DESIGN AND IMPLEMENTATION OF A MICROCOMPUTER, CONTROLLER FOR STABILIZING AN INVERTED PENDULUM MOUNTED ON A CART

by

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[Signature]
Major Professor
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CHAPTER I
INTRODUCTION

In this advanced technological age there are many useful systems which have a natural instability; for example missiles, helicopters, aircrafts, torpedos, etc. To stabilize these systems, for useful and safe operation, control systems are designed and built to suit their individual requirements.

In recent years, especially since the advent of the microcomputer, great advances have been made in the field of discrete control systems. These systems differ from conventional, continuous systems in that some of the signals are in the form of either discrete values or numerical codes.

With the development of sophisticated, fast, inexpensive computers and interfacing hardware, an impetus is given to the study and development of discrete control systems. The extensive arithmetical and logical capabilities of microcomputers make them suitable for executing complex and extensive control algorithms. Hence digital computers have gained significant importance as control elements in control systems.

The object of this thesis is to develop and implement a discrete control system for a naturally unstable system. An inverted pendulum mounted on a cart was chosen for this study. Controlling this system is similar to the problem of controlling the attitude of a rocket during the initial stages of the launch.

A d.c. motor was used to drive the cart. Power was tran-
smitted from the d.c. motor to the wheels of the cart by a belt and pulley drive system. The movement of the cart is limited to the X-direction and the displacement of the pendulum is restricted in the vertical plane along the X-direction.

This thesis is divided into eight major parts.

To know the dynamics of the system to be controlled the mathematical model of the system is developed in Chapter II. The parameters of the model were obtained by frequency response analysis.

The discrete data model was obtained by applying the Z-transform. The Z-transform analysis for the system is given in Chapter III.

The physical system has one mode of instability and one mode of neutral stability. The system is stabilized by a compensation technique. The compensation analysis is presented in Chapter IV.

The procedure and the results of a simulation of the system are given in Chapter V.

The system was controlled by a microcomputer. The program was written in assembly language. Emphasis was placed on making the program as efficient as possible due to the time constraint for the execution of the computation. The main program and its subroutines are explained in Chapter VI. The programs are listed in Appendix E.

The experimental arrangement and procedure are given in Chapter VII.
The results of the experiment are given in Chapter VIII. The conclusions and recommendations are presented in Chapter IX.
CHAPTER II

MATHEMATICAL MODELING

2.1 Introduction

The details of the system can be visualised from photographic Figure 2.1.

A d.c. motor is used to drive the cart which has separate excitations for the armature and the field. The armature voltage was varied to control the speed of the cart while a constant voltage was applied to the field. Power from the d.c. motor was transmitted to the wheels of the cart by belt drives with a gear ratio of 19.7:1. The position of the pendulum was measured by a potentiometer which was mounted on the pivot of the pendulum. The speed of the cart was measured by a tachometer which was coupled to the shaft of the d.c. motor.

In this chapter a mathematical model of the system is developed. To obtain a mathematical model it is necessary to analyze the relationship between the system variables. Since the system is dynamic, the descriptive equations are differential equations. Necessary assumptions were made to linearize the system, in order to simplify the analysis. The parameters of the system were determined by frequency response analysis.

2.2 Mathematical Model of the D.C. Motor, Drive and Cart.

A schematic representation of the d.c. motor and the drive system is shown in Figure 2.2. An armature controlled d.c. motor
Figure 2.1 Photograph of the Cart Pendulum System
Figure 2.2. Schematic Representation of the D.C. Motor, Drive and Cart
is used to drive the cart. The transfer function of the d.c motor is developed. Second order effects, such as hysteresis, inductance of the armature and the voltage drop across the brushes are neglected. The spring constant, $K_b$, of the belt drive is considered while the damping coefficient of the belt is neglected because it is very small.

The input voltage, $V_a$, is applied across the armature terminals. The back voltage, $V_b$, is proportional to the speed of the armature.

$$V_b(s) = K_s \cdot V_a(s),$$ (2.1)

where $K_s$ is the motor speed constant (volts/radian/sec), and $V_a$ is the speed of the armature (radians/sec).

The armature current, $I_a$, is related to the input voltage applied to the armature by

$$I_a(s) = \frac{V_a(s) - V_b(s)}{R_a},$$ (2.2)

where $R_a$ is the resistance of the armature.

The torque, $T_m$, produced by the motor is proportional to the current through the armature. The motor torque is given by the relation,

$$T_m(s) = K_m \cdot I_a(s),$$ (2.3)

where $K_m$ is the motor torque constant (Newton-m/ampere).

The motor torque is equal to the torque delivered to the load. Applying D'Alembert's principle to the armature shaft
rotation yields the equation,

\[ J_a \omega_a + f_a \omega_a + K_b (D_a - K_w D_c) = T_m, \]  

(2.4)

where \( J_a \) = moment of inertia of the armature (Kgm\(^m^2\)),  
\( f_a \) = coefficient of friction (Newton\(\cdot\)m\(\cdot\)sec),  
\( K_b \) = spring constant for the belt (Newton\(\cdot\)m),  
\( D_a \) = displacement of the armature (radian),  
\( D_c \) = displacement of the cart (m),  
\( W_c \) = speed of the cart (meters/sec), and  
\( K_w \) = transmission ratio (radians/m).

The Laplace transform of Equation 2.4 yields,

\[ (J_a s^2 + f_a s + K_b)\omega_a = T_m s + K_b K_w W_c. \]  

(2.5)

Similarly, applying D'Alembert's principle to the displacement of the cart yields,

\[ M_c \omega_c + f_c \omega_c + K_b K_w^2 (D_c - D_a / K_w) = 0. \]  

(2.6)

where \( M_c \) = mass of the cart (Kgm) and  
\( f_c \) = coefficient of friction (Newton/m/sec).

The Laplace transform of Equation 2.6 yields,

\[ (M_c s^2 + f_c s + K_b K_w^2) \omega_c = K_w K_b \omega_a. \]  

(2.7)

Rearranging the terms in Equation 2.7 yields the transfer function relating the speed of the cart and the armature rotation as,

\[ \frac{W_c(s)}{W_a(s)} = \frac{K_w K_b}{M_c s^2 + f_c s + K_b K_w^2}. \]  

(2.8)
Figure 2.3 is a block diagram representation of the elements of the d.c. motor, drive and the cart. Applying block diagram reduction techniques the reduced block diagram is shown in Figure 2.4. The transfer function relating the motor speed to the armature voltage is,

\[
\frac{\text{Wa}(s)}{\text{Va}(s)} = \frac{K_m(M_c s^2 + f_c s + K_b K_w^2)}{R_a(D_3 s^3 + D_2 s^2 + D_1 s + D_0)}
\]

where \(D_3 = J_a M_c\),
\(D_2 = M_c f_a + J_a f_c + M_c K_m K_s / R_a\),
\(D_1 = J_a K_b K_w^2 + f_a f_c + M_c K_b + f_c K_m K_s / R_a\), and
\(D_0 = K_b f_c + K_b f_a K_w^2 + K_b K_m K_s K_w^2 / R_a\)

The coefficients of the transfer function are determined by frequency response analysis, since it is very difficult to measure the individual parameters accurately.

2.3 Mathematical Model for the Inverted Pendulum

The inverted pendulum mounted on the cart is illustrated in Figure 2.5. The differential equation describing the motion of the pendulum is obtained by summing the moments about the pivot point. In order to make the equation linear, the assumption made is that the angle of rotation, \(\theta\), of the pendulum is small. The sum of the moments about the pivot point is,

\[
m l_1 \ddot{\theta} + m l_2^2 \dddot{\theta} - m l_1 g \theta = 0
\]

where \(m\) = mass of the pendulum,
\(l_1\) = length of the center of mass of the pendulum from the pivot point,
\(l_2\) = radius of gyration of the pendulum from the pivot point.
Figure 2.3. Block Diagram Representation of the D.C. Motor, Drive and Cart
Figure 2.4. Reduced Block Diagram of the D.C. Motor, Drive and Cart

Figure 2.5. A Cart and Inverted Pendulum
point, and

\[ g = \text{acceleration due to gravity} = 9.81 \text{ m/sec}^2. \]

A uniform pendulum of length, \( L = 0.59 \text{ m} \), was chosen for the system. Hence,

\[ l_1 = \frac{L}{2} \quad \text{and} \quad l_2 = \frac{L}{\sqrt{3}}. \]

Substituting the values of \( l_1 \) and \( l_2 \) in Equation 2.10 yields,

\[ \dot{W}_c + 2*\dot{L}^2/3 - g*P = 0 \quad (2.11) \]

or

\[ \dot{W}_c + 0.40*\dot{P} - 9.81*P = 0 \quad (2.12) \]

The Laplace transform of Equation 2.12 is

\[ \frac{P(s)}{W_c(s)} = \frac{-2.5*s}{s^2 - 25} \quad (2.13) \]

Equation 2.13 is the transfer function relating the dynamics of the pendulum to the speed of the cart.

The d.c. motor used to drive the cart has separate excitations for the armature and the field. The armature voltage is used for control of the speed of the cart and a constant voltage is applied to the field. Since the dynamic characteristics of the motor are affected by the value of the field voltage it is important to establish the value of the field voltage before carrying out the frequency response analysis to determine the coefficients of the transfer functions. An experiment to explore the effects of the value of the field voltage is described in the next section.
2.4 Dynamic Response of the Speed of the Motor due to Change of Voltage Across the Field Winding

In this experiment the cart is driven by applying a sinusoidal voltage to the armature of the d.c. motor. The field voltage is increased in increments of 1 volt for each successive observation. The voltage to the armature is a sinusoid with an amplitude of 5 volts peak to peak and a frequency of 1.2 radians/sec. The output from the tachometer is recorded on a strip chart recorder. The values of the tachometer signal amplitude and the frequency response amplitude ratio are given in Table 2.1 and plotted in Figure 2.6.

<table>
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<tr>
<th>Observation Number</th>
<th>Field Voltage</th>
<th>Amplitude of Tachometer Signal in Volts</th>
<th>Amplitude Ratio</th>
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</thead>
<tbody>
<tr>
<td>1)</td>
<td>6</td>
<td>2.61</td>
<td>0.52</td>
</tr>
<tr>
<td>2)</td>
<td>7</td>
<td>3.53</td>
<td>0.71</td>
</tr>
<tr>
<td>3)</td>
<td>8</td>
<td>4.75</td>
<td>0.95</td>
</tr>
<tr>
<td>4)</td>
<td>9</td>
<td>5.32</td>
<td>1.06</td>
</tr>
<tr>
<td>5)</td>
<td>10</td>
<td>5.70</td>
<td>1.14</td>
</tr>
<tr>
<td>6)</td>
<td>11</td>
<td>6.27</td>
<td>1.25</td>
</tr>
<tr>
<td>7)</td>
<td>12</td>
<td>6.67</td>
<td>1.33</td>
</tr>
<tr>
<td>8)</td>
<td>13</td>
<td>6.89</td>
<td>1.38</td>
</tr>
<tr>
<td>9)</td>
<td>14</td>
<td>7.03</td>
<td>1.41</td>
</tr>
<tr>
<td>10)</td>
<td>15</td>
<td>7.03</td>
<td>1.41</td>
</tr>
<tr>
<td>11)</td>
<td>16</td>
<td>7.03</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Table 2.1 Experimental Reading of the Amplitude Ratio of the D.C. Motor Versus Field Voltage.
From Figure 2.6 it is observed that the amplitude ratio increases as the field voltage is increased. After a certain value, the amplitude ratio does not continue to increase even if the field voltage is increased. This is due to saturation of the field windings.

For all future operation of the system the field voltage is maintained at 14 volts which is the value at which saturation first occurs. Small changes in field voltage setting will have little effect on the performance of the system.

2.5 Tachometer Gain

A d.c. motor is used to drive the wheels of the cart through a belt drive arrangement with a gear ratio of 19.7:1. The speed of the d.c. motor is obtained by a tachometer which is directly coupled to the d.c. motor. Hence, to determine the gain of the tachometer it is important to find a relationship between the speed of the cart and the voltage output from the tachometer.

In this experiment, the cart is supported such that the wheels do not touch the ground. A constant 14 volts is applied across the field of the d.c. motor. To vary the speed the voltage across the armature of the d.c. motor is varied. Various readings for the speed of the d.c. motor are obtained by a stroboscope. The voltage outputs from the tachometer are recorded on the strip chart recorder. The values of the speed of the d.c. motor and the voltage output from the tachometer are given in Table 2.2.
<table>
<thead>
<tr>
<th>Observation Number</th>
<th>R.P.M. of Armature</th>
<th>Speed of Armature in rad/sec</th>
<th>Voltage Output from the Tachometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>64</td>
<td>6.7</td>
<td>0.475</td>
</tr>
<tr>
<td>2.</td>
<td>130</td>
<td>13.6</td>
<td>1.105</td>
</tr>
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<td>3.</td>
<td>235</td>
<td>24.6</td>
<td>1.56</td>
</tr>
<tr>
<td>4.</td>
<td>420</td>
<td>44.0</td>
<td>3.20</td>
</tr>
<tr>
<td>5.</td>
<td>540</td>
<td>56.6</td>
<td>4.25</td>
</tr>
<tr>
<td>6.</td>
<td>990</td>
<td>103.7</td>
<td>7.0</td>
</tr>
<tr>
<td>7.</td>
<td>1300</td>
<td>136.1</td>
<td>9.77</td>
</tr>
<tr>
<td>8.</td>
<td>1460</td>
<td>152.9</td>
<td>10.87</td>
</tr>
<tr>
<td>9.</td>
<td>1530</td>
<td>160.2</td>
<td>11.37</td>
</tr>
<tr>
<td>10.</td>
<td>2180</td>
<td>228.3</td>
<td>16.00</td>
</tr>
</tbody>
</table>

Table 2.2 Experimental Data to find the Gain of the Tachometer.

A plot of the speed of the armature v/s output voltage from the tachometer is shown in Figure 2.7. It is observed that the graph is almost linear, so it can be concluded from the graph that the gain, $K_t$, of the tachometer is 0.072 volts/rad/sec.

The transmission ratio, $K_w$, of the cart is calculated by Equation 2.14,

$$K_w = \frac{gr}{rw} = \frac{19.7}{0.1035} = 190.34 \text{ radians/m.} \quad (2.14)$$

where $rw = \text{radius of the wheels of the cart} = 0.1035 \text{ m}$, and 

$gr = \text{gearing ratio between the armature of the motor and}$ 

$\text{the wheels of the cart} = 19.7:1$.

All physical systems have nonlinearities. It is a well established practice that providing a feedback around a system
Figure 2.7. Plot of Speed of the Armature versus Output Voltage from the Tachometer
reduces the nonlinear effects. Therefore, the tachometer output was amplified in an operational amplifier with a gain of 2.3 and was fed back, summed with the control signal, and amplified by a gain of 20 to provide the input signal to the power amplifier as shown in Figure 2.8. The signal $U$ is the control input and signal $V_b$ is the output signal from the summing amplifier which drives the power amplifier to produce a corresponding voltage across the armature of the d.c. motor. After applying feedback around the system, an experiment was conducted to determine the coefficients of the transfer function of the system.

2.6 Experimental Determination of the Transfer Function of the System Consisting of the D.C. Motor, Cart, Drive and Tachometer.

The first step in the analysis and design of a control system is to develop a mathematical model of the system. In Equation 2.9 the transfer function relating the motor speed to the armature voltage is given. Applying feedback around the system as shown in Figure 2.8, yields a transfer function relating the tachometer voltage to the control voltage of the form

$$\frac{V_t(s)}{U(s)} = \frac{K_c(M_c s^2 + f_c s + K_b K_w^2)}{C_3 s^3 + C_2 s^2 + C_1 s + C_0}$$

(2.15)

where $K_c = 46 K_a K_t K_m / R_a$,

$K_a =$ Power Amplifier gain $= 1$,

$C_3 = J_a M_c,$

$C_2 = f_a M_c + f_c J_a + M_c K_m K_s / R_a + K_c M_c,$

$C_1 = K_b M_c + f_a f_c + J_a K_b K_w^2 + f_c K_m K_s / R_a + K_c f_c,$ and
Figure 2.8 Experimental Setup for the Frequency Response Measurement of the System.
$$C_0 = f_c K_b + K_b f_a K_w^2 + K_m K_s K_w^2 / R_a + K_c K_b K_w^2.$$  

Observe that the form of this transfer function is the same as before, e.g. the feedback simply changes the values of the coefficients and reduces the effects of nonlinearities. In order to proceed with the analysis of the control system it is necessary to obtain numerical values for the coefficients of the transfer function. One way to do this is to make tests and measurements to determine the values of the individual parameters and compute the values of the coefficients. Several of the parameters are rather difficult to measure. Another approach is to make a set of measurements of the system and determine coefficient values which cause the transfer function model to fit the measured response. The frequency response method follows this approach.

The procedure involved to obtain a frequency response of the system is to excite the system with a sinusoidal input with measured amplitude. The output amplitude is measured and the amplitude ratio of the output to the input is computed. The measurement is repeated at different frequencies. A plot of the amplitude ratio versus frequency is called a Bode plot. The coefficients of the transfer function are determined by fitting the model to the Bode plot. An asymptotic log-magnitude curve consisting of several segments is built. With some judgement and experience it is usually possible to find a close fit to the experimental data.

Figure 2.8 shows the experimental arrangement for the fre-
frequency response measurement. The input voltage from the function generator to the summing operational amplifier is a sinusoid with an amplitude of 3.10 volts peak to peak, which is maintained at all frequencies. A constant 14 volts is applied to the field winding of the d.c. motor. The input signal, \( U \), from the function generator and the output signal, \( V_t \), from the tachometer are recorded on the strip chart recorder. Table 2.3 lists the frequencies and the corresponding output amplitudes and amplitude ratios for which data were collected.

A Bode diagram of the experimental data is plotted on Figure 2.9. The frequency response of the transfer function chosen to approximate the experimental data is shown on Figure 2.9. The transfer function of the system can be approximated by,

\[
\frac{V_t(s)}{U(s)} = \frac{0.92*500*(s^2 + 11.5s + 2285)}{(s + 500)*(s^2 + 21.4s + 2362)}.
\] (2.16)

The pole at \(-500\) is almost 50 times as far from the imaginary axis as the two complex poles and will have a negligible effect on the performance of the system. It will be ignored in the subsequent analysis.

2.8 Block Diagram Representation of the System

In the previous sections of this chapter mathematical models have been developed for each element of the system. Experiments have been described by which the coefficients of these models have been determined. In this section the form of the control will be developed in preparation for the establishment of specific compensators in subsequent chapters.
<table>
<thead>
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<th>No.</th>
<th>Frequency radians/sec</th>
<th>Output voltage peak to peak</th>
<th>Amplitude Ratio=A.R</th>
<th>20*Log(A.R)</th>
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<td>0.887</td>
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<td>3.</td>
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<td>2.75</td>
<td>0.887</td>
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<tr>
<td>4.</td>
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<td>0.887</td>
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<td>8.</td>
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<tr>
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</tr>
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<td>251.32</td>
<td>2.80</td>
<td>0.903</td>
<td>-0.89</td>
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</table>

Table 2.3  Experimental Frequency Response of the System.
Figure 2.9. Bode Plot of the Closed Loop System with Experimental Data and the Approximate Fit
Figure 2.10 shows a block diagram of the system. The blocks within the dashed line represent compensators implemented in the microcomputer. The rest of the blocks represent the physical elements of the inverted pendulum system. The value which results from the digital computation is placed in a latch, which drives a digital-to-analog converter. Each time the computations are performed a new value is placed in the latch. U represents the analog signal which is output by the digital-to-analog converter. The zero-order-hold (ZOH) represents the action of the latch and the digital-to-analog converter. The signals represented by Vt and Vp are the signals out of the tachometer and potentiometer respectively. These two digital signals are sampled by the computer and provide the signals upon which the digital computations are performed. The two digital values, Vt* and Vp* are produced by the analog-to-digital converters and correspond to the signals Vt and Vp.

The transfer function, Gl(s) relating the voltage output from the tachometer, Vt, to the input voltage, U, to the summing operational amplifier was given in Equation 2.16,

\[
\frac{Vt(s)}{Gl(s)} = \frac{0.92s^2 + 11.5s + 2285}{s^2 + 21.4s + 2362}
\]

(2.17)

The block following Gl(s), with a transfer function 1/Kt relates the speed of the armature of the cart with the output voltage from the tachometer. Kt is the tachometer gain equal to 0.072 volts/rad/sec.

Next, the values of the parameters in Equation 2.8 relating
Figure 2.10. Block Diagram Representation of the System.
Wc to Wa can be established using Equations 2.17 and 2.14. Therefore the next transfer function in the block diagram is,

\[ G_2(s) = \frac{12.0}{W_a(s)} = \frac{12.0}{s^2 + 11.5s + 2285} \quad (2.18) \]

The transfer function, \( G_3(s) \), relating the position of the pendulum, \( P \), to the speed of the cart, \( W_c \), is given by Equation 2.13,

\[ G_3(s) = \frac{P(s)}{W_c(s)} = \frac{-2.5s}{s^2 - 25} \quad (2.19) \]

The position of the pendulum is measured by a potentiometer which is mounted on the pivot of the pendulum. The gain of the potentiometer is represented by \( K_p \) in the block diagram. The value of \( K_p \) is adjustable.

The control strategy requires that the velocity of the cart be known. Since there is no direct measurement of the velocity of the cart it is necessary to compute it from the measured value of the speed of the motor. The digital model \( H_3(z) \), developed in Chapter III, provides a computed estimate of the velocity of the cart. \( H_1(z) \) and \( H_2(z) \) are compensation transfer functions which are developed in Chapter IV to stabilize the pendulum-cart system.
CHAPTER III
Z-TRANSFORM ANALYSIS

3.1 Introduction

The role of the z-transform to sampled data systems is similar to that of the Laplace transform to continuous data systems. The z-transform technique provides a mean of finding the output of a system at the k-th sample time, \( t=kT \), in terms of the input at the k-th sample time, \( t=kT \), and the input, and the output at previous sample times, \( t=(k-i)T \). This technique is applied to the problem of finding the voltage input to the d.c. motor in terms of the position of the pendulum and velocity of the cart. The z-transform technique is demonstrated below.

A time function \( x(t) \), shifted in time by an amount \( nT \) is \( x(t-nT) \). The Laplace transform is,

\[
L(x(t-nT)) = \int_{-\infty}^{\infty} x(t-nT) e^{-st} dt. \tag{3.1}
\]

Substituting, \( h = t-nT \), yields

\[
L(x(t-nT)) = \int_{-nT}^{\infty} x(h) e^{-s(h+nT)} dh. \tag{3.2}
\]

Since \( x(h) = 0 \) for \( h \leq 0 \), Equation 3.1 may be written as

\[
L(x(t-nT)) = e^{-snT} \int_{0}^{\infty} x(h) e^{-sh} dh.
\]

\[
= X(s)e^{-snT} \tag{3.3}
\]

The z-transform is obtained from the Laplace transform by substituting \( z = e^{st} \). Thus Equation 3.3 gives the z-transform of \( x(t-nT) \) as
\[ Z(x(t - nT)) = x(z)z^{-n}. \]  

(3.4)

Taking the inverse z-transform yields \( x(t-nT) = z^{-1}(x(z)z^{-n}) \) from Equation 3.4. This is known as the shifting theorem of z-transforms. The z-transfer function of a system, which represents the z-transform of the output over the z-transform of the input is a ratio of the polynomials in \( z^{-1} \) thus,

\[
\frac{X(z)}{Y(z)} = \frac{a_0 + a_1z^{-1} + a_2z^{-2} + \ldots + a_mz^{-m}}{1 + b_1z^{-1} + b_2z^{-2} + \ldots + b_nz^{-n}}
\]  

(3.5)

where \( X(z) \) and \( Y(z) \) are the transforms of the output and input respectively. Cross multiplying yields

\[
(1 + b_1z^{-1} + b_2z^{-2} + \ldots + b_nz^{-n})X(z) = (a_0 + a_1z^{-1} + a_2z^{-2} + \ldots + a_mz^{-m})Y(z)
\]  

(3.6)

Taking the inverse z-transform and using the shifting theorem yields,

\[
x(t) + b_1x(t-T) + b_2x(t-2T) + \ldots + b_nx(t-nT)
= a_0y(t) + a_1y(t-T) + a_2y(t-2T) + \ldots + a_my(t-mT)
\]  

(3.7)

The output function, \( x(t) \), at time \( t \) can be written in terms of the output and input at previous times \( t-iT \) as,

\[
x(t) = a_0y(t) + a_1y(t-T) + \ldots + a_my(t-mT)
- (b_1x(t-T) + b_2x(t-2T) + \ldots + b_nx(t-nT))
\]  

(3.8)
For \( t=kT \) Equation 3.8 may be written as,

\[
x(k) = a_0 \cdot y(k) + a_1 \cdot y(k-1) + \cdots + a_m \cdot y(k-m) \\
- (b_1 \cdot x(k-1) + b_2 \cdot x(k-2) + \cdots + b_n \cdot x(k-n))
\] (3.9)

where the sampling period \( T \) is understood.

3.2 Z-Transforms of the System

The z-transform of the system is found from a table of Laplace to z-transforms. The technique, used for transformation, is to first find the partial fraction expansion of a Laplace transfer function and then to obtain the z-transforms from the table.

The s-transfer functions whose z-transforms are required, are \( \frac{V_t(s)}{U^*(s)} \), \( \frac{V_p(s)}{U^*(s)} \) and \( \frac{W_c(s)}{V_t^*(s)} \), as shown in Figure 2.10. The Laplace transfer functions of the system have been derived in the previous chapter where,

- \( V_t \) = output signal from the tachometer,
- \( V_t^* \) = sampled value of \( V_t \),
- \( W_c \) = velocity of the cart,
- \( U \) = output voltage to the summing power amplifier,
- \( U^* \) = sampled signal of \( U \), the control signal, and
- \( V_p \) = output signal from the potentiometer.

3.2.1 Z-Transfer Function of \( \frac{V_t(s)}{U^*(s)} \)

The transfer function of \( \frac{V_t(s)}{U^*(s)} \) is equal to \( \text{ZOH}*G_l(s) \), as shown in Figure 2.10. Therefore,

\[
\frac{V_t(s)}{U^*(s)} = \frac{(1 - e^{-ST})}{s} \cdot \frac{0.92(s^2 + 11.5s + 2285)}{(s^2 + 21.4s + 2362)}
\] (3.10)
Taking the partial fraction expansion of Equation 3.10 gives,
\[
\frac{V_t(s)}{U^*(s)} = 0.92*(1 - e^{-sT})* \left[ \frac{A}{s} + \frac{B*s + C}{s^2 + 21.4*s + 2362} \right]
\]
(3.11)

where A, B and C are the coefficients of the expansion. These constants are, A = 0.9674, B = 0.0326 and C = -9.2024.

Hence,
\[
\frac{V_t(s)}{U^*(s)} = 0.92*(1 - e^{-sT})* \left[ \frac{0.9674}{s} + \frac{0.0326*s - 9.2024}{s^2 + 21.4*s + 2362} \right]
\]
(3.12)

The z-transform of Equation 3.12 is,
\[
\frac{V_t(z)}{U(z)} = 0.92*(1 - z^{-1})* \left[ \frac{0.9674*z}{z - 1} + \frac{0.0326*(z^2 - z* \cos(bT)*e^{-aT})}{z^2 - 2*z*\cos(bT)*e^{-aT} + e^{-2aT}} \right]
\]
\[
- \frac{0.2015*z*\sin(bT)*e^{-aT}}{(z^2 - 2*z*\cos(bT)*e^{-aT} + e^{-2aT})}
\]
(3.13)

where, \(a = 10.7\) and \(b = 47.41\).

Simplifying Equation 3.13 yields,
\[
\frac{V_t(z)}{U(z)} = \frac{0.92*(z^2 - N11*z + N10)}{(z^2 - D11*z + D10)}
\]
(3.14)

where,

\(N11 = 1.9674*\cos(bT)*e^{-aT} + 0.2017*\sin(bT)*e^{-aT} + 0.0326\),
\(N10 = 0.9674*e^{-2aT} + (0.0326*\cos(bT) + 0.2017*\sin(bT))*e^{-aT}\),
\(D11 = 2*\cos(bT)*e^{-aT}\), and
\(D10 = e^{-2aT}\).
3.2.2 Z-Transfer Function of $W_c(s)/U^*(s)$

The transfer function of $W_c(s)/U^*(s)$ is equal to $ZOH*G_1(s)*(1/Kt)*G_2(s)$ as shown in Figure 3.1.

Therefore,

$$
\frac{W_c(s)}{U^*(s)} = \frac{153.3*(1 - e^{-ST})}{s*(s^2 + 21.4*s + 2362)}
$$

The partial fraction expansion of Equation 3.15 is,

$$
\frac{W_c(s)}{U^*(s)} = \frac{153.3}{2362}*(1 - e^{-ST})*\left[ \frac{A}{s} + \frac{B*s + C}{s^2 + 21.4*s + 2362} \right]
$$

where, $A = 1$, $B = -1$ and $C = -21.4$.

Therefore,

$$
\frac{W_c(s)}{U^*(s)} = 0.0649*(1 - e^{-ST})*\left[ \frac{1}{s} - \frac{s + 21.4}{s^2 + 21.4*s + 2362} \right]
$$

The $z$-transform of the partial fraction expansion is obtained from the table as,

$$
\frac{W_c(z)}{U(z)} = 0.0649*(1-z^{-1})*\left[ \frac{z}{z-1} - \frac{z*(z-(\cos(bT) - 0.2256*\sin(bT))*e^{-aT})}{z^2 - 2*z*\cos(bT)*e^{-aT} + e^{-2aT}} \right]
$$

where, $a = 10.7$ and $b = 47.41$.

Simplifying Equation 3.18 as,
\[
W_c(z) = \frac{0.0649(N_{21}z - N_{20})}{U(z) \times (z^2 - D_{11}z + D_{10})}
\]

where
\[
N_{21} = 1 - (\cos(bT) + 0.2256\sin(bT))e^{-aT}, \quad \text{and}
\]
\[
N_{20} = e^{-2aT} - (\cos(bT) - 0.2256\sin(bT))e^{-aT}.
\]

3.2.3 Z-Transfer Function of \(V_p(s)/U^*(s)\)

The transfer function of \(V_p(s)/U^*(s)\) is equal to \(ZOH*G_l(s)*(1/K_t)*G_2(s)*G_3(s)*K_p\). Therefore,

\[
\frac{V_p(s)}{U^*(s)} = \frac{1 - e^{-ST}}{s} \times \frac{-0.92*(1/0.072)*12.0*2.5*K_p*s}{(s^2 + 21.4*s + 2362)*(s^2 - 25)}
\]

The partial fraction expansion of Equation 3.20 is,

\[
\frac{V_p(s)}{U^*(s)} = -383.5*K_p*(1-e^{-ST}) \times \left[ \frac{A}{s - 5} + \frac{B}{s + 5} + \frac{C*s + D}{s^2 + 21.4*s + 2362} \right]
\]

where the coefficients of expansion are, \(A = 4.0098*10^{-5}\), \(B = -4.3859*10^{-5}\), \(C = 3.7607*10^{-6}\) and \(D = -3.3937*10^{-4}\).

The z-transform of Equation 3.21 is,

\[
\frac{V_p(z)}{U(z)} = -383.5*K_p*(1-z^{-1}) \times \left[ \frac{A_1*z + A_2*z}{z-e^{5T}} + \frac{A_3*(z^2 - z*\cos(bT)*e^{-aT})}{z^2 - 2z*e^{-5T} + e^{-10aT}} \right]
\]

\[
\frac{A_4*z*\sin(bT)*e^{-aT}}{z^2 - 2z*\cos(bT)*e^{-aT} + e^{-2aT}} * 10^{-5}
\]
where, $A_1 = 4.0098$, $A_2 = -4.3859$, $A_3 = 0.3761$, $A_4 = 0.8006$, $a = 10.7$ and $b = 47.41$.

Simplifying Equations 3.22 as,

$$
\frac{V_p(z)}{U(z)} = \frac{+383.56 K_p 10^{-5} (z-1) (N32 z^2 - N31 z + N30)}{(z-e^{-5T})(z-e^{5T})(z^2 - D11 z + D10)}
$$

(3.23)

where,

$$
N32 = (A_1 + A_3) e^{-5T} + (A_2 + A_3) e^{5T} + 2(a + A_2) \cos(b T) e^{-aT} + (A_3 \cos(b T) + A_4 \sin(b T)) e^{-aT},
$$

$$
N31 = (A_1 + A_2) e^{-2aT} + A_3 + (e^{-5T} + e^{5T}) (A_3 \cos(b T) + A_4 \sin(b T)) e^{-aT} + 2(a_1 e^{-5T} + a_2 e^{-5T}) \cos(b T) e^{-aT}, \text{ and}
$$

$$
N30 = (A_1 e^{-5T} + A_2 e^{5T}) e^{-2aT} + (A_3 \cos(b T) + A_4 \sin(b T)) e^{-aT},
$$

3.3 Transfer Function for the Block $H_3(z)$

The symbol $W_c$ represents the velocity of the cart, and $W_c^*$ represents an estimate of the velocity of the cart that is generated by the microcomputer from the measurement of the tachometer voltage. In order to obtain the estimated velocity, the digital model $H_3(z)$ is developed. This digital model describes the relationship between the digital signal from the output of the tachometer, $V_t^*$, and the speed of the cart.

This relationship is obtained by dividing Equation 3.19 by Equation 3.14, thus,

$$
H_3(z) = \frac{W_c(z)}{V_t(z)} = \frac{0.0655 (N21 z - N20)}{0.92 (z^2 - N11 z + N10)}
$$

(3.24)
The next chapter contains the stability analysis of the system in the z-domain. The compensating transfer functions $H_1(z)$ and $H_2(z)$ are developed to stabilize the system.
CHAPTER IV

STABILITY ANALYSIS IN Z-DOMAIN

4.1 Introduction

A linear continuous feedback control system is stable if all the poles of the closed loop transfer function lie in the left half of the s-plane. The z-plane is related to the s-plane by the transformation

\[ z = e^{st} \]

or

\[ z = e^{(r+j\omega)t} . \]  \hspace{1cm} (4.1)

This relationship may also be written as

\[ |z| = e^{rt} . \] \hspace{1cm} (4.2)

For the left - half s-plane, \( r < 0 \), the relative magnitude of \( z \) varies between 0 and 1. Therefore, the imaginary axis of the s-plane corresponds to the unit circle in the z-plane and the inside of the unit circle corresponds to the left half of the s-plane. Thus a sample data system is stable if all the poles of the closed loop transfer function lie within the unit circle (6).

A sampling period of \( T = 0.01 \) secs is selected for the system. The frequency of sampling is 12 times the natural frequency of the drive system, and hence the reconstruction of the sampled signal is quite accurate.

The naturally unstable pendulum-cart system is stabilized by a feedback compensation technique. The stability analysis is given in the rest of this chapter.
4.2 Stability Analysis

The stability of a closed loop control system is directly related to the location of the roots of the characteristic equation in the z plane. Therefore, it is necessary to determine the roots of the characteristic equation for a check on stability. The root locus method was introduced by Evans in 1948 and has been developed and utilized extensively in control engineering. The root locus technique is a graphical method for drawing the locus of the roots in the s and z planes as a parameter is varied. If the root locations are not satisfactory the necessary parameter adjustment to stabilize the system can be readily deduced from the root locus.

The block diagram of the system is shown in Figure 4.1. The transfer functions \( \frac{V_t(z)}{U(z)} \), \( \frac{V_p(z)}{U(z)} \) and \( \frac{W_c(z)}{V_t(z)} \) are derived in Chapter III. Substituting \( T = 0.01 \) secs. into the above transfer functions yields,

\[
D_1(z) = \frac{V_t(z)}{U(z)} = \frac{0.92(z^2 - 1.688z + 0.89)}{z^2 - 1.599z + 0.807}, \tag{4.3}
\]

\[
D_2(z) = \frac{V_p(z)}{U(z)} = \frac{-3.835 \times 10^{-5} K_p (z-1) (1.562z^2 + 5.862z + 1.4033)}{(z-0.9521) (z-1.0513) (z^2 - 1.599z + 0.807)}, \tag{4.4}
\]

and

\[
D_3(z) = \frac{W_c(z)}{V_t(z)} = \frac{0.0077(z + 0.931)}{z^2 - 1.688z + 0.89}, \tag{4.5}
\]
Figure 4.1 Block Diagram of the System.
4.2.1 Stability Analysis of the Inner Loop.

The transfer function of the inner loop of the system shown in Figure 4.1 is,

\[
\frac{V_t(z)}{Cl(z)} = \frac{Dl(z)}{1 + Dl(z)\cdot H2(z)\cdot H3(z)}
\]  

(4.6)

Therefore, the characteristic equation of the inner loop is,

\[1 + Dl(z)\cdot H2(z)\cdot H3(z) = 0\]  

(4.7)

The compensating transfer function, \( H2 \), is unknown, and hence needs to be chosen to have a stabilizing effect on the inner loop of the system.

Substituting \( Dl(z) \) from Equation 4.3 and \( H3(z) \) from Equation 4.5 into Equation 4.7, yields,

\[\frac{H(z)\cdot 0.0077\cdot (z + 0.931)\cdot 0.92\cdot (z^2 - 1.688z + 0.89)}{1 + \frac{1}{(z^2 - 1.688z + 0.89)\cdot (z^2 - 1.599z + 0.807)}} = 0\]  

(4.8)

or

\[\frac{H2(z)\cdot 0.0071\cdot (z + 0.931)}{(z^2 - 1.599z + 0.807)} = 0\]  

(4.9)

If \( H2(z) \) is a gain compensation, then Equation 4.9 has a zero at \( z1 = -0.931 \) and a pair of complex conjugate poles at \( P1 = 0.7995 + 0.409j \) and \( P2 = 0.7995 - 0.409j \).

Therefore, the characteristic equation is,

\[z^2 - 1.599z + 0.807 + H2\cdot 0.0071\cdot (z + 0.931) = 0\]  

(4.10)

In order to determine a suitable gain, \( H2 \), the roots of the characteristic Equation 4.10 are plotted for \( 0 < H2 < \infty \). Figure 4.2 shows the location of the poles and the zero, as well as the
Figure 4.2 Root Locus of the Inner Loop of the System with a Gain Compensation
locus of the roots of the inner loop of the system for increasing gain, $H_2$. The roots have a tendency to move out of the unit circle as the gain $H_2$ is increased. This causes a destabilizing effect on the inner loop of the system. Therefore the approach at this point is to choose a transfer function, $H_2$, such that the roots do not have a tendency to move out of the unit circle. Hence $H_2$ is chosen as,

$$H_2(z) = \frac{C_2(z)}{W_c(z)} = \frac{K_1(z - 0.85)^2}{(z + 0.931)(z - 1)} \tag{4.11}$$

where $K_1$ is a variable gain which is determined by root locus analysis.

Choosing $H_2(z)$ above is also due to the following reasons:

a) A pole ($z + 0.931$) of the transfer function $H_2(z)$ cancels the zero of $H_3(z)$. This reduces the computation time in the real time controller and simplifies the analysis.

b) The pole ($z - 1$) in $H_2(z)$ provides digital integration of the velocity, $W_c^*$, of the cart, eg., a signal proportional to the displacement of the cart.

c) Second order zeros need to be placed such that the roots of the characteristic equation of the inner loop remain inside the unit circle. Hence the zeros $(z - 0.85)^2$ are chosen in the transfer function $H_2(z)$.

Substituting $H_2(z)$ in Equation 4.10 yields the characteristic equation of the inner loop as

$$(z^2 - 1.599z + 0.807)(z - 1) + K_1(z - 0.85)^2 = 0 \tag{4.12}$$

where $K_1 = K_1 \times 0.0071 \tag{4.13}$
Figure 4.3 Root Locus of the Inner Loop of the System in the Z-Plane with a Compensation Transfer Function
Figure 4.3 shows the locus of the roots of the characteristic equation of the inner loop of the system as the gain, Ki, is increased. The values of the roots of the characteristic equations for increasing gain, Ki, are given in Appendix A. From Figure 4.3 it can be deduced that the inner loop of the system has a tendency to stabilize, as the gain Ki is increased. A gain of Ki = 0.69 is selected for further analysis of the system. Therefore the value of Ki as deduced from Equation 4.13 is 97.18.

4.2.2 Stability Analysis Of The Entire System

The characteristic equation of the entire system shown in Figure 4.1 can be deduced as,

\[ 1 + D1(z)H2(z)H3(z) + D2(z)H1(z) = 0. \]  

(4.14)

The compensating transfer function, H1(z), is selected as

\[ \frac{C1(z)}{Vp(z)} = \frac{K2(z - 0.85)}{(z - 1.1)} \]  

(4.15)

where K2 is the variable gain which is determined by root locus analysis.

Substituting D1(z), H3(z), H2(z), D2(z) and H1(z) from Equations 4.3, 4.5, 4.11, 4.4 and 4.15 respectively into Equation 4.14 yields,

\[ 1 + \frac{0.69(z^2 - 1.7z + 0.7225)}{(z-1)(z^2 - 1.599z + 0.807)} \]

\[ Km*(1.562z^3 + 4.3z^2 - 4.4587z - 1.4033)*(z-0.85) \]

\[ + \frac{1}{(z-0.9512)(z-1.0512)(z^2-1.599z+0.807)(z-1.1)} = 0 \]  

(4.16)
Figure 4.4 Root Locus of the Entire System in the Z-Plane
where $K_m = -3.835 \times 10^{-5} K_p K_2$. \hspace{1cm} (4.17)

Simplifying Equation 4.16, the characteristic equation of the entire system is,

$$z^6 - 5.0506 z^5 + 10.5483 z^4 - 11.7148 z^3 + 7.3588 z^2 -$$
$$2.5133 z + 0.3716 + K_m (1.562 z^5 + 1.4097 z^4 -$$
$$11.8069 z^3 + 10.5019 z^2 - 1.194 z - 1.192) . \hspace{1cm} (4.18)$$

A plot of the root locus of the characteristic equation of the entire system is shown in Figure 4.4. As the gain $K_m$ is varied, the values of the roots are given in Appendix B. From Figure 4.4, it is observed, that the poles of the characteristic equation of the system lie within the unit circle in the $z$ plane for $0.005 < K_m < 0.020$. Hence the pendulum cart system is stable for values of $K_m$ within this range.

Therefore from Equation 4.17, $K_2 K_p$, can be deduced to lie within $-521.5 < K_2 K_p < -130$, for the system to be stable. With the assumed value of $K_2$ equal to $-256$, the range of $K_p$ is between 0. and 2.04 in order for the system to be stable. The next chapter discusses the simulation analysis, where the effect of varying the gain of the potentiometer, $K_p$, is studied.
CHAPTER V

SIMULATION OF THE SYSTEM

5.1 Introduction

If a model is available for a component or a system, a computer can be utilized to investigate the behaviour of the system. A computer model of the system in a mathematical form suitable for demonstrating the system's behavior may be utilized to investigate designs of a planned system without actually building the system itself. A computer simulation uses a model and the actual condition of the system being modeled and actual input commands to which the system will be subjected (6).

The simulation program was written on a Hewlett-Packard 9845B series desk top computer in Basic Language and is listed in Appendix C. This program was later modified and implemented in Assembly Language on a Z-80 microcomputer for the real time control.

The system equations are represented by recurrence equations by taking the inverse transformation as described in Section 3.1.

5.2 Recurrence Equations

The transfer functions of the system were presented in Chapter III and the compensating transfer functions to stabilize the system have been developed in Chapter IV. Figure 4.1 shows the block diagram of the system. The transfer functions are,

\[
\frac{V_t(z)}{D_l(z)} = \frac{0.92(z^2 - 1.688z + 0.89)}{U(z)} \frac{1}{(z^2 - 1.599z + 0.807)} \quad (5.1)
\]
where \( D_1(z) \) describes the relationship between the output signal of the tachometer and the control signal, \( U \).

\[
D_2(z) = \frac{-3.835 \times 10^{-5} K_p (1.562 z^3 + 4.30 z^2 - 4.4587 z - 1.4033)}{(z^4 - 3.601 z^3 + 5.009 z^2 - 3.216 z + 0.807)}
\]

(5.2)

where \( D_2(z) \) describes the relationship between the output signal from the potentiometer and the control signal, \( U \).

\[
H_1(z) = \frac{-256(z - 0.85)}{V_p(z)(z - 1.1)}
\]

(5.3)

where \( H_1(z) \) is the digital compensator which generates the values \( C_1 \), by sampling the signal, \( V_p \), of the potentiometer.

\[
H_2(z) * H(3) = \frac{C_2(z)}{V_t(z)} = \frac{0.75(z^2 - 1.7 z + 0.7225)}{(z^3 - 2.688 z^2 + 2.578 z - 0.89)}
\]

(5.4)

where \( H_2(z) * H(3) \) is the digital compensator which generates the values \( C_2 \), by sampling the signal, \( V_t \), from the tachometer.

In a recurrence form the above equations can be written as,

\[
V_t(k) = 1.599 V_t(k-1) - 0.807 V_t(k-2) + 0.92 (U(k) - 1.688 U(k-1) + 0.89 U(k-2))
\]

(5.5)

\[
V_p(k) = 3.601 V_p(k-1) - 5.009 V_p(k-2) + 3.216 V_p(k-3) - 0.8075 V_p(k-4) - 3.835 K_p 10^{-5} (1.562 U(k-1) + 4.30 U(k-2) - 4.487 U(k-3) - 1.403 U(k-4)).
\]

(5.6)

\[
C_1(k) = 1.1 C_1(k-1) - 256 V_p(k) + 217.6 V_p(k-1).
\]

(5.7)
\[ C_2(k) = 2.688 \cdot C_2(k-1) - 2.578 \cdot C_2(k-2) + 0.89 \cdot C_2(k-3) + 0.75 \cdot (V_t(k-1) - 1.7 \cdot V_t(k-2) + 0.7225 \cdot V_t(k-3)) \] (5.8)

The recurrence Equations 5.5 through 5.8 are used in the computer simulation.

5.3 Simulation

It is assumed that there is no delay between the time the data is sensed and the time the control signal is computed. The control signal, \( U \), is computed based on the signals available from the potentiometer, \( V_p \), and the tachometer, \( V_t \). Figure 4.1 shows the digital models of the components of the system. The control signal, \( U \), at instant \( k \) is the input to the block \( D_1(z) \). \( D_1(z) \) produces a signal representing the output from the tachometer, \( V_t \), and the next block, with a transfer function \( D_2(z)/D_1(z) \) produces, \( V_p \), both at time \( k \). \( V_t \) and \( V_p \) are processed by the digital compensators \( H_1(z) \) and \( H_2(z) \cdot H_3(z) \) respectively. The corresponding computed digital values \( C_1 \) and \( C_2 \) are sent to the summer which produces a new control signal, \( U \), at time \( k \). This process is repeated to describe the behavior of the system.

The simulation program took into consideration the saturation of the control signal, \( U \), equal to 10 volts. The discretization of the D/A and the A/D converters were also accounted for. The pendulum was given an initial angular displacement of 10° from the upright position. Results of the simulation were obtained for various gains of the potentiometer, \( K_p \).

From the simulation of the system, it was observed that the
system was stable for a gain of the potentiometer, \( K_p \), between 0.55 and 1.875 and unstable for all other gains. However, at the end of Chapter IV, it was concluded that the values of \( K_p \) should lie within 0.51 and 2.04 for stability. This reduced range of \( K_p \) is due to the discretization of the sampled data and the saturation effect of the control signal which have been taken into account in the computer simulation of the system.

The simulation results of the position of the pendulum, the output control voltage, speed of the d.c. motor and displacement of the cart vs. time were plotted as shown in Figures 5.1, 5.2, 5.3 and 5.4 for a variable gain, \( K_p \), equal to 0.55, 1.0 and 1.875.

From Figure 5.1 it is observed that the overshoot of the position of the pendulum for a gain \( K_p \) equal to 0.55 is large compared to a gain of \( K_p \) equal to 1. For a gain of \( K_p \) equal to 1.875, the position of the pendulum is observed to fluctuate more compared to gains of \( K_p \) equal to 0.55 and 1.0. For \( K_p \) equal to 1.0, and with an initial angular displacement of the pendulum equal to \( 10^\circ \), the pendulum is brought to within \( 1^\circ \) of the upright position within 0.75 sec, and is limited to this range for all greater time.

Figure 5.2 shows the simulation results for the output control voltage vs. time, for a variable gain, \( K_p \). From the graphs it can be deduced that for gains of potentiometer, \( K_p \), equal to 0.55 and 1.0 less control voltage is required to stabilize the system. However for gains of \( K_p \) greater than 1.875 the saturation of the control signal makes the system unstable.
Figure 5.1. Plot of the Angular Position of the Pendulum vs Time for potentiometer Gain Equal to 0.55, 1 and 1.875
Figure 5.2. The Plot of Control Voltage vs Time for a Potentiometer Gain of 0.55, 1 and 1.875
Referring to Figure 5.3, the plots of the speed of the d.c. motor shows a very small change for increasing Kp from 0.55 to 1.0, but for a value of Kp equal to 1.875 the response is oscillatory, which is consistent with the inference from Figures 5.1 and 5.2.

Figure 5.4 shows the results of the displacement of the cart vs. time. It is observed that changes in gain, Kp, do not have significant effects on this parameter.

From the above analysis of the plots, it was decided to set the values of the gain Kp equal to 1.0 for the experiment.

The next chapter deals with the development of the assembly language real time control program.
Figure 5.3. The Plot Of Speed of D.C. Motor (volts) vs Time for a Potentiometer Gain of 0.55, 1 and 1.875
Figure 5.4. The Plot of Displacement of the Cart (meters) vs Time for a Potentiometer Gain of 0.55, 1 and 1.875
CHAPTER VI

SOFTWARE

6.1 Introduction

The computer used in this research was a Z-80 from Digital Group. The real time control of the system was done by this computer. The control program was written in assembly language. The Assembler occupies addresses from 0 to 2FFF in hexadecimal. The programmer source and object code could be placed at any address above the assembler. Additional facilities were available to store and retrieve the programs from a floppy disk or an audio cassette. Listing of the program was possible on a printer.

The software developed for this research was one of the major tasks. The programming was done in assembly language. Since memory was limited, the software was divided into many parts and stored on the disk. The object code of each part was assembled and stored separately. These object code parts were later linked in the correct order. The memory used by the object code was 2.2 K. The object code was placed from address 3000 Hex to 38D3 Hex.

A floating point binary representation with a 15 bit mantissa and an 8 bit exponent was used for numerical calculations. This provided a resolution of 1 part in 32,764. By using this type of representation the accuracy of computation was maintained.

The control program consists of a main program and
subroutines. Memory addresses 3E00H to 3E7FH were used for storing the coefficients, variables, and parameters of the program. A memory map that shows the memory used in the first page of 3E is given in Appendix D. The starting addresses of the main program and the subroutines are also shown in Appendix D.

6.2 Main Program

A flow chart of the main program is given in Figure 6.1 and the assembly language program is listed in Appendix E. The memory used for the object code of the main program is 0.94 K. bytes.

The function of the main program is to read the digital signals of the speed of the d.c. motor and the position of the pendulum from the A/D converter, perform specific computations, and send a digital control signal to the D/A converter. The main program is divided into three parts:

1) The Initialization,
2) The Computations, and
3) The Updating of the Parameters.

6.2.1 Initialization

The flow chart of this program is shown in Figure 6-2. The first step of this program is to initialize the output voltage from the D/A converter to zero, so that the cart remains stationary. The next operation resets memory block 3E00H - 3E7FH to zero. Then the subroutine INITI and PENDIS are executed. By executing the INITI subroutine, the zero reference of the pendulum position and the bias are determined. The zero
**Figure 6-1 THE MAIN PROGRAM**

- ENTER
- INITIALIZATION
  - COMPUTATION
    - READ $V_t(4)$ AND $V_p(4)$
    - OUTPUT $V_a(4)$
  - UPDATING PARAMETERS

**Figure 6-2 THE INITIALIZATION PROGRAM**

- INITIALIZATION
  - SET OUTPUT VOLTAGE WHICH DRIVES D.C. MOTOR TO 0
  - INITIALIZE MEMORY LOCATION 3E00H TO 3E7FH TO 0
  - CALL INITI
  - CALL PENDIS
  - INITIALIZE ALL THE COEFFICIENTS
  - GO TO THE COMPUTATION PROGRAM
reference is the output voltage from the potentiometer corresponding to the vertical position of the pendulum. The subroutine PENDIS helps in fixing the limits of the angular displacement of the pendulum. Details of these two subroutine are given in Section 6.3.

The next operation stores all the coefficients and the variables required for the software program into appropriate memory locations.

The initialization program is executed once at the beginning of each experiment. The execution time of this program is not critical because it is not a part of the sampling period.

6.2.2 Computation

The computation program is the most important part of the main program. This program samples the velocity of the cart and the position of the pendulum from the A/D converter and performs computations of the equations shown below for generating a control signal $U(4)$.

$$C_1(4) = 1.1 * C_1(3) - 256 * V_p(4) + 217.6 * V_p(3)$$  \hspace{1cm} (6.1)

$$C_2(4) = 2.69 * C_2(3) - 2.58 * C_2(2) + 0.89 * C_2(1) + 0.75 * V_t(4)$$
$$\quad - 1.275 * V_t(3) + 0.542 * V_t(2)$$  \hspace{1cm} (6.2)

$$U(4) = C_1(4) - C_2(4).$$  \hspace{1cm} (6.3)

The coefficients in the above equations are obtained based on the computer simulation, as derived in the previous chapter. The values of the coefficients, which are in decimal, are
COMPUTATIONS

RESET THE TIMER TO COUNT 10 ms.

READ C2(3), C2(2), C2(1)
Vt(3), Vt(2) AND ITS COEFF
CALL MULT, EXP, STAD & COMP
X=AC*2^(-2)*C2(3)-AS*2^(-2)*C2(2)+
+39*2^(-2)*C2(1)-A3*2^(-3)*Vt(3)+
+BB*2^(-4)*Vt(2)

READ Vt(4)

READY ?

NO

READ Vt(4)
INITIALIZE A/D TO READ Vp(4)
CONVERT Vt(4) TO 2's COMP AND SUBTRACT THE BIAS

READ X, Vt(4) AND ITS COEFF
CALL MULT, STAD
C2(4)=X*2^(-4)+3*2^(-2)*Vt(4)

CALL DISPLAY
CALL STORE

READ C1(3), Vp(3) AND ITS COEFF
CALL MULT, EXP, STAD & COMP
Y=BO*2^(-2)*C1(3)+D9*Vp(3)

READ Vp(4)

READY ?

NO

YES

READ Vp(4)
INITIALIZE A/D TO READ Vt(4)
CONVERT Vt(4) TO 2's COMP AND SUBTRACT BIAS

READ X, Vt(4) AND ITS COEFF
C1(4)=Y-2^8*Vp(4)
U(4)=C1(4)-C2(4)

MODIFY U(4) AND OUTPUT TO D/A

GOTO THE UPDATING PARAMETER PROGRAM

Figure 6-3 THE COMPUTATION PROGRAM
converted to hexadecimal and stored in the memory location shown in Appendix D.

The equations are represented in hexadecimal as,

\[ C1(4) = 8D \times 2^{-7} \times C1(3) + D9 \times Vp(3) - 2^8 \times Vp(4) \quad (6.4) \]

\[ C2(4) = AC \times 2^{-2} \times C2(3) - A5 \times 2^{-2} \times C2(2) + 39 \times 2^{-2} \times C2(1) - \\
- A3 \times 2^{-3} \times Vt(3) + 8B \times 2^{-4} \times Vt(2) \times 2^{-4} + 3 \times 2^{-2} \times Vt(4). \quad (6.5) \]

\[ U(4) = C1(4) - C2(4). \quad (6.6) \]

The flow chart of the computation program is shown in Figure 6-3. The first step of this program resets the timer to count 10 ms. The next step performs the computation of,

\[ X = AC \times 2^{-2} \times C2(3) - A5 \times 2^{-2} \times C2(2) + 39 \times 2^{-2} \times C2(1) - \\
- A3 \times 2^{-3} \times Vt(3) + \\
+ 8B \times 2^{-4} \times Vt(2). \quad (6.7) \]

In the next operation, the A/D is initialized to read Vt(4) and the 'BUSY' signal of the A/D converter is checked to verify if the signal Vt(4) has settled down. If signal Vt(4) has settled, then it is read by the A/D converter. The A/D converter is initialized to read the signal of the position of the pendulum, Vp(4). The value of Vt(4) is converted to 2's complement form and adjusted by subtracting or adding the bias. Then, Vt(4) is stored in memory and computations are performed to yield,

\[ C2(4) = 2^{-4} \times X + 3 \times 2^{-2} \times Vt(4). \quad (6.8) \]
In the next operation Subroutine DISPLAY is executed to display the position of the pendulum, speed of the d.c. motor, and the output control signal on the CRT. Then Subroutine STORE is executed to store the data \( V_t(4), V_p(4), U(4), C_1(4), \) and \( C_2(4). \)

In the next operation the computation performed is,

\[
Y = 8D \times 2^{-7} \times C_1(3) + D9 \times V_p(3). \tag{6.9}
\]

The A/D is initialized to read \( V_p(4). \) Then the 'BUSY' signal is checked until the value of \( V_p(4) \) is ready to be read. The next operation reads \( V_p(4) \) from the A/D converter. The A/D is now initialized to read the speed of d.c. motor, \( V_t(4). \) The value of \( V_p(4) \) is converted to 2's complement form and adjusted by subtracting or adding the bias. Then \( V_p(4) \) is stored in memory and computations are performed to yield,

\[
C_1(4) = Y - 256 \times V_p(4). \tag{6.10}
\]

The control signal \( U(4) \) is obtained by solving the equation,

\[
U(4) = C_1(4) - C_2(4). \tag{6.11}
\]

The control signal calculated in Equation 6.11 is in 2's complement form with a 24 bit mantissa. This value is modified and sent to the D/A converter.

6.2.3 Updating Parameters

The parameters have to be updated before executing the computation program in the next sampled period. By executing
Figure 6-4 THE UPDATING PARAMETER PROGRAM
Subroutine SHIFT explained in Section 6.3.6, the values shift one step down; that is the data of \( V_t(3) \) shifts to \( V_t(2) \) and the computed values \( C2(3) \) and \( C2(2) \) to \( C2(2) \) and \( C2(1) \) respectively. The value of \( C1(4) \) and \( C2(4) \) are in 2's complement form with 24 bit mantissa. These values are converted into the 15 bit mantissa, a sign byte and an exponent byte by executing Subroutine TRANS which is explained in Section 6.3.5. The converted value of \( C1(4) \) is stored in memory locations \( C1(3) \) and \( C2(4) \) is stored in memory locations \( C2(3) \).

The next step in the program is to check if the 10 ms sampling time is over. If the sampling time is over, then the operation will return back to the computation program.

6.3 Subroutines

The main program is supported by eleven subroutines. They are, a fifteen by eight bit multiplication routine; a 24 bit signed addition routine; a routine for performing the 2's complement operation; a routine to shift data; a routine for converting a value from a fixed point representation to floating point; a routine to update values; a routine to halt the execution of the program; a routine to display data on the CRT; a routine to store the data in specific memory blocks; a routine to find the zero reference of the pendulum when it is in the upright position and a routine to adjust the range of the angular movement of the pendulum about the vertical axis.

6.3.1 Subroutine MULT

This subroutine performs the multiplication of a 15 bit by an
MULTIPLICAND (15 BITS) → DE
MULTIPLIER (8 BITS) → INDIRECTLY
ADDRESSED BY IX-REG.

ENTER

B → A, C, H, L

B → B (COUNTER)

SHIFT RIGHT MULTIPLIER
ADDRESSED BY IX-REG

C=1

Yes

AHL+CE → AHL REGISTERS

No

ROTATE C, D and E LEFT

SHIFT CARRY LEFT IN C-REG

DECREMENT COUNTER

COUNTER=0

Yes

EXIT

Figure 6.5 SUBROUTINE MULT
8 bit unsigned binary number. Figure 6.5 shows the flow chart of this subroutine. Basically, the multiplication routine is a series of tests and shifts of the multiplier and multiplicand.

To use this subroutine, the 15 bit multiplicand is loaded into the DE register pair and the 8 bit multiplier is indirectly addressed by the IX register A, B, C, D, E, H and L registers are utilized by this subroutine. The maximum time for performing a multiplication is 0.298 ms. The final result is placed in the CHL registers.

6.3.2 Subroutine STAD

This subroutine converts an unsigned 23 bit binary value stored in the CHL registers into a signed value. The sign bit of the 23 bit unsigned data is loaded in the least significant bit of the A register. Then this subroutine adds the two 24 four bit signed values which are loaded in the CHL and C'H'L' registers. The result of the adding operation is placed in the C'H'L' registers. A, C, D, E, H, L, C', D', E', H' and L' registers are utilized by this subroutine. A flow chart of this subroutine is given in Figure 6.6.

6.3.3 Subroutine COMPLEMENT

Figure 6.7 shows the flow chart of this subroutine. This subroutine converts a twenty-three bit binary number loaded in the CHL registers to 2's complement form. The result of this operation is placed in the CHL registers. A, C, H and L registers are utilized by this routine.
Figure 6.6 SUBROUTINE START

Figure 6.7 SUBROUTINE COMPLEMENT
6.3.4 Subroutine EXPONENT

The subroutine Exponent divides a twenty-three bit binary number stored in CHL registers by four. It then shifts the remaining value in the CHL registers to the left a number of bits depending on the value placed in the B register before this subroutine is executed. The final result is placed in the CHL registers. Figure 6.8 shows the flow chart of this subroutine.

6.3.5 Subroutine TRANS

This subroutine converts a signed three bytes value loaded in the CHL registers into a fifteen bit unsigned mantissa stored in the HL registers, a sign byte, and an exponent byte. The sign byte is placed in the memory location pointed to by the IY register and the exponent byte is placed in the memory location pointed to by IX register. Before calling this subroutine, the value is loaded in the CHL registers and the IX and IY are loaded. A, C, D, H and L registers are utilized by this subroutine. A flow chart of this subroutine is shown in Figure 6.9.

6.3.6 Subroutine SHIFT

Figure 6.10 shows the flow chart for this subroutine. The updating of the values in the program is done by this subroutine. This subroutine shifts the magnitude and sign of VT(3) to memory location for VT(2); the magnitude, sign and exponent byte of C2(2) to C2(1) and C2(3) to C2(2). A, H and L registers are utilized by this subroutine.
Figure 6.8 SUBROUTINE EXPONENT
Figure 6.9 SUBROUTINE TRANS
Figure 6.11 SUBROUTINE STOP

Figure 6.18 SUBROUTINE SHIFT
6.3.7 Subroutine STOP

This subroutine is used to halt the execution of the main program. The flow chart of this subroutine is shown by Figure 6.11.

If key 'S' is depressed from the keyboard when the main program is being executed, the entire execution of the program is terminated. At the same instant the control voltage output from the D/A converter, which is used to drive the d.c. motor, is initialized to 0. The past values of the position of the pendulum, velocity of cart and the control voltage remain displayed on the screen. If the key 'A' is also depressed, the system is reset and then the contents of the register are displayed on the CRT.

6.3.8 Subroutine DISPLAY

This subroutine displays the pendulum position, speed of the motor and the control voltage output from the D/A converter on the CRT. This was useful in debugging the software and to verify the correct functioning of the hardware. Signed binary values that represent the pendulum position, speed of the motor and the control voltage are converted to a hexadecimal ASCII codes and then displayed on the CRT with positive or negative signs. Figure 6.12 shows the flow chart of this subroutine.

6.3.9 Subroutine STORE

The store routine is written to store the data of Vp, Vt,
Figure 6.12 SUBROUTINE DISPLAY

Figure 6.13 SUBROUTINE STORE
C1, C2 and U in certain memory blocks. This routine is used for error diagnostics and data acquisition.

The signal from the tachometer, Vt, is stored from memory locations 3E80H to 4000H and the signal from the potentiometer, Vp, is stored from memory locations 2E80H to 3000H. Each of these data is stored in three bytes. The first two bytes contain the magnitude and the third byte contains the sign. For the sign byte, 01H indicates negative values while 00H indicates positive values.

Computed values C1(4) are stored from 3980H to 3B00H, C2(4) from 3B00H to 3C80H and U(4) from 3C80H to 3E80H. Each of these values is stored in three bytes and are signed numbers.

The subroutine also stops the execution of the program after a certain number of sampling events have passed. The desired number of sampling events are stored in memory locations 3E44H and 3E43H. A, D, H, and L registers are utilized by this subroutine. The flow chart of this subroutine is shown in Figure 6.13. When the subroutine is entered a pointer is decremented. The pointer gives the address off set for storing these values.

6.3.10 Subroutine INI

This subroutine is executed before the main program is executed. This subroutine has further access to three more subroutines CHECK, INC, and DEC. Figures 6.14 and 6.15 show the flow chart for the subroutines.

This subroutine initializes the reading of the position of the pendulum to 0 when the pendulum is almost vertical by a bias
Figure 4.14  SUBROUTINE INITI
Figure 6.15
from the D/A converter which is the input to the A/D converter. The A/D converter is set to be in differential mode with one input from the potentiometer and the other from the D/A converter.

The values of the current readings of the position of the pendulum and the reading of the velocity of the cart are displayed on the CRT. The program periodically executes subroutine CHECK which checks if key 'I', 'D' or 'S' is depressed. If the key 'I' is pressed then the output voltage from the D/A is increased for biasing the reading of the pendulum position by subroutine INC. If the key 'D' is pressed then the output voltage from the D/A is decreased for biasing the reading of the pendulum position by subroutine DEC. Since the value of the position of the pendulum is displayed, the biasing can be adjusted continuously. Since it is extremely difficult to get an exact 0 reading from the A/D converters when the cart is stationary, the zero error readings of the pendulum position and velocity of the motor are stored. These error readings are used as correction factors for correcting the readings of the pendulum position and the velocity of the armature when the main program is executed.

Key 'S' is depressed to return back to the main program.

6.3.11 Subroutine PENDIS

This subroutine is executed immediately after execution of subroutine INITI.
Figure 6.16 SUBROUTINE PENDIS
This subroutine displays the position of the pendulum on the CRT and hence helps in fixing the limits of the angular displacement of the pendulum about its mean position. The limits of the pendulum are fixed by adjusting the two screws at the bottom of the pendulum. After the desired adjustment is achieved, key 'R' is pressed to return back to the main program. Figure 6.16 shows the flow chart for this subroutine.
Chapter VII

EXPERIMENTAL PROCEDURE

7.1 Introduction

In this chapter the equipment used for experimentation will be explained first. The following section will contain a description of the testing procedure.

7.2 Equipment Arrangement

The inverted pendulum is mounted in two ball bearings on top of a light weight four wheeled cart. To measure the angular position of the pendulum, a micro torque potentiometer is attached to the pendulum shaft which provides a voltage, $V_p$, proportional to the angular position of the pendulum. A 10 volt power supply is used to energize the potentiometer.

The cart is driven by a d.c. servo motor, mounted on the bottom of the cart, through a speed reducing belt drive. The field of the motor is energized by a 14 volt d.c. power supply. The armature of the motor is controlled by the output of a d.c. power amplifier which is supplied by a $\pm 35$ volt d.c. power supply. A permanent magnet d.c. tachometer is attached to the motor shaft which provides a voltage, $V_t$, proportional to the speed of the motor. Electrical signals are passed to and from the cart by a flexible cord.

The Real time control of the system is accomplished by a Digital Group, Z-80 microcomputer. The analog signals $V_p$ and $V_t$ are sampled and digitized by a 12 bit A/D converter. The digital
control signal is delivered to the system by a 10 bit D/A converter. An 8253 programmable timer was used to time the 10 ms sampling period. Two operational amplifiers were used to provide feedback around the power amplifier and the d.c. motor. Figure 2.7 shows the arrangement of these amplifiers. Strip chart recorders were used for data collection.

7.3 Experimental Procedure

Two Z-80 microcomputers were used for this research. One was used as the real time controller described in the previous section. The other was used for program development. The program development computer operates under the DISKMON Operating System with two floppy disks for file storage. Both microcomputers are equipped with audio cassette recorder interfaces. By connecting the audio cassette output from one computer to the audio cassette input of the other, files can be transferred directly between the two microcomputers.

The assembler program is loaded into both the microcomputers from the disk and the cassette drive. The object code of the real-time control program is then loaded from the disk and is placed in memory locations starting from 3200H in the development computer. By executing the instruction 'SAVES' on the development microcomputer and the instructions 'LOADS' on the real-time control microcomputer, the object code program is transferred through the audio cassette interface to the real-time control microcomputer. At this point, the program is in the real-time controller and is ready to be executed. Before executing the
program the inverted pendulum is made vertical to find a zero reference voltage. This is accomplished by attaching a plumb line to the top of the inverted pendulum and adjusting the two screws at the bottom of the pendulum until the pendulum becomes vertical as checked by the plumb line. After removing the plumb line the real time control program is executed. By pressing keys 'I' and 'D' on the keyboard the digital value of the position of the pendulum, displayed on the CRT, is adjusted as close as possible to 800 H, which corresponds to a zero reading of the pendulum position. After biasing the potentiometer output the key 'S' is pressed. The position of the pendulum is displayed on the CRT. The angular displacement of the pendulum in either direction from the upright position is fixed by adjusting the two screws which are at the bottom of the inverted pendulum. The computation of the control signal to stabilize the system is started by depressing key 'R'.

It is not possible to adjust the pendulum to a perfectly upright position. Hence there is a bias present in the reading of the pendulum. This bias is defined as the reading of the position of the pendulum when it is in the perfectly upright position or the actual position of the pendulum when the reading is zero. It is evident from the model of the pendulum and cart, Equation 2.12, that the pendulum can be maintained at a nonvertical position by maintaining a constant acceleration of the cart. The larger the angle, the larger the required acceleration. Since it is not possible to completely eliminate the bias in the
pendulum position measurement there is a tendency for the cart to move in the direction of the bias. If a slight, upward curvature is built into each end of the surface on which the cart runs the effect of the bias is eliminated and the cart will find an equilibrium position about which it will operate.
Chapter VIII

PRESENTATION OF EXPERIMENTAL RESULTS

8.1 Introduction

Data obtained from the testing described in the previous chapter is discussed in this chapter. Data of the position of the pendulum, speed of the armature of the d.c. motor which drives the cart, distance of travel of the cart and the control signal from the D/A converter which control the speed of the d.c. motor are obtained on strip chart recorders at different chart speeds.

8.2.1 Results of Position of Pendulum versus Time

The pendulum was given an initial angular displacement of about $10^\circ$ from the upright position. The computation for the control signal to stabilize the pendulum was started by depressing key 'R' on the keyboard. The signal from the potentiometer which gives the angular displacement of the pendulum was recorded on the strip chart recorder for different speeds of 1 mm/sec, 5 mm/sec and 25 mm/sec. The sensitivity of the strip chart recorder was held constant at 50 mv/division. Figures 8.1.1, 8.1.2 and 8.1.3 show the results of the pendulum position versus time for the different chart speeds. The results indicate that at all times the position of pendulum is within $2.5^\circ$ from the upright position of the pendulum.
Figure 8.1.1  Position of Pendulum (degree) vs Time (minute), 1mm/sec chart speed.

Figure 8.1.2  Position of Pendulum (degree) vs Time (second), 5 mm/sec chart speed.

Figure 8.1.3  Position of Pendulum (degree) vs Time (second), 25 mm/sec chart speed.
8.2.2 Results of Speed of the D.C. Motor versus Time

The signal from the tachometer which is coupled to the shaft of the d.c. motor was recorded on the strip chart recorder for different chart speeds of 1 mm/sec, 5 mm/sec and 25 mm/sec. The sensitivity of the strip chart recorder was maintained at 200 mv/division. Figures 8.2.1, 8.2.2 and 8.2.3 show the speed of the d.c. motor versus time for different chart speeds. The maximum speed of the cart is 1.0 m/sec. It was observed that the output signal from the tachometer is periodic in nature with an almost constant amplitude.

8.2.3 Results of Distance Moved by Cart versus Time

The signal from the tachometer was integrated by an operational amplifier and was recorded on a strip chart recorder. The results thus obtained from the strip chart gave the distance moved by the cart from the starting position. The readings were recorded for chart speeds of 5 mm/sec and 25 mm/sec. The sensitivity of the strip chart was maintained at 500 mv/division. Figures 8.3.1 and 8.3.2 show the results of the distance moved by the cart versus time. For all times, the cart was observed to move back and forth through a distance of about 1 meter on either side of the mean position.

8.2.4 Result of Control Signal versus Time

The control signal from the D/A converter was recorded on the strip chart recorder for chart speeds of 5 mm/sec and 25 mm/sec. The sensitivity of the strip chart was maintained at 500
Figure 8.2.1 Speed of the D.C. Motor (200 mV/div) vs Time (1 mm/sec chart speed).

Figure 8.2.2 Speed of the D.C. Motor (200 mV/div) vs Time (5 mm/sec chart speed).

Figure 8.2.3 Speed of the D.C. Motor (200 mV/div) vs Time (25 mm/sec chart speed).
Figure 8.3.1  Distance moved by the cart (meters) vs Time (sec),
5 mm/sec chart speed.

Figure 8.3.2  Distance moved by the cart (meters) vs Time (sec),
25 mm/sec chart speed.
Figure 8.4.1 Control Signal from the A/D Converter vs Time, 5 mm/sec chart speed

Figure 8.4.2 Control Signal from the A/D Converter vs Time, 25 mm/sec chart speed
mv/division. The output control voltage from the D/A converter was limited to 10 volts. Figures 8.4.1 and 8.4.2 show the results of the control signal versus time. It may be observed that the output voltage from the D/A converter is large when the pendulum is initially at about 10° angular displacement from the mean position. Subsequently, when the cart moves and when the pendulum displacement becomes smaller, the output voltage also becomes corresponding smaller.
CHAPTER IX

CONCLUSIONS AND RECOMMENDATIONS

9.1 Introduction

This chapter provides a conclusion of this research and recommendations for further study.

9.2 Conclusions

From the experiment it is observed that the digital compensation is successful in stabilizing the naturally unstable cart pendulum system. A comparison of Figures 5.1 and 5.2 (for a gain \( K_p = 1.0 \)) with Figures 8.1 and 8.4 show that the response of the pendulum and the control voltage matches with the result of the computer simulation during an initial period of about 1.5 seconds. Subsequently, the effects of the bias, the non-linearities and the discretization of the sampled data makes the actual response deviate from the predicted response. It is believed that in the absence of these three effects, the actual response could match with the computer simulation for a much longer period.

9.3 Recommendations

There are several recommendations which can be made to improve upon the research conducted for this thesis, and to expand upon the system for further research.

A major time of this research was spent in writing and debugging the assembly language software control program. Hence it would be advantageous to work on a more advanced microproces-
sor (16 bits) or a microcomputer having a Fortran or Basic compiler.

If a higher stiffness belt is used, then the transfer function relating the speed of the cart to the speed of the d.c. motor can be modeled by a gain alone. This would simplify the stability analysis considerably.

Resolution of the accurate output control signal will provide smoother control. Hence, it is recommended that a 12 bit D/A converter be used instead of the 10 bit D/A converter used in the present research.

Two problems which were frequently encountered with the physical system were the drive belts and the potentiometer. Due to misalignment or fatigue, the life of the belts were short and hence needed replacement periodically. It would be worthwhile to modify the driving mechanism. The microtorque potentiometer, which was used to measure the position of the pendulum, failed. Therefore, a different method of measuring the angular displacement of the pendulum should be considered.

Since the mathematical model of the system is developed and adequate software is available, the technique of adaptive optimal control may be applied to the system.
REFERENCES


APPENDICES
APPENDIX A

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APPENDIX B

VALUES OF THE ROOTS OF THE ENTIRE SYSTEM
VALUES OF THE ROOTS OF THE ENTIRE SYSTEM

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APPENDIX C

THE SIMULATION PROGRAM
THE SIMULATION PROGRAM

100 T=.010 ! Sampling Interval.
110 Saturation=S=10 ! Maximum and minimum output voltage from the power supply to the cart.
120 Outresolution=.010 ! Resolution of the Output Control Signal 'U'
130 Velresolution=.010 ! Resolution of the Signal, Vt, from Tachometer.
140 Posresolution=.020 ! Resolution of the Signal, Vp, from Potentiometer.
150 Bias=.030 ! Bias for the reading of the pendulum position
160 K2=256 ! This gain is used in the microcomputer.
170 INTEGER Pi,Vt,Ci5
180 Vp(1)=Vp(2)=Vp(3)=Vp(4)=Vpi(3)=Vpi(4)=.3
190 Kp=1.45 ! Gain of the potentiometer.
200 G1d=1.1 ! Coefficients of Transfer Function H1(Z)
210 G1n=.85
220 K1=.75
230 H2d=.85
240 M=Z=C0(3)=C0(4)=C0(5)=C1(3)=C1(4)=C2(3)=C2(4)=U(1)=U(2)=0
250 U(3)=U(4)=U(5)=Vt(4)=Vt(5)=Us(1)=Us(2)=Us(3)=Us(4)=
260 PLOTTER IS 13,"GRAPHICS"
270 GRAPHICS
280 ! PLOTTER IS 7,5,"9872A"
290 PRINT "A55"
300 ! PRINT "VS5"
310 LOCATE 0,123,0,100
320 CSIZE 3.1,.45
330 SCALE -.5,.5,.32,.32
340 AXES 1,1,0,0
350 FRAME
360 PRINTER IS 16
370 FOR I=1 TO 5
380 MOVE I-.12,-.04
390 LABEL I
400 NEXT I
410 FOR I=-.3 TO .3 STEP .1
420 MOVE -.3,I
430 LABEL I
440 NEXT I

510 A=10.692
520 B=47.49293
530 N11=1.9673491*EXP(-A*T)*COS(B*T)+.2017993*EXP(-A*T)*SIN(B*T)+.032650
540 N10=1.9673491*EXP(-2*A*T)+EXP(-A*T)*(.0326509*COS(B*T)+.2017993*SIN(B*
550 D11=2*EXP(-A*T)*COS(B*T)
560 D10=EXP(-2*A*T)
570 A1=4.008916
580 A2=-4.385988
A3 = 0.375

R4 = 0.7998549

N2 = (A1 + A3) * EXP(-5 * T) + (A2 + A3) * EXP(5 * T) + (A1 + A2) * 2 * EXP(-A * T) * COS(B * T) + EXP(-A * T) * (A3 * COS(B * T) + A4 * SIN(B * T))

N1 = (A1 + A3) * COS(B * T) + (A2 + A3) * EXP(-5 * T) * COS(B * T) + (A1 + A2) * EXP(-2 * A * T) + A3 * COS(B * T) + A4 * SIN(B * T)) * EXP(-A * T)

N0 = (A1 + A3) * COS(B * T) + (A2 + A3) * EXP(-5 * T) * COS(B * T) + (A1 + A2) * EXP(-2 * A * T) + A3 * COS(B * T) + A4 * SIN(B * T))

D33 = EXP(-5 * T) + (A3 * COS(B * T) + 2 * EXP(-A * T) * COS(B * T))

D32 = 1 + (A3 * COS(B * T) + 2 * EXP(-A * T) * COS(B * T))

D31 = (A3 * COS(B * T) + 2 * EXP(-A * T) * COS(B * T)) + 2 * EXP(-A * T) * COS(B * T)

D30 = EXP(-2 * A * T)

N33 = N2

N32 = N2 + N1

N31 = N1 + N0

N30 = N0

N21 = 1 + EXP(-A * T) * COS(B * T) - 2 * EXP(-A * T) * COS(B * T)

N20 = EXP(-2 * A * T) - EXP(-A * T) * COS(B * T) - 2 * EXP(-A * T) * COS(B * T)

D21 = D11

D20 = D10

Statement 790 to 930 prints the transfer function of the system and the various constant of the transfer functions.

PRINT "V(x)/U(c) = .92(Z^-2-N11Z+N10)/(Z^-2-D11Z+D10)"


PRINT "Vc(z)/Vc(z) = 0.655009*(N21Z+N20)/(Z^-2-D21Z+D20)"


PRINT "Vp(z)/U(z) = 383.85*Kp*10^-5/(N33Z^-3-N32Z^-2+N31Z^-N30)/(Z^-4-D33Z^-3+D32Z^-2-D31Z^-D30)"


MOVE 1.7,.3

Statement 1040 TO 1220 prints the various practical limitation and the compensating transfer function used on the plot.

LABEL "SAMPLING TIME = "; T

LABEL "Saturation volts = "; Saturation

LABEL "Outresolution = "; Outresolution

LABEL "Velresolution = "; Velresolution

LABEL "Posresolution = "; Posresolution

LABEL "Bias in Pot volts = "; Bias

MOVE 1,-.12

LABEL "G1(x) = C1(z)/Vpi(z) = K2*(Z^2-G1n)/(Z-G1d)"

LABEL "H2(z) = C2(z)/Vti(z) = K1*(Z^2-2*H2d*Z+H2d^2)/(Z^2-C^2+1)"

LABEL "K2 = "; K2, "Kp = "; Kp

LABEL "G1n = "; G1n

LABEL "G1d = "; G1d

LABEL "K1 = "; K1

LABEL "H2de = "; H2d

The equations used from lines 1070 to 1220 are developed in Chapter 5.

1070 FOR X = 0 TO 300000
1080 FOR Y = 0 TO 10
1090 M = X + T
1100 C1(4) = G1d*C1(3) - K2*(Vpi(4) - G1n*Vpi(3))
1110 ! \( C2(4) = 2.69 \times C2(3) - 2.58 \times C2(2) + 0.82 \times C2(1) + K1 \times (Vt(4) - H2d^2 \times Vt(3)) \)
1120 Uc(4) = C1(4) - C2(4)
1130 Us(4) = U(4)
1140 ! Statements 1160 and 1170 simulates the saturation of the output voltage from the D/A converter.
1160 IF Us(4) > S THEN Us(4) = S
1170 IF Us(4) < -S THEN Us(4) = -S
1180 ! Statement 1220 to 1230 simulates the resolution of the D/A Convert
1190 Ui = Us(4) / Outresolution
1200 Us(4) = Ui / Outresolution
1210 Vt(5) = D11 \times Vt(4) - D10 \times Vt(3) + 0.92 \times (Us(4) - H11 \times Us(3) + N10 \times Us(2))
1220 P(5) = D33 \times P(4) - D32 \times P(3) + D31 \times P(2) - D30 \times P(1) - 383.5 \times Kp \times 10^{-5} \times (N33 \times Cs5(4) - N32 \times Cs5(3) + N31 \times Cs5(2) - N30 \times Cs5(1))
1230 ! Statements 1270 to 1310 simulates the resolution of the A/D converters for the reading of the velocity of the cart and pendulum position. The bias in the reading of the pendulum position is also included.
1270 Vpi = Vp(5) / Posresolution
1280 Vpi(5) = Vpi * Posresolution
1290 Vpi(5) = Vpi(5) + Bias
1300 Vti = Vt(5) / Velresolution
1310 Vti(5) = Vti * Velresolution
1320 ! Statements 1340 to 1550 shifts the values of the various variables from K+1 instant to K instant.
1340 Vpi(3) = Vpi(4)
1350 Vpi(4) = Vpi(5)
1360 Vti(2) = Vti(3)
1370 Vti(3) = Vti(4)
1380 Vti(4) = Vti(5)
1390 C1(3) = C1(4)
1400 Vp(1) = Vp(2)
1410 Vp(2) = Vp(3)
1420 Vp(3) = Vp(4)
1430 Vp(4) = Vp(5)
1440 Vt(2) = Vt(3)
1450 Vt(3) = Vt(4)
1460 Vt(4) = Vt(5)
1470 C2(1) = C2(2)
1480 C2(2) = C2(3)
1490 C2(3) = C2(4)
1500 U(1) = U(2)
1510 U(2) = U(3)
1520 U(3) = U(4)
1530 Us(1) = Us(2)
1540 Us(2) = Us(3)
1550 Us(3) = Us(4)
1570 PLOT M, Vp(5)
1580 NEXT Y
1590 PRINT USING 90; M, Vp(5), Vt(5), D, C1(4), C2(4), U(4), Us(4)
1600 NEXT X
1610 END
APPENDIX D

MEMORY MAP
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Memory Map of Page 3E
# LIST OF PROGRAMS AND SUBROUTINE

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APPENDIX E

MICROCOMPUTER PROGRAM LISTING
*  
** MAIN PROGRAM**

* THIS IS THE MAIN PROGRAM TO STABILIZE AN INVERTED PENDULUM MOUNTED ON A CART BY DIGITAL CONTROL. *

** EQU 3400H **

** EQU 341CH **

** EQU 342EH **

** EQU 3441H **

** EQU 3700H **

** EQU 3250H **

** ST 3001H ** # OPCODE STORED FROM 3001H

** LD A, 7FH ** # OUTPUT CONTROL VOLTAGE = 0 VOLTS

** LD (0FF3H), A **

** LD (0FF1H), A **

** LD A, 0 ** # MEMORY LOCATION 3E00H TO 3E07H

** LD B, 30H ** # IS INITIALIZED TO 0

** LD HL, 3E00H **

** LD (HL), A **

** INC HL **

** DNLZ LOOP **

** ID A, 12H ** # CHANNEL 1 AND AMPLIFICATION 4

** LD (3E03H), A ** # TO READ POSITION OF PENDULUM

** LD A, 0 ** # CHANNEL 0 AND AMPLIFICATION 1

** LD (3E0FH), A ** # TO READ VELOCITY OF CART

** CALL INITI ** # INITIALIZE PENDULUM POSITION

** CALL PENDIS ** # TO DISPLAY PENDULUM POSITION

** LD A, 0FFH ** # INITIALIZE COUNTER

** OUT 10H **

** CALL 00034E ** # CLEAR SCREEN

** LD A, 89H ** # STORE COEFF OF C1(3)

** LD (3E00H), A ** # STORE COEFF OF P(3)

** LD A, 09H ** # STORE COEFF OF C2(3)

** LD (3E01H), A ** # STORE COEFF OF C2(2)

** LD A, 0AH ** # STORE COEFF OF C2(1)

** LD (3E04H), A ** # STORE COEFF OF U(3)

** LD A, 3EH ** # STORE COEFF OF U(2)

** LD (3E05H), A ** # STORE COEFF OF U(2)

** LD A, 17H ** # INITIALIZE FOR SUBROUTINE STORE AND DISPLAY

** LD (3E50H) ** # SAMPLING EVENTS TO TAKE PLACE

** LD (3E40H), A ** # SAMPLING EVENTS TO TAKE PLACE

** HL, 0FF8H **

** LD (3E43H), HL **

** LD A, 03H ** # INITILISATION FOR SUB DISPLAY

** LD (3E4FH), A **

** LD A, 30H ** # TIMER COUNTER IS INITI IN MODE 0
0560 * ... CONT OF MAIN PROGRAM
0570 0580 OUT 17H
0590 LD A, 0
0600 OUT 14H
0610 LD A, 0
0620 OUT 14H
0630 MAIN LD BC, 0007H
0640 LD DE, 3E30H
0650 LD HL, 3E00H
0660 LDIR
0670 LD A, 0 # C'H'L' REGISTER IS INITIALIZED
0680 LD C, A # TO 0
0690 LD H, A
0700 LD L, A
0710 EXX
0720 LD DE, (3E24H) # LOADS MULTIPLICAND OF C2(3)
0730 LD IX, 3E32H # LOADS ADDRESS OF MULTIPLIER
0740 CALL MULT # MULTIPLY C2(3) AND COEFF OF C2(3)
0750 LD A, (3E19H) # LOADS EXPONENT OF C2(3)
0760 LD B, A
0770 INC B
0780 CALL EXP
0790 LD A, (3E12H) # LOADS SIGN OF C2(3)
0800 CALL STAD
0810 LD DE, (3E26H) # LOADS MULTIPLICAND OF C2(2)
0820 INC IX
0830 CALL MULT # MULTIPLY C2(2) AND COEFF OF C2(2)
0840 LD A, (3E19H) # LOADS EXPONENT OF C2(2)
0850 LD B, A
0860 INC B
0870 CALL EXP
0880 LD A, (3E13H) # LOAD SIGN OF C2(2)
0890 INC A
0900 CALL STAD
0910 LD DE, (3E28H) # LOAD MULTIPLICAND OF C2(1)
0920 INC IX # ADDRESS OF MULTIPLIER
0930 CALL MULT # MULTIPLY C2(1) AND COEFF OF C2(1)
0940 LD A, (3E1AH) # LOAD EXPONENT OF C2(1)
0950 LD B, A
0960 INC B
0970 CALL EXP
0980 LD A, (3E14H) # LOAD SIGN OF C2(1)
0990 CALL STAD
1000 LD DE, (3E2AH) # LOADS MULTIPLICAND OF V(3)
1010 INC IX # ADDRESS OF V(3)
1020 CALL MULT
1030 LD A, (3E15H) # LOADS SIGN OF V(2)
1040 INC A
1050 CALL STAD
1060 LD DE, (3E2CH) # LOADS MULTIPLICAND OF V(2)
1070 INC IX # ADDRESS OF MULTIPLIER IN IX REG
1080 CALL MULT
1090 LD A, (3E16H) # LOADS SIGN OF V(2)
1100 * ... CONTD OF MAIN PROGRAM
1110
1120
1130 STAD EQU 341CH
1140 SHIFT EQU 34A5H
1150 ST 3100H  # OPCODE STORED FROM 3100H
1160 CALL STAD
1170 EXX
1180 LD B, 05H  # LOAD COUNTER BY 5
1190 LD A, C  # CHECK IF SIGN +VE OR -VE
1200 AND 00H
1210 JR NZ LOOPG  # IF -VE THEN JUMP TO LOOPG
1220 LOOPI SPL C  # DIVIDE CHL REGISTER BY 32
1230 RR H
1240 RR L
1250 DJNZ LOOPI
1260 JR LOOPJ
1270 LOOPG NOP
1280 LOOPK SPL C  # DIVIDE CHL REGISTER BY 32
1290 RR H
1300 RR L
1310 DJNZ LOOPK
1320 LD A, C  # OR 0FCH BECAUSE CHL REGISTER IS
1330 OR 0FCH  # -VE AND IS IN 2' COMPLEMENT
1340 LD C, A
1350 LOOPJ LD (3E73H), HL
1360 LD A, C
1370 LD (3E75H), A
1380 CALL SHIFT
1390 LOOPE IN A, 10H  # WAIT IF 10 MILLI SECS (SAMPLING
1400 AND A  # TIME OVER)
1410 JR Z, LOOPE
1420 LD A, 30H  # INITIALISE TIMER COUNTER IN MODE 0
1430 OUT 17H
1440 LD A, 50H  # LOADS MOST SIGNIFICANT BYTE OF
1450 OUT 14H  # TIMER COUNTER AND THEN THE LEAST
1460 LD A, 60H  # SIGNIFICANT BYTE TO MAKE SAMPLING
1470 OUT 14H  # TIME 10 MILLI SECS.
1480 LD A, (3E0FH)  # INITIALISE A/D TO READ VELOCITY
1490 LD (0FFF3H), A
1500 LOOPF LD A, (0FFF1H)  # THIS AND THE NEXT LOOP CHECKS
1510 AND 80H  # WHETHER THE A/D CONVERSION IS
1520 JR Z, LOOPF  # OVER
1530 LOOPF LD A, (0FFF1H)
1540 AND 80H
1550 JR NZ LOOPF
1560 LD A, (0FFF2H)  # CHECK SIGN
1570 AND 08H
1580 JR Z, LAMP  # IF SIGN -VE THEN JUMP TO LAMP
1590 LD A, (0FFF0H)  # V(4) IS MULTIPLIED BY -1.
1600 LD L, A  # CONVERTS SHIFTED BINARY INTO
1610 LD A, (0FFF2H)  # 2' COMPLEMENT FORMAT.
1620 * ... CONTD OF MAIN PROGRAM.
1630 1640 AND 07H
1650 1660 LD H, A
1670 1680 LD A, (3E03H)
1690 1680 LD (OFFF3H), A
1690 1680 LD DE, (3E71H)
1690 1690 SBC HL, DE # SUBTRACT BIAS
1700 1710 LD A, H
1710 1710 AND 90H
1720 1720 JR Z, B1
1730 1730 ADD HL, DE
1740 1740 LD A, E
1750 1750 SUB L
1760 1760 LD L, A
1770 1770 JR B2
1780 B1 LD (3E2AH), HL # STORE MAGNITUDE OF U(4) IN U(3)
1790 1790 LD A, 0 # STORE +VE SIGN FOR U(3)
1800 1800 LD (3E15H), A
1810 1810 LD DE, (3E73H)
1820 1820 LD A, (3E75H)
1830 1830 ADD HL, DE
1840 1840 ADC 0
1850 1850 LD (3E73H), HL
1860 1860 LD (3E75H), A
1870 1870 JR LOOPP
1880 LAMP LD A, (OFFF0H) # IV U(4) IS -VE IT IS CONVETED IN
1890 1890 CPL # IN 2'COMPLEMENT
1900 1900 LD L, A
1910 1910 LD A, (OFFF2H)
1920 1920 CPL
1930 1930 AND 07H
1940 1940 LD H, A
1950 1950 LD A, (3E00H)
1960 1960 LD (OFFF1H), A
1970 1970 LD DE, (3E71H) # BIAS IS ADDED TO U(4)
1980 1980 ADD HL, DE
1990 B2 LD (3E2AH), HL # STORE THE MAGNITUDE OF U(4) IN
2000 2000 LD A, 01H # U(3), STORE -VE SIGN FOR U(4)
2010 2010 LD (3E15H), A # IN U(3)
2020 2020 LD A, L
2030 2030 CPL
2040 2040 LD L, A
2050 2050 LD A, H
2060 2060 CPL
2070 2070 LD H, A
2080 2080 LD DE, (3E73H)
2090 2090 LD A, (3E75H)
2100 2100 ADD HL, DE
2110 2110 ADC 0FFH
2120 2120 LD (3E73H), HL # STORE C2(4)
2130 2130 LD (3E75H), A
2140 LOOPP HOP
* CONTI OF MAIN PROGRAM.

MULT EQU 3400H
COMP EQU 342EH
STAD EQU 341CH
DISPLA EQU 3530H
STORE EQU 3E40H
STOP EQU 34DFH
ST 31EH
LD DE,(3E20H) # LOAD MULTIPLICAND OF C1(3)
LD IX,3E30H # LOAD ADDRESS OF C1(3) IN IX REG
CALL MULT # MULTIPLY C1(3) AND COEFF OF C1(3)
LD B,0F7H
LD A,(3E17H)
ADD B
CPL
LD B,A # LOAD COUNTER WITH EXPONENT
LOOP SRL C # DIVIDE THE RESULT OF C1(3)
RR H # BY THE REMAINDER.
RR L
DJNZ LOOP
LD A,(3E10H) # TEST WHETHER C1(3) IS +VE OR -VE
RRA # IF -VE THEN TAKE 2'C COMPLEMENT
JR NC, LOOP1 # IF NOT -VE THEN JUMP TO LOOP1
CALL COMP # RESULT IS STORED IN C'H'L' REG
LD DE,(3E22H) # LOADS MULTIPLICAND OF P(3)
INC IX # ADDRESS OF P(3) IN IX REGISTER
CALL MULT # MULTIPLY P(3) AND COEFF OF P(3)
LD B,03H # DIVIDE THE RESULT OF P(3) WHICH
LOOPM SRL C # IS STORED IN CHL REGISTER BY 8
RR H
RR L
DJNZ LOOPM
LD A,(3E11H)
CALL STAD
LOOPM
LD (3E70H),HL # MEMORY LOCATION 3E70H, 3E71H AND
LD A,C # 3E72H
LD (3E72H),A # 3E70H
HL (3E41H) # INITILISATION FOR SUB DISPLAY
LD A,00H # INITIALISE COUNTER FOR DELAY
LD A,(3E40H) # THIS PART OF THE PROGRAM STORES
INC A # AND DISPLAYS ALTERNATE VALUES OF
AND 01H # DATA ON SCREEN AND IN MEMORY
LD (3E40H),A
JR Z,SKIP
CALL DISPLA
CALL STORE
LD HL,(3E1BH)
INC HL
LD (3E1BH),HL
2680 * .... CONT'N OF MAIN PROGRAM.
2690
2700 LD B, 7EH # DELAY TIME SO THAT THE A/D HAS
2710 SKIP CALL STOP # ENOUGH TIME FOR SETTLING TO
2720 DJNZ SKIP # READ CHANNEL FOR PENDULUM POSI-
2730 LD A, (3E0BH) # TION. INITIATE A/D TO READ
2740 LD (0FF3H), A # PENDULUM POSITION.
2750 LOOPQ LD A, (0FF1H) # TEST THE MSB TO CHECK IF A/D
2760 AND #0H # CONVERSION IS OVER.
2770 JR Z, LOOPQ
2780 LOPQ LD A, (0FF1H) #
2790 AND #0H
2800 JR NZ, LOPQ
2810 NOP
2820 NOP
2830 NOP
2840 NOP
2850 NOP
2860 NOP
2870 ST #3270H
2880 TRANS EQU #345FH
2890 MAIN EQU #3069H
2900 LD A, (0FF2H) # TEST IF VALUE OF P(4) IS +VE OR
2910 AND #06H # -VE
2920 JR Z, LOOPR # IF P(3) -VE THEN JUMP TO LOOPR
2930 LD A, (0FF0H) # LOADS THE LEAST SIGNIFICANT BYTE
2940 LD L, A # OF P(3) IN L REG. LOAD THE MOST
2950 LD A, (0FF2H) # SIGNIFICANT NIBBLE IN H REGISTER
2960 AND #07H # RESET THE UPPER NIBBLE
2970 LD H, A
2980 LD A, (3E0FH) # INITIALISE THE A/D TO READ
2990 LD (0FF3H), A # VELOCITY OF CART
3000 LD DE, (3E7BH) # LOAD BIAS FOR PENDULUM POSITION
3010 SBC HL, DE # SUBTRACT BIAS FROM VALUE OF P(3)
3020 LD A, H # CHECK IF MAGNITUDE OF BIAS IS
3030 AND #00H # GREATER THAN THE VALUE OF P(3)
3040 JR Z, B3 # IF BIAS LESS THAN P(3) THEN B3
3050 ADD HL, DE # IF BIAS > THAN P(4) THEN
3060 LD A, E # SUBTRACT P(3) FROM BIAS AND S
3070 SUB L # STORE -VE SIGN FOR P(4)
3080 LD L, A
3090 JP B4
3100 B3 LD (3E22H), HL # STORE MAGNITUDE OF P(4)
3110 LD A, 0 # STORE +VE (00H) FOR SIGN OF P(4)
3120 LD (3E11H), A # MULTIPLY P(4) BY 8
3130 SLA L
3140 RL H
3150 SLA L
3160 RL H
3170 SLA L
3180 RL H
3190 LD A, L
3200 CPL
3210 LD L, A
2555  " CONTINUED MAIN PROGRAM."
2560  LD  DE, (3E70H)
3220  LD  A, H
3230  CPL
3240  LD  H, A
3250  LD  C, OFFH
3260  LD  DE, (3E70H)
3270  LD  A, (3E72H)
3280  ADD HL, DE
3290  ADC C
3300  LD  C, A
3310  LD  (3E70H), HL  "STORE THE VALUE OF C1(3) IN MEM--
3320  LD  (3E72H), A  "MEMORY LOCATION 3E72H, 3E71H AND
3330  LD  (3E70H), HL  "3E70H ALSO STORE IT IN 3E73H.
3340  LD  (3E72H), A  "3E77H AND 3E76H
3350  JR  LOOP
3360  LOOPR  LD  A, (0FFF0H)  "LOADS P(4) MOST SIGNIFICANT BYTE
3370  CPL  "SINCE A/D IS IN SHIFTED BINARY
3380  LD  L, A  "AND VALUE IS -VE COMPLEMENT
3390  LD  A, (0FFF2H)  "THE VALUE TO GET CORRECT MAGNIT--
3400  CPL  "DO THE SAME FOR UPPER
3410  AND 07H  "NIBBLE. RESET THE UPPER BYTE
3420  LD  H, A
3430  LD  A, (3E0FH)  "INITIALISE A/D TO READ VELOCITY
3440  LD  (0FFF3H), A  "OF CART
3450  LD  DE, (3E73H)  "LOAD BIAS OF PENDULUM POSITION
3460  ADD HL, DE  "SINCE P(4) IS -VE ADD BIAS T
3470  B4  LD  (3E22H), HL  "STORE MAGNITUDE OF P(4) IN P(3)
3480  LD  A, 01H  "SINCE SINE -VE STORE (01H) FOR
3490  LD  (3E11H), A  "SIGN OF P(4)
3500  SLA L  "MULTIPLY P(4) BY 8
3510  SLA L
3520  SLA L
3530  RL H
3540  SLA L
3550  RL H
3560  LD  DE, (3E70H)  "LOADS INCOMPLETE C3(4)
3570  LD  A, (3E72H)
3580  ADD HL, DE
3590  ADC 0  "TO GET VALUE OF C3(4)
3600  LD  (3E70H), HL  "STORE C1(4) IN 3E72H, 3E71H AND
3610  LD  (3E72H), A  "3E70H. ALSO STORE C1(4) IN 3E73H
3620  LD  (3E70H), HL  "3E77H AND 3E76H.
3630  LD  (3E72H), A
3640  LD  C, A
3650  LOOP  NOP
3660  LOOP  LD  DE, (3E73H)  "VA = C1(4) - C1(4). LOAD C2(4)
3670  LD  A, E  "GET VALUE OF C2(4)
3680  CPL  "TAKE COMPLEMENT AND ADD TO C1(4)
3690  LD  E, A  "C1(4) IS PRESENT IN CHL REGISTER
3700  LD  A, A
3710  CPL
3720  LD  D, A
CONTID OF MAIN PROGRAM

LD A, (3E7EH)
CPL
ADD HL, IE
ADC C
LD (3E62H), A
LD (3E60H), HL
LD C, A
AND 80H
JR NZ, LOOPU
LD A, C
AND 0FH
JR NZ, SAT
LD A, H
AND 0FH
JR NZ, SAT
LD A, L
CPL
AND 0FH
JR FINAL
SLA L
RL H
DJNZ NEG
LD L, A
LD A, H
CPL
AND 7FH
JR FINAL
LD A, C
CPL
AND 7FH
JR NZ, SAT1
LD A, H
CPL
AND 0FH
JR NZ, SAT1
LD A, L
CPL
AND 0CH
JR FINAL
SLA L
RL H
DJNZ NEG
LD L, A
LD A, H
CPL
OR 80H
JR FINAL
MOP
4222 * .... CONT'0 OF MAIN PROGRAM.
4223 *
4230 SAT1  LD  L, 0CH  # OUTPUT TO D/A IS=-10 VOLTS HENCE
4240  LD  A, 0FFH  # STORE ALL 1'S IN A AND C REGI
4250 FINAL  LD  (0FFF1H), A  # THE MSB 8 BITS OF THE D/A IS
4260  LD  H, A  # ADDRESSED BY 0FFF1H AND THE 2
4270  LD  A, (3E0DH)  # LSB BITS ARE THE THIRD AND THE
4280  OR  L  # FOURTH BITS. AND IT IS ADDRESSED
4290  LD  (0FF2H), A  # BY 0FF2H
4300  SRL  L
4310  SRL  L
4320  LD  (3E41H), HL  # STORE OUTPUT TO D/A
4330  LD  A, (3E75H)  # LOAD C2(3) IN AHL REGISTER
4340  LD  HL, (3E72H)
4350  LD  IX, 3E19H  # ADDRESS FOR EXPONENT OF C2(3)
4360  LD  IY, 3E12H  # ADDRESS FOR SIGN OF C2(3)
4370  CALL  TRANS
4380  LD  (3E24H), HL  # STORE MAGNITUDE OF C2(3)
4390  LD  A, (3E72H)  # LOAD THE VALUE OF C1(3) IN
4400  LD  HL, (3E70H)  # AHL REGISTER
4410  LD  IX, 3E17H  # ADDRESS FOR EXPONENT OF C1(3)
4420  LD  IY, 3E10H  # ADDRESS FOR SIGN OF C1(3)
4430  CALL  TRANS
4440  LD  (3E20H), HL  # STORE MAGNITUDE OF C1(3)
4450  JP  MAIN
4560
4570 **************************************************
* SUBROUTINE MULT.

This subroutine performs a 16 by 8 bit multiplication.
The 16 bit multiplicand is placed in DE register and
the 8 bit multiplier is indirectly addressed by IX reg.
Before this subroutine is called, the final result
of multiplication is placed in CHL register.

ST 3400H
STOP EQU 340FH
CALL STOP
LD B, 08H  # COUNTER IS INITIALLY.
LD A, 0    # INITIALISE A, H, L AND C REG TO 0.
LD H, A
LD L, A
LD C, A
LOOP1 SRL (IX+0)  # SHIFT RIGHT MULTIPLIER INDIRECTLY
IR NC, SKIP   # ADDRESSED BY IX REG.
ADD HL, DE
ADC C
SKIP SLA E    # THE MULTIPLICAND IS ROTATED LEFT
RL D         # THROUGH CARRY, THE CARRY IS SHIFTED
RL C         # LEFT IN THE 'E' REGISTER.
DINZ LOOP1
LD , C, A
RET

* SUBROUTINE STAD.

The sign bit of a 23 bit value is placed in 'A' reg.
And the magnitude is placed in CHL register. If the
sign is -VE then a 2' complement is taken by subroutine
COMP. It then adds CHL reg to C'H'L' reg. The result
is placed in C'H'L' register.

RRA  # CHECK SIGN.
JR NC, LOOPC  # IF SIGN +VE THEN JUMP TO LOOPC.
CALL 342EH  # REG. IF THE SIGN BIT IS -VE A
LOOPC LD A, C  # 2' COMP IS TAKEN OF THE CHL REGS
PUSH HL  # SAVE HL ON STACK.
EXX
POP DE     # GET VALUE FROM STACK IN DE.
ADD HL, DE
ADC C
LD C, A
EXX  # EXCHANGE B, C, D, E, H, AND L REG.
RET  # WITH B', C', D', E', H' AND L' REG.

5130 *
5140 *  
5150 SUBROUTINE COMP
5160 ****************************
5170 * THIS SUBROUTINE TAKES THE 2' COMPLEMENT OF CHL REGISTER
5180 * AND STORES IT BACK IN CHL REGISTER.
5190 ****************************
5200  
5210 LD A, L
5220 CPL
5230 ADD 01H
5240 LD L, A
5250 LD A, H
5260 CPL
5270 ADC 0
5280 LD H, A
5290 LD A, C
5300 CPL
5310 ADC 0
5320 LD C, A
5330 RET
5340  
5350 ****************************
5360  
5370  
5380  
5390 SUBROUTINE EXP
5400  
5410 ****************************
5420 * THIS SUBROUTINE DIVIDES THE CHL REGISTER BY 8. IT THEN
5430 * MULTIPLIES THE CHL REGISTER BY 2 EXPONENT(VALUE). THIS
5440 * (VALUE) IS PLACED IN CHL REGISTER. THE FINAL RESULT IS
5450 * STORED IN CHL REGISTER.
5460  
5470  
5480 ****************************
5490  
5500 SRL C  # CHL REGISTER IS DIVIDED BY 8.
5510 RR H
5520 RR L
5530 SRL C
5540 RR H
5550 RR L
5560 SRL C
5570 RR H
5580 RR L
5590 LOP
5600 SLA L
5610 RL H
5620 RL C
5630 DJNZ LOP
5640 RET
5650  
5660 ****************************
SUBROUTINE TRANS

5660 *
5670
5680 ****************************************************
5690 * CONVERTS Q# AND D# FROM 24 BITS AND 2'S COMPLEMENT WHICH
5700 * IS PLACED IN AHI REGISTER INTO 15 BIT MAGNITUDE,
5710 * SIGN BYTE AND EXPONENT BYTE.
5720 * THE SIGN BYTE IS INDIRECTLY ADDRESSED BY IY REGISTER AND
5730 * THE EXPONENT BYTE IS INDIRECTLY ADDRESSED BY IX REGISTER
5740 * THE 15 BIT MAGNITUDE IS STORED IN HL REGISTER.
5750
5760 ******************************************************
5770
5780     LD    D, A    # LOAD MOST SIGNIFICANT BYTE.
5790     SLA    D      # CHECK SIGN
5800     JR    NC, LOOPX  # IF +VE SIGN THEN JUMP TO LOOPX
5810     LD    A, 01H  # SIGN -VE AND HENCE STORE 01H
5820     LD    (IY+0), A    # INDIRECTLY ADDRESSED BY IY REGISTER
5830     LD    A, D
5840     RRCA
5850     LD    C, A    # TAKE 2'S COMPLEMENT OF AHI REGISTER
5860     LD    A, L    # TO GET MAGNITUDE.
5870     CPL
5880     ADD    01H
5890     LD    L, A
5900     LD    A, H
5910     CPL
5920     ADC    0
5930     LD    H, A
5940     LD    A, C
5950     CPL
5960     ADC    0
5970     AND    7FH
5980     JR    LOOP0
5990
6000     LOOPX     LD    D, A    # SIGN +VE AND HENCE STORE 00H
6010     LD    (IY+0), A    # INDIRECTLY ADDRESSED BY IY REG.
6020     LD    A, D
6030     LOOP     LD    C, A
6040     LD    D, A
6050     LD    A, 09H
6060     LD    B, 08H
6070     LOOPY    SLA    C
6080     JR    C, LOM
6090     DEC    A
6100     Dinz    LOOPY
6110     LOM     LD    B, A    # STORE EXPOHENT VALUE INDIRECTLY
6120     LD    (IX+0), A    # ADDRESSED BY IX REGISTER.
6130     LOOPZ    SRL    D    # IF MAGNITUDE GREATER THAN 15 BITS
6140     RR    H    # THEN SAVES THE FIRST 15 BITS IN HL
6150     RR    L    # REGISTER.
6160     Dinz    LOOPZ
6170     LO    RET
6180
6190 ******************************************************
**SUBROUTINE SHIFT**

6200 * THIS SUBROUTINE SHIFTS THE MAGNITUDE AND SIGN OF U(3)
6210 TO MEMORY LOCATION OF U(2). MAGNITUDE, SIGN AND EXPON
6220 OF C2(2) TO MEMORY LOCATION OF C2(1). MAGNITUDE, SIGN
6230 AND EXPONENT OF C2(3) TO MEMORY LOCATION OF C2(2)
6240 ********************************************************************
6250 LD HL, (7E2AH) # MAGNITUDE OF U(3) TO U(2)
6260 LD (7E2CH), HL
6270 LD HL, (7E2AH) # MAGNITUDE OF C4(2) TO C4(1)
6280 LD (7E2BH), HL
6290 LD HL, (7E24H) # MAGNITUDE OF C2(3) TO C2(2)
6300 LD (7E26H), HL
6310 LD A, (7E15H) # SIGN OF U(3) TO U(2)
6320 AND 01H
6330 LD (7E18H), A
6340 LD A, (7E17H) # SIGN OF C2(2) TO C2(1)
6350 AND 01H
6360 LD (7E14H), A
6370 LD A, (7E12H) # SIGN OF C2(3) TO C2(2)
6380 AND 01H
6390 LD (7E10H), A
6400 LD A, (7E19H) # EXPONENT OF C2(2) TO C2(1)
6410 AND 01H
6420 LD (7E17H), A
6430 LD A, (7E18H) # EXPONENT OF C2(3) TO C2(2)
6440 LD (7E14H), A
6450 LD A, (7E12H) # EXPONENT OF C2(3) TO C2(2)
6460 LD (7E10H), A
6470 RET

**SUBROUTINE STOP**

6500 ********************************************************************
6510 * IF THE LETTER 'S' IS PRESSED FROM THE KEYBOARD THEN
6520 THE EXECUTION OF THE PROGRAM IS STopped. AT THE SAME
6530 INSTANT THE OUTPUT VOLTAGE TO THE CARt IS MADE 0. THE
6540 PAST VALUES OF THE POSITION OF PENDULUM, VOLTAGE OUT TO
6550 THE CARt AND THE VELOCITY OF THE CARt ARE DISPLAYED ON
6560 THE SCREEN. IF KEY 'A' IS PRESSED THEN THE CONTROLS ARE
6570 * PRESSED TO RESTART.
6580 ********************************************************************
6590 LD A, (7E7FH) # REGISTER A IS SAVED
6600 IN 0.A # INPUT FROM KEY BOARD.
6610 CP 'S' # CHECK IF KEY 'S' IS DEPRESSED.
6620 JP Z, END # IF YES THEN JUMP TO END.
6630 LD A, (7E7FH) # LOADS REGISTER A WITH ORIGINAL
6640 VALUE.
6650 RET
6660 END IN 0.A # CHECK IF KEY 'A' IS DEPRESSED.
6670 LD A, 7FH # OUTPUT TO I/M TO CARt =0 VOLTS
6680 LD (OFFF1H), A
6690 IR NZ, END # IF KEY 'A' IS DEPRESSED THEN
6700 RST 48H # RST.
**SUBROUTINE DISPLAY**

6750* THIS SUBROUTINE DISPLAYS ON SCREEN THE CONTROL VOLTAGE
6760* OUTPUT (U), PENDULUM POSITION (P), SPEED OF THE A
6770* ARMATURE OF THE D.C. MOTOR IN HEXDEcimal. THE M
6780* VALUES ARE DISPLAYED WITH SIGNS.

6750~

6850 ST 257FH
6860 STOP EQU 240FH
6870 CALL STOP
6880 LD A, 00FH
6890 CALL 000772
6900 LD A, 08FH
6910 CALL 000772
6920 LD A, H
6930 AND 0FH
6940 TR NZ, POST # TEST SIGN OF CONTROL VOLTAGE
6950 TR NZ, POST # IS THE SIGN THEN JUMP TO POST
6960 LD A, H
6970 CPL
6980 AND 7FH
6990 LD A, H
7000 LI H, L
7010 LD A, 00FH
7020 TR NF, NET # ASCII CODE FOR '+'
7030 NET CALL 000772
7040 LD A, 04FH
7050 AND 6FH # ASCII CODE FOR '-'
7060 LD A, H
7070 CALL 000772
7080 LD A, H # DISPLAY OUTPUT CONTROL VOLTAGES
7090 AND 6FH # IN HEXA DECIMAL OF THE VALUE
7100 LD A, H # PRESENT IN HL REGISTER.
7110 SRA L
7120 SRA L
7130 SRA L
7140 CP 00H
7150 CP 00H
7160 CALL 000772
7170 SRL H
7180 SRL H
7190 SRL H
7200 SRL H
7210 SRL H
7220 SRL H
7230 SRL H
7240 SRL H
7250 SRL H
7260 CP 00H
7270 CP 00H
7360 CALL P.HEX
7366 CALL 000772
7370 LD A, B
7376 AND 0FH
7380 OR 0DH
7386 CP 0AH
7390 CALL P.HEX
7396 CALL 000772
7400 CALL 000772
7406 CALL STOP
7410 LD HL, (7E32H)  # LOADS MAGNITUDE OF POSITION
7416 LD A, 00H     # OF PENDULUM, ASCII CODE FOR 'P'
7420 CALL 000772
7426 LD A, 07H     # ASCII CODE FOR '='
7430 CALL 000772
7436 LD A, (7E11H)
7440 AND 01H  # TEST SIGN BIT
7446 JR NZ, POST1  # IF WE THEN DISPLAY '/' ON SCREEN
7450 LD A, 0BH     # ASCII CODE FOR '+'
7456 JR NET1
7460 POST1 LD A, 09H  # ASCII CODE FOR '-'
7466 NET1 CALL 000772
7470 LD A, H      # DISPLAY MAGNITUDE FOR POSITION OF
7476 AND 07H      # PENDULUM IN HEX OF THE VALUE &
7480 OR 00H       # STORED IN HL REGISTER.
7486 CALL 000772
7490 LD A, L
7500 SPL A
7510 SPL A
7520 SPL A
7530 OR 0204
7540 CP 00AH
7550 CALL P.HEX
7560 CALL 000772
7570 LD A, L
7580 AND 07H
7590 OR 0004
7600 CP 00AH
7610 CALL P.HEX
7620 CALL 000772
7630 LD A, L
7640 AND 07H
7650 OR 0004
7660 CP 00AH
7670 CALL P.HEX
7680 CALL 000772
7690 CALL 000772
7700 CALL STOP
7710 LD A, 007H    # ASCII CODE FOR 'S'
7716 CALL 000772
7720 LD A, 08H    # ASCII CODE FOR '='
7726 CALL 000772
7730 LD A, (7E15H) # LOADS SIGN OF VELOCITY
7736 AND 01H  # TEST SIGN
7740 JR NZ, POST2 # IF WE SIGN THEN DISPLAY '/'
7746 LD A, 0AH    # ASCII CODE FOR '+'
7750 JR NET2
7800  P0872  LD  A, 0AH    # ASCII CODE FOR '－'
7810  NET2  CALL  000272
7820  LD  HL, (7E2AH)
7830  LD  A, H      # DISPLAY THE MAGNITUDE FOR VE
7840  AND $07H    # VELOCITY OF CART IN HEX OF THE
7850  OR $08H    # VALUE PRESENT IN HL REGISTER.
7860  CALL  000272
7870  LD  A, L
7880  SPL A
7890  SPL A
78A0  SPL A
78B0  SPL A
78C0  SPL A
78D0  OR $08H
78E0  CP $09H
78F0  CALL  P.HEX
7900  CALL  000272
7910  LD  A, L
7920  AND $04H
7930  OR $06H
7940  CP $06H
7950  CALL  P.HEX
7960  CALL  000272
7970  CALL  000272
7980  LD  A, (7E4FH)
7990  DEC A
79A0  LD  (7E4FH), A
79B0  IR NZ, SKIP
79C0  CALL  000272
79D0  LD  A, 07H
79E0  LD  (7E4FH), A
79F0  CALL  STOP
8000  SKIP RET
8010  HEV ADD 7
8020  RET
8030  ***************
* SUBROUTINE STORE *

* STORE VALUES OF THE SPEED OF THE ARMATURE FROM 3E50H TO 3E80H *
* TO 4000H AND THE PENDULUM POSITIONS FROM 2E80H TO 3000H *
* EACH VALUE IS STORED IN 2 BYTES: THE FIRST TWO BYTES *
* CONTAIN THE MAGNITUDE AND THE THIRD CONTAINS THE SIGN. *
* 01H INDICATES A POSITIVE SIGN WHILE 00H INDICATE A NEGITIVE SIGN. *
* C1 IS STORED FROM 3900H TO 3B00H, C2 FROM 3100H TO 3300H *
* VA FROM 3000H TO 3200H. C1, C2, VA ARE STORED IN 3 *
* BYTES IN 2'S COMPLEMENT FORMAT. AFTER THE ENTIRE MEMORY *
* SPACE FOR THE VALUES ARE FULL, THE PROGRAM IS INITIALIZED TO STORE THE VALUES FROM THE BEGINNING ADDRESS *
* THIS SUBROUTINE ALSO STOPS THE EXECUTION OF THE MAIN *
* PROGRAM AFTER A FIXED NUMBER OF CYCLES. *

8340 STOP EQU 741FH
8350 END EQU 746CH
8360 ST 3E40H
8370 STORE LD A, (3E50H) # CHECK IF ENTIRE MEMORY IS
8380 DEC A # FILLED UP.
8390 LD (3E50H), A
8400 TRnz. INITI
8410 LD A, 30H
8420 LD (3E50H), A
8430 LD HL, 3E80H # INITIALIZE ADDRESS FOR U(3)
8440 LD (3E51H), HL
8450 LD HL, 2E80H # INITIALIZE ADDRESS FOR P(3)
8460 LD (3E52H), HL
8470 LD HL, 3F00H # INITIALIZE ADDRESS FOR C1
8480 LD (3E53H), HL
8490 LD HL, 3F00H # INITIALIZE ADDRESS FOR C2
8500 LD (3E54H), HL
8510 LD HL, 3F00H # INITIALIZE ADDRESS FOR VA
8520 LD (3E55H), HL
8530 INITI LD A, (3E15H) # LOADS SIGN OF U(3)
8540 LD DE,(3E25H) # LOAD MAGNITUDE OF U(3)
8550 LD HL,(3E14H) # LOADS ADDRESS OF U(3)
8560 LD (HL), E # STORE MAGNITUDE AND SIGN OF U(3)
8570 INC HL
8580 LD (HL), D
8590 INC HL
8600 LD (HL), A
8610 INC HL
8620 LD (3E51H), HL # STORE UPDATED ADDRESS BACK.
8630 CALL STOP
8640 LD A,(3E11H) # LOADS SIGN OF P(3)
8650 LD DE,(3E22H) # LOADS MAGNITUDE OF P(3)
8660 LD HL,(3E23H) # LOADS ADDRESS OF P(3)
8670 LD (HL), D # STORE MAGNITUDE AND SIGN OF P(3)
8680 INC HL
8690 LD (HL), D
8700 INC HL
8710 LD (HL), A
8720 INC HL
8730 LD (3E53H), HL # STORE UPDATED ADDRESS BACK
8740 LD A, (3E78H) # LOAD MOST SIGNIFICANT BYTE OF C1
8750 LD DE, (3E78H) # LOAD THE LAST TWO BYTES OF C1
8760 LD (HL), E # STORE MAGNITUDE OF C1
8770 LD (HL), E
8780 INC HL
8790 LD (HL), D
8800 INC HL
8810 LD (HL), A
8820 INC HL
8830 LD (3E55H), HL # STORE UPDATED ADDRESS OF C1
8840 CALL STOP
8850 LD A, (3E75H) # LOAD MS BYTE OF C2
8860 LD DE, (3E75H) # LOAD THE LAST TWO BYTES OF C2
8870 LD HL, (3E75H) # LOAD ADDRESS OF C2
8880 LD (HL), E # STORE VALUE OF C2
8890 INC HL
8900 LD (HL), D
8910 INC HL
8920 LD (HL), A
8930 INC HL
8940 LD (3E57H), HL # STORE UPDATED ADDRESS OF VA
8950 LD A, (3E52H) # LOAD M.S. BYTE OF VA
8960 LD DE, (3E52H) # LOAD LAST TWO BYTES OF VA
8970 LD (HL), E # LOAD ADDRESS OF VA
8980 LD (HL), E # STORE VALUE OF VA IN MEMORY
8990 INC HL
9000 LD (HL), D
9010 INC HL
9020 LD (HL), A
9030 INC HL
9040 LD (3E59H), HL
9050 CALL STOP
9060 LD HL, (3E42H) # LOAD COUNTER - NO OF SAMPLES
9070 LD (3E42H), HL # STORE BACK COUNTER
9080 LD A, H
9100 AND $FFH # CHECK IF COUNTER=0
9110 JR Z, END1 # IF COUNTER=0 THEN STOP THE
9120 RET
9170 END1 LD A, L # HALT THE EXECUTION OF PROGRAM
9140 AND $FFH
9150 JR Z, END2
9160 RET
9170 END2 JP END
9190
SUBROUTINE LAST

9200  "This subroutine is used to bias the pendulum position
9220  "reading and store the correction factor for the velocity
9240  "reading for the cart in stationary condition. The
9260  "biasing is done by depressing the key 'I' and 'D' which
9270  "increases and decreases the biasing voltage from the
9280  "D/A. To return to the main program key 'E' should be
9290  "depressed.
9300  "This subroutine has access to six more subroutine which
9310  "are 1) DEL1, 2) INC, 3) DEC, 4) END, 5) DELAY AND
9320  "6) SCREEN.

9330  "-----------------------------------------------
9340  "-----------------------------------------------

9350  ST  3700H
9360  LD  HL,01CDH  ; loads initial value for bias
9370  LD  (3E07H),HL
9380  LD  HL,0
9390  LD  (3E08H),HL
9400  LD  (3E10H),HL
9410  LOOP  LD  HL,(3E07H)  ; convert the last 10 bits in
9420  LD  B,L  ; HL reg in 8 bits in H reg and
9430  SLA  A  ; last 2 bits of L register.
9440  RL  H
9450  SLA  L
9460  RL  H
9470  SLA  L
9480  RL  H
9490  SLA  L
9500  RL  H
9510  SLA  L
9520  RL  H
9530  SLA  L
9540  RL  H
9550  LD  A,H
9560  LD  (0FF0H),A  ; output D/A
9570  LD  A,B
9580  LD  (0FF2H),A
9590  LD  A,0D0H  ; ASCII code for 'P'
9600  CALL  000372
9610  LD  A,0C2H  ; ASCII code for 'B'
9620  CALL  000372
9630  LD  A,0B1H  ; ASCII code for "=".
9640  CALL  000372
9650  LD  A,(3E08H)  ; initialise A/D to read pendulum
9660  LD  A,(0FF3H),A
9670  LOOP1  LD  A,(0FF1H)  ; check whether A/D conversion is
9680  AND  50H
9700  JR  Z,LOOP1
9710  LOOP2  LD  A,(0FF1H)
AND 80H
JR NZ, LOOP2
LD A, (OFFF2H)  # READ MOST SIGNIFICANT NIBBLE.
AND 0FH
LD H, A
LD A, (OFFFH)
CALL SCREEN
LD A, (3E0FH)  # INITIALISE A/D TO READ VELOCITY
LD (0FF3H), A
CALL DEL1
LD DE, 07FFH  # SUBTRACT BIAS AND ADD PREVIOUS
LD A, (3E7BH)  # BIAS, TO IT. THEN DIVIDE BY 2.
ADD 0
SBC HL, DE
ADD L
SRL A
LD (3E7BH), A  # STORE BIAS.
CALL DEL1
LD A, 0D6H  # ASCII CODE FOR 'V'
CALL 000372
LD A, 0C2H  # ASCII CODE FOR 'B'
CALL 000372
LD A, 02DH  # ASCII CODE FOR '='
CALL 000372
LD A, (3E0FH)  # INITIALISE A/D TO READ VELOCITY
LD (0FF3H), A
CALL DEL1
CALL DEL1
LD A, (OFFF2H)  # READ MOST SIGNIFICANT NIBBLE.
AND 0FH
LD H, A
LD A, (OFFFH)  # LOAD LEAST SIGNIFICANT BYTE
LD L, A
CALL SCREEN
LD DE, 07FFH  # SUBTRACT BIAS FOR VELOCITY. ADD
LD A, (3E7DH)  # PREVIOUS BIAS. DIVIDE BY 2
ADD 0
SBC HL, DE
ADD L
SRL A
LD (3E7DH), A  # STORE BIAS OF VELOCITY.
LD A, (3E0BH)  # INITIALISE A/D TO READ POSITION
LD (0FF3H), A  # OF PENDULUM.
CALL DEL1
CALL DEL1
CALL DEL1
CALL DEL1
JP LOOP
SUBROUTINE DEL1

0220
0230 * THIS SUBROUTINE CHECKS IF ANY INPUT FROM THE KEY BOARD
0240 * IS MADE TO EITHER INCREASE OR DECREASE THE BIAS VOLTAGE
0250 * FOR THE READING OF THE POSITION OF THE PENDULUM OR TO
0260 * RETURN BACK TO THE MAIN PROGRAM.
0270 DEL1  LD B, 3FH  # LOAD VALUE IN COUNTER.
0280 JM  IN 0, A
0290 CP 'D'  # CHECK IF KEY 'D' IS DEPRESSED.
0300 CALL Z, DEC
0310 IN 0, A
0320 CP 'I'  # CHECK IF KEY 'I' IS DEPRESSED
0330 CALL Z, INC
0340 IN 0, A
0350 CP 'E'  # CHECK IF KEY 'E' IS DEPRESSED.
0360 JR Z, END
0370 DJNZ JM  # DECREMENT COUNTER. IN NZ JP DEL1
0380 RET
0390
0400
0410 *

SUBROUTINE DEC

0420
0430 * DECREMENTS THE OUTPUT VOLTAGE FROM THE D/A WHICH IS USED
0440 * TO BIAS THE POSITION OF THE PENDULUM.
0450 DEC  LD HL, (3E07H)
0460 INC HL
0470 LD (3E07H), HL
0480 CALL DELAY
0490 RET
0500
0510
0520 *

SUBROUTINE INC

0530
0540 * INCREASES THE VOLTAGE OUTPUT FROM THE D/A TO BIAS THE
0550 * READING OF THE PENDULUM POSITION.
0560 INC  LD HL, (3E07H)
0570 DEC HL
0580 LD (3E07H), HL
0590 CALL DELAY
0600 RET
0610
0620
0630 *

SUBROUTINE END

0640
0650 * SAVE THE LAST TWO BITS OF THE BIAS OUTPUT FOR THE
0660 * BIAS OUTPUT VOLTAGE TO THE PENDULUM READING AND RETURNS
0670 * BACK TO THE MAIN PROGRAM.
0680 END  LD A, (3E07H)
0690 AND 03H
0700 JP 3021H
0710 DELAY  LD C, 4FH
0720 LAB  DEC C
0730 JR NZ, LAB
0740 RET
SUBROUTINE SCREEN

0770 * DISPLAYS ON SCREEN IN HEX THE VALUE WHICH IS PLACED
0780 * IN THE LAST 12 BITS OF THE HL REGISTER.
0790 SCREEN LD A, H
0795 AND $7F
079A OR $B0H
079F CP $BAH
0804 CALL P, HEX
0809 CALL 000372
0814 LD A, L
0819 SRL A
081E SRL A
0823 SRL A
0828 SRL A
082D OR $B0H
0832 CP $BAH
0837 CALL P, HEX
083C CALL 000372
0841 LD A, L
0846 AND $0F
084B OR $B0H
0850 CP $BAH
0855 CALL P, HEX
0860 CALL 000372
086F CALL 000370
0878 RET
087D HEX ADD $07H
0882 RET
1070 **********************************************
1075

SUBROUTINE PENDIS.

1120

1130 *********************************************************************************************
1140 * THIS SUBROUTINE DISPLAYS THE POSITION OF THE PENDULUM.
1150 * ON THE SCREEN. HENCE THE PENDULUM POSITION CAN BE
1160 * INITIALISED AT TIME T=0. IF KEY "R" IS DEPRESSED
1170 * THEN THE EXECUTION OF THE MAIN PROGRAM IS BEGUN.
1180
1190 *********************************************************************************************
1200
1210 SCREEN EQU 3814H
1220 ST 3850H
1230 LD A, ($3E0BH)  # SELECT CHANNEL R% AMPLIFICATION
1240 LD ($0FF3H), A  # TO READ POSITION OF PENDULUM.
1250 LD A, $05H
1260 DEL1 LD B, $0FFH  # DELAY.
1270 DEL DEP MOP
1280 DJNZ DEL
1290 DEC A
1300 JR NZ, DEL1
1310 AGAIN LD A, 000H  # ASCII CODE FOR FOR 'P'.
1320 CALL 000372  # ASCII CODE FOR '='.
1330 LD A, 0BDH
1340 CALL 000372
1350 LD A, (3E89H)
1360 LD (0FF3H), A
1370 POST LD A, (0FF1H)  # TEST THE MSB TO CHECK IF THE
1380 AND $80H  # A/D CONVERSION IS OVER.
1390 JR Z, POST
1400 POST1 LD A, (0FF1H)
1410 AND $08H
1420 JR NZ, POST1  # LOADS MOST SIGNIFICANT NIBBLE.
1430 LD A, (0FF2H)
1440 AND $08H
1450 JR Z, SKIP
1460 LD A, 0  # SAVE THE SIGN IN MEMORY 3E11H.
1470 LD (3E11H), A
1480 LD A, 0ABH  # ASCII CODE FOR '+'.
1490 CALL 000372
1500 LD A, (0FF2H)
1510 AND $07H
1520 LD H, A
1530 LD A, (0FF2H)  # LOAD LEAST SIGNIFICANT BYTE.
1540 LD L, A
1550 LD DE, (3E78H)
1560 SBC HL, DE
1570 LD A, H
1580 AND $80H
1590 JR Z, SKIP2
1600 LD HL, 0
1610 JR SKIP2
1620 SKIP LD A, 01H  # SAVE THE SIGN.
1630 LD (3E11H), A
1640 LD A, 0ADH  # ASCII CODE FOR '−'.
1650 CALL 000372
1660 LD A, (0FF2H)  # LOADS MOST SIGNIFICANT NIBBLE.
1670 CPL
1680 AND $07H
1690 LD H, A
1700 LD A, (0FF0H)  # LOADS LEAST SIGNIFICANT BYTE.
1710 CPL
1720 LD L, A
1730 LD DE, (3E78H)
1740 ADD HL, DE
1750 SKIP2 LD (3E22H), HL
1760 CALL SCREEN
1770 IN D, A  # LOADS THE INPUT FROM KEY-BOARD
1780 CP 'R'  # CHECK TO SEE WHETHER 'R' WAS
1790 JR Z, END  # DEPRESSED. IF TRUE THEN JUMPS
1800 JR AGAIN  # BACK TO THE MAIN PROGRAM.
1810 END RET
1820 *
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DESIGN AND IMPLEMENTATION OF A MICROCOMPUTER,
CONTROLLER FOR STABILIZING AN INVERTED PENDULUM
MOUNTED ON A CART.

by

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AN ABSTRACT OF A THESIS

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ABSTRACT

In recent years, especially since the advent of the micro-
computer, great advances have been made in the field of discrete
control systems. A real-time, discrete controller was developed
and implemented to stabilize a naturally unstable system. An
inverted pendulum mounted on a motor driven cart was used.

A mathematical model of the system was developed and the
parameters of the model were obtained by frequency response ana-
lysis. The discrete model was obtained by applying the Z-trans-
forms. The naturally unstable system was stabilized by using
compensators developed by the Z-plane analysis technique. Simu-
lation results were obtained for various values of the system
gain.

A Z-80 microcomputer, was used as the real-time controller
for the experimental investigation. The results of the experi-
mental investigation compare favorably with the result predicted
by the analysis and the simulation. Recommendations for further
study are provided.