THIS BOOK IS OF POOR LEGIBILITY DUE TO LIGHT PRINTING THROUGH OUT IT'S ENTIRETY.

THIS IS AS RECEIVED FROM THE CUSTOMER.

A WIENER-LEE TRANSFORM SCHEME FOR CALCULATING QUANTITIES THAT OBEY DISPERSION RELATIONS

by

David M. Zimmerman

B.S., Kansas State University, 1972

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Physics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1976

Approved by:

O. L. Weaver

Major Professor

TABLE OF CONTENTS

TABLE OF FIGURES

ACKNOWLEDGEMENTS

I am happy to express my gratitude to Dr. O. L. Weaver, who is the source of a good deal of my understanding concerning dispersion theory, and who has provided me with invaluable assistance throughout this work.

Drs. Williams, McGuire, Curnutte, and Pinkley have taken part in many stimulating discussions relative to this work and have helped clarify a number of problems.

Ken Laws has provided me with the program, FSTDFT, from his personal library and has given me numerous impromptu lectures on the theory of fast Fourier transforms. Dr. L. Pinkley has provided me with a program to do numerical Kramers-Kronig analysis.

I would also like to thank The Research Corporation for their financial assistance.

Introduction

Numerical analysis of dispersion relations or causal relations of the general form

$$Re(g(w)) = \frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{Im(g(w'))\ dw'}{w' - w}$$

has been used for years to calculate the Fourier transforms of causal functions. Precise calculations have necessarily been restricted to band-limited functions with well-known asymptotic behavior. And, since these are integral relations, a more computer-time efficient process than the Simpson's rule numerical integration process (usually used to solve such relations) would be convenient. Moreover, the integrals involved are principal-value integrals, which have their own inherent difficulties when done numerically, because one must banish integration over all singularities.

Recently, some research has aimed to circumvent these difficulties by returning to the basic assumptions upon which dispersion relations have been founded and deriving new relations which avoid the principal-value integrals. One such approach is an analysis which stays in the time domain. This method, advanced by Peterson and Knight (1973) and D.W. Johnson (1975), ultilizes the fast Fourier transform to find the real part of g from the imaginary part (or vice versa).

The research contained in this thesis also makes use of the fast Fourier transform. However, the approach herein is based on a conformal mapping of the region of analyticity of g(w) — the Fourier transformed causal function. This mapping, called the Wiener-Lee transform renders the region of analyticity finite and bounded.

The purpose of this research is to determine the usefulness of this Wiener-Lee transform method. This is assessed by applying the Wiener-Lee transform analysis to the absorption data of water and comparing the calculated results with those of the standard Kramers-Kronig numerical analysis.

This paper will first derive the Wiener-Lee transform and explain how this transform helps define a scheme for calculating quantities which also obey the causal relations. Then the connection between this process and the solution of the Kramers-Kronig dispersion relations for the index of refraction of water is established. A computer program written by the author which uses the Wiener-Lee transform technique is discussed (including a brief overview of fast Fourier transforms). It is then applied to the problem of finding the real part of the index of refraction of water from the absorption spectrum. Finally the results are compared with a standard Kramers-Kronig analysis used in the laboratory here at Kansas State and to the most accepted values for the refractive index to date. In addition, the traditional derivation of dispersion relations and some other applications of the causal relations are explored in the appendices.