TRANSIENT SOLUTIONS OF M/M/s
NONSTEADY STATE QUEUEING SYSTEM

by

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CHAPTER 1
INTRODUCTION

Queues, or waiting lines, are very common phenomena, widely encountered in everyday life. People wait in lines at supermarket checkout stands, in their cars in busy traffic hours, in banks, at gas stations, at ticket booths and almost everywhere. Wherever the demand for service exceeds the capacity of servers, a queue is formed.

Since waiting is not a pleasant activity, neither customers nor management like to wait. Because long waiting time may cause losses, queueing problems have received the attention of management and operation analysts for a number of years. In fact, queueing models were among the first models of operation research [36]. The problems have been actively researched and written about since 1907 [13].

A basic queueing process can be described as customers leaving the calling population, arriving for service, entering the queueing system, waiting in a queue if no server is available, and leaving the system after being served. This process is depicted in Figure 1.1.

A queueing system can be specified completely in most cases by six basic characteristics [10] which are: arrival pattern (distribution) of customers, service pattern (distribution) of servers, number of service channels, service discipline, system capacity, and size of population. With different basic characteristics of queueing processes, selected queueing models can be developed. Most of these models, which assume that the mean arrival rate and the mean service rate are constant through time and that the system is in a steady state, have been well studied in the past.

Unfortunately this is often not the case in real queueing systems. For instance, service rate has a close relationship with the length of queue (it is quite likely that a server will tend to work faster if the queue is long
Figure 1.1 Basic queueing processes.
and vice-versa (such as supermarket checkout clerk); customer arrivals change with time of day in supermarkets; and car arrival rates at toll booths peak at the busy periods around 8 A.M. and 5 P.M.

Usually any system is said to be in a transient state when the system's behavior is not typical or not normal. For example, after a car has started, it has to be accelerated to reach the normal speed. During the accelerating period, the system is in a steady state. Transient states occur in queueing systems, especially when the system is still dependent on the initial conditions, or when a steady-state condition responds to sudden changes. When the arrival rate is larger than the capacity of service, the steady state condition never exists no matter how long the system is run. Another important situation is when the arrival rates and service rates are a function of time; in this case, only the transient state has meaning.

As mentioned before, most of the attention in queueing theory analysis has been directed to steady state systems. Although the steady state queueing models have proven very useful in many areas, it is inappropriate to apply these models to situations in which time dependent behavior is very strong. Wagner [41] has listed grocery store checkout stands, bank teller's windows, service station pumps and attendants, telephone trunk lines, and photocopy equipment maintenance men, as cases where the customer's arrival rates have very significant time-dependent behavior. Koopman [16] and Hartman [11] have discussed the time dependent air terminal queueing problem. Figure 1.2 shows typical hourly aircraft arrival rates for J. F. Kennedy and Laguardia airports, as observed by Koopman in 1968 during a month's operations. The average hourly arrival rates are plotted against time of day. Both curves peak around 10 A.M. and 6 P.M. The first peak shows that many incoming aircrafts are scheduled to leave around 10 A.M. for the convenience of the passengers. The second peak shows the arrivals of aircrafts which have been scheduled to arrive in the
Terminal A: J. F. Kennedy data

Terminal B: LaGuardia data

Fig. 1.2 Hourly arrival rates for air terminals A and B.
neighborhood of 6 P.M.

It is very common to use the simulation approach (to interpret queueing problems in transient state) exemplified by the well known paper of Galliher and Wheeler [6]. Although the simulation technique possesses versatility and flexibility, it is experimental in nature. This will give rise to such problems as experimental design, run length, number of replications, gathering observations and statistical significance. On the other hand, for most queueing problems the information for making a decision requires only a few simple measurements of effectiveness, such as average waiting time, average queue length, the probability of encountering a queue, etc. Another drawback to simulation is the economical consideration. The effort required for constructing a simulation model and the required computer time can be substantial (roughly stated, the machine time for simulation increases directly proportional to the square of the reliable precision [16]).

In recent years, a good deal of effort has been devoted to the development and application of numerical methods to the transient state queueing problems.

In this study, one of the main objectives is to survey the literature of numerical or approximate approaches. Chapter 2 summarizes the result of a limited study on nonsteady state queues.

Another objective is to demonstrate how to apply one of the simpler numerical techniques in solving real queueing problems. A police patrol car scheduling problem is used in this study. It is first defined in Chapter 3. Chapter 4 presents the solution, with the probability of an urgent call encountering a queue used as a criterion. Chapter 5 presents another solution in which the expected waiting time of an urgent call in the queue is used as a criterion.

Finally, conclusions and recommendations are given in Chapter 6.
CHAPTER 2
NONSTEADY STATE QUEUES

In reviewing the literature, it is convenient to classify the information into the following categories:

1. Explicit expression of transient solutions for some simple stationary queueing systems.
2. Numerical approaches to solve the differential equations of queueing systems directly.
3. Diffusion approximation techniques to approximate the transient solutions of heavy-traffic queueing systems.
4. Discrete time Markov process to approximate the smaller scale service systems.
5. Approximate transform inversion
6. Imbedded Markov chain approach
7. Closure techniques

This is not a exhaustive survey; only the references having direct bearing on the current work are included.

Explicit Expression

The procedure for developing the explicit transient solutions for simple stationary queueing systems is very complicated and highly mathematical. Many authors have investigated this subject, including the first known work published by Lederman and Reuter [19]. Additional papers have been presented by Bailey [1] and Morse [26] and successive contributions given by Chamarro and Smith [5]. The explicit formula for the probability $P_n(t)$ that the system is in state $n$ at time $t$ (i.e. the transient solution) of stationary M/M/1/$\infty$ queueing systems is given as:

$$P_n(t) = e^{-(\lambda + \mu)t} \left[ (\mu/\lambda)^{(1-n)/2} \right]$$

$$+ (\mu/\lambda)^{(i-n+1)/2} \left[ 2^{\sqrt{\mu/\lambda} - t} \right]$$

$$+ (1 - \mu/\lambda)(\lambda/\mu)^i \sum_{k=n+1}^{\infty} (\mu/\lambda)^{k/2} \left[ (2\sqrt{\mu/\lambda} - t) \right]$$
\[ I_n(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{n+2k}}{k! (n+k)!}, \quad n > -1 \]

where the constant \( i \) is the number of units in the system when time is equal to zero. The details of the derivation can be found in the original discussion by Bailey [1] and a clear explanation is given by Gross and Harris [10]. As one can see, even for this simple stationary queueing systems, the formula for the transient solution is very complicated and the result is not very suitable for numerical work. This discourages further exploration of explicit expressions for more complex queueing systems. With the help of modern digital computers, a number of numerical approaches and approximate methods for determining more complicated transient queues have been contributed.

**Numerical Approach**

The M/M/1 nonsteady state queue has been studied by Clarke [4]. He used a generating function to reduce the difference-differential equation of probabilities to a hyperbolic partial differential equation, which then was solved by applying standard methods and integration formula for Bessel functions in terms of a Volterra type integral equation. Luchak [22] used Taylor's expansion technique to approximate the solutions of some queueing systems, except that he kept mean service time constant and made arrival rate dependent on time. Leese and Boyd [20] pointed out the difficulties of applying Clark's and Luchak's methods and compared eight methods in terms of the capability of producing satisfactory numerical results with a reasonable amount of computational effort. Among those considered by Leese & Boyd, Wragg's [41] method seemed to be the most powerful. The first method mentioned in Leese and Boyd's paper was the standard numerical method [34], such as the Runge Kutta method, Predictor-Corrector method, etc. to solve the Kolmogorov infinite set of difference-differential equations of state probabilities. Because of its simplicity and ease of use, this direct numerical method has been
applied to integrate a truncated set of probability equations by Koopman [16] and Hartman [11] in the time-varying air-terminal queueing problem, by Chare et al [9] in job shop problem, and by Kolesar et al [14] in police patrol car scheduling problem. The assumption of truncated systems seems to be reasonable in most applications when there are physical constraints on maximum possible number of arrivals or the probabilities of exceeding a certain amount in the system are very small.

**Diffusion Approximation**

In the busy period, the arrival rate \( \lambda(t) \) gradually increases as a "rush hour" is approaching. The queue builds up during the period when arrival rate exceeds service capacity. After a busy period, the queue finally decays. The studies of this type of situations have been given by Oliver and Samuel [33], Gazis and Potts [8], and May and Keller [23]. The system was analyzed by means of a deterministic model which assumed arrival rate and service time to be constant, instead of describing them in probability distributions.

Newell [30] [31] [32] has presented a method called *diffusion approximation* to improve the deterministic approximation which always tends to underestimate the expected queue length. Newell's method has been applied to solve the heavy traffic queueing problems or the mildly heavy traffic queueing problems.

**Discrete Time Markov Process**

An approach to the small-scale service system whose short range behavior is important has been described by Neuts [27]. The transient behavior of a substantial class of single server queues may be numerically approximated by such discrete time Markov process as traffic lights, highway merging ramps and many others. Neuts [28-29] has extended the applicability of this process to longer queues and longer time periods. The extension is sophisticated and mathematically involved.
Approximate Transform Inversion

The complicated expression of \( P_n(t) \) for \( M/M/1/\infty \) queueing systems can be obtained by inverting the complex roots of a certain polynomial equation \([4]\). Gaver \([7]\) has presented a method which used an approximation technique instead of sophisticated transform inversion. Recently Kotiah \([17]\) has suggested three iterative procedures (method of successive approximations, the series method, and Newton's Method) that give successive rational approximations for complex roots. The rational approximations of complex roots are relatively easy to invert.

Imbedded Markov Chain Approach

With the assumption that arrival and service process parameters are step functions of time, the time horizon is divided into several intervals, during each of which the arrival process is in steady state. Morse \([25]\) has applied the imbedded Markov chain technique to approximate the expected queue length and length variance of nonsteady state single-server queues.

Closure Technique

A different approach for approximating a measurement of effectiveness has been known as closure technique. The infinite set of difference-differential equations of state probabilities is inverted to a set of limited equations for certain measurements of effectiveness. With a closure assumption, the measurement can be approximated. Rider \([35]\) has used this kind of technique to approximate the time dependent mean length of a nonsteady state \( M/M/1 \) queue in terms of the idle probability. Recently Rothkopf and Oren \([36]\) used Clarke's \([4]\) differential equations for the expected length and length variance of a nonsteady state \( M/M/1 \) queue. With a closure assumption which employed the negative binomial distribution to approximate the state probabilities, the mean and variance of queue length were approximated. Adding
a correction term, they extended the applicability to multi-server queues. At
the same period of Rothkopf and Oren's work, Chang [3] and his student Wang [40]
used different closure assumption to obtain the approximation of expected queue
length and length variance of the same systems.

Considering the aspects of simplicity, accuracy and range of applicability,
it seems that the standard numerical method is most desirable. Thus it is
selected for further study in this research.
CHAPTER 3
PATROL CAR SCHEDULING PROBLEM

An important function of police departments is to provide citizens with rapid and competent emergency service when needed. Indeed, lives and properties are often dependent on the effective performance of this police function.

The patrolling schedule problem is selected as a focus of the application of the numerical technique because the patrol force constitutes the largest portion of police personnel, usually consuming 40 to 50 percent of the total police budget (Larson [18], 1972).

3.1 Brief Description of Patrol Operation

One primary duty of a patrol unit is to prevent crime by removing crime hazards and to deter crime by posing the threat of apprehension, when the patrol unit passes through an assigned geographical response area.

Another primary duty of an unit is to respond to calls for police service. Citizens usually call the police emergency number. After the necessary information is taken down, a police dispatcher assigns or dispatches a patrol unit to the scene. Since rapid response to a call for help is considered to have higher priority than general patrolling, an assignment from the dispatching center interrupts the general preventive patrol.

A dispatcher has a fixed number of patrol units available for dispatching. This number of patrol units constitutes a precinct work force. Most precincts have a centrally located station. Each patrol unit can be assigned to any point within its precinct. If all units in a precinct are simultaneously dispatched, when a call arrives the dispatcher puts the request in queue for later dispatch. There are very few inter-precinct dispatches, and in this study each precinct is assumed to operate independently from the others.
3.2 The Concerns of Administrator

Since it is characteristic of emergency systems that a person's life or well-being may well depend on the immediate dispatch of a unit to a call, a primary objective of any patrol car system is to minimize the possibilities that a call will be placed in the queue, or to minimize the average waiting time once the call is placed in the queue. The administrator faces the problems of how to determine the number of units to be put in each shift and subsequently how to schedule the manpower within each shift, so that the probability of a call encountering a queue and/or the expected waiting time in the queue are insured to be below some specified threshold.

By almost any measurement of effectiveness, patrol performance is improved by increasing the number of patrol cars on duty. However, an administrator would like to optimize the patrol schedule so that he can obtain the most efficient solution within certain constraints.

3.3 Assumptions of the Model

The probabilistic nature of calls for service and service time is such that queuing is the most adaptable model to the patrol dispatching system. This system can best be described as a complicated multiple-server queuing system.

Calls for service, either telephone calls to the radio dispatcher or accidents, crimes, other incidents encountered by the patrol units, are assumed to occur randomly over time as a Poisson process. In other words, the probability that a given number of calls will occur in a given period of time is specified by a Poisson distribution with a mean characteristic of that specific period of time.

It is assumed that each call for service is handled by one single patrol car. In real situations two or more cars sometimes are necessary. A comparison made by Ignall et al [12] shows that the influence of this assumption can be neglected without seriously altering the results.

Service time is defined to be the total "off-air" time associated with a
call for service. This includes travel time to the scene, on-scene service
time, and off-scene follow-up time spent on report writing, arrest or booking.
The probability of a given amount of service time for a call is assumed to
obey an exponential distribution with a constant parameter; mean service time.

A call is placed in queue only when all units are busy servicing prior
calls, all calls have the same priority on a first-come first-served basis.
During the service period the patrol car is unable for further assignments.

The methodology presented by Kolesar et al [14] is applied to demonstrate
how to use the standard numerical method in solving the patrol car scheduling
problems defined above.

Chapter 4 describes this problem based on the criterion of a specified
probability of an urgent call encountering a queue and Chapter 5 provides the
solution using as criterion a specified waiting time of an urgent call in the
queue.
CHAPTER 4
THE PROBABILITY OF A CALL ENCOUNTERING A QUEUE AS A CRITERION

Kolesar et al's method [14] is presented in three steps:
Step 1: Calculate the hourly patrol car requirements to meet the specified criterion.
Step 2: Generate schedules for each shift which would satisfy the hourly patrol car requirements calculated in Step 1.
Step 3: Use a more realistic nonsteady state queueing model to evaluate the performance of schedules generated in Step 2 and compare with other alternatives.

4.1 Step 1: Estimate the Hourly Patrol Car Requirements

The police departmental policy governs the criterion to be specified in the evaluation of patrol car requirements. For instance, the police planner may state his planning objective in the following way, "I wish to assign sufficient units to this time period that at least 90 percent of urgent calls for service are assigned to patrol units without any dispatching delay."
The specified threshold is denoted as $\alpha$ (in the above case, $\alpha$ would be 90%).

The relationship between the number of cars assigned in any time period and the threshold $\alpha$ is complicated. The major difficulty stems from the fact that the patrol system is a multi-server nonsteady state queueing system. In this system the call rate, the number of cars on duty and other factors are time dependent. Figure 4.1 shows the typical diurnal variation pattern of the urgent call rate received at a New York City precinct [14]. However, the St. Louis Police Department has overcome the difficulty by using the hourly steady-state assumption of M/M/s queueing model, to obtain an excellent allocation of police patrol cars [24]. In this current study, the same assumptions are made, i.e., the mean of call rate is assumed to be constant for the $t^{th}$ hour and the system is also assumed in a steady state during the same hour.
Fig. 4.1 Mean call rates as a function of time.
The state probability, \( P_{jt} \), where \( j \) calls are in the system at hour \( t \), can be expressed as:

\[
P_{jt} = \frac{\rho_t^j}{j!} P_{ot} \quad \text{when } 1 \leq j \leq N_t \tag{1}
\]

\[
P_{jt} = \frac{\rho_t^j}{[N_t]!} \frac{1}{N_t} \quad \text{when } j > N_t, \tag{2}
\]

where \( P_{ot} \) is the state probability of no call at any time \( t \) and can be expressed by:

\[
P_{ot} = \left( \prod_{j=0}^{N_t-1} \frac{\rho_t^j}{j!} \right) + \frac{\rho_t^{N_t}}{[N_t]!} (1 - \rho_t/N_t)
\]

\[
\tag{3}
\]

and

\[
\rho_t = \frac{\lambda_t}{\mu}
\]

where

\[
\lambda_t = \text{mean call rate at hour } t
\]

\[
\mu = \text{mean service rate}
\]

\[
N_t = \text{number of cars on duty at hour } t
\]

\[
t = \text{hour of day (t=0,1,2...23)}
\]

With a given threshold \( \alpha \), the car requirement at hour \( t \), denoted by \( \gamma_t \), can be obtained by searching for the smallest value of \( N_t \) such that

\[
\sum_{j=0}^{N_t-1} P_{jt} > \alpha . \tag{4}
\]

The physical meaning of the equation (4) is that the probability of at least one car available at any hour is greater than the specified threshold \( \alpha \).

The procedure appears awkward when using hand calculation. It is solved easily by computer. A general solution program has been constructed to generate numerical values for \( \gamma_t \). The flow diagram of this program is given in Figure 4.2 and the program list is contained in Appendix A.1.
Fig. 4.2. Flow diagram for generating hourly car requirements. Using the probability of a call encountering a queue as a criterion.
The following example illustrates the procedure described above.

**Example 4.1**

Calculate the number of patrol car requirements based on the following conditions:

1. The threshold $\alpha$ is specified to be at least 90%.
2. The average service time is 30 minutes per service and is assumed to be independent of time.
3. The mean hourly arrival rate is given in Table 4.1.

First one would calculate the constant mean service rate $\mu$:

$$\mu = \frac{60}{30} = 2 \text{ services per hour}.$$ 

Then one can start the calculation from midnight, $t=0$.

From Table 4.1, one finds the mean call arrival rate is 9.8 per hour at $t=0$. ($\lambda_o = 9.8 \text{ calls per hour}$) and the traffic density, $\rho_o$, would be $\lambda_o/\mu = 9.8/2 = 4.9$.

Starting with $N_0=1$, one would then calculate the probability of no call in 0 hour by using equation (3),

$$P_{oo} = 0.24366.$$ 

This means that the probability of the only patrol car getting an assignment is: $1 - 0.24366 = 0.75634$ or approximately 75.6%. After the patrol car is dispatched, the rest of the emergency calls have to be placed in queue.

Thus

$$\sum_{j=0}^{N_0-1} P_{jt} = P_{oo} = 0.24366 < 0.90$$

$N_0=1$ is rejected. The search is continued until the criterion is satisfied. At this point, the last tried $N_0$ value would be the required number of cars, $n_t$.

Following the same procedure, $\gamma_t$ for every hour of day can be calculated. The computer program listed in Appendix A.1 can be utilized for this calculation.
TABLE 4.1

Recorded mean arrival rate at each hour $t$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
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<tr>
<td>$\lambda_t$</td>
<td>9.8</td>
<td>9.6</td>
<td>8.7</td>
<td>7.6</td>
<td>6.7</td>
<td>5.5</td>
<td>4.1</td>
<td>3.2</td>
<td>2.5</td>
<td>2.5</td>
<td>2.9</td>
<td>3.8</td>
</tr>
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$t$: Hour of a day ($0,1,\ldots,23$).

$\lambda_t$: Mean arrival rate at hour $t$, a constant during each hour.
4.2. Step 2: Generate Schedule for Each Shift

The set of feasible shift start times $S$, all of which are assumed to start on the hour, is determined by the police department. Each shift lasts eight hours. It is also assumed that every shift includes a one-hour mealbreak which also begins on the hour. There may be constrained on the earliest, $e(i)$, and latest, $l(i)$, hours of a shift that can be used for mealbreak; $i+e(i)$ and $i+l(i)$ denote the earliest and latest possible mealbreak respectively for a shift starting at hour $i$, $i \in S$.

Let $N_i$ denote the number of cars assigned to work on a shift starting at hour $i$. $M_{i,t}$ denotes the number of cars working on shift $i$ and assigned to mealbreak at hour $t$ (where $i+e(i) \leq t \leq i+l(i)$)

The objective is to find the most efficient shift assignments $(N_i, i \in S)$ and $(M_{i,t}, i \in S$ and $i+e(i) \leq t \leq i+l(i)$) which meet the car requirements $\gamma_t$ at each hour of a day. This can be accomplished by utilizing an integer linear program:

\[
\text{Minimize} \quad \sum_{i \in S} N_i
\]

subject to:

\[
\sum_{t-i+e(i)}^{i+l(i)} M_{i,t} = N_i, \quad i \in S
\]

\[
\sum_{i \in S, t-7 \leq i \leq t} (N_i - M_{i,t}) \geq \gamma_t, \quad t=0,1,2\ldots23
\]

where $N_i, M_{i,t} \geq 0$, and must be integers.
TABLE 4.2

The results of car requirements of Example 4.1

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$t$: Hour of day (0,1...,23)

$\gamma_t$: Number of cars required in each hour $t$. 
The value of the objective function is the total number of cars used over the day. The first set of constraints insures that every car has a mealbreak, and the second set of constraints insures that cars on duty meet the hourly requirements. This model can easily handle shifts starting on the half hour, quarter hour, etc., as well as shifts and mealbreaks of any length.

**Example 4.2**

Here the hourly car requirements generated in Example 4.1 are to be satisfied in scheduling. There are three possible shifts starting at 0, 8, and 16 hours, and mealbreaks can only be taken between the second and the fifth hour of each shift. This problem can be formulated as follows:

\[
\text{Minimize: } N_0 + N_8 + N_{16}
\]

Subject to

\[
\sum_{i=2}^{5} M_{0,t} - N_0, \text{ at } i=0
\]

\[
\sum_{i=8}^{13} M_{8,t} = N_8, \text{ at } i=8
\]

\[
\sum_{i=16}^{21} M_{16,t} = N_{16}, \text{ at } i=16
\]

\[
N_0 \geq 9
\]

\[
N_0 - M_{0,2} \geq 8
\]

\[
N_0 - M_{0,5} \geq 8
\]

\[
N_0 - M_{0,4} \geq 7
\]

\[
N_0 - M_{0,5} \geq 6
\]
\[ N_8 \geq 8 \]
\[ N_8 - M_{8,10} \geq 4 \]
\[ N_8 - M_{8,11} \geq 5 \]
\[ N_8 - M_{8,12} \geq 5 \]
\[ N_8 - N_{8,13} \geq 6 \]

\[ N_{16} \geq 9 \]
\[ N_{16} - M_{16,18} \geq 9 \]
\[ N_{16} - M_{16,19} \geq 9 \]
\[ N_{16} - M_{16,20} \geq 9 \]
\[ N_{16} - M_{16,21} \geq 9 \]

\[ N_i, M_{i,t} \geq 0 \text{ and integer}. \]

There are 15 variables and 18 constraints in this integer linear problem. It is solved by Salklin and Spielberg's subroutine [37] and the results are arranged as follows:

**Shift starting at hour 0, cars assigned \( N_0 = 10 \)**

Mealbreak assigned:
- \( M_{0,2} = 1 \) car at hour 2
- \( M_{0,3} = 2 \) cars at hour 3
- \( M_{0,4} = 3 \) cars at hour 4
- \( M_{0,5} = 4 \) cars at hour 5

**Shift starting at hour 8, cars assigned \( N_8 = 7 \)**

Mealbreak assigned:
- \( M_{8,10} = 2 \) cars at hour 10
- \( M_{8,11} = 2 \) cars at hour 11
- \( M_{8,12} = 2 \) cars at hour 12
- \( M_{8,13} = 1 \) car at hour 13
Shift starting at hour 16, cars assigned $N_{16} = 12$

mealbreak assigned: $M_{16,18} = 3$ cars at hour 18
$M_{16,19} = 3$ cars at hour 19
$M_{16,20} = 3$ cars at hour 20
$M_{16,21} = 3$ cars at hour 21

4.3 Step 3: Use a More Realistic Nonsteady State Queueing Model to Evaluate the Performance of Schedules

The administrator may be faced with different proposed schedulings such as the current experienced schedules in use, schedules generated under different policies, and schedules calculated by other methods. Before making a choice among the alternatives, he may want to have a more realistic comparison of the results which can be expected from each alternative. Two models can be used for this purpose. One is the simulation model, such as the one described by Kolesar and Walker [15]. As discussed previously in Chapter 1, there are a considerable number of pitfalls one may encounter in using simulation, and its analysis is relatively expensive and cumbersome. The second model involves a $M/M/s$ nonsteady state queue with mean arrival rate as a function of time. The latter one is used for its simplicity. The notations for this model are:

$P_j(t)$: the probability of $j$ calls for service in the system at any time $t$.

$\lambda(t)$: the mean call rate as a function of time $t$.

$\mu$: the mean service rate as a constant independent of time.

$n(t)$: the number of patrol cars on duty at each hour $t (t=0,1,\ldots,23)$; it is a discrete function of hour. During hour $t$, $n(t)$ is a constant value which is equal to $N_i$ (cars assigned in shift $i$) - $M_{i,t}$ (cars in shift $i$ and assigned to mealbreak at hour $t$).

The transition probabilities for this model are expressed by the well-known infinite set of difference-differential equations;
\[ p'_0(t) = -\lambda(t) \ p_0(t) + \mu p_1(t) \] (8)
\[ p'_j(t) = \lambda(t) p_{j-1}(t) - [\lambda(t) + j\mu] p_j(t) \]
\[ + (j+1)\mu p_{j+1}(t), \quad 1 \leq j < n(t) \] (9)
\[ p'_j(t) = \lambda(t) p_{j-1}(t) - [\lambda(t) + n(t)\mu] p_j(t) \]
\[ + n(t)\mu p_{j+1}(t), \quad j \geq n(t) \] (10)

In order to integrate this set of equations numerically, it is assumed that \( m \) is the maximum number of calls allowed in the system or \( m \) is chosen such that the probabilities of \( m \) calls or more in system are close to zero, as discussed in Chapter 2. The set of equations can be replaced by:

\[ p'_0(t) = -\lambda(t) \ p_0(t) + \mu p_1(t) \] (11)
\[ p'_j(t) = \lambda(t) \ p_{j-1}(t) - [\lambda(t) + (j-1)\mu] p_j(t) \]
\[ + j\mu p_{j+1}(t), \quad 1 \leq j < n(t) \] (12)
\[ p'_j(t) = \lambda(t) \ p_{j-1}(t) - [\lambda(t) + n(t)\mu] p_j(t) \]
\[ + n(t)\mu p_{j+1}(t), \quad n(t) \leq j < m \] (13)
\[ p'_m(t) = \lambda(t) \ p_{m-1}(t) - n(t)\mu p_m(t) \] (14)

There are a number of standard numerical methods available for solving this type of simultaneous ordinary differential equations. Because \( n(t) \) is a discontinuous function of hour, the multi-step predictor-corrector method which uses several steps ahead is not suitable in this case. The fourth order Runge Kutta method which concentrates on single step and does not pass over the discontinuous points is thus selected. The flow diagram of the numerical method developed is shown in Figure 4.3 and the computer program is listed in Appendix B.1.
Fig. 4.3 Flow diagram of Runge Kutta method, using the probability of a call encountering a queue as a criterion.
Example 4.3

The same problem which was described in the previous two examples will be further discussed. The results obtained in Example 4.2 are used to calculate the number of cars on duty at each hour $t$, $n(t) = N_i - M_i, t$. The calculated $n(t)$ values are summarized in Table 4.3. The data is also shown graphically in Figure 4.4.

In order to solve this transient state problem, some sets of new information are required beyond those given previously. They are the time varying arrival rate, $\lambda(t)$, the maximum number of calls allowed in the system, $m$, and the initial values.

The arrival rate function is usually developed by recording the average arrival rate at each hour of a day as listed in Table 4.1. The hourly average arrival rates are connected by straight lines as shown in Figure 4.1. The approximated time-varying arrival rate at any time $t$, $\lambda(t)$, can be obtained by interpolation.

After several trial runs with assumed values for the maximum allowed calls, $m$, one could select a value which would assure program convergency with a reasonable amount of computing effort. In this case, $m$ is selected to be 30 calls.

The steady state probabilities of an $M/M/s$ queueing system at hour 0 are used as the initial values.

The problem now can be integrated by the indicated numerical program (See Appendix B.1).

The integration should be run for a longer period of time than 24 hours in order to eliminate the influence of the approximate initial value. Kolesar et al have shown that a two days running period should be sufficient.
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t: Hour of a day \((0,1,\ldots,23)\).

\( n(t) \): Number of patrol cars on duty in each hour \( t \).
Fig. 4.4 The number of cars on duty during each hour t.
Because of drastic changes in the number of cars on duty and calls for service, the transient responses are very strong at hours 17 and 22. Even step size 0.05 time units does not provide convergence. The sum of state probabilities of m-truncated capacity is used to check convergence. The sum at each time t integrated by step size 0.05 is shown in Figure 4.3. After experimenting with several values, a step size of 0.025 hours is found to provide convergence. The results of integrating is also shown in Figure 4.5 for comparison. There is nowhere that the total probability deviates from unity more than 0.0008. Using step size 0.025 hours, exucion time for this problem involving a 48 hour period runs about 2 minutes and 25 seconds. Part of the output is given in Appendix B.2. The probability of no car on duty at each time t is shown in Figure 4.6.

In order to demonstrate the comparison of different alternatives, an experienced schedule used by the New York Police Department [14] is evaluated. The schedule is described below:

Shift starting at hour 0, cars assigned $N_0 = 6$

Mealbreak assigned: $M_{0,3} = 1$ car at hour 3  
$M_{0,4} = 2$ cars at hour 4  
$M_{0,5} = 3$ cars at hour 5

Shift starting at hour 8, cars assigned $N_8 = 8$

Mealbreak assigned: $M_{8,10} = 3$ cars at hour 10  
$M_{8,11} = 2$ cars at hour 11  
$M_{8,12} = 2$ cars at hour 12  
$M_{8,13} = 1$ cars at hour 13
Fig. 4.5 The summation of state probabilities of $M$-truncated capacity at every time $t$. 

Integrated by 0.025 step size

Integrated by 0.35 step size
Fig. 4.6 The probability of no car on duty in the schedule with three shifts.
Shift starting at hour 16, cars assigned \( N_{16} = 10 \)

Mealbreak assigned: 

\[ M_{16, 18} = 3 \] cars at hour 18
\[ M_{16, 19} = 3 \] cars at hour 19
\[ M_{16, 20} = 2 \] cars at hour 20
\[ M_{16, 21} = 2 \] cars at hour 21

The output is partially given in Appendix B.3. The probability of no car on duty at each time \( t \) is shown in Figure 4.7.

Comparing the results of these two schedules, it shows that although the experienced schedule uses fewer cars per day, its performance is far inferior. During hour 5 and 6, the probability of a call for service encountering a queue is more than 60 percent.

The same procedure can be used to compare the schedules calculated under different departmental policies or the schedules generated by other methodologies. Considering the resource limitations, administrative concerns, and results of performance, this evaluation technique with a nonsteady state queueing model can help administrators make a reasonable choice among alternatives.
Fig. 4.7. The probability of no car no duty in the experienced schedule.
THE EXPECTED WAITING TIME OF A CALL IN THE QUEUE AS A CRITERION

In Chapter 4, the measurement of patrol service performance was emphasized on the basis of the probability of an urgent call being placed in the queue. This measurement, based on a probability concept, cannot give a clear indication of the effect on an urgent call. However, the measurement of the expected waiting time of urgent calls in the queue seems to be more relevant in gauging the effectiveness of a patrol scheduling system considering the time-critical nature of police services. In this chapter, the level of the time related measurement is used as a specified threshold to calculate the car requirements. The three-step methodology by Kolesar et al has to be modified. The necessary modifications are given in the next several sections.

5.1 Step 1: Estimate the Hourly Patrol Car Requirements

Except for the measurement of patrol performance, the conditions are the same as those in Section 4.1. The objective of the police planner in this case is to assure that the expected waiting time of a call in the queue is below some specific threshold.

The state probability $P_{ot}$ calculated from Equation (3), can be extended to estimate the expected number of calls to be placed in the queue. This number, denoted by $L_t$, can be calculated by:

$$L_t = \frac{n_t+1}{\rho_t \cdot P_{ot}} \cdot \left[ n_t ! \cdot \left( \frac{1 - \rho_t}{n_t} \right)^n \right]$$

(15)

The expected waiting time of a call in the queue, $\bar{W}_t$, can then be calculated by using Little's formula [21]:

$$\bar{W}_t = \frac{L_t}{\lambda_t}$$

(16)
With a given threshold \( \alpha \), the car requirement in hour \( t \) can be found by finding the smallest value of \( n_t \) which also satisfies the condition:

\[
W_t < \alpha
\]

(17)

The mathematical manipulation to solve these equations is quite complicated. A computer program which carries out the task is given in Appendix C.1. and its flow chart is shown in Figure 5.1.

**Example 5.1**

Here the specified threshold is that the expected waiting time of a call in the queue in any hour should not exceed one minute. The rest of the pertinent information is the same as in Example 4.1.

Again, we start solving the problem at midnight, \( t=0 \). The required number of cars would be searched following the procedure below:

Starting with \( n_0 = 1 \), \( P_{00} \) is calculated by using equation (3):

\[
P_{00} = 0.24366
\]

The expected number of calls in the queue in hour \( t=0 \) is given by Equation (15)

\[
L_0 = \left( \frac{9.8}{2} \right)^2 \left( 0.24366 \right) / \left( 1 - 0.8/z \right)^2 = 0.3846
\]

The expected waiting time of a call in the queue in hour \( t=0 \) then can be calculated from Equation (16).

\[
W_0 = \frac{0.3846}{9.8} = 0.0392 \text{ hours}
\]

This value is greater than the specified threshold value of 1 minute (or \( \alpha = 0.0017 \) hours). Thus, if only one car is on duty in hour \( t=0 \), the performance of the patrol service can not meet the required level. Continuing this trial and error process, one finds 9 is the first value of \( n_0 \) which meets the requirement.

\[
W_0 = 0.0088 < \alpha = 0.017
\]

Therefore, \( n_0 \) is estimated to be 9.
Fig. 5.1. Flow diagram for generating hourly car requirements, using the expected waiting of a call in the queue as a criterion.
Repeating the procedure, the hourly car requirements for the day can be eventually obtained. The final solution is summarized in Table 5.1.

5.2. Step 2: Generate Schedule for Each Shift

The assumptions on the set of feasible shift starting times and the constrains on the earliest and latest mealbreak hours are not changed here. The integer linear program described by Equations (5-7) is used in the same way in arranging shift assignments.

Example 5.2

It is to be pointed out the only difference between the $y_e$ values generated in Example 4.1 and Example 5.1 lies on $y_7$, the car requirement in the 7th hour. One more car is required in the latter case (comparing Tables 4.2 and 5.1). However, both $y_7$ values never enter the calculations involved for schedule generating. Therefore, the answers from Example 4.2 can be applied directly in this case.

5.3. Step 3: Use a More Realistic Nonsteady State Queueing Model to Evaluate the Performance of Schedules

The truncated nonstationary queueing model described in Equations (11-14) is again used for the evaluation.

With the new set of criteria, two additional notations are needed: $L(t)$ denotes the expected number of calls in the queue at any time $t$ and $W(t)$ denotes the expected waiting time of a call in the queue at any time $t$.

When Equations (11-14) are integrated by the Runge-Kutta numerical method, the call arrival and service rates are again assumed to be constant within each integrating step.

Equations (15) and (16) can be extended in solving nonsteady-state cases;
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t: Hour of day (0, 1, ..., 23)

$\gamma_t$: Number of cars required in each hour t.
\[ L(t) = \left[ \rho(t) \hat{n}(t) + 1 \right] P_0(t) / [n(t)!(1-\rho(t)/n(t))^2 n(t)] \]  
(18)

(\text{t}) = \lambda(t)/\mu \]  
(19)

and

\[ W(t) = L(t)/\lambda(t) \]  
(20)

The general program which calculates the transient probabilities and the expected waiting time of a call in the queue for the truncated M/M/s non-steady state queue is given in Appendix D.7 and its flow chart is shown in Figure 5.2.

Example 5.3

Since the schedule generated in Example 5.2 is the same as the one calculated in Example 4.2, the cars needed on duty in each hour \( t \) are shown in Table 4.2. With the same truncated capacity and initial values of state probabilities, the problem is integrated with a step size of 0.025 hours for a 48 hour period. The output of the solution is given partially in Appendix D.2. The execution time for running this program is approximately 2 minutes and 21 seconds. The character of this schedule is illustrated in Figure 5.3.

The experienced schedule used by a New York police precinct [14] is also evaluated. Part of the output is shown in Appendix D.3. Figure 5.4 shows the character of this experience schedule.

An additional case study is evaluated for further comparison. A more flexible schedule given by Kolesar et al [14] is employed here. This schedule has five start times - 0, 8, 12, 16 and 20. Meal times are allowed at any hour during a shift.
Fig. 5.2. Flow chart of Runge Kutta method, using the expected waiting time of a call in the queue as a criterion.
Fig. 5.3. The expected waiting time of a call in the queue in the schedule with three shifts.
Fig. 5.4. The expected waiting time of a call in the queue in the experienced schedule.
The schedule is duplicated here as follows:

Shift starting at hour 0; car assigned \(N_0 = 7\)
mealbreak assigned:
\[
\begin{align*}
M_{0,3} &= 1 \text{ car} \\
M_{0,5} &= 1 \text{ car} \\
M_{0,6} &= 2 \text{ cars} \\
M_{0,7} &= 3 \text{ cars}
\end{align*}
\]

Shift starting at hour 8 car assigned \(N_8 = 5\)
mealbreak assigned:
\[
\begin{align*}
M_{8,8} &= 1 \text{ car} \\
M_{8,9} &= 1 \text{ car} \\
N_{8,10} &= 1 \text{ car} \\
M_{8,12} &= 1 \text{ car} \\
M_{8,14} &= 1 \text{ car}
\end{align*}
\]

Shift starting at hour 12 cars assigned \(N_{12} = 2\)
mealbreak assigned:
\[
\begin{align*}
M_{12,12} &= 1 \text{ car} \\
M_{12,13} &= 1 \text{ car}
\end{align*}
\]

Shift starting at hour 16 cars assigned \(N_{16} = 8\)
mealbreak assigned
\[
\begin{align*}
M_{16,16} &= 2 \text{ cars} \\
M_{16,17} &= 2 \text{ cars} \\
M_{16,18} &= 1 \text{ car} \\
M_{16,19} &= 1 \text{ car} \\
M_{16,22} &= 1 \text{ car} \\
M_{16,23} &= 1 \text{ car}
\end{align*}
\]

Shift starting at hour 20 cars assigned \(N_{20} = 2\)
mealbreak assigned
\[
\begin{align*}
M_{20,20} &= 1 \text{ car} \\
M_{20,21} &= 1 \text{ car}
\end{align*}
\]
The evaluation of this more flexible schedule is illustrated in Figure 5.5. Part of its output is listed in Appendix D.4. The above discussions illustrate the effectiveness of using the nonstationary queueing model in evaluating alternative proposals for scheduling patrol cars. The schedule generated in Example 5.2 apparently performs better than the experienced schedule, although the criterion $\alpha = 0.017$ hrs is not satisfied entirely. One can see that for the experienced schedule, the average waiting time of a call in the queue is extremely high for most of the night time hours (see Figure 5.4). However, by simply relaxing the constraints on the number of shifts and the starting and ending hours for breaks, one can obtain a schedule with superior performance without increasing the total number of cars required. (See Figure 5.5)

A police administrator can with great ease use this tool, incorporating any departmental policy (constraints) to select the most cost-effective patrol car scheduling.
Fig. 5.5. The expected waiting time of a call in the queue in the schedule with five shifts.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

From this study, the following conclusions are made:

1. A police administrator can select a reasonable schedule by applying the \( M/M/s \) nonsteady state queueing model to evaluate patrol car scheduling alternatives.

2. The fourth order Runge Kutta method can be used to solve this non-steady state model with reasonable step size.

3. The example discussed in Chapters 4 and 5 showed two strong transient responses at hours 17 and 22, because of drastic changes in the number of available cars and call arrival rates. These strong responses required a much smaller step size in integration. A step size value of 0.025 hours was shown to be able to provide convergence.

4. Considering the information provided by this nonsteady state model, the execution time of approximate 2 minutes and 25 seconds seems to be acceptable.

5. The experienced schedule was evaluated both in Chapters 4 and 5. Two criteria were used in the judgment. The first was based on a threshold probability of an urgent call placed in queue. The second was based on the expected waiting time of a call in the queue. From both points of view, the experienced schedule is extremely inadequate.

Two suggestions are given for future research:

1. Since the strong transient responses occurred at only two hours, it is not very economical that a small step size 0.025 hours is used to cover the entire 48 hour period. Therefore, a variable step size algorithm may be advantageous.

2. The level of patrol performance was measured by the expected waiting time of a call in the queue in Chapter 5. In this case, the Rothkopf
and Oren's closure technique [36], which directly approximate
the expected number of calls and variance in the M/M/s nonsteady
state queueing system, may be used to replace the fourth order
Runge Kutta numerical method to solve the infinite set of difference-
differential equations. The expected waiting time of a call in the
queue can be calculated from the following equation:

\[ W(t) = \left[ (\text{expected waiting time of a call}) - \frac{\lambda(t)}{\mu} \right] / \lambda(t) \]

in the system at time t

Rothkopf and Oren have solved several nonsteady state problems with fixed
number of servers. In contrast, the patrol scheduling problem is typefied by
varying patrol car requirements on duty at different hours, thus a subroutine
for calculating the correction term at each step may be needed. This correction
term is used to remove the error caused by negative binomial closure assumption.
REFERENCES


APPENDIX A

M/M/s STEADY STATE QUEUE WITH
THE PROBABILITY OF A CALL ENCOUNTERING
A QUEUE AS A CRITERION
ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE
Program List

1 DIMENSION PT(100)
2 READ*,'N'
3 PRINT 5
4 5 FORMAT('1')
5 PRINT 'M',N,UI,AL
6 FORMAT('1*13.5A,11*13.5A,PERIOD OF CYCLE = 1/13*
7 1*13.5A,MEAN SERVICE RATE = M,UI,AL,SPECIFIED THRESHOLD = T,AL')
8 READ,RA,NT
9 READ,RA,NT
10 NT=1
11 10 NI=NI+1
12 20 NS=NS+1
13 FACT=1
14 SUM=1
15 IF(NT.EQ.11) GO TO 20
16 NT=NT+1
17 GO TO 10
18 FACT=FACT*NI
19 SUM=SUM+FACT/J/FACT
20 FACT=1
21 DD 33 1=1,AL
22 FACT=FACT*1
23 PLS=PLS*AL/N1(FACT*AL/RCA1/N1)
24 P10=P10/J/(SUM+PLS)
25 SUM=SUM+P10
26 IF(NT.EQ.11) GO TO 34
27 FACT=1
28 NI=NI-1
29 GO TO 10
30 FACT=FACT*J
31 PTL=P10*AL/N1(FACT*P10/FACT)
32 SUM=SUM+PTL
33 IF(SMPL.EQ.(1-AL)) GO TO 10
34 SUM=SUM+PTL
35 S

This program calculates the state probabilities of M/G/1
steady state Markov model and search the number of calls
required such that the probability of a call encountering
a queue is less than the specified threshold.

N: LENGTH OF CYCLE, IN THIS STUDY IT IS ONE DAY, STARTING
M: 24 HOURS TO 24 HOURS.
UI: MEAN SERVICE RATE, A CONSTANT.
N1: MEAN ARRIVAL RATE, CHANGE WITH HOUR.
AL: THE SPECIFIED THRESHOLD, THE PROBABILITY OF A CALL NOT
ENCOUNTERING A QUEUE.
M: EQUAL TO MEAN ARRIVAL RATE DIVIDED BY MEAN SERVICE RATE.
AL: NUMBER OF CALLS ON DUTY.
FAC1: FACTORIAL OF N1.
PTL: PROBABILITY OF j CALLS IN THE SYSTEM, P10.
SUM: PROBABILITY OF j CALLS IN THE SYSTEM, DD.

58
36  \text{n} = \text{n} + 1
37  \text{CO} \ 35 \ = \text{n} + 1, \text{HI}
38  \text{PI}[\text{J}] = \text{PI}[\text{I}] \times \text{PTO} / (\text{FLOAT(N)} - \text{J} - 1) \times \text{FAC(N)}
39  \text{SUM} \ = \text{SUM} + \text{PI}[\text{J}]
40  \text{PC} = \text{PC} - 1 \times \text{PI}[\text{J}]
41  \text{PRINT} 40, \text{K}, \text{N}, \text{PC}
42  40  \text{FORMI} \times \text{-} * * * \text{AT HOUR}, 13.5X, \text{THE NUMBER OF PATROL CARS REQUIRED}
43  \text{L} = \text{L} + 1, \text{H}, \text{THE PROBABILITY OF AC CARS CR URY} = \text{L} \times \text{F} \times \text{O} \times \text{O}
44  \text{PR} \text{IM} \times 5, \text{E} \times \text{U} \times \text{B} \text{L}
45  45  \text{FORMAT} \times * \times \text{PERS ARRIVAL RATE} = \text{F} \times \text{U} \times \text{O}
46  \text{PRINT} 40, \text{PI}, \text{L}, \text{PI}[\text{J}], \text{L} \times \text{L}
47  60  \text{FORMI} \times * \times \text{K(N)} = \text{L} \times \text{H} \times \text{L} \times \text{L} \times \text{L} \times \text{L} \times \text{L}
48  \text{PRINT} 70, \text{M}, \text{L}
49  70  \text{FORMI} \times * \times \text{PERIAL PROBABILITY FM 1 TO } * \times \text{L}, \text{CALLS IN THE SYSTEM}
50  \text{L} = \text{L} \times \text{F} \times \text{U} \times \text{O}
51  80  \text{CONTINUE}
52  85  \text{PRINT} 85
53  \text{ENTRY}
TRUNCATION CAPACITY = 36 PERIOD OF CYCLE = 24
MEAN SERVICE RATE = 2.00000 SPECIFIED THRESHOLD = 0.100000

AT HOUR 0 THE NUMBER OF PATROL CARS REQUIRED = 9 THE PROBABILITY OF NO CARS ON DUTY = 0.072512

MEAN ARRIVAL RATE = 5.96000
P(0) = 0.0007361 P(1) = 0.036073 P(2) = 0.048372 P(3) = 0.145441 P(4) = 0.176817 P(5) = 0.173281 P(6) = 0.141514
P(7) = 0.095059 P(8) = 0.060774 P(9) = 0.033033 P(10) = 0.017945 P(11) = 0.007992 P(12) = 0.003531
P(13) = 0.002902 P(14) = 0.001593 P(15) = 0.000660 P(16) = 0.000246 P(17) = 0.000055 P(18) = 0.000013
P(19) = 0.000096 P(20) = 0.000014 P(21) = 0.000002 P(22) = 0.000000 P(23) = 0.000000 P(24) = 0.000000
P(25) = 0.000002 P(26) = 0.000001 P(27) = 0.000000 P(28) = 0.000000 P(29) = 0.000000 P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR 1 THE NUMBER OF PATROL CARS REQUIRED = 9 THE PROBABILITY OF NO CARS ON DUTY = 0.065088

MEAN ARRIVAL RATE = 9.60000
P(0) = 0.0008145 P(1) = 0.034155 P(2) = 0.094376 P(3) = 0.156201 P(4) = 0.180741 P(5) = 0.173032 P(6) = 0.138625
P(7) = 0.094976 P(8) = 0.056552 P(9) = 0.030376 P(10) = 0.016700 P(11) = 0.008640 P(12) = 0.004638
P(13) = 0.002758 P(14) = 0.001911 P(15) = 0.000869 P(16) = 0.000373 P(17) = 0.000199 P(18) = 0.000136
P(19) = 0.000057 P(20) = 0.000030 P(21) = 0.000018 P(22) = 0.000009 P(23) = 0.000005 P(24) = 0.000002
P(25) = 0.000001 P(26) = 0.000001 P(27) = 0.000000 P(28) = 0.000000 P(29) = 0.000000 P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR 2 THE NUMBER OF PATROL CARS REQUIRED = 8 THE PROBABILITY OF NO CARS ON DUTY = 0.0988605

MEAN ARRIVAL RATE = 8.79000
P(0) = 0.012714 P(1) = 0.055305 P(2) = 0.077001 P(3) = 0.134414 P(4) = 0.185878 P(5) = 0.165018 P(6) = 0.149638
P(7) = 0.074484 P(8) = 0.040729 P(9) = 0.021022 P(10) = 0.011696 P(11) = 0.006499 P(12) = 0.003534
P(13) = 0.001952 P(14) = 0.001359 P(15) = 0.000868 P(16) = 0.000305 P(17) = 0.000116 P(18) = 0.000091
P(19) = 0.000018 P(20) = 0.000007 P(21) = 0.000002 P(22) = 0.000000 P(23) = 0.000000 P(24) = 0.000000
P(25) = 0.000001 P(26) = 0.000001 P(27) = 0.000000 P(28) = 0.000000 P(29) = 0.000000 P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR 3 THE NUMBER OF PATROL CARS REQUIRED = 8 THE PROBABILITY OF NO CARS ON DUTY = 0.045681

MEAN ARRIVAL RATE = 7.40000
P(0) = 0.022241 P(1) = 0.064515 P(2) = 0.165790 P(3) = 0.203400 P(4) = 0.193230 P(5) = 0.146855 P(6) = 0.093008
P(7) = 0.054542 P(8) = 0.022583 P(9) = 0.011352 P(10) = 0.005411 P(11) = 0.002570 P(12) = 0.001221
P(13) = 0.000980 P(14) = 0.000275 P(15) = 0.000131 P(16) = 0.000062 P(17) = 0.000030 P(18) = 0.000016
P(19) = 0.000007 P(20) = 0.000003 P(21) = 0.000002 P(22) = 0.000000 P(23) = 0.000000 P(24) = 0.000000
P(25) = 0.000000 P(26) = 0.000000 P(27) = 0.000000 P(28) = 0.000000 P(29) = 0.000000 P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000

AT HOUR 4 THE NUMBER OF PATROL CARS REQUIRED = 7 THE PROBABILITY OF NO CARS ON DUTY = 0.062656

MEAN ARRIVAL RATE = 6.70000
P(0) = 0.034776 P(1) = 0.116499 P(2) = 0.155136 P(3) = 0.217907 P(4) = 0.182493 P(5) = 0.122270 P(6) = 0.068268
P(7) = 0.032671 P(8) = 0.015635 P(9) = 0.007483 P(10) = 0.003581 P(11) = 0.001714 P(12) = 0.000820
<table>
<thead>
<tr>
<th>Hour</th>
<th>Mean Arrival Rate</th>
<th>Probability No Cars On Duty</th>
<th>Total Probability from 1 to 30 Calls in the System</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5.3992</td>
<td>0.060378</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4.1098</td>
<td>0.065273</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.2006</td>
<td>0.090760</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2.5050</td>
<td>0.042218</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2.5050</td>
<td>0.042218</td>
<td></td>
</tr>
</tbody>
</table>

For Hour 1:
- Mean Arrival Rate: 4.1278
- Probability No Cars On Duty: 0.03711
- Total Probability from 1 to 30 Calls in the System: 0.599595

For Hour 2:
- Mean Arrival Rate: 3.1940
- Probability No Cars On Duty: 0.03711
- Total Probability from 1 to 30 Calls in the System: 0.599595

For Hour 3:
- Mean Arrival Rate: 2.667
- Probability No Cars On Duty: 0.03711
- Total Probability from 1 to 30 Calls in the System: 0.599595

For Hour 4:
- Mean Arrival Rate: 2.266
- Probability No Cars On Duty: 0.03711
- Total Probability from 1 to 30 Calls in the System: 0.599595

For Hour 5:
- Mean Arrival Rate: 1.981
- Probability No Cars On Duty: 0.03711
- Total Probability from 1 to 30 Calls in the System: 0.599595

For Hour 6:
- Mean Arrival Rate: 1.7664
- Probability No Cars On Duty: 0.03711
- Total Probability from 1 to 30 Calls in the System: 0.599595
Mean Arrival Rate = 6.6000

At Hour 15: The number of patrol cars required = 7
The probability of no cars on duty = 0.058354

Mean Arrival Rate = 6.6000

At Hour 16: The number of patrol cars required = 8
The probability of no cars on duty = 0.052061

Mean Arrival Rate = 7.0000

At Hour 17: The number of patrol cars required = 8
The probability of no cars on duty = 0.083370

Mean Arrival Rate = 9.6000

At Hour 18: The number of patrol cars required = 9
The probability of no cars on duty = 0.058217

Mean Arrival Rate = 9.6000

At Hour 19: The number of patrol cars required = 9
The probability of no cars on duty = 0.072512

Mean Arrival Rate = 9.8000
<table>
<thead>
<tr>
<th>Time of Day</th>
<th>Number of Patrol Cars Required</th>
<th>Probability of No Cars On Duty</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>0.067218</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>0.049456</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>0.077382</td>
</tr>
<tr>
<td>13</td>
<td>6</td>
<td>0.047445</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>0.052830</td>
</tr>
</tbody>
</table>

**Mean Arrival Rate:**

- 2.5/25
- 1.8/22
- 4.3/21
- 5.6/20
- 5.9/22

**Total Probability from 1 to 30 Calls in the System:**

- 1.000000
- 1.000000
- 1.000000
- 1.000000
- 1.000000

**Examples of Calculations:**

- **Mean Arrival Rate:**
  - **Mean Arrival Rate = 2.5**
  - **Mean Arrival Rate = 1.8**
  - **Mean Arrival Rate = 4.3**
  - **Mean Arrival Rate = 5.6**
  - **Mean Arrival Rate = 5.9**

**Probability of No Cars On Duty:**

- **Probability of No Cars On Duty = 0.067218**
- **Probability of No Cars On Duty = 0.049456**
- **Probability of No Cars On Duty = 0.077382**
- **Probability of No Cars On Duty = 0.047445**
- **Probability of No Cars On Duty = 0.052830**
<table>
<thead>
<tr>
<th>Hour</th>
<th>Mean Arrival Rate</th>
<th>The Number of Patrol Cars Required</th>
<th>The Probability of No Cars On Duty</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10.2000</td>
<td>9</td>
<td>0.084102</td>
</tr>
<tr>
<td>21</td>
<td>10.4000</td>
<td>9</td>
<td>0.098307</td>
</tr>
<tr>
<td>22</td>
<td>10.2000</td>
<td>9</td>
<td>0.089102</td>
</tr>
<tr>
<td>23</td>
<td>10.0001</td>
<td>9</td>
<td>0.080515</td>
</tr>
</tbody>
</table>

**Examples of calculations:***
- **At Hour 20:**
  - Mean Arrival Rate = 10.2000
  - The Number of Patrol Cars Required = 9
  - The Probability of No Cars On Duty = 0.084102

- **At Hour 21:**
  - Mean Arrival Rate = 10.4000
  - The Number of Patrol Cars Required = 9
  - The Probability of No Cars On Duty = 0.098307

- **At Hour 22:**
  - Mean Arrival Rate = 10.2000
  - The Number of Patrol Cars Required = 9
  - The Probability of No Cars On Duty = 0.089102

- **At Hour 23:**
  - Mean Arrival Rate = 10.0001
  - The Number of Patrol Cars Required = 9
  - The Probability of No Cars On Duty = 0.080515

---

**Note:**
Each column represents a different hour, with the mean arrival rate, the number of patrol cars required, and the probability of no cars on duty. The calculations involve probability theory, specifically, the use of probability distributions to model the arrival of patrol cars and the probability of no cars being on duty at any given time.
APPENDIX B

M/M/s NONSTEADY STATE QUEUE WITH

THE PROBABILITY OF A CALL ENCOUNTERING

A QUEUE AS A CRITERION
B. 1 PROGRAM LIST

DIMENSION D1(100), D2(100), D3(100), D4(100), PK(100)
C
C THIS PROGRAM CALCULATES TRANSIENT STATE PROBABILITIES AND
C THE PROBABILITY OF NO CAR ON DUTY IN THE SYSTEM FOR A NK SERVER
C TRUNCATED MAXIMUM ALLOWED CAPACITY QUEUING SYSTEM BY THE
C FOURTH ORDER RUNGE KUTTA METHOD.
C
C N: MAXIMUM ALLOWED CAPACITY IN THE QUEUING SYSTEM.
C DELT: INTEGRATING STEP SIZE.
C T: TOTAL INTEGRATING LENGTH, 48 HOURS SUGGESTED.
C U: MEAN SERVICE RATE, A CONSTANT.
C PK: MEAN ARRIVAL RATE, CHANG WITH TIME.
C PKJ: NUMBER OF CARS ON DUTY, CHANG WITH TIME.
C PK(J): PROBABILITY OF J-1 CALLS IN SYSTEM.
C
C
1 100 READ,PK(J)
10 100 READ,NK,PK1,PK2
12 100 NK=1
14 100 TIME=0
15 100 FORMAT(1,1,H,DELT,T,U)
16 14 FORMAT(1,1,'TRUNCATED CAPACITY = ',T,13.5x,'STEP SIZE = ',F8.5
17 14 FORMAT(1,1,'INTEGRATED LENGTH = ',T,13.5x,'MEAN SERVICE RATE = ',F8.4)
18 15 FORMAT(1,1,'AT HOUR OF 5X,PATROL CARS ON DUTY = ',T,13.5x)
19 16 FORMAT(1,1,'PK(J) = ',F8.6)
20 70 TIME=TIME+DEL T
21 100 IF(KLT,NK) GO TO 20
22 100 READ,NK,PK3
24 100 PK2=PK3
25 100 NH=1
26 100 IF(NH.0GT.4N) NH=NH-24
27 100 IF(NH.GT.24) NH=NH-24
29 100 NK=INT(16,NK,RK)
30 16 FORMAT(1,1,'AT HOUR OF 5X,PATROL CARS ON DUTY = ',T,13.5x)
31 100 NH=1
32 20 NK=PK2-RK1)*FLOAT(NK)*DEL T+PK1
33 100 NM=NO+1
34 100 D1(J)=FKK1(J-1),PK(J),D1(J),U,RANDK,J)*DEL T
35 100 NK=NM+1
36 50 50 J=2,NK
37 100 IF(J.LT.NK) GO TO 30
38 100 D1(J)=FKK(J-1),PK(J),D1(J),U,RANDK,J)*DEL T

30 IF (J,F0,.O,M) GO TO 40
31 D1(J)=F0N(PK(J-1),PK(J),PK(J+1),U,RA%O,K,N)*DEL1
32 G0 TO 50
33 D0(J)=F0N(PK(J),PK(J+1),U,RA%O,K,N)*DEL1
34 CONTINUE
35 D2(J)=F0N(PK(J),1./2.,D1(J),PK(J),U,RA%O,K,N)*DEL1
36 C0 51 J=2,M1
37 IF (J,E<.N,K1) GO TO 81
38 IF (J,E<.KO,1,M) GO TO 84
39 DK(J)=FLN(PK(J-1),1./2.,D1(J-1),PK(J-1),U,RA%O,K,N)*DEL1
40 P0(J+1)=1./2.,D1(J+1),U,RA%O,K,N)*DEL1
41 GO TO 51
42 IF (J,E<.N1,J1) GO TO 81
43 D1(J)=F0N(PK(J-1),1./2.,D1(J-1),PK(J),U,RA%O,K,N)*DEL1
44 GO TO 51
45 CONTINUE
46 D0(J)=F0N(PK(J),1./2.,D1(J),PK(J),U,RA%O,K,N)*DEL1
47 D5 62 J=2,M1
48 IF (J,E<.N,K1) GO TO 82
49 D3(J)=FLN(PK(J-1),1./2.,D2(J-1),PK(J),U,RA%O,K,N)*DEL1
50 IF (J,E<.N1,K1) GO TO 83
51 GO TO 52
52 IF (J,E<.N1,K1) GO TO 84
53 D3(J)=F0N(PK(J-1),1./2.,D2(J-1),PK(J),U,RA%O,K,N)*DEL1
54 GO TO 52
55 CONTINUE
56 D4(J)=F0N(PK(J),1./2.,D2(J),PK(J),U,RA%O,K,N)*DEL1
57 GO TO 52
58 CONTINUE
59 D5 63 J=2,M1
60 IF (J,E<.N,K1) GO TO 85
61 D4(J)=F0N(PK(J),1./2.,D3(J),PK(J),U,RA%O,K,N)*DEL1
62 GO TO 53
63 CONTINUE
64 D6(J)=F0N(PK(J),1./2.,D3(J),PK(J),U,RA%O,K,N)*DEL1
65 GO TO 53
66 IF (J,E<.N,K1) GO TO 86
67 D4(J)=F0N(PK(J-1)+D3(J-1),PK(J)+D3(J),U,RA%O,K,N)*DEL1
68 GO TO 53
69 CONTINUE
70 D5 64 J=2,M1
71 D4(J)=F0N(PK(J-1)+D3(J-1),PK(J)+D3(J),PK(J),U,RA%O,K,N)*DEL1
72 GO TO 53
73 CONTINUE
74 D6(J)=F0N(PK(J),1,M1
75 PRINT 65,K,RAND(K-1),PK(J),J=1,P1
76 FORMAT('D1,J*',STEP*,*,5X,*MEAN ARRIVAL RATE = ',F8.4,'*,6(E13.1)),*F8.6))
77 SUM=0
78 DO 66 J=1,K
79 SUM=SUM+PK(J)
80 SUM=SUM
81 DO 68 J=1,K
82 SUM=SUM+PK(J)
83 PRINT 69,SUM
84 PRINT 69,F8.6
85 PRINT 69,F8.6
86 PRINT 69,F8.6
87 PRINT 69,F8.6
88 PRINT 69,F8.6
89 PRINT 69,F8.6
90 PRINT 69,F8.6
91 PRINT 69,F8.6
### An Analysis of Car Parking in a Duty Scenario

**Data for Duty Time 21**

<table>
<thead>
<tr>
<th>Step</th>
<th>Mean Arrival Rate</th>
<th>Arrival Times (in minutes)</th>
<th>Service Times (in minutes)</th>
<th>Total Probability of Car in Duty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00</td>
<td>1:00, 1:30, 1:45, 1:50, 2:00</td>
<td>0.12, 0.10, 0.08, 0.06, 0.04</td>
<td>0.004</td>
</tr>
<tr>
<td>2</td>
<td>4.50</td>
<td>1:00, 1:30, 1:45, 1:50, 2:00</td>
<td>0.10, 0.12, 0.08, 0.06, 0.04</td>
<td>0.005</td>
</tr>
<tr>
<td>3</td>
<td>5.00</td>
<td>1:00, 1:30, 1:45, 1:50, 2:00</td>
<td>0.10, 0.12, 0.08, 0.06, 0.04</td>
<td>0.005</td>
</tr>
</tbody>
</table>

**Data for Duty Time 23**

<table>
<thead>
<tr>
<th>Step</th>
<th>Mean Arrival Rate</th>
<th>Arrival Times (in minutes)</th>
<th>Service Times (in minutes)</th>
<th>Total Probability of Car in Duty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00</td>
<td>1:30, 1:45, 1:50, 2:00, 2:15</td>
<td>0.10, 0.12, 0.08, 0.06, 0.04</td>
<td>0.005</td>
</tr>
<tr>
<td>2</td>
<td>4.50</td>
<td>1:30, 1:45, 1:50, 2:00, 2:15</td>
<td>0.10, 0.12, 0.08, 0.06, 0.04</td>
<td>0.005</td>
</tr>
<tr>
<td>3</td>
<td>5.00</td>
<td>1:30, 1:45, 1:50, 2:00, 2:15</td>
<td>0.10, 0.12, 0.08, 0.06, 0.04</td>
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**Data for Duty Time 24**

<table>
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<th>Step</th>
<th>Mean Service Rate</th>
<th>Service Times (in minutes)</th>
<th>Total Probability of Car in Duty</th>
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</thead>
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<td>1:30, 1:45, 1:50, 2:00, 2:15</td>
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**B.2 Schedule with 3 Shifts**

---

69
STEP 1: MEAN ARRIVAL RATE = 9.4200

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<tr>
<td>PT 2</td>
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<tr>
<td>PT 12</td>
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<td>0.00046</td>
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<tr>
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<tr>
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TOTAL PROBABILITY = 1.00373

PROBABILITY NG CAP IN DUTY = 0.02474

A1 HOUR 1 PATROL CAPS IN DUTY = 10

STEP 2: MEAN ARRIVAL RATE = 5.1500

<table>
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<tr>
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TOTAL PROBABILITY = 1.00373

PROBABILITY NG CAP IN DUTY = 0.02620

A1 HOUR 2 PATROL CAPS IN DUTY = 9

STEP 3: MEAN ARRIVAL RATE = 8.7000

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<tr>
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<td>0.00053</td>
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TOTAL PROBABILITY = 1.00373

PROBABILITY NG CAP IN DUTY = 0.02973

A1 HOUR 3 PATROL CAPS IN DUTY = 9

STEP 4: MEAN ARRIVAL RATE = 8.1500

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<tr>
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<td>0.00000</td>
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TOTAL PROBABILITY = 1.00373

PROBABILITY NG CAP IN DUTY = 0.036657

A1 HOUR 4: MEAN ARRIVAL RATE = 7.6900

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<tr>
<td>PT 18</td>
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<tr>
<td>PT 30</td>
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</table>

TOTAL PROBABILITY = 1.00373

PROBABILITY NG CAP IN DUTY = 0.02620
### At Hour 3
**Patrol Cars on Duty = 8**

**Step 1:** Mean Arrival Rate = 7.1500
- **P(1) = 0.023711**
- **P(2) = 0.024750**
- **P(3) = 0.025437**
- **P(4) = 0.025831**
- **P(5) = 0.026012**
- **P(6) = 0.026063**
- **P(7) = 0.026005**
- **P(8) = 0.025831**

**Total Probability = 1.000000**

**Probability FC Cap on Duty = 0.044594**

**Step 2:** Mean Arrival Rate = 6.7000
- **P(1) = 0.037939**
- **P(2) = 0.059727**
- **P(3) = 0.107074**
- **P(4) = 0.180720**
- **P(5) = 0.235024**
- **P(6) = 0.266171**
- **P(7) = 0.271961**
- **P(8) = 0.250000**

**Total Probability = 1.000000**

**Probability FC Cap on Duty = 0.034713**

### At Hour 4
**Patrol Cars on Duty = 7**

**Step 1:** Mean Arrival Rate = 6.0000
- **P(1) = 0.045578**
- **P(2) = 0.058976**
- **P(3) = 0.062767**
- **P(4) = 0.066035**
- **P(5) = 0.068008**
- **P(6) = 0.068956**
- **P(7) = 0.069896**
- **P(8) = 0.070611**

**Total Probability = 1.000000**

**Probability FC Cap on Duty = 0.056796**

**Step 2:** Mean Arrival Rate = 5.5000
- **P(1) = 0.054790**
- **P(2) = 0.062870**
- **P(3) = 0.069760**
- **P(4) = 0.074490**
- **P(5) = 0.077520**
- **P(6) = 0.079199**
- **P(7) = 0.080032**
- **P(8) = 0.080600**

**Total Probability = 1.000000**

**Probability FC Cap on Duty = 0.054079**

### At Hour 5
**Patrol Cars on Duty = 6**

**Step 1:** Mean Arrival Rate = 4.7000
- **P(1) = 0.059359**
- **P(2) = 0.065525**
- **P(3) = 0.069711**
- **P(4) = 0.072146**
- **P(5) = 0.073996**
- **P(6) = 0.074800**
- **P(7) = 0.075108**
- **P(8) = 0.075108**

**Total Probability = 1.000000**

**Probability FC Cap on Duty = 0.039756**

### At Hour 6
**Patrol Cars on Duty = 6**

**Step 1:** Mean Arrival Rate = 4.7000
- **P(1) = 0.059359**
- **P(2) = 0.065525**
- **P(3) = 0.069711**
- **P(4) = 0.072146**
- **P(5) = 0.073996**
- **P(6) = 0.074800**
- **P(7) = 0.075108**
- **P(8) = 0.075108**

**Total Probability = 1.000000**

**Probability FC Cap on Duty = 0.062283**
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<th>Cumulative Probability</th>
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<td>P4</td>
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</tbody>
</table>

Total Probability = 1.000075

P(t) | Cumulative Probability |
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<thead>
<tr>
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<tbody>
<tr>
<td>P1</td>
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<td>P2</td>
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<tr>
<td>P3</td>
<td>0.1116</td>
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<tr>
<td>P4</td>
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Total Probability = 1.000075

At M/C 1, Mean Arrival Rate = 4.27

Step 1 | Mean Arrival Rate = 4.27 | P(t) | Cumulative Probability |
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Total Probability = 1.000075

P(t) | Cumulative Probability |
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Total Probability = 1.000075

At M/C 2, Mean Arrival Rate = 8.15

Step 1 | Mean Arrival Rate = 8.15 | P(t) | Cumulative Probability |
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Total Probability = 1.000075

P(t) | Cumulative Probability |
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Total Probability = 1.000075

At M/C 3, Mean Arrival Rate = 7.65

Step 1 | Mean Arrival Rate = 7.65 | P(t) | Cumulative Probability |
<table>
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<tbody>
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Total Probability = 1.000075

P(t) | Cumulative Probability |
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Total Probability = 1.000075

P(t) | Cumulative Probability |
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<tbody>
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<tr>
<td>PI</td>
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APPENDIX C

M/M/s STEADY STATE QUEUE WITH
THE EXPECTED WAITING TIME OF A
CALL IN THE QUEUE AS A CRITERION
This program calculates the state probabilities of a M/K/S steady state queueing model and searches the number of patrol cars required such that the average waiting time of a call in the queue is less than the specified threshold.

N: maximum number of calls allowed in system at same time.
M: length of cycle, in this study it is one day, starting from 0 hour to 24 hour.
U1: mean service rate, a constant.
N: the specified threshold, maximum average waiting time allowed in the queue for a call.
RA: mean arrival rate, change with hour.
RO: equal to mean arrival rate divided by mean service rate.
Nt: number of cars on duty.
FACTORIAL: factorial of Nt.
P: probability of zero call in the system, P(0).
PT(J): probability of J calls in the system, 0<J.
Q: average length of queue.
W: average waiting time in the queue.

1 DIMENSION PT(100)
2 READ,N,Nt,U1,AL
3 PRINT 5
4 5 FORMAT(12)
5 PRINT 6,N,Nt,U1,AL
6 6 FORMAT(‘0’,‘13,5X’,‘PERIOD OF CYCLE = ’,13/1
7 ’,‘MEAN SERVICE RATE = ’,8.4,‘5X’,‘SPECIFIED THRESHOLD = ’,8.6)
8 GO TO K=1,N
9 READ,RA
10 NT=N+1
11 NT=NT+1
12 RO=RAA/LT
13 FACTJ=1
14 SUM=1
15 IF NT-EC.1 NT+1 GT 30
16 NT1=NT-1
17 DO 25 J=1,NT1
18 FACTJ=FACTJ*J
19 25 SUM=SUM+ROA**J/FACTJ
20 NT1=1
21 DO 31 I=1,NT
22 FACTJ=FACTJ*I
23 PLUS=ROA**NT/FACTN*(1-ROA/NT)
24 PTO=1/SUM*PLUS
25 QUILT=ROA**NT/PT0/FACTN*(1-ROA/NT)**2*NT
26 IF QUILT.LT.0.1 GO TO 10
27 W=QUILT/PAD
28 IF W > QUIT.NUM AL GO TO 10
29 SUMI=P0
30 IF NT-EC.1 NT+1 GT 34
31 FACTJ=1
32 NT=NT+1
DU 33 J=1,NT2
FACTJ=FACTJ*J
PI(J)=RCA1**J*P10/FACTJ
SUMF=SUMF+PI(J)
N1I=POAT**NT*PI(N)/FACNT
SUM=SUMF+PI(N)
NT3=NT1+1
DO 35 J=NT3,K
PI(J)=ROAT**J*PI(J)/IFLOAT(N1)*E(J-N1)*FACNT
SUM=SUM+PI(J)
35 PRINT 40,K-1,AT,WAIT1N
40 FORMAT(8-,6*,A0,H0,1.3,5X,5X,THE NUMBER OF PATRCL CARS REQUIRED = 1,13,5X,5X,THE AVERAGE WAITING TIME IN THE QUEUE = 1,F0.6)
45 PRINT 45,RADA1
46 FORMAT(8-,6*,MEAN ARRIVAL RATE = 1,F0.4)
47 PRINT 60,PIE(J,J,J,J),J=1,H
48 FORMAT(8-,*,PI(0) = 1,F0.6/*,614*,*,PI(12,12) = 1,F0.6,2X)
49 PRINT 70,H,SUM
50 FORMAT(8-,*,TOTAL PROBABILITY FROM 1 IN 1,12,1 CALLS IN THE SYSTEM
1 = 1,F0.6)
51 CONTINUE
52 PRINT 05
53 FORMAT(1*)
54 STOP
55 END

$ENTRY
INDUCED CAPACITY = 30  PERIOD OF CYCLE = 25
MEAN SERVICE RATE = 2.0000  SPECIFIED THRESHOLD = 0.017000

AT HOUR 0  THE NUMBER OF PATROL CARS REQUIRED = 9  THE AVERAGE WAITING TIME IN THE QUEUE = 0.008843

MEAN ARRIVAL RATE = 9.6000
P(i) = 0.006743
P(11) = 0.013486
P(21) = 0.020221
P(31) = 0.026955
P(41) = 0.033689
P(51) = 0.040422
P(61) = 0.047155
P(71) = 0.053888
P(81) = 0.060621
P(91) = 0.067354
P(101) = 0.074087
TOTAL PROBABILITY FROM 1 TO 10 CARS IN THE SYSTEM = 0.999999

AT HOUR 1  THE NUMBER OF PATROL CARS REQUIRED = 9  THE AVERAGE WAITING TIME IN THE QUEUE = 0.002198

MEAN ARRIVAL RATE = 9.6000
P(i) = 0.008149
P(11) = 0.015997
P(21) = 0.023855
P(31) = 0.031713
P(41) = 0.039571
P(51) = 0.047429
P(61) = 0.055287
P(71) = 0.063145
P(81) = 0.071003
P(91) = 0.078861
P(101) = 0.086719
TOTAL PROBABILITY FROM 1 TO 10 CARS IN THE SYSTEM = 0.999999

AT HOUR 2  THE NUMBER OF PATROL CARS REQUIRED = 8  THE AVERAGE WAITING TIME IN THE QUEUE = 0.002138

MEAN ARRIVAL RATE = 9.6000
P(i) = 0.012134
P(11) = 0.019982
P(21) = 0.027830
P(31) = 0.035678
P(41) = 0.043526
P(51) = 0.051374
P(61) = 0.059222
P(71) = 0.067070
P(81) = 0.074918
P(91) = 0.082756
P(101) = 0.090594
TOTAL PROBABILITY FROM 1 TO 10 CARS IN THE SYSTEM = 0.999999

AT HOUR 3  THE NUMBER OF PATROL CARS REQUIRED = 8  THE AVERAGE WAITING TIME IN THE QUEUE = 0.005438

MEAN ARRIVAL RATE = 9.6000
P(i) = 0.022241
P(11) = 0.030089
P(21) = 0.037937
P(31) = 0.045785
P(41) = 0.053633
P(51) = 0.061481
P(61) = 0.069329
P(71) = 0.077177
P(81) = 0.085025
P(91) = 0.092873
P(101) = 0.099721
TOTAL PROBABILITY FROM 1 TO 10 CARS IN THE SYSTEM = 0.999999

AT HOUR 4  THE NUMBER OF PATROL CARS REQUIRED = 7  THE AVERAGE WAITING TIME IN THE QUEUE = 0.008583

MEAN ARRIVAL RATE = 9.6000
P(i) = 0.034716
P(11) = 0.042564
P(21) = 0.050412
P(31) = 0.058260
P(41) = 0.066108
P(51) = 0.073956
P(61) = 0.081804
P(71) = 0.089652
P(81) = 0.097499
P(91) = 0.095347
P(101) = 0.093195
TOTAL PROBABILITY FROM 1 TO 10 CARS IN THE SYSTEM = 0.999999
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<td>5</td>
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<td>Hour 8</td>
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</tr>
<tr>
<td>Hour 9</td>
<td>4</td>
<td>0.007676</td>
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\[
\begin{align*}
F(1) & = 0.000001 \quad F(10) = 0.000000 \quad F(20) = 0.000000 \quad F(30) = 0.000000 \quad F(40) = 0.000000 \\
F(2) & = 0.000000 \quad F(3) = 0.000000 \quad F(4) = 0.000000 \quad F(5) = 0.000000 \quad F(6) = 0.000000 \\
F(3) & = 0.000000 \quad F(5) = 0.000000 \quad F(7) = 0.000000 \quad F(8) = 0.000000 \quad F(9) = 0.000000 \\
F(4) & = 0.000000 \quad F(6) = 0.000000 \quad F(8) = 0.000000 \quad F(9) = 0.000000 \quad F(10) = 0.000000 \\
\text{TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM} & = 1.000000
\end{align*}
\]

**At Hour 10**

The number of patrol cars required = 4
The average waiting time in the queue = 0.013160

Mean arrival rate = 2.9000

**F(1) = 0.232958**

**F(2) = 0.233785**

**F(3) = 0.118211**

**F(4) = 0.042857**

**F(5) = 0.015914**

**F(6) = 0.006313**

**F(7) = 0.002041**

**F(8) = 0.000241**

**F(9) = 0.000000**

**F(10) = 0.000000**

**F(11) = 0.000000**

**F(12) = 0.000000**

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**F(97) = 0.000000**

**F(98) = 0.000000**

**F(99) = 0.000000**

**F(100) = 0.000000**

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</tr>
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</tr>
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</tr>
<tr>
<td>10</td>
<td>10</td>
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**Notes:**
- The table above shows the relationship between the number of patrol cars required and the total calls in the system, along with the mean arrival rate and mean waiting time in the queue.
- The mean arrival rate is calculated by dividing the number of calls by the time interval.
- The mean waiting time in the queue is calculated by subtracting the mean service time from the mean arrival time.
- The total calls in the system is calculated by multiplying the number of calls by the time interval.
At hour 23, the number of patrol cars required = 9
The average waiting time in the queue = 0.01423

Mean arrival rate = 10.200

Mean probability from 1 to 30 calls in the system = 0.99999

At hour 21, the number of patrol cars required = 9
The average waiting time in the queue = 0.012935

Mean arrival rate = 10.400

Mean probability from 1 to 30 calls in the system = 0.99998

At hour 22, the number of patrol cars required = 9
The average waiting time in the queue = 0.01423

Mean arrival rate = 10.200

Mean probability from 1 to 30 calls in the system = 0.99999

At hour 23, the number of patrol cars required = 9
The average waiting time in the queue = 0.010064

Mean arrival rate = 10.000

Mean probability from 1 to 30 calls in the system = 0.99999
APPENDIX D

M/M/s NONSTEADY STATE QUEUE WITH
THE EXPECTED WAITING TIME OF A
CALL IN THE QUEUE AS A CRITERION
THIS PROGRAM CALCULATES TRANSIENT STATE PROBABILITIES AND
THE AVERAGE WAITING TIME IN THE QUEUE FOR NK SERVER
TRUNCATED MAXIMUM ALLOWED CAPACITY QUEUEING SYSTEM BY THE
FOURTH ORDER RUNGE KUTTA METHOD.

M: MAXIMUM ALLOWED CAPACITY IN THE QUEUEING SYSTEM.
DELT: INTEGRATING STEP SIZE.
T: TOTAL INTEGRATING LENGTH, 48 HOURS SUGGESTED.
U: MEAN SERVICE RATE, A CONSTANT.
R: MEAN ARRIVAL RATE, CHANGE WITH TIME.
NK: NUMBER OF CAPS ON DUTY, CHANGE WITH TIME.
PK: J: PROBABILITY OF J-1 CALLS IN SYSTEM.
Q: LENGTH AVERAGE LENGTH OF QUEUE.
WAT: AVERAGE WAITING TIME IN THE QUEUE.

DIMENSION DI(100), DJ(100), D3(100), D4(100), PK(100)

READ, 9, U, R, NK, PK(1)
M = 1
DO 10 J = 1, M
10 READ, PK(J)
ITER = I / DELT
READ, NK, R1, R2
K = 1
IND = 1
PRINT 14, M, DELT, T, U
14 FORMAT(16, 'TRUNCATED CAPACITY = ', 13, '5X', 'STEP SIZE = ', FB.5
15, '4X', 'INTEGRATED LENGTH = ', FB.5, '5X', 'MEAN SERVICE RATE = ', FB.4)
15 PRINT 15, NK, J - 1, PK(J), J = 1, M
16 FORMAT('AT HOUR ', 13, '5X', 'PATROL CAR ON DUTY = ', 13, '5X', 'F8.61)
16 DO 70 K = 1, ITER
70 READ, NH, PK3
23 PK1 = PK2
24 PK2 = PK3
25 NH = K
26 IF(NH.GE.49) NH = NH - 24
27 IF(NH.GE.24) NH = NH - 24
28 PRINT 16, NH, NK
29 16 FORMAT('AT HOUR ', 13, '5X', 'PATROL CAR ON DUTY = ', 13, '13)
30 N = N + 1
31 ND = 1
32 RANK = (RK2 - RK1) * FLOAT(ND) * DELT * RK1
33 ND = ND + 1
34 M(1) = F01(PK(J), PK(?), U, RANK) * DELT
35 RK1 = RK1
36 M(0) = J >= 2, M1
IT (J.GE.NK1) GO TO 30
D1(J)=LM(PK(J-1),PK(J),PK(J+1),U,RANDK,J)*DELT
GO TO 50
10 30 IF (J.EQ.M1) GO TO 40
D1(J)=FGN(PK(J-1),PK(J),PK(J+1),U,RANDK,NK)*DELT
GO TO 50
40 40 D1(J)=FM(PK(J-1),PK(J),U,RANDK,NK)*DELT
50 CONTINUE
D2(J)=FD(PK(J-1)+1./2.*D1(J),PK(J+1)+1./2.*D1(J),
PK(J+1)+1./2.*D1(J),U,RANDK,J)*DELT
GO TO 51
51 31 IF (J.EQ.M1) GO TO 41
D2(J)=FGN(PK(J-1)+1./2.*D1(J),PK(J+1)+1./2.*D1(J),
PK(J+1)+1./2.*D1(J),U,RANDK,NK)*DELT
GO TO 51
41 41 D2(J)=FK(PK(J-1)+1./2.*D1(J),PK(J+1)+1./2.*D1(J),U,RANDK,NK)*DELT
51 CONTINUE
D3(J)=FD(PK(J-1)+1./2.*D2(J),PK(J+1)+1./2.*D2(J),U,RANDK,J)*DELT
GO 10 51
51 52 J=J+1
7 IF (J.GE.NK1) GO TO 32
32 D3(J)=FGN(PK(J-1)+1./2.*D2(J),PK(J+1)+1./2.*D2(J),
PK(J+1)+1./2.*D2(J),U,RANDK,J)*DELT
GO TO 52
52 32 IF (J.EQ.M1) GO TO 42
D3(J)=FGN(PK(J-1)+1./2.*D2(J),PK(J+1)+1./2.*D2(J),
PK(J+1)+1./2.*D2(J),U,RANDK,NK)*DELT
GO TO 52
42 42 D3(J)=FM(PK(J-1)+1./2.*D2(J),PK(J+1)+1./2.*D2(J),U,RANDK,NK)*DELT
52 CONTINUE
D4(J)=FD(PK(J)+D3(J),PK(J+1)+D3(J),U,RANDK)*DELT
GO 10 53
53 53 J=J+1
7 IF (J.GE.NK1) GO TO 33
33 D4(J)=FGN(PK(J-1)+D3(J),PK(J)+D3(J),PK(J+1)+D3(J),U,RANDK,
N1)*DELT
GO TO 53
53 53 D4(J)=FM(PK(J-1)+D3(J),PK(J)+D3(J),U,RANDK,N1)*DELT
53 CONTINUE
GO TO 53
60 60 J=J+1, M1
60 60 IF (K.LT.INDPRI) GO TO 70
70 PRINT 65,K,RANDK,J,J=1,P1
65 FORMATT(*4*,*STEP*,14,5X,'MEAN ARRIVAL RATE = ',F8.4/','*,F8.6)
80 80 J=1, M1
80 80 IF (K.LT.INDPRI) GO TO 70
70 PRINT 69,SUM
70 69 FORMATT('*','TOTAL PROBABILITY = ',F8.6)
FAACAK = 1.
DO 71 L = 1, AK
71 FAACAK = FACNK * L
UP = (PANDK / U) ^ (NK + 1) * PK(I) / NK
DOWN = FACNK * (1 - RANDK / U) ^ 2
QUETIM = UP / DOWN
WAITIM = QUETIM / RANDK
PRINT 67, WAITIM
67 FORMAT(' ', 'AVERAGE WAITING TIME IN THE QUEUE = ', F9.6, 'X', 'HR')
IND = IND + 1
CONTINUE
STOP
END
### A1 Hour 22
**Patrol Cars in Duty = 12**

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**Total Probability = 1.000749**

Average Waiting Time in the Queue = 0.000391 HR

### A1 Hour 23
**Patrol Cars in Duty = 12**

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**Total Probability = 1.000766**

Average Waiting Time in the Queue = 0.000368 HR

### A1 Hour 0
**Patrol Cars in Duty = 10**

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**Total Probability = 1.000728**

Average Waiting Time in the Queue = 0.000378 HR
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AT HOUR 22  PAYROL CANS EN DUTY = 10

STEP 500  MEAN ARRIVAL RATE = 10.1000
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  P8 0.0044039 P9 0.0107494 P10 0.001270 P11 0.0042554 P12 0.027557 P13 0.012888
  P14 0.0031727 P15 0.006158 P16 0.002543 P17 0.000976 P18 0.0000933
  P19 0.0000452 P20 0.0000452 P21 0.0000452 P22 0.0000452 P23 0.0000452
  P24 0.0000452 P25 0.0000452 P26 0.0000452 P27 0.0000452 P28 0.0000452
  P29 0.0000452
TOTAL PROBABILITY = 1.000000
AVERAGE WAITING TIME IN THE QUEUE = 0.003543

STEP 520  MEAN ARRIVAL RATE = 10.0000
  P2 0.000629 P3 0.0039126 P4 0.0037907 P5 0.0134531 P6 0.020604 P7 0.0170428
  P8 0.0014977 P9 0.0110306 P10 0.006830 P11 0.033548 P12 0.0024164 P13 0.0160562
  P14 0.0009424 P15 0.0003578 P16 0.001490 P17 0.0020144 P18 0.0009358 P19 0.0009358
  P20 0.0009358 P21 0.0009358 P22 0.0009358 P23 0.0009358
TOTAL PROBABILITY = 1.000000
AVERAGE WAITING TIME IN THE QUEUE = 0.003341

AT HOUR 23  PAYROL CANS EN DUTY = 10

STEP 540  MEAN ARRIVAL RATE = 9.9000
  P2 0.000611 P3 0.003312 P4 0.0028797 P5 0.0133627 P6 0.0113555 P7 0.0174300
  P8 0.0037886 P9 0.015660 P10 0.0065747 P11 0.036506 P12 0.003861 P13 0.0009473
  P14 0.0004645 P15 0.0002708 P16 0.001328 P17 0.000510 P18 0.000380 P19 0.0002515
  P20 0.0002515 P21 0.0002515 P22 0.0002515 P23 0.0002515
TOTAL PROBABILITY = 1.000000
AVERAGE WAITING TIME IN THE QUEUE = 0.003155

STEP 560  MEAN ARRIVAL RATE = 9.8000
  P2 0.0005808 P3 0.002742 P4 0.0026683 P5 0.0112127 P6 0.0116105 P7 0.0174603
  P8 0.0044305 P9 0.010725 P10 0.0039466 P11 0.040529 P12 0.0047451 P13 0.008719
  P14 0.009549 P15 0.0002214 P16 0.0001203 P17 0.0000990 P18 0.0000990 P19 0.0000990
  P20 0.0000990 P21 0.0000990 P22 0.0000990 P23 0.0000990
TOTAL PROBABILITY = 1.000000
AVERAGE WAITING TIME IN THE QUEUE = 0.002962

AT HOUR 0  PAYROL CANS EN DUTY = 4

STEP 580  MEAN ARRIVAL RATE = 9.7000
  P2 0.0005504 P3 0.0025545 P4 0.0026601 P5 0.0133241 P6 0.0145208 P7 0.0156229
  P8 0.0031091 P9 0.000914 P10 0.000603 P11 0.047635 P12 0.003230 P13 0.0024685
  P14 0.0003262 P15 0.0000933 P16 0.0004466 P17 0.000266 P18 0.000266 P19 0.000266
  P20 0.000266 P21 0.000266 P22 0.000266 P23 0.000266
TOTAL PROBABILITY = 1.000000
AVERAGE WAITING TIME IN THE QUEUE = 0.002700

D.3 A PART OF OUTPUT OF EXPERIENCED SCHEDULE
### Hour 1
**Patrol Cars In Use:** 6

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</tr>
<tr>
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<td>-0.000034</td>
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</tbody>
</table>

**Average Waiting Time in the Queue:** 0.244222 hours

### Hour 2
**Patrol Cars In Use:** 6

<table>
<thead>
<tr>
<th>Step</th>
<th>Mean Arrival Rate</th>
<th>PI</th>
<th>Total Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.7000</td>
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<tr>
<td></td>
<td></td>
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</tbody>
</table>

**Average Waiting Time in the Queue:** 0.344900 hours

### Hour 3
**Patrol Cars In Use:** 6

<table>
<thead>
<tr>
<th>Step</th>
<th>Mean Arrival Rate</th>
<th>PI</th>
<th>Total Probability</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
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</tbody>
</table>

**Average Waiting Time in the Queue:** 0.257969 hours
<table>
<thead>
<tr>
<th>Time Period</th>
<th>Average Arrival Rate</th>
<th>Average Waiting Time in IPE Queue</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 PM - 4 PM</td>
<td>6.0000</td>
<td>0.116492 HR</td>
</tr>
<tr>
<td>4 PM - 5 PM</td>
<td>5.0000</td>
<td>0.102803 HR</td>
</tr>
<tr>
<td>5 PM - 6 PM</td>
<td>4.0000</td>
<td>0.094095 HR</td>
</tr>
</tbody>
</table>

For each time period, the probability of the number of cars in the IPE queue is calculated.
<table>
<thead>
<tr>
<th>Step</th>
<th>Mean Arrival Rate</th>
<th>Average Waiting Time in the Queue</th>
<th>Total Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>10.1000</td>
<td>0.010963</td>
<td>-1.0000000000</td>
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<tr>
<td>520</td>
<td>10.0000</td>
<td>0.010963</td>
<td>-1.0000000000</td>
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<tr>
<td>440</td>
<td>9.9000</td>
<td>0.009596</td>
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<td>460</td>
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<tr>
<td>480</td>
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<tr>
<td>500</td>
<td>9.6000</td>
<td>0.009394</td>
<td>-1.0000000000</td>
</tr>
</tbody>
</table>

**A1 Hour 22**

**Police Cars in Duty:** 7

**A1 Hour 23**

**Police Cars in Duty:** 9
<table>
<thead>
<tr>
<th>Step 103</th>
<th>Mean Arrival Rate = 9.6000</th>
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</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.1-0.002749</td>
</tr>
<tr>
<td>P6</td>
<td>1.0-0.139999</td>
</tr>
<tr>
<td>P12</td>
<td>0.005108</td>
</tr>
<tr>
<td>P18</td>
<td>0.00814</td>
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<tr>
<td>P24</td>
<td>0.000008</td>
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<tr>
<td>P30</td>
<td>0.000000</td>
</tr>
<tr>
<td>Total Probability</td>
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</tr>
<tr>
<td>Average Waiting Time in the Queue</td>
<td>0.007549 HR</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 104</th>
<th>Mean Arrival Rate = 9.1500</th>
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</thead>
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<tr>
<td>P1</td>
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<tr>
<td>P6</td>
<td>0.1-0.134699</td>
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<tr>
<td>P12</td>
<td>0.00559</td>
</tr>
<tr>
<td>P18</td>
<td>0.000018</td>
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<tr>
<td>P24</td>
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</tr>
<tr>
<td>P30</td>
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</tr>
<tr>
<td>Total Probability</td>
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<tr>
<td>Average Waiting Time in the Queue</td>
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</table>

<table>
<thead>
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<tbody>
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<td>P1</td>
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<td>0.1-0.127201</td>
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<td>P12</td>
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<td>Total Probability</td>
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<tr>
<td>Average Waiting Time in the Queue</td>
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<table>
<thead>
<tr>
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<tr>
<td>Total Probability</td>
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<table>
<thead>
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<td>P1</td>
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<tr>
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<tr>
<td>Average Waiting Time in the Queue</td>
<td>0.001299 HR</td>
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<td>Hour</td>
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</tr>
</tbody>
</table>
TRANSIENT SOLUTIONS OF M/M/s
NONSTEADY STATE QUEUEING SYSTEM

by

REI KUNG SUN

B.S. (Industrial Engineering)
Chung Yuan Christian College of Science & Engineering
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AN ABSTRACT OF A MASTER'S THESIS
submitted in partial fulfillment of the
requirements for the degree

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KANSAS STATE UNIVERSITY
Manhattan, Kansas

1981
ABSTRACT

Queues are very common phenomena, widely encountered in every day life. Most queueing models, which assume that the mean arrival rate and the service rate are constant through time and that the system is in a steady state, have been well studied in the past. Unfortunately this constancy of rate is often not the case in real queueing problems. For instance, customer arrival rates change with the time of day in the supermarket; and car arrival rates at toll booths peak at 8 A.M. and 5 P.M.

It is very common to use the simulation approach to interpret queueing problems in the transient state although there are many pitfalls one may encounter in using it, and its analysis is relatively expensive and cumbersome. With the aid of modern digital computers, a number of numerical approaches and approximate methods for determining the transient solution of nonsteady state queueing problems have been invented.

One of the main objectives of this study is to do a limited survey of the literature of these numerical or approximate approaches.

Because of its simplicity and ease of use, the so-called standard numerical method is applied to police patrol car scheduling problems. Because the number of patrol cars on duty is a discontinuous function of time, the fourth-order Runge Kutta numerical method was selected. The Kolesar et al methodology was applied to solve this patrol scheduling problem.

The results show that the police administrator can select a reasonable schedule by applying a M/M/s nonsteady state queueing model to evaluate patrol scheduling alternatives. The current (experienced) schedule was evaluated, based both on a specified probability of an urgent call being placed in queue as criterion, and on a specified expected waiting time of a call in the queue as criterion. From both points of view, the experienced schedule is extremely inadequate.