ANALYSIS AND DESIGN P/C CYLINDRICAL SHELLS

by

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I. INTRODUCTION AND SCOPE

The first concrete shell was built in Europe as early as 1924, but there was little or no interest in this form of construction in the United States, until 1954, when The First Conference on Thin Shells which met at MIT may be said to have marked a definite turning point. Since that time, engineers and architects have become increasingly aware of the economic and aesthetic possibilities offered by shell roofs for enclosing large column-free spaces required for a variety of applications.

As the span of a cylindrical shell becomes large, the tension in the shell edge or the edge beam reaches a very high value demanding the provision of very heavy reinforcement. Additionally, the deflections become excessive and large transverse moments are created which at the crown can not be readily resisted. For these reasons, a design in reinforced concrete often tends to be uneconomical. Prestressing the shell is the answer.

The purpose of this report is to compare the design of a prestressed cylindrical shell without edge beams with that for a prestressed cylindrical shell with edge beams. A description of the methods of analysis is presented and comparisons made between the two designs in terms of stress distributions. Cost considerations for the two designs are also made.
II. LITERATURE SURVEY

History of Design and Construction of Shells

The thin reinforced-concrete shell, as we know it today, had its beginnings in Germany in the 1920s. Most of the early shells built were cylindrical barrels. In 1924, the first concrete shell roof was designed by Carl Zeiss and built in the Zeiss works in Jena, Germany.

History of Analysis Methods (7, 9)

The first analytical approach to the design of shells was presented by G. Lame' and E. Clapeyron, who in 1826 produced the "membrane analogy" in which a shell was considered capable of resisting external loads by direct stresses unaccompanied by any bending. The next important contribution in this field was made in 1892 by A. E. H. Love, who developed mathematical conceptions which made possible a more accurate analysis than could be achieved by membrane analogy. Around the year 1923 U. Finsterwalder and F. Dischinger were the first to develop a theoretical analysis applicable to reinforced concrete cylindrical shells. In the United States, H. Schorer further simplified the derivation of Finsterwalder (1936). Until about 1940, the cylindrical shell more or less dominated the scene. The beam theory for shell analysis was developed by H. Lundgren of Denmark in 1949. This consists of separate analyses in which the shell is considered first as a beam and secondly as an arch. The load balancing method, which was developed by T. Y. Lin, for prestressed concrete member offers a new approach and greatly simplifies the design of prestressed shells.

Design Consideration

(a) Selection of shell type (7). For covering very large areas, for hangars,
warehouses, etc., short shells are usually economical. The span of the short shell may be chosen as between one-sixth and one-third of the chord width. A practical limit on the span of long reinforced-concrete shells is about 100 ft. For longer shells, prestressing will prove economical.

(b) The radius of a cylindrical shell has (7) to be chosen keeping acoustic considerations in view. It is desirable to see that the center of curvature does not lie at the working level.

(c) Semicentral angle (7)

The practice is to keep the semicentral angle between 30° and 45°. If the angle exceeds 45°, concreting becomes difficult without the use of top forms. If the angle is below 40°, wind load can be ignored, because it causes only a suction on the shell.

(d) Thickness (7, 9)

The minimum thickness of reinforced-concrete cylindrical shells is governed by practical considerations such as accommodating reinforcement and providing adequate cover. According to a Dutch report (7) the usual recommended thickness is between 7 and 8 cm. A minimum of 4 cm is recommended by the Institute for Typification of the German Democratic Republic at Berlin

(e) Width of Edge Beam (7)

A width of two to three times the thickness of the shell would usually suffice. A minimum of 6" is demanded by practical considerations

(f) Design of Reinforcement (non-prestressed) (8)

a) The ratio of steel to concrete in any portion of the tensile zone should not be less than 0.35% of the cross sectional area of concrete.
b) The minimum temperature and shrinkage steel should not be less than 0.14% of the cross sectional area of concrete.

c) The maximum spacing of bars should not exceed 40 bar diameters nor five times the thickness of the shell.
III. METHOD OF ANALYSIS

1. Surface Geometry

The cylindrical shell with a circular directrix and radius $R$ is shown along with the coordinate system in Fig. 1 where the positive direction of the load components per unit area of the surface are also indicated.

2. Load Balancing Method

Consider the cylindrical shell without edge beams in Fig 2. The cables can be post-tensioned along the shell surface so that the vertical component of a cable will balance the gravity load. The prestressing forces, and its vertical component $W_v$ and horizontal component $W_h$ are given by (6)

$$H = -\frac{Wt^2}{8f_v}\frac{1}{2}, \quad (3-1a)$$

$$W_v = H\frac{8f_v}{L^2}, \quad (3-1b)$$

$$W_h = H\frac{8f_h}{L^2}, \quad (3-1c)$$

In these expressions $W$ is the uniform load of the shell per linear ft along the X axis, $L$ is the span of the shell, $f_v$ is the projected vertical sag of the parabolic cable and $f_h$ is the projected horizontal sag of the parabolic cable.

3. Beam Theory

The beam theory involves two analyses:

(A) The beam analysis (Fig. 3)

(i) The longitudinal stresses at any cross section of the shell are computed on the basis of the simple flexural theory
\[ N_x = \frac{M_c}{I} \]  

(3 - 2a)

where \( I \) is the moment of inertia of the shell cross-section about the axis yy. (Fig. 5) It may be shown that (7).

\[ I = R^3t [\phi_c + \sin \phi_c (\cos \phi_c - \frac{2\sin \phi_c}{\phi_c})] \]  

(3 - 2b)

The shearing stresses are computed by the corresponding simple expression

\[ N_{x\phi} = \frac{VQ}{I_b} \]  

(3 - 2c)

where \( Q \) is the first statical moment of the cross section up to the point under consideration about the axis yy and is (7)

\[ Q = 2 \cdot R^2t \cdot (\sin \phi - \frac{\phi \sin \phi_c}{\phi_c}) \]  

(3 - 2d)

(ii) For the case of the prestressing force acting on the shell (Fig. 4) the vertical component of the cable force is equal to some gravity load, therefore, the beam shear force and bending moment are zero for this load.

The longitudinal force is

\[ N_x = -\frac{2H}{A} \]  

(3 - 3a)

where \( A \) is the cross-sectional area of the shell.

The following equations for transverse forces and moments have been developed by using the free bodies shown in Fig. 6. The force \( F_y \) was obtained by summing vertical forces, and \( M_y \) by taking moments about the intersection of a radial line at angle \( \phi \) and the arc of the shell.

a) Forces due to Dead Load (Fig. 6a)
\[ F_V = g R (\phi - \phi_c), \]  
\[ M_\phi = -g R^2 \left[ \cos \phi - \cos \phi_c - \sin \phi (\phi - \phi_c) \right], \]  
\[ N_\phi = g R (\phi - \phi_c) \sin \phi, \]  
\[ Q_\phi = -g R (\phi - \phi) \cos \phi. \]  

\[ (3 - 4a) \]  
\[ (3 - 4b) \]  
\[ (3 - 4c) \]  
\[ (3 - 4d) \]  

b) Forces due to Snow Load (Fig. 6b)  
\[ F_V = p_s R (\sin \phi_c - \sin \phi), \]  
\[ M_\phi = \frac{1}{2} p_s R^2 (\sin \phi_c - \sin \phi), \]  
\[ N_\phi = p_s R \sin \phi (\sin \phi_c - \sin \phi)^2, \]  
\[ Q_\phi = -p_s R \cos \phi (\sin \phi_c - \sin \phi). \]  

\[ (3 - 5a) \]  
\[ (3 - 5b) \]  
\[ (3 - 5c) \]  
\[ (3 - 5d) \]  

c) Forces due to Prestressing Load (Fig. 6c)  
\[ M_\phi = -W_{h1} R \left[ \cos \phi - \cos \phi_i (\chi) \right] - W_{V1} R \left[ \sin \phi - \sin \phi_i (\chi) \right], \]  
\[ N_\phi = -W_{h1} \cos \phi - W_{V1} \sin \phi, \]  
\[ Q_\phi = -W_{h1} \sin \phi + W_{V1} \cos \phi, \]  

\[ (3 - 6a) \]  
\[ (3 - 6b) \]  
\[ (3 - 6c) \]  
in which \( M_\phi = N_\phi = Q_\phi = 0 \) when \( \phi \) is greater than \( \phi_i (\chi) \).  

(B) The Arch Analysis  
When forces acting on the shell are balanced by prestressing forces, there is no shear flow produced at all, which means no arch
action in the shell; but when the acting forces are unbalanced by the
prestressing forces, shear flow will appear and the arch action will
have an effect on the shell response. The vertical components of the
shear flow balance the load on the shell arch; the horizontal compon-
ents of the shear flow which are symmetrically disposed about the crown
balance themselves. The resulting internal forces are

\[ F_v = -f^{c}_\phi \frac{c_0^{c} R^2 c_0}{I} \sin \phi d\theta + gR(c_c - c) \]  \hspace{1cm} (3 - 8a)

\[ F_h = -f^{c}_\phi \frac{c_0^{c} R^2 c_0}{I} \cos \phi R d\theta \]  \hspace{1cm} (3 - 8b)

\[ Q = -F_v \cos \phi + F_h \sin \phi \]  \hspace{1cm} (3 - 8c)

\[ N = F_v \sin \phi + F_h \cos \phi \]  \hspace{1cm} (3 - 8d)

\[ M = \int^{\phi}_c c_0^{c} R^3 c_0^{c} Q(\theta) \sin \phi \left( \sin \theta \right) \left( \cos \phi \right) \left( -\sin \phi \right) d\theta \]

\[ \int^{\phi}_c c_0^{c} R^3 c_0^{c} Q(\theta) \cos \phi \left( -\cos \phi \right) \left( -\cos \phi \right) d\theta - \\ gR^2 \left[ \cos \phi - \cos c_c - \sin \phi \left( c_c - \phi \right) \right]. \]  \hspace{1cm} (3 - 8e)

4. Membrane Theory

The differential equations of equilibrium of a shell based on membrane
theory are given by

\[ \frac{2N_x}{\theta} + \frac{1}{R} \frac{2N_x}{\phi} + X = 0 \]  \hspace{1cm} (3 - 9a)
\[ \frac{1}{R} \frac{\partial^2 N\phi}{\partial \phi^2} + \frac{\partial N x\phi}{\partial x} + y = 0, \quad (3 - 9b) \]

\[ N\phi - Rz = 0, \quad (3 - 9c) \]

where the angle \( \phi \) is measured from the crown.

Substituting stress-strain relations and strain-displacement relations into the equilibrium equations, and combining gives the resultant forces.

(i) Stresses and Displacements under Dead Load (7)

A uniform load \( g \) is developed in a Fourier Series as

\[ g = \frac{4}{\pi} g \sum_{n=1,3,5,\ldots} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{n\pi x}{L}. \quad (3 - 10) \]

Taking only the first term into account, the components of the dead load are

\[ x = 0, \quad (3 - 11a) \]

\[ z = -\frac{4}{\pi} g \cos \frac{\pi x}{L} \cos \phi, \quad (3 - 11b) \]

\[ y = -\frac{4}{\pi} g \cos \frac{\pi x}{L} \sin \phi, \quad (3 - 11c) \]

Now from (3 - 9c),

\[ N\phi = -\frac{4}{\pi} Rg \cos \frac{\pi x}{L} \cos \phi, \quad (3 - 12a) \]

From (3 - 9b),

\[ N x\phi = -\int \left( \frac{1}{R} \frac{\partial^2 N\phi}{\partial \phi^2} + y \right) dx = \frac{8g}{\pi^2} g \sin \frac{\pi x}{L} \sin \phi, \quad (3 - 12b) \]
From (3 - 9a),

\[ N_x = - \frac{1}{R} \int \frac{\partial N_x}{\partial \phi} \, dx = - \frac{8g \ell^2}{R \pi^2} \cos \frac{\pi x}{L} \cos \phi, \quad (3 - 12c) \]

The corresponding displacements can be shown to be the following:

(7)

Transverse displacement,

\[ v = \frac{8g}{\pi \ell^2} \sin \phi \cos \left( \frac{\pi x}{L} \left( \frac{2}{k^2} + \frac{1}{R^2 k^4} \right) \right), \quad (3 - 12d) \]

where \( k = \pi / \lambda \), corresponding to the first Fourier term.

Normal displacement,

\[ w = \frac{\partial v}{\partial \phi} = - \frac{8g}{\pi \ell^2} \cos \phi \cos \left( \frac{\pi x}{L} \left( \frac{2}{k^2} + \frac{1}{R^2 k^4} \right) \right), \quad (3 - 12e) \]

Longitudinal displacement,

\[ u = - \frac{1}{E \ell} \frac{8g}{R k^3} \cos \phi \sin \frac{\pi x}{L} \]

(3 - 12f)

(ii) Stresses and Displacement under snow load (7)

As before, a snow load \( p_0 \) can be represented as
\[ p_0 = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{n-1} \frac{(-1)^{n-2}}{n} \cos \frac{n\pi x}{\ell} \]

Proceeding in the same manner as for the dead load the following expressions will be obtained (7),

\[ N_\phi = -\frac{4}{\pi} P_\circ R \cos \frac{\pi x}{\ell} \cos^2 \phi \quad (3 - 13a) \]

\[ N_x \phi = -\frac{6}{R^2} P_\circ \ell \sin \frac{\pi x}{\ell} \sin 2\phi \quad (3 - 13b) \]

\[ N_x = -\frac{12}{R^3} P_\circ \ell^2 \cos \frac{\pi x}{\ell} \cos 2\phi \quad (3 - 13c) \]

\[ v = +\frac{24}{E_\ell \pi^2} P_\circ R^2 \left( \frac{\ell}{R} \right)^4 \left[ (\frac{R \pi}{\ell})^2 + 2 \right] \sin 2\phi \cos \frac{\pi x}{\ell}, \quad \text{(3 - 13d)} \]

\[ w = \frac{24}{E_\ell \pi^2} P_\circ R^2 \left( \frac{\ell}{R} \right)^4 \left[ (\frac{R \pi}{\ell})^2 + 2 \right] \cos 2\phi \cos \frac{\pi x}{\ell}, \quad \text{(3 - 13e)} \]

and

\[ u = -\frac{12}{E_\ell \pi^2} P_\circ \ell^3 \cos 2\phi \sin \frac{\pi x}{\ell}. \quad \text{(3 - 13f)} \]

5. The Bending Theory

(A) The forces affecting the equilibrium of the shell element are set up by referring to Fig. 8.
\[ \frac{3N_x}{\partial x} + \frac{1}{R} \frac{3N_x \phi}{\partial \phi} = 0, \]  
\[ (3 - 14a) \]

\[ \frac{3N\phi}{\partial \phi} + R \frac{3N\phi x}{\partial x} = 0, \]  
\[ (3 - 14b) \]

\[ R \frac{3Qx}{\partial x} + \frac{3Q\phi}{\partial \phi} + N\phi = 0, \]  
\[ (3 - 14c) \]

\[ R \frac{3Mx\phi}{\partial \phi} + \frac{3M\phi}{\partial \phi} - R Qx = 0, \]  
\[ (3 - 14d) \]

\[ \frac{3M\phi x}{\partial \phi} + R \frac{3Mx}{\partial x} - R Qx = 0, \]  
\[ (3 - 14e) \]

\[ Nx\phi - N\phi x = 0. \]  
\[ (3 - 14f) \]

Substituting stress-strain relations and strain-displacement relations into the equilibrium equations, and combining leads to (1)

\[ \left[ R^2 \frac{3^8}{\partial x^8} + 4R^6 \frac{3^8}{\partial x^6 \partial \phi^2} + 6R^4 \frac{3^8}{\partial x^4 \partial \phi^4} + 4R^2 \frac{3^8}{\partial x^2 \partial \phi^6} + \frac{3^8}{\partial \phi^8} \right] + \frac{R^4}{A} \frac{3^8}{\partial x^4 \partial \phi^4} = 0. \]  
\[ (3 - 15) \]

This is Donnell's equation in \( w \).

Use the following formulation for \( w \) which satisfies the boundary conditions at the ends, i.e.

\[ w = He^{m\phi} \cos \frac{\lambda x}{R}, \]  
\[ (3 - 16) \]

in which \( H = \text{const}, \ m = \text{parameter}, \ \lambda = \frac{\pi R}{l}. \)
Substituting this into Eq. (3 - 15) leads to

\[(m^2 - \lambda n^2)^4 + \frac{\lambda n^4}{k} = 0, \quad (3 - 17)\]

where

\[k = t^2/(12 R^2).\]

The eight roots \(m_1, m_2, \ldots, m_8\) of the solution of Eq. (3 - 17) are

\[m_1 = a_1 + i\beta_1 \quad m_5 = -m_1\]

\[m_2 = a_1 - i\beta_1 \quad m_6 = -m_2\]

\[m_3 = a_2 + i\beta_2 \quad m_7 = -m_3\]

\[m_4 = a_2 - i\beta_2 \quad m_8 = -m_4\]

where

\[a_1 = \frac{\rho}{8^4} \left[ \sqrt{(1 + \kappa \sqrt{2})^2 + 1} - (1 + \kappa \sqrt{2}) \right]^{1/2},\]

\[a_2 = \frac{\rho}{8^4} \left[ \sqrt{(1 - \kappa \sqrt{2})^2 + 1} - (1 - \kappa \sqrt{2}) \right]^{1/2},\]

\[\beta_1 = \frac{\rho}{8^4} \left[ \sqrt{(1 + \kappa \sqrt{2})^2 + 1} + (1 + \kappa \sqrt{2}) \right]^{1/2},\]

\[\beta_2 = \frac{\rho}{8^4} \left[ \sqrt{(1 - \kappa \sqrt{2})^2 + 1} + (1 - \kappa \sqrt{2}) \right]^{1/2},\]

\[p = \frac{\rho}{4\sqrt{2}} = \left( \frac{3\pi^4}{t^2} \right)^{1/6}, \quad \kappa = \left( \frac{2\pi^4 \bar{2}}{3 \xi^4} \right)^{1/4}.\]
THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE. THIS IS AS RECEIVED FROM CUSTOMER.
From strain displacement relationships, stress-strain relationships and assuming Poisson's Ratio is zero, the force and moment displacement equations are obtained in the following matrix form.

\[
\begin{bmatrix}
N_x \\
M_\phi \\
u \\
w
\end{bmatrix} =
\begin{bmatrix}
\frac{4DRk^4}{P^2\kappa^2} \cos^2 \frac{x}{\lambda} & 0 & 0 & 0 \\
0 & \frac{-4DRk^4}{\kappa^2} \cos \frac{\pi x}{\lambda} & 0 & 0 \\
0 & 0 & \frac{4DRk^4}{E\kappa^3\ell} \sin \frac{\pi x}{\lambda} & 0 \\
0 & 0 & 0 & 2
\end{bmatrix}
\begin{bmatrix}
x \\
f_1 \\
f_2 \\
f_3 \\
f_4
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1 & 1+\kappa & 1 & 1-\kappa & 0 \\
1+\kappa & 1 & \kappa-1 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 \\
-1 & 1+\kappa & 1 & 1-\kappa & 0 \\
1 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
f_1 \\
f_2 \\
f_3 \\
f_4 \\
0
\end{bmatrix}
\begin{bmatrix}
A_n \\
B_n \\
C_n \\
D_n \\
0
\end{bmatrix}
\]

(3 - 19)
\[
\begin{bmatrix}
0 & \frac{4DK_3^3}{(v^2)^3p} \cos \frac{\pi x}{\lambda} & 0 & 0 & 0 \\
0 & 0 & \frac{2DK_3^3}{(v^2)^3p} \cos \frac{\pi x}{\lambda} & 0 & 0 \\
0 & 0 & 0 & \frac{2DK_3^3}{(v^2)^3p} \cos \frac{\pi x}{\lambda} & 0 \\
0 & 0 & 0 & 0 & \cos \frac{\pi x}{\lambda}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\beta_1 & a_1 & \beta_2 & a_2 & 0 \\
\alpha_1 - \beta_1 & a_1 + \beta_1 & a_2 + \beta_2 & a_2 - \beta_2 & 0 \\
\alpha_1 + \beta_1(1-\kappa) & -\alpha_2 + \beta_2(1+\kappa) & -\beta_2(1-\kappa) & -\beta_2(1+\kappa) & 0 \\
2a_1 + (\bar{R}B_1) & 2a_1 + (\bar{R}B_2) & 2a_1 + (\bar{R}B_3) & 2a_1 + (\bar{R}B_4) & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
f_1^1 \\
f_2^1 \\
f_3^1 \\
f_4^1 \\
f_5^1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
A_n \\
B_n \\
C_n \\
D_n \\
0
\end{bmatrix}
\]

Observe that one part of the coefficients \(B_1\), etc., for \(\theta\) are obtained from \(v\), that is, \((\bar{R}B_1) = \frac{4DK_3^3}{Et(\sqrt{v^2})^3p}[\alpha_1 + \beta_1(1-\kappa)]\).
in which $D = EI$, $f_1 = \cos \beta_1 \phi \cosh \alpha_1 \phi$, $f_2 = \sin \beta_1 \phi \sinh \alpha_1 \phi$, $f_3 = \cos \beta_2 \phi \cosh \alpha_2 \phi$, $f_4 = \sin \beta_2 \phi \sinh \alpha_2 \phi$, $f_1' = \cos \beta_1 \phi \sinh \alpha_1 \phi$, $f_3' = \cos \beta_2 \phi \sinh \alpha_2 \phi$, $f_4' = \sin \beta_2 \phi \cosh \alpha_2 \phi$.

The arbitrary constants $A_n$, $B_n$, $C_n$, and $D_n$.

(B) Formulation of boundary conditions for a P/C shell with edge beam

The boundary conditions applicable to this shell at its junction with the edge beam are stated as follows:

(a) The resultant horizontal force at shell edge is equal to zero, i.e.,

$$\left( N_\phi \right)_{0 b} \cos \phi_c - \left( Q_\phi \right)_{0 b} \sin \phi_c = 0 \quad (3 - 21a)$$

(b) The transverse moment $M_\phi$ at the shell edge is equal to zero, i.e.,

(assuming negligible torsion stiffness in the edge beam)

$$M_\phi = 0 \quad (3 - 21b)$$

(c) The vertical deflection of the shell edge is equal to the vertical deflection of the edge beam. This condition is formulated below.

1. The edge beam prestressed by a curved cable is shown in Fig. (9a). The effect of prestressing can be subdivided into three particular cases of loading (Fig. 9b): (4)

(d) An upward vertical force $W_\nu$ caused by the curvature of the cable,
\[ W_v = \frac{8 \, H(e + e_1)}{k^2} \]  

(3-21c-1)

(ii) End moments \( M \) due to eccentricity of anchorage,

\[ M = H e_1. \]  

(3-21c-2)

(iii) An axial compressive force \( H \) applied at both ends at the center of gravity of beam section.

For vertical displacement,

\[ \begin{align*} 
-(v)_{m+b} \sin \phi_c + (v)_{m+b} \cos \phi_c &= \frac{1}{K'} \left[ (N')_{m+b} \sin \phi_c + Q' \cos \phi_c - 
\right] \\
(Nx')_{m+b} k a_1 &= -\frac{4}{\pi} W + \frac{4}{\pi} x \frac{8H(e+e_1)}{k^2} - \frac{4}{\pi} H e_1 k^2 
\end{align*} \]  

(3-21c-3)

2. Maximum stresses in edge beam

Shear force,

\[ S_1 = \frac{1}{k}(Q' \cos \phi_c + N' \sin \phi_c - \frac{4}{\pi} W); \]  

(3-21c-4)

\[ q_c = -\frac{H'}{A} + \frac{H(e+e_1)a_1}{I} - \frac{M_{a_1}}{I}; \]  

(3-21c-5)

\[ q_b = -\frac{H'}{A} - \frac{H(e+e_1)a_1}{I} + \frac{M_{a_1}}{I}; \]  

(3-21c-6)

\( H' = H - F \) (tensile force due to shear at top of beam)

where \( a_1 \) is at top or bottom edge.

(d) Longitudinal displacement of the shell edge is equal to the longitudinal displacement of the edge beam at its junction with the shell, i.e.,
\[(u)_{m+b} = \{ (N_\phi)_{m+b} \sin \phi_c + (Q_\phi)_{m+b} \cos \phi_c \} a_1 \frac{1}{k^2 E I} k - \]

\[\left( \frac{N_{x\phi}}{k^2} \right)_{m+b} \frac{1}{AE} \right] - \left( (N_{x\phi})_{m+b} a_1^2 k^2 \frac{1}{k^2 E I} \right) - \]

\[a_1 \frac{1}{k^2 E I} k \frac{4}{\pi} W \}

(3 -21d)
IV. NUMERICAL SOLUTIONS AND DESIGN EXAMPLE

The P/C cylindrical shells without and with edge beam used for design examples are shown in Fig. 10.

Design Loads

Dead load $g = 25$ psf of surface area

Snow load $p = 20$ psf of horizontal projection

Maximum design load of Freyssinet System's cable=$54,000$ lb/cable

Material Parameters

Young's modulus $E = 360 \times 10^6$ psf

Poisson's ratio $\nu = 0$

1. Comparison between beam theory solution and bending theory solution for the P/C shell without edge beam under dead load, snow load and cable load. The results of computer calculations for the force resultants and the comparison between beam theory solution and bending theory solutions in which it is assumed that horizontal cable forces are acting on the shell edge are given Table 1. The results of beam solutions for dead load and cable load only are also shown in Table 2. Figs. 11, 12, and 13 illustrate the distribution of the force resultants.

Comments on comparison of these results.

(a) The values of $N_\phi$ show little difference at the crown between the two theories, but at the edge they are different.

(b) The values of $N_{\phi}$ are different. There is nothing in the beam solution because the gravity load is balanced by upward cable forces.

(c) The compressive forces $N_x$, are constant at all cross sections in the beam solutions because there are no beam bending moments produced.

(d) The transverse bending moment $M_\phi$ in the bending solutions are about 8
to 14 times of that in the beam solutions at midspan.

Generally, using a beam theory it is easier to analyze effects of the prestressing cables than using a bending theory in which it is assumed all the horizontal forces are acting on the shell edge.

2. Design Example for the shell without edge beam and the shell with edge beam (1, 8, 9)

(1) Reinforcement (using elastic method)

\[ f_s = 24,000 \text{ psi}, \quad f_y = 60,000 \text{ psi}, \quad f_c = 1,800 \text{ psi}, \]

\[ f_c' = 4,000 \text{ psi} \]

The minimum reinforcement ratio in the tensile zone at any portion should not less than 0.0035.

\[ A_s = 0.0035 (12 \text{ t}) \text{ in}^2/\text{ft} \]

for \( t = 3 \text{ in} \), \[ A_s = 0.126 \text{ in}^2/\text{ft} \]

Also, the maximum spacing of bars in any portion should not exceed 40 diameters or five times the thickness of the shell given in CRSI Design Handbook. In this design, this is 10".

(a) Longitudinal Steel for \( N_x \)

The requirement of steel area can be calculated by the following formula.

\[ A_{sl} = \frac{N_x}{f_s} , \text{ if } N_x \text{ is in tension} \]

In the examples, the values of \( N_x \) are compressive at all cross sections, therefore, the bar number and the spacings are chosen based on the minimum temperature and shrinkage steel requirements.

(b) Diagonal Steel for the in-plane Shear Force

As can be observed from Figs. 13 and 14, the maximum shear force
for both shells $N_\phi$ is at the end. Also note that shearing
stresses in the shell without edge beam are low enough such that
diagonal reinforcement is not necessary.

From determining the steel necessary to resist the tensile forces,
the principal forces obtained by combining direct forces and
tangential shears must be evaluated. This can be done by using
the governing equations that

$$T_p = \frac{N_x + N_\phi}{2} \pm \sqrt{\left(\frac{N_x - N_\phi}{2}\right)^2 + N_x^2}$$

in which

$T_p$ = the principal forces. The plane on which the first prin-
cipal force acts is given by

$$\tan 2\delta = \frac{2N_x \delta}{N_x - N_\phi}$$

in which, for positive values of $\tan 2\delta$, $\delta$ is measured in a
counter clockwise direction from the face on which $N_x$ acts.

With these, the steel cross sectional area as for the principal
tensile forces can be calculated by the following equation,

$$A_{sd} = \frac{T_p}{f_s \cos^2 (45^\circ - \delta)}$$

For this, bar numbers and spacings for the shell with edge beams
are given in Table 8.

(c) Transverse Steel for $N_\phi$, $M_\phi$

Figs. 11b, 12b, 15b, and 16b indicate that the maximum effects
for $N_\phi$, $M_\phi$ are in the midspan at the crown. For the shell with-
out edge beam $N_\phi = -1,319$ lb/ft, $M_\phi = -1,408$ lb-ft/ft, and for
the shell with edge beam $N_\phi = -1,447 \text{ lb/ft}$, $M_\phi = 815 \text{ lb-ft/ft}$.

Observing Figs. 20a and 20b, the transverse reinforcement $A_s$ for the maximum effect for $N_\phi$, $M_\phi$ is calculated as follows.

For the P/C shell without edge beam the eccentricity

$$e = \frac{N_\phi jd + M_\phi}{N_\phi} = 13.68 \text{ in},$$

$$A_{st} = \frac{N_\phi(e - jd)}{f_{s}jd} = \frac{1,319(13.68 - 0.875 \times 2.625)}{24,000 \times 0.875 \times 2.625}$$

$$= 0.2724 \text{ in}^2/\text{ft},$$

the spacing and bar number are shown in Table 6.

For the P/C shell with edge beam the eccentricity

$$e = \frac{N_\phi jd + M_\phi}{N_\phi} = 7.63 \text{ in},$$

$$A_{st} = \frac{N_\phi(e - jd)}{f_{s}jd} = \frac{1,447(7.633 - 0.875 \times 2.625)}{24,000 \times 0.875 \times 2.625}$$

$$= 0.13 \text{ in}^2/\text{ft},$$

Results giving bar spacings and number are given for both shells in Tables 6 and 7.

(d) Tensile Steel for Edge Beams

$$A_s = \frac{1/2 ft \times B x}{f_s} = \frac{1/2 \times 260 \times 6}{24,000} = 0.6165 \text{ in}^2$$

For this, bar numbers shown in Fig. 19.

(2) The adequacy of the design with respect to ACI 318-71 requirements

(a) The maximum steel area/per foot should be less than

$$\frac{7.2 \times f_c}{f_y} = \frac{7.2 \times 3 \times 4,000}{60,000} = 1.44 \text{ in}^2,$$

or $$\frac{29,000 \times 3}{60,000} = 1.45 \text{ in}^2$$

0.K.

(b) The maximum spacing
Because $4 \sqrt{f_C} = 227.68$ psi > the computed tensile stresses due to design load, the maximum spacing allowed could be greater than three times the thickness, $3h = 9$, but not farther apart than five times the thickness nor 18 in. (capacity-reduction factor $\phi = 0.9$)

(c) The ratio of the minimum reinforcement per foot to the concrete area is equal to 0.0014.

For #3 @15", $A_g/\text{ft} = 0.088 \text{ in}^2/\text{ft}$, and

$$\frac{\text{steel area}}{\text{concrete area}} = \frac{0.088}{12 \times 3} = 0.00244 > 0.0014$$

O.K.

(d) The minimum reinforcement ratio in the tensile zone at any portion shall not be less than 0.0035

For #3 @10", $A_g/\text{ft} = 0.13 \text{ in}^2/\text{ft}$

$$\frac{0.13}{12 \times 3} = 0.0036 > 0.0035$$

O.K.

3. Comparison between the P/C cylindrical shell without edge beam and the P/C cylindrical shell with edge beam.

The numerical solutions of the P/C shell without edge beam are based on beam theory, and the P/C shell with edge beam are based on the bending theory.

The comparisons of design results for these two shells are presented in Table 4. The results of the calculations for the force resultants of the P/C shell with edge beam are given in Table 3. Figs. 14, 15 and 16 illus-
strate the distribution of the force resultants. The stress resultants of the edge beam which are based on Eqs. (3-21c-4), (3-21c-5), and (3-21c-6) are also given in Table 5.

Comments on comparison of results.

(a) It is obvious that by the introduction of edge beam the shell as a unit has reduced forces and moments.

(b) The design results show that the P/C shell without edge beam requires more cables but less concrete; the P/C shell with edge beam needs more concrete and reinforcement but less cables.

4. Discuss the results shown in Figs. 17, 18 and 19.

The requirement of cables in the P/C shell without edge beam is twice that of the P/C shell with edge beam. The volume of concrete in the P/C shell without edge beam is about 40% less than that required in the P/C shell with edge beam, but the weight of steel used is almost the same for both cases.

Theoretically, there is no stirrup reinforcement required for the edge beam; usually some stirrup reinforcement are used to hold the cables in a stable position.
V. DISCUSSION AND CONCLUSIONS

1. The beam theory is easy and simple to use and also can be applied to shells with noncircular directrices.

2. The structural action of the shell using the beam theory is easily visualized.

3. In the bending solutions the force calculation are based upon only the first term of the Fourier Series.

4. The wind load was not taken into consideration in this report because the semicentral angle does not exceed 45°, and therefore the wind causes only a suction on the shell. This will result in a decrease in shell forces.

5. For the P/C shell with edge beam, the bending forces and longitudinal forces are small compared to those forces in the P/C shell without edge beam under dead load, snow load, and cable load.

6. It is much easier to layout the cables on the edge beam than those along the shell surface.

7. It has been shown that the P/C shell without edge beams needs more cables and reinforcement but less concrete, than the P/C shell with edge beam.

8. For design purposes the shell forces should be calculated under the action of dead load, snow load and cable load acting on the shell and dead load plus cable load only.

9. In order to eliminate cracks and reduce moments and deflections, prestress-
ing is often used in the long shell with edge beams. In this manner the difficulty in placing a great quantity of tension bars in the beams is avoided.
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8. "Building Code Requirement for Reinforced Concrete (ACI 318 - 71)", Ameri-
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tice - No. 31", ASCE, New York, N. Y. 1952. (Reprinted 1956 and 1960)
The following symbols are used in this report:

- $A$ = cross-sectional area;
- $A_n$ = arbitrary constant;
- $A_{sd}$, $A_{sl}$, $A_{st}$ = the reinforcement steel cross-sectional area;
- $a_1$ = height of edge beam;
- $B$ = width of beam;
- $B_1$, $B_2$, $B_3$, $B_4$ = numerical coefficient;
- $B_n$ = arbitrary constant;
- $b$ = thickness of section, bending theory;
- $C_n$ = arbitrary constants;
- $c$ = distance;
- $D$ = flexural rigidity, $D = E t^3/12(1-v^2)$;
- $D_n$ = arbitrary constant;
- $e$, $e_1$ = eccentricities of prestressing force with respect to the neutral axis of the beam;
- $E$ = Young's modulus;
- $F$ = tensile force;
- $F_v$, $F_h$ = vertical and horizontal reaction forces;
- $f_v$, $f_h$ = vertical and horizontal sag of cables;
- $f_1$, $f_2$, $f_3$, $f_4$, $f_5$, $f_6$, $f_7$, $f_8$ = exponential terms;
- $f_c$ = compressive working strength of concrete, $0.45 f'_c$;
- $f'_c$ = compressive strength of concrete;
- $f_s$ = working strength of steel, $0.4 f_y$;
- $f_y$ = ultimate strength of steel;
\( g \) = gravity load per unit area of the surface;

\( H \) = prestressing force;

\( I \) = moment of inertia;

\( k \) = \( \pi / L \);

\( L \) = longitudinal span;

\( M \) = bending moment;

\( M_\phi \) = transverse moment, considered positive when it produces tension in the outward fibers;

\( M_{x\phi} \) = twisting moment, considered positive when it produces tension in the outward fibers;

\( m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8 \) = the eight roots of Donnell's equation;

\( N_\phi \) = direct force component in the transverse direction, considered positive when tensile;

\( N_{x\phi} \) = tangential shearing force, considered positive when tensile;

\( N_x \) = direct force component in the longitudinal direction, considered positive when tensile;

\( n \) = no terms of Fourier's series;

\( P \) = constant;

\( P_0 \) = snow load per unit area of the horizontal projection;

\( Q \) = first moment about neutral axis of area of cross section;

\( Q_\phi \) = radial shearing force, considered positive when outward direction;

\( Q'_\phi \) = combining the radial shearing force and the twisting moment, \( Q'_\phi = Q_\phi + 3M_{x\phi} / \partial x \);

\( \bar{R} \) = numerical coefficient, \( \bar{R} = \frac{4DRk^3}{E(\nu)} \tau_{tp} \);
$R$  = radius of shell;

$S_1$ = shearing force;

$T_p$ = principal force;

$t$  = thickness of shell;

$V$  = shear contributed by gravity load;

$W$  = weight;

$W_{hi}$ = horizontal prestressing force at ith cable;

$W_{vi}$ = vertical prestressing force at ith cable;

$X, Y, Z$ = the forces per unit area acting on the shell in the longitudinal, tangential, and radial directions;

$x, y, z$ = coordinates of shell;

$\sigma_1, \sigma_2, \beta_1, \beta_2$ = numerical coefficient;

$\nu$ = Poisson’s ratio;

$u$  = longitudinal displacement of the shell, considered positive in the direction of increasing values of $x$;

$v$  = tangential displacement of the shell, considered positive in the direction of increasing values of $\phi$;

$w$  = radial displacement of the shell, considered positive in the toward direction;

$\theta$ = rotation of the shell, considered positive when the section rotates counterclockwise;

$\phi$ = angle measured from the crown;

$\phi_i(x)$ = angle of th cable as the founction of $x$;

$\phi_c$ = semicentral angle;

$\lambda$ = $\pi R / \ell$ ;

$\sigma_t$ = fiber stresses on the top of edge beam;

$\sigma_b$ = fiber stresses on the bottom of edge beam;
ACKNOWLEDGEMENTS

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and encouragements and for her patience and carefulness in typing this report.
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Table 1. Comparison Between Beam Theory Solution and Bending Theory Solution for the Shell without Edge Beam
Table 2. Beam Theory Solution for the Shell without Edge Beam

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<tr>
<td>35</td>
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<td>-48</td>
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<td>0</td>
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<tr>
<td>40</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>
Table 3. The resultant Forces For the Shell with Edge Beam (Bending Theory)

<table>
<thead>
<tr>
<th>$x$ (degree)</th>
<th>$N_x$ (lbf/ft)</th>
<th>$N_\phi$ (lbf/ft)</th>
<th>$Q_\phi$ (lbf/ft)</th>
<th>$M_\phi$ (lbf-ft/ft)</th>
<th>$N_x \phi$ (lbf/ft)</th>
<th>$N_x$ (lbf/ft)</th>
<th>$N_\phi$ (lbf/ft)</th>
<th>$Q_\phi$ (lbf/ft)</th>
<th>$M_\phi$ (lbf-ft/ft)</th>
<th>$N_x \phi$ (lbf/ft)</th>
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</thead>
<tbody>
<tr>
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<td>0</td>
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<td>0</td>
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<td>1</td>
<td>469</td>
<td>0</td>
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<td>826</td>
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<td>476</td>
<td>0</td>
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<tr>
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<td>0</td>
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<td>-6,439</td>
<td>-493</td>
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<td>427</td>
<td>0</td>
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<td>597</td>
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<td>-5,136</td>
<td>-374</td>
<td>-52</td>
<td>349</td>
<td>0</td>
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<tr>
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<td>-2,659</td>
<td>408</td>
<td>-142</td>
<td>361</td>
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<td>-4,243</td>
<td>-244</td>
<td>-82</td>
<td>213</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>441</td>
<td>-179</td>
<td>-206</td>
<td>0</td>
<td>0</td>
<td>-4,071</td>
<td>-106</td>
<td>-122</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2/4</td>
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<td>-1,023</td>
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<td>576</td>
<td>0</td>
<td>-7,965</td>
<td>-571</td>
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<td>330</td>
</tr>
<tr>
<td>5</td>
<td>-14,816</td>
<td>-1,004</td>
<td>1</td>
<td>579</td>
<td>-433</td>
<td>-7,994</td>
<td>-561</td>
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<td>-434</td>
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<td>-13,652</td>
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<td>0.4</td>
<td>584</td>
<td>-849</td>
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<td>-849</td>
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<td>584</td>
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<td>-6,527</td>
<td>-485</td>
<td>-2</td>
<td>336</td>
<td>-1,229</td>
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<tr>
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<td>-9,467</td>
<td>-742</td>
<td>-17</td>
<td>568</td>
<td>-1,561</td>
<td>-5,571</td>
<td>-423</td>
<td>9</td>
<td>328</td>
<td>-1,561</td>
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<tr>
<td>25</td>
<td>-6,828</td>
<td>-603</td>
<td>-36</td>
<td>520</td>
<td>-1,839</td>
<td>-4,553</td>
<td>-349</td>
<td>20</td>
<td>302</td>
<td>-1,839</td>
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<tr>
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<td>-4,184</td>
<td>-450</td>
<td>-64</td>
<td>422</td>
<td>-2,062</td>
<td>-3,632</td>
<td>-264</td>
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<td>247</td>
<td>-2,052</td>
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<td>-100</td>
<td>255</td>
<td>-2,242</td>
<td>-3,000</td>
<td>-173</td>
<td>58</td>
<td>150</td>
<td>-2,242</td>
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<tr>
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<td>0</td>
<td>312</td>
<td>-127</td>
<td>-146</td>
<td>0</td>
<td>-2,400</td>
<td>-2,879</td>
<td>-75</td>
<td>-86</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: For $x = 2/2$, the values remain unchanged.

Where $N_x$ is the axial force, $N_\phi$ is the hoop force, $Q_\phi$ is the shear force, $M_\phi$ is the bending moment, and $N_x \phi$ is the coupling moment.
Table 4. Comparisons of Design Results for the Shell with Edge Beam and without Edge Beam

<table>
<thead>
<tr>
<th>Shell</th>
<th>No. of Cables</th>
<th>Volume of Concrete, ft³</th>
<th>Weight of Steel, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>P/C shell without edge beam</td>
<td>20</td>
<td>1,090</td>
<td>5,148</td>
</tr>
<tr>
<td>P/C shell with edge beam</td>
<td>10</td>
<td>1,840</td>
<td>5,210</td>
</tr>
</tbody>
</table>

Table 5. Stress in Edge Beam

<table>
<thead>
<tr>
<th>Acting Load</th>
<th>Dead Load, Snow Load &amp; Cable Load</th>
<th>Dead Load &amp; Cable Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top fiber stress of edge beam</td>
<td>+ 33 psi</td>
<td>+260 psi</td>
</tr>
<tr>
<td>Bottom fiber stress of edge beam</td>
<td>-378 psi</td>
<td>-987 psi</td>
</tr>
</tbody>
</table>
Table 6. Bar Number and Spacing for Transverse Reinforcement for Shell without Edge Beam Corresponding to $N_\phi$ & $M_\phi$ at $x=0$

<table>
<thead>
<tr>
<th>Region</th>
<th>$y$ (ft)</th>
<th>Corresponding $N_\phi$ (lb/ft)</th>
<th>$M_\phi$ (lb-ft/ft)</th>
<th>$A_s$ (in$^2$/ft)</th>
<th>Bar No.</th>
<th>Spacing (in) c.c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-1,319</td>
<td>-1,408</td>
<td>0.29</td>
<td>3</td>
<td>4$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>4.363</td>
<td>-1,334</td>
<td>-1,365</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.726</td>
<td>-1,375</td>
<td>-1,138</td>
<td>0.29</td>
<td>3</td>
<td>4$\frac{1}{2}$</td>
</tr>
<tr>
<td>3</td>
<td>13.089</td>
<td>-1,435</td>
<td>-643</td>
<td>0.22</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>17.452</td>
<td>0</td>
<td>0</td>
<td>0.22</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 7. Bar Number and Spacing for Transverse Reinforcement for Shell with Edge Beam Corresponding to $N_\phi$ & $M_\phi$ at $x=0$

<table>
<thead>
<tr>
<th>Region</th>
<th>$y$ (ft)</th>
<th>Corresponding $N_\phi$ (lb/ft)</th>
<th>$M_\phi$ (lb-ft/ft)</th>
<th>$A_s$ (in$^2$/ft)</th>
<th>Bar No.</th>
<th>Spacing (in) c.c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-1,447</td>
<td>815</td>
<td>0.13</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>4.363</td>
<td>-1,361</td>
<td>826</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.726</td>
<td>-1,049</td>
<td>803</td>
<td>0.13</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>13.089</td>
<td>-635</td>
<td>597</td>
<td>0.11</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>17.452</td>
<td>-179</td>
<td>0</td>
<td>0.11</td>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

1 kg = 2.2046 lb
1 in = 2.54 cm
Table 8. Bar Number and Spacing for Diagonal Reinforcement for Shell with Edge Beam Corresponding to $T_{p1}$ at $\phi = 40^\circ$

<table>
<thead>
<tr>
<th>Region x (ft)</th>
<th>Corresponding $T_{p1}$ (lb/ft)</th>
<th>(in$^2$/ft)</th>
<th>Bar No.</th>
<th>(in) c.c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-179</td>
<td>0.13</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>15.625</td>
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</tr>
<tr>
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<td>23.4375</td>
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<td>0.13</td>
<td>3</td>
</tr>
<tr>
<td>39.0625</td>
<td>3,668</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>46.875</td>
<td>4,085</td>
<td>0.19</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>62.5</td>
<td>5,466</td>
<td>0.24</td>
<td>3</td>
</tr>
</tbody>
</table>
Fig. 1 Notation of Displacements and Forces

Fig. 2 Cylindrical Shell Prestressed for Load Balancing
Fig. 3 Longitudinal Stress and Transverse Shear Stress Distributions

Fig. 4 Longitudinal Stress and Transverse Shear Distributions with Prestressing Force Acting on the Shell

Fig. 5 Properties of Arch Cross Section

Fig. 6 Component Loads
Fig. 6 Component Loads

Fig. 7 Force Acting on the Arch
Fig. 8 Internal Tractions
Case 1.

Uniform vertical load $W_y$

Case 2.

Case 3.

Fig. 9 Principle of Load Transformation for Prestressing with a Curved Cable
(a) The P/C Cylindrical Shell without Edge Beam

(b) The P/C Cylindrical Shell with Edge Beam

Fig. 10
(a) Radial Shearing Force $Q_\phi$ (lb/ft)

(b) Transverse Moment $M_\phi$ (lb-ft/ft)

Fig. 11 Shearing Forces and Transverse Moments in Shell without Edge Beam

--- Dead load + snow load + cable load
--- Dead load + cable load
Fig. 12  Longitudinal and Transverse Forces in Shell without Edge Beam

--- Dead load + snow load + cable load

--- Dead load + cable load
Fig. 13 Tangential Shearing Force $N_{x\phi}$ (lb/ft)

in Shell without Edge Beam

Fig. 14 Tangential Shearing Force $N_{x\phi}$ (k/ft)

in Shell with Edge Beam

- Dead load + snow load + cable load
- Dead load + cable load
Fig. 15 Longitudinal and Transverse Forces in Shell with Edge Beam

- Dead load + snow load + cable load
- Dead load + cable load
Fig. 16 Radial Shearing Forces and Transverse Moments in Shell with Edge Beam

- Dead load + snow load + cable load
- Dead load + cable load
Fig. 17 Cables and Reinforcement Placement of P/C Shell without Edge Beams
Freyssinet system 5 cables, 12 wires of 0.196"

Fig. 19 Prestressing Cables in the Edge Beam of P/C Shell

Fig. 20 Shell Section with Transverse Moment and Thrust
ANALYSIS AND DESIGN P/C CYLINDRICAL SHELLS

by

WAIN-LAI KUO

Diploma, Taipei Institute of Technology, 1968

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirement for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1976
This report presents the analysis and design of P/C cylindrical shells with and without edge beams. These shells are of the same span, radius and semicentral angle. The analysis of the shell with edge beams is based on membrane theory and bending theory and the shell without edge beam is based on the load balancing method and beam theory.

Comparisons were made between the two designs in terms of stress distributions. From these analyses and designs, the following conclusions were reached:

(1) Beam theory is much easier and simpler to apply than bending theory.
(2) It is much easier to layout the cables on the edge beam than along the shell surface.
(3) The P/C shell without edge beams needs more cables and reinforcement but less concrete, than the P/C shell with edge beams.