

SCALE UP FACTORS IN THE DESIGN OF A HYDRAULIC STARCH MILL

by

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## INTRODUCTION

Because grain sorghums are able to withstand dry and hot weather, they have become a popular crop in the Great Plains area. They yield better and give a more reliable yield than other crops over a period of years, and they afford protection against wind erosion which other crops may not provide. The production of sorghum grains in Kansas was 31,878,000 bushels in 1955(11).

Since starch is the principal constituent of grains, being present to the extent of approximately 70 per cent, the production of starch from sorghums was selected as the most promising industrial use of this crop. A series of investigations on the feasibility of extracting starch from sorghum grain and on the utility of by-products was started at Kansas State College in the late 1930's.

In earlier investigations the starch was recovered from sorghum endosperm through operations quite similar to those used in the corn starch industry. Johnston (8) reported that a wet milling process was developed to obtain starch from sorghum at this college in 1942. Lately, similar reports were made by Zipf et al. (16), Watson et al. (15), and Kerr (12) on this subject. Taylor (14) and Hightower (5) reported in 1949 that the wet milling process for sorghum starch had been developed into a full industrial scale process at the Corpus Christie, Texas, plant of the Corn Products Refining Company.

With the purpose of producing starch granules of approximately the same size by avoiding direct compressing and shearing forces in grinding, Banowetz (1) and Drobot (4) made investigations of "hydraulic grinding" of sorghum grain instead of the conventional Buhrstone mill method. They

reported that a starch of good quality was obtained but no satisfactory yield was achieved, probably because of the excessive over grinding which is a characteristic of batch grinding. In order to overcome this defect, Fan (5) developed this process into a continuous operation. Higher recovery of starch, lower power consumption, smaller units for a given capacity, more uniform particle sizes, lower labor requirements, and good quality starch were reported.

Further research using the hydraulic mill for continuous milling was done by Chiang (3), Chai (2) and Hsieh (7). Chiang studied factors which affected the yield and quality of the product, such as steeping conditions, feeding rate, water rate, and screen capacity. Chai compared the use of sulfur dioxide solutions with plain water in steeping the grits, and also the operation of the hydraulic mill with a Buhr stone mill in the grinding process. He concluded that the use of sulfur dioxide in steeping sorghum grits was of little value. Hydraulic milling utilizes the principle of physical disruption in the presence of water to remove the starch from the grit structure. Mechanical pressure is not applied directly to the grits, instead, breakdown is induced by the action of high speed rotating blades through a slurry of grits and water contained in a mill casing. The grit particles are abraded as they strike the casing wall, rub each other and undergo exposure to a rapidly rotating liquid stream. It is believed that the advantages of the process are less tendency for local overheating of the slurry, less rupture of the starch granules, and more efficient removal of protein from the starch granules. These combine to produce a higher quality starch. Hsieh's work was devoted entirely to the design variables in the hydraulic mill.

Since no theories for the operation of this type of grinding device were available, trial and error methods were used by Hsieh. Different sizes and shapes of mill casings and different sizes and pitch of blades for milling were used at various shaft speeds and at various feeding rates to the mill.

Three different sizes and shapes of mill were used by Hsieh. The first was a clover-leaf shape, the second was a small, round casing made of 8-inch standard pipe, and the third was a large casing made of 12-inch standard pipe. Two different sizes of blades were used. One was 2-13/16 inch by 1 inch by 1/8 inch, and the other was 4-15/16 inch by 1 inch by 1/8 inch. Different pitches and arrangements of blades were used at various shaft speeds ranging from 1800 to 3100 rpm and at various feeding rates, ranging from 20 to 107 pounds per hour on dry grits.

The results showed that the small casing constructed from 8-inch standard pipe was the most effective in grinding and its energy consumption, which was the lowest of the three, ranged from 0.031 to 0.053 KWH per pound of starch produced.

Milling speeds of 2100 to 2400 rpm were satisfactory. The mill temperature and the power consumption increased with increased mill speed, while the grinding effectiveness was not improved by increasing mill speed.

Flat blades with zero degree pitch were found to be most effective and gave the lowest rise in mill temperature and the lowest power consumption. Small clearance between the blades and the casing was desirable for highly effective grinding.

An increase in feed rate reduced the energy consumption per pound of starch, while the mill temperature was independent of the feed rate.

The capacity of the screens and the debranner was an important variable in obtaining high starch recovery. An additional washing screen for the bran greatly reduced the loss of starch in the bran fraction. A starch recovery of 85 per cent of the starch fed was the maximum for all runs.

This work was a continuation of this series of studies at Kansas State University. The purpose was to evaluate the scale up factors for the design of a hydraulic starch mill. In order to do this, the variables in hydraulic milling had to be simplified. All the scale up factors (variables of the milling process) had to be measured under steady state conditions. Actually, the steady state of milling was not easily determined, since any small change in one of the variables also changed the equilibrium conditions. For this reason part of this work was devoted to an analysis of the time required for steady state operation to be achieved.

Scale-up is not only the "mechanical" act of taking the results of a small unit operation and by various methods and techniques designing a satisfactory full size unit operation. Scale up is also a philosophy. It embraces an entire range of thinking from pre-pilot plant considerations to considerations given to the operation of the full size commercial unit.

A pilot plant is unlikely to yield the maximum possible amount of information unless the "critical" components at least are designed and operated in accordance with model theory. The first step is to derive the similarity criteria which govern the operations of the processes to be studied. These may be obtained either by dimensional analysis or from the fundamental differential equations of the process.

The theory of similarity is concerned with the relations between physical systems of different sizes, and it is thus fundamental to the

scaling up or down of physical and chemical processes. The principle of similarity is usually coupled with dimensional analysis, yet they are quite distinct. The principle of similarity is a general principle of nature, dimensional analysis is only one of the techniques by which the principle may be applied to specific cases, the other technique being to start from the generalized equations of motion of the system.

Material objects and physical systems in general are characterized by three qualities, size, shape and composition. All three are independently variable, so that two objects may differ in size while having the same shape and chemical composition. The principle of similarity is more particularly concerned with the general concept of shape, or it may be stated that the spatial and temporal configuration of a physical system is determined by ratios of magnitudes within the system itself and does not depend upon the size or nature of the units in which these magnitudes are measured.

The chemical engineer is concerned with complex systems composed of solid bodies and fluids in which transfer of matter and energy may occur as well as chemical change. The concept of "shape" applied to these systems involves not only the geometrical proportions of their solid members and surfaces but also such factors as fluid-flow patterns, temperature gradients, time-concentration profiles, etc. Systems which have the same configuration in one or more of these respects are said to be similar.

According to Johnstone and Thring (9) similarity may be defined in two ways: specifying the ratios either of different measurements in the same body, or specifying the ratios of corresponding measurements in different bodies. Four similarity states are important in chemical engineering namely:

Geometrical Similarity

Mechanical Similarity

Thermal Similarity

Chemical Similarity

Two bodies are geometrically similar when to every point in the one body there exists a corresponding point in the other. In geometrically similar bodies, all the corresponding ratios (or shape factors) are constant, and this ratio is termed the "scale ratio". Mechanical similarity comprises static, or static-force similarity, kinematic similarity, and dynamic similarity. These are defined as follows:

Static similarity ---- Geometrically similar bodies are statically similar when under constant stress their relative deformations are such that they remain geometrically similar.

Kinetic similarity ---- Geometrically similar moving systems are kinematically similar when corresponding particles trace out geometrically similar paths in corresponding intervals of time.

Dynamic similarity ---- Geometrically similar moving systems are dynamically similar when the ratio of all corresponding forces are equal.

Systems are thermally similar when the corresponding temperature differences bear a constant ratio to one another and when the systems are geometrically similar and, if moving, kinematically similar. Systems are chemically similar when the corresponding concentration differences bear a constant ratio to one another, and when the systems are geometrically and thermally similar and, if moving, are kinematically similar.

It has been mentioned that mechanical, thermal, or chemical similarity between geometrically similar systems can be specified in terms of

criteria which are intrinsic ratios of measurements, forces, or rates within each system. In most scale problems a large number of variables are reduced to a few dimensionless groups by using dimensional analysis or introducing dimensionless variables into differential equations. Once the relevant variables of a given process have been chosen and rearranged into dimensionless groups, one may proceed along the following route (9) (10) (13).

If it has been shown that a general functional relationship must exist between a dimensionless group,  $D_1$ , and any number of other dimensionless groups  $D_2, D_3, \dots, D_n$ , i.e., if

$$D_1 = \phi_1 (D_2, D_3, \dots, D_n) \quad (1)$$

then, the value of  $D_1$  must be fixed by the numerical values of the other groups. Therefore, if one wishes to predict  $D_1$  for a certain piece of plant-scale equipment, a single pilot-plant test in which the values of  $D_2, D_3, \dots, D_n$  were identical to those which were to be used in the full size prototype, would suffice. Since  $D_2, D_3, \dots, D_n$  would be the same for both the pilot plant and the prototype, the numerical values of  $D_1$  must also be identical, regardless of the form of the function  $\phi_1$ . This theory-of-models approach represents the simplest possible use of pilot-plant data since it is not necessary to determine the actual functional relationships between the various dimensionless groups.

While such simplicity has certain merits, it must also be noted that no further understanding of the process at hand could be obtained from the experimental run. In particular, no knowledge of the relative importance of the various dimensionless groups,  $D_2, D_3, \dots, D_n$  could be obtained. To determine the precision of the data point, it might be desirable to repeat

the experiment a number of times. While such repeated determination of the value of  $D_1$  would determine its precision or reproducibility, no light would be shed on the true accuracy of the work in this manner.

According to the theory of similitude and the theory of models reviewed above, it was necessary to consider all the possible variables (factors) which would influence the milling effect. The relationship between these variables then had to be found by dimensional analysis in order to obtain a suitable scale-up equation for the design of hydraulic mill. In order to prove the fitness of the scale-up equation, a new mill which satisfied the conditions of the scale-up equation was designed and operated under certain specified conditions. Comparison of the performance of the two mills proved the validity of the scale-up equation.

## DEVELOPMENT OF THEORY

### Rate of Approach to Steady State

In order to simplify the experimental technique, once-through operation of the pilot plant, without recycle streams, was used. Even so, after starting a run the concentration of the contents of the mill changed with time. The rate of change of mill concentration decreased, however, and eventually the concentration became constant. In order to determine an empirical equation for the rate of approach to the steady state for once-through experiments, the following analysis was made:

Samples were taken from different levels in the mill at particular milling times,  $t$  (minutes), and the concentration of the samples,  $C$  (pounds per pound water), were measured. These concentrations were then plotted against mill length,  $L$  (inches, measured from the top of the mill).

The average mill concentration,  $C_m$  (lb. grits per lb. water), was then determined from the following equation:

$$C_m = \int_{L=0}^{L=35} \frac{CdL}{L} \quad (2)$$

The integral was evaluated by the trapezoidal rule.

The average mill concentration,  $C_m$ , was obtained at various milling times and a plot of  $C_m$  versus  $t$  was made. Smooth curves were obtained for each run. The slope of these curves at various times gave the rate of change of concentration of the mill contents,  $dC_m/dt$ , (lb grits/(lb water) (min), for a particular grits feed rate.

After a certain milling time had elapsed, the concentration in the mill remained constant. This concentration is defined as the concentration of the mill contents at steady state,  $C_s$ , (lb grits per lb water).

A plot of the values of  $dC_m/dt$  against  $(C_s - C_m)$  on log-log graph paper gave a straight line. The equation of this line could be written:

$$\log(dC_m/dt) = \log k + n \log(C_s - C_m) \quad (3)$$

or in the form of

$$dC_m/dt = k(C_s - C_m)^n \quad (4)$$

The method of averages was used to evaluate the constants of these straight lines.

The next step was to correlate the constants with the grits feed rate,  $F$  (lb dry grits per hr). When  $n$  and  $k$  were plotted versus  $F$  on log-log graph paper, two straight lines were obtained, whose equations could be written as:

$$n = aF^b$$

or  $\log n = \log a + b \log F$  (5)

and  $k = cF^d$

or  $\log k = \log c + d \log F$  (6)

The constants, a, b, c, d were evaluated by the method of averages.

The concentration at the steady state as a function of feed rate was obtained by plotting  $C_s$  against F on ordinary graph paper. Another straight line represented by the following equation was obtained:

$$C_s = e + fF \quad (7)$$

Again using the method of averages, the constants of this line were found. By substituting for n, k and  $C_s$  in equation (4), the following expression, giving the rate of change of mill concentration as a function of feed rate, F, and concentration,  $C_m$ , was obtained.

$$dC_m/dt = cF^d (e + fF - C_m)^{aF^b} \quad (8)$$

In practical use, it was difficult to find the time required to reach the steady state, from the  $C_m$  vs. t plot, but it was easy to find the value of  $C_s$ . The value of  $0.99C_s$  could then be calculated, and the corresponding milling time  $t_{0.99C_s}$  could then be found. The changes in concentration occurring after this interval were considered to be insignificant.

The empirical equation for the rate of approach to the steady state at any feed rate could then be integrated between  $t_{C_m} = 0$  and  $t_{C_m} = t_{0.99C_s}$  to obtain the time required to reach an essentially constant condition in the mill. This integration is shown below.

$$t_{0.99C_s} = \int_0^{0.99C_s} \frac{dC_m}{cF^d (e + fF - C_m) aF^b}$$

$$= \int_0^{0.99(e + fF)} \frac{dC_m}{cF^d (e + fF - C_m) aF^b} \quad (9)$$

$$= \int_0^{0.99(e + fF)} \frac{dC_m}{\phi(C_m)} \quad (10)$$

Once the feed rate was chosen, this integration could be evaluated either analytically or graphically. That is, the area under the curve formed by plotting  $1/\phi(C_m)$  against  $C_m$  at constant feed rate between  $C_m = 0$  and  $C_m = 0.99C_s$  would give the time to reach  $0.99C_s$  in average mill concentration. A plot of  $t_{0.99C_s}$  vs.  $F$  could then be constructed for practical use.

The same procedures were applied to evaluate the empirical equation for the rate of approach to the steady state for recycling experiments on the 8-inch hydraulic mill.

#### Scale-Up Equation

Selection of the Significant Variables Affecting the Mill Design. One of the important factors affecting sorghum starch production in the hydraulic mill is the steeping process. Previous investigations indicated that  $SO_2$  steeping was not superior to plain water steeping, and that short steeping times were suitable for sorghum grits. The suitable steeping

temperature range was from 120 to 160°F. The steeping conditions were fixed at

Steeping time	1 hour
Steeping temperature	130°F.
Steeping solution	plain water

Another important variable was the feed rate of steeped sorghum grits. As shown by Chai (2) an increase in feed rate reduced the energy consumption per pound of starch. The feed rates used in the present pilot plant hydraulic mill varied from 5 to 50 pounds dry grits per hour. Below this range, an excessive time was required to reach the steady state, while above this range the capacities of the debranner, flight conveyer, and the screens were insufficient.

The rpm of the mill shaft was a very important factor influencing the starch production from sorghum grits. Hsieh (7) showed that at low speeds, the rate of grinding increased rapidly with increasing speed rising from a rather low rate to 80 per cent ground per pass. As the speed of stirring increased, the rate of grinding became approximately a straight line function of the speed. The vortex at such speeds was scarcely noticeable, but, as the speed was further increased, it became deeper, and above a certain speed, the mill blades were partially uncovered. This resulted in lower efficiency so that the per cent ground per pass dropped off above a certain critical speed.

The mill temperature increased with time until equilibrium was reached. This equilibrium temperature also increased with mill speed. The even distribution of the heat generated caused no local overheating of the starch. When the feed water was at a temperature of 80°F, and the mill shaft speed

fixed at 2200 rpm, the mill temperature reached 83°F as a maximum.

Dimensional Analysis. In the hydraulic milling process, a large part of the applied energy is spent in the agitation of the sorghum grits in order to maintain them in suspension in water and to provide a turbulent flow pattern. The problem of the design of a hydraulic mill is therefore similar to the design of an agitator.

The first step in developing a suitable scale-up equation was to reduce the large number of variables to a few dimensionless groups by the use of dimensional analysis. The variables which affect the mill design were:

T Thrust exerted by the propeller,  $mlt^{-2}$

N Rate of revolution of propeller,  $t^{-1}$

D Inside diameter of mill casing, l

G Ascending speed of fluid in mill,  $lt^{-1}$

$\rho$  Density of slurry,  $ml^{-3}$

$\mu$  Viscosity of slurry,  $ml^{-1}t^{-1}$

g Gravity,  $lt^{-2}$

r' Ratio of mill diameter to mill height, dimensionless

r'' Ratio of mill diameter to the diameter of the propeller, dimensionless

r''' Ratio of mill diameter to height of outlet, dimensionless

Here m, l, and t represent the dimensions mass, length and time respectively.

By dimensional analysis, these variables were arranged in dimensionless groups as shown in equation (11).

$$\phi \left( \frac{\rho D^2 G^2}{T}, \frac{DN}{G}, \frac{\rho DG}{\mu}, \frac{Dg}{G^2}, r', r'', r''' \right) = 0 \quad (11)$$

The r's represent all ratios required to make the mills geometrically similar.

This equation may be rewritten as:

$$T = \rho D^2 G^2 \phi' \left( \frac{DN}{G}, \rho \frac{DG}{\mu}, \frac{Dg}{G^2}, r', r'', r''' \text{ ---} \right) \quad (12)$$

Discussion of the Theory of Models. Equation (12) shows without any experimentation at all, that if  $\frac{DN}{G}$ ,  $\rho \frac{DG}{\mu}$ , and  $\frac{Dg}{G^2}$  were held constant, the thrust of a propeller in a mill of any given shape ( $r'$ ,  $r''$ ,  $r'''$  constant) would be proportional to the density of the liquid, the square of mill diameter and the square of the ascending speed of liquid. Actually, it would be hard to keep  $\frac{DN}{G}$ ,  $\rho \frac{DG}{\mu}$ , and  $\frac{Dg}{G^2}$  constant. Hence, in order to design a large scale mill with dynamic similarity to the small model, the following procedure was adopted.

The principle of dynamic similarity, as discussed by Johnstone and Thring (9) and Murphy (13), states that in passing from one mill to a second in the same or another liquid any three of the quantities such as ( $\rho$ ,  $D$ ,  $G$ ) could be changed in any ratio whatever, and that the equation relating the thrust with the other variables would remain precisely the same if the value of the arguments of  $\phi'$  remained unchanged. In other words equation (12) could be written:

$$T = K \rho D^2 G^2 \quad (13)$$

where  $K = \phi' \left( \frac{DN}{G}, \rho \frac{DG}{\mu}, \frac{Dg}{G^2}, r', r'', r''' \text{ ---} \right)$

In order for  $K$  to be constant the values of  $\frac{DN}{G}$ ,  $\rho \frac{DG}{\mu}$ ,  $\frac{Dg}{G^2}$ ,  $r'$ ,  $r''$  and  $r'''$  must be the same in model and prototype.

The simplest of these requirements to satisfy was that the  $r$ 's should be constant, that is the two mills, whatever their diameter, should be geometrically similar.

The next simplest condition was that  $DN/G$  should remain constant. To accomplish this for a model mill with diameter  $D'$ , and a large mill of diameter  $D$ , with the ratio of diameters =  $r$ , or

$$D = rD' \quad (14)$$

the feed rate would have to be controlled so that the ascending rates of fluid in the model mill,  $G'$ , and in the large mill,  $G$ , were related by

$$G = \sqrt{r}G' \quad (15)$$

and the mill speed of the small mill,  $N'$  and of the larger mill,  $N$ , must be related by

$$N = N'/\sqrt{r} \quad (16)$$

Then the ratio of  $DN/G$  would be constant for the two mills:

$$\frac{DN}{G} = \frac{rD'N'/\sqrt{r}}{\sqrt{r}G'} = \frac{r}{\sqrt{r} \cdot \sqrt{r}} \frac{D'N'}{G'} = \text{constant} \quad (17)$$

If these restrictions were met, equation (12) could be reduced to

$$T = \rho D^2 G^2 \phi' \left( \frac{\rho DG}{\mu}, \frac{Dg}{G^2} \right) \quad (18)$$

This equation could be used for comparing two mills, providing  $\rho DG/\mu$  and  $Dg/G^2$  could be made constant for the two conditions. The gravitational constant  $g$ , is unchanged of course, and in addition  $\rho/\mu$  would be the same for the two mills, since we will be confined to water slurries at essentially the same temperature and concentration. This brings out an inherent difficulty, since it is obvious now that  $DG$  and  $D/G^2$  must both be constant for the two mills to be dynamically similar, and this is impossible.

It has been shown that for fluids in very turbulent motion, viscosity is not an important factor. The mechanical behavior of a fluid in very turbulent motion is much more dependent on density than on viscosity.

Since high turbulence is a characteristic of the hydraulic mill, it appears safe to assume that the Reynolds number,  $\rho DG/\mu$ , would not be an important factor in equation (18) and probably could be omitted. This assumption permits equation (18) to be further simplified to

$$T = \rho D^2 G^2 \phi'(Dg/G^2) \quad (19)$$

If the ratios expressed by equations (14) and (15) existed,

$$\frac{Dg}{G^2} = \frac{rD'g}{(\sqrt{r}G')^2} = \frac{D'g}{G'^2} = \text{constant} \quad (20)$$

The scale-up equation then reduces to

$$T = K \rho D^2 G^2 \quad (21)$$

and

$$T' = K \rho D'^2 G'^2 \quad (22)$$

The ratio of the thrust for the large mill to that for the model then becomes

$$\frac{T}{T'} = \frac{D^2 G^2}{D'^2 G'^2} = \frac{r^2 D'^2 (\sqrt{r} G')^2}{D'^2 G'^2} = r^3 \quad \text{or} \quad \frac{T}{T'} = \left(\frac{D}{D'}\right)^3 \quad (23)$$

This relationship was tested by running two mills of different size under the conditions specified in this analysis. These are:

1. The two mills must be geometrically similar. That is, all the ratios  $r'$ ,  $r''$ ,  $r'''$ , etc., must be the same for the two mills.
2. The ratio of the ascending speeds of the fluids in the two mills must be  $G/G' = \sqrt{r}$ , where  $r$  is the ratio of diameters, or  $r = D/D'$ .
3. The ratio of the speed of the small mill to that of the large mill must be  $\frac{N'}{N} = \sqrt{r}$ .

## MATERIAL AND METHODS

## Material

The only raw material used in this investigation was milo sorghum grits which were supplied by Grain Products, Incorporated, of Dodge City, Kansas. The term "grits" used here refers to the endosperm part of the milo kernel after the removal of bran and germ. As the grits arrived at the laboratory, they were stored in 55-gallon, open-head barrels. A 50-cc erlenmeyer flask containing carbon disulfide was placed in each container as an insecticide. The barrels were then covered tightly. A typical analysis of milo sorghum grits is shown below.

Table 1. Analysis of milo sorghum grits.

Component	Weight %
Protein	11.43
Ether extract	1.33
Crude fiber	1.06
Moisture	10.09
Ash	0.83
N-free extract	75.26
Carbohydrates	75.60
Starch	69.70

Both steeping and processing water were taken from the Manhattan City system. The analysis of water was reported as follows:

Table 2. Analysis of Manhattan city water.

Total hardness (parts of calcium carbonate per million)	76
Non-carbonate hardness (parts of calcium carbonate per million)	45
Total dissolved solids (parts per million)	218
pH	8.97

## Equipment

A photographic view and a flow sheet of the whole pilot plant are shown in Plates I and II. The essential equipment used in this pilot plant was the hydraulic starch mill. Two different sizes of mills were used in this experiment. One was an 8-inch mill used in once-through and recycling experiments, and the other was a 6-inch mill, which was designed and constructed according to the requirements of the scale-up equation and was used for comparing the dynamic similarity of the two different size mills.

The 8-inch hydraulic starch mill was constructed from 8-inch standard pipe mounted on a base carrying a vertical stainless steel shaft. The shaft carried horizontal blades and was driven from the bottom. All dimensions of the mill are shown in Plates III, IV, and V. The blades were made of 1/8-inch by 1-inch stainless steel bar stock and were 2-13/16 inches long. They screwed into threaded holes in the shaft and were held in place by a jam nut. The pitch of the blades was variable.

A cover carrying a second bearing for the shaft was mounted on top of the casing. The shaft was driven by a 10 horse-power, 1170 rpm Fairbanks-Morse induction motor through two V-belts. Various operating speeds were obtained by varying the ratio of the shaft and motor pulleys. A tachometer was used to indicate the shaft speed of the mill. The power consumption was measured by a General Electric polyphase watt-hour meter, having a watt-hour constant of 7.2, which was placed in the motor power input circuit.

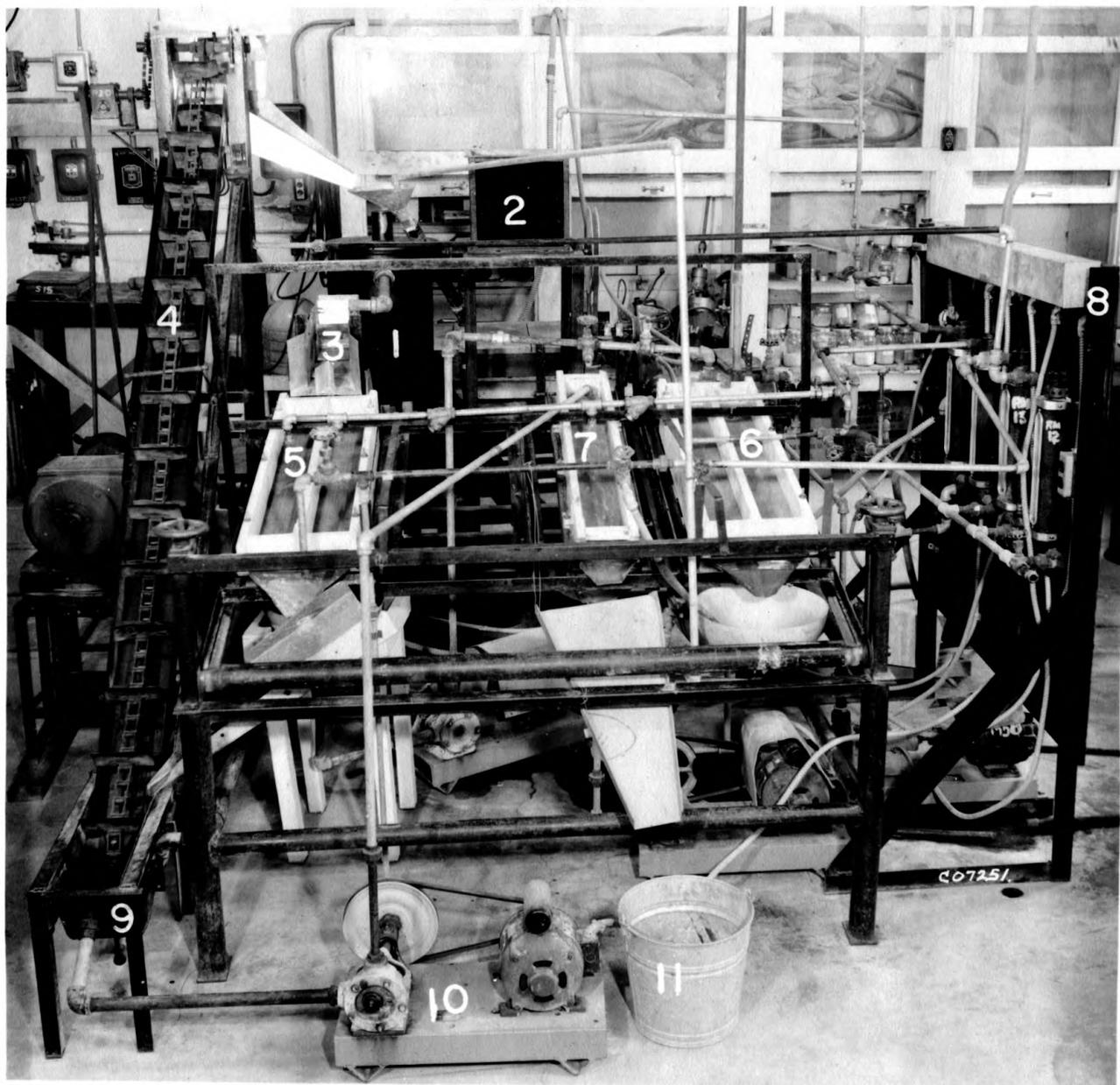
The 6-inch hydraulic mill was constructed from 6-inch standard pipe mounted on a base identical with that of the 8-inch mill. The same shaft was used for both mills, though the blades were changed for the 6-inch mill.

## EXPLANATION OF PLATE I

### View of Pilot Plant

1. Hydraulic mill
2. Feed hopper
3. Tipper
4. Flight conveyer
5. Coarse screen (40-mesh)
6. Fine screen (200-mesh)
7. Bran washing screen (200-mesh)
8. Control panel
9. Debranner
10. Bran pump
11. Bran receiver

# PLATE I

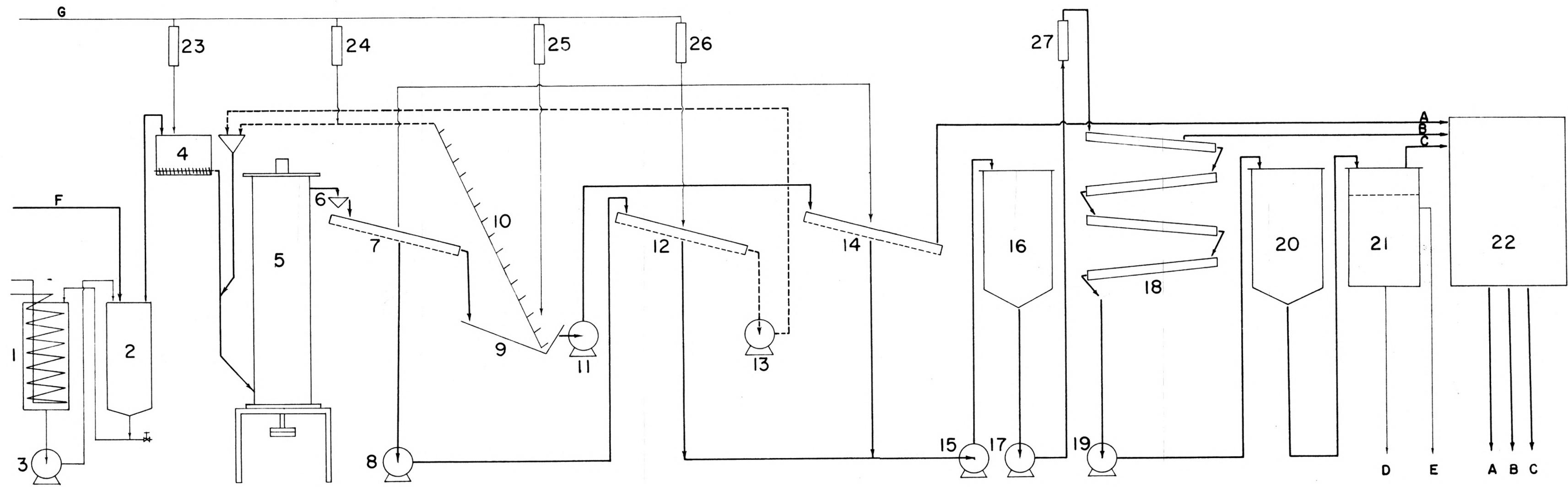


## EXPLANATION OF PLATE II

### Flow Diagram of the Pilot Plant

1. Steeping water heater
2. Steeping tank
3. Steeping water circulating pump
4. Sorghum grits feeder
5. Hydraulic starch mill
6. Tipper
7. Coarse screen (40-mesh)
8. Gear pump
9. Debranner
10. Flight conveyer
11. Gear pump
12. Fine screen (200-mesh)
13. Gear pump
14. Fine screen (200-mesh)
15. Centrifugal pump
16. Starch milk tank
17. Centrifugal pump
18. Starch table
19. Gear pump
20. Gluten settling tank
21. Vacuum filter
22. Compartment dryer
- 23, 24, 25, 26, 27. Flowraters

PLATE II



EXPLANATION OF PLATE III

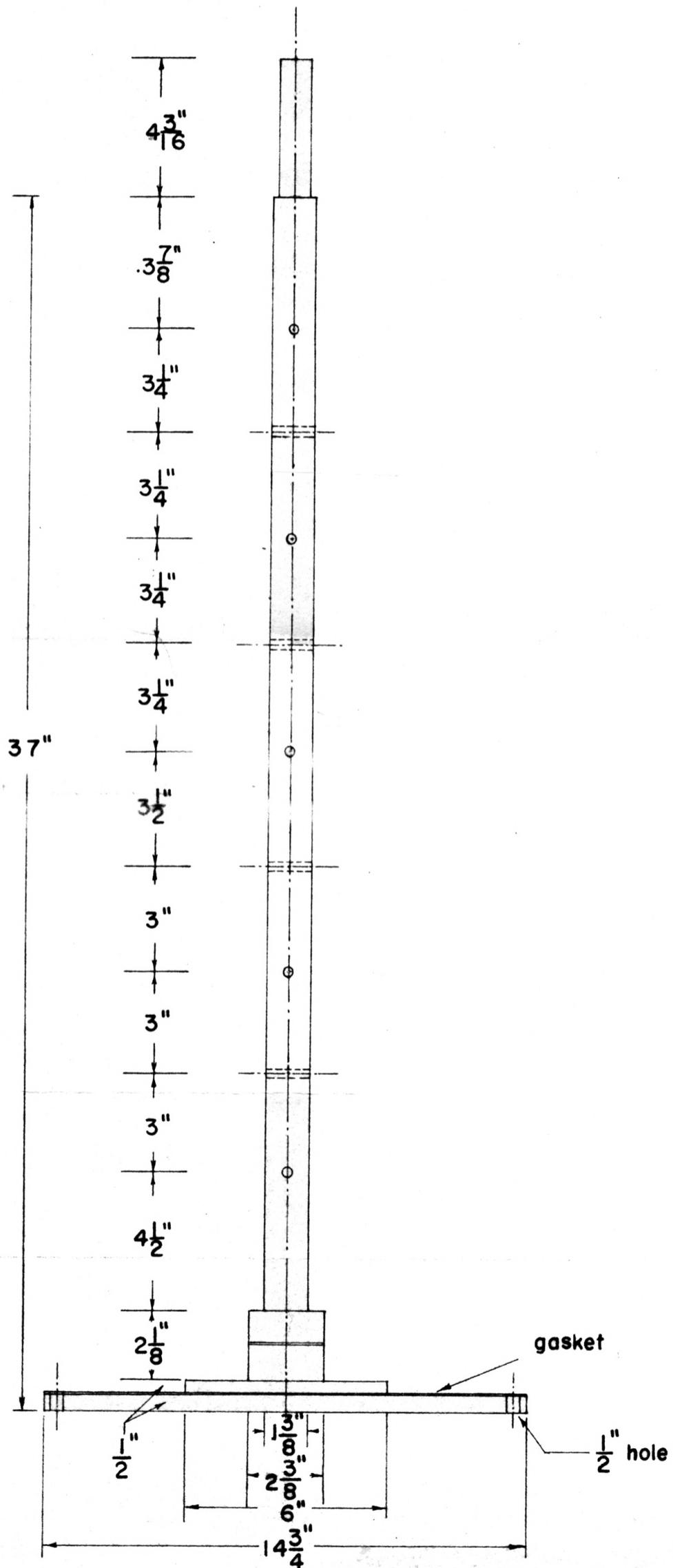
Drawing of 8-Inch Hydraulic Starch Mill Casing



EXPLANATION OF PLATE IV

Detailed Drawing of Mill Shaft for the 8-Inch Hydraulic  
Starch Mill Showing the Designation of Blade Positions.

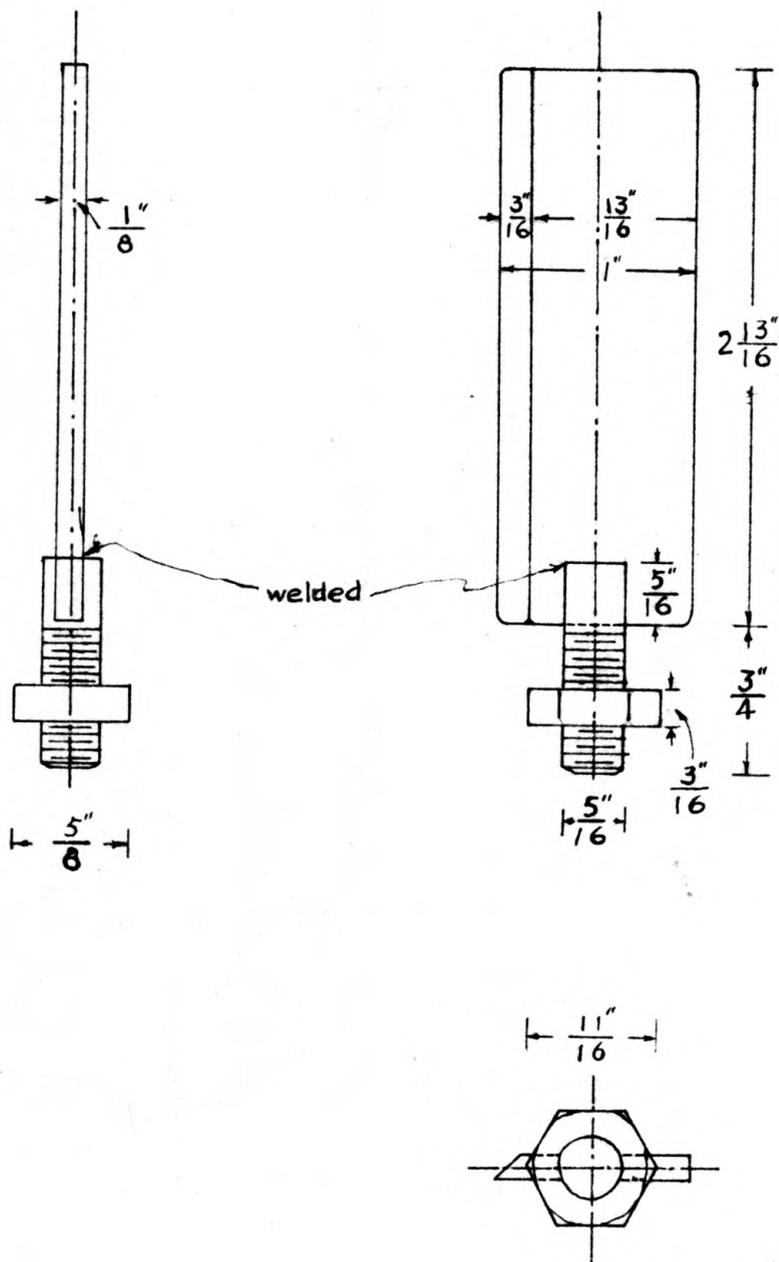
PLATE IV



EXPLANATION OF PLATE V

Detailed Drawing of Blade for 8-Inch Hydraulic Starch Mill

## PLATE V



All dimensions of the 6-inch mill are shown in Plates VI, VII, and VIII. The blades were made of 1/8-inch by 1-inch stainless steel bar stock and were 2-1/4-inches long. Each pair of the blades was welded on opposite sides of a stainless steel ring with zero degree pitch. The rings slipped over the shaft and the position of each ring could be fixed by two set screws. In order to have geometric similarity between the 8-inch and 6-inch hydraulic mills, the blade positions in the 6-inch mill had to be such that the ratio of the height of each blade to the diameter of the mill would be the same as that for the 8-inch mill. In other words, all the dimensions in the 6-inch mill had to be geometrically similar to those of the 8-inch mill. A spacer was inserted into the mill to reduce the height of the mill. The feed and overflow pipes were also located geometrically similar. Various operating speeds were obtained by varying the ratio of the shaft and motor pulleys.

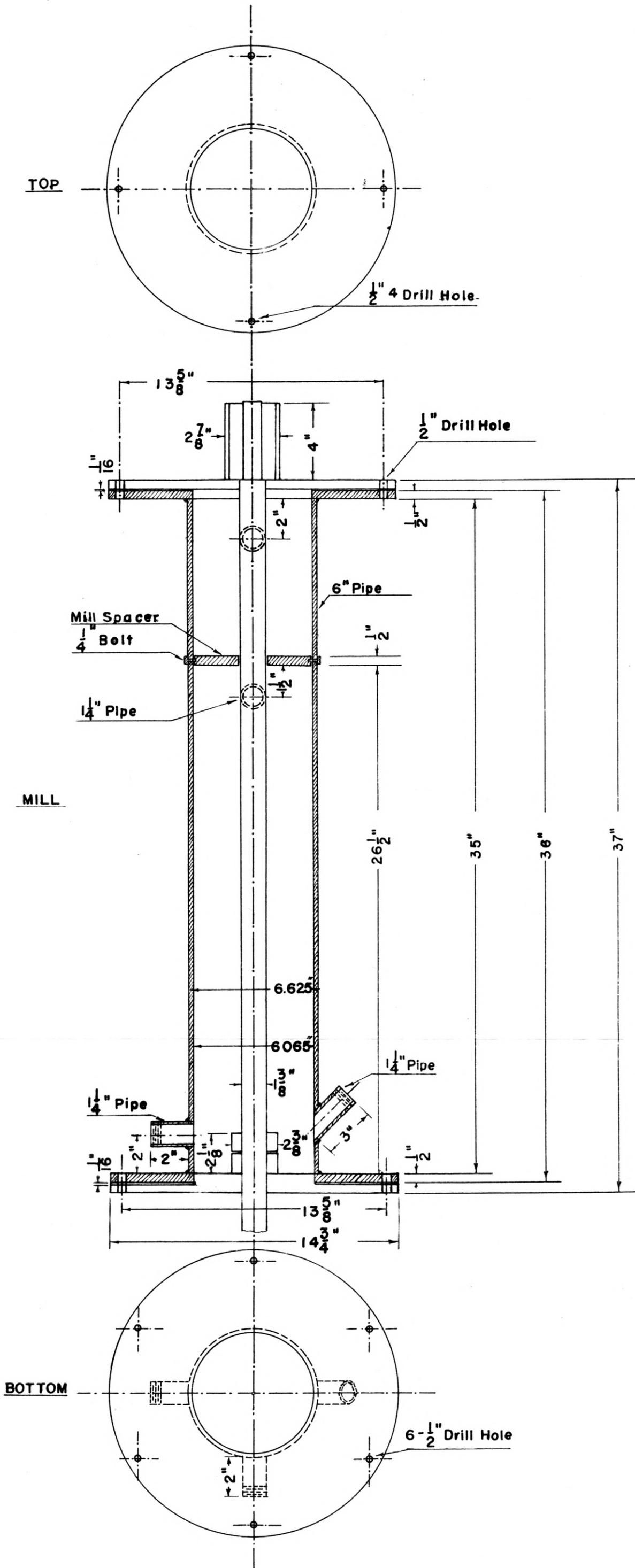
#### Methods

Once-Through Experiments. A certain amount of raw sorghum grits was weighed and put in the steeping tanks. The grits were steeped with warm water at 130°F for one hour. After the steeping water was drained, they were weighed and transferred to the feed hopper of the hydraulic mill.

Before starting the experiment, the mill was run empty and the power consumption at no load measured. Then the screw feeder in the hopper and the feed water were turned on simultaneously. All variables were held constant in all runs except the grits feed rate. The grits feed rate could be adjusted by varying the rpm of the screw feeder by means of the Reeves Varimotor.

EXPLANATION OF PLATE VI

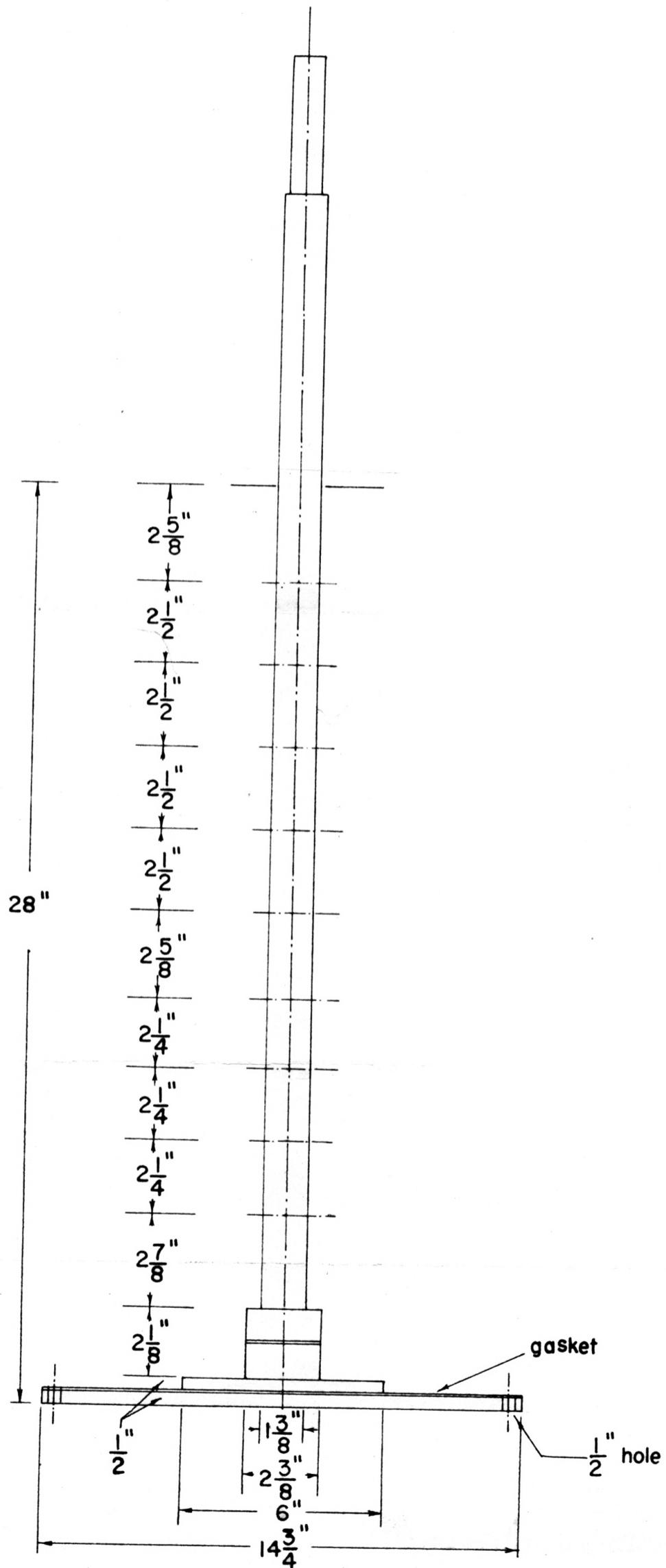
Drawing of 6-Inch Hydraulic Starch Mill Casing



EXPLANATION OF PLATE VII

Detailed Drawing of the Mill Shaft for the 6-Inch  
Hydraulic Starch Mill Showing the Positions of the Blades.

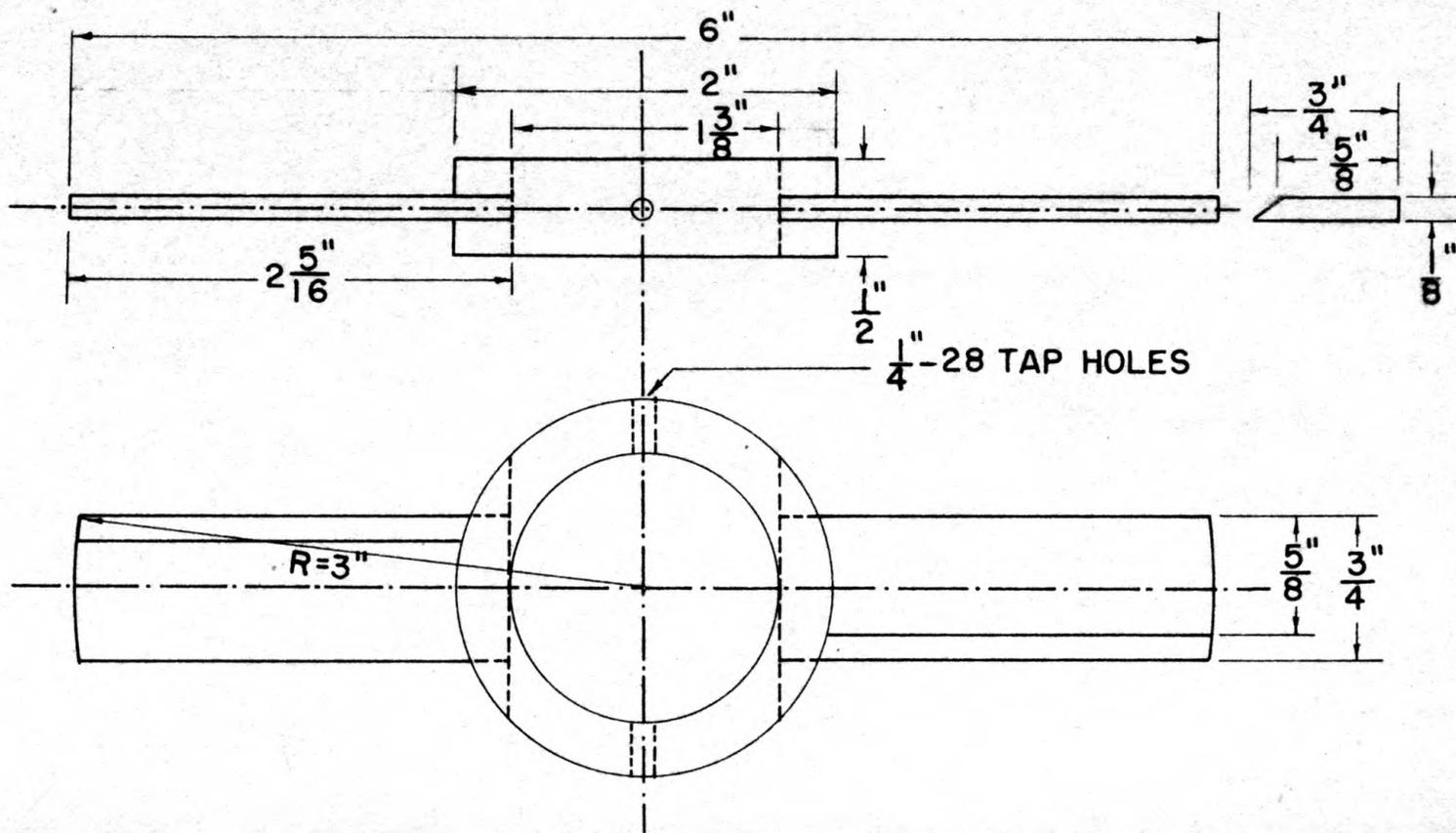
PLATE VII



EXPLANATION OF PLATE VIII

Detailed Drawing of the Blades for the 6-Inch Hydraulic Starch Mill

PLATE VIII



Once the liquid started to overflow from the top of the mill, samples of the mill content at certain time intervals were taken from the valves located in the side wall of the mill. Power consumption and temperature of the mill were also measured at definite intervals of time. The mill overflow stream was discarded to the sewer until the steady state was reached.

When the steady state was reached, the power consumption, the temperature of the mill, and the concentration of the mill overflow all remained constant. At that time measurements of the rate of overflow, the power consumption and the temperature of the mill were taken in order to compare the model and prototype hydraulic mills.

In once-through experiments using the 8-inch mill all the variables were kept constant at the following levels:

Steeping temperature	130 ± 5°F.
Steeping time	1 hour
Speed of mill shaft	1550 rpm
Feed water rate	0.91 gpm

All the samples for the determination of concentration of mill content were weighed, and filtered to remove the water, then dried by hot air at 125 ± 5°F for 20 hours. The mill overflow rate samples were treated in the same way. After all the samples were dried they were weighed and the concentration as pounds of grits per pound of water was calculated.

Recycling Experiments. All the procedures were the same as for the once-through experiments except that the partially ground grits in the mill over-flow were recycled back to the mill. The over-flow from the mill, poured into a tipper and flowed down to the coarse 40-mesh vibrating screen,

the over sized particles were washed into the debranner, and carried back by a conveyer to the mill for further grinding, while the underflow was pumped by a gear pump to the head of the 200-mesh screen. The portion coarser than 200-mesh which overflowed from the screen was also pumped back to the mill for further grinding by a gear pump, while the underflow suspension was pumped to a storage tank by a centrifugal pump as starch milk for tabling.

All variables for the recycling experiments were set at the same values for the once-through experiments. In addition, the recycling water rate was 0.4 gpm, the screen spray water rate was 0.3 gpm and debranning water rate was 0.2 gpm.

Comparison of the Dynamic Similarity Between Different Size Mills. A small-scale model simulating the performance of a large-scale prototype equipment was used in this experiment. A 6-inch hydraulic mill geometrically similar to the 8-inch mill was used as the small-scale model. All the experimental procedures were the same as those for once-through experiments on the 8-inch mill.

In order to run the 6-inch small mill in dynamic similarity to the 8-inch large mill, the variables were controlled according to the criteria developed below. In the following expressions, all primed letters refer to the 6-inch small mill, and plain letters to the original 8-inch large mill.

$$D = 7.981 \text{ inches}$$

$$D' = 6.065 \text{ inches}$$

$$r = \frac{D}{D'} = \frac{7.981}{6.065} = 1.34 \quad (24)$$

$$N = 1550 \text{ rpm}$$

$$\begin{aligned} N' &= \sqrt{r}N = \sqrt{1.34} (1550) = (1.157)(1550) & (25) \\ &= 1793 = 1800 \text{ rpm} \end{aligned}$$

$$G = \sqrt{r}G' \quad (26)$$

$$G' = G/\sqrt{r} = G/\sqrt{1.34} = G/1.157 \quad (27)$$

It was difficult to adjust the feed rate of grits and water to the exact requirements of the scale-up equation. This problem was overcome by adjusting the feed water to the rate required, and then running the mill at various grits feed rates. The actual overflow rate was then measured for comparison. A change in the grits feed rate caused but little change in the overflow rate, but a rather big change in the power consumption.

## EXPERIMENTAL RESULTS

The Rate of Approach to the Steady State  
in the 8-Inch Hydraulic Mill

Results on Once-through Experiments. A total of four runs were made.

The experimental data obtained are shown in Table 3.

Table 3. Distribution of concentration of mill slurry with distance below outlet at various milling times for once-through experiments.

Run No.	Feed rate lb/hr	Milling time min.	Concentration, lb grits per lb water at distances below top of mill, inches				
			2	9 3/4	17 1/2	25 1/4	33
0-1	19.6	8	0.007	0.008	0.007	0.018	0.154
		16	0.010	0.012	0.011	0.048	0.189
		35	0.027	0.032	0.026	0.189	0.256
		47	0.029	0.036	0.028	0.227	0.329
		76	0.036	0.046	0.032	0.281	0.362
		97	0.036	0.044	0.030	0.268	0.390
		113	0.029	0.037	0.027	0.234	0.398
0-2	24.0	8	0.007	0.009	0.007	0.022	0.172
		24	0.030	0.037	0.029	0.196	0.310
		43	0.040	0.049	0.039	0.291	0.376
		59	0.042	0.053	0.041	0.316	0.403
		79	0.046	0.055	0.043	0.351	0.436
		101	0.043	0.061	0.042	0.342	0.402
		125	0.042	0.053	0.041	0.321	0.438
0-3	26.3	7	0.007	0.008	0.007	0.025	0.145
		32	0.041	0.051	0.039	0.299	0.384
		57	0.047	0.059	0.045	0.366	0.411
		77	0.049	0.057	0.044	0.363	0.471
		108	0.051	0.066	0.049	0.395	0.436
		138	0.049	0.059	0.044	0.358	0.417
0-4	26.4	8	0.013	0.016	0.015	0.091	0.248
		30	0.049	0.055	0.044	0.340	0.400
		54	0.053	0.062	0.049	0.386	0.479
		71	0.052	0.060	0.045	0.385	0.484
		101	0.050	0.061	0.048	0.324	0.456

The sample concentrations,  $C$ , were plotted against mill length,  $L$ , at various milling times,  $t$ , for each run. These curves are shown in Figs. 1, 2, 3, 4. The area under each curve was determined by the trapezoidal rule. This area divided by the total mill length (35 inches) gave the average concentrations in the mill,  $C_m$ , in lbs of grits per lb of water, which are given in Table 4.

Table 4. Average mill concentration at various milling times for once-through experiments.

Run number	Feed rate lb/hr	Time min	$C_m$ lb grits per lb water
0-1	19.6	0	0.
		8	0.035
		16	0.050
		35	0.093
		47	0.125
		76	0.147
		97	0.147
		113	0.110
0-2	24.0	0	0
		8	0.040
		24	0.102
		43	0.146
		59	0.166
		79	0.183
		101	0.173
		125	0.173
0-3	26.3	0	0
		7	0.036
		32	0.158
		57	0.181
		77	0.191
		108	0.195
		138	0.181
		0-4	26.4
8	0.072		
30	0.173		
54	0.200		
71	0.199		
101	0.180		

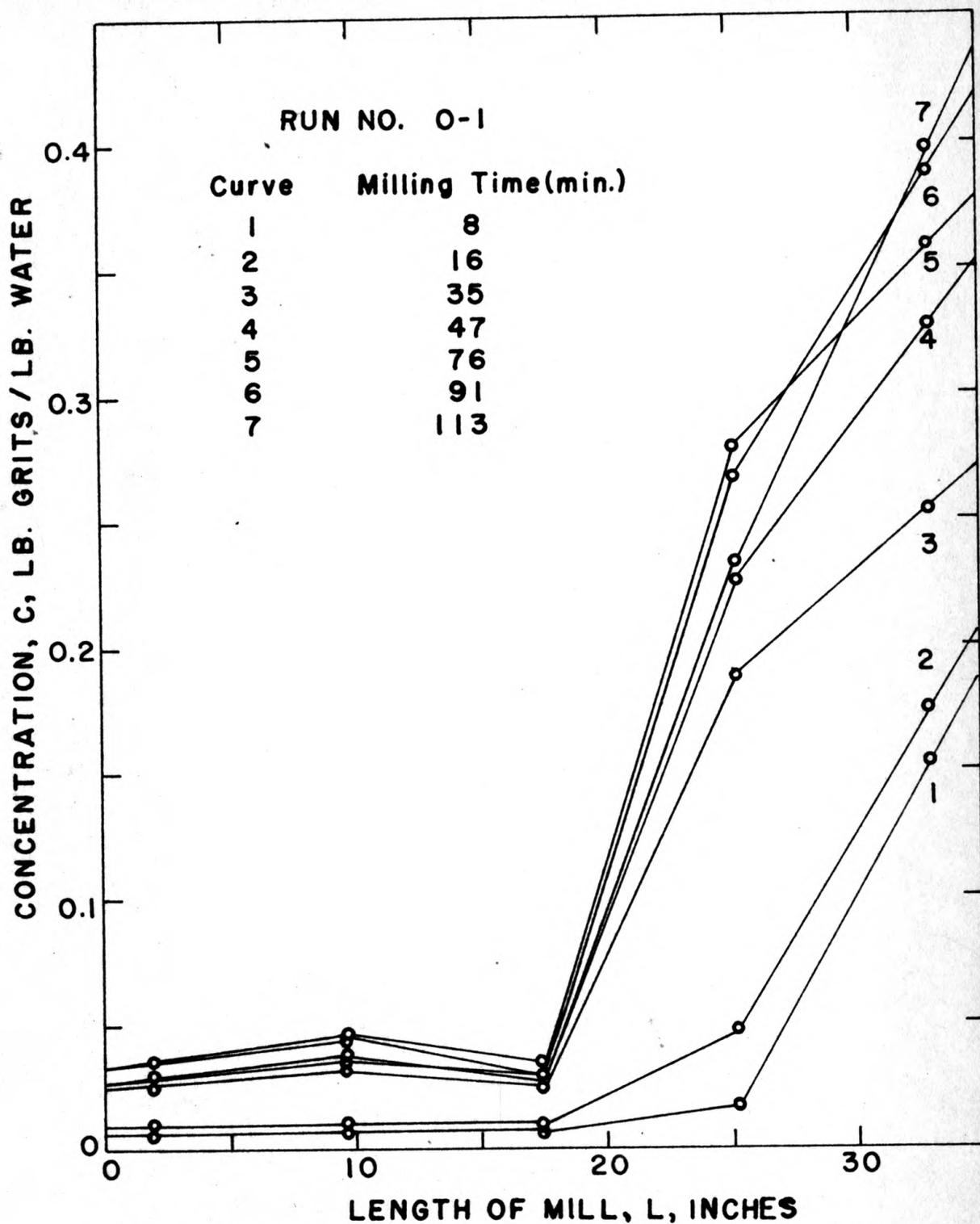


Fig. 1. Distribution of concentration in the mill with mill length at various milling times for once-through experiments.

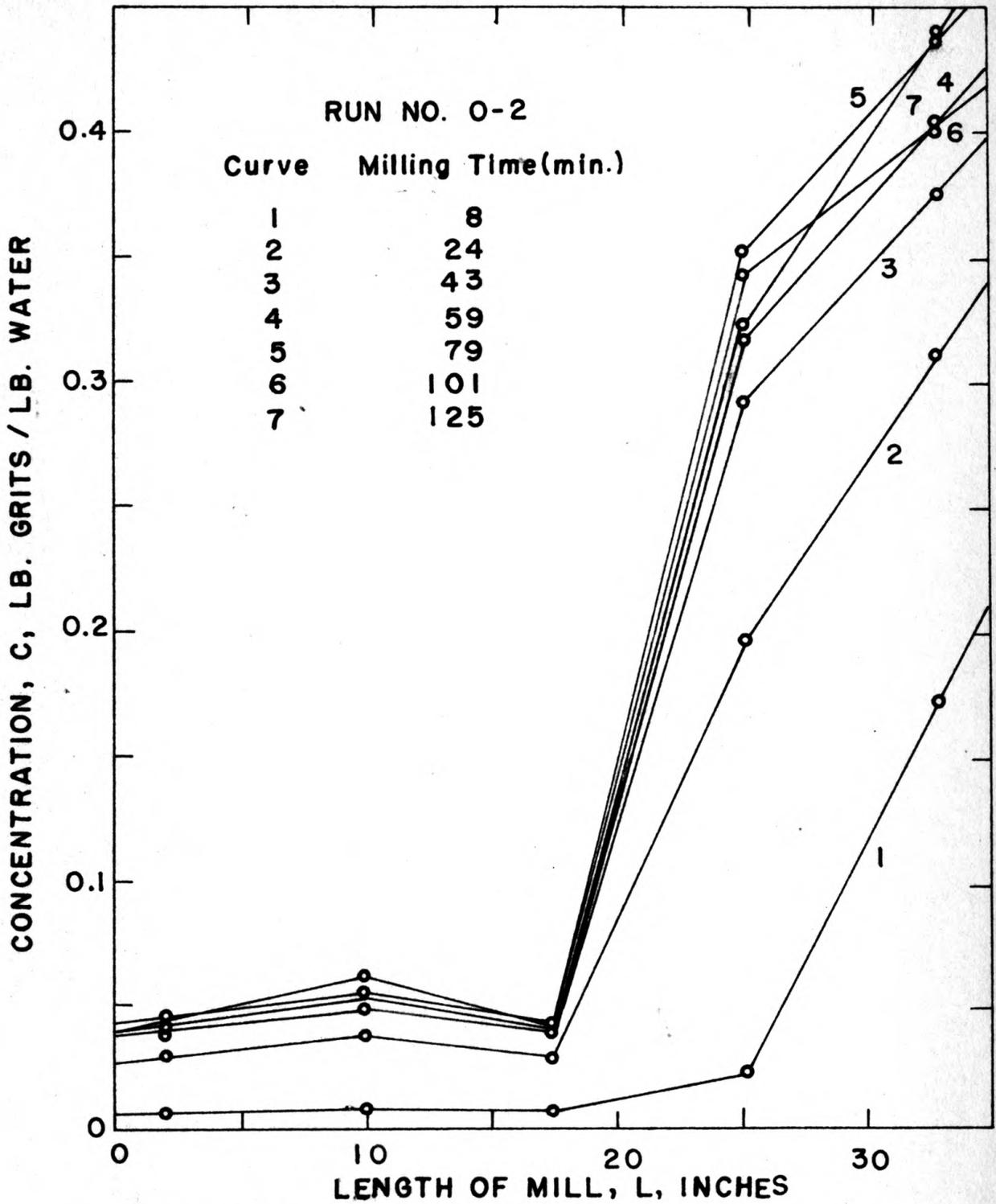


Fig. 2. Distribution of concentration in the mill with mill length at various milling times for once-through experiments.

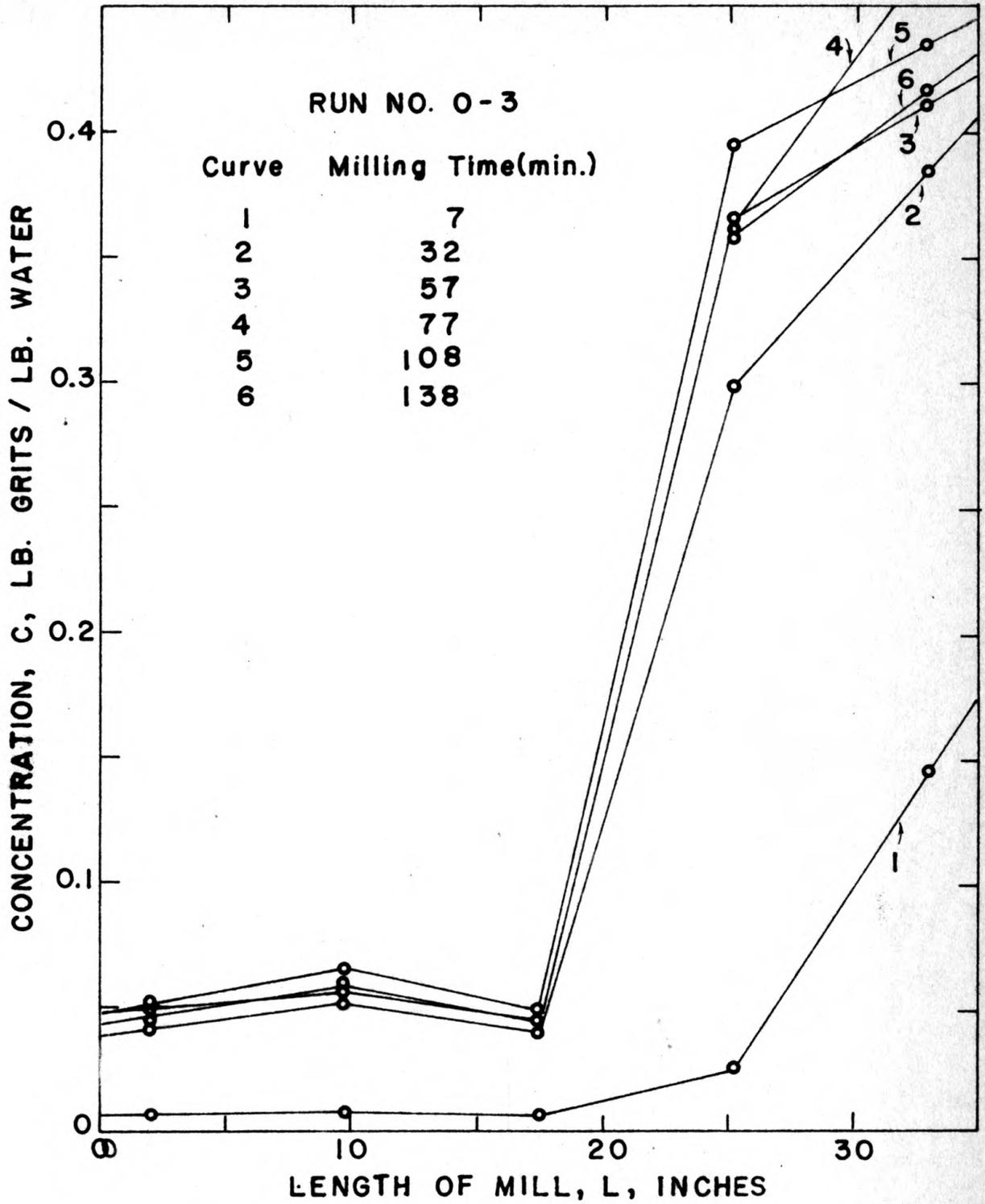


Fig. 3. Distribution of concentration in the mill with mill length at various milling times for once-through experiments.

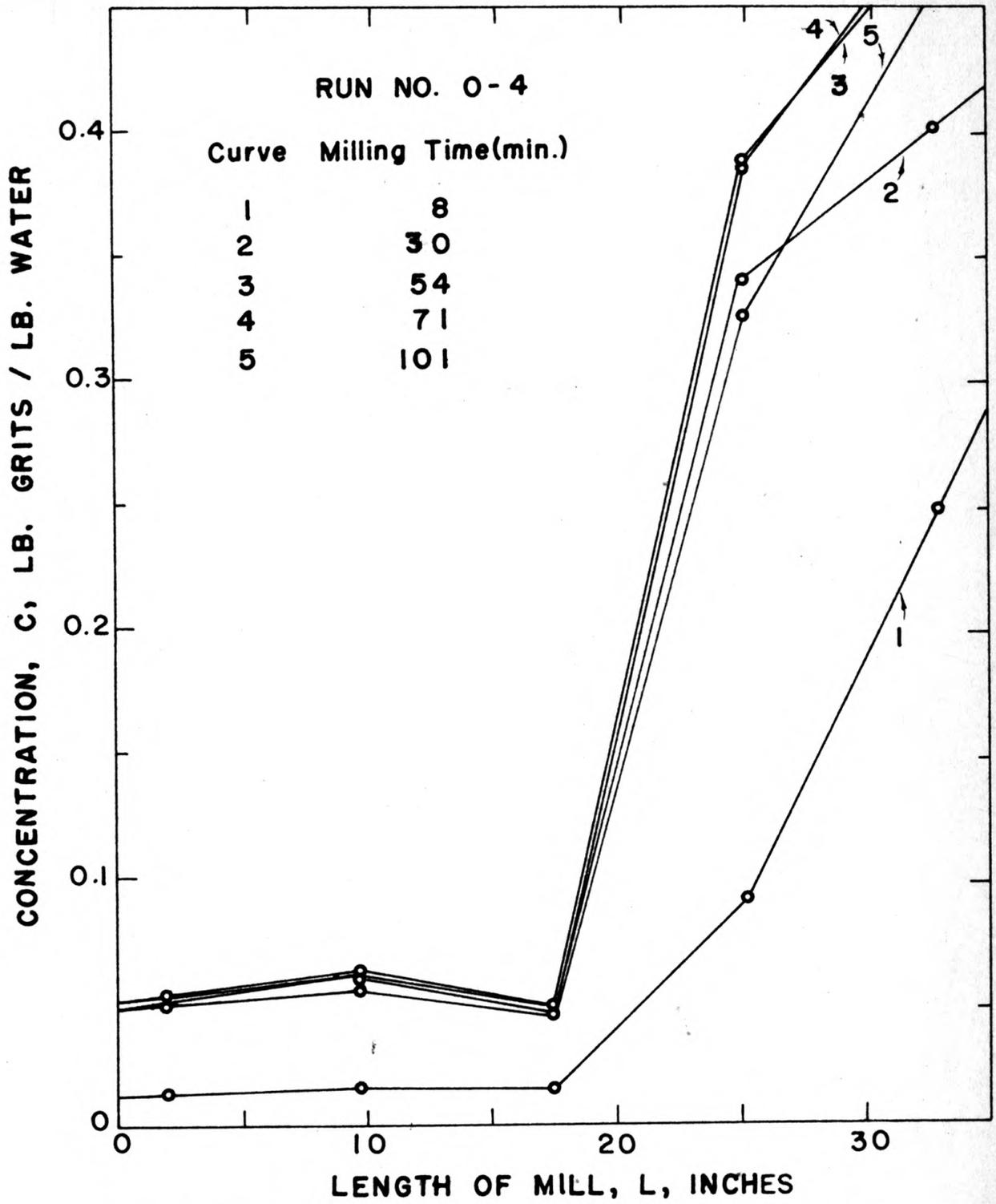


Fig. 4. Distribution of concentration in the mill with mill length at various milling times for once-through experiments.

The average concentration in the mill,  $C_m$ , was then plotted against milling time,  $t$ . Smooth curves, shown in Figs. 5, 6, 7, 8, were obtained. By measuring the slope of the curve at various times, the rate of change of the concentration,  $dC_m/dt$ , at these times was obtained. These rates are given in Table 5. The values of  $C_s$ , the average concentration in the steady state were also determined from Figs. 5, 6, 7, 8, and are given in Table 5.

Table 5. Rate of change of average concentration in mill for once-through experiments.

Run No.	$t$ , minutes	$C_m$ , lb grits lb water	$dC_m/dt$ , lb grits (lb water)(min)	$C_s$ , lb grits lb water	$C_s - C_m$ , lb grits lb water
0-1	10	0.036	0.0034	0.150	0.114
	20	0.066	0.0029		0.084
	40	0.112	0.00175		0.038
	60	0.136	0.00083		0.014
	80	0.146	0.00031		0.004
	90	0.149	0.00014		0.001
0-2	10	0.048	0.00442	0.178	0.130
	20	0.086	0.00340		0.092
	30	0.117	0.00276		0.061
	40	0.141	0.00200		0.037
	50	0.157	0.00132		0.021
	60	0.167	0.000778		0.011
	70	0.173	0.000526		0.005
	80	0.1765	0.000200		0.0015
0-3	10	0.058	0.00540	0.189	0.131
	20	0.108	0.00445		0.081
	30	0.146	0.00345		0.043
	40	0.170	0.00138		0.019
	50	0.180	0.00075		0.009
	60	0.186	0.000348		0.003
0-4	10	0.090	0.00650	0.196	0.106
	20	0.140	0.00384		0.056
	30	0.171	0.00208		0.025
	40	0.185	0.000975		0.011
	50	0.192	0.000506		0.004
	60	0.195	0.000210		0.001

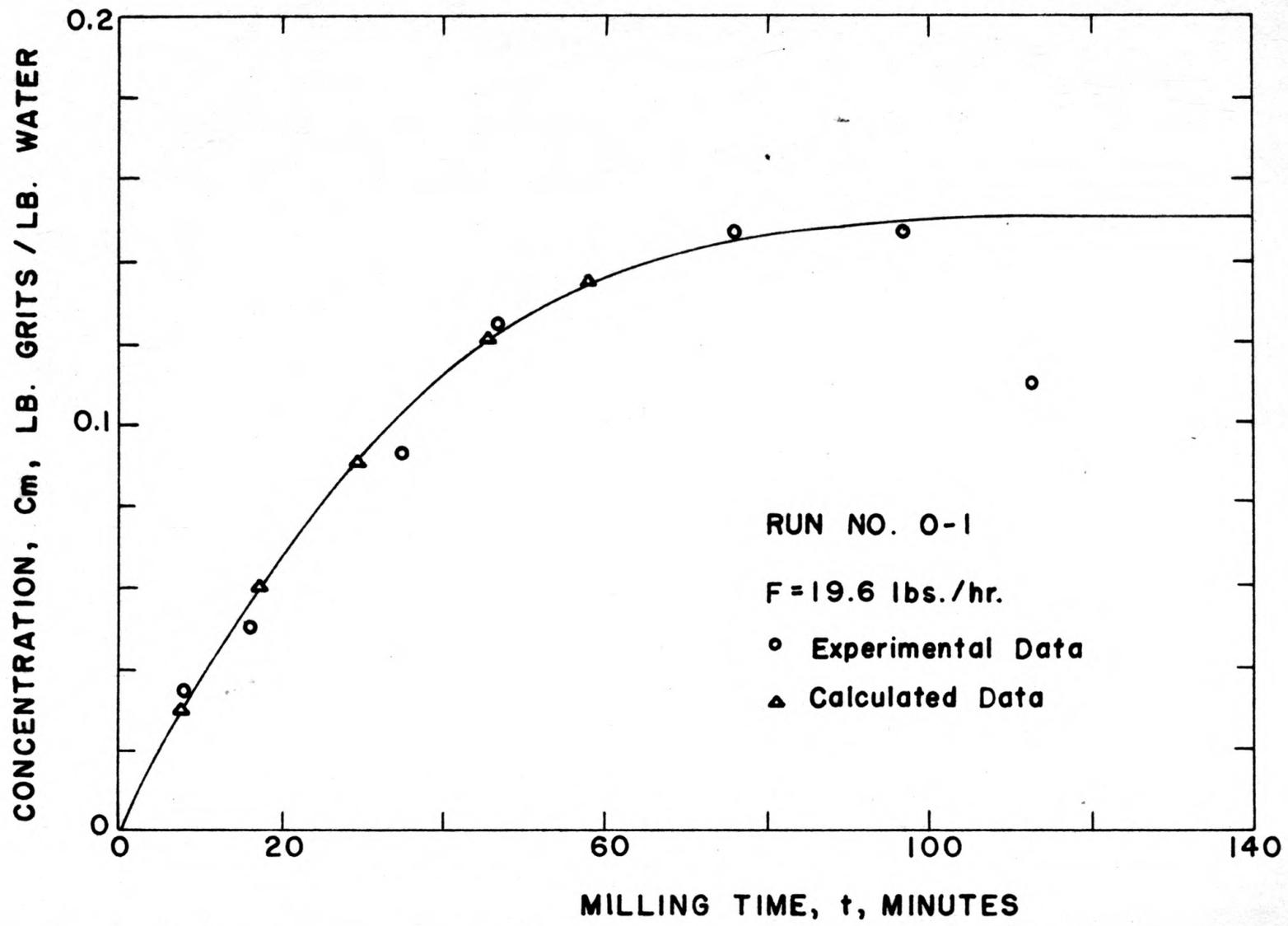


Fig. 5. Effect of milling time on average mill concentration.

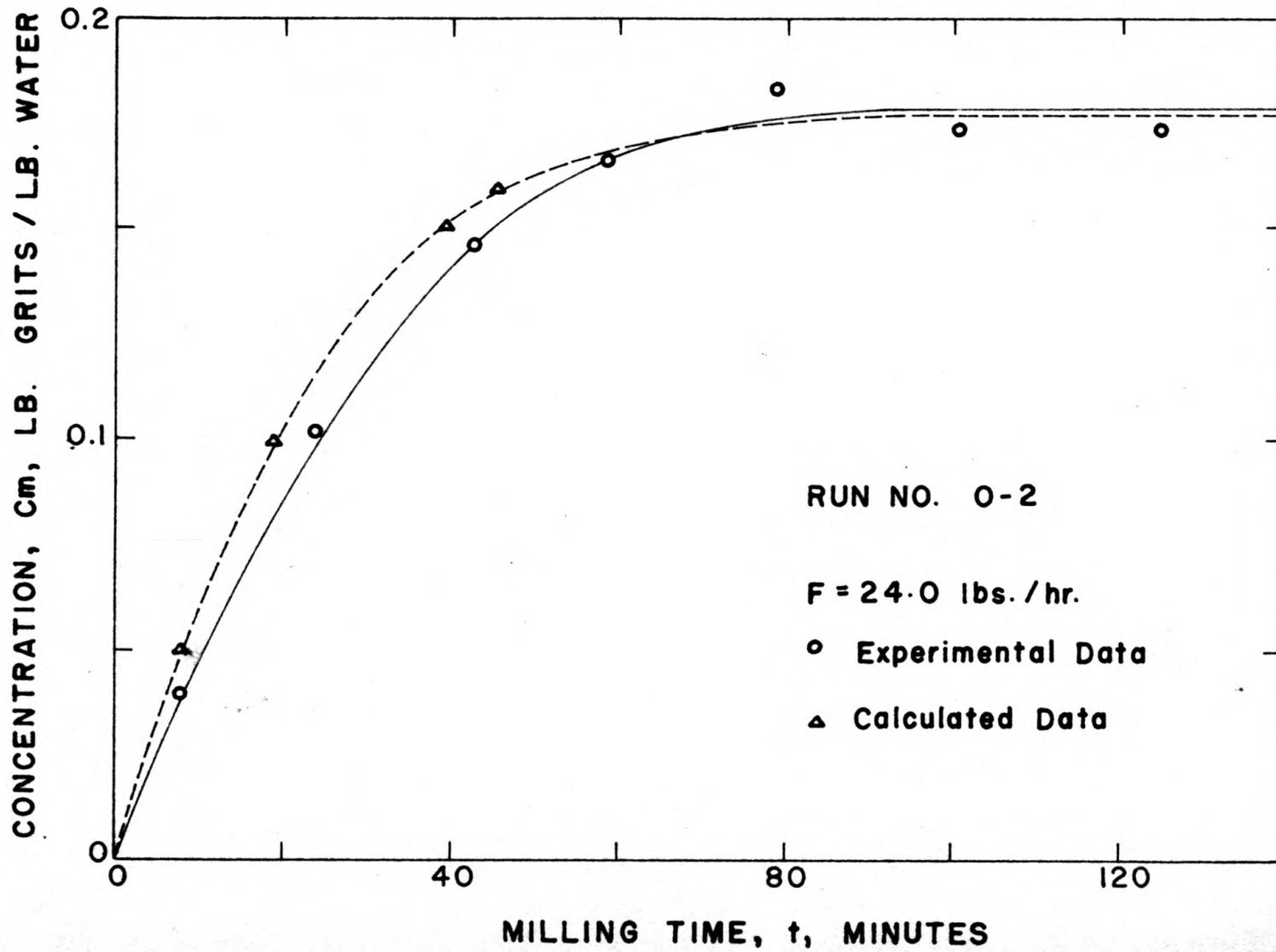


Fig. 6. Effect of milling time on average mill concentration.

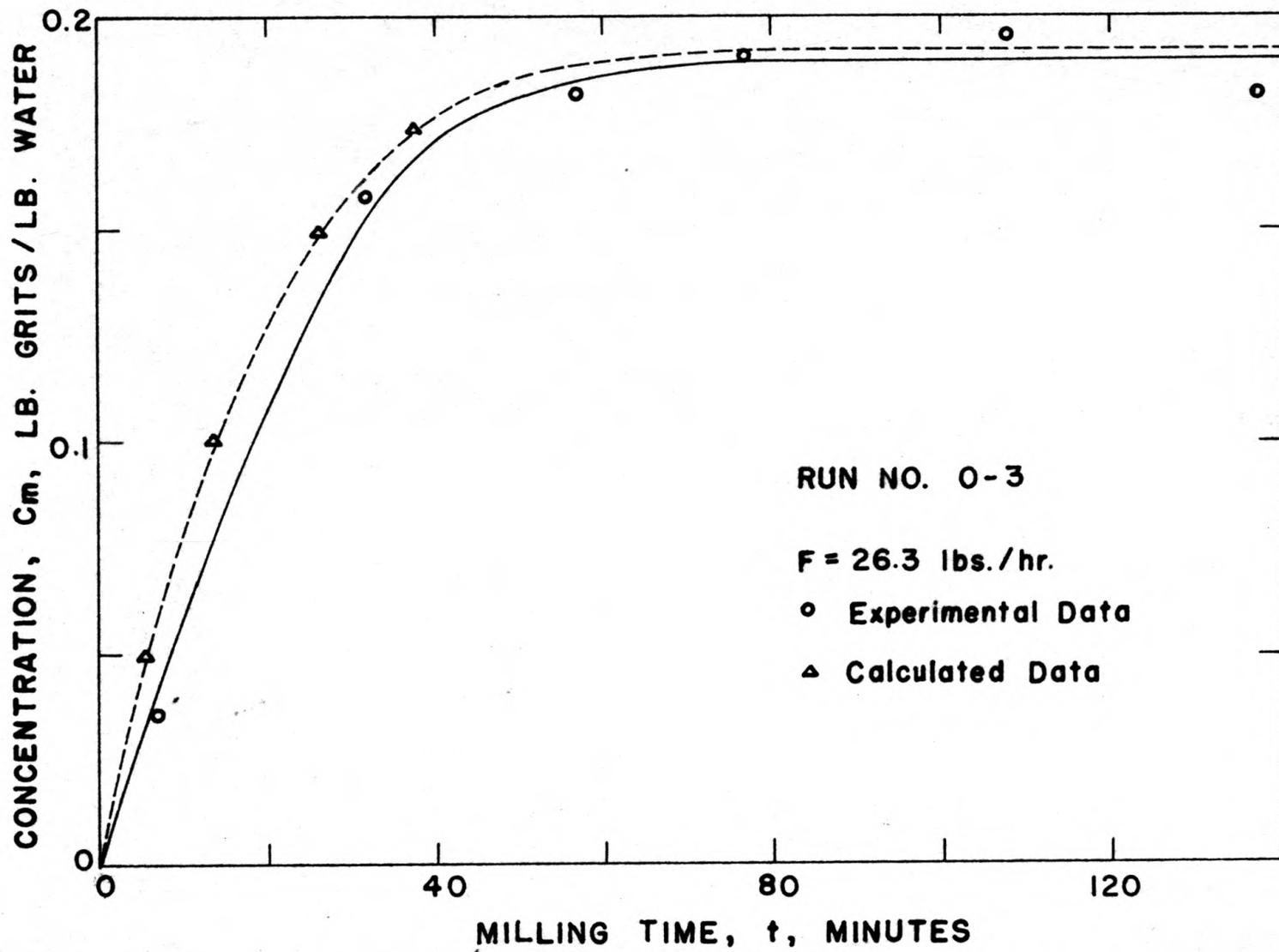


Fig. 7. Effect of milling time on average mill concentration.

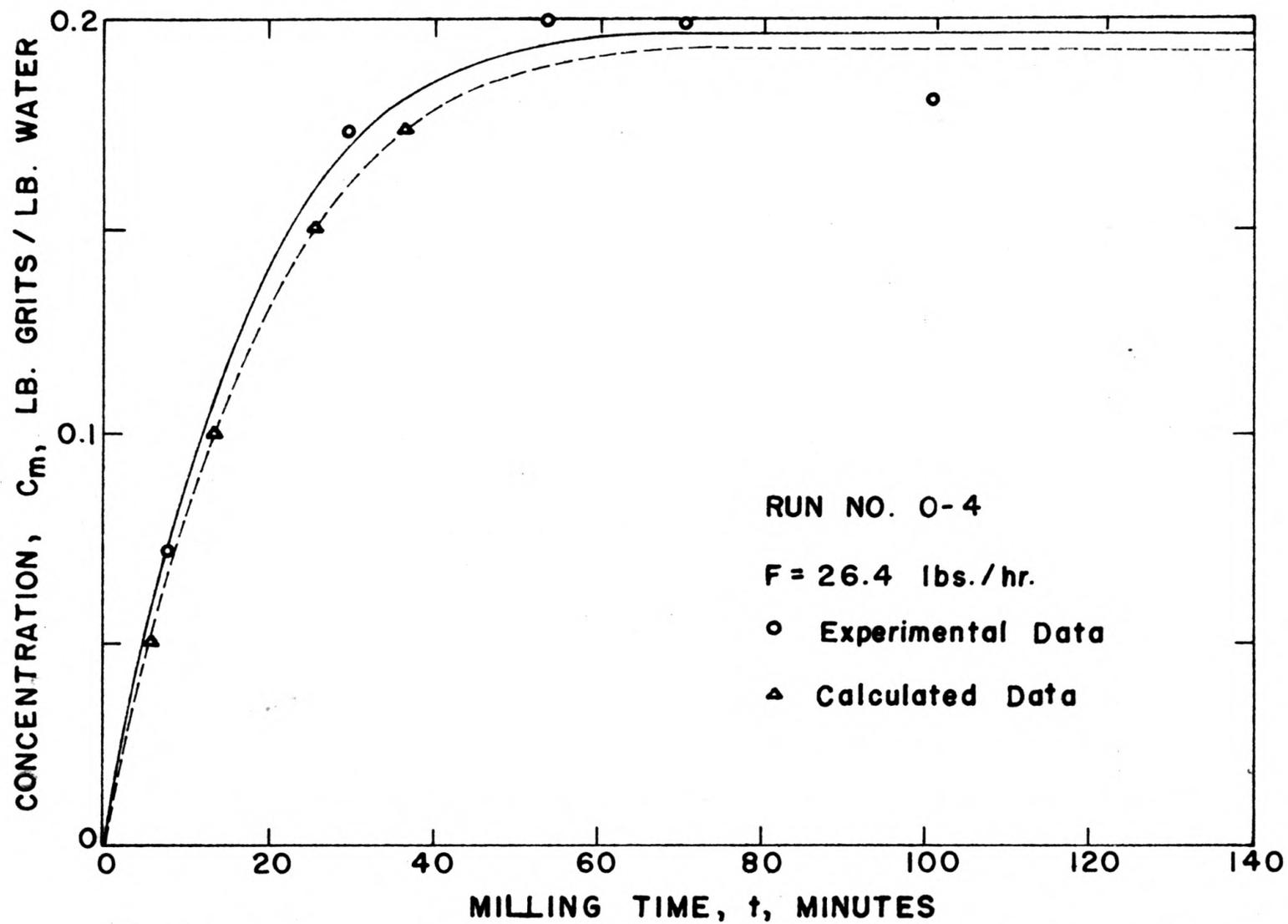


Fig. 8. Effect of milling time on average mill concentration.

In Figs. 9, 10, 11, 12, the rates,  $dC_m/dt$  are plotted on log-log scales against  $C_s - C_m$ . The equations of these straight lines were determined by the method of averages and are given in Table 6.

Table 6. Empirical equations for the rate of approach to the steady state for once-through experiments at various feed rates.

Run No. :	Feed rate lb grits per hr :	Empirical equation	
0-1	19.6	$\frac{dC_m}{dt} = 0.0165(C_s - C_m)^{0.703}$	(28)
0-2	24.0	$\frac{dC_m}{dt} = 0.0185(C_s - C_m)^{0.688}$	(29)
0-3	26.3	$\frac{dC_m}{dt} = 0.0366(C_s - C_m)^{0.800}$	(30)
0-4	26.4	$\frac{dC_m}{dt} = 0.0353(C_s - C_m)^{0.765}$	(31)

The constants in these equations were correlated with the feed rate,  $F$ . This was done by plotting the exponents,  $n$ , and the coefficients,  $k$ , against  $F$ , on log-log graph paper as shown in Figs. 13, 14. Two straight lines were obtained. By using the method of averages, the equations of these lines were determined as given in equations (32) and (33).

$$n = 0.108 F^{0.605} \quad (32)$$

$$k = 1.99 \times 10^{-7} F^{3.7} \quad (33)$$

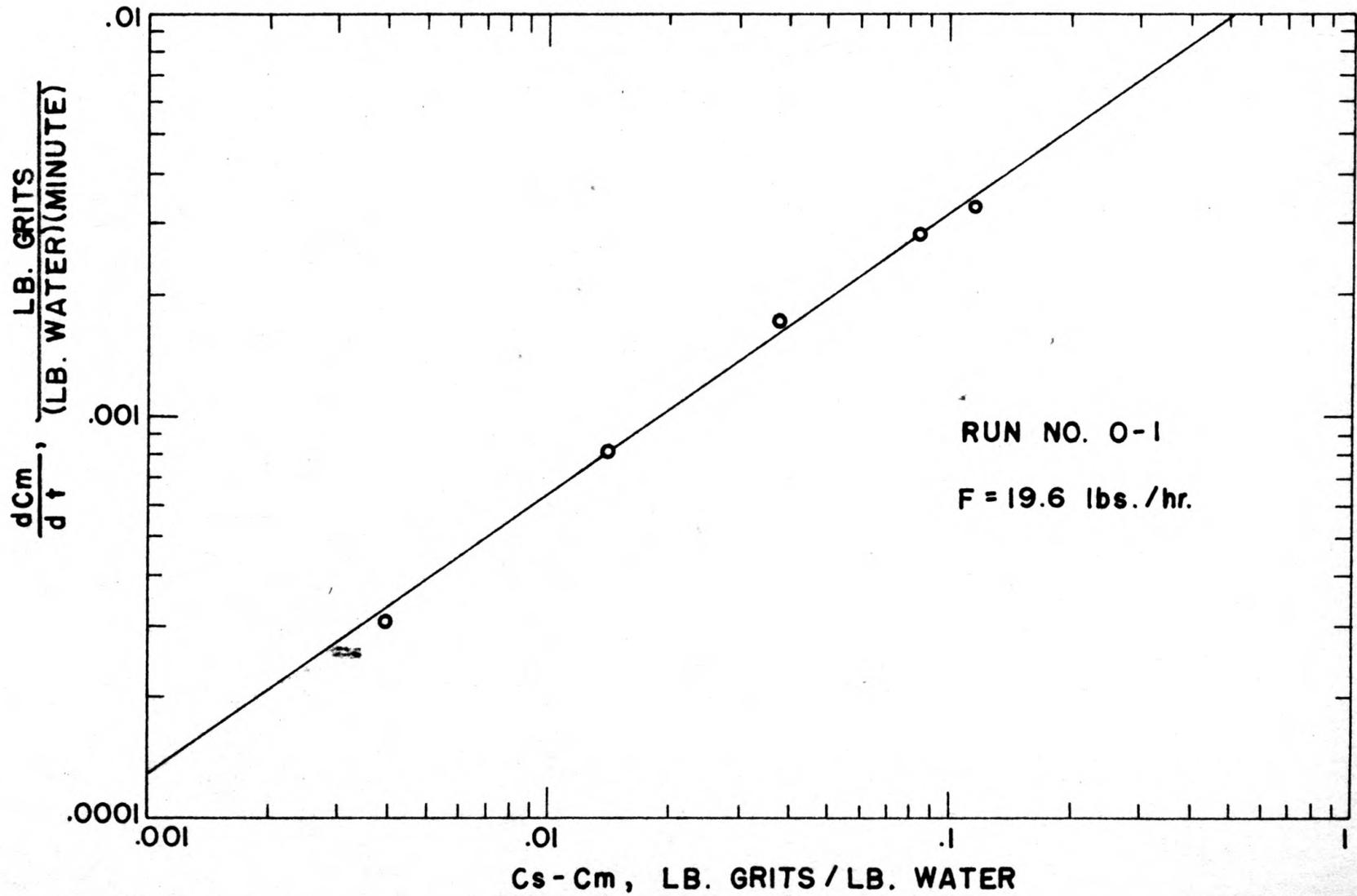


Fig. 9. Effect of  $C_s - C_m$  on the rate of change of average mill concentration.

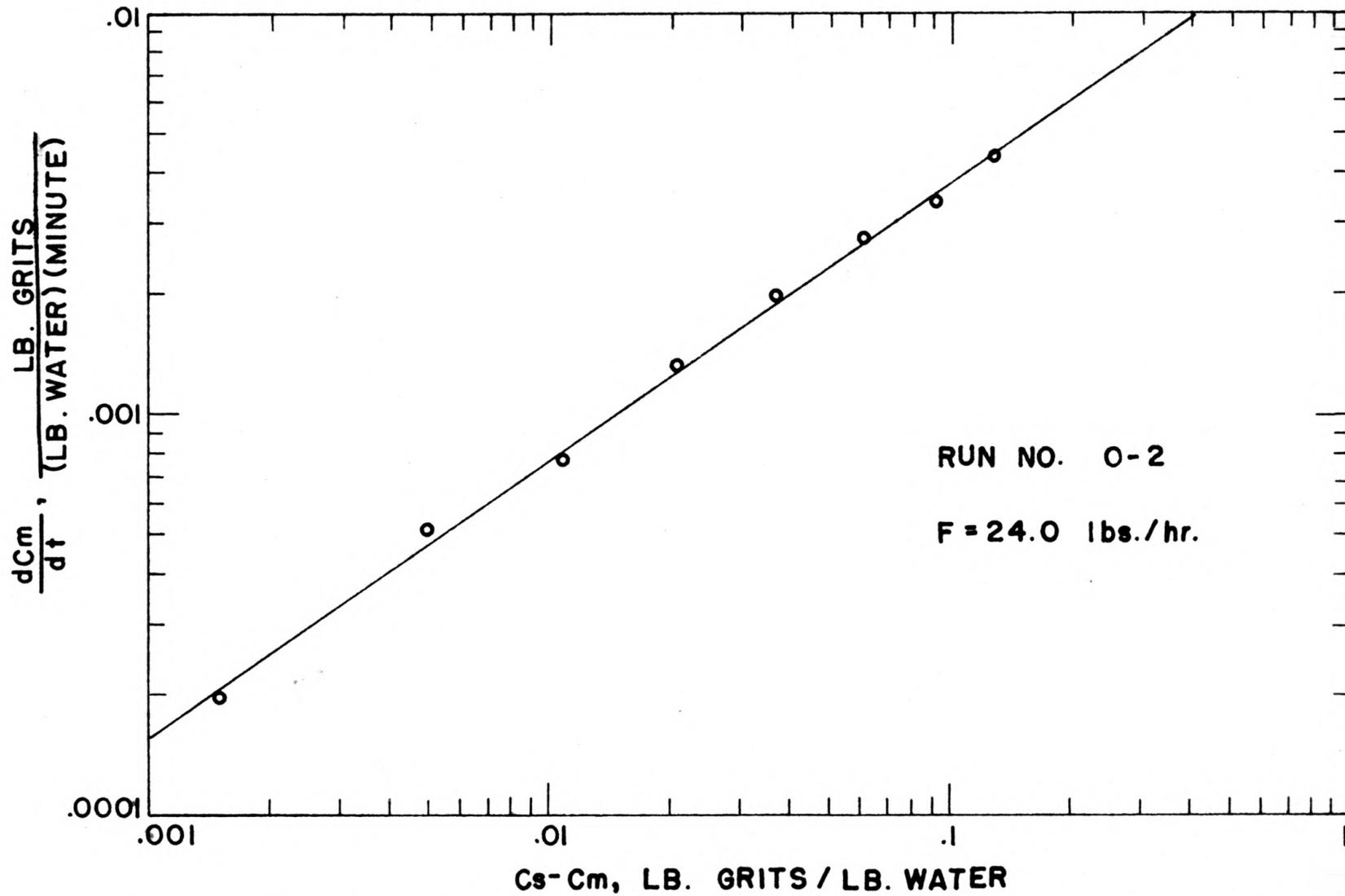


Fig. 10. Effect of  $C_s - C_m$  on the rate of change of average mill concentration.

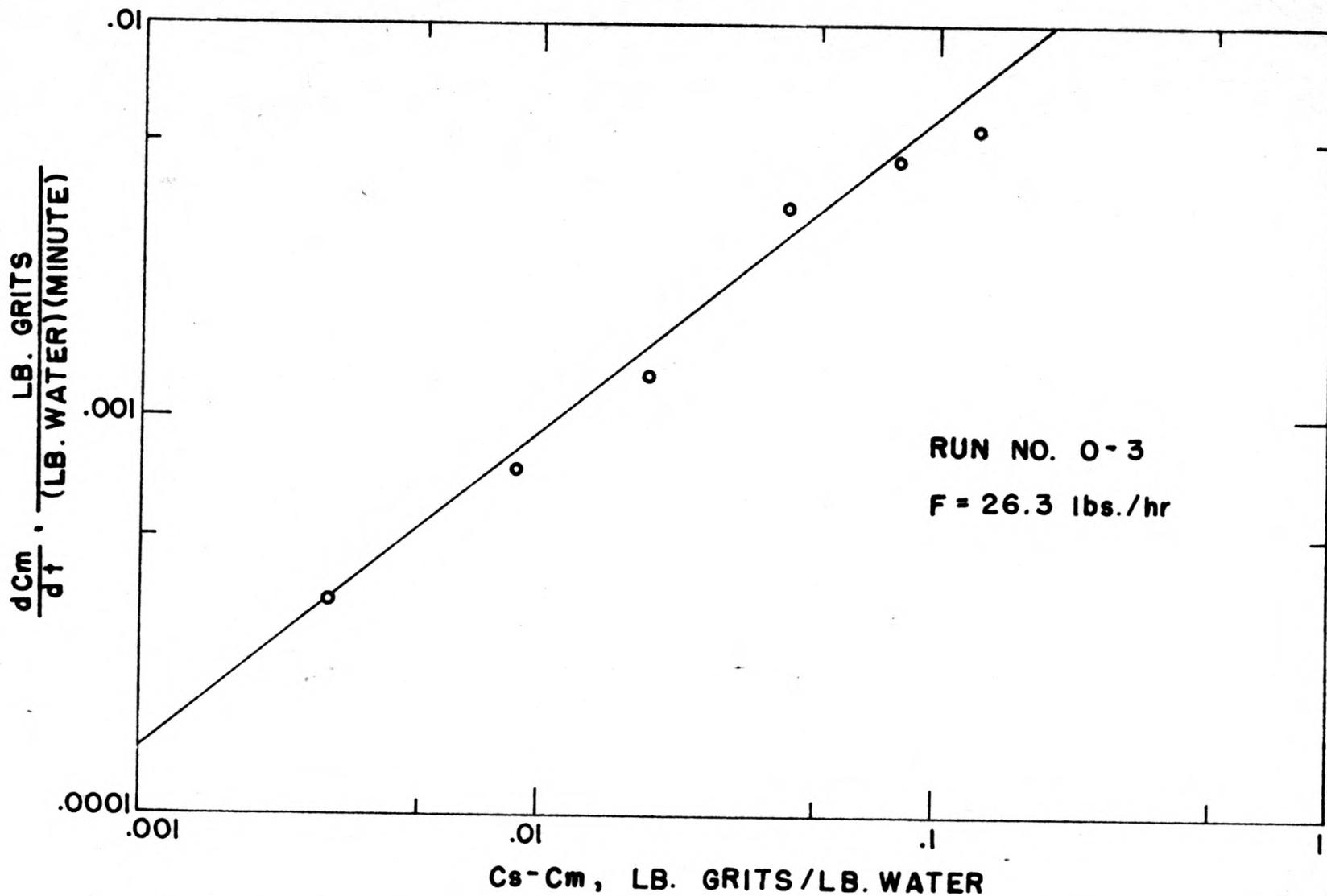


Fig. 11. Effect of  $C_s - C_m$  on the rate of change of average mill concentration.

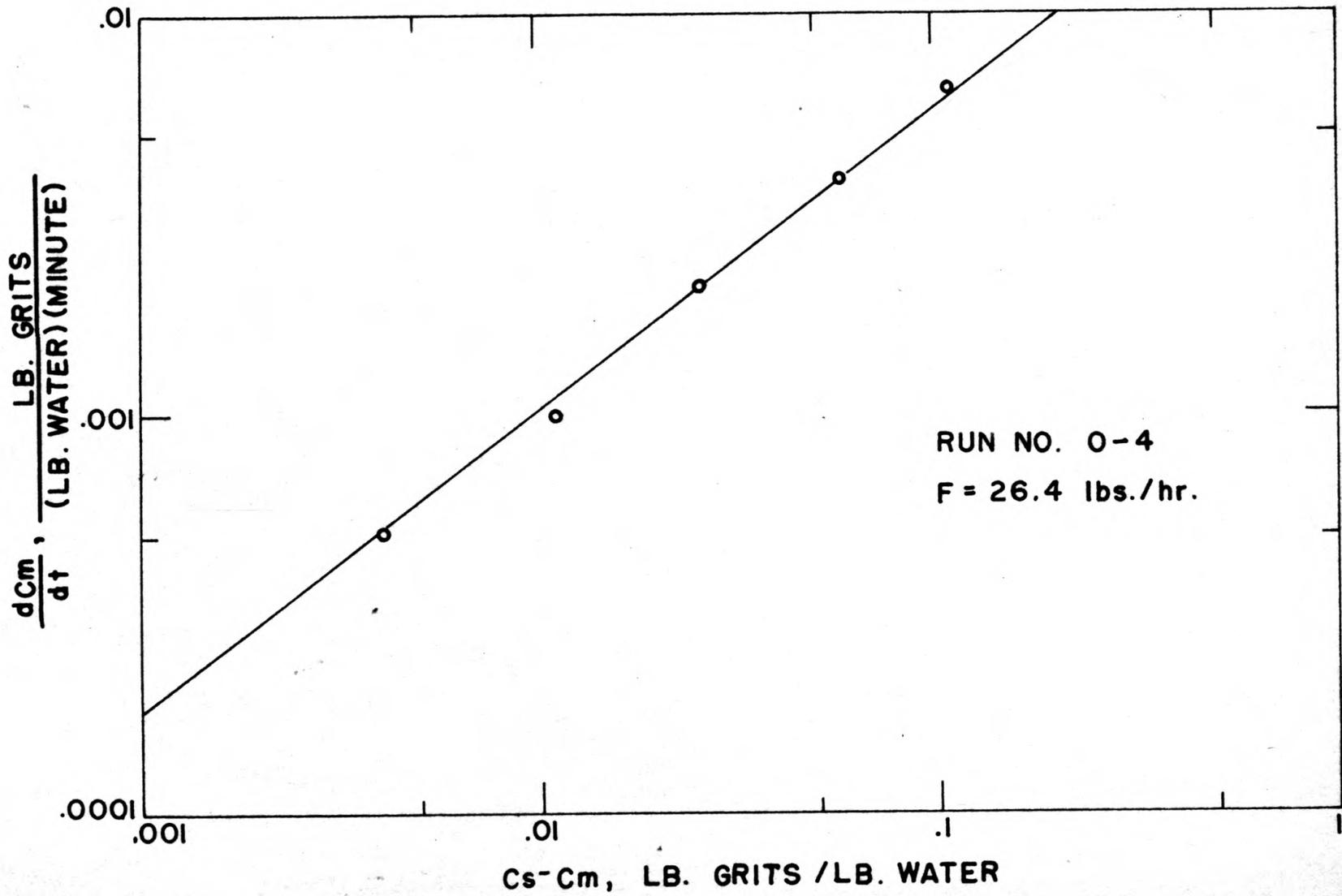


Fig. 12. Effect of  $C_s - C_m$  on the rate of change of average mill concentration.

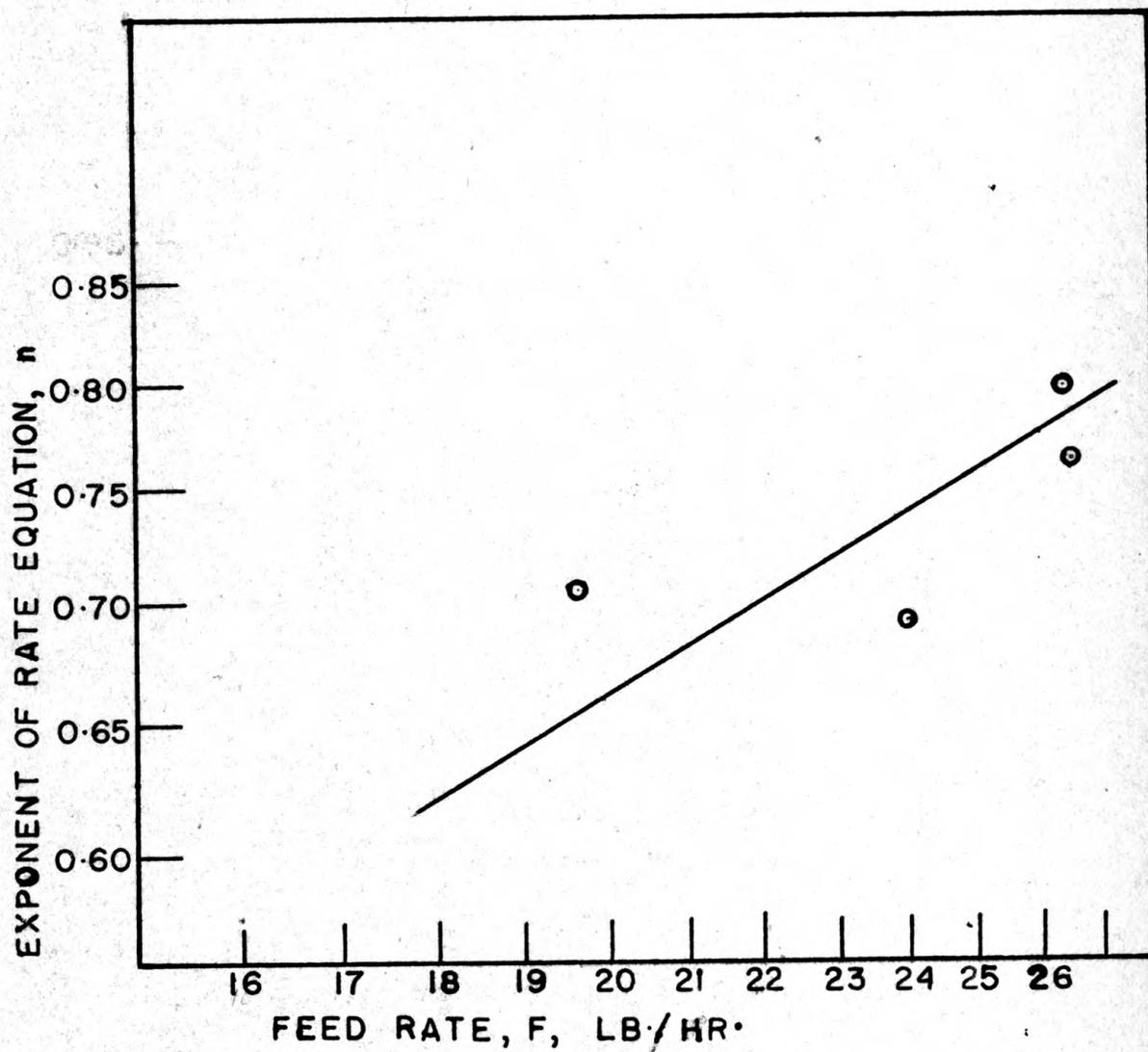


Fig. 13. Effect of feed rate on exponent of the rate equation,  $n$ .

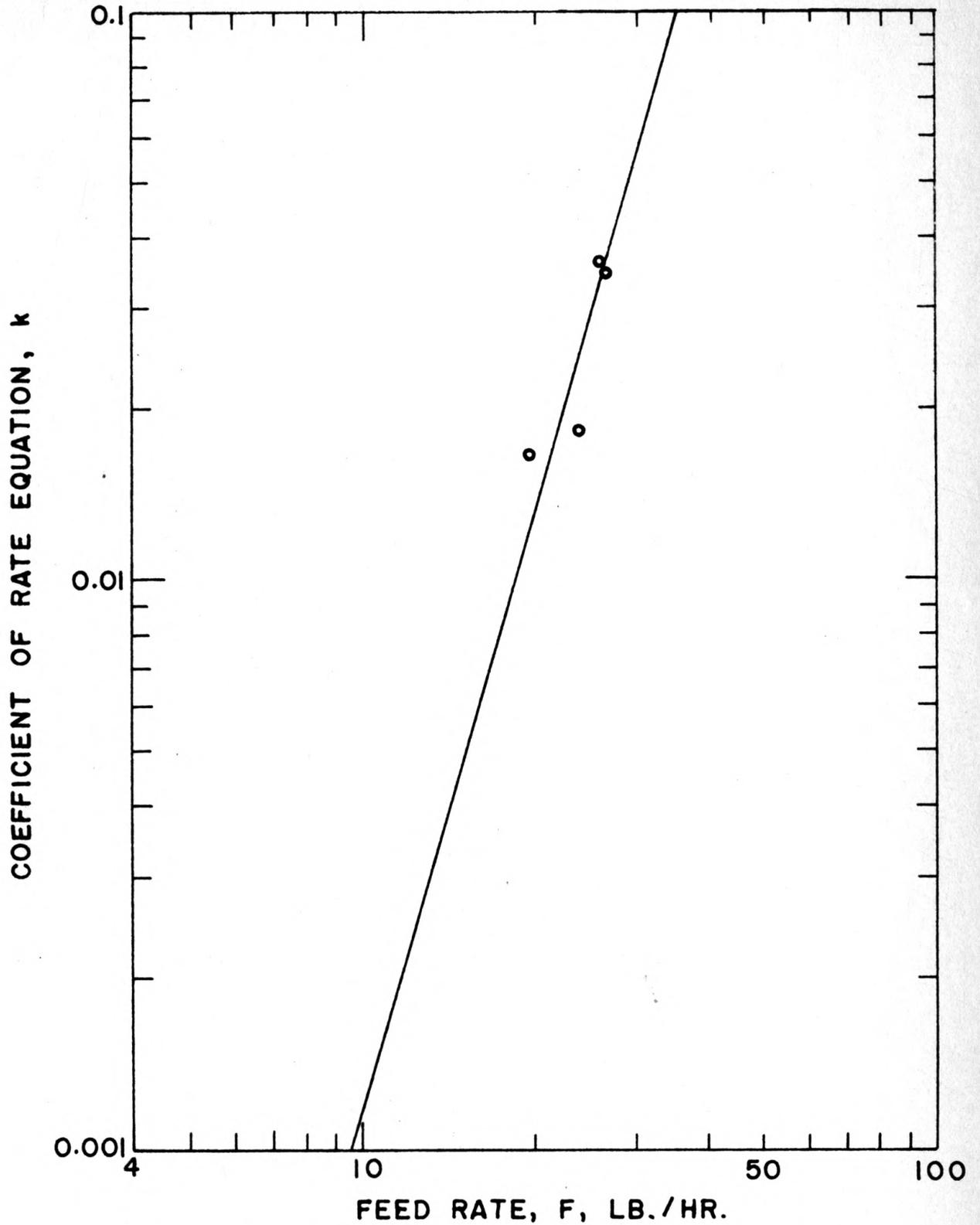


Fig. 14. Effect of feed rate on the coefficient of the rate equation,  $k$ .

The average mill concentration in the steady state,  $C_s$ , was also correlated with feed rate  $F$ . The straight line formed when  $C_s$  was plotted on arithmetic coordinates against  $F$  is shown in Fig. 15. The equation of this line is

$$C_s = 0.027 + 0.00626 F \quad (34)$$

These correlations now allow the evaluation of the time required to reach a concentration of  $0.99C_s$ , as expressed by equation (9). The substitution of the constants determined above into equation (9) gave

$$t_{0.99C_s} = \int_0^{0.99(0.027 + 0.00626 F)} \frac{dC_m}{1.99 \times 10^{-7} F^{3.7} (0.027 + 0.00626 F - C_m)^{0.108 F^{0.605}}} \quad (35)$$

The length of time required to reach an essentially constant concentration,  $t_{0.99C_s}$ , was determined by the application of the trapezoidal rule for numerical integration to values of  $1/\phi(C_m)$  for values of the feed rate,  $F$ , ranging from 10 to 50 lbs per hr. These calculations are summarized in Table 7. The values of  $t_{0.99C_s}$  are plotted against feed rate,  $F$ , in Fig. 16.

The time required to reach certain average concentrations in the mill was calculated by using the derived empirical rate equation at the feed rate used for each experiment. The calculated data are plotted on the  $C_m$  against  $t$ , plots of each experiment in Figs. 5, 6, 7 and 8 for comparison.

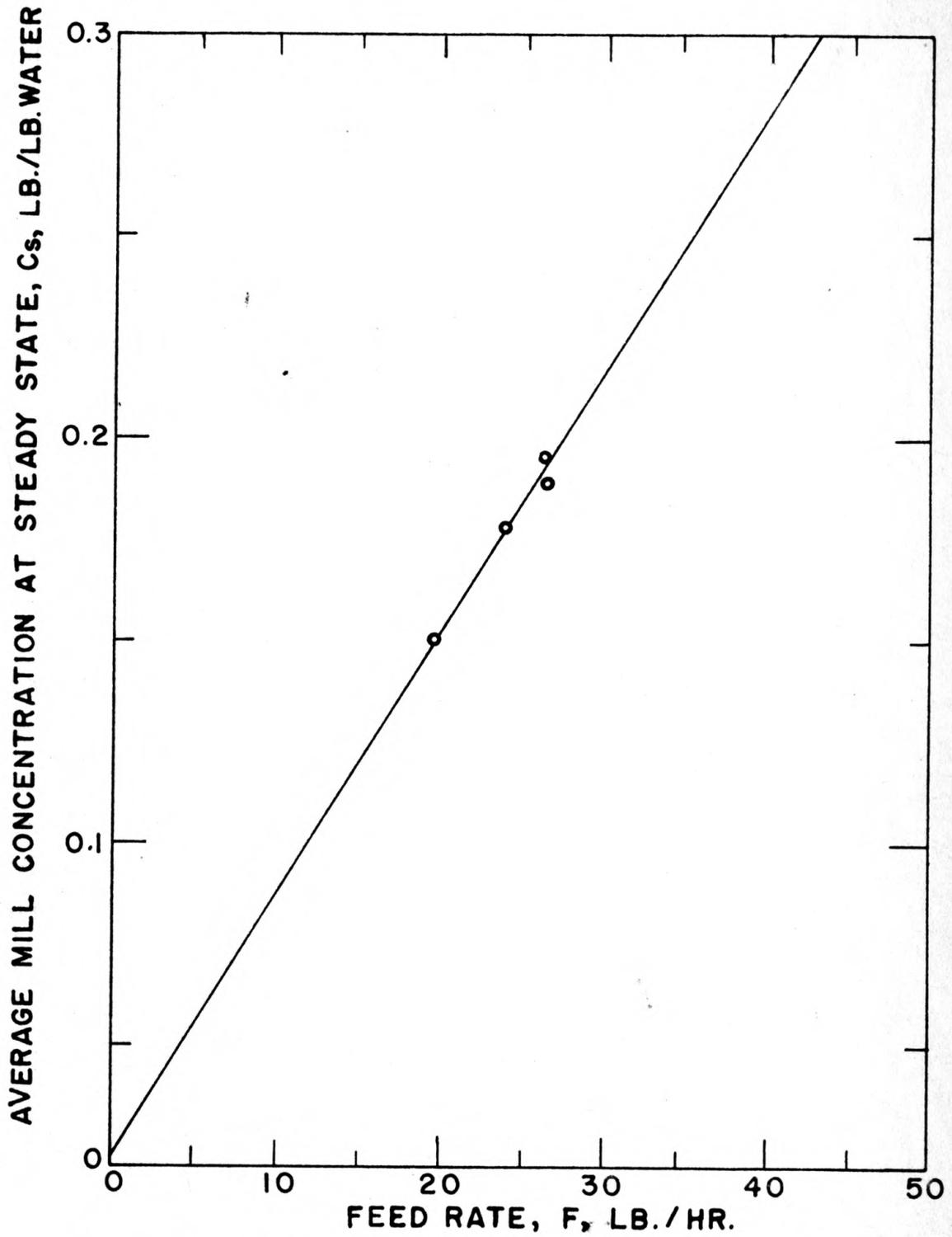


Fig. 15. Effect of feed rate on average mill concentration at steady state for once-through experiments.

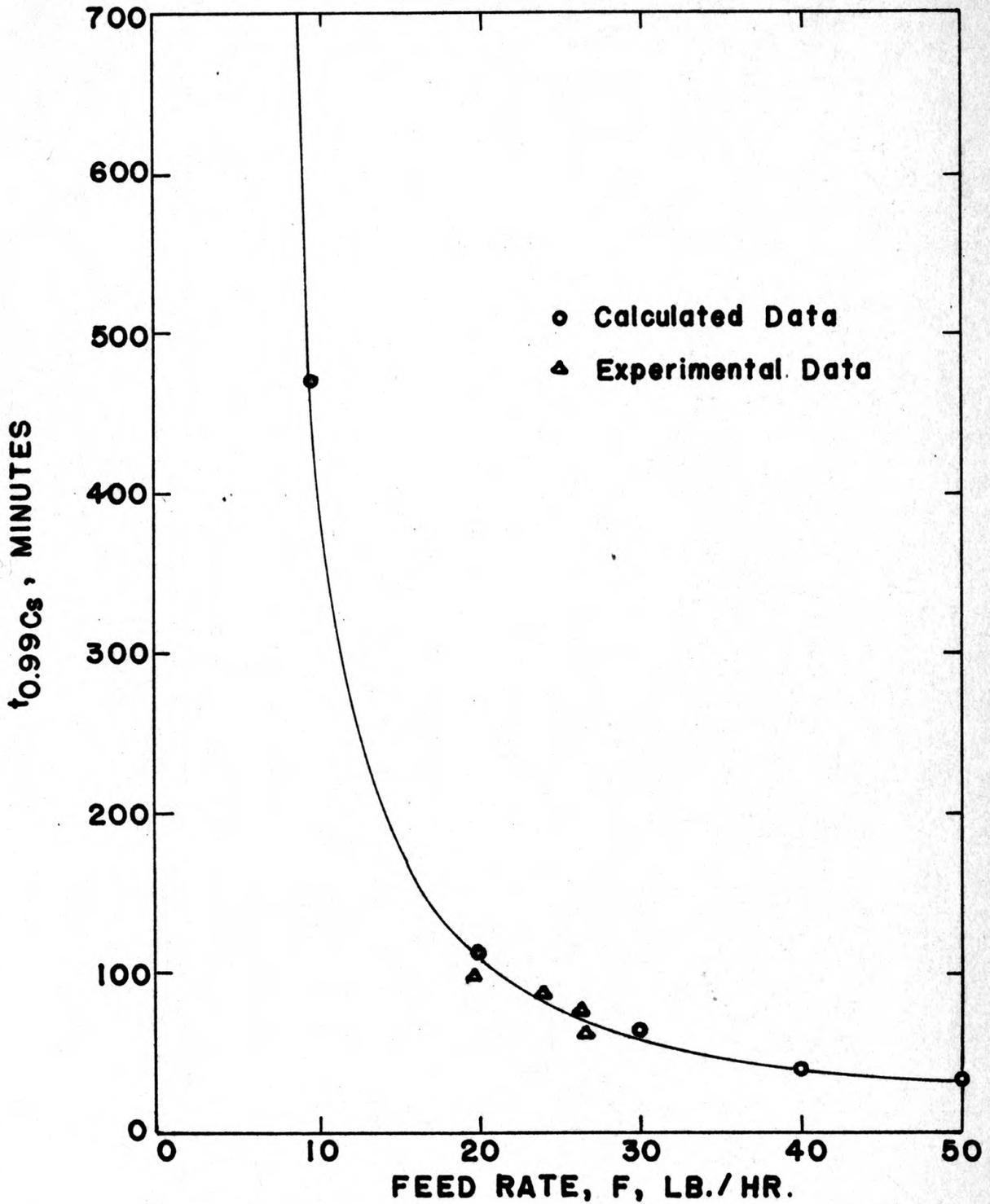


Fig. 16. Effect of feed rate on the time required to reach 0.99Cs for once-through experiments.

Table 7. Calculation of  $t_{0.99C_s}$  for once-through experiments.

F lbs/hr	$C_m$ lb/lb water	$C_s$ lb/lb water	$1/\phi(C_m)$	$t_{0.99C_s}$ minutes
10	0	0.0896	2940	471.7
	0.02		3290	
	0.04		3850	
	0.06		3850	
	0.08		8000	
	0.0887		21297	
20	0	0.152	254	112.1
	0.04		322	
	0.08		432	
	0.12		735	
	0.1505		5556	
30	0	0.215	64	65.8
	0.05		80	
	0.10		107	
	0.15		175	
	0.213		3333	
40	0	0.277	21.4	37.9
	0.05		26.2	
	0.10		33.8	
	0.15		47.2	
	0.20		82.0	
	0.274		2032	
50	0	0.34	9.0	32.5
	0.10		13.4	
	0.20		25.0	
	0.30		106.5	
	0.3366		1831	

Results on Recycling Experiments. A total of four runs were made.

The data obtained are shown in Table 8.

Table 8. Distribution of concentration of mill slurry with distance below outlet at various milling times for recycling experiments.

Run No.	: Feed rate, : : lb/hr	: Milling : : time, : : minutes :	Concentration, lb grits per lb water at distance below top of mill, inches				
			2	9 3/4	17 1/2	25 1/4	33
R-1	19.6	15	0.007	0.008	0.008	0.013	0.191
		45	0.021	0.024	0.019	0.087	0.207
		90	0.034	0.037	0.027	0.149	-
		115	0.038	0.050	0.033	0.209	-
		150	0.039	0.057	0.038	0.216	-
		165	0.037	0.056	0.035	0.215	-
R-2	18.1	5			0.006	0.007	0.121
		20	0.015	0.015	0.014	0.064	0.167
		30	0.019	0.021	0.017	0.113	0.247
		60	0.022	0.027	0.021	0.163	0.276
		90	0.031	0.041	0.027	0.239	0.312
		120	0.045	0.067	0.043	0.338	0.361
		148	0.047	0.074	0.047	0.349	0.387
R-3	29.0	20	0.012	0.030	0.028	0.145	0.236
		36	0.040	0.053	0.042	0.288	0.337
		58	0.052	0.075	0.052	0.366	0.362
		70	0.058	0.097	0.068	0.420	0.387
		82	0.057	0.082	0.070	0.438	0.388
R-4	31.7	10	0.019	0.022	0.021	0.116	0.217
		23	0.050	0.058	0.050	0.261	0.322
		35	0.059	0.074	0.060	0.382	0.372
		50	0.065	0.087	0.079	0.422	0.417
		65	0.078	0.126	0.099	0.492	0.444
		80	0.078	0.126	0.095	0.487	0.410

All the data were treated in the same way as for once-through experiments. The results are tabulated in Tables 9, 10, 11, and 12.

Table 9. Average mill concentration at various milling times for recycling experiments.

Run No.	Feed rate, lb/hr	Time, minutes	$C_m$ , lb grits per lb water
R-1	19.6	15	0.040
		45	0.067
		90	0.093
		115	0.119
		150	0.124
		165	0.103
R-2	18.1	5	-
		20	0.052
		30	0.084
		60	0.096
		90	0.127
		120	0.155
		148	0.188
R-3	29.0	20	0.087
		36	0.125
		58	0.185
		70	0.205
		82	0.204
R-4	31.7	10	0.075
		23	0.146
		35	0.186
		50	0.210
		65	0.246
		80	0.238

Table 10. Rate of change of average concentration in mill for recycling experiments.

Run No.	t, minutes	$C_m$ , lb grits lb water	$dC_m/dt$ , lb grits (lb water) (min)	$C_s$ , lb grits lb water	$C_s - C_m$ , lb grits lb water
R-1	20	0.038	0.00140	0.124	0.086
	40	0.067	0.00095		0.057
	60	0.084	0.00092		0.040
	80	0.097	0.00057		0.027
	100	0.107	0.00041		0.017
	120	0.114	0.00030		0.010
	140	0.120	0.00024		0.004
	160	0.123	0.00011		0.001
R-2	10	0.031	0.00252	0.156	0.125
	30	0.071	0.00155		0.085
	50	0.097	0.00114		0.059
	70	0.115	0.000781		0.041
	90	0.128	0.000590		0.028
	110	0.140	0.000530		0.016
	130	0.149	0.000364		0.007
	150	0.1545	0.000167		0.0015
R-3	10	0.042	0.00416	0.205	0.163
	20	0.082	0.00379		0.123
	30	0.117	0.00324		0.088
	40	0.147	0.00271		0.058
	50	0.172	0.00200		0.033
	60	0.191	0.00171		0.014
	65	0.1985	0.00111		0.0065
	70	0.2030	0.000564		0.0020
75	0.2045	0.000200	0.0005		
R-4	10	0.072	0.00615	0.250	0.178
	20	0.124	0.00445		0.126
	30	0.163	0.00333		0.087
	40	0.192	0.00252		0.058
	50	0.214	0.00196		0.036
	60	0.231	0.00145		0.019
	70	0.2425	0.00098		0.0075
	80	0.2490	0.00035		0.0010

Table 11. Empirical equations for the rate of approach to the steady state for recycling experiments of various feed rates.

Run No.	Feed rate lb grits per hr	Empirical equation
R-1	19.6	$\frac{dC_m}{dt} = 0.00577(C_s - C_m)^{0.606}$ (36)
R-2	18.1	$\frac{dC_m}{dt} = 0.00677(C_s - C_m)^{0.605}$ (37)
R-3	29.0	$\frac{dC_m}{dt} = 0.0100(C_s - C_m)^{0.466}$ (38)
R-4	31.7	$\frac{dC_m}{dt} = 0.0170(C_s - C_m)^{0.546}$ (39)

The coefficients,  $k$ , and the exponents,  $n$ , of these equations were again correlated with feed rate,  $F$ , to obtain equations (40) and (41).

$$k = 6.8 \times 10^{-5} F^{1.54} \quad (40)$$

$$n = 1.88 F^{-0.385} \quad (41)$$

In addition,  $C_s$  was correlated with  $F$  to obtain

$$C_s = 0.005 + 0.00765 F \quad (42)$$

Substitution of these equations into equation (9) gave the final integral equation for the time required to reach a concentration equal to  $0.99C_s$  in terms of the feed rate

$$t_{0.99C_s} = \int_0^{0.99(0.005 + 0.00765 F)} \frac{dC_m}{6.8 \times 10^{-5} F^{1.54} (0.005 + 0.00765 F - C_m)^{1.88 F^{-0.385}}} \quad (43)$$

This integral was also evaluated by the trapezoidal rule to obtain the time to reach an essentially steady state,  $t_{0.99C_s}$ . These values are shown in Table 12. Fig. 17 gives  $t_{0.99C_s}$  plotted versus feed rate,  $F$ , for recycling experiments.

Table 12. Calculation of  $t_{0.99C_s}$  for recycling experiments.

$F$ lb/hr	$C_m$ , lb/lb water	$C_s$ , lb/lb water	$1/\phi(C_m)$	$t_{0.99C_s}$ , minutes
10	0	0.0815	2910	743.2
	0.02		4130	
	0.04		4950	
	0.06		8330	
	0.0806		95240	
20	0	0.158	440	176.8
	0.05		550	
	0.10		800	
	0.1564		7000	
30	0	0.2345	162	75.6
	0.05		183	
	0.10		214	
	0.15		270	
	0.20		425	
	0.232		1625	
40	0	0.311	85.5	48.6
	0.05		92.6	
	0.10		102.0	
	0.15		115.0	
	0.20		137.0	
	0.25		178.0	
	0.308		706.0	
50	0	0.3875	52.7	35.2
	0.10		59.6	
	0.20		71.5	
	0.30		98.0	
	0.3836		368.0	

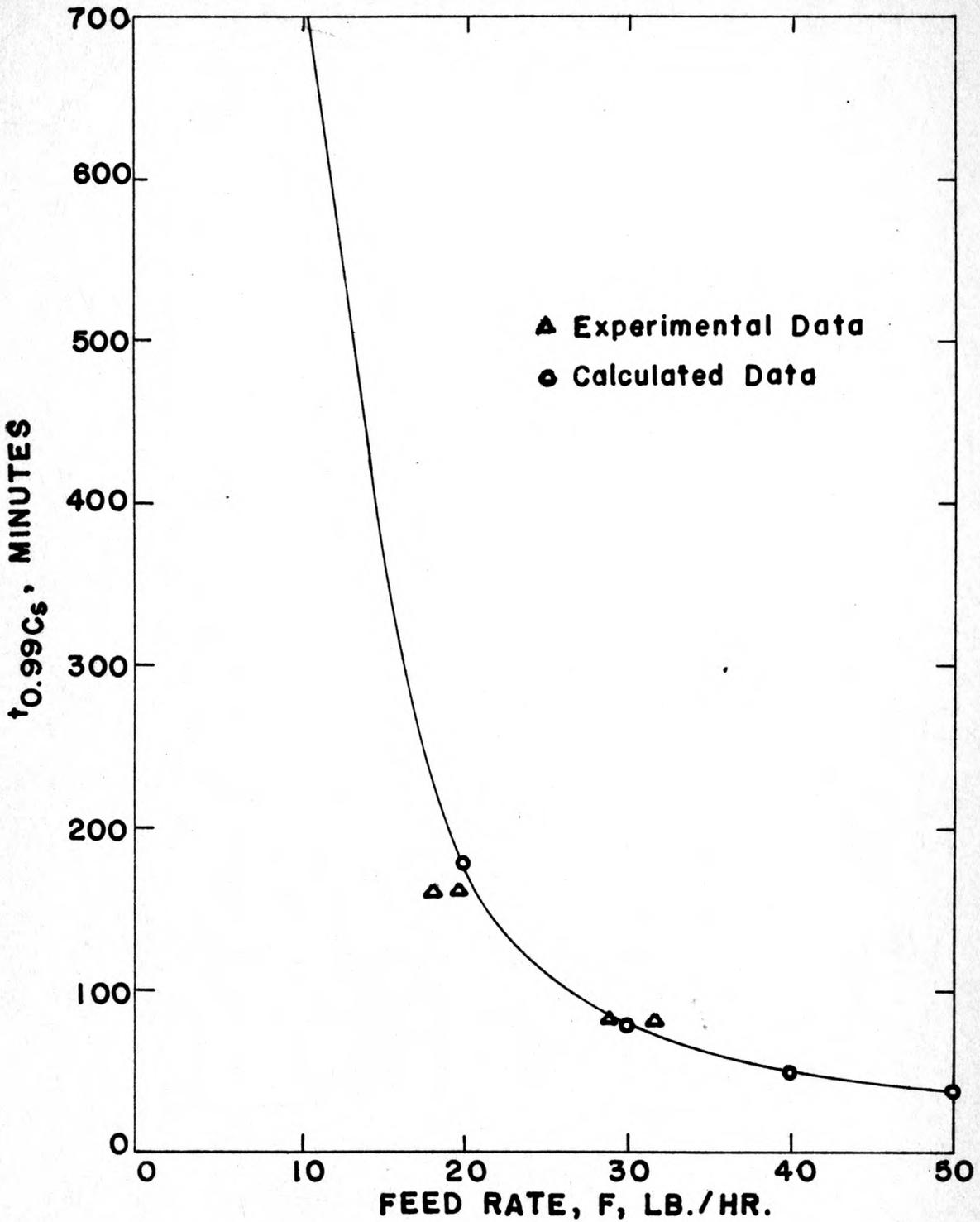


Fig. 17. Effect of feed rate on the milling time required to reach 0.99C<sub>s</sub> for recycling experiments.

### Experimental Test of the Scale-Up Equation

For this purpose, the relationships between the grits feed rate and the power consumption and the volume of the mill overflow were obtained. The data for once-through experiments on the 8-inch mill are given in Table 13.

Table 13. Power consumption, overflow rate and grits feed rate in once-through experiments for the 8-inch mill.

Grits feed rate (dry basis), lb/hr	Net power consumption, KW	Overflow rate, cc/10 sec
19.6	0.538	596
24.0	0.562	603
26.3	0.600	604
26.4	0.575	605

The power consumption,  $P$ , (in KW), is plotted against feed rate in Fig. 18. The equation of the straight line formed was obtained by the method of averages, as given in equation (44).

$$P = 0.37 + 0.0082 F \quad (44)$$

The mill overflow rate,  $V$ , (in cc/10 sec) is plotted against feed rate,  $F$ , in Fig. 19. The equation of this line was found to be

$$V = 577 + 1.04 F \quad (45)$$

The water feed rate for 6-inch mill was controlled in such a way that the volume of the mill overflow satisfied the following requirement

$$G' = G/1.157 \quad (27)$$

or

$$\frac{V'}{186.4} = \frac{V}{334.5} \times \frac{1}{1.157}$$

which reduces to

$$V = 2.07V' \quad (46)$$

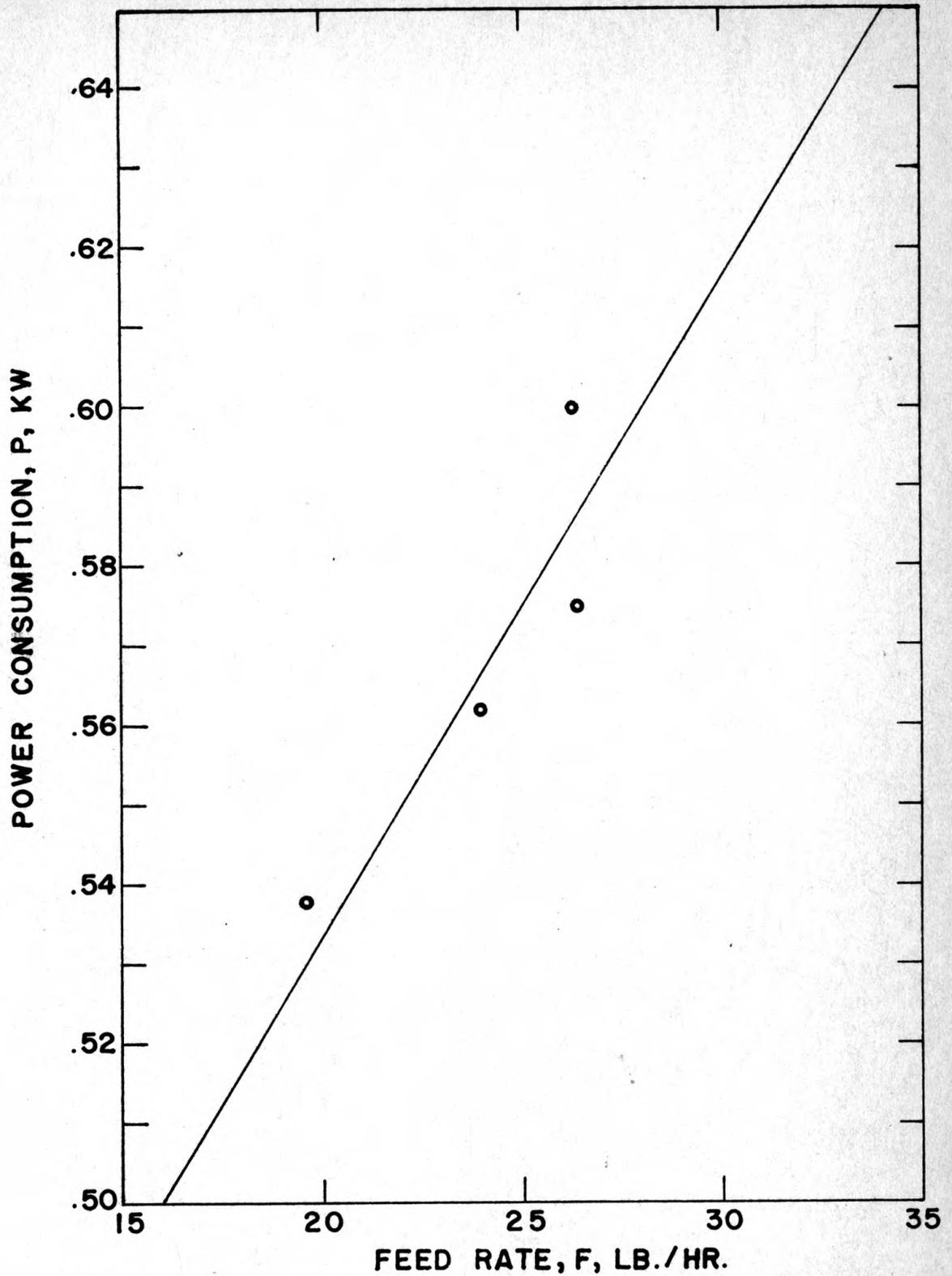


Fig.18. Effect of grits feed rate on power consumption for once-through experiments on 8-inch mill.

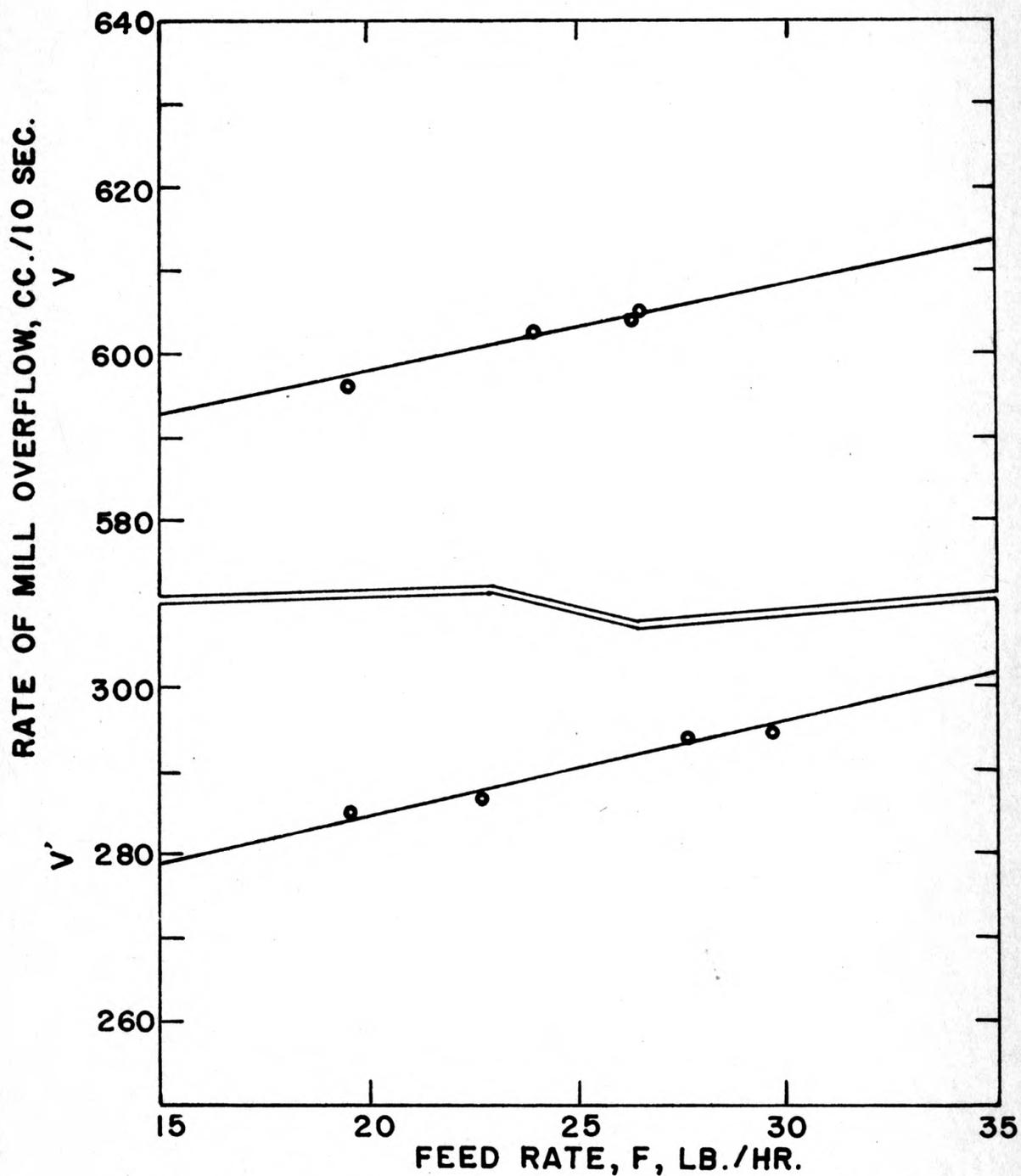


Fig.19. Effect of grits feed rate on rate of mill overflow for once-through experiments.

The feed water rate for the 8-inch mill was 0.91 gal/min. For the 6-inch mill the feed water rate was set at 0.42 gal/min, a figure obtained from equation (46). The actual feed rate of grits was then adjusted to give correct values for comparison with the 8-inch mill, as determined by equation (46). This was, of course, a means of controlling the residence times in the two mills to give dynamic similarity.

The power consumption and overflow rate data for the 6-inch mill are given in Table 14.

Table 14. Power consumption, overflow rate and grits feed rate in once-through experiments for the 6-inch mill.

Grits feed rate (dry basis), lb/hr	Net power consumption, KW	Overflow rate, cc/10 sec
19.6	0.225	285
22.7	0.240	287
27.6	0.248	294
29.7	0.248	295

The overflow rates for the 8-inch and the 6-inch mills are shown in Fig. 19. The two lines are approximately related by equation (46). Exact agreement was difficult to obtain because of the difficulty of setting exact values of the feed water rates and the grits feed rate. The equation of the line for the 6-inch mill was found to be

$$V' = 262 + 1.13F' \quad (47)$$

The power consumption for the 6-inch mill is plotted as a function of feed rate on Fig. 20. The equation of this line is

$$P' = 0.189 + 0.00206F' \quad (48)$$

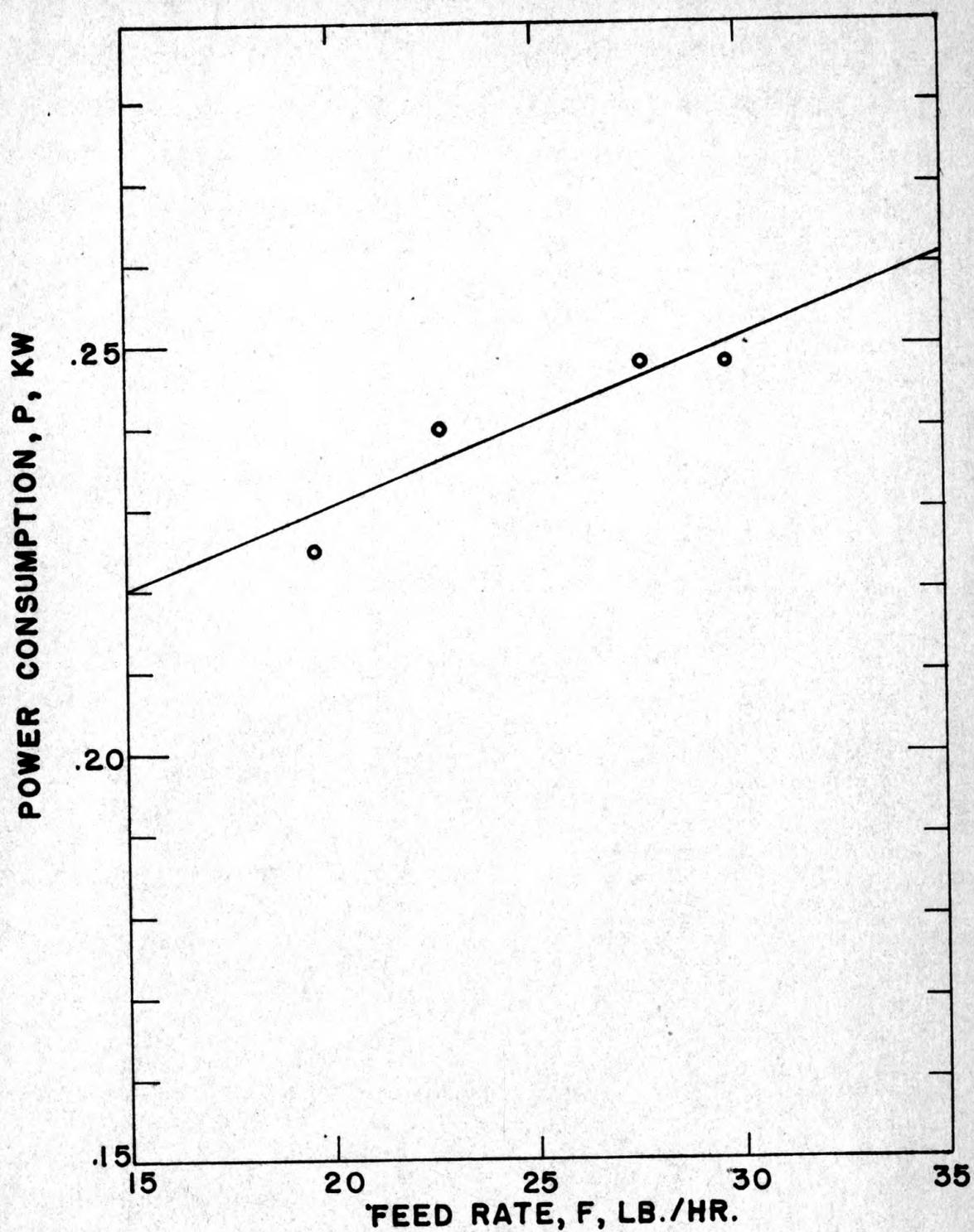


Fig.20. Effect of grits feed rate on power consumption for once-through experiments on 6-inch mill.

In order to compare the power consumption of the two mills with the scale-up equation (23), the following procedure was used:

Using the experimental values of the feed rate,  $F'$ , for the 6-inch mill, the mill overflow rate,  $V'$ , was determined either from Fig. 19, or from equation (47). The value of the corresponding overflow rate for the 8-inch mill,  $V$ , was then determined from equation (46). Then the slope of equation (45) and (47) should be the same, and the relationship between the feed rates should be the same as for the overflow rates, or

$$F = 2.07 F' \quad (49)$$

The net power consumed by the 8-inch mill at the feed rate calculated from equation (49) was found then from Fig. 18, or computed from equation (44).

The experimental net power consumed in the 6-inch mill and the net power computed by this procedure for the 8-inch mill should be related by the scale-up equation, or, assuming that thrust is proportional to net power consumption,

$$\frac{P}{P'} = \frac{T}{T'} = r^3 = 1.34^3 = 2.42 \quad (50)$$

The comparison is summarized in Table 15.

Table 15. Comparison of the power consumed by the two mills with the scale-up equation.

Feed rate for 6-inch mill, lb/hr	19.6	22.7	27.6	29.7
$P'$ , measured, KW	0.225	0.240	0.248	0.248
Feed rate for 8-inch mill, lb/hr	40.6	47.0	57.2	61.5
$P$ , calculated, KW	0.703	0.755	0.838	0.874
$P/P'$	3.12	3.14	3.38	3.52

## DISCUSSION AND CONCLUSIONS

The empirical equations derived for the rate of change of average mill concentration with time, and the integrated form giving the time to reach a concentration equal to 0.99 of the final equilibrium concentration, are useful in giving an insight into the time required to reach equilibrium, or steady state, operation. These equations, one for once-through operation, the other for recycling, are valid only for one condition of all the variables, except feed rate. If these fixed variables, such as mill rpm and feed water rate, were changed, the equations would change also. Other correlations are therefore possible, and it should be possible to find the conditions which would give the fastest rate of approach to equilibrium, with the least water and power consumptions.

In comparing the times required to approach equilibrium in once-through and recycling experiments, it appears that at the same feed rate the time required is larger for recycling than for once-through experiments. This is caused by the greater hold-up in the recycling process.

The ratios of power consumptions in the 8-inch and 6-inch mills, given in Table 15, are greater than the ratio derived by theoretical considerations by a factor of about 30 per cent. This is not surprising in view of the many simplifications made in developing the theory. The fact that the ratio appears to be fairly constant indicates that a considerable part of the difference may be due to a fixed difference between the two mills, not accounted for in the theory. Such a difference may be the result of one or many geometrical non-similarities in the two mills. Among such possibilities are differences in the roughness of the inner surfaces of the

mill and differences in the clearance between the blades and the mill casing. Since both casings were made from standard iron pipe, close control of the inner dimensions of the casings was not possible.

In deriving the scale-up equation it was assumed that the effect of viscosity was negligible. This might not be valid if one system was using water and another some other liquid.

The mill speed chosen for the 8-inch mill was lower than that used by some previous workers. This low speed was selected purposely to reduce the effect of vortexing in the mill, since the speed of the smaller mill is larger than for the larger mill.

The feed rate of the 8-inch mill is 2.07 times larger than that for the 6-inch mill when they were operated in dynamic similarity. The net power consumption increased with feed rate. Therefore, the minimum net power consumption per unit feed grits could be found, also the optimum mill size could be determined.

In actually designing a full scale mill, other factors would have to be considered also. Among these are the strength of the propeller blades. Since the thrust will vary as the cube of the ratio of mill diameters, the stress on the blades will increase rapidly. Another characteristic of the fluid, surface tension, may also be a factor under certain conditions.

The scale-up of equipment such as the mill used here can become very complicated if all possible factors are considered. In addition, to those discussed cost may also be a factor. It is usually necessary to make some simplifying assumptions, such as were used here, in order to keep the expenditure of time and money to a reasonable level.

## LIST OF SYMBOLS

a	constant
b	constant
c	constant
C	concentration of sample, lb grits/lb water
$C_m$	average concentration in mill, lb grits/lb water
$C_s$	average concentration in mill at equilibrium, lb grits/lb water
d	constant
D	inside diameter of mill casing, inches. D' refers to the small mill.
$D_1, D_2, \dots, D_n$	dimensionless groups
e	constant
f	constant
F	sorghum grits feed rate, dry basis, lb/hr. F' refers to small mill.
g	acceleration of gravity, $l\ t^{-2}$
G	ascending speed of slurry in mill, $l\ t^{-1}$ , G' refers to small mill.
k	constant value of function, $\phi'$ , in rate equation
l	dimension of length
L	length of mill, measured from top of mill, inches
m	dimension of mass
n	exponent of the rate equation, constant
N	r. p. m. of mill blades, 1/minute
P	net power consumed, KW. P' refers to small mill
r	ratio of diameters, $D/D'$
$r'$	ratio of diameter to height of mill
$r''$	ratio of diameter of mill to diameter of blades

$r'''$	ratio of diameter of mill to height of overflow
$t$	milling time, minutes
$t_{0.99C_s}$	time required to reach a concentration of $0.99C_s$ , minutes
$T$	thrust exerted by the blades, $m \ l \ t^{-2}$ , $T'$ refers to small mill
$V$	volume rate of mill overflow, cc/10 sec. $V'$ refers to small mill
$\phi, \phi', \phi_1$	function
$\rho$	density, $m \ l^{-3}$
$\mu$	viscosity, $m \ l^{-1} \ t^{-1}$

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SCALE UP FACTORS IN THE DESIGN OF A HYDRAULIC STARCH MILL

by

YUNG-LING KO

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This work was a continuation of a series of studies on the production of starch from sorghum grits at Kansas State University. The purpose was to evaluate the scale-up factors for the design of a hydraulic starch mill. In order to do this, the variables in hydraulic milling had to be simplified. All the scale-up factors (variables of the milling process) had to be measured under steady state conditions. Actually, the steady state of milling was not easily determined, since any small change in one of the variables also changed the equilibrium conditions. For this reason part of this work was devoted to an analysis of the time required for steady state operation to be achieved.

Two empirical equations for rate of approach to the steady state were obtained:

For once-through experiments,

$$dC_m/dt = 1.99 \times 10^{-7} F^{3.7} (0.027 + 0.00626F - C_m)^{0.108} F^{0.605}$$

For recycling experiments,

$$dC_m/dt = 0.000068 F^{1.54} (0.005 + 0.00765F - C_m)^{1.88} F^{-0.385}$$

where  $C_m$  = average mill concentration, lb grits/lb water

$t$  = time in minutes

$F$  = grits feed rate, dry basis, lb/hr

These empirical rate equations were integrated graphically for various particular feed rates. In this experiment, the times required to reach 99 per cent of the average mill concentration at the steady state were calculated and plots of  $t_{0.99C_s}$  versus  $F$  were constructed.

From the previous investigations and an analysis of the hydraulic milling process, significant variables for the design of the hydraulic mill were chosen. These were organized into dimensionless groups by dimensional analysis, to give

$$T = \rho D^2 G^2 \phi' \left( \frac{DN}{G}, \frac{\rho DG}{\mu}, \frac{Dg}{G^2}, r', r'', r''' \right).$$

Here	T	= thrust exerted by the blades, $\text{mlt}^{-2}$
	$\rho$	= density, $\text{ml}^{-3}$
	D	= inside diameter of mill, l
	G	= ascending speed of slurry in mill, $\text{lt}^{-1}$
	$\phi'$	= function
	N	= rpm of mill, $\text{t}^{-1}$
	$\mu$	= viscosity, $\text{ml}^{-1}\text{t}^{-1}$
	g	= acceleration of gravity, $\text{lt}^{-2}$
	$r'$	= ratio of mill diameter to mill height, dimensionless
	$r''$	= ratio of mill diameter to diameter of blades, dimensionless
	$r'''$	= ratio of mill diameter to height of mill outlet, dimensionless

This function was simplified by assuming that the mill was operated in highly turbulent conditions, so that the effect of viscosity might be omitted. When two different sizes of hydraulic mill have geometric similarity and their diameters are related by

$$D = rD'$$

and are operated at such conditions of rpm and feed rate

that  $N = N' / \sqrt{r}$

and  $G = G' \sqrt{r}$

$\phi'$  would remain constant, and a relationship of variables independent of the mill size was obtained

$$T = k \rho D^2 G^2 \quad \text{or} \quad T/T' = r^3$$

where  $k$  is the constant value of  $\phi'$  and  $T'$  is the thrust in the smaller mill.

This equation was checked experimentally by designing a 6-inch mill, geometrically similar to the 8-inch mill previously used in the pilot plant. These mills were run at various feed rates and when conditions of feed rate and rpm were such as to produce dynamic similarity, the rates of power consumed by the two mills was found to be in close agreement with the relationship for thrust expressed above.