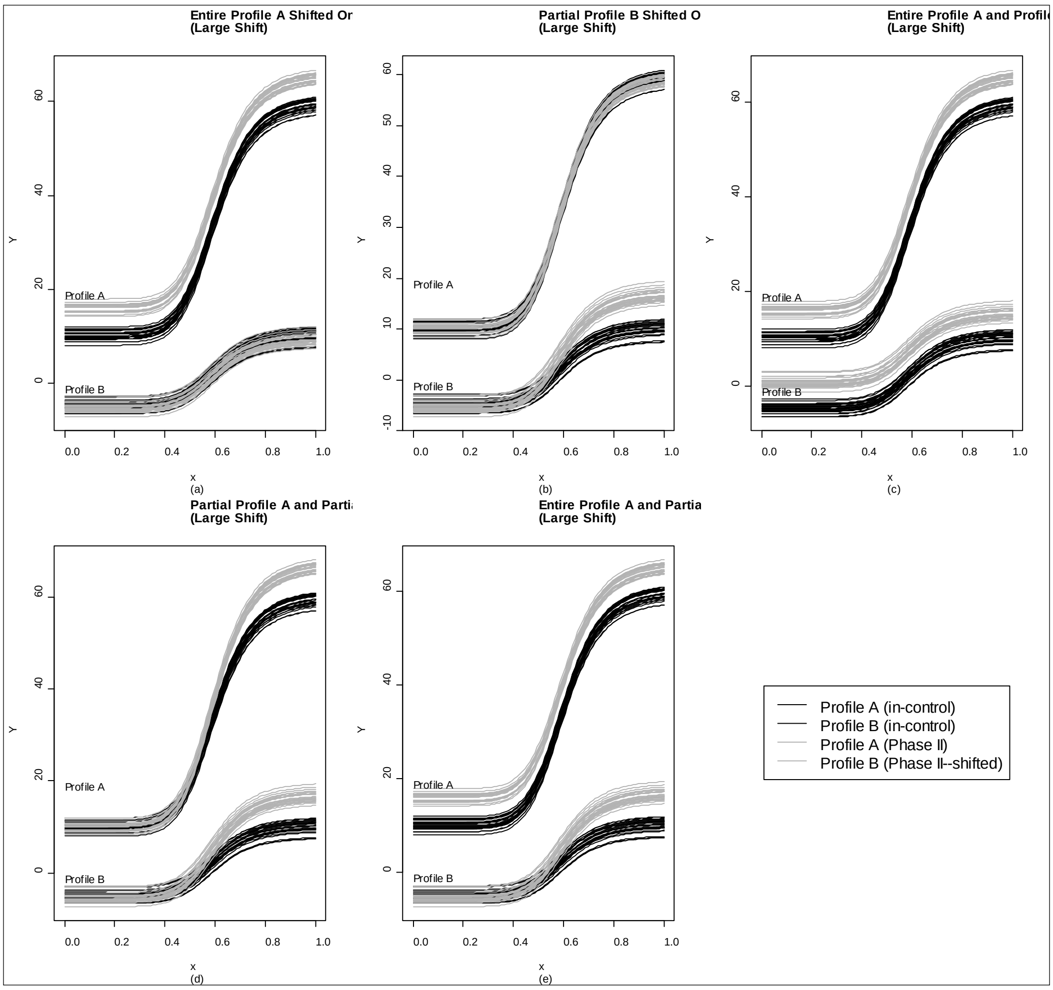


Figure 6 Scenarios of simulation study, (a) entire Profile A shifted only; (b) partial Profile B shifted only; (c) both Profile A and Profile B shifted entirely; (d) both Profile A and Profile B changed partially; (e) entire Profile A changed and partial Profile B changed.

Simulation Results and Discussion

Simulations according to the scenarios 1 to 5 described in the previous section are conducted. The property of method I and method II is characterized by both in-control and out-of-control ARL values. We fixed the in-control ARL_0=200 for both methods. The out-of-control ARL, denoted by ARL_1, is the smaller the better. The smallest ARL_1 of both methods for all scenarios are shown in Table 1, which can also be used for MEWMA design since λ and H are readily available. In Table 1, the values of ARL_1 of both methods are close to each other when the correlation coefficient, ρ, is low or moderate. However, once ρ is high, the ARL_1 is dramatically decreased in all scenarios. For example, the ARL_1 for the case of small shift of scenario 2 is 36.349 when $\boldsymbol{\rho}$=0.3, but it decreases to 19.9006 when $\boldsymbol{\rho}$=0.9. This crucial result confirms the research hypothesis that highly correlated profiles enhance detection power. This result also justifies the merit of the use of multivariate control charts to monitor all profiles simultaneously rather than monitoring one profile at a time.

Table 1 The smallest ARL_1 method I method II under different correlation structures in all scenarios.

		ρ=0.3		ρ=0.5		ρ=0.9	
		Method I	**Method II**	**Method I**	**Method II**	**Method I**	**Method II**
Scenario	Shift Size	λ=0.1 H=14.03	λ=0.1 H=25.90	λ=0.1 H=14.49	λ=0.1 H=28.28	λ=0.1 H=14.49	λ=0.1 H=33.01
1	Small	**2.981**	3.994	**2.922**	4.140	**1.695**	2.103
	Medium	**1.480**	1.827	**1.468**	1.873	**1.205**	1.259
	Large	**1.101**	1.233	**1.107**	1.253	**1.061**	1.081
2	Small	36.349	**2.477**	37.816	**2.562**	19.901	**1.706**
	Medium	18.203	**1.480**	18.407	**1.555**	9.160	**1.069**
	Large	10.474	**1.136**	10.470	**1.171**	5.333	**1.018**
3	Small	**1.753**	2.369	**1.753**	2.604	**1.513**	1.914
	Medium	**1.070**	1.195	**1.055**	1.244	**1.006**	1.260
	Large	**1.001**	1.006	**1.000**	1.006	**1.000**	1.001
4	Small	18.390	**1.385**	20.123	**1.552**	25.526	**2.181**
	Medium	9.389	**1.054**	10.341	**1.131**	13.240	**1.471**
	Large	5.696	**1.003**	6.339	**1.015**	8.163	**1.173**
5	Small	2.907	**1.730**	2.968	**1.791**	2.048	**1.151**
	Medium	1.476	**1.101**	1.497	**1.120**	1.208	**1.039**
	Large	1.103	**1.004**	1.110	**1.006**	1.044	**1.006**

Moreover, method I performs better (i.e. smaller ARL1 values) in scenario 1 and scenario 3 comparing to method II, while the method II has lower ARL_1 in scenario 2, 4 and 5 than method I. In summary, when the shape of a profile change entirely, such as, scenario 1 and 3, method I is recommended. Otherwise, method II is recommended for the other cases, such as, scenarios 2, 4, and 5, where profiles shift partially. In practice, one may not know whether a profile may shift entirely or partially. It is up to the quality engineers to collect process data and identify the majority of the scenarios during a phase I study. Details of the proposed methods property, ARL_0 and ARL_1, for all five scenarios in the simulation study are shown in Table A1 and Table A2 in the Appendix.

We suggest the following strategy for users to implement the proposed methods. First, users should determine whether segmentations of profiles are appropriate or not. Second, they should also examine whether a linear model or nonlinear model can be used to fit the entire profile or a segment of a profile. For example, the shape of temperature profiles in the curing process consists of three stages the heat-up stage, the curing stage and the cool-down stage. If users need to know at what stage a profile may be out of control, then profiles need to be divided into three segments. A linear model is adequate for the curing stage since the profile shape in this stage is a straight line. However, nonlinear models are needed to fit the warm-up stage and the cool-down stage. If users are confident in their process is stable and they only intent to know whether the process is in-control or not, method I is capable of fulfilling this need with its simplicity. Nonlinear models should be used to fit the entire temperature and pressure profiles.

If the number of segments times the number of profile types is larger than 10, the conventional multivariate control chart may lose its effectiveness (Montgomery, 2009) such that the proposed methods work worse than the nominal performance. To deal with this issue, users can combine method I and method II together. In other words, users can apply the hybrid method to separate process monitoring into stages if the process stages are well defined. Each stage can be treated as one complete profile period and method I can be applied to each stage. For instance, if the process can be divided into three segments, users can construct three multivariate control charts for the process. Note that each multivariate control chart is associated to each stage. The advantage of this method is that users can monitor the process and diagnose a potential problem at the end of a stage instead of the end of a process. This hybrid method maintains the effectiveness of the multivariate control chart.

In summary, the charting frameworks of method I, method II, and the hybrid method are summarized here for any general multiple profile problem with p profiles and m segments. We assume that all profiles can be segmented at the same locations. For method I, one MEWMA chart is maintained with the plotting statistics of a $p \times 1$ vector. Each element of this vector represents an average sum of deviation**s** of a profile from its nominal profile. Method II also maintains one MEWMA chart but with the plotting statistics of a $mp \times 1$vector. Each element of this vector is the average sum of deviation of a segment instead of a profile. Finally, the hybrid method maintains m EWMA charts with the plotting statistics of a $p \times 1$vector. Each element of this vector is the average sum of deviation of a segment. All segments are from their respective profiles. Each MEWMA is used during a particular segment only.

A Case Study: a Curing Process of High-Pressure Hose Products

In this section, the proposed charting framework for monitoring multiple nonlinear correlated profiles is applied to a curing process that consists of temperature and pressure profiles for high-pressure hose products. PH Corporation seeking opportunities for improvement provides 154 air temperature and pressure profiles of phase I data. All 154 profiles are superimposed in Figure 1. Also, based on the PLC setting for temperature, each profile was divided into three segments, the heat-up stage, curing stage~~,~~ and cool-down stage. Therefore, p=3 is given for method II in the process. Since two kinds of profiles are involved, m=2 in equation (9).

Those 154 profiles are prepared for phase I control charting. In Figure 1, it is easy to visually identify some out-of-control profiles due to their shapes are significantly different from the major group. Figure 7 shows all abnormal profiles, characterized by solid lines, as well as in-control profiles, shown in shadow. Note that the abnormal profiles are excluded from the phase I control charting. Figure 8 shows the MEWMA control charts on the cleaned data set using (a) method I and (b) method II. From these figures, we observe that all data points are in control with its associated control limit H. Therefore, the control limit H, mean vector, and variance-covariance matrix generated from method I and method II are used in phase II process for further process monitoring.

The issue of losing effectiveness using the multivariate control chart caused by the number of input variables larger than 10 may happen when the number of segments and number of profile types are large. To deal with this problem, the hybrid method is used to construct~~s~~ three MEWMA charts for heat-up, curing, and cool-down stage, respectively. Figure 9 shows the phase I MEWMA control charts for these three stages using the hybrid methods. Note that, since the number of profile types in each control chart is two, users can use the same control limit H as in method I, i.e., H=14.03. By using this hybrid method, users can not only diagnose which stage is responsible for the out-of-control signal, but also examine the stability at the end of the stage instead of at end of the entire process. For example, if the MEWMA control chart in the heat-up stage signal for out-of-control, operators can determine to stop the curing process or not, and start to diagnose potential causes at the end of that heat-up stage rather than at the end of the curing process. Early detection of process faults provides energy savings and assures product quality.

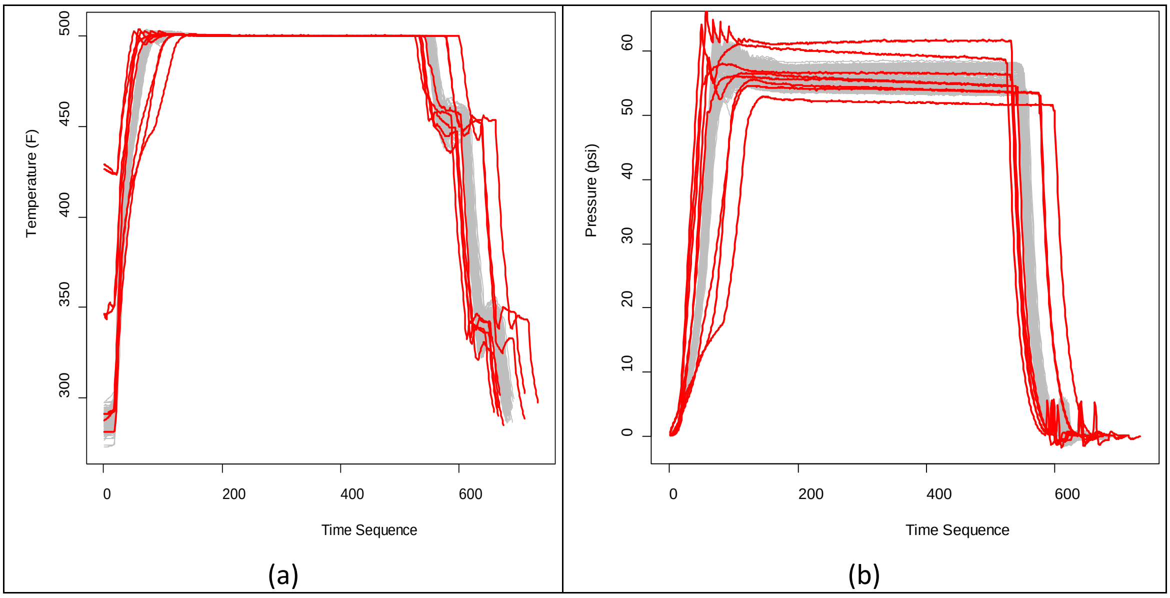
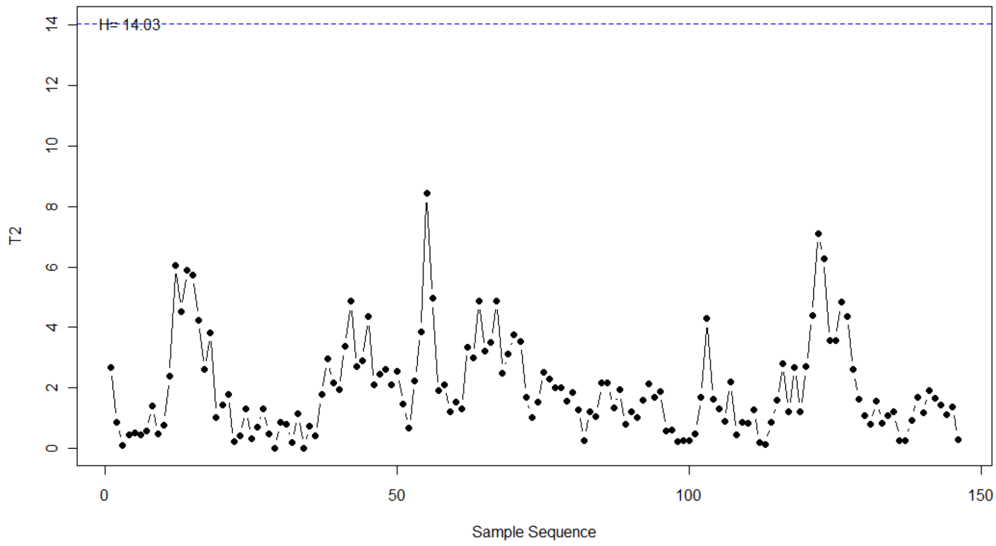
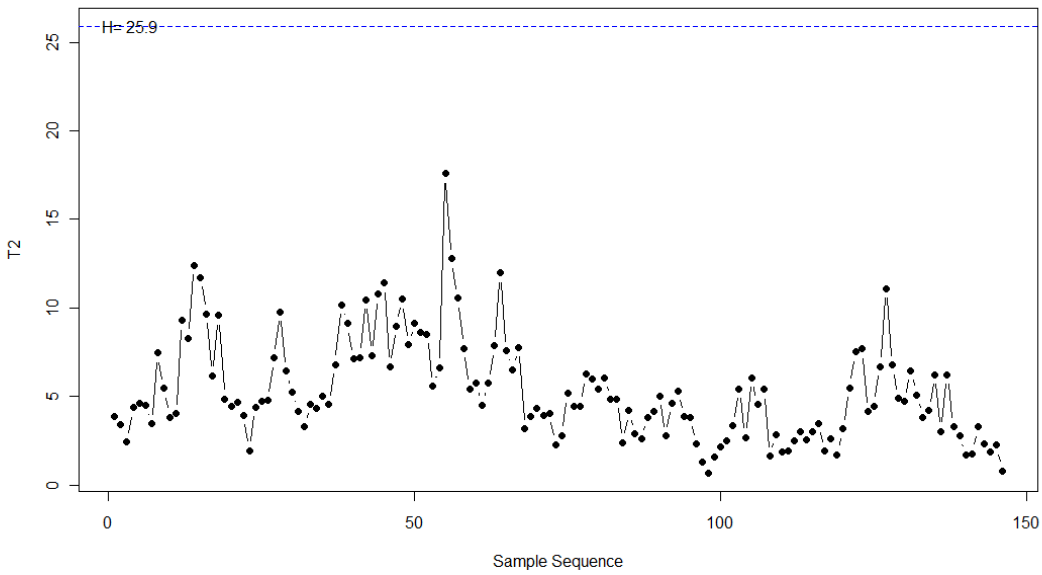


Figure 7 Abnormal (a) temperature profiles and (b) pressure profiles superimposed on in-control profiles.



(a)



(b)

Figure 8 The phase I process MEWMA control charts of curing process of high-pressure hose products using (a) method I and (b) method II.

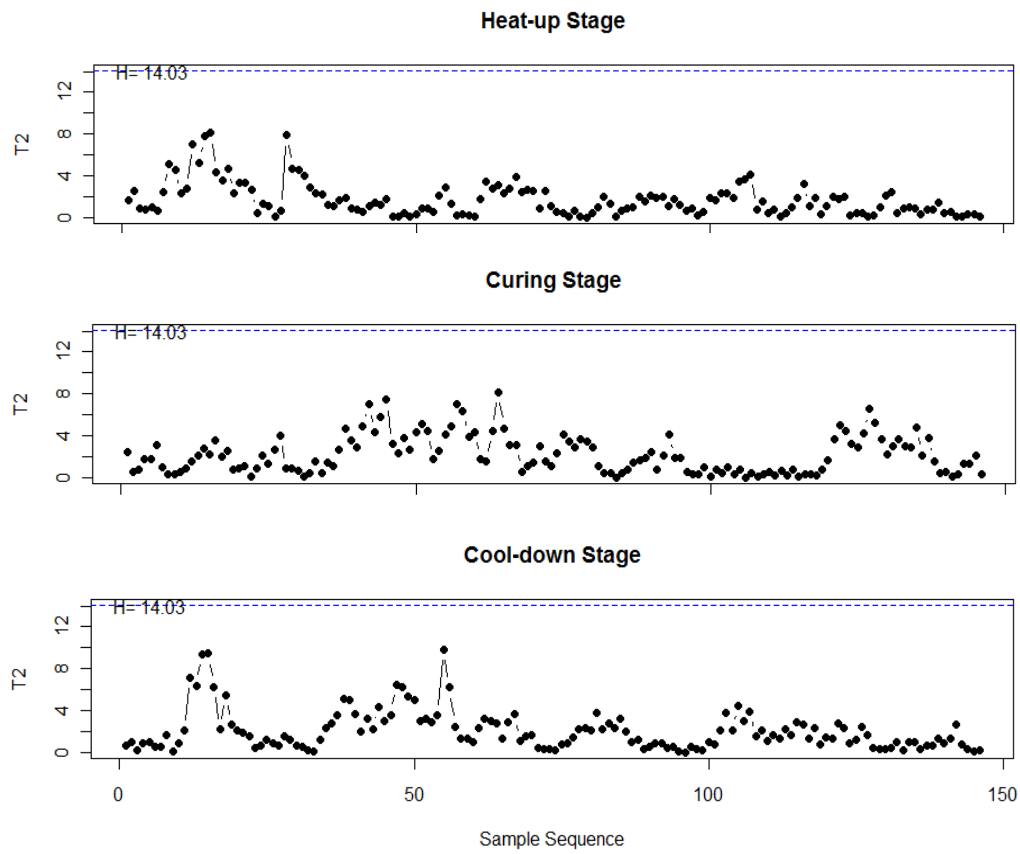


Figure 9 phase I control charts of the curing process using the hybrid method.

Conclusion and Future Study

Most profile analysis research up to date deals with the monitoring of single profile, but applications, such as, the hydraulic press machine and the curing process of high-pressure hose products, contain multiple correlated profiles for process monitoring. Although the monitoring of each type of profiles independently may be considered as an approach for detecting abnormal ones, high false alarm and low detecting power may be a consequence when given profiles are highly correlated. This study aims to tackle this problem by providing a framework for the process monitoring when the process consists of multiple linear or nonlinear correlated profiles. The proposed method I applies the modified Chang and Yadama's method that extends one profile process monitoring method (with m segments) to a multiple profile solution (each profile representing a segment). The proposed method II divides each profile into p segments first, and then applies the modified Chang and Yadama's method to each segment. Therefore, the multiple-profile problem can be considered as a multivariate statistical process control problem with the number of input variables $m \times p$. According to the simulation study, the method I has better performance in terms of out-of-control ARL values when the shape of profiles is changed entirely in the process, and method II is more sensitive when the process experiences partial profiles changes.

Additionally, the simulation results show that when profiles are highly correlated, the detecting power of the proposed method is much better than those cases with lower correlations. In a real-world case study, both method I and method II are capable of constructing the phase I process control chart if the product of the number of segments and the number of profile types is smaller than 10. The hybrid method that combines method I and method II are also investigated to provide quality engineers a broader view of diagnosis and maintains the effectiveness of detecting power. Moreover, since the proposed methods are sharing the common treatment of Chang and Yadama's (2010) method, they can handle the different shapes from other applications as long as profiles in the process have their own desired shapes with gold reference profiles existed. For example, the shape of temperature and pressure profiles generated from a curing process of high-pressure hose products follow their respective shapes, and the proposed methods all construct the phase I control chart without generating any false alarm signals.

Although the proposed methods show capability of dealing with multiple nonlinear profiles on monitoring products' quality perspective, they have some limitations. First, although the method I is capable of detecting entire shape shifted scenario, it lack of diagnosis ability when the process goes out-of-control. Second, even though the method II provides quality engineers more information in regards to diagnostic purpose, it will be less efficient if the number of segments times the profile types is over 10. Finally, the hybrid method takes care of those disadvantages that provided by the method I and method II, but it will be distract if there are too many segments have been defined. The future study of profile analysis should be extended to cover all the disadvantages of the proposed methods and to be applied for more complicated data sets, such as, the data sets of images and spatial surface.

Reference

Chang, S.I. and Chou, S.H., 2009. A Study of Using Wavelet Transformation and B-Spline Approximation for Nonlinear Profiles Monitoring, Proceedings of the 2009 Industrial Engineering Research Conference (IERC), 1567-1572, Miami, Florida, May 30-June 03.

Chang, S.I. and Chou, S.H., 2010. A Visualization Decision Support Tool for Multivariate SPC Diagnosis Using Marginal CUSUM Glyphs, *Quality Engineering*, 22(3), 182-198.

Chang, T. C., Gan, F. F., 2006. Monitoring Linearity of Measurement Gauges. *Journal of Statistical Computation and Simulation*, 76(10), 889-911.

Chang, S.I., Tsai, T.R., Lin, D.K.J., Chou, S.H., and Lin, Y.S., 2012. Statistical Process Control for Monitoring Nonlinear Profiles: A Six Sigma Project on Curing Process, *Quality Engineering*, 24(2), 251-263.

Chang, S.I. and Yadama, S., 2010. Statistical Process control for Monitoring Non-linear Profiles using Wavelet Filtering and B-Spline Approximation, *International Journal of Production Research*, 48(4), 1049-1068.

Chicken, E. Pignatiello, Jr., J., andSimpson, J.R., 2009. Statistical Process Monitoring of Nonlinear Profiles Using Wavelets, *Journal of Quality Technology*, 41(2), 198-212.

Chen, S. and Nembhard, H.B., 2011. A High-dimensional Control Chart for Profile Monitoring, *Quality and Reliability Engineering International*, 27(4), 451-464.

Ding, Y., Zeng, L., and Zhou, S., 2006. Phase I Analysis for Monitoring Nonlinear Profiles in Manufacturing Processes, *Journal of Quality Technology*, 38(3), 199-216.

Holmes, D. S. and Mergen, A. E., 1993. Improving the Performance of the T2 Control Chart, *Quality Engineering*, 5(4), 619-625.

Jensen, W.A. and Birch, J.B., 2009. Profile Monitoring Via Nonlinear Mixed Models, *Journal of Quality Technology* 41(1), 18-34.

Jensen, W.A., Birch, J.B., and Woodall, W.H., 2008. Monitoring Correlation within Linear Profiles using Mixed Models, *Journal of Quality Technology*, 40(2), 167-183.

Jin, J., and Shi, J., 1999. Feature-Preserving Data Compression of Stamping Tonnage Information Using Wavelets, *Technometrics* 41(4), 327–339.

Kazemzadeh, R.B., Noorossana, R., Amiri, A., 2008. Phase I Monitoring of Polynomial Profiles, *Communications in Statistics—Theory and Methods*, 37(10), 1671-1686.

Kang, L.; Albin, S. L., 2000. On-Line Monitoring when the Process Yields a Linear Profile, *Journal of Quality Technology*, 32(4), 418-426.

Kim, K., Mahmoud, M. A. and Woodall, W. H., 2003. On the Monitoring of Linear Profiles, *Journal of Quality Technology*, 35, 317-328.

Lowry, C.A., Woodall, W.H., Champ, C.W., and Rigdon, S.E., 1992. A Multivariate Exponentially Weighted Moving Average Control Chart, *Technometrics*, 34(1), 46-53.

Mahmoud, M. A., 2008. Phase I Analysis of Multiple Regression Linear Profiles, *Communications in Statistics - Simulation and Computation*, 37(10), 2106-2130.

Mason, R.L., Tracy, N.D., and Young, J.C., 2001. A Practical Approach for Interpreting Multivariate T2 Control Chart Signals, *Journal of Quality Technology*, 29 (4), 396 – 406.

Montgomery, D. C., 2009. *Introduction to statistical quality control*. New York, NY: John Wiley & Sons.

Noorossana, R., Amiri, A., and Soleimani, P., 2008. On the Monitoring of Autocorrelated Linear profiles, *Communications in Statistics-Theory and Methods*, 37(3), 425-442.

Noorossana, R, Eyvazian, M., Vaghefi, A., 2010. Phase II Monitoring of Multivariate Simple Linear Profiles, *Computers & Industrial Engineering*, 58(4), 563-570.

Noorossana, R., Saghaei, A., and Amiri, A., 2011. *Statistical Analysis of Profile Monitoring*, Hoboken, New Jersey: John Wiley & Sons, Inc.

Paynabar, K. and Jin, J., 2011. Characterization of Non-linear Profiles Variations using Mixed-effect Models and Wavelets, *IIE Transactions*, 43(4), 275-290.

Prabhu, S.S. and Runger, G.C., 1997. Designing a Multivariate EWMA Control Chart, *Journal of Quality Technology*, 29(1), 8-15.

Qiu, P., Zou, C. and Wang, Z., 2010. Nonparametric Profile Monitoring by Mixed Effects Modeling, *Technometrics*, Vol. 52(3):265-277.

Reis, M.S., SARAIVA, P.M., 2006. Multiscale Statistical Process Control of Paper Surface Profiles, *Quality Technology and Quantitative Management*, 3(3), 263-282.

Shiau, J.J.H., Huang, H.L., Lin, S.H., and Tsai, M.Y., 2009. Monitoring Nonlinear Profiles with Random Effects by Nonparametric Regression, *Communications in Statistics-Theory and Methods*, 38(10), 1664-1679.

Walker, E. and Wright, S. P., 2002. Comparing Curves Using Additive Models, *Journal of Quality Technology*, 34(1), 118-129.

Williams, J. D., Woodall, W. H., and Birch, J. B., 2007. Statistical Monitoring of Nonlinear Product and Process Quality Profiles, *Quality & Reliability Engineering International*, 23(7), 925-941.

Woodall, W.H., 2007. Current Research on Profile monitoring, *Produção* 17(3), 420-425.

Zou, C.,Tsung, F., and Wang, Z., 2007. Monitoring General Linear Profiles Using Multivariate Exponentially Weighted Moving Average Schemes, *Technometrics*, 49(4), 395-408.

Appendix

TableA 1 The average run length of method I with three different correlation structures.

Method I									
ρ=0.3									
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
H	14.03	15.81	16.97	17.86	18.61	19.13	19.55	19.87	19.98
ARL_0	200.170	199.925	199.809	200.167	199.617	199.829	199.840	200.167	199.809
Scen. 1 (small)	2.981	3.203	3.400	3.614	3.896	4.242	4.705	5.275	5.987
Scen. 1 (medium)	1.480	1.554	1.608	1.665	1.717	1.760	1.821	1.897	2.014
Scen. 1 (large)	1.101	1.125	1.142	1.162	1.174	1.184	1.195	1.205	1.214
Scen. 2 (small)	36.349	49.035	59.258	68.725	76.818	83.557	88.074	94.022	99.801
Scen. 2 (medium)	18.203	24.604	30.988	37.398	43.991	49.290	55.851	62.033	66.700
Scen. 2 (large)	10.474	13.208	16.867	21.015	23.316	29.945	34.551	39.603	44.195
Scen. 3 (small)	1.753	1.886	1.976	2.053	2.163	2.277	2.432	2.662	2.886
Scen. 3 (medium)	1.070	1.091	1.107	1.122	1.135	1.144	1.155	1.164	1.173
Scen. 3 (large)	1.001	1.001	1.002	1.002	1.003	1.004	1.004	1.004	1.004
Scen. 4 (samll)	18.390	23.135	27.833	32.099	36.519	40.005	43.883	47.784	51.197
Scen. 4 (medium)	9.389	11.178	13.228	15.711	18.580	21.055	23.962	27.048	29.549
Scen. 4 (large)	5.696	6.504	7.522	8.668	10.082	11.653	13.457	15.642	18.002
Scen. 5 (small)	2.907	3.135	3.301	3.493	3.743	4.047	4.386	4.935	5.499
Scen. 5 (medium)	1.476	1.545	1.594	1.641	1.696	1.732	1.785	1.864	1.953
Scen. 5 (large)	1.103	1.128	1.145	1.163	1.175	1.186	1.194	1.205	1.217
ρ=0.5									
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
H	14.49	16.26	17.55	18.52	19.28	19.83	20.3	20.57	20.72
ARL_0	199.870	199.829	200.106	200.249	199.916	199.809	200.152	200.166	199.870
Scen. 1 (small)	2.922	3.163	3.369	3.571	3.834	4.165	4.638	5.218	5.852
Scen. 1 (medium)	1.468	1.541	1.592	1.646	1.701	1.748	1.811	1.886	1.983
Scen. 1 (large)	1.107	1.132	1.155	1.172	1.185	1.198	1.210	1.221	1.238
Scen. 2 (small)	37.816	49.977	61.750	71.478	80.470	86.518	92.114	96.633	102.777
Scen. 2 (medium)	18.407	24.791	32.102	39.083	45.808	51.963	58.493	64.189	69.712
Scen. 2 (large)	10.470	13.175	17.043	21.470	26.123	31.187	36.361	41.304	46.366
Scen. 3 (small)	1.753	1.891	2.002	2.115	2.255	2.419	2.654	2.930	3.265
Scen. 3 (medium)	1.055	1.079	1.096	1.111	1.125	1.134	1.148	1.160	1.171
Scen. 3 (large)	1.000	1.000	1.001	1.001	1.001	1.001	1.001	1.001	1.001
Scen. 4 (samll)	20.123	25.168	30.797	35.059	40.091	43.785	47.510	51.021	55.109
Scen. 4 (medium)	10.341	12.377	14.897	17.851	20.570	23.577	26.695	29.542	32.576
Scen. 4 (large)	6.339	7.271	8.485	9.909	11.521	13.280	15.450	17.942	20.349
Scen. 5 (small)	2.968	3.192	3.398	3.596	3.857	4.185	4.604	5.130	5.797
Scen. 5 (medium)	1.497	1.569	1.623	1.677	1.727	1.770	1.837	1.913	2.026
Scen. 5 (large)	1.110	1.135	1.154	1.174	1.186	1.194	1.210	1.220	1.236
ρ=0.9									
lambda	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
H	14.49	15.95	16.99	17.81	18.4	18.9	19.16	19.38	19.43
ARL_0	199.995	199.809	199.743	199.731	199.840	200.167	200.167	200.152	200.098
Scen. 1 (small)	1.695	1.741	1.767	1.792	1.814	1.825	1.843	1.869	1.881
Scen. 1 (medium)	1.205	1.219	1.229	1.236	1.244	1.250	1.253	1.259	1.261
Scen. 1 (large)	1.061	1.067	1.073	1.076	1.078	1.082	1.084	1.085	1.085
Scen. 2 (small)	19.901	24.360	29.931	35.278	41.769	48.012	53.213	58.377	62.349
Scen. 2 (medium)	9.160	10.493	12.289	14.643	17.444	20.958	24.490	28.552	32.204
Scen. 2 (large)	5.333	5.852	6.479	7.379	8.526	10.127	11.763	13.906	16.345
Scen. 3 (small)	1.513	1.590	1.652	1.713	1.762	1.827	1.888	1.941	1.972
Scen. 3 (medium)	1.006	1.020	1.038	1.049	1.062	1.073	1.081	1.088	1.091
Scen. 3 (large)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Scen. 4 (samll)	25.526	31.831	37.762	43.219	48.015	52.195	55.907	59.355	62.502
Scen. 4 (medium)	13.240	16.166	19.269	22.902	26.093	29.443	32.566	35.929	39.076
Scen. 4 (large)	8.163	9.380	10.997	12.814	14.774	17.386	19.817	22.445	25.021
Scen. 5 (small)	2.048	2.136	2.196	2.258	2.326	2.405	2.481	2.603	2.747
Scen. 5 (medium)	1.208	1.233	1.246	1.258	1.269	1.275	1.283	1.294	1.300
Scen. 5 (large)	1.044	1.051	1.054	1.058	1.060	1.062	1.062	1.064	1.064

TableA 2 The average run length of method II with three different correlation structures.

Method II									
***ρ*=0.3**									
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
H	25.9	30.85	35.68	40.12	44.13	47.45	50.14	52.09	53.3
ARL_0	199.958	199.829	199.809	199.967	200.173	199.989	199.905	199.910	200.157
Scen. 1 (small)	3.994	4.754	5.920	7.896	11.420	17.129	26.333	39.057	55.486
Scen. 1 (medium)	1.827	2.076	2.350	2.655	3.075	3.673	4.565	6.151	8.515
Scen. 1 (large)	1.233	1.323	1.426	1.538	1.668	1.789	1.966	2.217	2.549
Scen. 2 (small)	2.477	2.812	3.152	3.583	4.008	4.425	4.776	5.004	5.128
Scen. 2 (medium)	1.480	1.659	1.828	1.995	2.169	2.378	2.592	2.823	3.011
Scen. 2 (large)	1.136	1.229	1.331	1.431	1.524	1.619	1.726	1.873	2.035
Scen. 3 (small)	2.369	2.807	3.319	4.021	5.273	7.464	11.209	17.554	27.241
Scen. 3 (medium)	1.195	1.297	1.417	1.552	1.696	1.848	2.044	2.332	2.833
Scen. 3 (large)	1.006	1.015	1.031	1.057	1.083	1.105	1.132	1.153	1.175
Scen. 4 (samll)	1.385	1.544	1.672	1.821	1.964	2.101	2.241	2.368	2.470
Scen. 4 (medium)	1.054	1.099	1.155	1.209	1.262	1.319	1.371	1.416	1.463
Scen. 4 (large)	1.003	1.009	1.016	1.029	1.044	1.063	1.083	1.107	1.111
Scen. 5 (small)	1.730	1.964	2.198	2.440	2.740	3.050	3.459	3.829	4.169
Scen. 5 (medium)	1.101	1.166	1.247	1.323	1.403	1.476	1.560	1.648	1.752
Scen. 5 (large)	1.004	1.010	1.021	1.037	1.053	1.068	1.085	1.100	1.113
***ρ*=0.5**									
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
H	28.28	34.4	40.14	45.58	50.21	54.05	56.94	59.14	60.46
ARL0	200.002	199.826	199.918	199.918	200.157	199.890	200.026	200.261	200.009
Scen. 1 (small)	4.140	5.068	6.520	9.160	13.793	22.487	36.628	55.881	78.656
Scen. 1 (medium)	1.873	2.185	2.487	2.860	3.376	4.108	5.312	7.541	10.969
Scen. 1 (large)	1.253	1.363	1.477	1.615	1.760	1.917	2.143	2.431	2.876
Scen. 2 (small)	2.562	2.946	3.353	3.806	4.288	4.704	5.004	5.217	5.377
Scen. 2 (medium)	1.555	1.753	1.937	2.134	2.326	2.542	2.799	3.039	3.189
Scen. 2 (large)	1.171	1.296	1.421	1.524	1.632	1.751	1.894	2.062	2.246
Scen. 3 (small)	2.604	3.188	3.963	5.355	8.145	13.603	23.815	39.324	60.158
Scen. 3 (medium)	1.244	1.412	1.593	1.786	2.009	2.284	2.727	3.555	5.149
Scen. 3 (large)	1.006	1.020	1.051	1.092	1.140	1.194	1.242	1.294	1.352
Scen. 4 (samll)	1.552	1.727	1.910	2.078	2.243	2.385	2.536	2.684	2.778
Scen. 4 (medium)	1.131	1.206	1.280	1.371	1.438	1.505	1.577	1.644	1.710
Scen. 4 (large)	1.015	1.033	1.061	1.102	1.141	1.177	1.204	1.231	1.263
Scen. 5 (small)	1.791	2.059	2.315	2.584	2.927	3.329	3.785	4.211	4.630
Scen. 5 (medium)	1.120	1.204	1.296	1.384	1.481	1.567	1.661	1.774	1.910
Scen. 5 (large)	1.006	1.016	1.030	1.050	1.070	1.093	1.113	1.133	1.151
***ρ*=0.9**									
lambda	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
H	33.01	42.39	51.24	58.85	65.58	71.08	75.29	78.35	79.98
ARL_0	199.989	199.850	200.026	200.289	199.826	199.936	200.164	199.826	199.826
Scen. 1 (small)	2.103	2.590	3.096	3.683	4.451	5.813	8.575	13.740	23.184
Scen. 1 (medium)	1.259	1.352	1.463	1.582	1.705	1.825	1.968	2.143	2.359
Scen. 1 (large)	1.081	1.118	1.157	1.188	1.214	1.234	1.261	1.286	1.303
Scen. 2 (small)	1.706	2.172	2.520	2.906	3.377	4.022	4.677	5.181	5.428
Scen. 2 (medium)	1.069	1.187	1.377	1.567	1.749	1.891	2.056	2.335	2.686
Scen. 2 (large)	1.018	1.029	1.049	1.081	1.133	1.193	1.259	1.308	1.350
Scen. 3 (small)	1.914	2.386	2.934	3.507	4.032	4.585	5.046	5.445	5.723
Scen. 3 (medium)	1.260	1.453	1.673	1.897	2.208	2.610	3.038	3.462	3.849
Scen. 3 (large)	1.001	1.035	1.151	1.297	1.409	1.500	1.617	1.826	2.137
Scen. 4 (samll)	2.181	2.479	2.773	3.091	3.366	3.569	3.665	3.737	3.777
Scen. 4 (medium)	1.471	1.660	1.842	1.998	2.151	2.312	2.433	2.520	2.556
Scen. 4 (large)	1.173	1.308	1.436	1.538	1.640	1.751	1.881	2.010	2.116
Scen. 5 (small)	1.151	1.269	1.459	1.643	1.808	1.984	2.187	2.464	2.876
Scen. 5 (medium)	1.039	1.052	1.065	1.075	1.084	1.093	1.099	1.104	1.109
Scen. 5 (large)	1.006	1.012	1.015	1.018	1.022	1.025	1.026	1.028	1.029