

THREE ESSAYS ON HUMAN CAPITAL

by

XIAOYAN CHEN YOUDERIAN

B.A., Huazhong University of Science and Technology, P.R. China, 2007

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Economics
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Manhattan, Kansas

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Abstract

The first essay considers how the timing of government education spending influences the intergenerational persistence of income. We build a life-cycle model where human capital is accumulated in early and late childhood. Both families and the government can increase the human capital of young agents by investing in education at each stage of childhood. Ability in each dynasty follows a stochastic process. Different abilities and resultant spending histories generate a stochastic steady state distribution of income. We calibrate our model to match aggregate statistics in terms of education expenditures, income persistence and inequality. We show that increasing government spending in early childhood education is effective in lowering intergenerational earnings elasticity. An increase in government funding of early childhood education equivalent to 0.8 percent of GDP reduces income persistence by 8.4 percent. We find that this relatively large effect is due to the weakening relationship between family income and education investment. Since this link is already weak in late childhood, allocating more public resources to late childhood education does not improve the intergenerational mobility of economic status. Furthermore, focusing more on late childhood may raise intergenerational persistence by amplifying the gap in human capital developed in early childhood.

The second essay considers parental time investment in early childhood as an education input and explores the impact of early education policies on labor supply and human capital. I develop a five-period overlapping generations model where human capital formation is a multi-stage process. An agent's human capital is accumulated through early and late childhood. Parents make income and time allocation decisions in response to government expenditures and parental leave policies. The model is calibrated to the U.S. economy so that the generated data matches the Gini index and parental participation in education expenditures. The general equilibrium environment shows that subsidizing private education spending and adopting paid parental leave are both effective at increasing human capital. These two policies give parents incentives to increase physical and time investment, respectively. Labor supply decreases due to the introduction of paid parental leave as intended. In addition, low-wage earners are most responsive to parental leave by working less and spending more time with children.

The third essay is on the motherhood wage penalty. There is substantial evidence that women with children bear a wage penalty of 5 to 10 percent due to their motherhood status. This

wage gap is usually estimated by comparing the wages of working mothers to childless women after controlling for human capital and individual characteristics. This method runs into the problem of selection bias by excluding non-working women. This paper addresses the issue in two ways. First, I develop a simple model of fertility and labor participation decisions to examine the relationships among fertility, employment, and wages. The model implies that mothers face different reservation wages due to variance in preference over child care, while non-mothers face the same reservation wage. Thus, a mother with a relatively high wage may choose not to work because of her strong preference for time with children. In contrast, a childless woman who is not working must face a relatively low wage. For this reason, empirical analysis that focuses only on employed women may result in a biased estimate of the motherhood wage penalty. Second, to test the predictions of the model, I use 2004-2009 data from the 1997 National Longitudinal Survey of Youth (NLSY97) and include non-working women in the two-stage Heckman selection model. The empirical results from OLS and the fixed effects model are consistent with the findings in previous studies. However, the child penalty becomes smaller and insignificant after non-working women are included. It implies that the observed wage gap in the labor market appears to overstate the child wage penalty due to the sample selection bias.

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Approved by:

Major Professor
William F. Blankenau

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Chapter 1 - Early childhood education expenditures and the intergenerational persistence of income

1 Introduction

Children from lower income families tend to earn less in adulthood than children from wealthier families. This intergenerational persistence of income reflects more than inherited traits. Children from wealthier families are provided with more and better education, a socioeconomic environment more suitable for human capital accumulation, and greater workplace opportunities through networking. As a result, children of equal ability at birth can enter the job market with vastly different prospects.

Government education spending can mitigate this to some degree by weakening the link between parental income and the educational opportunities of their offspring. In the United States, government plays the predominant role in education funding beginning with primary school. Through primary and secondary school, government provides more than 90 percent of all expenditures. Learning opportunities, however, arise much earlier. A wealth of evidence shows that a child's learning environment prior to primary education can have a substantial effect on academic achievement and beyond. Government support of education through these years is substantial but much lower. Government provides less than 40 percent of the expenditures for early childhood education.

This paper considers the extent to which increased public funding of early childhood education could reduce the intergenerational persistence of income. We distinguish funding in early childhood from late childhood spending because of the different government spending patterns across these levels and the singular role of early childhood in the development of skills. We develop a model where the relatively low level of government funding in early childhood causes a relatively high disparity in resources devoted to children's education at this level. As a result, differences in ability and skills are developed before primary education begins. The ability gap means a disparity in the preparedness of students to acquire human capital through additional education. With differences in learning ability, even relatively egalitarian primary and secondary education further widens the achievement gap between children from poor and wealthy families. This achievement gap becomes a wage gap as students become workers.

The magnification of ability differences in the pre-primary years is made more severe by the critical role of early childhood education in skill accumulation. Compelling evidence demonstrates that skills attained early in life form the foundation of later achievement. Cunha et al. (2005) show that disparities in ability across young children account for much of the variation in socioeconomic outcomes as adults. Knudsen et al. (2006) cite evidence from economics, neurobiology and sociology to show that different abilities and skills are formed in different stages of the life cycle and that some essential skills are developed in early childhood. They emphasize that one cannot easily substitute later education for earlier education. Heckman (2006) summarizes two key roles of early learning that account for this lack of substitutability. First, it causes children to value acquired skills which motivates additional learning. Second, mastering cognitive, social and emotional skills early in life makes later learning more efficient.¹

Based on this evidence, we follow Cunha et al. (2007) and model human capital accumulation as a multi-stage process where the timing of education investment is critical to its effectiveness. Education at one stage enhances productivity in the next and later investment increases the value of earlier investment. We model early childhood education as being both relatively productive and a strong complement to late childhood education. This raises the stakes for any missed opportunity to invest in early childhood.

We show that increments to public funding of education in early childhood have much larger effects on persistence than increments to funding in late childhood. We then explore which features of early childhood education explain this. Part of the difference in policy effectiveness stems from the pivotal role of early childhood education. When we allow early childhood education to play the same role as education at other levels, increasing early spending causes a smaller change in persistence. However, it is still more effective than increasing late spending. The remaining difference in policy effectiveness is explained by differences in how education at the different levels is funded.

In our model, both families and government can provide funding at each stage of education. The key to decreasing income persistence is to weaken the link between total spending on education and family income. The key to weakening this link through increased government funding of education is the responsiveness of family spending to government

¹ Related work includes Carneiro and Heckman (2003) and Currie (2001).

spending. In our model, families value the human capital of their children but also value consumption. In the case where private and public spending are substitutable, increased government spending on education results in decreased family spending. When family spending is low, however, there is scarce room for such crowding out. Since low income families spend relatively little on education, a modest amount of government spending can drive family spending near zero. At the same level of government spending, a wealthy family may devote considerable private resources to education. An increment to government spending then lowers private spending more for the wealthy than for the poor. Equivalently, increments to government expenditures increase total spending more for low income families than for high income families. As a result, income and education spending are less closely linked and persistence of income diminishes.

At the primary and secondary levels, the link between income and spending is weak at current levels of government spending. Since government provides most of the resources, family spending is near zero for most families. In essence, the potential to decrease persistence through this channel has been nearly fully exploited. Since families provide more spending in early childhood, there is more scope for increased government funding to decrease the spending gap. In turn there is more scope to decrease income persistence.

Since public spending mostly crowds out private spending at the upper end of the income distribution, spending at the lower end of the distribution is more effective in reducing persistence. We show that considerable decreases in persistence arise from allocating early childhood spending progressively. This is helpful in understanding the success of such programs as the Perry Preschool Project, the Abecedarian Project (see Cunha et al. (2007)), and Head Start (see Currie (2001)). These are programs targeted directly at the early development of children from low income families and each has arguably been highly effective.

Our work is related to recent papers by Restuccia and Urrutia (2004) and Holter (2010). Restuccia and Urrutia also consider the role of education at different stages on the intergenerational persistence of earnings. However, they focus on a two-stage education process modeled after early and college education where early education encompasses all of education prior to college. Features that distinguish these two levels of education are quite different than those that distinguish early and late childhood education. Thus they consider related but distinct questions. They find that increasing funding of pre-college education is more effective than funding for college. In this sense our work can be seen as a refinement of this prescription. We

argue that when increasing pre-college funding, it is best to focus these additional resources on the pre-primary period.

Holter (2010) builds a model in a similar vein in order to understand better the sources of differing levels of intergenerational income persistence in Western economies. He also considers how persistence in the U.S. would change upon implementing Danish policies. He finds that the required increased progressivity of taxes would have a larger effect on persistence than the required spending changes. Holter models education as a multistage process. While there are several periods prior to college, education is assumed to begin at age 5 and the pre-college periods contribute to human capital in a symmetric fashion. Also, there are not sharp funding differences across the pre-college periods. Thus the paper also addresses a distinct set of questions.

Our work is also related to Abington and Blankenau (2011) and the model in that paper is the starting point for this work. Abington and Blankenau consider circumstances under which the current government funding structure, i.e. focusing on later childhood, can be appropriate despite the importance of early childhood education. In that model, however, agents perfectly inherit the ability of their parents. In a steady state, prodigy income is the same as parental income so persistence is one. Thus substantial modifications to the model are required to facilitate an investigation of income persistence.

2 The model

We consider an overlapping generations model where agents live for fifteen periods and each period lasts five years. The first period is spent in early childhood, the next three in late childhood, the fifth as a worker and the parent of a child in early childhood, the next three as a worker and the parent of a child in late childhood, and the remainder in work and retirement as empty nesters. There are $4N$ distinct dynasties where N is large. Each dynasty has a child every four periods. The dynasties are staggered so that a child is born into one fourth of the dynasties in each period. At the beginning of any period t , then, N agents comprising generation t are born into early childhood as the prodigy of the current young parents. In the subsequent period, as their parents move to late parenthood, the offspring move to late childhood. Generation t reaches parenthood in period $t+4$ with offspring in early childhood. As they transition to late parenthood in period $t+5$, their children transition to late childhood. At the end of period $t+8$,

they are empty nesters and continue working until period $t+12$. In periods $t+13$ and $t+14$ they are retired and subsequently exit the economy.

2.1 Production of human capital

We focus on four distinct life stages: early and late childhood and early and late parenthood. Early childhood and early parenthood each last one period whereas late childhood and late parenthood each last three periods. While the fifteen-period structure is convenient for our calibration, distinctions across these life stages are key to our results.

As agents enter parenthood, they are heterogeneous in human capital. The root cause of this heterogeneity is exogenous ability shocks in each dynasty. Let $a_{i,t}$ be the ability parameter of the child born to dynasty i in period t and $a_{i,t-4}$ be the ability parameter of this child's parent who was born to dynasty i in period $t-4$. For each dynasty, the sequence of abilities across generations follows a first order autoregressive process in logarithms. More succinctly

$$\ln(a_{i,t}) = \bar{a} + \rho \ln(a_{i,t-4}) + \varepsilon_{i,t}, \varepsilon_{i,t} \sim N(0, \sigma_a^2).$$

Here \bar{a} is a constant and ρ is the intergenerational correlation of innate abilities. Accordingly, the mean of $\ln(a_{i,t})$ is $\frac{\bar{a}}{1-\rho}$. The parameter $\varepsilon_{i,t}$ is a random shock term with a normal distribution of mean 0 and variance σ_a^2 . Ability differences partly explain income differences in most empirical and theoretical work. Thus it is natural to consider persistence in ability in investigations of income persistence. Persistence of this sort is a feature in both Restuccia and Urrutia (2004) and Holter (2010).

There is a high notational cost for precision in our model. We opt to limit this where possible and adopt a shorthand². We drop the i notation fully, with the understanding that the productivity measure, a , and many endogenous items pertain to a dynasty and should have subscript i . We also eliminate time subscripts. Instead, items with no qualifiers refer to adults and the notation refers to their children. This allows the restatement

$$\ln(\hat{a}) = a + \rho \ln(a) + \varepsilon, \varepsilon \sim N(0, \sigma_a^2). \quad (1.1)$$

An agent's human capital accumulates according to

$$\hat{h} = \hat{a} e^\mu h^\nu. \quad (1.2)$$

² The model with complete notations is presented in Appendix 1.

Here e is a measure of education expenditures on the agent's behalf, h is parental human capital and the parameters $\mu, \nu \in [0,1]$ are elasticities of human capital accumulation with respect to these inputs. Human capital has a genetic component through a , a socioeconomic component through h , and a means to modify these preordained inputs through education expenditures through e . In our model the three components are reinforcing in that the more able tend to have higher parental human capital and receive more education expenditures in equilibrium. Government spending can moderate this.

Variants of equation (1.2) pervade the literature on human capital accumulation. For example, Becker and Tomes (1986) propose that human capital is formed by education expenditures and natural endowments, which are genetically inherited from parents.³ It is our specification of e that puts us in a more narrow literature where the multi-stage nature of human capital accumulation plays a key economic role. We define

$$e = (\gamma i_e^\Phi + (1 - \gamma) i_l^\Phi) \quad (1.3)$$

where i_e and i_l are investment in education in early and late childhood.⁴ These inputs to early and late childhood form e in a constant elasticity of substitution (c.e.s.) production function with constant return to scale. The parameter $\gamma \in [0,1]$ gauges their relative importance.

Human capital accumulation is a hierarchical process when early and late spending aggregates are not perfect substitutes; both the sum of these aggregates and their timing are important to the outcome. The parameter $\Phi \leq 1$ governs the substitutability of investment in early and later childhood. We have imperfect substitutability so long as $\Phi < 1$ and substitutability decreases with Φ . When Φ approaches 0, equation (1.3) converges to Cobb-Douglas form $e = i_e^\gamma i_l^{1-\gamma}$ with unit elasticity of substitution. Our specification is similar to that in Abington and Blankenau (2011), Caucutt and Lochner (2011) and Cunha and Heckman (2008).

Investment in late childhood itself is a function of investment in the three periods of late childhood and given by

$$i_l = (i_2 i_3 i_4)^{\frac{1}{3}}. \quad (1.4)$$

³ See also Becker and Tomes (1979), Su (2004), Restuccia and Urrutia (2004) and Holter (2011).

⁴ Examples of similar human capital production functions include Cunha et al. (2005), Abington and Blankenau (2011), and Caucutt and Lochner (2011).

Here i_2, i_3 , and i_4 are investment in the second through fourth periods.

Both government and parents may invest in the education of children. As in Abington and Blankenau (2011), we specify

$$i_e = (\alpha f_e^\eta + (1 - \alpha)g_e^\eta)^{\frac{1}{\eta}}, \quad i_k = (\alpha f_k^\eta + (1 - \alpha)g_k^\eta)^{\frac{1}{\eta}}, \quad k \in (2,3,4). \quad (1.5)$$

Here f_e and g_e are parental and government education spending when the child is in early childhood. The item i_e in its entirety is a measure of early childhood inputs to education for the agent. Symmetrically, f_k and g_k , $k \in \{2,3,4\}$ are parental and government education spending when the child is in late childhood and i_k is a measure of late childhood inputs to education in the k^{th} period of childhood. Investment in each period is a c.e.s. combination of public and private expenditures where $\alpha \in [0,1]$ gauges their relative importance and $\eta \leq 1$ gauges their substitutability. We allow government spending to be different in early and late childhood and their relative sizes will be a primary object of our investigation.

For much of our investigation government spending within a period is the same for all agents but we also consider progressive government spending in early childhood. Parameters other than a are common to all. From, equations (1.2)-(1.5), then, heterogeneity arises from differences in a, h, f_e , and f_k and sometimes g_e . We have described the exogenous process governing \hat{a} . As described in the following section, h, f_e , and f_k will be endogenous responses to the series of shocks and government spending experienced by the dynasty.

This human capital production process allows several distinctions between early and late childhood. The aggregate i_l weighs the three stages of late childhood equally and makes them equally substitutable. In contrast, we can gauge the weight of early childhood and its substitutability with late childhood education through γ and Φ .

Of course the various periods of later childhood might be similarly distinguished. Quality education, for example, might be more important in the early primary years than later. Also, the implied unit elasticity of these later stages implied by equation (1.4) may be too restrictive. However, we opt for this simpler specification as it more succinctly captures the singular role of early childhood.

2.2 Agents' decisions

Children make no economic choices. Parents allocate resources across consumption in the eleven periods of adulthood and education spending in the early and late childhood of their offspring in order to maximize

$$U = \sum_{j=1}^{11} \beta^{j-1} \frac{c_{4+j}^\sigma}{\sigma} + \xi \frac{\hat{h}^\sigma}{\sigma}. \quad (1.6)$$

Utility depends on consumption through adulthood. We use T to indicate the number of periods an agent has been an adult, so c_{4+j} is consumption in the j^{th} period of an agent's adulthood. The discount rate on consumption is $\beta < 1$ and $\sigma < 1$ gauges marginal utility. Agents also value the human capital of their offspring, \hat{h} , and ξ indicates the relative importance of child's accumulated human capital to the parent. Education spending influences accumulated human capital through equations (1.2)-(1.5).

We consider borrowing constraints only in the first period of adulthood so the budget constraints can be written as

$$c_5 + f_e + b_5 = wh(1 - \tau) \quad (1.7)$$

$$\sum_{j=2}^{11} \frac{c_{4+j}}{r^{j-1}} + \sum_{j=2}^4 \frac{f_j}{r^{j-1}} = \sum_{j=2}^9 \left(\frac{z}{r}\right)^{j-1} wh(1 - \tau) + rb_5.$$

Here $w > 0$ is the wage rate per unit of human capital, $\tau \in [0,1]$ is the rate at which labor income is taxed and b_5 is bond holdings at the end of early adulthood. In general these can be positive or negative. However, we will consider also the impact of borrowing constraints in our sensitivity analysis. This will require $b_5 \geq 0$. Each bond is a claim to r units of output in the subsequent period. We assume that r is set in a world economy which accommodates any net saving or borrowing. As such, it is exogenous to our model. Income from bonds is untaxed. This simplifies the model without appreciably altering the results.

We have three motivations for considering borrowing constraints only in early adulthood. First, evidence by Cunha et al. (2005) suggests that borrowing constraints are relevant to educational outcomes only early in the education process. Second, as discussed below, private spending on education is highest in early childhood. Government provides most education expenditures beyond that so there is little scope for constraints to restrain private

spending. Third, we find below that even constraints in early adulthood are of minor importance. As such more constraints do not warrant the additional complexity.

The first line of equation (1.7) shows that in early adulthood, income net of taxes is allocated across current consumption, current education spending, and savings. The second line requires that the present discounted value of additional consumption and education spending must equal the present discounted value of additional income plus the return on first period savings. Through experience, agents in later life can be more productive. Though we do not model this process explicitly, we allow the possibility by including $z \geq 1$ as the experience premium.

Output is linear in human capital employed. This convenience makes the wage rate exogenous. Since the wage equals the marginal product of labor in a competitive labor market, we use the same notation for each. With leisure unvalued in utility, all human capital is used in production. Total output then is

$$Y = w \left(\sum h_{-4} + \sum zh_{-5} + \sum z^2 h_{-6} + \dots + \sum z^8 h_{-12} \right) = wH. \quad (1.8)$$

We again use simplified notation. Y indicates total output at time t and H indicates the measure of human capital currently employed across dynasties. The first summation is over the human capital of workers currently in the first period of adulthood and the subscript indicates that they were born in period $t-4$. The second summation is over the human capital of workers in the second period of adulthood. This is scaled by z to reflect the gain from experience. Since nine generations are working, we have nine similar terms reflecting when the working generations were born and their current productivity.

2.3 Government

Government taxes labor income at rate τ , collecting revenue equal to τY . Holter (2010) shows that progressivity in the tax code can be important for reducing persistence. However, since we are focusing more narrowly on spending issues, we opt for this uniform taxation. Revenue is allocated across education expenditures for the N agents currently in early childhood and the N agents currently in each period of late childhood. Government must balance its budget in each period giving

$$\sum g_e + 3Ng_l = \tau Y$$

where each g value has the t subscript. The summation is across the N agents in early childhood in period t indicating that government spending at this level can differ across agents. This allows us to consider the case of progressive spending at this level. Spending in late childhood is constant across agents so N indicates the number of agents at a particular stage receiving amount g_t and the 3 indicates that three stages are funded.

The value τ measures not only the tax rate but also the share of output allocated to government education spending. We investigate the effects of tax level τ and the allocation of tax across early and late childhood, thus we define

$$\zeta_e = \frac{\sum g_e}{Y}, \zeta_l = \frac{Ng_l}{Y}$$

such that ζ_e is the share of output allocated by government to early childhood and ζ_l is the share allocated to each of the periods of late childhood.

3 Calibration

We calibrate parameter values to form a baseline economy that matches the U.S. data. When available, we use empirical counterparts to the parameters of our model. In other cases, we choose parameters so that features of the generated data match features of the U.S. economy. Table 1-1 shows our choice of directly calibrated parameters.

Table 1-1. Parameters set exogenously

Description	Parameters	Values
Innate ability scalar	\bar{a}	1
Intergenerational persistence of innate ability	ρ	0.25
Weight of private spending	α	0.5
Returns on educational expenditures parameter	μ	0.159
Substitutability parameter of private and public expenditures	η	0.95
Substitutability parameter of early and late childhood expenditures	Φ	-1.8
Discount rate	β	0.815
Intertemporal preference parameter	σ	0
Wage rate	w	1
Wage growth rate	z	1.054
Interest rate	r	1.05
% of GDP on early childhood education by government	ζ_e	0.4
% of GDP on late childhood education per period by government	ζ_l	1.2

The first two parameters govern the transference of ability. The first serves only to scale the economy so we normalize $a = 1$. Our specification of ρ is meant to capture only the genetic

aspects of ability transference. Since direct observations of ability are not available, researchers consider proxies. For example, Black et al. (2008) show that the intergenerational persistence of IQ scores is 0.32. However, the IQ measure is a combination of innate ability (nature) and malleable ability (nurture). For our purposes, this measure may overstate the persistence of ability. As such, we consider 0.32 to be an upper bound in a later sensitivity analysis and set $\rho = 0.25$ in the baseline economy. This is in line with work by Restuccia and Urrutia (2004) and Holter (2010). These authors calibrate ρ so that the intergenerational correlation of earnings in generated data matches observed values and arrive at values ranging from 0.2 to 0.332.

The next four rows consider parameters of human capital production. Of these, the first two are share parameters and the others gauge elasticities. We set $\alpha = 0.5$ so that neither public nor private spending is inherently more productive in generating human capital. It can be argued that since families know better the needs of their children, their expenditures are more targeted and effective; i.e. $\alpha > 0.5$. Alternatively, it can be argued that government has more accumulated experience in providing human capital and thus spends more effectively; i.e. $\alpha < 0.5$. Our choice reflects neutrality on this issue.

The parameter μ gauges the importance of education quality in generating human capital. With $w = 1$, earnings in our model is $wh = h$. So μ also governs the importance of education quality for earnings. Card and Krueger (1996) provide a comprehensive summary of the empirical literature relating education quality and earnings. With quality being measured by expenditures per student, researchers estimate the impact of quality by regressing earnings on education expenditures. Card and Krueger calculated that the estimates have a mean of 0.159, with an interquartile ranging from 0.085 to 0.195. We follow their results and choose $\mu = 0.159$. Restuccia and Urrutia (2004) calibrate this parameter in the context of their model and use $\mu = 0.24$. We take this value as an upper bound in our sensitivity analysis.

It is common in the literature to treat public spending as highly substitutable with private spending. This is because for many education inputs, the source of funding is irrelevant to its productivity. For example, whether government or a family purchases a computer for coursework should have no impact on its effectiveness. Some researchers assume these inputs are perfectly substitutable, i.e. $\eta = 1$.⁵ However, families might allocate some of their

⁵ See Restuccia and Urrutia (2004), Holter (2010) and Abington and Blankenau (2011).

expenditures to items complementary to public education spending such as software for a computer or some types of tutoring. In this case, substitution is not perfect; i.e. $\eta < 1$. This assumption is also common.⁶ We follow the precedent of assuming government and family inputs to be highly substitutable by setting $\eta = 0.95$. One advantage of setting $\eta < 1$ is that in equilibrium, private education spending is positive for all families. Nordblom (2003) finds that for most families, parents provide basic school supplies, lodging, and "within the family" education prior to college. Even the poorest families incur some expenses for such informal investment. We consider alternative values of η in our sensitivity analysis. We show that the effectiveness of public education spending in reducing persistence decreases as η falls and is eliminated at $\eta = 0$.

As mentioned above, evidence suggests that education quality in late childhood is not a good substitute for education quality in early childhood. Estimates of elasticity of substitution between early and late childhood are scarce but support the case of complementarity. Cunha et al. (2010) estimate a parameter somewhat analogous to our Φ . They consider substitutability of investment during early childhood and later investment through age 12 in forming cognitive skills. They provide an estimate of $\Phi = -1.373$ and indicate that the value is decreasing with age. This is because spending becomes less effective in improving acquired skills as children age. Because we are considering a period including children to age 20, we use this as an upper bound in our sensitivity analysis and set $\Phi = -1.8$ in our baseline case. This value is further motivated by the work of Caucutt and Lochner (2011). They calculate a complementarity parameter over two twelve year periods of $\Phi = -1.67$. Since our distinction is between two more starkly different learning episodes, it is reasonable that input in our model are more complementary.

For preference parameters we set the discount factor β to 0.815 and the elasticity parameter σ to 0. The discount rate corresponds to the commonly used annual rate of 0.96. The elasticity parameter corresponds to log preferences. Log preferences are common in the literature and are in line with empirical estimates.⁷ Other estimates range between $\sigma = -0.5$ and -2 so we consider $\sigma = -2$ as a lower bound in the sensitivity analysis.⁸

⁶ Studies that specify inputs as imperfect substitutes include Arcalean and Schiopu (2010), Bearnse et al. (2005) and Nordblom (2003).

⁷ See Beaudry and Wincoopn (1996) and Vissing-Jørgensen (2002).

We next consider items that determine the present value of lifetime income, w , z , and r . The wage rate only scales the economy so we normalize it to 1. Our value for z comes from Heckman et al. (2006). Using their estimates from a Mincer earnings regression, we calculate that earnings increased by 52.63 percent with 40 years of working experience. This corresponds to $z = 1.054$ in the context of our model. This value is somewhat larger than the estimate by Restuccia and Urrutia (2004). They use the data from the 1990 Panel Study of Income Dynamics (PSID) and obtain a 15 year wage growth rate of 12 percent. For our 5 year periods this corresponds to $z = 1.039$. We take this value as a lower bound in our sensitivity analysis. The first-order conditions in our model reduce to $\frac{c_{t+1}}{c_t} = r\beta$. We choose interest rate $r = 1.05$ to target a consumption growth rate of 5.2 percent calculated by Lee et al. (2006).

The final parameters of Table 1-1 are the policy parameters ζ_e and ζ_l . They represent public expenditures on early childhood and each stage of late childhood as a percentage of GDP. Education at a Glance (2007) shows that public expenditures on primary education, lower secondary and upper secondary education represent 1.7, 0.97 and 0.93 percent of GDP. Upon adjusting for the number of students at each level, per pupil expenditures are the highest for upper secondary students. However, the difference is small and to avoid notational complexity, we assume in our baseline economy that public spending on late childhood education is equally distributed across three periods totally 15 years. Thus we set ζ_l constant across the three stages of late childhood using the average of these values; i.e. $\zeta_l = 0.012$. Heymann et al. (2004) report and compare government expenditures on early childhood education and care by the United States and its European peers. They find that 0.4 percent of U.S. GDP is spent by government on early education. Accordingly, we set $\zeta_e = 0.004$.

The four remaining parameters are chosen so that the model generates observed features of the U.S. economy. Table 1-2 lists these parameters and values as well as the targets.

Table 1-2. Parameters set endogenously

Description		Value	Target	Data	Model
s.e. of random shocks to innate ability	σ_a	0.775	Gini index	0.470	0.465
Contribution of parental human capital	v	0.142	Persistence	0.440	0.442
Weight of early childhood	γ	0.475	$f_1/(f_1 + g_1)$	0.60	0.56
Discount rate on children's human capital	ξ	1.3	$\sum f_k / (3g_l + \sum f_k)$	0.086	0.085

⁸ See Keane and Wolpin (2001) and Hubbard et al. (1994).

The parameter σ_a has a strong influence on the Gini coefficient of our generated data. As such we choose it to match the Gini coefficient reported by the U.S. Census Bureau. Our model generates a value of 0.47 when we set $\sigma_a = 0.775$.

We choose ν to target the intergenerational earnings persistence. Corak (2006) reviews empirical work on earnings persistence and offers a cross country comparison of earnings mobility. The study shows that income persistence varies significantly among developed countries. The United States has a high intergenerational elasticity of earnings of 0.47 (see Grawe (2004)), while this number is much smaller in European countries.⁹ Other researchers find the value of θ to be around 0.4.¹⁰ We choose a midpoint and target earnings persistence of 0.44.

Persistence is generally estimated in the following regression

$$\ln y = \beta_0 + \beta_1 \ln y_{-1} + \varepsilon$$

where y is adult income of children and y_{-1} is parent income. Here β_0 is a constant term and the error term ε includes other factors that influence children's income. The estimate of interest β_1 reflects the intergenerational persistence of earnings. Upon generating data we run the same regression with $y = wh$ and $y_{-1} = wh$.

Persistence in earnings has three sources in our model. First, rich parents are more likely to have high ability. Their children inherit this in part if $\rho > 0$. Second, parents with higher earnings tend to invest more in children's education. The third source of persistence is through a direct socioeconomic effect of parental human capital when $\nu > 0$. Becker and Tomes (1979, 1986) and Solon (2004) include a fixed endowment in human capital accumulation and allow the endowment to be partly inherited by children. Explaining this inheritable endowment, Roemer (2004) points out that beyond innate ability, parents can positively influence children through a family culture that values skills and discipline and through social connections that facilitate access to jobs and opportunities. There is also some evidence that parents with higher human capital affect children's earnings positively through a better family environment and

⁹ Intergenerational persistence of earnings is 0.15 in Denmark, 0.17 in Norway, 0.18 in Finland, and 0.32 in Germany (see Corak, (2006)).

¹⁰ Examples include Solon (1992), Hyson (2003), and Levine and Mazumder (2002).

higher quality parenting.¹¹ We target persistence through this third channel and generate persistence equivalent to 0.442 with $\nu = 0.142$.

We choose ζ and γ jointly to match observed patterns of private participation in funding education. Total private expenditures are influenced strongly by ζ and its allocation is influenced by γ . Barnett and Masse (2003) estimate that about 60 percent of early childhood spending is private so we target $\frac{f_1}{g_1+f_1} = 0.6$. Using data from Education at a Glance (2007), we find that expenditures on primary and secondary education by households represent 8.6 percent of total expenditures from all sources. Therefore we target $\frac{\sum f_k}{3g_l+\sum f_k} = 0.086$, $k \in \{2,3,4\}$.

We are able to get near the target values by setting $\gamma = .475$ and $\zeta = 1.3$. There are not clear counterparts to these measures in the literature but both appear to be reasonable values. At $\gamma = .25$ all stages of education are weighed equally. Evidence on the importance of early childhood spending suggests that γ should exceed this value by a considerable amount. Government spending on early education in the U.S. is relatively low compared to European Countries, so the share of parental spending is relatively high. This may result in an overestimated value of γ . We consider a lower bound of $\gamma = 0.4$ in the sensitivity check. With $\zeta > 0$, parents value children's human capital that transfers to income in the labor market. With $\zeta = 1.3$, parents' altruism is defensible since human capital affects children's labor market outcome through the following 9 working periods. The relatively high level of U.S. government spending in late childhood generates a small share of parental spending. Thus the corresponding discount rate of children's education, ζ , may be underestimated. We consider $\zeta = 1.5$ in the sensitivity check.

4 Current Policy

Education in early childhood differs from late childhood through the human capital production function and through government funding. Absent these distinctions we should expect changes in government spending at the different levels to yield similar results. In this section we show that in fact increased expenditures in early childhood have much larger effects on persistence. We then investigate which distinctions drive this result. Insights from this investigation suggest that increased government funding of early childhood at the lower end of

¹¹ See McLanahan (2004) and Cunha and Heckman (2010).

the income distribution is key to reducing persistence. This motivates an investigation of a progressive allocation of government spending.

To set the stage for this investigation, we first demonstrate the impact of current government policy. To do this, we compare the economy in the case where government does not spend on education ($\zeta_e = \zeta_l = 0$) to the case where it spends at current levels. The results of this experiment are reported in Table 1-3. In the third through seventh rows we have normalized the measure by the value it takes in our baseline model.

Comparing the first and second column, we see that government spending has a substantial effect on persistence. Spending at current levels yields intergenerational persistence of income equal to 0.442. Absent government intervention in education, this would rise by 17 percent to 0.518. Spending also influences the distribution of income within a generation. The second row of Table 1-3 shows that absent government spending on education, the Gini coefficient would be about 4 percent higher.

Table 1-3. Policy experiments

Variable	Baseline	No public spending	% Change
Persistence of earnings	0.442	0.518	17
Gini coefficient	0.465	0.485	4
Output	1	0.86	-14
Utility	1	0.89	-11
Education expenditure	1	0.49	-51
f_1	1	1.13	13
$f_2+f_3+f_4$	1	4.92	394

Decreased persistence and inequality of income are accompanied by a higher level of average output and utility. The third row shows that output would be 13 percent lower without government spending. The loss in utility is equivalent to a decrease of 11 percent in consumption at each period of adulthood. Since dynasties are homogeneous except for random productivity shocks, we can think of this average utility in the steady state as an expected utility. It is instructive to consider why expected utility increases. Since the tax is proportional to income and all agents receive the same government spending, government spending is a transfer from wealthier families to less wealthy families. Prior to the realization of shocks, dynasties do not know if they will be poor in any period, so the current policy has an insurance aspect to it, which serves to increase expected utility. A second effect arises if families do not highly value the human capital of their children. At the extreme, consider the case where $\zeta = 0$. Parents do

not value children's education at all. Therefore human capital and consumption in the steady state are zero absent government involvement. Any increment to government spending would have an infinite effect on utility. With our setting of $\zeta = 1.3$, this effect is still at play, though to a lesser extent. We find that agents from rich families also obtain higher utility with government spending. At any given period, government spending lowers their utility since it is a net transfer from them to the poor. Intertemporally, however, they are better off since the policy in the prior period yielded more spending on their behalf.

Digging a bit deeper, we see that these effects result from changes in education funding. Absent government spending, education spending would be 51 percent lower in total. Family spending in early childhood would be 13 percent higher and family spending in late childhood would be nearly 5 times as high. The larger change for later childhood reflects that government spends more at this level.

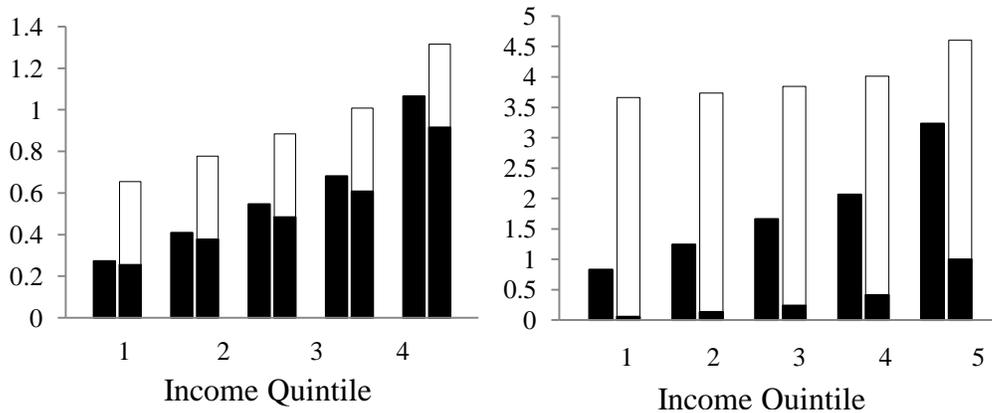


Figure 1-1. Family and government spending on early and late childhood education

Figure 1 helps to explain how government spending reduces persistence through weakening the link between family income and education investment. The first panel shows the weakening of this relationship for early childhood. The first bar in each pairing shows family spending in early childhood for an income quintile of the population and the second bar shows total spending at this level when government spending is positive. It further decomposes this total into family and government spending, where government spending is the same in each quintile. The first pairing considers families in the lowest quintile of the population and subsequent pairings consider other quintiles in ascending order. Absent government spending, families are the only spenders on education. Family spending in the upper quintile is 4 times

higher than that in the lowest quintile. With government spending, total expenditure in the upper quintile is only twice that of the lowest quintile.

The narrowing of these ratios has two causes. The first is simply algebraic. A common amount of government spending has a smaller proportional effect when family spending is high. The other cause is differences in the responsiveness of family spending to government spending. From comparing the solid bars in each pairing, we see that government spending displaces more family spending for the wealthy than for the poor. There are two causes of this difference in crowding out. When government spends on a level of education, families can opt to lower their own spending and allocate more of their income to consumption or to the other level of education. However, for the families that spend little on education absent government spending, private spending can at most be driven to zero by significant government spending, so crowding out cannot be large. The other limiting force on crowding out for poor families arises from decreasing marginal productivity of education spending. When total spending on early childhood is low, its marginal product is high. Thus cutting back on spending yields greater decreases in human capital for the poor than for the wealthy. This makes family spending by the poor less responsive to government spending.

With early childhood spending, none of the crowding out is severe. Even for the highest income families, the decrease in private spending is only 37 percent of the increase in government spending. As a result, the relationship between family income and total expenditure is weakened, but by a relatively modest amount. The second panel shows more dramatic effects along these lines for late childhood education. With no government spending, the most wealthy quintile spends about 4 times as much as the least wealthy. With government spending this ratio is reduced to 1.26. Crowding out of private spending is severe at each quintile. In the highest quintile the decrease in private spending is now 62 percent of the increase in government spending. In the lowest quintile, private spending is driven close to zero. The larger crowding out for late childhood is largely due to the higher level of government spending at this stage.

Figure 2 tells the story from another perspective. In the first panel, the solid curve represents the Lorenz curve for private spending on early childhood education f_1 in the baseline economy, and the dashed curve is the Lorenz curve for f_1 with no government spending. Since the solid curve is further to the right, the introduction of government spending is shown to make private spending more unequal as measured by the Gini coefficient. The greatest shift occurs in the low end of the spending distribution and this is due principally to spending being driven to

almost zero for nearly 20 percent of the population. This shift of family spending distribution translates to more equal total spending. First, all those with almost zero family spending experience the same total spending. Secondly, as shown above, private spending falls more for the wealthier so the gap in spending among the more wealthy and less wealthy falls.

The second panel shows a more dramatic shift of the Lorenz curves for private spending on late childhood education. Comparing the solid and dashed curves, we find government spending in late childhood drives private spending to zero for more than 80 percent of the population. Each of these families has same level of total spending. Given this, the link between total investment in late childhood education and family income is broken for the majority of families. This is the main cause for a lower intergenerational persistence in earnings.

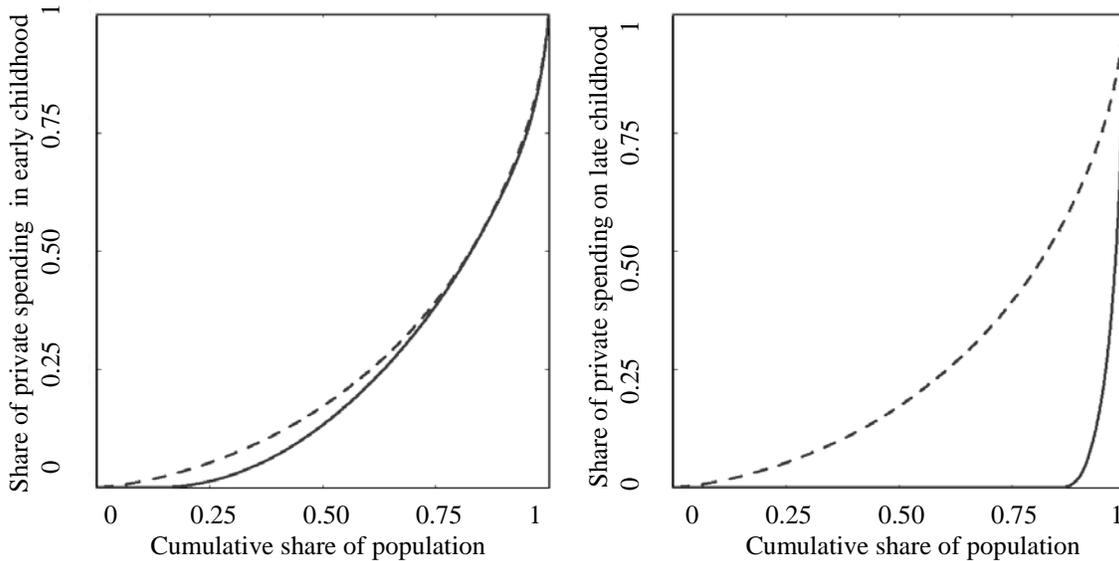


Figure 1-2. The Lorenz curves of private spending in early and late childhood before and after introducing current policy

5 Policy experiments

We establish above that government education funding at observed levels has a number of positive effects. We now consider the impact of changing government funding from its baseline levels. We first consider the effects of equalizing government spending across all stages with no net increase in spending. This requires increasing early childhood spending and decreasing late childhood spending. Since currently $\zeta_e + 3\zeta_l = 0.04$, we set $\zeta_e = \zeta_l = (0.04/4) = 0.01$. We then consider equalizing early and late spending by increasing government spending

in early childhood and keeping government spending in late childhood unchanged. This requires that early childhood funding be increased by 0.8 percent of GDP. In the third experiment, we consider increasing government spending on late childhood while leaving expenditures in early childhood unchanged. To facilitate a comparison, we again increase total spending by 0.8 percent of GDP but now allocate the increased expenditures equally across the three periods of late childhood. We find that an increase of education expenditures on early childhood has larger effects on earnings persistence, income inequality and total output. To further examine this case, we consider progressive government spending on early education in the last experiment. We increase government expenditures on early education by 0.8 percent of GDP again but allocate the funding only to the lowest income quintile.

5.1 Results

Table 1-4 shows the impact of these policy changes. The first column of data reiterates the features of baseline economy. The second shows the effects of redistributing the current level of spending. The third and fourth columns show the effects of increasing early and late childhood spending holding the other constant. The last column shows the effects of subsidizing the lowest income group through an increase in early childhood spending.

Consider first the effects on the persistence of earnings. A reallocation decreases persistence from 0.442 to 0.412. Thus persistence can be considerably altered with no increase in total spending. The third data column shows that by spending on early childhood at the rate in late childhood, persistence drops further to 0.405. If this same increment is spent on late childhood education, the result is not a decrease in persistence, but rather an increase. The final column again considers the same increment to early childhood. However, here the increased expenditures are allocated to the poorest 20 percent of the population. The persistence of earnings is further reduced to 0.398.

Before looking deeper into the results regarding persistence, we first consider some other ways in which the different experiments change the economy. The second row shows that the Gini index is most effectively reduced by an increment to spending on early childhood. With both broad-based and progressive spending, the measure drops from .465 to .458. In contrast, spending more on late childhood has almost no effect. A simple reallocation has a modest effect in lowering this measure of inequality.

Table 1-4. Impact of policy changes

Variable	Current spending		Current spending+0.8% GDP		
	Baseline	Reallocation	Early	Late	Early
Persistence	0.442	0.412	0.405	0.445	0.398
Gini index	0.465	0.460	0.458	0.466	0.458
Ed.	1	0.96	1.14	1.18	1.18
Ed. quality	1	1.11	1.35	1.11	1.23
Output	1	1.02	1.05	1.02	1.03
Utility	1	1.05	1.09	1.02	1.05
f_1	1	0.30	0.25	1.02	1.01
$f_2+f_3+f_4$	1	1.27	1	0.91	0.98

*The increased 0.8 percent of GDP on early childhood is allocated to the first quintile.

The third row considers total education expenditures. When government spending increases, as in the final three columns, the result is an increase in total education spending. However when government spending is held constant but reallocated toward early childhood, family spending falls. With government spending unchanged, total spending falls, as shown in the second column. This is mostly due to the crowding out in early childhood. The next line shows that despite the fall in total spending, a reallocation of expenditures toward early childhood results in an increase in education quality. In the baseline model, early education spending is relatively low and thus has a relatively high marginal return. In contrast, late education is relatively abundant and has a lower marginal return. The policy decreases total spending but allocates it more efficiently. More specifically, education quality, as defined in equation (1.3), increases by 11 percent in the case of reallocation. The relative scarcity of early childhood spending explains also why an increment to government spending in late childhood has a smaller effect on quality than an equal spending increment on early education. This is shown in columns three and four.

Output and utility both rise across all columns. Spending on late childhood has positive but the lowest effects to the third decimal. While broad-based spending in early childhood has the largest effect on output, it is less effective than progressive spending in reducing persistence. This suggests a trade-off between persistence and output.

From row seven we see that family spending in early childhood drops off sharply when expenditures are reallocated and even more when new expenditures are directed to early childhood. In the latter case family spending is only 25 percent as much as the spending level in the baseline economy. However, private spending increases slightly when these same

expenditures are allocated progressively. This is because the extra resources are being directed almost exclusively to families who spend nearly nothing on early childhood. This shuts down the channel for crowding out while the increment to income has a positive effect on spending. The result is increased private spending despite increased public spending. The subsequent row shows that increased government spending in late childhood leads to a reduction of private spending at the same stage. However, these effects are smaller than the analogous changes in the previous row.

5.2 Persistence

We now consider the effect on persistence more deeply. Since a reallocation is both an increase in early spending and a decrease in late spending, we gain insights into the reallocation from first considering the other two cases. The third column shows that increasing ζ_e to current ζ_l levels decreases persistence by 8.4 percent to 0.405. While this is a general equilibrium adjustment, three factors are key to explaining this relatively large effect.

First, an increase in government spending at this stage crowds out private spending mostly for the wealthiest agents. For the poorest agents, private spending is small, so there is little scope for crowding out. Hence, increasing government spending causes a larger increment in total spending for the poor families. This weakens the link between income and education quality and hence reduces the income persistence.

In addition, an increase in government spending also increases the share of the population that receives the same level of total spending. This is the second key to reducing persistence and is clear from the first panel of Figure 3 below. The first panel of Figure 3 shows how the Lorenz curve of family spending on early childhood changes when government spending increases. The curve shifts substantially to the right. This is largely due to an expansion of the region where agents spend nearly zero. Stated differently, the policy expands significantly the share of the population for whom education spending in early childhood is not dependent upon private income but rather upon government finance. This further weakens the relationship between expenditures and income. The more equal education spending is shown in the third panel of Figure 3. The third panel of Figure 3 shows the shift of the Lorenz curve of total spending on early education. The leftward movement of the curve implies that education spending becomes almost unvarying across the population.

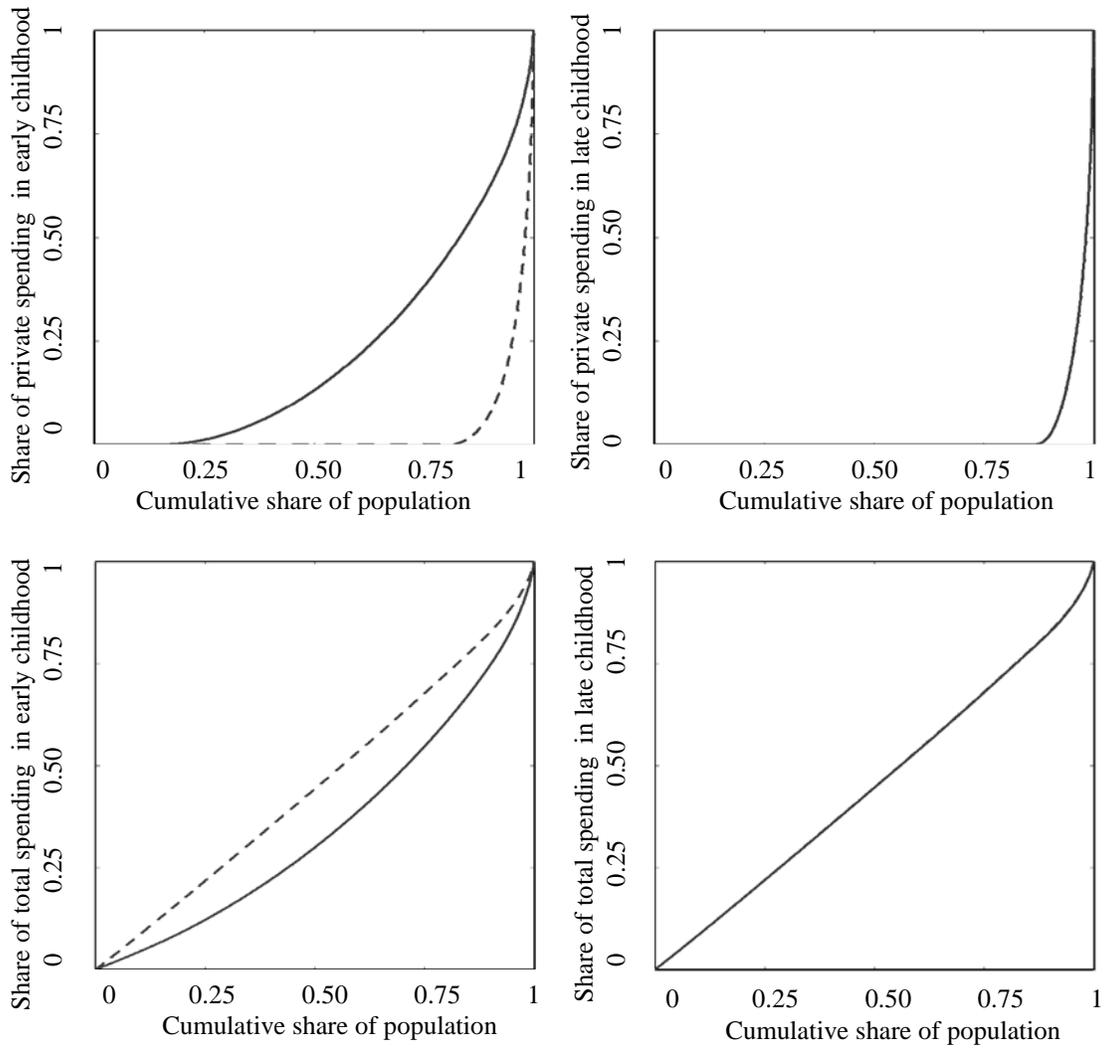


Figure 1-3. The Lorenz curves of private spending and total spending on early and late childhood education before and after an increase in government spending on early childhood education

The third key factor works counter to this. Education expenditure directly increases education quality and higher education quality is transformed to higher human capital. The productivity of education quality, though, varies by children's ability and parental human capital. Higher income families tend to have higher ability children with higher parental human capital, so a unit increment in education quality is more productive for the wealthier families. As a result, an increase in government spending can strengthen the link between income across generations. However, in considering early childhood spending, this effect is relatively small and thus public expenditure at this level is still effective in reducing persistence.

The fourth column in Table 1-4 shows that increased government spending in late childhood can increase persistence. The same three factors are key to understanding this. Again, an increase in government spending at this stage crowds out private spending mostly for the wealthiest agents. However, for around 80 percent of the population, private spending is near zero at this stage. When government spends more, there is no change in the distribution of education spending across most of the population. Thus, the effect of the first factor is diminished. Also, since most agents are already spending near zero, there is only a slight increase in the number of agents supported by the spending floor. This is shown in the second panel of Figure 4 above. An increase in government expenditures on late childhood shifts the Lorenz curve of private spending to the right, causing more agents to spend near zero. However, in comparison with the first panel in Figure 3, the change is fairly small. Thus the second effect is also weakened. The third (negative) effect is larger in the case of late childhood spending. A common increase in spending on late childhood education is more productive for the children from rich families, not only due to their higher value of innate ability and higher parental human capital, but also due to their higher education quality in early childhood. Extra spending on later education only widens the income gap that was developed through unequal early education quality.

The second and third experiments explain the effect on persistence of the first experiment. The increase in spending in early childhood decreases persistence significantly. Following from the results of increasing spending in late childhood, a decrease in spending would reduce persistence slightly. All told, then, the reallocation leads to lower persistence. Equalizing expenditure through a pure increase in early childhood spending has an even larger effect.

In the fourth experiment, we allocate the 0.8 percent of GDP on early education to agents in the first quintile of the income distribution. It generates a lower persistence than broad-based spending. This is due to the larger effect of the first factor. With more government spending on each family in the poorest group, crowding out is smaller and the floor on early education spending is raised to a higher level compared to the case of broad-based spending. With less agents affected by the extra spending, the effect of the second factor is smaller. However, overall, the stronger effect of the first factor dominates and it induces a larger change in persistence.

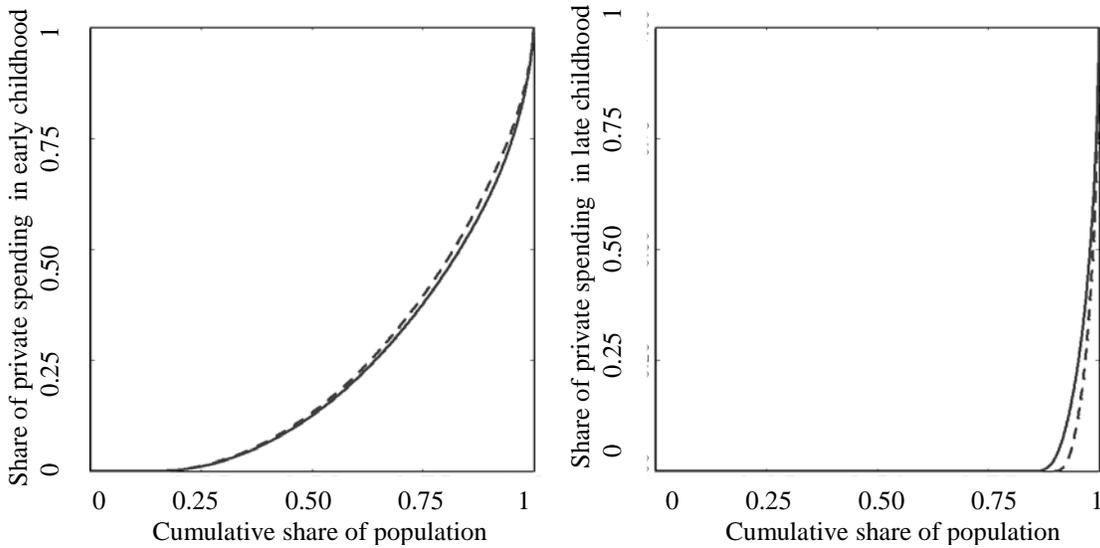


Figure 1-4. The Lorenz curves of private spending on early and late childhood education before and after an increase in government spending on late childhood education

5.3 Driving factors

To this point, we have established that government spending on early childhood is more effective in reducing intergenerational persistence than government spending on late childhood. Our explanation has focused largely on differences in the level of government education funding across these two levels. However, we have modeled early childhood education as having two distinct features. First, we have made it relatively more important in generating human capital than later education. This is due to our setting $\gamma = 0.475$. If all expenditures were of equal importance we would have $\gamma = 0.25$. Secondly, we have made early and later education complements in production by setting $\Phi = -2$. If early education shared unit elasticity of substitution with education at other levels we would have $\Phi = 0$.

In this subsection, we evaluate whether these modeling distinctions are important for our results. To do this, we investigate the extent to which spending on early childhood influences persistence when we remove these distinctions. Table 1-5 reports the results.

The first row is our baseline case and is repeated for ease of comparison. In the second row we maintain the relative importance of early childhood but set $\Phi = 0$. In the third row, we maintain $\Phi = -2$ but set $\gamma = 0.25$. In the final row, we remove both distinctions. The first data column gives the level of persistence for the new parameterization with baseline funding. The next column calculates the persistence when early childhood expenditures are increased and the final column gives the percentage change.

Table 1-5. Impact on persistence with $\Phi=0, \gamma=0.25$

Setting	Baseline economy	Early childhood+0.8%	% Change
$\Phi = -2, \gamma = 0.475$	0.442	0.405	8.4
$\Phi = 0, \gamma = 0.475$	0.433	0.403	7.0
$\Phi = -2, \gamma = 0.25$	0.427	0.402	7.0
$\Phi = 0, \gamma = 0.25$	0.413	0.403	2.4

The key message is that each feature of our model contributes to making early childhood spending an effective means of reducing persistence. With both features in place, increased funding reduces persistence by 8.4 percent. Removing any one feature changes the reduction to about 7 percent. With both features removed, increased funding would reduce persistence by only 2.4 percent.

We conclude that each feature is important and the combination of these features is especially important in establishing the effectiveness of early childhood education. However, when we remove these features, spending on early childhood is still more effective than spending on late childhood. We attribute this to differences in the level of education funding at early and late stages.

5.4 Sensitivity analysis

In this section we calibrate the model and comment on some alternative parameterization. In this section we evaluate the extent to which these alternative parameters influence our results. The first column of Table 1-6 below gives the alternative values of various parameters. The following values in parenthesis are the baseline parameter values. The next column shows persistence with the alternative parameter at baseline spending. The subsequent

columns show how this value changes with an increment to early and late childhood education spending.

The first row repeats the baseline case. The key result from the remainder of the table is that these alternative parameter choices have only modest effects on both persistence and the effectiveness of policy in reducing persistence. The second row shows that higher persistence in ability (ρ) leads to higher persistence in income. A consequence of this unsurprising result is that policy is less effective. This is because a greater share of persistence is attributable to immutable inherited ability. When the return on education investment (μ) is larger, so is persistence and policy effectiveness. The substitutability of early and late childhood (Φ), the intertemporal preference parameter (σ), wage growth (z), the share of early education (γ) and the discount rate of children's human capital all have modest effects on policy effectiveness.

Table 1-6. Sensitivity check

Parameters	Persistence	Early childhood	Late childhood
Baseline	0.442	-8.4	+0.7
$\rho=0.32$ (0.25)	0.488	-7.2	+0.4
$\mu=0.24$ (0.159)	0.482	-9.1	-0.2
$\Phi=-1.373$ (-1.8)	0.441	-8.1	+0.3
$\sigma=-2$ (0)	0.500	-7.8	-0.1
$z=1.039$ (1.054)	0.443	-8.3	+0.5
$\gamma=0.4$ (0.475)	0.432	-8.2	+0.1
$\xi=1.5$ (1.3)	0.436	-8.2	+0.0
Borrowing	0.442	-8.4	+0.5

We proceed and consider borrowing constraints in early adulthood. This requires bond holdings in the first period of adulthood to be non-negative, i.e. $b_5 \geq 0$ in equation (1.7). We find that with the parameter setting in our baseline model, borrowing constraint is binding, i.e. $b_5 < 0$ for some agents, not all. As shown in the last row of Table 1-6, having borrowing constraints does not affect persistence or the effectiveness of early spending.

A more crucial parameter in our model is the substitutability of government and private spending. Crowding out of private spending by government spending is a key to our results. When these inputs are less substitutable, we should expect our results to be less pronounced. In the baseline economy, we assume that government and private expenditure on early childhood education are close to perfect substitutes with $\eta = 0.95$. Table 1-7 below shows that this setting is not required for our results to hold. However, considerable substitutability is needed.

Table 1-7. Changes in persistence in more complementary cases

Substitutability	Persistence	Early childhood	Late childhood
$\eta=0.95$	0.442	-8.4	+0.5
$\eta=0.90$	0.442	-7.8	+0.4
$\eta=0.5$	0.448	-2.8	-0.3
$\eta=0$	0.454	0	0
$\eta=-1$	0.456	+3.7	+0.2

The first row of Table 1-7 reiterates the baseline case, showing the generated persistence in the first column and the percentage change in persistence from increasing early and late spending in the next two columns. In each case the increment is again 0.8 percent of GDP. From this and the subsequent row we see that moving away modestly from $\eta=0.95$ has only a small effect on our results. Persistence does not change and early spending is a bit less effective in reducing persistence. As we move further to less substitutability, the changes become more pronounced. At $\eta = 0.5$ persistence rises to 0.448, and increasing early spending decreases persistence by only 2.8 percent. At $\eta = 0$ the effect on persistence is fully eliminated and in the complementary case where $\eta = -1$, spending on early childhood increases persistence.

This pattern emerges because of the way in which public spending affects private spending. When public and private spending are close to perfect substitutes, more government spending decreases the marginal product of private spending. Thus more public spending yields less private spending. We have established that this crowding out is more severe among the rich. When public and private spending are sufficiently complementary, more government spending increases the marginal product of private spending. This increases private spending and again the effect is more pronounced among the wealthy. With spending higher for the wealthy, income persistence is reinforced.

6 Conclusion

In the United States, there is a strong correlation between the income of parents and their offspring. With intergenerational persistence of income equal to about .44, the mobility of economic status across generations is lower in the U.S. than in most other OECD countries.¹²

¹² See Corak (2006).

Education is one source of persistence. Wealthier families provide a better education to their offspring. This results in more human capital and a higher wage. Public education can lower persistence by weakening the link between parental income and the education quality received by their offspring. However, public expenditure is the dominant source of education funding only after a child enters primary school. Through the formative years of early childhood, education quality can differ considerably and depend strongly on parental income. Relatively equal spending through the primary and secondary years serves in part to amplify differences developed prior to the first day of school.

This paper considers changes to public education funding when early childhood education plays a unique role in the development of human capital and is funded primarily by families. We embed these features into a life cycle model where human capital is accumulated throughout early childhood and three periods of late childhood. Building on studies of early childhood, we capture this unique role by making early education highly complementary with later education and giving it a relatively high weight in the human capital production function. These features aggravate the negative lifetime consequences of poor education quality in early childhood.

We calibrate the model to replicate features of the U.S. economy and run experiments to examine the effects of education policy on income persistence. We find that increasing government spending on early education is most effective in reducing persistence. Increasing public spending on later education may even increase persistence. The unique role of early education and the relative paucity of public funding at this level combine to explain the difference in policy effectiveness. An increment to early public childhood spending largely offsets private spending for wealthier families so that total spending changes little. For lower income families, in contrast, this same spending increment results in higher total spending. The resulting tighter spending distribution weakens the income/expenditure link and lowers persistence. With early childhood playing a central role in human capital production, this effect is amplified. In contrast, most spending on later childhood education is already provided publicly. Except for the very high end of the income distribution, there is little scope for further equalization of expenditures. As such, there is little scope for further reducing persistence.

Since government spending is most effective in raising education investment for agents at the low end of the income distribution, we consider progressive spending on early childhood. Our results imply a larger change in persistence from progressive spending than from broad-

based spending. However, the lower persistence is accompanied by a smaller increase in output. This is because the less wealthy, on average, have lower productivity in generating human capital. Targeting the least wealthy directs resources from agents with a higher return to education quality to agents with a lower return.

Since college education has been considered by both Restuccia and Urrutia (2004) and Holter (2010), we have chosen to focus on the dichotomy between early and late childhood. In this paper we have considered only physical investment at these different stages. However, parents make considerable time investments in their children. This investment contributes to the development of human capital, especially in early childhood. In future work, we will consider the effects of government spending and tax policy when parents invest both income and time. Since government spending cannot substitute for parental time, this could add an interesting additional layer to policy analysis. While crowding out private spending, policy may also alter the incentives of parents to work less and spend more time with children.

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Chapter 2 - Parental time investment in early education and the labor supply of young parents

1 Introduction

There is compelling evidence from economics and sociology that early education plays a unique role in human capital accumulation.¹³ Quality learning in early years has long-term effects on educational attainment and labor market outcomes. Missed opportunities to invest in young children can not be easily compensated by later investment. Economists capture this feature by modeling human capital formation in a hierarchical fashion, where early and later education interact in producing human capital.¹⁴ Most studies consider expenditures by parents and the government as the only education input.¹⁵ However, many parents invest large amounts of time teaching their young children words, numbers and manners by reading and talking to them. Such investments are an indirect cost of early education that is not negligible. Haveman and Wolfe (1995) show that the monetary cost of mothers' day care time accounts for 18 percent of total parental investment in early education.

This parental time investment is valuable to early development. Numerous developmental studies show that parenting in early childhood is crucial to the growth of children's cognitive and social skills.¹⁶ Landry et al. (2003) argue that parent-child interactions help children to understand that others respond to their interests and needs. With that understanding, children are more likely to explore and learn in an active way. An active learning experience in early years provides a good foundation for later learning. Carneiro et al. (2007) find that noncognitive skills, such as perseverance, persistence, self-esteem and discipline are

¹³ See Shonkoff and Phillips (2000), Carneiro and Heckman (2003), Currie and Blau (2006), Knudsen et al. (2006) and Schweinhart et al. (2005).

¹⁴ See Cunha et al (2005), Cunha and Heckman (2007), Abington and Blankenau (2011), Caucutt and Lochner (2011) and Blankenau and Youderian (2012).

¹⁵ Such expenditure is used for day care and physical tools that help children to learn, for example, toys, games, books and software.

¹⁶ See Landry et al. (2001), Landry et al. (1997), Ainsworth et al. (1978) and Bornstein and Tamis-LeMonda (1989).

important to academic achievements and economic success.¹⁷ Empirical work shows that non-parental care is associated with children's vocabulary and behavior problems (see Belsky et al. (2007)). This result is consistent with the positive influences parents may have on children's noncognitive skills. Experience with parents builds a foundation for children to foster trust and a sense of security as well as to teach them effective ways to communicate and draw attention (see Grusec and Goodnow (1994)).

Given the significant size and effect of parental time, I include time investment as an education input in this paper. As in Caucutt and Lochner (2011), human capital development is modeled as a multi-stage process. Children receive private and public education expenditures in both early and late childhood, while they receive time investment from parents only in early childhood. Parents not only choose income allocation between education investment and consumption, they also decide how much time to invest in young children. The cost of time investment has two parts. First, parents forgo income by working less. Second, parents accumulate less work experience which reduces future wages. By including the parental time allocation choice, I am able to explore how early education policy affects human capital and labor supply.

In my model, only parents with children in early childhood choose time allocation, while parents with older children provide one unit of inelastic labor supply. I choose this specification for two reasons. First, empirical evidence demonstrates that the presence of preschool children has a substantially larger impact on parental work time than the presence of older children.¹⁸ Second, parental involvement in early years is found to be most influential to children's learning.

The interaction between time investment and education expenditure determines variance in labor supply across individuals. Parental time can be substituted by education spending to some degree. For example, working parents can send children to day care or hire an in-home nanny. In addition, parents can invest in devices such as tablet computers that can read a book or play interactive games with children. Some of the traditional home teaching by parents can be replaced by physical tools. Due to this substitutability, the model shows that higher wage

¹⁷ Related studies include Heckman and Rubinstein (2001), Mobius and Rosenblat (2005), Waddell (2006) and Borghans et al. (2006).

¹⁸ See Kalenkoski et al. (2005) and Lefebvre and Merrigan (2008).

earners work more and spend more on education to compensate for lower time investment. This negative relationship between wages and day care time is also found in empirical studies.¹⁹

Compared to European countries, the US government is less involved in children's early education. This is shown through less public spending on early education. European governments pay 70 to 90 percent of the total cost of early education, while US governments pay 40 percent of the total cost (see Witte and Trowbridge (2005)). The low public funding in the U.S. is also shown by the coverage of early education programs. For example, the Head Start program was designed to prepare children from disadvantaged families to read and succeed in school. However, less than 40 percent of eligible families receive services from the Head Start due to limited government budget. Other than generous spending, European countries also provide paid parental leave to encourage parents to devote more time to their newborn children. Mothers in Europe are entitled to 14 to 20 weeks of maternity leave with 70 to 100 percent pay (see Rossin-Slater et al. (2011)). In contrast, the US does not provide nationwide paid leave. Most states only offer unpaid maternity leave of 12 weeks.²⁰

This paper investigates the impact of adopting European-style early education policies would have on human capital and labor supply across income groups. First, I consider an increment in government expenditures on early education. In the United States, government spending on early education accounts for 0.04 percent of GDP. Adjusted for the lengths of early and late childhood, the corresponding government spending in late childhood is 0.14 percent of GDP. I run counterfactual experiments by allowing the government to fund early education at the same level as late education. This leads to an increment in government expenditures equivalent to 0.1 percent of GDP. In the first two experiments, the government allocates increased public resources to respectively direct government spending and subsidizing private spending. Second, I introduce paid parental leave to the economy. The percentage of wages replaced during parental leave is chosen at a level such that the policy is funded by 0.1 percent of GDP. Therefore, my analysis provides comparable macroeconomic effects across policy experiments. I show that increasing direct government spending is least effective at increasing human capital. This is because increased government spending crowds out private spending, especially for wealthy families. Consequently, there is no sizable increase in total spending as

¹⁹ See Friedberg and Webb (2005) and Kooreman and Kapteyn (1987).

²⁰ California, New Jersey and Washington have introduced partially paid maternity leave of 5 to 6 weeks.

intended and thus the policy does not produce higher human capital. Subsidizing private spending and adopting paid parental leave have significantly larger effects on human capital. This is achieved because these two policies substantially increase education expenditures and parental time investment, respectively.

Labor supply, on average, increases modestly in response to both more direct public spending and the private spending subsidy. The key to this result is the substitutability between physical and time investment. Both policies bring up total education spending. Higher education spending crowds out parental time which leads to higher labor supply. This prediction is confirmed by empirical work from Canada. In 1997, the Quebec government initiated a policy to subsidize day care by 5 dollars per day per child. Researchers show that this policy has significantly positive effects on both labor force participation and work weeks (see Lefebvre and Merrigan (2008) and Baker et al. (2008)). In contrast, paid parental leave as intended decreases labor supply substantially.²¹

Moreover, I show that low wage earners are most responsive to paid leave due to a lower opportunity cost of missing work. Rossin-Slater et al. (2011) examine the adoption of paid maternity leave in California and show the same pattern. They find that the California program has doubled overall maternity leave use with the largest increase among less advantaged women.²²

My work is related to the study by Nordblom (2003). She introduces "within-the-family education" (WFE) as direct parental investment. Such investment includes both parental time and effort as well as monetary investments. Examples of WFE would include teaching children manners, disciplining them, helping them on homework or the purchase of computers and software. Nordblom argues that parental investment is complementary to public spending and more public spending helps children with better educated parents more. As a result, further investment in public schooling may widen the educational inequality. I include parental investment in a different way by separating time investment and physical investment. As in Nordblom (2003), in my model parenting is more effective when parents have higher human capital. However, parenting is also more costly to wealthy parents. I capture this feature and

²¹ Empirical evidence of this result includes Baker and Milligan (2008), Burgess et al. (2008) Dustmann and Schonberg (2008) and Carneiro et al. (2010).

²² Carneiro et al. (2010) show similar results using data from Norway.

treat parental time as a substitute with parental spending. While Nordblom does not distinguish education in different periods, I treat human capital formation as a multi-stage process and focus on education in the early stage.

This study is related to Abington and Blankenau (2011) who also model human capital in a hierarchical fashion. They explore the effects of different government funding structures on human capital, given the crucial role of early education. My work departs from theirs by including parental time allocation choice. Moreover, I evaluate paid parental leave policy and provide a comparative analysis.

2 Model

I develop an overlapping generations model where agents live five periods. In each period, new agents are born into early childhood. In the subsequent period, agents are in late childhood. Throughout the first two periods of life, agents do not make economic decisions. They receive human capital investments from parents and the government. After accumulating human capital, agents enter early adulthood in the third period. In this period, they also have children in early childhood. As parents and workers, agents allocate time and income as specified below. The fourth period of life is spent in late adulthood where agents face a different set of allocation decisions and their offspring are in late childhood. Agents become empty nesters in the last period where their children enter adulthood as workers and parents.

The economy is formed by $2N$ dynasties. Given any time period t , N dynasties have children in early childhood, agents in early parenthood and agents in the last period of life. While the other N dynasties have children in late childhood and parents in late parenthood. Income tax is collected from $3N$ working agents to fund education for N children in early childhood and N children in late childhood.

2.1 Formation of human capital

Agents entering childhood have heterogeneous innate ability. I assume that through nature and nurture, children perfectly inherit innate ability from parents. The innate ability parameter a_i measures the productivity of children in dynasty i of transforming physical investment, such as day care and schooling, into human capital. With different a_i and private education investments, agents enter adulthood with heterogeneous human capital.

For simplicity I remove subscript i indicative of dynasty, taking it as given that the innate ability parameter a and all endogenous variables pertain to a dynasty. In addition, I avoid subscript t indicative of the time period. Rather, variables with notation $\hat{\cdot}$ refer to the generations in childhood and the ones without $\hat{\cdot}$ refer to the parents.

An agent's human capital, \hat{h} , is accumulated as follows

$$\hat{h} = ((ae)^\theta + (ht)^\theta)^{\frac{\nu}{\theta}}. \quad (2.1)$$

Here human capital is a function of a physical investment component, ae , and a time investment component, ht . The first component has two parts, innate ability, a , and a measure of education expenditures, e . As stated earlier, a is a productivity measure for transferring education expenditures into human capital and e is a measure of the education expenditures devoted by both parents and the government. The second component also has two parts, parental human capital, h , and parental time investments in early education, t . Active parenting in early years helps to develop children's learning ability, so time spent with young children, t , contributes to their human capital, h . Lefebvre and Merrigan (1998) use data from Canada and show the positive effect of parental human capital on children's cognitive development. They argue that this may be the result of quality parent-children interaction, such as frequency of reading and efficient parenting. This argument is supported by Behrman et al. (1999) based on data from rural India. Their estimates suggest that maternal schooling increases the productivity of home teaching, and thus increases the schooling of next generation. Accordingly, I allow parents with more human capital h to be more effective in nurturing the offspring. The parameter $\theta \leq 1$ governs the substitutability between physical and time investments. When the parameter $\theta = 0$, the production function is reduced to a Cobb-Douglas function and the elasticity of substitution equals 1. The parameter $\nu \in (0, 1)$ governs the curvature of the function. With $\nu < 1$, the human capital production function exhibits diminishing returns.

The measure of education expenditures is formed in a hierarchical fashion. As in Blankenau and Youderian (2011) and Cunha and Heckman (2008), I define

$$e = (\gamma i_e^\Phi + (1 - \gamma) i_l^\Phi)^{\frac{1}{\Phi}} \quad (2.2)$$

where i_e and i_l are education investment in early and late childhood. The measure of education expenditures, e , is formed by i_e and i_l in a c.e.s production function, where $\gamma \in (0, 1)$ gauges the weight of investment on early education and $\Phi \leq 1$ gauges the substitutability between

early and late investment. When $\Phi \in (0, 1]$, the elasticity of substitution exceeds 1. I refer to this as early and late investments being substitutes. When $\Phi < 0$, the elasticity of substitution is smaller than 1. I refer to this as early and late investments being complements.

Education investment in each period of childhood depends on parental and government expenditures on education. I specify

$$i_e = (f_e^\eta + g_e^\eta)^{\frac{1}{\eta}}, i_l = (f_l^\eta + g_l^\eta)^{\frac{1}{\eta}} \quad (2.3)$$

where f_e and g_e are private and public spending on early childhood education, and f_l and g_l are private and public spending on late childhood education. The parameter $\eta \leq 1$ governs the degree of substitutability between private and public spending. This specification is similar to that in Restuccai and Urrutia (2004) and Arcalean and Schiopu (2008).

2.2 Labor market outcome

Worker productivity in this economy depends on accumulated human capital and labor market work experience. More specifically, the wage is determined as follows

$$w_j = h e^{\alpha x_j}, j \in (1,2,3). \quad (2.4)$$

I use j to indicate the number of periods an agent has been an adult. So w_1, w_2 , and w_3 are respectively the wages agents face when they are young parents, old parents and empty nesters. Wages depend on an agent's human capital, h , and accumulated work experience, x_j . The experience parameter $\alpha \geq 0$ corresponds to the coefficient before the work experience variable in the Mincer wage equation. A higher α indicates that work experience is more valuable for labor productivity. As an agent enters early parenthood, work experience is $x_1 = 0$, and thus $w_1 = h$. During early parenthood, the agent is endowed with one unit of time and chooses time allocation between work and children. Given the time spent with young children t , the agent's work experience accumulated during early adulthood is $x_2 = 1 - t$. As a result, $w_2 = h e^{\alpha(1-t)}$. The labor supply is inelastic when the agent is in late adulthood, hence the accumulated work experience is $x_3 = 2 - t$ when one becomes an empty nester and keeps working. So the wage in the last period of life is $w_3 = h e^{\alpha(2-t)}$.

2.3 The agent's problem

Children are passive economic agents. Parents are endowed with one unit of time in each period of adulthood. They choose time allocation between work and time spent with

children in early parenthood, consumption through adulthood and education spending in their offspring's early and late childhood in order to maximize

$$\sum_{j=1}^3 \beta^{j-1} \frac{c_j^\sigma}{\sigma} + \xi \frac{\hat{h}^\sigma}{\sigma}. \quad (2.5)$$

Here c_j denotes consumption in the j^{th} period of an agent's adulthood and $\beta < 1$ discounts future consumption. Utility also depends on the human capital of an agent's offspring, \hat{h} .²³ The term ξ scales the importance of child's human capital and σ gauges marginal utility of c_j and \hat{h} . The utility function exhibits log preferences when $\sigma = 0$. An agent's choice of time allocation and education spending on children affects \hat{h} through equations (2.1)-(2.3). Time investments in children's early education, t , incurs the cost of wages forgone in the current period. In addition, a lower labor supply in early adulthood results in lower wages in the subsequent two periods through less work experience. Defining r to be the interest rate and τ to be the income tax rate, the agent's budget constraints can be written as

$$\sum_{j=1}^3 \frac{c_j}{r^{j-1}} + f_e + \frac{f_l}{r} = \left(w_1(1-t) + \frac{w_2}{r} + \frac{w_3}{r^2} \right) (1-\tau), \quad (2.6)$$

$$c_1, c_2, c_3 \geq 0,$$

$$f_e, f_l \geq 0,$$

$$t \in [0, 1].$$

2.4 Government

In any given period, the aggregate income of agents in early parenthood across N dynasties is $\sum w_1(1-t)$. The aggregate income of agents in following two stages of life are $\sum w_2$ and $\sum w_3$, respectively. Using the simplified notation again, I specify total output Y as

$$Y = \sum (w^1(1-t) + w^2 + w^3). \quad (2.7)$$

Government taxes labor income with a common tax rate τ to fund early and late childhood education. In the baseline model, government supports education directly through government expenditure. Defining G as total government spending, the budget relationship is

$$\sum g_e + \sum g_l = \tau Y = G. \quad (2.8)$$

This study focuses on government policies regarding early childhood education, thus I define

²³ See Becker and Tomes (1979, 1986) and Behrman et al. (1995) for similar specification of utility function.

$$\zeta_e = \frac{\sum g_e}{Y} \tag{2.9}$$

$$\zeta_l = \frac{\sum g_l}{Y}$$

where $\zeta_e, \zeta_l \in [0, 1]$ are respectively the share of output allocated to government spending on early and late childhood education.

2.5 Equilibrium

The analysis focuses on steady state equilibria of the model. At such a steady state, individual decision rules solve the utility maximization problem and the government balances the budget. Decision rules and economic outcomes within each dynasty are time-invariant after convergence. A detailed definition of equilibrium with complete subscripts is presented in the appendix 1.

3 Calibration

The model is calibrated to the U.S. economy. Many parameters can be obtained without solving the model. I calibrate these parameters to their empirical counterparts. The remaining parameters are estimated to match several aggregate statistics. Table 2-1 summarizes the values of directly calibrated parameters.

Innate ability is exogenous and time-invariant for each dynasty. In the initial period, I assign a random draw from a log normal distribution to each dynasty as the ability parameter, a . The first two parameters in Table 2-1 capture the main features of the distribution. I normalize the mean and standard deviation to 1, $a = \sigma_a = 1$.

The next parameter, θ , governs the degree of substitutability between physical and time investment. Herbst (2010) uses CPS and SIPP data between 1990 and 2004 to study the wage effects on the time allocation of single mothers. He finds that mothers with higher wages work more after controlling for the price of day care. I argue that this finding is supportive of the relative substitutability between education expenses and time investment. When parental time is substitutable, higher wage earners work more and with more income, they spend more to compensate for less time investment. However, I exclude the extreme case of perfect substitutability, since attention from parents is necessary for children to develop emotional and social skills. Therefore, I choose $\theta = 0.5$ for the benchmark economy.

Table 2-1. Parameters set exogenously

Description	Parameters	Values
Mean of innate ability	a	1
Standard deviation of innate ability	σ_a	1
Substitutability parameter of physical and time investment	θ	0.500
Substitutability parameter of private and public expenditures	η	0.950
Weight on early education	γ	0.750
Discount rate	β	0.815
Intertemporal preference parameter	σ	0
Interest rate	r	1.056
Experience parameter	α	0.054
% of GDP on early childhood education by government	ζ_e	0.004
% of GDP on late childhood education by government	ζ_l	0.014

The parameter η measures the substitutability between private and public spending. Since the source of education expenditure does not affect its productivity, I treat private and public spending as highly substitutable by setting $\eta = 0.95$. This assumption is common in the literature.²⁴ I consider a less substitutable case in the sensitivity check by setting $\eta = 0.5$. Studies on early childhood intervention suggest that the skills that affect schooling and labor market outcomes are shaped in early years.²⁵ Their finding reflects the relative importance of investing early in children's education. Cunha et al. (2005) summarize the main findings in the human capital literature and reach a similar conclusion. They find the return to investments in early childhood is high while the return to investments in late childhood and beyond is low. Blankenau and Youderian (2012) calibrate a similar model of human capital and their estimate of the weight of early education is 0.475. In their paper, late childhood lasts three times long as early childhood. Adjusted for the relative lengths of early and late childhood in this paper, the corresponding value γ is 0.75. Accordingly, I set $\gamma = 0.75$ in the benchmark economy and consider a lower bound of γ as 0.65 in the sensitivity check.

I take the commonly used annual discount rate of 0.96 and compute the corresponding discount rate $\beta = 0.815$. Empirical estimates for the other preference parameter, σ , are available

²⁴ See Becker and Tomes (1986), Glomm and Kaganovich (2003) and Abington and Blankenau (2011).

²⁵ See Shonkoff and Phillips(2000) and Carneiro and Heckman (2003).

and they range from -2 to -0.5 in the literature.²⁶ For simplicity, I choose $\sigma = 0$ which is consistent with log preferences.

Next I consider the variables that capture the economic environment. Holter (2011) calculates the average 3-month T-bill rates minus inflation during period 1947-2008. I take it as the annual interest rate and set $r = 1.056$ accordingly. The experience parameter α can be interpreted as the wage increase, on a percentage basis, due to additional working experience of five years. I adopt Heckman et al. (2006)'s estimates on the Mincer earnings regression and derive the value of α in the context of this model.

The policy parameters are ζ_e and ζ_l , which represent the share of GDP allocated by the government to early and late education, respectively. Heymann et al. (2004) show that the United States spends 0.4 percent of total output in childhood. Based on their finding, I set $\zeta_e=0.004$. Education at a Glance (2007) reports that total public spending on primary, lower secondary and upper secondary education accounts for 3.6 percent of GDP. As a modeling choice in this study, late childhood is symmetric to early childhood in length. Given that late childhood takes 5 years, I compute the corresponding GDP share allocated to that period and choose $\zeta_l =0.014$.

I calibrate the remaining parameters simultaneously so that the model generates an economy similar to the U.S. economy, regarding income inequality and parental participation in education investment. Table 2-2 displays the parameters and their targets. As shown in the fourth column of Table 2-2, I target the Gini index and the share of private spending in early and late education. The model is able to generate data that precisely matches these statistical targets.

Table 2-2. Parameters set endogenously

Description		Value	Target	Data
Curvature parameter	ν	0.606	Gini index	0.470
Discount rate on children's human capital	ξ	0.177	$f_e/(f_e + g_e)$	0.600
Substitutability between early and late edu.	Φ	-0.201	$f_l/(f_l + g_l)$	0.086

Notice that the substitutability parameter Φ is negative, indicating that early investment is complementary to later investment. Caucutt and Lochner (2011) and Cunha et al. (2010) use different estimation and calibration strategies to examine the complementarity/substitutability

²⁶ See Keane and Wolpin (2001), Beaudry and Wincoopn (1996) and Hubbard et al. (1994).

between investments across periods. They agree that investments in different periods of childhood are complements. Heckman (2006) explains this complementarity using evidence from child development studies. He argues that acquired skills and abilities in early years motivate children to learn more and make later learning more efficient.²⁷ Blankenau and Youderian (2012) calibrate this parameter in a similar human capital accumulation model and set $\Phi = -1.8$. I choose a more complementary case in the sensitivity check by setting $\Phi = -0.8$.

4 Policy experiments

4.1 Current policy

In this section, I examine the economic outcomes in the benchmark economy where government spending on early and late education is 0.4 and 1.4 percent of GDP, respectively. For comparison purposes, I divide the population by income tercile (low, medium, high) and compute mean innate ability, a , accumulated human capital, h , life-time income, $w_1(1-t) + \frac{w_2}{r} + \frac{w_3}{r^2}$, labor supply in the first working period, $l-t$, the share of total income spent on early education, $\frac{f_e}{w_1(1-t) + \frac{w_2}{r} + \frac{w_3}{r^2}}$ and the share of total income spent on on later education, $\frac{f_l}{w_1(1-t) + \frac{w_2}{r} + \frac{w_3}{r^2}}$ for each income group. Table 2-3 presents the results. Innate ability, human capital and income are normalized by the average value in the lowest-income group.

Table 2-3. The benchmark economy by earnings tercile

	Low	Medium	High
Innate ability	1	2.772	9.434
Private spending in early childhood	0.005	0.014	0.022
Private spending in late childhood	0.000	0.000	0.004
Labor supply in early adulthood	0.945	0.965	0.982
Human capital	1	1.973	6.485
Income	1	1.981	6.574

Innate ability is exogenously determined, more specifically, it is randomly drawn from a log normal distribution. In this model, variance in ability is the root of variance in parental decisions and economic outcomes across agents. The second and third rows of Table 2-3 show that private spending on education increases progressively with family income. The lowest-

²⁷ Related studies include Cameron (2004) and Knudsen et al. (2006).

income group spends almost nothing on children's education. While the highest-income group, on average, allocates 2.6 percent of total income to education. Parents spend more on early education than on later education. This is because government spending on later education is higher, resulting in a lower marginal return to private spending in this stage.

The labor supply of wealthier agents is higher than that of less wealthy agents. On average, a parent in the high-income group works 4 percent more than a parent in the low-income group. This is because higher wage earners incur a higher opportunity cost of taking time off for child care, so they work more. Part of this extra income is allocated to education to compensate for a lower time investment by parents. This pattern is found in many empirical studies on working mothers.²⁸

A higher level of private spending on both early and late education by wealthier families generates a higher level of human capital and income for their children. This is shown in the last two rows in Table 2-3. However, the income gap between the poor and rich is smaller than the ability gap. This is because parents in the low-income group spend more time with children and that counteracts the low level of physical investment.

4.2 Increasing government spending

Compared to other OECD countries, the U.S. government spends a smaller share of GDP on early education. To investigate the policy effects of more public spending in this area, I consider increasing government expenditures in early childhood from 0.4 to 1.4 percent of GDP. In this case, the government allocates the same amount of public resources to early and late education, and the policy parameter setting becomes $\zeta_e = \zeta_l = 0.014$. Figure 1 shows the impact of this policy on average human capital, labor supply and income for each earnings tercile. All measures are presented relative to their counterparts in the benchmark economy. For example, the very first bar indicates that on average, human capital is 32 percent higher for low wage earners when $\zeta_e = 0.014$ relative to $\zeta_e = 0.004$.

Notice that human capital increases noticeably for both low- and medium-income groups, while the change in human capital for the high-income group is miniscule. This result can be explained by the crowding out effect of government spending. Since public and private spending are very substitutable, when government spends more on early education, parents

²⁸ See Hetherington et al. (1983), Haveman et al. (1991) and Freidberg and Webb (2005).

spend less and allocate more resources to consumption and late education. This crowding out effect varies with family income. In the benchmark economy, poor families spend almost nothing on children's early education, so there is not much room to crowd out private spending. In contrast, rich families spend a fair share of total income in early childhood, and hence, more government spending reduces parental spending by a larger amount for them. A given increment of government spending results in a larger increase in total spending for the poor, which results in a larger increase in human capital.

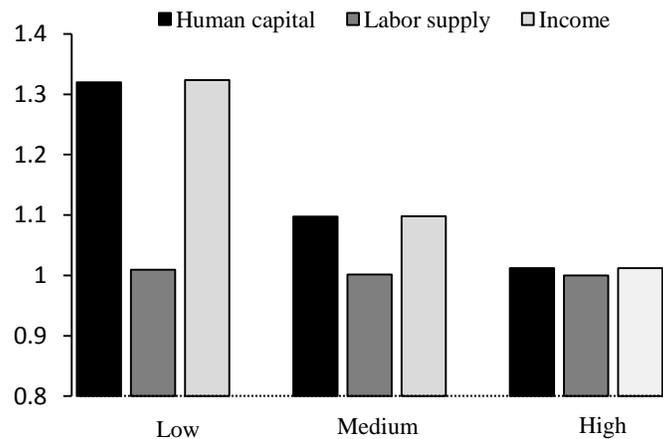


Figure 2-1. Policy effects of increasing government spending

In terms of labor supply, the low-income group is the most responsive to increased government spending. An increase in government spending affects labor supply through two channels. First, more government spending increases total physical investment in education, which crowds out time investment and leads to an increase in labor supply. Second, a higher level of government spending is accompanied by a higher income tax rate, which causes labor supply to drop. For the low-income group, the measure of education expenditures, e , doubles. Therefore, the first effect dominates the second effect and poor families work more. While for the high-income group, the second effect dominates, since the same increase in tax rate corresponds to a larger amount of income loss for them. As a result, parents in the highest earnings tercile work less in response to an increase in ζ_e .

Combining the effects on human capital and labor supply, it is easy to understand why income grows most, on a percentage basis, for the lower-income agents. This result is shown by the third bar for each income group.

4.3 Subsidizing parental spending

Due to the crowding out effect of direct government spending, parents spend less on early education. The same public resources can be allocated to early education by subsidizing parental spending, in which case parents are encouraged to spend more. In this experiment, I consider a government subsidy on private spending in early childhood. With a subsidy rate $s_f \in [0, 1]$, government provides a subsidy of $s_f f_e$ to parents to spend on early education. So equations (2.3) can be rewritten as

$$i_e = (\alpha((1 + s_f)f_e)^\eta + (1 - \alpha)g_e^\eta)^{\frac{1}{\eta}}, i_l = (\alpha f_l^\eta + (1 - \alpha)g_l^\eta)^{\frac{1}{\eta}} \quad (2.10)$$

and the budget relationship becomes

$$\sum g_e + \sum g_l + s_f \sum f_e = \tau Y = G.$$

I define

$$\zeta_f = \frac{s_f \sum f_e}{Y}.$$

Here policy parameter ζ_f is the share of total output allocated to subsidize private spending. With a given subsidy rate, s_f , there is a corresponding tax rate τ that balances the government budget. In the previous experiment, the tax rate is increased to 0.028 in order to fund the increase in government expenditures on early education. In order to generate comparable policy effects, I set $s_f = 0.375$ in this experiment to match $\tau = 0.028$. The policy parameter setting becomes $\zeta_e = 0.004$, $\zeta_l = 0.014$, $s_f = 0.375$. Figure 2 shows the average human capital, labor supply and income in each income group. Again, the values are relative to their counterparts in the benchmark economy.

Human capital increases substantially for all agents and the effect increases with family income. Private spending is subsidized which is equivalent to an increase in the marginal return to private spending. So all parents spend more in early childhood which further increases the total spending on early education. Due to the high degree of complementarity between early and later education, parents also allocate more resources in late childhood. As a result, the measure of education expenditures, e , increases by 53, 58 and 61 percent respectively for the low-, medium- and high-income group. The government subsidy is proportional to private spending.

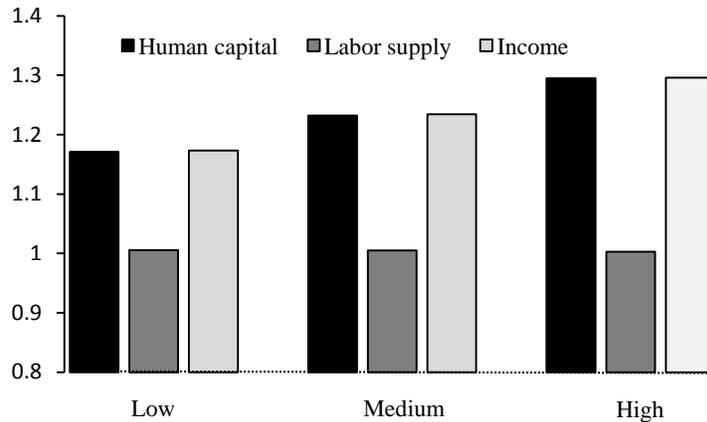


Figure 2-2. Policy effects of subsidizing private spending

With considerably more private spending, wealthy families receive more subsidy than less wealthy families. Consequently, the high-income group benefits from the policy with most increase in total education spending which leads to most increase in human capital.

The policy affects labor supply through the same channels as discussed earlier. Without the crowding out effect, wealthy families also observe a large increase in physical investment in education which crowds out time investment. So the positive effect on labor supply is strengthened for the wealthy and it leads to an increase of labor supply.

The percentage change in income is positively related to family income, as shown by the bars of income in Figure 2.

4.4 Paid parental leave

As a part of early childhood education and care policy, paid parental leave is available in many European countries. In general, European countries have more generous parental leave programs than the U.S.. New parents in the U.S. are entitled to take 12 weeks of unpaid job-protected parental leave, while some European countries provide up to 26 weeks of paid parental leave. In this experiment, I explore the impact of a paid parental leave program, where parents are compensated by the government when they take time off for child care. The compensation is proportional to parents' earnings lost by not working. I let $s_t \in [0, 1]$ to be the percentage of forgone income subsidized by the government. The subsidy for young parents is $w_1 t s_t$. Agents' budget constraints become

$$\sum_{j=1}^3 \frac{c_j}{r^{j-1}} + f_e + \frac{f_l}{r} = \left(w_1(1-t) + w_1 t s_t + \frac{w_2}{r} + \frac{w_3}{r^2} \right) (1-\tau)$$

and the government budget relationship is

$$\sum g_e + \sum g_l + s_t \sum w_1 t = \tau Y = G.$$

To facilitate the investigation on the change in taxation, I define

$$\zeta_t = \frac{s_t \sum w_1 t}{Y}$$

where ζ_t is the share of total output spent by the government on paid parental leave. Any given s_t is strictly mapped to a τ that balances the government budget. Again, I choose the value of s_t to target $\tau = 0.028$ so that government reallocates the same share of GDP to this program as it does in the previous experiments. I find that when stay at home parents receive 31.8 percent of the earnings forgone, the tax rate is increased to 0.024. The policy parameter setting in this experiment is $\zeta_e = 0.004$, $\zeta_l = 0.014$, $s_f = 0.493$. Figure 3 shows the results.

Compared to the previous policies, parental leave is targeting time investment rather than physical investment in early education. By providing incentives to parents to spend more time with young children, the paid parental leave policy generates higher human capital for all families. As depicted in Figure 3, the greatest change in human capital occurs to the least wealthy families. This is because poor parents are most responsive to the policy by working less and investing more time in early childhood. More specifically, the labor supply decreases by 7.6 percent for the low-income group, while it decreases by 3.4 percent for the high-income group.

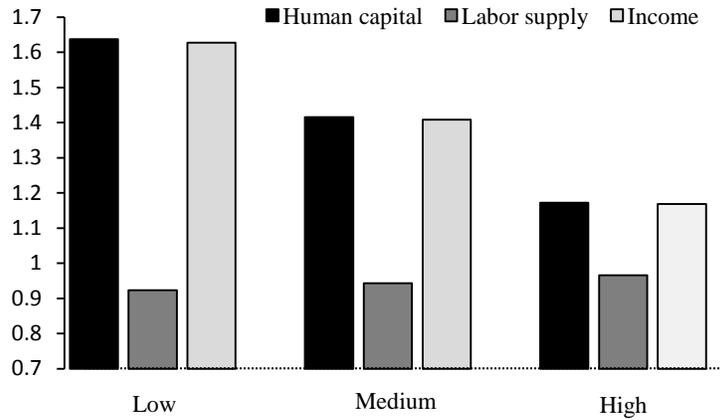


Figure 2-3. Policy effects of paid parental leave

The cost of taking time off from the labor market has two parts. First, parents endure the earnings forgone in the current period by not working. Second, parents lose some future earnings due to less working experience accumulated. A parental leave program helps young parents to cover some of the direct cost but not the indirect cost due to less experience. The cost of forgone experience is higher for wealthy parents since they obtain high human capital. This explains why they are not as responsive as less wealthy parents to the parental leave program. This is in the line with empirical findings by Rossin-Slater et al. (2011). They study the first paid family leave program in the United States using data from California.²⁹ Their estimates show that the overall use of maternity leave increased from 3 to 4 weeks to 6 to 7 weeks. Moreover, the effect is most significant among less-educated, unmarried and black mothers.

The change in income exhibits the same pattern as the change in human capital. Lower-income families end up with a greater increase in total income than higher-income families. The increment in human capital is not fully transmitted to extra income, because agents work less in response to the policy.

4.5 Aggregate effects

In this section, I compare the aggregate effects of the three policies. For each experiment, I compute the average human capital, labor supply and utility of the population. Total output and the Gini index are also calculated to show the effects on aggregate income and income inequality. Table 2-4 shows the results. In the first through fourth rows, all measures are normalized to the benchmark economy.

Table 2-4. Aggregate effects of policy experiments

	Benchmark	Increasing ζ_e	Subsidizing f_e	Paid parental
Human capital	1	1.062	1.269	1.272
Labor supply	1	1.004	1.004	0.945
Total output	1	1.062	1.270	1.266
Utility	1	1.151	1.208	1.421
Gini index	0.470	0.425	0.488	0.401

²⁹ The paid family leave program in California took place in 2004. Parents of new born children are entitled to a partially paid leave 6 weeks. Wage replacement rate is 55% with a weekly wage ceiling of \$959.

Comparing the average human capital across columns, It appears that increasing government expenditures is least effective at increasing human capital. This is due to the crowding out effect of public spending. Subsidizing private spending and introducing paid parental leave increase the average human capital by 26.9 and 27.2 percent, respectively. The former policy increases physical investment in early education by providing incentives to parents to spend more. While the latter policy increases parental time investment by incentivizing parents to work less.

Labor supply is not very responsive to the first two policies, as shown in the second row. This is because both experiments affect labor supply indirectly through higher education expenditures and a higher tax rate. Through these two channels, economic forces create positive and negative effects on labor supply, so the change in labor supply depends on the relative sizes of the two forces. In contrast, the paid parental leave program provides incentives to young parents to work less and that directly affects labor supply. On average, young parents work 5.5 percent less paid parental leave is available.

Notice the change in total output is similar to the change in human capital. Labor supply drops when the paid parental leave is introduced and that makes total output grow less than human capital in the last experiment. The fourth row shows the change in utility for all policy experiments. This is measured by the equivalent variation. For example, increasing government expenditures on early education increases average utility. The change in utility is equivalent to a 10 percent increase in consumption in all periods. The average utility increases across columns, while the paid parental leave policy generates the largest effect. This is because the policy produces the largest increase in human capital which is valued by parents.

The last row shows the change in income inequality measured by the Gini index. Increasing government spending on early education brings down the Gini index to 0.425. As discussed earlier, the smaller crowding out effect among the poor leads to a larger increase in total education spending for them, and hence they see a larger increase in human capital and income than the rich. As a result, income inequality is reduced. Subsidizing private spending on early education has the opposite effect on the Gini index which increases to 0.488. Since rich parents spend more on early education than poor parents, a proportional subsidy policy results in the largest increase in total education spending for rich parents. Consequently, they also experience the largest increase in human capital and income which strengthens the income inequality. Paid parental leave is most effective at reducing the Gini index as it drops to 0.401 in

the last experiment. This is due to the fact that low-income families are more responsive to the policy. They allocate substantially more time with young children and that contributes to a larger increase in human capital and subsequently income.

4.6 Sensitivity check

I consider alternative values of a few parameters in this section. Table 2-5 shows the values of these parameters in sensitivity analysis and the benchmark economy (in parenthesis) as well as the corresponding policy effects.

Table 2-5. Sensitivity check

	Benchmark	Increasing ζ_e	Subsidizing f_e	Paid parental
$\gamma = 0.65$ (0.75):				
Total output	1	1.066	1.228	1.285
Gini index	0.458	0.413	0.476	0.386
$\eta = 0.5$ (0.95):				
Total output	1	1.300	1.119	1.210
Gini index	0.463	0.436	0.471	0.417
$\Phi = -0.8$ (-0.2)				
Total output	1	1.058	1.235	1.275
Gini index	0.461	0.419	0.477	0.392

The relative importance of early education, γ , and the degree of complementarity between early and late childhood education, Φ seem to have modest impact on the relative effects of the three policies. It is shown again that subsidizing private spending and paid parental leave are much more effective at increasing GDP than a direct increase in public spending. However, subsidizing private spending causes more income inequality while paid parental leave reduces the Gini index.

When public and private spending are less substitutable, $\eta = 0.5$, increasing government spending becomes more effective than the other two policies. This is not surprising because the crowding out effect is smaller when private spending is less substitutable by government spending. The smaller crowding out effect results in larger increase in total spending, and consequently larger increase in human capital and output.

5 Conclusion

Parent-child interactions are considered to be vital in early childhood, as it is during this period that important cognitive and social skills develop. Quality interactions with parents improve children's focus, foster trust and build a foundation for intellectual curiosity and active learning. This paper contributes to the literature by including parental time investment as an education input in early childhood. My purpose is to explore the effects of different expenditure policies and paid parental leave on human capital, labor supply and income inequality. I build an overlapping generations model where human capital is formed in early and late childhood. The model is calibrated to the U.S. economy so that the generated data matches parental participation in education expenditures and the Gini index. With the parameter setting in the benchmark economy, I run counterfactual experiments and evaluate different policies in the general equilibrium.

I show that increasing government spending is least effective at increasing human capital. This is because public spending crowds out private spending and thus total spending increases less than intended. In addition, the reduction of private spending is larger for wealthier families since they spend relatively more and there is more room for crowding out. This also explains why the income gap between the poor and rich narrows, as shown by a lower Gini index. Subsidizing private spending instead, motives parents to spend more. The increase in total spending is the largest in the high-income group since as they spend more privately, they receive more government subsidy. This contributes to increased income inequality as the Gini index rises to 0.488. Paid parental leave induces the largest increase in human capital by increasing parental time investment. This result implies the importance of parental-care to children's early development and later achievement.

Paid family leave is designed to encourage more time allocation to parental-care. My model shows that adopting paid parental leave with a wage replacement rate of 49.3 percent, on average, reduces labor supply by 5.5 percent. The comparison analysis indicates that poor parents are most responsive and they work 7.6 percent less. This finding is supported by empirical studies, such as Rossin-Slater et al. (2011) and Carneiro et al. (2010). They use data from the U.S. and Norway, respectively. Both studies reach the conclusion that less advantaged mothers are more likely to take advantage of maternity leave if it includes wage compensation. The driving economic factor is the opportunity cost of parental-care. Higher-wage earners not

only bear a higher income loss, they also endure a higher cost of future wage cuts due to less work experience. Consequently, rich parents reduce labor supply by a smaller amount. Their smaller increase in time investment results in a smaller increase in human capital and income. This explains why paid parental leave is most effective in reducing income inequality indicated by a lower Gini index of 0.401.

This paper considers parental time allocation choice as the only determinant of employment. However, demand forces in the labor market may also be affected by the adoption of paid parental leave. A promising extension of current research is a more comprehensive analysis that includes the responses of employers. I also abstract from fertility choices by assuming one child is born into each family. Incorporating endogenous fertility choices would create another interesting layer of policy analysis.

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Chapter 3 - The motherhood wage penalty and non-working women: theory and evidence

1 Introduction

It is well documented that mothers, on average, earn lower wages than women without children. The traditional human capital model (see Mincer and Polachek (1974)) offers three explanations for this wage gap. First, human capital stops growing when women take a break from the labor market due to childbearing and maternity leave. Second, human capital depreciates when women are off work. Third, women who expect to have children and focus more on within-family production invest less in human capital. However, after controlling for educational attainment, work experience and individual heterogeneity, the wage gap remains positive and significant.³⁰

In the literature, this wage gap is also called the child/motherhood wage penalty and is obtained by regressing log wage on a dummy variable indicating motherhood status (i.e. 1-mother; 0-not a mother). Productivity and demographic variables are also included in the regression to control for human capital and the labor market environment. The estimate of the motherhood variable is interpreted as the wage effect of having children. Researchers originally studied the wage effect of childbirth by focusing on employment breaks. Mincer and Polachek (1974) use the 1967 National Longitudinal Survey (NLS) data and show that the child penalty due to work interruptions is not statistically significant.

The cross-sectional estimates are biased if unobserved worker characteristics are correlated with both wages and motherhood variables. To address the heterogeneity problem, further work examines the child penalty using panel data.³¹ Waldfogel (1998) uses data from the National Longitudinal Surveys of Youth (NLSY79) in the United States and National Development Survey in Britain. She adopts the fixed effects model to capture unobserved individual attributes such as perseverance and leadership skills. However, controlling for heterogeneity does not significantly change the size of the wage penalty in this study. Anderson

³⁰ See Budig and England (2001), Phipps et al. (2001) and Waldfogel (1998).

³¹ See Baum (2002), Gupta and Smith (2002) and Waldfogel (1997, 1995).

et al. (2003) and Amuedo-Dorantes and Kimmel (2005) both use U.S. data but find that the child penalty is significantly reduced in the fixed effects model.

In search of other causes of the child wage penalty, some researchers study mothers' job choices after childbirth and show that mothers' self-selection into family-compatible occupations partly explains the motherhood penalty.³² Beblo et al. (2009) use data on mothers and non-mothers in Germany to examine the pay gap between families with children and families without children. They find that the wage cut due to the first birth is reduced from 26 percent to 19 percent after controlling for occupation-specific effects. Simonsen and Skipper (2006) use a 1997 cross-sectional subsample of Danish women in the 20-40 age bracket. Controlling for sector-specific effects and using matching estimators in the analysis, they find that the direct causal effect of motherhood on wages is small while the total effect is significantly negative. In addition, they suggest that mothers self-sort into the public sector due to time flexibility and large non-pecuniary benefits. Baum (2002) finds that the wage effect of work interruptions due to childbirth is reduced if mothers return to their pre-childbirth jobs.

Most studies on the child penalty estimate the wage difference between employed mothers and non-mothers, excluding all non-working women due to the lack of wage data. However, if the observed wage difference is not representative of the wage gap between all mothers and non-mothers, analysis that excludes the non-working population may incur a selection problem and lead to biased estimates of the motherhood effect. To address the issue, I develop a simple model of fertility and work decisions to examine the relationships among fertility, employment and wages. The model implies that given the same non-labor income, all childless women face the same reservation wage and a childless woman chooses to work only when her wage exceeds that reservation wage. As a result, the working childless women are higher wage earners among all childless women. In contrast, the reservation wage of mothers depends on their motherhood preference. A mother with strong preference over child care faces a higher reservation wage. So a non-working mother may face a relatively high wage but chooses not to work due to her interest in child care. Excluding all non-working women is equivalent to selecting childless women with lower wages, mothers with lower wages and potentially mothers with higher wages out of the sample. Consequently, ignoring all non-working women may result in an overestimated wage child penalty.

³² See Nielsen et al. (2004) and Herry and Wolframz (2009).

To test the predictions of the model and examine the potential selection bias empirically, I use data from the 1997 National Longitudinal Surveys of Youth (NLSY97). I follow the literature and adopt both cross-sectional OLS and the fixed effects models to examine the child wage penalty. To account for heterogeneous human capital and selection into occupations, I control for educational attainment, work experience, and occupation in the wage equation. By doing so, I am able to isolate the contribution of each variable to the wage gap between mothers and non-mothers. In addition, the two-stage Heckman selection model is adopted to include non-working women and I find that the child wage penalty becomes small and insignificant. This finding supports the existence of selection bias implying that the child penalty is typically overstated.

To my knowledge, there are few studies that address this selection problem. One exception is the work by Amuedo-Dorantes and Kimmel (2005). They first estimate a probit model of likelihood to work and then include the inverse Mill's ratio (λ) constructed from the probit model in the wage regression. To predict the likelihood of working, Amuedo-Dorantes and Kimmel include the highest degree completed by the respondent's parents, a dummy variable indicating whether the respondent lived with her parents by age 18 and the respondent's years of schooling in the first stage. The coefficient on the motherhood variable is reduced by more than 40 percent after non-working women are included in the analysis. Their result suggests that the child penalty is overstated due to selection bias. I use the same approach but include a different set of control variables in the first stage. The control variables I include are the respondent's general health condition, a dummy indicator of presence of toddlers, total transfer from a government program, and a dummy indicator of presence of a spouse/partner with positive income. I find that the presence of toddlers is a stronger predictor of a woman's employment choice.

My work is also related to recent studies that take a quantitative approach to study the interactions between fertility decisions and labor market outcomes. Erosa et al. (2002) build and calibrate a model of fertility and labor market choices to study the cause of gender difference in employment and wages. They find that fertility decisions generate a long-lasting employment ratio difference and wage gap between males and females. Da-Rocha and Fuster (2005) develop a two-stage theoretical model to explain the positive association between fertility rates and female employment among OECD countries. The calibrated model implies that labor market frictions lead women to postpone child birth, which explains why countries with lower female

employment ratio also have lower fertility rates. Other theoretical work on women's labor market outcomes focus more on marriage issues rather than fertility decisions.³³ Compared to previous work, my model is relatively simple and focuses on a different set of questions. In my model, women make employment and fertility decisions simultaneously and their choice of working depends on their labor market prospects as well as motherhood status. I show that the reservation wage is different for mothers and non-mothers. This finding indicates the importance of accounting for non-working women in the analysis on the child penalty.

My research contributes to the literature in three ways. First, I develop a simple theoretical model that offers a framework to examine fertility and labor market decisions. The implications of the model are consistent with the empirical results. Second, I focus on a young cohort aged 24 to 30 in 2009 using more recent data (2004-2009) from the 1997 National Longitudinal Surveys of Youth (NLSY97). I compare my results with studies based on other U.S. data sets (i.e. NLSY79 and NLSYW) and examine the trend in the child wage penalty over time. Third, in addition to cross-sectional OLS and the fixed effects model, I adopt the two-stage Heckman selection model to correct for selection bias. The empirical results verify the existence of selection bias and show that the child penalty is overestimated due to this bias.

The paper proceeds as follows. In section 2, I present the model with its predictions. Section 3 introduces econometric approaches and reports the empirical results. Section 4 concludes.

2 The Model

2.1 A model of fertility and labor supply

Consider a model where women choose time allocation among work, child care and leisure. Time spent on work, children and leisure can be denoted by t_w , t_c and t_l , respectively. If a woman chooses not to work, $t_w = 0$. If a woman chooses not to have children, $t_c = 0$. A positive t_c denotes the total time a woman spends with children. I don't model the number of children in this study, so t_c can be distributed across more than one child. Women gain utility from consumption, time spent with children and leisure. Women gain utility from consumption,

³³ See Eckstein and Wolpin (1989), Caucutt et al. (2002) and Chade and Ventura (2002).

time spent with children and leisure. The utility function is concave in these three inputs as shown below

$$U = \ln(w(h, e, d)t_w + n) + \ln(t_c + \rho) + \ln(t_l)$$

where w is the wage rate which is a function of human capital, h , which can be gained from education and experience; labor market environment, e , i.e. the wage premium of certain occupations and contracts (part-time versus full-time) and demographic variables, d . Term $w(h, e, d)t_w$ is labor income. Consumption has a second part, $n \geq 0$, which is non-labor income, i.e. transfer payment from government, spouse or parents. Term $\rho > 0$ denotes women's preference for motherhood. This motherhood preference is affected by marital status, family background and environment. A lower ρ indicates more interest in having and raising children. Consider the extreme case when $\rho \rightarrow 0$, the marginal utility of having children is close to infinite when $t_c = 0$, so a woman chooses to be a mother. I define $T = (t_c, t_w, t_l)$ and let the time endowment to be 1. Women choose T to solve the following problem

$$\max_T \ln(w(h, e, d)t_w + n) + \ln(t_c + \rho) + \ln(t_l)$$

subject to

$$\begin{aligned} t_w + t_c + t_l &\leq 1 \\ t_c, t_w &\geq 0, t_l > 0. \end{aligned}$$

Rewrite the woman's problem as the following Lagrangian

$$\max_T L = \ln(w(h, e, d)t_w + n) + \ln(t_c + \rho) + \ln(t_l) + \lambda(1 - t_w - t_c - t_l).$$

The time constraint always binds, $\lambda > 0$, since utility is increasing in all time input. The Kuhn-Tucker conditions are

$$t_w: \frac{w}{wt_w + n} - \lambda \leq 0, t_w \geq 0, \text{ and } \left(\frac{w}{wt_w + n} - \lambda \right) t_w = 0 \quad (3.1)$$

$$t_c: \frac{1}{t_c + \rho} - \lambda \leq 0, t_c \geq 0, \text{ and } \left(\frac{1}{t_c + \rho} - \lambda \right) t_c = 0 \quad (3.2)$$

$$t_l: \frac{1}{t_l} = \lambda \text{ and } t_l > 0 \quad (3.3)$$

$$\lambda: 1 - t_w - t_c - t_l = 0. \quad (3.4)$$

The model can be solved analytically and there are four cases regarding women's choice of t_w and t_c . I adopt superscript $*$ to indicate a positive value. For example, solution $T = (t_w^*, 0, t_l^*)$ shows a woman's choice of positive work time, zero time investment in children and positive leisure time. This solution suggests that a woman is working, but chooses not to have

children. Since $t_l \in (0, 1]$, so I remove term t_l^* in all cases with the understanding that leisure time is positive. The four cases are listed below and they are linked to observed employment and motherhood status

- Case 1: working childless women. Let $T = (t_w^*, 0)$, equations (3.1) and (3.3) into equation (3.4) and the assumption $t_c = 0$ give

$$t_w = t_w^* = \frac{w - n}{2w}; t_c^* = 0; t_l = t_l^* = \frac{w + n}{2w};$$

- Case 2: working mothers. Let $T = (t_w^*, t_c^*)$, equations (3.1)-(3.3) into equation (3.4) gives

$$t_w = t_w^* = -\frac{2n}{3w} + \frac{1 + \rho}{3}; t_c = t_c^* = \frac{n}{3w} + \frac{1 - 2\rho}{3}; t_l = t_l^* = \frac{n}{3w} + \frac{1 + \rho}{3};$$

- Case 3: Non-working mothers. Let $T = (0, t_c^*)$, equations (3.2) and (3.3) into equation (3.4) and the assumption $t_w = 0$ give

$$t_w = 0; t_c = t_c^* = \frac{1 - \rho}{2}; t_l = t_l^* = \frac{1 + \rho}{2};$$

- Case 4: Non-working childless women. Let $T_4 = (0, 0)$, equation (3.3) into equation (3.4) and the assumption $t_w = t_c = 0$ give

$$t_w = 0; t_c = 0; t_l^* = 1.$$

2.2 Some predictions

The model yields the following results.

Proposition 1: $\rho \leq (1/2)$ is a sufficient condition for the choice of being a mother, $t_c^* > 0$; $\rho \geq 1$ is a sufficient condition for the choice of being childless, $t_c^* = 0$; When $\rho \in (\frac{1}{2}, 1)$, a woman chooses to be a mother, only if the wage is low enough, $w \leq \frac{n}{2\rho - 1}$.

Proof: consider conditions for case 2: $t_w = t_w^* = -\frac{2n}{3w} + \frac{1 + \rho}{3} \geq 0$ and $t_c = t_c^* = \frac{n}{3w} + \frac{1 - 2\rho}{3} \geq 0$. Using $\rho > 0$, these constraints can be written as

$$w \in \left[\frac{2n}{\rho + 1}, \frac{n}{2\rho - 1} \right] \text{ and } \rho \in \left(\frac{1}{2}, 1 \right]; \tag{3.5}$$

$$\text{or } w \geq \frac{2n}{\rho + 1} \text{ and } \rho \in \left(0, \frac{1}{2} \right].$$

Next consider conditions for case 3: $t_c = t_c^* = \frac{1-\rho}{2} \geq 0$ and $\frac{w}{wt_w+n} - \lambda \leq 0$ when $t_w = 0$. Combine $\lambda = \frac{1}{t_l^*} = \frac{1}{\frac{1+\rho}{2}}$ into these constraints

$$w \leq \frac{2n}{\rho+1} \text{ and } \rho \leq 1; \quad (3.6)$$

Combining equations (3.5) and (3.6) gives the conditions for the choice of having children, $t_c > 0$. Given any wage rate, w , women with $\rho \leq \frac{1}{2}$ always choose to be a mother.

Consider conditions for case 1: $t_w^* = \frac{w-n}{2w} \geq 0$ and $\frac{1}{t_c+\rho} - \lambda \leq 0$ when $t_c = 0$. Using $\lambda = \frac{1}{t_l^*} = \frac{1}{\frac{w+n}{2w}}$, I derive

$$w \geq \max\left(n, \frac{n}{2\rho-1}\right) \text{ and } \rho \geq \frac{1}{2} \quad (3.7)$$

Next consider conditions for case 4: $\frac{w}{wt_w+n} - \lambda \leq 0$ when $t_w = 0$ and $\frac{1}{t_c+\rho} - \lambda \leq 0$ when $t_c = 0$. Given $\lambda = \frac{1}{t_l^*} = 1$, these constraints can be combined into

$$w \leq n \text{ and } \rho \geq 1 \quad (3.8)$$

Equations (3.7) and (3.8) give the conditions for the choice of having no children, $t_c^* = 0$. Given any wage rate, w , women with $\rho \geq 1$ always choose to have no children.

Equations (3.5) and (3.7) show that wage is relevant to a woman's fertility decision when ρ falls in the middle range. When $\rho \in (\frac{1}{2}, 1)$, if $w \leq \frac{n}{2\rho-1}$, a woman chooses have no children. Otherwise, she chooses to be childless. ■

Proposition 1 indicates the importance of women's preference for motherhood on their fertility choice. With a high enough ρ (the lower bound is 1), women choose to have no children regardless of their potential wage. While when ρ is low enough (the higher bound is 1/2), women with all possible wages choose to be a mother. The wage rate is relevant to women's fertility choice only when ρ is in a middle range, i.e. $\rho \in (\frac{1}{2}, 1)$.

Proposition 2: A mother chooses to work only if $w > \frac{2n}{\rho+1}$. A childless woman chooses to work only if $w > n$.

Proof: equations (3.7) and (3.8) suggest that for a woman who chooses not to have children, if the wage exceeds n , she chooses to work. Otherwise, she chooses not to work.

Equations (3.5) and (3.6) together show that for a woman who chooses to have children, if the wage exceeds $\frac{2n}{\rho+1}$, she chooses to work. Otherwise she stays outside of the labor market.

Proposition 2 shows the reservation wage of mothers and non-mothers. Reservation wage here means the wage rate at which a woman is indifferent between working and not working. The reservation wage is $\frac{2n}{\rho+1}$ and n for mothers and childless women, respectively.

When $\rho < 1$, a mother faces a higher reservation than a non-mother, and vice versa. ■

Notice that given the same n , all childless women face the same reservation wage, while the mothers' reservation wage varies with the preference term, ρ . Mothers with stronger preference for child care face a higher opportunity cost of working, and thus their wage needs to exceed a higher threshold in order for them to work. For this reason, it is possible that a non-working mother has a higher potential wage than a working mother but still chooses not to work, because she prefers child care more. In contrast, a non-working childless woman always has a lower potential wage than a working childless woman.

Figure 3-1 helps to explain this finding. Each of the four areas in the graph represents the combinations of wage, w , and preference parameter, ρ , that correspond to women's employment and motherhood choice (i.e. A-working non-mothers; B-non-working non-mothers; C-working mothers; D-non-working mothers). For example, area A shows that when both wage and preference parameter are large enough, a woman chooses to work but have no children. Compare area A and B which corresponds to working and non-working childless women, respectively. Any point in A is above B, which means non-working childless women all face a lower potential wage than working childless woman. The same is not true for area C and D which corresponds to working and non-working mothers, respectively. There are points in D that are above some points in C. It suggests that some non-working mothers may face a higher potential wage than some working mothers, but they choose not to work due to a low value of ρ . This finding is supported by the empirical results. I show that the mean potential wage of non-working mothers is not significantly different than the mean wage of working mothers, while non-working childless women would have earned much less than their working counterparts.

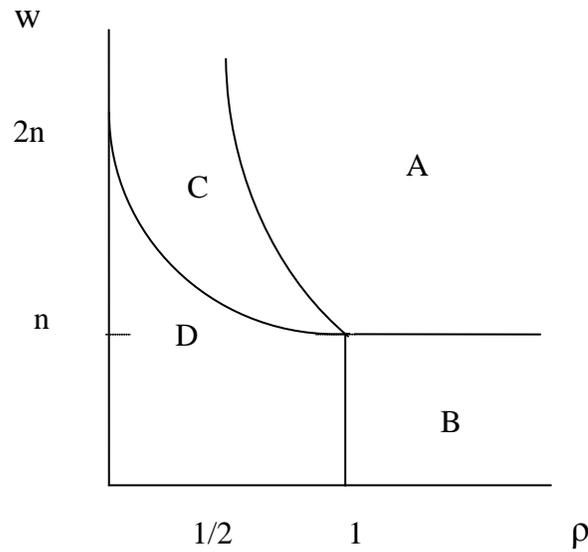


Figure 3-1. Women’s fertility and employment choice

In estimating the wage gap between mothers and non-mothers, excluding all non-working women is equivalent to removing both area B and D out of the picture. This may cause selection bias, because some potential high wage earners among mothers are excluded while only low wage earners among childless women are excluded. Consequently, the wage gap between mothers and non-mothers is overestimated. To correct for the bias, I include non-working women in the empirical analysis and show that the wage gap is overstated due to the existence of selection bias.

3 Empirical Analysis

3.1 Methodology

I use a panel sample of young women aged 24 to 30 in 2009 in the empirical analysis. The panel dataset enables me to control for unobserved worker characteristics, such as work ethics and ambition. If women with children are more devoted to home production and less focused on work than women without children, the cross-sectional estimates tend to overstate the children effect. I compare estimates in cross-sectional and panel analysis to examine the impact of individual heterogeneity on the child penalty.

The wage equation estimated is as follows

$$\ln W_{i,t} = \alpha + M_{i,t}\beta + X_{i,t}\gamma + v_i + u_{i,t} \quad (3.9)$$

where $W_{i,t}$ is self-reported hourly monetary compensation in 2009 dollars of respondent i 's primary job in year t for all $i=1,2,\dots,1240$ and $t=2004,2005,\dots,2009$. $M_{i,t}$ is a set of motherhood variables for respondent i in year t . In the first specification, $M_{i,t}$ is a dummy variable indicating the presence of children. In the second specification, $M_{i,t}$ are dummy variables indicating the number of children, i.e. zero, one, or two or more. $X_{i,t}$ is a vector of productivity and demographic variables of respondent i in year t . Specifically, I control the highest degree, AFQT scores, work experience, tenure, age, race, part time, marital status, occupation and urban residence. Term v_i is the unobserved individual-specific effect, which is invariant over time; $u_{i,t}$ is the standard error term. The estimate of interest is β . Having wages in the natural log form as the dependent variable, I interpret β as the impact of motherhood variables on wage expressed on a percentage basis.

If unobserved worker attributes are correlated with control variables, random effect estimators provide inconsistent estimates. Since individual characteristics that are correlated with wages are usually related with education choices, i.e. perseverance and self-discipline, I use a fixed effects estimator.

Wages are only available for employed women. Therefore, the wage regression incurs a sample selection bias. If there is systematic difference between the reservation wage of mothers and that of non-mothers, the child penalty can be overstated or understated due to the exclusion of non-working population. To address the problem, I use the Heckman two-stage method. In the first stage, I estimate a probit model of the likelihood of working and obtain the inverse Mill's ratio (λ). In this regression, I not only include all the productivity and demographic variables in $X_{i,t}$ in equation (3.9), I also control the respondent's general health condition, a dummy indicator of presence of children under the age of 3, total amount of money received from a government program excluding unemployment insurance and worker's compensation, and a dummy indicator of presence of a spouse/partner with positive income during the last period. In the second stage, I add the inverse Mill's ratio (λ) as an explanatory variable in the wage regression (3.9).

3.2 Data

The panel data used in this study is from the 1997 National Longitudinal Surveys of Youth (NLSY97). These surveys cover a nationally representative youth sample of 6,748

individuals aged 12 to 18 as of December 31, 1996. The first round survey was conducted in 1997, and the follow up surveys took place on a yearly basis. The NLSY97 was designed to document youths' transition from schools to work and to adulthood. It contains extensive information on respondents' demographic characteristics, schooling experiences, family background, employment and fertility. I work with an unbalanced panel dataset on women from the 2004-2009 surveys. In my empirical analysis, I restrict my sample to observations for which information is available regarding demographics, education, fertility, employment and marriage across years. Since the purpose of this study is to examine the labor outcomes of women in early career, I exclude all the women who were enrolled in school during 2004-2009 survey years. After the restrictions above, I have 1,460 women (1,240 working women and 220 non-working women) in the 2009 survey and 4,027 women in the five-year panel.

The wage included in this study is hourly monetary compensation of respondents' primary jobs in 2009 dollars. I exclude the sample with extreme reports (hourly wage being lower than 5 dollars or higher than 200 dollars). I measure work experience by the number of working weeks accumulated through years. Tenure is a variable indicating the number of weeks of total tenure at the primary job to the survey date. Other than the highest degree received, I also include the Armed Forces Qualification Test (AFQT) scores to control for women's academic achievement. The Armed Forces Qualification Test is used by the U.S. armed forces as a measure of ability. The raw AFQT scores are problematic when used for comparison across ages. To correct for that problem, the NLSY97 reports revised AFQT scores adjusted for age.

Table 3-1 presents statistics summary of the sample of working women in the 2009 survey. The first column shows statistics of all working women, and the second and third columns report descriptive information for working mothers and non-mothers, respectively. The 1,240 working women aged from 24 to 30 in 2009 indicating that most of them were at the early stage of their career. On average, they earn approximately 17 dollars per hour. Regarding education, 6 percent of the sample has no degree and 24 percent of them have only a high school diploma or a GED. 37 percent of the sample graduated from college. The rest received either a master or Ph.D. degree. In my sample, 47 percent of the women are mothers and around half of the mothers have two or more children.

Comparing the second and third columns in Table 3-1, a few differences are worthy of note. Women without children, on average, receive higher education and AFQT scores, obtain more work experience and are more likely to work full-time than mothers. The wage gap

between mothers and childless women is prominent in the descriptive evidence. It is suggested that mothers earn 23 percent less than childless women per hour.

Table 3-1. Variables, means and standard deviation in the year, 2009

Variable	(Standard Deviation for Continuous variables in Parentheses)		
	All working women	Mothers	Non-mothers
Wage (2009 \$)	17.24 (11.9)	14.93 (10.39)	19.32 (12.89)
Mother	0.47	1	0
One child	0.22	0.46	0
Two or more children	0.25	0.54	0
Age	26.26 (1.47)	26.47 (1.45)	26.07 (1.46)
Black	0.17	0.22	0.12
White	0.72	0.57	0.77
Other race	0.11	0.21	0.11
Married	0.37	0.47	0.27
Less than high school	0.06	0.11	0.01
High school	0.24	0.34	0.14
Some college	0.26	0.31	0.23
College	0.37	0.22	0.50
Graduate school	0.07	0.02	0.12
AFQT score	52.67 (28.36)	42.86 (26.69)	61.50 (26.91)
Work experience in	382.06 (140.07)	371.13 (143.86)	391.88 (135.93)
Tenure in weeks	138.90 (123.56)	138.19 (126.94)	139.54 (120.54)
Part-time	0.31	0.37	0.25
Urban	0.79	0.75	0.83
Professional and	0.17	0.11	0.24
Sales	0.12	0.15	0.09
Clerical	0.31	0.37	0.27
Laborers	0.05	0.05	0.04
Services	0.25	0.22	0.28
Operatives (workers)	0.21	0.25	0.16
Number of	1240	587	653

I include non-working women in the analysis to control for selection bias. Table 3-2 shows statistics of working women and non-working women and offers a comparison on productivity indicators. Note that non-working women are disproportionately black, married and mothers of two or more children than working women. In terms of educational background, working women tend to receive higher degrees and higher AFQT scores than women who are not in the labor market. The comparison between the first and second columns reflects that non-working women are less competitive wage earners than working women.

Table 3-2. Working women and non-working women, 2009

(Standard Deviation for Continuous variables in Parentheses)		
Variable	All working women	Non-working women
Mother	0.47	0.83
One child	0.22	0.22
Two or more children	0.25	0.61
Black	0.17	0.22
White	0.72	0.67
Other race	0.11	0.11
Married	0.37	0.50
Less than high school	0.06	0.22
High school	0.24	0.32
Some college	0.26	0.30
College	0.37	0.15
Graduate school	0.07	0.01
AFQT score	52.67 (28.36)	37.31 (27.89)
Work experience in weeks	382.06 (140.07)	218.28 (152.51)
Number of observations	1,240	220

3.3 Results

3.3.1 Baseline estimates

Table 3-3 shows the estimation results of pooled ordinary least square (OLS) regressions using the sample from 2004-2009 surveys. In specification (1), I only control for basic demographic variables (i.e. age, race, marital status and urban residence) and leave out productivity variables. Data column (1) shows the results. Mothers earn approximately 16 percent less than childless women per hour. After controlling for education and ability with the highest degree completed and AFQT scores in specification (2), the wage gap is substantially reduced to 5.4 percent. This indicates that the observed wage gap between mothers and women without children can be mostly explained by the disparity in education and ability. Including work experience and quadratic work experience in the regression, I move on to specification (3) and find that the child penalty falls by another 1.8 percentage points. Column (4) reports the estimate of the child penalty after tenure and quadratic tenure are controlled. The result implies that mothers still bear a wage penalty of 3.2 percent. As shown in column (5), controlling for occupation does not have a large impact on the wage penalty.

I switch to the other set of motherhood variables indicative of the presence of one or more children in the next specification. Column (6) in Table 3-3 shows the results. After controlling for all productivity and demographic variables, there is wage penalty of 3.2 percent

for both having one child and having two or more children. However, the result for mothers with one child is not statistically significant.

Table 3-3. Pooled OLS estimation results of wage equations, NLSY97, 2004-2009

Variables	Log hourly monetary compensation					
	(1)	(2)	(3)	(4)	(5)	(6)
Mother	-0.159*** (0.015)	-0.054*** (0.015)	-0.036** (0.015)	-0.032** (0.015)	-0.032** (0.015)	
One child						-0.032 (0.021)
Two or more						-0.032* (0.016)
Highest degree		×	×	×	×	×
AFQT score		×	×	×	×	×
Work experience			×	×	×	×
Tenure				×	×	×
Occupation					×	×
Observations	4,027	4,027	4,027	4,027	4,027	4,027
R-squared	0.11	0.20	0.21	0.23	0.25	0.25

1. × means the variables are included in the regression. 2. Age, age squared, ethnicity, urban, part-time and marital status are controlled in all specifications. 3. Standard error in parentheses; * significant at 10% ** significant at 5% *** significant at 1%.

Compared to the estimates in other studies on U.S. data, with the same method and control variables, I derive smaller wage effects.³⁴ It is largely due to the younger age of my sample.³⁵ The comparison suggests that the wage gap is widened over years. There are several explanations for this trend. First, human capital may grow faster with more experience for childless women than for mothers. Thus, part of the motherhood penalty appears with time and should not affect my younger sample. Second, from the signaling theory's perspective, having no children at an older age is a stronger signal of commitment and concentration to work to employers and the stronger signal results in a higher wage premium.

3.3.2 Correcting for heterogeneity and selection bias

³⁴ Anderson et al. (2003) use NLSYW68-88 data and show that the wage effect of one child is 5.6% and that of two or more children is 8.2%. Kimmel et al (2005) use 19 rounds of NLSY79 surveys and their estimated one child effect is 4.5% and two or more children effect is 8.9%.

³⁵ The average age of my sample is 26 at the last survey. Kimmel et al (2005) studies a group of women with a mean age of 40. In Anderson et al (2003), women aged from 34 to 44 in the year when last survey took place.

To control for individual characteristics that may explain the wage penalty, I estimate the wage gap in the fixed effects model. I then address the selection problem by including non-working women and using a two-stage Heckman selection model. Table 3-4 shows the results.

I report the results from the pooled OLS again in the first and second columns of Table 3-4 for the ease of comparison. As shown in column (3) of Table 3-4, the child penalty becomes 1.5 percentage points larger but less significant after accounting for heterogeneity. The following column shows in the fixed effects model that having one child results in a wage penalty of 5.7 percent, while having two or more children does not significantly affect women's wages. With the larger estimates in the fixed effects model, I conclude that individual effects don't contribute to the child penalty. This finding is consistent with Waldfogel (1998)'s results.

Including non-working women increases the sample size by around 25 percent. The final two columns in Table 3-4 present the results from the Heckman selection model. In each of the two specifications, the estimates of children effects are small and statistically insignificant after correcting for selection bias. This result implies that excluding non-working women causes an upward bias on the estimated child wage penalty.

Table 3-4. Estimation results in the fixed effects model and Heckman selection model

Variables	Log hourly monetary compensation					
	Pooled	Fixed		Heckman		
Mother	-0.032** (0.015)		-0.047* (0.026)		-0.004 (0.018)	
One child		-0.032 (0.021)		-0.057** (0.027)		0.003 (0.025)
Two children		-0.032* (0.016)		-0.036 (0.028)		-0.007 (0.019)
Observations	4027	4027	4027	4027	4027	4027

1. Race and AFQT scores drop out in the fixed effects model. Productivity and demographic variables all have the expected signs in both fixed effects model and Heckman selection model. 2. Standard error in parentheses;

* significant at 10% ** significant at 5%.

Using the estimates of the wage equation from the Heckman selection model, I calculate the estimated mean wage of women by fertility and working status. Table 3-5 shows the statistics. I interpret the wage of non-working women as the wage they would have earned if they chose to work. First compare the mean wages across rows. It is shown that among working women, mothers earn lower wages than non-mothers. The estimated wage differential is \$1.22. The same pattern exists among non-working women. Mothers, on average, would have earned lower wages than non-mothers if both groups chose to work. However, the wage gap is as small

as \$0.49. This explains why the estimated motherhood wage penalty becomes smaller and insignificant when non-working women are included.

Table 3-5. Estimated mean wage from the Heckman selection model

	(Standard Deviation in Parentheses)	
	Working	Non-working
Mothers	13.713 (0.057)	13.282 (0.082)
Non-mothers	14.933 (0.073)	13.771 (0.160)

Comparing the mean wages across columns, I find that working women, on average, earn a higher wage than non-working women. However, the wage gap is much larger among non-mothers than that among mothers. This finding can be explained by Proposition 2 of the model that mothers face various reservation wages. Mothers who select out of the labor market may have relatively high wages, but since they have strong preference for child care, they choose not to work. This explains the small wage gap between working mothers and non-working mothers. On the contrary, all non-mothers face the same reservation wage. If a childless woman chooses not to work, that is only because her wage is too low. This is why there is a large wage gap between working and non-working childless women.

3.3.3 Estimating the likelihood of working

In the first stage of the Heckman selection model, I estimate the likelihood of working with all the control variables in the wage regression and an additional set of variables. The additional variables include the respondent's general health condition and presence of toddlers. Table 3-6 reports the estimates of interest in the probit model.

The first data column in Table 3-6 shows the estimates in specification (1), in which motherhood variable is a dummy indicating whether the respondent has children. In specification (2), motherhood variables are dummies indicating the number of children (the choices are 0, 1, and 2 or more). The estimates in specification (2) are shown in the second column in Table 3-6.

The motherhood variables are all negative and significant in each of the specifications suggesting that mothers are less likely to work. The following row shows the impact of having toddlers at home on the likelihood of working. With the significantly negative estimates in both specifications, it is suggested that mothers with toddlers are less likely to work compared to childless women and other mothers. This result is not surprising, because children in early years

are physically vulnerable and they need care from adults, especially their own mothers for nursing. Mothers with toddlers have a higher opportunity cost of working. As a result, they have a lower probability of working even after controlling for labor market related variables.

Table 3-6. Estimation results in the probit model for working decisions

Variables	Likelihood of working	
	(1)	(2)
Mother	-0.115* (0.062)	
One child		-0.141* (0.077)
Two or more children		-0.103 (0.066)
Toddlers	-0.153*** (0.058)	-0.151*** (0.058)
Excellent health condition	0.156** (0.071)	0.156** (0.071)
Good health condition	0.240*** (0.075)	0.239*** (0.075)
Amount from a government	-0.001 (0.001)	-0.001 (0.001)
Positive income of	-0.040 (0.057)	-0.040 (0.057)
Observations	4958	4958

1. Married women are less likely to work than single women; women with higher degrees and more experience are more likely to work. 2. Standard error in parentheses; * significant at 10% ** significant at 5%; *** significant at 1%.

Self-reported health condition is also controlled in the probit model. The baseline dummy is poor health condition. As shown in Table 3-6, compared with women in poor health conditions, women in excellent and good health conditions are more likely to work. The estimates are at 1 percent level of significance. It indicates that health condition is a strong predictor of the likelihood of working.

The model in Section 2 predicts that women with higher non-labor income face a higher reservation wage. To test for this prediction, I include total transfer from a government program in the probit model. The estimates in both specifications appear to have the expected sign. However, the results are not significant at 10 percent level. Since there is no variable in NLSY97 on direct transfer from parents or spouse, I control a dummy indicating whether the respondent's spouse/partner made positive income during the last period. The coefficient is negative as predicted but not statistically significant.

4 Conclusion

In the literature, the motherhood wage penalty is estimated using wage data on working mothers and non-mothers. Such analysis excludes non-working women and thus it potentially results in biased estimates due to sample selection. To include non-working women in the examination of the wage gap, I build a simple model of women's employment and fertility choices and derive the reservation wage of mothers and childless women. The model implies that all childless women with the same non-labor income face the same reservation wage, while mothers with different preference over motherhood face different reservation wages. Since the reservation wages of mothers and non-mothers have different forms, the wage gap observed in the labor market is a biased measure of the child wage effect. To empirically test for the existence of selection bias, I use data from the NLSY97 (2004-2009) and adopt the two-stage Heckman selection model. The women included in the analysis are aged 24 to 30 in 2009. The relatively young age of the sample enables me to focus on women's labor market outcomes in their early career.

The estimates in the pooled OLS and the fixed effects model suggest a statistically significant wage penalty for mothers. More specifically, a mother's wage is 3.2 to 4.7 percent lower. This finding is consistent with the literature on the child wage penalty. However, the wage gap is eliminated in the Heckman selection model. This implies that selection bias does exist and it causes an overestimated child penalty. I calculate the potential wage of non-working women using estimates from the Heckman selection model. There is no statistically significant wage gap between non-working mothers and non-working childless women. This explains why the child penalty becomes smaller and insignificant when non-working women are included.

The model presented in this paper provides insights on women's fertility and employment choices that are supported by the empirical results. However, women make work and fertility decisions simultaneously in my model while it is possible that these decisions are actually made sequentially. In addition, for simplicity, my analysis does not account for marriage decisions which may also have an impact on the labor market outcomes. I leave multi-stage models and accounting for marriage decisions for future research.

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Appendix 1

Definition of equilibrium

To facilitate the definition of an equilibrium in Chapter 1, we introduce more precise notation. With this notation, equation (1.5) is

$$i_{i,t,t} = (\alpha f_{i,t,t}^\eta + (1 - \alpha)g_{i,t,t}^\eta)^{\frac{1}{\eta}}, i_{i,t,t+j} = (\alpha f_{i,t,t+j}^\eta + (1 - \alpha)g_{i,t,t+j}^\eta)^{\frac{1}{\eta}}, j \in \{1,2,3\}, \quad (1.9)$$

where $i_{i,t,t}$ is the measure of human capital investment in early childhood in period t (third subscript) on behalf of the generation t (second subscript) member of dynasty i . This is a function of family spending, $f_{i,t,t}$, and government spending $g_{i,t,t}$ on this agent in early childhood. Similarly, $i_{i,t,t+j}$, $f_{i,t,t+j}$, and $g_{i,t,t+j}$ are human capital investment, family spending, and government spending in late childhood on behalf of the same agent in period $t + j$, $j \in \{1,2,3\}$.

The aggregate of late childhood spending in equation (1.4) is

$$i_{i,t,l} = (i_{i,t,t+1}i_{i,t,t+2}i_{i,t,t+3})^{\frac{1}{3}} \quad (1.10)$$

and this combines with $i_{i,t,t}$ to give a more precise statement of equation (1.3)

$$e_{i,t} = (\gamma i_{i,t,t}^\Phi + (1 - \gamma)r_{i,t,l}^\Phi)^{\frac{1}{\Phi}}. \quad (1.11)$$

This measure of education quality combines with the ability of the generation t member of dynasty i , $a_{i,t}$, and the human capital of this agent's parents, $h_{i,t-4}$, to generate the human capital of this agent, $h_{i,t}$. Specifically

$$h_{i,t} = a_{i,t} e_{i,t}^\mu h_{i,t-4}^\nu. \quad (1.12)$$

The generation $t - 4$ member of dynasty i chooses family education spending on the generation t member of dynasty i in that member's early childhood and the three periods of late childhood. This is denoted by $f_{i,t,t+j}$, $j \in \{0,1,2,3\}$. The agent also chooses own consumption in each of the 11 periods of adulthood, $c_{i,t-4,t+j}$, $j \in \{0,1,\dots,10\}$, and bond holdings in each of these periods other than the last, $b_{i,t-4,t+j}$, $j \in \{0,1,\dots,9\}$. These choices are made to maximize

$$\sum_{j=0}^{10} \beta^j \frac{c_{i,t-4,t+j}^\sigma}{\sigma} + \xi \frac{h_{i,t}^\sigma}{\sigma} \quad (1.13)$$

subject to

$$\sum_{j=0}^{10} \frac{c_{i,t-4,t+j}^\sigma}{r^j} + \sum_{j=0}^3 \frac{f_{i,t,t+j}}{r^j} = \sum_{j=0}^8 \left(\frac{z}{r}\right)^j w h_{i,t} (1 - \tau) \quad (1.14)$$

and the relationships in equations (1.9)-(1.12). There are non-negativity constraints on $f_{i,t,t+j}$, $j \in \{0,1,2,3\}$ and $c_{i,t-4,t+j}$, $j \in \{0,1,\dots,10\}$. However, these do not bind in equilibrium and are therefore ignored. In the case of borrowing constraints, however we have the additional constraint

$$b_{i,t-4,t} \geq 0. \quad (1.15)$$

Output in period t is

$$Y_t = w \left(\sum_{i=1}^N h_{i,t-4} + \sum_{i=1}^N z h_{i,t-5} + \sum_{i=1}^N z^2 h_{i,t-6} + \dots + \sum_{i=1}^N z^8 h_{i,t-12} \right) = w H_t \quad (1.16)$$

and with G_t being total government spending, the budget relationships are

$$\sum_{i=1}^N g_{i,t,t} + N(g_{t-1,t} + g_{t-2,t} + g_{t-3,t}) = \tau Y_t = G_t \quad (1.17)$$

There are initially N dynasties in each of the 15 life cycle stages. The agents initially in early childhood are the first to have the full fifteen periods in the economy. Initial conditions must be specified for all other generations. For example, the original agents in the first period of late childhood must have an initial endowment of $i_{i,t,t}$ and those originally in the second period

of late childhood must have endowments of $i_{i,t,t}$ and $i_{i,t,t+1}$. All adults in initial periods have equivalent endowments of human capital. Parents of the agents originally in early childhood make all the decisions described above. Other adults have an abbreviated set of choices to make. For example, parents of the agents in the first period of late childhood do not choose family spending on children in early childhood or consumption in their first period of adulthood. Empty nester and retiree choose only consumption in the remaining periods of their lives. For this reason, a different set of problems and constraints exist for initial agents. For brevity, these are not presented here and are ignored in the definition of an equilibrium. However, they are straightforward to derive and are accounted for in programs which solve the model.

An equilibrium is comprised of the sets of agents' choices and outcomes for $\{c_{i,t-4,t+j}, j \in \{0,1,\dots,10\}, b_{i,t-4,t+j}, j \in \{0,1,\dots,9\}, f_{i,t,t+j}, j \in \{0,1,2,3\}, h_{i,t}\}$ for all $i \in \{1,2,\dots,N\}$ and all $t \geq 0$, and government policy parameters $\{\tau, g_{i,t,t}, g_{t-1,t}, g_{t-2,t}, g_{t-3,t}\}$ for all $i \in \{1,2,\dots,N\}$ and all $t \geq 0$ such that

1. Taking own human capital and government policy as given, the agent from dynasty i born in period $t - 4, t \geq 4$ chooses $c_{i,t-4,t+j}, j \in (0,1,\dots,10), b_{i,t-4,t+j}, j \in (0,1,\dots,9)$, and $f_{i,t,t+j}, j \in (0,1,2,3)$ to maximize equation (1.13) subject to equation (1.14) and in the case of borrowing constraints subject equation (1.15).
2. The government sets taxes and expenditures to satisfy equations (1.16) and (1.17).
3. Human capital accumulates as in equations (1.9)-(1.12).
4. Surpluses and shortages in the goods market are accommodated by the international bond market.
5. The labor market clears in each period.

Solving the model

Since budget constraints will bind in equilibrium we define $x_{i,t-4} = c_{i,t-4,t+j}, j \in \{0,1,\dots,10\}$ $f_{i,t,t+j}, j \in \{0,1,2,3\}$ and write the agent's problem as the following Lagrangian.

$$L = \max_{x_{i,t-4}} \sum_{j=0}^{10} \beta^j \frac{c_{i,t-4,t+j}^\sigma}{\sigma} + \xi \frac{h_{i,t}^\sigma}{\sigma} + \lambda \left(\sum_{j=0}^8 \left(\frac{z}{r}\right)^j w h_{i,t} (1 - \tau) - \sum_{j=0}^{10} \frac{c_{i,t-4,t+j}^\sigma}{r^j} - \sum_{j=0}^3 \frac{f_{i,t,t+j}}{r^j} \right).$$

As mentioned above, there are reduced sets of choices for the 10 oldest original generations. First order conditions reduce to

$$\sum_{j=0}^{10} \frac{c_{i,t-4,t+j}^\sigma}{r^j} = \sum_{j=0}^8 \left(\frac{z}{r}\right)^j w h_{i,t} (1 - \tau) - \sum_{j=0}^3 \frac{f_{i,t,t+j}}{r^j}$$

$$c_{i,t-4,t} = (r\beta)^{\left(\frac{j}{\sigma-1}\right)} c_{i,t-4,t+j}, j \in \{1,2 \dots 10\}$$

$$h_{i,t} = c_{i,t-4,t} \left(\xi \frac{\partial h_{i,t}}{\partial e_{i,t}} \frac{\partial e_{i,t}}{\partial i_{i,t,e}} \frac{\partial i_{i,t,e}}{\partial f_{i,t,t}} \right)^{\frac{1}{1-\sigma}}$$

$$h_{i,t} = c_{i,t-4,t+j} \left(\frac{\xi}{\beta^j} \frac{\partial h_{i,t}}{\partial e_{i,t}} \frac{\partial e_{i,t}}{\partial i_{i,t,l}} \frac{\partial i_{i,t,l}}{\partial f_{i,t,t+j}} \frac{\partial i_{i,t,t+j}}{\partial f_{i,t,t+j}} \right)^{\frac{1}{1-\sigma}}, j \in \{1,2 \dots 10\}$$

In general, the model must be solved numerically. We generate an $N \times T$ matrix of errors drawn from a normal distribution with standard deviation 0.775 and mean 0. Here T is large and is the total number of time periods in which new agents enter the economy and N is the number of agents born in each period. We also create an $N \times 3$ matrix of original lagged values of $a_{i,t}$. We use these matrices in equation (1.1) to generate an $N \times T$ matrix of $a_{i,t}$ values.

The first order conditions for agent i can be reduced to a system of four equations in $f_{i,t-4,t+j}, j \in \{0,1,2,3\}$. In order to choose family spending in each of these four periods, the agents must know government spending in each period of her offspring's childhood. In the initial period, the human capital and thus the income of all adults is known; i.e. specified as an initial condition. For a given tax rate then, total revenue collected is known and for a given distribution rule government spending on all students is known. The parents of the N children initially in the final stage of late childhood need only to choose family spending in this period; i.e. $f_{i,0-7,0}$. Upon solving for spending for each agent, the income of their offspring in the subsequent period is determined since previous inputs to their human capital are specified as initial conditions. Since these will be the only entrants to the labor market in period 1, income of all workers in period 1 is known. From this we can find government spending on all agents in period 1. The parents of children initially in the second period of late childhood now have all information required to solve for their family spending in periods 0 and 1; i.e. $f_{i,0-6,0+j}, j \in \{0,1\}$. This will be sufficient to know income and thus government spending in period 2 as their offspring enter the labor force. Thus the parents of the agents initially in the first period of late childhood can solve for family spending in the three remaining periods of their offspring's childhood; i.e. $f_{i,0-5,0+j}, j \in \{0,1,2\}$. This will give income and thus government spending in

period 3, allowing the parents of agents initially in early childhood to solve for $f_{i,0-4,0+j}, j \in \{0,1,2,3\}$ given their own human capital and the ability level of their offspring.

Beyond this initial period, we only need to solve for the decisions of one generation in each period. For this generation we solve for their spending across four periods of financing education. Specifically, the parents of the agents in early childhood observe the productivity of their offspring and then choose $f_{i,t-4,t+j}, j \in \{0,1,2,3\}$. They are able to do this because government spending on their offspring will depend only on the income of adults in the current and subsequent three periods. Since we know the family spending education of the preceding generations, we know the income of the current adults and those who will become adults over the next three periods while the current young are in school. With this we can find government spending in each period that they are in school.

In this way, we calculate $f_{i,t-4,t+j}, j \in \{0,1,2,3\}$ for all $t \in \{0,1,2 \dots T\}$ sequentially given our matrix of a_{it} values. With this and government spending known, we can find other items such as consumption from equation (1.18). To eliminate the effects of our choice of starting values, we consider only the final T' periods in our calculations. We choose T' such that only periods where the economy is in a stochastic steady state are considered. In a stochastic steady state, the income of each generation varies through time as an endogenous response to the stochastic stream of abilities. However the distribution of income is consistent through time and tends to stability as N and T' become large. We use the generated data to compute income persistence and other items of interest.

Appendix 2

I first provide more precise statements regarding human capital formation, agents' decision rule and government budget balance and then define a general equilibrium with full notations. The last part of this appendix describes the computation procedure.

Restating the model

The measure of education expenditures in early and late childhood can be written as

$$i_{i,t,e} = (f_{i,t,e}^\eta + g_{t,e}^\eta)^{\frac{1}{\eta}}, \quad i_{i,t,l} = (f_{i,t,l}^\eta + g_{t,l}^\eta)^{\frac{1}{\eta}} \quad (2.11)$$

where $i_{i,t,e}$ is the measure of education expenditures in early childhood (represented by e) on behalf of the generation t member (born in period t) of dynasty i . This is a function of private

spending, $f_{i,t,e}$ and government spending, $g_{t,e}$. Since government spending is uniform across dynasties in each period, there is no subscript i in $g_{t,e}$. This child transition into late childhood (represented by l) in the next period when the measure of education expenditures becomes $i_{i,t,l}$. This is again a function of private spending, $f_{i,t,l}$, and government spending, $g_{t,l}$.

With this more precise notation, equation (2.2) is

$$e_{i,t} = (\gamma i_{i,t,e}^\Phi + (1 - \gamma) i_{i,t,l}^\Phi)^{\frac{1}{\Phi}} \quad (2.12)$$

and this combines with this agent's innate ability, a_i , parental human capital, $h_{i,t-2}$, and parental time investment received in early childhood, $t_{i,t}$ to generate the human capital in equation (2.1)

$$h_{i,t} = \left((a_i e_{i,t})^\theta + (h_{i,t-2} t_{i,t})^\theta \right)^{\frac{1}{\theta}}. \quad (2.13)$$

The generation $t-2$ member of dynasty i chooses time investment in children's early education, $t_{i,t}$. With one unit of time endowment in each period, the agent's labor supply in early parenthood is $1 - t_{i,t}$ accordingly. Give the wage function specified as below

$$w_{i,t-2,j} = h_{i,t-2} e^{\alpha x_{i,t-2,j}}, j \in (1,2,3) \quad (2.14)$$

where $w_{i,t-2,j}$ is the generation $t-2$ member of dynasty i 's wage in the j^{th} period of adulthood and $x_{i,t-2,j}$ is the agent's accumulated work experience at the beginning of that period, the life-time income of the agent is $w_{i,t-2,1}(1 - t_{i,t}) + w_{i,t-2,2} + w_{i,t-2,3}$.

The generation $t-2$ member of dynasty i also chooses consumption in each period of adulthood, $c_{i,t-2,j}, j \in \{1, 2, 3\}$ and family education spending early childhood, $f_{i,t,e}$ and family education spending in late childhood, $f_{i,t,l}$. The choices are made to maximize

$$\sum_{j=1}^3 \beta^{j-1} \frac{c_{i,t-2,j}^\sigma}{\sigma} + \xi \frac{h_{i,t}^\sigma}{\sigma}. \quad (2.15)$$

subject to

$$\sum_{j=1}^3 \frac{c_{i,t-2,j}}{r^{j-1}} + f_{i,t,e} + \frac{f_{i,t,l}}{r} = \left(w_{i,t-2,1}(1 - t_{i,t}) + \frac{w_{i,t-2,2}}{r} + \frac{w_{i,t-2,3}}{r^2} \right) (1 - \tau), \quad (2.16)$$

$$c_{i,t-2,1}, c_{i,t-2,2}, c_{i,t-2,3} \geq 0,$$

$$f_{i,t,e}, f_{i,t,l} \geq 0,$$

$$t_{i,t} \in [0, 1].$$

As mentioned earlier, there are $2N$ dynasties in the economy. In any given period t , N dynasties have agents in the first and third period of adulthood. Their total income is

$\sum_{i=1}^N (w_{i,t-2,1}(1 - t_{i,t}) + w_{i,t-4,3})$. The other N dynasties have agents in the second period of adulthood and their total income $\sum_{i=N+1}^{2N} w_{i,t-3,2}$. Therefore, output in period t is

$$Y = \sum_{i=1}^N (w_{i,t-2,1}(1 - t_{i,t}) + w_{i,t-4,3}) + \sum_{i=N+1}^{2N} w_{i,t-3,2} \quad (2.17)$$

With G_t being government expenditures, the budget relationship is

$$Ng_{t,e} + Ng_{t,l} = \tau Y_t = G_t. \quad (2.18)$$

Steady state equilibrium

A steady state equilibrium is comprised of the sets of agents' allocational and educational outcomes $\{c_{i,t-2,j}, j \in \{1, 2, 3\}, f_{i,t,e}, f_{i,t,l}, t_{i,t}, h_{i,t-2}, h_{i,t}\}$ for all $i \in \{1, 2, \dots, 2N\}$ and all $t \geq 0$, and government policy parameters $\{\tau, g_{t,e}, g_{t,l}\}$ for all $t \geq 0$ such that

1. Human capital formations satisfy equations (2.10)-(2.13).
2. Taking own human capital, $h_{i,t-2}$, child's ability, a_i , and government policy as given, the generation $t-2$ member of dynasty i chooses $c_{i,t-2,j}, j \in \{1, 2, 3\}, f_{i,t,e}, f_{i,t,l}, t_{i,t}$ to maximize equation (2.15) subject to equation (2.16).
3. The government sets taxes and expenditures to satisfy equations (2.17) and (2.18).
4. Surpluses and shortages in the goods market are accommodated by the international bond market.
5. The labor market clears in each period.
6. $h_{i,t-2} = h_{i,t}$ and similarly other generation specific variables are constant.

Solving the model

I define $y_{i,t-2} = c_{i,t-2,j}, j \in \{1, 2, 3\}, f_{i,t,e}, f_{i,t,l}, t_{i,t}$ and write the agent's problem as the following Lagrangian

$$L = \max_{y_{i,t-2}} \sum_{j=1}^3 \beta^{j-1} \frac{c_{i,t-2,j}^\sigma}{\sigma} + \xi \frac{h_{i,t}^\sigma}{\sigma} \\ + \lambda_i \left(w_{i,t-2,1}(1 - t_{i,t}) + \frac{w_{i,t-2,2}}{r} + \frac{w_{i,t-2,3}}{r^2} \right) (1 - \tau) - \sum_{j=1}^3 \frac{c_{i,t-2,j}}{r^{j-1}} - f_{i,t,e} - \frac{f_{i,t,l}}{r}.$$

First-order conditions reduce to

$$\sum_{j=1}^3 \frac{c_{i,t-2,j}}{r^{j-1}} = \left(w_{i,t-2,1}(1 - t_{i,t}) + \frac{w_{i,t-2,2}}{r} + \frac{w_{i,t-2,3}}{r^2} \right) (1 - \tau) - f_{i,t,e} - \frac{f_{i,t,l}}{r} \quad (2.19)$$

$$c_{i,t-2,1} = (r\beta)^{\frac{j}{\sigma-1}} c_{i,t-2,j}, j \in \{2,3\} \quad (2.20)$$

$$h_{i,t} = c_{i,t-2,1} \left(\xi \frac{\partial h_{i,t}}{\partial e_{i,t}} \frac{\partial e_{i,t}}{\partial i_{i,t,e}} \frac{\partial i_{i,t,e}}{\partial f_{i,t,e}} \right)^{\frac{1}{1-\sigma}} \quad (2.21)$$

$$h_{i,t} = c_{i,t-2,1} \left(r \xi \frac{\partial h_{i,t}}{\partial e_{i,t}} \frac{\partial e_{i,t}}{\partial i_{i,t,l}} \frac{\partial i_{i,t,l}}{\partial f_{i,t,l}} \right)^{\frac{1}{1-\sigma}} \quad (2.22)$$

$$h_{i,t} = c_{i,t-2,1} \left(\xi \frac{\partial h_{i,t}}{\partial i_{i,t}} \right)^{\frac{1}{1-\sigma}} \left(w_{i,t-2,1} - \frac{\partial w_{i,t-2,2}}{\partial t_{i,t}} - \frac{\partial w_{i,t-2,3}}{\partial t_{i,t}} \right)^{\frac{1}{1-\sigma}} \quad (2.23)$$

For the stationary equilibrium, I choose and update $f_{i,t,e}, f_{i,t,l}, t_{i,t}, h_{i,t-2}$ until convergence. More detailed steps are as follows

1. Take initial guesses of $f_{i,t,e}, f_{i,t,l}, t_{i,t}, h_{i,t-2}$.
2. Given $t_{i,t}$ and $h_{i,t-2}$, obtain $g_{t,e}$ and $g_{t,l}$ from the government budget relationship.
3. Given $a_i, f_{i,t,e}, f_{i,t,l}, t_{i,t}, h_{i,t-2}, g_{t,e}$ and $g_{t,l}$, obtain $h_{i,t}$ from human capital formation and $c_{i,t-2,j}, j \in \{1,2,3\}$ from equations (2.19)-(2.20).
4. Check the first-order conditions in equations (2.21)-(2.23) and convergence, $h_{i,t} = h_{i,t-2}$. If all equations hold, stop; otherwise update initial guesses and iterate from step 1 until convergence.