A STUDY OF THE CHARACTERISTICS OF SAND MOVEMENT BY WIND

by

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INTRODUCTION

Large areas of farm land in the southern and eastern portions of the High Plains of the United States have soils of a sandy texture. These soils are relatively unstable during the periods of intense wind movement indigenous to the Plains. When they are used under intensive systems of agriculture, a wind erosion problem is presented. Sand moved by the wind results in damage to crops. Accumulations of sand along field boundaries, fence rows, windbreaks, and building sites are common. Again, the formation of hummocks and irregularities in fields present leveling and drainage problems. In general, it may be stated that the blowing of sandy material on agricultural lands results in both direct and indirect damages. In addition, excessive amounts of dust are capable of affecting the health and well-being of peoples.

Drifting sand has a tendency to accumulate into dune masses. The vast sand seas of the Sahara in North Africa represent the extreme of this phenomenon in which the sands have accumulated through ages and appear to have migrated to locations where they are more or less hemmed in by the general wind movement.

Active sand dune areas in the United States are not extensive. In the High Plains they are confined mainly to the lands bordering the larger streams. Upon occasion, these sand masses become a hazard to transportation facilities and to highly developed agricultural or urban areas. They may then be of economic consequence sufficient to warrant stabilization measures.

A knowledge of the characteristics of sand movement is basic to the design of measures to cope with the wind-sand problem. The need for the scientific approach tends to become greater with the passing of time and the increasing need to control and utilize our resources to the fullest.
THE INVESTIGATION

The object of this investigation was to obtain knowledge of the dynamic characteristics of sand movement by air. An artificial air stream developed by a laboratory wind tunnel was used to secure data on the subject. Information on the following phases of the phenomenon was obtained:

1. Nature of sand movement.
2. Velocity distribution and shear of the wind over stable sand surfaces.
3. Velocity distribution and shear of the wind over drifting sand surfaces.
5. Theoretical considerations of particle movement.
6. Distribution of sand flow above the bed.
7. Rates of sand flow.
8. Average height of saltation.

REVIEW OF LITERATURE

A perusal of the literature on the transport of solid material by fluids shows that research on the mechanics of the transport of sand and soil material by wind has received little attention compared to the study of sediment transport by water. Both water and wind phenomena may be classified under the general subject of fluid mechanics and since the two phenomena are allied closely, many of the findings where water has been used as the fluid medium are applicable to air.

It is not the purpose of this thesis to give a comprehensive review of the development of sedimentation mechanics. An excellent review of the
subject has been given by Vanoni (19). According to him, the science had its beginning with the work of DuBoys, who introduced the idea of a critical tractive force for bed movement in 1878. Gilbert (9) in 1914 described very clearly the modes of sand movement in water. He observed that for incipient motion, the grains of sand are moved by rolling and sliding along the bed. As the velocity increases, he noted that the grains tend to take short jumps in curved orbits. He described this mode of transportation as saltation. As the flow was increased further, he noted that material was carried in suspension.

The problem of transportation of material by streams was attacked vigorously following World War I. The flume and model were used extensively. The result was a number of bed load formulas for estimating movement. All were of an empirical or semi-empirical nature. Johnson (11) compared a number of these formulas and noted that many yielded estimates of the same order. He concluded that the choice of equations could be made on the basis of the convenience in measuring the variables appearing in the formula.

Starting about 1935, developments in basic fluid mechanics following a rapid development of aeronautics were applied to the problem of sediment transport. The approach involved dimensionless parameters. An example is the critical dimensionless tractive force of Shields (18). This approach utilized a form of the Reynolds number and a dimensionless function of grain and force characteristics required for movement to occur.

In 1942 Einstein (6) applied a new concept in developing a formula to describe bed load movement of particles in water. The formula is not based on a critical tractive force or condition. The sediment is assumed to move in steps or jumps and movement to be dependent on the length of the step and the probability that the lift force will exceed the weight of the particle in water.
The activity of the wind as a geological agent long has been recognized and forms the extensive literature of eolian geology. In 1911, Free (8) attempted to bring together, correlate, and summarize the known data on eolian geology from the viewpoint of the student of soils. The bibliography of eolian geology appended to the publication of Free contains 2,475 references. Information available at that time on the mechanics of wind translocation, including detachment, abrasion, sorting, and transport of soil material by the wind, was reviewed and summarized.

Malina (15) published a review paper in 1942 entitled "Recent Developments in the Dynamics of Wind Erosion." This paper contains 59 references and is an excellent summary of developments to that date.

A summary of pertinent information relating to the several wind tunnels constructed at various locations and the techniques used to study wind-erosion phenomena is contained in a paper by Zingg and Chepil (26).

A work devoted wholly to the subject of sand transport by wind was published by Bagnold (2) in 1943. The approach involved wind tunnel techniques and the gathering of information in the Liberian desert. Chepil (3, 4, 5) subsequently carried out researches on the movement of soil by wind, utilizing several of the relationships developed by Bagnold. Some characteristics of aeolian sand movement were reported by Zingg (25) in 1951.

Several theoretical analyses of some of the phenomena of sand movement by wind have been published. Worthy of note is a work by Kalinski (13) concerning turbulence and the transport of sand and silt by wind. Von Kármán (21) has also made a speculative analysis of the phenomenon of surface ripples.
DESCRIPTION OF EXPERIMENTAL EQUIPMENT

Wind Tunnel

A laboratory wind tunnel was employed to provide an air stream for the study of moving sand. Briefly, the wind-making equipment comprises a governor-controlled gasoline engine and a heavy-duty axial-flow type ventilating fan. Control of air movement is effected partly by changing the speed of the engine and also by an adjustable vane inlet. For normal operating ranges the engine is run at a constant speed and control is obtained with the adjustable air-intake device. The air flow from the fan blades is redistributed and evened out by a series of screens in a metal transition section connecting the fan to the duct. A honeycomb-type air straightener is located in the upper portion of the duct immediately below this transition section. It is fabricated from 324 two-inch aluminum tubes, each 1 foot in length.

The duct of the tunnel is 56 feet long and 3 feet square. The sides of the duct are comprised of alternate plywood and plate glass panels. The alternate wood panels are removable to give easy access to the test space. Removable also is the top of the duct which is made from 3/4-inch plywood. In addition the top panels slide along the horizontal length of the duct to provide ingress for air measuring and other devices at desired points. Air flowing from the end of the tunnel discharges into a room with a floor area of 300 square feet and a height of 13 feet. It is recirculated to the fan by a 4-foot square duct on the second floor of the building housing the tunnel. A photograph of the wind-making equipment and the tunnel duct is shown in Plate 1.

The characteristics of velocity distribution developed throughout the length of the duct have been described previously (26).
EXPLANATION OF PLATE I

View of laboratory wind tunnel. The orientation of the engine and fan is shown in the left foreground. The transition, or diffusion section, connects the blower with the 3-foot square tunnel duct. Variable density screening is contained in the transition section. Air straightening tubes are located in the tunnel duct adjacent to the transition section.
Air Measuring Devices

A special Pitot tube was constructed to measure wind velocities above sand surfaces in the tunnel duct. The impact portion of the tube was a 0.5 mm inside diameter hypodermic needle. The static tube was separated a horizontal distance of 1 cm from the impact tube. Both pressure devices were mounted at the same vertical level on horizontal brass tubing. Control of height for traverses above sand surfaces was accomplished by clamping the vertical brass tubing to a graduated vernier-type point gage.

An alcohol manometer inclined at a 1:20 slope was used to register air velocity pressures. This manometer was constructed in the laboratory for general use. It was calibrated by use of a Dwyer hook gage employing an air stream to develop velocity pressures. Values of wind velocity obtained are for standard temperature and atmospheric pressure conditions; i.e., 70° F. at sea level.

Apparatus for Direct Measurement of Bed Shear

One of the variables in sand movement by wind is the value of $\tau_0$, the average shear or drag of the wind on the surface. Its indirect determination from velocity traverse is often difficult or problematic. As an aid in determining its true value for a given condition, equipment was constructed to measure it directly. A 4-inch layer of 2-6 mm gravel comprised the "floor" of the tunnel. A metal tank 18 inches wide and 6 feet long was recessed into this bed of nonerodible gravel. A second tank having approximately $\frac{3}{5}$-inch clearance on the sides and $\frac{3}{4}$-inch clearance on the ends was also constructed. This second tank was 3 inches in depth and was placed in the larger tank. Its surface area was 11.5 square feet. Water was used to float the smaller tank.
It was partially filled with insulating board to give it buoyancy sufficient to carry a layer of experimental sand bed. When properly weighted the floating tank formed a "shear tray," simulating an integral portion of the bed. The horizontal drag of the wind on the tray was transmitted to a small spring scale placed outside the tunnel by use of thread passing over fiber pulleys. The scale registered the value of $\tau_o$ in grams. A view of the floating shear tray comprising the lower 8 feet of a bed of sand on the floor of the tunnel is shown in Plate II.

Sand Sampler

Equipment to collect quantitative samples of airborne sediment at four heights was developed. The device was designed for use either in the laboratory or field. The sampler has been described in a previous publication (24). Briefly, the average velocity of intake of 0.92 inch square sampling tubes is controlled to equal the velocity of the sediment laden air stream at the level of sampling. Separation of sediment is made by a system of filters. A view of the sampler oriented at the end of a portable wind tunnel duct in the field is shown in Plate III. The installation of the 4-sampling tubes at the end of a sand bed in the laboratory tunnel is shown in Plate II. The standard 110 volt-60 cycle electrical system is used to power the sampler in the laboratory. A motor generator is required for field use away from power distribution lines.

Lighting for Photographic Purposes

Intense illumination was required for photographic study of sand movement by wind. Natural sunlight was used for the purpose, employing procedures similar to those of Chepil (3). Mirrors were utilized to transfer the parallel
EXPLANATION OF PLATE II

View of inside of 3-foot square wind tunnel duct looking downwind over an experimental sand bed and the shear tray. The floating tray occupies the lower 8 feet of the tunnel floor. A nonerodible gravel forms the bed outside the 18-inch experimental width. A Pitot tube is mounted on a point gage directly over the center of the shear measuring device. The four intake tubes of the sand sampler are shown mounted at fixed heights above the end of the bed and shear tray. The top of the duct has been removed and artificial lighting installed to provide illumination for photographic purposes.
EXPLANATION OF PLATE III

View of airborne sediment sampler in use at the end of a portable wind tunnel in the field. Increment samples of sediment-laden air are drawn through the four intake tubes of the device. The average velocity of air intake of each sampling tube is controlled to equal the velocity of duct air flow at the height of sampling. Air control and filter devices are mounted on a table. The equipment is portable for use in either the laboratory or field.
rays of the sun to a lens of 45-inch focal length. The lens is 18 inches long and 10 inches wide. It was constructed from $\frac{\text{3}}{16}$-inch plate glass, a curved piece being sealed to a flat one and the enclosed space filled with water. Near the focal length the beam of light was concentrated to a width of approximately $\frac{1}{8}$ inch. The lens was located above the laboratory wind tunnel duct at a position 3 feet from its lower end. When the long narrow beam of light was focused a short distance above the bed, the action of a restricted and intensely illuminated portion of the sand flow could be observed and photographed readily.

PROCEDURE

Several cubic feet of sand of varying size was obtained by dry sieving a water-deposited sand obtained near the Arkansas River in south-central Kansas. The sands are predominately quartz. The grains tend to be smooth, well rounded, and approximate a spherical shape. The five size separations obtained are shown in Table 1.

<table>
<thead>
<tr>
<th>Screen separation size ranges (mm)</th>
<th>Average size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15-0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>0.25-0.30</td>
<td>0.275</td>
</tr>
<tr>
<td>0.30-0.42</td>
<td>0.36</td>
</tr>
<tr>
<td>0.42-0.59</td>
<td>0.505</td>
</tr>
<tr>
<td>0.59-0.84</td>
<td>0.715</td>
</tr>
</tbody>
</table>

In addition to the sands of the above size ranges, a natural dune sand was secured from a location near Lamar, Colorado. Limited studies of this
natural sand of wide size distribution were made to clarify the phenomenon of surface rippling. The average diameter of the natural dune sand was 0.3 mm.

The sands were placed on trays of 18-inch width along the center line of the 3-foot bottom of the tunnel. The horizontal length of exposed sand varied from 16 feet for the smallest to 32 feet for the largest size used. The lower 8 feet of the exposed sand area was contained on the shear measuring device.

Wind velocities above a point near the center of the shear tray were obtained by use of the small impact and static tube described under the section on "Experimental Equipment." Sand passing the lower end of the shear tray was sampled by use of the sand sampler described previously.

Reference to Plate II shows the orientation of all experimental equipment. A sand is in place for testing. The nonerodible gravel occupies the 9-inch space on either side of the test sand.

Tests were first made on each of the graded-sand surfaces when stabilized to drifting from wind action by use of a fine water spray. Velocity traverses were made over each stabilized sand surface for five levels of wind movement. An arbitrary total pressure measured in the transition section between the fan and the duct provided an index for the systematic variation in the level of wind movement. The location of this arbitrary pressure, designated $P_1$, and its relation to velocity and bed shear has been described and published previously (27). Direct measurements of the horizontal component of shear on the sand surface were obtained for each level of wind applied.

The sands were next studied in a dry state by subjecting them to several wind velocities wherein movement occurred over the entire bed. The surface was permitted to erode at a given velocity for a time of two minutes. During the two-minute period a velocity traverse was made over the moving bed above the shear tray. The four-increment sampler was operated during each two-
minute period to determine the quantitative distribution of moving particles with height and to provide an estimate of the rate of movement at different force levels.

Four observers were required to operate the tunnel and obtain experimental data over the moving sand surfaces. The first operated the wind tunnel and controlled the level of wind movement. The second made a traverse of the sand laden air stream. The third read and recorded velocities during the traverse above the bed and the drag indicated by the dial of the shear measuring scale. The fourth was required to control the air intake of the sand sampler.

Insofar as possible the experiment was carried out on a routine basis. After each two-minute test period the sand bed was leveled, the shear tray re-oriented, and the bed elevation determined for a subsequent test and velocity traverse. The sand collected by the sampler was also removed and weighed after each test.

During the first few tests on erodible sands the average height and spacing of surface ripples was determined. The procedure proved too time consuming to be retained as part of the routine experiment. The height and spacing of ripples for given levels of \( \tau_0 \), therefore, was determined subsequently utilizing relationships of \( P_1 \) to \( \tau_0 \) developed from the routine experiment.

Photography to study the nature of particle movement was also carried out separately from the routine tests. Both still and motion picture photography were employed. The time exposures with still photography varied from \( \frac{1}{1000} \) to \( \frac{1}{25} \) second. High speed motion photography of moving sand particles was at an exposure rate of 3,000 frames per second. The exposure time of an individual frame was \( \frac{1}{15,000} \) second. High speed motion photography was confined to sizes
of sand larger than those used in the routine experiment. This use of relatively large sand grains was necessary due to the limits of speed of photography. Approximately two pictures must be obtained per diameter of grain motion to make possible an apparent continuous view of the action of a grain.

RESULTS

Nature of Particle Movement

Bagnold (2) has classified movement of solid material by wind into suspension, saltation, and surface creep. The materials capable of transport by suspension are those having a terminal velocity of fall less than the upward eddy currents of the surface wind. The sands used in this study were above the size suspended by the turbulence of the air stream and their movement was of an intermittent nature either on the bed or a short distance above the bed.

The spinning of a few saltating sand grains was detected in photographs by Bagnold. He states, however, that rotation does not seem to be the rule. Chepil (3) in researches with soils in Canada found in 1945 that soil grains carried in saltation rotated at speeds of 200 to 1,000 revolutions per second. He states, "Photographs show clearly that about 50 percent of the grains carried in saltation are spinning, while another 25 percent exhibit relatively indistinct rotation. It is possible, however, that more than 75 percent of the grains were rotating but that this could not be detected without greater intensity of illumination."
A majority of the results of the present experiment have been given previously by the author (25) before an International Colloquium in Algiers. The following findings reported in this section constitute a recapitulation of these materials with some modification and condensation.

Elements of typical grain paths as viewed horizontally and at right angles to the direction of flow are shown in Fig. 1, Plate IV. This photograph was obtained from a 0.2 second exposure of a quartz sand ranging in size from 0.59-0.84 mm. Inch graduations on the ruler to the right of the photograph orient the scale of the phenomenon. The drag velocity of the wind, \( \sqrt{\frac{D}{\rho}} \), driving the particles was approximately 2.1 miles per hour or 92 cm per second. Studies of this photograph show that the average angle of descent of the grains to the bed after they have gained forward momentum from the wind is within the range of 8 to 12 degrees from the horizontal. The light paths of individual grains are characterized by a dashed appearance indicative of rotation. All grains, even those rising only a few grain diameters above the bed, are rotating, although this fact cannot be discerned clearly from the original 9 by 12 cm photograph without magnification. Counts of the dashes in the grain paths are indicative of rotation up to a rate of 1,500 revolutions per second. The rate appears to vary greatly, however. A few of the grain paths at heights of 1 to 3 inches above the bed appear to be very short for the exposure time of 0.2 second. This fact led to the hypothesis that many of the particles have a component of motion at right angles to the general direction of air movement and are simply passing through the narrow beam of light which parallels the forward direction of flow. Assuming this to be the case, it is apparent that the rate of grain rotation cannot be estimated accurately from counting the number of similar reflections present in the visible paths of the traveling grains.
EXPLANATION OF PLATE IV

Fig. 1. Horizontal view of the saltation movement of quartz sand of a size range 0.59-0.84 mm. The drag velocity of the wind is 92 cm per second. Sand movement from left to right.

Fig. 2. Vertical view of the flow of sand in a plane approximately 2 inches above the bed. Sand size and wind force identical to that described for Fig. 1. Direction of sand flow is from lower to upper portion of photograph. Distance from lower to upper portion of photograph is 18 inches.
Fig. 1

Fig. 2
To test the hypothesis that the saltating grains have components of motion in the Z as well as the X and Y directions, the phenomenon was photographed from a vertical position. Figure 2 of Plate IV is a vertical view of a plane of motion 2 inches above the bed. The sand and flow conditions are identical to those described for Fig. 1. In this instance, the beam of sunlight was introduced horizontally at right angles to the forward direction of air and sand flow. It was focused at the center of the 36-inch width of the tunnel. The dispersion of light from the sand particles increases the depth of view considerably and the bed itself is visible faintly. From Fig. 2 it will be noted that paths of individual grains near the center of the tunnel depart laterally from the forward direction of motion by angles as large as 30°. The average angle of departure from the X, or forward direction of motion, is approximately 8°.

Similar photographic studies of drifting beds of sand grains, varying in diameter from 0.20 to 6.0 mm, have demonstrated that the phenomenon of rotation is common to the saltation movement. The sand grains evidently leave the bed in varying X, Y, and Z directions. This suggests that their rise from the bed is associated with and governed by the phenomena of impact once grain movement is initiated by direct wind pressure. The photograph of the motion of a large sand material of size range 2 to 6 mm is shown in Plate V. The nature of the movement is easily discerned for grains of pebble size.

Still photography does not permit an evaluation of the axes of rotation of the grains or the direction of rotation about such axes. Neither does it tell us when the rotation is gained. It seems reasonable to assume, however, that these factors are related to the impact phenomena of grains returning to the bed after deriving forward momentum from the wind.
EXPLANATION OF PLATE V

View of the movement of large size sand grains having a diameter range of 2 to 6 mm. Exposure time was approximately \(\frac{1}{200}\) second. Velocity of the air stream at a height of 3 inches was approximately 50 mph. Motion is from left to right.
High-speed motion photography of the action of saltating sand grains as they strike other grains on the bed demonstrated the following:

1. The characteristics of movement of an individual grain are governed by complex forces.

2. The angular velocity is variable and is gained upon impact with other grains either upon the bed or by collision in air.

3. The axes of rotation are variable and rotation may be in either direction.

4. The axis of rotation tends to remain constant as a given particle is translated through the air, i.e., the phenomenon is gyroscopic.

5. In general, when particles are nearly uniform in size, they do not impinge into the bed to form impact craters. Appreciable movement appears to be limited to a few particles on or near the immediate surface.

6. No distinct line of demarcation can be drawn between saltation and bed load movement of grains. All particles which move appreciably leave the immediate surface of the bed and experience rotation even though they may move only a few grain diameters.

Assume a saltating sand grain 1 to be near the end of its saltation path. Assume further that it is descending to strike a surface grain 2 in a packed position among other grains on the bed. At the instant before impact, grain 1 has kinetic energy

\[ P = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} I_1 w_1^2 \]

where \( P \) is the total kinetic energy due to the velocity of translation \( u_1 \) of a particle of given mass \( m_1 \), plus the kinetic energy associated with the rotation of the particle. The latter is proportional to the moment of inertia, \( I_1 \), of the particle about an axis passing through its center of gravity,
times the square of the angular velocity $w_1$. The vectorial quantity $\frac{1}{2} m_1 w_1^2$ may have components in the X, Y, and Z directions. In a similar way the energy $\frac{1}{2} I_1 w_1^2$ may be associated with rotation of the particle in either direction about any axis passing through its center of gravity. The ratio of angular to linear velocity, $\frac{w_1}{u_1}$, is dimensionless when $w_1$ is measured in radians per unit time and $u_1$ in number of grain diameters per unit time. For sand particles of a size larger than 0.2 mm photographs indicate that the ratio of angular velocity to the velocity associated with translation is in the neighborhood of $\frac{w_1}{u_1} = \pi$ to $\frac{\pi}{2}$. In other words, many of the saltating particles at the time of impact with the bed rotate about one revolution for each two to four diameters of translational grain movement. Immediately after impact values of $\frac{w_1}{u_1}$ often exceed $2\pi$.

For sand particles of the sizes studied, the ratio of $\frac{w_1}{u_1}$ appears to be very nearly independent of particle size and the drag force of the wind driving the sand. It is also of interest that studies of photographs do not show appreciable retardation of the angular velocity of a given particle during its saltation. This indicates that the drag of the air around the particle tending to retard spinning is of low magnitude. Assuming this to be the case, spinning of the grains has little to do with the grains' rise from the bed or their paths of saltation.

Considering the sand particles to be spherical in shape and $I_1$ to be approximated by $\frac{m_1 d^2}{10}$, where $d$ is the diameter of the particle, the ratio of the kinetic energy of rotation to that of translation may be determined. If we assume a saltating particle to have a ratio of angular to translatory velocity equal to $\frac{w}{d}$ as it strikes the bed,
Thus, the kinetic energy associated with the spinning of grains moving in wind by the saltation process would be about equal to the translational kinetic energy. Assuming the ratio of velocities to be \( \frac{v}{2d} \), the ratio of kinetic energy of rotation to that of translation would be about one-fourth. Further study will be necessary to describe the phenomenon accurately and to determine the condition associated with sand grains smaller than 0.2 mm. It is obvious that rotation of the grains will cease or be of little consequence for conditions of transport by suspension.

The momentum conservation equation for grains 1 and 2 for the instant before and after impact may be written as follows:

\[
\begin{align*}
K.E. \text{ (Rotation)} &= \frac{m_1 d^2 w_1^2}{10 m_1 u_1^2} = \frac{d^2 (w_1)^2}{10 u_1} = \frac{d^2 (\tau)^2}{10 d} = \tau^2 \\
K.E. \text{ (Translation)} &= \frac{m_1 d^2 u_1^2}{10 m_1 u_1^2} = \frac{d^2 (u_1)^2}{10 u_1} = \frac{d^2 (\tau)^2}{10 d} = \tau^2
\end{align*}
\]

where the subscript numbers refer to grains 1 and 2, \( v \) and \( V \) are the linear velocities of grains before and after impact, and \( w \) and \( W \) are the angular velocities of grains before and after impact. \( F \) represents the energy lost due to molecular and surface friction. The above equation assumes two-dimensional impact. When grain 1 strikes grain 2, a frequent occurrence is for grain 1 to rebound almost vertically from grain 2. Since the actual phenomenon involves oblique eccentric impact, little advantage is likely to accrue from solving the equation in its two-dimensional form. The coefficient of restitution, friction, angles of impact, and irregular geometry of the particles enter into the solution for values of \( V \) and \( W \).
After impact of grains 1 and 2, grain 2 collides with other grains on the bed. Since grain 2 was supported originally by its packing among surrounding grains on the bed, little if any angular momentum was imparted to it by grain 1. In some instances grain 2 will rebound from the bed through secondary impacts. Its rebound will be somewhat delayed due to the elastic nature of the bed of supporting particles. The same momentum relations given above could be used to describe this action. The delayed rise of grains from the bed after saltating grains have rebounded from them appears to be an important finding associated with high-speed motion photography. It helps to explain the process by which grains in a packed position on the bed rise to become entrained in the saltation flow.

**Velocity Distribution and Shear of the Wind Over Stable Sand Surfaces**

As presented by Rouse (17), the steady and uniform flow of fluids over rough surfaces is approximated by the general equation

$$\bar{u} = C \log \frac{y}{y_1}$$

where $\bar{u}$ is the velocity at any height $y$, and $y_1$ is a reference parameter equal to the value of $y$ at which the curve intersects the $y$-axis. The coefficient $C$ represents the slope of the curve and according to Von Kármán's (22) development is equal to $\frac{2\alpha^2}{k} \sqrt{\frac{\tau_0}{\rho}}$, where the term $\sqrt{\frac{\tau_0}{\rho}}$ is the velocity gradient or friction velocity, $\tau_0$ is the average surface drag of the wind per unit area of surface, and $\rho$ is the mass fluid density. $k$ is the universal constant for turbulent flow. Experiments by Nikuradse (16) have shown the value of $k$ to be 0.4 for clear fluids.
The roughness elements of the surface have been found by Goldstein (10) and White (23) to be within a laminar sublayer when

$$ R_c = \frac{d \sqrt{\frac{\tau_0}{\rho}}}{\nu} \leq 3.5 \text{ to } 4.0 $$

where \( d \) is their height, \( \nu \) is the kinematic viscosity of the fluid and \( R_c \) is the critical Reynolds number. The laminar layer is disrupted completely when the critical value of \( R_c \) is exceeded, and the rough boundary equation appears to hold within the region of the roughness elements comprising the surface.

The present research considers only flow and surface conditions where the critical value of \( R_c \) is exceeded. The experimental results provide the opportunity to check all factors in the rough boundary equation for surface roughness composed of several size-ranges of sand.

Values of the arbitrary fan pressure, \( P_1 \), measured in inches, the measured drag on the 11.5 square feet of the shear tray in grams, and the value of the apparent drag calculated from the velocity distribution over the shear tray are given in Table 2.

A sample plotting of the distribution of velocity over the 0.30-0.42 mm sand surface is given in Fig. 1. The family of lines drawn through the velocities obtained with height for varying wind forces are shown to converge to a value of \( y_1 \) equal to 0.0009 feet, or 0.0274 mm. The apparent shearing force on the surface is calculable from the Von Kármán equation by its use in the form

$$ \bar{u} = 5.75 \sqrt{\frac{\tau_0}{\rho} \log \frac{y}{y_1}} $$

When \( y = 30 \, y_1 \), the equation becomes

$$ u_{30y_1} = 8.5 \sqrt{\frac{\tau_0}{\rho}} \text{ or } \tau_0 = c_1 (u_{30y_1})^2 $$
Table 2. Basic data for water stabilized surfaces.

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>$P_1$</th>
<th>$\tau_o$ (Grams/11.5 sq.ft.)</th>
<th>Apparent shear (Grams/11.5 sq.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.200</td>
<td>0.23</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>0.275</td>
<td>0.41</td>
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<td></td>
<td>0.81</td>
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</tr>
<tr>
<td></td>
<td>0.855</td>
<td>45</td>
<td>51</td>
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</table>
Fig. 1. Relationship of wind velocity to height above a 0.30-0.42 mm stabilized sand surface within the turbulent boundary layer.
\( \tau_0 \), or surface drag, was read in units of grams per 11.5 square foot area of the shear measuring device, and velocities were recorded in miles per hour for the experiment. To determine the apparent shear for these dimensional units

\[
\tau_0 = 0.362 (\bar{u} y_1) \]

By reading values of \( \bar{u} y_1 \) from the lines drawn through the points of Fig. 1, \( \tau_0 \) is readily calculable. The recorded values of apparent shear tabulated in Table 2 were derived by this procedure.

The agreement between measured and calculated shear provides a check on the validity of the Von Kármán equation when the value of \( k = 0.40 \). This agreement is shown graphically in Fig. 2. Calculated and measured shear are equal for an average of the data. The standard deviation between them is \( \pm 2 \) grams. The standard error or the range within which the true value lies is \( \tau_0 (\text{measured}) = \tau_0 (\text{calculated}) \pm 0.41 \) grams.

It has been assumed by Bagnold (2) that the value of \( y_1 \) is equal approximately to \( \frac{1}{30} \) the diameter of sand grains on the bed, White (23), however, has obtained values of approximately \( \frac{1}{9} \). The results given in Table 3 were obtained in the present experiment.

Table 3. Values of \( y_1 \) and \( \frac{d}{y_1} \) for various sizes of sand.

<table>
<thead>
<tr>
<th>Average diameter of sand (d) (mm)</th>
<th>Value of ( y_1 ) (mm)</th>
<th>( \frac{d}{y_1} ) (ratio)</th>
</tr>
</thead>
<tbody>
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<td>0.20</td>
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</tr>
<tr>
<td>0.715</td>
<td>0.0437</td>
<td>14.7</td>
</tr>
</tbody>
</table>
Fig. 2. Comparison of apparent and measured shear values obtained from stabilized sand surfaces of different size ranges of sand.
A plotting of the above data is shown in Fig. 3. The value of $y_1$ apparently varies as the log of grain diameter. An average relationship is

$$y_1 = 0.81 \log \frac{d}{0.18}$$

The fact that the value of $\frac{d}{y_1}$ decreases rapidly with grain sizes above 0.2 mm may account for the variable results obtained by various investigators. A reversal from a minimum value of approximately 1 is shown to take place as the grain diameter increases beyond 0.5 mm. The projected value of $\frac{d}{y_1} = \infty$ at a grain diameter of approximately 0.18 mm is possibly indicative of the transition from rough to smooth boundary layer flow. Stabilizing the bed with a water spray has the effect of smoothing the dry sand roughness. For this reason, the value of $d = 0.18$ mm is above the size where a viscous layer develops over a dry, cohesionless sand.

### Velocity Distribution and Shear of the Wind Over Drifting Sand Surfaces

The distribution of wind velocity above a moving bed of sand has been found by Bagnold (2) to be of the type

$$\bar{u} = C \log \frac{y}{y_n} + u_n$$

where $\bar{u}$ is the velocity at any height $y$, and $y_n$ is the height at which the velocity distribution curves project to a focal point at a velocity $u_n$. As over the stable sand surfaces, $C$ is a coefficient representing the slope of the lines and has been assumed by Bagnold to be equal to $5.75 \sqrt{\frac{g}{\rho}}$.

From studies of the results of the present experiment it became apparent early that the velocity distribution curves approximated a curved shape near the bed. This result is at variance with the velocity profiles obtained by Bagnold. From studies of photographs similar to Fig. 2 of Plate IV, it was
Fig. 3. Relationship of $y_1$ and $\frac{d}{y_1}$ to average grain diameter $d$. 

$$RATIO = \frac{d}{y_1}$$
also evident that drifting sand had lateral components of movement and that a considerable portion of it was drifting beyond the 18-inch width of the sand bed. This dispersion and nonuniform distribution of drifting sand across the width of the tunnel duct obviously affected the direct measurements of \( \tau_0 \) on the 18-inch floating tray at the lower end of the sand bed. A secondary experiment was performed subsequently to clarify the phenomenon. A 16-foot length of the 0.15-0.25 mm sand was placed to occupy the full 36-inch width of the tunnel. Velocity traverses over the drifting sand were made not only in the center but at locations 3 inches from either side of the shear tray. These values were averaged for purposes of calculating shear. The procedure was repeated four times, holding the fan pressure constant, making a total of 12 traverses. The sand was leveled before each test and the measured level of shear on the tray was obtained 12 times.

Velocity traverse data from the supplementary experiment are given in Fig. 4. An average velocity distribution curve has been fitted to the data. The average of measured shear values was 24.1 grams per 11.5 square foot area of shear tray. The standard deviation of the measured value was ± 1.92 grams. Calculating shear from the straight upper portion of the velocity distribution curve, using a value of \( C = 5.75 \frac{\sqrt{\tau}}{p} \), wherein \( k \), the universal constant, is assumed to be equal to 0.4, yielded an apparent shear value of 27.4 grams per 11.5 square feet area of shear tray. The measured and apparent values of \( \tau_0 \), therefore, differ significantly. It is apparent that a value of \( k < 0.40 \) is indicated by the results of the experiment. An approximate value of \( k = 0.375 \) is in close agreement with fact and is used subsequently in calculating apparent shear for the main body of experimental data obtained for sand drifting conditions. Employment of the value of \( k = 0.375 \) modifies the value of \( C \).
Fig. 4. Average distribution of wind velocity with height above a 0.15–0.25 mm drifting sand surface, showing the range of values obtained for repeated tests at one pressure level of the fan.
in the velocity distribution equation to \( 6.13 \sqrt{\frac{\tau_0}{\rho}} \) and the equation becomes

\[
\bar{u} = 6.13 \sqrt{\frac{\tau_0}{\rho}} \log \frac{y}{y_n} + u_n
\]

Values of the pressure \( P_1 \) and experimental values of both measured and apparent shear are given for the five size-grades of drifting sand in Table 4. A plotting of all velocity distribution data from traverses above the surfaces is presented in Fig. 5. The velocity profiles over a given sand for varying wind forces do not follow the straight-line semi-logarithmic relationship for values of \( y \) below an elevation of approximately 0.05 feet, or 1.5 cm, above the average elevation of the bed. They tend to show a curved convergence, without crossing, to values of \( y \) much nearer the bed.

The projected focal point \((y_n, u_n)\) appears to bear a relationship to grain size. In Fig. 5 it has been located with values of \( y_n = 10d \) and \( u_n = 20d \), where \( y_n \) and the grain diameter \( d \) are measured in millimeters, and \( u_n \) is in miles per hour. These values with the exception of those for the largest size sand appear to fit the data quite well. The four traverses made on the 0.59-0.84 mm sand are not conclusive in defining a possible "focal point" for the velocity distribution with height. The experimental data are limited to four traverses associated with a limited range of wind force. Additional data were not secured due to the extreme difficulty of maintaining control of elevations of the shear measuring device with this relatively large size of drifting sand.

A comparison of apparent shear values calculated from the velocity distribution curves with the measured shear on the 11.5 square foot floating tray is given in Fig. 6. The force measurements for the drifting surfaces are subject to a relatively large error due to the mechanical difficulties
Table 4. Basic data for drifting sand surfaces.

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>P₁ (Inches of water)</th>
<th>τ₀ measured (Grams/11.5 sq.ft.)</th>
<th>τ₀ Apparent (Grams/11.5 sq.ft.)</th>
</tr>
</thead>
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</tr>
<tr>
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Table 4 (concl.).

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<th>d (mm)</th>
<th>( P_1 ) (Inches of water)</th>
<th>( \tau_0 ) (Grams/11.5 sq.ft.)</th>
<th>( \tau_0 ) (Grams/11.5 sq.ft.)</th>
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Fig. 5. Distribution of wind velocity with height over drifting surfaces of varying diameter showing estimated values of the projected focal point \((U_x, y_0)\).
Fig. 6. Comparison of apparent and measured shear values obtained from drifting sand surfaces composed of different size ranges of sand grains.
involved in floating a tray in the environment of moving sand. It was found that the measured horizontal components of shear averaged 76.7 percent of the values obtained indirectly from the velocity profiles. The coefficient of variation was ± 11.6 percent. This difference between apparent and measured shear appears reasonable in view of the dispersion of sand flow beyond the 18-inch limit of the bed. A few measurements of the proportion of sand traveling outside the limits of the shear measuring tray at the end of the bed showed it to be within the range of 15 to 29 percent.

During the process of sand drifting it appears that little of the direct force of the wind is expended on the bed. It appears, rather, that the energy of the wind is transmitted to the entrained sand grains above the approximate elevation of the projected focal point. The sand grains in turn transmit a portion of the energy they have gained from the wind to the bed. The velocity obtained by the grains propelled from the bed to the upper portion of the sand cloud is greater than the velocity of the wind near the bed. As these faster-moving grains descend to the bed they tend to speed up the relatively slow-moving air. This appears to be a cause of the superfluity of velocity over the straight-line projection obtained from the relationship existing at greater heights. Again it is possible that the effective density of the air stream is increased by the entrained sand.

Initiation of Particle Movement

There has been much confusion concerning the velocity or force required to initiate particle or bed movement of sand. Since nearly all graded sands have particles varying somewhat in size, shape, and density, this situation is to be expected. Bagnold (2) gives "impact" and "fluid" threshold values
for various sizes of sand. His "impact" threshold was obtained by initiating a sustained saltation movement by dropping sand on the bed near the upwind end of a tunnel. The value of adding kinetic energy to the bed to initiate particle movement and to determine a "threshold" value associated with it is not clear. Chepil (4) recognizes the spread of values that may be obtained for visual determinations of "fluid" and "impact" thresholds and uses the descriptions minimal and maximal to define various phases of the phenomenon.

In working with the relatively large beds of sand of various size grading it was apparent that the movement of particles on or above the bed was a quite variable and unsteady phenomenon near the force at which bed movement occurred. In an attempt to secure a definable value at which bed movement occurred the device of determining trend lines of force over stabilized and drifting beds was used. The arbitrary fan pressure used as an index to tunnel operation, $P_1$, was found to be directly proportional to the force $\tau_0$ over give stable or drifting surfaces, as determined for the velocity distribution over the surfaces. These basic data are contained in Table 4. Trend lines giving the relationship between $P_1$ and $\tau_0$ for the varying stable and drifting surfaces are given in Fig. 7.

The trend lines of Fig. 7 were determined by multiple regression procedures employing the method described by Ezekiel (7). The equation approximating the relationship over stable surfaces is

$$\tau_0 = 71.4 P_1 d^{-4}$$

where $\tau_0$ is in grams of force per 11.5 square feet of bed, $P_1$ is the fan pressure in inches of water, and $d$ is the average diameter of the sand in mm. The index of correlation is $R = 0.917$. 
Fig. 7. Trends of apparent shear $\tau_0$ over stable and drifting sand surfaces in relation to an arbitrary fan pressure $P_1$. The intersection of the trend lines for a given size range of sand yields the saltation threshold shear $\tau_s$. 
The relationship obtained over the drifting surfaces was

$$
\tau_0 = 175 P_1 - 54 d^{51}
$$

where the units of measurement are the same as given for the stable surfaces.

The index of correlation for the drifting surface was $R = 0.824$.

The point of intersection of the force lines for stable and drifting surfaces appears to represent the best estimate obtainable of a "saltation threshold" for grains of a given diameter. Since it was obtained from force levels where sustained drifting occurred, it would be somewhat in excess of values where a few particles drift or roll intermittently on the bed.

Values of the "saltation threshold" determined from the intersection of the force lines of Fig. 7 are shown in Fig. 8. The plotted values are in units of $\tau_s$ equivalent to pounds per square foot of bed-area and $d$ in mm. The approximate relationship is

$$
\tau_s = 7(10^{-3})d
$$

Bagnold (2) and Chepil (4) have used an experimental coefficient in a dimensionless formula to describe a threshold velocity. The expression is

$$
u_t = A \sqrt{\frac{\gamma g d}{\rho}}
$$

where $v_t$ equals the threshold velocity $\sqrt{\frac{\tau_s}{\rho}}$, $\gamma$ is the apparent density ratio $\frac{\sigma - \rho}{\rho}$ where $\sigma$ equals density of grain and $\rho$ equals density of air, and $A$ is an experimental coefficient. The value of $A$ was found by Bagnold (2) to be 0.1 for nearly uniform sand grains of diameters $> 0.2$ mm. Chepil (5) obtained values of $A$ ranging from 0.09 to 0.11 for the "maximal condition." The value of $A$ for the "saltation" threshold described in the present experiment is 0.116.
Fig. 8. Apparent shear $\tau_s$ required to initiate and sustain sand movement for a range of grain diameters exceeding 0.20 mm.
Shields' (18) value of $\infty$ in the expression

$$\infty = \frac{\tau_0}{\rho} (\nu g d)$$

is equivalent to the square of the coefficient $A$ in the formula used by Bagnold. Shields' plot of the dimensionless function $\infty$ to the Reynolds number of the flow around grains in water gave values of $\infty = \text{approximately} 0.032$ to $0.05$ in the turbulent range. A value of $A = 0.1$ is equivalent to $\infty = 0.01$. For reasons which have not been explained, the values of $\infty$ found in water are greater than those found in air. A discussion of this subject is given by Bagnold (1).

Theoretical Considerations of Particle Movement

The stability of a sand grain when laying on a bed of grains is dependent on several factors. A schematic representation of the factors involved and the nomenclature used for their description are given in Fig. 9.

The maximum shear on a grain on the bed may be expressed as

$$\tau_g = \frac{T \nu_0 \tau d^2}{\rho}$$

where $T \nu_0$ approximates the maximum force associated with velocity fluctuations common to large scale turbulence and possible flow separation around the grain. The moment of $\tau_g$ about the point of support is then

$$\tau_g \theta = \frac{T \nu_0 \tau d^2}{\rho} \left[ \frac{d}{2} \cos \theta + K d - \frac{d}{2} \right]$$

The mass of the grain may be expressed as

$$w_g = \frac{\pi d^3}{6} (\rho - \rho') g$$

The moment of $w_g$ about the point of support is

$$w_g \theta = \frac{\pi d^3}{6} (\rho - \rho') g \frac{d}{2} \sin \theta$$
\[ \tau_g = \frac{T \cdot \tau_o \cdot \pi d^2}{4} \]

\[ W_g = \frac{\tau}{G} d^3 (\rho - \rho') g \]

\[ \tau_g = \text{Shear on grain} \]

\[ W_g = \text{Mass of grain} \]

\[ \phi = \text{Angle of repose of grain} \]

\[ O = \text{Point of grain support} \]

\[ d = \text{Diameter of particle} \]

\[ K = \text{Level at which the fluid force acts as a proportion of } d \]

\[ \rho = \text{Apparent density of particle} \]

\[ \rho' = \text{Density of air} \]

\[ g = \text{Acceleration of gravity} \]

\[ \tau_o = \text{Average shear per unit area of bed as determined by direct measurement} \]

\[ \text{in accord with } \frac{\partial u}{\partial y} \]

\[ T = \text{Turbulence factor} \]

\[ N = \text{Number of particles per unit area that are effective in taking shear} \]

\[ \text{and equals } \frac{P}{\pi d^2} \text{, where } P \text{ is the proportion of the bed taking the fluid shear.} \]

Fig. 9. Schematic arrangement of forces acting on a grain on the top of a sand bed.
The critical value of fluid shear at which grain movement will occur may be found by equating I and II and solving for \( \tau_0 \), thus

\[
\tau_0 = \frac{P g d^2 (\rho - \rho') \sin \theta}{3T \left( \frac{d}{2} \cos \phi + Kd - \frac{d}{2} \right)}
\]  

Equation III is a general one in which a specific height at which the force acts has not been fixed. White (23) has shown that the lift or vertical force is quite negligible for conditions of bed movement, and that the exact location of the fluid force is dependent on the Reynolds number, \( R_g \), of the grain. Where the Reynolds number is greater than 3.5 the fluid forces acting normally on the particle are greater than the tangential. The resultant force then passes approximately through the center of gravity of the particle. For conditions of the present experiment it may be assumed that the latter condition is applicable. Substituting a value \( K = 0.5 \) in equation III yields the expression

\[
\tau_0 = \frac{2}{3} T^{-1} P g d (\rho - \rho') \tan \theta
\]  

In working with the beds of sand it appeared that the linear dimension of roughness of the surface was equivalent to \( d \) and that the spacing of the top grains taking the shear was about 3 \( d \). The value of \( P \) for this condition is approximately 0.10. The angle of repose of the loose sand beds was approximately 30°, which may be taken as the approximate value of \( \phi \). The value of \( (\rho - \rho') \) may be taken as approximately 2.65. Substitution of these values in equation IV yields the expression

\[
\tau_0 = 0.021 T^{-1} d
\]

where \( \tau_0 \) is in pounds per square foot of bed and \( d \) is in mm.

The value of \( T \) is associated with the fluctuations of force on the particle. It results from both the small scale turbulence in the wake of
the particle and the fluctuations caused by large scale eddies in the boundary layer above the bed. Both Shields (18) and White (23) have found the maximum force associated with the turbulent wake to be about twice the average. The fluctuations due to large scale turbulence have been found by White (23) to again double the force on the grain. Shields' (18) experimental results in a small rectangular flume gave the value of the maximum at 2.25 times the average. Kalinski (13) has given experimental results indicating the maximum force from large scale turbulence near the bottom of rivers to be about 3 times the average. For conditions of the present experiment it will be assumed that fluctuations in the turbulent boundary layer yield maximum values of 2.25. In other words, the maximum force on the grain due to both turbulence in the wake of the grain and large scale turbulence would be about 4.5 times the average.

When a value of $T = 4.5$ is substituted in equation IV the critical bed shear for particle movement becomes $\tau_0 = 0.0047d$. This is about $\frac{2}{3}$ of the value obtained for the "saltation" threshold through experiment. The value seems reasonable due to the fact that a few grains will roll intermittently along the bed before sustained saltation movement occurs.

In the foregoing analysis the value of $P = 0.10$ is an estimate only. It is much lower than values previously found for sand in water. It is, however, possible that the packing of grains on the bed is closer in water than in air.

Distribution of Sand Flow Above the Bed

A plot of the weight of sand collected in a two-minute period by the 0.92 inch square openings of the collection device over the 0.20 mm sand is shown in Fig. 10. The plottings in the upper half of the figure are for
Fig. 10. Plot of the weight of sand collected by the sampler at four heights for varying pressure levels of the fan.
heights \( y \) in inches above the average bed level given on the right hand vertical scale. These heights of the midpoint of the sample tubes were 0.625, 2.875, 5.625, and 9.625 inches, respectively. Approximate trend lines for 12 levels of \( P_1 \) are drawn.

An approximate power function of sand loss with height is

\[
x = \left[ \frac{a}{y + c} \right]^n
\]

where \( x \) = grams of sand collected by the 0.92 inch square sampling tube in two minutes, \( a \) is a coefficient of variation, \( n \) is the slope of the sand loss-height function, and \( y + c \) is height in inches above the bed plus a constant. The value of \( c \) required to satisfy the above equation was obtained by use of residual equations as described by Lipka (14). A value of \( c = 0.891 \) inch satisfies the equation approximately for the 0.20 mm sand. A plot of values of \( x \) at \( y + 0.891 \) is shown in the lower half of Fig. 10. It will be noted that a poor fit of the lines is obtained at the level \( y + c = 10.52 \) inches. Due to the expanded nature of the scale this involves little error upon subsequent integration of the function. It is of interest that the family of lines for different values of \( P_1 \) has a variable slope. This demonstrates that as the flow of sand increases a greater proportion of it is carried at given heights above the bed.

Values of \( n \), \( a \), and \( c \) found in the generalized formula for all sizes of sand are shown in Table 5. Values of both \( n \) and \( a \) increase with sand size,
Table 5. Basic data for drifting sand surfaces.

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<th>d</th>
<th>P₁</th>
<th>τ₀</th>
<th>n</th>
<th>a</th>
<th>c</th>
<th>q</th>
<th>Ya</th>
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Table 5. (concl.).

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<th>n</th>
<th>a</th>
<th>c</th>
<th>q</th>
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</table>

d = Average grain diameter in mm.

P$_1$ = Arbitrary fan pressure in diffuser section in inches of water.

$\tau_0$ = Average surface drag in lbs. per sq.ft. of bed area, based on a value of $k = 0.375$.

n, a, and c = values in equation $x = (\frac{a}{y + c})^n$ where $x$ = grams of sand collected per 0.92 inch square sampling tube per two minute period at height y in inches above bed.

q = Rate of sand flow in lbs. per ft. width of bed per hour.

ya = Average height of sand flow in inches.

Q = Percent of total sand flow carried below a height of 10 $y_a$. 
Rates of Sand Flow

The area-under curves similar to those of Fig. 10 will be proportional to sand loss for a vertical section equal to the width of the 0.92 inch square sampling tubes. This area, A, may be found by integration of the expression

\[ x = \left[ \frac{a}{y + c} \right]^\frac{1}{n} \]

when written in the form

\[ y = \frac{a}{x^n} - c \]

for limits between \( x = 0 \) and \( x = (\frac{a}{c})^\frac{1}{n} \). Thus,

\[ A = \int_0^{(\frac{a}{c})^\frac{1}{n}} \frac{a}{x^n} \, dx - c \int_0^{(\frac{a}{c})^\frac{1}{n}} \, dx \]

\[ A = \left[ \frac{ax^{1-n}}{1-n} - cx \right]_0^{(\frac{a}{c})^\frac{1}{n}} \]

When A is in units of inch-grams obtained directly from the plotting of data as shown in Fig. 10,

\[ q = \frac{1.875 \, A}{t} \]

where q is in units of pounds of sand flow per foot width of bed per hour and \( t \) is the time of sampling in minutes.
Rates of sand flow calculated by the method described are given for experimental levels of \( P_1 \) and \( \tau_0 \) in Table 5. They are also plotted in Fig. 11 in relation to \( \tau_0 \). The average trend of sand loss with \( \tau_0 \) is indicated by the straight lines drawn through an average of the points of Fig. 11. The equation of this family of lines, as determined by multiple regression, is

\[
q = 175(10^3) d^{3/4} \tau_0^{3/2}
\]

where \( q \) is in units of pounds per foot width of bed per hour, \( d \) is in mm, and \( \tau_0 \) is in pounds per square foot of bed area. The value of the index of correlation is 0.977.

Bagnold (2) found the rates of sand flow in a wind tunnel for grains from 0.1 to 1 mm in diameter to be

\[
q = C \left( \frac{d}{D} \right)^{1/3} \frac{d}{\rho} \left( \frac{\tau_0 \rho}{\rho} \right)^{3/2}
\]

where \( q \) represents the weight of sand moving along a lane of unit width per unit time, \( \frac{d}{D} \) is the ratio of sand of a given diameter \( d \) to the diameter of a 0.25 mm standard sand. His values of \( C \) were 1.5 for a nearly uniform sand, 1.8 for a naturally graded sand found in dunes, and 2.8 for a sand with a very wide range of grain size. Chepil (5) found that the value of \( C \) in the above formula developed by Bagnold varied from 1 to 3.1 when applied to rates of flow of soil fraction \(< 0.84 \) mm in diameter.

The present experimental results when expressed in the form of an equation similar to that of Bagnold (2) yield the expression

\[
q = C \left( \frac{d}{D} \right)^{3/4} \frac{d}{\rho} \left( \frac{\tau_0 \rho}{\rho} \right)^{3/8}
\]

The value of \( C = 0.83 \). This result indicates a considerably less rate of sand flow than has been obtained from previous experiments. This study has
Fig. 11. Rate of flow of sand in relation to grain size and apparent shear.
taken into consideration saltation movement only, and integration of the power function used to estimate rates of flow may miss some sand moving at or near the bed level. Again, it is believed that the writer's interpretation of $\tau_0$ is such that greater values are obtained for a given condition. This would have the apparent effect of making the rate of sand flow relatively low.

It is of interest that the increase in the rate of sand flow with grain diameter has been found to be greater than indicated by previous investigators. In this connection it should be remarked that the turbulent boundary layer has not been developed to the maximum height of sand movement in any research to date. In the present experiment a few grains of the 0.715 mm sand bounded to the top of the 3-foot tunnel duct. It seems obvious that the size and geometry of the tunnel duct as well as the depth of the turbulent boundary layer are factors affecting experimental results concerning rates of sand flow.

Average Height of Saltation

The height above the bed, $y_a$, in inches above and below which equal amounts of sand were transported at the end of the bed may be estimated. The area under the curves of Fig. 10 may be expressed as

$$A = a^\frac{1}{n} \left[ \frac{(y + c)^{1 - \frac{1}{n}}}{1 - \frac{1}{n}} \right] y$$

We are interested in the value of $y$ for $\frac{A}{2}$. By substitution of limits and solving for $y$

$$y_a = \text{antilog} \left\{ \left( \frac{1}{n - 1} \right) \log \left[ \frac{A(1 - \frac{1}{n})}{a^\frac{1}{n}} + c^{1 - \frac{1}{n}} \right] \right\} - c$$
Values of $y_a$ obtained from the above expression are given in Table 5. They are plotted in relation to $\tau_0$ in Fig. 12.

An equation for the average height of saltation and sand movement is

$$y_a = 7.7 \, d^{2/3} \, \tau_0^{1/4}$$

where $y_a$ is the average height in inches, $d$ is grain diameter in mm, and $\tau_0$ is in units of pounds per square foot of bed area. A dashed line indicating the value of the saltation threshold, $\tau_s$, above which the expression is applicable is also shown in Fig. 12.

Characteristics of Surface Ripples

Results of the measurement of spacing and height of surface ripples for each of the size ranges of sand are given in Figs. 13 to 17. Values of $\tau_0$ were determined for these data from the expression

$$\tau_0 = 175 \, P_1 - 54 \, d^{0.51}$$

A range of fan pressures, $P_1$, was applied and ripple spacing and height measurements continued until sufficient data were at hand to draw trend lines. An inspection of the plotted points of Figs. 13 to 17 demonstrates the fact that the error of measurement is large.

For the range of $\tau_0$ applied to the sand beds it was found that the spacing and height of the surface ripples increased uniformly with $\tau_0$ on both the 0.20 and 0.275 mm sand. When values of $\tau_0 - \tau_s$ are plotted versus spacing or height of ripples formed in the 0.20 mm sand the relationships are exponential. They are as follows:

$$\lambda = 12(\tau_0 - \tau_s)^{0.6}$$

and
Fig. 12. Average height of saltation sand movement in relation to apparent shear and sand size.
Fig. 13. Spacing and height of surface ripples formed in a 0.15-0.25 mm sand at various levels of apparent shear.
Fig. 14. Spacing and height of surface ripples formed in a 0.25-0.30 mm sand at various levels of apparent shear.
Fig. 15. Spacing and height of surface ripples formed in a 0.30-0.42 mm sand at various levels of apparent shear.
Fig. 16. Spacing and height of surface ripples formed in a 0.42-0.59 mm sand at various levels of apparent shear.
Fig. 17. Spacing and height of surface ripples formed in a 0.59-0.84 mm sand at various levels of apparent shear.
$h = 0.55(\tau_0 - \tau_s)^{0.6}$

where $l$ and $h$ are ripple spacing and height, respectively, measured in inches.

For this relationship $\frac{l}{h} = \text{constant} = 22$, approximately.

A marked change in the functional relationship of $l$ or $h$ to $\tau_0$ is evident in the results obtained with sizes of sand averaging 0.36 mm or greater. With these larger sands the rate of increase of either $l$ or $h$ is variable. The ripples continue to grow to a certain height and spacing after which they break up and become very irregular. At a somewhat greater shear the spacing again tends to become regular and the rate of increase of spacing and height becomes relatively large for increases in $\tau_0$.

The ratio of ripple spacing and height $\frac{l}{h}$ apparently tends to remain constant for a given sand regardless of the size of ripples. A plot of average values of $\frac{l}{h}$ for each average sand diameter $d$ in mm is shown in Fig. 18. While subject to considerable error the approximate relationship is as follows:

$$\frac{l}{h} = 75 d^3$$

The five sizes of sand studied have relatively small size-ranges and possible grading during the short period of tests appears to be limited. Dune sands in the field have a wider distribution of particle size and it is common to see large grains on the crests of ridges and smaller ones in the valleys between them. To determine if the results obtained with the five experimental sands are applicable to a naturally graded sand, limited tests of a natural Colorado dune sand were made in the wind tunnel. The result of application of three forces for various time-periods is shown in Fig. 19. Ripple spacing varied with average bed shear, time, and the percentage of sand material $> 0.42$ mm contained in the ridges.
Fig. 18. Ratio of ripple spacing to height for sands of varying grain diameter.
Fig. 19. Relationship between composition of ridges and ripple spacing in a Colorado dune sand for 3 wind forces for different time periods.
The size-distribution of grain diameter for the Colorado dune sand for one level of wind force is shown in the "percent finer" diagram of Fig. 20. These graphic data show the size composition of the original dune material. They also show the progressive change in the diameter of material contained in the crests of ridges or ripples. It is apparent that the condition is not one of equilibrium sand movement but of removal.

During the process of ripple growth a change in the immediate surface of the bed material over which the ripples traveled was noted. The immediate surface of the bed between ripples was a shade darker than either the ridge or original bed material. Samples of this layer were removed with a spatula and sieved. It was found that its average size composition was little different from the original bed material. It did contain, however, more extremely fine and more large material than the original bed. A typical size-distribution curve superimposed on that of the original bed is shown in Fig. 20. A study of the fine grade dark-colored material showed it to be of greater specific gravity than quartz, which is approximately 2.65. It was identified as magnetite with a specific gravity in the neighborhood of 5.00.

It is apparent that the drifting of dune sand is accompanied by separation phenomena in relation to both size and relative density of the composite material. A diagram of the phenomena of this particular experiment is given in the sketch of Fig. 21. The lee side of the ripples appears to be sheltered somewhat from the paths of the descending saltation. The largest sand is found near the crest while the front slope of the ripple contains sand of progressively lesser size as the low point of the valley is approached. The dark or black color of the valleys is caused by the exposure of what has here
Fig. 20. Size distribution of sand ripple material in bed and ridges of drifting sand.
Direction of wind and ripple travel

Point of erosion in density sheet and bed material.

Descending saltation paths

Density sheet or zone of accumulation of magnetite.

Composite bed material

Ripple material (all of which is involved in movement processes). Material is largest near crest and smallest in valley near point of removal of bed material.

Fig. 21. Sketch of ripple formation and grading of bed material under removal conditions.
been termed a density layer. In general, these heavy minerals were in the size-range < 0.149 mm.

**DISCUSSION OF THE RESULTS**

The scope of the experiment and results is such that many characteristics of sand movement by wind are covered. Any one of the nine phases of the phenomenon reported upon is worthy of detailed study far in excess of the present treatment.

The laboratory study of a natural phenomenon similar to the one carried out here represents a highly artificial condition. The characteristics of an artificial air stream influence the results. The level and state of turbulence may differ greatly between wind tunnels and from the condition of natural wind. Again, the boundary conditions resulting from a square duct vary greatly from those associated with atmospheric conditions. The results indicate that equilibrium conditions are approached over stable sand surfaces. This is not the case over the drifting surfaces as the height reached by the saltating grains exceeds the depth of the turbulent boundary layer. It would be impractical or impossible to build a tunnel wherein the field phenomenon could be approximated closely.

The studies concerning the nature of sand movement have been concentrated on rotational and impact characteristics of grains. The fact that the axis of rotation of saltating grains is variable and that rotation is in either direction is a new discovery. Again, the present research is the first in which the lateral component of grain motion has been cited.

The technique of directly measuring shear over stable sand surfaces proved successful. The value of the universal constant $k = 0.40$ in the rough
boundary equation for clear fluids was confirmed experimentally. It is of interest that the reference parameter $y_1$, or the height at which the logarithmic equation intersects the $y$ axis at a projected point of zero velocity, was found to be a logarithmic function of grain diameter. This fact tends to explain the variable results obtained by different investigators in past research. It is also apparent that fixing the surface with a water spray may result in a decrease of the effective sand roughness and be reflected in the data.

The shape of the velocity profiles obtained over drifting surfaces varies from those secured in past investigations. The shape, however, parallels the results secured in water sedimentation research by Vanoni (20). It appears that the value of the universal constant $k$ in the logarithmic equation for flow over drifting surfaces is $< 0.40$. While a value of $k = 0.375$ has been used in the analysis of data for this experiment, it is possible that the actual value varies with the rate of sand flow and for beds of grains of different diameter.

A standard method of determining a definable saltation threshold velocity or shear was developed. Past methods have been dependent upon visual observation and subject to personal error or judgment. The general level of the saltation threshold approximated that found in research by others. It is interesting that a greater bed shear is apparently required to move beds of sand grains in water than in air. It is believed that the packing of loose sand beds in air differs from the conditions common to water. The average spacing of the top grains taking the shear appears to be greater in air than in water.
Equations were developed to define the distribution of drifting sand with height above the bed. This is the first time a functional relationship to describe the phenomenon has been presented. Integration of the weight of sand-height function provides a method for estimating the total saltation sand flow. The procedure, however, may miss some sand flow characterized as bed movement. Since quantities of flow are substantially below those obtained by prior investigators this is possibly the case. It appears that the only way to determine the total movement accurately is to make direct weighings of the bed before and after movement occurs. This is virtually impossible for the large beds of sand used in the present experiment. Again, it would be desirable and possible profitable to re-run the experiment with sand occupying the entire 36-inch width of the tunnel. The lateral dispersion of drifting sand from the 18-inch experimental width was not recognized when the experiment was designed.

The average height of sand transported above the bed was found to vary with grain size and wind force. The present results are the first for which a functional relationship has been obtained.

Experimental results obtained from measuring the spacing and height of surface ripples show the ratio of spacing to height to approximate a constant for a given uniform sand. This ratio was found to vary directly with the three-fourths power of grain diameter. The spacing of ripples increased uniformly with shear for sand sizes of average diameter ≤ 0.275 mm. With larger grains the spacing did not increase at a uniform rate with shear.

Ripples or ridges formed in a naturally graded Colorado dune sand continued to grow with the time a given force was applied to the bed. The phenomenon of drifting in the wind tunnel represented a non-uniform and
unsteady condition. The process resulted in sorting and removal of the fractions of smaller size and density. The largest grains tended to accumulate near the top of ridges. Material of relatively small size and great density accumulated in a sheet upon which the ripples or ridges were superimposed.

One of the difficulties associated with the experimental procedure was the rather involved technique of measuring a large number of variables simultaneously. Any one of the several measurements of the various phenomena are subject to rather large experimental error. There is further possibility of systematic error in the basic data or its interpretation.
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A STUDY OF THE CHARACTERISTICS OF SAND MOVEMENT BY WIND

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ABSTRACT

The object of the investigation was to obtain information on the dynamic characteristics of sand movement by wind. A knowledge of the characteristics of sand movement is basic to the design of measures to cope with the wind-sand problem. The need for the scientific approach tends to become greater with time and the increasing need to control and utilize our resources to the fullest.

A review of the literature on the transport of solid materials by fluids shows that research on the mechanics of the transport of sand and soil material by wind has received little attention in relation to the study of sediment transport by water. Both wind and water phenomena may be classified under the general subject of fluid mechanics. Since the two phenomena are allied closely, many of the findings where water has been used as the fluid medium are applicable to either science.

A laboratory wind tunnel and supplementary equipment was used in the study. The sands for which movement characteristics were obtained ranged in size from 0.2 to 6.0 mm. For conditions of movement by wind these sizes are above the range where viscosity of air is a factor. The surfaces are, therefore, aerodynamically "rough."

Results were obtained on the following:

1. Nature of sand movement.
2. Velocity distribution and shear of the wind over stable sand surfaces.
3. Velocity distribution and shear of the wind over drifting sand surfaces.
5. Theoretical considerations of particle movement.
6. Distribution of sand flow above the bed.
7. Rates of sand flow.
8. Average height of saltation.

The studies concerning the nature of sand movement were concentrated on rotational and impact characteristics of grains. The fact that the axis of rotation of saltating grains is variable and that rotation is in either direction is a new discovery. Again, the present research is the first in which the lateral component of grain motion has been cited.

A technique of directly measuring shear over stable sand surfaces proved successful. The value of the universal constant $k = 0.40$ in the rough boundary equation for clear fluids was confirmed experimentally. It is of interest that the reference parameter $y_1$, or the height at which the logarithmic equation intersects the y axis at a projected point of zero velocity, was found to be a logarithmic function of grain diameter. This fact tends to explain the variable results obtained by different investigators in past research. It is also apparent that fixing the surface with a water spray may result in a decrease of the effective sand roughness and be reflected in the data.

The shape of the velocity profiles obtained over drifting surfaces varies from those secured in past investigations. The shape, however, parallels the results secured in some water sedimentation research. It appears that the value of the universal constant $k$ in the logarithmic equation for flow over drifting surfaces is $< 0.40$. While a value of $k = 0.375$ has been used in the analysis of data for this experiment, it is possible that the actual value is variable with the rate of sand flow and for beds of grains of different diameter.
A standard method of determining a definable saltation threshold velocity or shear was developed. Past methods have been dependent upon visual observation and subject to personal error or judgment. The general level of the saltation threshold approximated that found in research by others. It is interesting that a greater bed shear is apparently required to move beds of sand grains in water than in air. It is believed that the packing of loose sand beds in air differs from the conditions common to water. The average spacing of the top grains taking the shear appears to be greater in air than in water.

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