

**AN OPTIMIZATION MODEL: MINIMIZING  
FLOUR MILLERS' COSTS OF PRODUCTION  
BY BLENDING WHEAT AND ADDITIVES**

by

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## **ABSTRACT**

Grands Moulins d'Abidjan (GMA) is a flour milling company operating in Côte d'Ivoire. It wishes to determine the optimal blend of wheat and additives that minimizes its costs of production while meeting its quality specifications. Currently, the chief miller selects the mix of ingredients. The management of the company would like to dispose of a scientific tool that challenges the decisions of the chief miller.

The thesis is about building and testing this tool, an optimization model.

GMA blends up to six ingredients into flour: soft wheat, hard wheat, gluten, ascorbic acid and two types of enzyme mixes. Quality specifications are summarized into four flour characteristics: protein content, falling number, Alveograph W and specific volume of a baguette after four hours of fermentation. GMA blending problem is transformed into a set of equations. The relationships between ingredients and quality parameters are determined with reference to grains science and with the help of linear regression.

The optimization model is implemented in Microsoft Office Excel 2010, in two versions. In the first one (LP for Linear Programming model), it is assumed that weights of additives can take any value. In the second one (ILP for Integer Linear Programming model), some technical constraints restrain the set of values that weights of additives can take.

The two models are tested with Premium Solver V11.5 from Frontline Systems Inc., against four situations that actually occurred at GMA in 2011 and 2012,.

The solutions provided by the model are sensible. They challenge the ones that were actually implemented. They may have helped GMA save money.

The optimization model can nevertheless be improved. The choice of relevant quality parameters can be questioned. Equations that link ingredients and quality parameters, and particularly those determined with the help of linear regression, should be further researched. The optimization model should also take into account some hidden constraints such as logistics that actually influence the decision of GMA chief miller. Finally, sensitivity analyses may also be used to provide alternative solutions.

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## CHAPTER I: INTRODUCTION

The profitability of a firm depends upon both the quality of its outputs and the costs of its inputs. In the flour milling industry, to be profitable, a firm must produce flour that meets the needs of its customers by choosing the correct blend of wheat and additives that is as cheap as possible.

The second element of this statement is of particular importance. Wheat and additives represent more than eighty percent of the total production costs of flour millers. However, if cheap production prices result in flour of poor quality, it will have adverse effects on operational efficiency.

Economists have designed tools that deal with such issues. Operations research and optimization techniques simplify economic reality by using mathematical models in order to find an optimal solution and inform decision making.

The present thesis is about the implementation of an optimization model.

### 1.1 Thesis objective

The objective of the thesis is to determine the optimal economic blend of wheat and additives that minimizes flour miller's cost of production while meeting quality requirements. The modeling effort is based on facts and figures provided by Grands Moulins d'Abidjan (GMA), a flour milling company operating in Côte d'Ivoire in West Africa.

**Figure 1.1: GMA Logo**



GMA processes about 250,000 tons of wheat per year. Ninety percent of GMA flour is sold to small bakeries, which almost exclusively produce baguettes, a French type bread. Much smaller percentages of GMA flour are used to produce pan bread, cookies and pastries. The present thesis will focus on bakery flour designed for making baguettes.

**Figure 1.2: Baguettes at GMA test bakery**



Since wheat does not grow in Côte d'Ivoire, GMA has to import it by sea vessels from other areas of production. Quite logically, French soft wheat is well adapted to the production of French type bread. For many years, GMA only imported French wheat in order to produce its flour.

However, over time, in order to satisfy the needs of Ivorian bakers, as well as to keep pace with market developments, GMA has started to blend other ingredients.

Hard wheat from North America brings higher protein content and strength to GMA flour. Additives such as gluten, ascorbic acid or enzyme mixes modify flour characteristics. From a technical point of view, such additives are complementary products to wheat. From an economic point of view, hard wheat and additives can, to some extent and for some characteristics, be considered as soft wheat substitutes. When some desired characteristics of soft wheat are not available at hand, hard wheat or additives can be used as replacements.

The specific operating conditions of GMA reinforce the importance of the issue of blending wheat and additives.

Every year, GMA receives about 15 vessels, each of them carrying an average of 15,000 tons of soft wheat. The quality of wheat of each cargo varies from ship to ship. Due to this variation, in order to maintain quality standards, GMA has to deal with blending problems about every three weeks, whenever it ends up with one cargo of wheat and switches to the next one.

GMA is located far away from wheat production areas and wheat cannot be delivered except by sea vessels. It takes at least four weeks between the moment an order is placed and the moment wheat is delivered to Abidjan. When the expected specifications of a cargo are not met, GMA may ask for some refund from its suppliers, but it must nevertheless process the wheat that has actually been received and wait several weeks for another shipment. Unfortunately, such a problem occurs from time to time. The only solution is to design an appropriate mix of ingredients, at short notice, to meet needed standards.

The chief miller is responsible for the blending decision. He knows the different specifications and characteristics of ingredients, wheat and additives, in his possession. He knows what type of flour must be produced. Capitalizing upon his experience, he designs a satisfactory blend. This way of doing things has proved to be quite efficient over the years. However, the management of the company believes this process can be improved.

An optimization model could help GMA define the mix of wheat and additives that both meets the needs of its customers, while being the least expensive. The optimum defined by this program should not replace the decision of the chief miller. However, based upon a scientific approach, it could challenge his proposal and give rise to a hopefully fruitful discussion before a final decision is made.

**Figure 1.3: General view of GMA silos and flour mill**



## **1.2 Limitations**

Flour milling has to deal with blending techniques. Flour millers purchase wheat from different geographical origins or from different classes or grades. Out of these different inputs, they wish to produce flour of consistent quality. To do so, they use two main techniques: blending wheat or blending flour. The two techniques have pros and cons. We focus only on wheat blending here as GMA's mill layout favors wheat rather than flour blending.

It is also important to make it clear from the beginning that this study is only about economic optimization. We will not talk about flour milling techniques. Of course, flour millers, with the help of various processes and machines, optimize the wheat blending process as well as the use of additives. All these techniques are beyond the scope of the present thesis. We will focus on optimizing the blending process through economic tools and techniques.

### **1.3 Framework**

The economic optimization of wheat and additives blending is a crucial issue for flour millers. As regards GMA, an optimization model may lead to saving significant amounts of money. The thesis objective will be therefore to design and build a model which can efficiently address this issue.

Another interest of the present thesis is that it provides an opportunity to apply another technique, optimization, to a GMA business issue. As such, it fits quite adequately with the purpose of an executive education program such as the Master of Agri-Business at Kansas State University.

The present study is organized as follows: definition of objective; literature review; data and methods; results and conclusion. In addition, the process takes account of the pragmatic five-step optimization modeling process identified by Ragsdale (2008): identifying the problem, mathematically analyzing the problem, implementing the problem on computer, solving the problem using software tools and, finally, testing the results.

The present thesis will comprise 6 chapters. In the present Chapter 1 “Introduction”, the thesis objective is identified and is defined. In Chapter 2, the "Literature Review" describes previous papers or studies on similar or related subjects. It outlines how the present project differs from these previous works. Chapter 3 "Data and Methods 1.Mathematical Analysis" explains how actual business conditions are transformed into a set of equations and inequalities. Chapter 4 "Data and Methods 2.Computer Implementation", depicts how the equations of the model are captured on a spreadsheet. In Chapter 5 "Results", optimal solutions given by the model are compared with actual decisions made by GMA. Finally, Chapter 6 "Summary and Conclusion" draws conclusions and suggests ideas for further research and improvement of the optimization model.

## CHAPTER II: LITERATURE REVIEW

The objective of the present thesis is to use wheat and additives blending as a means of minimizing flour millers' costs of production while still meeting quality requirements.

This is a common issue among flour millers. Fowler (2009, p. 62-66) summarizes the economic reasons why millers blend wheat and add ingredients to flour. They want to deliver a consistent or a unique product and they want to minimize their raw material cost.

The way to achieve this objective is through optimization techniques, particularly linear programming. Blending problems are traditional applications of linear programming. Some of the earliest to be addressed were the nut-mix problem (Charnes et al. 1953) and the sausage-blending problem (Steuer 1986).

Niernberger (1973) was certainly the first to formulate and evaluate a wheat blending model in order to maximize profit from flour milling operations. He designed a computerized linear programming model that determined the optimum blend of different lots of wheat and maximized profit, under several technical and economic constraints. Niernberger's model's purpose is close to the objectives of the present thesis. There are nevertheless significant differences between the two efforts. Niernberger's objective was to optimize the flour miller's profit originating from all its products: patent flour, 1<sup>st</sup> clear flour, 2<sup>nd</sup> clear flour, as well as mill feed. The objective of the present thesis is only to minimize the cost of production of one type of flour, designed for making French type bread, baguettes. Other differences derive from geographical contexts. Niernberger's model only considers types of hard winter wheat. He uses Brabender Farinograph data to build constraints and the flour produced is designed to make pan-bread. In the present thesis, different wheat varieties from Europe and North America are mixed. The addition of additives that may influence the price, as well as the characteristics of flour is also considered. Flour is used to make baguettes. Finally, Chopin Alveograph is used instead of Brabender Farinograph.

Hayta and Cakmakli (2001) used linear programming to optimize the blending of wheat lots. Using linear regression, they identify three criteria that characterize wheat lots and that



are significantly correlated with loaf volume: particle size index, dough volume and falling number. Then they design a linear programming model that determines the most economic wheat mix. Hayta and Cakmali focus on the selection of quality criteria rather than on the optimization problem itself. They work on wheat and flour characteristics that are different from those used in West Africa. In addition, they do not take account of additives.

In addition to published literature, the idea of the present thesis was triggered by two other pieces of work.

The International Grains Program (IGP) organizes short courses for flour millers, in association with Kansas State University. The 2006 Flour Milling short course included a lesson on spreadsheet solutions by Bryan Shurle and Mark Fowler. Among other things, this lesson displayed an example of a wheat blending problem worked out by Microsoft Office Excel Solver. However, although quite realistic, this spreadsheet had to be adapted in order to meet actual constraints and become an effective tool.

In the 2000's, Peter Lloyd of US Wheat Associates (USW) also designed a Microsoft Office Excel spreadsheet that helped millers determine the most profitable blends of wheat. All millers visited by US Wheat Associates can request this spreadsheet, specifically in Africa since Peter Lloyd is based out of Casablanca, Morocco. Millers enter in the spreadsheet several inputs such as wheat characteristics, type of flour produced, prices of wheat, prices of flour, operating costs, etc. They choose a specific blend of wheat and the spreadsheet enables them to compare the characteristics of this blend with what they expect in terms of flour quality, as well as gross margin. Solver and Goal Seek functions are used to fine tune the wheat blend. The USW spreadsheet is more ambitious than the present thesis project: it is designed to compute flour millers' gross margins and not only minimize production costs. However, it takes into account only the rheological characteristics of the flour produced. The present thesis will also consider bread-making characteristics of flour. As all other works, the USW model does not take account of additives.

## **CHAPTER III: DATA AND METHODS 1 MATHEMATICAL ANALYSIS**

In the introductory chapter, the thesis objective was identified as the minimization of flour millers' production costs by blending wheat and additives, while meeting flour standards. In the present chapter, this objective as well as GMA constraints will be analyzed and transformed into a mathematical model to be optimized.

The optimization model and its different components: variables, equations and inequalities will be defined in section 3.1. In the subsequent sections, the different elements of the model will be reviewed. In section 2, the decision variables, i.e. the different ingredients of the GMA mix will be considered. In section 3, technical constraints will be identified and described in mathematical terms. In sections 4 to 6, quality constraints will be identified, given limits and put into equations. Finally, the whole optimization model will be displayed in section 7.

### **3.1 Optimization of wheat and additives blending**

In the modeling approach, the blending problem is translated into equations and/or inequalities. The mathematical formulation of the problem requires definition of decision variables, objective function, and constraints.

#### *3.1.1 The Decision Variables*

Decision variables represent the choice to be made: the quantities the researcher wishes to determine. For the GMA model, decision variables ( $W_1, W_2, \dots, W_i$ ) are the actual weights of the different ingredients that are blended in order to produce flour of a desired and consistent quality.

It must be stated from the beginning of the thesis that, since Côte d'Ivoire has adopted the metric system, all weights are expressed in metric tons (t) or kilograms (kg). And in order to keep things simple, it is assumed that, in the present optimization model, the total weight of all ingredients is equal to one thousand metric tons. The price of 1,000 tons of a mix of wheat and additives is large enough to be significant. Using weights instead of respective proportions of ingredients in the mix, for instance, makes it easier to compute prices since unit prices are expressed in CFA francs per metric ton. The CFA franc (FCFA) is the West

African Economic and Monetary Union (WAEMU) currency and is worth about 0.002 US dollars.

As regards wheat, either hard or soft, each  $W_i$  represents a weight which is associated to one sea vessel. This is how GMA differentiates lots of wheat. Wheat from each vessel is consistent since cargoes are homogenized in port elevators before loading. They are handled and stored separately in GMA silos after reception at Abidjan. Last but not least, to each and every vessel corresponds a specific unit price of wheat.

### 3.1.2 The Objective Function

The objective function is a function of the decision variables that the researcher wishes to maximize or minimize. For GMA, the objective function of the optimization model is to minimize the cost of the blend of wheat and additives processed by the mill.

**Table 3.1: Objective Function Formula**

<b>Min: <math>\Sigma W_i P_i</math></b>
<b>where:</b>
<ul style="list-style-type: none"> <li>• <b><math>W_i</math> is the weight of wheat or any additive used in the mix, the total of which amounts to one thousand metric tons ;</b></li> <li>• <b><math>P_i</math> is the price of the corresponding ingredient, expressed in CFA francs per metric ton (FCFA/t).</b></li> </ul>

### 3.1.3 The Constraints

Constraints are other functions of the decision variables. In a world of limited resources, they are restrictions on the solutions available to any business. Constraints can be stated mathematically as follows:

**Table 3.2: Constraint Formulas**

$f(W_1, W_2, \dots, W_n) \leq \alpha$ , or $f(W_1, W_2, \dots, W_n) \geq \alpha$ , or $f(W_1, W_2, \dots, W_n) = \alpha$
<b>where:</b>
<ul style="list-style-type: none"> <li>• <b><math>W_i</math> is the weight of wheat or additive used in a mix, the total of which amounts to one thousand metric tons ;</b></li> <li>• <b><math>\alpha</math> is the limit value of the constraint.</b></li> </ul>

In order to determine the optimal mix of wheat for GMA, the chief miller has to face three categories of constraints: constraints that bind the decision variables themselves,

constraints that are imposed by technical considerations and, finally, constraints that concern the quality of flour.

There are two constraints that bind the decision variables themselves. Weights of wheat and additives cannot be negative. And, as already mentioned above, the total weight of wheat and additives is one thousand metric tons.

Other constraints are imposed by technical considerations. Proportions of additives in the mix should be compatible with the dosing scales of the flour mill. Incorporation rates may be recommended by suppliers of these ingredients. The technical constraints are considered in section 3.3.

Sections 3.4 to 3.6 deal with quality constraints. Relevant quality constraints parameters must be selected. Specifications must be defined for these constraints. Finally, the mathematical functions that link the ingredients of the mix and the selected quality constraints parameters must be identified.

#### *3.1.4 Linearity*

In principle, objective function and constraints can have any mathematical form. The important point is that they should accurately describe the problem which is to be solved.

However, preferably, functions representing the objective function and constraints should be linear. According to Studenmund (2006, p. 207-208), a function can be linear in the variables and/or linear in the coefficients. A function is linear in the variables “if plotting the function in terms of X and Y generates a straight line”. A function is linear in the coefficients “if the coefficients appear in their simplest form – they are not raised to any powers (other than one), are not multiplied or divided by other coefficients, and do not themselves include some sort of function (like logs or exponents)”.

Solving a set of linear functions is easier and is more reliable than a set of non-linear functions. When using only linear functions, operations research is often termed linear programming (LP). In the course of the present thesis, one non-linear function will be tested but only linear functions will eventually be used in the optimization model.

**Table 3.3: Linear Constraint Formulas**

$\beta_0 + \beta_1 W_1 + \beta_2 W_2 + \dots + \beta_n W_n \leq \alpha, \text{ or}$ $\beta_0 + \beta_1 W_1 + \beta_2 W_2 + \dots + \beta_n W_n \geq \alpha, \text{ or}$ $\beta_0 + \beta_1 W_1 + \beta_2 W_2 + \dots + \beta_n W_n = \alpha$
<p><b>where:</b></p> <ul style="list-style-type: none"><li>• <math>W_i</math> is the weight of wheat or any additive used in a mix, the total of which amounts to one thousand metric tons ;</li><li>• <math>\beta_i</math> is the technical coefficient attached to <math>W_i</math> ;</li><li>• <math>\alpha</math> is the limit value of the constraint.</li></ul>

### **3.2 Decision variables: ingredients of the mix, wheat and additives**

In order to make flour, GMA can mix up to six ingredients: soft wheat, hard wheat, gluten, ascorbic acid and two types of enzyme mixes.

In further developments, flour made out of some or all of these ingredients will be referenced to by letters ‘FLR’. For instance, the price of soft wheat will be labeled  $P_{FLR}$ .

#### *3.2.1 Soft wheat*

Soft wheat is the main ingredient of GMA flour designed for making baguettes. The total mix usually includes up to 90% or 95% soft wheat. Soft wheat processed by GMA is imported mostly from France. However, GMA also exploits market opportunities and, from time to time, imports soft wheat from other origins such as the Black Sea region, Germany or Argentina.

GMA collects data on soft wheat for every vessel that comes to Abidjan, at various stages of the supply process.

Samples of wheat are tested in the port of loading silos as well as later, when the ship is unloaded in Abidjan. These analyses provide data about physical (dockage, moisture etc.) as well as rheological (protein content, falling number, Alveograph etc.) characteristics of every cargo of wheat.

Upon arrival, a sample of soft wheat from every vessel is also processed and transformed into flour in GMA mills. Milling and rheological characteristics of this flour are analyzed. It is also baked and transformed into bread and graded at the GMA test bakery.

Altogether, GMA can characterize every cargo of soft wheat with some twenty parameters.

The GMA accounting system computes a price for every shipment of wheat. This price is expressed in CFA francs per ton (FCFA/t). It comprises the Cost, Insurance and Freight (CIF) price plus all forwarding costs involved until wheat is stored in bins and ready for milling.

In recent periods of time, the price of soft wheat has suffered from high volatility. Prices recorded by GMA follow the fluctuations of world market prices with a few weeks delay due to transportation time. In addition, they are affected by fluctuations in freight rates. In January 2010, the price of soft wheat at GMA was 124,688 FCFA/t. It was relatively stable until July 2010. Then it started to increase rapidly and went from 202,844 FCFA/t in September 2010 to 229,343 FCFA/t in March 2011. It remained at high levels until September 2011. Then the price went down, but it is still subject to significant fluctuations. In March 2012, GMA price for soft wheat was 197,575 FCFA/t.

Soft wheat will be referred to by the letters 'sw'. The weight of soft wheat in the mix of ingredients will be labeled  $W_{sw}$  and the unit price of soft wheat will be labeled  $P_{sw}$ .

### *3.2.2 Hard wheat*

At a low incorporation rate, hard wheat, with its higher protein content, brings many interesting properties that are appreciated by GMA customers: baking strength, tolerance, bread volume, etc. However, high percentages of incorporation of hard wheat can have negative effects, which do not suit the production of baguettes.

Hard wheat is imported by GMA from North America. In the past years, GMA has imported mostly Canada Western Red Spring (CWRS) wheat. CWRS is hard red spring wheat of superior milling and baking quality.

When GMA purchases hard wheat, it performs the same tests as on soft wheat. These tests provide data on physical, as well as rheological characteristics of the wheat. In addition, on every shipment, GMA processes a few kilograms of hard wheat in a laboratory mill. The rheological, as well as milling characteristics of this flour are tested

However, GMA does not transform this sample of flour into bread. The weight of flour obtained from the laboratory mill is too small. Moreover, it is well known that 100% hard wheat flour does not fit the production of baguettes. Consequently, unlike soft wheat, GMA does not record the baking characteristics of its hard wheat supplies.

The price of hard wheat is usually higher than the price of soft wheat. It is computed by the GMA accounting system in exactly the same way as soft wheat. This price has also been subject to significant fluctuations in recent periods of time. It actually ranged from 163,682 FCFA/t in November 2009 to 253,491 FCFA/T in November 2011.

Hard wheat will be referred to by the letters 'hw'. The weight of hard wheat in the mix of ingredients will be labeled  $W_{hw}$  and the unit price of hard wheat will be labeled  $P_{hw}$ .

### *3.2.3 Gluten*

Gluten is made of water insoluble proteins, glutenins and gliadins. Gluten can be found in wheat kernels. It is also marketed on its own.

GMA incorporates gluten in the mix whenever soft wheat lacks protein content. Gluten can be seen as a substitute for hard wheat. However, its effects have a more limited range.

The price of gluten is linked to the price of wheat but is nevertheless more stable. GMA recorded a price of gluten at 1,286 FCFA/kg in October 2010. It reached a peak in September 2011 at 1,618 FCFA/kg and went down to 1,205 FCFA/kg in January 2012.

Gluten will be referred to by the letters 'GLT'. The weight of gluten in the mix of ingredients will be labeled  $W_{GLT}$  and the unit price of gluten will be labeled  $P_{GLT}$ .

### *3.2.4 Ascorbic acid*

Ascorbic acid is incorporated into flour essentially because of its functionality properties. It is an oxidizing agent that favors the baking process. It increases dough extensibility.

Ascorbic acid price varies significantly according to its origin. In 2011, GMA purchased ascorbic acid from Europe at 12,186 FCFA/kg and from China at 5,246 FCFA/kg.

Ascorbic acid will be referred to by the letters ‘AAC’. The weight of ascorbic acid in the mix of ingredients will be labeled  $W_{AAC}$  and the unit price of ascorbic acid will be labeled  $P_{AAC}$ .

### *3.2.5 Enzyme mixes*

There are many different kinds of enzymes that flour millers incorporate in their mixes: amylases, proteases, lipases, glucose-oxidases, etc. These products act as catalysts. They trigger or enhance chemical reactions during the baking process. Flour millers use enzymes to correct wheat deficiencies and help provide for consistent quality flour.

Knowledge about the effects of these different enzymes has dramatically improved in past years. It is very difficult for a flour miller like GMA to keep up to date with progresses made in this domain of research. As a consequence, GMA is not able to formulate by itself relevant enzyme mixes that can address its quality issues. GMA refers to specialized firms that design its enzyme mixes. The formulas of these enzyme mixes are kept confidential by the supplier and GMA does not know the composition exactly.

In 2011 and 2012, GMA used two different enzyme mixes. The price of Enzyme Mix 1 varied from 26,504 FCFA/kg in December 2010 to 27,256 FCFA/kg in February 2011. The price of Enzyme Mix 2 is equal to 24,752 FCFA/kg and is unique since GMA has purchased only one lot of it.

The first enzyme mix and the second enzyme mix will be referred to as ‘EN1’ and ‘EN2’, respectively. Weights of EN1 and EN2 in the total mix of ingredients will be labeled  $W_{EN1}$  and  $W_{EN2}$ , respectively. Unit prices of EN1 and EN2 will be labeled  $P_{EN1}$  and  $P_{EN2}$ , respectively.

### **3.3 Technical constraints**

In order to find a relevant solution to the optimization problem, it is necessary to consider the technical constraints of the mill. The milling process, the capabilities of dosing scales, as well as suppliers’ advice have an impact on the incorporation of ingredients.



In the case of wheat, the relative proportions of soft and hard wheat can be affected. In the case of additives, the set of weights that can actually be incorporated in a mix of one thousand metric tons is restricted to certain values.

### *3.3.1 GMA milling process and the incorporation of ingredients*

Wheat is unloaded on the quays of Abidjan harbor and is directed by conveyors to GMA elevators.

**Figure 3.1: Ship unloading wheat at GMA facilities**



After a period of storage, soft wheat and hard wheat are blended in a silo bin. The blend is then conveyed to the flour mill. It is cleaned, tempered and put to rest. Flour milling theory teaches that soft wheat and hard wheat should be treated differently, as regards the amount of water that is added to wheat and the time it is allowed to rest. However, for decades, GMA has not respected these differences and is used to blending and treating soft wheat and hard wheat together.

Afterwards, the blend of wheat goes through a series of roller mills and sifters in order to separate endosperm from bran and to reduce endosperm particles in the flour. Flour is collected and goes through conveyors to flour bins. Dosing scales are implemented on these conveyors so that GMA can put additives, gluten, ascorbic acid and enzyme mixes, into the flour.

**Figure 3.2: GMA dosing scales**



After flour has been stored in bins, it is extracted, put into bags and finally delivered to customers.

### *3.3.2 Incorporation of wheat*

At GMA, soft wheat and hard wheat are blended together in a silo bin. The relative proportions of soft wheat and hard wheat that are directed to this silo bin are pre-determined by scales which are computer-controlled. The precision of these scales is of half a percent.

It means that in a lot of 1,000 metric tons of wheat, weights of soft wheat and hard wheat can only be multiples of 5 tons.

However, when additives are added to the mix, respective weights of soft wheat and hard wheat can assume other values. If, for instance, 4 tons of gluten are added into the mix, the weight of wheat amounts to 996 tons in a total of 1,000 metric tons and 0.5% of this weight represents 4.98 tons. If, for instance, 1 ton of gluten and 56 kilograms of enzyme mix are added into the mix, the total weight of wheat amounts to 998.944 tons in a total of 1,000 tons and 0.5% of this weight represents 4.99472 tons.

Since weights of soft wheat and hard wheat can take such different values in a mix of one thousand metric tons, it will be assumed in the optimization model that these variables are continuous.

### *3.3.3 Incorporation of additives*

#### *a) Additives: Incorporation rates and increments*

When it comes to additives, one has to consider both limitations and sensibilities of dosing scales but also recommendations from suppliers of ingredients.

GMA dosing scales are able to add gluten into flour at a rate which ranges between 0.1% and 1.0% with increments of 0.1%.

Ascorbic acid is usually added to flour at rates which can vary between 0 to 100 parts per million (ppm). Because of GMA dosing scales capabilities, this rate of incorporation can only increase by steps of 10 ppm.

According to its supplier, enzyme mix 1 is to be incorporated at a rate of 70 ppm. It also recommends that enzyme mix 2 should be mixed into flour at rates of 5, 10, 15 or 20 ppm. Incorporation rates may vary but with increments of 5 ppm and a maximum limit of 20 ppm.

The above rates and increments are computed, as is usual in a flour mill, upon the basis of flour weights. In the optimization model, these rates and increments need to be recalculated upon the basis of the weight of the total mix of ingredients.

#### *b) Additives: Incorporation rates denominator*

Two steps are necessary to change the denominator of incorporation rates of additives. First, they must be computed over weights of wheat instead of weights of flour. Then, they must be calculated over the total weight of wheat and additives instead of the weight of wheat only.

The rate of flour extraction out of wheat depends on many different parameters ranging from wheat characteristics: dockage, moisture, hardness etc., to the milling process: length

of roller mills, flour ash rate etc. It is difficult to predict precisely what an extraction rate of flour out of wheat will be. However, GMA statistical records show that, on the long run, its extraction rate is, on the average, equal to 80%.

Such an extraction rate may appear quite high to US millers which process hard wheat. Soft wheat extraction rates are generally higher than hard wheat. In addition, GMA flour mills have been designed to provide a high extraction rate.

When computed on wheat rather than flour, the above incorporation rates and increments should therefore be multiplied by 80%. If the incorporation rate of gluten is, for instance, of 0.7% on flour, it is equal to  $(0.7\% \times 80\%) = 0.56\%$  on wheat. With this formula, incorporation rates on flour can be transformed on incorporation rates upon the basis of the wheat blend.

However, what is needed is incorporation rates computed on the weight of the total mix, wheat and additives included.

If, for instance, gluten is the only additive that is incorporated in the mix, then 0.56% on wheat is equal to  $0.56 / (100 + 0.56) = 0.5569\%$  when computed on the weight of the total mix. In another example, 0.8% of gluten and 50ppm of ascorbic acid and 56 ppm of enzyme mix 1 are added to a basis of wheat. When calculated with reference to the weight of the total mix, these incorporation rates become, respectively,  $0.8 / (100 + 0.8 + 0.005 + 0.0056) = 0.7936\%$  of gluten and  $0.005 / (100 + 0.8 + 0.005 + 0.0056) = 49.6\text{ppm}$  of ascorbic acid and  $0.0056 / (100 + 0.8 + 0.005 + 0.0056) = 55.5\text{ppm}$  of enzyme mix 1.

In the following table, all additives are incorporated at their maximum rate and the differences between incorporation rates calculated on the mix of wheat or on the total mix are at their maximum.

**Table 3.4: Impact of calculations on incorporation rates of additives**

	Incorporation rates computed over weight of flour	Incorporation rates computed over weight of wheat (A)	Weights (metric tons)	Weights (for a total of 1,000 metric tons)	Incorporation rates computed over weight of the mix (B)	Difference (A-B)
Wheat			1,000.0000	991.9139		
Gluten	1.0000%	0.8000 %	8.0000	7.9353	0.7935%	0.0065%
Ascorbic acid	100.0000 ppm	80.0000 ppm	0.0800	0.0794	79.3531 ppm	0.6469 ppm
Enzyme mix 1	70.0000 ppm	56.0000 ppm	0.0560	0.0555	55.5472 ppm	0.4528 ppm
Enzyme mix 2	20.0000 ppm	16.0000 ppm	0.0160	0.0159	15.8706 ppm	0.1294 ppm
<b>TOTAL</b>			<b>1,008.1520</b>	<b>1,000.0000</b>		

The maximum relative difference on incorporation rates calculated on the weight of wheat and incorporation rates calculated on the weight of the total mix is equal to  $(0.8000 - 0.7935) / 0.8000 = (80.0000 - 79.3531) / 80.0000 = (56.0000 - 55.5472) / 56.0000 = (16.0000 - 15.8706) / 16.0000 = 0.8086\%$ .

This error term is not significant. It is below the sensitivity limits of dosing scales. Increments defined by the manufacturers of these dosing scales are much higher than this error term. In addition, the uncertainty implied by the use of 80% as the average extraction rate of GMA is, by far, larger.

As a consequence, in order to simplify the model, the difference between incorporation rates upon the basis of wheat and incorporation rates upon the basis of the total mix will be neglected. Incorporation rates computed on the weight of wheat will be used without change in the optimization model.

*c) Additives: Weight sets*

Gluten is incorporated in the mix at a rate  $n_{GLT}$ , calculated on the weight of flour, which ranges between 0.1% and 1.0% with increments of 0.1%. On wheat, with an extraction rate of 80%, the set of relevant incorporation rates becomes:  $n_{GLT} \in \{0.00\%; 0.08\%; 0.16\%; 0.24\%; 0.32\%; 0.40\%; 0.48\%; 0.56\%; 0.64\%; 0.72\%; 0.80\%\}$ .

Ascorbic acid is incorporated in the mix at a rate,  $n_{AAC}$ , which ranges between 0 and 100 ppm with increments of 10 ppm, on the weight of flour. The set of relevant incorporation

rates on the weight of wheat is:  $n_{AAC} \in \{0\text{ppm}; 8\text{ppm}; 16\text{ppm}; 24\text{ppm}; 32\text{ppm}; 40\text{ppm}; 48\text{ppm}; 56\text{ppm}; 64\text{ppm}; 72\text{ppm}; 80\text{ppm}\}$ .

Supplier recommends that enzyme mix 1 is incorporated at a rate,  $n_{EN1}$  of 70ppm on the weight of flour. The set of relevant incorporation rates on the weight of wheat is:  $n_{EN1} \in \{0\text{ppm}; 56\text{ppm}\}$ .

Supplier recommends that enzyme mix 2 is incorporated at a rate  $n_{EN2}$  between 5 and 20ppm with increments of 10ppm on the weight of flour. The set of relevant incorporation rates on the weight of wheat is:  $n_{EN2} \in \{0\text{ppm}; 4\text{ppm}; 8\text{ppm}; 12\text{ppm}; 16\text{ppm}\}$ .

Assuming that incorporation rates on wheat are not significantly different from incorporation rates on the total mix of ingredients, they can be transformed into sets of relevant weights for additives when the weight of the total mix is equal to 1000 tons. All weights are expressed in metric tons.

**Table 3.5: Additives Weight Sets**

<b>Gluten</b>	$W_{GLT} \in \{0.0; 0.8; 1.6; 2.4; 3.2; 4.0; 4.8; 5.6; 6.4; 7.2; 8.0\}$
<b>Ascorbic Acid</b>	$W_{AAC} \in \{0.000; 0.008; 0.016; 0.024; 0.032; 0.040; 0.048; 0.056; 0.064; 0.072; 0.080\}$
<b>Enzyme Mix 1</b>	$W_{EN1} \in \{0.000; 0.056\}$
<b>Enzyme Mix 2</b>	$W_{EN2} \in \{0.000; 0.004; 0.008; 0.012; 0.016\}$

These sets of relevant weights are technical constraints of the optimization model. They have a significant impact on the optimization model since they change the model from a Linear Programming (LP) model to an Integer Linear Programming (ILP) model.

### 3.4 Quality constraints: selection

GMA is very concerned about the quality of its products. It records many different data about its flour quality: physical, rheological, milling characteristics as well as baking characteristics. Altogether, GMA can display at least twenty series of data about each lot of flour manufactured.

It is not desirable however to build twenty constraints in an optimization model. The higher the number of constraints, the more time and IT resources consuming the

optimization model is. Some of these constraints may be irrelevant or redundant. In addition, with too many constraints, a feasible solution may become difficult to find. The model is more robust when it has only a few constraints.

In order to select relevant quality constraints, two types of references will be used: previous literature and econometrics.

#### *3.4.1 Previous literature*

The parameters that were selected as constraints in previous literature are not the same from one work to another.

Niernberger (1973) used 9 characteristics as quality constraints. The IGP model is based upon 4 constraints. The US Wheat Associates model uses 8 constraints. In these different works, the way quality constraints were selected is not explicit. On the other hand, Hayta and Cakmali (2001) use econometrics techniques to select 3 constraints that are highly correlated to loaf volume of bread.

The following table summarizes the parameters that were selected as constraints in these works.

**Table 3.6: Quality parameters in previous literature**

	Niernberger (1973)	IGP model	US Wheat Associates model	Hayta & Cakmali (2001)
<b>Physical Wheat Traits</b>				
Test Weight		X		
Moisture		X		
Wheat protein		X		
Falling number			X	X
<b>Milling and Rheological Traits</b>				
Wet Gluten		X	X	
Flour protein	X		X	
Alveo P			X	
Alveo L			X	
Alveo W			X	
Alveo P/L			X	
Flour ash	X		X	
Particle Size Index				X
Far. Absorption	X			
Far. Arrival time	X			
Far. Development time	X			
Far. Valorimeter	X			
Starch Damage	X			
<b>Baking Data</b>				
Dough volume				X
Loaf volume	X			
Total score	X			

No single quality parameter has been selected by more than two authors. Only four of them have been selected by two authors: Falling number, Wet Gluten, Flour protein and Flour ash.

However, one must note that four characteristics selected by Niernberger (1973) and four other characteristics selected in the US Wheat Associates model measure the same thing but with a different device. Alveograph is widely used in France and is rather dedicated to soft wheat. Farinograph is widely used in other countries and is rather dedicated to hard wheat. Both Alveograph and Farinograph are laboratory devices that test the physical traits of dough.



### 3.4.2 *Econometrics*

In order to minimize the number of constraints in the optimization model, redundant characteristics should be excluded.

Econometricians search for redundant variables in order to avoid multicollinearity in regression functions. They consider that two variables are redundant when their coefficient of determination is high. A high coefficient of determination between two variables means that one of them is largely determined by the other. There is no universally admitted definition of what is a high  $R^2$  coefficient. However,  $R^2$  ranging between 0 and 1, one may admit that when  $R^2$  is higher than 0.5, data are highly correlated and therefore redundant.

The coefficient of determination  $R^2$  between twenty quality parameters has been computed for every cargo of soft wheat received by GMA during the year 2010. The tables showing these twenty parameters for every vessel and their coefficients of determination are displayed in Appendix A.

Eight parameters out of twenty have coefficients of determination higher than 0.5. These relatively high correlation coefficients between characteristics make sense.

The P and G measures from the Alveograph are correlated with P/L. Actually, P/L is computed by dividing P by L and L is a function of G ( $G = 2.226 \sqrt{L}$ ).

It makes sense that the volume of bread after 3 hours of fermentation is highly correlated with the loaf weight and that the volume of bread after 4 hours of fermentation is highly correlated with the volume of bread after 3 hours of fermentation.

The total score of bread is also highly correlated with the bread volume, the dough grade, the bread grade and the crumb grade. Actually the total score is the sum of all the other characteristics.

All these parameters should not be selected together as quality constraints of the optimization model.

### *3.4.3 Quality constraints selection*

The objective of the present work is to minimize production costs while still meeting requirements on flour quality. It therefore makes sense to focus on final products: flour and bread. Wheat quality parameters, although important when it comes to procurement, may be considered as less relevant in the optimization model.

In order to minimize the number of parameters selected as constraints of the optimization model, it also makes sense to consider aggregates rather than their components.

In addition, flour ash, a parameter that was selected as a quality constraint by two previous works, is irrelevant. In Côte d'Ivoire, it is a law requirement that bakery flour should have an ash content between 0.50% and 0.60%. All bakery flours from GMA are at 0.60%.

The parameters that have been selected as constraints of the optimization model are:

1. Flour protein content
2. Flour falling number
3. Alveograph W
4. Specific volume of baguette after 4 hours of fermentation.

These parameters have already been selected by previous authors; they are not highly correlated with each other; they concern the final product, flour; and they cover the whole range of flour characteristics:

- Flour Physical Traits: protein content and falling number
- Milling Properties: Alveograph W
- Baking Properties: specific volume of baguette after 4 hours of fermentation.

There are good reasons to select these four quality parameters as constraints of the optimization model. Their choice nevertheless remains at least partly subjective. One will have to keep in mind that the selection of better quality parameters will remain a way to improve the optimization model.

### **3.5 Quality constraints: specifications (RHS)**

In the optimization model, quality constraints are represented by inequalities. In the current section, the focus will be on the Right Hand Side (RHS) or  $\alpha$  of such inequalities: the specifications or the limits GMA assigns to quality parameters.

#### *3.5.1 Flour protein content*

A kernel of wheat is composed of some 83% of endosperm, 14.5% of bran and 2.5% of germ. Basically, wheat milling consists in separating endosperm from bran and germ and reducing endosperm into a fine powder called flour. Wheat flour is therefore essentially made of the components of endosperm: starch, moisture and protein. Protein contents of flour vary from 7% to 16%. They are essentially determined by wheat genetics, milling techniques and environment.

Proteins are essential components in human food. They have also important characteristics when it comes to flour functionality. Wheat proteins include glutenins, gliadins, globulins, albumins, glycoproteins and others. While albumins and globulins contain some functional enzymes, glutenins and gliadins account for gluten formation. Gluten is water insoluble and it forms when wheat flour is mixed with water. It impacts dough elasticity and gives dough gas retaining ability. Protein content is therefore a major parameter of flour quality.

There are different ways to measure flour protein content. However, all methods are based upon the fact that proteins contain nitrogen. Standard methods are known as Kjeldahl or Dumas. GMA uses a quicker method: infrared spectroscopy. A small quantity of flour is put into a device called Infraneo, manufactured by Chopin Technologies ([www.chopin.fr](http://www.chopin.fr)). It instantaneously reads nitrogen content and converts it into protein content. Although less reliable than Kjeldahl or Dumas, this method is widely used by flour millers, because it is very quick. GMA experience of the market has shown that flour protein content between 11% and 13% is optimal for the production of baguettes in Côte d'Ivoire.

**Figure 3.3: Flour Protein Content test**



In further equations, flour protein content will be labeled ‘FPC’ with subscript characters indicating which product is concerned. For instance,  $FPC_{sw}$  will mean protein content of flour made out of soft wheat only and  $FPC_{FLR}$  will mean protein content of flour made out of a mix of ingredients.

### *3.5.2 Flour falling number*

Enzymes are catalysts in the chemical reactions that occur during the baking process. Wheat kernels contain different types of enzymes. Among them, alpha-amylases trigger the breakdown of starch into sugar during fermentation. The level of alpha-amylase activity is therefore an important parameter of flour quality.

Alpha-amylase activity is measured by Hagberg falling number, with a device manufactured by Perten ([www.perten.com](http://www.perten.com)). The falling number actually records the time it takes a piston to sink through a paste made of boiling water and flour. The higher the falling number is, the lower the enzyme activity. A certain level of enzyme activity is necessary for the baking process. However, too much enzyme activity would produce adverse effects.

**Figure 3.4: Falling Number test**



GMA standards in terms of falling number are in between 350 and 500 seconds.

In further equations, flour falling number will be labeled ‘FLN’ with subscript characters indicating which product is concerned. For instance,  $FLN_{sw}$  will mean falling number of flour made out of soft wheat only and  $FLN_{FLR}$  will mean falling number of flour made out of a mix of ingredients.

### *3.5.3 Alveograph W*

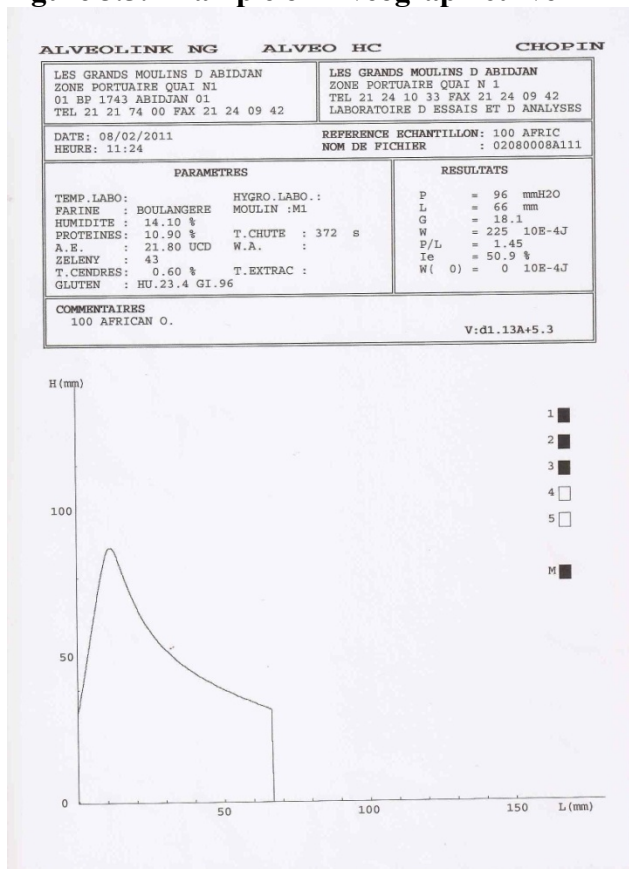
Protein content and Falling number measure physical and chemical characteristics of flour. However the quality of flour also relies upon the physical characteristics of the dough that is made with it. In French baking traditional areas, millers generally use a device called Alveograph, manufactured by Chopin Technologies ([www.chopin.fr](http://www.chopin.fr)), to test dough properties.

A sample of flour is mixed with a salt solution to form dough. It is then extruded, sheeted and cut into disks that are allowed to rest in the Alveograph under controlled heat conditions. Then the Alveograph blows air into a dough disk. This dough disk expands into

a bubble until it eventually breaks. During this process, pressure variations on the dough bubble are recorded and printed as a curve on a graph.

Four main figures come with this curve: P, L, Ie and W. P, for pressure, represents the highest point of the curve. It measures tenacity or the resistance to pressure of the dough. L, for length, represents the width of the curve from the beginning of the process until the breaking point. It measures the extensibility of the dough. Ie is the Index of elasticity, the ability of dough to regain its initial form. W, for work, represents the area below the curve. It is an indicator of the baking strength of dough and the quality of proteins. W gives a global view of the baking strength of dough. It is particularly influenced by protein quantity and quality, the amount of damaged starch and the enzymatic activity of dough.

**Figure 3.5: Example of Alveograph curve**



As regards W, GMA sets its objectives at values higher than 230.

In further equations, Alveograph W will be labeled 'ALW' with subscript characters indicating which product is concerned. For instance,  $ALW_{sw}$  will mean Alveograph W of flour made out of soft wheat only and  $ALW_{FLR}$  will mean Alveograph W of flour made out of a mix of ingredients.

#### *3.5.4 Specific volume of baguette after 4 hours of fermentation*

Baking tests are eventually the only ones that can predict the end product performance. At GMA, they are performed at a trial bakery upon the basis of the BIPEA protocol. The BIPEA (Bureau Inter-Professionnel d'Etudes Analytiques) is a French society that sets up industry standards. It has designed baking tests that are widely used in French mills. GMA has adapted these tests in order to take greater account of the requirements of Ivorian bakers.

Experience has shown that the most important criterion for Ivorian bakers is the volume of baguette after four hours of fermentation. Ivorian bakers are looking for high volumes of bread. They also appreciate tolerant dough which can stand for long hours of fermentation under tropical climate.

**Figure 3.6: Volumeter test**



Because the weights of baguettes are not always the same, this quality characteristic is measured by a specific volume: the volume, in cubic centimeters, of one gram of baguette. Volumes of baguettes are measured in a device called a “Volumeter” and their weights are read on a laboratory balance.

According to GMA standards, the specific volume of a baguette after 4 hours of fermentations should be higher than 11.5 cubic centimeters per gram.



**Figure 3.7: Weighing baguettes**



In further equations, the specific volume of a baguette after 4 hours of fermentation will be labeled ‘BVL’ with subscript characters indicating which product is concerned. For instance,  $BVL_{SW}$  will mean specific volume of bread made out of soft wheat only and  $BVL_{FLR}$  will mean specific volume of bread made out of a mix of ingredients.

The following table summarizes GMA objectives as regards quality constraints.

**Table 3.7: GMA quality specifications**

Quality Parameter	Minimum	Maximum
Flour Protein Content	11%	13%
Flour Falling Number	350 s.	500 s.
Flour Alveograph W	230	
Specific volume of baguette after 4 hours of fermentation	11.5 cm <sup>3</sup> /gram	

These specifications reflect the requirements of the Ivorian market in 2011/2012. They may evolve in the future.

### 3.6 Quality constraints: equations (LHS)

The current section deals with the Left Hand Side (LHS) of the constraint equations: the relationships between ingredients and quality parameters.

Grains science is the major source of information for defining these quality constraint equations. Actually, most relationships between wheat, additives and flour characteristics have already been studied and documented by grain scientists.

However, some specific relationships in the optimization model remain unknown. This is the case when it comes to the specific volume of baguette. This is also the case when it comes to enzymes mixes, because GMA has no precise information on their contents. In such cases, regression analysis will be used in order to determine the relationships between ingredients and flour quality parameters.

According to Ragsdale (2008, p. 409), “the goal in regression analysis is to identify a function that describes, as closely as possible, the relationship between these (independent and dependent) variables so that we can predict what value the dependent variable will assume given specific values for the independent variables”. In other words, regression analysis helps determine what the technical coefficients,  $\beta_{is}$ , are in the constraints.

**Table 3.8: Regression Analyses  $\beta$  coefficients**

$\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n \leq \alpha, \text{ or}$ $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n \geq \alpha, \text{ or}$ $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_n X_n = \alpha$
<p><b>where:</b></p> <ul style="list-style-type: none"><li>• <math>X_i</math> are the independent variables ;</li><li>• <math>\alpha</math> is the dependent variable.</li></ul>

In the optimization model,  $X_i$  will represent some characteristics of soft wheat, hard wheat, gluten, ascorbic acid or enzyme mixes and the different  $\alpha$ s will stand for GMA specifications as regards protein content, falling number, Alveograph W and baguette specific volume.

Regression analyses will be performed on data collected by GMA in the past. GMA has achieved tests of flour quality that were specially designed at gaining a better understanding of the impacts of different inputs on the final product. Altogether 73 tests were conducted in 2010 and 2011 with varying incorporation rates of soft wheat, hard wheat, gluten, different enzyme mixes and/or ascorbic acid. Values of independent variables and of corresponding dependent variables from all these tests are displayed in Appendix B.

In the present thesis, regression analysis equations are determined using the Ordinary Least Squares method, with the help of Microsoft Excel functions.

### 3.6.1 Flour protein content

Flour milling theory teaches that the flour protein content of a mix of wheat is the weighted average of the flour protein contents of the different types of wheat that have been blended.

Flour millers also know that, in the range of protein contents used by GMA, the addition of x% of gluten in flour will result in an increase of 0.8x% of protein content in the mix.

Accordingly, with an extraction rate of 80%, the addition of y% of gluten over wheat, will result in an increase of  $(80\% \times 0.8y\%) = 0.64y\%$  of protein content in the mix.

Consequently, the protein content of a flour made out of soft wheat, hard wheat and gluten is mathematically determined by the following equation.

**Table 3.9: Equation FPC1 – Flour Protein Content**

$(W_{sw}/1000) FPC_{sw} + (W_{hw}/1000) FPC_{hw} + 0.64 (W_{GLT}/1000) = FPC_{FLR}$
<p>where:</p> <ul style="list-style-type: none"> <li>• <math>W_{sw}</math>, <math>W_{hw}</math> and <math>W_{GLT}</math> represent the weights in metric tons of respectively soft wheat, hard wheat and gluten used in a mix, the total of which amounts to one thousand metric tons ;</li> <li>• <math>FPC_{sw}</math> and <math>FPC_{hw}</math> and <math>FPC_{FLR}</math> represent the protein contents of flours produced out of respectively soft wheat and hard wheat and the final mix.</li> </ul>

This equation has been tested against 10 analyses achieved by GMA of protein contents of flours made exclusively out of wheat and gluten. The comparison of predicted flour protein contents with actual ones is displayed in Appendix C.

The coefficient of correlation  $R$  between the two sets of data is equal to 0.87. Their coefficient of determination  $R^2$  is equal to 0.75, meaning that 75% of actual flour protein content is explained by Equation FPC1. And the adjusted  $R^2$  is equal to 0.72. All these figures are high, confirming close correlation between flour protein contents predicted by equation FPC1 and actual figures. In addition, a Student's  $t$  test has been performed on the two sets of data and concludes that the means of the two sets of data are the same (see Appendix C).

This confirms that, when there are no other inputs than wheat and gluten, equation FPC1 above is valid.

Equation FPC1 has also been tested against other data, when other inputs, acid ascorbic and different enzyme mixes, had been incorporated in the mix in addition to wheat and gluten. Protein contents of 55 different flours made out of various ingredients were compared to the results of equation FPC1. This test is displayed in Appendix D.

The coefficient of correlation  $R$ , the coefficient of determination  $R^2$  and the adjusted  $R^2$  of the two new sets of data drop down to, respectively 0.80, 0.65 and 0.64. Such coefficients are still high. However, the hypothesis stating that the means of the two sets of data are the same, is not confirmed by a Student's  $t$  test.

The drop in coefficients may be explained by the presence of ascorbic acid or enzyme mixes. However, incorporation of ascorbic acid should have no effect on flour protein content. Ascorbic acid does not contain proteins. As regards enzyme mixes, they may contain protein but their rate of incorporation to the blend is so low that they should not have a significant impact.

Consequently and because it is theoretically sound, FPC1 will be used as the flour protein content constraint equation of the optimization model.

### 3.6.2 Flour falling number

Grains science has shown that the falling number of flour made out of a mix of wheat is not the weighted average of the falling numbers of flours made out of its wheat components. However, milling scientists have identified a proxy, the liquefaction number, which has this desired characteristic. If FLN is the falling number, then the corresponding liquefaction number LNR is equal to  $(6,000 / (FLN+50))$ .

The relationship between the liquefaction number of flour made from a mix of wheat and the liquefaction numbers of flours made out of its wheat components can be written as follows.

**Table 3.10: Equation LNR1 – Liquefaction Number**

$(n_{w1}LNR_{w1} + n_{w2}LNR_{w2} + \dots + n_{wn}LNR_{wn}) = LNR_{FLR}$
where:
<ul style="list-style-type: none"><li>• <math>n_{w1}</math>, <math>n_{w2}</math> and <math>n_{wn}</math> represent the relative proportion of <math>n</math> lots of wheat in the mix, the sum of <math>n_{wi}</math> being equal to 100% ;</li><li>• <math>LNR_{w1}</math>, <math>LNR_{w2}</math> ... <math>LNR_{wn}</math> represent the liquefaction numbers of flours produced out of the respective lots of wheat 1,2 or <math>n</math> ;</li><li>• <math>LNR_{FLR}</math> represents the liquefaction number of flour made out of the mix of wheat.</li></ul>

Because it is much easier to use linear equations, liquefaction number will be used instead of falling number in the equation of the optimization model. In Section 3.4, GMA falling number specifications were fixed at 350 and 500 seconds. These standards now become respectively 15.000 and 10.909 in terms of liquefaction numbers.

In further equations, liquefaction number will be labeled 'LNR' with subscript characters indicating which product is concerned. For instance,  $LNR_{sw}$  will mean liquefaction number of flour made out of soft wheat only and  $LNR_{FLR}$  will mean liquefaction number of flour made out of a mix of ingredients.

For a blend weighing 1,000 metric tons that is made exclusively out of one lot of soft wheat and one lot of hard wheat, equation LNR1 becomes:

**Table 3.11: Equation LNR1 – Liquefaction Number – Soft wheat and hard wheat only**

$$(W_{sw}/1000) LNR_{sw} + (W_{hw}/1000) LNR_{hw} = LNR_{FLR}$$

where:

- $W_{sw}$  and  $W_{hw}$  represent the weights in metric tons of respectively soft wheat and hard wheat used in a mix, the total of which amounts to one thousand metric tons ;
- $LNR_{sw}$  and  $LNR_{hw}$  represent the liquefaction numbers of flours produced out of respectively soft wheat and hard wheat ;
- $LNR_{FLR}$  represents the liquefaction number of flour made out of the mix of wheat.

This equation has been tested against 9 series of data GMA has recorded on falling numbers or liquefaction numbers of flours made exclusively out of soft wheat and hard wheat. The comparison between predicted liquefaction numbers and actual ones is displayed in Appendix E.

The coefficient of correlation R between the two sets of data is equal to 0.52. Their coefficient of determination  $R^2$  is equal to 0.28, meaning that 28% of actual liquefaction number is explained by the theoretical equation. And the adjusted  $R^2$  is equal to 0.17. A Student's t test, performed on the two sets of data, concludes that the means of the two sets of data are the same. All these figures seem to confirm that there is a correlation between liquefaction numbers predicted by equation LNR1 and actual figures. However, this correlation is not very strong.

Differences between predictions from equation LNR1 and actual liquefaction numbers may arise from many different sources. If wheat lots are not homogeneous enough, liquefaction numbers from one sample may not represent the value of the whole lot. Because the test of enzymatic activity is relatively sophisticated, the person who performs the test may also influence the results. The devices with which tests are performed may also cause errors: manufacturers of such devices acknowledge that tests performed on similar samples do not always give the same results and the margin of error may be as high as five percent.

Although  $R^2$  is smaller than expected, it can reasonably be assumed that, when there are no other inputs than soft wheat and hard wheat, equation LNR1 above is confirmed by tests.

Equation LNR1 has also been tested against other data, when other ingredients such as gluten, ascorbic acid or enzyme mixes have been incorporated in the mix in addition to wheat. The comparison of the liquefaction numbers of 49 flours made out of different ingredients and the results of equation LNR1 is displayed in Appendix F.

Surprisingly, coefficients  $R$ ,  $R^2$  and adjusted  $R^2$  increase to 0.64, 0.41 and 0.39, respectively. And a Student's  $t$  test confirms that the means of the two sets of data are the same. The fact that this second correlation is stronger than the previous one without additives may come from the fact that it is tested against a larger dataset. However, other ingredients should have no impact on falling number and, consequently, on liquefaction number.

Gluten is composed of proteins and does not contain alpha-amylases. Ascorbic acid is not an enzyme. The presence of these ingredients does not affect flour liquefaction number.

Enzyme mixes should increase the alpha-amylase activity of dough, as long as they contain alpha-amylases. In their presence, falling number should decrease and liquefaction number should increase. GMA has no information about the presence of alpha-amylases in its enzyme mixes.

Regression analysis has been used in order to assess the relationship between enzyme mixes and the proportion of flour liquefaction number which is not explained by wheat in equation LNR1 above. The details of the regression analysis are shown in Appendix G. The following table summarizes the Ordinary Least Squares estimates.

**Table 3.12: Equation LNR2 – Liquefaction Number**

<b>LNR<sub>res</sub> =</b>	<b>1.7711</b>	<b>- 27.3652 W<sub>EN1</sub></b>	<b>- 153.5047 W<sub>EN2</sub></b>
<b>Standard deviation</b>		<b>21.0408</b>	<b>104.8242</b>
<b>t-statistic</b>		<b>- 1.3006</b>	<b>- 1.4644</b>
<b>Adjusted R<sup>2</sup> = 0.0153</b>	<b>n = 39</b>		

**where:**

- **LNR<sub>res</sub> is the amount of liquefaction number that is not explained by the liquefaction numbers of the mix of wheat ;**
- **W<sub>EN1</sub> and W<sub>EN2</sub> are the weights, in metric tons, of respectively enzyme mix 1 and enzyme mix 2, used in the mix, the total of which amounts to one thousand metric tons.**

The adjusted R<sup>2</sup> is very low in the regression equation. Given their t-statistics, the coefficients of W<sub>EN1</sub> and W<sub>EN2</sub> are not statistically significant at a level of 10%. In addition, they are surprisingly negative. This poor regression equation may mean that there are no alpha-amylases in the enzyme mixes used by GMA.

Consequently, equation LNR1 will be retained as the constraint equation of the optimization model as regards flour liquefaction number.

### 3.6.3 Alveograph W

According to Chopin Technologies, the company that manufactures the Alveograph, W of flour made out of a mix of wheat is equal to the weighted average of Ws of flours made out of these different types of wheat. This can be mathematically translated as follows.



**Table 3.13: Equation ALW1 – Alveograph W**

$(n_{w1}ALW_{w1} + n_{w2}ALW_{w2} + \dots + n_{wn}ALW_{wn}) = ALW_{FLR}$
<p>where:</p> <ul style="list-style-type: none"> <li>• <math>n_{w1}</math>, <math>n_{w2}</math> and <math>n_{wn}</math> represent respectively the relative proportion of n types of wheat in the mix, the sum of <math>n_{wi}</math> being equal to 100% ;</li> <li>• <math>ALW_{w1}</math>, <math>ALW_{w2}</math> ... <math>ALW_{wn}</math> represent Alveograph Ws of flours produced out of the respective types of wheat 1, 2 or n ;</li> <li>• <math>ALW_{FLR}</math> represents the Alveograph W of flour made out of the mix of wheat.</li> </ul>

When flour is made exclusively out of a blend of soft wheat and hard wheat and when the total mix weighs 1,000 metric tons, equation ALW1 becomes.

**Table 3.14: Equation ALW1 – Alveograph W – Soft wheat and hard wheat only**

$(W_{sw}/1000)ALW_{sw} + (W_{hw}/1000)ALW_{hw} = ALW_{FLR}$
<p>where:</p> <ul style="list-style-type: none"> <li>• <math>W_{sw}</math> and <math>W_{hw}</math> represent the weights in metric tons of respectively soft wheat and hard wheat used in a mix, the total of which amounts to one thousand metric tons ;</li> <li>• <math>ALW_{sw}</math> and <math>ALW_{hw}</math> represent Alveograph Ws of flours produced out of, respectively soft wheat and hard wheat ;</li> <li>• <math>ALW_{FLR}</math> represents the Alveograph W of flour made out of the mix of wheat.</li> </ul>

This equation has been tested against 9 series of data GMA has recorded on flours made exclusively out of soft wheat and hard wheat. The comparison between predicted Ws and actual ones is displayed in Appendix H.

The coefficient of correlation R between the two sets of data is equal to 0.97. Their coefficient of determination  $R^2$  is equal to 0.93, meaning that 93% of actual Alveograph W is explained by the equation ALW1. And the adjusted  $R^2$  is equal to 0.93. All these figures confirm that there is a strong correlation between Alveograph W numbers predicted by equation ALW1 and actual figures. A Student's t test has been performed on the two sets of data and it concludes that the means of the two sets of data are the same.

Equation ALW1 has also been tested against other data, when additives, gluten, acid ascorbic or enzyme mixes had been incorporated into flour in addition to wheat. The comparison of 55 Alveograph Ws from flours made out of different inputs and the results of equation ALW1 is displayed in Appendix I.

The coefficient of correlation R, the coefficient of determination R<sup>2</sup> and the adjusted R<sup>2</sup> between the two new sets of data drop down to, respectively 0.93, 0.87 and 0.87. These coefficients nevertheless remain high. A Student's t test confirms that the means of the two sets of data are the same.

Theory supporting equation ALW1 is strong and is reinforced by tests on actual data.

Some additives may nevertheless have a further impact on Alveograph W. Gluten reinforces pressure and extensibility of dough although this is generally considered as not significant. Experience teaches that enzyme mixes may influence the strength of dough and consequently Alveograph W. However, their impact is nevertheless difficult to forecast.

Regression analysis has been used in order to assess the relationship between gluten, enzyme mixes and residual W, the amount of Alveograph W which is not explained by wheat mixes. The details of the regression analysis are shown in Appendix J. The following table summarizes the Ordinary Least Squares estimates.

**Table 3.15: Equation ALW2 – Alveograph W**

<b>ALW<sub>res</sub> =</b>	<b>-0.6236</b>	<b>+ 0.4938 W<sub>GLT</sub></b>	<b>+ 110.3011 W<sub>EN1</sub></b>	<b>+ 272.3460 W<sub>EN2</sub></b>
<b>St. deviation</b>		<b>0.4324</b>	<b>77.7021</b>	<b>398.1873</b>
<b>t-statistic</b>		<b>1.1421</b>	<b>1.4195</b>	<b>0.6840</b>
<b>Adjusted R<sup>2</sup> = 0.0868</b>		<b>n = 46</b>		
<b>where:</b>				
• <b>ALW<sub>res</sub> is the amount of W that is not explained by the Ws of the mix of wheat ;</b>				
• <b>W<sub>GLT</sub>, W<sub>EN1</sub> and W<sub>EN2</sub> represent the weights, in metric tons, of respectively gluten, enzyme mix 1 and enzyme mix 2 used in a mix, the total of which amounts to one thousand metric tons ;</b>				

Adjusted R<sup>2</sup> is low at 0.0868. Signs of  $\beta$  coefficients are as expected. Only the  $\beta$  coefficient of  $W_{EN1}$  is statistically significant at a level of 10%, according to the Student's t-test. Altogether this regression equation is not very satisfactory. But it is theoretically sound.

Consequently, ALW3, a mix of equations ALW1 and ALW2, will be used as the constraint equation for Alveograph Ws in the optimization model.

**Table 3.16: Equation ALW3 – Alveograph W**

$(W_{sw}/1000)ALW_{sw} + (W_{hw}/1000)ALW_{hw} - 0.6236 + 0.4938 W_{GLT} + 110.3011 W_{EN1} + 272.3460 W_{EN2} = ALW_{FLR}$
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#### 3.6.4 Specific volume of baguette after 4 hours of fermentation

Unlike the other quality parameters, there is no readily available theoretical model that links ingredients and flour as regards the specific volume of baguette after 4 hours of fermentation. Only experience gives some hints.

Soft wheat, as the most important component of GMA mix is obviously a major influence on the volume of baguettes. This influence is expressed in the specific volume of baguettes made exclusively out of the soft wheat lot under review.

Incorporation of hard wheat at a relatively small percentage increases the volume of baguettes. However, when this percentage is too high, it has an adverse effect. Stronger networks of protein hinder the growth of dough. As already mentioned earlier, GMA does not make baguettes out of its cargoes of hard wheat. In order to represent hard wheat influence in the baguette specific volume constraints equation, a proxy, Alveograph W,  $ALW_{hw}$ , the baking strength of flour made exclusively out of hard wheat will be used.

Gluten has a similar effect as hard wheat on baguette volume. It brings higher gas retaining power in dough. At relatively low incorporation rates, it favors high volume of bread. At higher incorporation rates, it has an adverse effect.

The major reason for incorporating ascorbic acid into the mix is to increase bread volume. Ascorbic acid brings oxygen in dough and helps breaking the protein network. It enhances extensibility of dough, i.e. the ability of dough to expand while retaining gas.

Different enzymes may have different effects on the volume of bread. For instance, glucose-oxidases favor bread volume while some proteases don't. However, GMA requests its supplier to elaborate enzyme mixes that increases bread volumes. One should therefore expect that the effect of at least one of its enzyme mixes is positive when it comes to the volume of baguettes.

In absence of a theoretical model, regression analysis is used in order to determine a mathematical relationship between all these inputs and flour as regards the specific volume of baguette after 4 hours of fermentation. Details of this analysis are displayed in Appendix K. The following table summarizes the Ordinary Least Estimates.

**Table 3.17: Equation BVL1 – Specific Volume of Baguette**

<b>BVL<sub>FLR</sub> =</b>	<b>-1.0237</b>	<b>+ 1.0482 (W<sub>sw</sub>/1000) BVL<sub>sw</sub></b>	<b>+ 0.0295 (W<sub>hw</sub>/1000) ALW<sub>hw</sub></b>
<b>St. deviation</b>		<b>0.1523</b>	<b>0.0053</b>
<b>t-statistic</b>		<b>6.8839</b>	<b>5.5289</b>
	<b>+ 0.00159 W<sub>GLT</sub></b>	<b>+ 6.9306 W<sub>AAC</sub></b>	<b>+ 23.0209 W<sub>EN1</sub></b>
<b>St. deviation</b>	<b>0.0654</b>	<b>5.5136</b>	<b>5.9985</b>
<b>t-statistic</b>	<b>0.2437</b>	<b>1.2570</b>	<b>3.8378</b>
	<b>+ 15.0943 W<sub>EN2</sub></b>		
<b>St. deviation</b>	<b>31.3419</b>		
<b>t-statistic</b>	<b>0.4816</b>		
<b>Adjusted R<sup>2</sup> = 0.6198</b>		<b>n = 66</b>	
<b>where:</b>			
<ul style="list-style-type: none"> <li>• <b>BVL<sub>FLR</sub> is the specific volume (volume divided by weight) expressed in cubic centimeters divided by grams, of baguettes after 4 hours of fermentation ;</b></li> <li>• <b>BVL<sub>sw</sub> is the specific volume (volume divided by weight) expressed in cubic centimeters divided by grams, of baguettes, after 4 hours of fermentation, made from soft wheat only ;</b></li> <li>• <b>ALW<sub>hw</sub> represents the Alveograph W of flour produced out of hard wheat only ;</b></li> </ul>			

- $W_{sw}$ ,  $W_{hw}$ ,  $W_{GLT}$ ,  $W_{AAC}$ ,  $W_{EN1}$  and  $W_{EN2}$  represent the weights in metric tons of respectively soft wheat, hard wheat, gluten, ascorbic acid, enzyme mix 1 and enzyme mix 2 used in a mix, the total of which amounts to one thousand metric tons.

Adjusted  $R^2$  is quite high at 0.62. Signs of  $\beta$  coefficients are positive as expected, except for gluten.  $\beta$  coefficients are also statistically significant, according to Student's t tests, except for gluten, ascorbic acid and enzyme mix 2. Altogether this regression equation is relatively satisfactory.

In between the limits of the technical constraints identified in Section 3.3 above, the adverse effects of high incorporation rates of hard wheat and gluten should not be felt. However, another way to take account of adverse effects is to use other functional forms in the regression model. Equations with quadratic functions applied to hard wheat and gluten have been tested. This regression analysis is documented in Appendix L and gives the following results.

**Table 3.18: Equation BVL2 – Specific Volume of Baguette**

<b>BVL<sub>F<sub>LR</sub></sub> =</b>	<b>-1.1560</b>	<b>+ 1.0488 (W<sub>sw</sub>/1000) BVL<sub>sw</sub></b>	<b>+</b>
<b>St. deviation</b>	<b>0.1543</b>		
<b>t-statistic</b>	<b>6.7952</b>		
	<b>+ 0.0374 (W<sub>hw</sub>/1000) ALW<sub>hw</sub></b>	<b>+ 0.0001 ((W<sub>hw</sub>/1000) ALW<sub>hw</sub>)<sup>2</sup></b>	<b>+</b>
<b>St. deviation</b>	<b>0.0117</b>	<b>0.0001</b>	
<b>t-statistic</b>	<b>3.1835</b>	<b>0.7581</b>	
	<b>- 0.0047 W<sub>GLT</sub></b>	<b>- 0.0050 (W<sub>GLT</sub>)<sup>2</sup></b>	<b>+ 6.9548 W<sub>AAC</sub></b>
<b>St. deviation</b>	<b>0.1427</b>	<b>0.0246</b>	<b>5.6151</b>
<b>t-statistic</b>	<b>- 0.0331</b>	<b>- 0.2021</b>	<b>1.2386</b>
	<b>+ 23.9619 W<sub>EN1</sub></b>	<b>+ 18.5955 W<sub>EN2</sub></b>	
<b>St. deviation</b>	<b>6.1970</b>	<b>32.1070</b>	
<b>t-statistic</b>	<b>3.8667</b>	<b>0.5792</b>	

Adjusted R<sup>2</sup> = 0.6105

n = 66

where:

- **BVL<sub>FLR</sub>** is the specific volume (volume divided by weight) expressed in cubic centimeters divided by grams, of baguettes after 4 hours of fermentation ;
- **BVL<sub>sw</sub>** is the specific volume (volume divided by weight) expressed in cubic centimeters divided by grams, of baguettes, after 4 hours of fermentation, made from soft wheat only ;
- **ALW<sub>hw</sub>** represents the Alveograph W of flour produced out of hard wheat only ;
- **W<sub>sw</sub>, W<sub>hw</sub>, W<sub>GLT</sub>, W<sub>AAC</sub>, W<sub>EN1</sub> and W<sub>EN2</sub>** represent the weights in metric tons of respectively soft wheat, hard wheat, gluten, ascorbic acid, enzyme mix 1 and enzyme mix 2 used in a mix, the total of which amounts to one thousand metric tons.

Adjusted R<sup>2</sup> in equation BVL2 is slightly lower than in equation BVL1. Signs of coefficients of gluten are unexpectedly negative. According to Student's t tests, the coefficients of gluten, ascorbic acid and enzyme mix 2 are not statistically significant. Coefficients of negative squared weight of hard wheat W as well as negative squared weight of gluten are also not statistically significant.

Because of the insignificance of the non-linear terms in equation BVL2, equation BVL1 has been preferred as the specific baguette volume constraint in the optimization model.

### 3.7 The optimization model

After identifying the objective function, the constraints, their limits and their equations, the blending problem of GMA can be expressed in mathematical terms. The optimization model includes an objective function and three types of constraints: self-binding constraints, technical constraints and quality constraints.

#### 3.7.1 The Objective Function

**Table 3.19: Optimization Model – Objective Function**

MIN:  $(W_{sw} P_{sw}) + (W_{hw} P_{hw}) + (W_{GLT} P_{GLT}) + (W_{AAC} P_{AAC}) + (W_{EN1} P_{EN1}) + (W_{EN2} P_{EN2})$

Prices are expressed in CFA francs per metric tons (FCFA/t) and weights are expressed in metric tons (t).

### 3.7.2 Self-binding constraints

Decision variables cannot be negative. The total weight of the mix is equal to 1,000 metric tons

**Table 3.20: Optimization Model - Self-binding Constraints**

<b>Non negativity</b>	$W_{sw} \geq 0 ; W_{hw} \geq 0 ; W_{GLT} \geq 0 ; W_{AAC} \geq 0 ; W_{EN1} \geq 0 ; W_{EN2} \geq 0$
<b>Total Weight</b>	$W_{sw} + W_{hw} + W_{GLT} + W_{AAC} + W_{EN1} + W_{EN2} = 1,000$

### 3.7.3 Technical constraints

GMA milling process, technical specifications of dosing scales or suppliers' advice affect incorporation rates and their increments. Weights of additives, in metric tons, can take only a limited set of values.

**Table 3.21: Optimization Model - Technical Constraints**

<b>Gluten</b>	$W_{GLT} \in \{0.0; 0.8; 1.6; 2.4; 3.2; 4.0; 4.8; 5.6; 6.4; 7.2; 8.0\}$
<b>Ascorbic Acid</b>	$W_{AAC} \in \{0.000; 0.008; 0.016; 0.024; 0.032; 0.040; 0.048; 0.056; 0.064; 0.072; 0.080\}$
<b>Enzyme Mix 1</b>	$W_{EN1} \in \{0.000; 0.056\}$
<b>Enzyme Mix 2</b>	$W_{EN2} \in \{0.000; 0.004; 0.008; 0.012; 0.016\}$

### 3.7.4 Quality constraints

The third set of constraints set limits on flour quality parameters.

**Table 3.22: Optimization Model – Quality Constraints**

<b>Flour Protein Content</b>	
<b>FPC1</b>	$11.0 \leq (W_{sw}/1000) FPC_{sw} + (W_{hw}/1000) FPC_{hw} + 0.64 (W_{GLT}/1000) \leq 13.0$
<b>Flour Liquefaction Number, as a proxy of Flour Falling Number</b>	
<b>LNR1</b>	$10.909 \leq (W_{sw}/1000) LNR_{sw} + (W_{hw}/1000) LNR_{hw} \leq 15.000$
<b>Alveograph W</b>	
<b>ALW3</b>	$230 \leq (W_{sw}/1000) ALW_{sw} + (W_{hw}/1000) ALW_{hw} - 0.6236 + 0.4938 W_{GLT} + 110.3011 W_{EN1} + 272.3460 W_{EN2}$

**Specific volume of baguette after 4 hours of fermentation**

$$\mathbf{BVL1} \quad \mathbf{11.5 \leq -1.0237 + 1.0482 (W_{sw}/1000) BVL_{sw} + 0.0295 (W_{hw}/1000) ALW_{hw} + 0.0159 W_{GLT} + 6.9306 W_{AAC} + 23.0209 W_{EN1} + 15.0943 W_{EN2}}$$

The object of Chapter 3 was to transform GMA blending problem into a set of equations. The real difficulty that appeared in this process was to make choices. The selection of quality parameters, of their specifications (RHS), of the form of their equations (LHS) is at least partly subjective and questionable. These choices do impact the results of the optimization model.



## **CHAPTER IV: DATA AND METHODS COMPUTER IMPLEMENTATION**

Nowadays, many spreadsheets provide tools, called solvers that easily solve optimization model such as the one that has been identified in the previous Chapter.

In the first section of the present Chapter, one of these solvers will be considered. Then the optimization model data will be entered on templates designed in Microsoft Excel.

### **4.1 Solver**

Operational research and optimization techniques were first developed for military purposes during World War II. Since then, these techniques have met an increasing success. The different methods developed in order to solve an optimization model come down to testing different solutions and selecting the optimal one. Efficient techniques like the Simplex method allow for a low the number of iterations before finding the optimum solution. However, solving a complex optimization problem nevertheless requires a significant computing power. As a consequence, what really generalized the use of operational research was the development of information systems and particularly personal computers in the last decades of the 20<sup>th</sup> century. Spreadsheets and their solvers have made it easy and simple to solve optimization problems.

Eventually, the implementation of the problem on computer has become a necessary and ordinary step of the optimization modeling process. It is the third step of the five identified by Ragsdale (2008) and it constitutes the fourth chapter of the present thesis.

Solvers are computer programs that are designed to find the values of certain cells, called variable cells, which maximize or minimize the value of another cell, called a target cell, while meeting problem constraints listed in other cells of the spreadsheet. In other terms, solvers provide solutions to optimization problems.

There is a wide range of solver software available on the market nowadays. Some of them are supplied on their own. Most often, they are included in spreadsheet packages. And

nowadays, any spreadsheet commonly integrates more or less sophisticated solver functions.

For the purpose of the present thesis, the optimization model will be implemented on Microsoft Office Excel 2010. Excel includes a solver function which was developed by a company named Frontline Systems Inc. ([www.solver.com](http://www.solver.com)).

However another solver, also developed by Frontline Systems Inc., and that work as an Excel add-in will be preferred. Premium Solver V11.5 is more powerful than Excel Solver. It includes a guided mode and it can handle larger and more complex models. It can be purchased at a price of USD 4,000 which is worth about 2 million FCFA. This is cheap in comparison of the price of one thousand tons of wheat. The cost of acquiring Premium Solver V11.5 will therefore be neglected in the optimization model.

A drawback of Premium Solver V11.5 in a French-speaking country like Côte d'Ivoire is that it is only available in English and it they must be added to the English version of Microsoft Excel. The Excel Solver, on the other hand, is available on the French version of Microsoft Excel.

Premium Solver V11.5 will be used and tested on two different models.

## **4.2 Models**

In the previous Chapter, three types of constraints were identified: self-binding, quality and technical constraints. The technical constraints limit the values that the weights of additives can take. They drastically restrict the set of possible solutions to the optimization model. Such constraints are equivalent to integrality conditions: decision variables can assume only integer values.

A standard linear programming (LP) problem, where all variables are assumed to be continuous has an infinite number of feasible solutions. An integer linear programming (ILP) problem has only a finite set of feasible solutions. Integrality conditions may even lead to infeasibility.

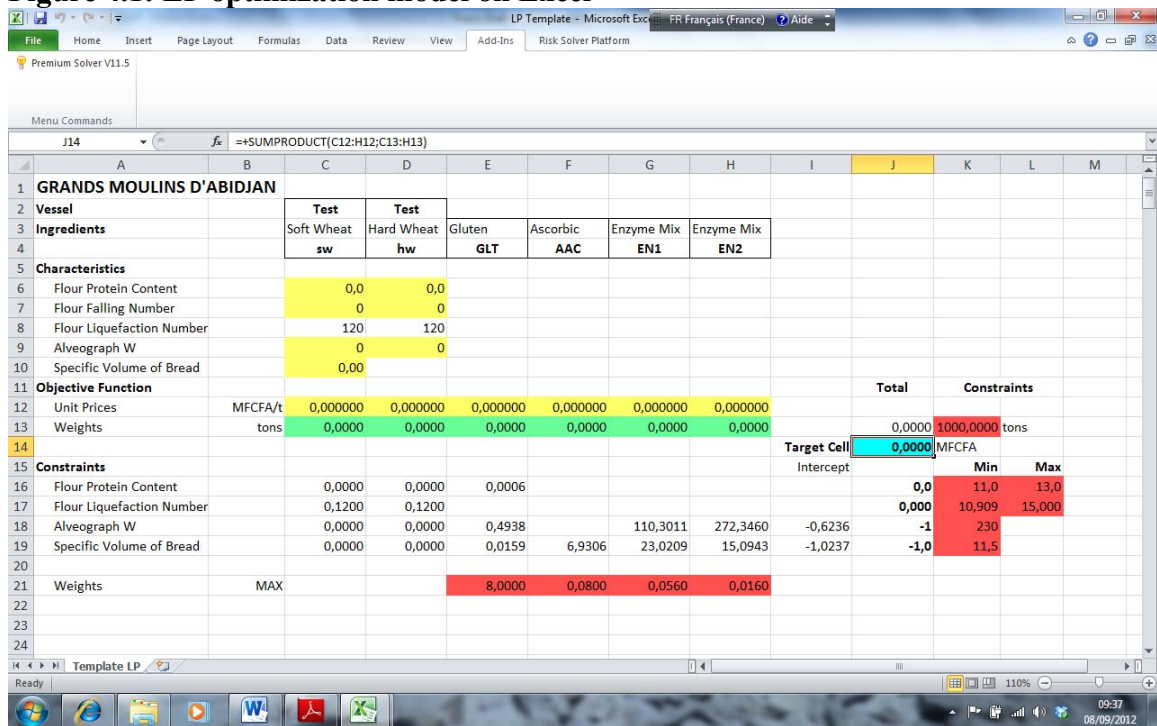
In order to deal with this issue, two models will be built: an ILP model where all integrality conditions are met, but also a LP model where the technical constraints are not considered. If the ILP model does not find any solution, it can be relaxed. LP model solutions will then be considered and may serve as substitutes.

Because it is easier to design and to implement, the LP model will be designed first. Integrality conditions on additives will then be introduced in the ILP model

#### 4.2.1 The LP Model

Microsoft Excel offers many ways to implement a LP model. Figure 4.1 shows one of them.

**Figure 4.1: LP optimization model on Excel**



Quality parameters and prices of the different ingredients of the flour mix are inputs of the model. They are highlighted in yellow. Variable cells, i.e. the weights of the different ingredients in a mix of 1,000 metric tons, are highlighted are green. The target cell, the

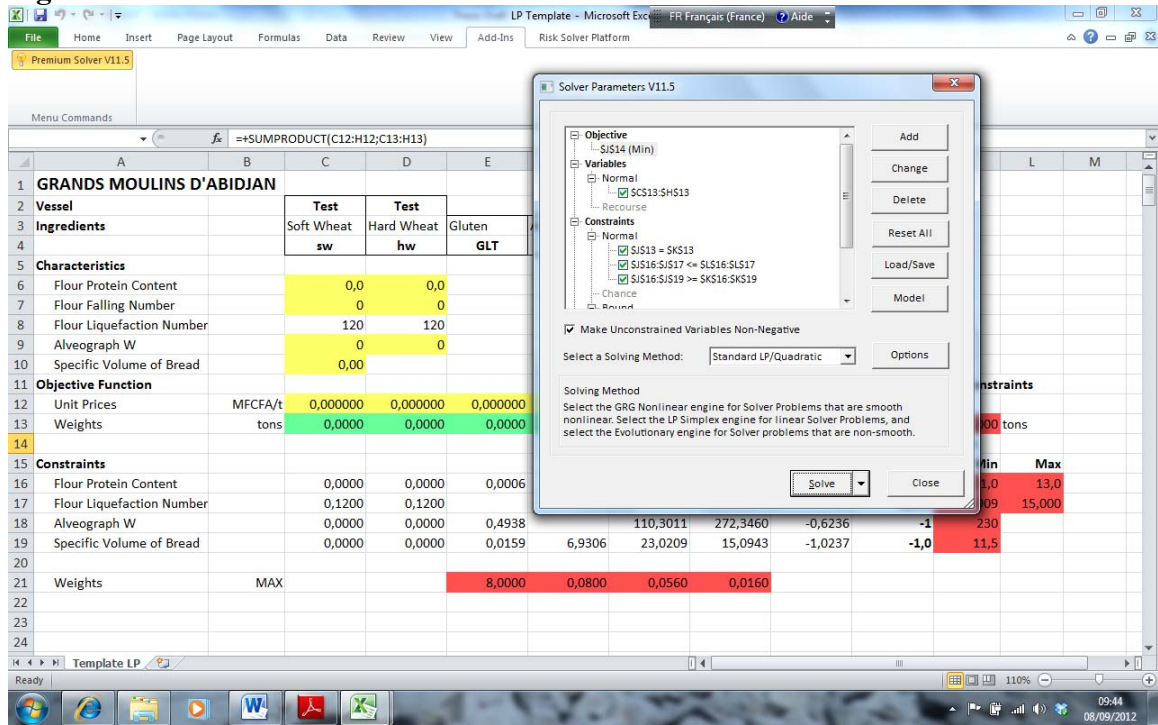
price of the mix of 1,000 metric tons, is highlighted in blue. And finally, constraints are highlighted in red.

Formula for cell J14, the target cell, is the sum of the prices of the different components of the mix. Such prices are the products of unit prices (in row 12) by weights (in row 13). In order to limit scaling problems, all prices are expressed in millions of CFA francs (MFCFA).

The total weight of the mix is assumed to be equal to 1,000 metric tons, which is the first constraint shown in cell K13. Data in cells C16:I19 record the different components (LHS) of the quality constraints equations. The results of these equations are displayed in cells J16:J19. These figures should be higher than constraints limits shown in cells K16:K19 and lower than constraints limits shown in cells L16:L17 (RHS). Finally, cells E21:H21 show the upper limits of the weights of additives in the mix.

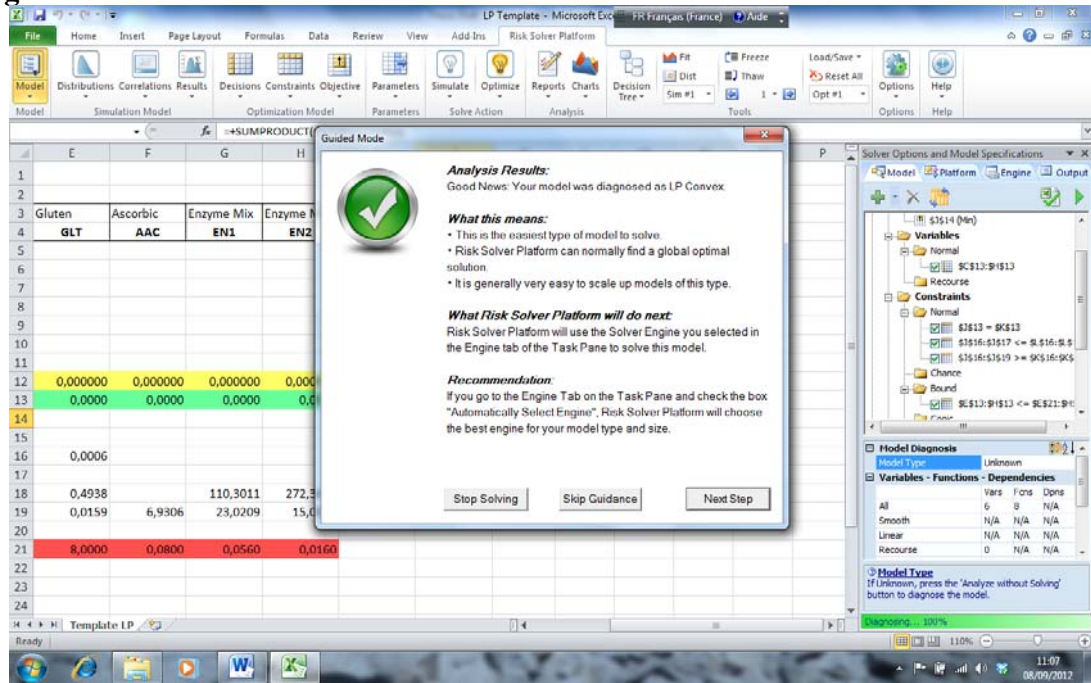
In Excel, Premium Solver V11.5 is available in the Add-Ins menu. Target cell, variable cells and constraints are entered into the Solver Parameters box, as displayed in Figure 4.2.

**Figure 4.2: LP Premium Solver V11.5 Parameters box**



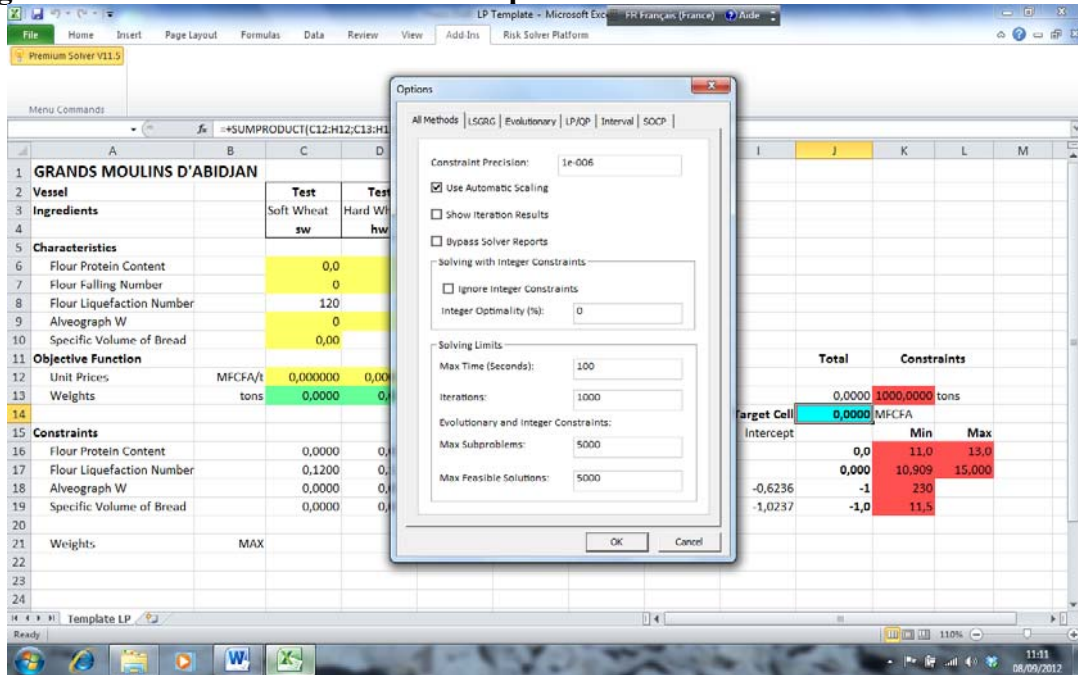
Premium Solver V11.5 guided mode confirms that the model is LP convex.

Figure 4.3: LP Premium Solver V11.5 Guided mode



Several options can be defined in the Options box of Premium Solver V11.5.

Figure 4.4: LP Premium Solver V11.5 Options Box



Automatic Scaling is always useful although some of the scaling issues have already been dealt with by using millions of CFA francs for prices.

Until now, technical constraints described in Section 3.3 have not been implemented in the model.

#### 4.2.3 The ILP model

The ILP model template is built on the basis of the LP model template. However, in order to deal with the technical constraints, variables representing additives weights have been redefined so that their respective increments correspond to one unit. With this conversion, the model becomes an integer one: variable cells can assume only integer values. To do so, variables  $W_i$  are replaced by their proxies  $W_i'$ .

**Table 4.1: LP/ILP model - Units Correspondence Table**

Ingredients	Formulas	$W_i$ units	$W_i'$ units
Gluten	$W_{GLT}' = W_{GLT} \times 1.25$	1 metric ton	800 kg
Ascorbic acid	$W_{AAC}' = W_{AAC} \times 125$	1 metric ton	8 kg
Enzyme Mix 1	$W_{EN1}' = W_{EN1} \times 1,000 / 56$	1 metric ton	56 kg
Enzyme Mix 2	$W_{EN2}' = W_{EN2} \times 250$	1 metric ton	4 kg

With such transformations, the technical constraints become:

**Table 4.2: LP/ILP model - Technical Constraints Correspondence Table**

Ingredients	Technical Constraints with $W_i$	Technical Constraints with $W_i'$
Gluten	$W_{GLT} \in \{0.0; 0.8; 1.6; 2.4; 3.2; 4.0; 4.8; 5.6; 6.4; 7.2; 8.0\}$	$W_{GLT}' \in \{0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10\}$
Ascorbic acid	$W_{AAC} \in \{0.000; 0.008; 0.016; 0.024; 0.032; 0.040; 0.048; 0.056; 0.064; 0.072; 0.080\}$	$W_{AAC}' \in \{0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10\}$
Enzyme Mix 1	$W_{EN1} \in \{0.000; 0.056\}$	$W_{EN1}' \in \{0; 1\}$
Enzyme Mix 2	$W_{EN2} \in \{0.000; 0.004; 0.008; 0.012; 0.016\}$	$W_{EN2}' \in \{0; 1; 2; 3; 4\}$

Unit prices of ingredients are modified as well.

**Table 4.3: LP/ILP model - Unit Prices Correspondence Table**

Ingredients	Formulas	$W_i$ units	$W_i'$ units
Gluten	$P_{GLT}' = P_{GLT} / 1.25$	1 metric ton	800 kg
Ascorbic acid	$P_{AAC}' = P_{AAC} / 125$	1 metric ton	8 kg
Enzyme Mix 1	$P_{EN1}' = P_{EN1} / 1,000 \times 56$	1 metric ton	56 kg
Enzyme Mix 2	$P_{EN2}' = P_{EN2} / 250$	1 metric ton	4 kg

Coefficients of quality constraints also change, where weights of additives are concerned.

**Table 4.4: LP/ILP model – Quality Constraints Correspondence Table**

Quality Constraint	$W_i$ equations	$W_i'$ equations
Flour protein Content	$11.0 \leq (W_{sw}/1000) FPC_{sw} + (W_{hw}/1000) FPC_{hw} + 0.64 (W_{GLT}/1000) \leq 13.0$	$11.0 \leq (W_{sw}/1000) FPC_{sw} + (W_{hw}/1000) FPC_{hw} + 0.512 (W_{GLT}' / 1000) \leq 13.0$
Flour Liquefaction Number	$10,909 \leq (W_{sw}/1000) LNR_{sw} + (W_{hw}/1000) LNR_{hw} \leq 15,000$	$10,909 \leq (W_{sw}/1000) LNR_{sw} + (W_{hw}/1000) LNR_{hw} \leq 15,000$
Alveograph W	$230 \leq (W_{sw}/1000) ALW_{sw} + (W_{hw}/1000) ALW_{hw} - 0.6236 + 0.4938 W_{GLT} + 110.3011 W_{EN1} + 272.3460 W_{EN2}$	$230 \leq (W_{sw}/1000) ALW_{sw} + (W_{hw}/1000) ALW_{hw} - 0.6236 + 0.3950 W_{GLT}' + 6.1769 W_{EN1}' + 1.0894 W_{EN2}'$
Specific Volume of Baguette	$11.5 \leq -1.0237 + 1.0482 (W_{sw}/1000) BVL_{sw} + 0.0295 (W_{hw}/1000) ALW_{hw} + 0.0159 W_{GLT} + 6.9306 W_{AAC} + 23.0209 W_{EN1} + 15.0943 W_{EN2}$	$11.5 \leq -1.0237 + 1.0482 (W_{sw}/1000) BVL_{sw} + 0.0295 (W_{hw}/1000) ALW_{hw} + 0.0127 W_{GLT}' + 0.0554 W_{AAC}' + 1.2892 W_{EN1}' + 0.0604 W_{EN2}'$

Finally, the sum of the weights of ingredients, which is assumed to be equal to 1,000 metric tons, was straightforward in the LP model. It now becomes.

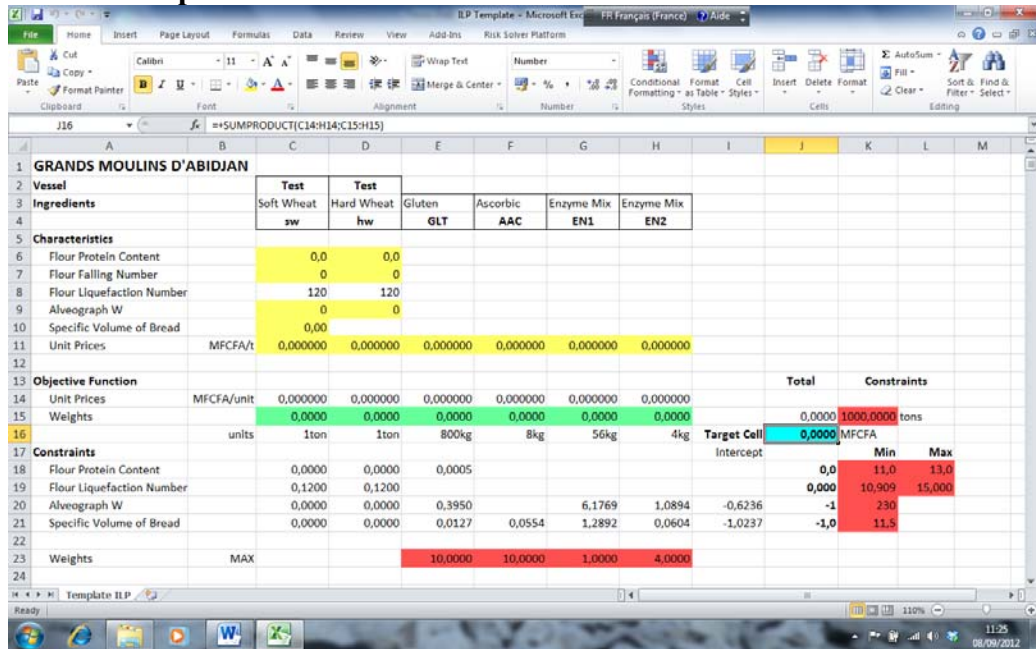
**Table 4.5: LP/ILP model – Sum of Weights Correspondence Table**

LP model / $W_i$ equations	ILP model / $W_i'$ equations
$W_{sw} + W_{hw} + W_{GLT} + W_{AAC} + W_{EN1} + W_{EN2} = 1,000$	$W_{sw} + W_{hw} + (W_{GLT}' / 1.25) + (W_{AAC}' / 125) + (W_{EN1}' \times 56 / 1000) + (W_{EN2}' / 250) = 1,000$

This new ILP model can be implemented in Excel as seen in Figure 4.5.

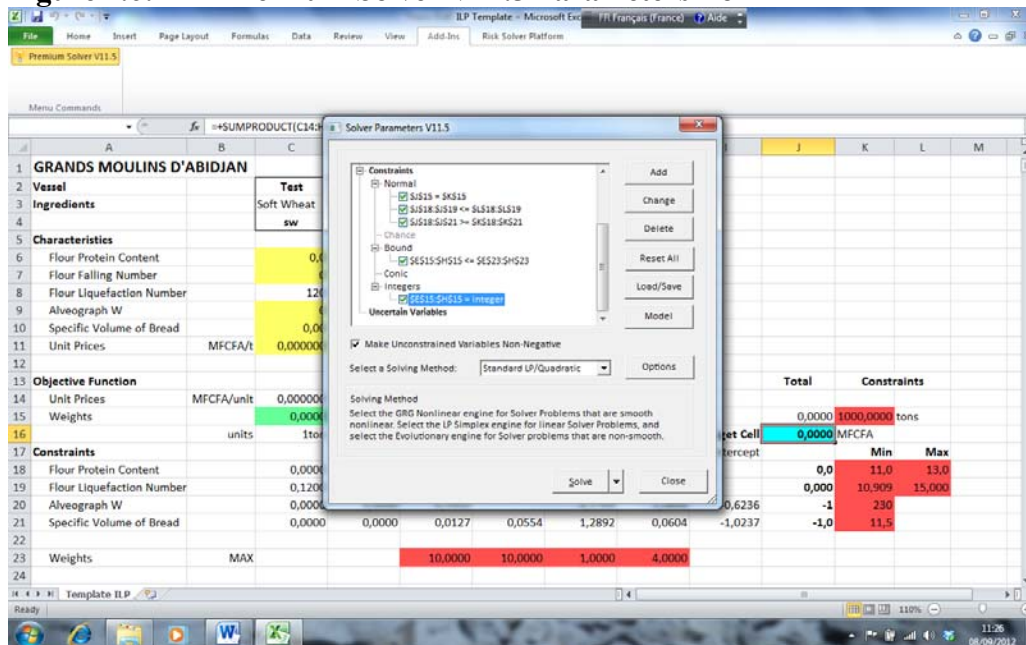


Figure 4.5: ILP optimization model on Excel



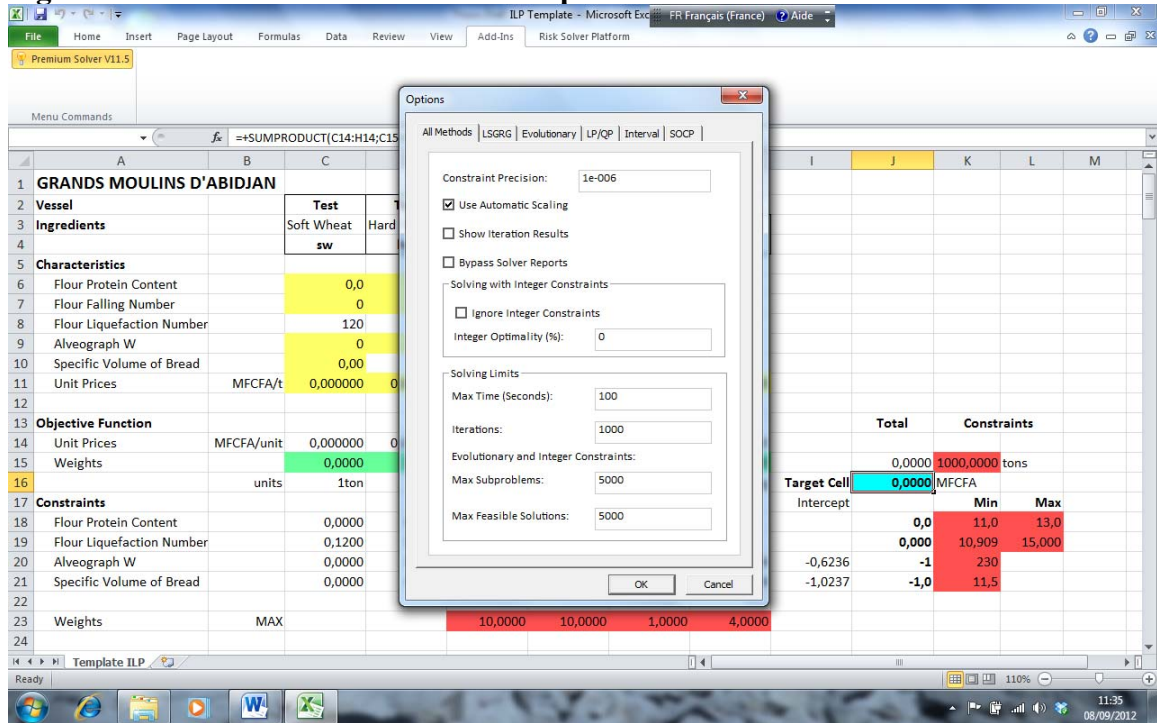
Unit prices, total weight, constraints formulas and limits have been changed. And a new constraint has been introduced: additives weights must be integer figures.

Figure 4.6: ILP Premium Solver V11.5 Parameters Box



The integer options box has also been fulfilled and integer optimality set to 0% as shown in Figure 4.7. With this option, the model does not allow any tolerance on the constraints.

**Figure 4.7: ILP Premium Solver V11.5 Options Box**



Unlike the LP Model, the ILP model integrates all constraints.

The set of equations and inequalities of the optimization model has been translated into two spreadsheet templates. Premium Solver V11.5 has been fed with the model parameters. The optimization model, in its two versions, LP and ILP, is now ready to be tested with actual data.

## CHAPTER V: RESULTS

The GMA blending problem has been put into a mathematical programming model and implemented on computer. The optimization model is ready to be solved and tested.

Four months when GMA has made blending decisions have been selected: February 2011, May 2011, August 2011 and February 2012. Corresponding data have been entered in the model templates.

In the first section, the optimization model solutions will be described. In the second section, they will be discussed. Finally, in the third section, a special attention will be given to the quality constraints equations that were identified in Chapter 3.

### 5.1 Results

In the months being considered, GMA processed soft wheat and hard wheat with the following characteristics.

**Table 5.1: Soft wheat quality parameters**

<u>Period</u>	<u>Feb. 2011</u>	<u>May 2011</u>	<u>Aug. 2011</u>	<u>Feb. 2012</u>
Vessel	African Orchyd	Silva-plana	Lavaux	Monte Azul
Flour Protein Content (%)	10.7	10.8	10.8	11.1
Flour Falling Number (s.)	372	325	354	339
Alveograph W	225	207	235	250
Specific weight of baguette after 4 hours of fermentation (cm <sup>3</sup> /g)	11.88	11.40	10.52	12.55

**Table 5.2: Hard wheat quality parameters**

<u>Period</u>	<u>Feb. 2011</u>	<u>May 2011</u>	<u>Aug. 2011</u>	<u>Feb. 2012</u>
Vessel	Amorita	Greenwing	Federal Leda	Neptune Pioneer
Flour Protein Content (%)	15.7	15.6	15.4	15.8
Flour Falling Number (s.)	468	424	598	430
Alveograph W	457	415	387	475

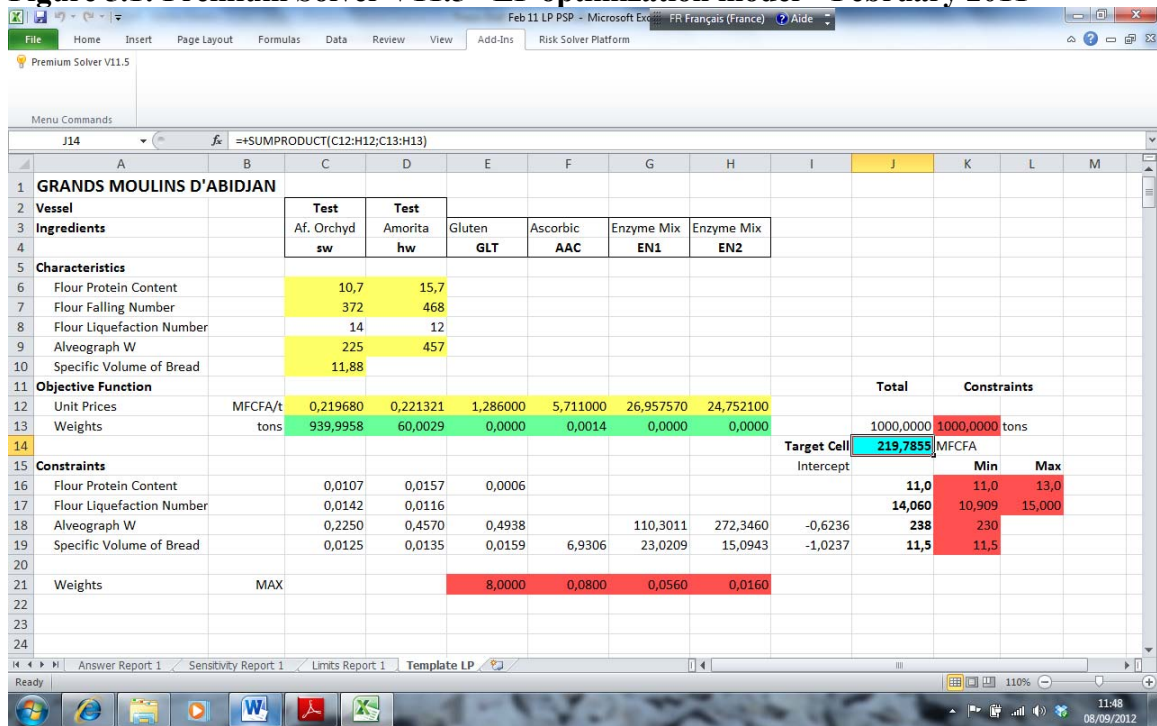
Prices of the different ingredients, in FCFA/t, as recorded in GMA books, were as follows.

**Table 5.3: Unit prices of ingredients**

Prices (FCFA/t)	Feb. 2011	May 2011	Aug. 2011	Feb. 2012
Soft Wheat	219,680	229,302	212,644	188,669
Hard Wheat	221,321	241,044	230,980	245,530
Gluten	1,286,000	1,238,000	1,579,000	1,204,900
Ascorbic acid	5,711,000	5,245,570	5,245,570	4,415,180
Enzyme mix 1	26,957,570	26,957,570	27,255,950	27,255,950
Enzyme mix 2	24,752,100	24,752,100	24,752,100	24,752,100

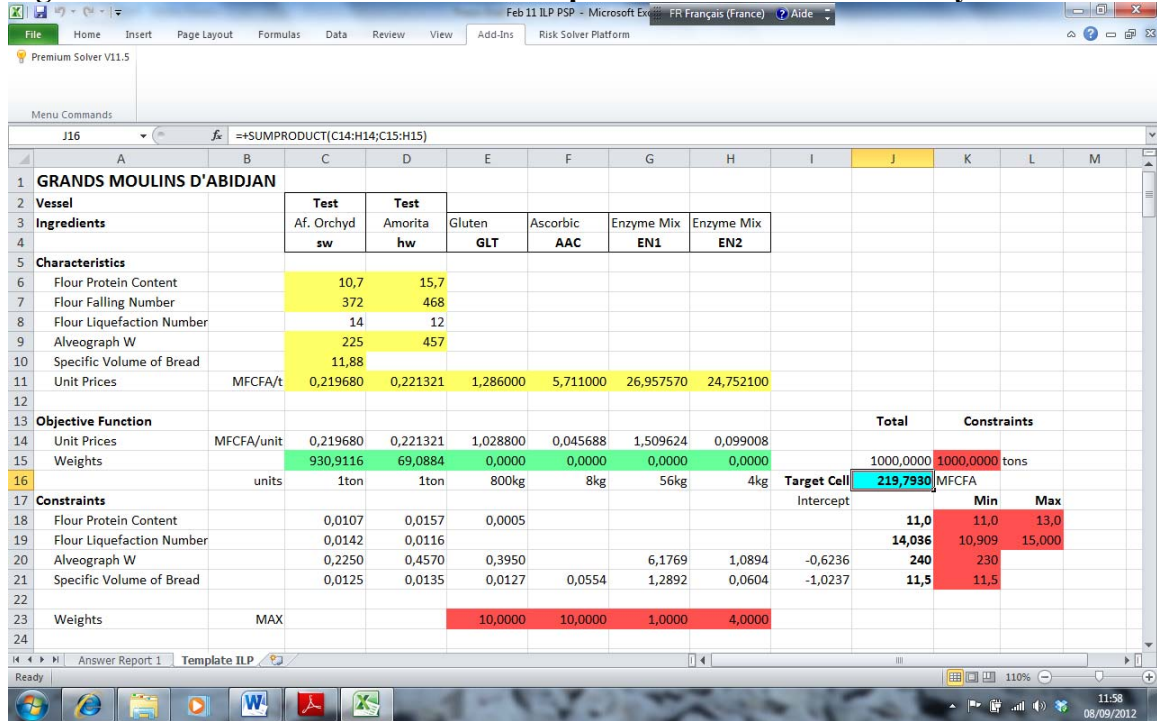
With these data as inputs, Premium Solver V11.5 gives the following LP optimal solution, as regards February 2011.

**Figure 5.1: Premium Solver V11.5 - LP optimization model – February 2011**



Premium Solver V11.5 gives a different solution to the ILP model.

**Figure 5.2: Premium Solver V11.5 - ILP optimization model – February 2011**



Figures 5.1 and 5.2 are screen captures from Microsoft Excel. Premium Solver V11.5 can also display optimal solutions as an Answer Report.

**Figure 5.3: Premium Solver V11.5 - ILP Answer Report – February 2011**

Microsoft Excel 14.0 Answer Report

Worksheet: [Feb 11 ILP PSP.xlsx]Template ILP

Report Created: 9/8/2012 11:57:40 AM

Result: Solver found a solution. All constraints and optimality conditions are satisfied.

Engine: Standard LP/Quadratic

Solution Time: 01 Seconds

Iterations: 0

Subproblems: 0

Incumbent Solutions: 0

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$J\$16	Target Cell Total	219,7930115	219,7930115

Decision Variable Cells

Cell	Name	Original Value	Final Value	Type
\$C\$15	Weights sw	930,9116	930,9116	Normal
\$D\$15	Weights hw	69,0884	69,0884	Normal
\$E\$15	Weights GLT	0,0000	0,0000	Normal
\$F\$15	Weights AAC	0,0000	0,0000	Normal
\$G\$15	Weights EN1	0,0000	0,0000	Normal
\$H\$15	Weights EN2	0,0000	0,0000	Normal

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$J\$15	Weights Total	1000,0000	\$J\$15=\$K\$15	Binding	0
\$J\$18	Flour Protein Content Total	11,0	\$J\$18<=\$L\$18	Not Binding	1,954557754
\$J\$19	Flour Liquefaction Number Total	14,036	\$J\$19<=\$L\$19	Not Binding	0,96403844
\$J\$18	Flour Protein Content Total	11,0	\$J\$18>=\$K\$18	Not Binding	0,045442246
\$J\$19	Flour Liquefaction Number Total	14,036	\$J\$19>=\$K\$19	Not Binding	3,126870651
\$J\$20	Alveograph W Total	240	\$J\$20>=\$K\$20	Not Binding	10,40492022
\$J\$21	Specific Volume of Bread Total	11,5	\$J\$21>=\$K\$21	Binding	0
\$E\$15	Weights GLT	0,0000	\$E\$15<=\$E\$23	Not Binding	10
\$F\$15	Weights AAC	0,0000	\$F\$15<=\$F\$23	Not Binding	10
\$G\$15	Weights EN1	0,0000	\$G\$15<=\$G\$23	Not Binding	1
\$H\$15	Weights EN2	0,0000	\$H\$15<=\$H\$23	Not Binding	4

The following tables compare, for each month under review, the LP model solutions, the ILP model solutions and the blend that was actually implemented by GMA.

**Table 5.4: LP and ILP optimal solutions vs. actual ones**

<u>February 2011</u> (tons)	LP Model	ILP model	Actual Blend
Soft Wheat	939.9958	930.9116	947.6668
Hard Wheat	60.0029	69.0884	49.8772
Gluten			2.4000
Ascorbic acid	0.0014		
Enzyme Mix 1			0.0560
Enzyme Mix 2			
<b>Total Weight</b>	<b>1,000.0000</b>	<b>1,000.0000</b>	<b>1,000.0000</b>
<b>Price MFCFA</b>	<b>219.7855</b>	<b>219.7930</b>	<b>223.8180</b>

<u>May 2011</u> (tons)	LP Model	ILP model	Actual Blend
Soft Wheat	701.0240	701.0303	949.9886
Hard Wheat	298.9057	298.8977	49.9994
Gluten			
Ascorbic acid	0.0703	0.0720	
Enzyme Mix 1			
Enzyme Mix 2			0.0120
<b>Total Weight</b>	<b>1,000.0000</b>	<b>1,000.0000</b>	<b>1,000.0000</b>
<b>Price MFCFA</b>	<b>233.1643</b>	<b>233.1725</b>	<b>230.1830</b>

<u>August 2011</u> (tons)	LP Model	ILP model	Actual Blend
Soft Wheat	956.1192	956.2271	947.6668
Hard Wheat	43.7606	43.6849	49.8772
Gluten			2.4000
Ascorbic acid	0.0800	0.0320	
Enzyme Mix 1	0.0402	0.0560	0.0560
Enzyme Mix 2			
<b>Total Weight</b>	<b>1,000.0000</b>	<b>1,000.0000</b>	<b>1,000.0000</b>
<b>Price MFCFA</b>	<b>214.9376</b>	<b>215.1207</b>	<b>218.3525</b>

<b>February 2012</b> <b>(tons)</b>	<b>LP Model</b>	<b>ILP model</b>	<b>Actual Blend</b>
<b>Soft Wheat</b>	<b>854.9451</b>	<b>854.9451</b>	<b>899.9496</b>
<b>Hard Wheat</b>	<b>145.0549</b>	<b>145.0549</b>	<b>99.9944</b>
<b>Gluten</b>			
<b>Ascorbic acid</b>			
<b>Enzyme Mix 1</b>			<b>0.0560</b>
<b>Enzyme Mix 2</b>			
<b>Total Weight</b>	<b>1,000.0000</b>	<b>1,000.0000</b>	<b>1,000.0000</b>
<b>Price MFCFA</b>	<b>196.9173</b>	<b>196.9173</b>	<b>195.8709</b>

The solutions provided by the optimization model are alternatives to the blends that were actually implemented by GMA. They make sense and, in two cases out of four, are cheaper than actual blends. However, they must be considered in more depth.

## **5.2 Discussion**

The different solutions provided by the optimization models need to be assessed. In the following paragraphs, the following points will be addressed:

- Different LP model solutions and ILP model solutions;
- Optimization model solutions and actual blends.

### *5.2.1 Different optimization model solutions: LP vs. ILP*

In February 2012, solutions of the LP model and of the ILP model are the same. In the other 3 months, solutions of the LP model are, logically, cheaper than solutions of the ILP model since ILP models include more constraints (technical constraints) than the LP models.

An alternative way to introduce the technical constraints into the optimization model would be to round the decision variables of the LP model solutions to the next values that belong to the set of admitted weights for additives.

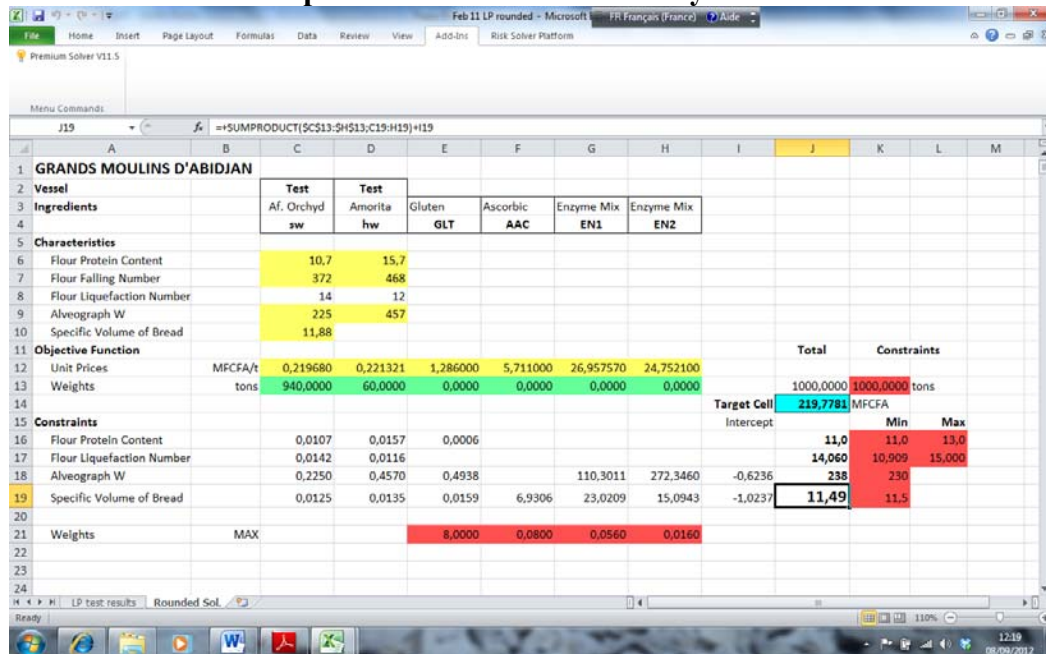


**Table 5.5: Rounded LP optimal solutions**

Weights in tons	Feb. 2011	May 2011	Aug. 2011
Soft Wheat	940.000	701.0172	956.1040
Hard Wheat	60.000	298.9028	43.7600
<b>Gluten</b>			
Ascorbic acid		0.0800	0.0800
Enzyme Mix 1			0.0560
Enzyme Mix 2			
<b>Total Weight</b>	<b>1,000.0000</b>	<b>1,000.0000</b>	<b>1,000.0000</b>
<b>Total Price (MFCFA)</b>	<b>219.7781</b>	<b>233.2127</b>	<b>215.3637</b>

The rounded LP optimal solution for February 2011 (219.7781 MFCFA) is cheaper than both LP optimal solution (219.7855 MFCFA) and ILP optimal solution (219.7930 MFCFA). However, with this rounded LP optimal solution, the specific volume of baguette after 4 hours of fermentation is predicted to go down to 11.49 cm<sup>3</sup> against a minimum fixed at 11.50 cm<sup>3</sup>.

**Figure 5.4: Rounded LP optimization model – February 2011**



When it comes to May 2011, all quality constraints are met by the rounded LP optimal solution. However, its total price is more expensive (233.2127 MFCFA) than both the LP optimal solution (233.1643 MFCFA) and ILP optimal solution (233.1725 MFCFA).

The same thing happens in August 2011. All quality constraints are met but the total price of the rounded LP solution (215.3637 MFCFA) is more expensive than both the LP optimal solution (214.9376 MFCFA) and ILP optimal solution (215.1207 MFCFA).

LP solutions and rounded LP solutions, although theoretically questionable, are nevertheless interesting. They require less computing power from solvers than ILP: the Excel Solver is powerful enough to provide the same solutions as Premium Solver V11.5. However, rounding of LP solutions may end up with solutions that do not respect all quality constraints or that are not optimal.

### *5.2.2 Optimization model solutions vs. actual blends*

Blends that were actually implemented by GMA never correspond to optimal solutions of the model, whether LP or ILP. Different reasons may explain this fact.

#### *a) Routine thinking*

One can note that the blend that has actually been implemented in August 2011 is the same as the one that had already been implemented in February 2011. This may be the effect of some routine thinking. The chief miller may use solutions that have worked previously rather than take risks with a new blend. If this assumption is true, it reinforces the interest of the optimization model for GMA management since the optimization model may be more imaginative than the chief miller.

#### *b) Hidden constraints*

It is also remarkable that the weights of hard wheat that are suggested by the optimization model for May 2011 and February 2012 are much larger than in actual blends. However, if GMA had applied these solutions, it would have had to order a vessel of hard wheat four or five times earlier than scheduled. One may assume that the chief miller does not want to be short of hard wheat and that, when he makes a decision, he takes account of the inventory of supplies.

Something similar happens with enzyme mixes. Their incorporation is suggested by the optimization model in August 2011 only. On the other hand, the chief miller has actually used such mixes in all four months. This may be because enzyme mixes have limited shelf life. If GMA does not use these enzyme mixes, it will have to throw them away.

These two considerations show that there are some hidden constraints that are not taken into account by the optimization model. One must not forget that modeling is a process that simplifies reality and sometimes reality is more complex than expected.

*c) Potential savings*

The following tables compare the prices of the ILP optimal solutions and of the actual blends.

**Table 5.6: Price of optimal solutions vs. actual blends**

Price (MFCFA/1,000 t.)	Feb. 2011	May 2011	Aug. 2011	Feb 2012	Total
<b>ILP model</b>	<b>219.7930</b>	<b>233.1725</b>	<b>215.1207</b>	<b>196.9173</b>	<b>865.0035</b>
<b>Actual blend</b>	<b>223.8180</b>	<b>230.1830</b>	<b>218.3525</b>	<b>195.8709</b>	<b>868.2244</b>
<b>Difference (Actual – ILP)</b>	<b>4.0250</b>	<b>- 2.9895</b>	<b>3.2318</b>	<b>- 1.0464</b>	<b>3.2209</b>

The purchase of 4,000 metric tons of ingredients, i.e. 1,000 metric tons in each month of February 2011, May 2011, August 2011 and February 2012 according to the suggestions of the ILP optimization model would have cost 865.0035 MFCFA against 868.2244 MFCFA actually paid by GMA. The difference  $(868.2244 - 865.0035) = 3.2209$  corresponds to 0.805 MFCFA per thousand tons. Since GMA processes some 250,000 tons of ingredients per year, one can infer that the optimization model could enable GMA to save some 201.3 MFCFA per year. This sum is worth about 400,000 US dollars. The optimization model may indeed help GMA reduce its costs of production.

*d) Quality specifications, binding constraints and sensitivity analyses*

Optimization model solutions are not always the cheapest ones. In May 2011 and February 2012, actual blends are cheaper than optimization model solutions. This happens because

all constraints are not met by actual blends. The following tables show the values of the different quality parameters as computed by equations defined in Chapter 3 and applied to the different mixes.

**Table 5.7: Quality constraints of optimal vs. actual solutions**

<u>February 2011</u>	<u>Limits</u>	<u>LP model</u>	<u>ILP model</u>	<u>Actual</u>
Flour Protein Content	$11.0\% \leq x \leq 13.0\%$	11.0%	11.0%	10.9%
Flour Falling Number	$350s. \leq x \leq 500s.$	377s.	377s.	377s.
Alveograph W	$230 \leq x$	238	240	243
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$	$11.5 \text{ cm}^3$	$11.5 \text{ cm}^3$	$12.8 \text{ cm}^3$

<u>May 2011</u>	<u>Limits</u>	<u>LP model</u>	<u>ILP model</u>	<u>Actual</u>
Flour Protein Content	$11.0\% \leq x \leq 13.0\%$	12.2%	12.2%	11.0%
Flour Falling Number	$350s. \leq x \leq 500s.$	350s.	350s.	329s.
Alveograph W	$230 \leq x$	269	269	220
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$	$11.5 \text{ cm}^3$	$11.5 \text{ cm}^3$	$11.1 \text{ cm}^3$

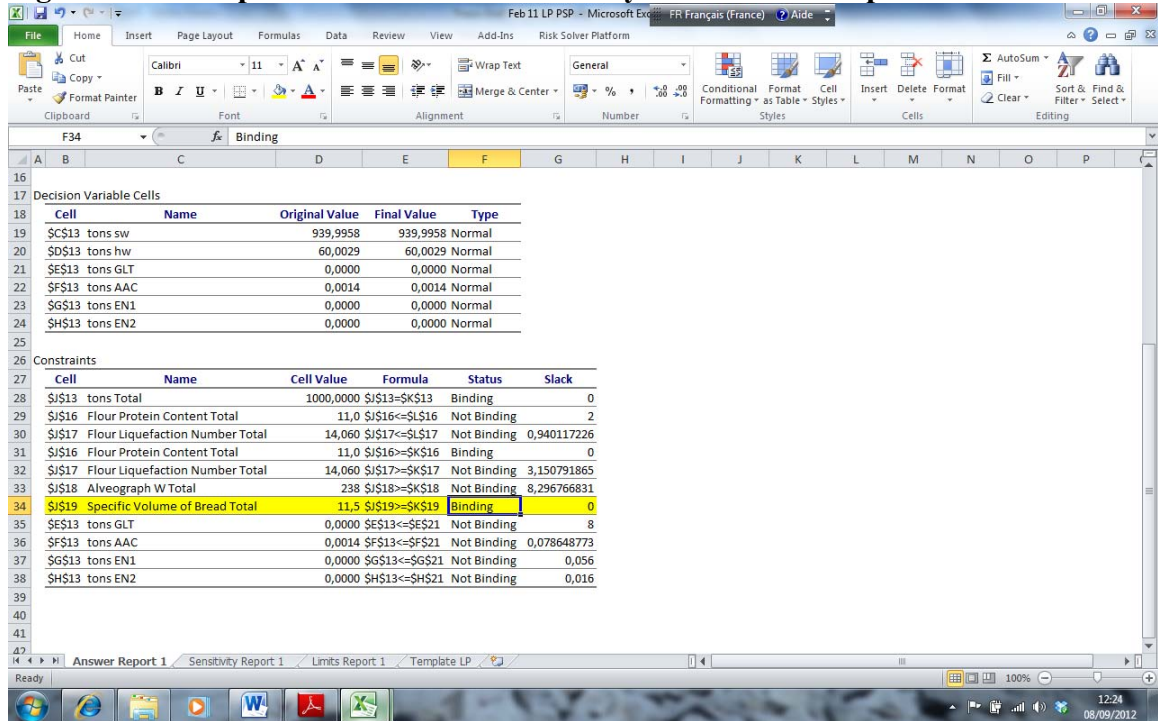
<u>August 2011</u>	<u>Limits</u>	<u>LP model</u>	<u>ILP model</u>	<u>Actual</u>
Flour Protein Content	$11.0\% \leq x \leq 13.0\%$	11.0%	11.0%	11.0%
Flour Falling Number	$350s. \leq x \leq 500s.$	361s.	361s.	363s.
Alveograph W	$230 \leq x$	245	247	249
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$	$11.5 \text{ cm}^3$	$11.5 \text{ cm}^3$	$11.3 \text{ cm}^3$

<u>February 2012</u>	<u>Limits</u>	<u>LP model</u>	<u>ILP model</u>	<u>Actual</u>
Flour Protein Content	$11.0\% \leq x \leq 13.0\%$	11.8%	11.8%	11.6%
Flour Falling Number	$350s. \leq x \leq 500s.$	350s.	350s.	347s.
Alveograph W	$230 \leq x$	282	282	278
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$	$12.3 \text{ cm}^3$	$12.3 \text{ cm}^3$	$13.5 \text{ cm}^3$

When it comes to actual blends, parameters highlighted in yellow do not respect GMA quality specifications.

In addition to optimal solutions, solvers provide information about binding constraints, i.e. constraints which are strictly satisfied in the optimal solution, with no slack. The following table displays, for instance, the Answer report for February 2011 ILP Model. This report outlines the fact that the specific volume of baguette is a binding constraint.

**Figure 5.5: ILP optimization model – February 2011 Answer Report**



The following tables compare the binding quality constraints of the LP and the ILP models and the constraints that were not met by actual blends.

**Table 5.8: Optimal solutions binding constraints and quality parameters of actual blends**

February 2011	Constraints	LP model	ILP model	Actual
Flour Protein Content	$11.0\% \leq x$ $x \leq 13.0\%$	X		X
Flour Falling Number	$350s. \leq x$ $x \leq 500s.$			
Alveograph W	$230 \leq x$			
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$	X	X	

<u>May 2011</u>	Constraints	LP model	ILP model	Actual
Flour Protein Content	$11.0\% \leq x$			
	$x \leq 13.0\%$			
Flour Falling Number	$350s. \leq x$	X	X	X
	$x \leq 500s.$			
Alveograph W	$230 \leq x$			X
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$	X		X

<u>August 2011</u>	Constraints	LP model	ILP model	Actual
Flour Protein Content	$11.0\% \leq x$	X	X	
	$x \leq 13.0\%$			
Flour Falling Number	$350s. \leq x$			
	$x \leq 500s.$			
Alveograph W	$230 \leq x$			
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$	X		X

<u>February 2012</u>	Constraints	LP model	ILP model	Actual
Flour Protein Content	$11.0\% \leq x$			
	$x \leq 13.0\%$			
Flour Falling Number	$350s. \leq x$	X	X	X
	$x \leq 500s.$			
Alveograph W	$230 \leq x$			
Specific volume of baguette	$11.5 \text{ cm}^3 \leq x$			

In all four months of the sample, constraints that were not met by actual blends correspond to optimization models binding constraints. Optimization models can effectively identify the most sensitive constraints.

But solvers can go further than that. They provide Sensitivity reports for LP models. These reports give information about the consequences of relaxing binding constraints.

## Figure 5.6: LP optimization model – February 2011 Sensitivity Report

Microsoft Excel 14.0 Sensitivity Report

Worksheet: [Feb 11 LP PSP.xlsx]Template LP

Report Created: 9/8/2012 11:47:19 AM

Engine: Standard LP/Quadratic

Objective Cell (Min)

Cell	Name	Final Value
\$J\$14	Target Cell Total	219,7855172

Decision Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$13	tons sw	939,9958	0,0000	0,2196796	0,000825019	1,160254621
\$D\$13	tons hw	60,0029	0,0000	0,22132114	1,702429678	0,000824896
\$E\$13	tons GLT	0,0000	1,0652	1,286	1E+30	1,065242224
\$F\$13	tons AAC	0,0014	0,0000	5,711	2,547523653	5,494835186
\$G\$13	tons EN1	0,0000	8,4707	26,95757	1E+30	8,470695015
\$H\$13	tons EN2	0,0000	12,5590	24,7521	1E+30	12,55903251

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$J\$13	tons Total	1000,0000	0,2080	1000	0,912217206	53,0960267
\$J\$16	Flour Protein Content Total	11,0	0,0	13	1E+30	2
\$J\$17	Flour Liquefaction Number Total	14,060	0,000	15	1E+30	0,940117226
\$J\$16	Flour Protein Content Total	11,0	0,2	11	0,045442246	0,178840744
\$J\$17	Flour Liquefaction Number Total	14,060	0,000	10,90909091	3,150791865	1E+30
\$J\$18	Alveograph W Total	238	0	230,6236	8,296766831	1E+30
\$J\$19	Specific Volume of Bread Total	11,5	0,8	12,5237	0,544276976	0,00935096

In the example of February 2011, binding constraints identified by the LP optimization model are "Flour Protein Content" and "Specific Volume of Bread". As shown on Figure 5.8, the Shadow Prices for these constraints are, respectively, equal to 0.2 and 0.8 million FCFA, with constraints (RHS) limits fixed at, respectively, 11.0% and 11.5 cm<sup>3</sup> per gram. It means that if GMA decides to relax a constraint and to accept, for instance, a flour with a protein content of 10.9% instead of 11.0%, one tenth less than before, the price of the mix will drop down by one tenth of 0.2 million FCFA, i.e. 0.02 million FCFA, all other coefficients remaining constant.

GMA may have to consider such constraints modifications. Premium Solver V11.5 provides all the relevant information that is necessary to make such a decision. The optimization model enables GMA management to take such a decision with full knowledge of its consequences, in terms of price as well as in terms of quality.

Up till now, the values of quality constraint parameters have been computed by the equations of the optimization model as they were determined in Chapter 3. Obviously, such values are true only if these quality constraint equations hold. It is therefore important to test these quality constraint equations.

### **5.3 Optimization model quality constraints equations.**

GMA laboratory performs flour tests on a daily basis. At least one sample of flour produced per work shift is tested on its rheological and milling properties. At least one sample of flour per working day is transformed into bread in the test bakery.

The results of these tests for February 2011, May 2011, August 2011 and February 2012 are displayed in Appendix M.

Results of laboratory tests have been compared with the results of the equations of the optimization model. They have also been used to check whether actual blends respect GMA quality specifications.

#### *5.3.1 Test of quality constraint equations*

Quality parameters of actual flour samples are analyzed by GMA laboratory and GMA test bakery.

**Table 5.9: Quality parameters of actual samples of flour**

<u>February 2011</u>	Average	Standard Deviation	Number of tests
Flour Protein Content	10.9%	0.2	24
Flour Falling Number	368s.	12	24
Alveograph W	240	15	24
Specific volume of baguette	12.46 cm <sup>3</sup> /g	0.42	9



<u>May 2011</u>	Average	Standard Deviation	Number of tests
Flour Protein Content	11.0%	0.1	35
Flour Falling Number	364s.	15	35
Alveograph W	225	17	35
Specific volume of baguette	12.23 cm <sup>3</sup> /g	0.62	15

<u>August 2011</u>	Average	Standard Deviation	Number of tests
Flour Protein Content	11.1%	0.2	67
Flour Falling Number	360s.	13	67
Alveograph W	244	15	67
Specific volume of baguette	12.37 cm <sup>3</sup> /g	0.31	29

<u>February 2012</u>	Average	Standard Deviation	Number of tests
Flour Protein Content	11.3%	0.1	39
Flour Falling Number	347s.	8	39
Alveograph W	273	19	39
Specific volume of baguette	13.01 cm <sup>3</sup> /g	0.35	12

These tests have been conducted on samples. If one assumes that the four quality parameters are normally distributed, then the 99.74 percent confidence interval of the population means is determined by the following formula.

**Table 5.10: Normal Distribution Confidence Intervals**

$$[ m - 3\sigma/\sqrt{n} ; m + 3\sigma/\sqrt{n} ]$$

where:

- $m$  is the average of the sample ;
- $\sigma$  is the standard deviation of the sample ;
- $n$  is the size of the sample.

The following table compares, for each period and each quality parameter:

- values computed by optimization model equations and
- confidence intervals of the population means, determined upon the basis of sample tests.

**Table 5.11: Quality parameters: computed figures vs. confidence intervals**

<u>February 2011</u>	Optimization model equations	Confidence Interval lower limit	Confidence Interval upper limit
Flour Protein Content	10.9%	10.8%	11.0%
Flour Falling Number	377s.	361s.	376s.
Alveograph W	243	231	249
Specific volume of baguette	12.80 cm <sup>3</sup> /g	12.04 cm <sup>3</sup> /g	12.88 cm <sup>3</sup> /g

<u>May 2011</u>	Optimization model equations	Confidence Interval lower limit	Confidence Interval upper limit
Flour Protein Content	11.0%	10.9%	11.0%
Flour Falling Number	329s.	356s.	372s.
Alveograph W	230	217	234
Specific volume of baguette	11.10 cm <sup>3</sup> /g	11.74 cm <sup>3</sup> /g	12.71 cm <sup>3</sup> /g

<u>August 2011</u>	Optimization model equations	Confidence Interval lower limit	Confidence Interval upper limit
Flour Protein Content	11.0%	11.0%	11.1%
Flour Falling Number	363s.	356s.	365s.
Alveograph W	249	238	249
Specific volume of baguette	11.30 cm <sup>3</sup> /g	12.19 cm <sup>3</sup> /g	12.54 cm <sup>3</sup> /g

<u>February 2012</u>	Optimization model equations	Confidence Interval lower limit	Confidence Interval upper limit
Flour Protein Content	11.6%	11.2%	11.3%
Flour Falling Number	347s.	343s.	350s.
Alveograph W	278	264	282
Specific volume of baguette	13.50 cm <sup>3</sup> /g	12.71 cm <sup>3</sup> /g	13.31 cm <sup>3</sup> /g

Values highlighted in yellow are outside of the confidence intervals. The following table summarizes the cases when values computed by optimization model equations fall into or outside the limits of the confidence intervals.

**Table 5.12: Quality parameters: computed figures vs. confidence intervals - Summary**

	Feb. 2011	May 2011	Aug. 2011	Feb. 2012
Flour Protein Content	IN	IN	IN	OUT
Flour Falling Number	OUT	OUT	IN	IN
Alveograph W	IN	IN	IN	IN
Specific volume of baguette	IN	OUT	OUT	OUT

Altogether, computed figures are in between the limits of the confidence intervals in nine cases out of sixteen.

Flour Protein Content equation FPC1 is exclusively based upon Grains Science knowledge. This equation gives results that fall within confidence intervals limits, in three out of four cases.

Flour Falling Numbers in the optimization model are computed with Flour Liquefaction Numbers equation LNR1. This equation has been built upon theory because econometrics did not bring significant results. However, the correlation between the equation results and GMA data was not very strong. Only two out of four results are inside the confidence intervals.

The Alveograph W equation ALW3 is designed out of both theory and econometrics. The model equation gives four results that are inside confidence intervals.

The specific volume of baguette equation BVL1 is determined exclusively by econometrics. The results of this equation lie outside the limits of the confidence intervals three times out of four.

These results outline the need for GMA to improve the optimization model by enhancing the validity and robustness of the quality constraints equations. This is particularly true when regression analysis is involved. Further econometrics research should be made more specifically on “Flour falling number” and on “Specific volume of baguette after 4 hours of fermentation”.

### 5.3.2 Actual flour and quality specifications

Actual flour quality parameters, measured by confidence intervals, have also been tested against GMA quality specifications.

**Table 5.13: Flour quality standards vs. actual**

	GMA specifications	Feb. 2011	May 2011	Aug. 2011	Feb. 2012
<b>Flour Protein Content</b>	$11.0\% \leq x \leq 13.0\%$	[10.8% - 11.0%]	[10.9% - 11.0%]	[11.0%- 11.1%]	[11.2%- 11.3%]
<b>Flour Falling Number</b>	$350s. \leq x \leq 500s.$	[361 – 376]	[356-372]	[356-365]	[343-350]
<b>Alveograph W</b>	$230 \leq x$	[231 – 249]	[217-234]	[238-249]	[264-282]
<b>Specific volume of baguette</b>	$11.5 \text{ cm}^3 \leq x$	[12.04-12.88]	[11.74–12.71]	[12.19-12.54]	[12.71-13.31]

At worst, confidence intervals of actual flour quality parameters have common limits with GMA specifications. These worst cases are highlighted in yellow. Under such circumstances, one cannot reject the claim that GMA flour respects its quality standards. The chief miller’s experience may be a better predictor of flour quality than the optimization model quality equations.

GMA managers must be aware that the optimization model is no more valid than its assumptions. This observation leads to two remarks.

First, there is a need to improve quality constraints equations. Actual blends of May 2011 and February 2012 are cheaper than ILP optimization model solutions and, although quality constraints equations tell another story, one cannot prove that this happens because quality parameters are not respected.

Then, one must not forget that all the conclusions of this section are subject to the assumption that the four quality parameters are normally distributed. This may be true but laboratory tests are subject to biases.

## CHAPTER VI :SUMMARY AND CONCLUSIONS

The objective of the present thesis, as it is defined in Chapter 1 is to determine the optimal blend of wheat and additives that minimizes flour millers' cost of production while meeting quality requirements.

This objective has been achieved. The objective function and the constraints of GMA have been translated into mathematical equations. The set of equations and inequalities has been implemented in Microsoft Office Excel 2010. Premium Solver V11.5 has found optimal solutions to several examples of actual business situations.

**Figure 6.1: GMA flour mill staff**



These optimization model solutions do question the habits of the chief miller, without any prejudice. And it can be inferred from these examples that the implementation of these optimal solutions would overall have saved money for GMA when compared with actual blends. However, on a case by case basis, money saving is not always true.

Some observations need to be made and several limitations remain.

### *Choosing a solver: ILP vs. LP model*

The optimization model takes account of technical constraints such as dosing scales capacities or additives suppliers' advice. They limit the set of values that additives weights can take in the blend and transform the model into an Integer Linear Programming (ILP) problem instead of a simpler Linear Programming (LP) one. ILP models require powerful solvers. However, Premium Solver V11.5 is effective at solving GMA ILP optimization model.

If technical constraints are neglected and only quality constraints are considered, Excel Solver is sufficient to solve the LP optimization model. Excel Solver has several advantages: it is easy to implement, it is free of additional charge and it is available in French. On the other hand, solutions provided by the LP optimization model may be irrelevant. Rounding of LP solutions may lead to solutions that do not respect quality constraints.

### *Assessing the assumptions*

In order to build quality constraints, several important assumptions were made. These assumptions should not be taken for granted. They need to be questioned and periodically revised. The following considerations must be taken account of:

#### *1. Selecting quality parameters*

Four quality parameters were selected to represent the expectations of GMA customers: flour protein content, flour falling number, Alveograph W and the specific volume of baguettes after 4 hours of fermentation. The choice of these parameters is supported by previous literature, some econometrics and the experience of the Ivorian market. It is nevertheless at least partly subjective and should be reassessed from time to time.

#### *2. Setting limits (RHS) to quality parameters*

The limits that are assigned to quality parameters are designed in order to fit with market requirements. They should reflect the evolution of the market.

#### *3. Determining quality (LHS) constraint equations*

The LHS of the quality equations describe the way ingredients impact flour quality parameters. Equations have been determined with reference to grains science theory and

with the help of econometrics. The comparison of the results of these equations and test analyses of actual flour shows that quality constraint equations should be further researched and improved.

Improved quality constraint equations are particularly important when GMA wants to assess, with the help of sensitivity reports from solvers, the possibility of relaxing its quality specifications.

*Keeping hidden constraints in mind*

The comparison between optimization model solutions and actual blends shows that the model does not take account of some hidden constraints such as the delivery program of hard wheat or expiration dates for consumption of ingredients. GMA management should be cautious about the possible existence of such hidden constraints when considering the solutions provided by the optimization model.

More generally, the main problems encountered during this thesis did not lie with optimization techniques. The most important issues boil down to modeling the economic reality. Reality is often too complex to be easily and fully grasped into an economic model. However, although perfectible, the optimization model designed in the present thesis has proven to be of interest for GMA in providing challenging ideas for minimizing costs of production while still meeting quality requirements.

The next step will be to implement the model and to use it as frequently as possible when blending decisions are to be made. This way, the advantages but also the limitations and imperfections of the model will be revealed. Hopefully GMA will save money with the help of this model and this will enhance the interest in correcting its imperfections.



## WORKS CITED

Charnes, A., W.W. Cooper and A. Henderson. *An Introduction to Linear Programming*. Wiley, New York, 1953.

Chopin Technologies <http://www.chopin.fr> (accessed September 8, 2012°)

Del Frate, Régis. “Mieux Connaître la Farine.” *Les Nouvelles de la Boulangerie Pâtisserie* 85 (January 2005): 2-15

Fowler, Mark. “Blending for Value.” *World Grain*, October 2009.

Frontline Systems Inc. – Risk Solver Platform V11.5 2012 - <http://www.solver.com> (accessed September 1, 2012).

Hayta, Mehmet and Cakmakli Ünsal. “Optimization of Wheat Blending to Produce Breadmaking Flour.” *Journal of Food Process Engineering* 24 (2001): 179-192

Lloyd, Peter. “Wheat Blend Calculator.” Spreadsheet model presented by US Wheat Associates, Casablanca, May 2005

Niernberger, Floyd F. “Blending Wheat to Meet Product Specifications.” *Bulletin-Association of Operative Millers* (September 1973): 3395-3400

Perten Instruments <http://www.perten.com> (accessed September 8, 2012)

Ragsdale, Cliff T. *Spreadsheet Modeling & Decision Analysis*. Thomson-South Western, 2008

Schurle, Bryan and Mark Fowler. “Spreadsheet Solutions.” Lesson of the Flour Milling Short Course, International Grains Program, Kansas State University, June 5-16, 2006

Steuer, R.E. *Multi-Criteria Optimization: Theory, Computation, and Application*. John Wiley & Sons, New York, 1986

Studenmund, A.H. *Using Econometrics*. Pearson-Addison Wesley, 2006

# APPENDIX A: ANALYSIS OF REDUNDANCY (CORRELATION) OF QUALITY PARAMETERS

## 1. Analyses Data

ANALYSES DATA		01/2010	02/2010	03/2010	04/2010	05/2010	06/2010	07/2010	08/2010	09/2010	11/2010	12/2010	13/2010	14/2010	15/2010	17/2010	18/2010	19/2010	21/2010
Reference	Date	20-janv-10	02-fevr-10	03-mars-10	04/2010	05/2010	06/2010	07/2010	08/2010	09/2010	11/2010	12/2010	13/2010	13-août-10	28-août-10	21-sept-10	12-oct-10	03-nov-10	23-déc-10
Ship	Ansia	Sylweta V3	Alfija	de Sa Praetorius	Laura	Sylweta V4	Bombi	Ever Regal	Shapona	Thais	Silweta V5	Andra	Voge Eria	Expolius	PanBless	Silweta V6	Great Succ	African Hawk	
WHEAT	Aver. Moisture	13,7	13,8	13,8	13,5	13,7	13,6	13,7	13,5	13,6	13,6	13,4	13,3	13,6	13,6	13,7	13,8	13,2	13,1
WHEAT	Aver. Protein Cont.	11,9	11,9	11,8	11,7	11,8	11,5	11,5	11,7	11,4	11,8	11,7	11,4	11,2	11,4	11,4	11,5	11,3	12,1
WHEAT	Aver. Falling Nber	327			342				343	336	348	338	326	336	331	345	335	341	339
WHEAT	Aver. Test Weight	79,6	79,1	80,2	79,3	80,1	79,4	80,3	80,0	80,4	80,4	80,3	80,3	80,3	81,4	80,5	80,3	81,2	80,6
FLOUR	Protein Content	10,7	10,5	10,5	10,6	10,5	10,6	10,6	10,8	10,4	10,3	10,5	10,8	11,0	10,9	10,9	10,6	10,9	10,8
FLOUR	Water Content	13,8	14,2	14,2	13,9	13,9	13,9	13,4	13,6	14,2	14,3	14,4	14,5	14,2	14,0	14,2	14,2	14,4	14,0
FLOUR	Ash Rate	0,62	0,60	0,60	0,62	0,60	0,61	0,60	0,68	0,63	0,66	0,58	0,61	0,62	0,59	0,62	0,59	0,62	0,61
FLOUR	Falling Number	360	349	349	347	385	350	345	341	344	331	340	335	312	354	339	324	341	343
FLOUR	Alveo. P	78	90	90	84	94	92	95	90	94	97	94	89	96	98	102	91	98	103
FLOUR	Alveo. G	18,60	18,20	18,20	18,90	17,80	17,90	17,90	18,20	17,40	18,60	19,00	19,40	17,50	16,80	17,40	19,50	18,50	16,00
FLOUR	Alveo. P.L	1,11	1,34	1,34	1,17	1,47	1,42	1,46	1,34	1,54	1,39	1,29	1,17	1,55	1,72	1,67	1,18	1,42	2,00
FLOUR	Alveo. W	177	205	205	203	210	204	214	198	197	223	222	223	203	204	220	241	233	202
BREAD	Water Absorption	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66
BREAD	Weight	221	225	225		200	213	211	213	213	214	215	213	215	213	213	210	212	213
BREAD	Vol.1	7	5,63	5,53	8,38	9,45	8,42	8,65	7,43	8,07	7,86	7,45	7,98	8,67	9,41	9,5	9,68	9,62	9,59
BREAD	Vol.2	9,49	8,78	8,78	8,31	9,55	9,32	10,8	8,81	8,79	9,58	8,16	9,4	10,08	10,5	11,24	11,1	10,52	11,07
BREAD	Dough Grade	77,5	86,5	86,5	77,5	82,0	83,5	89,5	82,0	76,0	86,5	83,5	86,5	86,5	89,5	89,5	89,5	89,5	86,5
BREAD	Bread Grade	85,0	85,0	85,0	82,0	85,0	85,0	91,0	85,0	82,0	85,0	80,5	85,0	79,0	85,0	85,0	85,0	88,0	88,0
BREAD	Crumb Grade	85,0	85,0	85,0	85,0	85,0	85,0	88,0	85,0	85,0	85,0	88,0	85,0	85,0	88,0	88,0	88,0	88,0	88,0
BREAD	Total Score	247,5	256,5	256,5	244,5	252,0	253,5	266,5	252,0	243,0	256,5	252,0	256,5	250,5	262,5	262,5	262,5	265,5	262,5



# APPENDIX B: QUALITY TESTS ON DIFFERENT BLENDS OF WHEAT AND ADDITIVES

## 1. Independent Variables

		INDEPENDENT VARIABLES																			
		INPUT																			
		Soft French Wheat		CWRS																	
Test Nbr.	Date	Test nbr.	Vessel	Vessel	Weight Soft Wheat	F Flour Protein content	F Falling Number	F Liquefaction Number	F Alveo W	F Saguette Vol./g	Weight hard wheat	C Flour Protein content	C Falling Number	F Liquefaction Number	C Alveo W	Weight gluten	Weight Acid ascorbic	Weight enzyme mix 1	Weight enzyme mix 2	Total Weight	
					W <sub>SW</sub>	FPC <sub>SW</sub>	FLN <sub>SW</sub>	LNR <sub>SW</sub>	ALW <sub>SW</sub>	BVL <sub>SW</sub>	W <sub>HW</sub>	FPC <sub>HW</sub>	FLN <sub>HW</sub>	LNR <sub>HW</sub>	ALW <sub>HW</sub>	W <sub>git</sub>	W <sub>aac</sub>	W <sub>en1</sub>	W <sub>en2</sub>	W <sub>TOT</sub>	
1	22/06/2010	1	Silvaplana	Durban Bulker	919.92	10.4	344	15.23	197	8.79	79.99	14.8	493	11.05	559		0.080			0.008	1 000.00
2	22/06/2010	2	Silvaplana	Durban Bulker	899.92	10.4	344	15.23	197	8.79	99.99	14.8	493	11.05	559		0.080			0.008	1 000.00
3	22/06/2010	3	Silvaplana	Durban Bulker	879.92	10.4	344	15.23	197	8.79	119.99	14.8	493	11.05	559		0.080			0.008	1 000.00
4	17/08/2010	1	Vogue Eva	N/A	999.92	11.0	312	16.57	203	10.08								0.024	0.056		1 000.00
5	17/08/2010	2	Vogue Eva	Durban Bulker	919.93	11.0	312	16.57	203	10.08	79.99	14.8	493	11.05	559			0.024	0.056		1 000.00
6	17/08/2010	3	Vogue Eva	Durban Bulker	899.93	11.0	312	16.57	203	10.08	99.99	14.8	493	11.05	559			0.024	0.056		1 000.00
7	17/08/2010	4	Vogue Eva	Durban Bulker	879.93	11.0	312	16.57	203	10.08	119.99	14.8	493	11.05	559			0.024	0.056		1 000.00
8	17/08/2010	5	Silvretta	Durban Bulker	949.92	10.9	329	15.83	218	9.58	50.00	14.8	493	11.05	559			0.024	0.056		1 000.00
9	18/08/2010	1	Andra	Durban Bulker	949.92	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559			0.024	0.056		1 000.00
10	18/08/2010	2	Andra	Durban Bulker	950.00	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559						1 000.00
11	18/08/2010	3	Andra	Durban Bulker	949.95	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559			0.040		0.008	1 000.00
12	18/08/2010	4	Andra	Durban Bulker	949.97	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559			0.024		0.012	1 000.00
13	26/08/2010	1	Vogue Eva	Durban Bulker	949.97	11.0	312	16.57	203	10.08	50.00	14.8	493	11.05	559			0.024		0.012	1 000.00
14	26/08/2010	2	Vogue Eva	N/A	999.96	11.0	312	16.57	203	10.08									0.024	0.012	1 000.00
15	26/08/2010	3	Vogue Eva	N/A	997.57	11.0	312	16.57	203	10.08							2.4	0.024		0.012	1 000.00
16	26/08/2010	4	Vogue Eva	N/A	995.38	11.0	312	16.57	203	10.08						4.0	0.024		0.012	1 000.00	
17	31/08/2010	1	Andra	Durban Bulker	949.97	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559			0.024		0.012	1 000.00
18	31/08/2010	2	Andra	Durban Bulker	950.00	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559						1 000.00
19	31/08/2010	3	Andra	Durban Bulker	949.95	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559				0.056		1 000.00
20	31/08/2010	4	Andra	Durban Bulker	949.92	10.9	323	16.09	212	9.76	50.00	14.8	493	11.05	559			0.024	0.056		1 000.00
21	15/09/2010	1	Explorius	N/A	1 000.00	10.9	354	14.85	204	10.50											1 000.00
22	15/09/2010	2	Explorius	N/A	999.92	10.9	354	14.85	204	10.50								0.024	0.056		1 000.00
23	15/09/2010	3	Explorius	N/A	999.92	10.9	354	14.85	204	10.50								0.024	0.056		1 000.00
24	15/09/2010	4	Explorius	Durban Bulker	899.93	10.9	354	14.85	204	10.50	99.99	14.8	493	11.05	559			0.024	0.056		1 000.00
25	15/09/2010	5	Explorius	Durban Bulker	849.93	10.9	354	14.85	204	10.50	149.99	14.8	493	11.05	559			0.024	0.056		1 000.00
26	16/09/2010	1	Explorius	Durban Bulker	849.97	10.9	354	14.85	204	10.50	149.99	14.8	493	11.05	559			0.024		0.012	1 000.00
27	16/09/2010	2	Explorius	Durban Bulker	849.93	10.9	354	14.85	204	10.50	149.99	14.8	493	11.05	559			0.024	0.056		1 000.00
28	16/09/2010	3	Explorius	Durban Bulker	849.92	10.9	354	14.85	204	10.50	149.99	14.8	493	11.05	559			0.024	0.056	0.012	1 000.00
29	16/09/2010	4	Explorius	Durban Bulker	849.93	10.9	354	14.85	204	10.50	149.99	14.8	493	11.05	559			0.024	0.056		1 000.00
30	16/09/2010	5	Explorius	Durban Bulker	849.91	10.9	354	14.85	204	10.50	149.98	14.8	493	11.05	559			0.048	0.056		1 000.00
31	16/09/2010	6	Explorius	Durban Bulker	849.89	10.9	354	14.85	204	10.50	149.98	14.8	493	11.05	559			0.072	0.056		1 000.00
32	07/10/2010	1	Pan Bless	Durban Bulker	949.91	10.9	339	15.42	220	11.24	50.00	14.8	493	11.05	559			0.040	0.056		1 000.00
33	07/10/2010	2	Pan Bless	Durban Bulker	948.39	10.9	339	15.42	220	11.24	49.92	14.8	493	11.05	559	1.6	0.040	0.056		1 000.00	
34	07/10/2010	3	Pan Bless	Durban Bulker	946.12	10.9	339	15.42	220	11.24	49.80	14.8	493	11.05	559	4.0	0.040	0.056		1 000.00	
35	15/12/2010	1	Great Success	Federal Kumano	949.91	10.9	341	15.35	233	10.52	50.00	15.4	535	10.26	599			0.040	0.056		1 000.00
36	15/12/2010	2	Great Success	Federal Kumano	947.63	10.9	341	15.35	233	10.52	49.88	15.4	535	10.26	599	2.4	0.040	0.056		1 000.00	
37	15/12/2010	3	Great Success	Federal Kumano	946.88	10.9	341	15.35	233	10.52	49.84	15.4	535	10.26	599	3.2	0.040	0.056		1 000.00	
38	31/01/2011	1	African Hawk	N/A	999.90	10.8	343	15.27	202	11.07								0.040	0.056		1 000.00
39	31/01/2011	2	African Hawk	N/A	995.92	10.8	343	15.27	202	11.07							4.0	0.040	0.056		1 000.00
40	01/02/2011	1	African Hawk	N/A	999.90	10.8	343	15.27	202	11.07								0.040	0.056		1 000.00
41	01/02/2011	2	African Hawk	N/A	991.97	10.8	343	15.27	202	11.07							7.9	0.040	0.056		1 000.00
42	24/03/2011	1	African Orchid	N/A	1 000.00	10.7	372	14.22	225	11.88											1 000.00
43	24/03/2011	2	African Orchid	GreenWing	950.00	10.7	372	14.22	225	11.88	50.00	15.6	424	12.66	415						1 000.00
44	24/03/2011	3	African Orchid	GreenWing	920.00	10.7	372	14.22	225	11.88	80.00	15.6	424	12.66	415						1 000.00
45	24/03/2011	4	African Orchid	GreenWing	900.00	10.7	372	14.22	225	11.88	100.00	15.6	424	12.66	415						1 000.00
46	24/03/2011	5	African Orchid	GreenWing	880.00	10.7	372	14.22	225	11.88	120.00	15.6	424	12.66	415						1 000.00
47	24/03/2011	6	African Orchid	GreenWing	850.00	10.7	372	14.22	225	11.88	150.00	15.6	424	12.66	415						1 000.00
48	24/03/2011	7	African Orchid	N/A	999.95	10.7	372	14.22	225	11.88								0.040		0.012	1 000.00
49	24/03/2011	8	African Orchid	GreenWing	949.95	10.7	372	14.22	225	11.88	50.00	15.6	424	12.66	415			0.040	0.012	1 000.00	
50	24/03/2011	9	African Orchid	GreenWing	919.95	10.7	372	14.22	225	11.88	80.00	15.6	424	12.66	415			0.040	0.012	1 000.00	
51	24/03/2011	10	African Orchid	GreenWing	899.95	10.7	372	14.22	225	11.88	99.99	15.6	424	12.66	415			0.040	0.012	1 000.00	
52	24/03/2011	11	African Orchid	GreenWing	879.95	10.7	372	14.22	225	11.88	119.99	15.6	424	12.66	415			0.040	0.012	1 000.00	
53	24/03/2011	12	African Orchid	GreenWing	849.96	10.7	372	14.22	225	11.88	149.99	15.6	424	12.66	415			0.040	0.012	1 000.00	
54	26/05/2011	1	Anu Princess	GreenWing	947.63	10.9	363	14.53	224	11.35	49.88	15.6	424	12.66	415	2.4	0.040	0.056		1 000.00	
55	26/05/2011	2	Anu Princess	GreenWing	947.63	10.9	363	14.53	224	11.35	49.88	15.6	424	12.66	415	2.4	0.040	0.056		1 000.00	
56	26/05/2011	3	Anu Princess	GreenWing	950.00	10.9	363	14.53	224	11.35	50.00	15.6	424	12.66	415						1 000.00
57	26/05/2011	4	Anu Princess	GreenWing	946.22	10.9	363	14.53	224	11.35	49.80	15.6	424	12.66	415	4.0					1 000.00
58	26/05/2011	5	Anu Princess	GreenWing	946.12	10.9	363	14.53	224	11.35	49.80	15.6	424	12.66	415	4.0	0.040	0.056		1 000.00	
59	08/06/2011	1	Anu Princess	GreenWing	947.63	10.7	367	14.39	220	11.24	49.88	15.6	424	12.66	415	2.4	0.040	0.056		1 000.00	
60	08/06/2011	2	Anu Princess	GreenWing	899.91	10.7	367	14.39	220	11.24	99.99	15.6	424	12.66	415			0.040	0.056		1 000.00
61	01/09/2011	1	Maori Maiden	Federal Leda	947.63	11.6	346	15.15	212	10.73	49.88	15.4	598	9.26	387	2.4	0.040	0.056		1 000.00	
62	01/09/2011	2	Maori Maiden	N/A	997.47	11.6	346	15.15	212	10.73								2.4	0.080	0.056	1 000.00
63	01/09/2011	3	Maori Maiden	N/A	997.49	11.6	346	15.15	212	10.73								2			

## 2. Dependent Variables

Test Nbr.	Date	Test nbr.	Soft French Wheat		DEPENDENT VARIABLES OUTPUT					
			Vessel	Vessel	Flour Protein content	Flour Falling Number	F			Baguette Vol./g
							Liquefaction Number	Flour Alveo W		
FPC <sub>FLR</sub>	FLN <sub>FLR</sub>	LNR <sub>FLR</sub>	ALW <sub>FLR</sub>	BVL <sub>FLR</sub>						
1	22/06/2010	1	Silvapiana	Durban Bulker	10,6	340	15,38	225	9,42	
2	22/06/2010	2	Silvapiana	Durban Bulker	10,8	341	15,35	235	10,30	
3	22/06/2010	3	Silvapiana	Durban Bulker	11,0	353	14,89	242	9,25	
4	17/08/2010	1	Vogue Eva	N/A	11,0	312	16,57	205	10,08	
5	17/08/2010	2	Vogue Eva	Durban Bulker	11,2	340	15,38	240	11,45	
6	17/08/2010	3	Vogue Eva	Durban Bulker	11,3	331	15,75	245	12,51	
7	17/08/2010	4	Vogue Eva	Durban Bulker	11,3	335	15,58	250	11,19	
8	17/08/2010	5	Silvretta	Durban Bulker	11,0	328	15,87	247	9,94	
9	18/08/2010	1	Andra	Durban Bulker	10,7	343	15,27	230	12,11	
10	18/08/2010	2	Andra	Durban Bulker	10,7	315	16,44	228	9,50	
11	18/08/2010	3	Andra	Durban Bulker	10,7	307	16,81	230	9,68	
12	18/08/2010	4	Andra	Durban Bulker	10,7	305	16,90	227	8,92	
13	26/08/2010	1	Vogue Eva	Durban Bulker	11,0	340	15,38	225	9,30	
14	26/08/2010	2	Vogue Eva	N/A	10,7	328	15,87	206	9,52	
15	26/08/2010	3	Vogue Eva	N/A	10,9	330	15,79	205	9,65	
16	26/08/2010	4	Vogue Eva	N/A	11,0	332	15,71	205	9,95	
17	31/08/2010	1	Andra	Durban Bulker	11,0	339	15,42	231	10,52	
18	31/08/2010	2	Andra	Durban Bulker	11,0	371	14,25	225	9,19	
19	31/08/2010	3	Andra	Durban Bulker	11,0	353	14,89	235	10,56	
20	31/08/2010	4	Andra	Durban Bulker	11,0	339	15,42	240	12,11	
21	15/09/2010	1	Explorius	N/A					10,60	
22	15/09/2010	2	Explorius	N/A					10,61	
23	15/09/2010	3	Explorius	N/A					10,58	
24	15/09/2010	4	Explorius	Durban Bulker					10,92	
25	15/09/2010	5	Explorius	Durban Bulker					12,45	
26	16/09/2010	1	Explorius	Durban Bulker					4,62	
27	16/09/2010	2	Explorius	Durban Bulker					12,56	
28	16/09/2010	3	Explorius	Durban Bulker					13,32	
29	16/09/2010	4	Explorius	Durban Bulker					12,38	
30	16/09/2010	5	Explorius	Durban Bulker					11,38	
31	16/09/2010	6	Explorius	Durban Bulker					12,07	
32	07/10/2010	1	Pan Bless	Durban Bulker	11,0	350	15,00	240	12,44	
33	07/10/2010	2	Pan Bless	Durban Bulker	11,3	347	15,11	245	11,41	
34	07/10/2010	3	Pan Bless	Durban Bulker	11,3	345	15,19	245	12,35	
35	15/12/2010	1	Great Success	Federal Kumano	10,8	309	16,71	255	10,81	
36	15/12/2010	2	Great Success	Federal Kumano	11,0	291	17,60	260	11,04	
37	15/12/2010	3	Great Success	Federal Kumano	11,2	333	15,67	257	12,75	
38	31/01/2011	1	African Hawk	N/A	10,8	330	15,79	205	11,10	
39	31/01/2011	2	African Hawk	N/A	11,0	322	16,13	210	12,32	
40	01/02/2011	1	African Hawk	N/A	10,8	329	15,83	209	11,88	
41	01/02/2011	2	African Hawk	N/A	11,6	338	15,46	209	12,22	
42	24/03/2011	1	African Orchid	N/A	10,7	362	14,56		227	
43	24/03/2011	2	African Orchid	GreenWing	10,8	359	14,67		230	
44	24/03/2011	3	African Orchid	GreenWing	11,0	357	14,74		235	
45	24/03/2011	4	African Orchid	GreenWing	11,2	366	14,42		243	
46	24/03/2011	5	African Orchid	GreenWing	11,4	357	14,74		250	
47	24/03/2011	6	African Orchid	GreenWing	11,6	395	13,48		256	
48	24/03/2011	7	African Orchid	N/A	10,8	378	14,02		228	
49	24/03/2011	8	African Orchid	GreenWing	11,0	349	15,04		237	
50	24/03/2011	9	African Orchid	GreenWing	11,0	376	14,08		250	
51	24/03/2011	10	African Orchid	GreenWing	11,1	381	13,92		247	
52	24/03/2011	11	African Orchid	GreenWing	11,2	362	14,56		249	
53	24/03/2011	12	African Orchid	GreenWing	11,3	362	14,56		259	
54	26/05/2011	1	Ainu Princess	GreenWing	11,1	369	14,32		235	
55	26/05/2011	2	Ainu Princess	GreenWing	11,1	353	14,89		238	
56	26/05/2011	3	Ainu Princess	GreenWing	10,9	354	14,85		232	
57	26/05/2011	4	Ainu Princess	GreenWing	11,3	369	14,32		235	
58	26/05/2011	5	Ainu Princess	GreenWing	11,0	374	14,15		241	
59	08/06/2011	1	Ainu Princess	GreenWing	11,0	355	14,81		238	
60	08/06/2011	2	Ainu Princess	GreenWing	11,0	347	15,11		245	
61	01/09/2011	1	Maori Maiden	Federal Leda					12,95	
62	01/09/2011	2	Maori Maiden	N/A					11,85	
63	01/09/2011	3	Maori Maiden	N/A					12,71	
64	02/09/2011	1	Maori Maiden	Federal Leda					12,95	
65	02/09/2011	2	Maori Maiden	Federal Leda					13,08	
66	02/09/2011	3	Maori Maiden	N/A					11,73	
67	02/09/2011	4	Maori Maiden	N/A					12,77	
68	17/09/2011	1	Maori Maiden	Global Glory	11,2			230	12,14	
69	17/09/2011	2	Maori Maiden	Global Glory	11,6			233	11,89	
70	17/09/2011	3	Maori Maiden	Global Glory	11,9			232	13,01	
71	22/09/2011	1	Moleson	Global Glory	11,3			224	12,24	
72	22/09/2011	2	Moleson	Global Glory	11,6			243	13,33	
73	22/09/2011	3	Moleson	Global Glory	11,7			250	13,45	
73	Observations									
	Average				11,08	346,40	15,19	234,50	11,49	
	Standard Deviation				0,29	21,70	0,85	14,79	1,46	
	Minimum				10,60	291,00	13,48	205,00	4,62	
	Maximum				11,90	395,00	17,60	260,00	13,45	

# APPENDIX C: TEST OF EQUATION FPC1 ON FLOURS MADE OF WHEAT AND GLUTEN ONLY

Test Nbr	Date	INDEPENDENT VARIABLES										FPC <sub>glu</sub>	FPC <sub>glu</sub> / W <sub>glu</sub>	FPC <sub>glu</sub> / W <sub>glu</sub> × (W <sub>glu</sub> / W <sub>tot</sub> ) × 64	Predicted FPC <sub>glu</sub>	DEPENDENT VARIABLE OUTPUT			
		Test nbr	Weight Sort/Wheat content	F Flour Protein	Weight hard wheat content	C Flour Protein	Weight gluten	Weight Acid ascobic	Weight enzyme mix.1	Weight enzyme mix.2	Total Weight						W <sub>glu</sub>	FPC <sub>glu</sub>	W <sub>glu</sub> / W <sub>tot</sub>
10	18/08/2010	2	950.00	10.9	50.00	14.8								10.36	0.74	11.10	10.7	10.7	
18	31/08/2010	2	950.00	10.9	50.00	14.8								10.36	0.74	11.10	11.0	11.0	
42	24/03/2011	1	1000.00	10.7										10.70		10.70	10.7	10.7	
43	24/03/2011	2	950.00	10.7	50.00	15.6								10.17	0.78	10.95	10.8	10.8	
44	24/03/2011	3	920.00	10.7	80.00	15.6								9.84	1.25	11.09	11.0	11.0	
45	24/03/2011	4	900.00	10.7	100.00	15.6								9.63	1.56	11.19	11.2	11.2	
46	24/03/2011	5	880.00	10.7	120.00	15.6								9.42	1.87	11.29	11.4	11.4	
47	24/03/2011	6	860.00	10.7	150.00	15.6								9.10	2.34	11.44	11.6	11.6	
56	26/05/2011	3	950.00	10.9	50.00	15.6								10.36	0.78	11.14	10.9	10.9	
57	26/05/2011	4	948.22	10.9	49.80	15.6								10.31	0.78	11.35	11.3	11.3	
10	Observations																		
	Average		929.62	10.78	89.99	15.42	0.40							10.02	1.08	11.13	11.06	11.06	
	Standard Deviation		43.03	0.10	43.22	0.35	1.26							0.68	0.68	1.27	0.31	0.31	
	Minimum		860.00	10.70		14.80								9.10		9.10	10.70	10.70	
	Maximum		1000.00	10.90	150.00	15.60	3.98							10.70	2.34	13.29	11.60	11.60	
SUMMARY OUTPUT																			
Regression Statistics																			
Multiple R	0.8613871																		
R Square	0.7320355																		
Adjusted R Square	0.7290812																		
Standard Error	0.1523617																		
Observations	10																		
ANOVA																			
	df	SS	MS	F	Significance F														
Regression	1	0.6323367	0.6323367	34.0365175	0.00138269														
Residual	8	0.2107453	0.0263432																
Total	9	0.844																	
		Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 90%	Upper 90%	Lower 80%	Upper 80%	Lower 70%	Upper 70%	Lower 60%	Upper 60%	Lower 50%	Upper 50%			
Intercept		1.15249617	2.8866328	0.0066632	0.1385712	-0.9202401	3.5806028	-3.4726121	2.8726348										
Predicted FPC <sub>glu</sub>		1.1744551	0.2536329	4.9200007	0.00118219	0.6739626	1.8709827	0.7886571	1.2521452										

# APPENDIX D: TEST OF EQUATION FPC1 ON FLOURS MADE OF WHEAT AND ADDITIVES

## 1. Data

Test No.	Date	Temp	INDEPENDENT VARIABLES INPUT				DEPENDENT VARIABLES OUTPUT									
			Weight Soft Wheat content	F Flour Protein content	Weight hard wheat content	C Flour Protein content	Weight Flour content	Weight Additives	Weight Flour	Weight Total						
			$W_{SW}$	$FPC_{SW}$	$W_{HW}$	$FPC_{HW}$	$W_{AS}$	$W_{AS}$	$W_{AS}$	$W_{TOT}$	$(W_{HW}/W_{TOT}) \times FPC_{HW}$	$(W_{SW}/W_{TOT}) \times FPC_{SW}$	$(W_{HW}/W_{TOT}) \times FPC_{HW}$	$(W_{SW}/W_{TOT}) \times FPC_{SW}$	Flour Protein content	FPC <sub>SW</sub>
1	22-06-2010		10.4	79.99	14.8	0.08	0.01	1000.00	0.01	1000.00	9.67	1.18	11.3	10.6		
2	22-06-2010		10.4	79.99	14.8	0.08	0.01	1000.00	0.01	1000.00	9.67	1.18	11.3	10.6		
3	22-06-2010		10.4	79.99	14.8	0.08	0.01	1000.00	0.01	1000.00	9.67	1.18	11.3	10.6		
4	17-08-2010		11.0	79.99	14.8	0.08	0.06	1000.00	0.06	1000.00	11.00	1.18	11.0	11.0		
5	17-08-2010		11.0	79.99	14.8	0.08	0.06	1000.00	0.06	1000.00	11.00	1.18	11.0	11.0		
6	17-08-2010		11.0	79.99	14.8	0.08	0.06	1000.00	0.06	1000.00	11.00	1.18	11.0	11.0		
7	17-08-2010		11.0	79.99	14.8	0.08	0.06	1000.00	0.06	1000.00	11.00	1.18	11.0	11.0		
8	17-08-2010		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.35	0.74	11.09	11.0		
9	17-08-2010		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.35	0.74	11.09	11.0		
10	18-08-2010		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.35	0.74	11.09	11.0		
11	18-08-2010		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.35	0.74	11.09	11.0		
12	18-08-2010		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.35	0.74	11.09	11.0		
13	20-08-2010		11.0	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	11.00	0.74	11.00	11.0		
14	20-08-2010		11.0	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	11.00	0.74	11.00	11.0		
15	20-08-2010		11.0	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	11.00	0.74	11.00	11.0		
16	31-01-2011		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.95	0.74	11.13	10.9		
17	31-01-2011		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.95	0.74	11.13	10.9		
18	31-01-2011		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.95	0.74	11.13	10.9		
19	31-01-2011		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.95	0.74	11.13	10.9		
20	31-01-2011		10.9	50.00	14.8	0.08	0.06	1000.00	0.06	1000.00	10.95	0.74	11.13	10.9		
21	07-10-2010		10.9	49.52	14.8	1.60	0.04	1000.00	0.04	1000.00	10.34	0.74	11.18	11.3		
22	07-10-2010		10.9	49.52	14.8	1.60	0.04	1000.00	0.04	1000.00	10.34	0.74	11.18	11.3		
23	07-10-2010		10.9	49.52	14.8	1.60	0.04	1000.00	0.04	1000.00	10.34	0.74	11.18	11.3		
24	07-10-2010		10.9	49.52	14.8	1.60	0.04	1000.00	0.04	1000.00	10.34	0.74	11.18	11.3		
25	15-12-2010		10.9	49.52	15.4	2.39	0.04	1000.00	0.04	1000.00	10.33	0.77	11.25	11.0		
26	15-12-2010		10.9	49.52	15.4	2.39	0.04	1000.00	0.04	1000.00	10.33	0.77	11.25	11.0		
27	15-12-2010		10.9	49.52	15.4	2.39	0.04	1000.00	0.04	1000.00	10.33	0.77	11.25	11.0		
28	31-01-2011		10.8	49.84	15.4	3.88	0.04	1000.00	0.04	1000.00	10.80	0.80	10.80	10.8		
29	31-01-2011		10.8	49.84	15.4	3.88	0.04	1000.00	0.04	1000.00	10.80	0.80	10.80	10.8		
30	01-02-2011		10.8	49.84	15.4	3.88	0.04	1000.00	0.04	1000.00	10.80	0.80	10.80	10.8		
31	01-02-2011		10.8	49.84	15.4	3.88	0.04	1000.00	0.04	1000.00	10.80	0.80	10.80	10.8		
32	01-02-2011		10.8	49.84	15.4	3.88	0.04	1000.00	0.04	1000.00	10.80	0.80	10.80	10.8		
33	01-02-2011		10.8	49.84	15.4	3.88	0.04	1000.00	0.04	1000.00	10.80	0.80	10.80	10.8		
34	01-02-2011		10.8	49.84	15.4	3.88	0.04	1000.00	0.04	1000.00	10.80	0.80	10.80	10.8		
35	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.71	0.78	11.22	11.0		
36	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.71	0.78	11.22	11.0		
37	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.71	0.78	11.22	11.0		
38	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.71	0.78	11.22	11.0		
39	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.71	0.78	11.22	11.0		
40	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.71	0.78	11.22	11.0		
41	01-02-2011		10.9	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
42	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
43	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
44	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
45	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
46	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
47	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
48	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
49	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
50	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
51	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
52	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
53	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
54	24-03-2011		10.7	50.00	15.6	7.94	0.04	1000.00	0.04	1000.00	10.94	1.25	11.09	11.0		
55	26-02-2011		10.9	49.88	15.6	2.39	0.04	1000.00	0.04	1000.00	10.33	0.78	11.26	11.2		
56	26-02-2011		10.9	49.88	15.6	2.39	0.04	1000.00	0.04	1000.00	10.33	0.78	11.26	11.2		
57	26-02-2011		10.9	49.88	15.6	2.39	0.04	1000.00	0.04	1000.00	10.33	0.78	11.26	11.2		
58	26-02-2011		10.9	49.88	15.6	2.39	0.04	1000.00	0.04	1000.00	10.33	0.78	11.26	11.2		
59	08-06-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
60	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
61	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
62	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
63	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
64	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
65	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
66	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
67	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
68	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
69	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
70	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
71	17-05-2011		11.6	50.00	15.6	2.39	0.04	1000.00	0.04	1000.00	10.14	0.78	11.07	11.0		
72	22-05-2011		11.2	99.99	15.1	0.04	0.06	1000.00	0.06	1000.00	10.30	1.21	11.95	11.9		
73	22-05-2011		11.2	99.99	15.1	0.04	0.06	1000.00	0.06	1000.00	10.30	1.21	11.95	11.9		
74	22-05-2011		11.2	99.99	15.1	0.04	0.06	1000.00	0.06	1000.00	10.30	1.21	11.95	11.9		
75	22-05-2011		11.2	99.99	15.1	0.04	0.06	1000.00	0.06	1000.00	10.30	1.21	11.95	11.9		
76	22-05-2011		11.2	99.99	15.1	0.04	0.06	1000.00	0.06	1000.00	10.30	1.21	11.95	11.9		

## 2. Regression Analysis

SUMMARY OUTPUT										
<b>Regression Statistics</b>										
Multiple R	0,803591949									
R Square	0,64576002									
Adjusted R Square	0,639076247									
Standard Error	0,171475191									
Observations	55									
<b>ANOVA</b>										
		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>				
Regression	1	2,840874443	2,840874443	2,840874443	96,61608784	1,53063E-13				
Residual	53	1,558398284	0,029403741							
Total	54	4,399272727								
		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90,0%</i>	<i>Upper 90,0%</i>	
Intercept	1,504985596	0,974029574	1,545112835	0,128269637	-0,448670275	3,458641468	-0,125653128	3,135624321		
Predicted FPCFLR	0,855292325	0,087014144	9,829348292	1,53063E-13	0,680764059	1,029820592	0,709620534	1,000964116		







## 2. Regression Analysis

SUMMARY OUTPUT									
<i>Regression Statistics</i>									
Multiple R	0.637157201								
R Square	0.405969298								
Adjusted R Square	0.393303047								
Standard Error	0.663873913								
Observations	49								
<i>ANOVA</i>									
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>				
Regression	1	14,15641756	14,15641756	32,12048966	8,5446E-07				
Residual	47	20,71424292	0,440728573						
Total	48	34,87066048							
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90.0%</i>	<i>Upper 90.0%</i>	
Intercept	5,4524267	1,729352498	3,152871786	0,002815112	1,973418274	8,931435126	2,550699931	8,354153469	
Predicted LNR	0,649469325	0,1145955	5,667494125	8,5446E-07	0,418932918	0,880005732	0,457186473	0,841752177	



## 2. Regression Analysis

SUMMARY OUTPUT									
<b>Regression Statistics</b>									
Multiple R	0,259094211								
R Square	0,06712981								
Adjusted R Square	0,015303688								
Standard Error	0,725210309								
Observations	39								
<b>ANOVA</b>									
		<b>df</b>	<b>SS</b>	<b>MS</b>	<b>F</b>	<b>Significance F</b>			
Regression		2	1,362462764	0,681231382	1,295289092	0,28627398			
Residual		36	18,93347972	0,525929992					
Total		38	20,29594248						
		<b>Coefficients</b>	<b>Standard Error</b>	<b>t Stat</b>	<b>P-value</b>	<b>Lower 95%</b>	<b>Upper 95%</b>	<b>Lower 90,0%</b>	<b>Upper 90,0%</b>
Intercept		1,771134482	1,166673093	1,518106907	0,137721334	-0,594988203	4,137257167	-0,198557011	3,740825975
Wen1		-27,36518431	21,04084017	-1,300574696	0,201668415	-70,03798573	15,30761712	-62,88838624	8,158017624
Wen2		-153,5047368	104,8241764	-1,464402031	0,151762843	-366,0980187	59,08854507	-330,4791521	23,4696785



# APPENDIX I: TEST OF EQUATION ALW1 ON FLOURS MADE OF WHEAT AND ADDITIVES

## 1. Data

Test Nbr.	Date	Testnbr.	INDEPENDENT VARIABLES										INDEPENDENT VARIABLES INPUT		DEPENDENT VARIABLE OUTPUT	
			Weight Soft Wheat	F. Alveo W	hand wheat	C. Alveo W	Weight gluten	Weight Add isozyme	Weight enzyme max 1	Weight enzyme max 2	Total Weight	$(W_{aw} / W_{tot}) \times ALW_{pre}$	$(W_{pre} / W_{tot}) \times ALW_{pre}$	Predicted ALWFLR	Flour Alveo W	ALWFLR
1	22/09/2010	1	915.92	197	79.99	559	0.08	0.01	1000.00	0.01	1000.00	181	45	205	235	
2	22/09/2010	2	895.92	197	99.99	559	0.08	0.01	1000.00	0.01	1000.00	177	07	203	235	
3	22/09/2010	3	875.92	197	119.99	559	0.08	0.01	1000.00	0.01	1000.00	173	07	240	242	
4	17/09/2010	4	869.92	203	79.99	559	0.02	0.06	1000.00	0.06	1000.00	203	45	203	205	
5	17/09/2010	5	895.92	203	99.99	559	0.02	0.06	1000.00	0.06	1000.00	197	45	231	240	
6	17/09/2010	6	895.92	203	119.99	559	0.02	0.06	1000.00	0.06	1000.00	183	56	239	248	
7	17/09/2010	7	875.92	218	50.00	559	0.02	0.06	1000.00	0.06	1000.00	207	28	240	250	
8	17/09/2010	8	845.92	212	50.00	559	0.02	0.06	1000.00	0.06	1000.00	201	28	250	257	
9	15/09/2010	9	845.92	212	50.00	559	0.04	0.04	1000.00	0.04	1000.00	201	28	239	238	
10	15/09/2010	10	845.92	212	50.00	559	0.02	0.04	1000.00	0.02	1000.00	201	28	229	230	
11	15/09/2010	11	845.92	203	50.00	559	0.02	0.04	1000.00	0.02	1000.00	201	28	227	227	
12	15/09/2010	12	845.92	203	50.00	559	0.02	0.04	1000.00	0.02	1000.00	203	28	231	225	
13	25/09/2010	13	895.92	203	50.00	559	0.02	0.04	1000.00	0.02	1000.00	203	28	209	209	
14	25/09/2010	14	895.92	203	50.00	559	0.02	0.04	1000.00	0.02	1000.00	202	28	205	205	
15	25/09/2010	15	895.92	203	50.00	559	0.02	0.04	1000.00	0.02	1000.00	202	28	202	205	
16	25/09/2010	16	895.92	203	50.00	559	0.02	0.04	1000.00	0.02	1000.00	201	28	208	208	
17	31/09/2010	17	845.92	212	50.00	559	0.02	0.04	1000.00	0.02	1000.00	201	28	208	208	
18	31/09/2010	18	845.92	212	50.00	559	0.02	0.04	1000.00	0.02	1000.00	201	28	208	208	
19	31/09/2010	19	845.92	212	50.00	559	0.02	0.04	1000.00	0.02	1000.00	201	28	208	208	
20	31/09/2010	20	845.92	212	50.00	559	0.02	0.04	1000.00	0.02	1000.00	201	28	209	240	
21	07/10/2010	21	845.92	220	50.00	559	0.04	0.06	1000.00	0.04	1000.00	209	28	237	240	
22	07/10/2010	22	845.92	220	49.92	559	1.60	0.04	1000.00	1.60	1000.00	209	28	237	245	
23	07/10/2010	23	845.12	220	49.92	559	3.88	0.04	1000.00	3.88	1000.00	208	28	239	248	
24	07/10/2010	24	845.12	220	49.92	559	3.88	0.04	1000.00	3.88	1000.00	208	28	239	248	
25	15/11/2010	25	845.92	233	50.00	599	2.39	0.04	1000.00	2.39	1000.00	221	30	251	255	
26	15/11/2010	26	845.92	233	49.88	599	2.39	0.04	1000.00	2.39	1000.00	221	30	251	255	
27	15/11/2010	27	845.92	233	49.88	599	3.19	0.04	1000.00	3.19	1000.00	221	30	251	255	
28	31/01/2011	28	869.92	203	49.94	599	3.19	0.04	1000.00	3.19	1000.00	201	30	202	202	
29	31/01/2011	29	869.92	202	49.94	599	3.88	0.04	1000.00	3.88	1000.00	201	30	202	202	
30	01/02/2011	30	869.92	202	49.94	599	3.88	0.04	1000.00	3.88	1000.00	202	30	202	202	
31	01/02/2011	31	869.92	202	49.94	599	7.84	0.04	1000.00	7.84	1000.00	200	200	200	209	
32	01/02/2011	32	869.92	202	49.94	599	7.84	0.04	1000.00	7.84	1000.00	200	200	200	209	
33	24/03/2011	33	1000.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	215	225	225	227	
34	24/03/2011	34	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	214	214	214	235	
35	24/03/2011	35	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	207	207	207	235	
36	24/03/2011	36	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	203	203	203	240	
37	24/03/2011	37	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
38	24/03/2011	38	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
39	24/03/2011	39	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
40	24/03/2011	40	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
41	24/03/2011	41	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
42	24/03/2011	42	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
43	24/03/2011	43	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
44	24/03/2011	44	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
45	24/03/2011	45	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
46	24/03/2011	46	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
47	24/03/2011	47	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
48	24/03/2011	48	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
49	24/03/2011	49	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
50	24/03/2011	50	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
51	24/03/2011	51	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
52	24/03/2011	52	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
53	24/03/2011	53	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
54	24/03/2011	54	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
55	24/03/2011	55	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
56	24/03/2011	56	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
57	24/03/2011	57	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
58	24/03/2011	58	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
59	24/03/2011	59	960.00	225	50.00	415	0.00	0.06	1000.00	0.00	1000.00	196	196	196	248	
60	08/04/2011	60	947.03	220	49.88	415	2.39	0.04	1000.00	2.39	1000.00	208	21	233	238	
61	08/04/2011	61	895.92	220	49.88	415	2.39	0.04	1000.00	2.39	1000.00	198	41	239	248	
62	08/04/2011	62	895.92	212	50.00	337	0.00	0.06	1000.00	0.00	1000.00	198	17	218	230	
63	17/09/2011	63	845.92	212	49.99	337	0.00	0.06	1000.00	0.00	1000.00	196	27	222	233	
64	17/09/2011	64	845.92	212	49.99	337	0.00	0.06	1000.00	0.00	1000.00	196	27	222	233	
65	22/09/2011	65	845.92	212	49.88	337	0.00	0.06	1000.00	0.00	1000.00	191	24	214	224	
66	22/09/2011	66	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	209	27	208	204	
67	22/09/2011	67	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	209	27	208	204	
68	22/09/2011	68	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
69	22/09/2011	69	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
70	22/09/2011	70	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
71	22/09/2011	71	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
72	22/09/2011	72	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
73	22/09/2011	73	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
74	22/09/2011	74	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
75	22/09/2011	75	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
76	22/09/2011	76	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
77	22/09/2011	77	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
78	22/09/2011	78	845.92	227	49.88	337	0.00	0.06	1000.00	0.00	1000.00	204	34	253	250	
79	22/0															

## 2. Regression Analysis

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0,93313057							
R Square	0,87073266							
Adjusted R Square	0,868293654							
Standard Error	5,366939993							
Observations	55							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	10283,13108	10283,13108	357,0030221	3,31971E-25			
Residual	53	1526,614379	28,80404489					
Total	54	11809,74545						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90,0%</i>	<i>Upper 90,0%</i>
Intercept	14,52045148	11,66545287	1,24473963	0,218703532	-8,877483343	37,91838631	-5,008872574	34,04977554
Predicted ALWFLR	0,953525494	0,050465707	18,8945236	3,31971E-25	0,852304107	1,054746882	0,869040034	1,038010954





## 2. Regression Analysis

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0,384311734							
R Square	0,147695509							
Adjusted R Square	0,086816617							
Standard Error	4,943635564							
Observations	46							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	3	177,8749126	59,29163753	2,42605448	0,07889624			
Residual	42	1026,460369	24,43953259					
Total	45	1204,335281						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90,0%</i>	<i>Upper 90,0%</i>
Intercept	-0,623622097	4,354141948	-0,143225027	0,886797422	-9,41063619	8,163391997	-7,947081413	6,69983722
Wglt	0,49384076	0,432405153	1,142078806	0,259890635	-0,378788156	1,366469676	-0,233444106	1,221125626
Wen1	110,3011424	77,70213402	1,419538135	0,163122125	-46,50811069	267,1103955	-20,39014514	240,9924299
Wen2	272,3459866	398,1872629	0,683964586	0,497754127	-531,2284335	1075,920407	-397,3860192	942,0779924



## 2. Regression Analysis

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0,809258544							
R Square	0,654899391							
Adjusted R Square	0,619804414							
Standard Error	0,743173194							
Observations	66							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	6	61,83878472	10,30646412	18,66077268	4,99257E-12			
Residual	59	32,5860774	0,552306397					
Total	65	94,42486212						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90,0%</i>	<i>Upper 90,0%</i>
Intercept	-1,02368998	1,598446392	-0,640428096	0,524373888	-4,222173795	2,174793836	-3,694842608	1,647462649
(Wsw / WTOT) x BVLsw	1,048220541	0,152271909	6,883873391	4,25263E-09	0,743525158	1,352915924	0,793760015	1,302681066
(Whw / WTOT) x ALWhw	0,029520273	0,005339277	5,528889854	7,70595E-07	0,018836405	0,040204141	0,020597845	0,038442701
Wglt	0,015928725	0,065370127	0,243669784	0,808331866	-0,114876596	0,146734046	-0,093310839	0,125168289
Waac	6,930560558	5,513647052	1,25698299	0,213713261	-4,102221616	17,96334273	-2,283256614	16,14437773
Wen1	23,02090359	5,998494894	3,837779976	0,000305001	11,01794314	35,02386405	12,99686057	33,04494662
Wen2	15,09429886	31,34194498	0,481600579	0,63187128	-47,62078766	77,80938539	-37,28100703	67,46960475



## 2. Regression Analysis

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0,81144489							
R Square	0,65844281							
Adjusted R Square	0,610504959							
Standard Error	0,752207168							
Observations	66							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	8	62,17337157	7,771671446	13,73534261	7,18822E-11			
Residual	57	32,25149055	0,565815624					
Total	65	94,42486212						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90,0%</i>	<i>Upper 90,0%</i>
Intercept	-1,155983662	1,627365011	-0,710340738	0,480389167	-4,414725861	2,102758536	-3,876984973	1,565017648
(Wsw / WTOT) x BVLsw	1,048789	0,1543427	6,79519667	6,97607E-09	0,739723077	1,357854923	0,790723547	1,306854453
(Whw / WTOT) x ALWhw	0,037374935	0,011740049	3,183541703	0,002357669	0,013865893	0,060883976	0,017745234	0,057004635
-((Whw / WTOT) x ALWhw) <sup>2</sup>	0,00010226	0,000134884	0,758136989	0,451492256	-0,00016784	0,000372361	-0,000123269	0,00032779
Wglt	-0,004726989	0,142739632	-0,033116163	0,973697647	-0,29055817	0,281104192	-0,243391777	0,2339378
-Wglt <sup>2</sup>	-0,004964337	0,024562627	-0,20210937	0,840550849	-0,05415015	0,044221475	-0,04603376	0,036105085
Waac	6,954831844	5,615083137	1,238598196	0,22057102	-4,289178103	18,19884179	-2,433749375	16,34341306
Wen1	23,96187908	6,197020818	3,866677196	0,000285425	11,55255904	36,37119912	13,60028125	34,32347691
Wen2	18,59552486	32,10703875	0,579172841	0,564754516	-45,69771074	82,88876046	-35,08837147	72,27942118

## APPENDIX M: QUALITY TESTS OF ACTUAL FLOURS

### 1. February 2011

FEBRUARY 2011					
Date	code	Prot	TCH	W	Vol.
04/02/11	1	10,8	354	243	
04/02/11	2	10,6	347	248	
04/02/11	3	10,6	361	252	12,18
05/02/11	1	10,9	372	235	
05/02/11	2	11,1	385	241	12,12
07/02/11	1	10,9	378	224	
07/02/11	2	11,0	381	222	12,67
08/02/11	1	10,9	370	256	
08/02/11	2	11,1	368	245	
08/02/11	3	11,2	377	235	12,24
09/02/11	1	11,0	372	232	
09/02/11	2	11,0	372	225	
09/02/11	3	11,1	381	236	12,57
10/02/11	1	10,9	384	230	
10/02/11	2	11,0	371	238	
10/02/11	3	11,2	376	223	12,57
11/02/11	1	10,8	368	234	
11/02/11	2	10,9	377	246	
11/02/11	3	11,0	377	239	13,43
12/02/11	1	10,8	354	240	
12/02/11	2	10,9	362	233	
12/02/11	3	10,9	355	224	12,29
14/02/11	1	11,0	349	278	
14/02/11	2	11,0	352	278	12,09
	Average	10,9	368	240	12,46
	Standard deviation	0,2	12	15	0,42
	n	24	24	24	9
3	$Av-(3std/n^{1/2})$	10,8	361	231	12,04
3	$Av+(3std/n^{1/2})$	11,0	376	249	12,88
	MODEL EQUATIONS	10,9	377	243	12,80
			14,052		
		OK	ERR	OK	OK

2. May 2011

MAY 2011					
Date	code	Prot	TCH	W	Vol.
14/05/11	1	10,9	368	199	
14/05/11	2	11,0	385	202	
14/05/11	3	11,0	394	209	11,79
16/05/11	1	11,0	387	228	12,67
17/05/11	1	10,9	351	205	
17/05/11	2	11,0	379	212	12,84
18/05/11	1	10,9	352	190	
18/05/11	2	11,0	374	201	
18/05/11	3	11,1	336	211	11,76
19/05/11	1	10,9	388	236	
19/05/11	2	11,0	349	238	
19/05/11	3	11,2	378	241	12,29
20/05/11	1	10,9	395	232	
20/05/11	2	10,9	355	225	
20/05/11	3	11,0	369	222	12,44
21/05/11	1	10,8	357	237	
21/05/11	2	10,9	362	237	
21/05/11	3	11,0	374	207	11,47
22/05/11	1	10,9	351	232	
22/05/11	2	10,9	368	265	
22/05/11	3	11,0	365	243	11,76
23/05/11	1	10,9	348	225	
23/05/11	2	11,0	357	234	11,92
24/05/11	1	10,9	361	232	
24/05/11	2	11,1	368	256	12,91
25/05/11	1	10,9	372	215	
25/05/11	2	11,0	354	231	11,38
26/05/11	1	10,9	339	227	
26/05/11	2	11,1	344	245	12,89
27/05/11	1	10,9	352	210	
27/05/11	2	11,0	356	223	12,83
28/05/11	1	10,8	352	222	
28/05/11	2	11,0	365	243	11,35
30/05/11	1	11,0	364	225	13,08
31/05/11	1	11,0	373	224	
	Average	11,0	364	225	12,23
	Standard deviation	0,1	15	17	0,62
	n	35	35	35	15
3	Av-(3std/n <sup>1/2</sup> )	10,9	356	217	11,74
3	Av+(3std/n <sup>1/2</sup> )	11,0	372	234	12,71
	MODEL EQUATIONS	11,0	329	230	11,10
			15,833		
		OK	ERR	OK	ERR



### 3. August 2011

AUGUST 2011					
Date	code	Prot	TCH	W	Vol.
23/08/11	1		11,1	361	234
23/08/11	2		11,1	358	241
23/08/11	3		11,1	367	248
24/08/11	1		11,1	349	227
24/08/11	2		11,2	354	241
24/08/11	3		11,4	370	265
25/08/11	1		11,2	357	238
25/08/11	2		11,2	354	238
25/08/11	3		11,3	351	259
26/08/11	1		11,2	353	236
26/08/11	2		11,2	367	226
26/08/11	3		11,4	363	238
27/08/11	1		11,2	390	282
27/08/11	2		11,3	377	231
29/08/11	1		11,4	387	266
31/08/11	1		11,0	375	262
31/08/11	2		11,2	364	254
01/09/11	1		11,1	384	264
01/09/11	2		11,2	376	264
02/09/11	1		11,2	373	234
02/09/11	2		11,3	376	232
03/09/11	1		11,2	375	224
03/09/11	2		11,4	346	267
05/09/11	1		11,3	361	233
05/09/11	2		11,4	329	242
06/09/11	1		11,0	358	250
06/09/11	2		11,2	374	231
07/09/11	1		11,0	357	229
07/09/11	2		11,1	372	228
08/09/11	1		10,8	362	226
08/09/11	2		11,0	362	243
09/09/11	1		10,8	357	252
09/09/11	2		10,9	361	265
09/09/11	3		11,0	356	223
10/09/11	1		10,8	367	235
10/09/11	2		11,0	366	241
10/09/11	3		11,1	346	247
12/09/11	1		10,9	366	244
12/09/11	2		11,0	384	261
12/09/11	3		11,1	381	246
13/09/11	1		11,0	354	240
13/09/11	2		11,1	368	246
13/09/11	3		11,1	358	262
14/09/11	1		10,9	361	230
14/09/11	2		11,0	358	263
14/09/11	3		11,2	354	260
15/09/11	1		11,0	349	249
15/09/11	2		10,9	352	250
15/09/11	3		11,2	348	260
16/09/11	1		10,8	376	235
16/09/11	2		10,8	372	209
16/09/11	3		11,0	364	229
17/09/11	1		10,8	362	255
19/09/11	1		11,0	346	261
20/09/11	1		10,6	350	240
20/09/11	2		10,8	351	244
20/09/11	3		10,9	345	240
21/09/11	1		10,8	352	246
21/09/11	2		10,8	342	219
21/09/11	3		11,0	346	242
22/09/11	1		10,8	339	221
22/09/11	2		10,9	351	227
22/09/11	3		11,1	348	244
23/09/11	1		10,8	360	228
23/09/11	2		10,9	365	281
24/09/11	1		10,9	332	237
26/09/11	1		10,9	363	238
	Average		11,1	360	244
	Standard deviation		0,2	13	15
	n		67	67	67
3	Av-(3std/n <sup>1/2</sup> )		11,0	356	238
3	Av+(3std/n <sup>1/2</sup> )		11,1	365	249
	MODEL EQUATIONS		11,0	363	249
				14,536	
			OK	OK	OK
					ERR

#### 4. February 2012

FEBRUARY 2012					
Date	code	Prot	TCH	W	Vol.
01/02/12	1	11,3	349	274	
01/02/12	2	11,0	345	242	
01/02/12	3	11,1	348	236	
01/02/12	4	11,2	347	256	12,55
01/02/12	5	11,3	337	293	
02/02/12	1	11,4	337	272	
02/02/12	2	11,4	339	279	
02/02/12	3	11,3	348	256	12,59
02/02/12	4	11,2	343	283	
03/02/12	1	11,4	340	278	
03/02/12	2	11,4	344	265	
03/02/12	3	11,3	346	239	12,61
06/02/12	1	11,5	347	275	
06/02/12	2	11,5	347	275	
06/02/12	3	11,3	338	286	12,98
07/02/12	1	11,4	340	239	
07/02/12	2	11,4	353	304	13,18
08/02/12	1	11,3	352	273	
08/02/12	2	11,3	345	286	
08/02/12	3	11,3	355	252	12,92
09/02/12	1	11,3	339	279	
09/02/12	2	11,3	341	255	
09/02/12	3	11,3	361	307	13,06
10/02/12	1	11,3	347	267	
10/02/12	2	11,4	355	254	
10/02/12	3	11,3	368	306	13,07
11/02/12	1	11,2	351	267	
11/02/12	2	11,3	345	262	
11/02/12	3	11,2	357	279	13,25
11/02/12	4	11,2	357	279	
14/02/12	1	11,3	347	293	
14/02/12	2	11,5	351	281	
14/02/12	3	11,2	349	300	12,96
15/02/12	1	11,2	347	291	
15/02/12	2	11,3	336	260	
15/02/12	3	11,2	342	295	13,83
16/02/12	1	11,1	329	291	
16/02/12	2	11,2	351	260	
16/02/12	3	11,2	341	254	13,13
	Average	11,3	347	273	13,01
	Standard deviation	0,1	8	19	0,35
	n	39	39	39	12
3	Av-(3std/n <sup>1/2</sup> )	11,2	343	264	12,71
3	Av+(3std/n <sup>1/2</sup> )	11,3	350	282	13,31
	MODEL EQUATIONS	11,6	347	278	13,50
			15,131		
		ERR	OK	OK	ERR