

AN INVESTIGATION OF THE PERFORMANCE OF
CROSS-FLOW HEAT EXCHANGERS USED IN AIR-CONDITIONING

by

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INTRODUCTION

Air-conditioning requires apparatus for transferring heat to or from the air. A commonly used type of heat exchanger consists of rows of tubes carrying the refrigerant or heating agent. Air flowing perpendicular to the tubes is thus cooled or heated. Such units are referred to as cross-flow heat exchangers.

The evaluation or prediction of the performance of these units is fundamentally a problem in heat transfer. For the case of heating air, heat must pass from the heating agent through the metal pipe and to the air. It is known that most of the temperature drop between the steam and the pipe wall occurs across a thin film of the steam adjacent to the metal. Similarly, a thin air film is the controlling resistance to heat flow on the outside of the pipe. The conductivity of the air film is much less than that of the steam film, so that it is desirable to increase the heat flow area on the air side. For this purpose, metal fins are placed on the outside of the pipes. While the fins increase the capacity of the heat exchanger, they complicate the design of such units appreciably.

The original purpose of this investigation was to set

up a rational design procedure for cross-flow heat exchangers employing finned tubes with particular reference to that type of heat exchanger used in air-conditioning work. It was found advisable, however, to limit the investigation to a study of the phenomenon of heat transfer from the fin surfaces to air being heated.

In order to solve the problem of establishing a rational design procedure for cross flow heat exchangers it appeared advisable to proceed as follows:

1. Obtain an expression for the temperature at any point on the fin, considering conduction of heat in the fin and energy transfer by convection to or from the fin surface.

2. Obtain an expression for the temperature at any point on the surface of a circular pipe bearing fins, considering conduction to or from the cooling or heating fluid inside the pipe, and through the fluid film, the pipe metal, and the outside fluid film; also convection in the outside fluid mass.

3. Utilize the above expressions to obtain the rate of heat transfer to or from any fin-tube combination, for a given refrigerant at any initial temperature and for any flow conditions.

4. Obtain an expression for the rate of heat transfer to or from the side walls or headers of the unit.

5. Extend (1) above, to include the effect of wet fins and tubes (i.e. moisture deposits due to dehumidification of the air when the unit is used as a cooler).

6. Obtain an expression for the condition of air passing over the finned tubes and along the walls or headers, the expression to hold for any initial air conditions (velocity, temperature, and moisture content).

7. Extend (6) to tube banks.

The first problem, then, was to obtain an expression for the temperature at any point on a fin. In order to avoid the complications introduced when dehumidification occurs, the heating problem was considered. Heat passes from the fluid in the pipe through the pipe and fin to the air. Under these conditions, the fins would at all times be dry. It has been verified experimentally that the direction of heat flow does not affect the value of the air film heat transfer coefficient for this case of no dehumidification (Grimison, 1937).

If in attempting to solve this problem, one considers first the flow of heat in the fin and second the flow of heat in the air, the result is a complication. As Dryden (Durand, 1936, p. 229) points out, one would have to know the flow of heat within the fin for all possible rates of heat loss at the boundary of the fin and the flow of heat

within the air for all rates of supply of heat at the boundary of the fin. Then those solutions would be selected for which the boundary conditions agree. This fully general problem has not been solved (Durand, 1936, p. 229). The main difficulty in obtaining an analytical solution lies in mathematically expressing the air condition for a given velocity of flow and with fixed boundary conditions. Before a solution is attempted, the basic flow phenomena should be understood.

There exist in nature two radically different kinds of flow, laminar and turbulent. By laminar flow is meant a flow in which the fluid moves in laminar layers which do not mix. For such flow the path of a particle of fluid can be defined mathematically. In turbulent flow, on the other hand, there is turbulent mixing of the fluid, so that the particles move in random and unpredictable paths which cannot be defined mathematically.

The criterion of transition from laminar to turbulent flow, for geometrically similar systems and for a given initial turbulence, is the Reynold's Number. Its value at the transition point is the critical Reynold's Number. The critical Reynold's Number is decreased if the initial turbulence is increased. In fact, there is a functional relation between the critical Reynold's Number and the initial turbulence.

Both types of flow may occur, and may exist simultaneously in different parts of a given field of flow. This makes the problem of obtaining expressions for quantities such as the total heat transfer a complicated one. To avoid the complications that arise when an attempt is made to study heat transfer in the presence of both laminar and turbulent flow it is usual to treat the theory of heat transfer for turbulent flow apart from that for laminar flow.

In commercial heat exchangers, the air is usually turbulent, and it is for this type of flow that an analytical expression would be exceedingly useful. Attempts to establish such a formula have been made by Reynolds, Stanton, Boussinesq, G. I. Taylor, Prandtl, von Karman and others, but without satisfactory results.

The flow of air over a finned tube is a three dimensional problem. It was, however, expected that the variation in the local heat transfer coefficient on the fin would have the same pattern as the local heat transfer coefficient on a cylinder, particularly at points close to the cylinder. Accordingly, it was considered advisable to investigate the flow over a cylinder in some detail.

The nature of the flow about a cylinder depends on the value of the Reynold's Number $R = \frac{V_0 D}{\nu}$, where V_0 = speed

of the fluid stream at a distance from the cylinder, $D =$ the diameter and $\nu =$ the kinematic viscosity. The various types of flow may be tabulated as follows:

- $R = 1$ The fluid closes in completely behind the cylinder and flow is everywhere laminar.
- $R = 3$ Stationary eddies develop behind the cylinder. As R increases the eddies move away from the cylinder and become unstable.
- $R = 100$ Eddies form periodically in the wake, arranging themselves in the Karman vortex street. Over the forward part of the cylinder, the flow is still laminar; it does not close in behind the cylinder, however, but separates from the surface. There is some evidence that the flow remains of a laminar character for some distance beyond the separation point before eddy motion develops.
- $R = 30,000$ Flow remains laminar up to the point of separation but becomes turbulent almost immediately afterward.
- $R = 200,000$ Flow becomes turbulent before separation and the process of separation is delayed, the drag coefficient falling rapidly. For $30,000 < R < 200,000$, the drag is constant; the speed just outside the boundary layer increases from zero at the upstream stagnation point to a maximum of about 1.55 times the speed of the approaching stream, the boundary layer accordingly being subjected to a pressure gradient in the direction of flow. Dryden (Durand, 1936, p. 277) mentions that the pressure drop reduces the thickness of the boundary layer very materially except near the stagnation point, and hence, as shown by experiment, increases the skin friction by a factor of two or more as compared with that on a thin flat plate set parallel to the flow. Evidently, the increased

velocity gradient gives a higher shearing stress, and thus a higher skin friction in the case of the tube.

When the angle ϕ , as defined on Fig. 1, becomes about 70° , the flow separates from the surface. Immediately behind this point of separation the air near the surface is moving in a direction opposite to that of the main stream.

The process of separation is described in the following way. The particles near the wall are dragged along by the friction of the neighboring faster moving particles but are retarded by the pressure. As the layer thickens, the retarding force predominates and this finally causes a reversal of the flow near the surface. The reversal of flow causes an amount of stagnant fluid to accumulate at the boundary with the result that the actual path of flow recedes from the surface. When the Reynold's Number is increased above about 200,000, the flow in the boundary layer is eddying before separation, and the point of separation advances to a larger ϕ . In the eddying flow there is a more thorough mixing of the particles of air, and the driving action of the outer air on the fluid near the surface is greater. Hence the fluid near the surface can proceed farther against the pressure gradient. Dryden (Durand, 1936, p. 279) states that the exact mechanism of flow at separation is not known. Even in this case, then, the

condition of the air cannot be expressed mathematically.

This general picture applies to a cylinder with its outer surface at constant temperature. Dryden (Durand, 1936, p. 278) states that studies of the heat transfer from a heated strip on a cylinder as a function of ϕ are "of interest" -- but "they give no information on the local rate of transfer of heat from a cylinder whose entire outer surface is maintained at constant temperature". In view of other results this statement appears too strong. Elias (1931) found that the change in speed of air at any point in its passage over a skin friction plate did not exceed 2 or 3% for a temperature rise of about 35° C and that the point of transition from laminar to turbulent flow was practically unaffected. It seems reasonable, therefore, to expect that the flow characteristics would not be affected if the heated strip were not at too high a temperature relative to the air temperature. Although the variation in temperature over the cylinder undoubtedly has some effect on the flow, since a change of temperature causes a change in μ , the results should closely approximate those for isothermal conditions, particularly if the temperature variation is small (say less than 95° F, or 35° C) and the heated strip is thin.

The possibility of using the analogy between heat transfer and fluid friction (McAdams, 1933, p. 158) was considered in attempting to obtain an analytical solution to the problem. The problem of heat transfer from surfaces is closely related to skin friction and boundary layer theory (Biermann and Pinkel, 1934). The same mechanism that transfers heat through a boundary layer also transfers momentum. Dryden (Durand, 1936, p. 259) showed, however, that the analogy was not universally applicable. He demonstrated that it was valid only for the case of flow near a skin friction plate of a fluid for which Prandtl's Number was equal to one. The analogy will not hold, therefore, when separation occurs, as in the case of flow at right angles to a circular cylinder.

As is usual in engineering practice when theory is not sufficiently developed, it was found necessary to resort to dimensional analysis. Previous investigators have used this method of analysis to correlate experimental determinations of the average heat transfer coefficient for a fin as a function of air velocity.

The general equation for the surface coefficient in forced convection, derived by dimensional analysis, is:

$$\frac{hD}{K} = c \left(\frac{VD\rho}{\mu} \right)^n \left(\frac{Cp\mu}{K} \right)^m \quad (1)$$

where h = film coefficient.

D = linear dimension

V = velocity of flow

ρ , c_p , μ , K refer to fluid properties - density, specific heat, absolute viscosity, and thermal conductivity.

c = proportionality constant

n and m = experimentally determined constants.

For the forced convection of air, with moderate ranges of temperature, Prandtl's Number $\frac{c_p \mu}{K}$ and likewise $\frac{K}{\mu}$ may be considered constant (Tuve, 1934). Then

$$h = \frac{BG^n}{D^{1-n}}$$

where

G = VA = mass velocity

B = constant (experimental)

n = constant (experimental)

Numerous investigators have employed this formula in correlating experimental data. Some of the results are reproduced in Table 1, arranged by Tuve and McKeeman (1934).

Similar data are shown by King and Knaus (1934). It will be noted that the only dimension included in these expressions is that of the tube diameter. It is obvious that other fin dimensions, such as spacing and thickness and ratio of air side to refrigerant side areas will affect the expression for h. The velocity distribution and the

Table 1. Showing empirical expressions for the heat transfer coefficient h in B.T.U./ $(hr)(ft^2)(^{\circ}F)$

Formula	Investi- gators	Appli- cations	Heat Transfer Coefficient for Heating Air at 100° F with Pipe Diameter of 1 inch and Velocities Indicated	
			V = 500 f.P.M.	V = 1500 f.P.M.
1. $h = .0035 \frac{G^{0.8}}{D^{0.2}}$	Nusselt, Josse, Royds, Campbell	Flow inside 3/4" to 1" smooth pipes	2.6	6.4
2. $h = .0183 \frac{G^{0.69}}{D^{0.31}}$	Reiher, Carrier and Busey, Rietschel	Flow over several rows 5/8" to 1 1/4" smooth tubes, staggered	7.8	16.8
3. $h = .033 \frac{G^{0.56}}{D^{0.44}}$	Hughes, Reiher, Vornehm, Gibson	Flow over single tubes or pipes, 1/4" to 2" O.D.	6.7	13.3
4. $h = .061 \frac{G^{0.51}}{D^{0.49}}$	T. E. Schmidt	Flow over 6 rows 7" O.D. fin tubes, 2" fins, 15 per foot	9.0	15.7

temperature gradients between tube surfaces and fin sections will change for different ratios of these significant dimensions.

Tuve and McKeeman (1934) recommend the following approximate expressions:

1. Large plain tubes, staggered arrangement or with flow disturbers $h = .022 \frac{G^{0.7}}{D^{0.3}}$
2. Common large fin tubes, staggered arrangement or with flow disturbers $h = .04 \frac{G^{0.6}}{D^{0.4}}$
3. Small tubes with large fins, or small plain tubes widely spaced; smooth air flow $h = .05 \frac{G^{0.5}}{D^{0.5}}$

Due to the difficulty of obtaining the effect of such variables as fin dimensions, air velocity, turbulence, etc. on the air side, and the necessity for working with similar variables on the refrigerant side, it has been found more convenient to use overall coefficients based on air side surface area and a log mean temperature difference between air and refrigerant. A large amount of experimental data have been compiled on this basis (Tuve, 1934). The overall coefficient is plotted as a function of the velocity, viz.

$$u = cv^n \quad (3)$$

The only justification for this procedure is that it actually gives a straight line on log - log paper for most of the common cases of dry-coil heat transfer.

Much of the design work on finned tube heat-exchangers has been carried out by the use of the above formula. The desirability for experimental checks on the performance of any particular design employing this approximate relationship is apparent.

The usual design procedure has been to employ values of h obtained experimentally on similar equipment, and to use some simple step-by-step integration to obtain the heat transfer in the fin (Swart, 1938). Mathematical expressions for the heat flow by conduction in the fin had been worked out using various approximations (Harper and Brown, 1923), but designers seemed reluctant to avail themselves of the rather involved solutions. Recently, a relatively exact solution was obtained by Murray (1938), the most serious assumption being that the heat transfer coefficient was constant over the fin. He also derived a dimensionless effectiveness factor, defined as the ratio of the heat transferred by a given fin to the heat which would be transferred by the pipe area under the fin if the fin were removed. This factor could be obtained directly from a chart for given values of the heat transfer coefficient and for fixed fin dimensions. It enables one to determine the effect that adding a given size of fin has on the heat transfer from a fin-tube combination. As fins are added to a tube, however,

the fin spacing changes, and with it the value of the heat transfer coefficient. It is of importance, therefore, to determine if the coefficient is sufficiently constant to justify the use of the effectiveness factor¹.

The review of literature bearing on the problem convinced the writer that a satisfactory method of solution required a more accurate treatment of the heat flow in a fin than was at present available. This involved a knowledge of the variation in the heat transfer coefficient over the fin surface at various temperatures and for different velocities of flow and fin spacings. Previous investigators (Biermann and Pinkel, 1934) concluded that the value of the local heat transfer coefficient varies mainly with the air velocity and fin spacing, the effect of the other fin dimensions being small. It was decided to investigate the variation in the local heat transfer coefficient with air velocity and fin spacing, and equipment was constructed for that purpose.

Furthermore, the data obtained were used in an attempt to develop an expression for the heat transfer from a fin. The other parts of the original problem were not considered.

¹Personal communication from R. H. Norris, General Electric Co., Schenectady, N. Y.

MATERIAL AND METHODS

It was desired to obtain an experimental expression for the heat transfer coefficient, h , as a function of air velocity and fin spacing, with air conditions in the range encountered in air-conditioning practice.

If the air velocity were constant over the surface of the fin, then the temperature θ would be the same at every point on the fin which was at the same radius - i.e., circular isothermals would be formed. In the actual case, however, if two elements of surface at equal radii are considered, the heat transfer coefficient h may not be the same for each element. For the element with the lower h , the surface temperature will be the higher, and the set of isothermals will be distorted from the circular pattern. But the radial heat flow rate will also change, due to the change in the temperature gradient between the element and the heat source at the base of the fin. It is not correct, therefore, to consider the deviation of the isothermals from the circular pattern as a measure of the change in h alone.

If the temperature at the tube wall did not vary with angular position, it would be possible to determine the total heat flow rate from the fin. Then dividing this total heat rate by the total area, one would obtain an average

rate of heat flow per unit area. The fin could be divided up into a number of small areas, the average temperature for each area determined, and h thus computed for each area. In the actual case, however, the values of h thus found would be in error by an amount which depends on the variation in the temperature at the tube wall.

It was decided to construct a fin of a type of fiber board available in the Woodworking Laboratories. All fins used were 7 inches in diameter and approximately 0.3 inch thick, and were mounted on a 2.25-inch outside diameter iron pipe. A hollow copper disc was made by punching thin copper sheeting into the form of cups, $3/8$ inch in outside diameter and about 0.15 inch deep. Two of these cups, when placed together, formed a hollow copper disc. One of these discs was placed in each fin at a certain radius, different for each fin. The disc faces were flush with the fin sides. Figure 1 shows the details of the experimental fin. A length of number 23 "Comet" (nickel alloy) resistance wire was coiled and placed between thin mica sheets inside the copper disc, to act as a heating element. Potential leads were soldered to the resistance wire, and all leads were brought out of the heating element through the inside of the fiber fin, into the pipe, and through the pipe to the outside. A Weston Ammeter, with a scale from 0 to 3 amperes and a least count of 0.05 ampere, and a Jewell voltmeter

with a scale from 0 to 3 volts and a least count of .05 volts, were used to measure the power input to the heating element. The power input was varied from 0.24 to 0.73 watts during the course of the tests.

Number 28 gauge copper and constantan wires were peened into the copper as thermocouples and were insulated from the heating element circuit. A copper sheet was placed between the heating element and the thermocouple leads in order to avoid direct radiation on the hot junction. The calibration curve for the thermocouples is shown in Fig. 2. In all tests, the cold junction was kept at 32° F. The Leeds and Northrup potentiometer used to measure the thermal electromotive forces had a least count of .01 millivolt (corresponding to about 0.5° F temperature difference) in the range from 0 to 2 millivolts, and a least count of 0.1 millivolt on the high scale reading from 0 to 20 millivolts. Readings could be duplicated on the low scale to .002 millivolt.

With all leads passing from the heating disc on the inside of the fin and then through the inside of the pipe, no wires were left in the airstream. The faces of the heating disc were ground down flush with the fin faces, and all fins were given approximately the same surface finish. One fin containing a heating element was placed at the center of the

pipe, and a movable dummy fin with no heating element was placed on each side of this central fin. The spacing could thus be set at any desired value. It was realized that all of the energy released in the heater element would not pass to the air through the faces of the heater disc. There was radial heat flow into the fiber fins as well as radiation losses. Radial conduction introduced by far the larger error and was difficult to evaluate (Discussion, p. 47). The radiation losses were minimized by polishing the copper surfaces to give a high reflectivity. The upper limits to the radiation loss can be computed by considering the heating disc as radiating to a black body.

$$q = \alpha_1 \sigma_s A_1 (T_1^4 - T_2^4) \quad (4)$$

where

q = heat transfer rate (B.T.U./hr.)

α_1 = absorptivity of disc

$\sigma_s = 17.3 \times 10^{-10} \frac{\text{B.T.U.}}{(\text{ft}^2)(\text{hr})(^\circ\text{F absolute})}$

A_1 = disc surface area (ft^2)

T_1 = disc temperature ($^\circ\text{F absolute}$)

T_2 = temperature of surroundings ($^\circ\text{F absolute}$)

Assuming $\alpha_1 = .03$ (McAdams, 1933, p. 45), $T_1 = 620^\circ\text{F}$

absolute and $T_2 = 520^\circ \text{ F}$ absolute, the loss is .00175 watts, or about 0.4% of the input. This is, of course, negligible.

The fin tube assembly was set at the throat of a small wind tunnel. The whole assembly could be rotated from 0 to 180° (as defined on Fig. 1), the angle being set by means of a protractor and a pointer lined up with the heating disc on the fin. A traverse was made of only half of the fin because of the existence of a plane of symmetry passing through the 0 and 180 degree points (Biermann and Pinkel, 1934). All leads were brought out to a small switchboard on the outside of the wind tunnel.

The air speed was varied by means of rheostats in the armature and field circuits of the direct current motor which was belted to the propeller. A small dynamo with a linear voltage-speed characteristic was belted to the motor shaft, and leads were passed from its terminals to a voltmeter. By checking the voltmeter readings against the air velocities measured by a vane anemometer placed at the throat in the center of the wind tunnel cross-section, a calibration curve for the air speed as a function of the voltmeter reading was obtained and plotted on Fig. 3. The air velocity was varied from 150 to 1250 feet per minute. Figures 4, 5, 6, and 7 show the details of the experimental equipment.

It was desirable that the turbulence of the wind tunnel and the velocity distribution across the tunnel throat be known. The turbulence could be obtained from sphere tests (Durand, 1936, p. 263) except that the necessary scale balance was not available. The velocity distribution could be obtained by the use of an instrument measuring velocities at a point or over a small area, such as some form of hot wire anemometer. This latter instrument was not available, but a pitot tube was made according to N.A.C.A. specifications and used in the tunnel. Unfortunately, a manometer with a least count of .01 inch of water was the only one available. This corresponded to a velocity of about 300 feet per minute. The variations in velocity were less than 300 feet per minute over the greater part of the cross section of the throat, as no movement of the manometer could be discerned.

Three different fins, referred to in the following as Fin No. 1, Fin No. 2, and Fin No. 3, were used in the tests, with heating discs set at radii of 2.32, 1.44, and 3.50 inches respectively, measured from the center of the pipe. With each fin tested, two spacings of 0.5 and 1.0 inch were used. The spacings were measured from center to center of the fins.

Air temperature was measured by two thermometers, each with a least count of 0.1° F. One thermometer was placed

EXPLANATION OF PLATE I

- Fig. 4. General arrangement of apparatus.
- Fig. 5. View showing the position of the fins in the wind tunnel.

PLATE I

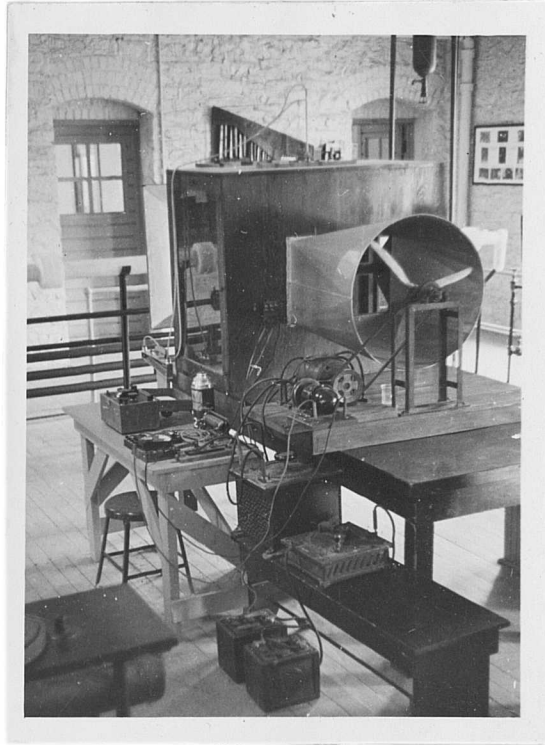


Fig. 4

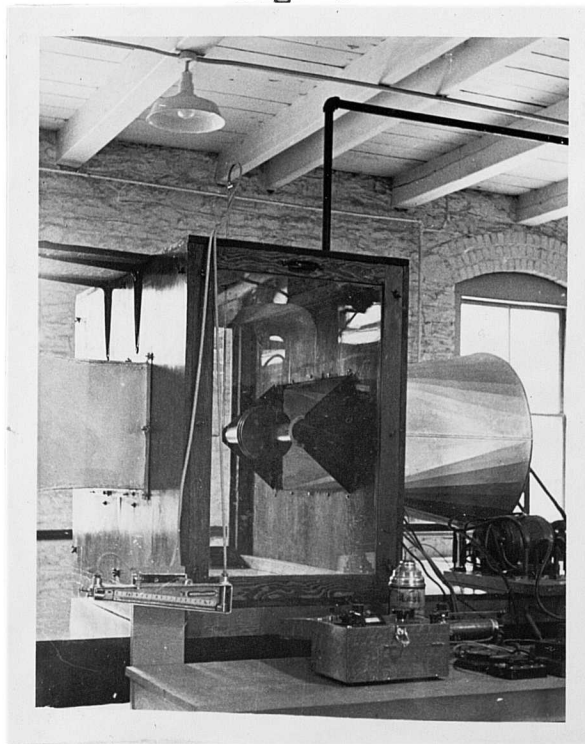


Fig. 5

EXPLANATION OF PLATE II

Fig. 6. Close-up view of the fins in the wind tunnel.

Fig. 7. View showing the position of the vane anemometer in the entrance section of the tunnel for air velocity measurements.

PLATE II

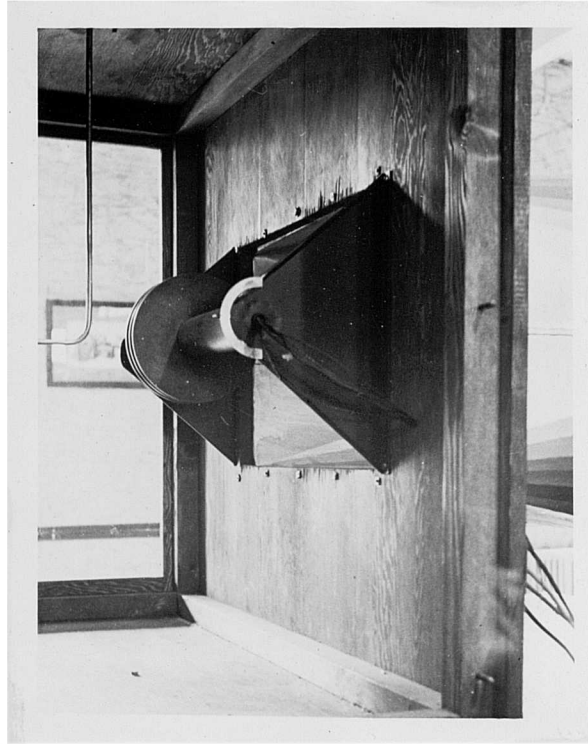


Fig. 6

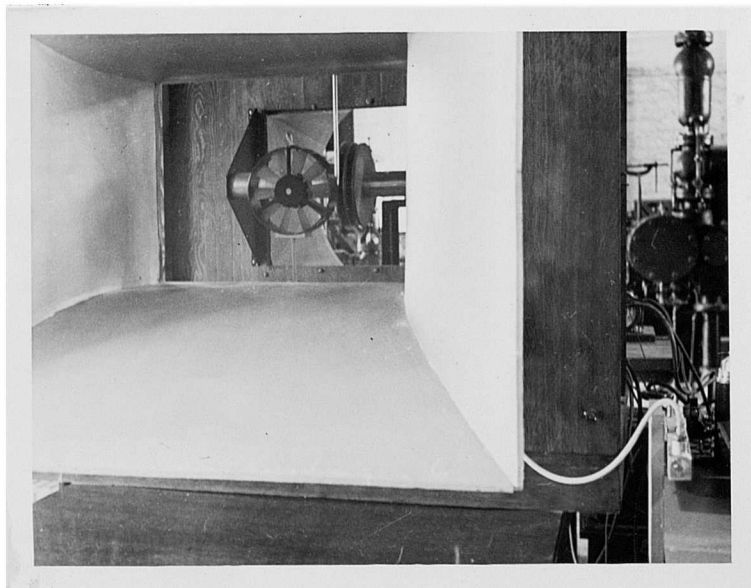


Fig. 7

about 5 feet in front of the wind tunnel, the other directly in the outlet air stream. These thermometers checked within one degree at all times, the difference being ascribed to radiation from the room lights. The thermometer in the air-stream was shielded from radiation, and its reading was taken as the air temperature.

Sample calculations for a typical run follow.

(1)	(2)	(3)	(4)	(5)	(6)
134	1210	1.747	107.1	88.4	18.7
(7)	(8)	(9)	(10)	(11)	(12)
97.8	.0364	0.497	1.695	46.6	60°
(13)	(14)	(15)	(16)	(17)	(18)
0.50	75°	70°	3.96	13.15	6.12
(19)	(20)	(21)	(22)	(23)	
.0702	84.9	1.30	1.443	20.4	

Columns (1), (2), (3), (5), (9), (12), (13), (14) are experimental data. The column headings and units employed above are as follows:

- (1) Run number
- (2) Air flow (ft. per min.)
- (3) Average thermocouple reading (millivolts)

- (4) Disc temperature ($^{\circ}\text{F}$)
- (5) Air temperature ($^{\circ}\text{F}$)
- (6) Temperature difference, air and disc ($^{\circ}\text{F}$)
- (7) Mean temperature, air and disc ($^{\circ}\text{F}$)
- (8) Heater area x temperature difference ($\text{ft}^2 \times ^{\circ}\text{F}$)
- (9) Watts input
- (10) Rate of heat transfer (B.T.U./hr.)
- (11) Heat transfer coefficient "h" $\left(\frac{\text{B.T.U.}}{(\text{hr})(\text{ft}^2)(^{\circ}\text{F})} \right)$
- (12) Angle ϕ (degrees)
- (13) Spacing S (inches)
- (14) Wet bulb air temperature ($^{\circ}\text{F}$)
- (15) Dew point of air ($^{\circ}\text{F}$)
- (16) Thermal conductivity of air "K" $\times 10^6 \left(\frac{(\text{B.T.U.})(\text{ft})}{\text{ft}^2 ^{\circ}\text{F Sec}} \right)$
- (17) $\frac{\text{Pipe Diameter}}{K} \left(\frac{(\text{hr})(\text{ft}^2)(^{\circ}\text{F})}{\text{B.T.U.}} \right)$
- (18) Nusselt's Number $\frac{"hD"}{K} \times 10^{-2}$ -- dimensionless
- (19) Mass density of air " ρ " ($\text{lb. mass}/\text{ft}^3$)
- (20) Air velocity $\times \rho \left(\frac{(\text{lb. mass})}{(\text{min})(\text{ft}^2)} \right)$
- (21) Absolute air viscosity $\mu \times 10^5 \left(\frac{\# \text{ mass}}{(\text{sec})(\text{ft})} \right)$
- (22) $\frac{D}{\mu} \times 10^{-4} \left(\frac{\text{ft}^2 \text{ sec}}{\# \text{ mass}} \right)$
- (23) Reynold's Number $\frac{VD\rho}{\mu} \times 10^{-3}$ -- dimensionless

Column (3) is referred to as an average thermocouple reading since the temperature was not exactly the same on each side of the heating element. The difference varied from zero to a maximum of 7° F. As can be seen from the sample tabulation of data, the watts input to the heater was converted to B.T.U. per hour and divided by the area of the heating element times the temperature difference between the heating disc and the air, to obtain the heat transfer coefficient h . The physical properties of air (density, thermal conductivity, absolute viscosity, and specific heat) were obtained from the sources and in the manner described in the Appendix. The density was evaluated at air temperature, while the other properties were taken at a temperature defined as the mean of the heating disc and air temperatures. This procedure has been found to give the most satisfactory correlation of data (Grimison, 1937; Boelter, 1937).

Since the data were found to be in error, none of the computations based on the experimental data were included in this thesis, but have been filed with the department of mechanical engineering, Kansas State College of Agriculture and Applied Science.

THEORY AND RESULTS

Dimensional Analysis

The use of h in convection problems arises from the concept of a thermally resistant film at the surface of the convecting material, across which the entire temperature drop between the surface and the air mass is assumed to occur. The heat transfer across the film is by conduction. Heat is removed at the air side of the film by a process of mass transfer. Air adjacent to the film absorbs the energy passing through the film. As a result the temperature of this air rises. This air, in turn, is supplanted by fresh air at the same original temperature, and the process is repeated. Comparison of the equations for conduction and convection shows that h may be defined as K/l , where K is the thermal conductivity of the air and l is the length (in the direction of heat flow) of the air film. h is therefore dependent upon the state of the air only. If it changes with a change in the rate of heat flow q , it does so only insofar as q affects the temperature, and hence the values of K and l of the air film. This is the fundamental assumption made in setting up h as a function of dimensionless criteria which do not include q or the temperature of the

convecting material. Thus,

$$h = F (V, D, S, W, T, C_p, \mu, K, \rho, r, r\phi)$$

where

V = air velocity

D = pipe diameter

S = distance between fins

W = fin diameter

T = fin thickness

C_p = specific heat of air at constant pressure

μ = absolute viscosity of air

K = thermal conductivity of air

ρ = density of air

r, ϕ = coordinates of a point on the fin surface (in consistent units)

$$h = V^a D^b S^c W^d T^e C_p^f \mu^g K^h \rho^k r^j (r\phi)^k$$

Four units will be used: M, L, T, θ

Where

M = mass

T = time

L = distance

θ = temperature difference

$$\frac{M}{T^3 \theta} = \left(\frac{L}{T}\right)^a L^b L^c L^d L^e \left(\frac{L^2}{T^2 \theta}\right)^f \left(\frac{M}{LT}\right)^g \left(\frac{ML}{T^3 \theta}\right)^h \left(\frac{M}{L^3}\right)^k L^j L^k$$

$$(1) \quad a + b + c + d + e + 2f - g + h - 3i + j + k = 0$$

$$(2) \quad g + h + i = 1$$

$$(3) \quad a + 2f + g + 3h = 3$$

$$(4) \quad f + h = 1$$

Solve for all exponents in terms of c, d, e, f, and i.

$$h = 1 - f$$

$$g = 1 - f + i = 1$$

$$f = f - i$$

$$a = 3 - 3h - g - 2f = 3 - 3 + 3f - f + i - 2f$$

$$a = i$$

$$b = 3i - 1 + f + f - i - 2f - e - d - c - i$$

$$b = i - e - d - c - 1 - j - k$$

$$h = v^i D^{1-e-d-c-1} s^c w^d T^e c_p^f \mu^{f-i} K^{1-f} \rho^i r^j (r\phi)^k$$

$$\frac{hD}{K} = \beta \left[\left(\frac{vD\rho}{\mu} \right) \left(\frac{c_p\mu}{K} \right) \left(\frac{s}{D} \right) \left(\frac{w}{D} \right) \left(\frac{T}{D} \right) \left(\frac{r}{D} \right) \phi \right]$$

where β is an unknown function, the form of which must be determined by experiment. This is as far as dimensional reasoning takes us. However, in this type of problem it has been found experimentally that Nusselt's Number $\left(\frac{hD}{K} \right)$ may usually be expressed as a constant times Reynold's $\left(\frac{vD\rho}{\mu} \right)$ and Prandtl's $\left(\frac{c_p\mu}{K} \right)$ Numbers to some power. The expression will therefore be written in that way, pending experimental verification.

$$Nu = C (Re)^a (Pr)^b \left(\frac{s}{D} \right)^c \left(\frac{w}{D} \right)^d \left(\frac{T}{D} \right)^e \left(\frac{r}{D} \right)^f \quad (5)$$

where

C, a, b, c, d, e, and f = constants.

As expected, this analysis indicates that a correlation of experimental data can be obtained by use of Nusselt's, Reynold's, and Prandtl's Numbers, plus certain ratios of linear dimensions, as dimensionless criteria.

There should also be some sort of a roughness factor included above, to allow evaluation of the effect of surface finish. As written, the effect of turbulence is not included. That is, if the above correlation is used on two different systems, the turbulence conditions must be assumed the same for each.

In general, the accuracy of the relations obtained by dimensional analysis depends upon how correctly and how completely the pertinent variables have been set down. Thus, in writing h as a function of certain variables, we are tacitly assuming that these are the only variables which affect h . Dimensional analysis shows us how these variables must be related, but it says nothing as to how complete our assumptions are.

It will be noted that, in this particular case, it has been assumed that free convection is negligible (hence Grashof's Number does not appear), and that the temperatures involved are such that radiation is not important (i.e., h is a function of $(\theta - \theta_a)$, and not of θ and θ_a separately).

Assuming, as before, that θ is a power function of these groups:

$$\theta = U \cdot \theta_0 \left(\frac{hr}{K} \right)^b \left(\frac{r\phi}{r} \right)^e ,$$

$$\frac{\theta}{\theta_0} = U \left(\frac{hr}{K} \right)^b (\phi)^e \quad (6)$$

where

$U, b, e = \text{constants.}$

This expression affords a mean of obtaining the temperature distribution in a fin if h is known at every point. With h known at any value of r and ϕ , θ and θ_0 could be measured experimentally, and the constants $U, b,$ and e determined. The equation would be, in effect, an experimental solution of the equation developed later for heat transfer in a fin. It is of interest, since the exact solution of the mathematical equation has not been obtained.

Equations of Heat Transfer in a Fin

Consider a circular metal fin, which is thin enough so that the temperature gradient in the direction of the thickness can be neglected, and a two dimensional solution can be considered. The following assumptions are made:

1. The fin material is homogeneous and isotropic.

2. There are no internal heat sources.
3. There is no temperature gradient across the fin width (in the x direction).
4. The rate of heat transfer from the metal surface to the air is proportional to the temperature difference between the metal and the air and to h .
5. The temperature distribution is symmetric with respect to the $0 - 180^\circ$ plane.
6. The conductivity coefficient K is constant throughout the fin.
7. The temperature at the fin base (where $r = r_0$) is a function of the angle ϕ .
8. The temperature at any point on the fin is independent of time.

Assumption 5 has been verified experimentally (Schey and Rollin, 1934). The other assumptions appear reasonable from physical considerations.

Let

θ = temperature at any point on fin surface

θ_a = air temperature

K = thermal conductivity of fin material

h = convection transfer coefficient

r and ϕ = polar coordinates of a point on the fin

$2x$ = fin thickness

Consider an element of fin volume, $(A_r dr)$. The rate of heat flow to and from the element is as follows:

Conduction into element:

$$q_1 = - K A_r \left(\frac{\partial \theta}{\partial r} \right)_r$$

$$q_2 = - K dr \cdot 2x \left(\frac{\partial \theta}{\partial (r\phi)} \right)_\phi$$

Conduction out of element:

$$q_3 = - K A_{(r + dr)} \left(\frac{\partial \theta}{\partial r} \right)_{(r + dr)}$$

$$q_4 = - K dr \cdot 2x \left(\frac{\partial \theta}{\partial (r\phi)} \right)_{(\phi + d\phi)}$$

Convection from element:

$$q_5 = 2h (rdrd\phi) (\theta - \theta_a)$$

Rate of heat flow in = rate of heat flow out (steady state)

$$\begin{aligned} & - K A_r \left(\frac{\partial \theta}{\partial r} \right)_r + K A_{r+dr} \left(\frac{\partial \theta}{\partial r} \right)_{r+dr} - K \cdot dr \cdot 2x \left(\frac{\partial \theta}{\partial (r\phi)} \right)_\phi \\ & + K \cdot dr \cdot 2x \left(\frac{\partial \theta}{\partial (r\phi)} \right)_{\phi + d\phi} = 2h (rdrd\phi) (\theta - \theta_a) \\ & K \cdot \frac{d}{dr} \left(A \frac{\partial \theta}{\partial r} \right) dr + K \cdot dr \cdot 2x \cdot \frac{1}{r} \frac{d}{d\phi} \left(\frac{\partial \theta}{\partial \phi} \right) d\phi = \\ & 2h (rdrd\phi) (\theta - \theta_a) \end{aligned}$$

$$\frac{d}{dr} \left(A \frac{d\theta}{dr} \right) = \frac{dA}{dr} \cdot \frac{d\theta}{dr} + A \frac{d^2\theta}{dr^2}$$

$$A = r \cdot d\phi \cdot 2x \qquad \frac{dA}{dr} = 2x \cdot d\phi$$

$$K \cdot 2x \cdot d\phi \cdot \frac{d\theta}{dr} dr + Krd\phi \cdot 2x \frac{d^2\theta}{dr^2} dr + Kdr \cdot 2x \frac{1}{r}$$

$$\frac{d^2\theta}{d\phi^2} \cdot d\phi = 2h (rdrd\phi)(\theta - \theta_a)$$

$$\frac{d\theta}{dr} + r \frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d^2\theta}{d\phi^2} = \frac{hr}{Kx} (\theta - \theta_a)$$

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} + \frac{1}{r^2} \frac{d^2\theta}{d\phi^2} = \frac{h}{Kx} (\theta - \theta_a) \quad (7)$$

which is the desired relationship.

The air becomes progressively warmer as it passes over an actual fin, so that the heat transfer from the trailing section should be poorer than for the front section. As an approximation, however, in the interests of a solution of the equation, the variation in the air temperature over the fin surface will be neglected. θ_a may then be taken as zero, and θ becomes the fin temperature measured above this arbitrary datum.

h is not a constant, and it evidently must be expressed as a function of the independent variables r and ϕ if a solution is to be obtained. When air flows perpendicular to the tube, a vortical section is set up behind the pipe, in which region the heat transfer is less than for a surface over which a boundary layer exists.

It would therefore be expected that h would be a minimum near 180° . Reiher (1926) obtained data which would seem to favor this distribution. However, measurements made by Drew and Ryan (1931) of the average rates of heat transfer from longitudinal strips of a cylinder, the walls of which were maintained at a constant temperature, showed h to be a maximum for $\phi = 0^\circ$ and 180° , while it is a minimum at about $\phi = 90^\circ$. The surface from $\phi = 180^\circ$ to $\phi = 360^\circ$ gave similar results. These data were checked by Lohrisch (McAdams, 1933, p. 214) with a different experimental technique. The experiments made by the writer gave distributions which conformed more closely with Reiher's results. The discrepancy has not been explained, although it was mentioned by Drew and Ryan. The results of the experimental work done in connection with this thesis will therefore be used, and the heat transfer coefficient will be written as

$$h = Ar \left(1 - b \sin^2 \frac{\phi}{2} \right) \quad (8)$$

where A and $b = \text{constants}$.

The solution of equation (7) with this value for h substituted could not be obtained in the time available. It is doubtful that it can be obtained in terms of known functions². A method of approximations was also attempted, and it failed in this case.

The next possibility considered was to express h as a function of r only. This held with fair accuracy over small segments of the fin. The fin could be considered as split up into several segments, over each of which a different functional relationship between h and r might be assumed to hold. The heat transfer for each segment could be obtained, and the results added to give the total heat transfer. It was found experimentally (Fig. 8) that h could be expressed as a power function of r .

With this assumption, the equation for heat transfer in the fin becomes

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} - cr^n \theta = 0 \quad (9)$$

The solution could not be obtained in terms of known functions, but a series expression was found. The details of the solution are shown in the Appendix.

²Personal communication from Professor C. H. Morrey, Mathematics Department, University of California, Berkeley.

$$\theta = \frac{M\theta_1}{MN - 0} \left[1 + \frac{cr^{n+2}}{(n+2)^2} + \frac{c^2 r^{2(n+2)}}{2^2(n+2)^4} + \frac{c^3 r^{3(n+2)}}{2^2 \cdot 3^2(n+2)^6} + \dots \right]$$

$$- \frac{\theta_1}{MN-0} \left[r + \frac{c^2 r^{2n+5}}{(n+3)^2(2n+5)^2} + \frac{c^3 r^{3n+7}}{(n+3)^2(2n+5)^2(3n+7)^2} + \dots \right] \quad (10)$$

Then

$$dQ = 2h \theta \, dA$$

and

$$Q = 2c\phi\theta_1 \left\{ \frac{M}{MN-0} \left[\frac{r_o^{n+2} - r_i^{n+2}}{n+2} + \frac{c(r_o^{2(n+2)} - r_i^{2(n+2)})}{2(n+2)^3} \right. \right.$$

$$+ \left. \frac{c^2(r_o^{3(n+2)} - r_i^{3(n+2)})}{2^2 \cdot 3(n+2)^5} + \dots \right] - \frac{1}{MN-0} \left[\frac{r_o^{n+3} - r_i^{n+3}}{n+3} \right.$$

$$+ \left. \frac{c(r_o^{3n+7} - r_i^{3n+7})}{(n+3)^2(2n+5)^2(3n+7)} + \frac{c^2(r_o^{4n+9} - r_i^{4n+9})}{(n+3)^2(2n+5)^2(3n+7)^2(4n+9)} + \dots \right] \quad (11)$$

where

r_o = outer fin radius

r_i = inner fin radius

$M, N, 0$ = constants defined as follows:

$$M = \frac{1 + \frac{c^2 r_o^2 (n+2)}{(n+3)^2 (2n+5)} + \frac{c^3 r_o^3 (n+2)}{(n+3)^2 (2n+5)^2 (3n+7)} + \dots}{\frac{c r_o^{n+1}}{n+2} + \frac{c^2 r_o^{2n+3}}{2(n+2)^3} + \frac{c^3 r_o^{3n+5}}{2^2 \cdot 3(n+2)^5} + \dots}$$

$$N = 1 + \frac{c r_i^{n+2}}{(n+2)^2} + \frac{c^2 r_i^2 (n+2)}{2^2 (n+2)^4} + \frac{c^3 r_i^3 (n+2)}{2^2 \cdot 3^2 (n+2)^6} + \dots$$

$$O = r_i + \frac{c^2 r_i^{2n+5}}{(n+3)^2 (2n+5)^2} + \frac{c^3 r_i^{3n+7}}{(n+3)^2 (2n+5)^2 (3n+7)^2} + \dots$$

If h is considered constant over the fin, the equation for heat transfer is simpler still.

The general equation for heat transfer by conduction is (Boelter, 1937):

$$\gamma c_p \frac{d\theta}{dt} = K \Delta^2 \theta + W$$

where

γ = specific weight of conducting material

c_p = specific heat of conducting material

t = time

W = heat flow per unit volume due to an internal source or sink

$\Delta^2 \theta$ = the Laplacian operator.

For steady state conditions, the equation as applied to a fin reduces to

$$\frac{K}{\gamma c_p} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) - \frac{1}{\gamma c_p} \frac{h \theta}{x} = 0$$

where

$$- \frac{h \theta}{x} = W$$

Clearing terms, we have

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} - \frac{h}{Kx} \theta = 0 \quad (13)$$

$$\text{Let } B^2 = \frac{h}{Kx}$$

The solution of equation (13) is (Gray, Matthews, and MacRobert, 1931, p. 161)

$$\theta = A_1 I_0 (Br) + B_1 K_0(Br) \quad (14)$$

where

$I_0 (Br)$ = modified Bessel function of the first kind and order zero.

$K_0 (Br)$ = modified Bessel function of the second kind and order zero.

Using the boundary conditions

$$r = r_1 \quad \theta = \theta_1 \quad (15)$$

$$r = r_0 \quad \frac{\partial \theta}{\partial r} = 0$$

and evaluating the constants A_1 and B_1 , the solution becomes

$$\theta = \theta_1 \left[\frac{K_1 (Br_0) I_0(Br) + I_1(Br_0) K_0(Br)}{K_1 (Br_0) I_0(Br_1) + K_0(Br_1) I_1(Br_0)} \right] \quad (16)$$

Then the rate of heat transfer is

$$Q = 2 \int_A h \theta \, dA$$

and substituting for θ from equation (16) we obtain

$$Q = \frac{4 \pi h \theta_1 r_1}{B} \left[\frac{I_1(Br_0) K_1(Br_1) - K_1(Br_0) I_1(Br_1)}{I_0(Br_1) K_1(Br_0) + K_0(Br_1) I_1(Br_0)} \right] \quad (17)$$

It will be noted that the expression in brackets is dimensionless. When the temperature θ_1 and coefficient of heat transfer h are assumed to be constant, Q becomes a function of r only.

Murray (1938) obtained a solution assuming θ_1 to be a function of ϕ . The expression was complicated, and he substituted $\theta_1 = \text{constant}$ into his general expression and obtained equation (17). He also pointed out the approximate boundary condition (15) above, which simplifies the solution greatly. The actual boundary condition is that all heat taken up to the outer edge of the fin by conduction is carried away by convection, or mathematically

$$K \frac{d\theta}{dr} + h\theta = 0 \quad (18)$$

The error introduced by boundary condition (15) depends upon the magnitude of gradient at $r = r_0$. If it is small, the error is not large.

Numerical computations with equations (11) and (17) are shown in the Appendix.

Murray also pointed out that a fin effectiveness factor may be defined as the ratio of the heat transferred by the fin to the heat which would be transferred by the pipe area under the fin if the fin were not present. Using equation (17) as the expression for the heat transferred by the fin, the fin effectiveness n becomes

$$n = \frac{2}{Bx} \left[\frac{I_1(Br_0)K_1(Br_1) - K_1(Br_0)I_1(Br_1)}{I_1(Br_0)K_0(Br_1) - K_1(Br_0)I_0(Br_1)} \right] \quad (19)$$

It will be noted that the expression is dimensionless and is independent of the temperature at the inner fin radius. Murray obtained a solution for the case where the temperature at the inner fin radius is not constant, but varies with angular position. The fin effectiveness reduced to equation (19) for that case also, and he concluded that the effectiveness was independent of the temperature distribution around the inner fin radius.

EXPERIMENTAL FINDINGS

Figures 9 and 10 show Nusselt's Number $\frac{hD}{K}$ plotted as a function of Reynold's Number $\frac{VD\rho}{\mu}$ for Fin No. 1. Figures 11 and 12 are cross-plotted from these and from similar curves for the other fins. They show the variation in Nusselt's Number, and therefore the approximate variation in the heat transfer coefficient, with fin position and fin spacing.

DISCUSSION

Figures 9 and 10 indicate that a flow transition occurs between Reynold's Numbers of about 6000 and 9000. The nature of the transition is not clear. Similar experiments have never been attempted, to the writer's knowledge, and there is therefore no comment in the literature concerning the phenomenon. It is known that the flow over a cylinder changes in character at a Reynold's Number of about 30,000, while the flow over a skin friction plate changes at a Reynold's Number, based on the distance from the leading edge, of from 100,000 to 1,000,000 (depending on the initial turbulence of the air stream). The fin was not a skin friction plate, due to its appreciable thickness. It is likely that the transition is more closely related to the thin plate phenomenon than to that of flow over a cylinder,

particularly since data obtained for the front half of the fin shows the same effect. This may be seen from a study of Fig. 9 and Fig. 10.

Figure 12 indicates that the effect of fin spacing is negligible over the range of spacings encountered in commercial work.

The average values of $\frac{hD}{K}$ for the 0.5-inch spacing may be expressed as a function of Reynold's Number as follows:

$$\text{Fin No. 1} \quad \frac{hD}{K} = 270 \left(\frac{VD\rho}{\mu} \right)^{0.21} \quad (20)$$

$$\text{Fin No. 2} \quad \frac{hD}{K} = 318 \left(\frac{VD\rho}{\mu} \right)^{0.22} \quad (21)$$

$$\text{Fin No. 3} \quad \frac{hD}{K} = 230 \left(\frac{VD\rho}{\mu} \right)^{0.27} \quad (22)$$

The effect of fin spacing was found to be small, and equations (20), (21), and (22) would be expected to hold for all spacings in the commercial range.

A Reynold's Number greater than 7000 is seldom encountered in air conditioning work (Tuve and Siegel, 1938). The values of h are not reported in the literature in terms of Nusselt's Number over this range, but are given in the form

$$h = CV^n$$

where C and n are constants. Equations (20), (21), and (22) therefore could not be checked directly. It was noted, however, that the experimental values of h were from two to three times those given in the literature. Also, it was known that the slopes of the $\frac{hD}{K}$ versus $\frac{VD\rho}{\mu}$ lines should be approximately 0.55 (McAdams, 1933, p. 233). These facts indicated that the loss of heat by radial conduction from the heating disc into the fiber fin was far from negligible as had been supposed. The results as summarized by equations (20), (21), and (22), therefore, cannot be used for design purposes.

The losses can be computed by means of equation (17). A sample computation will be shown using the data for run 185. For an air velocity of 500 feet per minute, Kent (1936, p. 3:30) shows $h = 6$ for tubes and $h = 3$ for smooth plane surfaces. It would be expected, therefore, that h would be approximately 4 for Fin No. 1. Using this value, we have:

$$r_i = \frac{0.375}{2} = 0.188 \text{ inches} \quad r_o = 0.661 \text{ inches}$$

$$B = \sqrt{\frac{h}{Kx}} = \sqrt{\frac{2 \times 4}{.03 \times 0.27 \times 12}} = 9.07 \text{ inch}^{-1} \text{ units}$$

$$K = .03 \frac{(\text{B.T.U.})(\text{ft})}{(\text{hr})(\text{ft}^2)(^{\circ}\text{F})} \quad (\text{McAdams, 1933, p. 316})$$

$$\text{Br}_1 = 1.71 \qquad \text{Br}_0 = 6.0$$

$$\theta_1 = 115^{\circ} \text{ F}$$

$$Q = \frac{4\pi h \theta_1 r_1}{B} \cdot \frac{I_1(\text{Br}_0)K_1(\text{Br}_1) - K_1(\text{Br}_0)I_1(\text{Br}_1)}{I_0(\text{Br}_1)K_1(\text{Br}_0) + K_0(\text{Br}_1)I_1(\text{Br}_0)}$$

$$= \frac{4\pi \times 4 \times 115 \times 0.188}{9.07 \times 144} \cdot \frac{61.34 \times 0.1853 - .001340 \times 1.208}{1.876 \times .001340 + 0.1478 \times 61.34}$$

$$= 0.832 \times 1.255 = 1.05 \text{ B.T.U./hr.}$$

Similar computations were made for other values of r_0 , and the results are shown on Fig. 13. It will be noted that the increase in the heat loss is small if r_0 is larger than 0.45 inches. Therefore this value will be taken as defining the area of fin around the heating element from which the heat loss is important. From run 185, we have

$$Q = 1.757 \text{ B.T.U./hr.}$$

$$A\Delta t = .0504 \text{ ft}^2 \text{ } ^{\circ}\text{F.}$$

Then the actual heat transfer through the disc, assuming as a first approximation that h for the fiber is 4, is

$$Q_D = 1.757 - 1.05 = 0.71 \text{ B.T.U./hr.}$$

and

$$h = \frac{QD}{A\Delta t} = \frac{0.71}{.0504} = 14.1 \frac{\text{B.T.U.}}{(\text{hr})(\text{ft}^2)(^{\circ}\text{F})}$$

If we assume that the area from which the heat loss is important remains the same for different values of h over the range to be considered, other values of h may be used to obtain the heat loss. When the h computed as above checks the assumed h , we may assume the value used to be correct. Following this procedure, we have

<u>Assumed h</u>	<u>Heat Loss</u>	<u>Computed h</u>
8.0	1.40	7.1
7.0	1.322	8.6

Evidently the correct value of h under these conditions is between 7 and 8.

It may be seen, therefore, that the losses into the fiber fin were of great importance, and the reason for the incorrectness of the data is apparent. Theoretically, all of the data could be corrected by the above procedure. Actually, the procedure is excessively complicated and tedious; also, the correction is only an approximation.

The fact that the losses were large does not, however, nullify the value of Fig. 12 in showing the effect of fin spacing on h . For when it was found experimentally that at a given velocity and for a constant value of heat input to

the element the fin temperature remained constant with changes in fin spacing, then it was concluded that h actually was constant under these conditions. That is, while the data showing the variation in h was distorted due to losses, the data showing h to be constant meant that h actually was constant. Unfortunately, only two fin spacings were used. There is, however, no reason to expect that a maximum or minimum value of h will occur at intermediate spacings, and it is believed that the conclusions reached are valid.

Figure 8, obtained by cross-plotting from Fig. 12, shows h plotted as a function of r at a given Reynold's Number. The fact that the graph is a straight line on log - log paper indicates that h can be represented by an equation of the form

$$h = cr^n$$

at a given Reynold's Number and for a given angular position. For the particular conditions shown on Fig. 8, different constants could be used for the segments from 0° to 30° , from 30° to 90° , from 90° to 150° , and from 150° to 180° (and for the corresponding segments from 180° to 360°). It must be emphasized again that, although the data do show that h can be expressed as a function of r under these conditions, the constants obtained from these graphs are in

error and cannot be used in equation (11) directly.

CONCLUSIONS

The theoretical and experimental findings are summarized as follows.

The heat transfer coefficient h for a fin may be correlated in terms of dimensionless groupings including Nusselt's Number, Reynold's Number, Prandtl's Number, and ratios of dimensions (p. 31).

The general solution of the equation for heat transfer

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} + \frac{1}{r^2} \frac{d^2\theta}{d\phi^2} = \frac{h}{Kx} \theta$$

where

$$h = A(1 - b \sin^2 \frac{\phi}{2})$$

with the boundary conditions

$$\begin{aligned} r = r_1 & \quad \theta = f(\phi) \\ r = r_0 & \quad K \frac{d\theta}{dr} + h\theta = 0 \end{aligned}$$

is excessively complicated if not unattainable. Assuming the boundary conditions to be

$$\begin{aligned} r = r_1 & \quad \theta = \theta_1 \\ r = r_0 & \quad \frac{d\theta}{dr} = 0 \end{aligned}$$

and taking $h = cr^n$, the solution may be obtained (p. 40).

With these approximate boundary conditions, and with the further assumption of $h = \text{constant}$, the resulting solution (p. 43) is relatively simple and convenient to use.

The fin effectiveness factor (p. 44) may be used for design purposes. The experimental data indicated that the effect of fin spacing on the factor is negligible over the range of spacings used commercially.

The experimental data show a flow transition occurring for values of Reynold's Number between 7000 and 9000. This is the upper limit of the range used commercially, but it indicates that the slope of the Nusselt's Number versus Reynold's Number curve must be changed for a Reynold's Number above 9000. The thickness of the fin will doubtless have an effect on the air velocity at which the transition occurs. For true similarity, the model fin should perhaps have the same thickness as the prototype. It is probable that the transition range will be different for an actual fin.

If further tests on the local heat transfer coefficients on fins are attempted, it is suggested that

1. The whole fin be maintained at the same temperature as the test section. This procedure will obviate the losses which have been seen to exert a large influence on the data.

2. The model fin be made the same thickness as the prototype.

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APPENDIX

The Effect of Water Vapor in the Air

Although this research was intended for applications in air-conditioning work, the method of attack used was perfectly general and any results obtained could be used for all heat transfer problems involving finned surfaces. In considering air-conditioning work, however, the effect of the added variable of moisture in the air must be considered. The problem will be discussed under two main headings: Case I -- No Dehumidification; Case II -- Dehumidification.

Case I -- No Dehumidification. Any air stream contains a certain amount of moisture. For a given pressure and with a fixed weight of moisture in a sample of air, there is a definite temperature at which the air becomes saturated with water vapor, and further removal of heat at that temperature results in condensation of the moisture. In commercial coils used for comfort air-conditioning, at least a part of the coils are covered with moisture, this process of condensing water vapor being referred to as dehumidification of the air. Under Case I, it will be assumed that the initial air conditions and the fin temperatures

are such that there is no loss of water vapor -- i.e., dehumidification does not occur. It then becomes important to determine if the presence of moisture in the air has any effect on the heat transfer coefficient under these conditions.

The heat transfer coefficient has been set up as a function of certain variables. Of the groupings obtained by dimensional analysis, only two are a function of the air temperature; Reynold's and Prandtl's Numbers. These involve c_p , μ , K , and ρ , the specific heat, viscosity, thermal conductivity, and density of the air, respectively. The variation in these quantities will fix the variation in h for differing air moisture contents. No additional variables will be introduced, and it is therefore concluded that the dimensionless criteria already set up will adequately determine the behavior of h when no dehumidification occurs. It will, however, be necessary to consider the effect of water vapor upon the values of c_p , μ , K , and ρ .

It is pointed out, also, that there should be some sort of roughness factor included to allow evaluation of the effect of different surface finishes. However, most of the coils used commercially have about the same surface roughness, and it is expected that the experimental results on one coil may be used for purposes of design, with the

introduction of only a small error.

Case II -- Dehumidification. When dehumidification occurs, the cooling surface becomes covered with moisture, and h will obviously be changed from its value for a dry surface. The moisture may also condense so as to form a film (referred to in the literature as film condensation) or to leave irregularly sized drops (dropwise condensation). It is to be expected that the heat transfer coefficient will be different for each of these cases.

Computation of the Properties of Humid Air

It was necessary to determine c_p , μ , ρ , and K for the air-water vapor mixtures which were encountered.

Specific Heat c_p . Values for the humid specific heat c_p are tabulated by Goodman (1938). They have been calculated for a saturated mixture at the dew point by means of the following formula:

$$c_p = 0.24 + 0.45w$$

where

c_p = B.T.U. per lb. dry air per $^{\circ}$ F.

w = lbs. water vapor per lb. of dry air.

The first term is recognized as the average value of the specific heat of dry air over the temperature range to

be used (McAdams, 1933, p. 337). The coefficient of the second term is empirical, and is based upon the Keenan and Keyes (1936) steam tables. It is evidently an average value of the specific heat of saturated vapor in the lower temperature range. Thus when the dew point is selected, the value of c_p is fixed, and may be used over the dry bulb temperature range as an average value.

Density ρ . Values of the specific volume of dry air, based on the perfect gas laws, are tabulated by Goodman (1938). Also, values of a factor by which the specific volumes of dry air are to be multiplied to obtain specific volumes of humid air are given for various dew point temperatures. These factors were computed by determining the partial pressures of air and water vapor for various mixtures and correcting the specific volume of the dry air according to the perfect gas laws.

The density was computed as $\rho = \frac{1}{V}$.

Absolute Viscosity μ . Values of μ for dry air and for water vapor are given by McAdams (1933, p. 341) and by Keenan and Keyes (1936, p. 76). Humid air is a solution of water vapor in air, and its viscosity was computed on the assumption that it is an ideal solution, and that viscosity is a property which depends only on the number of molecules

per unit volume of the solute and solvent. This may be subject to error, but the water vapor is only a small percentage of the total weight of solution; furthermore, the viscosities of air and water vapor are of the same order of magnitude. Hence the following formula was used:

$$\mu_{\text{mixture}} = (\mu_{\text{air}}) (N_{\text{air}}) + (\mu_{\text{water vapor}})(N_{\text{w.v.}})$$

where N is the mole fraction of the substance.

Thermal Conductivity K. Data on the thermal conductivity of gases and vapors is meager, and is probably accurate within only 7% (McAdams, 1933, p. 323; Keenan and Keyes, 1936, p. 23). K for air is given by McAdams within the desired temperature limits as:

$$K = K_{32} \frac{492 + 225}{T + 225} \left(\frac{T}{492} \right)^{\frac{3}{2}}$$

where

$$K_{32} = .0129 \frac{(\text{B.T.U.})(\text{ft})}{(\text{ft}^2)(^{\circ}\text{F})(\text{hr})}$$

T = temperature in $^{\circ}\text{F}$ absolute

Only two values of K for water vapor are given:

$$K_{115} = 0.0104$$

$$K_{212} = 0.0126$$

Assuming that K for water vapor varies directly with

temperature (as is true for liquid water), the formula is:

$$K_{w.v.} = At + B$$

where A and B are determined from the known values of K_{115} and K_{212} , giving:

$$K_{w.v.} = .0000227 t + .0078$$

where t is temperature in $^{\circ}\text{F}$.

Making the same assumptions as to the nature of this property and of the solution as in determining μ , the equation becomes:

$$K \text{ mixture} = K \text{ air } N \text{ air} + K_{w.v.} N_{w.v.}$$

General. In all cases, the total pressure was taken as 14.7 lbs. per sq. in.; all quantities were computed for dry bulb temperatures from 40°F to 100°F , and for a dew point range from 30°F to 80°F .

The units used were as follows:

$$v = \frac{\text{ft}}{\text{sec}}$$

$$\rho = \frac{\# \text{ mass}}{\text{ft}^3}$$

$$\mu = \frac{\# \text{ mass}}{(\text{sec})(\text{ft})}$$

$$c_p = \frac{(\text{B.T.U.})}{(\# \text{ mass})(^{\circ}\text{F})}$$

$$K = \frac{(\text{B.T.U.})(\text{ft})}{(^{\circ}\text{F})(\text{ft}^2)(\text{sec})}$$

Results of Calculations. Calculations for μ showed that for low moisture contents (Dew Point = 30° F) the viscosity of the mixture was equal to that of the air, to the number of significant figures shown. Also, the variation in viscosity with moisture content over a range of dew points from 30° to 80° F was found to be less than 3%. Hence, the following values for μ taken for a mixture with a 50° F dew point were used over the entire experimental range.

Dry Bulb (°F)	50	60	70	80	90	100	110	120
$\mu \times 10^5$	1.22	1.24	1.26	1.28	1.29	1.30	1.31	1.32

It was also found that K for the mixture differed from K_{air} by a maximum of less than 1%. Values of K_{air} were used over the entire range of dew-point temperatures.

Calculations showed that the variation of $\frac{c_p \mu}{K}$, Prandtl's Number, is negligible for a large change in dry bulb temperatures. The following values were used for a dry bulb temperature range of 50 to 80° F.

Dew Point (°F)	30	40	50	60	70	80
Prandtl's Number	0.80	0.80	0.81	0.81	0.82	0.83

For a dry bulb temperature range of 80° to 100° F:

Dew Point (°F)	30	40	50	60	70	80
Prandtl's Number	0.80	0.80	0.80	0.81	0.81	0.82

Values of the kinematic viscosity $\frac{\mu}{\rho}$ are plotted on Fig. 14 for dry bulbs from 40° to 100° F and for dew points from 30° to 80° F, in 10° F intervals.

In computing experimental data, however, it was found advisable to compute ρ at the average air temperature, and c_p , μ , and K at the mean temperature of the air and the fin surface (Grimison, 1937). Accordingly, Fig. 14 was not used in the calculations.

Solution of the Heat Transfer Equation
with $h = a$ Function of r

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} - cr^n \theta = 0$$

Let

$$\theta = \sum a_i r^i$$

Then

$$\theta'' + \frac{1}{r} \theta' - cr^n \theta = \sum a_i i^2 r^{i-2} - ca_i r^{n+1} = 0$$

$$\therefore a_{n+i+2} = \frac{Ca_i}{(n+i+2)^2}$$

Take $i = 0$

$$a_{n+2} = \frac{Ca_0}{(n+3)^2} \qquad a_{2n+4} = \frac{Ca_{n+2}}{2(n+2)^2} = \frac{C^2 a_0}{2^2 (n+2)^4}$$

Take $i = 1$

$$a_{n+3} = \frac{Ca_1}{(n+3)^2}, \quad a_{2n+5} = \frac{Ca_{n+3}}{(2n+5)^2} = \frac{C^2 a_1}{(n+3)^2 (2n+5)^2}$$

$$\theta = a_0 \left[1 + \frac{Cr^{n+2}}{(n+2)^2} + \frac{C^2 r^{2n+4}}{2^2 (n+2)^4} + \frac{C^3 r^{3n+6}}{2^2 3^2 (n+2)^6} \dots \right]$$

$$+ a_1 \left[r + \frac{C^2 r^{2n+5}}{(n+3)^2 (2n+5)^2} + \frac{C^3 r^{3n+7}}{(n+3)^2 (2n+5)^2 (3n+7)^2} \dots \right]$$

Using boundary conditions

$$r = r_1 \quad \theta = \theta_1$$

$$r = r_0 \quad \frac{\partial \theta}{\partial r} = 0$$

$$a_0 = -a_1 \left[\frac{1 + \frac{C^2 r_0^{2n+4}}{(n+3)^2 (2n+5)^2} + \frac{C^3 r_0^{3n+6}}{(n+3)^2 (2n+5)^2 (3n+7)^2} \dots}{\frac{C r_0^{n+1}}{n+2} + \frac{C^2 r_0^{2n+3}}{2(n+2)^3} + \dots} \right]$$

$$= -a_1 (M)$$

$$\theta_1 = -a_1 M \left[1 + \frac{Cr_1^{n+2}}{(n+2)^2} + \frac{C^2 r_1^{2n+4}}{2^2 (n+2)^4} + \dots \right]$$

$$+ a_1 \left[r_i + \frac{c^2 r_i^{2n+5}}{(n+3)^2 (2n+5)^2} + \frac{c^3 r_i^{3n+7}}{(n+3)^2 (2n+5)^2 (3n+7)^2} + \dots \right]$$

$$= a_1 (0 - MN)$$

$$a_1 = \frac{\theta_i}{0 - MN}$$

$$\theta = \frac{M\theta_i}{MN-0} \left[1 + \frac{Cr^{n+2}}{(n+2)^2} + \frac{c^2 r^{2n+4}}{2^2 (n+2)^4} + \dots \right]$$

$$- \frac{\theta_i}{MN-0} \left[r + \frac{c^2 r^{2n+5}}{(n+3)^2 (2n+5)^2} + \frac{c^3 r^{3n+7}}{(n+3)^2 (2n+5)^2 (3n+7)^2} + \dots \right]$$

$$Q = 2 \int_A h \theta dA = 2C\phi \int_{r_i}^{r_o} \theta r^{n+1} dr$$

$$Q = 2C\phi\theta_i \left\{ \frac{M}{MN-0} \left[\frac{r_o^{n+2} - r_i^{n+2}}{n+2} + \frac{c(r_o^{2(n+2)} - r_i^{2(n+2)})}{2(n+2)^3} \right. \right.$$

$$\left. \frac{c^2(r_o^{3(n+2)} - r_i^{3(n+2)})}{2^2 \cdot 3(n+2)^5} \right] + \frac{1}{MN-0} \left[\frac{r_o^{n+3} - r_i^{n+3}}{n+3} + \right.$$

$$\left. \frac{c^2(r_o^{3n+7} - r_i^{3n+2})}{(n+3)^2 (2n+5)^2 (3n+7)} + \frac{c^3(r_o^{4n+9} - r_i^{4n+9})}{(n+3)^2 (2n+5)^2 (3n+7)^2 (4n+9)} + \dots \right\}$$

Comparison of Heat Transfer Equations

From Fig. 8, $h = 17.0 (r)^{-0.2}$ for $\phi = 180^\circ$

$$C = \frac{17.0}{Kx} = \frac{17.0 \times 12}{225 \times .01} = 90 \text{ feet}^{-1} \text{ units}$$

$$Cr^n = 90r^{-0.2}$$

$$r_o = .0625 \text{ feet}$$

$$r_i = .0252 \text{ ft.}$$

$$M = \frac{1 + \frac{8100 \times .0000461}{7.7 \times 4.6} + \frac{729,000 \times .000000313}{7.7 \times 2.1 \times 6.4}}{\frac{90 \times 0.109}{1.8} + \frac{8100 \times .00074}{2 \times 5.80} + \frac{729,000 \times 0.953 \times 10^{-7}}{12 \times 18.75}}$$

$$= \frac{1 + 0.00104 + .000219}{6.05 + 0.517 + .00031} = \frac{1.0013}{6.567} = 0.1523$$

$$N = 1 + 0.0371 + 0.00035 = 1.037$$

$$O = 0.0252 + 0.00026 = 0.0255$$

$$O - MN = .0255 - (0.1523)(1.037) = - 0.1325$$

$$Q = 2 \times 90 \times \phi \times \theta_1 \left[\frac{0.1523}{0.1325} (.00304 + .000618 + .0000113) \right.$$

$$\left. - \frac{1}{0.1325} (.000134 + .000000008) \right]$$

$$= 2 \times 90 \times \phi \times \theta_1 (.00321)$$

For the sake of comparison, let $\phi = 2\pi$ and $\theta_1 = 200^\circ\text{F}$. It is only desired to demonstrate that the two equations give results of the same order of magnitude.

$$Q = 2 \times 90 \times 2\pi \times 200 \left[.00321 \right] = 730 \text{ B.T.U./hr.}$$

Equation (17), with h equal to a constant which is the arithmetic mean of the maximum and minimum values of the variable h, yields

$$Q = \frac{4\pi h \theta_1 r_1}{B} \left[\frac{I_1(\text{Br}_0)K_1(\text{Br}_1) - K_1(\text{Br}_0)I_1(\text{Br}_1)}{I_0(\text{Br}_1)K_1(\text{Br}_0) + K_0(\text{Br}_1)I_1(\text{Br}_0)} \right]$$

Let $h = 22$

Then

$$B = \sqrt{\frac{h}{Kx}} = \sqrt{\frac{22 \times 12}{225 \times .01}} = 10.85 \text{ feet}^{-1} \text{ units}$$

$$\text{Br}_0 = 0.733$$

$$\text{Br}_1 = 0.296$$

$$Q = \frac{4\pi \times 22 \times 200 \times .0252}{10.85} (0.68)$$

$$= 870 \text{ B.T.U./hr.}$$

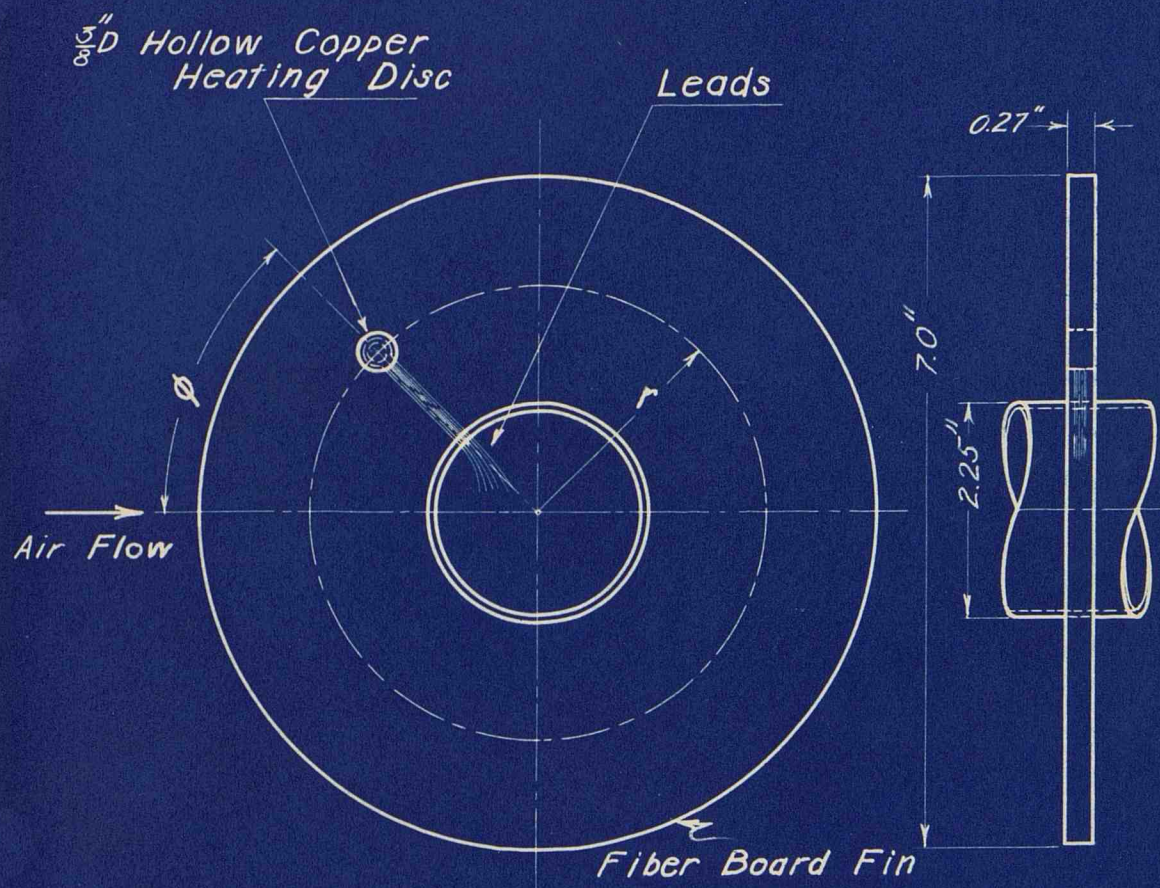


Fig. 1. The experimental fin.

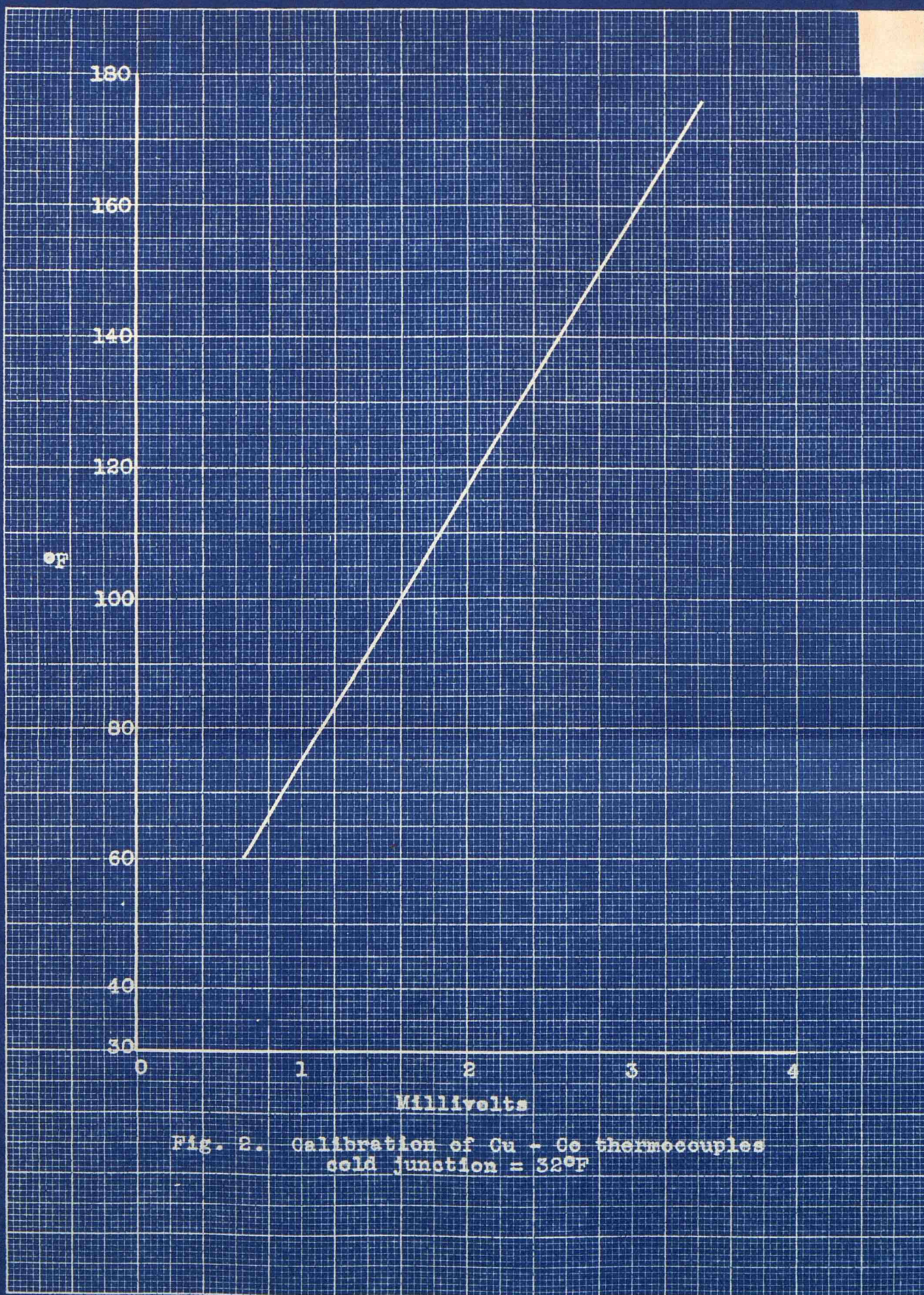


Fig. 2. Calibration of Cu - Co thermocouples
cold junction = 32°F

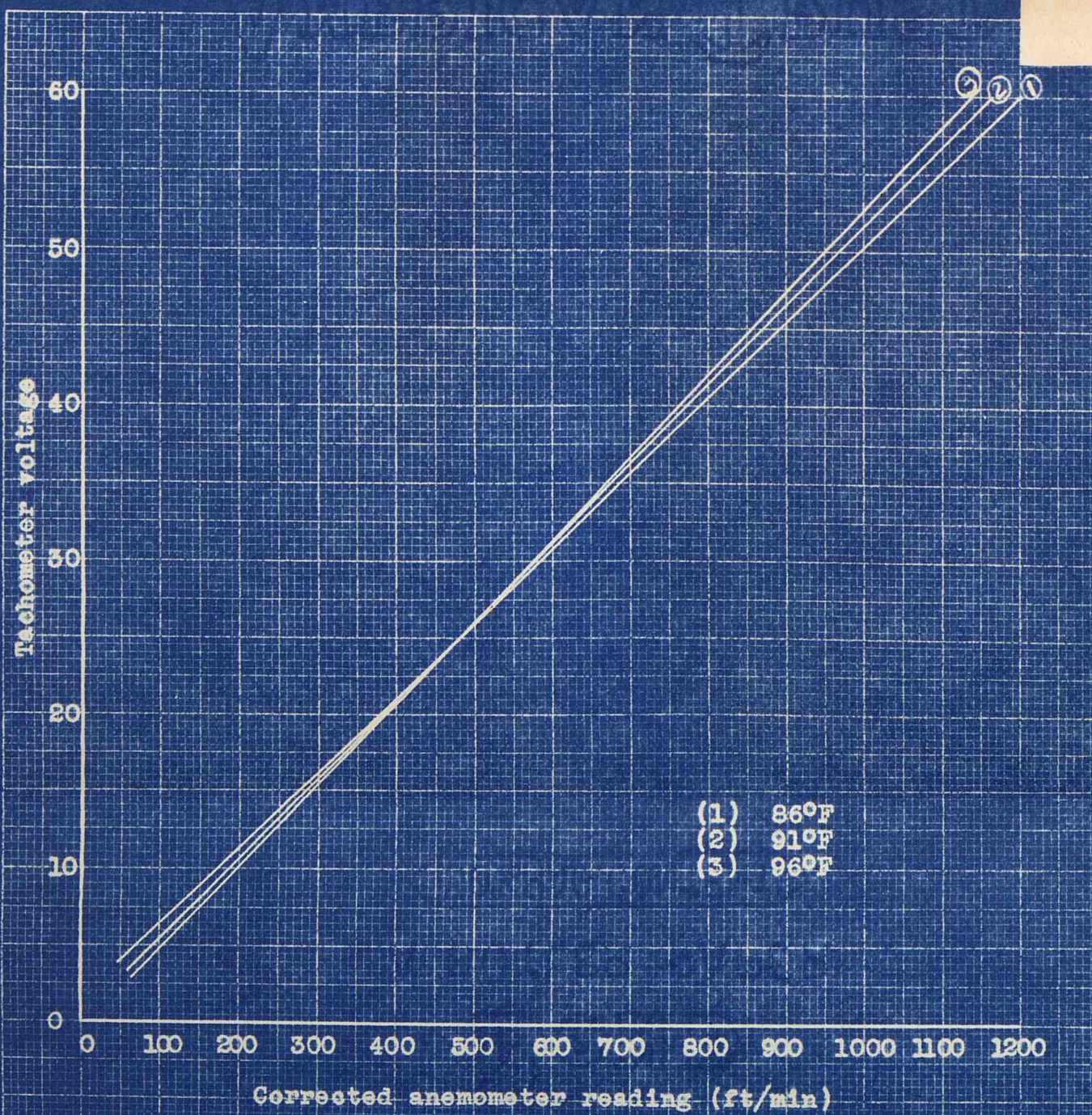


Fig. 3. Anemometer - tachometer calibration.

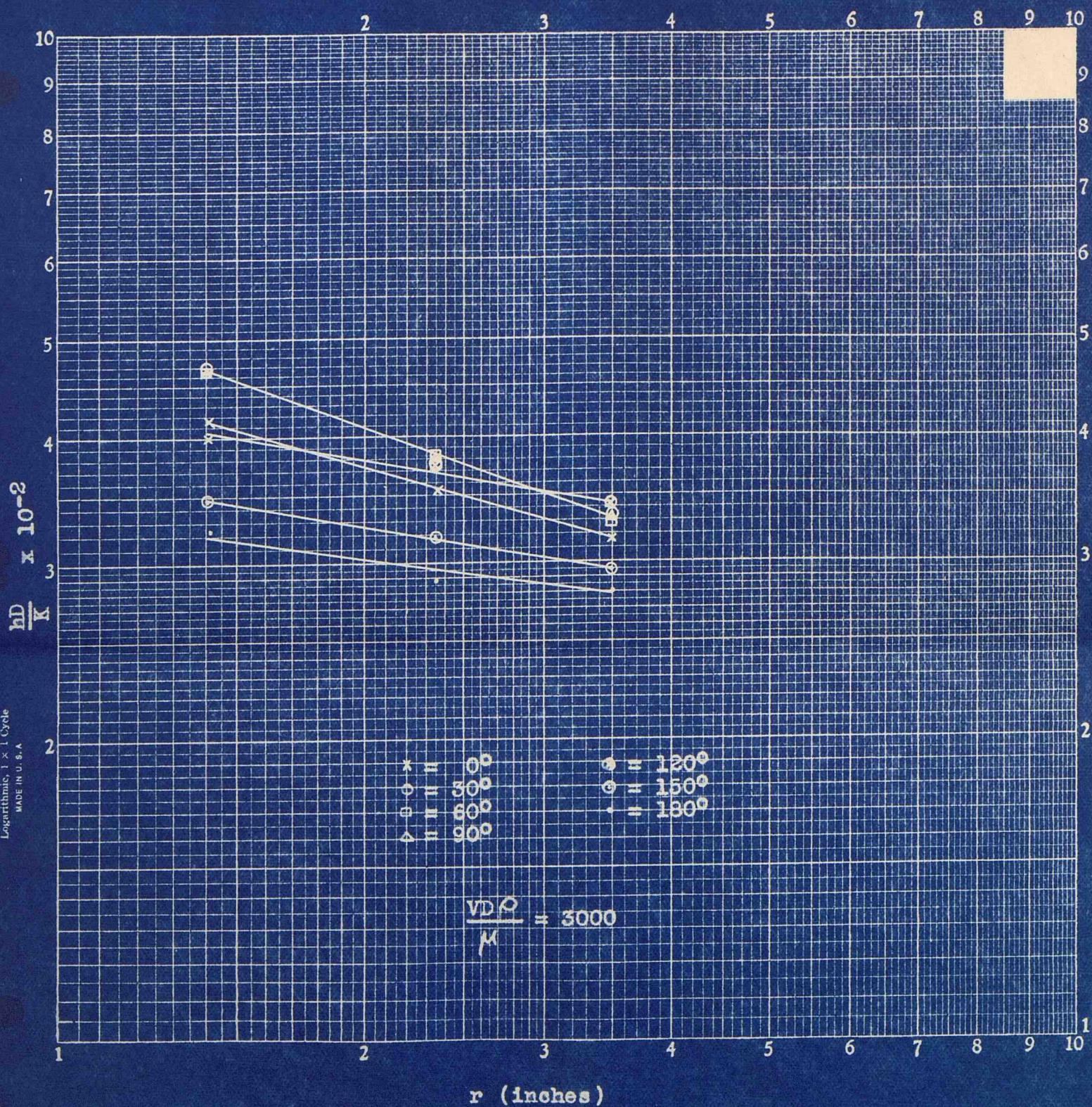


Fig. 8. The variation in Nusselt's Number with radial position on the fin.

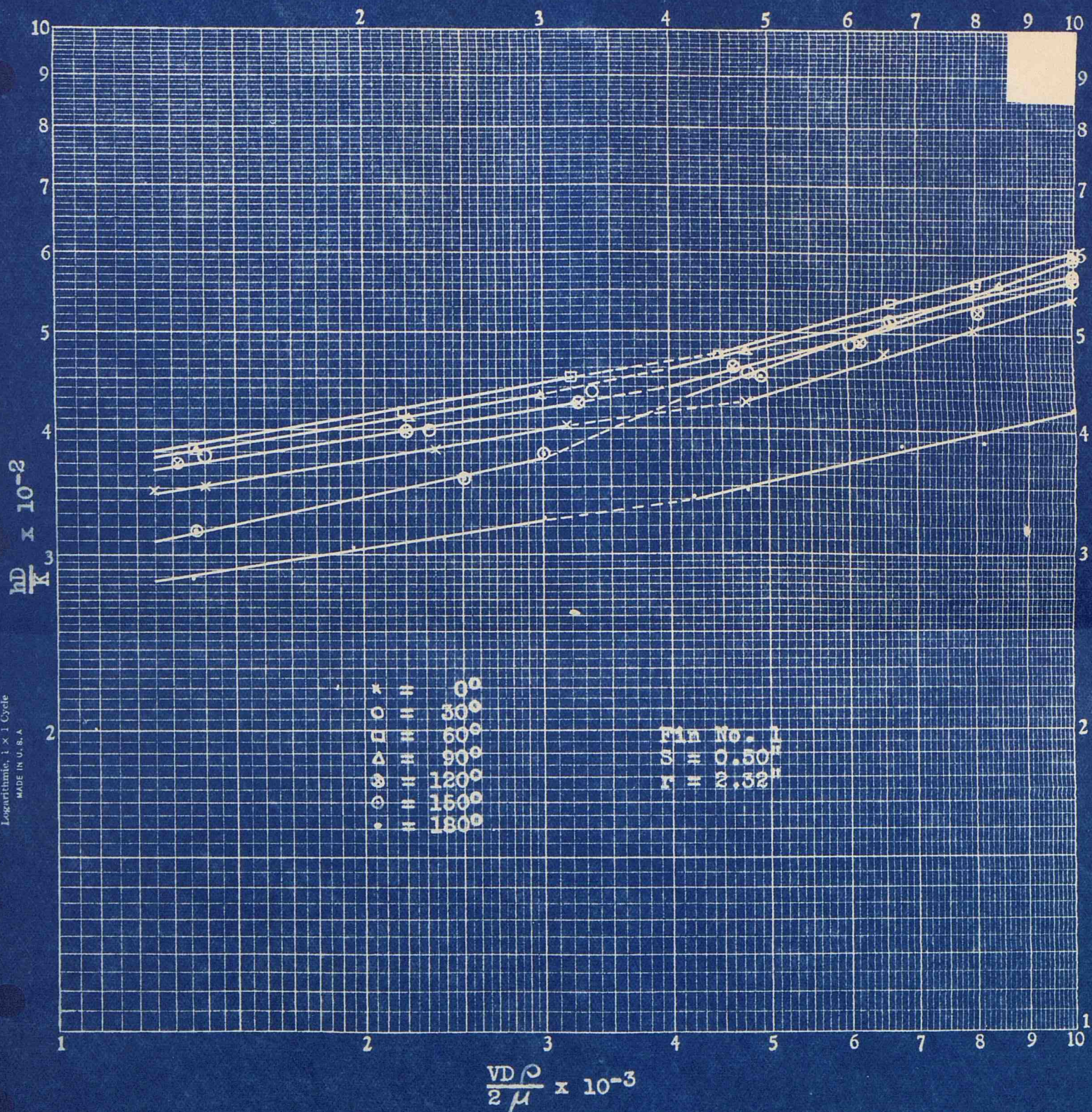


Fig. 9. Variation in Nusselt's Number with Reynold's Number for various angular positions on the fin.

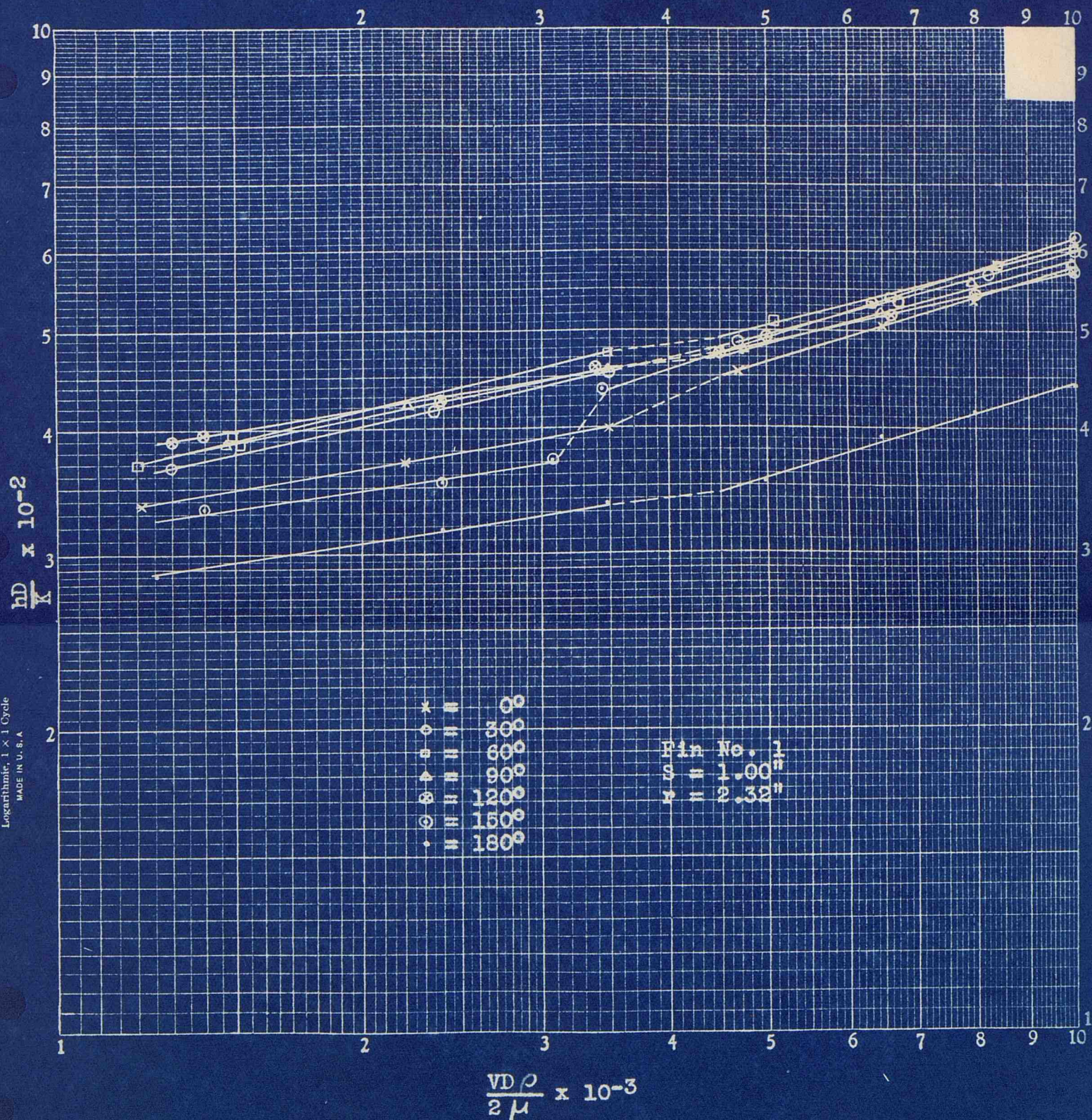
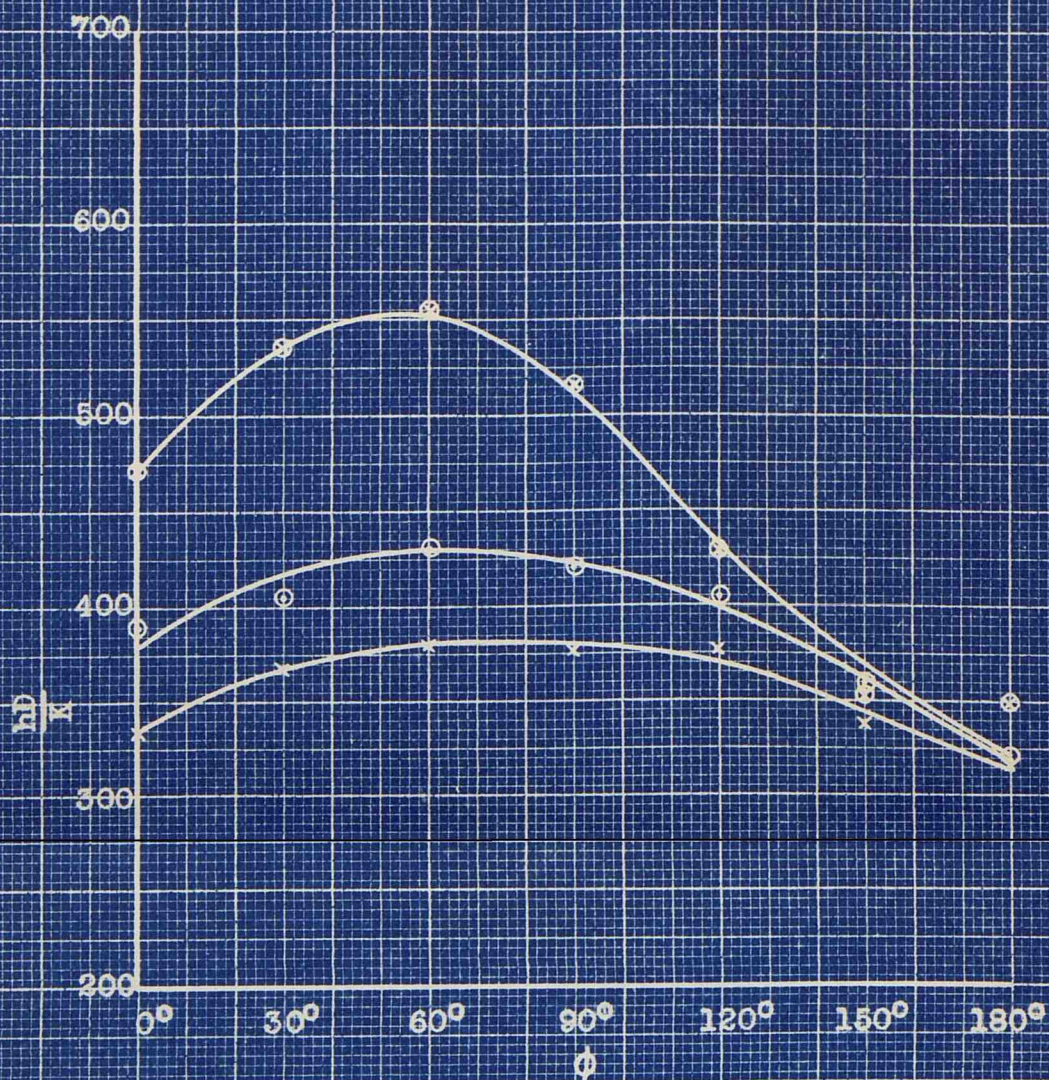


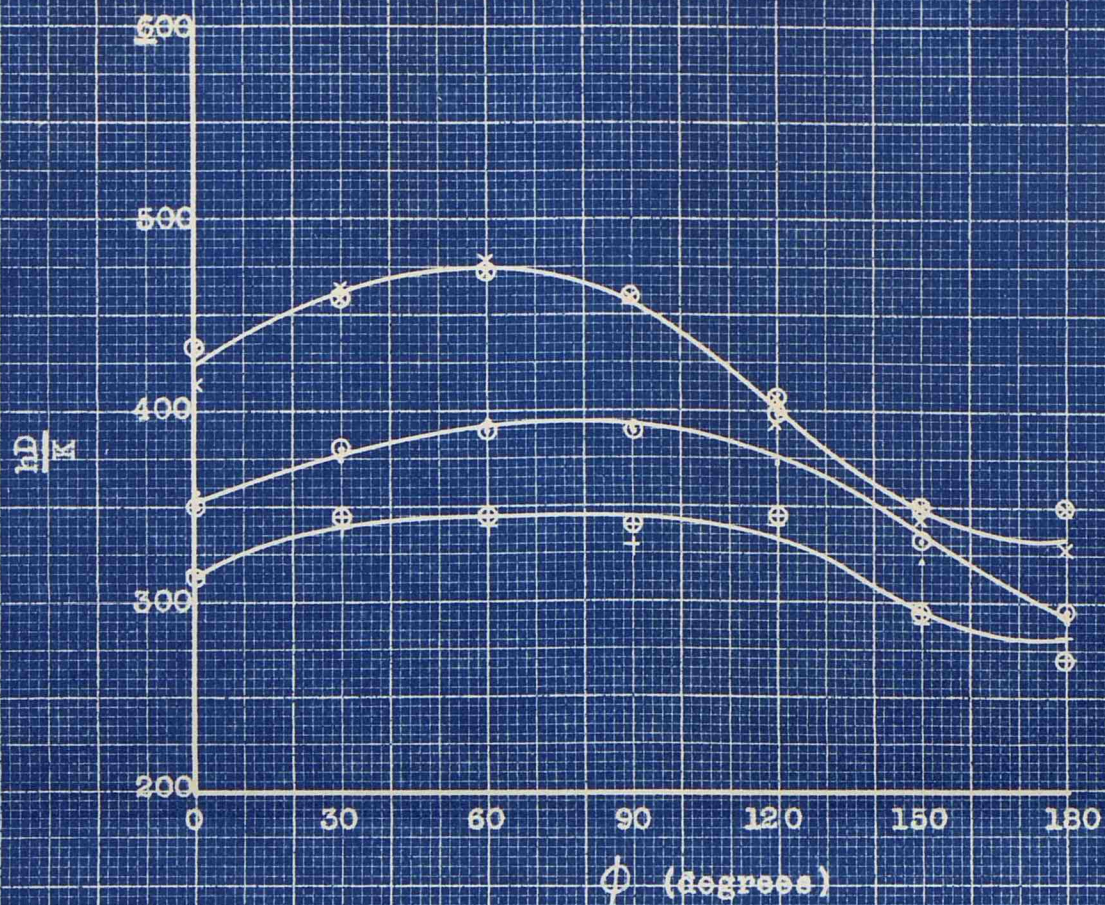
Fig. 10. Variation in Nusselt's Number with Reynold's Number for various angular positions on the fin.



Fin No. 2 $r = 1.44''$
Fin No. 1 $r = 2.32''$
Fin No. 3 $r = 3.50''$

$S = 0.50''$
 $\frac{VD \rho}{\mu} = 5000$

Fig. 11. The variation in the local heat transfer coefficient with angular position on the fin.



Fin No. 1	S = 0.5"	r = 2.32"	Fin No. 1	S = 1.0"	r = 2.32"
Fin No. 2	S = 0.5"	r = 1.44"	Fin No. 2	S = 1.0"	r = 1.44"
Fin No. 3	S = 0.5"	r = 3.50"	Fin No. 3	S = 1.0"	r = 3.50"

Fig. 12. The variation in the local heat transfer coefficient with angle ϕ , radius r , and spacing S , at a Reynold's Number of 3000.

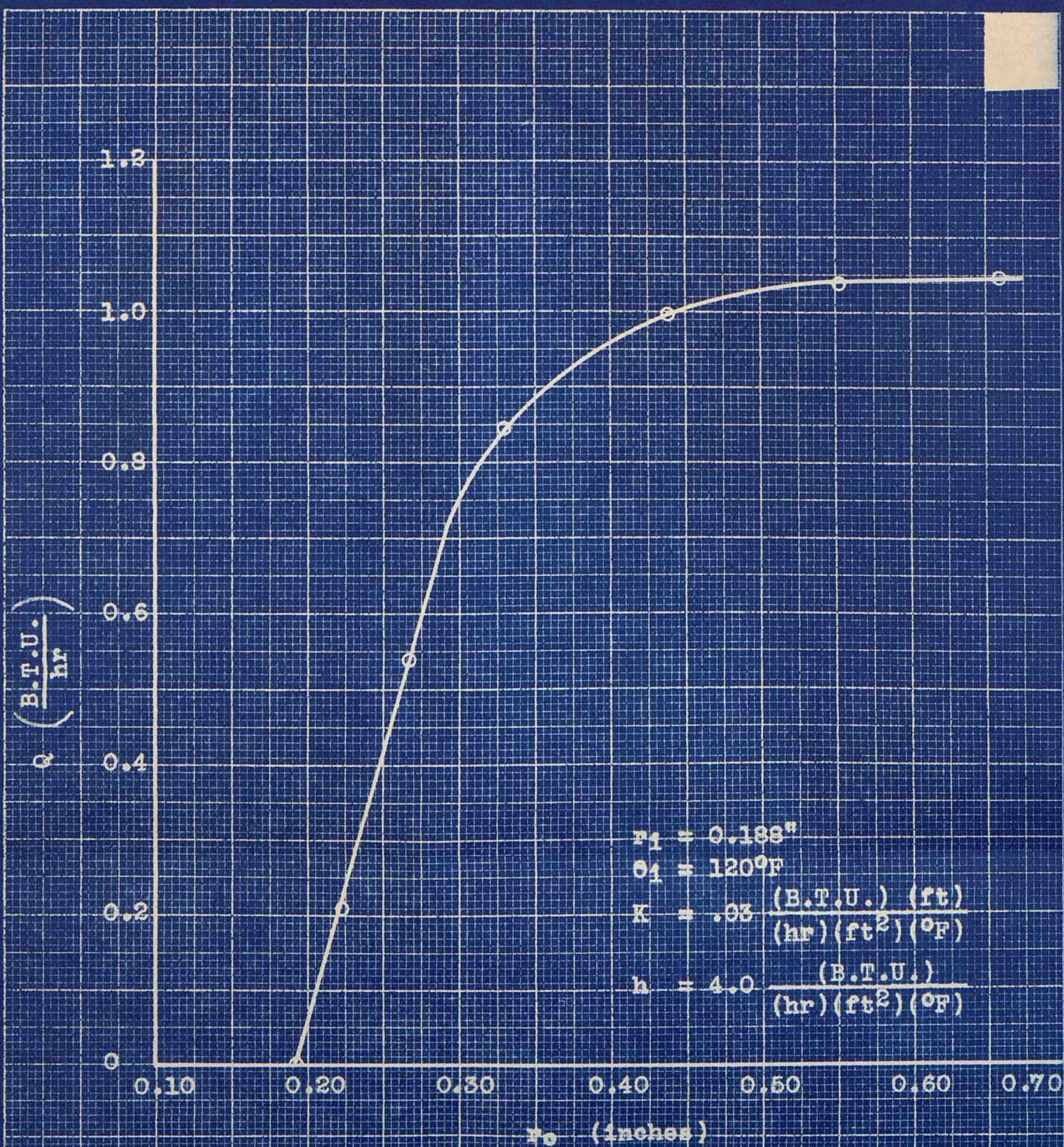


Fig. 13. Results of calculations of the radial heat loss in the fin by equation 17.

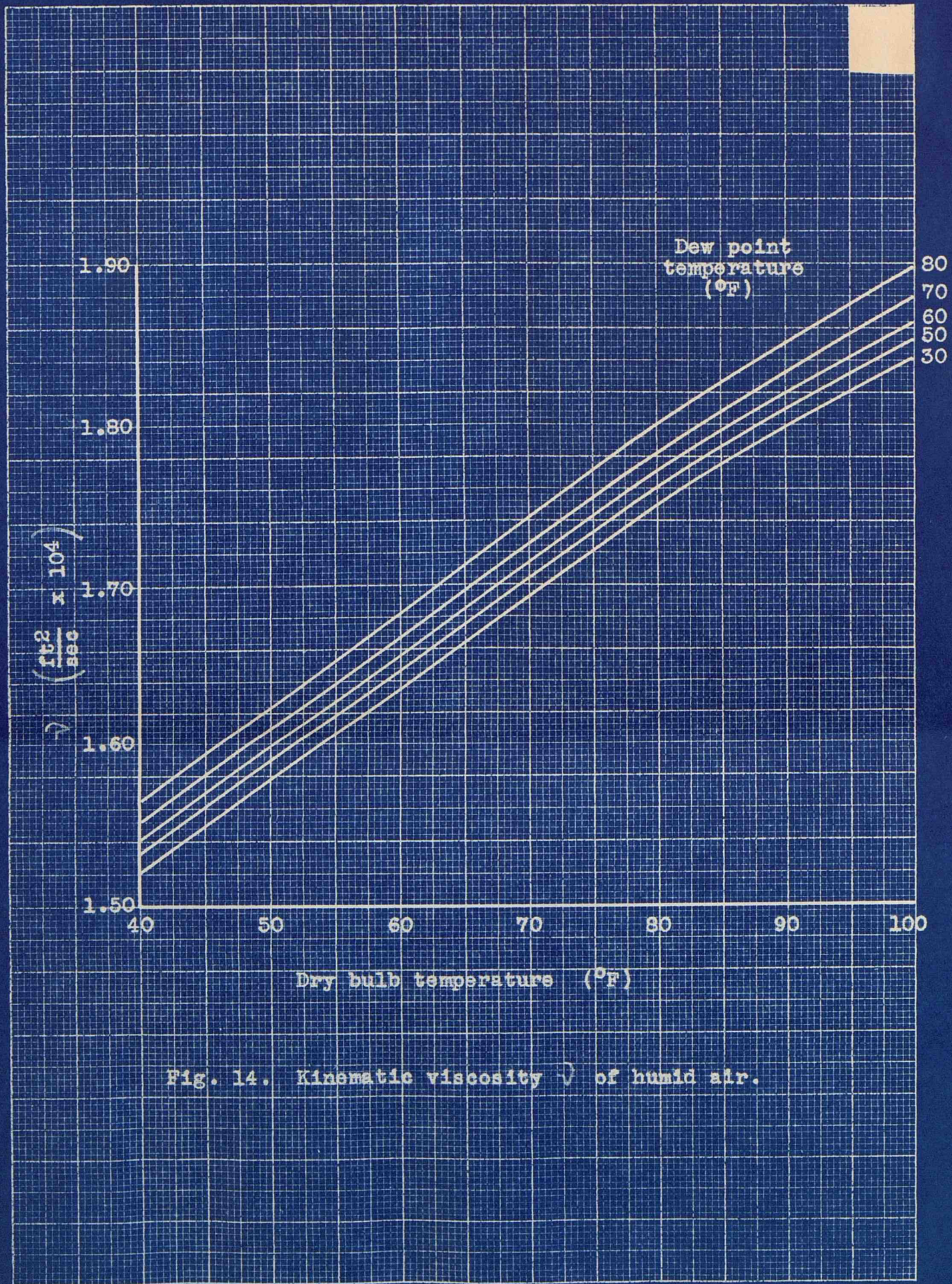


Fig. 14. Kinematic viscosity ν of humid air.