THE ROBUSTNESS OF CONFIDENCE INTERVALS FOR EFFECT SIZE IN ONE WAY DESIGNS WITH RESPECT TO DEPARTURES FROM NORMALITY

by

DAVID HEMBREE

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Department of Statistics
College of Arts and Sciences

KANSAS STATE UNIVERSITY
Manhattan, Kansas

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Approved by:

Major Professor
Dr. Paul Nelson

Abstract

Effect size is a concept that was developed to bridge the gap between practical and statistical significance. In the context of completely randomized one way designs, the setting considered here, inference for effect size has only been developed under normality. This report is a simulation study investigating the robustness of nominal 0.95 confidence intervals for effect size with respect to departures from normality in terms of their coverage rates and lengths. In addition to the normal distribution, data are generated from four non-normal distributions: logistic, double exponential, extreme value, and uniform.

The report discovers that the coverage rates of the logistic, double exponential, and extreme value distributions drop as effect size increases, while, as expected, the coverage rate of the normal distribution remains very steady at 0.95. In an interesting turn of events, the uniform distribution produced higher than 0.95 coverage rates, which increased with effect size. Overall, in the scope of the settings considered, normal theory confidence intervals for effect size are robust for small effect size and not robust for large effect size. Since the magnitude of effect size is typically not known, researchers are advised to investigate the assumption of normality before constructing normal theory confidence intervals for effect size.

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Acknowledgements

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I would also like to thank the members of my committee, Dr. Gary Gadbury and Dr. Abigail Jager. Both of these professors are incredible teachers, and they were extremely helpful to me during my pursuit of this degree.

Chapter 1

Introduction to Effect Size

Effect size is a concept that was developed to bridge the gap between practical and statistical significance. Consider, for example, the problem of comparing *t* treatments based on a completely randomized, one-way design. It is often assumed that the observations from the i^{th} treatment are normally distributed with mean μ_i (for $i = 1, 2, \ldots, t$), and that all of the distributions have a common unknown variance, denoted σ^2. The treatments are compared by testing the following hypotheses:

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_t \qquad \text{vs.} \qquad H_1 : \text{at least two means differ} \tag{1.1}$$

A problem with this approach is that it is almost always known a priori that H_0 is false, and rejecting it does not directly address the following question:

> Are differences among the responses to the treatments large enough to be of practical value? (1.2)

For example, a new treatment, in the context described above, that extends mean human life by one hour, so that H_0 is false, would generally not be considered an improvement. Confidence intervals for contrasts among the means do aid in answering the question raised in (1.2). However, the sizes of these intervals depend on the scale of measurement, e.g. feet, miles, kilometers, and when *t* is more than three, do not, in general, provide a simple, concise answer to (1.2). Cohen (1988), Murphy and Myors (2004), and Steiger (2004), among others, when sample sizes are equal and denoted by *n*, proposed constructing a confidence interval for what is commonly called *effect size*, defined in the setting described above by

$$ES = \sum_{i=1}^{t} \frac{(\mu_i - \overline{\mu})^2}{\sigma^2}. \tag{1.3}$$

Note that *nES* is the location-scale invariant, non-centrality parameter of the distribution of $F = \dfrac{MST}{MSE}$, the statistic used to test (1.1) based on independent random samples of common sample size *n* from normal distributions having the same unknown variance. Cohen (1988), using a combination of empirical studies and subjective judgment, proposed benchmark values for *ES*, denoting small, middle, and large effect sizes. Although power and effect size are related under normality, they are inherently different concepts. A test based on very large samples may have high power for detecting a small difference among the means in a setting where the effect size is very small. Some psychologists have advocated estimating effect size instead of testing for equality of means. The standard procedure for constructing a confidence interval for *ES*, described in chapter 2, is valid under the assumption of normality. The purpose of this report is to assess the performance of these intervals when normality does not in fact hold.

The following is a concrete example showing that a test based on a large sample size can have high power for detecting negligible differences among the means when the effect size is small.

Example 1

Suppose $X_i \sim N(\mu_i, \sigma^2)$ for $i = 1,2,3$. Let $\mu_1 = 100.1$, $\mu_2 = 100.2$, $\mu_3 = 100.3$. Note that in this situation, $ES = \dfrac{0.02}{\sigma^2}$. Letting $\sigma = 1$ so that $ES = 0.02$, the overlapping densities in Figure 1.1 show that responses sampled from three different normal distributions can be very similar when the means differ but the effect size is small. To illustrate the misleading inferences that could be drawn in this admittedly extreme and artificial case, I used R to generate independent random samples from these distributions, each of size n = 428, a value arrived at by trial and error to cause rejection of the hypothesis of equal means at the 0.05 type I error rate. The data are summarized in the almost coincident side by side boxes of the box plot in Figure 1.2 and given in their entirety in Appendix E. The normal theory test for equality of means yields *F* = 3.029, corresponding to a p-value of 0.0487. Using the method described in chapter 2, a 95% confidence interval for effect size is (0.000, 0.039), indicating a small effect according to conventional standards.

Figure 1.1

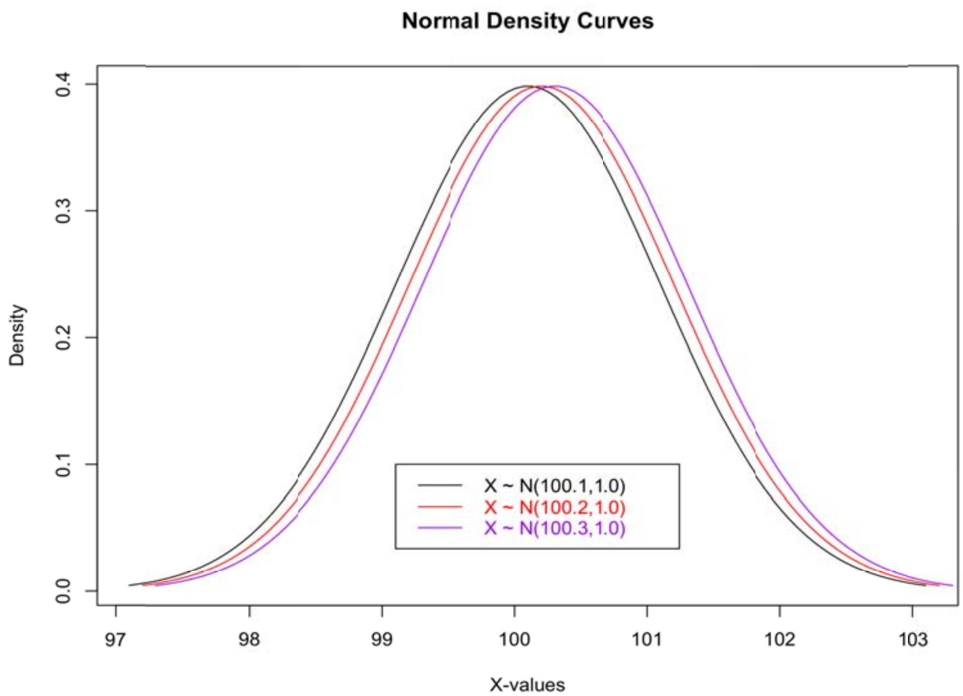
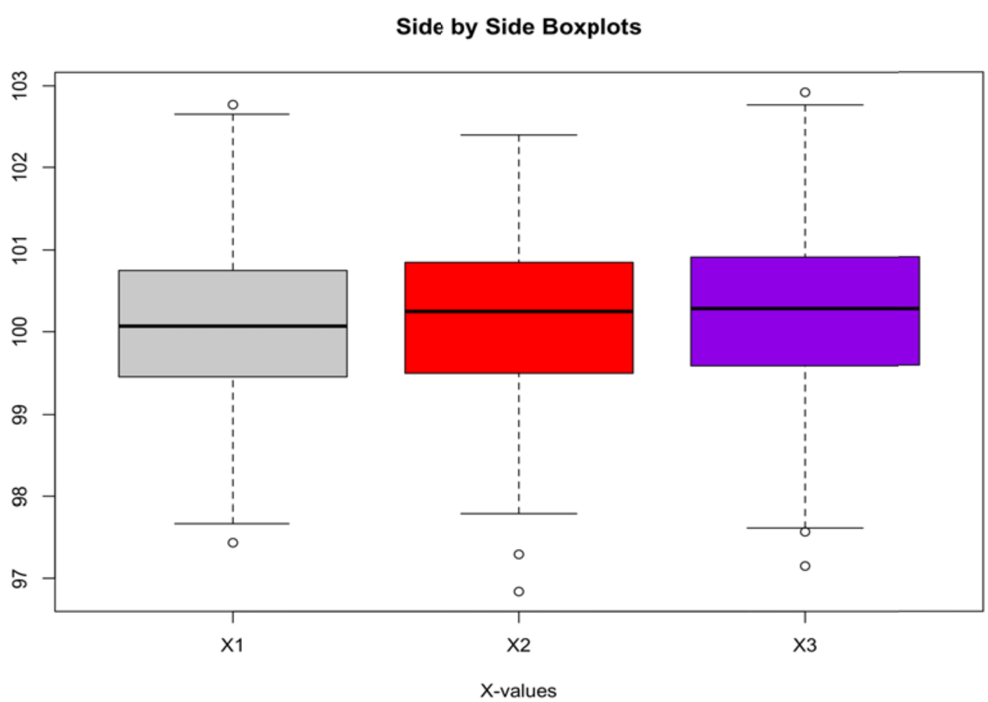


Figure 1.2



Side by Side Box Plots
Independent Random Samples from N(100.1,1), N(100.2,1) , N(100.3,1)
n = 428 Observations from each Distribution

Now we consider what happens when the standard deviation σ is decreased from 1.0 to 0.1. Again, I generated n = 428 independent random samples from the normal distributions with the same means, but smaller standard deviation = 0.1. The densities are plotted in Figures 1.3 and 1.4, and the data, summarized in Figure 1.5, is given in its entirety in Appendix E. These plots illustrate that the curves have now separated, indicating an increasing probability that the responses of the three populations will systematically differ in the same order as the means. The normal theory test for equality of means yields $F = 432.0376$, corresponding to a p-value essentially equal to 0.0. The 95% confidence interval for effect size becomes (1.559, 2.114), indicating a much larger effect size, according to conventional standards, than when standard deviation was equal to 1.0.

Figure 1.3

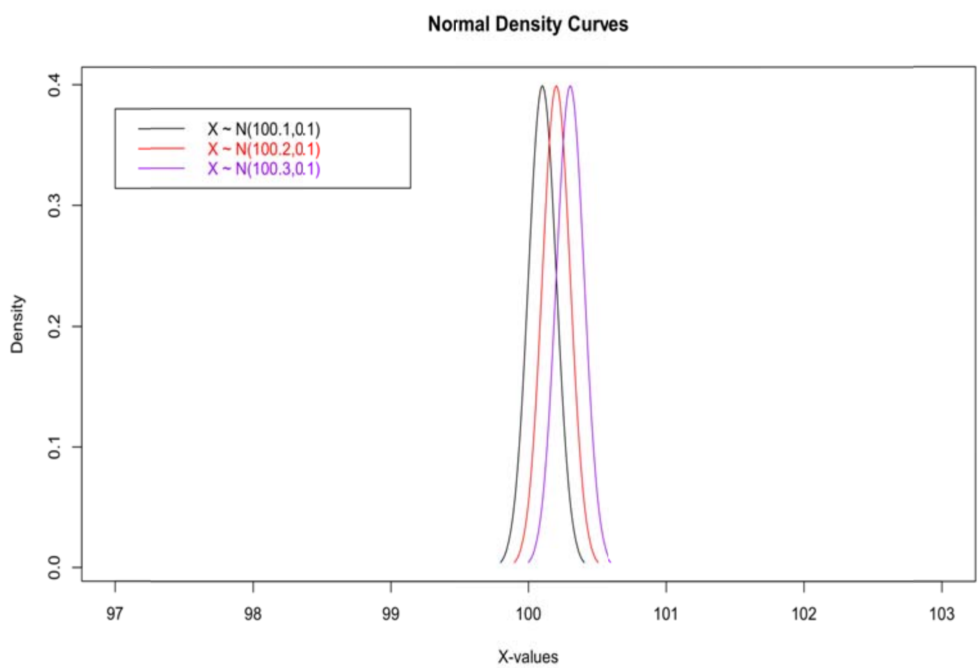


Figure 1.3 has the same axis notation as the case when σ = 1.0. You can already notice the separation among the three distributions. However, when the picture is blown up, you can especially see the separation among the curves.

Figure 1.4

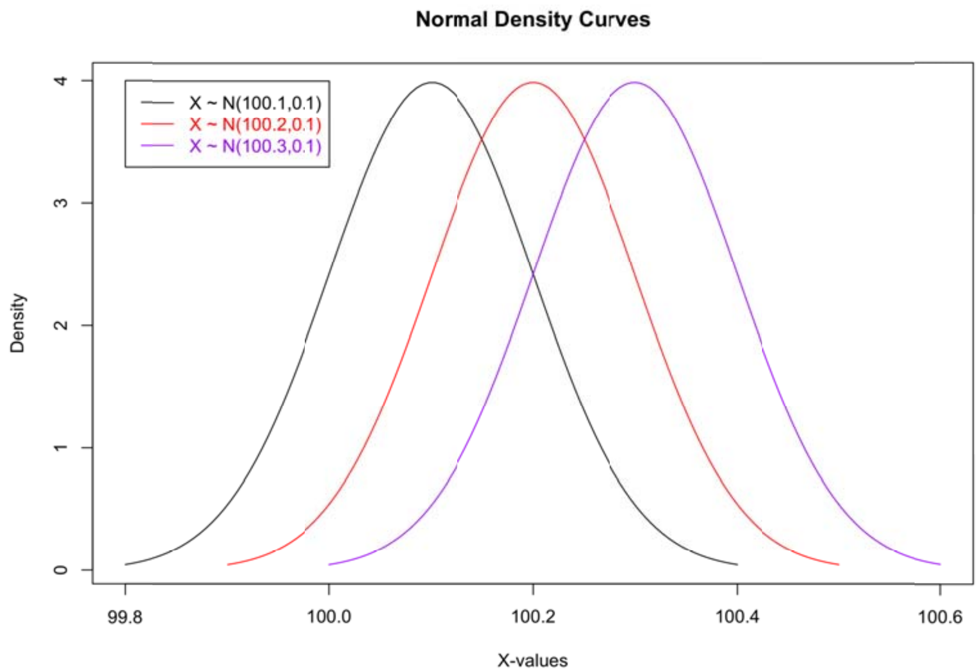
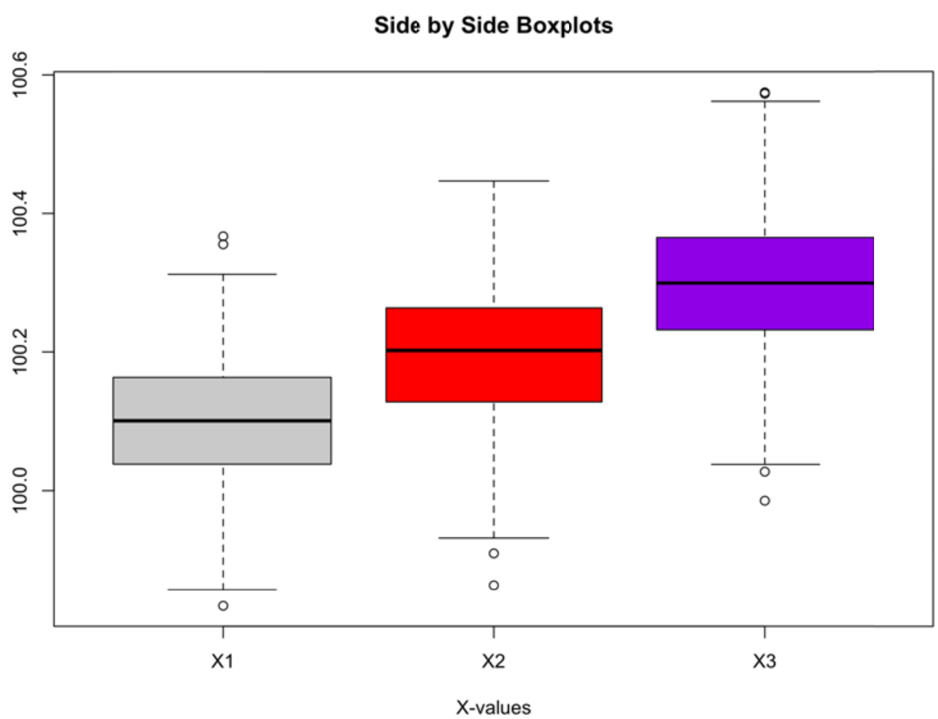


Figure 1.5



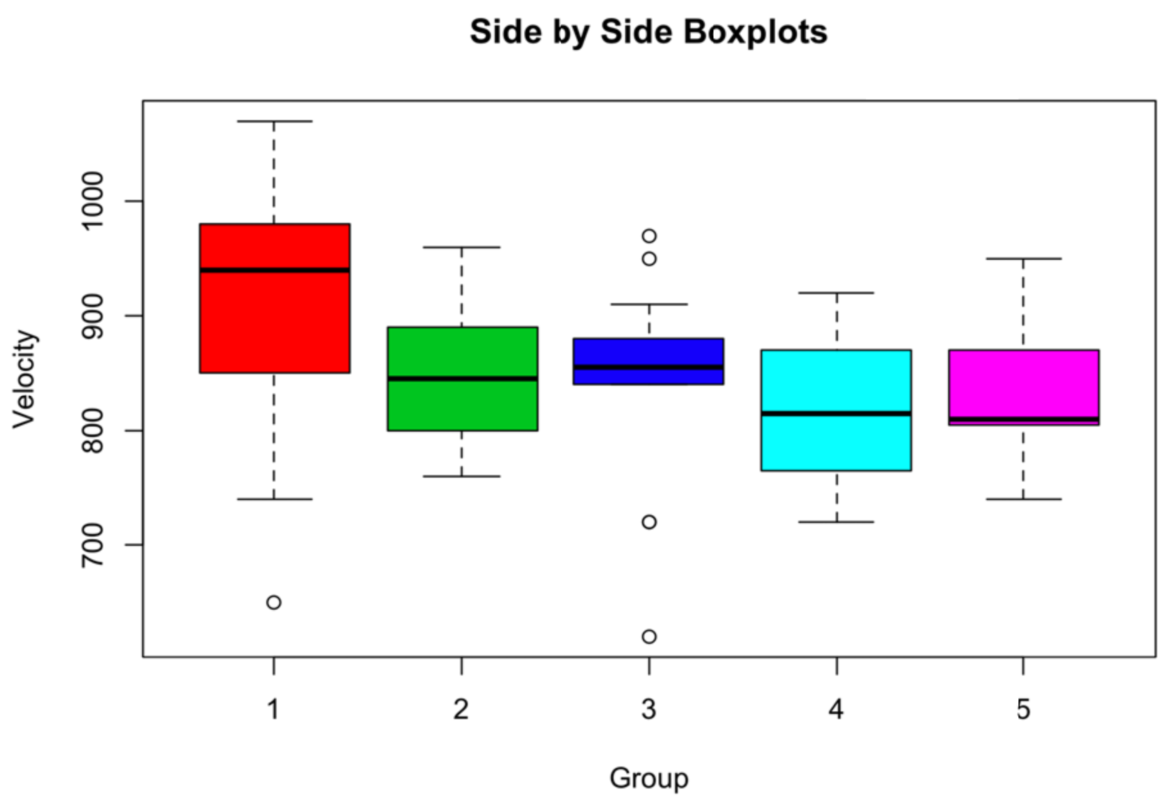
Side by Side Box Plots
Independent Random Samples from N(100.1,0.1), N(100.2,0.1) , N(100.3,0.1)
n = 428 Observations from each Distribution

The following example, based on data taken from Stigler (2007), illustrates how complicated assessing the separation among distributions can be. The data, given in Appendix F, consists of one hundred coded determinations of the speed of light divided into $t = 5$ groups.

Example 2

Figure 1.6 presents side by side box plots of the 5 groups. These plots show considerable overlap among several of the groups and separation among others. The F-test for the equality of means gives a test statistic $F = 4.2878$, yielding a p-value of 0.0031. This implies that there is strong evidence for a difference between the means of at least two of the groups. However, again using the methodology presented below, a 95% confidence interval for effect size in this example is (0.1092, 1.7276), typically considered a range of small to moderate effect sizes. Note that these results assume normality and equal variances, the latter being questionable.

Figure 1.6



Chapter 2

Confidence Intervals for Effect Size

Under normality and equal sample sizes, the standard method for constructing a confidence interval for *ES* is a special case of the method known as *inverting a test*, which in the general case is carried out as follows.

Suppose interest lies in constructing a confidence interval for the parameter θ lying in Θ and for all $\theta_0 \in \Theta$ there is an exact, non-randomized size α test of

$$H_0 : \theta = \theta_0 ,$$

$$H_1 : \theta \neq \theta_0 ,$$

given by the rule: Having observed $\underline{X} = \underline{x}$, reject H_0 if and only if $\underline{x} \in C_{\theta_0}$, where C_{θ_0} is a subset of the sample space called the rejection region of the test. Thus, for all $\theta \in \Theta$, $P_\theta(\underline{X} \in C_\theta) = \alpha$. Correspondingly, for all $\underline{x}$ in the sample space, let $A_{\underline{x}} = \{\theta, \underline{x} \in C_\theta^c\} \subset \Theta$, where C_θ^c denotes the complement of C_θ. Then, for all $\theta \in \Theta$, $P_\theta(\theta \in A_{\underline{X}}) = P_\theta(X \in C_\theta^c) = 1-\alpha$, which asserts that $A_{\underline{x}}$ is a $1-\alpha$ confidence set for θ.

Using this procedure to invert the *F*- test for $\theta = ES$, having observed $F = F_0$ based on independent random samples of size *n* from normal distributions having the same unknown variance, a *two* sided, $1 - \alpha$ confidence interval [L,U] for *nES* is obtained by solving the equations:

$$H(F_0; t-1, t(n-1), L) = \frac{\alpha}{2} \tag{2.1}$$

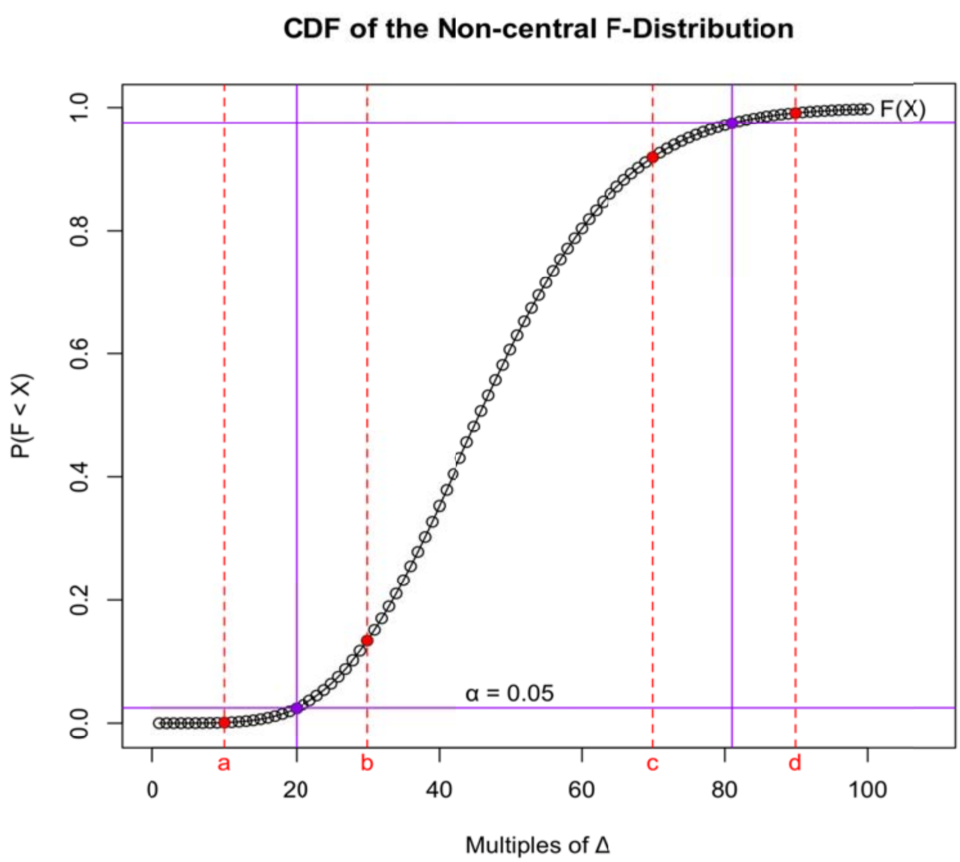
$$H(F_0; t-1, t(n-1), U) = 1-\frac{\alpha}{2}$$

where $H(x; t-1, t(n-1), nES) = P(F \leq x)$.

Inverting a Test

In order to achieve this, one must find the points where the distribution of $F = \dfrac{MST}{MSE}$ reaches $\dfrac{\alpha}{2}$ and $1-\dfrac{\alpha}{2}$. One of the most efficient ways of accomplishing this is through the method of bisection. Since F(x), the cumulative density function of the non-central F-distribution, is an increasing function of its noncentrality parameter for fitted x, we can start with a multiple of Δ (point a on figure 2.1) such that $F(a) < \frac{\alpha}{2}$. Now find a multiple of Δ (point b on figure 2.1) such that $F(b) > \frac{\alpha}{2}$. Therefore, the multiple of Δ where the distribution of $F = \frac{MST}{MSE}$ reaches $\frac{\alpha}{2}$ will be between *a* and *b*. Now, if $F\left(\frac{a+b}{2}\right) > \frac{\alpha}{2}$, then let *a* remain the same and $b = \frac{a+b}{2}$. However, if $F\left(\frac{a+b}{2}\right) < \frac{\alpha}{2}$, then let $a = \frac{a+b}{2}$ and *b* remain the same. Repeat this process as many times as desired. When iterated *m* times, the final estimation will have error less than 2^{-m}. We can find the point where the distribution of $F = \frac{MST}{MSE}$ reaches $1 - \frac{\alpha}{2}$ in a similar manner (see figure 2.1 with points *c* and *d*). Finally, divide these two confidence bounds by *n* to obtain a $1-\alpha$ confidence interval for *ES*.

Figure 2.1



Chapter 3

Report Topic

I carried out a simulation study of the *robustness* of the intervals in (2.1) with respect to departures from normality. Specifically, I investigated the performance of the intervals in terms of mean length, median length, and coverage rate when the data are sampled from the normal and non-normal, location-scale families of densities $\{f^{(i)}\}$ having finite variances, where in each case the scale parameter is functionally independent of the location parameter, denoted by μ_f. In this general setting, I define effect size by

$$ES^{(f)} = \sum_{i=1}^{t} \frac{(\mu_i^{(f)} - \overline{\mu}^{(f)})^2}{\mathrm{var}(f)}. \tag{3.1}$$

The normal family was included to provide a basis of comparison.

Coverage Rate: Let I be a confidence interval for a parameter θ such that under assumptions A, $P_A(\theta \in I) = 1 - \alpha$. Suppose that conditions B, under which the data are sampled, differ from A. Then, under B, $1 - \alpha$ is the *nominal* coverage rate of I and $P_B(\theta \in I)$ is the *actual* coverage rate. Similarly, mean length is defined by $E_B(U\text{-}L)$.

Models: Let X_{ij} denote the random variable representing the jth observation in treatment *i*, for $i = 1, 2, \ldots, t$; $j = 1, 2, \ldots, n$ and $f_i(\cdot)$ its continuous density function. Assume that $\{X_{ij}\}$ are jointly independent with

$$f_i(x;\mu,\sigma) = \frac{1}{\sigma} g\left(\frac{x-\mu_i}{\sigma}\right), \tag{3.2}$$

where $g(\cdot)$ is a known density function, σ is an unknown positive scale parameter, and $\{\mu_i\}$ are unknown location parameters. When $g(\cdot)$ is a standard normal density, assumption A, the interval in (2.1) is exact. I will use simulation to study the behavior of the intervals in (2.1) when assumptions B hold:

B: $g(\cdot)$ is logistic, double exponential, extreme value and uniform.

Densities Used for Simulation Study

- Normal: $$f(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\},$$
- Logistic: $$f(x;\mu,\theta)=\frac{\exp\left\{-(x-\mu)^2/\theta\right\}}{\theta(1+\exp\{-(x-\mu)/\theta\})^2},$$
- Double Exponential: $$f(x;\mu,\beta)=\frac{1}{2\beta}\exp\left\{\frac{|x-\mu|}{\beta}\right\},$$
- Extreme Value: $$f(x;\mu,\omega)=\frac{1}{\omega}\exp\left[\left(\frac{x-\mu}{\omega}\right)-\exp\left(\frac{x-\mu}{\omega}\right)\right],$$
- Uniform: $$f(x;\mu,\gamma)=\frac{1}{2\gamma}I_{(-1,1)}\left(\frac{x-\mu}{\gamma}\right).$$

In order to facilitate comparisons among these distributions, the scale parameters of the logistic, double exponential, extreme value and the uniform distributions will be selected so that they have the same inter-quartile range as the corresponding normal distribution. The scale parameters of each distribution are described in terms of σ, the standard deviation of the normal distribution, in the following table. In addition, the variances are given, which is the denominator of *ES* for each distribution.

Table 3.1

Distribution	Inter-Quartile Range	Scale Parameter	Variance
Normal	1.349σ	σ	σ^2
Logistic	2.197θ	θ = (1.349 / 2.197)σ	$(1.349\sigma/2.197)^2*(\pi^2/3)$
Double Exponential	2ln(2)β	β = [1.349 / 2ln(2)]σ	$2*(1.349\sigma/2\ln(2))^2$
Extreme Value	ω[ln(ln(4))-ln(ln(4/3))]	ω = (1.349 / 1.5725)σ	$(1.349\sigma/1.5725)^2*(\pi^2/6)$
Uniform	0.5γ	γ = (2*1.349)σ	$(2*1.349)^2*\sigma^2$ / 12

Description of the Simulation Study

First, I selected parameter settings for my simulation experiment using a factorial design. I decided to look specifically at six different effect sizes, *ES* = 0.3, *ES* = 0.5, *ES* = 1.0, *ES* = 5.0, *ES* = 10.0, and *ES* = 20.0. In order to understand how the number of treatments (t) effects coverage rate, I looked at three different numbers of treatments, t = 2, t = 3, and t = 5. Without loss of generality, I took the scale parameter to be one (σ = 1) so that the common interquartile range of 1.35 provides a rough benchmark for assessing confidence interval length. Due to time constraints, I was only able to look at $\alpha = 0.05$. In a future study, one may want to include $\alpha = 0.10$ and $\alpha = 0.01$ to see if it effects coverage rates. In addition, I took the population means to be equally spaced starting with $\mu_1 = 0$ such that the effect size is equal to the six numbers listed above. Finally, I looked at five different sample sizes (n) per treatment combination, $n = 5$, $n = 10$, $n = 20$, $n = 50$, and $n = 100$.

Next, I generated observations from the five distributions above using R, and computed the F-statistic for a one-way analysis of variance.

Having set these parameters, I conducted a simulation experiment in the form of a fully crossed three factor, factorial design with 1000 replicates for each of the ninety parameter settings. The use of a random number generator justifies the design as being completely randomized. Specifically, I generated $N = 1000$ independent data sets for each parameter setting, and constructed the interval in (2.1) for each data set. I then recorded the length of the interval, and whether or not it contained the true value of ES.

In the following chapter, I will summarize the results of the study in terms of estimated actual coverage rates, and estimated mean and median lengths of the confidence intervals for each distribution.

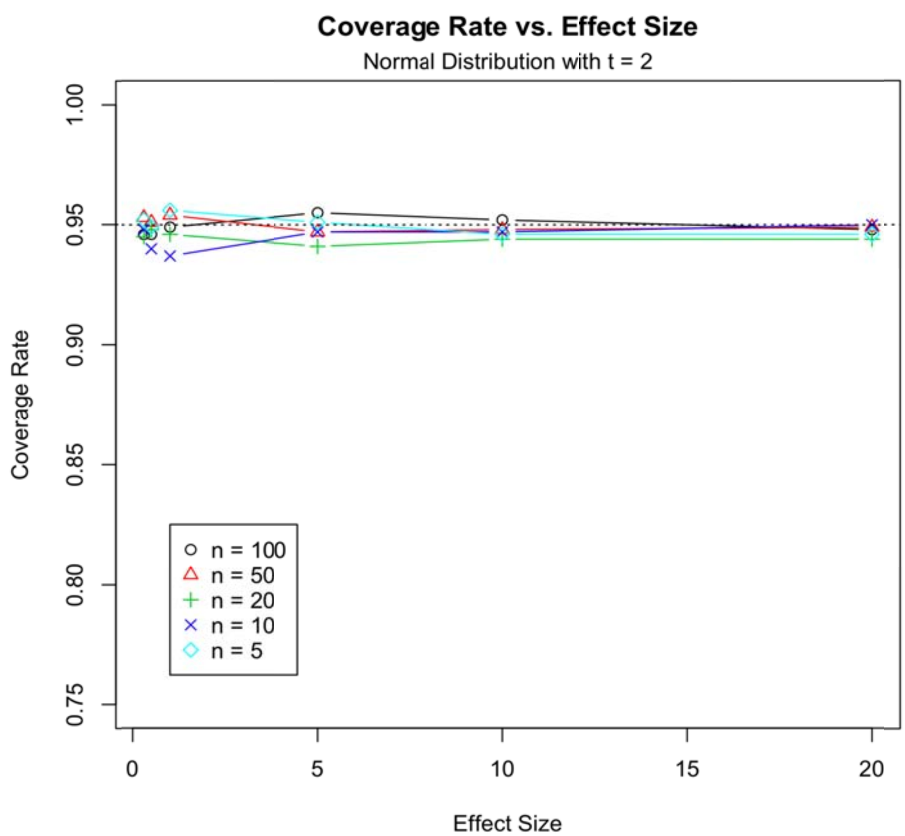
Chapter 4

My simulation results are summarized in 18 tables and 15 coverage rate plots. The tables and plots not mentioned in this chapter can be found in Appendix B and C, respectively. I will focus my discussion of the results on the case where the number of treatments (t) is equal to 2. The cases when $t = 3$ and $t = 5$ show similar results to those mentioned in this chapter.

Results for the Normal Distribution

As seen in Figure 4.1, the coverage rates for the normal distribution remained right around 0.95. This suggests that the coverage rate will stay around $1-\alpha$ regardless of the true value of effect size. The normal distribution was included in this study to provide a basis of comparison for the other four distributions.

Figure 4.1



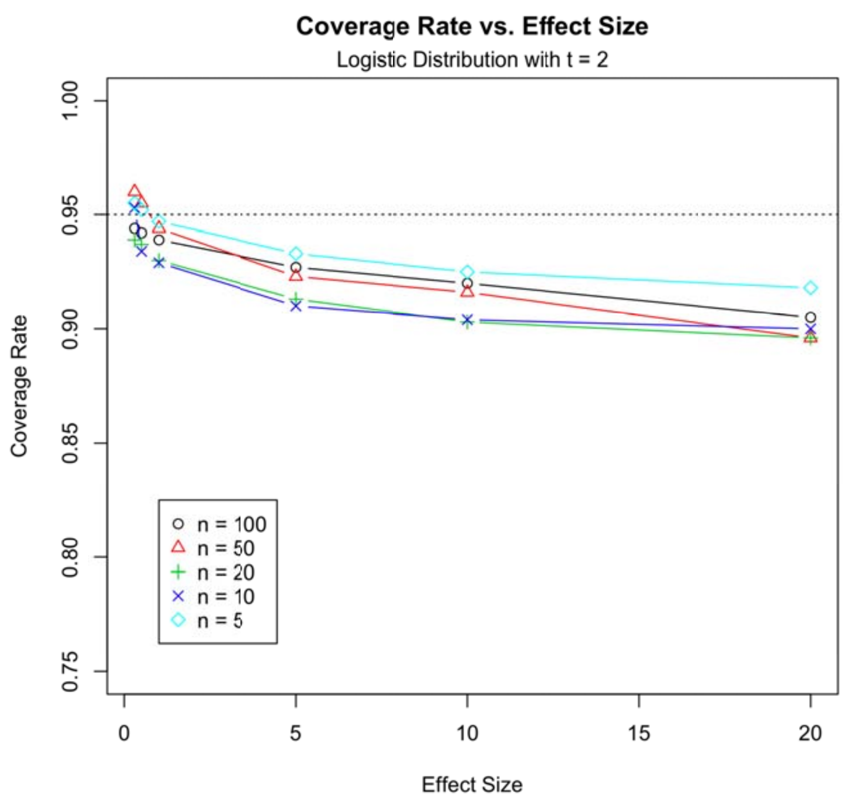
As can be seen in Appendix B, the mean and median confidence interval length decreases as effect size increases. When t = 2, the lowest coverage rate (0.937) was seen when *ES* = 1.0

and $n = 10$. In this case, the lower bound was too large to include *ES* 33 times, while the upper bound was not large enough to include *ES* 30 times. The mean interval length of the case when the lower bound was too large was equal to 7.5311. This interval was plenty large to include *ES*; it was simply located in the wrong place. Conversely, the mean interval length of the case when the upper bound was too small was equal to 0.7755. This would suggest that the length of the interval was simply too small to encompass the true value of *ES*.

Results for the Logistic Distribution

Estimated coverage rates for the logistic distribution appear to decrease as effect size increases, although they never fell lower than 0.880 (see Table B.12 in Appendix B). As seen in Figure 4.2, the coverage rate seems to drop pretty quickly as *ES* increases from 0.30 to 1.0. The decline continues from *ES* = 1.0 until *ES* = 5.0, however appears to have only a very slight decline for *ES* greater than 5.0.

Figure 4.2



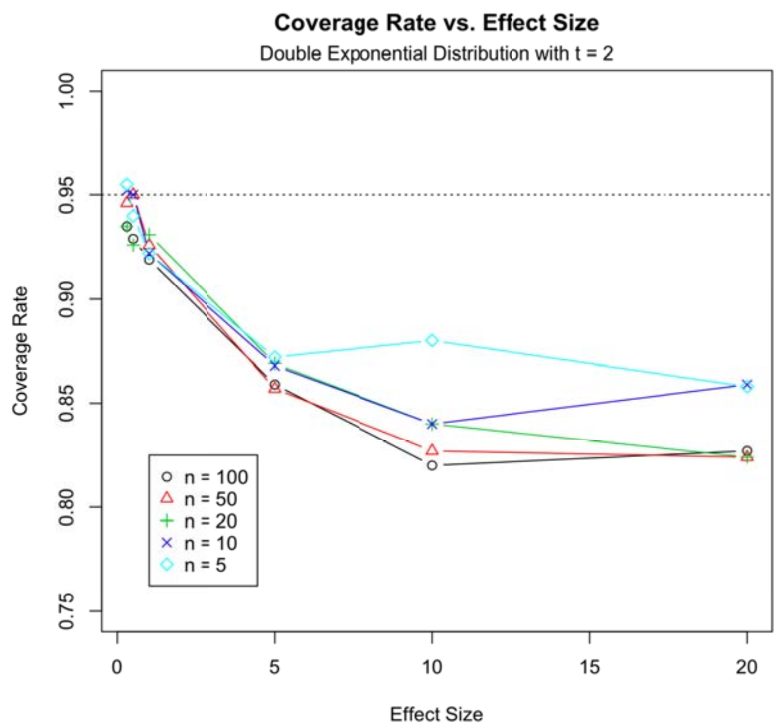
Similar to the normal distribution, the logistic distribution's average and median confidence interval lengths decrease as effect size increases. Looking only at $t = 2$, I noticed the smallest coverage rates occur when n = 50 and *ES* = 20 (coverage rate equal to 0.896). When the coverage rate dipped, was it because the intervals were too narrow or because they were located in the wrong place? The lower bound of the interval was not small enough to include *ES* 48 times, while the upper bound of the interval was not large enough to include *ES* 56 times. The mean of the confidence interval lengths of the 48 times when the lower bound was not small enough was 18.0085, while the mean of the confidence interval lengths of the 56 times when the upper bound was not large enough was 8.7714. Similarly, the overall mean interval length is 12.59614. Therefore, I conclude that when the lower bound was not small enough, the interval appeared to be plenty large, but simply located in the wrong place. Whereas, when the upper bound was not large enough to include *ES*, the interval was too small to include *ES* = 20 on a consistent basis.

Results for the Double Exponential Distribution

The double exponential distribution showed the lowest estimated coverage rates of any of the five distributions studied. The minimum estimated coverage rate seen was 0.796, which occurred when *ES* = 20, t = 3 and n = 100 (see Table B.12 in Appendix B). Similar to the logistic distribution, coverage rates for the double exponential appear to drop as effect size increases, as seen in Figure 4.3. The sharpest decline occurs when *ES* moves from 0.3 to 1.0, but it still drops quickly when *ES* moves from 1.0 to 5.0. With the exception of n = 5, the coverage rates continue to decrease as *ES* reaches 10.0, but they level off for all five sample sizes when *ES* shifts from 10.0 to 20.0.

When t = 2, the lowest coverage rate that occurred for the double exponential distribution was 0.820 when *ES* = 20 and n = 20. That is to say that the interval missed 180 times out of 1000. Specifically, the lower bound was not small enough to include *ES* 100 times, while the upper bound was not large enough to include *ES* 80 times. Of those 100 cases when the lower bound was not small enough to encompass *ES*, the average confidence interval length was 39.65146. These intervals were plenty large; they were simply located in the wrong place. Of the 80 cases when the upper bound was not large enough to include *ES*, the average confidence interval length was 17.25802. This suggests that the interval may not be large enough to include *ES* = 20 on a consistent basis.

Figure 4.3

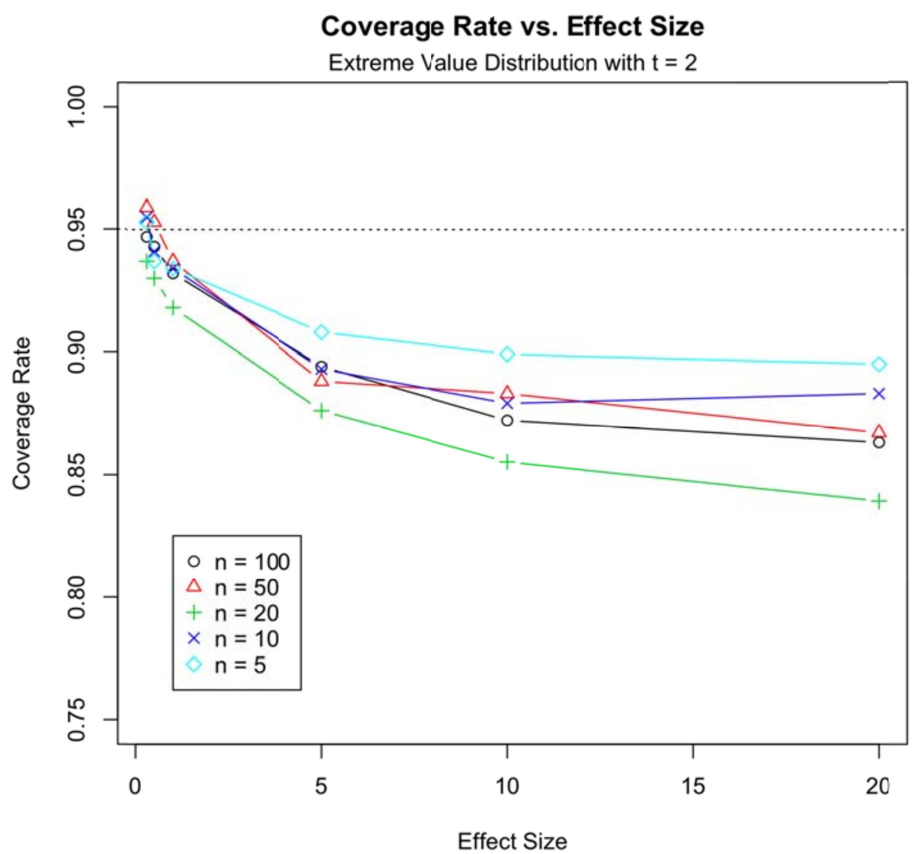


Results for the Extreme Value Distribution

With the exception of the double exponential, the extreme value distribution performed the worst of the five distributions in terms of estimated coverage rates. The minimum estimated coverage rate observed was 0.830, which occurred when *ES* = 20, *t* = 3, and *n* = 100 (see Table B.12 in Appendix B). Similar to the other non-normal distributions, the extreme value coverage rates drop quickly from *ES* = 0.3 to *ES* = 5.0, as can be seen in Figure 4.4. You can still see a slight decline when *ES* shifts from 5.0 to 10.0, however the rates appear to level off as *ES* increases to 20.

For the case when *t* = 2, the lowest coverage rate was 0.839, which again occurred when both *n* and *ES* are equal to 20. In this case, the lower bound was not small enough 91 times, while the upper bound was not large enough 70 times to encompass *ES*. The mean interval length of those times when the lower bound was not small enough was equal to 37.50748. The overall mean interval length is equal to 21.78565. This suggests that the failed intervals are plenty large to include *ES*, however they are located in the wrong place. Similarly, the mean interval length of those instances when the upper bound was not large enough to include *ES* was equal to 11.29301. This suggests that the interval may simply not be large enough to include *ES*.

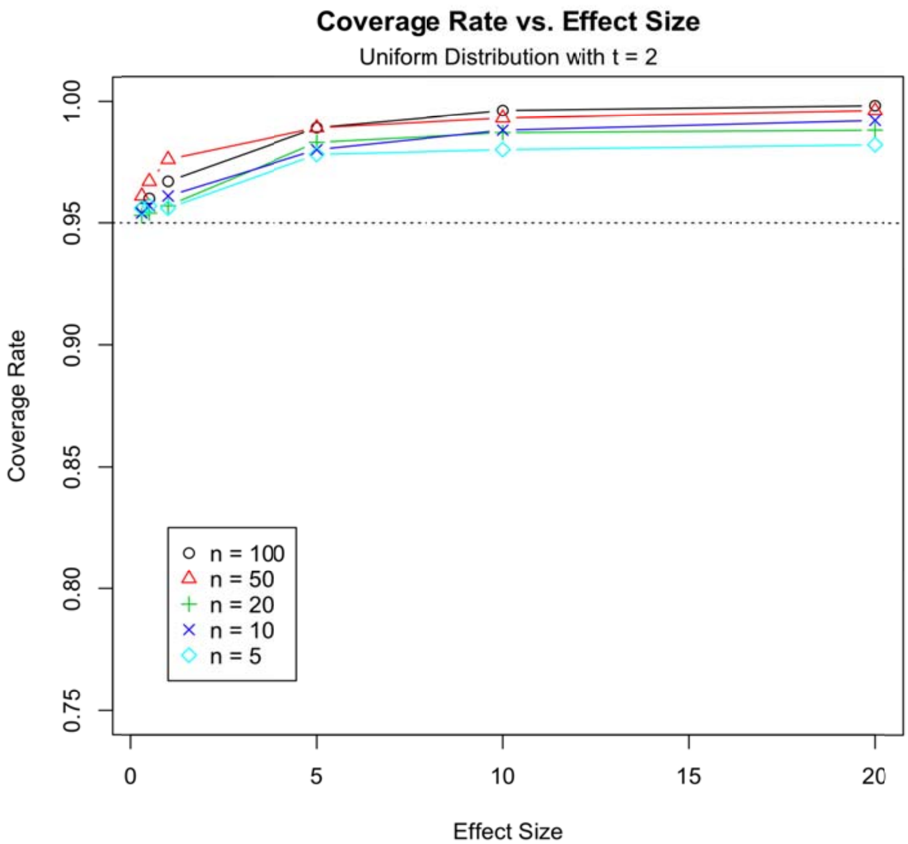
Figure 4.4



Results for the Uniform Distribution

The uniform distribution showed considerably different results than the other four distributions. The lowest estimated coverage rates were seen when *ES* was very small. The minimum value is 0.9400 when *ES* = 0.3, *t* = 5, and *n* = 50 (see table B.13 in Appendix B). However, as seen in Figure 4.5, as *ES* increases, so does the coverage rate, up to a maximum value of 0.998. The rates seem to increase quickly from *ES* = 0.3 up to *ES* = 5.0, then increase less quickly for *ES* = 5.0 to *ES* = 10.0. Finally, the coverage rates level off as *ES* moves from 10.0 to 20.0.

Figure 4.5



Looking specifically at $t = 2$, the lowest coverage rate seen was when $n = 20$ and $ES = 0.3$. However, this value is still 0.953. That is to say that the confidence interval did not contain the true value of *ES* 47 times out of 1000. Of those 47 failures, 22 were because the lower bound of the interval was not small enough, while 25 were a result of the upper bound not being large enough to include *ES*. The mean interval length of those 22 cases when the lower bound was not small enough was 2.391769. This seems like a very small interval, however the overall mean interval length was 1.035068. Based on the overall mean length, I would conclude that this interval length is plenty large enough to include $ES = 0.3$, but the intervals were simply located in the wrong place. However, the mean interval length of those 25 cases when the upper bound was not large enough was 0.208165. This interval may not be large enough to include $ES = 0.3$ at a consistent rate.

Relative Confidence Interval Length

Another way to summarize confidence interval length is to average the mean interval length / *ES* over the six different effect sizes studied for each distribution. Then you can chart these values on the y-axis with sample size on the x-axis plotting a different curve for each distribution. Figure 4.6 shows this graph for the case when $t = 2$. The plots for $t = 3$ and $t = 5$ are similar, and can be found in Appendix D.

Figure 4.6

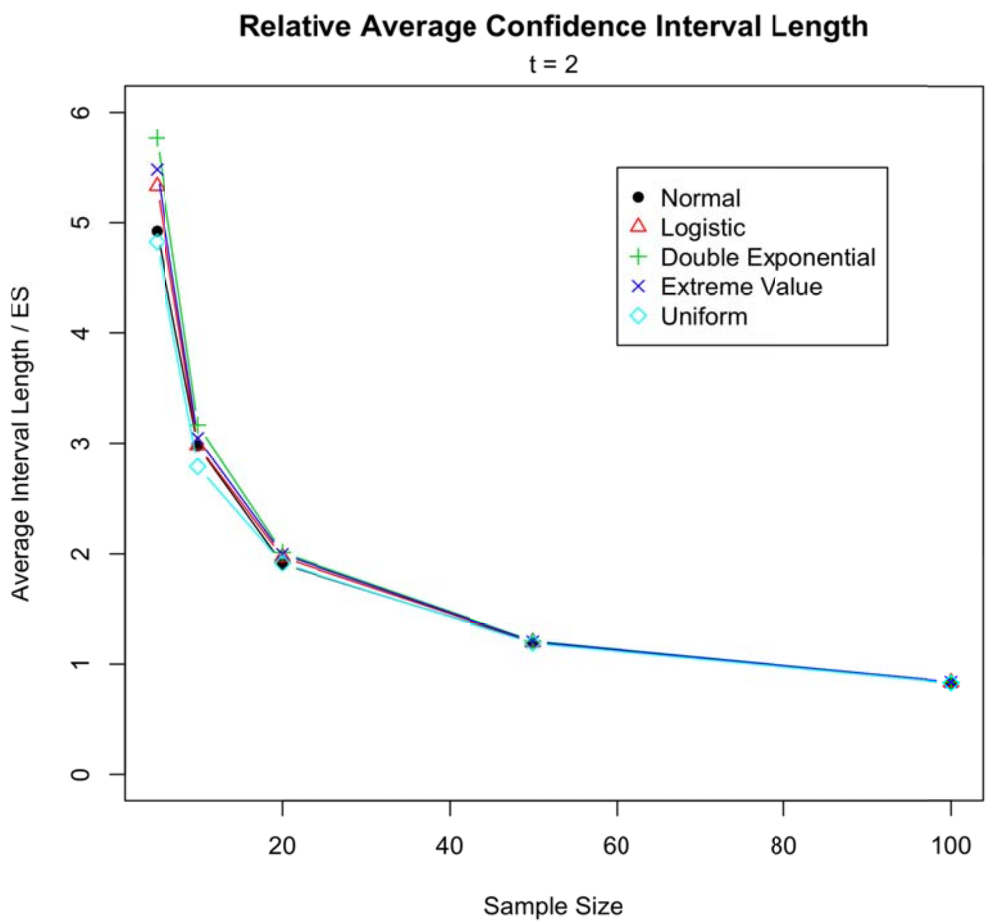


Figure 4.6 shows that there isn't a large difference between the five distributions studied when mean interval length is averaged across the six effect sizes studied. I can use a similar technique with median interval length / *ES*. Figure 4.7 shows the average of median interval length / *ES* over the six different effect sizes plotted against sample size for the case when $t = 2$. The cases when $t = 3$ and $t = 5$ can again be found in Appendix D.

Figure 4.7

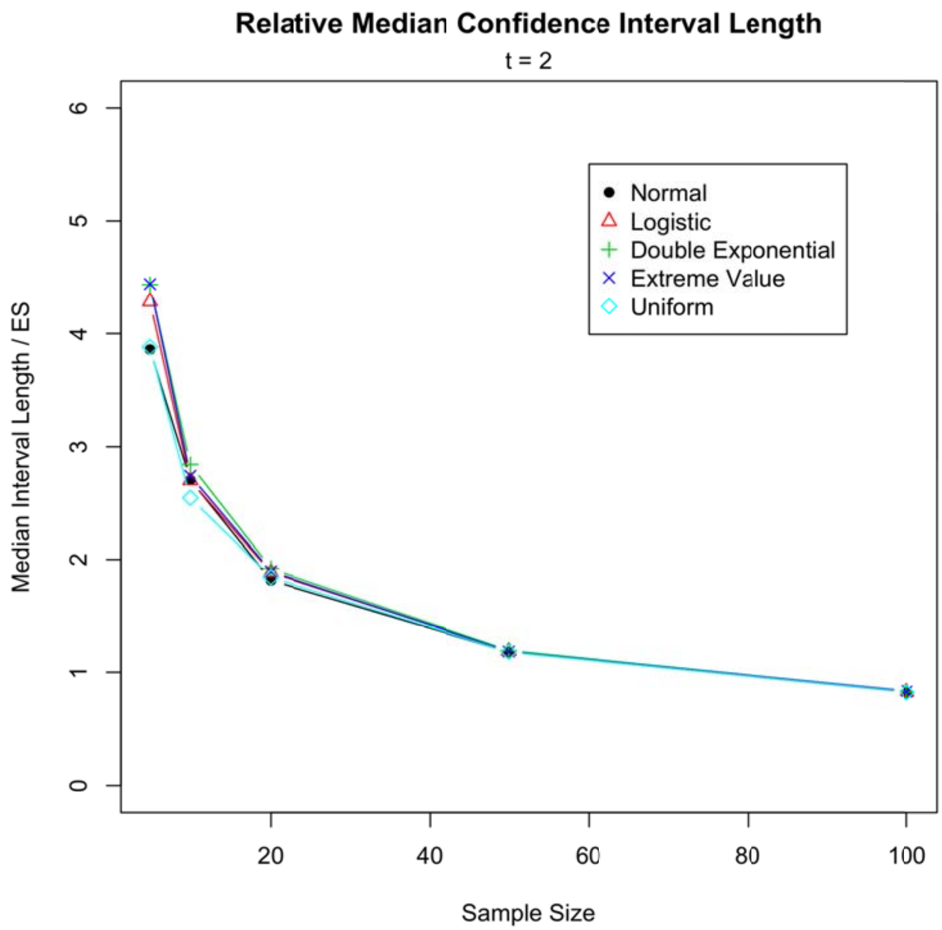


Figure 4.7 shows that median interval length / *ES* is generally shorter than mean interval length / *ES*, but is again very similar for the five distributions studied.

Chapter 5 – Recommendations

There are several noteworthy conclusions that come from this simulation study. Specifically, the intervals in (2.1) are relatively robust for small effect sizes (*ES* = 0.30). However, as *ES* increases we see coverage rates drop for the logistic, double exponential, and extreme value distributions. This suggests that the intervals are not robust with respect to departures from normality as *ES* grows. The results found from the uniform distribution were very surprising. It appears that coverage rates get better than 0.95 as effect size increases.

Ling and Nelson (2012) develop and explore tests and confidence intervals under normality for effect size without requiring equal sample sizes or equal variances. Future studies should be carried out to investigate the robustness of their methods with respect to departures from normality.

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Appendix A – R Program

```
############################################
#                    Normal Distribution with t = 2
#  Obtaining (k=1000) CIs with n = 5, 10, 20, 50, 100
############################################

set.seed(544)
j=1
P=f=L1=L=U=Length=rep(0,1000)
Prop=AvgLength=Median=rep(0,5)
Lower=Upper=Lengths=matrix(0,nrow=1000,ncol=5)
alpha=.05
mu1=0
mu2=1
mu=c(mu1,mu2)
mubar=mean(mu)
sig=1.0
total=1000
ES = sum( (mu-mubar)^2 ) / (sig^2)
for( n in c(5,10,20,50,100))
{
for( k in seq(total) )
{
        Y=cbind(c(rnorm(n,mu1,sig),rnorm(n,mu2,sig)))
        X=cbind(rep(1,2*n),c(rep(0,n),rep(1,n)))
        f[k]=anova(lm(Y~X))$F[1]
        NumDF=anova(lm(Y~X))$Df[1]
        DenDF=anova(lm(Y~X))$Df[2]
        e=try((L2=uniroot(function(x) 1-pf ( f[k] , NumDF , DenDF , x) - alpha/2 , c(0,100000),
                tol=10^-10)), silent=TRUE)
        if (class(e) == "try-error") {L1=0}
        else {L1=L2$root}
        g=try((U2=uniroot(function(y) 1-pf ( f[k] , NumDF , DenDF , y) - (1-alpha/2) ,
                c(0,100000) , tol=10^-10)), silent=TRUE)
        if (class(g) == "try-error") {U1=0}
        else {U1=U2$root}
        L[k] = L1 / n
        U[k] = U1 / n
        Length[k]=U[k]-L[k]
        if (L[k] <= ES && ES <= U[k]) {P[k]=1}
        else {P[k]=0}
        Lengths[k,j]=Length[k]
        Lower[k,j] = L1
        Upper[k,j] = U1
}
```

```
Prop[j]=sum(P)/total
AvgLength[j]=mean( c(Length) ) / ES
Median[j]=median( c(Length) ) / ES
j=j+1

}

results=rbind(Prop,AvgLength,Median)
dimnames(results)=list(c("Coverage Rate" , "Average CI Length / ES" , "Median CI Length /
        ES"),c(5,10,20,50,100))
results
```

Appendix B – Simulation Results

Table B.1

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size				
				n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 0.30	t = 2	Coverage Rate	0.9520	0.9480	0.9450	0.9530	0.9460
		t = 2	Avg. CI Length / ES	9.0749	5.3990	3.4188	2.1464	1.4850
		t = 2	Med. CI Length / ES	6.8516	4.7751	3.2244	2.1099	1.4760
Logistic	≈ 0.30	t = 2	Coverage Rate	0.9550	0.9530	0.9390	0.9600	0.9440
		t = 2	Avg. CI Length / ES	9.7350	5.3368	3.5051	2.1533	1.5042
		t = 2	Med. CI Length / ES	7.7319	4.7244	3.3641	2.1424	1.4910
Double Exponential	≈ 0.30	t = 2	Coverage Rate	0.9550	0.9520	0.9350	0.9460	0.9350
		t = 2	Avg. CI Length / ES	10.2551	5.5306	3.5659	2.1671	1.5110
		t = 2	Med. CI Length / ES	8.1488	4.8633	3.4063	2.1561	1.4929
Extreme Value	≈ 0.30	t = 2	Coverage Rate	0.9530	0.9550	0.9370	0.9590	0.9470
		t = 2	Avg. CI Length / ES	9.8769	5.4090	3.5292	2.1564	1.5055
		t = 2	Med. CI Length / ES	7.7673	4.7820	3.3642	2.1211	1.4889
Uniform	≈ 0.30	t = 2	Coverage Rate	0.9560	0.9540	0.9530	0.9610	0.9560
		t = 2	Avg. CI Length / ES	9.2092	5.0777	3.4421	2.1391	1.4932
		t = 2	Med. CI Length / ES	7.2111	4.4914	3.2926	2.1265	1.4850

Table B.2

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size				
				n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 0.50	t = 2	Coverage Rate	0.9490	0.9400	0.9480	0.9510	0.9460
		t = 2	Avg. CI Length / ES	6.7506	4.1581	2.6918	1.6999	1.1791
		t = 2	Med. CI Length / ES	5.2350	3.8007	2.5507	1.6803	1.1729
Logistic	≈ 0.50	t = 2	Coverage Rate	0.9520	0.9340	0.9370	0.9550	0.9420
		t = 2	Avg. CI Length / ES	7.3065	4.1286	2.7675	1.7049	1.1924
		t = 2	Med. CI Length / ES	5.8956	3.7785	2.6724	1.6895	1.1820
Double Exponential	≈ 0.50	t = 2	Coverage Rate	0.9400	0.9500	0.9260	0.9500	0.9290
		t = 2	Avg. CI Length / ES	7.7829	4.4201	2.8066	1.7216	1.1818
		t = 2	Med. CI Length / ES	5.9034	4.1686	2.6890	1.6898	1.1661
Extreme Value	≈ 0.50	t = 2	Coverage Rate	0.9370	0.9410	0.9300	0.9530	0.9430
		t = 2	Avg. CI Length / ES	7.4531	4.2086	2.7943	1.7094	1.1947
		t = 2	Med. CI Length / ES	6.0646	3.7943	2.6796	1.6808	1.1836
Uniform	≈ 0.50	t = 2	Coverage Rate	0.9570	0.9570	0.9540	0.9670	0.9600
		t = 2	Avg. CI Length / ES	6.8062	3.9156	2.7099	1.6931	1.1840
		t = 2	Med. CI Length / ES	5.3613	3.5530	2.6040	1.6821	1.1775

Table B.3

Distribution	Effect Size	Number of treatments	Coverage Rate Avg. CI Length / ES Med. CI Length / ES	Sample Size n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 1.00	t = 2	Coverage Rate	0.9560	0.9370	0.9460	0.9540	0.9490
		t = 2	Avg. CI Length / ES	4.9290	3.0840	2.0118	1.2675	0.8807
		t = 2	Med. CI Length / ES	3.9421	2.8306	1.9160	1.2562	0.8740
Logistic	≈ 1.00	t = 2	Coverage Rate	0.9470	0.9290	0.9300	0.9440	0.9390
		t = 2	Avg. CI Length / ES	5.3647	3.0898	2.0753	1.2717	0.8899
		t = 2	Med. CI Length / ES	4.3953	2.8281	1.9915	1.2578	0.8851
Double Exponential	≈ 1.00	t = 2	Coverage Rate	0.9220	0.9220	0.9310	0.9260	0.9190
		t = 2	Avg. CI Length / ES	5.7610	3.2125	2.1201	1.2680	0.8886
		t = 2	Med. CI Length / ES	4.4926	2.9408	2.0459	1.2521	0.8851
Extreme Value	≈ 1.00	t = 2	Coverage Rate	0.9340	0.9340	0.9180	0.9370	0.9320
		t = 2	Avg. CI Length / ES	5.5094	3.1499	2.1004	1.2751	0.8915
		t = 2	Med. CI Length / ES	4.6120	2.8923	1.9995	1.2531	0.8851
Uniform	≈ 1.00	t = 2	Coverage Rate	0.9560	0.9610	0.9570	0.9760	0.9670
		t = 2	Avg. CI Length / ES	4.8554	2.9038	2.0222	1.2608	0.8829
		t = 2	Med. CI Length / ES	3.9251	2.6888	1.9538	1.2513	0.8794

Table B.4

Distribution	Effect Size	Number of treatments	Coverage Rate Avg. CI Length / ES Med. CI Length / ES	Sample Size n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 5.00	t = 2	Coverage Rate	0.9510	0.9470	0.9410	0.9470	0.9550
		t = 2	Avg. CI Length / ES	3.1631	1.9121	1.2251	0.7638	0.5308
		t = 2	Med. CI Length / ES	2.5610	1.7575	1.1793	0.7538	0.5292
Logistic	≈ 5.00	t = 2	Coverage Rate	0.9330	0.9100	0.9130	0.9230	0.9270
		t = 2	Avg. CI Length / ES	3.4561	1.9466	1.2774	0.7676	0.5365
		t = 2	Med. CI Length / ES	2.7707	1.7807	1.2073	0.7573	0.5336
Double Exponential	≈ 5.00	t = 2	Coverage Rate	0.8720	0.8680	0.8690	0.8570	0.8590
		t = 2	Avg. CI Length / ES	3.8867	2.1635	1.3182	0.7830	0.5406
		t = 2	Med. CI Length / ES	2.9169	1.8516	1.2526	0.7643	0.5357
Extreme Value	≈ 5.00	t = 2	Coverage Rate	0.9080	0.8930	0.8760	0.8880	0.8940
		t = 2	Avg. CI Length / ES	3.6077	2.0012	1.3036	0.7709	0.5380
		t = 2	Med. CI Length / ES	2.9804	1.8188	1.2212	0.7554	0.5372
Uniform	≈ 5.00	t = 2	Coverage Rate	0.9780	0.9800	0.9830	0.9890	0.9890
		t = 2	Avg. CI Length / ES	2.9575	1.7803	1.2245	0.7564	0.5302
		t = 2	Med. CI Length / ES	2.4635	1.6593	1.1867	0.7489	0.5274

Table B.5

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 10.00	t = 2	Coverage Rate	0.9460	0.9470	0.9440	0.9480	0.9520
		t = 2	Avg. CI Length / ES	2.8836	1.7111	1.0870	0.6750	0.4689
		t = 2	Med. CI Length / ES	2.3297	1.5801	1.0528	0.6635	0.4674
Logistic	≈ 10.00	t = 2	Coverage Rate	0.9250	0.9040	0.9030	0.9160	0.9200
		t = 2	Avg. CI Length / ES	3.1489	1.7510	1.1382	0.6789	0.4742
		t = 2	Med. CI Length / ES	2.5258	1.5905	1.0797	0.6668	0.4712
Double Exponential	≈ 10.00	t = 2	Coverage Rate	0.8800	0.8400	0.8400	0.8270	0.8200
		t = 2	Avg. CI Length / ES	3.5404	1.9005	1.1579	0.6921	0.4787
		t = 2	Med. CI Length / ES	2.6751	1.6791	1.1015	0.6728	0.4711
Extreme Value	≈ 10.00	t = 2	Coverage Rate	0.8990	0.8790	0.8550	0.8830	0.8720
		t = 2	Avg. CI Length / ES	3.3043	1.8061	1.1651	0.6822	0.4757
		t = 2	Med. CI Length / ES	2.6665	1.6308	1.0957	0.6701	0.4747
Uniform	≈ 10.00	t = 2	Coverage Rate	0.9800	0.9880	0.9870	0.9930	0.9960
		t = 2	Avg. CI Length / ES	2.6466	1.5858	1.0845	0.6674	0.4679
		t = 2	Med. CI Length / ES	2.2359	1.4875	1.0490	0.6612	0.4658

Table B.6

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 20.00	t = 2	Coverage Rate	0.9460	0.9500	0.9440	0.9490	0.9480
		t = 2	Avg. CI Length / ES	2.7352	1.6010	1.0110	0.6258	0.4347
		t = 2	Med. CI Length / ES	2.2294	1.4841	0.9795	0.6165	0.4346
Logistic	≈ 20.00	t = 2	Coverage Rate	0.9180	0.9000	0.8960	0.8960	0.9050
		t = 2	Avg. CI Length / ES	2.9813	1.6444	1.0618	0.6298	0.4397
		t = 2	Med. CI Length / ES	2.3907	1.4958	1.0084	0.6206	0.4356
Double Exponential	≈ 20.00	t = 2	Coverage Rate	0.8580	0.8590	0.8240	0.8240	0.8270
		t = 2	Avg. CI Length / ES	3.3828	1.7684	1.1097	0.6365	0.4390
		t = 2	Med. CI Length / ES	2.4603	1.5399	1.0333	0.6265	0.4340
Extreme Value	≈ 20.00	t = 2	Coverage Rate	0.8950	0.8830	0.8390	0.8670	0.8630
		t = 2	Avg. CI Length / ES	3.1393	1.7001	1.0891	0.6331	0.4412
		t = 2	Med. CI Length / ES	2.5358	1.5273	1.0216	0.6226	0.4389
Uniform	≈ 20.00	t = 2	Coverage Rate	0.9820	0.9920	0.9880	0.9960	0.9980
		t = 2	Avg. CI Length / ES	2.4761	1.4795	1.0073	0.6181	0.4334
		t = 2	Med. CI Length / ES	2.0842	1.3883	0.9791	0.6120	0.4321

Table B.7

Distribution	Effect Size	Number of treatments	Coverage Rate Avg. CI Length / ES Med. CI Length / ES	Sample Size				
				n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 0.30	t = 3	Coverage Rate	0.9480	0.9380	0.9590	0.9600	0.9540
		t = 3	Avg. CI Length / ES	9.6021	5.4671	3.5192	2.1402	1.4798
		t = 3	Med. CI Length / ES	7.9556	4.9084	3.4870	2.1230	1.4675
Logistic	≈ 0.30	t = 3	Coverage Rate	0.9530	0.9550	0.9480	0.9450	0.9500
		t = 3	Avg. CI Length / ES	9.9424	5.5092	3.4720	2.1383	1.4918
		t = 3	Med. CI Length / ES	8.3670	5.1152	3.3685	2.1525	1.4907
Double Exponential	≈ 0.30	t = 3	Coverage Rate	0.9490	0.9470	0.9430	0.9390	0.9470
		t = 3	Avg. CI Length / ES	10.2151	5.6089	3.5078	2.1510	1.4956
		t = 3	Med. CI Length / ES	8.7474	5.1287	3.3837	2.1840	1.4952
Extreme Value	≈ 0.30	t = 3	Coverage Rate	0.9580	0.9480	0.9540	0.9400	0.9500
		t = 3	Avg. CI Length / ES	10.0101	5.5784	3.4978	2.1379	1.4894
		t = 3	Med. CI Length / ES	8.4393	5.1193	3.3699	2.1425	1.4821
Uniform	≈ 0.30	t = 3	Coverage Rate	0.9520	0.9640	0.9520	0.9530	0.9610
		t = 3	Avg. CI Length / ES	9.5947	5.3990	3.4310	2.1190	1.4858
		t = 3	Med. CI Length / ES	8.0245	5.0142	3.3473	2.1365	1.4806

Table B.8

Distribution	Effect Size	Number of treatments	Coverage Rate Avg. CI Length / ES Med. CI Length / ES	Sample Size				
				n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 0.50	t = 3	Coverage Rate	0.9470	0.9480	0.9590	0.9590	0.9520
		t = 3	Avg. CI Length / ES	6.8895	4.0861	2.7181	1.6749	1.1626
		t = 3	Med. CI Length / ES	5.6685	3.7710	2.7099	1.6527	1.1547
Logistic	≈ 0.50	t = 3	Coverage Rate	0.9440	0.9560	0.9510	0.9420	0.9500
		t = 3	Avg. CI Length / ES	7.1311	4.1491	2.6936	1.6768	1.1703
		t = 3	Med. CI Length / ES	6.0996	3.9152	2.6316	1.6835	1.1651
Double Exponential	≈ 0.50	t = 3	Coverage Rate	0.9430	0.9430	0.9460	0.9340	0.9390
		t = 3	Avg. CI Length / ES	7.4015	4.2416	2.7242	1.6875	1.1735
		t = 3	Med. CI Length / ES	6.4018	3.9481	2.6321	1.6976	1.1693
Extreme Value	≈ 0.50	t = 3	Coverage Rate	0.9470	0.9570	0.9440	0.9440	0.9440
		t = 3	Avg. CI Length / ES	7.2132	4.2168	2.7157	1.6782	1.1701
		t = 3	Med. CI Length / ES	6.2450	3.9661	2.6414	1.6783	1.1649
Uniform	≈ 0.50	t = 3	Coverage Rate	0.9480	0.9650	0.9560	0.9550	0.9620
		t = 3	Avg. CI Length / ES	6.8066	4.0433	2.6605	1.6618	1.1664
		t = 3	Med. CI Length / ES	5.7905	3.8480	2.5878	1.6682	1.1643

Table B.9

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 1.00	t = 3	Coverage Rate	0.9510	0.9410	0.9620	0.9590	0.9450
		t = 3	Avg. CI Length / ES	4.7443	2.9237	1.9756	1.2237	0.8525
		t = 3	Med. CI Length / ES	4.0627	2.7657	1.9708	1.2069	0.8491
Logistic	≈ 1.00	t = 3	Coverage Rate	0.9500	0.9420	0.9510	0.9400	0.9430
		t = 3	Avg. CI Length / ES	4.9251	2.9849	1.9689	1.2281	0.8573
		t = 3	Med. CI Length / ES	4.3944	2.8661	1.8973	1.2286	0.8548
Double Exponential	≈ 1.00	t = 3	Coverage Rate	0.9370	0.9310	0.9360	0.9270	0.9250
		t = 3	Avg. CI Length / ES	5.1740	3.0651	1.9933	1.2366	0.8597
		t = 3	Med. CI Length / ES	4.6014	2.9025	1.9194	1.2346	0.8571
Extreme Value	≈ 1.00	t = 3	Coverage Rate	0.9440	0.9480	0.9410	0.9370	0.9320
		t = 3	Avg. CI Length / ES	5.0022	3.0430	1.9832	1.2289	0.8573
		t = 3	Med. CI Length / ES	4.4752	2.8815	1.9212	1.2247	0.8542
Uniform	≈ 1.00	t = 3	Coverage Rate	0.9530	0.9690	0.9650	0.9630	0.9640
		t = 3	Avg. CI Length / ES	4.6231	2.8903	1.9411	1.2160	0.8544
		t = 3	Med. CI Length / ES	4.0425	2.7986	1.8877	1.2129	0.8561

Table B.10

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 5.00	t = 3	Coverage Rate	0.9530	0.9370	0.9600	0.9530	0.9460
		t = 3	Avg. CI Length / ES	2.6688	1.6477	1.1055	0.6856	0.4786
		t = 3	Med. CI Length / ES	2.3826	1.5529	1.0865	0.6757	0.4771
Logistic	≈ 5.00	t = 3	Coverage Rate	0.9310	0.9280	0.9220	0.9260	0.9090
		t = 3	Avg. CI Length / ES	2.7933	1.7007	1.1104	0.6912	0.4811
		t = 3	Med. CI Length / ES	2.4677	1.6085	1.0753	0.6900	0.4803
Double Exponential	≈ 5.00	t = 3	Coverage Rate	0.8940	0.8910	0.8830	0.8910	0.8550
		t = 3	Avg. CI Length / ES	3.0143	1.7703	1.1309	0.6980	0.4830
		t = 3	Med. CI Length / ES	2.5660	1.6470	1.0907	0.6959	0.4807
Extreme Value	≈ 5.00	t = 3	Coverage Rate	0.9190	0.9130	0.9030	0.9100	0.8710
		t = 3	Avg. CI Length / ES	2.8686	1.7492	1.1212	0.6920	0.4817
		t = 3	Med. CI Length / ES	2.5471	1.6546	1.0830	0.6882	0.4812
Uniform	≈ 5.00	t = 3	Coverage Rate	0.9730	0.9760	0.9760	0.9780	0.9800
		t = 3	Avg. CI Length / ES	2.5261	1.6130	1.0857	0.6811	0.4785
		t = 3	Med. CI Length / ES	2.2726	1.5516	1.0625	0.6783	0.4780

Table B.11

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 10.00	t = 3	Coverage Rate	0.9520	0.9390	0.9550	0.9470	0.9440
		t = 3	Avg. CI Length / ES	2.3140	1.4174	0.9458	0.5856	0.4086
		t = 3	Med. CI Length / ES	2.0648	1.3408	0.9237	0.5784	0.4066
Logistic	≈ 10.00	t = 3	Coverage Rate	0.9210	0.9270	0.9140	0.9150	0.8930
		t = 3	Avg. CI Length / ES	2.4336	1.4701	0.9526	0.5914	0.4108
		t = 3	Med. CI Length / ES	2.1410	1.3890	0.9230	0.5919	0.4091
Double Exponential	≈ 10.00	t = 3	Coverage Rate	0.8690	0.8560	0.8650	0.8580	0.8170
		t = 3	Avg. CI Length / ES	2.6544	1.5394	0.9728	0.5981	0.4127
		t = 3	Med. CI Length / ES	2.2096	1.4391	0.9395	0.5958	0.4088
Extreme Value	≈ 10.00	t = 3	Coverage Rate	0.9020	0.8930	0.8750	0.8930	0.8510
		t = 3	Avg. CI Length / ES	2.5109	1.5182	0.9630	0.5923	0.4115
		t = 3	Med. CI Length / ES	2.1940	1.4155	0.9320	0.5879	0.4098
Uniform	≈ 10.00	t = 3	Coverage Rate	0.9810	0.9830	0.9820	0.9890	0.9870
		t = 3	Avg. CI Length / ES	2.1663	1.3815	0.9280	0.5815	0.4082
		t = 3	Med. CI Length / ES	1.9466	1.3385	0.9046	0.5805	0.4070

Table B.12

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 20.00	t = 3	Coverage Rate	0.9510	0.9420	0.9520	0.9440	0.9420
		t = 3	Avg. CI Length / ES	2.1159	1.2878	0.8551	0.5286	0.3688
		t = 3	Med. CI Length / ES	1.9008	1.2226	0.8402	0.5228	0.3668
Logistic	≈ 20.00	t = 3	Coverage Rate	0.9070	0.9080	0.9080	0.9100	0.8800
		t = 3	Avg. CI Length / ES	2.2334	1.3406	0.8632	0.5347	0.3707
		t = 3	Med. CI Length / ES	1.9326	1.2712	0.8403	0.5334	0.3688
Double Exponential	≈ 20.00	t = 3	Coverage Rate	0.8500	0.8350	0.8460	0.8410	0.7960
		t = 3	Avg. CI Length / ES	2.4546	1.4101	0.8835	0.5413	0.3726
		t = 3	Med. CI Length / ES	2.0050	1.3087	0.8498	0.5374	0.3694
Extreme Value	≈ 20.00	t = 3	Coverage Rate	0.8890	0.8790	0.8640	0.8770	0.8300
		t = 3	Avg. CI Length / ES	2.3125	1.3889	0.8734	0.5355	0.3715
		t = 3	Med. CI Length / ES	2.0082	1.2886	0.8453	0.5308	0.3700
Uniform	≈ 20.00	t = 3	Coverage Rate	0.9850	0.9910	0.9880	0.9940	0.9960
		t = 3	Avg. CI Length / ES	1.9651	1.2509	0.8384	0.5248	0.3682
		t = 3	Med. CI Length / ES	1.7865	1.2171	0.8183	0.5255	0.3671

Table B.13

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 0.30	t = 5	Coverage Rate	0.9400	0.9440	0.9480	0.9400	0.9490
		t = 5	Avg. CI Length / ES	10.5090	5.9448	3.6808	2.1659	1.4948
		t = 5	Med. CI Length / ES	9.3627	5.5774	3.5263	2.1395	1.4948
Logistic	≈ 0.30	t = 5	Coverage Rate	0.9450	0.9590	0.9470	0.9300	0.9540
		t = 5	Avg. CI Length / ES	10.6667	5.8889	3.7035	2.1659	1.5021
		t = 5	Med. CI Length / ES	8.9405	5.5419	3.6503	2.1596	1.4925
Double Exponential	≈ 0.30	t = 5	Coverage Rate	0.9370	0.9570	0.9480	0.9310	0.9530
		t = 5	Avg. CI Length / ES	10.8519	5.9451	3.7223	2.1703	1.5023
		t = 5	Med. CI Length / ES	9.1056	5.5882	3.6984	2.1557	1.4915
Extreme Value	≈ 0.30	t = 5	Coverage Rate	0.9470	0.9520	0.9590	0.9300	0.9460
		t = 5	Avg. CI Length / ES	10.7009	5.9544	3.7034	2.1636	1.5001
		t = 5	Med. CI Length / ES	9.0318	5.6489	3.7047	2.1488	1.4948
Uniform	≈ 0.30	t = 5	Coverage Rate	0.9500	0.9460	0.9570	0.9400	0.9550
		t = 5	Avg. CI Length / ES	10.3910	5.8059	3.6850	2.1599	1.5034
		t = 5	Med. CI Length / ES	9.0755	5.3395	3.6193	2.1699	1.4969

Table B.14

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 0.50	t = 5	Coverage Rate	0.9480	0.9410	0.9470	0.9420	0.9540
		t = 5	Avg. CI Length / ES	7.2265	4.3114	2.7850	1.6717	1.1583
		t = 5	Med. CI Length / ES	6.4648	4.0857	2.7646	1.6611	1.1593
Logistic	≈ 0.50	t = 5	Coverage Rate	0.9450	0.9560	0.9490	0.9320	0.9560
		t = 5	Avg. CI Length / ES	7.3954	4.2826	2.8071	1.6730	1.1641
		t = 5	Med. CI Length / ES	6.3222	4.0590	2.8433	1.6576	1.1607
Double Exponential	≈ 0.50	t = 5	Coverage Rate	0.9450	0.9550	0.9450	0.9260	0.9530
		t = 5	Avg. CI Length / ES	7.5631	4.3331	2.8230	1.6769	1.1643
		t = 5	Med. CI Length / ES	6.4530	4.1059	2.8325	1.6578	1.1570
Extreme Value	≈ 0.50	t = 5	Coverage Rate	0.9480	0.9490	0.9550	0.9280	0.9440
		t = 5	Avg. CI Length / ES	7.4445	4.3370	2.8081	1.6728	1.1630
		t = 5	Med. CI Length / ES	6.4195	4.1822	2.8471	1.6669	1.1615
Uniform	≈ 0.50	t = 5	Coverage Rate	0.9540	0.9490	0.9550	0.9440	0.9530
		t = 5	Avg. CI Length / ES	7.1448	4.2062	2.7878	1.6663	1.1640
		t = 5	Med. CI Length / ES	6.2507	3.9542	2.7890	1.6633	1.1584

Table B.15

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 1.00	t = 5	Coverage Rate	0.9490	0.9430	0.9510	0.9480	0.9500
		t = 5	Avg. CI Length / ES	4.7005	2.9678	1.9684	1.1949	0.8331
		t = 5	Med. CI Length / ES	4.2718	2.9091	1.9381	1.1848	0.8324
Logistic	≈ 1.00	t = 5	Coverage Rate	0.9390	0.9550	0.9480	0.9310	0.9550
		t = 5	Avg. CI Length / ES	4.8339	2.9532	1.9821	1.1959	0.8356
		t = 5	Med. CI Length / ES	4.2627	2.9064	1.9640	1.1843	0.8331
Double Exponential	≈ 1.00	t = 5	Coverage Rate	0.9350	0.9450	0.9410	0.9260	0.9460
		t = 5	Avg. CI Length / ES	4.9805	2.9953	1.9954	1.1997	0.8361
		t = 5	Med. CI Length / ES	4.3772	2.9295	1.9867	1.1901	0.8302
Extreme Value	≈ 1.00	t = 5	Coverage Rate	0.9460	0.9490	0.9510	0.9330	0.9400
		t = 5	Avg. CI Length / ES	4.8916	2.9928	1.9849	1.1972	0.8354
		t = 5	Med. CI Length / ES	4.3119	2.9743	1.9766	1.1862	0.8324
Uniform	≈ 1.00	t = 5	Coverage Rate	0.9560	0.9550	0.9570	0.9540	0.9530
		t = 5	Avg. CI Length / ES	4.6304	2.8964	1.9683	1.1912	0.8359
		t = 5	Med. CI Length / ES	4.1299	2.8161	1.9488	1.1821	0.8334

Table B.16

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 5.00	t = 5	Coverage Rate	0.9550	0.9480	0.9510	0.9580	0.9490
		t = 5	Avg. CI Length / ES	2.3177	1.4867	1.0036	0.6175	0.4331
		t = 5	Med. CI Length / ES	2.1686	1.4569	0.9855	0.6157	0.4316
Logistic	≈ 5.00	t = 5	Coverage Rate	0.9290	0.9410	0.9340	0.9270	0.9360
		t = 5	Avg. CI Length / ES	2.4043	1.4981	1.0110	0.6198	0.4336
		t = 5	Med. CI Length / ES	2.1982	1.4477	1.0028	0.6165	0.4319
Double Exponential	≈ 5.00	t = 5	Coverage Rate	0.8980	0.9090	0.8980	0.8940	0.9060
		t = 5	Avg. CI Length / ES	2.5121	1.5295	1.0215	0.6228	0.4342
		t = 5	Med. CI Length / ES	2.2558	1.4734	1.0090	0.6214	0.4327
Extreme Value	≈ 5.00	t = 5	Coverage Rate	0.9230	0.9220	0.9270	0.9150	0.9110
		t = 5	Avg. CI Length / ES	2.4483	1.5206	1.0130	0.6207	0.4333
		t = 5	Med. CI Length / ES	2.2185	1.4711	0.9997	0.6167	0.4322
Uniform	≈ 5.00	t = 5	Coverage Rate	0.9690	0.9740	0.9780	0.9700	0.9670
		t = 5	Avg. CI Length / ES	2.2628	1.4559	0.9984	0.6156	0.4332
		t = 5	Med. CI Length / ES	2.1043	1.4111	0.9896	0.6119	0.4319

Table B.17

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size				
				n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 10.00	t = 5	Coverage Rate	0.9490	0.9570	0.9480	0.9540	0.9450
		t = 5	Avg. CI Length / ES	1.8952	1.2095	0.8190	0.5040	0.3537
		t = 5	Med. CI Length / ES	1.7702	1.1829	0.8049	0.5032	0.3526
Logistic	≈ 10.00	t = 5	Coverage Rate	0.9180	0.9260	0.9230	0.9160	0.9290
		t = 5	Avg. CI Length / ES	1.9713	1.2256	0.8253	0.5065	0.3539
		t = 5	Med. CI Length / ES	1.7891	1.1942	0.8173	0.5051	0.3526
Double Exponential	≈ 10.00	t = 5	Coverage Rate	0.8710	0.8910	0.8680	0.8660	0.8840
		t = 5	Avg. CI Length / ES	2.0756	1.2559	0.8354	0.5093	0.3545
		t = 5	Med. CI Length / ES	1.8456	1.2094	0.8213	0.5075	0.3537
Extreme Value	≈ 10.00	t = 5	Coverage Rate	0.9110	0.9040	0.9100	0.8910	0.8940
		t = 5	Avg. CI Length / ES	2.0144	1.2467	0.8275	0.5074	0.3537
		t = 5	Med. CI Length / ES	1.8359	1.2015	0.8123	0.5043	0.3521
Uniform	≈ 10.00	t = 5	Coverage Rate	0.9750	0.9800	0.9840	0.9810	0.9780
		t = 5	Avg. CI Length / ES	1.8363	1.1851	0.8129	0.5024	0.3534
		t = 5	Med. CI Length / ES	1.7070	1.1558	0.8071	0.5000	0.3525

Table B.18

Distribution	Effect Size	Number of treatments	Coverage Rate / Avg. CI Length / ES / Med. CI Length / ES	Sample Size				
				n = 5	n = 10	n = 20	n = 50	n = 100
Normal	≈ 20.00	t = 5	Coverage Rate	0.9440	0.9540	0.9540	0.9570	0.9460
		t = 5	Avg. CI Length / ES	1.6522	1.0475	0.7101	0.4365	0.3065
		t = 5	Med. CI Length / ES	1.5248	1.0260	0.6998	0.4354	0.3055
Logistic	≈ 20.00	t = 5	Coverage Rate	0.9020	0.9190	0.9080	0.9040	0.9170
		t = 5	Avg. CI Length / ES	1.7215	1.0672	0.7159	0.4392	0.3065
		t = 5	Med. CI Length / ES	1.5592	1.0333	0.7067	0.4371	0.3056
Double Exponential	≈ 20.00	t = 5	Coverage Rate	0.8590	0.8700	0.8410	0.8400	0.8670
		t = 5	Avg. CI Length / ES	1.8243	1.0972	0.7260	0.4420	0.3071
		t = 5	Med. CI Length / ES	1.6226	1.0460	0.7132	0.4402	0.3068
Extreme Value	≈ 20.00	t = 5	Coverage Rate	0.8860	0.8970	0.8890	0.8660	0.8740
		t = 5	Avg. CI Length / ES	1.7651	1.0877	0.7183	0.4402	0.3063
		t = 5	Med. CI Length / ES	1.6082	1.0511	0.7044	0.4372	0.3045
Uniform	≈ 20.00	t = 5	Coverage Rate	0.9830	0.9850	0.9890	0.9890	0.9920
		t = 5	Avg. CI Length / ES	1.5897	1.0269	0.7034	0.4351	0.3059
		t = 5	Med. CI Length / ES	1.4754	0.9993	0.6972	0.4319	0.3054

Appendix C – Coverage Plots

Figure C.1

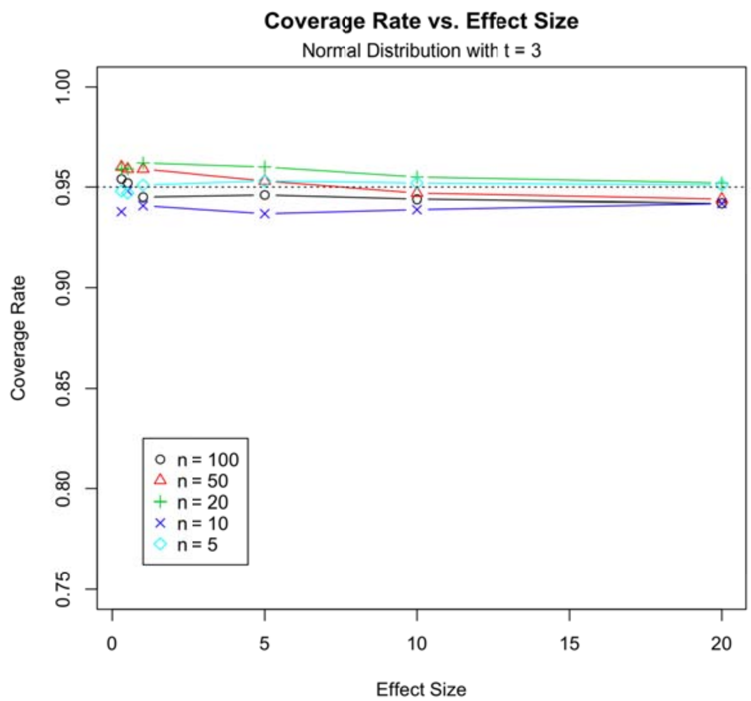


Figure C.2

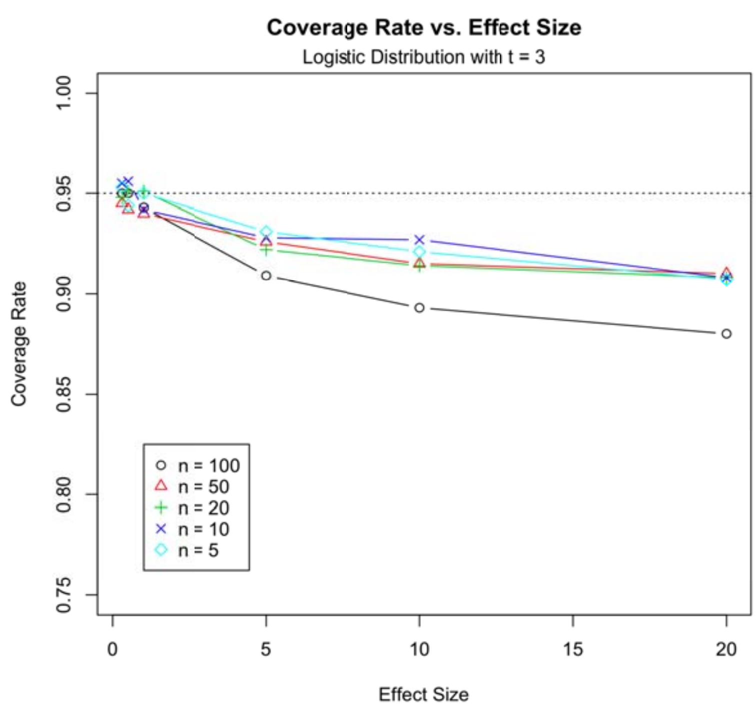


Figure C.3

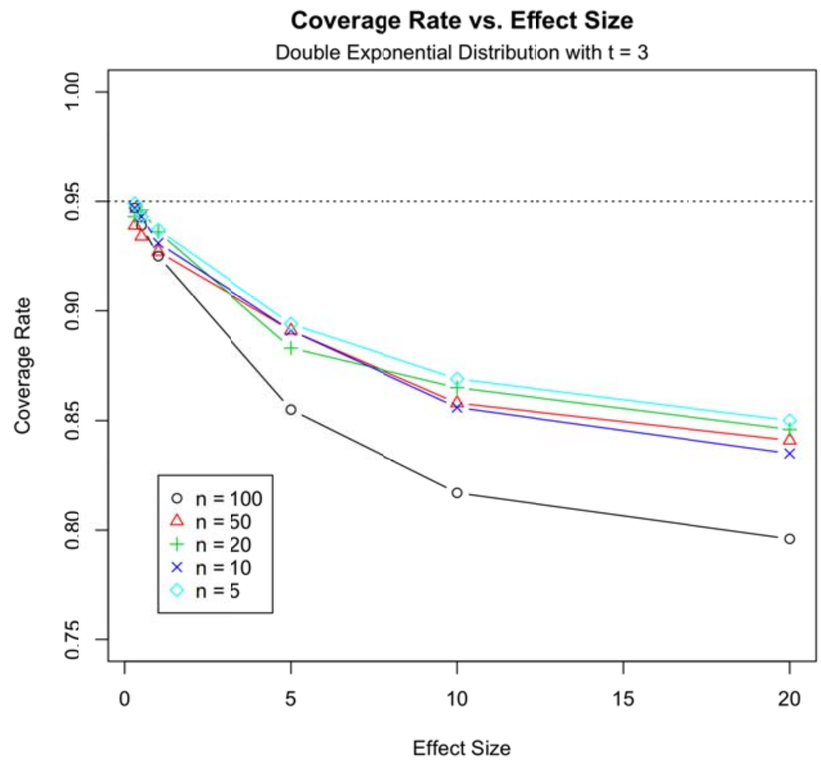


Figure C.4

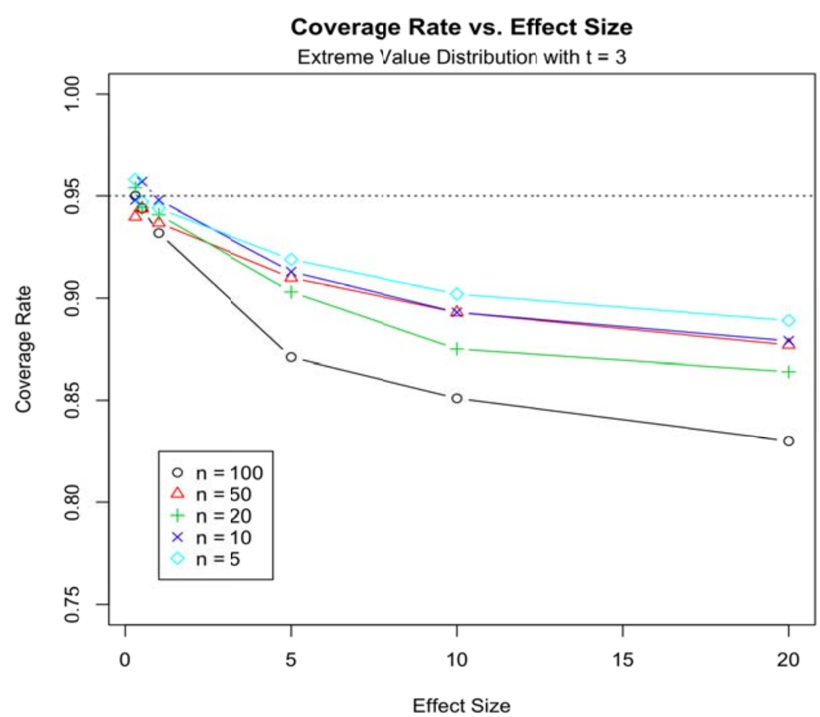


Figure C.5

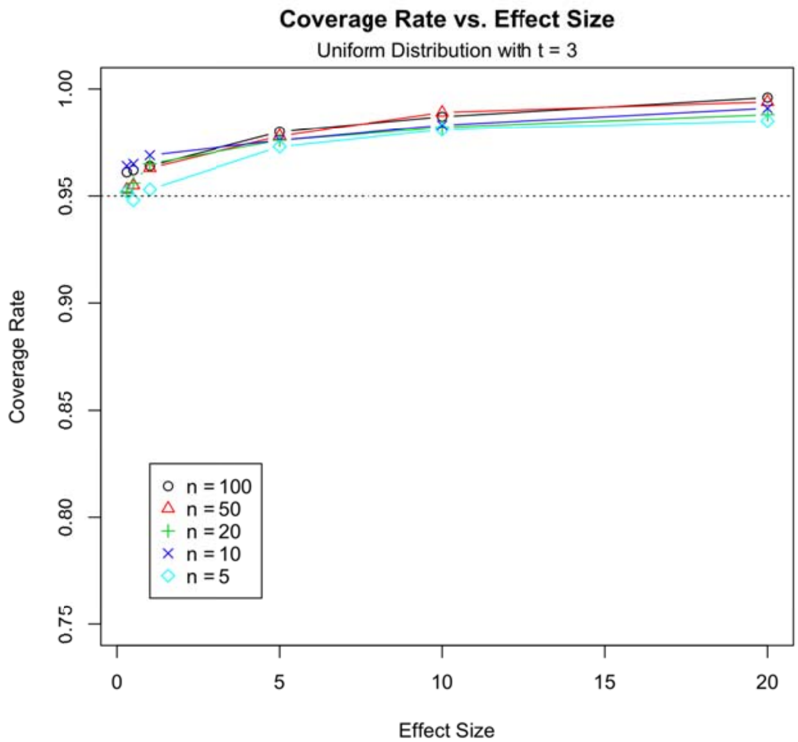


Figure C.6

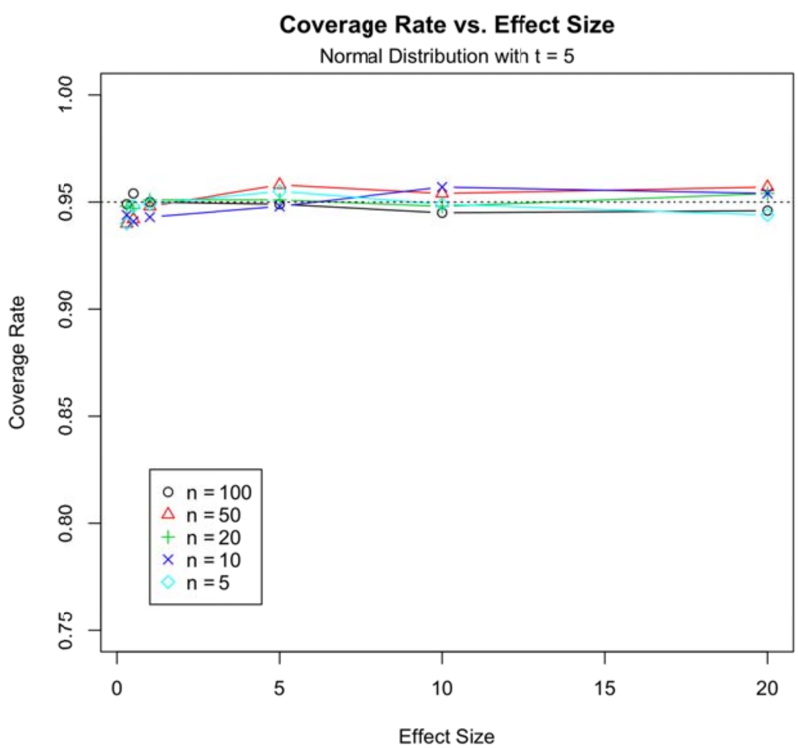


Figure C.7

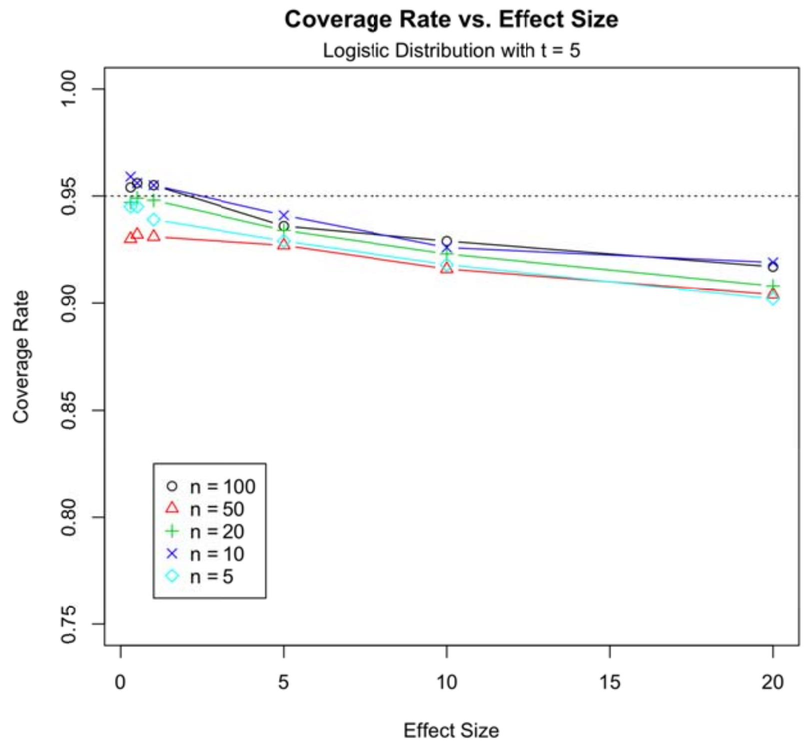


Figure C.8

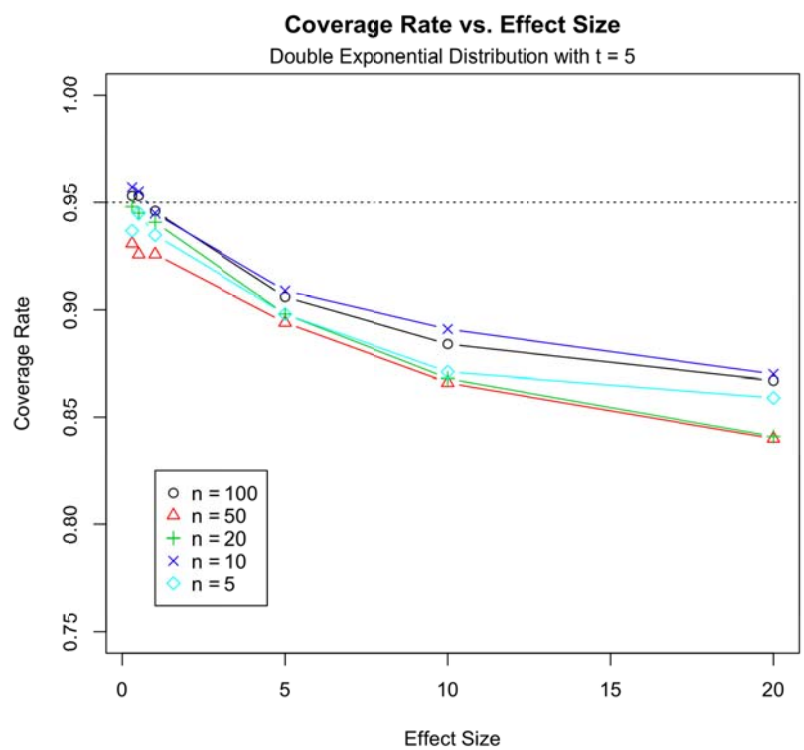


Figure C.9

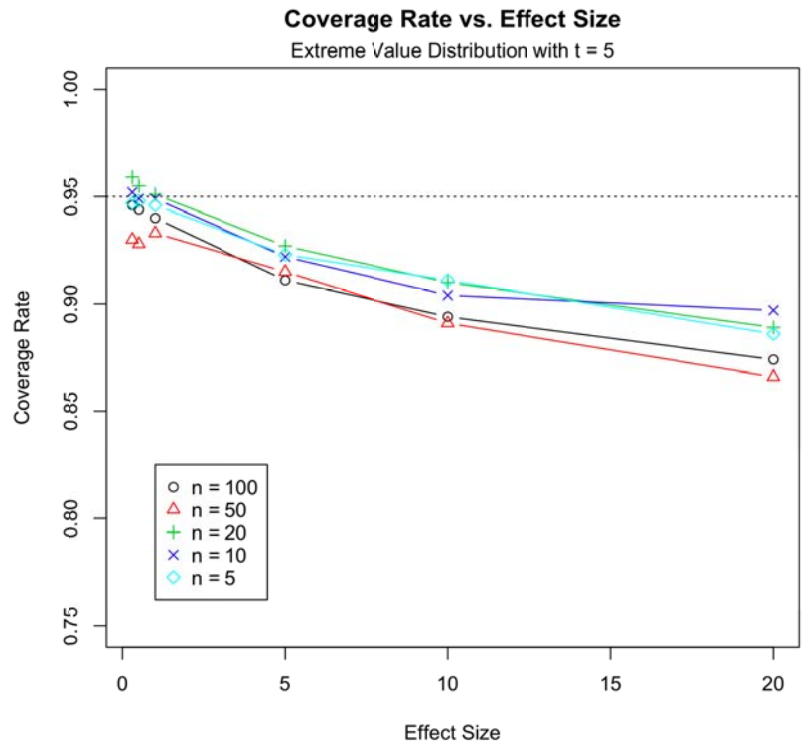
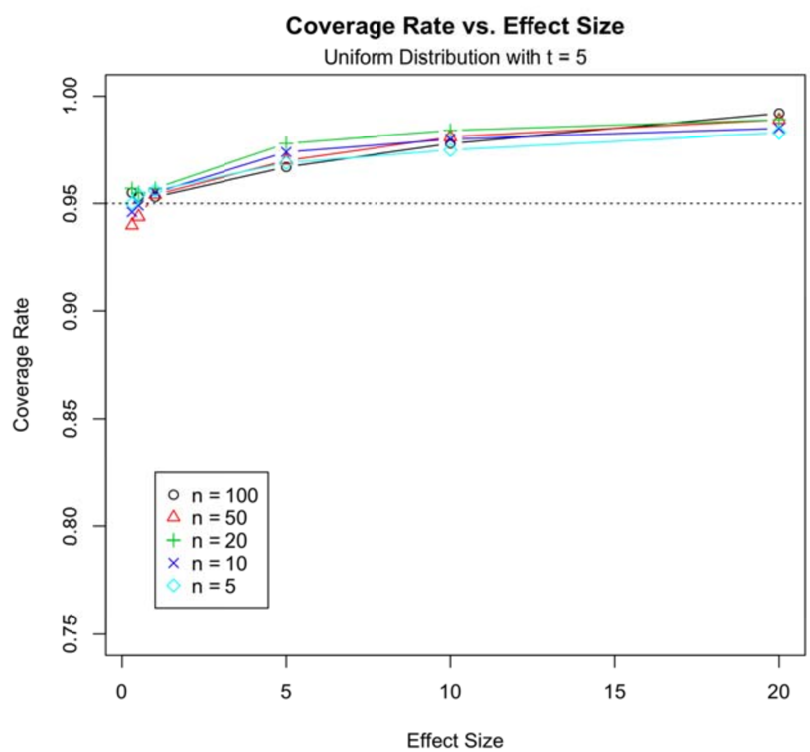


Figure C.10



Appendix D – Relative Length Plots

Figure D.1

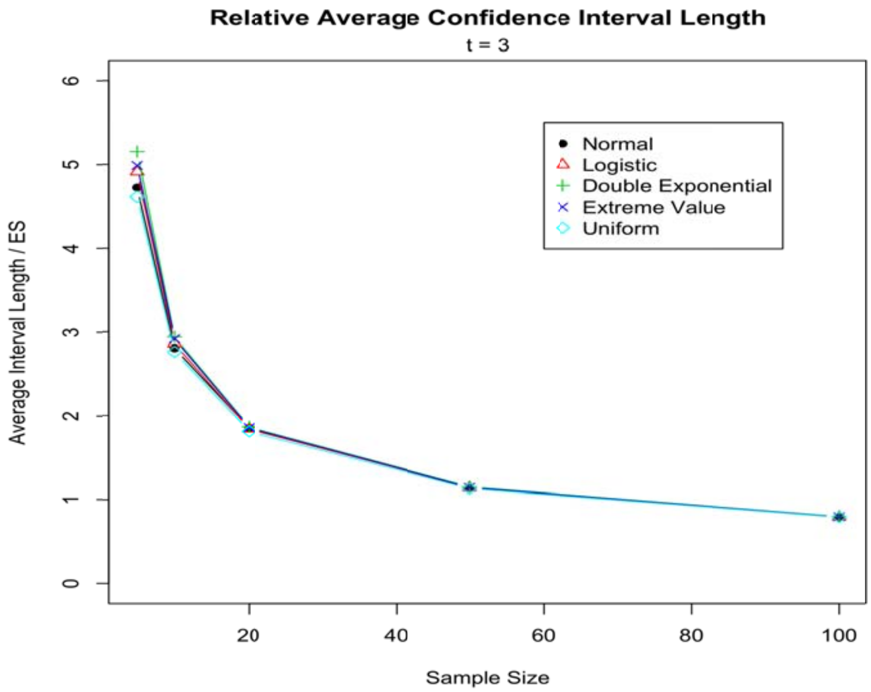


Figure D.2

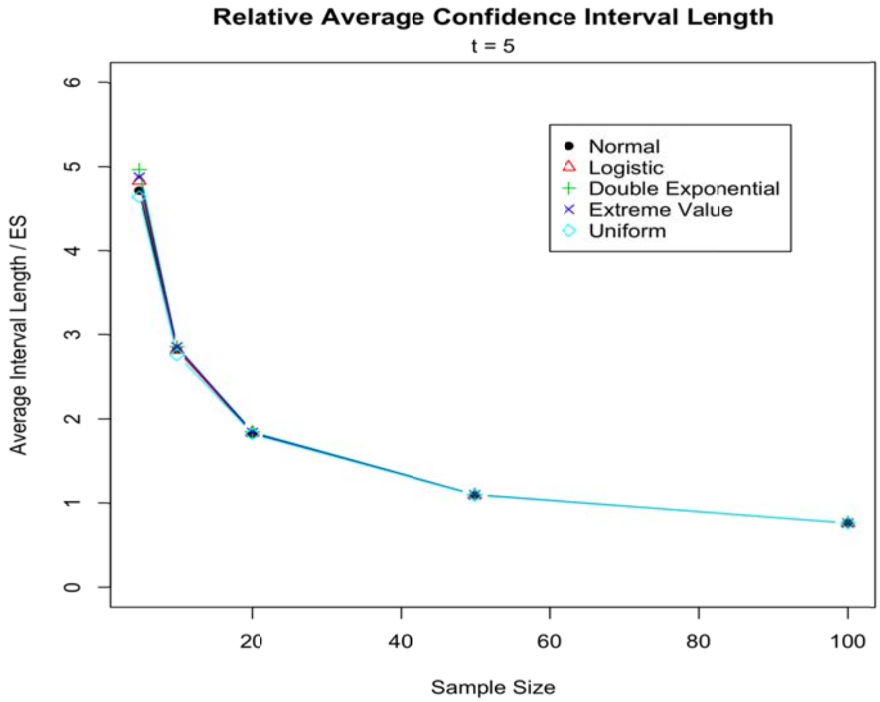


Figure D.3

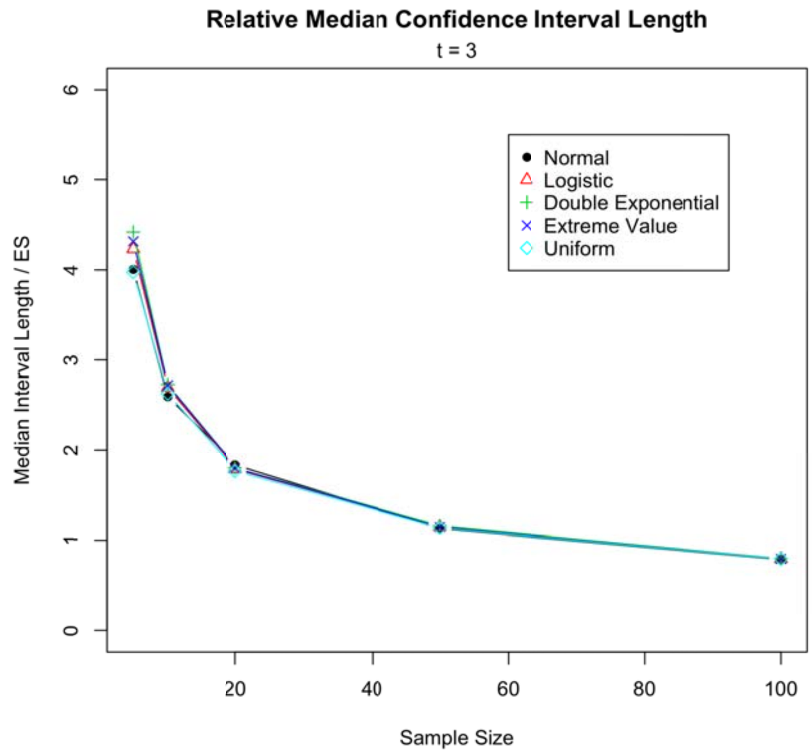
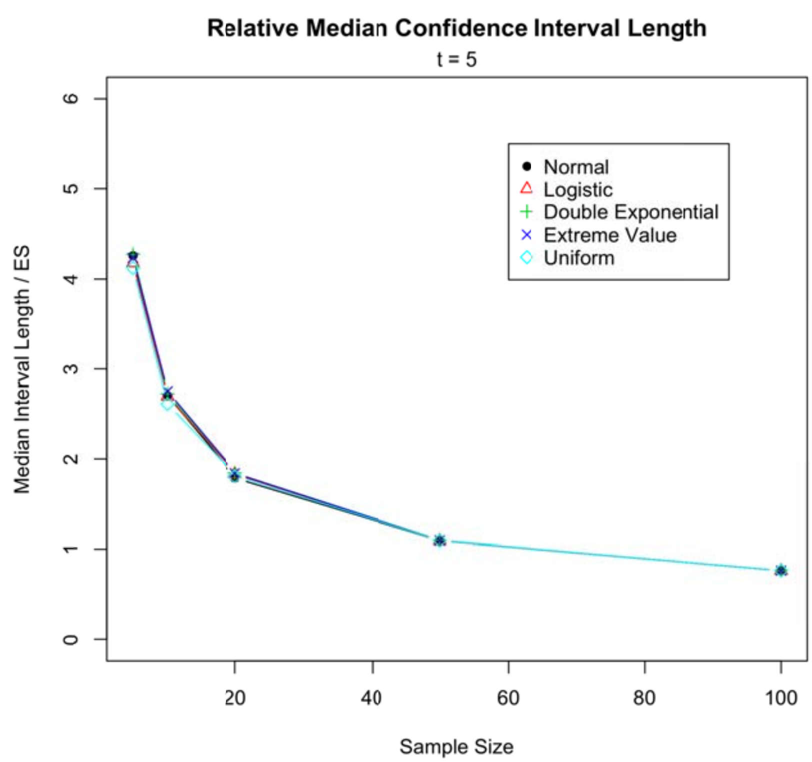


Figure D.4



Appendix E – Data from Example 1

$X_1 \sim N(\mu_1 = 100.1, \sigma = 1.0)$									
99.65	100.29	100.51	99.27	99.47	99.92	100.68	98.63	100.68	99.72
98.89	100.06	99.98	98.77	100.1	100.9	101.01	99.91	100.03	99.1
100.14	99.88	100.03	100.32	100.46	99.96	99.74	99.79	101.12	100.77
100.74	101.56	97.78	99.42	99.33	98.04	100.7	100.17	100.96	101
99.31	100.33	99.05	99.59	99.92	99.38	99.48	99.78	99.58	101.53
99.71	100.21	102.16	100.89	99.28	98.47	100.33	99.62	99.37	100.14
99.62	98.72	102.07	101.14	98.8	100.06	98.29	100.73	101.09	98.88
100.82	99.13	98.18	100.28	98.9	99.6	101.16	99.05	99.93	101.83
100.08	100.35	100.56	100.09	102	99.69	99.45	100.15	100.99	99.4
98.73	99.01	99.94	99.59	101.32	99.63	98.76	99.98	100.72	99.49
99.12	100.5	100	101.22	100.33	98.58	99.74	101.13	100.5	99.95
99.55	99.1	100.57	98.62	99.91	99.3	102.18	99.55	100.79	101.33
100.22	100.2	100.54	98.71	101.78	98.45	101.98	98.34	99.24	101.77
99.98	101.05	100.93	100.2	101.13	100.78	100.23	99.01	99.69	98.39
98.76	98.31	99.71	100.36	101.6	101.46	100.26	99.23	99.69	100.18
100.27	100.41	102.12	100.59	101.21	100.11	100.84	99.14	98.66	98.83
100.26	102.65	98.73	99.29	100.6	98.59	101.78	100.11	99.09	100.89
100.12	99.24	100.26	101	100.15	100.61	100.39	101.39	100.8	99.74
98.05	100.64	99.99	98.92	99.58	100.94	98.71	97.44	99.64	99.88
100.31	99.71	100.96	99.75	100.92	99.57	100.82	100.25	100.2	100.06
102.77	101.34	100.23	101.32	101.2	99.24	100.96	100.12	101.67	99.88
98.87	101.3	100.29	101.8	99.71	99.27	99.64	99.82	99.36	101.35
100.93	99.6	100.49	100.13	99.74	102.1	99.88	99.5	99.36	99.44
100.63	99.81	97.87	100.81	98.94	98.16	99.4	99.99	100.63	99.92
99.45	99.53	101.13	99.15	99.39	99.49	99.87	98.91	99.73	100.05
100.7	101.72	101.3	100.9	100.45	100.44	99.67	101.43	99.82	100.17
98.32	99.14	100.85	100.2	100.53	100.49	99.52	99.65	99.42	100.63
100.43	100.14	99.81	101.44	101.76	99.18	101.05	100.79	100	100.53
100.66	98.6	99.87	100.07	100.68	102.2	100.55	101.49	98.92	100.06
101.32	100.76	99.97	98.28	100.24	99.17	101.16	98.65	99.23	99.73
99.89	100.39	101.91	99.1	100.44	99.3	99.27	100.71	99.39	100.36
100.8	101.49	99.61	100.41	99.17	100.82	99.91	101	100.79	100.04
99.39	99.94	100.39	100.23	101.02	100.06	101.21	100.69	99.43	100.13
99.63	99.64	98.6	99.02	100.83	100.81	101.88	100.21	100.85	100.31
98.33	100.27	101.72	100.29	98.42	100.05	101.78	101.71	98.79	98.7
100.29	101.5	100.34	101.02	99.87	100.21	98.88	99.64	101.5	99.98
99.73	100.83	100.67	102.1	99.44	100.59	102.14	98.28	98.25	99.26
101.16	100.44	99.78	99.8	100.02	100.57	99.81	102.22	101.58	99.8
99.36	101.27	100.56	99.67	100.26	100.21	99.05	100.56	102.14	100.63
98.75	100.35	101.17	99.79	100.01	99.03	99.6	99.4	101.09	99.61
99.58	99.74	100.42	100.51	101.06	100.69	99.6	100.96	100.84	99.28
101.51	101.48	99.43	100.56	99.47	99.99	98.32	99.58	101.53	99.75
99.53	100.66	100.29	99.88	99.07	100.34	98.91	97.67		

X$_2$ ~ N(μ_2 = 100.2, σ = 1.0)									
98.87	100.5	100.02	100.12	100.05	100.6	99.71	100.26	100.07	100.14
99.47	100.17	99.79	100.89	100.09	99.18	99.21	98.92	98.72	100.31
99.27	100.24	101.2	99.38	100.96	100.25	101.28	100.27	100.69	99.67
101.11	100.47	101.44	100.46	98.96	101.96	99.8	98.78	101.11	100.49
100.79	100.1	98.66	99.88	99.61	100.65	98.92	100.51	101.16	100.29
100.27	98.85	99.76	100.57	98.16	100.84	99.48	99.16	99.37	97.99
100.27	100.05	99.78	99.07	99.64	100.81	101.88	98.5	99.09	100.76
100.72	100.47	100.38	99.98	101.46	101.31	101.64	100.91	99.92	98.98
100.55	99.8	100.62	99.85	99.55	99.29	98.9	99.26	100.5	100.77
101.33	100.56	100.85	102.12	101.1	100.31	100.83	100.33	98.43	99.55
100.86	99.32	98.2	100.65	99.05	100.12	101.98	100.65	101.49	98.06
99.83	98.76	99.27	100.9	100.36	100.57	101.07	98.69	99.83	101.68
99.67	101.31	100.55	100.63	101.08	100.75	98.39	100.39	100.51	101.74
100.47	100.53	100.89	101.05	100.99	101.17	99.7	99.37	99.46	99.78
99.1	100.71	99.25	100.63	99.46	101.78	101.44	101.4	100.62	100.49
100.87	100.4	101.73	100.98	99.15	100.12	98.92	100.85	100.68	99.78
99.45	101.91	100.59	100.92	99.92	99.57	100.3	101.02	99.4	102.07
99.65	101.91	100.57	100.81	99.82	102.39	100.05	100.44	98.12	101.88
101.69	100.68	100.62	99.05	100.58	99.88	99.22	100.59	100.33	100.98
100.36	100.1	100.97	98.63	100.1	99.26	99.54	101.19	101.69	101.16
101.8	99.66	99.01	100.41	100.78	101.47	100.35	98.54	99.52	99.42
99.42	99.89	102.09	99.97	100.18	100.11	99.48	99.42	100.79	99.31
101.81	100.43	99.84	101.01	99.56	98.46	101.29	100	100.56	101.15
100.74	100.09	100.31	101.21	99.06	100.8	99.85	100.03	100.5	101.03
100.54	98.26	99.83	102.18	100.83	101.36	100.01	100.75	100.93	100.7
100.53	100.31	99.46	101.29	100.5	99.02	98.31	100.68	100.91	98.41
100.13	100.88	100.19	98.76	100.74	100.09	98.72	99.69	97.3	100.82
100.62	99.81	101.64	99.64	101.2	100.39	100.81	98.23	99.76	101.21
102.14	99.45	101.49	102.03	100.53	99.31	100.09	100.45	101.59	101.84
99.73	100.89	100.7	100.24	102.39	99.26	100.57	99.36	99.08	98.79
100.62	100.59	98.65	100.5	100.12	99.68	102.31	101.23	99.83	100.55
98.82	100.11	98.06	101.55	101.12	100.79	100.77	102	100.13	99.04
100.29	98.25	100.22	99.97	100.16	99.79	99.65	102.18	100.81	100.07
101.42	99.53	98.81	100.88	98.36	100.88	100.63	100.87	100.08	102.24
100.85	100.21	100.76	100.34	100.14	101.97	100.93	101.04	100.34	98.42
99.73	100.62	99.94	98.47	100.14	98.74	101	99.25	99.45	99.57
99.76	99.48	100.23	99.74	99.55	100.95	99.06	99.04	100.86	101.31
100.69	100.63	99.76	100.32	97.79	99.7	99.76	99.78	98.82	101.1
98.47	100.18	100.51	96.84	99.98	99.68	101.23	101.12	99.54	98.33
101.61	99.7	98.62	99.1	98.77	100.03	100.05	101.62	98.7	98.73
98.74	98.61	99.67	101.51	101.13	99.43	101.66	99.65	99.45	100.25
100.2	101.8	99.01	100.3	98.63	99.43	102.17	101.44	99.54	101.17
100.06	100.72	98.77	98.63	101.51	99.57	98.22	100.08		

$X_3 \sim N(\mu_3 = 100.3, \sigma = 1.0)$									
99.09	98.63	100.13	101.28	99.12	100.76	100.34	100.39	100.44	101.02
100.32	100.06	100.66	100.15	99.47	102.26	99.43	100.76	100.05	99.01
101.06	99.02	101.35	100.08	99.58	100.26	100.05	99.9	100.85	100.43
100.18	100.81	100.55	101.55	99.85	99.5	100.47	100.41	100.86	101.14
99.18	101.8	100.8	98.96	102	100.51	101.07	99.74	100.17	98.55
100.83	99.37	99.91	101.37	99.55	100.54	100.6	101.47	99.61	100.73
100.15	100.32	101.19	101.75	99.16	98.83	99.75	101.29	100.44	101.46
100.37	99.52	101.22	100.45	100.87	99.61	99.32	100.95	100.19	99.47
98.69	101.6	101.09	98.74	100.67	101.09	99.67	99.04	100.43	102.28
99	100.53	97.16	100.21	99.09	100.73	99.65	100.45	100.75	101.75
101.78	100.81	97.68	99.28	100.01	100.33	101.4	99.49	98.03	102.13
98.71	100.9	102.92	100.6	98.73	100.25	101.07	99.91	102.44	101.66
98	99.27	99.2	99.76	101.39	101.89	99.08	99.53	98.9	101.92
100.44	99.15	100.69	101.44	101.43	98.63	99.7	101.15	100.94	99.67
98.6	99.58	100.77	100.49	99.08	101.6	102.27	99.07	100.68	100.26
100.54	100.06	99.57	102.38	100.28	100.85	101.07	101.53	100.66	100.15
100.76	100.44	97.7	99.08	100.12	100.24	99.93	100.24	99.56	98.74
99.17	100.28	99.48	101.16	100.55	101.35	99.19	98.86	98.86	100.31
100.4	100.04	99.91	100.94	100.29	99.74	102.14	100.07	100.88	99.34
101.14	99.35	100.83	99.01	99.79	102.02	99.88	99.74	100.34	100.45
99.56	100.7	101.12	100.25	98.97	98.86	99.89	99.95	99.59	99.66
101.25	100.64	101.04	100.69	101.64	98.86	100.81	100.92	100.06	99.28
102.77	100.26	100.63	101.01	98.9	100.39	98.6	99.48	99.91	101.26
101	100.12	100.87	100.14	101.19	98.49	99.83	100.15	98.76	100.33
100.4	100.77	99.53	100.32	99.92	100.39	101.54	99.18	99.54	99.69
100.28	100.06	98.74	102.69	101.35	97.57	100.05	99.48	98.97	100.06
98.98	99.56	98.06	100.13	102.35	98.03	100.26	101.24	99.64	101.23
100.5	97.62	99.7	99.63	100.56	99.6	100.87	101.46	100.05	99.48
100.01	99.7	99.86	98.53	100.71	102.21	99.95	99	101.18	99.4
100.92	100.48	100.86	99.85	101.71	97.97	100.22	100.71	101.07	100.65
101.55	100.14	101.86	100.62	100.65	101.15	101.08	101.02	101.12	99.6
99.73	101.44	101.43	101.92	100.09	99.74	99.88	99.66	102.55	99.64
100.57	102.2	99.73	99.53	99.9	98.92	101.06	101.8	101.46	99.77
99.92	99.92	99.7	100.34	98.85	100.21	98.95	102.36	100.5	100.34
99.76	101.32	100.78	100.88	101.68	99.92	100.73	100.91	99.81	101.05
99.54	100.69	100.45	100.93	99.15	99.69	100.62	98.32	100.18	100.17
101.2	100.77	102.36	100.36	100.5	99.94	99.52	100.03	100.97	101.43
98.89	100.34	101.56	101.52	100.16	100	101.86	100.5	99.29	99.72
102.05	99.88	101.34	100.24	98.99	100.27	101.23	101.04	100.86	100.54
102.02	101.39	99.31	100.39	100.39	99.6	99.41	99.21	100.73	101.66
100.76	98.83	99.58	99.58	99.26	98.93	101.17	101.29	99.11	100.49
100.12	101.15	100.74	100.08	100.88	100.46	99.14	100.7	98.91	99.77
100.28	100.58	101.68	99.69	99.38	99.63	100.02	100.33		

$X_1 \sim N(\mu_1 = 100.1, \sigma = 0.1)$									
100.06	100.10	100.09	100.03	100.08	99.94	99.92	99.99	100.19	100.03
99.98	100.08	99.87	100.05	100.02	100.10	100.21	100.10	100.16	100.04
100.10	100.25	100.00	100.18	99.97	100.05	100.04	100.09	100.14	100.09
100.16	100.12	100.31	100.20	99.98	100.06	99.97	100.20	100.17	100.22
100.02	100.11	100.30	100.12	100.29	100.05	100.06	100.04	100.01	100.27
100.06	99.96	99.91	100.10	100.22	99.95	100.31	99.92	100.06	99.93
100.05	100.00	100.15	100.05	100.12	100.02	100.29	99.99	100.06	100.11
100.17	100.13	100.08	100.21	100.08	99.93	100.11	100.01	99.96	99.97
100.10	99.99	100.09	99.95	100.27	100.17	100.12	100.00	100.00	100.18
99.96	100.14	100.15	99.96	100.20	100.24	100.17	100.10	100.17	100.06
100.00	100.00	100.14	100.11	100.25	100.10	100.27	100.23	100.05	100.08
100.04	100.11	100.18	100.13	100.21	99.95	100.13	99.83	100.11	100.10
100.11	100.20	100.06	100.15	100.15	100.15	99.96	100.11	100.26	100.08
100.09	99.92	100.30	100.02	100.10	100.18	100.17	100.10	100.03	100.23
99.97	100.13	99.96	100.19	100.05	100.05	100.19	100.07	100.03	100.03
100.12	100.36	100.12	99.98	100.18	100.01	100.05	100.04	100.15	100.08
100.12	100.01	100.09	100.06	100.21	100.02	100.08	100.09	100.06	100.09
100.10	100.15	100.19	100.22	100.06	100.30	100.03	99.98	100.07	100.11
99.90	100.06	100.11	100.27	100.06	99.91	100.08	100.23	100.03	100.15
100.12	100.22	100.12	100.10	99.98	100.04	100.06	100.05	100.09	100.14
100.37	100.22	100.14	100.17	100.03	100.13	100.04	100.17	99.98	100.10
99.98	100.05	99.88	100.00	100.13	100.14	100.20	100.24	100.01	100.06
100.18	100.07	100.20	100.18	100.14	100.01	100.15	99.95	100.03	100.13
100.15	100.04	100.22	100.11	100.27	100.31	100.21	100.16	100.17	100.09
100.04	100.26	100.18	100.23	100.16	100.01	100.02	100.19	100.03	100.10
100.16	100.00	100.07	100.10	100.11	100.02	100.08	100.16	100.18	100.12
99.92	100.10	100.08	99.92	100.13	100.17	100.21	100.11	99.97	99.96
100.13	99.95	100.09	100.00	100.01	100.10	100.28	100.26	100.24	100.09
100.16	100.17	100.28	100.13	100.19	100.17	100.27	100.05	99.92	100.02
100.22	100.13	100.05	100.11	100.17	100.10	99.98	99.92	100.25	100.07
100.08	100.24	100.13	99.99	99.93	100.11	100.30	100.31	100.30	100.15
100.17	100.08	99.95	100.12	100.08	100.15	100.07	100.15	100.20	100.05
100.03	100.05	100.26	100.19	100.03	100.15	99.99	100.03	100.17	100.02
100.05	100.12	100.12	100.30	100.09	100.11	100.05	100.19	100.24	100.06
99.92	100.24	100.16	100.07	100.12	99.99	100.05	100.05	100.06	99.86
100.12	100.17	100.07	100.06	100.09	100.16	99.92	100.16	100.00	99.98
100.06	100.13	100.15	100.07	100.20	100.09	99.95	100.09	100.17	100.12
100.21	100.22	100.21	100.14	100.04	100.16	100.08	100.20	100.19	100.00
100.03	100.12	100.13	100.15	100.08	100.19	100.07	100.19	100.24	100.08
99.97	100.06	100.03	100.04	100.18	100.06	100.11	100.05	100.10	100.12
100.05	100.24	100.02	100.10	100.09	100.16	100.07	100.03	99.98	100.16
100.24	100.14	99.97	100.14	99.89	100.04	100.05	100.20	100.27	100.04
100.12	100.09	100.12	100.02	100.03	100.12	100.16	100.08		

$X_2 \sim N(\mu_2 = 100.2, \sigma = 0.1)$									
100.07	100.23	100.18	100.19	100.18	100.24	100.15	100.21	100.19	100.19
100.13	100.20	100.16	100.27	100.19	100.10	100.10	100.07	100.05	100.21
100.11	100.20	100.30	100.12	100.28	100.20	100.31	100.21	100.25	100.15
100.29	100.23	100.32	100.23	100.08	100.38	100.16	100.06	100.29	100.23
100.26	100.19	100.05	100.17	100.14	100.24	100.07	100.23	100.30	100.21
100.21	100.07	100.16	100.24	100.00	100.26	100.13	100.10	100.12	99.98
100.21	100.19	100.16	100.09	100.14	100.26	100.37	100.03	100.09	100.26
100.25	100.23	100.22	100.18	100.33	100.31	100.34	100.27	100.17	100.08
100.24	100.16	100.24	100.17	100.13	100.11	100.07	100.11	100.23	100.26
100.31	100.24	100.26	100.39	100.29	100.21	100.26	100.21	100.02	100.13
100.27	100.11	100.00	100.25	100.08	100.19	100.38	100.24	100.33	99.99
100.16	100.06	100.11	100.27	100.22	100.24	100.29	100.05	100.16	100.35
100.15	100.31	100.23	100.24	100.29	100.25	100.02	100.22	100.23	100.35
100.23	100.23	100.27	100.28	100.28	100.30	100.15	100.12	100.13	100.16
100.09	100.25	100.11	100.24	100.13	100.36	100.32	100.32	100.24	100.23
100.27	100.22	100.35	100.28	100.09	100.19	100.07	100.27	100.25	100.16
100.12	100.37	100.24	100.27	100.17	100.14	100.21	100.28	100.12	100.39
100.14	100.37	100.24	100.26	100.16	100.42	100.19	100.22	99.99	100.37
100.35	100.25	100.24	100.08	100.24	100.17	100.10	100.24	100.21	100.28
100.22	100.19	100.28	100.04	100.19	100.11	100.13	100.30	100.35	100.30
100.36	100.15	100.08	100.22	100.26	100.33	100.22	100.03	100.13	100.12
100.12	100.17	100.39	100.18	100.20	100.19	100.13	100.12	100.26	100.11
100.36	100.22	100.16	100.28	100.14	100.03	100.31	100.18	100.24	100.30
100.25	100.19	100.21	100.30	100.09	100.26	100.16	100.18	100.23	100.28
100.23	100.01	100.16	100.40	100.26	100.32	100.18	100.26	100.27	100.25
100.23	100.21	100.13	100.31	100.23	100.08	100.01	100.25	100.27	100.02
100.19	100.27	100.20	100.06	100.25	100.19	100.05	100.15	99.91	100.26
100.24	100.16	100.34	100.14	100.30	100.22	100.26	100.00	100.16	100.30
100.39	100.12	100.33	100.38	100.23	100.11	100.19	100.23	100.34	100.36
100.15	100.27	100.25	100.20	100.42	100.11	100.24	100.12	100.09	100.06
100.24	100.24	100.04	100.23	100.19	100.15	100.41	100.30	100.16	100.24
100.06	100.19	99.99	100.33	100.29	100.26	100.26	100.38	100.19	100.08
100.21	100.01	100.20	100.18	100.20	100.16	100.14	100.40	100.26	100.19
100.32	100.13	100.06	100.27	100.02	100.27	100.24	100.27	100.19	100.40
100.27	100.20	100.26	100.21	100.19	100.38	100.27	100.28	100.21	100.02
100.15	100.24	100.17	100.03	100.19	100.05	100.28	100.10	100.12	100.14
100.16	100.13	100.20	100.15	100.14	100.27	100.09	100.08	100.27	100.31
100.25	100.24	100.16	100.21	99.96	100.15	100.16	100.16	100.06	100.29
100.03	100.20	100.23	99.86	100.18	100.15	100.30	100.29	100.13	100.01
100.34	100.15	100.04	100.09	100.06	100.18	100.18	100.34	100.05	100.05
100.05	100.04	100.15	100.33	100.29	100.12	100.35	100.14	100.12	100.21
100.20	100.36	100.08	100.21	100.04	100.12	100.40	100.32	100.13	100.30
100.19	100.25	100.06	100.04	100.33	100.14	100.00	100.19		

$X_3 \sim N(\mu_3 = 100.3, \sigma = 0.1)$									
100.18	100.13	100.28	100.40	100.18	100.35	100.30	100.31	100.31	100.37
100.30	100.28	100.34	100.29	100.22	100.50	100.21	100.35	100.28	100.17
100.38	100.17	100.41	100.28	100.23	100.30	100.27	100.26	100.36	100.31
100.29	100.35	100.32	100.43	100.25	100.22	100.32	100.31	100.36	100.38
100.19	100.45	100.35	100.17	100.47	100.32	100.38	100.24	100.29	100.13
100.35	100.21	100.26	100.41	100.22	100.32	100.33	100.42	100.23	100.34
100.29	100.30	100.39	100.44	100.19	100.15	100.24	100.40	100.31	100.42
100.31	100.22	100.39	100.32	100.36	100.23	100.20	100.36	100.29	100.22
100.14	100.43	100.38	100.14	100.34	100.38	100.24	100.17	100.31	100.50
100.17	100.32	99.99	100.29	100.18	100.34	100.23	100.32	100.35	100.45
100.45	100.35	100.04	100.20	100.27	100.30	100.41	100.22	100.07	100.48
100.14	100.36	100.56	100.33	100.14	100.29	100.38	100.26	100.51	100.44
100.07	100.20	100.19	100.25	100.41	100.46	100.18	100.22	100.16	100.46
100.31	100.18	100.34	100.41	100.41	100.13	100.24	100.38	100.36	100.24
100.13	100.23	100.35	100.32	100.18	100.43	100.50	100.18	100.34	100.30
100.32	100.28	100.23	100.51	100.30	100.36	100.38	100.42	100.34	100.28
100.35	100.31	100.04	100.18	100.28	100.29	100.26	100.29	100.23	100.14
100.19	100.30	100.22	100.39	100.32	100.40	100.19	100.16	100.16	100.30
100.31	100.27	100.26	100.36	100.30	100.24	100.48	100.28	100.36	100.20
100.38	100.21	100.35	100.17	100.25	100.47	100.26	100.24	100.30	100.32
100.23	100.34	100.38	100.30	100.17	100.16	100.26	100.26	100.23	100.24
100.39	100.33	100.37	100.34	100.43	100.16	100.35	100.36	100.28	100.20
100.55	100.30	100.33	100.37	100.16	100.31	100.13	100.22	100.26	100.40
100.37	100.28	100.36	100.28	100.39	100.12	100.25	100.29	100.15	100.30
100.31	100.35	100.22	100.30	100.26	100.31	100.42	100.19	100.22	100.24
100.30	100.28	100.14	100.54	100.41	100.03	100.27	100.22	100.17	100.28
100.17	100.23	100.08	100.28	100.50	100.07	100.30	100.39	100.23	100.39
100.32	100.03	100.24	100.23	100.33	100.23	100.36	100.42	100.27	100.22
100.27	100.24	100.26	100.12	100.34	100.49	100.27	100.17	100.39	100.21
100.36	100.32	100.36	100.26	100.44	100.07	100.29	100.34	100.38	100.33
100.43	100.28	100.46	100.33	100.34	100.39	100.38	100.37	100.38	100.23
100.24	100.41	100.41	100.46	100.28	100.24	100.26	100.24	100.52	100.23
100.33	100.49	100.24	100.22	100.26	100.16	100.38	100.45	100.42	100.25
100.26	100.26	100.24	100.30	100.16	100.29	100.17	100.51	100.32	100.30
100.25	100.40	100.35	100.36	100.44	100.26	100.34	100.36	100.25	100.37
100.22	100.34	100.32	100.36	100.19	100.24	100.33	100.10	100.29	100.29
100.39	100.35	100.51	100.31	100.32	100.26	100.22	100.27	100.37	100.41
100.16	100.30	100.43	100.42	100.29	100.27	100.46	100.32	100.20	100.24
100.47	100.26	100.40	100.29	100.17	100.30	100.39	100.37	100.36	100.32
100.47	100.41	100.20	100.31	100.31	100.23	100.21	100.19	100.34	100.44
100.35	100.15	100.23	100.23	100.20	100.16	100.39	100.40	100.18	100.32
100.28	100.38	100.34	100.28	100.36	100.32	100.18	100.34	100.16	100.25
100.30	100.33	100.44	100.24	100.21	100.23	100.27	100.30		

Appendix F – Data from Example 2

Group	Velocity	Group	Velocity	Group	Velocity	Group	Velocity	Group	Velocity
1	850	2	960	3	880	4	890	5	890
1	740	2	940	3	880	4	810	5	840
1	900	2	960	3	880	4	810	5	780
1	1070	2	940	3	860	4	820	5	810
1	930	2	880	3	720	4	800	5	760
1	850	2	800	3	720	4	770	5	810
1	950	2	850	3	620	4	760	5	790
1	980	2	880	3	860	4	740	5	810
1	980	2	900	3	970	4	750	5	820
1	880	2	840	3	950	4	760	5	850
1	1000	2	830	3	880	4	910	5	870
1	980	2	790	3	910	4	920	5	870
1	930	2	810	3	850	4	890	5	810
1	650	2	880	3	870	4	860	5	740
1	760	2	880	3	840	4	880	5	810
1	810	2	830	3	840	4	720	5	940
1	1000	2	800	3	850	4	840	5	950
1	1000	2	790	3	840	4	850	5	800
1	960	2	760	3	840	4	850	5	810
1	960	2	800	3	840	4	780	5	870