IRRIGATION SCHEDULING FOR A CORN CROP RESPONSE MODEL BY DYNAMIC PROGRAMMING

by

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CHAPTER 1

INTRODUCTION

Great Plains states are experiencing a rapidly increasing demand for agricultural water. In 1965, 2.35 million acre-feet (MAF), or 71% of the total Kansas water usage, was allocated to agricultural purposes. By the beginning of the Twenty-First century the gross agricultural water requirements are expected to increase to 10.9 MAF, 88% of the total Kansas water demand. Data collected in 1965 shows that 96% of the agricultural water allocation was for irrigation. That percentage is expected to increase in the future [14].

With the rapid growth of irrigation, the water table has dropped in many areas of Kansas. Lowered ground water tables require more energy to lift water from greater depths. Also, a declining water table renders wells unable to deliver at the desired rate. The consequence is a reduction in crop yields associated with higher production costs. Net farm incomes are much more sensitive to the effects on crop yield resulting from insufficient water than to the increased costs of maintaining well yields [24].

Also, excessive water usage leads to considerable leaching of soluble nutrients from the soil profile. Consequently demand for fertilizer increases to maintain equivalent grain yields [13].

Irrigation scheduling is a widely accepted procedure for preventing water shortages and conserving soil nutrients [10]. This procedure schedules the timely delivery of water to the crop root zone.

Biere and Perng [3] presented a corn growth model which related soil moisture, evapotranspiration, and scheduled irrigation water. This model assumed that corn growth was a single stage process throughout the plant development. The relative growth rate was expressed by an empirical function. The soil moisture to crop response had many relations.

Morgan [16] modified Biere's model and added more experimental data. He divided the single corn growth function into two: one for vegetative growth and another for ear development. Each function had its own growth rate. Furthermore, the crop response to soil moisture was functionalized. This study used a modification of the model developed by Thomas [16,17]. Changes which were incorporated included the ET model by Kanemasu et al. [12], and the use of two levels of irrigation water for each stage. The irrigation treatments were either no water or three inches of water. The irrigation schedule which optimized corn growth was developed through dynamic programming.

Irrigation scheduling is a multi-stage decision problem. Many multi-stage decision processes can be treated as combinatorial problems. Consider an N-stage process in which k decisions can be made at each stage. For each possible decision made in stage N, there are k possible decisions in stage N-1. For each possible decision in stage N-1, there are k possible decisions in stage N-2, and so on. A total of k^N possible decision paths must be considered at the beginning of the process to find the optimal solution. This is possible in principle but it becomes a prohibitively expensive approach for even the fastest computer. Dynamic programming eliminates the need for examining k^N paths at one time by taking each stage as it comes and choosing the best decision out of the k available at each stage. In other words, dynamic programming reduces a combinatorial problem involving k^N choices to a problem requiring only Nk choices, a significant reduction in problem size and difficulty.

CHAPTER 2

THE DEVELOPMENT OF A CORN CROP RESPONSE MODEL FOR AN ECONOMIC IRRIGATION SCHEDULING MODEL

2.1 INTRODUCTION

Irrigation scheduling is becoming a more important topic because of frequent irrigation water shortages and increasing costs. Many papers have been published in this field but few have considered the economic factors, or were related to the level of soil moisture depletion [10]. Because the ultimate purpose of irrigation scheduling is profit maximization we need to know the marginal revenue of yield and the marginal cost of additional water.

A fault common to most current crop models is their static character. A dynamic growth function is required to adequately describe plant growth processes in the real world. That is, the effects of water on plant growth vary in different stages.

In this chapter we shall introduce the development of a dynamic corn crop response model for an irrigation scheduling use.

2.2 SYSTEM ANALYSIS OF IRRIGATION CROP PRODUCTION

Before analyzing the whole system of plant growth processes we shall explain the importance of water.

Practically all the dry mater of higher plants originates from photosynthesis. This is a process by which plants utilize solar radiation energy, water, carbon dioxide and a catalyst, chlorophyll, to produce carbohydrates. The chemical equation for photosynthesis is:

$$CO_2 + H_2O + \text{energy} \rightarrow (CH_2O) + O_2 - 112{,}000 \text{ calories} \quad (2.1)$$

where CH_2O is a remainder in the plant [5].

Evapotranspiration (ET) is the combined evaporation from all surfaces and the transpiration of plants. Since water is consumed by the plant through ET, water must be constantly resupplied to the root zone. Hot weather increases the demand for water. This increase is due to higher level of plant transpiration and surface evaporation. Plant transpiration increases to reduce the effects of excessively high temperature. Thus, water supply is time related.

Figure 1 [3] indicates the plant growth process. The soil moisture is supplied from two sources: (1) natural precipitation and (2) irrigation. The natural precipitation generally occurs in the form of rainfall. Most of precipitation runs off the land before incorporation. Actually only a small portion of available water can be absorbed by the plant root zone. Similarly, most irrigation water is lost through surface run off or is percolated below the root zone area.

The quantity of moisture held in the root zone at a given time is determined by the net additions to soil moisture in the root zone along with existing soil moisture. A plant's ability to absorb this water is inversely related to the soil moisture tension. This concept, soil moisture tension, is the equivalent pressure required to bring soil water into hydraulic equilibrium through a permeable wall or membrane with a pool of water of the same composition. Hence the plant is less capable of absorbing water as the moisture level declines.

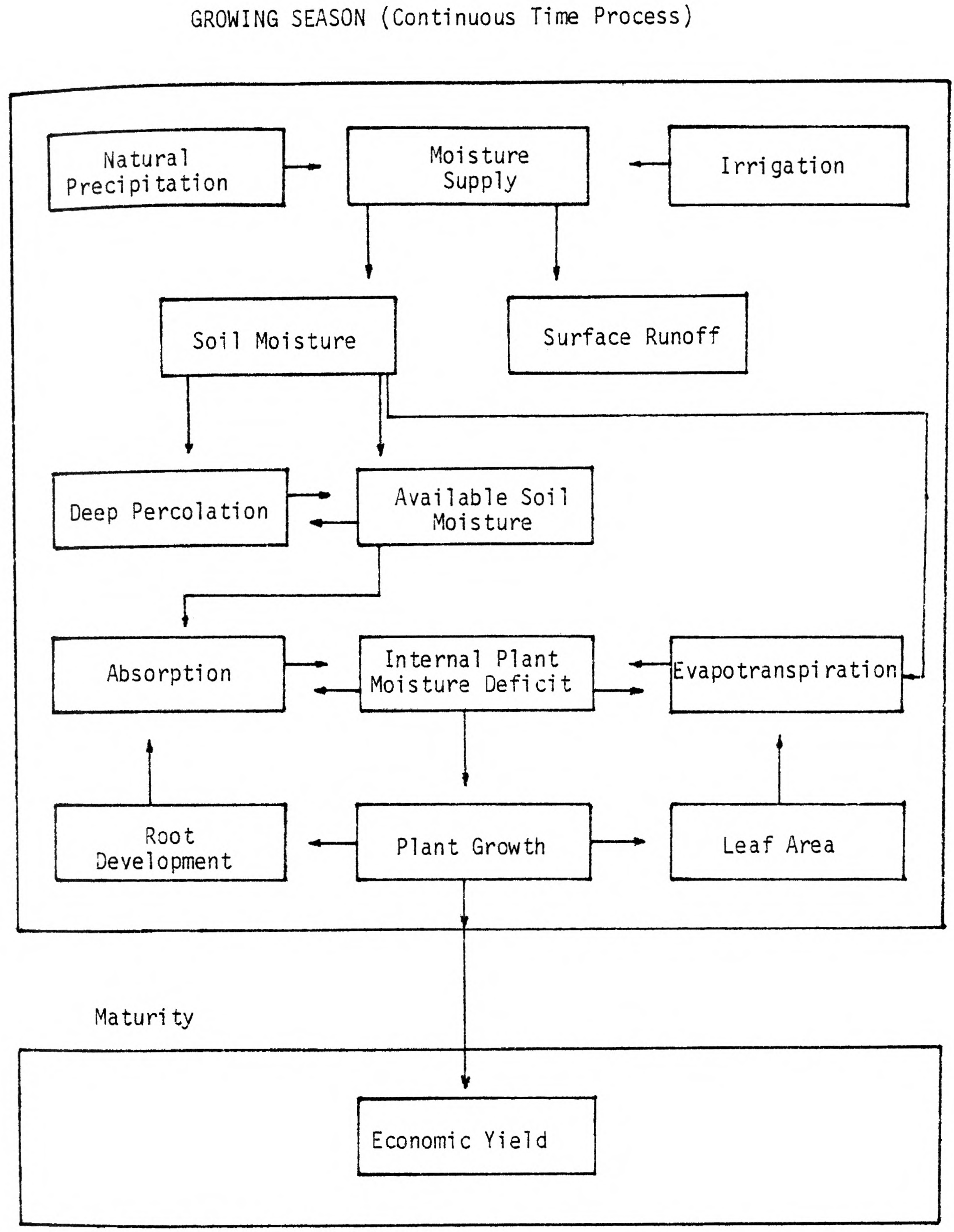


Figure 1. Flow chart for irrigated crop production.

ET is related to solar radiation, precipitation, leaf-area index, air temperature etc. While many methods have been published for estimation of ET, most require excessive measurements not supplied by the National Weather Station.

Crop response models involve the interrelationship between water, plant growth and ET. The following sections will discuss this interrelationship in detail.

2.3 ESTIMATION OF AVAILABLE SOIL MOISTURE

Available soil moisture is the amount of water held in the soil which is available to the plant for absorption. Much work has been done in soil moisture measurements. These techniques can be categorized as: measurements of moisture tension and measurements of water content. Moisture tension techniques utilize tensionmeters and electric methods. Water content measurements include oven drying methods and neutron scattering methods.

These methodologies can not be satisfactorily utilized by the farmer or the researcher because they are operationally expensive, time-consuming and require interpretation. Thus, it was necessary to develop a simple method amenable to routine field operations. The water balance technique has been successfully applied to estimation of soil moisture. The water balance equation can be stated as:

$$\begin{aligned}\text{Changes in soil moisture} &= \text{Precipitation} + \text{Irrigation} \\ &\quad - \text{Evapotranspiration} - \text{Percolation} - \text{Runoff} \qquad (2.2)\end{aligned}$$

This approach has many advantages, including low equipment costs, simplicity of measurement and reasonable accuracy of the results [5].

Using water balance technique, Biere and Perng [3] demonstrated a method of estimating soil moisture.

Here we shall explain some terminologies. The Permanent Wilting Point, is the soil moisture tension at which the plant first undergoes complete wilting without recovery in a saturated atmosphere. The Permanent Wilting Point is a useful approximation of the lower limit of availability of soil moisture in the root zone of plants. Field capacity is the moisture content of a well drained soil 2 to 3 days after saturation. Field capacity is a useful approximation of maximum soil water retention capacity.

Now we present Biere's method of estimating available soil moisture:

$$AM_t = \frac{W_t}{Z_t} = \frac{W_{t-1}-ET_{t-1}-P_{t-1}-RI_{t-1}+R_{t-1}+I_{t-1}}{Z_t} \tag{2.3}$$

where AM_t is the ratio of maximum available soil moisture in period **t**. Z_t is the maximum water capacity in the root zone in period t. W_t is the amount of available water actually held in period t. This quantity, W_t, equals that amount held in period t-1, minus the evapotranspiration, ET_{t-1}, minus the percolation, P_{t-1}, minus the runoff, RI_{t-1}, plus the rainfall, R_{t-1}, and plus the amount of irrigation water, I_{t-1}.

We assume that the percolation and runoff do not happen. We also assume the probability of rainfall is rather low for those areas in which scheduled irrigation are most benefical. After dropping P_{t-1}, RI_{t-1} and R_{t-1} from equation (2.3) we obtain

$$AM_t = \frac{W_t}{Z_t} = \frac{W_{t-1}-ET_{t-1}+I_{t-1}}{Z_t} \tag{2.4}$$

We shall use equation (2.4) to estimate the percent of available moisture in the soil profile.

2.4 THE DEVELOPMENT OF EVAPOTRANSPIRATION MODEL

Many empirical evapotranspiration (ET) models require meteorological measurements, such as vapor pressure gradients, which are not routinely measured by the National Weather Service. Therefore we need a model that requires minimum measurements, yet which will accurately estimate daily evapotranspiration rates.

Kanemasu et al. [12] developed a model requiring these minimum measurements: net radiation (or solar radiation), temperature, and Leaf-area index. The data of net radiation and temperature can be obtained from nearby weather stations. We can measure or estimate Leaf-area index (LAI) from leaf growth models [1]. Finally, the model has estimated daily ET from soybeans and sorghum in Manhattan, Kansas. Comparison of the results with lysimetric observations demonstrated that this model is simple, reliable and accurate.

Rosenthal et al. [23] used the model developed by Kanemasu et al. [12] to estimate the parameters required for a corn-crop model. The study was done at the Scandia Irrigation Experiment Field and the Evapotranspiration Research Field at Manhattan.

The model divides daily ET rates into transpiration (T) and evaporation (E_s).

By modifying the Priestly and Taylor [18] equation we estimate daily maximum evapotranspiration (ET_{max}) during predominantly nonadvective conditions.

$$ET_{max} = \alpha[S/(S+\gamma)]\ R_n \tag{2.5}$$

Where S is the slope of the saturation vapor pressure curve for a weighted average temperature $(3\ \frac{T_{max} + T_{min}}{4})$; γ is the psychrometric constant; R_n is daily net radiation; and α is a coefficient, dependent on the climate and type of corn. We chose $\alpha = 1.35$ from previous studies [12,25].

Since R_n estimates are not usually available we employed daily solar radiation data to formulate an estimate, R_n. The equations are:

$$R_n = 0.861\ R_s - 103.92 \quad \text{for LAI} < 3.0 \tag{2.6a}$$

$$R_n = 0.848\ R_s - 144.49 \quad \text{for LAI} > 3.0 \tag{2.6b}$$

$$R_n = 0.766\ R_s - 99.89 \quad \text{for LAI} > 3.0 \text{ and after blister stage.} \tag{2.6c}$$

Where R_s is daily solar radiation. LAI was determined from leaf area and plant population. Leaf area was obtained by an optical area meter and correlated with the maximum length and width of each leaf, which resulted in the equation:

$$LA \doteq 0.73 \sum_{i=1}^{n} (L_i \times W_i) \tag{2.7}$$

where LA is the total plant leaf area and n is the number of leaves per plant. L_i and W_i are the maximum length and width of each leaf. The coefficient in equation (2.7), 0.73, is identical to the results by Mckee [15].

Evaporation from the soil surface includes two stages: the constant (E_{s1}) and falling rate (E_{s2}) stages [17]. The equations are given by:

$$E_{s1} = \tau[S/(S+\gamma)]\ R_n \tag{2.8}$$

$$E_{s2} = ct^{1/2} - c(t-1)^{1/2} \tag{2.9}$$

where τ (1) is the ratio of net radiation reaching the ground (R_{ns} : R_n) and (2) is a function of LAI; c is a constant whose value depends on soil hydraulic properties and in this study c equals 3.6 mm $(\text{day})^{-1/2}$; t is the number of days from the beginning of E_{s2}.

The amount of available soil water in the root zone determines the transpiration rates. The maximum amount of available water in the 150-cm profile (Z) for Manhattan and Scandia is 335 mm and 183 mm, respectively.

Transpiration depends on LAI or the percentage of the ground shaded by the crop (cover percentage) when soil water is not limited. The following equations can be used to estimate the cover percentage.

$$\text{cover percent} = \left(\frac{LAI}{3.0}\right) \times 100\% \quad \text{for } LAI < 3.0 \tag{2.10a}$$

$$\text{cover percent} = 100\% \quad \text{for } LAI > 3.0 \tag{2.10b}$$

$$\text{cover percent} = 40\% \quad \text{for } LAI > 1.8 \text{ and after silking stage.} \tag{2.10c}$$

For less than 50% cover

$$T = \alpha_v(1-\tau)[S/(S+\gamma)]\ R_n \tag{2.11}$$

For greater than 50% cover

$$T = (\alpha-\tau)[S/(S+\gamma)]\ R_n \tag{2.12}$$

where $\alpha_v = (\alpha-0.5)\ /\ 0.5$

Tanner and Ritchie [26] determined the critical amount of available soil water affecting transpiration for a number of soils and crops to be 0.3 Z. After the soil water content has been depleted to 0.3 Z,

$$T = \frac{W}{0.3\ Z}\ \alpha_v(1-\tau)[S/(S+\gamma)]\ R_n \tag{2.13a}$$

or

$$T = \frac{W}{0.3\ Z}\ (\alpha-\tau)[S/(S+\gamma)]\ R_n \tag{2.13b}$$

where W is the actual amount of available soil water.

Actual ET is greater than ET_{max} under advective conditions [22]. An equation given by Kanemasu et al. [12] for sorghum was used to estimate the advective contribution (A):

$$A = 0.1\ T \quad \text{for } T_{max} > 33^{o}c. \tag{2.14}$$

Calculating A, T, and E_s gives:

$$ET = E_s + T + A. \tag{2.15}$$

The results from Scandia and Manhattan demonstrated the high accuracy of this model. A detailed description of the model was given by Kanemasu et al. [12] and Rosenthal et al. [23].

2.5 THE DEVELOPMENT OF THE GROWTH FUNCTION

Jensen [11] has proposed a growth function

$$Y = Y_o \prod_{t=1}^{n} \sigma_t^{\lambda_t} \tag{2.16}$$

$$\sigma_t = \ln(AM_t + 1) / \ln(101) \tag{2.17}$$

where Y is actual yield, Y_o is yield under optimum water; σ_t, which equals one at field capacity and zero at permanent wilting point. This σ_t indicates the plant's ability to uptake water when the soil moisture stress varies and λ_t is the relative growth rate in period t. A multiplicative growth function is used because the current stage of plant size is dependent on the previous stages.

The drawback to the use of this model is the paucity of information available for estimation of the λ_t's.

Biere and Perng [3] introduced another growth function related to the crop coefficient. They assumed the crop coefficient could represent relative growth and proposed:

$$X_t = \left[\frac{B(t)}{B(t-1)}\right]^{A_t} X_{t-1} \tag{2.18}$$

and

$$B(t) = \int_0^t \beta(v)\, dv \tag{2.19}$$

where β is a crop coefficient which indicates the relationship between plant size and evapotranspiration; A_t is some function of AM_t, and X_t is the actual level of plant development in period t.

Equation (2.18), the growth function, was also questioned because the results of simulating the model had a yield of 108 bushels per acre even though available soil moisture was exhausted during the last six weeks.

Morgan, Kanamasu and Biere [17] developed a corn crop model with water as the input and grain yield as the output. The relative growth rate and all other variables except water were estimated. A mid-season hybrid that silks at 66 days after emergence was used to test this model. All ordinary corn plants tend to follow the same general pattern of development. But the duration between stages and number of leaves developed may vary among different hybrids [8]. The plant growth process must be divided into two segments because the effect of water on plant growth varies in the different stages. Figure 2 [8] illustrates the development of corn at the different segments.

2.5.1 THE DEVELOPMENT OF THE VEGETATIVE FUNCTION

The period for the vegetative development is the first 60 days after emergence. During this stage, the plant builds its photosynthetic factory which will produce the grain after silking. The corn plant develops slowly at first and increases gradually as more leaves are exposed to sunlight. The leaf development is complete between 42 days and 49 days after plant emergence and the rate of growth is rapid thereafter [8].

The exponential equation was used because of its compatible shape (Figure 3) [16]. The equation is:

$$G_t = e^{r + \delta t} \tag{2.20}$$

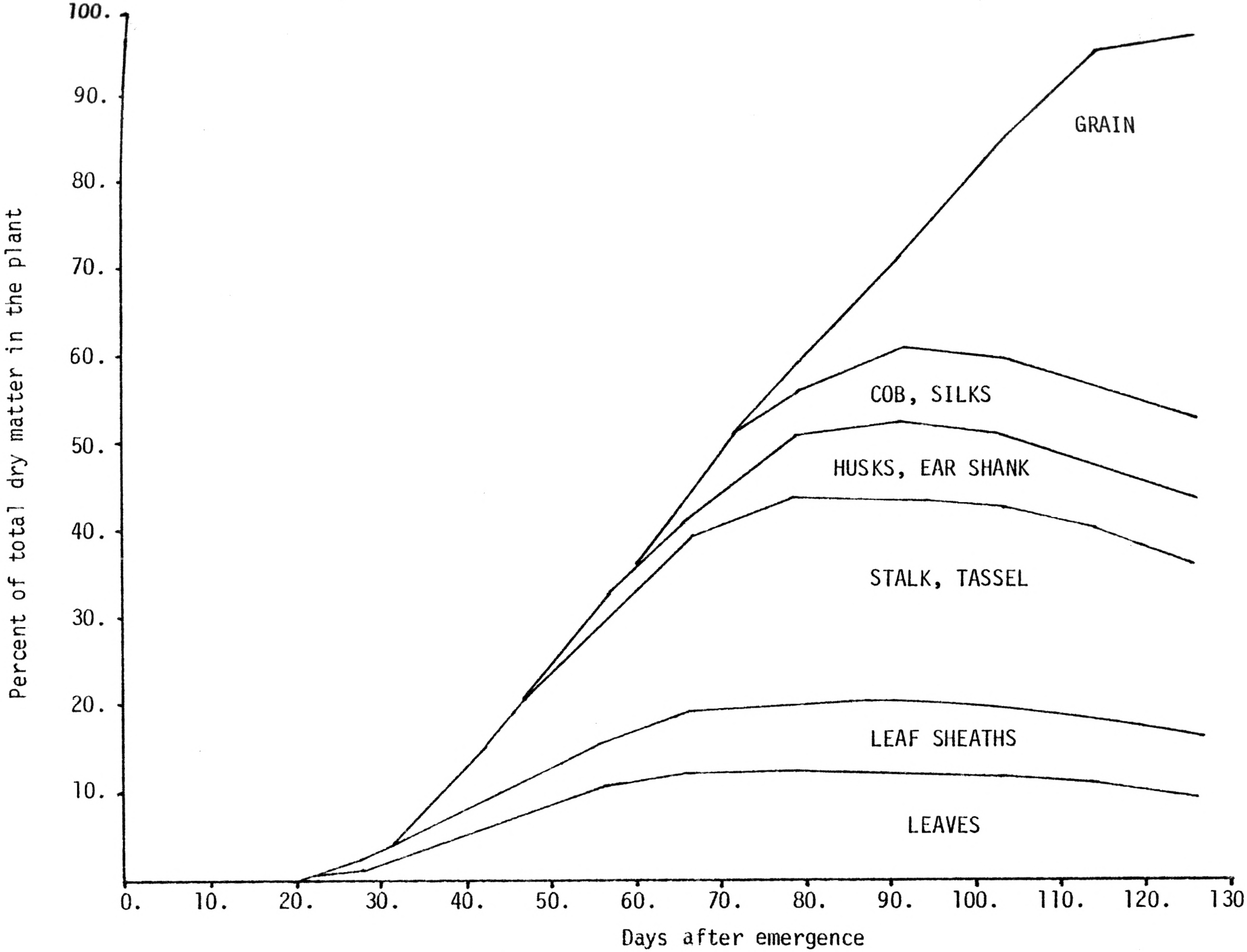


Figure 2. Dry matter accumulation in the corn plant reproduced using data from Hanway

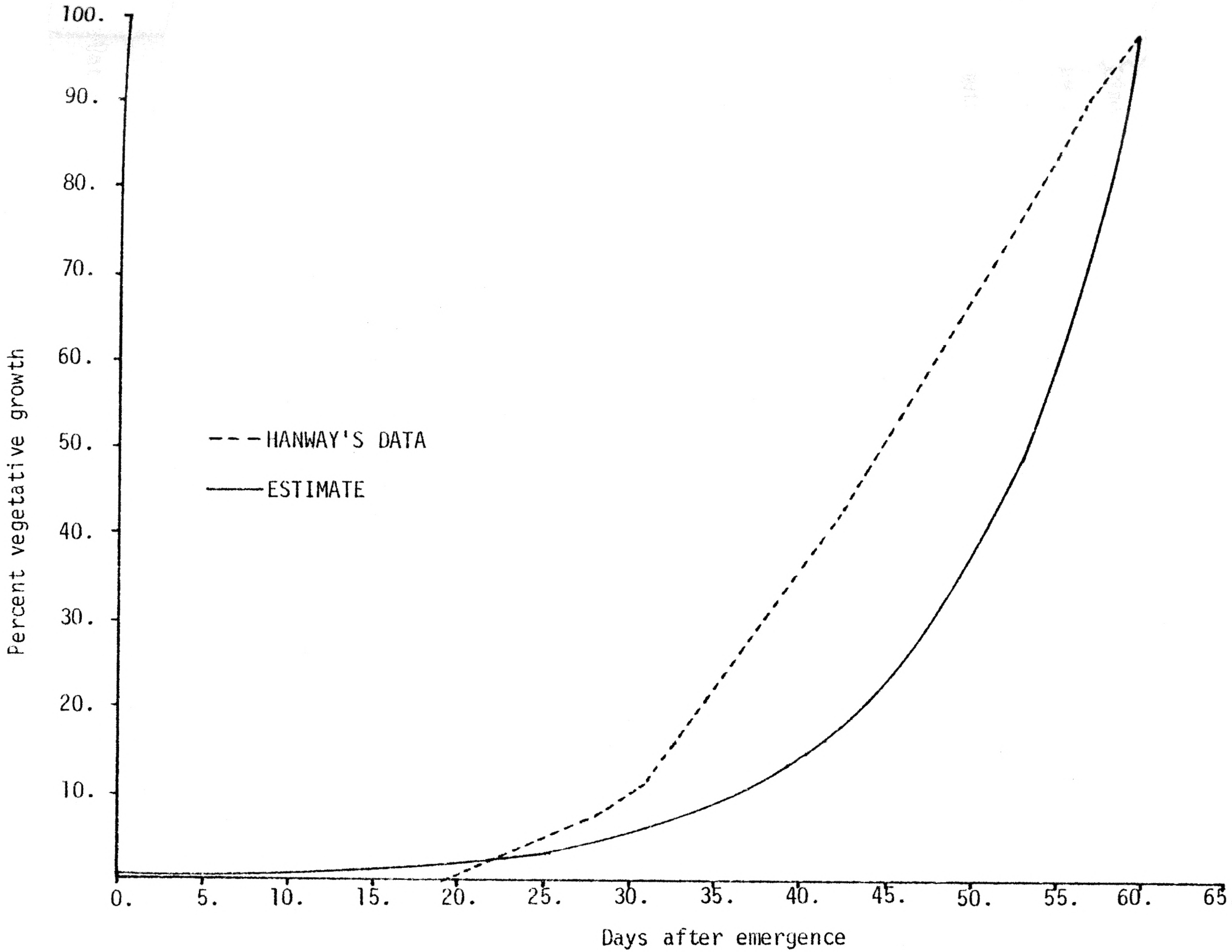


Figure 3. Vegetative growth function for corn and estimate using exponential equation and data from Hanway.

where G_t is corn growth at day t ($0 \leq t \leq 60$), r and δ are unknown constants and were estimated from Hanway's data (Figure 2) by taking the log transform of equation (2.20) and using a least-squares regression. The estimation for r and δ is -1.7 and 0.094 respectively with the regression $R^2 = 0.947$.

We employed a recursive form for equation (2.20) since the plant growth in stage t is dependent on the previous stage, t-1.

$$G_t = e^{r+\delta t} = e^{\delta} e^{r+\delta(t-1)} = e^{\delta} G_{t-1} \qquad (2.21a)$$

In this function G_0 is a very small number less than one since there is very little dry matter accumulation at emergence.

By substituting δ into equation (2.21a) we obtain:

$$G_t = e^{0.094}\ G_{t-1} \qquad (2.21b)$$

Using equation (2.21b) as the growth function, we develop the following as the growth relationship between water and vegetative for the first 60 days after emergence:

$$G_t = (e^{0.094})^{\sigma(AM_t)}\ G_{t-1} \qquad (2.22)$$

where $\sigma(AM_t)$ is defined as σ_t. The derivation of $\sigma(AM_t)$ will be given following section 2.5.2.

2.5.2 THE DEVELOPMENT OF EAR GROWTH FUNCTION

Hanway [8] found the development of the cob, silks and grain is related to the harvested yield. Therefore a combined dry matter accumulation curve (Figure 4) [16] was used to estimate the ear development function.

The function originates with the ear development (61 days after emergence) and spans to maturity (126 days after emergence). This relationship is dependent on the accumulated development throughout the vegetative stages and on the available water thereafter. Plant growth has the property of a reducing growth as the plant approaches some maturity. A modified logistics equation was used to estimate the ear development function.

The differential equation for grain develpment, H_t, assuming a logistic curve is:

$$\frac{dH_t}{dt} = \delta' H_t\left(\frac{K-H_t}{K}\right) \tag{2.23}$$

where $0 < t \leq 66$. The term $\left(\frac{K-H_t}{K}\right)$ shows the growth rate when H_t (accumulated ear development at day t) approaches K (the maximum amount of ear development attainable).

Substituting T for K and t for H into the term $\left(\frac{K-H_t}{K}\right)$ in equation (2.23) we have:

$$\frac{dH_t}{dt} = \delta' H_t\left(\frac{T-t}{T}\right) \tag{2.24}$$

where T is the day of maturity and t is the time in ear development stage.

Rewriting equation (2.24) we have:

$$\frac{dH_t}{H_t} = \delta'\left(\frac{T-t}{T}\right) dt = \delta' dt - \delta' \frac{t}{T} dt. \tag{2.25}$$

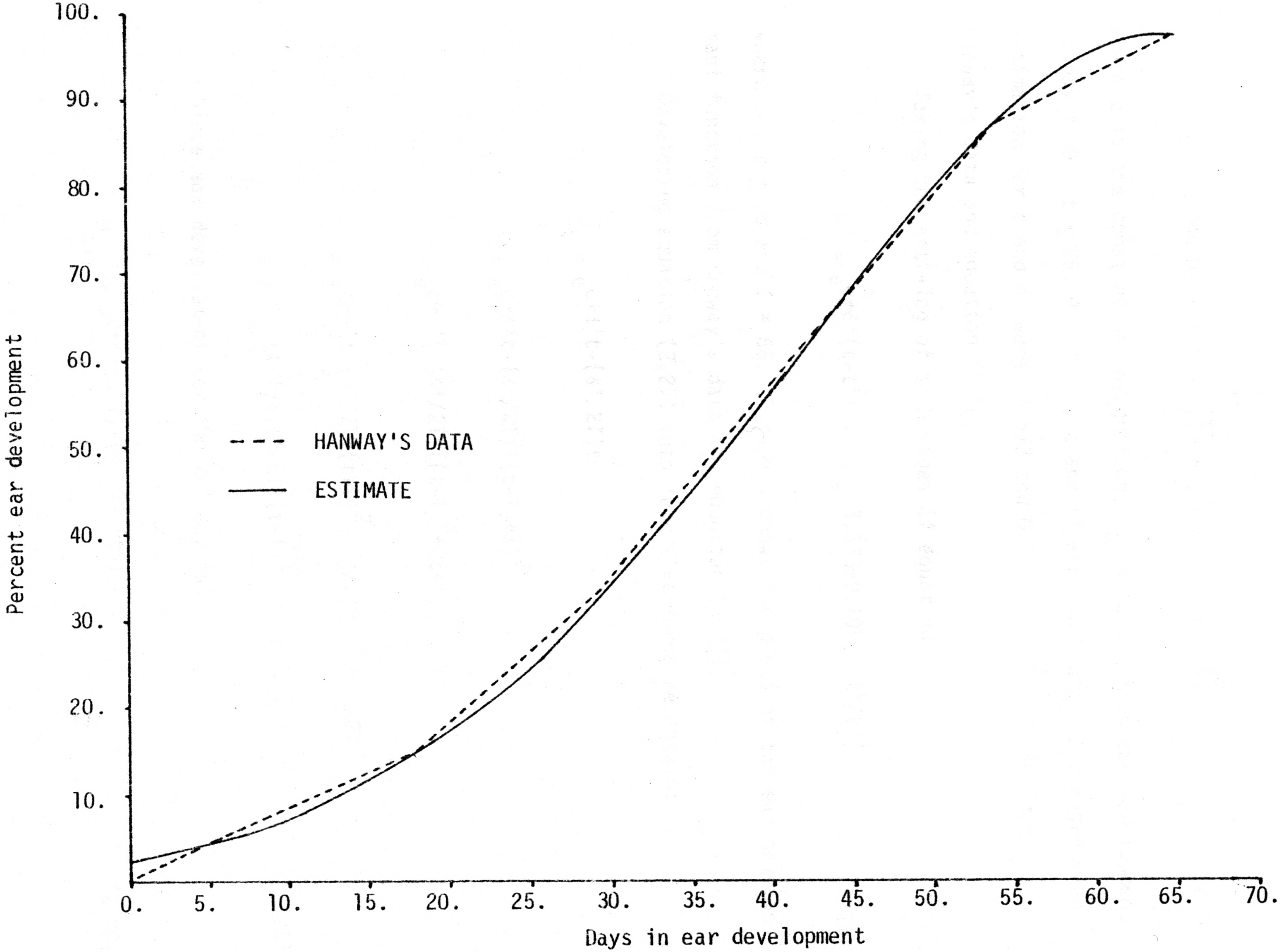


Figure 4. Ear development function for corn and estimate using modified logistics equation and Hanway's data.

Integrate both sides to obtain:

$$\log H_t = \delta' t - \frac{\delta' t^2}{2T} + c = c + \delta'(t - \frac{t^2}{2T}) \tag{2.26}$$

where c is the constant of integration, H_t is accumulated ear development at day t, $0 < t \leq 66$, $0 < H < 1$; c and δ' are constants. The regression estimation for c and δ' were -3.573 and 0.109 with $R^2 = 0.994$ from Hanway's data and equation (2.26).

Taking the anti-log of both sides of equation (2.26) we have:

$$H_t = e^{c+\delta'(t-t^2/2T)} = e^{-3.573+0.109(t-t^2/2T)} \tag{2.27}$$

where $0 < t \leq 66$ and $T = 66$. Figure 4 shows the plots of the ear development function from Hanway's data and equation (2.27).

Developing equation (2.27) into recursive forms we obtain:

$$\begin{aligned} H_t &= e^{c+\delta' t-(\delta'/2T)t^2} \\ &= e^{c+\delta' t-(\delta'/2T)((t-1)+1)^2} \\ &= e^{c+\delta' t-(\delta'/2T)((t-1)^2+2t-1)} \\ &= e^{c+\delta' t-(\delta'/2T)(t-1)^2-(\delta'/2T)(2t)+\delta'/2T} \\ &= e^{c+\delta'(t-1)-(\delta'/2T)(t-1)^2}\ e^{\delta'-\delta' t/T+\delta'/2T} \end{aligned} \tag{2.28}$$

Since ear development for the t-1 day is:

$$H_{t-1} = e^{c+\delta'(t-1)-(\delta'/2T)(t-1)^2} \tag{2.29}$$

We can substitute equation (2.29) into equation (2.28) to get:

$$H_t = H_{t-1}\, e^{\delta' - \delta' t/T + \delta'/2T}$$

$$= e^{\delta' + \delta'/2T}\, e^{-\delta' t/T}\, H_{t-1} \tag{2.30}$$

Equation (2.30) is used for the ear development function in the growth relationship for $0 < t \leq 66$,

$$H_t = [e^{\delta' + \delta'/2T}\, e^{-\delta' t/T}]^{\sigma(AM_t)}\, H_{t-1} \tag{2.31}$$

When $t = 1$, H_0 equals the total accumulated vegetative development. T, the date of maturity, is 66.

By substituting δ' and T into equation (2.31) we have:

$$H_t = [e^{0.1098}\, e^{-0.00165t}]^{\sigma(AM_t)}\, H_{t-1} \tag{2.32}$$

where $0 < t \leq 66$

We rewrite equation (2.32) to obtain:

$$H_t = [e^{0.1098}\, e^{-0.00165(t-60)}]^{\sigma(AM_t)}\, H_{t-1} \tag{2.33}$$

where $60 < t \leq 126$

Substituting X for G and H into equation (2.22) and equation (2.33) we rewrite them as:

$$X_t = (e^{0.094})^{\sigma(AM_t)}\, X_{t-1} \qquad 0 < t \leq 60 \tag{2.34}$$

$$X_t = [e^{0.1098}\ e^{-0.00165(t-60)}]^{\sigma(AM_t)}\ X_{t-1} \quad 60 < t \leq 126$$

(2.35)

2.6 THE ESTIMATION FUNCTION OF CROP'S RESPONSE TO SOIL MOISTURE

Since we measure the harvestable grain yield X_T instead of the intermediate grain yield X_t (when $t \neq T$) we rewrite equation (2.30) in product form to obtain:

$$X_T = \prod_{t=1}^{T} [U_t]^{\sigma(AM_t)}\ X_0 \tag{2.36}$$

where X_T is the percent of maximum grain yield at maturity, X_0 is plant development at the beginning of the ear development stage. The growth rate, U_t, is given by:

$$U_t = \exp[\delta'(1+1/2T - t/T)] \tag{2.37}$$

Taking the log transformation of equation (2.36) we have:

$$\log X_T = \sum_{t=1}^{T} \sigma(AM_t) \log U_t + \log X_0 \tag{2.38}$$

We shall use equation (2.38) to estimate $\sigma(AM_t)$.

The daily available soil moisture during the vegetative development was close to field capacity for all test plots and had little effect on grain yield variability. $\sigma(AM_t)$ was estimated only for the ear development stage. Estimation was restricted to this stage because the available soil moisture was varied only during ear development [16].

The functional form of $\delta(AM_t)$ was decomposed into a piecewise linear function of available soil moisture consisting of three linear segments.

$$\sigma(AM_t) = \begin{cases} a_1 + b_1 AM_t & \text{for } 0.000 \leq AM_t \leq 0.333 \\ a_2 + b_2 AM_t & \text{for } 0.333 \leq AM_t \leq 0.667 \\ a_3 + b_3 AM_t & \text{for } 0.667 \leq AM_t \leq 1.000 \end{cases}$$

where the a's and b's are to be estimated.

The crop data for estimating these coefficients were collected on test plots of the agronomy experiment fields at Manhattan, Kansas in 1974, 1975 and 1976, and at Scandia, Kansas in 1974 and 1975. Temperature and solar radiation measurements were obtained from the closest weather station [17]. The daily available soil moisture was estimated from the collected data by using a model developed by Rosenthal et al. [23].

Using spline regression to estimate equation (2.38) we obtain the following results:

$$\sigma(AM_t) = \begin{cases} 2.464 AM_t & \text{for } 0.000 \leq AM_t \leq 0.333 \\ 0.755 + 0.199 AM_t & \text{for } 0.333 \leq AM_t \leq 0.667 \\ 0.663 + 0.337\, AM_t & \text{for } 0.667 \leq AM_t \leq 1.000 \end{cases} \tag{2.39}$$

2.7 THE OBJECTIVE FUNCTION

The ultimate purpose of irrigation scheduling is to maximize net returns from irrigation. This model is only a partial analysis. It assumes that decisions concerning such variable as seeding, fertilizer costs are constants. Therefore, the objective function is:

$$\text{Maximize} \quad V = p_c X_T' - \sum_{t=1}^{T} h(I_t) \tag{2.40}$$

and

$$X_T' = f(X_T) = 140 \times X_T \tag{2.41}$$

where p_c is the price of corn received; X_T is the percent of optimum yield at maturity T. The harvestable yield, X_T', is obtained by multiplying X_T by the maximum possible yield. In this study we assume the maximum possible yield is 140 bushels/acre and the unit price, p_c, is \$2.00/bushel; $h(I_t)$ is the irrigation cost in period t; I_t is the depth of irrigation water in the field.

The equation which gives irrigation cost is:

$$h(I_t) = \$1.00 \qquad \text{when } I_t = 3'' \tag{2.42a}$$

or

$$h(I_t) = \$0 \qquad \text{when } I_t = 0 \tag{2.42b}$$

where equation (2.42a) means once we make decision to irrigate, we pump three inches of water onto the field and the associated cost is one dollar. Equation (2.42b) indicates that no irrigation existed, thus no cost was incurred.

2.8 SUMMARY

Now we summarize the whole model

$$\text{Maximize} \quad V = p_c X_T' - \sum_{t=1}^{T} h(I_t) \tag{2.40}$$

where

$$h(I_t) = \$1.00 \quad \text{(irrigate)} \qquad \text{when } I_t = 3'' \tag{2.42a}$$

$$h(I_t) = \$0 \qquad \text{(do not irrigate)} \quad \text{when } I_t = 0'' \tag{2.42b}$$

and

$$X'_T = f(X_T) = 140 \times X_T \qquad T = 126 \tag{2.41}$$

P_c = unit price of per bushel of grain yield

X_T = percent of maximum yield at maturity T

subject to

$$X_t = (e^{0.094})^{\sigma(AM_t)} X_{t-1} \qquad t = 1,2,\ldots,60 \tag{2.34}$$

$$X_t = (e^{0.1098}\, e^{-0.00165(t-60)})^{\sigma(AM_t)} X_{t-1} \qquad t = 61,62,\ldots,126 \tag{2.35}$$

$$AM_t = \frac{W_t}{Z_t} = \frac{W_{t-1} - ET_{t-1} + I_{t-1}}{Z_t} \tag{2.4}$$

$$ET_t = E_{St} + T_t + A_t \tag{2.15}$$

$$\sigma(AM_t) = \begin{cases} 2.464AM_t & \text{for } 0.000 \le AM_t \le 0.333 \\ 0.755 + 0.199AM_t & \text{for } 0.333 \le AM_t \le 0.667 \\ 0.663 + 0.337AM_t & \text{for } 0.667 \le AM_t \le 1.000 \end{cases} \tag{2.39}$$

2.9 DISCUSSION

The current corn response model has several advantages. (1) It considers the dynamics of the continuous growth process, yielding a good approximation to corn growth in the real world. (2) It needs minimum meteorological measurements which can be easily obtained from nearby National Weather Station or field site. (3) It is simple, reliable and accurate.

CHAPTER 3

FORMULATING THE CORN RESPONSE MODEL FOR ECONOMIC IRRIGATION SCHEDULING BY DYNAMIC PROGRAMMING

3.1 INTRODUCTION

Dynamic Programming was developed by Bellman [2] and has been applied extensively to many fields, such as production and Inventory problems, control problems in the field of chemical engineering etc.

Dynamic Programming is concerned with the general area of multi-stage decision processes. There are many problems that can be efficiently treated as multi-stage problems in the real world. The decisions of how to spend one's monthly income to optimize life-style between paydays is a common multi-stage decision process.

The ultimate purpose of a multi-stage decision process is to make a decision at each stage in order to obtain an optimal decision over the entire process.

Bellman's Principle of Optimality is a cornerstone of dynamic programming. It states "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision". Using the Principle of Optimality guarantees that the decision made at each stage is the best decision in light of the whole process.

3.2 DYNAMIC PROGRAMMING; FORMULATION OF THE IRRIGATION SCHEDULING PROBLEM

Equation (2.40) of the objective function shows that irrigation scheduling is a terminal problem. We divide the whole growth process into 126 days, each a day long. The crop growth is used as the state variable. The amount of water applied in a field is the decision variable. Each stage has two decisions which can be made.

The Principle of Optimality of dynamic programming allows us to decompose our model into a series of recursive equations starting from the end of the process. We break the time interval (0,T) into N equal increments of Δ duration so $N\Delta = T$. Time is counted forward so that T = 0 represents the initial time of plant development. The stages of the process are counted backward so that N = 1 refers to one stage remaining, and N = N refers to the beginning of the process, as given in Figure 5.

The initial condition is converted to:

$$X_{N+1} = c. \tag{3.5}$$

Let us define:

$g_N(c)$ = the maximum return function over the N remaining stages of the process, starting in state c, subject to equation (3.5) and using an optimal policy.

The optimal policies are functionalized by:

$$g_N(C) = \max_{I_N} \{g_{N-1}(X_N) - h(I_N)\} \tag{3.6}$$

$$g_1(C) = \max_{I_1} \{p_c x 140 x X_1 - h(I_1)\}; \tag{3.7}$$

where $N \geq 2$; X_1 is the precent of maximum grain yield at stage 1; $h(X_1)$ and $h(X_N)$ are the irrigation costs at the first stage and Nth stage. Return at each stage is obtained by calculating gross revenue from the previous stages and substracting the irrigation cost from the current stage.

3.3 DISCUSSION

Dynamic programming can bring about a tremendous reduction in computational difficulty and problem size. If the state variables exceed three the basic dynamic programming will encounter dimensionality. Consequently, we must use another approach, such as state increment dynamic programming to find the solution.

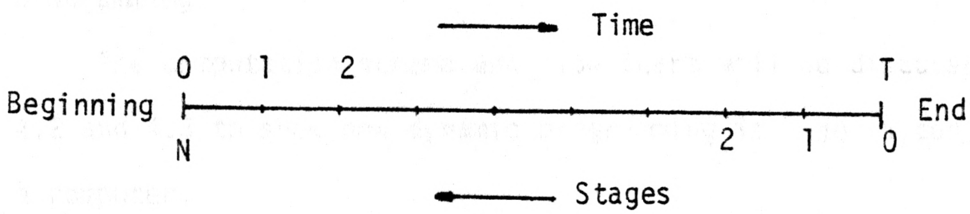


Figure 5. The backward process.

distinct from these grid points will be obtained by linear interpolation. The equation is:

if

$$k\Delta < c < (k+1)\Delta$$

we have

$$g_N(c) = g_N(k\Delta) + (c-k\Delta)[g_N(k+1)\Delta - g_N(k\Delta)]/\Delta \qquad (4.2)$$

The maximization of equation (3.6) and (3.7) is performed by a direct enumeration of cases, and a comparison of values without reliance upon calculus.

Let us discuss the procedures in greater detail. Since this is a terminal problem the formulation of the objective function at the first stage is different from the subsequent stages. For the one-stage process, the optimal policy is determined for the single decision variable, I_1, by the solution of

$$g_1(c) = \max_{I_1} \{P_c \times 140 \times X_1 - h(I_1)\} \qquad (4.3)$$

where c is the ratio of maximum grain yield is bounded by 0 and 1; p_c equals \$2 per bushel; $h(I_1)$ is the irrigation cost in stage 1 and its value can be obtained from equations (2.42a) and (2.42b). Applying equation (2.39) to the Raney and Kanemasu [19] data on soil moisture for the irrigation and no-irrigation conditions we were able to obtain the value of $\sigma(AM_1)$. Substituting $\sigma(AM_1)$ into equation (4.7) we obtain the value of X_1. A better return value is chosen from the comparison of the irrigation and no-irrigation cases and is stored in a computer for subsequent stage use.

For the two-stage process, the optimal policy is obtained by:

$$g_2(c) = \max_{I_2} \{g_1(X_2) - h(I_2)\} \tag{4.4}$$

where $g_1(X_2)$ can be calculated from the first stage table previously in computer. The calculation for all other factors is the same as in Stage 1.

For the j-stage process, the recursive functional equation is:

$$g_j(c) = \max_{I_j} \{g_{j-1}(X_j) - h(I_j)\} \tag{4.5}$$

Now, if the optimal policy including the stages, N-1, N-2, ..., and 1, is known, then stage N can be optimized by solving the maximum problem for the single decision variable I_N. That is,

$$g_N(c) = \max_{I_N} \{g_{N-1}(X_N) - h(I_N)\} \tag{4.6}$$

It should be noted that there are two different growth periods in the entire process we must use a particular transformation equation for a specific period, i.e.,

$$X_n = [e^{0.1098}\, e^{-0.00165(67-n)}]^{\sigma(AM_n)}\, C \tag{4.7}$$

where $1 \leq n \leq 66$

and

$$X_n = (e^{0.094})^{\sigma(AM_n)}\, C \tag{4.8}$$

where $67 \leq n \leq 126$

Now the optimal policy has been found. We use the direct enumeration from the beginning of growth development to the end associated with the optimal policy to obtain the final grain yield. The following transformation equations can be used in this calculation:

$$X_n = (e^{0.094})^{\sigma(AM_n)} X_{n-1}$$

where $1 \leq n \leq 60$

and

$$X_n = [e^{0.1098} e^{-0.00165(n-60)}]^{\sigma(AM_n)} X_{n-1}$$

where

$61 \leq n \leq 126$

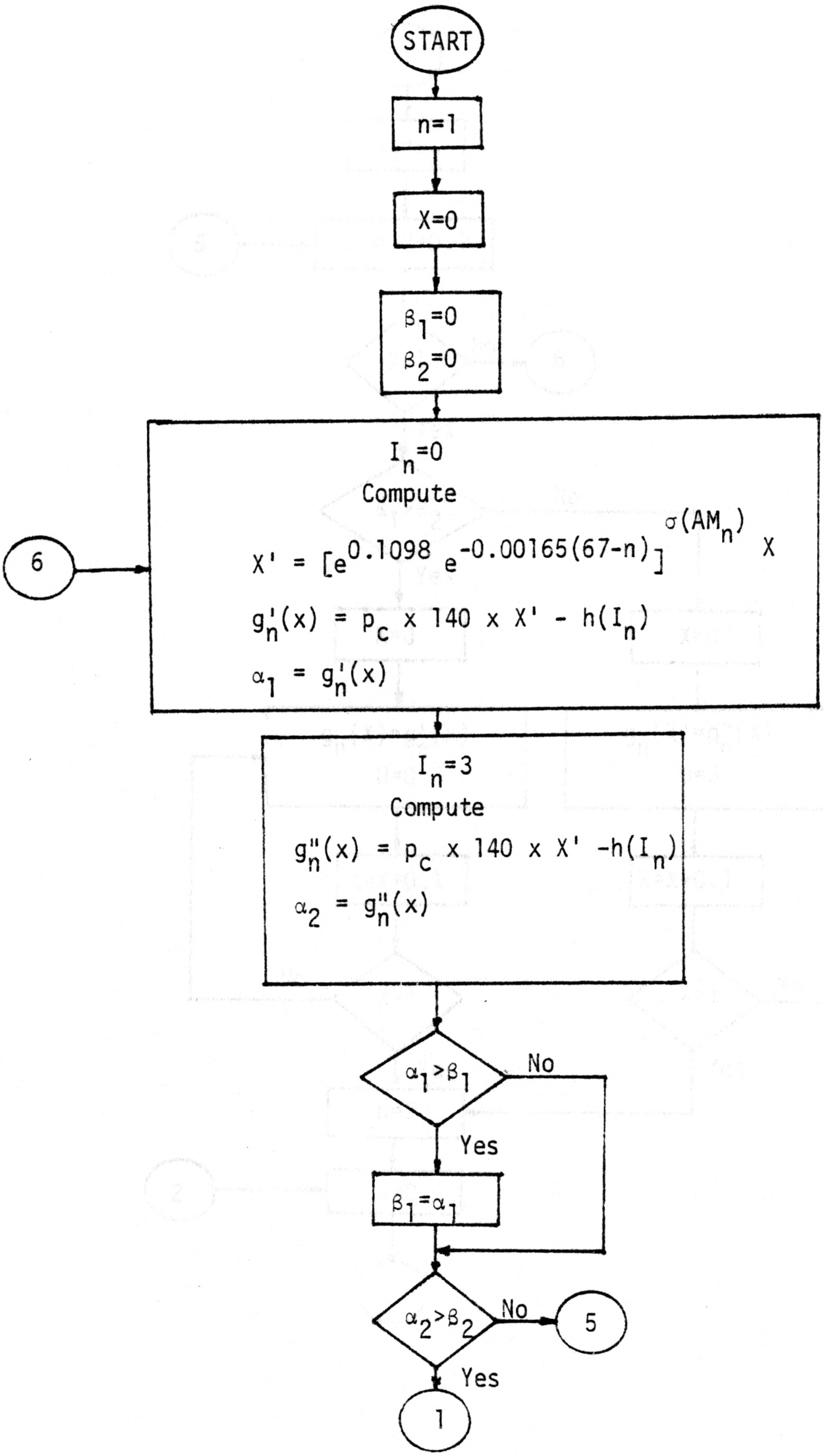
4.3 FLOW CHART ANALYSIS

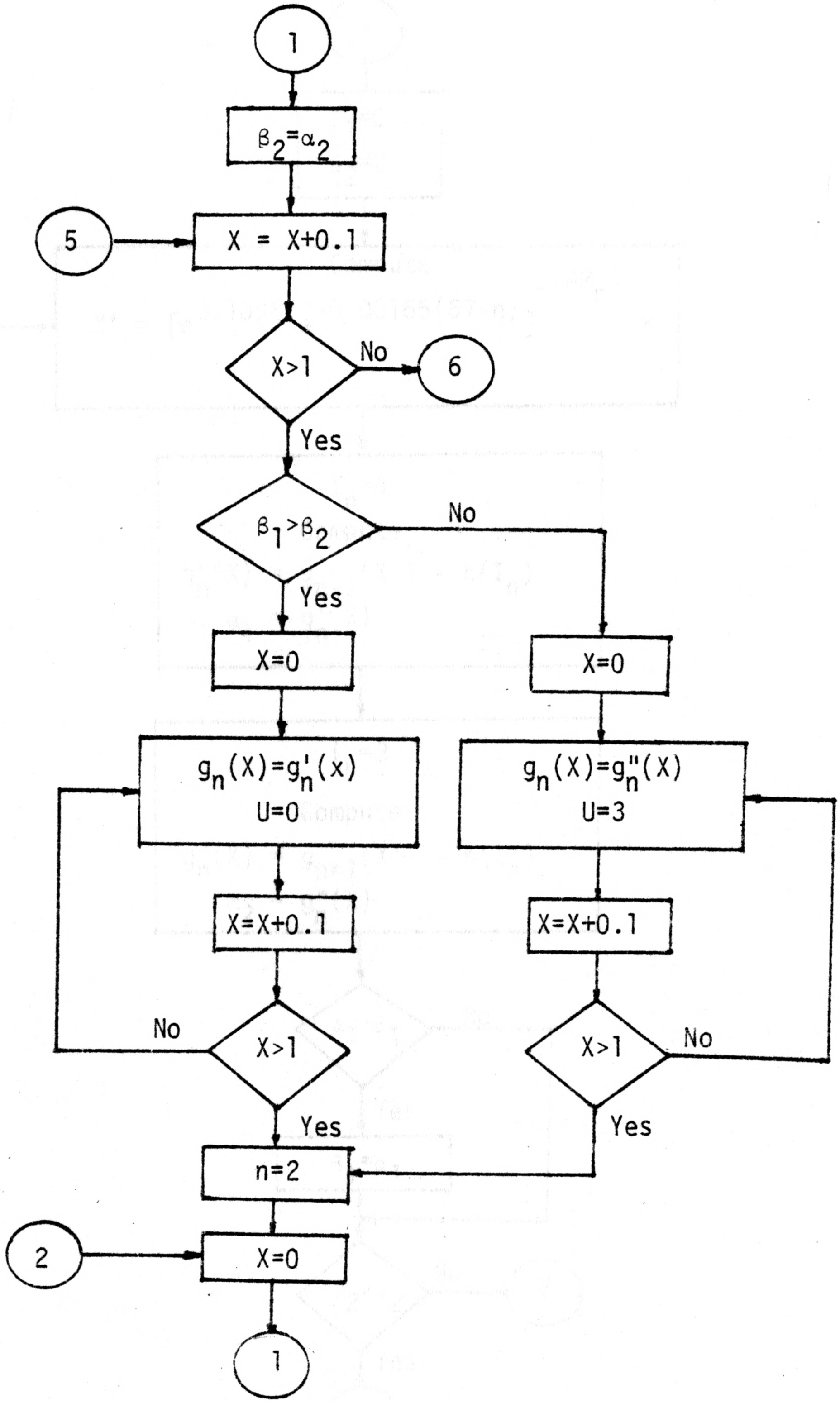
Figure 6 encompasses the whole computational procedure as a flow chart. The explanations are summarized as follows:

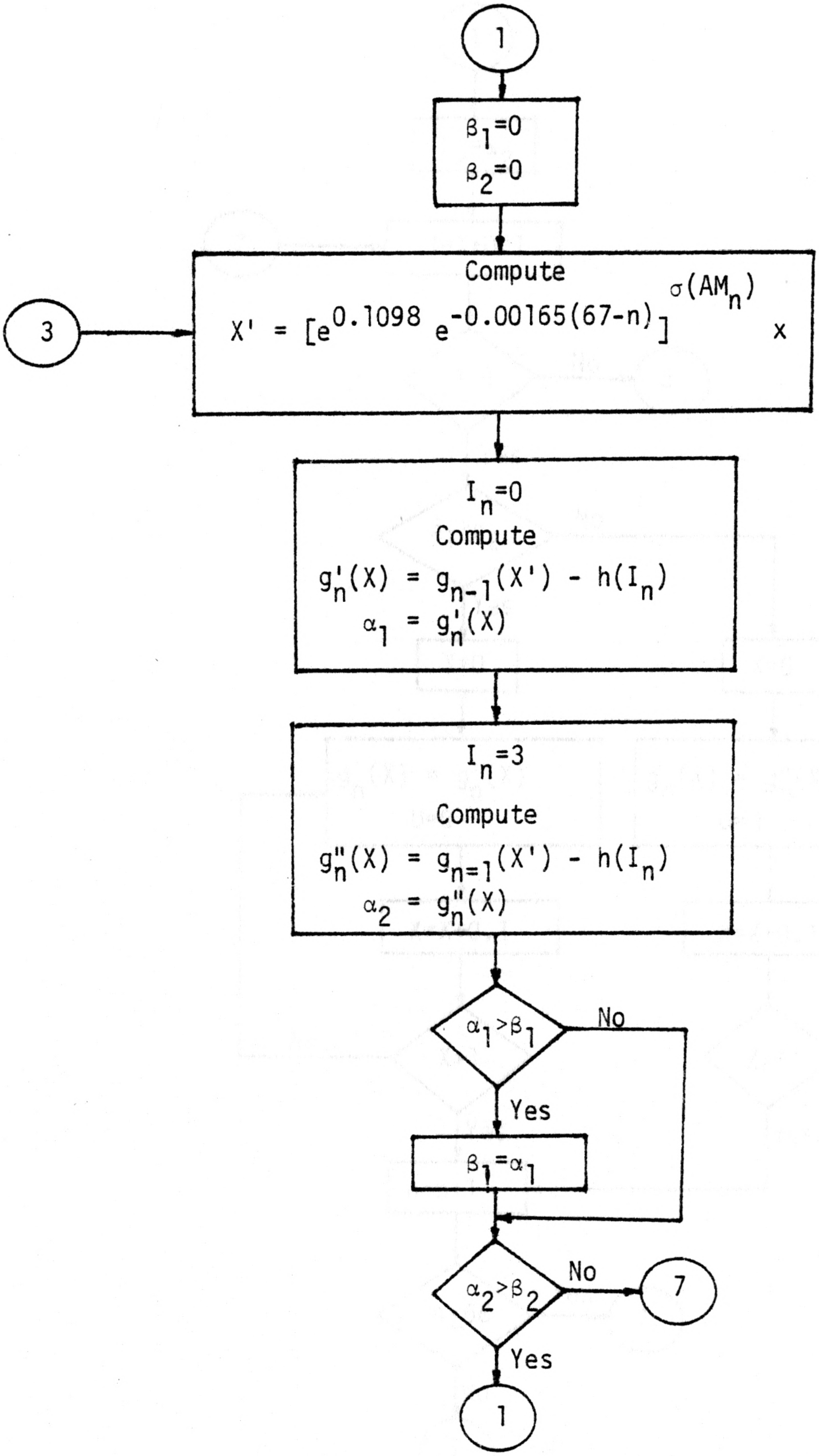
Step 1. The index n will indicate the number of activities that are under consideration. Initially, only single activity problems are dealt with. As the calculation proceeds the index n will be increased.

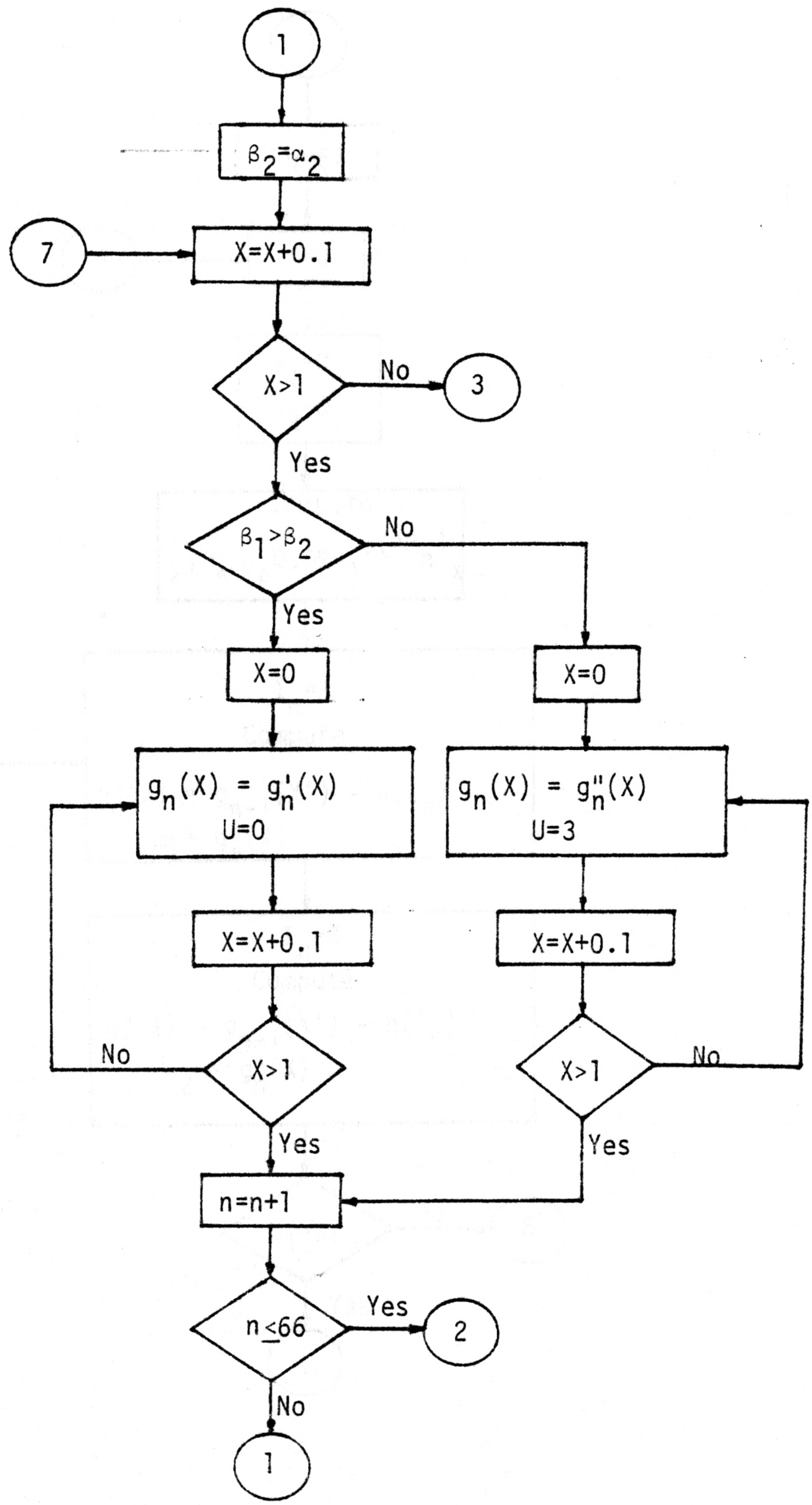
Step 2. A table of values representing the function $g_1(x)$ at grid points will be computed. X = 0 is the first argument for which $g_1(x)$ is derived. After computing $g_1(0)$, $g_1(\Delta)$, and then $g_1(2\Delta)$, and so on the values are stored until the table is completed.

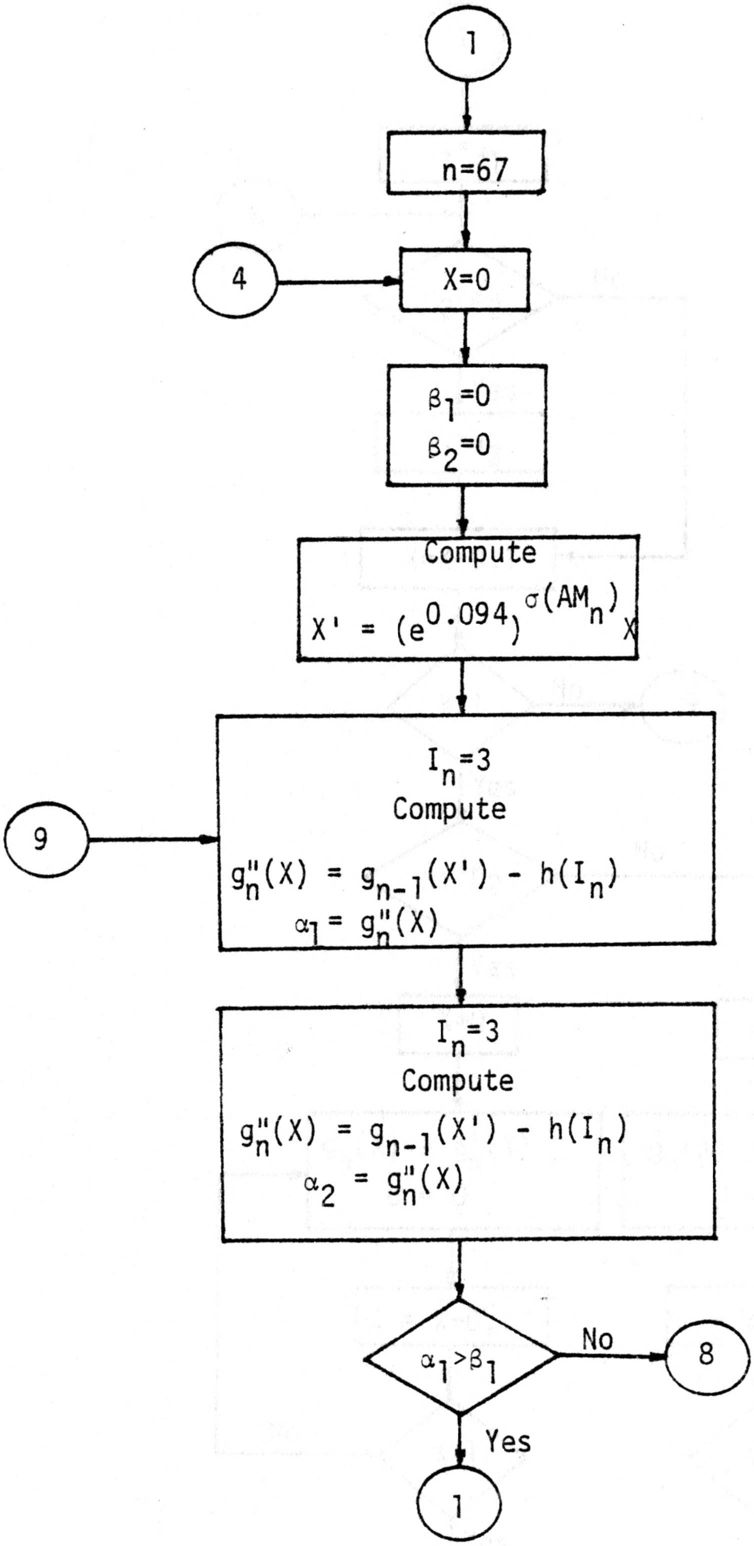
Step 3. The internal working locations β_1 and β_2 will contain the maximum returns of no irrigation and irrigation cases to the present time.

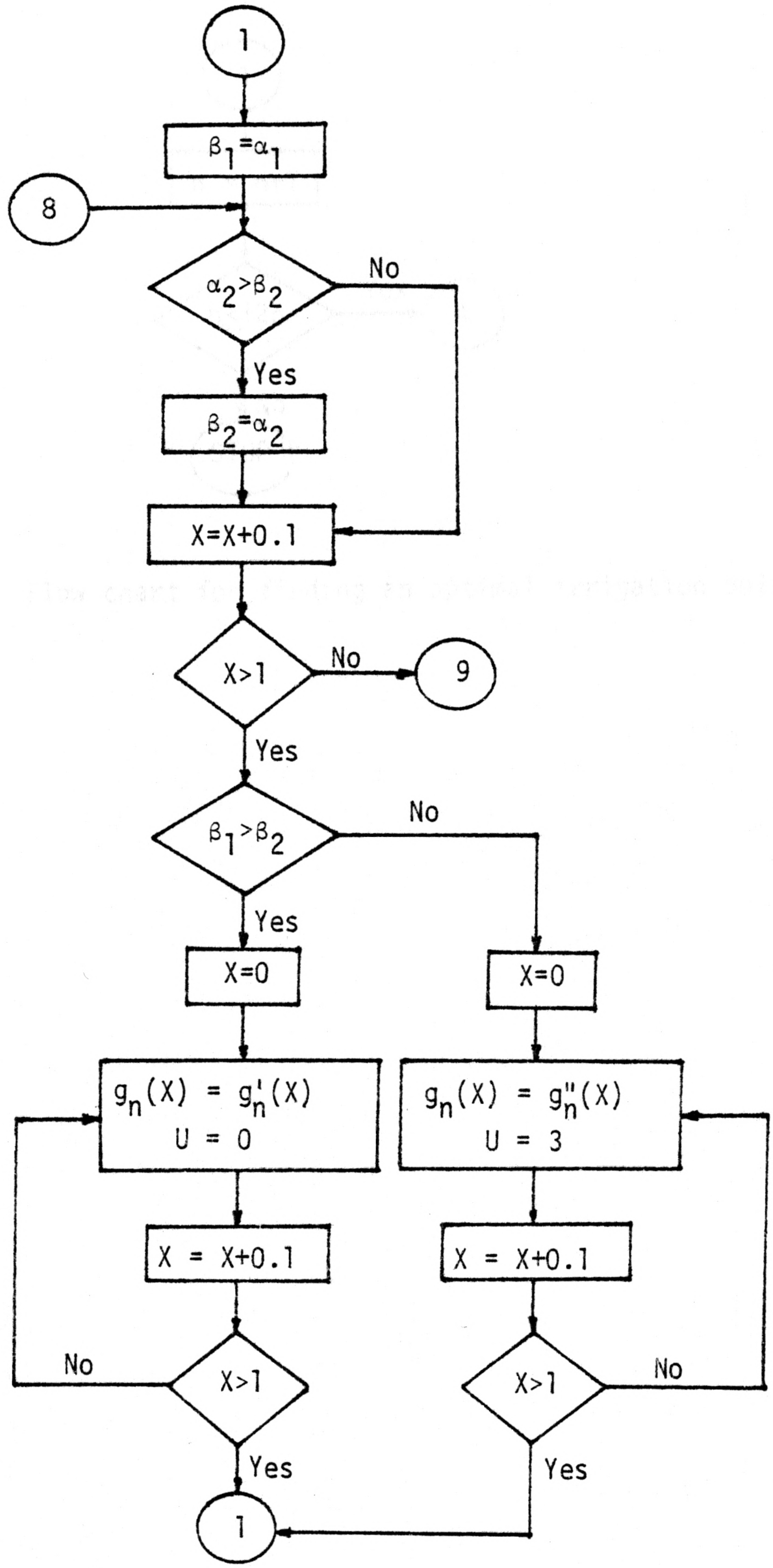

START
$n=1$
$X=0$
$\beta_1=0$
$\beta_2=0$
$I_n=0$
Compute
$X' = [e^{0.1098}\, e^{-0.00165(67-n)}]^{\sigma(AM_n)}\, X$
$g_n'(x) = p_c \times 140 \times X' - h(I_n)$
$\alpha_1 = g_n'(x)$
6
$I_n=3$
Compute
$g_n''(x) = p_c \times 140 \times X' - h(I_n)$
$\alpha_2 = g_n''(x)$
$\alpha_1 > \beta_1$
No
Yes
$\beta_1 = \alpha_1$
$\alpha_2 > \beta_2$
No
5
Yes
1


1
β2=α2
5
X = X+0.1
X>1
No
6
Yes
β1>β2
No
Yes
X=0
X=0
gn(X)=g'n(x)
U=0
gn(X)=g"n(X)
U=3
X=X+0.1
X=X+0.1
No
X>1
X>1
No
Yes
Yes
n=2
2
X=0
1


1
$\beta_1=0$
$\beta_2=0$
Compute
$X' = [e^{0.1098}\ e^{-0.00165(67-n)}]^{\sigma(AM_n)}\ x$
3
$I_n=0$
Compute
$g_n'(X) = g_{n-1}(X') - h(I_n)$
$\alpha_1 = g_n'(X)$
$I_n=3$
Compute
$g_n''(X) = g_{n=1}(X') - h(I_n)$
$\alpha_2 = g_n''(X)$
$\alpha_1>\beta_1$
No
Yes
$\beta_1=\alpha_1$
$\alpha_2>\beta_2$
No
7
Yes
1


1
$\beta_2=\alpha_2$
7
X=X+0.1
X>1
No
3
Yes
$\beta_1>\beta_2$
No
Yes
X=0
X=0
$g_n(X) = g_n'(X)$
U=0
$g_n(X) = g_n''(X)$
U=3
X=X+0.1
X=X+0.1
No
X>1
X>1
No
Yes
Yes
n=n+1
$n\leq 66$
Yes
2
No
1


1
n=67
4
X=0
β1=0
β2=0
Compute
X' = (e^0.094)^σ(AMn) X
9
In=3
Compute
g"n(X) = gn-1(X') - h(In)
α1 = g"n(X)
In=3
Compute
g"n(X) = gn-1(X') - h(In)
α2 = g"n(X)
α1>β1
No
8
Yes
1


1
$\beta_1=\alpha_1$
8
$\alpha_2>\beta_2$
No
Yes
$\beta_2=\alpha_2$
X=X+0.1
X>1
No
9
Yes
$\beta_1>\beta_2$
No
Yes
X=0
X=0
$g_n(X) = g_n'(X)$
U = 0
$g_n(X) = g_n''(X)$
U = 3
X = X+0.1
X = X+0.1
No
X>1
X>1
No
Yes
Yes
1

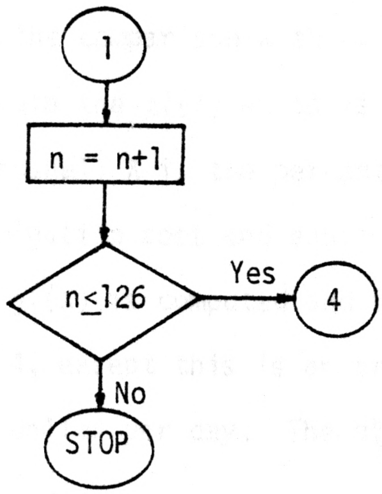


Figure 6. Flow chart for finding an optimal irrigation policy.

Assigning these cells an initial value of zero new maximum returns for two cases will result from the comparison with β_1 and β_2.

Step 4. We shall compute the $g_1'(X)$ which is the no irrigation case, where P_c is the price per bushel; X is the percent of optimum grain yield and $0 \leq X \leq 1$; $h(I_1)$ is irrigation cost and equals zero in the no irrigation case. The value of $g_1'(X)$ is computed and stored in location α_1.

Step 5. Same as step 4, except this is an irrigation case, and the irrigation cost equals one dollar per day. The $g_1''(X)$ is computed and stored in location α_2.

Step 6. Compare the number in location α_1 with the number in cell β_1. If $\alpha_1 > \beta_1$, go to next step; if it is not, go to step 8.

Step 7. Replace the contents of cell β_1 by the greater return that has just been stored in cell α_1.

Step 8. Compare the number in location α_2 with the number in cell β_2. If $\alpha_2 > \beta_2$, go to step 9; otherwise, perform step 10.

Step 9. Replace the contents of cell β_2 by the greater return that has been stored in cell α_2.

Step 10. Test the X which is larger than the previous X by Δ.

Step 11. If X exceeds its upperbound perform the next step; otherwise, go back to step 3 and proceed.

Step 12. Compare the number in cell β_1, the best return from the non-irrigation case for this stage, with the number in cell β_2, the best return from the irrigation case for this stage. If $\beta_1 > \beta_2$, go to next step; otherwise, go to step 17.

Step 13 to Step 16. This is a block transfer. We transfer the number in location $g_1'(X)$ into location $g_1(X)$ and store the decision yielding this return, U.

Step 17 to step 20. We transfer the number in location $g_1''(X)$ into location $g_1(X)$ and store the decision yielding this return, U.

From Step 21 to Step 44, the computation procedure from the 2nd stage to the 66th stage. This computation procedure is the same as the first stage, except that the return function is changed because of the character of terminal problem.

Step 45 to the final step, these routines are the computation procedure from the 67th stage to the 126th stage. It should be noted only the transformation equation differs from the previous computation procedure (Step 21 ~ Step 44).

To this point, we have obtained the optimal policy from previous calculations. Now, we present another flow chart in Figure 7 to show how to find the final optimal grain yield through direct enumeration.

Step 1. The procedure starts from stage 1. This is an actual beginning stage of growth development equaling the last stage in Figure 6.

Step 2. The initial value is given here to produce X_1, the percent of maximum grain yield at the first stage.

Step 3. This step is a calculation for X_1 through the transformation equation.

Step 4. The computer performs the second stage calculations.

Step 5. If $n \leq 60$ go back to step 3; otherwise, go to next step.

Step 6 ~ Step 9. These steps result in the calculations of ear development from stage 61 to stage 126. The computational procedure is the same as for the vegetative development period except that a different transformation equation is employed.

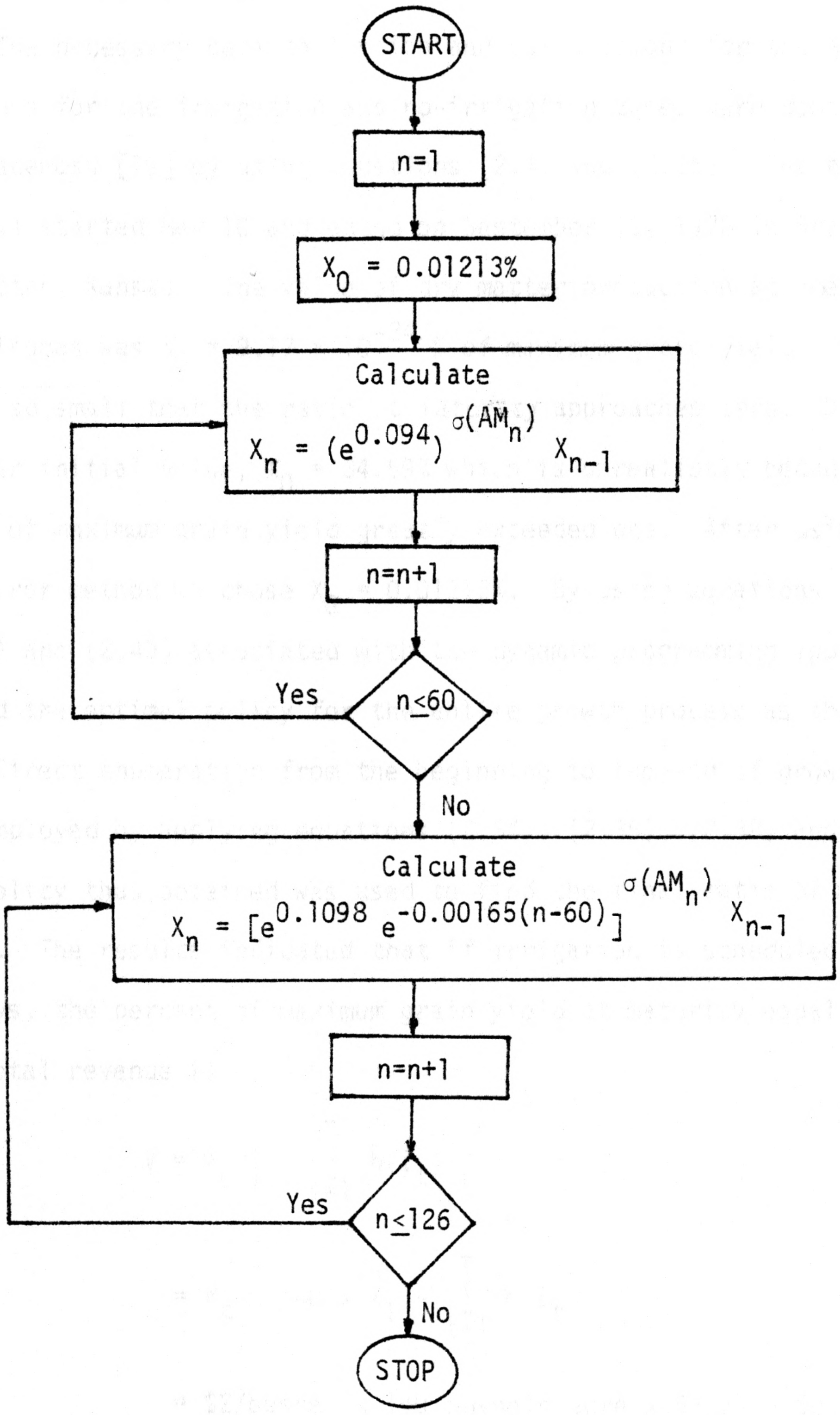


Figure 7. Flow chart for finding the percent of optimal grain yield at maturity.

4.4 NUMERICAL RESULTS

The necessary data collection and calculations for the amount of soil moisture for the irrigation and no-irrigation cases were done by Raney and Kanemasu [19] by using equations (2.4) and (2.15). The entire growth process started May 10 and ended on September 12, 1978 in Scandia and Manhattan, Kansas. The value of dry matter production at emergence obtained from Thomas was $X_0 = 9.17 \times 10^{-24}$ % of maximum grain yield. This initial value so small that the ratio at maturity approaches zero. Biere gave us another initial value, $X_0 = 34.59\%$ which is unrealistic because the final ratio of maximum grain yield greatly exceeded one. After using a trial-and-error method we chose $X_0 = 0.01213\%$. By using equations (2.34), (2.35), (2.39) and (2.40) associated with the dynamic programming approach we obtained the optimal policy for the entire growth process as shown in Figure 8.

Direct enumeration from the beginning to the end of growth development was employed by applying equations (2.34), (2.35), (2.39) and (2.40). The policy thus obtained was used to find the final ratio of maximum grain yield. The results indicated that if irrigation is scheduled on each of 50 days, the percent of maximum grain yield at maturity equals 63.3, and the total revenue is:

$$V = P_c X_T' - \sum_{i=1}^{T} h(I_t)$$

$$= P_c \times 140 \times X_T - \sum_{t=1}^{T} h(I_t)$$

$$= \$2/\text{bushel} \times 140 \text{ bushels/acre} \times 63.3\% - 50 \text{ days} \times \$1/\text{acre-day}$$

$$= \$127.24/\text{acre}$$

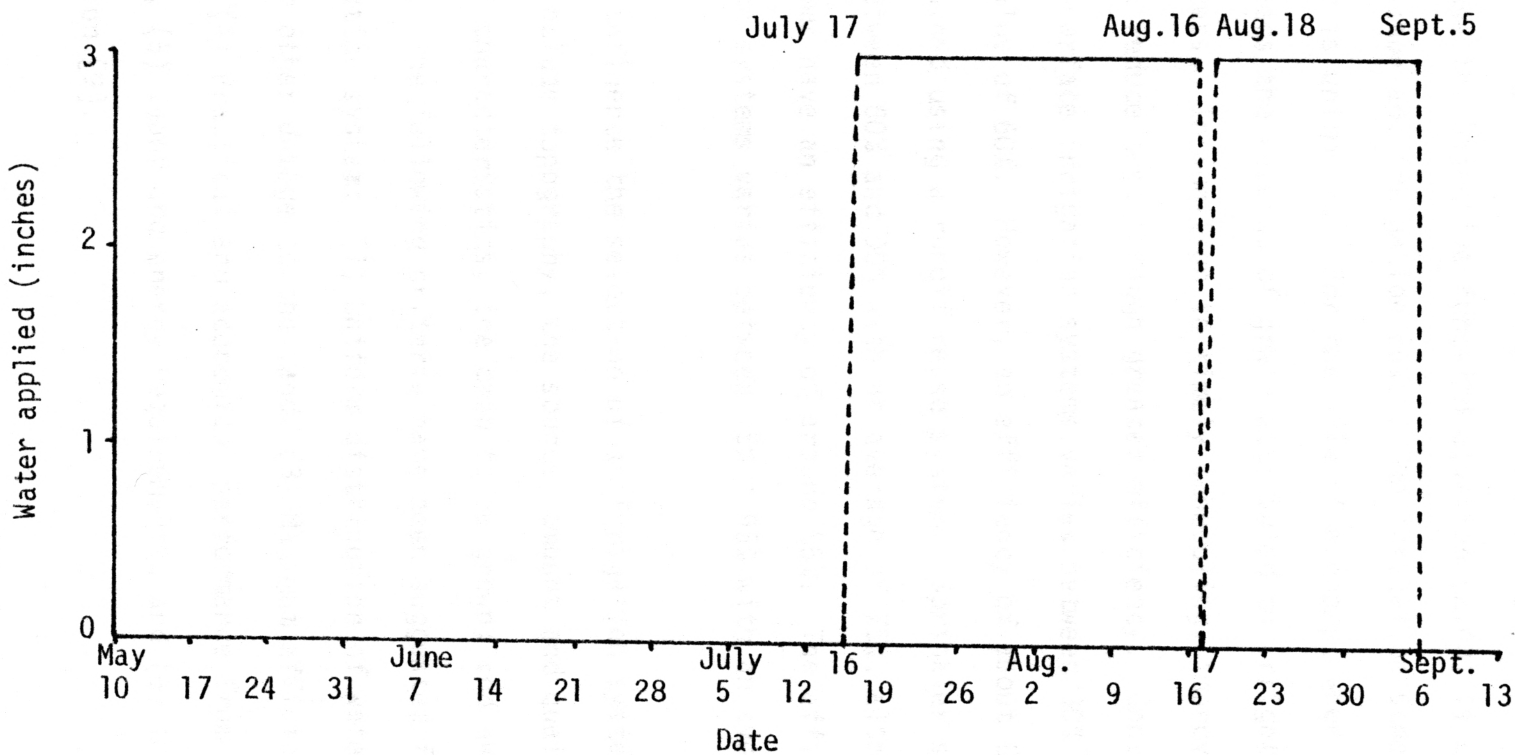


Figure 8. The optimal irrigation schedules.

4.5 DISCUSSION

The policy of irrigation scheduling may vary among different irrigation costs because the objective function equation (2.40) is composed of two parts: revenue and irrigation costs. No irrigation scheduling is needed if water is unlimited. For the sake of economy, when the irrigation cost exceeds the revenue of grain attributed to irrigation, we terminate the process. In addition to timely scheduling, improved irrigation systems can reduce cost through greater efficiency. Generally, the efficiency of surface irrigation systems varies between 30% and 70%, with an average value of 60%. However, an efficiency of about 85% or higher can be obtained using a runoff reuse system. Sprinkler system efficiencies vary between 60% and 90% with an average of 75%, although center-pivot systems have an efficiency of around 85%. The efficiency of trickler irrigation systems varies between 75% ~ 95% with an average of about 90% [7].

Many factors influence the selection of an irrigation system. Some of these factors include topography; the source, amount and quality of water supply; soil characteristics; the crop to be grown; and available labor and capital. The following criteria have been suggested for evaluation of an irrigation system: (1) Uniform distribution of water; (2) Minimum erosion or other damage to the land; (3) Maximum efficiency in the use of water; (4) Practical and economical performance from the standpoint of the crop, (5) labor and energy requirements, and (6) the costs of land proeparation [9].

CHAPTER 5

CONCLUSION

A dynamic corn response model is used for an irrigation scheduling problem. The model has several advantages. (1) It considers the dynamics of a continuous growth process enabling it to better represent corn growth in the real world. (2) It needs minimum meteorological measures all of which are easily obtained from any nearby National Weather Station or field site. (3) It is simple, reliable and accurate.

The dynamic programming approach is used here because the irrigation scheduling problem involves the optimization of many interrelated stages.

Using the Principle of Optimality of dynamic programming guarantees that the decision made at each stage is the best decision in light of the entire process.

Another principal advantage of dynamic programming to this, or any study, is the reduction in calculations required. In this case dynamic programming reduced a combinatorial problem involving 2^{126} choices to a problem requiring only 2 x 126 choices.

Future research efforts might well consider rainfall factors, probabilities of rainfall and adopt a stochastic dynamic programming procedure. An adequate irrigation system should also be considered.

The basic problem associated with the dynamic programming approach is that of dimensionality if the state variables exceed three. A modified approach, such as state increment dynamic programming appears reasonable as a means of eliminating this difficulty.

Although dynamic programming has been extensively used in many fields, including production and inventory problems, transportation problems, and control problems in chemical engineering, irrigation scheduling is a new application. It seems likely that many other agricultural applications are possible.

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APPENDIX

COMPUTER PROGRAM USED IN CHAPTER 4

```
C      *******************************************************************
C      IRRIGATION SCHEDULING FOR A CORN CROP RESPONSE MODEL
C      BY DYNAMIC PROGRAMMING
C
C      NOTATION
C
C      F------- GRID POINT
C      AM1----- AVAILABLE SOIL MOISTURE FOR NO-IRRIGATION CASE
C      AM2----- AVAILABLE SOIL MOISTURE FOR IRRIGATION CASE
C      U------- IRRIGATION POLICY
C      Y1------ GROWTH DEVELOPMENT FROM NO-IRRIGATION CASE
C      Y2------ GROWTH DEVELOPMENT FROM IRRIGATION CASE
C      G1------ RETURN FUNCTION FOR NO-IRRIGATION CASE
C      G2------ RETURN FUNCTION FOR IRRIGATION CASE
C      C------- BLOCK TRANSFER FOR RETURN FUNCTION
C      X------- BLOCK TRANSFER FOR GROWTH DEVELOPMENT
C      SMALL--- IRRIGATION COST
C      P------- UNIT PRICE OF CORN
C
C      THIS PROBLEM IS TO OBTAIN AN ECONOMIC IRRIGATION
C      SCHEDULES.
C
C      THIS PROGRAM WAS WRITTEN BY JAMES C. CHAO, DEPARTMENT
C      OF INDUSTRIAL ENGINEERING, KANSAS STATE UNIVERSITY,
C      MANHATTAN, KANSAS, MAY, 1979.
C
C      *******************************************************************
C
C      THE MAIN PROGRAM
C
       DIMENSION C(12),X(12),F(12),G1(12),G2(12),Y1(12),Y2(12
      C),AM1(126),AM2(126)
       P=2.
       SMALL=1.
       NN=11
       NNM=NN-1
       F(1)=0
       DO 6 J=1,NNM
       F(J+1)=F(J)+0.1
     6 CONTINUE
   115 FORMAT(7X,I3,7X,F7.4,4X,F7.4,5X,F7.4,6X,F7.3)
C
C      READ IN DATA
C
       READ 7,(AM1(L),L=1,126)
       READ 7,(AM2(L),L=1,126)
     7 FORMAT(12F6.3)
C
C      PRINT OUT THE TITLES FOR EACH STAGE
C
```

```
      PRINT 1111
 1111 FORMAT('1')
      PRINT 1
    1 FORMAT(5X,'L=',10X,'U=',7X,'F=',8X,'Y=',10X,'G=')
C
C     STAGE 1
C
      L=1
C
C     CALCULATE THE RETURN FUNCTION OF NO-IRRGGATION CASE
C     AND IRRIGATION CASE
C
      DO 14 J=1,NN
      IF(AM1(L).LT.0) A=2.464*AM1(L)
      IF((AM1(L).GE.0).AND.(AM1(L).LE.0.333)) A=2.464*AM1(L)
      IF((AM1(L).GT.0.333).AND.(AM1(L).LE.0.667)) A=0.755+0.
     C199*AM1(L)
      IF((AM1(L).GT.0.667).AND.(AM1(L).LE.1)) A=0.663+0.337*
     CAM1(L)
      IF(AM1(L).GT.1) A=0.663+0.337*AM1(L)
      Y1(J)=((EXP(0.1098)*EXP(-(0.00165*(67-L))))**A)*F(J)
      G1(J)=P*Y1(J)*140
      IF(AM2(L).LT.0) A=2.464*AM2(L)
      IF((AM2(L).GE.0).AND.(AM2(L).LE.0.333)) A=2.464*AM2(L)
      IF((AM2(L).GT.0.333).AND.(AM2(L).LE.0.667)) A=0.755+
     C0.199*AM2(L)
      IF((AM2(L).GT.0.667).AND.(AM2(L).LE.1)) A=0.663+0.337*
     CAM2(L)
      IF(AM2(L).GT.1) A=0.663+0.337*AM2(L)
      Y2(J)=((EXP(0.1098)*EXP(-(0.00165*(67-L))))**A)*F(J)
      G2(J)=P*Y2(J)*140-SMALL
   14 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE IN NO-IRRIGATION CASE
C
      MAX1=G1(1)
      DO 9 J=1,NN
      IF(G1(J).LE.MAX1) GO TO 9
      MAX1=G1(J)
    9 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE IN IRRIGATION CASE
C
      MAX2=G2(1)
      DO 8 J=1,NN
      IF(G2(J).LE.MAX2) GO TO 8
      MAX2=G2(J)
    8 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE FROM NO-IRRIGATION AND
```

```
C       IRRIGATION CASES, AND STORE IT
C
        IF(MAX1-MAX2) 3,3,2
      2 DO 10 J=1,NN
        U=0.
        PRINT 115,L,U,F(J),Y1(J),G1(J)
        C(J)=G1(J)
        X(J)=Y1(J)
     10 CONTINUE
        GO TO 12
      3 DO 11 J=1,NN
        U=3.
        PRINT 115,L,U,F(J),Y2(J),G2(J)
        C(J)=G2(J)
        X(J)=Y2(J)
     11 CONTINUE
C
C       STAGE 2 TO STAGE 66
C
     12 L=2
     34 PRINT 1
C
C       CALCULATE THE RETURN FUNCTION OF NO-IRRUGATION CASE
C       AND IRRIGATION CASE
C
        DO 16 J=1,NN
        IF(AM1(L).LT.0) A=2.464*AM1(L)
        IF((AM1(L).GE.0).AND.(AM1(L).LE.0.333)) A=2.464*AM1(L)
        IF((AM1(L).GT.0.333).AND.(AM1(L).LE.0.667)) A=0.755+
       C0.199*AM1(L)
        IF((AM1(L).GT.0.667).AND.(AM1(L).LE.1)) A=0.663+0.337*
       CAM1(L)
        IF(AM1(L).GT.1) A=0.663+0.337*AM1(L)
        Y1(J)=((EXP(0.1098)*EXP(-(0.00165*(67-L))))**A)*F(J)
        IF((J.EQ.NN).AND.(Y1(J).GT.X(J))) GO TO 18
        IF(Y1(J).LT.X(J)) GO TO 118
        IF((Y1(J).EQ.X(J)).OR.((Y1(J).GT.X(J)).AND.(Y1(J).LT.X
       C(J+1)))) GO TO 19
C
C       LINEAR INTERPOLATION
C
     18 GDIF1=C(J)+(C(J)-C(J-1))*(Y1(J)-X(J))/(X(J)-X(J-1))
        G1(J)=GDIF1
        GO TO 22
    118 GDIF1=C(J)+(Y1(J)-X(J))*(C(J)-C(J-1))/(X(J)-X(J-1))
        G1(J)=GDIF1
        GO TO 22
     19 GDIF1=C(J)+(Y1(J)-X(J))*(C(J+1)-C(J))/(X(J+1)-X(J))
        G1(J)=GDIF1
     22 IF(AM2(L).LT.0) A=2.464*AM2(L)
```

```
      IF((AM2(L).GE.0).AND.(AM2(L).LE.0.333)) A=2.464*AM2(L)
      IF((AM2(L).GT.0.333).AND.(AM2(L).LE.0.667)) A=0.755+
     CO.199*AM2(L)
      IF((AM2(L).GT.0.667).AND.(AM2(L).LE.1)) A=0.663+0.337*
     CAM2(L)
      IF(AM2(L).GT.1) A=0.663+0.337*AM2(L)
      Y2(J)=((EXP(0.1098)*EXP(-(0.00165*(67-L))))**A)*F(J)
      IF((J.EQ.NN).AND.(Y2(J).GT.X(J))) GO TO 24
      IF(Y2(J).LT.X(J)) GO TO 240
      IF((Y2(J).EQ.X(J)).OR.((Y2(J).GT.X(J)).AND.(Y2(J).LT.X
     C(J+1)))) GO TO 25
C
C     LINEAR INTERPOLATION
C
  240 GDIF2=C(J)+(Y2(J)-X(J))*(C(J)-C(J-1))/(X(J)-X(J-1))
      G2(J)=GDIF2-SMALL
      GO TO 16
   24 GDIF2=C(J)+(C(J)-C(J-1))*(Y2(J)-X(J))/(X(J)-X(J-1))
      G2(J)=GDIF2-SMALL
      GO TO 16
   25 GDIF2=C(J)+(Y2(J)-X(J))*(C(J+1)-C(J))/(X(J+1)-X(J))
      G2(J)=GDIF2-SMALL
   16 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE IN NO-IRRIGATION CASE
C
      MAX1=G1(1)
      DO 20 J=1,NN
      IF(G1(J).LE.MAX1) GO TO 20
C
C     CHOOSE THE MAXIMUM ONE IN IRRIGATION CASE
C
      MAX1=G1(J)
   20 CONTINUE
      MAX2=G2(1)
      DO 26 J=1,NN
      IF(G2(J).LE.MAX2) GO TO 26
      MAX2=G2(J)
   26 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE FROM NO-IRRIGATION AND
C     IRRIGATION CASES, AND STORE IT
C
      IF(MAX1-MAX2) 28,28,29
   29 DO 30 J=1,NN
      U=0.
      PRINT 115,L,U,F(J),Y1(J),G1(J)
      C(J)=G1(J)
      X(J)=Y1(J)
   30 CONTINUE
```

```
      GO TO 31
   28 DO 32 J=1,NN
      U=3.
      PRINT 115,L,U,F(J),Y2(J),G2(J)
      C(J)=G2(J)
      X(J)=Y2(J)
   32 CONTINUE
   31 L=L+1
      IF(L.LE.66) GO TO 34
C
C     STAGE 67 TO STAGE 95
C
      L=67
   74 PRINT 1
C
C     CALCULATE THE RETURN FUNCTION OF NO-IRROGATION CASE
C     AND IRRIGATION CASE
C
      DO 51 J=1,NN
      IF(AM1(L).LT.0) A=2.464*AM1(L)
      IF((AM1(L).GE.0).AND.(AM1(L).LE.0.333)) A=2.464*AM1(L)
      IF((AM1(L).GT.0.333).AND.(AM1(L).LE.0.667)) A=0.755+
     C0.199*AM1(L)
      IF((AM1(L).GT.0.667).AND.(AM1(L).LE.1)) A=0.663+0.337*
     CAM1(L)
      IF(AM1(L).GT.1) A=0.663+0.337*AM1(L)
      Y1(J)=(EXP(0.094)**A)*F(J)
      IF((J.EQ.NN).AND.(Y1(J).GT.X(J))) GO TO 53
      IF(Y1(J).LT.X(J)) GO TO 533
      IF((Y1(J).EQ.X(J)).OR.((Y1(J).GT.X(J)).AND.(Y1(J).LT.X
     C(J+1)))) GO TO 54
C
C     LINEAR INTERPOLATION
C
   53 GDIF1=C(J)+(C(J)-C(J-1))*(Y1(J)-X(J))/(X(J)-X(J-1))
      G1(J)=GDIF1
      GO TO 57
  533 GDIF1=C(J)+(Y1(J)-X(J))*(C(J)-C(J-1))/(X(J)-X(J-1))
      G1(J)=GDIF1
      GO TO 57
   54 GDIF1=C(J)+(Y1(J)-X(J))*(C(J+1)-C(J))/(X(J+1)-X(J))
      G1(J)=GDIF1
   57 IF(AM2(L).LT.0) A=2.464*AM2(L)
      IF((AM2(L).GE.0).AND.(AM2(L).LE.0.333)) A=2.464*AM2(L)
      IF((AM2(L).GT.0.333).AND.(AM2(L).LE.0.667)) A=0.755+
     C0.199*AM2(L)
      IF((AM2(L).GT.0.667).AND.(AM2(L).LE.1)) A=0.663+0.337*
     CAM2(L)
      IF(AM2(L).GT.1) A=0.663+0.337*AM2(L)
      Y2(J)=(EXP(0.094)**A)*F(J)
```

```
      IF((J.EQ.NN).AND.(Y2(J).GT.X(J))) GO TO 59
      IF(Y2(J).LT.X(J)) GO TO 599
      IF((Y2(J).EQ.X(J)).OR.((Y2(J).GT.X(J)).AND.(Y2(J).LT.X
     C(J+1)))) GO TO 60
C
C     LINEAR INTERPOLATION
C
   59 GDIF2=C(J)+(C(J)-C(J-1))*(Y2(J)-X(J))/(X(J)-X(J-1))
      G2(J)=GDIF2-SMALL
      GO TO 51
  599 GDIF2=C(J)+(Y2(J)-X(J))*(C(J)-C(J-1))/(X(J)-X(J-1))
      G2(J)=GDIF2-SMALL
      GO TO 51
   60 GDIF2=C(J)+(Y2(J)-X(J))*(C(J+1)-C(J))/(X(J+1)-X(J))
      G2(J)=GDIF2-SMALL
   51 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE IN NO-IRRIGATION CASE
C
      MAX1=G1(1)
      DO 48 J=1,NN
      IF(G1(J).LE.MAX1) GO TO 48
      MAX1=G1(J)
   48 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE IN IRRIGATION CASE
C
      MAX2=G2(1)
      DO 47 J=1,NN
      IF(G2(J).LE.MAX2) GO TO 47
      MAX2=G2(J)
   47 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE FROM NO-IRRIGATION CASE AND
C     IRRIGATION CASES, AND STORE IT
C
      IF(MAX1-MAX2) 68,68,69
   69 DO 70 J=1,NN
      U=0.
      PRINT 115,L,U,F(J),Y1(J),G1(J)
      C(J)=G1(J)
      X(J)=Y1(J)
   70 CONTINUE
      GO TO 71
   68 DO 72 J=1,NN
      U=3.
      PRINT 115,L,U,F(J),Y2(J),G2(J)
      C(J)=G2(J)
      X(J)=Y2(J)
   72 CONTINUE
```

```
   71 L=L+1
      IF(L.LE.95) GO TO 74
C
C     STAGE 96 TO STAGE 125
C
      L=96
  274 PRINT 1
C
C     CALCULATE THE RETURN FUNCTION OF NO-IRROGATION CASE
C     AND IRRIGATION CASE
C
      DO 251 J=1,NN
      IF(AM1(L).LT.0) A=2.464*AM1(L)
      IF((AM1(L).GE.0).AND.(AM1(L).LE.0.333)) A=2.464*AM1(L)
      IF((AM1(L).GT.0.333).AND.(AM1(L).LE.0.667)) A=0.755+
     C0.199*AM1(L)
      IF((AM1(L).GT.0.667).AND.(AM1(L).LE.1)) A=0.663+0.337*
     CAM1(L)
      IF(AM1(L).GT.1) A=0.663+0.337*AM1(L)
      Y1(J)=(EXP(0.094)**A)*F(J)
      IF(Y1(J).EQ.X(J)) GO TO 254
  254 GDIF1=C(J)
      G1(J)=GDIF1
      IF(AM2(L).LT.0) A=2.464*AM2(L)
      IF((AM2(L).GE.0).AND.(AM2(L).LE.0.333)) A=2.464*AM2(L)
      IF((AM2(L).GT.0.333).AND.(AM2(L).LE.0.667)) A=0.755+
     C0.199*AM2(L)
      IF((AM2(L).GT.0.667).AND.(AM2(L).LE.1)) A=0.663+0.337*
     CAM2(L)
      IF(AM2(L).GT.1) A=0.663+0.337*AM2(L)
      Y2(J)=(EXP(0.094)**A)*F(J)
      IF(Y2(J).EQ.X(J)) GO TO 260
  260 GDIF2=C(J)
      G2(J)=GDIF2-SMALL
  251 CONTINUE
      MAX1=G1(1)
      DO 248 J=1,NN
      IF(G1(J).LE.MAX1) GO TO 248
C
C     CHOOSE THE MAXIMUM ONE IN NO-IRRIGATION CASE
C
      MAX1=G1(J)
  248 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE IN IRRIGATION CASE
C
      MAX2=G2(1)
      DO 247 J=1,NN
      IF(G2(J).LE.MAX2) GO TO 247
      MAX2=G2(J)
```

```
  247 CONTINUE
C
C     CHOOSE THE MAXIMUM ONE FROM NO-IRRIGATION AND
C     IRRIGATION CASES, AND STORE IT
C
      IF(MAX1-MAX2) 268,268,269
  269 DO 270 J=1,NN
      U=0.
      PRINT 115,L,U,F(J),Y1(J),G1(J)
      C(J)=G1(J)
      X(J)=Y1(J)
  270 CONTINUE
      GO TO 271
  268 DO 272 J=1,NN
      U=3.
      PRINT 115,L,U,F(J),Y2(J),G2(J)
      C(J)=G2(J)
      X(J)=Y2(J)
  272 CONTINUE
  271 L=L+1
      IF(L.LE.125) GO TO 274
C
C     STAGE 126
C
      L=126
      PRINT 1
C
C     CALCULATE THE RETURN FUNCTION OF NO-IRROGATION CASE
C     AND IRRIGATION CASE
C
      J=4
      F(J)=0.3469
      IF(AM1(L).LT.0) A=2.464*AM1(L)
      IF((AM1(L).GE.0).AND.(AM1(L).LE.0.333)) A=2.464*AM1(L)
      IF((AM1(L).GT.0.333).AND.(AM1(L).LE.0.667)) A=0.755+
     C0.199*AM1(L)
      IF((AM1(L).GT.0.667).AND.(AM1(L).LE.1)) A=0.663+0.337*
     CAM1(L)
      IF(AM1(L).GT.1) A=0.663+0.337*AM1(L)
      Y1(J)=(EXP(0.094)**A)*F(J)
      IF((Y1(J).EQ.X(J)).OR.((Y1(J).GT.X(J)).AND.(Y1(J).LT.X
     C (J+1)))) GO TO 84
C
C     LINEAR INTERPOLATION
C
   84 GDIF1=C(J)+(Y1(J)-X(J))*(C(J+1)-C(J))/(X(J+1)-X(J))
      G1(J)=GDIF1
      IF(AM2(L).LT.0) A=2.464*AM2(L)
      IF((AM2(L).GE.0).AND.(AM2(L).LE.0.333)) A=2.464*AM2(L)
      IF((AM2(L).GT.0.333).AND.(AM2(L).LE.0.667)) A=0.755+
```

```
     C0.199*AM2(L)
      IF((AM2(L).GT.0.667).AND.(AM2(L).LE.1)) A=0.663+0.337*
     CAM2(L)
      IF(AM2(L).GT.1) A=0.663+0.337*AM2(L)
      Y2(J)=(EXP(0.094)**A)*F(J)
      IF((Y2(J).EQ.X(J)).OR.((Y2(J).GT.X(J)).AND.(Y2(J).LT.X
     C(J+1)))) GO TO 90
C
C     LINEAR INTERPOLATION
C
   90 GDIF2=C(J)+(Y2(J)-X(J))*(C(J+1)-C(J))/(X(J+1)-X(J))
      G2(J)=GDIF2-SMALL
C
C     CHOOSE THE MAXIMUM ONE FROM NO-IRRIGATION AND
C     IRRIGATION CASES
C
      IF(G1(J)-G2(J)) 108,108,109
  109 U=0.
      PRINT 115,L,U,F(J),Y1(J),G1(J)
      GO TO 101
  108 U=3.
      PRINT 115,L,U,F(J),Y2(J),G2(J)
  101 STOP
      END

$ENTRY
```

IRRIGATION SCHEDULING FOR A CORN CROP RESPONSE MODEL BY DYNAMIC PROGRAMMING

by

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ABSTRACT

Dynamic programming is a useful technique for solving multi-stage problems. The principal advantage of this approach is the computation reduction. For a 126-stage example with 2 decisions at each stage, a significant reduction in problem size and difficulty can be obtained by reducing this combinatorial problem involving 2^{126} choices to a problem requiring only 2 x 126 choices.

The purpose of this work is to modify a dynamic corn crop response model for scheduling irrigation use and obtain the maximum profit from irrigated production by using dynamic programming.

A dynamic corn response model has several advantages. (1) It considers the dynamics of a continuous growth process so that it represents a good approximation to corn growth in the real world. (2) It needs minimum meteorological measurements which can be obtained easily from nearby National Weather Station. (3) The model is simple, reliable and accurate. Most ordinary corn plants tend to have a pattern of development similar to this model, except that the duration of stages may vary among different hybrids.

The irrigation scheduling problem involves the optimum of many interrelated stages, hence, it is a multi-stage problem. Since dynamic programming has been powerfully demonstrated in dealing with multi-stage problems we apply it to the irrigation scheduling problem.

In this study we consider two decisions at each stage: irrigate 3-inch water per day or do not irrigate. The relevant irrigation costs are 1 dollar per day or zero, respectively. The numerical results indicate a 50-day-irrigation schedule for a mid-season hybrid corn which requires 126 days to mature. Policies may vary among different irrigation costs because our irrigation scheduling model considers the economic factor.

Although dynamic programming has been extensively used in many fields, this agriculture application is new.