ALLOCATIVE AND TECHNICAL EFFICIENCY OF
TRADITIONAL AGRICULTURE IN NORTHERN NIGERIA

by

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B.A., Kansas State University, 1978

A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Economics
Agricultural Economics

KANSAS STATE UNIVERSITY
Manhattan, Kansas
1980

Approved by:

[Signature]
Major Professor
This is how we know what love is: Jesus Christ laid down his life for us. And we ought to lay down our lives for our brothers. If anyone has material possessions and sees his brother in need but has no pity on him, how can the love of God be in him? Dear children, let us not love with words or tongue but with actions and in truth.

I John 3:16-18
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ACKNOWLEDGMENTS

I wish to express a sincere appreciation and thanks to my major professor, Dr. David W. Norman, Professor of Agricultural Economics, for his professional knowledge and personal commitment in the area of international agricultural development. These were extremely valuable in the development of this thesis. Appreciation is also extended to Dr. Bryan Schurle for his time and assistance on some difficult areas of this work. The encouragement he gave to me will be remembered the most. I would also like to thank Dr. E. Wayne Nafziger for his interest in me and for his role in my program.

Many others had influence on me during my program and thanks are extended to all. Fellow graduate students George New and Mike Roth need to be mentioned for their assistance and for the scholarly discussions they promoted which helped to develop my thinking.

A very special thanks goes to my wife, Marilyn, for her patience and encouraging love. She, too, spent many hours typing.

Finally, I am grateful to God for allowing me to pursue this area of study and for blessing me with the successful completion of my program.
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CHAPTER I

INTRODUCTION

1.1. STATEMENT OF PROBLEM

A preliminary report of a presidential commission indicated recently that a global hunger crisis is likely over the next twenty years. If major grain-producing nations experienced two successive years of poor harvests then "widespread famine and political disorder" could occur in poor countries and "would severely disrupt a fragile world economy already weakened by energy shortages and rampant inflation" (Jackson, Associated Press, December 10, 1979). The future appears bleak when one thinks of the problems which might occur as a result of a world food crisis.

The Less Developed Countries (LDC's) will be the ones to suffer the most, yet they seem to be the ones who have the least ability to try to head off the problem. LDC's are characterized by, among other things, high population growth rates, methods of production which are largely "traditional" in nature and make little use of capital, a very low marginal productivity of labor in agriculture, a labor force of about seventy to eighty percent of the population which is engaged in agriculture, and foreign exchange problems brought on by earnings which fluctuate widely from year to year because of dependency on
production of primary products (Hagen, 1975, pp. 75-82). Because LDC's have limited capital resources with which to abate the problem, it is of the utmost importance that policy-making decisions be based on a meaningful and accurate description of the situation in order to most effectively use these resources. Although some disagree that a world food crisis is eminent,¹ good policy-making, based on facts, will go a long way in helping to make the rift in incomes between the people of more developed versus less developed countries more equitable.

The country of interest for this study is Nigeria, a developing country with a population of 80 million, approximately twenty percent of the population of the continent of Africa. In spite of Nigeria's oil revenues, agriculture is still the most important sector of the economy as it is a source of income and employment for about seventy percent of the population (Mijindadi, 1980, p. 1). Yet, the performance of the agriculture sector has been questioned during the last decade (Mijindadi, 1980, p. 1; Etuk, 1979, p. 1; Wells, 1974, p. 55). Because of rising per capita incomes, a population growth rate of about 2.5 percent, and an accelerated rate of urbanization, food production has not kept up with increases in the demand for food. Using 1964/65 as a base year, the index of production of basic food crops in Nigeria has not reached the one hundred percent level in the nine year period between 1967 and 1975 (Table I-1).

¹Everett E. Hagen has noted that "Now there is no zero probability of the abrupt exhaustion of the world's mineral resources and only a slight probability, given present and prospective human efforts, that food will be in shorter supply relative to demand in the future than in the past" (Hagen, 1975, p. 1).
<table>
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<th>Index of Quantity of All Food Imports into Nigeria (1965 = 100)</th>
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As a result of declining farm productivity there have been frequent and sharp rises in food prices. Thus, Nigeria has had to divert "scarce foreign exchange to the importation of basic staples which had previously been produced locally and in sufficient quantities to satisfy domestic needs" (Etuk, 1979, p. 2; Table I-1).

The implications of this trend should seem obvious. Although Nigeria is extremely fortunate to have found oil within its borders, the importance of increased agricultural production has been recognized by the Nigerian government as evidenced by its support of a number of agricultural research institutes over the years (Norman et al., 1979, p. 1). Because it is unlikely that increases in acres under cultivation will be an important source of output growth in the future (Etuk, 1979, p. 5), the relevant question to the research institute is, "How can one increase output per unit of input?"

I.2. WHAT ARE THE ANSWERS?

There are three kinds of opportunities which lead to increases in output per unit of input for an individual farm firm. One is to reorganize the productive inputs within a given production possibility curve. By reorganizing or reallocating the inputs which are available to this farm firm, one is attempting to find an optimum combination of inputs and enterprises which are consistent with the goal of profit maximization. When prices are introduced, this optimum allocation is referred to as price (allocative) efficiency and the goal becomes one of equating the marginal value product of each input to its cost.
The second opportunity is to change the production function surface. This means that a change in the types and numbers of parameters of the production function would occur and this is usually by the introduction of new kinds of inputs into the production base. As a convenient label, I will call the new specifications in the production function "technological change" (Yotopoulos, 1968, p. 133; Schultz, 1964, p. 132).

The third opportunity seems to have received far less theoretical treatment in the economic literature, but, as Peter Timmer says, it is "potentially more important quantitatively (in terms of wasted resources)" (Timmer, 1970, p. 99). By maximizing this third opportunity, one will receive the greatest output for a given set of inputs. This is a result of maximizing technical efficiency. Technical efficiency is essentially an engineering or physical concept which studies the measurable output from a given combination of measurable inputs. A firm is said to have greater technical efficiency than another if it consistently produces larger quantities of output from the same quantities of measurable inputs. By examining firms which are judged as not technically efficient when compared to the standard of firms using the "best practice" in an industry, one finds that many times there are potentials which exist for increased productivity. One potential might be realized by simply removing constraints which bind the less productive farmer. Suppose, for example, the farmer showing technical inefficiencies had a shortage of cash which prevented him from performing a farming operation (weeding) at a required time and optimum level because he had to give
priority to off-farm employment in order to meet his immediate needs (cash for food). The farmer may very well have, on an annual basis, maximized his allocative (price) efficiency with respect to labor, but exhibited technical inefficiencies as farm work was precluded by the off-farm employment. Other factors, including environmental conditions, motivation, managerial ability, entrepreneurship, and other non-measurable fixed factors of production, also tend to influence differences in output with a given quantity of measurable inputs.

Before proceeding, it should be realized that these opportunities should not be thought of as mutually exclusive, but as complementary. Each assumes a potential which will, if pursued with relevant, organized, and specific programs, lead to increased productivity. The question addressed here is, "Which of these opportunities affords the greatest potential for realizing increased output per unit of input, both in the long and short term?"

The first opportunity, attempting to increase allocative efficiency, does not seem to hold the answer. Studies show that there is little potential for increasing productivity, neither in the long nor short term, by attempting to increase allocative efficiency. Schultz (1964) concluded that on the average traditional farmers were allocatively efficient, thus coining the phrase that the traditional farmer was "poor but efficient."
The second alternative, that of initiating technological change, seems to be one of the keys for increased productivity, at least in the long term. A quick look at more developed countries shows that all have made use of new and improved inputs into agriculture in order to increase productivity. As incomes and population pressures rise, technological change seems to offer the only opportunity for agriculture to keep up with the increasing demand for food. One should realize, though, that in few cases great increases in productivity will be seen in the short term, especially if one is concerned, as we are, about equitable development over the whole of the population. Therefore, technological change should imply introduction of relevant improved inputs which can be implemented by a majority of the sector.

Unfortunately, technological change has implied the use of a capital intensive means of production, such as improved bio-chemical inputs and tractors. These non-traditional inputs must then be delivered by an uncertain and underdeveloped infrastructure (Malton and Newman, 1979, p. 1). Thus, these inputs would tend to go to the village heads, the large land-owners, and the richer, more progressive farmers. This is not bad in itself, but the tendency is that only a few would benefit.

However technological change is introduced into a society, whether it occurs with introduction of capital intensive inputs or with relevant improved inputs, change does occur. When change occurs the equilibrium experienced by a traditional society is disrupted and inefficiencies
result. More recently there have been studies which focus on technical inefficiency that indicate there are significant differences in technical efficiency between groups of farms. If one is able to identify the reasons why some farmers show less technical efficiency than others then there seems to be potential, at least in the short run, to receive significant increases in production. These increases in production might possibly be realized with relatively little investment and can improve the return on the investment of new and improved inputs introduced into the society.

Therefore, it becomes important for policy-makers to have a clear understanding of the relationships of the factors of production and what motivates a man to produce, in order to make the wisest decision and optimize the very limited and valuable resources a country has to work with. But, to get to this relevant information necessitates very detailed interdisciplinary studies in order to determine casual relationships. A farming systems approach is needed where not only economic constraints are important, but also environmental, agronomic, religious, psychological, and cultural constraints, too. This kind of approach is new, yet the importance can be clearly seen.

I.3. OBJECTIVES OF STUDY

In this analysis I plan to look at only a few economic relationships, realizing the vast interdependencies and correlations among economic, environmental, agronomic, religious, psychological, and cultural conditions. The primary goal is to determine whether or not
the traditional agricultural system in the study areas of northern Nigeria is economically efficient. If it is not, then are the observed inefficiencies due to a lack of allocative efficiency or technical efficiency or both? The objective then becomes one of first testing the "poor but efficient" hypothesis by equating the marginal value product of each variable input to its factor cost. If allocative inefficiencies do not exist, which is what would be hypothesized, then the test becomes one of finding if technical inefficiencies exist. If technical inefficiencies exist, then one can assume that there is a potential, at least in the short run, for increased productivity. In order to verify that there is indeed a potential, where that potential is, and how to make policies which take advantage of that potential, one will need to isolate the factors which contribute to technical efficiency. This particular aspect of analysis is beyond the scope of the paper. I will simply perform a test to find if indeed there are areas of potential growth as indicated by a lack of allocative and/or technical efficiencies.

1.4. OUTLINE OF SUBSEQUENT CHAPTERS

In the second chapter, I outline the evolution of theories of productivity in economic development and how we have come to present day methodology in the analysis of economic efficiency. A few studies and the empirical methodology used in measuring efficiency will be examined, with an emphasis on looking at technical efficiency. With this the justification will be given for the empirical approach used. In Chapter III is the derivation of the model developed by Lau and
Yotopoulos which uses a profit function and corresponding factor demand functions. Then in Chapter IV, I describe the data, define the variables used, build the model, and then explain Zellner's Seemingly Unrelated Regression—the statistical method of estimation used. In Chapter V, the profit function and labor demand function are jointly estimated and tests of hypotheses are made which determine if inefficiencies are present. Policy-making implications and some comparisons with previous studies are made in the final chapter, Chapter VI.
CHAPTER II

THE REVIEW OF LITERATURE

I have divided this chapter into two sections. In the first section, the evolution of theories of productivity in economic development and how we have come to our present day thinking will be outlined. This leads to the second section where a few recent studies will be examined and one will see how present day thinking has influenced the empirical methodology used in measuring efficiency. Here, the justification of the empirical approach used will be made.

II.1. EVOLUTION OF THEORIES OF PRODUCTIVITY

The literature on productive efficiency in traditional agriculture can be grouped, it seems, into three phases. The first phase, expounded in the fifties and early sixties, developed as a result of people who saw that the agricultural sectors of many developing countries were densely populated. Studies seem to show that the same level of production could be maintained within the agricultural sector with fewer people being employed (Leibenstein, 1978, p. 57). As a result, the concept of surplus labor or disguised unemployment in agriculture was developed. Many, including Rosenstein-Rodan (1957), Nurske (1955), Lewis (1954), and Ranis and Fei (1961), used this concept as an important part of their early work on development theories. W. Arthur
Lewis, in justifying his two sector model of the subsistence sector versus the capitalist sector, asserts as an empirical fact that in many LDC's there are "large sectors of the economy where the marginal productivity of labor is negligible, zero, or even negative" (Lewis, 1954, p. 141). Lewis offers no evidence but simply says that "it is obviously the relevant assumption for the economies of Egypt, of India, or of Jamaica," though not true for some LDC's where there is an acute shortage of male labor as in parts of Africa and Latin America (Lewis, 1954, p. 140). This surplus labor, consequently, represents a resource that could be substituted for capital in the capitalist sector of the economy. Because the surplus labor is essentially free and capital is costly, this simple substitution will then encourage development.

Looking to traditional agriculture, this implies that it is within the administrative and financial capabilities of a small farm family to produce the same amount of food with less labor. Thus the migrant labor force is supplied with the surplus food and can be utilized to increase production of non-agricultural products. The main point is that development consists of reallocating surplus agricultural labor whose contribution was zero, or at least negligible, to the capitalist (industrial) sector where they become a productive part of the labor force.

Schultz's (1964) work seems to be regarded as the signpost which marked the beginning of the second phase. Schultz analyzed the effects in India of the severe influenza epidemic of 1918-19. He concluded, based on a percentage drop in the labor force, a corresponding drop in
acreage planted, and a computed figure for marginal productivity of labor, that the fall in acreage was about the percentage decline which would be expected from the percentage decline in the labor force. Therefore, Schultz argues that there was no appreciable surplus labor. Although Hagen disagrees that the example is not conclusive,¹ Schultz’s efforts can be appreciated. Using this and other data, Schultz supported his view that although the marginal productivity of labor in agriculture was low, it was indeed not zero.

Schultz then advances his "poor but efficient" hypothesis by citing work done by Tax in a village in Guatemala (Schultz, 1964, p. 41, fn. 1) and Hopper on a village in North Central India (Schultz, 1964, p. 46, fn. 11; Hopper, 1965). He quotes Hopper, "There is no evidence that an improvement in economic output could be obtained by altering the present allocations as long as the village relies on traditional resources and technology." Thus, it was concluded that the factors of production available to the people of this village (Senapur) were allocated efficiently (Schultz, 1964, p. 47).

Schultz says that, "the community is poor because the factors on which the economy is dependent are not capable of producing more under existing circumstances" (Schultz, 1964, p. 48). Schultz notes that

¹Hagen suggests that calculations of rainfall and marginal productivity of labor differed much more among provinces than Schultz assumed and that, "perhaps most important, the deaths from the epidemic probably were not evenly distributed." This leads to the point that the fall in acreage would be expected even if the previous labor supply was redundant or surplus (Hagen, 1975, p. 221).
when one states that there are no significant inefficiencies in the allocation of factors of production, these factors included more than the classical land, labor, and capital inputs. Also included is the "state of the arts, or the techniques of production, that are an integral part of the material capital, skills and technical knowledge of a people" (Schultz, 1964, p. 48).

Two implications would seem obvious from Schultz's analysis. One is that the observed poverty of traditional agricultural communities cannot be improved by the reshuffling or a reallocation of the factors of production which are at the disposal of the farmer. The second implication is that in order to increase agricultural production new inputs need to be introduced into the system including new techniques of production.

As a result of the work by Schultz, Hopper, and others (Yotopoulos, 1968; Norman, 1970) development planners shifted their emphasis away from programs which reallocated the inputs available to traditional farmers in order to achieve higher incomes, to programs which emphasized the introduction of improved technologies, many based primarily on the use of improved bio-chemical and petroleum-using inputs--inputs not traditionally found in these areas. Collinson (1972, p. 75) has characterized this approach as a "transformation" which involves a structural change in the farming system in order to secure higher production. During the 1960's development planners in Nigeria turned to the idea of a transformation which took the form of large settlement
schemes. But, these schemes failed (Etuk, 1979, p. 8). Yet, the transformation idea seemed to work in several countries in Asia and Latin America which implemented "green revolution" technology on a large scale and dramatically increased production. Although the success of the Green Revolution should not be underestimated, Norman (1978, p. 2) points out that "the 'top-down' approach, together with the primary emphasis on production, tended to ignore the potential distribution problems." Furthermore, many believe that the major breakthroughs in technology exemplified by the Green Revolution may now have been exhausted. Thus, although the gains resulting from capital intensive, non-traditional inputs may have been sufficient to outweigh the social costs involved, there is "increasing likelihood that further quantum jumps might not occur" (Norman, 1978, p. 3).

Mijindadi (1980, p. 10) notes that Schultz uses a very restrictive definition of traditional agriculture which has been often overlooked. Schultz meant his thesis to apply only to a people of a particular community where the states of the arts, preferences, and motives for holding and acquiring sources of income have remained constant over a long period of time. Floods or droughts followed by famine, new roads or a new railroad, irrigation schemes, significant political changes, large changes in relative prices of products, and advances in relevant knowledge useful to agricultural production can all be sources of disequilibrium and initiate a significant alteration to which the system must adjust. "Any poor agricultural community that is adjusting its production to one or more of these circumstances is excluded
from traditional agriculture to which the 'efficient but poor hypothesis' applies" (Schultz, 1964, pp. 37-38). I would contend that today there are few remaining communities which meet this restrictive definition of traditional agriculture.

Thus, more recently the literature on productive efficiency in traditional agriculture has entered into a third phase. This phase can be characterized by the importance given to studying differences in technical efficiency between farmers. Some of the recent studies have indicated that there are significant efficiency differences among farmers with different farm size (Khan and Maki, 1979; Yotopoulos and Lau, 1973), among farmers of different income strata (Matlon and Newman, 1979), and among farmers differentiated by indices of "general modernization" (Shapiro and Müller, 1977). This recent focus on technical efficiency is important in that it implies if there are substantial technical inefficiencies within traditional agriculture, then there may be potential for an increase in output, possibly simply by removing the constraints which some of the traditional farmers face. A relevant question becomes, "How can inputs be used in such a way as to imitate the 'best practice' farmers and thus secure maximum output?" This removal of constraints may be vastly superior to the idea of further developing and introducing modern, capital-intensive technologies into a society of low income farmers. Great costs, both monetary and social, could be incurred from the use of these expensive inputs and from insuring that these technologies would be implemented equitably throughout the society. These problems would be avoided by increasing the
relevant inputs which are currently available to the "best practice" farmers yet are constraining to another group of farmers. Yotopoulos notes that "by increasing the quantities of complementary factors of production (plant, equipment, improved seeds, knowledge, etc.), one may expect that the production possibility curve will be pushed outward and a new equilibrium will be obtained at higher levels of marginal productivity for all factors" (Yotopoulos, 1968, p. 133).

Collinson (1972, p. 75) characterizes this strategy as "improvement" which involves intensification of relevant technology usually associated with better seed, improved cultural practices, increased knowledge. Norman (1978, pp. 3-4) explains this strategy as a "bottom-up" approach to the development of relevant technology. Because the small farmer is the central and most important figure, schemes should be developed which are compatible with the objectives, resources, and production environment of the small farmer. A study done by Abalu concludes that even though complementary factors of production (he refers to them as relevant technologies) are "simple and unspectacular, they do nevertheless, involve significant jumps from the traditional technological levels of production" (Abalu, p. 1).

However, in order to make the best use of resources available to assist farmers, studies need to be made which not only identify that technical efficiency differences exist among groups of farmers, but what the causes of efficiency variation are. Recent studies by Matlon
and Newman (1979) and Mijindadi (1980) attempt to isolate some of these casual relationships.

II.2. ALTERNATIVE EMPIRICAL METHODOLOGIES

In this section I examine a few of the empirical approaches that have developed as a result of the way thinking has evolved. The approaches looked at will include the average production function, linear programming, the frontier production function, and the normalized profit function.

II.2.1. THE AVERAGE PRODUCTION FUNCTION

The production function is a tool which describes, given the known environment of the organization, the way in which inputs are transferred into outputs. Physical scientists study production functions by holding some factors as fixed and varying the levels of intensity of others to predict the amount of output from the input. For example, as fertilizer is applied to a single acre of land in varying quantities, he studies the corresponding changes in output, given all other factors unchanged. This is a measure of technical efficiency and is purely an engineering concept. Economists have largely concerned themselves with the nature of the production function when prices or values are introduced. Economists say that rather than view the production function in a purely physical sense we can relate the value of the input to the value of the product and thus make comparisons of the marginal value of product (Heady, 1952, p. 707). This is looking at allocative or price efficiency.
Using a strict production function approach when measuring efficiency seems to have several problems. First, the production function approach makes use of averages. Averages can be useful when comparing one farm to average levels, but they are not so useful when comparing the efficiency differentials between farms or groups of farms. Second are the statistical problems which are encountered. Most studies of farm productivity use a single equation with Ordinary Least Squares estimating techniques. This approach leads to problems with 1) simultaneous equations bias—bias that occurs when the value of one of the independent variables is itself a function of the dependent variable (Kelegian and Oates, 1974, p. 227), and 2) specification bias—bias that occurs when some variables are left out of the estimated equation (Rao and Miller, 1971, p. 29). Another problem originates from the fact that when one is wishing to examine economic efficiency, one needs to look at its two components of technical efficiency and price efficiency simultaneously and not separately. Finally, the exact specification of the production function is many times in question.

II.2.2. LINEAR PROGRAMMING

Linear programming, an analytical tool developed since World War II, is another approach one could use when analyzing economic efficiency. With a given supply of available resources and a number of given production activities, one is able to optimize the combination of activities and inputs into outputs. The advantage with programming is that one can test a wide range of alternative adjustments and analyze their consequences. But linear programming does have limitations. One is
the validity of comparing the behavior of a "representative" or "average" farm with its given supply of resources to the behavior of individual farms. Farmers do not possess the same quantities or kinds of resources nor do they respond alike to different situations, so the estimate of an optimum plan may not be applicable to all farmers (Mijindadi, 1980, p. 86). Secondly, one of the assumptions of programming is that each additional unit of output corresponds to an additional unit of input. Therefore, taking diminishing returns into account can be troublesome. Also, linear programming explains what should be the optimum allocation with a given situation, but it does not explain why farmers follow the patterns of production they do.

II.2.3. THE FRONTIER PRODUCTION FUNCTION

A third methodological approach was developed by Farrell. In an early work (1957), he realized the problem of attaining a satisfactory measure of economic efficiency—one which took account of the components of allocative and technical efficiency and one which avoided the conventional production function approach that would represent only average levels of efficiency. Thus, Farrell developed the use of the frontier production function as the standard of "perfect" efficiency by which the observed performance of a firm can be measured.

Figure II-1 shows firms plotted with regard to their inputs used per unit of output produced on an isoquant diagram with the frontier (most efficient) production function estimated by the isoquant SS'.
THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE. THIS IS AS RECEIVED FROM CUSTOMER.
All points illustrate firms producing the same quantity of output.
Thus, less efficient firms can be compared to those on the frontier.

**FIGURE II-1. DETERMINATION OF THE PRODUCTION FRONTIER**
Assuming that $SS'$ is known, Farrell then develops a measure of price and technical efficiency (Figure II-2).
SS' is a simple isoquant which represents, under this simplified version, the most efficient combination of inputs $X_1$ and $X_2$ that derive a given unit output. Point P represents the inputs of the two factors, per unit of output, that a firm is observed to use, while point Q represents an efficient firm using the two factors in the same ratio as firm P and producing the same output as P. While Q is producing the same output as P, Q uses only a fraction ($OQ/OP$) as much of each factor. This ratio is a measure of the relative technical efficiency of firm P.

AA' is the price line intercepting SS' at point Q' which represents the point at which profits are maximized. Q' and not Q illustrates the optimal method or production for although each shows perfect technical efficiency, the costs of producing at Q' will be a fraction ($OR/OQ$) of the costs of producing at Q. Therefore, this ratio is a measure of the relative allocative efficiency of Q.

Furthermore, it should be noted that the ratio $QR/OQ$ is also a measure of the price efficiency of P, since firms P and Q use the same combinations of inputs. If the observed firm P were perfectly efficient, both in terms of technical and price efficiency (i.e., produce at Q'), then an overall economic efficiency ratio can be obtained by taking the product of the technical and price efficiencies.

$$\frac{OQ}{OP} \times \frac{OR}{OQ} = \frac{OR}{OP}$$

T.E. $\times$ P.E. = OVERALL EFFICIENCY
Farrell's work has undergone some criticism and evolution. Timmer (1970, p. 111) notes the work of Aigner and Chu (1968) saying that the "envelope approach used by Aigner and Chu does not operate in isoquant space like the Farrell frontier, but in total output--input space. The advantage of this is that the assumption of constant returns to scale need not be made." Another criticism is that only marginal data are used since the bulk of the firms do not enter the frontier (Mijindadi, 1980, p. 97). Therefore, Timmer developed a "chance constrained" frontier to be estimated which ensures that the frontier is not determined by these questionable observations alone. Effectively this is done by discarding one at a time the "efficient" firms (which may be "efficient" because of errors of observation or other problems) until the resulting estimated coefficients stabilize.

II.2.4. THE NORMALIZED PROFIT FUNCTION

The normalized profit function (also referred to as the Unit-Output-Price Profit function) was developed by Lau and Yotopoulos (1971, 1972, 1973) as an alternative approach to measuring economic efficiency. The model is represented by a profit equation and a series of corresponding derived demand functions for the variable inputs. McFadden (1978) shows that a profit function can express a firm's maximum profit as a function of the prices of output and variable inputs of production and of the quantities of fixed factors of production. The variable input demand functions are obtained by differentiating the profit function with respect to the normalized price of that input. Firm or group specific technical efficiency
parameters and firm or group specific and variable input specific price efficiency parameters are then introduced to account for the two components of economic efficiency. By putting the profit function into Cobb-Douglas formation and jointly estimating all equations, one is able to simply examine the coefficient of a group (or firm) dummy variable in order to determine relative economic efficiency differentials of the two. Absolute values for price and technical efficiency parameters can also be computed.

The Lau and Yotopoulos model is the methodological approach I used. A detailed explanation of the derivation is given in the following chapter. This approach is considered because it addresses specifically the concepts of allocative efficiency and technical efficiency and, with relative ease, comparisons in differentials of efficiency can be made. There have been several recent studies which have made use of this analytical approach (O'Conner and Hammonds, 1976; Trosper, 1978; Yotopoulos et al., 1976; Sidhu, 1974; Yotopoulos and Lau, 1973). Therefore, the approach seemed valid. Finally, a recent doctoral dissertation by Mijindadi (1980) made use of Timmer's approach of a "chance constrained" frontier production function on the same data set that was used in this analysis. Comparisons between studies may be interesting and, hopefully, help to solve questions for policy-making purposes.
CHAPTER III

THE UNIT-OUTPUT-PRICE PROFIT FUNCTION MODEL

III.1. REQUIREMENTS OF A MODEL

There are deficiencies which exist in a number of existing approaches to measuring efficiency. Lau and Yotopoulos suggest that there are certain minimum requirements which must be met in order to effectively test for efficiency. An effective approach should:

1) Account for firms that produce different quantities of output from a given set of measured inputs of production. This is the component of differences in technical efficiency. 2) It should account for firms that succeed to varying degrees in maximizing their profits, that is, equating the value of the marginal product of each variable factor of production to its price. This is the component of price or allocative efficiency. 3) The approach should take into account that firms operate at different sets of market prices. Profit maximization will lead to profits, quantities of outputs supplied, and quantities of variable inputs demanded as a function (among other things) of input prices. Two firms of equal technical efficiency and price efficiency facing different input prices will yield different profits (Lau and Yotopoulos, 1971, p. 95).
The model suggested by Lau and Yotopoulos satisfies these minimum requirements. The general case will be considered first, then the model will be expanded to a Cobb-Douglas production function which is the functional form of interest. In the following explanation much use has been made of the 1971 and 1972 articles by Lau and Yotopoulos and the 1973 article by Yotopoulos and Lau.

III.2. THE GENERAL FORM

Consider two firms (or groups of firms\(^1\)) which have identical production functions,

\[ V^1 = F(X_1, \ldots, X_m; Z_j, \ldots, Z_n) \]
\[ V^2 = F(X_1, \ldots, X_m; Z_j, \ldots, Z_n) \]  

where \( V = \) output

\( X_1, \ldots, X_m = \) variable inputs, \( i = 1, 2, \ldots, m \)

\( Z_j, \ldots, Z_n = \) fixed inputs, \( j = 1, 2, \ldots, n \)

and the superscripts denote the firm.

\(^1\)The analysis can be expanded to looking at groups of firms instead of individual firms.
III.2.1.1. THE TECHNICAL EFFICIENCY PARAMETER INTRODUCED

One firm is considered to have greater technical efficiency than another if, given the same quantity of measurable inputs, one firm consistently produces more output. There are many sources of differences in technical efficiency and these are caused by a number of interdependent factors, some of which are external constraints on the farmer and some of which are reflections on his own abilities.

Norman (1978, pp. 4-5) has explained some of the possible underlying determinants of the farming system which is helpful in analyzing technical efficiency. The total environment in which a farmer works in and makes decisions can be divided into two parts—the technical element and the human element. The technical element is one source of technical inefficiencies as it includes physical and biological factors which influence the farming system. These factors are largely external constraints on the farmer which reflect what the potential farming system can be, yet the farmer has a limited ability to modify these factors. Examples are the development of irrigation schemes to improve water availability and chemical fertilizers to improve soil quality.

The human element is another source of differences in technical efficiency and can be characterized by two types of factors—exogenous and endogenous. Exogenous factors are the social environment which the farmer lives in and is essentially outside of his control. These social factors will influence what he does, which may not result in him producing his maximum. The endogenous factors, on the other hand, are
controlled by the farmer himself. Here is where interfarm differences in managerial and entrepreneurial abilities are reflected. Simmons, Hays, and Norman (1980, p. 49) also point out that the "historical dimension—the status, income and liquidity position which the farmer inherits from previous periods—can determine access to resources and thus influence the production and employment strategy the farmer adopts."

Leibenstein (1979, 1966), in his explanation of X-efficiency or technical (non-allocative) efficiency, discusses other sources of differences in technical efficiency. He believes that there is a great potential for increasing output by improving these efficiencies. One potential is looking at how efficiency is affected by motivation. How people behave in firms depends on their degrees of motivation and these degrees of motivation can be influenced. He states that although the basic human input is "effort," what is actually purchased is labor time and not effort. Under the proper motivation, managers and workers could admonish themselves to produce closer to optimality, while under other conditions they may be motivated to move further away from optimality. These sources of technical inefficiency seem to be a bit more intangible and cannot be measured outright.

---

Leibenstein makes a definite distinction between X-efficiency and technical efficiency (Leibenstein, 1977). The definition of X-efficiency theory is outside the framework of traditional neoclassical economic theory. For the discussion here, the two terms are treated as synonymous for the sake of simplicity.
The sources of technical inefficiency do seem to be varied and ambiguous. Yet all of these sources will be incorporated into a single technical efficiency parameter. To illustrate that firm 1 has greater technical efficiency than firm 2, the parameter \( A \) is introduced.

\[
\begin{align*}
    v^1 &= A^1 F(X_1, \ldots, X_m; Z_j, \ldots, Z_n) \\
    v^2 &= A^2 F(X_1, \ldots, X_m; Z_j, \ldots, Z_n)
\end{align*}
\] (2)

Firm 1 is more technically efficient than firm 2 if \( A^1 \) is greater than \( A^2 \).

**III.2.1.2. THE ALLOCATIVE EFFICIENCY PARAMETER INTRODUCED**

A firm which maximizes its profits, that is, equates the value of the marginal product of each variable input to its own price, can be considered price-efficient. If the firm does not achieve maximum profits, then it is price-inefficient. The marginal productivity conditions for a profit maximizing firm (including the technical efficiency parameter) are

\[
\frac{p \partial A F(X; Z)}{\partial X_i} = c_i'
\] (3)

where

- \( A = \) technical efficiency parameter
- \( X = \) variable inputs
- \( Z = \) fixed inputs
- \( X_i = i^{th} \) variable input
- \( p = \) unit price of output
- \( c_i' = \) unit cost of the \( i^{th} \) input
There are two complications which influence the definition of price efficiency as we consider comparing two firms. First, the costs of inputs may be different for each firm. Therefore, a maximizing firm equates the value of the marginal product of each input to its input- and firm-specific cost. Second, each firm may not maximize profits. For these firms the usual marginal conditions do not hold. Therefore, we will assume that these firms maximize by equating the value of the marginal product of each factor to a constant proportion (which may be firm- and factor-specific) of the respective firm- and factor-specific prices. For firm 1,

$$\frac{\partial A^1 F(X; Z)}{\partial X_i} = k_{i1}^{1}c_{i1}^{1}$$ (4)

The $k_{i1}^{1}$'s may be thought of as the "effective costs" where $k_{i1}$ is an index of the decision rule that describes firm 1's profit-maximizing behavior with respect to factor $i$. Notice that perfect or absolute profit maximization is a special case that occurs when $k_{i1}' = 1$, for all $i$. Notice also that two firms are equally price-efficient when $k_{i1}^{1} = k_{i2}^{2}$, for $i = 1, \ldots, m$.

By defining $c_{i1} = c_{i1}'/p$, we create the normalized cost of the $i^{th}$ input and can write (4) as

$$\frac{\partial A^1 F(X; Z)}{\partial X_i} = k_{i1}^{1}c_{i1}^{1}$$ (5)
Lau and Yotopoulos explain the \( k' \)'s by saying that they reflect a decision rule that gives the profit-maximizing marginal productivity conditions as a special case. They explain further:

That the decision rule for the firm consist of equating the marginal product to a constant times the normalized price of each input may be rationalized as follows: i) Consistent over- or under-valuation of the opportunity costs of the resources by the firm; ii) Satisfying behavior; iii) Divergence of expected and actual normalized prices; iv) Divergence of the subjective probability distribution of the normalized prices from the objective distribution of normalized prices; v) The elements of \( k' \) may be interpreted as the first-order coefficients of a Taylor's series expansion of arbitrary decision rules of the type

\[
\frac{\partial F}{\partial x_j} = f_j'(c_j), \quad i = 1, 2; \quad j = 1, \ldots, m
\]

where \( f_j'(0) = 0 \) and \( f_j'(c_j) \geq 0 \). A wide class of decision rules may be encompassed under v) (Lau and Yotopoulos, 1971, p. 99).

Thus, by including in the production function a technical efficiency parameter "A", and including in the marginal productivity conditions an allocative (or price) efficiency parameter "k" with the realization that firms operate at different sets of market prices, then it has been shown that this analysis will meet the minimum requirements as stated earlier which must be met in order to effectively test for economic efficiency.

Economic efficiency is a term which combines both the components of technical efficiency and price efficiency. Given two firms faced with the same prices yet with varying degrees of technical and price efficiency, the firm which receives the higher profits is the firm
which is relatively more economic-efficient. This is regardless of the
relative values of technical and price efficiency of one firm compared
to another. This is true only within a certain range of prices and
fixed inputs. Outside this range the whole structure of the production
function may change.

III.2.2.1. THE PRODUCTION FUNCTION APPROACH

The next step in building the model will be the derivation of the
profit function, which is the tool used in order to test for relative
economic efficiency of two firms. First, the profit function will be
developed from the production function, then the firm-specific technical
efficiency parameter and the firm- and input-specific price efficiency
parameters will be added. In the next section, it will be shown that
the production (or supply) function can be found from the profit func-
tion and how this is the better approach to be used in establishing
tests for efficiency.

Assuming a production function (with the neoclassical properties
that firms are profit-maximizers and price-takers, and that the produc-
tion function is concave in the variable inputs, i.e., there exists
decreasing returns in the variable inputs),

\[ V = F(X_1, \ldots, X_m; Z_j, \ldots, Z_n) \]  \hspace{1cm} (6)

where \( V = \) output

\( X_1, \ldots, X_m = \) variable inputs, \( i = 1, 2, \ldots, m \)

\( Z_j, \ldots, Z_n = \) fixed inputs, \( j = 1, 2, \ldots, n \)
Profit (defined as current revenues less current total variable cost) can be found by

\[ P' = pF(X_1, \ldots, X_m; Z_j, \ldots, Z_n) - \sum_{i=1}^{m} c_i'X_i \]  

(7)

where \( P' \) = profit (which is technically gross margin but will be referred to as profit)
\( p \) = unit price of output
\( c_i' \) = unit cost of \( i \)th input

By assuming maximization, the optimal quantities of the variable inputs, denoted \( X_i^* \), can be found by equating the marginal value product of each input to its cost. This is written

\[ \frac{p \partial F(X; Z)}{\partial X_i} = c_i' \quad \text{for } i = 1, 2, \ldots, m \]  

(8)

where \( X \) and \( Z \) denote vectors of the inputs.

As stated before, by definition

\[ c_i = \frac{c_i'}{p} \]  

(9)

where \( c_i \) is the normalized price of the \( i \)th variable input.

Using equation (9), equation (8) can be rewritten so that

\[ \frac{\partial F(X; Z)}{\partial X_i} = c_i \]  

(10)
By solving for $X_i$ in the equation (10), the maximum profit quantities of variable inputs, $X_i^{*}$'s, can be found. The resultant $X_i^{*}$'s can be expressed as a function of the $c_i$'s (the normalized prices of the variable inputs) and the $Z_j$'s (the quantities of fixed inputs), so that

$$X_i^{*} = f(c ; Z) \text{ for } i = 1, 2, \ldots, m$$  \hspace{1cm} (11)

where $c$ and $Z$ denote vectors.

From equation (7) the profit equation can be written which assumes profit maximization levels of the variable inputs.

$$P' = p \left[ F(X_1^{*}, \ldots, X_m^{*} ; Z_j, \ldots, Z_n) \right] - \sum_{i=1}^{m} c_i'X_i^{*}$$ \hspace{1cm} (12)

for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$

where $p$ = unit price of output

$P'$ = profit

$X_i^{*}$ = profit maximizing levels of the variable input $X_i$

$Z_j$ = quantity of fixed input $j$

$c_i'$ = cost of variable input $i$

Since the terms in the square brackets are essentially a function of $c$ and $Z$, equation (12) can be rewritten into the profit function

$$\pi' = pG(c_1, \ldots, c_m ; Z_j, \ldots, Z_n)$$  \hspace{1cm} (13)

One should notice the distinction between the profit equation (12) and the profit function (13).
By normalizing values (13) can be rewritten

$$\tau = \frac{\pi'}{p} = G(c_1, \ldots, c_m; Z_j, \ldots, Z_m)$$ \hspace{1cm} (14)

where $\tau$ = normalized profit
$p$ = unit price of output
$c_i$ = normalized cost of variable input $i$
$Z_j$ = quantities of fixed input $j$

This equation is called the Unit-Output-Price Profit (UOPP) function.

**III.2.2.2. INCORPORATION OF EFFICIENCY PARAMETERS**

Having derived the profit function in its general form, the firm-specific technical efficiency parameter and the firm- and input-specific price efficiency parameters will now be added. The firm-specific technical efficiency parameter is $A^a$, where $a =$ firm 1 or firm 2.

Given one of the productions functions shown at (2), there exists the corresponding UOPP function:

$$v^1 = A^1 F(X_i, \ldots, X_m; Z_j, \ldots, Z_m)$$

$$\pi^1 = A^1 G\left( \frac{c_i}{A^1}, \ldots, \frac{c_m}{A^1}; Z_j, \ldots, Z_m \right)$$ \hspace{1cm} (15)

$A^1$ appears in the denominator because it can be canceled out of the numerator, as it is a constant, when it is included with the marginal productivity conditions of equation (10).

In the previous section there was an index for price efficiency (the $k'$s). Here the $k'$s are introduced with the (firm-specific)
normalized input prices for variable inputs (the $c_i$'s) in order to account for firm- and input-specific price efficiency. Therefore, equation (15) would be written

$$\pi^1 = A^1 G\left( \frac{k_i^1 c_i^1}{A^1}, \ldots, \frac{k_m^1 c_m^1}{A^1}; Z_j, \ldots, Z_n \right)$$

(16)

where $i =$ variable inputs, $i = 1, 2, \ldots, m$

$j =$ fixed inputs, $j = 1, 2, \ldots, n$

III.2.3.1. THE IMPORTANCE OF DUALITY—THE PROFIT FUNCTION APPROACH

This equation (16) is called the Unit-Output-Price Profit function with efficiency parameters or the "behavioral" UOPP function which represents how the firm should behave. For proof to the theorem that this profit function ($\pi = G(c;Z)$) can be found from the given production function of equation (6) ($V = F(X;Z)$), one can refer to McFadden (1978). Yet, McFadden also proved that, given certain conditions, the production function can be found from a given profit function. A dual relationship can be shown.

There exist certain conditions of regularity on the production and profit functions to show duality which are explained in detail in McFadden. But the condition of concavity of the production function, and conversely, convexity of the profit function, is of most concern for the empirical applications here. The finer details will not be discussed. Lau and Yotopoulos note that it is sufficient to say that "almost all continuous production functions in current use which are
concave will give rise to a well-behaved profit function" (Lau and Yotopoulos, 1972, p. 11, fn. 2.).

Therefore, McFadden has shown that there exists a one-to-one relationship between concave production functions and convex profit functions, that is, for every concave production function there exists a dual which is a convex profit function, and vice versa. This means that when one is studying the behavior of profit-maximizing, price-taking firms, one need only consider the profit function in the analysis without an explicit specification of the corresponding production function. A great deal of flexibility is achieved.

To step back, let us look at the importance of this dual relationship to the empirical application here. Eventually the goal is to derive and estimate a system of simultaneous equations composed of a profit function and corresponding factor demand functions. There are basically two ways to approach this problem. The first is to estimate, by some procedure, the underlying production function for some activity and then calculate the factor demand functions. From this the profit function (and the cost function—which is the more studied dual of the production function) can be found. This is what has been shown in the previous section. Silberberg says that this is "a very arduous procedure (as) production functions are largely unobservable" (Silberberg, 1978, p. 312).
The second approach is to estimate the profit function directly which would seem to make more sense. Knowledge found about the production function is used to derive implications for input usage and cost and profit considerations, so why not estimate these functions directly from the profit function. Because of duality this is possible. Once estimated, the profit function can be used to derive the maximum profit values of $X_{i^*}$ by using the Shephard-Uzawa-McFadden Lemma. The factor demand functions and the firm's supply function (with efficiency parameters included) are obtained from Shephard's (1953, 1970) Lemma given as $^3$

$$X_{i^*} = -A^a \frac{\partial G(\frac{k_{c,a}^a}{A^a}; Z^a)}{\partial k_{1^*}^a}$$

$$X_{i^*} = -\frac{A^a}{k_{1^*}^a} \frac{\partial G(\frac{k_{c,a}^a}{A^a}; Z^a)}{\partial c_{i^*}^a}$$

(17)

for $i = 1, 2, \ldots, m$

where $a =$ firm 1, 2

---

$^3$ $\frac{k_{c,a}^a}{A^a}$ and $Z$ are vectors for the variables in the function.
\[ v^{a} = A^{a} G \left( \frac{k^{a}}{A^{a}} ; z^{a} \right) - A^{a} \sum_{i=1}^{m} k^{a} c_{i}^{a} \frac{\partial G \left( \frac{k^{a} c_{i}^{a}}{A^{a}} ; z^{a} \right)}{\partial k^{a} c_{i}^{a}} \]

\[ v^{a} = A^{a} G \left( \frac{k^{a}}{A^{a}} ; z^{a} \right) - A^{a} \sum_{i=1}^{m} c_{i}^{a} \frac{\partial G \left( \frac{k^{a} c_{i}^{a}}{A^{a}} ; z^{a} \right)}{\partial c_{i}^{a}} \]  \( (18) \)

where \( a = \text{firm 1,2} \)

The previous section illustrated the production function approach which necessitated the explicit specification of the production function in order to derive the (maximized profit) levels of input \( X_{i}^{*} \) by equation (10). But now (because of duality) I shall assume that the profit function given in (16) can be estimated directly and represents how firm 1 should behave. Equation (17) represents firm 1's factor demand functions or the actual quantities of inputs demanded by firm 1 given the firm-specific efficiency parameters. The actual quantities of output supplied for firm 1 are shown in (18). By combining the actual demand and supply functions, the "actual" UOPP function (for any firm \( a \)) is derived from the profit equation (12).
\[ \pi^a_{\text{actual}} = V^a - \sum_{i=1}^{m} c_i^a x_i^a \]

\[ = A^a G \left( \frac{k^a c_i^a}{A^a} ; z^a \right) - A^a \sum_{i=1}^{m} k_i^a c_i^a \frac{\partial G \left( \frac{k^a c_i^a}{A^a} ; z^a \right)}{\partial k_i^a c_i^a} \]

\[ - \sum_{i=1}^{m} c_i^a \left( -A^a \frac{\partial G \left( \frac{k^a c_i^a}{A^a} ; z^a \right)}{\partial k_i^a c_i^a} \right) \]

\[ = A^a G \left( \frac{k^a c_i^a}{A^a} ; z^a \right) - A^a \sum_{i=1}^{m} \frac{k_i^a c_i^a}{k_i^a} \frac{\partial G \left( \frac{k^a c_i^a}{A^a} ; z^a \right)}{\partial c_i^a} \]

\[ - (-A^a) \sum_{i=1}^{m} c_i^a \frac{k_i^a}{k_i} \frac{\partial G \left( \frac{k^a c_i^a}{A^a} ; z^a \right)}{\partial c_i^a} \]

\[ = A^a G \left( \frac{k^a c_i^a}{A^a} ; z^a \right) - A^a \sum_{i=1}^{m} \left( \frac{k_i^a}{k_i} \right) c_i^a \frac{\partial G \left( \frac{k^a c_i^a}{A^a} ; z^a \right)}{\partial c_i^a} \]

\[ + A^a \sum_{i=1}^{m} \left( \frac{1}{k_i} \right) c_i^a \frac{\partial G \left( \frac{k^a c_i^a}{A^a} ; z^a \right)}{\partial c_i^a} \]

\[ \pi^a_{\text{actual}} = A^a G \left( \frac{k^a c_i^a}{A^a} ; z^a \right) + A^a \sum_{i=1}^{m} \left( \frac{1-k_i^a}{k_i} \right) c_i^a \frac{\partial G \left( \frac{k^a c_i^a}{A^a} ; z^a \right)}{\partial c_i^a} \]

\[ = \text{firm 1,2} \quad (19) \]
III.2.3.2. ADVANTAGES OF THE PROFIT FUNCTION APPROACH

Some of the advantages of working with the Unit-Output-Price
Profit function, as opposed to the traditional method of estimating
a production function, will be quickly summarized. First, the Theory
of Duality, and more specifically the Shephard-Uzawa-McFadden Lemma,
makes it possible to derive the factor demand functions and the supply
function directly from a UOPP function.⁴ One does not need to satisfy
the marginal productivity conditions (equations (4) and (5)), hence
there is not needed an explicit specification of the production func-
tion, which is many times difficult to obtain. In all, this provides
a great deal of flexibility in empirical analysis.

Second, by initially using a profit function, one is assured by
duality that the system of profit, supply, and factor demand functions
is obtainable for any firm with a "production function concave in the
variable inputs subject to given fixed inputs and under competitive
markets" (Lau and Yotopoulos, 1972, p. 12). Every concave production
function has a dual which is a convex profit function, and vice versa.
Profit maximization (subject to the price efficiency indices \( k_i \) for
\( i = \text{variable inputs } 1, 2, \ldots, m \)) and price-taking behavior is assumed.

Third, the system of profit, supply, and factor demand functions
are derived from variables that are normally considered to be determined

⁴This is a UOPP function which needs to be decreasing and convex
in the normalized prices of the variable inputs and increasing in the
fixed inputs.
independently of the firm's behavior (i.e., prices of inputs and output and quantities of fixed inputs). These variables are then exogenously determined. Thus, by estimating these variables directly the problem of simultaneous equations bias can be avoided. Other advantages are outlined in Lau and Yotopoulos (1972) and O'Conor and Hammonds (1975).

III.2.3.3. THE TEST FOR ECONOMIC EFFICIENCY

From equation (19) the "actual" UOPP function is found when the actual prices, inputs, and efficiency parameters are incorporated for the firms. Lau and Yotopoulos make three observations from the "actual" UOPP function.

i) $\frac{\partial \pi^a}{\partial A^a} > 0$, i.e., actual profit always increases with the level of technical efficiency for given normalized input prices and $k^a$;

ii) When $k_i^a = 1$ for $i = 1, 2, \ldots, m$, i.e., the firm is a true profit maximizer, the actual (19) and behavioral (16) UOPP functions coincide;

iii) When $A^1 = A^2$ and $k^1 = k^2$, the actual UOPP functions of the two firms coincide with each other (Lau and Yotopoulos, 1971, p. 100).

Observation iii) above notes that the "actual" UOPP functions of two firms will coincide if $A^1 = A^2$ and $k^1 = k^2$. This is with the assumption that has been carried along that the two firms have similar production functions and a specified range of normalized prices for variable inputs and quantities of fixed inputs.
From these observations of the "actual" UOPP function, one can see that the profit function approach is very useful in the determination of relative economic efficiency. When the appropriate functional forms are specified for the UOPP function for two firms, then a test of hypothesis can be made for determining equal relative economic efficiency. This test is equivalent to testing for significant differences between the profit functions. If one firm's profit function is significantly different from another's, then one can say that their relative economic efficiency is also different. By comparing the actual values of the profits ($\pi^a$, where $a = \text{firm 1 or firm 2}$) of the two firms, a statement can be made as to which firm has greater economic efficiency. If $\pi^1$ is greater than or equal to $\pi^2$ for all normalized prices within a specified range, then firm 1 is relatively more efficient than firm 2. One should remember that two firms showing equal relative economic efficiency should not lead one to the conclusion that there also is equal technical and price efficiency. Economic efficiency is made up of the independent components of technical and price efficiency.

Two other tests of hypotheses may be relevant to some studies. One shows that the fixed inputs command equal rent in the two firms. This is done by computing the first derivative of the "actual" UOPP function with respect to the fixed inputs and then testing for their equality.

The other test is for constant returns to scale to all factors of production (Yotopoulos and Lau, 1973, p. 220). This is done by
adding the elasticities of production of the fixed inputs and testing to see if they are equal to one. For both tests, the use of a Cobb-
Douglas production function and a logarithmic form profit function facilitate the setting up of the hypothesis. This will be further explained in a later section.

These tests make implications about the optimal form of economic organization in terms of the distribution of fixed inputs between the two firms.

III.2.3.4. THE TEST FOR TECHNICAL AND PRICE EFFICIENCY

The profit function, as discussed, will yield a test for determining relative economic efficiency. In order to separately identify the components of economic efficiency a test has been developed by Lau and Yotopoulos which incorporates information from the derived input demand functions (17). Because the "actual" UOPP function and the derived demand functions have parameters which are common to both, the functions should be estimated jointly. Then, when the appropriate functional forms are used (for this analysis a Cobb-Douglas production function and a logarithmic profit function will be used), it will be possible to test the hypotheses of equal technical efficiency ($A^1 = A^2$), of equal price efficiency ($k^1 = k^2$), and of absolute price efficiency ($k^a = 1$).
III.3. THE COBB–DOUGLAS FORM OF THE PROFIT AND PRODUCTION FUNCTIONS

Up until now I have discussed the model for determining relative economic efficiency and its components of technical and price efficiency in general terms. The Cobb–Douglas form of the profit function, by expanding to a logarithmic profit function, will allow great versatility in performing the empirical tests of hypothesis. Other functional forms of the profit function are outlined in McFadden (1978) and Lau (1978).

In this section, it will be shown how the Cobb–Douglas form of the profit equation is derived from the Cobb–Douglas production function. Then, by duality, it will be assumed that the supply and factor demand functions can be found from an initial profit function. These, then, will be used to find the final estimating equations from which various tests of hypotheses can be made in order to evaluate efficiency.

III.3.1. THE PRODUCTION FUNCTION APPROACH

A Cobb–Douglas production function of a profit-maximizing, price-taking firm showing decreasing returns in the variable inputs⁵ (i.e., the production function is concave) is given by

\[ V = A \left( \prod_{i=1}^{m} X_i^{\alpha_i} \right) \left( \prod_{j=1}^{n} Z_j^{\beta_j} \right) \]

\[ \text{where } \sum_{i=1}^{m} \alpha_i = \mu < 1 \]

⁵ μ less than 1 needs to be specified to designate decreasing returns in the variable inputs. This is important since constant or increasing returns in the variable inputs are inconsistent with profit maximization.
where \( V = \) output

\[ A = \text{technical efficiency parameter indicating differences in non-measurable fixed factors of production} \]

\( X_i = \) variable inputs, \( i = 1,2, \ldots, m \)

\( Z_j = \) fixed inputs, \( j = 1,2, \ldots, n \)

\( \alpha_i = \) elasticity of production of variable input \( X_i \)

\( \beta_j = \) elasticity of production of fixed input \( Z_j \)

Profit (defined as current revenues less current total variable cost) can be found by

\[
p' = p \left[ A \left( \sum_{i=1}^{m} X_i^{\alpha_i} \right) \left( \sum_{j=1}^{n} Z_j^{\beta_j} \right) \right] - \sum_{i=1}^{m} c_i X_i
\]

where \( p' = \) profit

\( p = \) unit price of output

\( c_i' = \) unit cost of \( i^{th} \) input

By assuming maximization, the optimal quantities of the variable inputs, denoted \( X_i^* \), can be found by equating the marginal value product of each input to its cost. This is written

\[
p \frac{\partial V}{\partial X_i} = c_i' \quad \text{for } i = 1,2, \ldots, m
\]

where \( X \) and \( Z \) denote vectors of the inputs.
Dividing by the unit price of output, (22) becomes

\[ \frac{\partial V}{\partial x_i} = c_i \]

(23)

where \( c_i \) = normalized cost of \( i^{th} \) input.

By solving for \( x_i \) in this equation, the maximum profit quantities of variable inputs, \( x_i^* \)’s, can be found. In order to simplify notation, it will be assumed that there are only two variable inputs \( (x_1 \text{ and } x_2) \) and two fixed inputs \( (z_1 \text{ and } z_2) \).

\[ \frac{\partial V}{\partial x_1} = A \alpha_1 x_1^{\alpha_1-1} x_2 \alpha_2 z_1 \beta_1 z_2 \beta_2 = c_1 \]

\[ \frac{\partial V}{\partial x_2} = A x_1 \alpha_2 x_2^{\alpha_2-1} z_1 \beta_1 z_2 \beta_2 = c_2 \]

\[ x_1^* = c_1 \frac{1}{\alpha_1-1} + \frac{1}{\alpha_1-1} \frac{1}{\alpha_1-1} - \frac{1}{\alpha_1-1} \frac{1}{\alpha_1-1} - \frac{1}{\alpha_1-1} \]

\[ x_2^* = c_2 \frac{1}{\alpha_2-1} + \frac{1}{\alpha_2-1} \frac{1}{\alpha_2-1} - \frac{1}{\alpha_2-1} \frac{1}{\alpha_2-1} - \frac{1}{\alpha_2-1} \]

(24)
The resultant \( X_1^* \) and \( X_2^* \) are expressed in terms of \( c_1 \)'s (the normalized prices of the variable inputs), the \( Z_j \)'s (the quantities of fixed inputs), and other \( X_1 \)'s. \( X_1^* \) and \( X_2^* \) can be expressed as a function of the \( c_1 \)'s and \( Z_j \)'s by simply substituting, for example, \( X_2^* \) for the \( X_2 \) term of the right-hand side of the \( X_1^* \) equality. After solving for \( X_1^* \), this is substituted into the right-hand side of the \( X_2^* \) equality. Essentially, one is solving for two variables in two equations.

The resultant \( X_1^* \) and \( X_2^* \), expressed in terms of \( c_1 \)'s and \( Z_j \)'s, are

\[
X_1^* = A \left( \frac{1}{1-\mu} \right) \left( \frac{\alpha_2}{\alpha_1} \right) \left( \frac{c_2}{c_1} \right) \left( \frac{1-\mu}{1-\mu} \right) \left( \frac{1}{1-\mu} \right) Z_1 \frac{\beta_1}{1-\mu} \frac{\beta_2}{1-\mu} \\
X_2^* = A \left( \frac{1}{1-\mu} \right) \left( \frac{\alpha_1}{\alpha_2} \right) \left( \frac{c_1}{c_2} \right) \left( \frac{1-\mu}{1-\mu} \right) \left( \frac{1}{1-\mu} \right) Z_1 \frac{\beta_1}{1-\mu} \frac{\beta_2}{1-\mu} 
\]

(25)

where \( \mu = \alpha_1 + \alpha_2 = \Sigma \alpha_1, \ i = 1,2 \)

From equation (21) the profit equation can be written which assumes profit maximization levels of the variable inputs.

\[
P' = p \left[ A X_1^{*\alpha_1} X_2^{*\alpha_2} Z_1^{\beta_1} Z_2^{\beta_2} \right] - c_1' X_1^* - c_2' X_2^* 
\]

(26)
By substituting (25) into (26) and dividing by the unit price of output \((p)\), the Unit-Output-Price Profit function is found.

\[
\pi = \frac{\pi'}{p} = A X_1^{*\alpha_1} X_2^{*\alpha_2} Z_1^{\beta_1} Z_2^{\beta_2} - c_1 X_1^{*} - c_2 X_2^{*} \tag{27}
\]

For the general case

\[
\pi = A \sum_{i=1}^{m} \alpha_i X_i^{*\alpha_i} \sum_{j=1}^{n} \beta_j Z_j^{\beta_j} - \sum_{i=1}^{m} c_i X_i^{*} \tag{28}
\]

This UOPP function is an important one to understand and remember for it is from this original UOPP function that two other UOPP functions will be derived and estimated. These two will be the "behavioral" and the "actual" UOPP functions.

### III.3.2. THE PROFIT FUNCTION APPROACH

So far, with the Cobb-Douglas formation of the production function, the maximum profit quantities of the variable inputs, \(X_i^{*}\), have been found by equating the value of the marginal product of the input to its factor cost. The \(X_i^{*}\)'s were found to be a function of the normalized costs of the variable inputs and quantities of fixed inputs. From this a profit function can be found. If one will remember, the eventual goal in this analysis is to find knowledge about input usage and cost and profit considerations. But since there has been found to be a dual relationship between the production function and profit function, one can simply start with a profit function in order to find the desired considerations (Figure III-1). Thus, the analysis continues assuming
PRODUCTION FUNCTION

MARGINAL PRODUCTIVITY CONSIDERATIONS
(assume maximum profit)

\[ MVP_{X_i} = FC_{X_i} \]

MAXIMUM PROFIT USAGE

LEVELS OF VARIABLE INPUTS

\[ X_i^* \quad i = 1, 2, \ldots, m \]

PROFIT FUNCTION

Profit in terms of normalized costs of variable inputs and quantities of fixed inputs.

FIGURE III-1. DUALITY BETWEEN PRODUCTION AND PROFIT FUNCTIONS
that a profit function can be found and used to derive the information of interest.

III.3.2.1. THE "ACTUAL" UOPP FUNCTION

From equation (28) the "behavioral" and "actual" UOPP functions come forth. The "actual" profit function is one which shows how the actual normalized profit of the firm is found, which is the production function in terms of the maximized profit levels of $X_i$ less the variable costs incurred in terms of $X_i^*$. The "actual" profit function will be used to estimate desired parameters.

Although maximizing behavior is assumed, maximum profits are not always achieved. As pointed out earlier, $k_i^a$'s represent an index of the decision rule that describes firm a's profit maximizing behavior with respect to factor i. If a decision-maker has imperfect knowledge of prices and markets then, although he shows maximizing behavior, he does not maximize his profits. Thus, for the "maximum profit" levels of the $X_i^*$'s, $k_i$'s will be included with $c_i$'s in order to explain the fact of imperfect knowledge availability. The $k_i c_i$'s identify normalized "effective costs" which the producer faces and are shown (from equation (25)) for $X_1^*$ as

$$X_1^* = A(1-\mu)^{-1} \left( \frac{k_1 c_1}{a_1} \right) \frac{a_2 - 1}{1-\mu} \left( \frac{k_2 c_2}{a_2} \right) \frac{a_2}{1-\mu} \frac{\beta_1}{1-\mu} \frac{\beta_2}{1-\mu}$$

The "actual" UOPP function, for the two variable input--two fixed input case, will then appear as
\[
\pi = A x_1^{\alpha_1} x_2^{\alpha_2} z_1^{\beta_1} z_2^{\beta_2} - c_1 x_1^{*} - c_2 x_2^{*} \\
= A \left[ \begin{array}{ccc}
\frac{1}{1-\mu} & \alpha_2 & -\alpha_2 \\
\frac{k_1 c_1}{a_1} & \frac{1-\mu}{1-\mu} & \frac{1-\mu}{1-\mu} \\
\frac{1}{1-\mu} & \alpha_1 & -\alpha_1 \\
\frac{k_2 c_2}{a_2} & \frac{1-\mu}{1-\mu} & \frac{1-\mu}{1-\mu}
\end{array} \right]^{\alpha_1} \\
\times \left[ \begin{array}{cc}
z_1 & z_2 \\
\frac{k_1 c_1}{a_1} & \frac{1-\mu}{1-\mu} \\
\frac{k_2 c_2}{a_2} & \frac{1-\mu}{1-\mu}
\end{array} \right]
\]

\[
\pi = A \left[ \begin{array}{ccc}
\frac{1}{1-\mu} & \alpha_2 & -\alpha_2 \\
\frac{k_1 c_1}{a_1} & \frac{1-\mu}{1-\mu} & \frac{1-\mu}{1-\mu} \\
\frac{1}{1-\mu} & \alpha_1 & -\alpha_1 \\
\frac{k_2 c_2}{a_2} & \frac{1-\mu}{1-\mu} & \frac{1-\mu}{1-\mu}
\end{array} \right]^{\alpha_1} \\
\times \left[ \begin{array}{cc}
z_1 & z_2 \\
\frac{k_1 c_1}{a_1} & \frac{1-\mu}{1-\mu} \\
\frac{k_2 c_2}{a_2} & \frac{1-\mu}{1-\mu}
\end{array} \right]
\]

\[
\pi = A \left[ \begin{array}{ccc}
\frac{1}{1-\mu} & \alpha_2 & -\alpha_2 \\
\frac{k_1 c_1}{a_1} & \frac{1-\mu}{1-\mu} & \frac{1-\mu}{1-\mu} \\
\frac{1}{1-\mu} & \alpha_1 & -\alpha_1 \\
\frac{k_2 c_2}{a_2} & \frac{1-\mu}{1-\mu} & \frac{1-\mu}{1-\mu}
\end{array} \right]^{\alpha_1} \\
\times \left[ \begin{array}{cc}
z_1 & z_2 \\
\frac{k_1 c_1}{a_1} & \frac{1-\mu}{1-\mu} \\
\frac{k_2 c_2}{a_2} & \frac{1-\mu}{1-\mu}
\end{array} \right]
\]

Equation (30) simplifies, in the general case, to

\[
\pi = (1-\mu)^{-1} \left( 1 - \sum_{i=1}^{m} \frac{\alpha_i}{k_i} \right) \left( \prod_{i=1}^{m} \left( \frac{k_i c_i}{a_i} \right) \right) \left( \prod_{j=1}^{n} \beta_j (1-\mu)^{-1} \right)
\]

where \( \mu = \sum_{i=1}^{m} \alpha_i \)

This is the "actual" UOPP function with efficiency parameters A and \( k_i \) included.
By expansion, (31) can be written

\[
\pi = A^{-1}(1-\sum_{i=1}^{m} \frac{a_i}{k_i})(\prod_{i=1}^{m} \frac{-a_i(1-\mu)^{-1}}{k_i})
\]

\[
(\prod_{i=1}^{\pi} c_i)(\prod_{i=1}^{\pi} a_i)(\prod_{j=1}^{n} \beta_j(1-\mu)^{-1})
\]

(32)

By defining

\[
A^* = A^{-1}(1-\sum_{i=1}^{m} \frac{a_i}{k_i})(\prod_{i=1}^{m} \frac{-a_i(1-\mu)^{-1}}{k_i})(\prod_{i=1}^{\pi} a_i)
\]

(33)

\[
a_i^* = -a_i(1-\mu)^{-1}
\]

(34)

\[
\beta_j^* = \beta_j(1-\mu)^{-1}
\]

(35)

I can write (32) as

\[
\pi^a = A^*a(\prod_{i=1}^{m} c_i a_i^*)(\prod_{j=1}^{n} \beta_j^*)
\]

(36)

where \(a = \text{firm } 1, 2\)
III.3.2.2. THE "BEHAVIORAL" UOPF FUNCTION--THE VARIABLE INPUT DEMAND FUNCTIONS

Now, the other estimating equation(s)--derived input demand function(s)--will be found by looking at the "behavioral" UOPF function. Equation (28) can be written, after substituting in for the $X_i^{**}$'s and making several algebraic manipulations (see Appendix III-1 for the derivation), as

$$
\pi = A^{(1-\mu)^{-1}} (1-\mu) \left( \frac{c_1}{\alpha_1} \right)^{-\alpha_1(1-\mu)^{-1}} \left( \frac{c_2}{\alpha_2} \right)^{-\alpha_2(1-\mu)^{-1}} \beta_1(1-\mu)^{-1} \beta_2(1-\mu)^{-1} \frac{Z_1}{Z_2} 
$$

(37)

This is for the two variable input--two fixed input case. For the general case, the UOPF function is written

$$
\pi = A^{(1-\mu)^{-1}} (1-\mu) \left( \frac{1}{\mu} \right)^{-\sum_{i=1}^{m} \alpha_i} \left( \frac{1}{\pi} \right)^{-\sum_{j=1}^{n} \beta_j} \left( \frac{1}{Z_i} \right)^{-\sum_{i=1}^{m} \alpha_i} \left( \frac{1}{Z_j} \right)^{-\sum_{j=1}^{n} \beta_j} 
$$

(38)

where $\mu = \sum_{i=1}^{m} \alpha_i$

$\pi$ = normalized profit

At this point the "behavioral" UOPF function can be found by including the firm- and input-specific "effective costs," the $k_i^{a}$'s. The $k_i$'s represent a firm's profit maximizing behavior so that when $k_i = 1$, then maximum profits with respect to factor $i$ have been
achieved. Because there is a disagreement between perfect profit maximization and how a manager "behaves" many times, the $k_i$'s should be incorporated into the UOPP function.

$$
\pi = A (1-\mu)^{-1} \left( \prod_{i=1}^{m} \left( \frac{k_i c_i}{a_i} \right)^{-\alpha_i (1-\mu)^{-1}} \right) \left( \prod_{j=1}^{n} \beta_j (1-\mu)^{-1} \right)
$$

(39)

Shephard's Lemma (equation (17)) is now used, just as before, to derive the demand function equation for each variable input. This is done by differentiating the "behavioral" UOPP function with respect to the normalized "effective costs" of the inputs.

$$
X_i^* = \frac{-\partial \pi}{\partial k_i c_i}
$$

$$
X_i^* = \frac{-1}{k_i} \frac{\partial \pi}{\partial c_i}
$$

$$
X_i^* = \frac{-1}{k_i} \left( (1-\mu)^{-1} \left( \prod_{i=1}^{m} \left( \frac{k_i}{a_i} \right)^{-\alpha_i (1-\mu)^{-1}} \right) \left( \prod_{j=1}^{n} \beta_j (1-\mu)^{-1} \right) \left( \prod_{k=i+1}^{m} c_k \right) \left( \prod_{j=1}^{n} \beta_j (1-\mu)^{-1} \right) \left( \prod_{i=1}^{m} \left( \frac{k_i}{a_i} \right)^{-\alpha_i (1-\mu)^{-1}} \right) \left( \prod_{j=1}^{n} \beta_j (1-\mu)^{-1} \right) \left( \prod_{k=i+1}^{m} c_k \right) \right)
$$

(40)
But since
\[ c_i = c_i, \quad c_i = c_i, \quad c_i = \frac{-\alpha_i(1-\mu)^{-1}}{c_i} \]

(41)

then we can combine terms

\[ X_i^* = -A (1-\mu)^{-1} \left( \frac{1}{k_i c_i} \right) (1-\mu) \left( \sum_{i=1}^{m} k_i \right) \]

\[ \left( \sum_{i=1}^{m} \alpha_i (1-\mu)^{-1} \right) \left( \sum_{i=1}^{m} \frac{-\alpha_i (1-\mu)^{-1}}{c_i} \right) \]

\[ \left( \sum_{j=1}^{n} \beta_j (1-\mu)^{-1} \right) \left( \sum_{j=1}^{n} \frac{-\alpha_j (1-\mu)^{-1}}{1-\mu} \right) \]

(42)

A series of identities have been defined. From (33), (34), and (35), one remembers

\[ A^* = A (1-\mu)^{-1} \left( 1 - \sum_{i=1}^{m} \frac{\alpha_i}{k_i} \right) \left( \sum_{i=1}^{m} \frac{-\alpha_i (1-\mu)^{-1}}{k_i} \right) \left( \sum_{i=1}^{m} \frac{\alpha_i (1-\mu)^{-1}}{k_i} \right) \]

\[ \alpha_i^* = -\alpha_i (1-\mu)^{-1} \]

\[ \beta_j^* = \beta_j (1-\mu)^{-1} \]

A new identity is defined,

\[ k^* = \left( 1 - \sum_{i=1}^{m} \frac{\alpha_i}{k_i} \right) (1-\mu)^{-1} \]

(43)
These identities are substituted into equation (42) to get

\[ X_1^* = -A^* \left( \frac{1}{1 - \sum_{i=1}^{m} \frac{\alpha_i}{k_i}} \right) (1-\mu) \alpha_1^* k_1^{-1} c_1^{-1} \]

\[ \left( \begin{array}{c} m \\ i=1 \end{array} \frac{\alpha_i^*}{c_1} \right) \left( \begin{array}{c} n \\ j=1 \end{array} \frac{\beta_j^*}{Z_j} \right) \]

\[-c_1^* X_1^* = A^* \alpha_1^* (k^*)^{-1} (k_1^*)^{-1} \left( \begin{array}{c} m \\ i=1 \end{array} \frac{\alpha_i^*}{c_1} \right) \left( \begin{array}{c} n \\ j=1 \end{array} \frac{\beta_j^*}{Z_j} \right) \]

(44)

This is then divided by the "actual" UOPP function of equation (36).

\[-\frac{c_1^* X_1^*}{\pi} = \frac{A^* \alpha_1^* (k^*)^{-1} (k_1^*)^{-1} \left( \begin{array}{c} m \\ i=1 \end{array} \frac{\alpha_i^*}{c_1} \right) \left( \begin{array}{c} n \\ j=1 \end{array} \frac{\beta_j^*}{Z_j} \right)}{A^* \left( \begin{array}{c} m \\ i=1 \end{array} \frac{\alpha_i^*}{c_1} \right) \left( \begin{array}{c} n \\ j=1 \end{array} \frac{\beta_j^*}{Z_j} \right)} \]

\[-\frac{c_1^* X_1^*}{\pi^a} = \alpha_1^* (k^a)^{-1} (k_1^a)^{-1} = \alpha_1^* a \]

(45)

for \( i = 1, 2, \ldots, m \)

\[ a = \text{firm } 1, 2 \]

These are the factor demand functions which will be jointly estimated with the "actual" UOPP function.
By describing

Total input cost for $X_1 = c_1 X_1 = MVP_{X_1} X_1$

$$= P_y \frac{\partial Y}{\partial X_1} X_1 = P_y \frac{3Y}{3X_1} X_1 = \frac{3Y}{3X_1} \frac{X_1}{Y} Y P_y$$

Since elasticity of product $E_p = \frac{3Y}{3X_1} \frac{X_1}{Y} = \alpha_1$ in the Cobb-Douglas production function, then

$$\frac{3Y}{3X_1} \frac{X_1}{Y} Y P_y = \alpha_1 Y P_y$$

Also,

Total input cost for all $X_i$'s $= \sum_{i=1}^{m} c_i X_i = \sum_{i=1}^{m} X_i MVP_{X_i}$

$$= \sum_{i=1}^{m} X_i P_y MVP_{X_i} = \sum_{i=1}^{m} X_i P_y \frac{3Y}{3X_i} = \sum_{i=1}^{m} \frac{X_i}{Y} \frac{3Y}{3X_i} P_y Y$$

Since $E_p = \frac{3Y}{3X_i} \frac{X_i}{Y} = \alpha_i$ for all $i = 1, 2, \ldots, m$

in the Cobb-Douglas production function, then

$$\sum_{i=1}^{m} \frac{3Y}{3X_i} \frac{X_i}{Y} P_y Y = \sum_{i=1}^{m} \alpha_i P_y Y$$

(47)

Also,

Revenue $= Y P_y$ (48)
Therefore, by (46), (47), and (48)

\[
\frac{\text{Total input cost for } X_1}{\text{Revenue - Total input cost for all } X_1's} = \frac{a_1 \ Y \ P_y}{Y \ P_y - Y \ P_y \ \sum a_i} = \frac{a_1}{1 - \sum a_i}
\]  

(49)

Because \( k_1 = 1 \) (profit maximization) equation (45) is written

\[
\frac{-c_1X_1^*}{\pi} = \alpha_1^*
\]

(50)

Remembering that from (34)

\[
\alpha_1^* = -\alpha_1(1-\mu)^{-1}
\]

where \( \mu = \sum a_i \)

one can write

\[
\frac{-c_1X_1^*}{\pi} = \frac{-\alpha_1}{1-\mu}
\]

\[
\frac{c_1X_1^*}{\pi} = \frac{\alpha_1}{1-\mu}
\]

(51)

By substitution of (51) into (49), it is found that

\[
\frac{\text{Total input cost for } X_1}{\text{Revenue - Total input cost for all } X_1's} = \frac{c_1X_1^*}{\pi} = \frac{a_1}{1-\mu}
\]

(52)

This helps to clarify more explicitly the meaning of equation (45).
III.3.2.3. TESTING EFFICIENCY

Equation (45) is actually a family of equations representing the input demand functions. All of the demand functions differ across firms by only a constant. These equations and a logarithmic form of equation (36), the "actual" UOPP function, are the ones to be estimated. They are estimated jointly since there are terms common to all. Tests of hypotheses, then, can be made to test for relative economic efficiency, and its components of allocative (price) efficiency and technical efficiency. Tests for absolute allocative efficiency, or profit maximization, and constant returns to scale in all inputs of the production function can also be made. Performing these tests of hypotheses and realizing the implications from them are the goal of the last chapter. Before this, the empirical framework for the specific case of agriculture in northern Nigeria must be built. This is the next objective.
CHAPTER IV

THE EMPIRICAL METHOD

In this chapter, with the use of the Lau-Yotopoulos Unit-Output-Price Profit function model, I describe the data, define the variables to be used in the profit function, build the model, and explain the statistical method of estimation.

IV.1. THE DATA

This empirical analysis makes use of the data obtained from farm management studies carried out by the Rural Economy Research Unit of the Institute of Agricultural Research, Ahmadu Bello University. The studies were conducted in three states in Nigeria—Kaduna, Sokoto, and Bauchi—and centered around the towns of Zaria, Sokoto, and Bauchi, respectively, which are considered major markets for the areas. Three villages within each study area were selected based on the criteria of differing ease of access to the main city.

A random sample of 340 farmers were selected and interviewed twice weekly throughout a cropping year to obtain information on labor use, farm expenses, farm incomes, selected sociological variables, and many other factors. For a detailed explanation of the survey methodology, see Norman (1973). The Zaria and Sokoto studies have been published.
For a detailed summary of the studies, refer to Norman, Pryor, and Gibbs (1979).

The studies were undertaken between 1966 and 1968. Mijindadi (1980, p. 23) presents two reasons why it is felt that the data are not outdated at this point in time. First, previous work based on the data (Norman, 1972, 1976) gives results which are in general agreement with those from studies based on more recently collected data also from northern Nigeria (Matlon, 1977). Second, it is believed, as evidenced by official statistics, that the agricultural production base has remained very much the same over the decade, although there have been substantial increases in food prices. As noted in Chapter I (Table I-1), the Central Bank of Nigeria reported that the index of food production failed to reach 100 in the nine year period between 1967 and 1975 with 1964/1965 being the base period (Ojo, 1977). Therefore, it is believed that analysis of this data can produce findings relevant to northern Nigeria's current agricultural situation.

IV.2. THE PROFIT FUNCTION

The profit function for each agricultural household is assumed to be of the Cobb-Douglas form. I have characterized profit (or production) as being a function of three variables: labor being a variable input and land and capital being inputs which are fixed on the short run. This is done for several reasons, two of which are 1) for the ease of comparison with previous studies and 2) for the fact that
labor in a largely traditional agricultural setting is by far the major variable input.

**IV.2.1. LABOR**

Labor is the only variable input used in the profit function. The profit function calls for a price per unit of input which is the wage rate. Finding a single value per farm for the wage rate is complicated by the problem of imputing a wage rate for family labor. The problem is further complicated when considering that in this part of Nigeria, which is included into a wider, tribal orientated area called Hausaland, there can be identified three distinct types of hired labor—kwadago, jinga, and gayya. Hill defines kwadago as "work done for wages"; jinga as "farm-laboring undertaken on contract;" and gayya as "collective farmwork performed for cash remuneration lower than the prevailing wage rate for reasons of obligation, friendship, pity, enjoyment, etc." (Hill, 1972). Kwadago labor is the most significant type of hired labor used in the study areas.

In order to identify a single value for family labor wage rate, a hired labor wage rate was first imputed, which was a weighted average of values reported for each of kwadago, jinga, and gayya. This was done by summing over all farms all costs of the three kinds of hired labor and then dividing by the sum over all farms of the total hours of hired labor worked. This resulted in a single value for wage rate of hired labor.
The wage rate for family labor can now be computed. There are different methods of determining how to value family labor, which averaged 80 percent of total labor (Norman, 1976, p. 110). Sidhu (1974) values family labor services in wheat farms in the Indian Punjab as equivalent to those of the annual contract labor for each farm. Khan and Maki (1979) state that the family wage rate on farms in Pakistan is that chosen by the respondent and was equal, in most cases, to the hired labor rate. Matlon and Newman (1979) in their efficiency study of traditional farmers in northern Nigeria arbitrarily use one-half the observed hired labor wage rate in figuring an imputed cost for household labor. This analysis will assume that the cost which can be assigned to family labor is equivalent to the (weighted) cost assigned to hired labor.

The data include three categories for hours of family labor on the farm—male adult, female adult, and large children. Before one can establish wage costs for family labor, a common denominator is needed to quantify these three categories into a single value. There has been much controversy over this issue of labor equivalence and the relative weights one should use in deriving a single value. The weights used were those designed by Norman. They are as follows: young child (less than seven years old) = 0.00 of a male adult equivalent; large child (age seven to fourteen) = 0.50 of a male adult equivalent; female adult (more than fourteen years old) = 0.75 of a male adult equivalent. Norman states that the approach used may be criticized as being too simplistic (Norman et al., 1979, p. 32), yet
Spencer and Byerlee (1977) found that the mean national wage rates in Sierra Leone during their 1974/1975 study worked out to be weighted in the same man-equivalent ratios of 1.00, 0.75, and 0.50 for men, women, and children, respectively (Norman et al., 1979, p. 32; Mijindadi, 1980, p. 18). Therefore, although recognizing some of the shortcomings of this definitional problem, the contention is that the computational advantages outweigh these disadvantages.

Finally, a wage rate, unique to each farm, can be calculated which is a weighted average of the costs of labor—both family and hired—divided by the hours of labor used—both family and hired. This is written

\[
\text{Wage rate for farm } i = \frac{(W_F H_{Fi}) + W_{K_i} + W_{J_i} + W_{G_i}}{H_{Fi} + H_{K_i} + H_{J_i} + H_{G_i}}
\]  

(1)

where \( i = \text{farm 1}, 2, \ldots, 340 \)

\( W_F \) = family labor wage rate which is equal to a weighted average over the whole sample of the hired labor wage rate. This value is 0.40 Naira/hour

\( H_{Fi} \) = hours family labor given in man equivalents for farm \( i \)

\( W_{K_i} \) = wage costs of kwadago labor for farm \( i \)

\( W_{J_i} \) = wage costs of jinga labor for farm \( i \)

\( W_{G_i} \) = wage costs of gayya labor for farm \( i \)

\( H_{K_i} \) = hours of kwadago labor for farm \( i \)

\( H_{J_i} \) = hours of jinga labor for farm \( i \)

\( H_{G_i} \) = hours of gayya labor for farm \( i \)
IV.2.2. CAPITAL

The variable inputs of improved seeds, fertilizer, land rent, interest on fixed capital, and other costs of production should also be treated as variable in production, yet this is very difficult under the profit function formulation as there is a lack of specific "prices per unit" of these other costs. It is contended, though, that it is rational not to include these other costs as variable because they tend to be insignificant. Examples of previous studies (Khan and Maki, 1979; Sidhu, 1974; Yotopoulos and Lau, 1973) will be followed, though, which do include some of these "other costs" (other than labor) into the fixed input of capital.

Capital is considered a fixed factor input and is the sum of the fixed costs of depreciation, repairs, tools, and equipment plus one half of the sum of operating expenses. This total is multiplied by an interest charge of 12 percent per annum. Operating costs are the costs of sales and marketing (cash and in-kind payments); costs of seeds and cuttings, including an imputed cost for seed stock held over from last year's production; costs of inorganic fertilizer purchased; costs, both actual and imputed, of organic fertilizer applied; plus an interest charge (12%) on fixed capital.

The inclusion of operating costs follows previous studies, as mentioned. In this analysis, they were halved because of the significantly greater value operating costs had over fixed costs. Without halving, the fixed factor input of capital would take on more of the
appearance of a variable factor input. A 12 percent interest charge on fixed capital included in operating costs accounts for the opportunity cost of money used for fixed costs. It is quite small in value. The sum of fixed costs and one half of operating costs is charged a 12 percent interest rate in order to better approximate the amount of capital input used for production in a single year only.

Yotopoulos (1967) has noted that production theory is exclusively concerned with the current services provided by the capital input variable rather than that portion which provides for increases in future production. Therefore, "only the current service flow of capital good properly belongs as an input to the production function" (1967, p. 476). By defining the capital input as a quantity or stock of capital, as was done, the implicit assumption that the definition of capital stock is proportional to the service flow of capital is made. As Yotopoulos proves, this proportionality property can be satisfied only under the most restrictive assumptions, one being completely linear relationships. Although the service flow concept is conceptually the most correct input to use in the production function, the capital stock concept is almost exclusively used.

IV.2.3. LAND

Land is the other fixed factor input. There are two main types of agricultural land which can be differentiated within the studies. They are: gona or upland fields, which are rainfed and therefore can only be cultivated during the wet season, and fadama or lowland fields,
which are located where the water-table is near the soil surface and can support crops throughout the year. Generally it can be said that low value per acre crops, such as millet, guineacorn, cowpeas, red sorrel, and groundnuts, are grown on gona, whereas high value per acre crops, like rice, sugarcane, and calabashes, are grown on fadama. Although fadama has a much higher potential productivity as compared to gona, its availability is limited and its distribution is uneven among farmers. For example, in two villages within the Sokoto study area fadama accounted for only about two percent of the land on an average farm. Fadama accounted for about 38 percent of total farm land in the third village of the study area (Norman et al., 1976, p. 18).

Because the presence of fadama was significant and because some farmers had a zero acreage of fadama (thus complicating the computational work when using the logarithmic profit function because of the problem of taking the natural logarithm of zero which is undefined), cultivable land was homogenized into gona land-equivalents. This was done by looking at the average net returns from crops grown on fadama and gona and setting up a ratio. It was determined that fadama is approximately three times more productive than gona, therefore one acre of fadama would equal three acres in gona land-equivalents (Table IV-1).

This method of obtaining a common denominator of gona land-equivalents seems to be valid, as Yotopoulos, Lau, and Lin (1976)
TABLE IV-1. WEIGHTED AVERAGE NET RETURNS OF CROP ENTERPRISES ON GONA AND FADAMA: ZARIA AND SOKOTO

<table>
<thead>
<tr>
<th>Study</th>
<th>Average Net Return (shillings/acre)</th>
<th>Excluding Labor</th>
<th>Costing All Labor¹</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zaria</td>
<td>Gona</td>
<td>223</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>Fadama</td>
<td>651</td>
<td>383</td>
</tr>
<tr>
<td></td>
<td>F/G</td>
<td>2.92</td>
<td>3.45</td>
</tr>
<tr>
<td>Sokoto</td>
<td>Gona</td>
<td>157</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>Fadama</td>
<td>412</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td>F/G</td>
<td>2.62</td>
<td>2.86</td>
</tr>
<tr>
<td>Bauchi</td>
<td>Gona</td>
<td>Not</td>
<td>Not</td>
</tr>
<tr>
<td></td>
<td>Fadama</td>
<td>Available</td>
<td>Available</td>
</tr>
</tbody>
</table>

¹Family labor is costed at the hired labor wage rate.


combined paddy land and dry land into paddy land-equivalents in their analysis of agriculture in the Province of Taiwan.

IV.3. THE MODEL

In the previous chapter, the "actual" Unit-Output-Price Profit function in the Cobb-Douglas framework was derived (equation III-36). The profit function, in logarithmic form, is written

\[ \ln \pi = \ln A^\star + \sum_{i=1}^{m} \alpha_i^\star \ln c_i + \sum_{j=1}^{n} \beta_j^\star \ln z_j \]

(2)

where

\[ A^\star = A^{(1-\mu)^{-1}} \left( 1 - \sum_{i=1}^{m} \frac{\alpha_i}{K_i} \right) \left( \prod_{i=1}^{m} k_i^{1-\alpha_i(1-\mu)^{-1}} \right) \]

\[
\begin{pmatrix}
\alpha_i(1-\mu)^{-1} \\
\prod_{i=1}^{m} \alpha_i
\end{pmatrix}
\]

\[ \alpha_i^\star = -\alpha_i (1-\mu)^{-1} < 0 \]

\[ \beta_j^\star = \beta_j (1-\mu)^{-1} > 0 \]

The specified production function for this analysis is written

\[ \ln \pi = \ln A^\star + \alpha^\star \ln w + \beta_1^\star \ln L + \beta_2^\star \ln K \]

(3)

where \( \pi \) = normalized UOPProfit (total revenue less total variable cost, divided by the price of output)

\( w \) = cost of variable input which is the normalized wage rate (wage rate divided by price of output)

\( L \) = quantity of fixed input—cultivable land in gona land-equivalent acres

\( K \) = quantity of fixed input—fixed capital
The labor demand function found from (III-45) is written

\[-\frac{w}{\pi} \text{LAB} = \alpha^* \tag{4}\]

where \( \text{LAB} = \) quantity of all labor employed
\( w = \) normalized wage rate
\( \pi = \) normalized profit

At this point a problem arises. The UOP Profit and labor demand functions are given in terms of normalized prices, yet only money profits and wage rates are available because a single value for price of output would be very difficult to obtain. Fortunately, (3) may be rewritten

\[
\ln \left( \frac{\pi'}{P} \right) = \ln A^* + \alpha^* \ln \left( \frac{w'}{P} \right) + \beta_1^* \ln L + \beta_2^* \ln K
\]

\[
\ln \pi' - \ln p = \ln A^* + \alpha^* \ln w' - \alpha_1^* \ln p + \beta_1^* \ln L + \beta_2^* \ln K
\]

\[
\ln \pi' = \ln A^* + (1-\alpha^*) \ln p + \alpha_1^* \ln w' + \beta_1^* \ln L + \beta_2^* \ln K \tag{5}
\]

where \( \pi' = \) money profit
\( w' = \) money wage rate per hour
\( p = \) price of output

Observe also that (4) holds independently of the price of output so that

\[-\frac{w}{\pi} \text{LAB} = -\frac{w'}{\pi} \text{LAB} = \alpha_1^* \tag{6}\]
At this point, there is interest in comparing two groups of farms. This is accomplished while still using the equations that have been formulated by utilizing group dummy variables. The interest here is in comparing the economic efficiency of large farms versus small farms. Large farms are defined as those farms which are in the upper 30 percent bracket of land area as determined by the number of gona land-equivalent acres. Small farms are those farms which are in the lower 30 percent bracket of land area. Although 40 percent of the observations in each study will be excluded, it was felt that this would aid in establishing distinct differences, if they existed, between the two groups in the population. The superscript "a" will now be represented by an "L" or an "S".

One assumption of the model is that the production functions of the large and small farms are identical except for a neutral efficiency parameter. This implies that the coefficients for \( \ln w' \), \( \ln L \), and \( \ln K \) will be equal for large and small farms. Another assumption made here is that prices of outputs are constant within study areas. This implies that \( \pi \) also does not vary and will be the same for large and small farms. Therefore, the differences which arise between large and small farms will be due to differences in efficiency, which are a function of the \( k^a \)s and \( A^a \)s in the \( A^* \) term of equation (5).
\( D_L \) and \( D_S \) are dummy variables which take the value of 1 for large and small farms, respectively, and zero otherwise. The estimating equation for the profit function can now be written as

\[
\ln \pi' = \ln A^{*L} + \alpha^* \ln w' + \beta_1^* \ln L + \beta_2^* \ln K \\
+ (1-\alpha^*) \ln p
\]

\[
\ln \pi' = \ln A^{*L} - \ln A^{*S} + \ln A^{*S} + \alpha^* \ln w' + \beta_1^* \ln L \\
+ \beta_2^* \ln K + (1-\alpha^*) \ln p
\]

\[
\ln \pi' = \ln A^{*S} + \ln \left( \frac{A^{*L}}{A^{*S}} \right) + \alpha^* \ln w' + \beta_1^* \ln L \\
+ \beta_2^* \ln K + (1-\alpha^*) \ln p
\]

\[
\ln \pi' = \lambda + d_L D_L + \alpha^* \ln w' + \beta_1^* \ln L + \beta_2^* \ln K \quad (7)
\]

where \( \lambda = \ln A^{*S} + (1-\alpha^*) \ln p \)

\[
d_L = \ln \left( \frac{A^{*L}}{A^{*S}} \right)
\]

\( d_L \) will be used for testing whether or not large and small farms have equal relative economic efficiency. This is shown later.

The estimating equation for the labor demand function can now be written as

\[
-\frac{w'_{LAB}}{\pi'} = \alpha^L D_L + \alpha^S D_S \quad (8)
\]

Equations (7) and (8) are the final estimating equations.
IV.4. THE STATISTICAL METHOD OF ESTIMATION

The given assumptions of the profit function model were that farm households were profit-maximizing and price-taking, that the production function was concave in the variable inputs, and that the fixed assets of land and capital showed short-run constancy. These assumptions lead to the fact that output and the quantities of labor input are the firm's decision variables (which become the dependent variables) with the prices of output and labor as well as the quantities of fixed inputs being variables predetermined (which become the independent variables). Thus in equations (7) and (8) the left-hand sides consist of jointly dependent variables and the right-hand sides have only predetermined variables. Under these conditions, ordinary least squares (OLS) could be applied to each equation separately and a consistent estimation could be found. Yet, because $\alpha^*$ appears in both equations, OLS would yield inefficient results because of simultaneous equations error as $\alpha^*$ is considered independently in both equations. A method which would jointly estimate the equations is needed.

An implicit assumption made under the classical normal linear regression model is that there exists no other regression equation with a disturbance term ($e_j$) that would be correlated with the disturbance term ($e_i$) of the first equation (Kmenta, 1971, p. 202, fn. 2). If there exists some other "piece of information" that has not been taken into account when estimating the parameters of the regression equation, then the result, using OLS, can no longer be considered unbiased and efficient.
For this analysis, there will not be the assumption that the disturbance terms of the regression equations are not correlated; that is, for the same farm, the correlation of the disturbance terms is allowed to be non-zero. However, it is assumed that the correlations of the disturbance terms of either equation corresponding to different farms are always zero. Another assumption is that there is an additive error term with zero expectation and finite variance for each of the equations. This is an admittedly ad hoc practice yet it is widely done (Lau and Yotopoulos, 1971, 1972; Yotopoulos and Lau, 1973; Yotopoulos, Lau and Lin, 1976; Sidhu, 1974; Khan and Maki, 1979).

Given these specifications, Zellner's method of Seemingly Unrelated Regression (1962), which is asymptotically efficient, is the method used. The Seemingly Unrelated Regression method can be easily performed with the use of the SAS (Statistical Analysis System) computer package (SAS Institute, 1979). One makes use of Seemingly Unrelated Regressions (SUR) method of estimation to increase the efficiency of the estimates when there is believed to be a significant correlation existing between the error terms of the equations in a system of equations. One must take into account the correlation between equations by using a systems method of estimation (Pindyck and Rubinfeld, 1976). To check if SUR does give more efficient estimates than OLS, then one should examine the standard errors for each estimated coefficient under OLS and SUR. If SUR standard errors are uniformly less, then there are uniformly more efficient estimates
obtained under SUR. One should note, however, that some of the t-ratios may become smaller as a result of the reduction in magnitude of the estimated coefficient (Marcis and Reed, 1974, p. 264, fn. 6).
CHAPTER V

RESULTS OF ESTIMATION AND TESTS OF HYPOTHESES

This chapter is involved with examining the results of the Seemingly Unrelated Regression technique on the data set for each of the three study areas. Tests of hypotheses are then made to determine significant difference in efficiency between the large versus the small farms. The sources of differences in economic efficiency are studied from these tests. From the parameter estimates, indirect estimates of the production function elasticities can be made.

V.1. THE ESTIMATES

The parameter estimates for each study area are presented in Tables V-1, V-2, and V-3 for Zaria, Sokoto, and Bauchi, respectively. The single equation Ordinary Least Squares method of regression is included for comparison with the Zellner's Seemingly Unrelated Regression method. One should notice that the standard errors reported for each parameter under the SUR method are uniformly less in comparison to the standard errors reported for the OLS method. This is one indication of the validity of using the SUR method as although the profit function and labor demand function are "seemingly unrelated" to each other, there seems to be a correlation of the error terms
### TABLE V-1. JOINT ESTIMATION OF THE PROFIT FUNCTION AND LABOR DEMAND FUNCTION: ZARIA

<table>
<thead>
<tr>
<th>Profit Function</th>
<th>Parameter</th>
<th>Single Equation OLS&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Seemingly Unrelated Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$\lambda$</td>
<td>3.993 (1.041)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>4.274 (.697)</td>
</tr>
<tr>
<td>Dummy--Large Farms</td>
<td>$d_L$</td>
<td>0.046 (.361)</td>
<td>0.159 (.278)</td>
</tr>
<tr>
<td>Wage Rate</td>
<td>$\alpha^*$</td>
<td>-0.094 (1.073)</td>
<td>-0.052 (.711)</td>
</tr>
<tr>
<td>Land</td>
<td>$\beta_1^*$</td>
<td>0.267 (.264)</td>
<td>0.372 (.175)</td>
</tr>
<tr>
<td>Capital</td>
<td>$\beta_2^*$</td>
<td>0.861 (.212)</td>
<td>0.641 (.140)</td>
</tr>
<tr>
<td>Labor Demand Function</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy--Large Farms</td>
<td>$\alpha^L$</td>
<td>-1.224 (1.592)</td>
<td>-1.224 (1.592)</td>
</tr>
<tr>
<td>Dummy--Small Farms</td>
<td>$\alpha^S$</td>
<td>-3.618 (1.681)</td>
<td>-3.618 (1.681)</td>
</tr>
</tbody>
</table>

<sup>a</sup>OLS refers to Ordinary Least Squares regression.

<sup>b</sup>Numbers in parentheses are estimates of standard errors.
<table>
<thead>
<tr>
<th>Profit Function</th>
<th>Parameter</th>
<th>Single Equation OLS&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Seemingly Unrelated Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$\lambda$</td>
<td>4.176 (.811)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>4.372 (.514)</td>
</tr>
<tr>
<td>Dummy—Large Farms</td>
<td>$d_L$</td>
<td>0.479 (.338)</td>
<td>0.447 (.248)</td>
</tr>
<tr>
<td>Wage Rate</td>
<td>$\alpha^*$</td>
<td>-0.545 (.493)</td>
<td>-0.110 (.306)</td>
</tr>
<tr>
<td>Land</td>
<td>$\beta_1^*$</td>
<td>0.857 (.210)</td>
<td>0.831 (.131)</td>
</tr>
<tr>
<td>Capital</td>
<td>$\beta_2^*$</td>
<td>-0.066 (.212)</td>
<td>0.030 (.132)</td>
</tr>
<tr>
<td>Labor Demand Function</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy—Large Farms</td>
<td>$\alpha^L$</td>
<td>-0.545 (.292)</td>
<td>-0.545 (.292)</td>
</tr>
<tr>
<td>Dummy—Small Farms</td>
<td>$\alpha^S$</td>
<td>-1.676 (.319)</td>
<td>-1.676 (.319)</td>
</tr>
</tbody>
</table>

<sup>a</sup>OLS refers to Ordinary Least Squares regression.

<sup>b</sup>Numbers in parentheses are estimates of standard errors.
<table>
<thead>
<tr>
<th>Profit Function</th>
<th>Parameter</th>
<th>Single Equation OLS&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Seemingly Unrelated Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>( \lambda )</td>
<td>2.428 ( (.861)^b )</td>
<td>3.503 ( (.554) )</td>
</tr>
<tr>
<td>Dummy--Large Farms</td>
<td>( d_L )</td>
<td>-0.627 ( (.566) )</td>
<td>0.211 ( (.397) )</td>
</tr>
<tr>
<td>Wage Rate</td>
<td>( \alpha^* )</td>
<td>-1.039 ( (.723) )</td>
<td>-0.503 ( (.451) )</td>
</tr>
<tr>
<td>Land</td>
<td>( \beta_1^* )</td>
<td>1.067 ( (.365) )</td>
<td>0.521 ( (.228) )</td>
</tr>
<tr>
<td>Capital</td>
<td>( \beta_2^* )</td>
<td>0.269 ( (.184) )</td>
<td>0.249 ( (.115) )</td>
</tr>
</tbody>
</table>

**Labor Demand Function**

| Dummy--Large Farms | \( \alpha^L \) | -3.532 \( (.905) \) | -3.532 \( (.905) \) |
| Dummy--Small Farms | \( \alpha^S \) | -3.392 \( (1.030) \) | -3.392 \( (1.030) \) |

<sup>a</sup>OLS refers to Ordinary Least Squares regression.

<sup>b</sup>Numbers in parentheses are estimates of standard errors.
across the two functions as indicated by the decreased standard errors for the estimated coefficients.

Some initial points are of interest. As predicted by economic theory, the profit function is decreasing and convex in the variable input wage rate for each of the studies. This is indicated by the negative coefficients and shows that profits will decrease as more is paid for labor. Also following economic theory, the coefficients for land and capital are positive showing the profit function increasing in quantities of the fixed inputs.

The final estimating equations are (from (IV-7) and (IV-8))

\[
\ln \pi' = \lambda + d_L D_L + \alpha^* \ln w' + \beta_1^* \ln L + \beta_2^* \ln K \tag{1}
\]

where \( \lambda = \ln A^{*S} + (1-\alpha^*) \ln p \)

\[
d_L = \ln \left( \frac{A^{*L}}{A^{*S}} \right)
\]

\[
-\frac{w' \, LAB}{\pi'} = \alpha^* D_L + \alpha^* S D_S \tag{2}
\]
V.2. THE HYPOTHESES

The first hypothesis that will be tested within this analytical framework will be that of equal relative economic efficiency. As one remembers, economic efficiency $A^*$, is made up of the components of technical efficiency $A^a$ and allocative efficiency $k^a$. Note that it is possible for large farms and small farms to have equal economic efficiency without having equal technical efficiency or equal allocative efficiency. The test implies that

$$A^*L = A^*S$$

or, as derived in equation (IV-7),

$$d_L = \ln \frac{A^*L}{A^*S} = 0$$  \hspace{1cm} (3)

The results of the tests of hypotheses are recorded in Table V-4. This hypothesis is not rejected for Zaria and Bauchi at the 10 percent significance level.\(^1\) For Sokoto, it is rejected at the 10 percent level but cannot be rejected at the 5 percent significance level. Although there are positive coefficients for $d_L$ for each of the study areas, indicating that large farms have greater economic efficiency relative to small farms, these differences are not significant for Zaria and Bauchi, and only marginally significant for the Sokoto farms.

---

\(^1\)These significance levels correspond to the probability of making a Type I error, or an error committed if the null hypothesis is rejected when it is true.
<table>
<thead>
<tr>
<th>Hypotheses Tested</th>
<th>Computed F-ratios with Corresponding Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zaria</td>
</tr>
<tr>
<td>1) ( d_L = 0 )</td>
<td>( F(1,141) = 0.339 )</td>
</tr>
<tr>
<td>2) ( \alpha^L = \alpha^S )</td>
<td>( F(1,141) = 1.070 )</td>
</tr>
<tr>
<td>3) ( d_L = 0 ) and ( \alpha^L = \alpha^S )</td>
<td>( F(2,141) = 0.537 )</td>
</tr>
<tr>
<td>4) ( \alpha^L = \alpha^* )</td>
<td>( F(1,141) = 0.455 )</td>
</tr>
<tr>
<td>5) ( \alpha^S = \alpha^* )</td>
<td>( F(1,141) = 3.844^\dagger )</td>
</tr>
<tr>
<td>6) ( \beta_1^* + \beta_2^* = 1 )</td>
<td>( F(1,141) = 0.011 )</td>
</tr>
</tbody>
</table>

^\dagger - shows significant difference at 10 percent level.

^{\dagger\dagger} - shows significant difference at 5 percent level.

^{\dagger\dagger\dagger} - shows significant difference at 1 percent level.
The second statistical test is that of testing for equal relative allocative efficiency of $k^L = k^S$. From equation (III-45) it was noted that

\[
- \frac{c_i^{\alpha_X \alpha}}{\pi^a} = \alpha_i^* (k_i^a)^{-1} (k^a)^{-1} = \alpha_i^{*a}
\]

for \( i = 1, 2, \ldots, m \)

\[ a = \text{firm 1, 2} \]

From equation (2) $\alpha^{*L}$ and $\alpha^{*S}$ can be estimated.

One can observe that (in the one variable input case), if and only if $k^L = k^S$, then

\[ \alpha^{*L} = \alpha^{*S} \]  \hspace{1cm} (4)

Thus, a test of equal relative allocative efficiency consists of testing the hypothesis in (4).

This hypothesis is not rejected for Zaria and Bauchi at the 10 percent significance level. For Sokoto, it is rejected at the 5 percent level but cannot be rejected at the 1 percent significance level. One can conclude that large and small farms in Zaria and Bauchi have equal allocative efficiency, that is, they maximize their profits to the same degree, while in Sokoto allocative efficiency is not equal.

A third statistical test can be made which tests for equal relative technical and allocative efficiency. The first test tested
for equal economic efficiency, while the second tested for equal allocative efficiency. If these two tests hold jointly, then it is implied that farms also have equal technical efficiency. The hypothesis used comes from the two hypotheses presented in equations (3) and (4) above, yet they are tested jointly.

\[ d_L = 0 \]

\[ \alpha^{*L} = \alpha^{*S} \]  

(5)

As may have been expected, this hypothesis cannot be rejected for either Zaria and Bauchi at the 10 percent significance level. It is rejected, however, for Sokoto at the 5 percent level. This actually can be anticipated in view of the previous rejection. The conclusion is that small and large farms in Zaria and Bauchi have not only equal allocative efficiency but also equal technical efficiency as well.

The fourth hypothesis states that large farms have absolute allocative efficiency, that is, they maximize their profits relative to the labor input. Maximum profits are found by equating the value of marginal product of labor to the factor cost or wage rate. Observe that perfect profit maximization results when \( k = 1 \), thus equation (III-45) shows that

\[ \alpha^{*} = \alpha^{*a} \]

where \( a = \text{farm L or S} \)
This $\alpha^*$, which is derived from the labor demand function and gives an indication of the cost of the input labor, is the same $\alpha^*$ that is represented in the profit function (1), which gives an indication of the value of labor's marginal product. Therefore, the test for large farms becomes one of analyzing the hypothesis that

$$\alpha^L = \alpha^*$$ (6)

This hypothesis is rejected for Bauchi but cannot be rejected for Zaria and Sokoto. This implies that large farms in Zaria and Sokoto do tend to maximize profits, while Bauchi large farms do not.

The fifth hypothesis can be written the same for small farms:

$$\alpha^S = \alpha^*$$ (7)

This hypothesis is derived in the same manner as the fourth one for large farms. The results show this hypothesis is rejected for all three study areas at the 10 percent significance level thus indicating that small farms generally do not maximize profits.

A sixth test can be made which examines constant returns to scale in all inputs. For the Cobb-Douglas profit function case, it is a test of the hypothesis that the sum of the elasticities of the fixed factors of production are equal to one. The derivation of this test is explained in Lau (1978) and Lau and Yotopoulos (1972, p. 14) and will not be discussed here. For the relevant model, the hypothesis is
\[ \beta_1^* + \beta_2^* = 1 \] (8)

This hypothesis cannot be rejected in any of the study areas thus indicating that within the agricultural framework of each study area, there exist constant returns to scale. This implies that there are no immediate economic advantages associated with economies of scale. This seems especially true for Zaria.

The results of these tests of hypotheses are interesting in that they show large and small farms from Zaria and Bauchi have no significant differences between them with regards to overall economic efficiency and its components of allocative and technical efficiency. Yet, for the Sokoto villages, while large and small farms show that they have (marginally) equal economic efficiency, large farms tend to maximize allocative efficiency and small farms do not. This leads to an implication that small farms must show a greater technical efficiency than larger farms in order to have "equivalent" economic efficiency. Unfortunately, there is no empirical test that can be performed to verify this within the framework.

The Sokoto area was looked at closer to determine why the seemingly inverse correlation between farm size and technical efficiency. Norman et al. (1976, p. 18) point out that fadama was generally available only in limited quantities. For example, in the villages of Takatuku and Gidan Karma, fadama only accounted for about two percent of the land in an average farm. But, for Kaura Kimba, the percentage of fadama
was about 38 percent, which offset the fact that farmers had generally smaller farms than in the other two villages. Although in the present analysis an attempt was made to take into account the higher productivity per acre of fadama, the model was run separately for each of the three Sokoto villages in hopes of finding if there were significant differences in the technical efficiency of small versus large farms. In this analysis small farms were defined as those in the lower 40 percent and large farms as those in the upper 40 percent of land area as determined by gona land-equivalent acres. The results are shown in Table V-5.

Here again, each of the regression results followed economic theory showing a negative coefficient for wage rate and positive coefficients for land and capital. Table V-6 illustrates the results of the tests of hypotheses for the three villages. Assuming a significance level of 5 percent, in each village the hypotheses of equal economic efficiency and equal allocative efficiency cannot be rejected. Thus, the third hypothesis of equal allocative and equal technical efficiency is also not rejected. The fourth and fifth hypotheses indicated that for all villages both large and small farms maximize profits, that is exhibit absolute allocative efficiency. Lastly, the hypothesis of constant returns to scale is not rejected either.

Therefore, these findings explain, just as was shown for the Zaria and Bauchi study areas, that there seems to be little difference in allocative efficiency and technical efficiency between large and small farms.
<table>
<thead>
<tr>
<th>Profit Function</th>
<th>Parameter</th>
<th>Takatuku SUR&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Kaura Kimba SUR</th>
<th>Gidan Karma SUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>λ</td>
<td>2.693 (.059)</td>
<td>4.181 (.586)</td>
<td>4.136 (.687)</td>
</tr>
<tr>
<td>Dummy--Large Farms</td>
<td>d&lt;sub&gt;L&lt;/sub&gt;</td>
<td>0.447 (.551)</td>
<td>-0.200 (.319)</td>
<td>0.513 (.325)</td>
</tr>
<tr>
<td>Wage Rate</td>
<td>α*</td>
<td>-1.710 (.808)</td>
<td>-0.111 (.363)</td>
<td>-0.586 (.605)</td>
</tr>
<tr>
<td>Land</td>
<td>β&lt;sub&gt;1&lt;/sub&gt;*</td>
<td>0.238 (.541)</td>
<td>0.884 (.149)</td>
<td>0.262 (.302)</td>
</tr>
<tr>
<td>Capital</td>
<td>β&lt;sub&gt;2&lt;/sub&gt;*</td>
<td>0.289 (.181)</td>
<td>0.223 (.173)</td>
<td>0.447 (.200)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Labor Demand Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dummy--Large Farms</td>
</tr>
<tr>
<td>Dummy--Small Farms</td>
</tr>
</tbody>
</table>

<sup>a</sup>SUR refers to Seemingly Unrelated Regression technique.

<sup>b</sup>Numbers in parentheses are estimates of standard errors.
## TABLE V-6. TESTS OF STATISTICAL HYPOTHESES: THREE SOKOTO VILLAGES

<table>
<thead>
<tr>
<th>Hypotheses Tested</th>
<th>Computed F-ratios with Corresponding Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Takatuku</td>
</tr>
<tr>
<td>$d_L = 0$</td>
<td>$F(1,35) = 0.784$</td>
</tr>
<tr>
<td>$\alpha^*L = \alpha^*S$</td>
<td>$F(1,35) = 1.582$</td>
</tr>
<tr>
<td>$d_L = 0$ and $\alpha^*L = \alpha^*S$</td>
<td>$F(2,35) = 0.812$</td>
</tr>
<tr>
<td>$\alpha^<em>L = \alpha^</em>$</td>
<td>$F(1,35) = 0.229$</td>
</tr>
<tr>
<td>$\alpha^<em>S = \alpha^</em>$</td>
<td>$F(1,35) = 0.687$</td>
</tr>
<tr>
<td>$\beta_1^* + \beta_2^* = 1$</td>
<td>$F(1,35) = 1.190$</td>
</tr>
</tbody>
</table>

$^+$ shows significant difference at 10 percent level.
Although there is shown to be no significant difference in allocative efficiency between large and small farms, it is interesting to note the trends that occur. One notices that for all farms, except for large farms in the two village samples of Takatuku and Gidan Karma, the absolute value of $\alpha^a$ is greater than the absolute value of $\alpha$ in the profit function. As $\alpha^a$ is considered a measure of the cost of the variable input labor (refer back to equation (III-52) for explanation), and $\alpha$ (from the profit function) is considered a measure of the marginal value product of labor, one can say that the general trend is to find that the factor cost exceeds the MVP of labor.

This trend seems to be contradictory to the notion that because 1) there is a shortage of labor available for hiring at the times needed and 2) there exist social constraints such as kinship ties, and the traditional law of reciprocity in farm help, there exists a situation which should show a high MVP of labor relative to its wage rate (Mijindadi, 1980, p. 133). This "contradiction" can be rationalized from the following. First, Minindadi (1980, p. 133) illustrates in his analysis that total family labor for Sokoto and Bauchi did show equality between MVP and factor cost and for Zaria, MVP was less than factor cost by half (with family labor valued at the average of hired labor wage rate). Second, Norman et al. (1979, p. 43) indicate that labor use reaches seasonal peaks during the summer months which tends to be the constraining factor for increased production. Certainly during these months one would notice a high MVP for labor relative to its cost. The present analysis was unable to identify seasonal
variations, but instead looked at figures for the year which indicated that on the average there was an opposite relationship. A third contributing factor to a high factor cost for labor may be the result of using a higher opportunity cost for family labor than is actually appropriate. Family labor constitutes 84 percent of the total labor input on the farm for the study areas (Norman et al., 1979, p. 33). This would lead one to question what the true valuation of farm family labor should be. This question is outside the scope of the present analysis.

Another trend is interesting to note. Although not statistically significant, for the Zaria study area and each of the three Sokoto villages, small farms showed greater allocative inefficiency than large farms. This is not surprising when one realizes that the bulk of labor used by small farmers makes use of a proportionally greater amount of family labor than large farms and that family labor time may be over-valued. Mijindadi (1980, p. 134) states that this "observation is especially true in a situation where some family members are not free to enter the labor market (e.g. the practice of aurem kulle, the seclusion of wives common among Hausa Muslims, who constitute the bulk of the sample of farmers participating in this study)."

An alternative explanation for the trend that large farms exhibit greater allocative efficiency than small may be because the small farm manager's first goal is one of security and not maximum profits. Small farms, by virtue of their size, produce fewer food products, and it may
be that the farmer simply wants to insure that there will be adequate food for the year before he enters the market to sell any food.

The Bauchi study area, on the other hand, shows a trend that large farms have greater allocative inefficiency (although this is not significant). This may be explained by the fact that because family labor was given a value, sometimes individual farms experienced negative profits when the regression was performed. When this occurred these observations were excluded from the analysis. For Bauchi, out of 116 observations (farms), one large farm and eight small farms were excluded from the analysis (in addition to the middle 40 percent of farms which were excluded by definition). Of the 35 potential small farms which could have been included in the analysis, only 27 were actually used. Those farms showing negative profit were excluded, thus possibly giving results which biased the economic standing of small farms upward. The allocative efficiency of large farms might possibly be greater than small farms if these negative-profit small farms could have been included.

V.3. INDIRECT ESTIMATES OF PRODUCTION FUNCTION ELASTICITIES

From the estimated parameters one can compute the indirect estimates of the production function elasticities of labor, capital, and land by using the identities shown in equation (III-34) and (III-35). These indirect estimates are listed in Table V-7 for the three study areas. Conceptually, these indirect estimates of production coefficients are statistically consistent, as opposed to those obtained
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Zaria</th>
<th>Sokoto</th>
<th>Bauchi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor $\alpha$</td>
<td>.049</td>
<td>.099</td>
<td>.335</td>
</tr>
<tr>
<td>Land $\beta_1$</td>
<td>.354</td>
<td>.749</td>
<td>.346</td>
</tr>
<tr>
<td>Capital $\beta_2$</td>
<td>.610</td>
<td>.027</td>
<td>.166</td>
</tr>
<tr>
<td>Sum of Elasticities $(\alpha + \beta_1 + \beta_2)$</td>
<td>1.013</td>
<td>.875</td>
<td>.847</td>
</tr>
</tbody>
</table>
directly from the production function by ordinary least squares. OLS will generally lead to inconsistent estimates because of the existence of simultaneous equations bias. However, the indirect estimates obtained in Table V-7 seem to show some incongruities with previously estimated production function elasticities (Norman, 1972, p. 114; Norman et al., 1976, p. 144). For Zaria, the elasticity of capital seems to be overstated, while the elasticity of labor seems to be understated. For Sokoto, the elasticity of land seems to be overstated, while the elasticity of capital seems to be understated. The discrepancies might be a result of the simplistic specification of production, especially noting that total labor was considered as the only variable input.

Given reasonable estimates of production elasticities, the parameter for allocative efficiency can also be estimated. From equations (III-34), (III-43), and (III-45) the k term can be explicitly found. For example, for the Bauchi study area k_L is found to be 0.430 and k_S is 0.434. This indicates (as does Table V-4) that neither large nor small farms maximize profits (k^2 = 1) and that small farms and large farms show statistically equivalent allocative efficiency, which is consistent with earlier findings.
VI.1. COMPARISONS WITH PREVIOUS STUDIES

The results of this study generally conclude that there seems to be no significant difference between the economic efficiency of small farms and large farms, as they are defined, within the areas under study. Also, there seems to be no significant difference between the components of economic efficiency—allocative efficiency and technical efficiency—of small and large farms.

Mijindadi (1980) performed an analysis on this same set of data using a "chance-constrained" frontier production function approach developed by Timmer (1970). His analysis indicated that (pp. 168-171):

1) For Zaria, acres of gona (upland), although tending to be positively related to technical efficiency, showed no significant differences between technical efficiency classes. Acres of fadama (lowland) did show significant (at the 5% level) increases in technical efficiency as acres of fadama increased.

2) For Sokoto, as gona acres increased there was a decrease in technical efficiency, although not significant. Fadama acres showed no trend, although farmers with the smallest amount of fadama were rated in the highest technical efficiency class.
3) For Bauchi, although both *gona* and *fadama* acres tended to be positively related to technical efficiency, these differences were non-significant.

One can see that the results of the present analysis, where land was aggregated into *gona* land-equivalent acres, generally follow Mijindadi's findings. Although Mijindadi's results on the Sokoto area study seem to conflict with his findings for Zaria and Bauchi, as they did with my initial analysis, the contention is that if he would have disaggregated the Sokoto study into the three separate villages, then the Sokoto results would have been more in line with his other findings.

Other results by Mijindadi conclude that crop yields and technical efficiency can be increased by hiring more labor. Yet, since there is no landless class, when hired labor is needed the most, all available labor is likely to be busy on family farms. Furthermore, these periods of labor bottlenecks tend to coincide with periods when cash resources on the farm are low, thus further inhibiting the chance to hire more labor (Mijindadi, p. 224).

Also, Mijindadi points out that observed differences in technical efficiency may in part be due to differences in the quantities and qualities of the factors of production used. It is possible that farmers do not have equal access to factors of production as inequalities in input distribution were shown (Mijindadi, p. 225).
The implication of the two points above is that some farmers may be placed in a situation where their observed technical inefficiencies are in part related to constraints imposed by the environment in which the farmer makes his decisions. These seem to be factors which the farmer can do very little about.

Mijindadi did show that some factors, which are under the control of the farmer, do contribute to significant technical efficiency. The indigenous farm practices of fallowing (which is a measure for crop rotations) and use of crop mixtures were positively related to technical efficiency. Therefore, when designing recommendations for increased production, these indigenous practices should be included in research as they are techniques which are familiar to the indigenous farming system (Mijindadi, p. 226).

Yotopoulos and Lau (1973), in using this model to test efficiencies between small farms (farms less than ten acres) and large farms in India, conclude that the test of relative economic efficiency was in favor of the small farms as they had higher profits within the range of output studied at given output and input prices and fixed quantities of land and capital. The relative economic efficiency of small farms was then found to be the result of superior technical efficiency (by approximately 20%) of the small farms, and not because of greater allocative efficiency. In fact, they found that both large and small farms showed absolute allocative efficiency. Constant returns to scale were also found to exist in Indian agriculture.
Khan and Maki (1979), in their study of small farms (farms less than 12.5 acres) and large farms in the Punjab and Sind of Pakistan, report that large farms exhibit greater economic efficiency than small farms in both regions. Furthermore, they show that large farms and small farms maximize profits relative to the labor input in the Punjab, while in Sind large farms maximized profits but small farms do not. Also, there exists increasing returns to scale in both provinces. The implication is that large farms are more advantageous (economically) and therefore policies of land consolidation, if efficiency is the sole criteria, should be encouraged. But Khan and Maki point out that large farms may have a comparative advantage in obtaining and utilizing information, credit, and new inputs and that this advantage may be due "to previous public policy decisions and agrarian structures, not to inherent characteristics of 'size'" (Khan and Maki, 1979, p. 67).

Sidhu (1974) examined the wheat enterprise on farms in the Indian Punjab, using more recent data than Lau and Yotopoulos, and found no differences in economic efficiency, or its components of allocative and technical efficiency, of small and large farms growing wheat. He also made comparisons of the economic performance of old Indian wheat varieties with Mexican varieties, and tractor-operated and non-tractor-operated wheat farms. The results of these comparisons indicate that the new varieties of wheat were economically more efficient compared to the old wheats and that tractor-operated farms were no better off in terms of their economic performance than non-tractor-operated farms.
The conclusions reached by Sidhu differ from those of Lau and Yotopoulos, who showed that small farms exhibited greater economic efficiency. He proposes a possible explanation for this discrepancy by explaining that the Lau-Yotopoulos findings pertain to the mid-1950's at a time when Indian agriculture could be characterized as traditional and in a state of equilibrium with available technology. The superior technical efficiency of the small farms was a result of more labor being available per unit of land as compared to larger farms. Perhaps, then, this labor surplus was used to increase the general level of fertility through more intensive land improvement programs which thus resulted in higher productivity. Also, the managerial input becomes more intensive on smaller farms. Sidhu's research was conducted more recently when improved inputs—fertilizers and other chemicals—were introduced thus changing the equilibrium. Where the level of land fertility was previously dependent on the level of labor input, the improved inputs helped to reduce the fertility differences of land on small and large farms. Therefore, as Sidhu showed, technical efficiency differentials would decrease with the greater use of these improved inputs, causing technical efficiency on small and large farms to become equivalent.

This line of thinking does not seem to relate to this analysis of agriculture in northern Nigeria. The assumption made here is that agriculture is traditional and is in a state of equilibrium (Schultz, 1964), yet small farms showed no differences in economic efficiency nor technical efficiency as compared to large farms. This is explained
as follows: 1) Norman et al. (1979, p. 24) notes that in Hausaland, people have usufructuary rights to the use of land within the community where they reside, which implies that ownership of land is largely communal in nature. This system leads to a lack of security of land title which may, in turn, discourage farmers from initiating any long-term land improvements. 2) Because family land is partitioned on inheritance, cultivated land has tended to be fragmented. Land improvement and conservation measures may be much more difficult with excessive land fragmentation because of the need for cooperation among neighbors (Norman et al., 1979, p. 28).

Thus, one can see that because of the system of land tenure and a result of land fragmentation, small farms in northern Nigeria do not have the advantages exhibited by small farms in India in the Yotopoulos and Lau study. Therefore, the relative technical efficiency of small farms is equivalent to that of the large farms.

Matlon and Newman (1979) made a study of data obtained in a twelve month farm survey conducted during 1974-75 in three villages of Kano State in northern Nigeria. Their results indicated significant efficiency differentials between groups of farmers stratified by income per consumer man-equivalent. Then, after finding this correlation they went on to separate economic efficiency into the allocative and technical components. They found that, on the average, farmers showed allocative efficiency, thus, there were variations in technical efficiency present. These were identified in a regression analysis
examining the effects of differences in location, factor quality, and key management practices. Their work was different from the present analysis in that they went beyond the mere correlation of efficiency differentials to attempt to identify the causes of these inefficiencies.

Matlon and Newman (1979, p. 29) separated the factors they saw as affecting technical efficiency into an external set and an internal set. A factor considered internally determined (within the management decision scope of the farmer) would be the degree of intercropping, as reflected by the number of crops per mixture. Because intercropping was shown to be positively related to technical efficiency and because farmers in the lowest income class made a greater use of this management technique than other classes, poorer households reflected more technical efficiency.

The externally determined set of factors, factors over which the farmer had no control, were shown to be the least conducive to economic efficiency for the poorest households. These factors which constrained the farmer included village location, field tenure status, and family organization. Also included in such factors would be the differential accesses farmers may have to information, credit, and new inputs where large farms may have (as was the case for Khan and Maki in Pakistan) a comparative advantage in obtaining and utilizing these.
When identifying interfirn differences in technical efficiency, Matlon and Newman (1979, p. 28) argue that these differences are neither a necessary nor sufficient condition to demonstrate interfirn differences in management quality. This inference is valid only when managers face the same choice and range of factors—both internally determined and externally determined. That is, they must operate in similar environments. Factors such as the status, income, and liquidity position which a farmer inherits from a previous period, may play greatly in determining his access to resources which thus influences his economic efficiency.

The implication is that the poorest farmers may not have been the worse managers or suffered from motivational problems when compared to the higher income groups, but instead were victims of a much more constrained environment. It might even be found that these farmers would indeed be better managers if they had available to them the same range of production choices and constraints. But what actually emerges is a circular pattern of causation where the characteristics related to the poorest households are actually factors which constrain the management options of households who are already in the lowest income group. This pattern limits their technical efficiency. For example, the poorer families tended to plant the crops of millet, sorghum, and cowpeas somewhat later than average resulting in decreased yields. Matlon and Newman contend that this was not an indication of a lack of managerial competence but was deliberate as a shortage of cash and seed necessitated planting after the rains were established and not fallowed by a dry spell.
VI.2. FINAL COMMENTS

In the case of traditional agriculture in northern Nigeria for this study, there seem to be a few limited possibilities for growth by improving allocative efficiency to small farms. This may be done by providing better market information to small farmers so they could better make decisions. One should recognize, though, that the goal of the traditional farmer may not be necessarily one of maximizing profits but may be one of risk aversion. Small and large farms show equivalent technical efficiency; therefore, no growth would be realized by examining technical efficiency differentials. It seems that optimum output for a given set of inputs using a given level of technology has been achieved in this traditional setting for both large and small farms. Therefore, the most important source of economic growth would be the growth obtained from a change in the production function surface through the introduction of improved kinds of inputs into the production base. The introduction of technical change seems to be the answer for growth both in the short-term and long-term.

Because there are no differences in technical and allocative efficiency parameters for the farms in this study, policies curtailing or influencing farm size will be based only on social and political considerations. It should be noted, however, that once change is initiated (either through politics or technological implementation) and the traditional equilibrium is upset, the analysis of economic efficiency becomes very important as differentials are much more likely to exist in a changing system as opposed to the static one
studied here. When political changes are initiated into the traditional system, certain groups may tend to be elevated and receive preferential treatment. Differential power of access to some inputs then may help to create inefficiencies in the system.

Not only is the power of access to some inputs biased towards certain groups of people, but also new technologies may be introduced into the traditional system which have a narrow scope and not a broad base of application. Some farmers may be placed in a binding situation as the technology is too specific for them to utilize. A narrow range of dates for planting, weeding, and applying chemicals to improved crop varieties is an example of technology which is too specific. Inclusion of certain indigenous farming practices—crop mixtures and fallowing (crop rotation)—into new technologies will help aid in lessening the differentials which could develop in technical efficiency.

It should be recognized that technologies which call for timely application of improved methods do in fact reap the highest yields, therefore tend to maximize technical efficiency for those who adopt these practices. Thus there is a trade off which must be faced—maximum technical efficiency for a few with the introduction of technologies which are constraining to many or more equivalent levels of technical efficiency for all with the introduction of technologies which are more flexible and broad based. This is many times a hard choice.
The differences in allocative and technical efficiency are caused by a number of complex and interdependent factors which go beyond basic economic variables to include technical and sociological factors. Unless these factors are appropriately identified and the differences corrected by relevant strategies, increased inequality seems to be the only result. Only through the efforts of interdisciplinary work—using economists, anthropologists, sociologists, and technical scientists—can agricultural policy decisions in less developed countries be made to be cognizant of both allocative and technical efficiency considerations.
APPENDIX III-1
DERIVATION OF EQUATION (III-38) FROM EQUATION (III-27)

\[ \pi = A^* X_1^{\alpha_1} X_2^{\alpha_2} Z_1^{\beta_1} Z_2^{\beta_2} - c_1 X_1^* - c_2 X_2^* \]

\[ \pi = A \begin{bmatrix} \frac{1}{1-\mu} (c_1)_{a_1}^{1-\mu} & -\alpha_2 & \beta_1 & \beta_2 \\ \frac{1}{1-\mu} (c_2)_{a_2}^{1-\mu} & \frac{1}{1-\mu} & \frac{1}{1-\mu} & \frac{1}{1-\mu} \end{bmatrix} \]

\[ \pi = A \begin{bmatrix} \frac{1}{1-\mu} (c_1)_{a_1}^{1-\mu} & -\alpha_1 & \beta_1 & \beta_2 \\ \frac{1}{1-\mu} (c_2)_{a_2}^{1-\mu} & \frac{1}{1-\mu} & \frac{1}{1-\mu} & \frac{1}{1-\mu} \end{bmatrix} \]

\[ \pi = A \begin{bmatrix} \frac{1}{1-\mu} (c_1)_{a_1}^{1-\mu} & -\alpha_1 & \beta_1 & \beta_2 \\ \frac{1}{1-\mu} (c_2)_{a_2}^{1-\mu} & \frac{1}{1-\mu} & \frac{1}{1-\mu} & \frac{1}{1-\mu} \end{bmatrix} \]

\[ \pi = A \frac{1-\mu+\alpha_1+\alpha_2}{1-\mu} \frac{a_1 a_2 - a_1 a_2}{a_1 a_2 + a_2 a_1 - a_2} \frac{a_1 a_2 + a_2 a_1 - a_2}{a_1 a_2 + a_2 a_1 + a_2} \frac{a_1 a_2 + a_2 a_1 + a_2}{a_1 a_2 + a_2 a_1 + a_2} \]
Since $\mu = \Sigma \alpha_i$, $i = 1, 2$ or $\mu = \alpha_1 + \alpha_2$, then

$$\pi = A \frac{1}{1 - \mu} \left( \frac{c_1}{\alpha_1} \right)^{1 - \mu} \left( \frac{c_2}{\alpha_2} \right)^{1 - \mu} \frac{\beta_1}{1 - \mu} \frac{\beta_2}{1 - \mu}$$

$$- A \frac{1}{1 - \mu} \frac{1}{c_1} \left( \frac{1}{\alpha_1} \right)^{1 - \mu} \left( \frac{c_2}{\alpha_2} \right)^{1 - \mu} \frac{\alpha_2 - 1}{1 - \mu} \frac{\alpha_2 - 1}{1 - \mu} \frac{- \alpha_2}{1 - \mu} \frac{\beta_1}{1 - \mu} \frac{\beta_2}{1 - \mu}$$

$$- A \frac{1}{1 - \mu} \frac{1}{c_2} \left( \frac{1}{\alpha_2} \right)^{1 - \mu} \left( \frac{c_1}{\alpha_1} \right)^{1 - \mu} \frac{\alpha_1 - 1}{1 - \mu} \frac{\alpha_1 - 1}{1 - \mu} \frac{- \alpha_1}{1 - \mu} \frac{\beta_1}{1 - \mu} \frac{\beta_2}{1 - \mu}$$

By factoring out the $A$ and $Z$'s, then

$$\pi = A \frac{1}{1 - \mu} \frac{\beta_1}{1 - \mu} \frac{\beta_2}{1 - \mu} \left[ \left( \frac{c_1}{\alpha_1} \right)^{1 - \mu} \left( \frac{c_2}{\alpha_2} \right)^{1 - \mu} \right]$$

$$+ \frac{1 - \alpha_1 - \alpha_2 + \alpha_2 - 1}{1 - \mu} \frac{\alpha_2 - 1}{1 - \mu} \frac{- \alpha_2}{1 - \mu} - \frac{1 - \alpha_1 - \alpha_2 + \alpha_1 - 1}{1 - \mu} \frac{\alpha_1 - 1}{1 - \mu} \frac{- \alpha_1}{1 - \mu}$$

$$- c_1 \left( \frac{1}{\alpha_1} \right)^{1 - \mu} \left( \frac{c_2}{\alpha_2} \right)^{1 - \mu} - c_2 \left( \frac{1}{\alpha_2} \right)^{1 - \mu} \left( \frac{1}{\alpha_1} \right)^{1 - \mu}$$

$$\pi = A \frac{1}{1 - \mu} \frac{\beta_1}{1 - \mu} \frac{\beta_2}{1 - \mu} \left[ \left( \frac{c_1}{\alpha_1} \right)^{1 - \mu} \left( \frac{c_2}{\alpha_2} \right)^{1 - \mu} \right]$$

$$- c_1 \left( \frac{1}{\alpha_1} \right)^{1 - \mu} \left( \frac{1}{\alpha_2} \right)^{1 - \mu} - c_2 \left( \frac{1}{\alpha_2} \right)^{1 - \mu} \left( \frac{1}{\alpha_1} \right)^{1 - \mu}$$

$$- c_2 \left( \frac{1}{\alpha_2} \right)^{1 - \mu} \left( \frac{1}{\alpha_2} \right)^{1 - \mu} \left( \frac{1}{\alpha_1} \right)^{1 - \mu} \left( \frac{1}{\alpha_1} \right)^{1 - \mu}$$
\[ \pi = A \frac{1}{1-\mu} z_1 \frac{\beta_1}{1-\mu} z_2 \frac{\beta_2}{1-\mu} \left[ \begin{array}{cc} \frac{-\alpha_1}{\alpha_1} & \frac{-\alpha_2}{\alpha_2} \\ \frac{c_1}{1-\mu} & \frac{c_2}{1-\mu} \end{array} \right] \]

\[ \pi = A \frac{1}{1-\mu} \left( \frac{c_1}{\alpha_1} \right)^{-\frac{1}{1-\mu}} \left( \frac{c_2}{\alpha_2} \right)^{-\frac{1}{1-\mu}} z_1 \frac{\beta_1}{1-\mu} z_2 \frac{\beta_2}{1-\mu} \left[ 1 - \frac{1}{1-\mu} \right] \]

\[ \pi = A \frac{1}{1-\mu} (1-\mu)^{-\frac{1}{1-\mu}} \left( \frac{c_1}{\alpha_1} \right)^{-\frac{1}{1-\mu}} \left( \frac{c_2}{\alpha_2} \right)^{-\frac{1}{1-\mu}} z_1 \frac{\beta_1}{1-\mu} z_2 \frac{\beta_2}{1-\mu} \left[ 1 - \frac{1}{1-\mu} \right] \]

Or, for the general case, the UOPP function is written

\[ \pi = A (1-\mu)^{-\frac{1}{1-\mu}} \left( \sum_{i=1}^{m} \frac{\pi}{c_i} \frac{1}{\alpha_i} \right)^{-\frac{1}{1-\mu}} \left( \sum_{j=1}^{n} \frac{\beta_j}{Z_j} \right)^{-\frac{1}{1-\mu}} \]

where \( \mu = \sum_{i=1}^{m} \frac{\pi}{c_i} \frac{1}{\alpha_i} \)

\( \pi = \text{normalized profit} \)
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ALLOCATIVE AND TECHNICAL EFFICIENCY OF
TRADITIONAL AGRICULTURE IN NORTHERN NIGERIA

by

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AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

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Manhattan, Kansas
1980
There is an increasing concern in Nigeria that agricultural production is lagging behind the food needs of a growing population. With approximately one-fifth of the population of the continent of Africa within its borders, this is a significant problem. This study aims at assessing the potential for increasing crop output by examining the economic efficiency—and its components of allocative and technical efficiency—of the traditional farming system. If inefficiencies are found to exist, then there is potential for increasing output using existing technology.

The data used in the analysis were obtained from farm management studies undertaken between 1966 and 1968 in villages surrounding the central cities of Zaria, Sokoto, and Bauchi in northern Nigeria. The 340 farmers in the total sample were interviewed twice weekly to gather information on such factors as farm income, expenses, and land and labor use.

A model used to measure and compare the performance of farm firms was developed by Lau and Yotopoulos which makes use of a Unit-Output-Price, or normalized, profit function and its corresponding factor demand functions. Relative differences in economic efficiency between groups of firms (in this study between large and small farms) may result from variations in allocative efficiency (the degree of equating the value of marginal product of each variable input to its cost) and technical efficiency (larger output with equal amounts of inputs).

With the use of the Zellner's Seemingly Unrelated Regression method, a normalized profit function and a labor demand function
were jointly estimated in this analysis to obtain estimates of the parameters for wage rate, land, and capital and for dummy variables included for large and small farms. A series of tests of hypotheses indicated that for the Zaria and Bauchi study areas both large and small farms seemed to have statistically equal economic efficiency with equal allocative and technical efficiency indicated. Although the Sokoto area study showed equal economic efficiency with unequal allocative and technical efficiency, by examining each of the three villages in the Sokoto study separately, small farms showed no differences in allocative nor technical efficiency, hence there was determined equal economic efficiency.

It was hypothesized that since the agricultural system in this analysis was truly traditional and thus at an equilibrium, it was not unusual to come up with these findings. Therefore, there seem to be few possibilities for growth by improving economic efficiency. The most important source of economic growth would be that obtained from a change in the production function surface through the introduction of improved inputs. Once change is initiated, this analysis of economic efficiency becomes even more important as differentials are much more likely to exist in a changing system as opposed to a static one. Once differentials are found, an interdisciplinary approach is needed in order to identify the inefficiencies, the constraints on efficiency, and how they might be dealt with.