TIME DEPENDENCE OF THE MAGNETIC FIELD IN A RECTANGULAR TOROID

by

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1.0 INTRODUCTION

The purpose of this thesis is to formulate an equation which describes the time dependence of the flux distribution in a rectangular toroid of ferromagnetic material subject to given boundary conditions and to provide a numerical solution to the equation. This is of interest because it allows one to predict the time necessary to release a no-work magnet in an electromechanical system or define the switching time of a toroid used in the core plane of many modern digital computers. An analytical solution also provides a convenient tool for optimizing the many problem variables or studying the effect of changing one or more of the variables on the magnetic performance of a given electromagnetic circuit.

Development of theory and assumptions necessary to derive such an equation for the special case considered, i.e., for the toroid of rectangular cross section as illustrated in Fig. 7, or the electromechanical system as illustrated in Fig. 2, are given in section 2.0. The resulting equation obtained by manipulation of Maxwell's equations was found to resemble the diffusion equation when subject to the assumptions required for the problem of interest. The Hysteretic Diffusion Equation, equation (2-28), expresses the desired relationship for specifying the magnetic field intensity H as a function of position in the core and time. It should be noted that H is written as simply H for convenience and represents $H(x,y,z,t)$.

$$\frac{\partial H}{\partial t} = \frac{(C_2 + H)^2}{\sigma C_1 C_2}\left[\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right] \qquad (2\text{-}28)$$

where H is the magnetic field intensity (z component)

σ is the material conductivity

C_1 is a constant specifying material properties

C_2 is a constant specifying material properties.

A discussion of the effects of eddy currents and concepts used to confirm the existence of a flux distribution in the cross section during a transient condition are given to aid the reader in understanding the problem.

The task of evaluating H was accomplished by using the Modified Euler method of numerical integration in which the rectangular cross section was thought of as being divided into a grid. Each grid represented a toroid with a time-variant field intensity. This allowed one to express the Hysteretic Diffusion Equation in its finite difference form and approximate $\partial H/\partial t$ by numerical techniques. Once this quantity was known, Euler's method was used to predict an approximate value for the field intensity of each grid area at a time Δt later, thus an approximate solution was obtained for H as a function of time for each grid area of the cross section. Values of flux density B, were obtained by substitution of H in an equation approximating the B-H relationship for the given ferromagnetic material.

One must remember the numerical process yields only an approximation to the actual values. Accuracy increases as the grid size decreases; however, the process becomes very slow for very small grid sizes because the maximum time increment allowable to insure convergence of the Modified Euler process is dependent on many of the problem variables as shown by equation (3-44).

$$0 < \Delta t < \frac{\sigma C_1 C_2 h^2}{2(C_2 + H_{max})^2} \qquad (3\text{-}44)$$

Consideration of conditions for convergence and accuracy of the process are given in section 3.4.

The last two sections of the thesis describe the program and various input variables needed to execute the process. Each input variable and its function in the program is discussed in section 4.2B. A sample problem with a typical input data set and the corresponding output data is given in section 5.0. Results of the data indicate a flux distribution and flux decay as predicted by sections 2.3 and 2.4.

2.0 DERIVATION OF THE HYSTERETIC DIFFUSION EQUATION FOR A MAGNETIC BOUNDARY VALUE PROBLEM

2.1 Introduction

A problem of interest to many is the solution of the diffusion equation sometimes called the heat equation. It is useful in describing the temperature distribution in an iron bar as a function of time, or in the magnetic case, the time dependence of the flux distribution in a toroid of magnetic material subject to given boundary conditions. It is the latter case which will be considered in detail in this thesis. The solution of this particular boundary value problem is important in predicting performance objectives and analysis of many components in modern digital computers as well as being of use for analytical evaluation of heat transfer characteristics and other problems in many other fields described by the equation. For example, the memory of most modern computers consists of core planes in which rectangular toroids are affixed at the junction of two write windings as illustrated in Fig. 1.

To magnetize the toroid one-half of the current needed must flow in the proper direction through both write wires threaded through the core; thus the core is magnetized in one particular direction which designates a "1" bit and the opposite direction which designates a "0" bit. During a read out of the memory, the direction of magnetization determines the direction of current flow induced in a read winding and in turn determines the

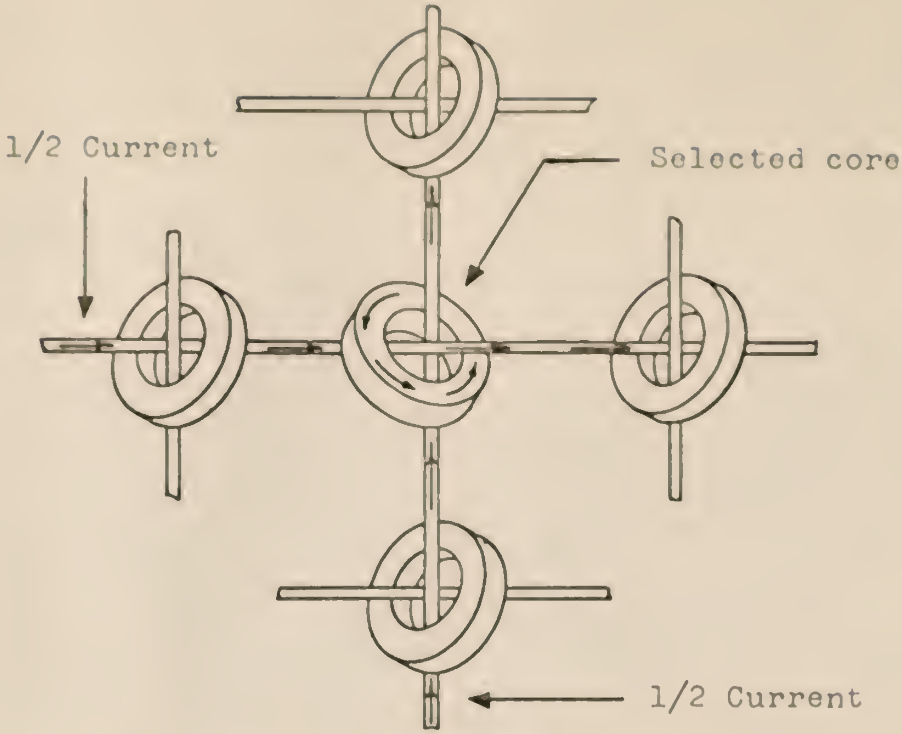

Fig. 1. Section of a memory core plane.

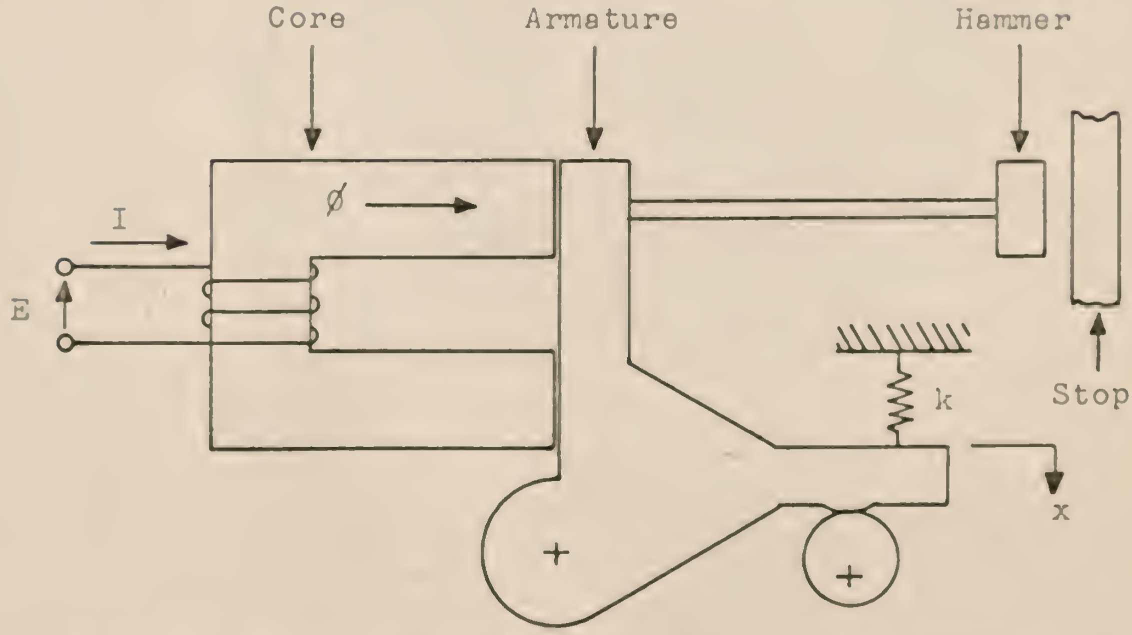

Fig. 2. Electromechanical device (no-work magnet).

presence of a "0" or "1" bit. One can see that it may be desirable to know the time to accomplish flux reversal during the write time. Furthermore, an analytical method of determining the reversal time for various cores of different sizes, shapes, materials, etc., would be convenient since this time could be a significant factor in determining the speed of the computer.

The solution could also provide information useful for improvements in the design of present-day input-output equipment for computation systems. Many high-speed card punches and printers use electromechanical devices in various punching and printing techniques, all of which require elaborate sequential timing and mechanical movement to accomplish the desired result.

Let us consider the electromechanical device in Fig. 2. Suppose a voltage E causes a current I to flow through the coil which produces a flux ϕ sufficient to overcome the force kx and hold the core and armature together. When I is removed (i.e., $E = 0$), the holding force is removed and the spring force allows the hammer to impact the stop. An extension of this principle is used for modern printers which can print at the rate of twelve hundred lines per minute with one hundred twenty characters per line. The time required to release the magnet becomes of prime importance since sequential timing and logic circuits required to release the armature cause a release and hold operation cycle to occur at very high rates of speed. A typical release time might be one millisecond. This same mechanism is used for obtaining punched cards and the same discussion could apply.

Due to effects of mechanical inertia, eddy currents generated by the rapidly changing boundary conditions on the magnetic circuit, and other factors, it may be desirable to provide an analytical solution which would account for changes in the parameters affecting the electromagnetic performance of an electromechanical system as outlined in the previous discussion. Assuming that the junction of the armature and core assembly of Fig. 2 does not provide an additional reluctance to impede the flow of flux across it other than that of the material itself, one can use the results of the following procedure to gain insight on effects that parameter changes produce on the electromagnetic performance.

2.2 Magnetic Circuit Concepts

Magnetic circuit considerations are closely analagous to those of resistive electrical circuits; however, the cause and effect relationship in the magnetic case is nonlinear, i.e., the reluctance of a d-c magnetic circuit depends on the flux in the circuit, while for the d-c electrical case resistance is relatively unaffected by the amount of current.

If one considers a toroid of magnetic material with a coil of wire wound tightly and distributed uniformly around it, a magnetic circuit problem is encountered. (See Fig. 3.)

The voltage E produces a flux ϕ in the magnetic material. The flux lines are perpendicular to a cross section of the toroid and should be uniformly distributed over the cross section in

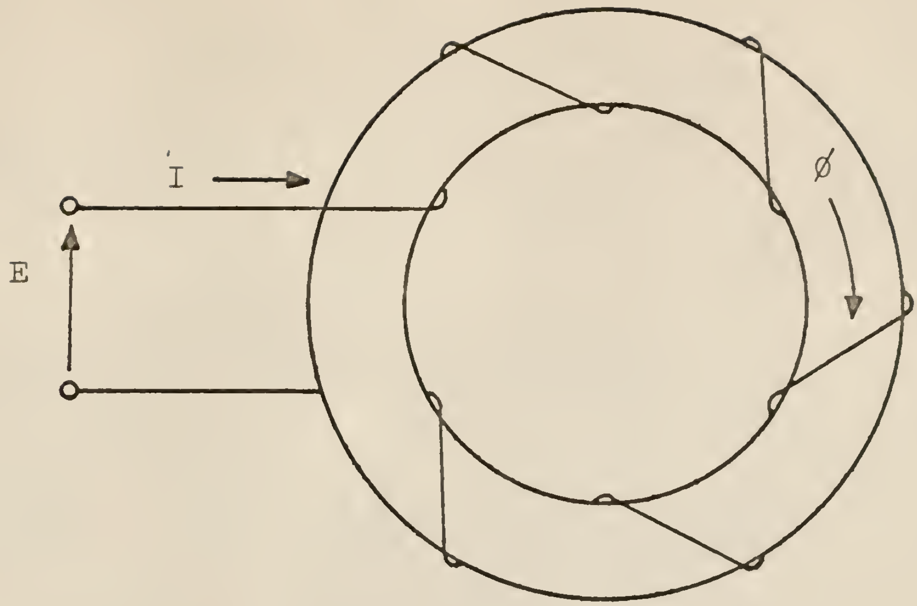

Fig. 3. Toroidal magnetic circuit.

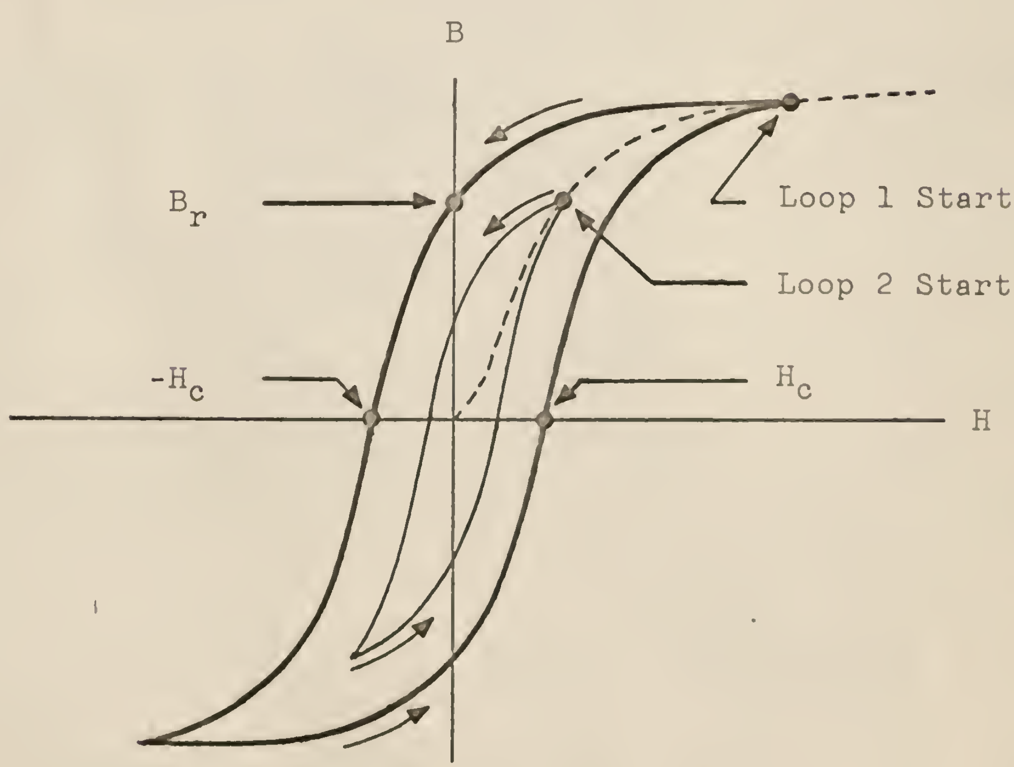

Fig. 4. Hysteresis loops and magnetization curve.

the steady-state case. To insure uniform boundary conditions produced by the current I, assume the cross-sectional diameter of the ring is very small when compared to the inside and outside diameters of the ring. Under these conditions the following relationships hold:

(2-1) $\mathcal{F} = NI = Hl = R\phi$

(2-2) $B = \mu H = \phi/A$

(2-3) $R = \mu l/A$

(2-4) $\text{Force (F)} = B^2A/2\mu_0$

The above also assumes that no leakage occurs and that one is only considering the steady-state solution; however, the transient operation is of major importance when an electromagnet or rectangular toroid is used in the applications as outlined in section 2.1.

If one desires to know the value of the flux at several discrete points in time during a build-up or decay cycle, one must use a different method of analysis than presented previously. During the transient state a relationship exists between flux and magnetic field intensity as illustrated by the hysteresis curve of Fig. 4. This curve specifies the magnetic properties of the ferromagnetic material.

One can also see that during a flux build-up or decay residual magnetism B_r, and coercive force H_c, depend on the original values of the flux density B and magnetic field intensity H. A typical ferrite material in the memory of a computer exhibits what is known as a square-loop property. This means that the material's B-H characteristic has a square or rectangular hysteresis loop, which allows an immediate change in the direction of magnetization once the proper value of H is present. In all cases the value of B for a corresponding H follows the upper curve if in the decay portion of the cycle and follows the lower curves if in the build-up portion of the cycle. Furthermore, during either build-up or decay, a flux distribution exists across the cross section of the magnetic core due to eddy currents generated from the changing flux; i.e., the outside of the core reaches the new value of flux density immediately while the center of the core remains at the original value and gradually attains the same value specified by the boundary in the steady-state condition. A more complete discussion of the eddy-current problem and its relation to the flux distribution is given in the next section.

2.3 Eddy Currents

Eddy currents are generated by a change of flux in the magnetic material of a core as shown in Fig. 3, and become a very important factor in the analysis of the transient behavior of high-speed electromagnetic circuits. These currents affect the change in flux by tending to produce an opposing flux, thus changing the flux distribution across the pole face in addition to delaying flux build-up or decay. Flux build-up is not affected nearly so significantly as flux decay because the eddy-current opposition flux is a small part of the total flux applied. Since a no-work magnet uses flux decay to accomplish its function, the eddy currents play a large part in controlling the release operation; thus they can limit the speed of operation for many of the electromechanical systems used in modern input-output machines of digital computation systems.

One can visualize the effect of eddy currents on the flux distribution by considering a cylindrical core to be made up of concentric shells, each shell being a hollow cylinder of differential thickness. Each shell constitutes a short-circuited turn enclosing part of the core flux. The outside shells link all the flux while the inner shells link only a small part of it. A voltage is induced in the shells upon a change of flux. The magnitude of the induced emf is determined by the amount of flux linked by the coil; thus the outer shells have larger emf's and eddy currents than the inner shells. Induced currents apply a magnetomotive force to the part of the core that lies within it; thus the center of the core is subject to the magnetomotive force of all eddy currents while the surface is subject to none. This accounts for the distribution present in the transient state. Induced currents tend to oppose any change in flux; thus they will tend to sustain a decreasing field and oppose an increasing field. It can be seen that the flux distribution is altered by the eddy currents and is not only a function of time but also of core radius. An approximate flux distribution with radius is shown in Figs. 5 and 6. Figure 5 illustrates the effect of eddy currents during the transient flux build-up condition while Fig. 6 illustrates the effect of eddy currents

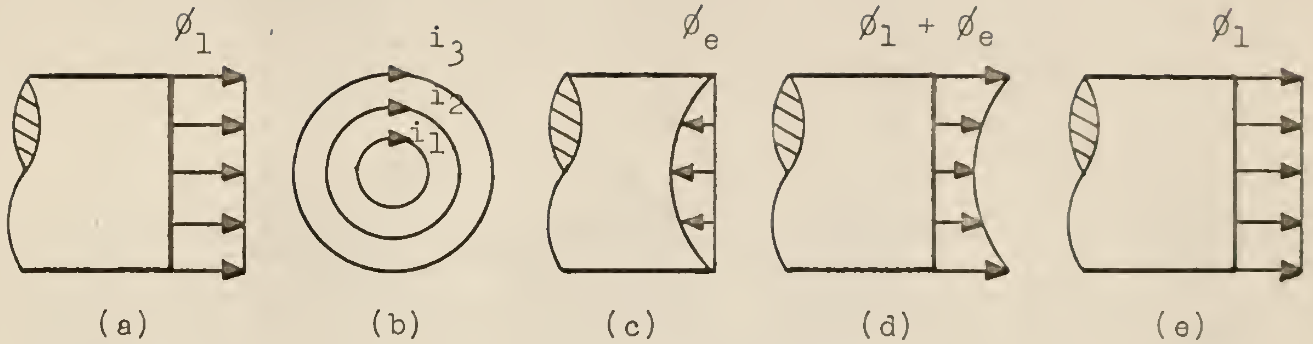

Fig. 5. Effect of eddy currents on flux buildup in a ferromagnetic material.

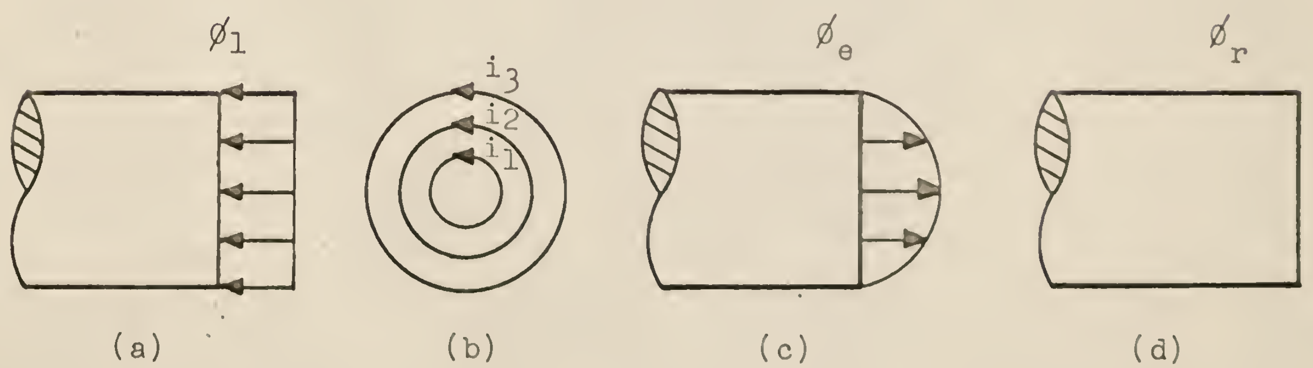

Fig. 6. Effect of eddy currents on flux decay in a ferromagnetic material.

during flux decay.

Let us represent the applied field intensity with vectors in the direction of the applied field. We may then illustrate the flux at $t = 0^+$ by Fig. 5a. Once the field is applied, eddy currents i_1, i_2, i_3, . . ., are set up in the core as illustrated by Fig. 5b. These eddy currents induce an opposing flux ϕ_e shown in Fig. 5c. The resultant flux at $t = 0^+$ is then $\phi_1 + \phi_e$ and is illustrated by Fig. 5d. Since the eddy currents depend on a changing flux they will decrease as time increases, thus ϕ_e will approach zero in the steady state and the steady-state flux will be equal to the applied flux ϕ_1.

Upon release the same phenomenon occurs; however, the core is originally in the magnetized state and eddy currents will be induced opposite in direction to that of the previous example. This follows from Lenz's Law. The only flux present at $t = 0^+$ is that induced by the eddy currents as shown in Fig. 6b. As time increases, ϕ_e decreases until the flux returns to zero. Actually it will return to a residual value ϕ_r.

2.4 Flux Distribution in the Rectangular Toroid

With the preceding discussion of eddy-current phenomena in mind, one can readily visualize a flux distribution existing in a toroid of rectangular cross section. Furthermore, one knows that the flux distribution is changing with respect to time and values of flux at any point within the core depend on the distance from the outer edge of the core at which the boundary condition is applied, the boundary condition itself, the material of which the toroid is made, and time. If one considers a rectangular cross section to be divided into many small squares of nearly infinitesimal area, the value of flux density in a given square within the core is a function of time and is different for each area. Time dependence of the flux density may be obtained if one can find a relationship sufficient to specify the value of magnetic field intensity for this area as a function of time. After division into small areas one may then consider each area separately as a rectangular toroid with uniform flux density provided the area is very small in comparison to the total cross-section area. Values of H, B, and ϕ for each area can then be calculated for each value of time according to the relationships given in section 2.7. One might note that once H is determined, it is a simple matter to evaluate B from the hysteresis loop. Multiplication of B and the area corresponding to the value of B calculated will yield a value of flux for that particular area. It then becomes possible to calculate total values of H, B, and ϕ for the entire cross section. If the above calculations are made for each value of time in the transient state, curves relating the time dependence of the flux distribution and total flux can be computed for various boundary conditions and the effect of parameter changes can be analyzed theoretically. Since no literature was found that expressed the magnetic field intensity as a function of position and time for the case in question, equation (2-28) was derived. It will be referred to as the "Hysteretic Diffusion Equation" and its derivation is included in the next section.

2.5 Derivation of the Hysteretic Diffusion Equation

The derivation presented requires only basic electromagnetic theory and simply applies Maxwell's Equations to the special case being considered. To refresh the reader's memory, the time variant Maxwell Equations are listed below in both point and integral form. Overbarred symbols designate vector quantities in the cartesian co-ordinate system. This system was the most convenient for the rectangular cross section being considered. A co-ordinate transformation of the resulting equation could be used if different cross-section shapes are studied.

Maxwell's Equations

	(Point form)	(Integral form)
(2-1)	A. $\bar{\nabla} \times \bar{E} = -\partial\bar{B}/\partial t$	B. $\oint \bar{E} \cdot d\bar{l} = (\partial/\partial t)\int_S \bar{B} \cdot d\bar{S}$
(2-2)	A. $\bar{\nabla} \times \bar{H} = \bar{i} + \partial\bar{D}/\partial t$	B. $\oint \bar{H} \cdot d\bar{l} = \int_S (\bar{i} + \partial\bar{D}/\partial t) \cdot d\bar{S}$
(2-3)	A. $\bar{\nabla} \cdot \bar{B} = 0$	B. $\oint \bar{B} \cdot d\bar{S} = 0$
(2-4)	A. $\bar{\nabla} \cdot \bar{D} = \rho$	B. $\oint \bar{D} \cdot d\bar{S} = \int_v \rho \cdot dv$

Consider a toroid with a very large inside diameter in comparison to the bar diameter or an infinite bar of ferromagnetic material which undergoes a change of magnetomotive force on its boundary due to a change of the exciting current I supplied from a voltage E. (See Fig. 7.)

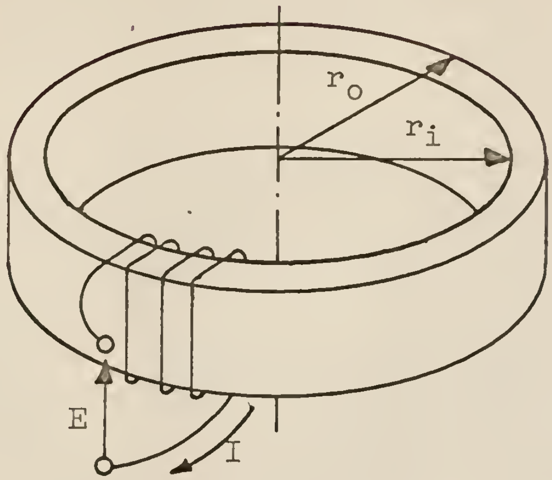

Fig. 7a. Rectangular toroid.

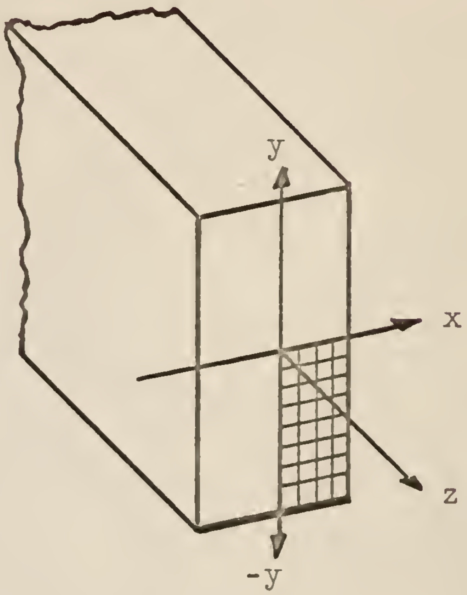

Fig. 7b. Cross section of a rectangular toroid.

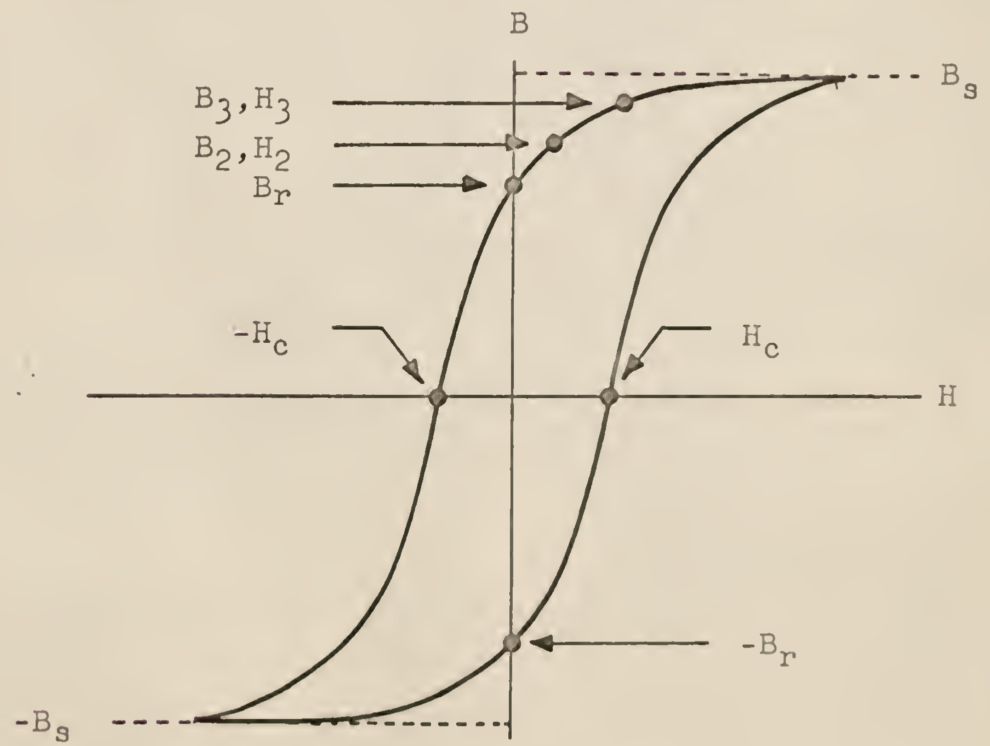

Fig. 8. Hysteresis loop.

It is known with some certainty that a flux distribution exists within the core due to eddy currents while the magnetization of the core is in the transient state, thus steady-state techniques cannot be applied; however, Maxwell's Equations hold in the transient state as well as for the steady state, thus an equation relating the effects of $\overline{H}$ as it varies with x, y, z, and t can be derived.

First we may assume that an mmf has been applied. When applied, a magnetic field $\overline{H}$, an electric field $\overline{E}$, and a flux density $\overline{B}$, exist in the toroid, and all are governed by Maxwell's Equations. If the electric field current density $\overline{D}$, is neglected, Maxwell's second equation becomes

(2-5) $$\overline{\nabla} \times \overline{H} = \overline{i}$$

Since the current flow in a conductor is in the direction of the applied electric field and perpendicular to an incremental surface $\overline{dS}$, the total current I is obtained by integrating the current density $\overline{i}$ over the surface; i.e.,

(2-6) $$I = \int_S \overline{i} \cdot \overline{dS} = \int_S (\overline{\nabla} \times \overline{H}) \cdot \overline{dS}$$

These currents are the eddy currents present in the transient state and can be represented by the scalar multiplication

(2-7) $$\overline{i} = \sigma \overline{E}$$

Thus the relationship between the magnetic field intensity $\overline{H}$, electric field intensity $\overline{E}$, material conductivity σ, and the conduction current density $\overline{i}$, is determined since

(2-8) $$I = \int_S \overline{i} \cdot \overline{dS} = \int_S (\overline{\nabla} \times \overline{H}) \cdot \overline{dS} = \sigma \int_S \overline{E} \cdot \overline{dS}$$

Therefore one can now show that

(2-9) $$\overline{\nabla} \times \overline{H} = \sigma \overline{E} = \overline{i}$$

A relationship between the flux density $\overline{B}$ and magnetic field intensity $\overline{H}$ can be found by combining equation (2-5) and Maxwell's first equation, equation (2-10).

(2-10) $$\overline{\nabla} \times \overline{E} = - \partial \overline{B}/\partial t$$

Combining equation (2-9) with equation (2-10) results in

(2-11) $$\overline{\nabla} \times \overline{\nabla} \times \overline{H} = - \partial \overline{B}/\partial t$$

In the general case $\overline{B}$, $\overline{H}$, and $\overline{E}$ are functions of x, y, z, and t and should be written $\overline{B}(x,y,z,t)$, $\overline{H}(x,y,z,t)$, and $\overline{E}(x,y,z,t)$. It is also known that a nonlinear relationship exists between $\overline{B}$ and $\overline{H}$; thus one can say that

(2-12) $$\overline{B} = f(\overline{H})$$

(2-13) $$\partial \overline{B}/\partial t = (\partial \overline{B}/\partial \overline{H})(\partial \overline{H}/\partial t)$$

Further examination of the functions $\overline{B}(x,y,z,t)$ and $\overline{H}(x,y,z,t)$ illustrates the fact that total derivatives may be used since

(2-14) $$\frac{d\overline{B}}{dt} = \frac{\partial \overline{B}}{\partial x}\frac{dx}{dt} + \frac{\partial \overline{B}}{\partial y}\frac{dy}{dt} + \frac{\partial \overline{B}}{\partial z}\frac{dz}{dt} + \frac{\partial \overline{B}}{\partial t}$$

$$(2\text{-}15)\qquad \frac{d\bar{H}}{dt} = \frac{\partial\bar{H}}{\partial x}\frac{dx}{dt} + \frac{\partial\bar{H}}{\partial y}\frac{dy}{dt} + \frac{\partial\bar{H}}{\partial z}\frac{dz}{dt} + \frac{\partial\bar{H}}{\partial t}$$

and the derivatives of distance with respect to time are equal to zero if the toroid is stationary with respect to the exciting mmf. Thus we have

$$\partial\bar{B}/\partial t = d\bar{B}/dt$$

$$\partial\bar{H}/\partial t = d\bar{H}/dt$$

$$(2\text{-}16)\qquad d\bar{B}/dt = (d\bar{B}/d\bar{H})(d\bar{H}/dt) = U(\bar{H})d\bar{H}/dt$$

$$(2\text{-}17)\qquad \partial\bar{B}/\partial\bar{H} = d\bar{B}/d\bar{H} = U(\bar{H})$$

The quantity $U(\bar{H})$ represents the relationship between $\bar{B}$ and $\bar{H}$ as illustrated by the slope of the hysteresis loop of Fig. 8. Combining equations (2-17), (2-13), and (2-11), we find that the following equation exists.

$$(2\text{-}18)\qquad \partial\bar{H}/\partial t = -(1/\sigma U(\bar{H}))\ (\bar{\nabla} \times \bar{\nabla} \times \bar{H})$$

Solution of equation (2-18) will provide an expression for $\bar{H}$ as a function of the position within the rectangular toroid and time; however, a solution is very hard to obtain. If one assumes the only component of field intensity present is along the z axis (see Fig. 7) and that this value of field intensity $\bar{H}$ is the same value at all points on the z axis, equation (2-18) reduces to a form generally recognized as the diffusion equation in two dimensions; i.e.,

$$(2\text{-}19)\qquad \partial\bar{H}/\partial t = \bar{\nabla}^2\,\bar{H}/\sigma U(\bar{H})$$

The above assumptions require that

$$H_x = H_y = \frac{\partial H_z}{\partial z} = 0$$

where in general $\overline{H}$ is given by the vector equation

$$\overline{H} = H_x\overline{i} + H_y\overline{j} + H_z\overline{k} \ .$$

Thus equation (2-18) may be written as

$$\frac{\partial \overline{H}}{\partial t} = - \frac{1}{\sigma U(H)} (\overline{\nabla} \times \overline{\nabla} \times H_z\overline{k}) \tag{2-20}$$

Expanding the expression $(\overline{\nabla} \times \overline{\nabla} \times \overline{H})$ of equation (2-18) with $\overline{H} = H_z\overline{k}$ and noting that

$$\overline{\nabla} \cdot \overline{H} = \left[\frac{\partial}{\partial x} \overline{i} + \frac{\partial}{\partial y} \overline{j} + \frac{\partial}{\partial z} \overline{k}\right] \cdot (H_z\overline{k}) = \frac{\partial \overline{H}_z}{\partial z} = 0$$

we may evaluate the expression $(\overline{\nabla} \times \overline{\nabla} \times \overline{H})$, i.e.,

$$\overline{\nabla} \times \overline{\nabla} \times \overline{H} = \overline{\nabla} (\overline{\nabla} \cdot \overline{H}) - (\overline{\nabla} \cdot \overline{\nabla})\overline{H}$$

$$\overline{\nabla} \times \overline{\nabla} \times \overline{H} = \overline{\nabla} (\ 0 \ ) - \overline{\nabla}^2 \overline{H}$$

$$\overline{\nabla} \times \overline{\nabla} \times \overline{H} = - \overline{\nabla}^2 H_z\overline{k} \tag{2-21}$$

Thus equation (2-18) is now reduced to the diffusion equation for the two-dimensional case, i.e.,

$$\frac{\partial H}{\partial t} = \frac{1}{\sigma U(H)} \left[\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2}\right] \tag{2-22}$$

An approximate solution of the above equation may be accomplished through use of numerical integration techniques once a suitable approximation for U(H) is determined. The remainder of the thesis will deal with the special case outlined above. The vector notation may be deleted since the only component present is H_z and the equation was solved only for this particular case.

The only remaining unknown is the functional relationship existing between B and H. One knows this relationship is expressed by the hysteresis curves of various magnetic materials; thus an equation which approximates the specific curve of interest would be desirable. Development of such an equation is undertaken in the next section.

2.6 Approximation of the B-H Curve for U(H)

Because of the flux distribution within the toroidal core, each small area as defined in section 2.4, will have a different value of field intensity and flux density at any given time in the transient condition; thus an approximation would allow ϕ to be calculated directly for each area once H for that area is known. This may be accomplished if an equation can be found which approximates the particular B-H curve of interest. Since this particular curve is determined by the initial values of H for a given material, an approximation of only this B-H curve would be sufficient to compute B once H is known.

A modified Froelich approximation equation as given by equation (2-23) can be used for generation of one-fourth of a given hysteresis loop.

$$B = \frac{C_1 H}{C_2 + H} + B_r \tag{2-23}$$

where B is the flux density

B_r is the residual flux density

H is the field intensity

C_1 constant specifying material properties

C_2 constant specifying material properties.

Values of C_1 and C_2 determine the shape of the curve and may be varied to generate an approximation to most hysteresis loops. They may be calculated from B_r and selection of two values, B_1, H_1, and B_2, H_2, taken from near the knee of the decay portion of a given hysteresis loop representative of the operating range in a given material. (See Fig. 8.)

Since both points selected must lie on the curve, a simple simultaneous solution of two equations formed by substituting B_1, H_1, and B_2, H_2, in equation (2-23) will be sufficient to specify C_1 and C_2. Performing this operation yields

$$B_1 = \frac{C_1 H_1}{C_2 + H_1} + B_r \tag{2-23a}$$

$$B_2 = \frac{C_1 H_2}{C_2 + H_2} + B_r \tag{2-23b}$$

Solving for C_1 and C_2 we have the following relationships:

$$C_2 = \frac{H_2 H_3 (B_3 - B_2)}{(B_2 - B_r)H_3 - (B_3 - B_r)H_2} \tag{2-24}$$

$$(2\text{-}25) \qquad C_1 = \frac{(B_2 - B_r)(C_2 + H_2)}{H_2}$$

After determining C_1 and C_2, one-fourth of the hysteresis loop can be plotted using the values calculated from equation (2-22). If there is a reasonable correspondence between the original curve and the curve plotted using the approximation equation, one can assume C_1 and C_2 are sufficiently accurate to specify the B-H relationship. Reflection and translation of this quarter section of the hysteresis loop generates the remaining portions of the loop. Calculation of B for values of $H < -H_c$ for flux decay and $H > H_c$ for flux build-up are considered in section 4.3.

This method of approximation is perhaps rather crude; however, it does suffice in this case. More accurate approximations are no doubt possible although maybe not practical since the magnetic properties of a given material will vary and cause larger errors than those due to the approximation.

2.7 Hysteretic Diffusion Equation

If we now use the approximation equation (2-22) for evaluating $U(H)$, we have

$$(2\text{-}26) \qquad U(H) = \frac{dB}{dH} = \frac{d}{dH}\left[\frac{C_1 H}{C_2 + H} + B_r\right]$$

Performing the differentiation we have

$$(2\text{-}27) \qquad U(H) = \frac{C_1 C_2}{(C_2 + H)^2}$$

Combining equations (2-22) and (2-27), we obtain the hysteretic form of the diffusion equation.

$$(2\text{-}28) \qquad \frac{\partial H_z}{\partial t} = \frac{(C_2 + H_z)^2}{\sigma C_1 C_2} \left[\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} \right]$$

It is this equation whose solution will yield the time dependence of the flux distribution pattern in the rectangular toroid subject to boundary conditions as outlined in section 2.5.

The remainder of this thesis is concerned with the numerical methods for obtaining this solution, implementation of these methods, and demonstration of the numerical process by actually obtaining a solution of equation (2-28) with its boundary values.

3.0 NUMERICAL SOLUTIONS WITH THE MODIFIED EULER METHOD

3.1 Euler Method for Solving an Ordinary Differential Equation

Numerous methods of obtaining solutions to ordinary and partial differential equations have become practical since the advent of the high-speed digital computer. The Modified Euler Method was chosen for this problem because of its simplicity, although other methods may provide a more accurate approximation to the solution or take less computation time. Before showing the application of Euler's Method and its modification to equation (2-28), let us consider the solution of an ordinary differential equation of first order. In symbolic form we may write

$$\frac{dH}{dt} = f(t,H) = D \tag{3-1}$$

The integral of equation (3-1) gives H as a function of time; thus we have $H = F(t)$. A graph of $F(t)$ is a curve in the H-t plane which may be approximated by a series of short line segments provided the curve is continuous; thus we have the approximation relation (see Fig. 9)

$$\Delta H = \Delta t \tan \theta = \left[\frac{dH}{dt}\right]_0 \Delta t = D_0 \Delta t \tag{3-2}$$

$$H_1 = H_0 + D_0 \Delta t \tag{3-3}$$

If we let $\Delta t = h = t_{i+1} - t_i$, we can express the approximation by

$$(3\text{-}4) \qquad H_{i+1} = H_i + D_i h \qquad (i = 0, 1, 2, \ldots 1)$$

This is known as Euler's Method. However, if h is taken small enough to yield sufficient accuracy the method is too slow; if h is larger, inaccuracies will cause the approximation to be unsatisfactory; furthermore, if the graph is monotonic, the approximation will diverge from the actual curve for any value of h chosen. A modification of this method tends to eliminate the divergence.

3.2 Modified Euler Method for Solving an Ordinary Differential Equation

Starting with an initial value H_0 one can approximate H_1 in the same manner as before to yield

$$(3\text{-}5)^1 \qquad H_1^{(1)} = H_0 + D_0 h$$

Substituting $H_1^{(1)}$ into equation (3-1), one obtains an approximation for dH/dt at the end of the first interval, i.e.,

$$(3\text{-}6)^1 \qquad D_1^{(1)} = f(t_1, H_1^{(1)})$$

An improved value of H is then found by multiplying h by the average of the values of dH/dt at the ends of the interval t_0 to t_1; thus we have

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[1]Note that D_i represents $(dH/dt)_i$. This notation will be used throughout the remainder of this section.

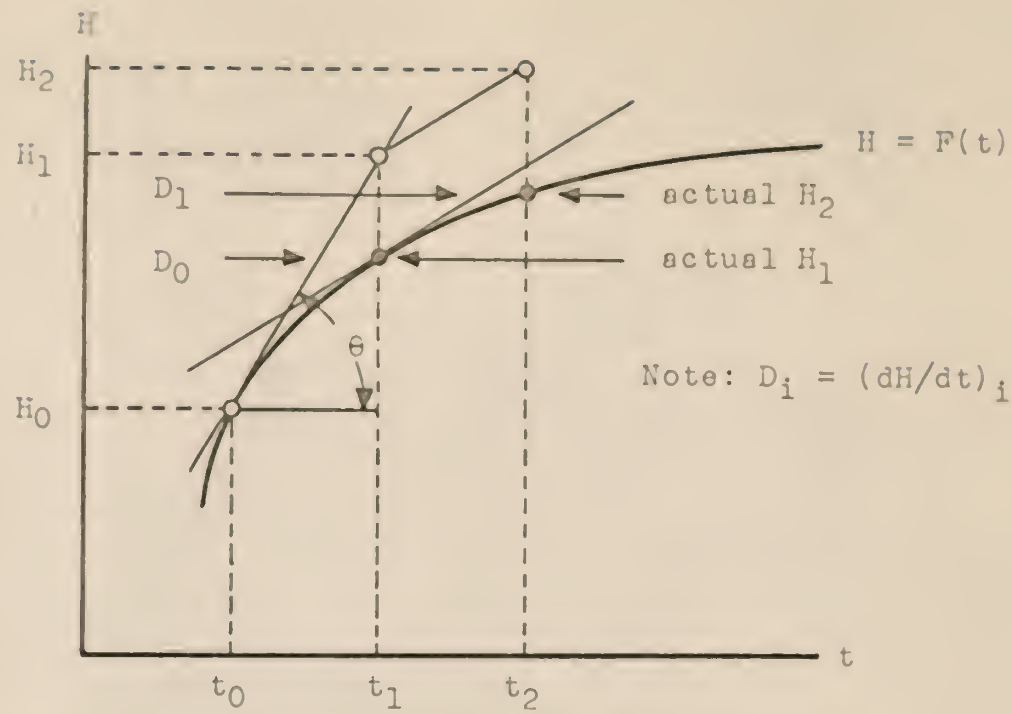

Fig. 9. Approximation with Euler method.

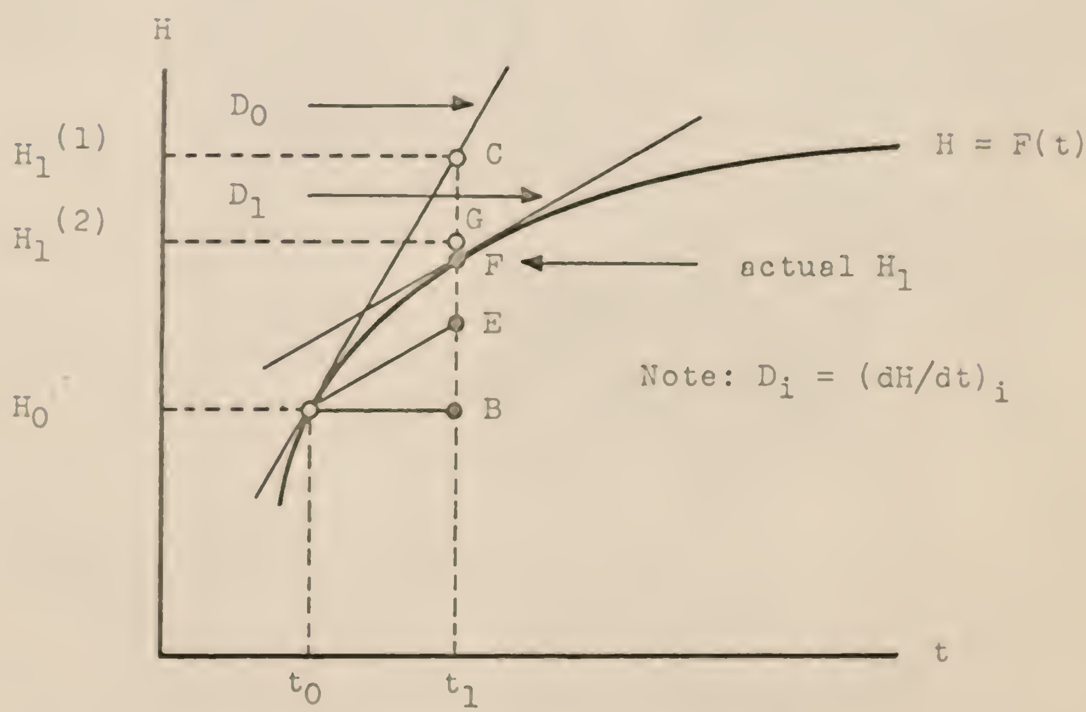

Fig. 10. Approximation with modified Euler method.

$$(3\text{-}7) \qquad \Delta H = \frac{1}{2} h(D_0 + D_1^{(1)})$$

A more accurate value of H_1 can now be calculated and will be denoted as follows:

$$(3\text{-}8)^{1} \qquad H_1^{(2)} = H_0 + \frac{1}{2} h(D_0 + D_1^{(1)})$$

If we look at Fig. 10, it is evident that this value of $H_1^{(2)}$ is more accurate than $H_1^{(1)}$. $H_1^{(1)}$ is represented by the line H_0 + BC if calculated according to Euler's formula. Substitution of $H_1^{(1)}$ in equation (3-1) gives an approximation of the slope represented by the tangent at point F. If a value of H_1 were calculated using the slope at the end of the interval, we would have $H_1 = H_0 + BE$. When the average of the slopes at the ends of the interval are used in place of D_0 we find that $H_1^{(2)} = H_0 + BG$ which is definitely a better approximation to the real value of H_1 at t_1 than the first value of H_1 computed. This process may be represented symbolically as

$$(3\text{-}9) \qquad \Delta H = \frac{1}{2} h(D_0 + D_1^{(1)})$$

$$\Delta H = \frac{1}{2} (BE + BC) = \frac{1}{2} (BE + BE + EC)$$

$$(3\text{-}10) \qquad \Delta H = BE + \frac{1}{2} EC$$

$$(3\text{-}11) \qquad H_1^{(2)} = H_0 + \Delta H = H_0 + (BE + \frac{1}{2} EC)$$

---

[1]The superscript (k) on $H_1^{(k)}$ indicates the k^{th} value of H_1 where H_1 is the approximation to the actual value of H_1 at a time $t = t_1 = 0 + \Delta t$.

The new value is much closer to the actual value of $H_1 = (H_0 + BF)$ than before. Continuation of this process by again calculating an approximation to the slope at the end point of the interval and by substituting $H_1^{(2)}$ in equation (3-1), will yield a more accurate approximation to the slope at point F. A new value of $H_1^{(3)}$ can then be calculated.

$$(3\text{-}12) \qquad H_1^{(3)} = H_0 + \frac{1}{2}h(D_0 + D_1^{(2)})$$

$H_1^{(3)}$ will be more accurate than $H_1^{(2)}$ since the approximation of the slope will be improved; i.e., $D_1^{(2)}$ is more accurate than $D_1^{(1)}$. Further continuation of the process will yield successively more accurate approximations to the actual value of H_1. The process may be continued until the value of $H_1^{(k+1)} = H_1^{(k)}$, where $H_1^{(k)}$ is the approximation of the actual value of H_1. It must be noted that $H_1^{(k)}$ is only an approximation to the actual H_1. As successive values of $H_1^{(k)}$ are generated, $H_1^{(k)}$ will converge to a value $H_1^{(n)}$ which will not be the same as the actual value of H_1. To force the approximation $H_1^{(k)}$ to converge to H_1 we must make h very small. The approximate solution will approach the exact solution as $h \to 0$; however, as $h \to 0$ the number of calculations increases and the computation time becomes very large. It then becomes necessary to determine the magnitude of errors permissible in relation to the time available and adjust h accordingly.

To illustrate the fact that the limit of $H_i^{(k)}$ does not approach the actual H_i as $k \to \infty$, let us consider the following example. Suppose $f(t, H)$ is expressed by equation (3-13) and the initial conditions $t_0 = 0.0$ and $H_0 = 1.0$.

(3-13) $$D = f(t, H) = t + H$$

Substituting these values of t_0 and H_0 in equation (3-13) we obtain

$$D_0 = f(t_0, H_0) = t_0 + H_0 = 1.0$$

If we select $h = 0.05$, we may then write

$$H_1^{(1)} = H_0 + D_0 h = 1.05$$
$$D_1^{(1)} = t_1 + H_1^{(1)} = 1.10$$

The second approximation to H_1 and D_1 are

$$H_1^{(2)} = H_0 + \frac{1}{2} h(D_0 + D_1^{(1)}) = 1.0525$$

$$D_1^{(2)} = t_1 + H_1^{(2)} = 1.1025$$

Continuing we have successive approximations to H_1 and D_1 as follows.

$$H_1^{(3)} = H_0 + \frac{1}{2} h(D_0 + D_1^{(2)}) = 1.05256$$

$$D_1^{(3)} = t_1 + H_1^{(3)} = 1.10256$$

$$H_1^{(4)} = H_0 + \frac{1}{2} h(D_0 + D_1^{(3)}) = 1.05256$$

Since $H_1^{(4)} = H_1^{(3)}$ we should now stop the process if we desire $H_1^{(k)}$ to agree with $H_1^{(k+1)}$ only to the fifth decimal place. Further values of $H_1^{(k)}$ for $3 \leq k \leq \infty$ will yield the same value for $H_1^{(k)}$ in the first five decimal positions as $H_1^{(3)}$. This points out the fact that the approximation $H_1^{(k)}$ will not converge to the actual H_1 as $k \to \infty$ since H_1 actual = 1.05254. We now take

$$H_1 = H_1^{(3)} = 1.0526$$
$$D_1 = D_1^{(3)} = 1.1026$$

Continuing, we can calculate the first approximation for H_2 and D_2.

$$H_2^{(1)} = H_1 + D_1^{(1)}h = 1.1077$$
$$D_2^{(1)} = t_2 + H_2^{(1)} = 1.2077$$

Then we calculate second and third approximations for H_2 and D_2 (i.e., $H_2^{(2)} H_2^{(3)}$ and $D_2^{(2)}$, $D_2^{(3)}$) and stop since $D_2^{(3)} = D_2^{(2)}$. Consequently $H_2^{(3)}$ agrees with $H_2^{(2)}$ and we have

(3-14) $$H_2 = H_2^{(3)} = 1.1104$$
(3-15) $$D_2 = D_2^{(3)} = 1.2104$$

Further continuations will yield successive values of H_3, D_3, H_4, D_4, etc.

To evaluate the accuracy of the method let us compare the approximated values of H_1, H_2, etc., to those calculated from the solution to $dH/dt = t + H$, which is

(3-16) $$H = 2e^t - t - 1$$

These are given in Table 1. Accuracies of H indicated in Table 1 can be improved only by using a smaller value for h.

Table 1. Comparison of approximate and exact solution of $dH/dt = t + H = D$.

i	t	H_i act	H_i approx	D_i act	D_i approx
0	0.00	1.00000	1.0000	1.00000	1.0000
1	0.05	1.05254	1.0526	1.10254	1.1026
2	0.10	1.11034	1.1104	1.21034	1.2104

In general, execution of the Modified Euler's method for the equation $D = f(t, H)$ is as follows.

Step 1. Obtain the initial conditions

$$t = t_0$$

$$h = \text{constant} = \Delta t$$

$$H = H_0$$

Step 2. Evaluate the approximations to H_1 by generating the approximations

$$D_0,\ H_1^{(1)},\ D_1^{(1)},\ H_1^{(2)},\ D_1^{(2)} \ldots H_1^{(k)},\ D_1^{(k)},$$

where

(3-17) $$D_0 = f(t_0, H_0)$$

(3-18) $$H_1^{(1)} = H_0 + D_0 h$$

(3-19) $$D_1^{(k)} = f(t_1, H_1^{(k)})$$

(3-20) $$H_1^{(k)} = H_0 + \frac{1}{2} h \left(D_0 + D_1^{(k-1)}\right)$$

for $k = (1, 2, 3, \ldots, p)$ and p an integer such that

(3-21) $$D_1^{(p)} = D_1^{(p-1)}$$

(3-22) $$H_1^{(p+1)} = H_1^{(p)}$$

When this condition occurs take

(3-23) $D_1 = D_1^{(p-1)}$

(3-24) $H_1 = H_1^{(p)}$

Then proceed to Step 3.

<u>Step 3</u>. Evaluate the approximations to H_2 from the calculated approximation to H_1 and D_1 obtained as the result of Step 2 and generation of

$$D_2^{(1)},\ H_2^{(2)},\ D_2^{(2)},\ .\ .\ .\ H_2^{(k)},\ D_2^{(k)}$$

where

(3-25) $H_2^{(k)} = H_1 + D_1 h$

(3-26) $D_2^{(k)} = f(t_2, H_2^{(k)})$

(3-27) $H_2^{(k)} = H_1 + \frac{1}{2} h \ (D_1 + D_2^{(k-1)})$

for $k = 1, 2, 3, . . ., q)$ and q an integer where

(3-28) $D_2^{(q)} = D_2^{(q-1)}$

(3-29) $H_2^{(q+1)} = H_2^{(q)}$

When this condition occurs take

(3-30) $D_2 = D_2^{(q-1)}$

(3-31) $H_2 = H_2^{(q)}$

Then proceed to Step 4.

<u>Step 4</u> will generate the approximation to H_3 and D_3 after which we proceed to <u>Step 5</u> for an approximation to H_4 and D_4 and so on until we have an approximation for H_n and

D_n. The process should stop when the n^{th} approximation has been evaluated. The value chosen for n will depend on the accuracy desired and maximum value of Δt to be used. A flow chart of the process is given in Fig. 11 for the function $D = f(t, H)$.

If the function $f(t, H)$ is specified by the Hysteretic Diffusion Equation and we apply the Modified Euler Method as before, we can solve for the value of $H(x, y, z, t)$ as specified by the partial differential equation (2-28).

3.3 Numerical Solution of the Hysteretic Diffusion Equation with the Modified Euler Method

When applying the Modified Euler Method to the Hysteretic Diffusion Equation, equation (2-28), we find that evaluating successive values of $\partial H/\partial t$ becomes somewhat more complex since $f(t, H)$ now depends on many variables; i.e.,

$$(3\text{-}32) \qquad f(t, H) = \frac{(C_2 + H_z)^2}{\sigma C_1 C_2} \left[\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} \right]$$

Because of this it becomes more difficult to visualize the physical significance of $\partial H/\partial t$ in terms of equation variables; however, no problem should exist in evaluating $\partial H/\partial t$ if we express the equation in its finite difference form and establish initial and boundary conditions. The quantity $\partial H/\partial t$ may now be evaluated simply by a series of numerical operations. Successive values of $\left.\partial H/\partial t\right|_{t=t_i}$ can be found through the recurrence relations as outlined in the flow chart of the Modified Euler

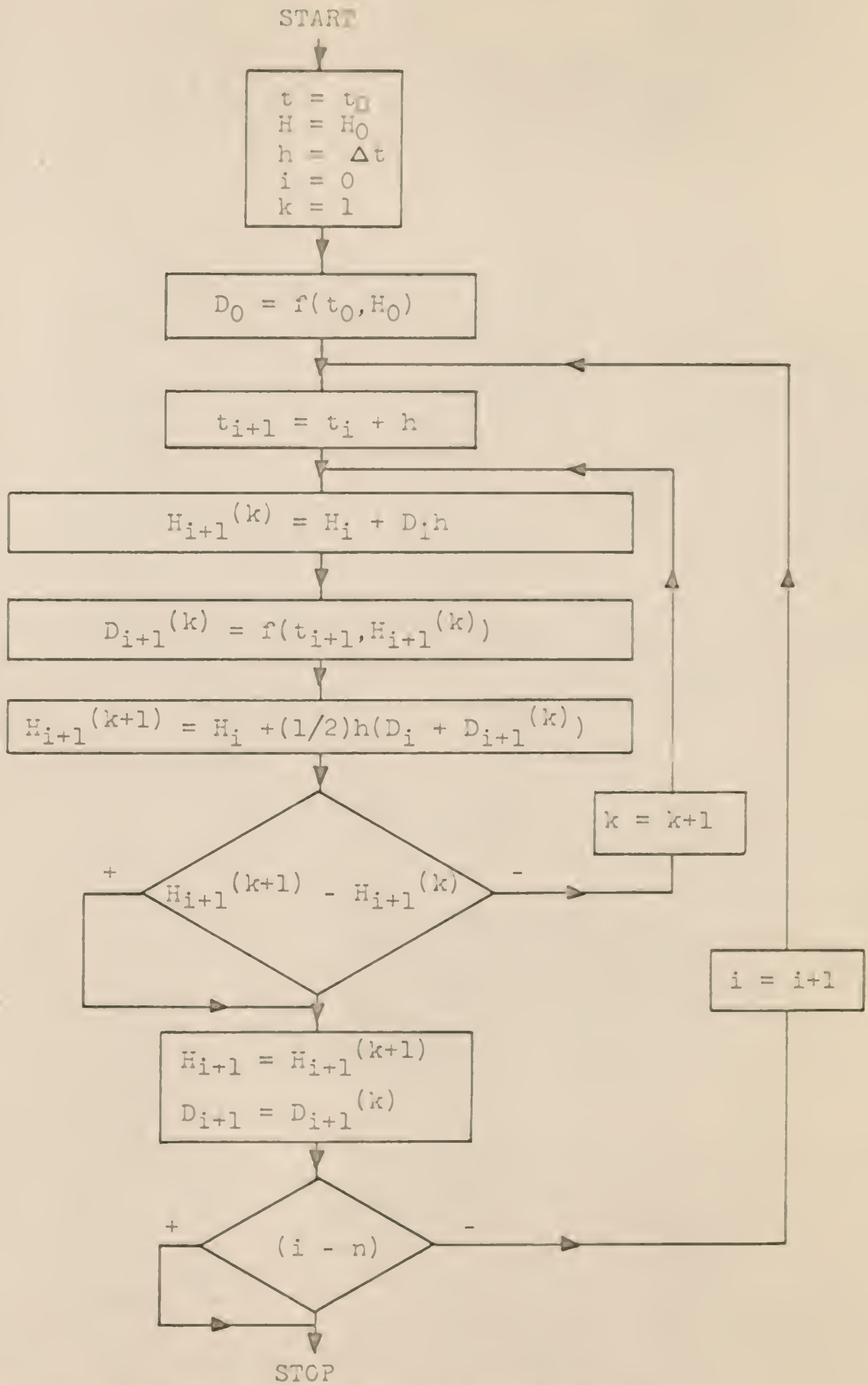

Fig. 11. Flow chart of modified Euler method for $dH/dt = f(t,H) = D$.

Process. Although some care must be exercised to limit the size of Δt to insure that the

$$(3\text{-}33) \qquad \lim_{k \to \infty} (H_i^{(k+1)} - H_i^{(k)}) \to 0$$

the problem is a straightforward procedure as outlined by the flow chart of Fig. 11.

In the following discussion, the magnetic field intensity for the I, J^{th} point in the grid of Fig. 12 is specified by H(I, J); the i^{th} approximation to H at the I, J^{th} point is denoted by $H(I, J)_i$; and the k^{th} value of the i^{th} approximation of H for the I, J^{th} point is $H(I, J)_i^{(k)}$. Note that the superscript does not indicate that $H(I, J)_i$ is raised to a power or that it is the k^{th} derivative of H. Let us also denote $(\partial H/\partial t)_i$ by P_i.

3.4 Conditions for Convergence

Conditions exist under which the process will not yield a solution. This is due to the fact that $\lim_{k \to \infty} (H_i^{(k+1)} - H_i^{(k)})$ will not approach zero; however, we can establish a relationship between the time increment and other parameters such as grid size, C_1, C_2, σ , and H_{max} to insure convergence of the process and existence of a solution.

Let us divide a rectangular cross section of the toroid considered in section 3.0 into a rectangular grid network as shown in Fig. 12. Each grid is of equal area h^2 and H(I, J)

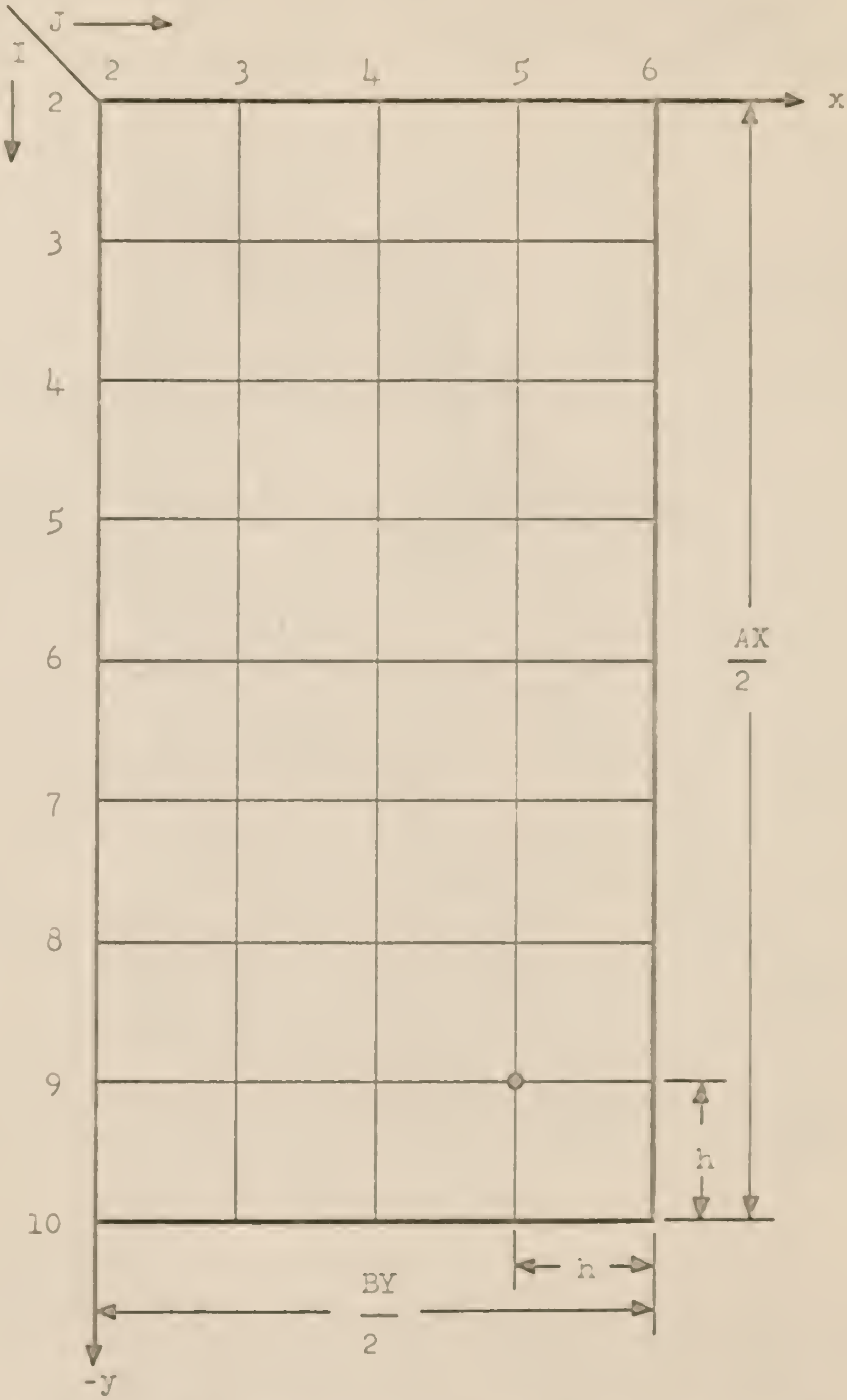

Fig. 12. One-fourth of rectangular toroid cross section for sample problem given in section 5.0. (See Fig. 7b.) Note: NI = 4, NJ = 8, NY = 6, NX = 10.

is the field intensity at the intersection of the I^{th} row and J^{th} column of the grid.

The Hysteretic Diffusion Equation for the I,J^{th} point is given by equation (3-34).

$$P(I,J) = \frac{(C_2 + H(I,J))^2}{\sigma C_1 C_2} \left[\frac{\partial^2 H(I,J)}{\partial x^2} + \frac{\partial^2 H(I,J)}{\partial y^2} \right] \tag{3-34}$$ [1]

Expressed in its finite difference form the equation becomes

$$V(I,J) = H(I + 1,J) - 2H(I,J) + H(I - 1,J)$$

$$W(I,J) = H(I,J + 1) - 2H(I,J) + H(I,J - 1)$$

$$P(I,J) = \frac{(C_2 + H(I,J))^2}{\sigma C_1 C_2} \left[\frac{W(I,J)}{h^2} + \frac{V(I,J)}{h^2} \right] \tag{3-35}$$ [1]

If we wish to evaluate P(I,J) at I = 5, J = 3, we obtain

$$V(5,3) = H(6,3) - 2H(5,3) + H(4,3)$$

$$W(5,3) = H(5,4) - 2H(5,3) + H(5,2)$$

$$P(I,J) = \frac{(C_2 + H(5,3))^2}{\sigma C_1 C_2} \left[\frac{W(5,3)}{h^2} + \frac{V(5,3)}{h^2} \right] \tag{3-36}$$ [1]

3.5 Relationship Between Δt and Other Parameters to Insure Convergence

To investigate the relationship between Δt and other parameters of the problem let us consider a sample calculation of the

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[1] Note that P_i represents $(\partial H/\partial t)_i$.

approximation to H at a corner position of the grid not on the boundary, i.e., the position $I = NY - 1$, $J = NX - 1$. (See Fig. 7 and Fig. 12, position $(I,J) = (5,9)$.)

Let us apply initial conditions and boundary conditions when all points inside the boundary are at $H_0 = H_b$ and all points on the boundary are at $H_0 = -mH_b$ where m is some real constant.

If we examine the Hysteretic Diffusion Equation expressed in its finite difference form (equation 3-48), we find that the largest value of P will occur at the $I = NY - 1$, $J = NX - 1$ position. To approximate a value of H at this position after a time Δt, we use the equation

(3-37) $$H(NY - 1, NX - 1)_1^{(1)} = H(NY - 1, NX - 1)_0 + P_0(NY - 1, NX - 1)\,\Delta t$$

Since we are considering only one position in the grid we may drop the subscripts; thus

$$H_1^{(1)} = H_b + P_0\,\Delta t$$

Evaluating P_0 we find that

$$P_0 = \frac{(C_2 + H_{max})^2}{\sigma C_1 C_2}\left[\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right]$$

since $\dfrac{\partial^2 H}{\partial x^2} = \dfrac{\partial^2 H}{\partial y^2}$ at this position we have

(3-38) $$P_0 = 2x_0(H(NY, NX - 1) - 2H(NY - 1, NX - 1) + H(NY - 2, NX - 1))$$

$$P_0 = 2x_0(-mH_b - 2H_b + H_b)$$
$$P_0 = -2x_0(m + 1)\ H_b$$

(3-39) $$H_1^{(1)} = H_b - 2x_0(m + 1)\ H_b\ \Delta t$$

We know the approximation H_1 is between $-mH_b$ and $+H_b$, thus $2x_0(m + 1)\ H_b\ \Delta t$ has to be chosen such that

(3-40) $$-mH_b \leq (H_b - 2x_0(m + 1)H_b\ \Delta t) \leq H_b\ .$$

We can therefore limit the value of Δt to satisfy this condition; i.e.,

(3-41) $$0 \leq 2x_0(m + 1)H_b\ \Delta t \leq (m + 1)H_b\ .$$

Thus the possible values of Δt become

(3-42) $$0 \leq \Delta t \leq 1/2x_0$$

where $$x_0 = \frac{(C_2 + H_{max})^2}{\sigma C_1 C_2 h^2}$$

Let us now investigate a few special cases.

<u>Case I</u>. For $\Delta t = 1/2x_0$ we have

$$P_0 = -2x_0(m + 1)H_b$$
$$H_1^{(1)} = H_b - 2x_0(m + 1)H_b\ \Delta t = -mH_b$$
$$P_1^{(1)} = +2x_0(m + 1)H_b$$
$$H_1^{(2)} = H_b$$
$$P_1^{(2)} = -2x_0(m + 1)H_b$$
$$H_1^{(3)} = -mH_b$$

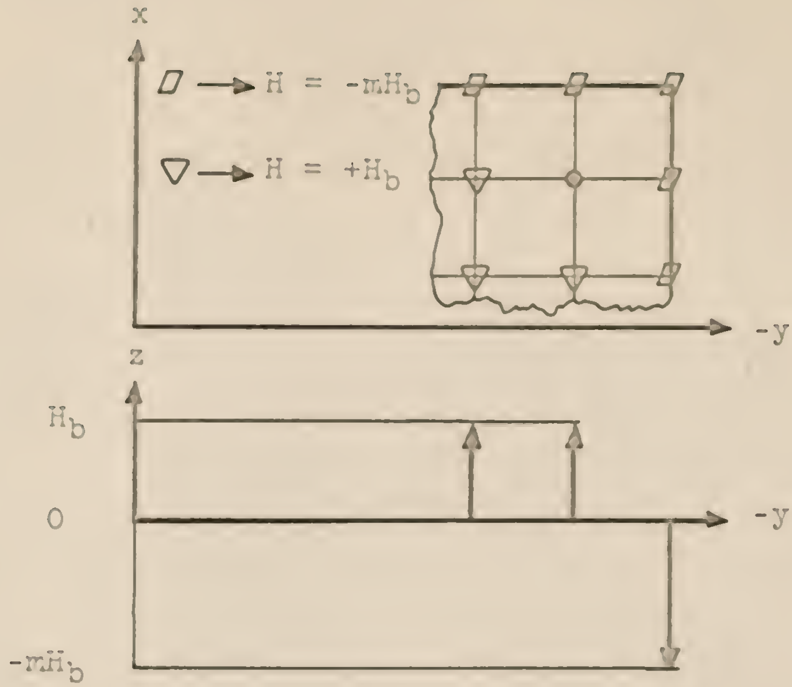


Fig. 13. Grid section of Fig. 12. Indicates values of H_z adjacent to $H_z(9,5)$ at $T = 0^+$.

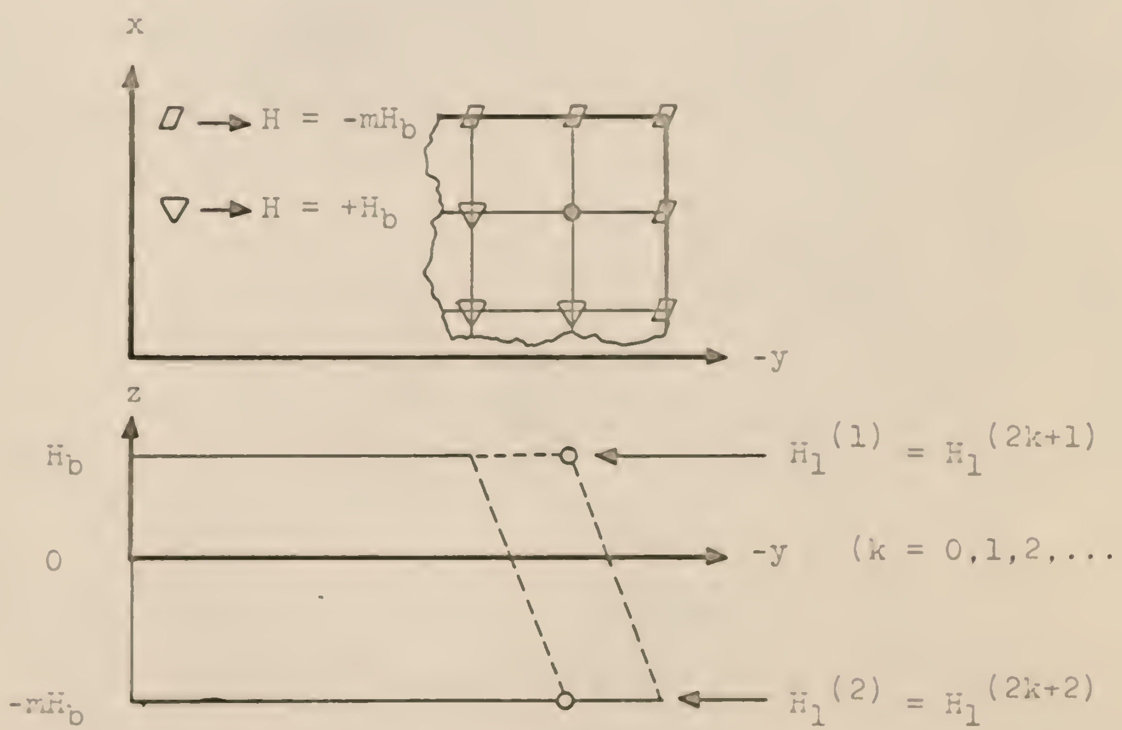


Fig. 14. Grid section of Fig. 12. Indicating values of $H_z(9,5)_1^{(k)}$, $k = 1,2,3, \ldots, p$; for $H_{boundary} = -mH_b$, $\Delta t = 1/2x_0$.

$$P_1^{(3)} = +2x_0(m + 1)H_b$$
$$H_1^{(4)} = H_b$$

Thus we can see that an oscillation between H_b and $-mH_b$ occurs and the approximation to H_1 can never be found since the $\lim_{k \to \infty} (H_1^{(k+1)} - H_1^{(k)}) \to 0$ is not satisfied.

Case II. For $\Delta t = 1/4x_0$ we have

$$P_0 = -2x_0(m + 1)H_b$$
$$H_1^{(1)} = H_b - 2x_0(m + 1)H_b \,\Delta t = \frac{H_b}{2}(1 - m)$$
$$P_1^{(1)} = 0$$
$$H_1^{(2)} = H_b - x_0(m + 1)H_b \,\Delta t = \frac{H_b}{4}(3 - m)$$
$$P_1^{(2)} = -x_0(m + 1)H_b$$
$$H_1^{(3)} = H_b - (3/2)x_0(m + 1)H_b \,\Delta t = \frac{H_b}{8}(5 - 3m)$$
$$P_1^{(3)} = -x_0/2(m + 1)H_b$$
$$H_1^{(4)} = H_b - (5/4)x_0(m + 1)H_b \,\Delta t = \frac{H_b}{16}(11 - 5m)$$

Thus the process is converging to some value $H_1^{(n)}$ and $\lim_{k \to \infty} (H_1^{(k+1)} - H_1^{(k)}) \to 0$ is satisfied. One can easily visualize convergence by considering H to be zero on the boundary (i.e., $m = 0$), and establishing approximations $H_1^{(k)}$ as k increases. This process is shown in Fig. 16.

In order to have convergence we must not include zero or $1/2x_0$ in the permissible values; thus

$$(3\text{-}43) \qquad 0 < \Delta t < 1/2x_0$$

Selection of the Δt to be used is dependent on the computation time available and accuracies desired. As before, large

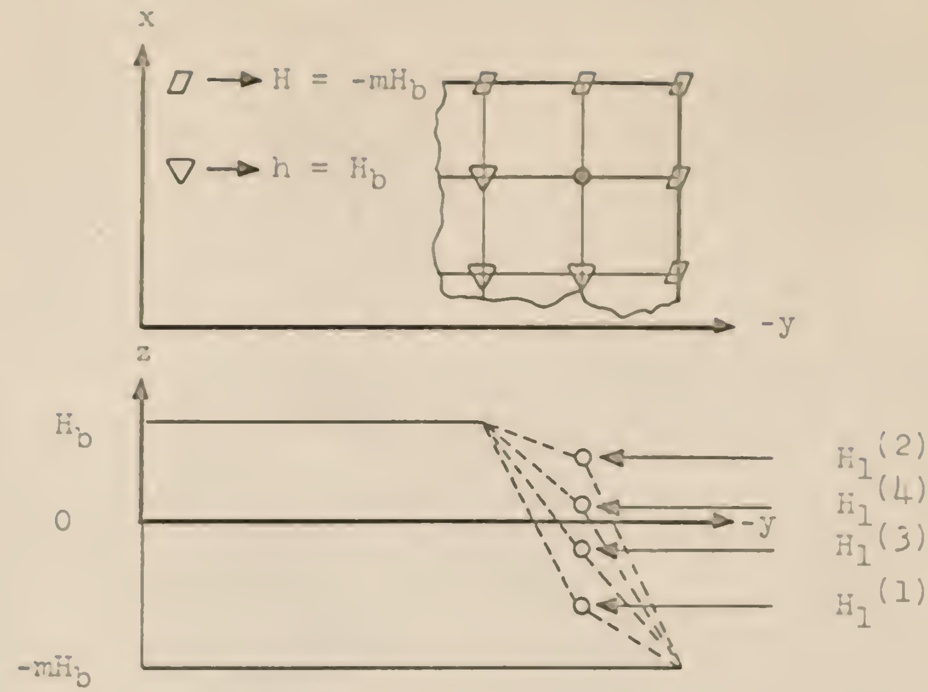


Fig. 15. Grid section of Fig. 12. Indicating values of $H_z(9,5)_1^{(k)}$, $k = 1,2,3,4$, for $H_{boundary} = -mH_b$, $\Delta t = 1/4x_0$.

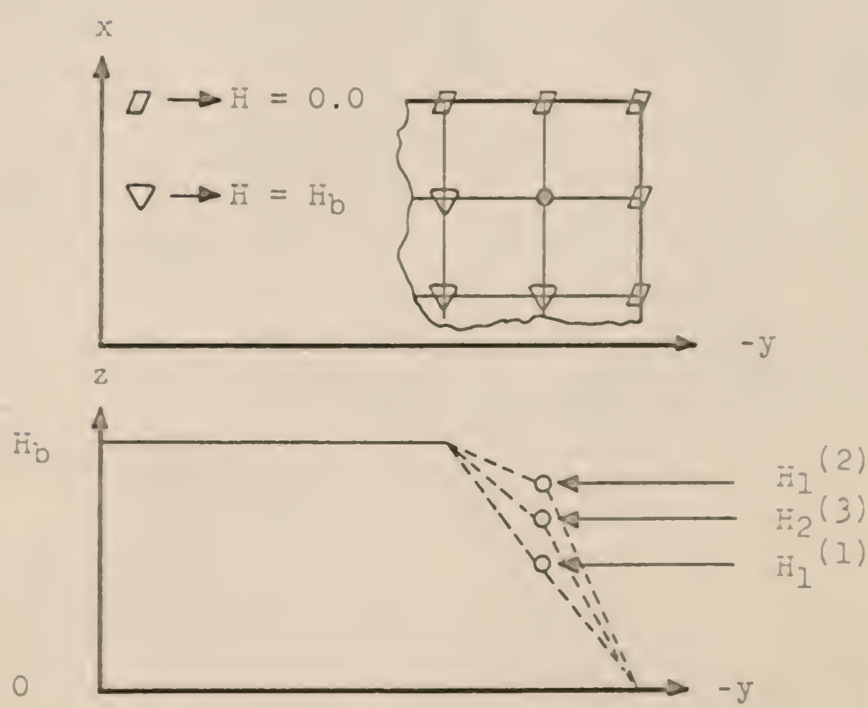


Fig. 16. Grid section of Fig. 12. Indicating values of $H_z(9,5)_1^{(k)}$, $k = 1,2,3$, for $H_{boundary} = 0.0$, $\Delta t = 1/4x_0$.

Δt's introduce inaccuracies; however, they may allow evaluation of the maximum value of H more rapidly. This is not necessarily the case since more iterations are necessary to evaluate approximations when Δt is large, and it becomes possible to use more time in evaluating these approximations than to evaluate H in smaller steps of Δt. Selection of a permissible Δt for convergence requires that

$$(3\text{-}44) \qquad 0 < \Delta t < \frac{\sigma C_1 C_2 h^2}{2(C_2 + H_{max})^2} \; .$$

If we force the corner position to converge all other positions along the boundary will converge since they only require that

$$(3\text{-}45) \qquad 0 < \Delta t < \frac{\sigma C_1 C_2 h^2}{(C_2 + H_{max})^2} \; .$$

To insure convergence we may write

$$(3\text{-}46) \qquad 0 < \Delta t < C_0 \frac{\sigma C_1 C_2 h^2}{(C_2 + H_{max})^2}$$

where the parameters of equation (3-46) are defined as follows:

C_0 = constant and is always < 0.5

σ = material conductivity

C_1 = constant for B-H approximation (see section 2-6)

C_2 = constant for B-H approximation (see section 2-6)

h = the grid size selected (see Fig. 12)

H_{max} = the magnitude of the maximum value of H(I,J) specified by the initial conditions of boundary conditions at time $t = 0^+$.

It is now evident that Δt has to be selected in accordance with the conditions of equation (3-44) and is not an arbitrary variable. It might also be noted that smaller grid areas h^2 will require smaller time increments. This condition specifies the accuracy of the method in the same manner as limiting the value of Δt to be small in the example of section 3.2.

With the various limitations on Δt now specified we can proceed to implement the solution to equation (2-28) by constructing a program to perform the almost endless number of numerical calculations necessary. Discussion of this task is given in the next section.

4.0 PROGRAM FOR SOLUTION OF THE HYSTERETIC DIFFUSION EQUATION

4.1 Introduction

The basic function of the program is to execute the Modified Euler Method of numerical integration to solve the Hysteretic Diffusion Equation and to obtain the magnetic field intensity distribution pattern in the cross section of a rectangular toroid subject to given initial time and boundary conditions. Before this process may be executed we must first define the problem in terms of variable names in accordance with the FORTRAN IV programming language, read in the input data, and establish output data formats. A method of completing this task is given in the next section. It is assumed that the reader is familiar with the FORTRAN IV language.

Discussion of the program is given in various sections with card numbers indicating locations of the sections in the program listing of Appendix C. Notation and symbolism used in the program follow closely that used in previous discussion, although some modifications were necessary to satisfy programming requirements.

4.2 Main Program EMEX

This program controls all numerical operations and may be subdivided to indicate the particular functions performed.

4.2A Comments (MG90000 - MG90106)

The cards serve as a partial list of variable names in the program and specify various numerical values to provide given print-out formats.

4.2B Input Data (MG90122 - MG90138) and (MG90262)

The first five cards of each data set give all data necessary to execute a given problem except the initial and boundary conditions which are contained on the remaining cards 6 through n.

Any variable name beginning with I, J, K, L, M, or N is fixed point and read according to an I5 format. Variable names beginning with other letters are floating point quantities and are read according to an E16.8 format. The E16.8 format allows any of the common systems of units to be used without alteration of the input-output format specifications. Input variables are read in the sequential order as specified below.

Card 1	EO, HSO, BR, HMAX, CUO
Card 2	TO, TKM, COND, AX, BY
Card 3	CDT, KZ, NI, NJ, K, KO, KI, KG
Card 4	B2, H2, B3, H3
Card 5	HIO, DHX, I3, I4, I5
Card 6	
.	
.	((HO(I,J), J = 2,NX), I = 2,NY)
[1]Card n	

[1]The value of n = 5 + (NI + 1)(NJ + 1)/5, NX = NJ + 2, NY = NI + 2.

Definition of the variables and methods of obtaining numerical values for them are given as follows.

1. BR - Residual flux density specified as the flux density for the magnetic field intensity H = 0.0.

2. B2, H2 B3, H3 - These are values of flux density and magnetic field intensity taken near the knee of the decay portion of the hysteresis curve. (See section 2.6.)

3. HIO, DHX I3 - Variables used for control information which specify the number of points on the approximate hysteresis loop. For a symmetrical hysteresis loop as shown in Fig. 22, choose I3 = 1 + 2(HIO/DHX).

   a. HIO - Specifies the maximum value of H to be plotted on the hysteresis loop approximation.

   b. DHX - Specifies the increments of H plotted on the hysteresis loop approximation.

   c. I3 - Specifies the number of points on the decay portion of the hysteresis loop approximation. The maximum value is 100.

4. HMAX - Specifies the maximum value of field intensity applied where H = $\mathfrak{F}/l$ = NI/l. (See equations 2-1, 2, 3, 4.)

5. CUO - Constant to allow for units conversion when computing values of force. Force = $B^2A/2\mu_0$ where $\mu_0 = 4\pi$CUO.

6. COND - Conductivity of ferromagnetic material.

7. AX - "X" dimension of the rectangular cross section as shown in Fig. 12.

8. BY - "Y" dimension of the rectangular cross section as shown in Fig. 12.

9. NI - Number of divisions of width CX in the "X" direction. (See Fig. 12, h = CX)

10. NJ - Number of divisions of width CX in the "Y" direction. (See Fig. 12, h = CX.)

11. TO - Specifies the initial value of time T(K).

12. K - Subscript indicating iterations. Appears as T(K).

13. I5 - Specifies execution for flux decay or flux buildup run. (See Appendix C for values.)

14. CDT - Specifies length of time increment. Same as C_0 of section 3.5. Use CDT < 0.5 to insure convergence, smaller for higher accuracy.

15. KZ - Specifies physical position of data set in relation to the last set; e.g., if four data sets are being run, KZ = 1 for the last set to be run, KZ = 4 for the first set to be run, etc.

16. KI - Specifies increments of time for which data is to be plotted; e.g., if we have 4000 values of total force or flux the printer need not plot every point; thus it plots every KI^{th} point. (Use KI such that KO/KI < 800.)

17. KG - Specifies increments of time for which a print-out of the flux distribution pattern is desired; i.e., a print-out for T(0), T(KG), T(2KG), etc.

18. EO - Specifies maximum error norm E as given by equation (4-1) to be less than EO; i.e., $E \leq EO$.

19. HSO
TKM
KO - Specify halt condition. A halt will occur immeately after any one of the conditions given below are met. One should also note that $T(K) = K\,\Delta t$.

   a. HSO - HS is the sum of H(I,J) for all points in one-fourth of the lattice. Halt occurs for $HS \geq HSO$, buildup; $HS \leq HSO$, decay.

   b. TKM - Specifies maximum time T(K) to be considered. Halt occurs when $T(K) \geq TKM$.

   c. KO - Specifies maximum number of increments desired. Halt occurs when $K \geq KO$.

20. I4 - Specifies data selected for output and format specifications to be used. A detailed discussion of I4 is given in section 4.2G.

21. HO(I,J) - Specifies numerical value of magnetic field intensity at every point in one-fourth of the lattice at $T(0) = 0^+$. This applies the boundary conditions and initial conditions at T(0) = 0. Quantities are "read" according to the instruction ((HO(I,J), J = 2,NX), I = 2,NY).

This completes the definition of data set constituents. Further information may be obtained by studying the sample problem given in section 5.0.

4.2C Evaluation of the Hysteresis Loop Approximation Formula (MG90140 - MG90250)

This section evaluates the constants C_1 and C_2 as given equations (2-24) and (2-25), then calculates the B-H relationship as illustrated by the hysteresis loop of Fig. 22. A graphical and printed output of an approximation to the material hysteresis loop is available at the beginning of each set of output data. To accomplish this task a duplication of the subroutine FLUD was included; however, the variables were changed and an output routine was added. Refer to section 4.3 for discussion of the procedure.

4.2D Calculation of Constants and Page Heading Routine

These cards are used to place a heading at the beginning of each output data set which defines calculated variables specifying operations to be performed; provide a print-out of data read from the input data set; check for input data errors; etc. Page 76 of section 5.2 was generated by cards contained in sections 4.2D and 4.2E.

4.2E Initial Conditions Print-out Routine (MG90724 - MG90874)

This section provides a print-out of the initial input data and corresponding initial values of flux density and force at $T(0) = 0^{+}$. Cards MG90724 - MG90784 provide the output in a list form as shown for Tape 9. The remaining cards give a matrix output format as shown for Tape 15.

In addition to the print-out instructions total values of flux (FLUXT) and theoretical force (FORCT) are calculated and printed following listing of B(I,J), H(I,J), and FORCE(I,J).

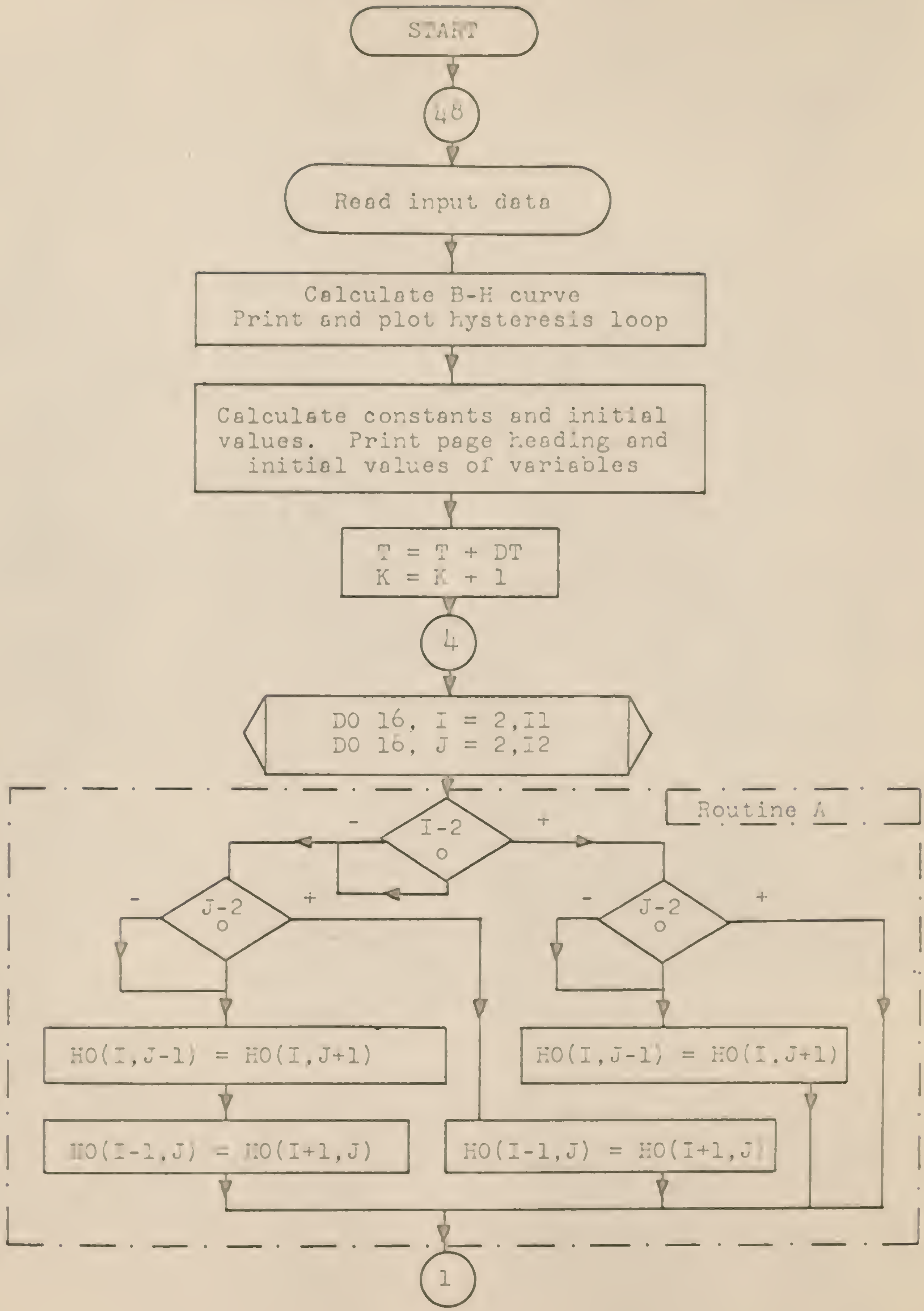
4.2F Programming the Modified Euler Process for the Hysteretic Diffusion Equation (MG90876 - MG91060)

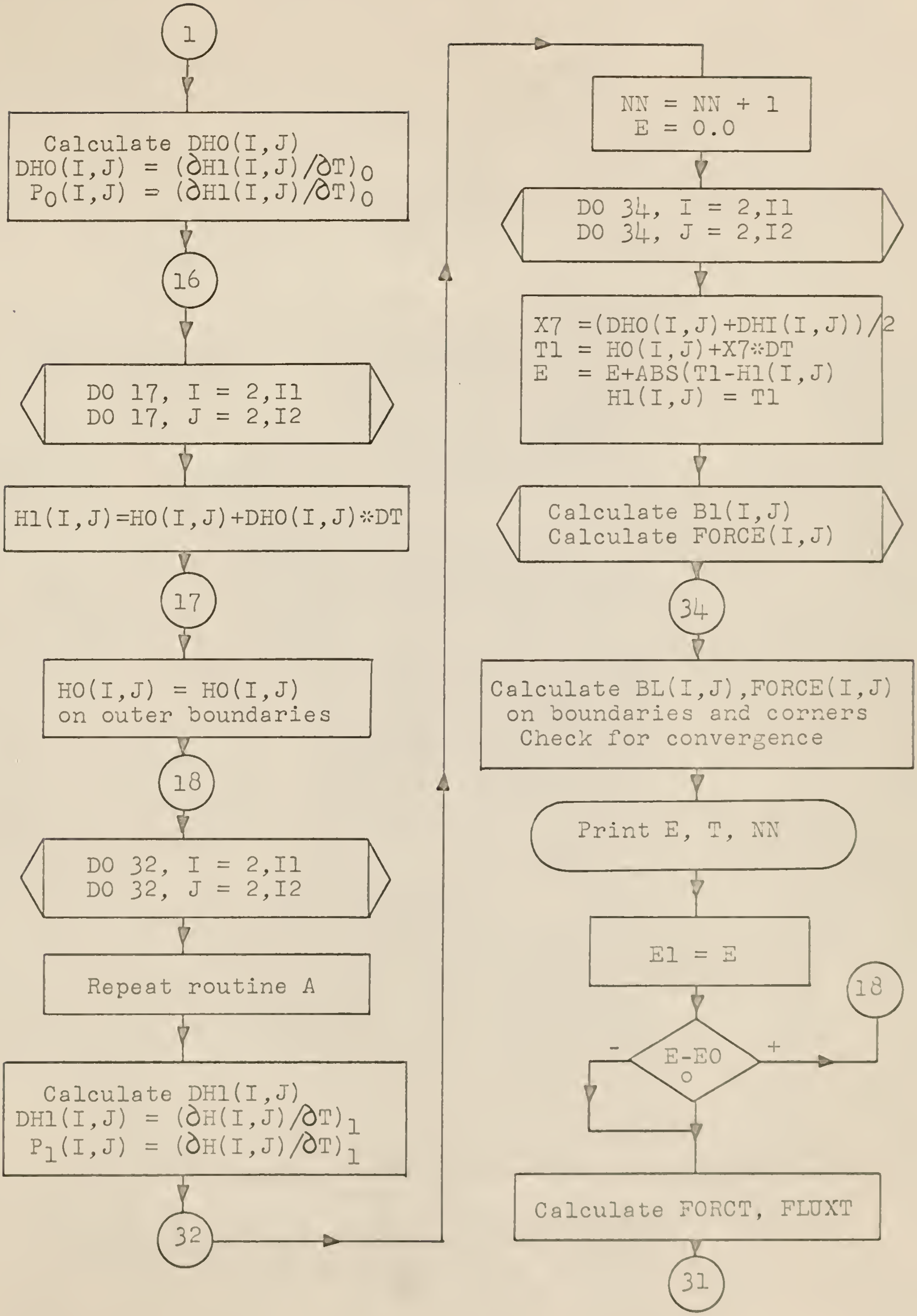
Execution of the process is nearly the same as described in section 3.2 and as outlined by the flow chart of Fig. 11; however, the condition $(r_o - (r_o/r_i)) >> 1$ (see Fig. 7a) allows use of one-fourth of the cross-sectional area because symmetrical boundary conditions are present when the magnetic field is applied.

The finite difference representation of the Hysteretic Diffusion Equation (equation 3-36) requires that H for all points in the lattice adjacent to the point under consideration be known. We must compute H only at all points inside the outer boundary since the boundary conditions require flux on the boundary to remain constant. Because of symmetry, values of H immediately to the left of the Y-axis are equal to those immediately to the right and values of H immediately below the X-axis are equal to those directly above. (See Fig. 12.) Cards MG90884 - MG90900 and MG90946 - MG90962 were coded to accomplish this task when values of I and J placed the point in question on the inner boundaries. The desired values of P_0 were computed for every I and J as were the approximations $H_1^{(1)}$, $P_1^{(1)}$, $H_2^{(2)}$, . . ., etc. While computing successive approximations to H, an accumulative error norm, E was generated. This error is expressed by equation (4-1).

$$(4\text{-}1) \qquad E = \sum_{I=2}^{NX-1} \sum_{J=2}^{NY-1} \left| H(I,J)_i^{(k+1)} - H(I,J)_i^{(k)} \right|$$

This method was chosen in order that new approximations for the surrounding points would be considered when calculating successive approximations. The maximum error is specified by E0 and should be small if a high degree of accuracy is desired. Successive approximations for H will continue until the matrix error norm, $E < E0$. When a convergence condition is satisfied, NN (variable specifying number of successive approximations to H, i.e., the same as (k) in $H_i^{(k)}$) is returned to a value of 1; values of flux density, total force, and total flux are computed; and a new approximation of $H = H_{i+1}$ is begun. If the error is not converging rapidly enough to yield a solution in a reasonable amount of time or if the error is diverging, an error message will alert the operator and a machine halt will occur. A flow diagram of the Modified Euler Method for the special problem considered is given in Fig. 17.


START
48
Read input data
Calculate B-H curve
Print and plot hysteresis loop
Calculate constants and initial values. Print page heading and initial values of variables
T = T + DT
K = K + 1
4
DO 16, I = 2,I1
DO 16, J = 2,I2
Routine A
I-2
0
J-2
0
J-2
0
HO(I,J-1) = HO(I,J+1)
HO(I-1,J) = HO(I+1,J)
HO(I,J-1) = HO(I,J+1)
HO(I-1,J) = HO(I+1,J
1


1
Calculate DHO(I,J)
DHO(I,J) = (∂H1(I,J)/∂T)0
P0(I,J) = (∂H1(I,J)/∂T)0
16
DO 17, I = 2,I1
DO 17, J = 2,I2
H1(I,J)=HO(I,J)+DHO(I,J)*DT
17
HO(I,J) = HO(I,J)
on outer boundaries
18
DO 32, I = 2,I1
DO 32, J = 2,I2
Repeat routine A
Calculate DH1(I,J)
DH1(I,J) = (∂H(I,J)/∂T)1
P1(I,J) = (∂H(I,J)/∂T)1
32
NN = NN + 1
E = 0.0
DO 34, I = 2,I1
DO 34, J = 2,I2
X7 =(DHO(I,J)+DHI(I,J))/2
T1 = HO(I,J)+X7*DT
E = E+ABS(T1-H1(I,J)
H1(I,J) = T1
Calculate B1(I,J)
Calculate FORCE(I,J)
34
Calculate BL(I,J),FORCE(I,J)
on boundaries and corners
Check for convergence
Print E, T, NN
E1 = E
18
-
E-EO
o
+
Calculate FORCT, FLUXT
31

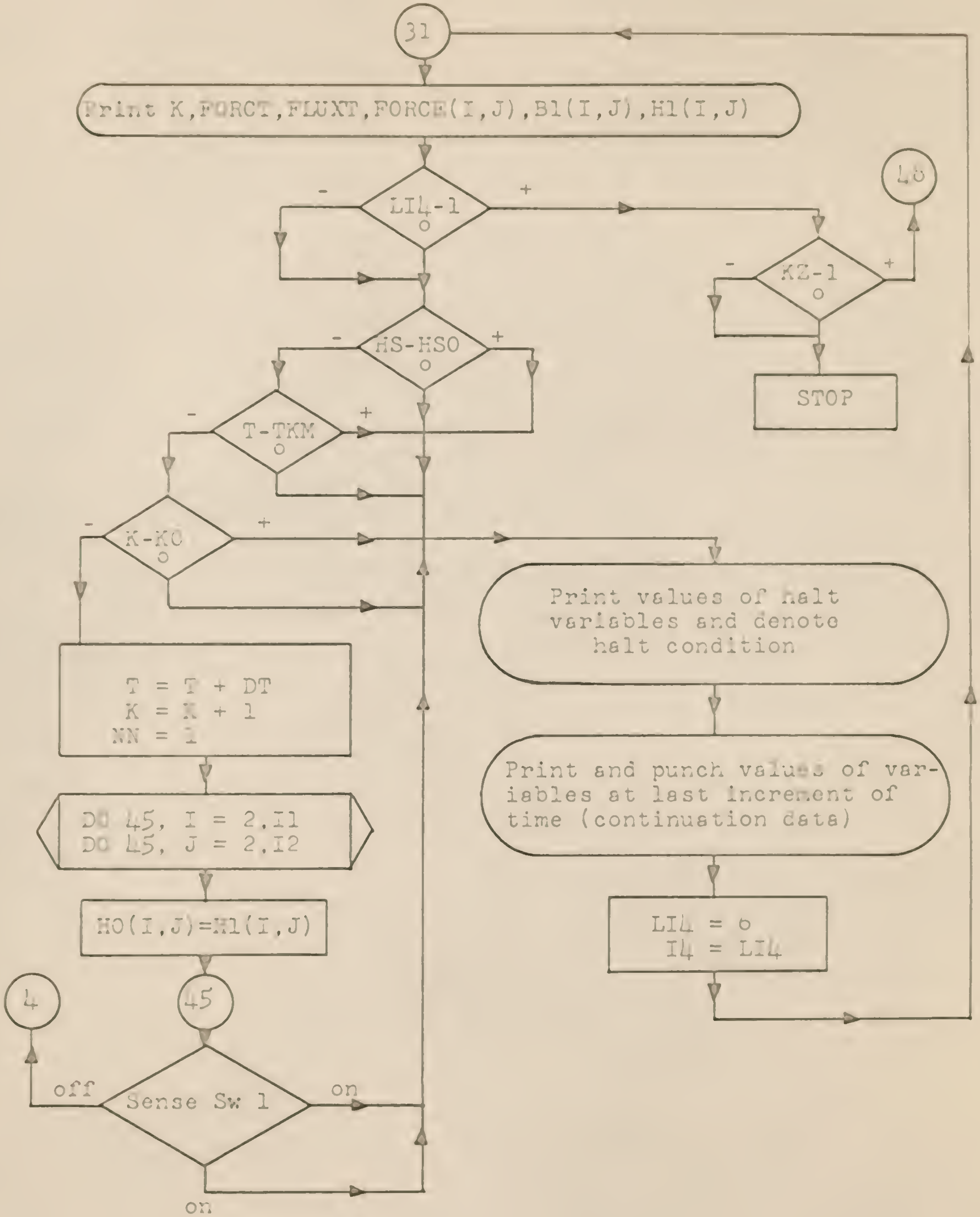


Fig. 17.
Flow chart of EMEX.

4.2G Output Data (MG91078 - MG91410)

The remaining cards are related to output data and appropriate headings for identification. Other special control functions are performed to specify the halt condition and to print-out all existing data at halt time.

Output data are available on Tapes 6, 9, 15, and 16. A sample of the output is given for Data Set 3 in section 5.2. Each tape has an output data heading for identification and prints initial values of H, B, FORCE, total force, total flux, and boundary values. Output data contained on the various tapes are as given below.

1. Tape 6 - Contains approximation to hysteresis loop and lists total force, total flux, and time. (See section 5.2A, Output Tape 6.)

2. Tape 9 - Lists all values of FORCE, B, and H with the corresponding time. The number of iterations required to satisfy the condition for convergence is also listed with the associated error norm E, i.e., lists values of E, T(K), and NN.

3. Tape 15 - Provides the same information as Tape 9 except that the output appears in a matrix form as shown in section 5.2C. Variables to be printed are selected by the value of I4 chosen. (See Table 2.)

4. Tape 16 - Data on this tape is used to punch a deck which may be used as input to a new run continuing from the existing values of variables at the completion of the first run. A print-out of the same data occurs on the output of Tape 6. One must change the value of TKM, HS0, and K0 to continue.

Desired output data can be selected by specifying different values of I4. These data are not written on the tapes if not desired since the write time is very large when compared to the computation time. I4 may range from 1-7 and the output can be determined from Table 2. If I4 = 7, only Tape 6 will have a useful output.

Table 2. Values of I4 and related output data.

Tape 9 (list)				:	Tape 15 (matrix)		
I4	H(I,J)	B(I,J)	FORCE(I,J)	NN	H(I,J)	B(I,J)	FORCE(I,J)
1	x	x	x	x	x	x	x
2	x	x	x	x			
3				x		x	
4				x	x		
5				x			x
6				x	x	x	x
7				x			

4.3 Subprogram FLUD

Subprogram FLUD computes a value of B for a corresponding H. The approximation is given by equation (2-22); however, the value of B is computed by four different methods depending on the value of H and whether the flux decay or flux buildup problem is being considered. We assume the hysteresis loop to be approximated by reflecting the decay portion of the loop and translating the entire decay loop to the right by $2H_c$. (See Fig. 18.) Let us consider the following cases to illustrate the regions and four different methods of computing B. Figure 18 illustrates the regions considered.

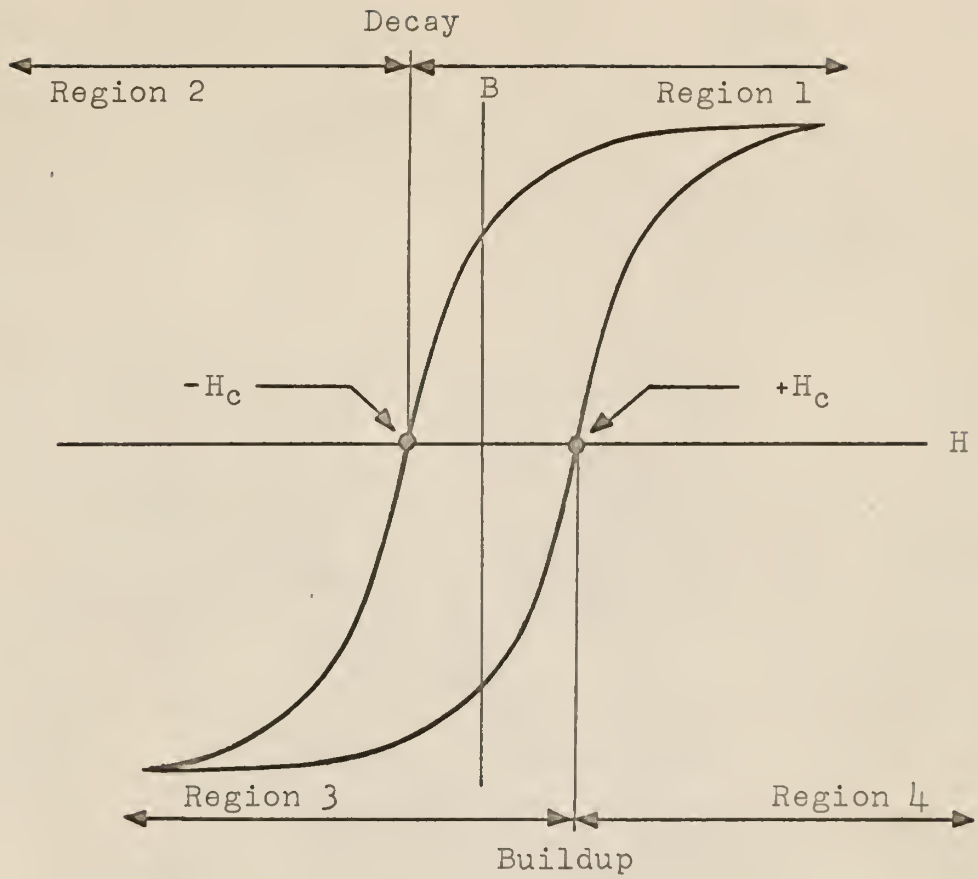


Fig. 18. Approximating the hysteresis loop.

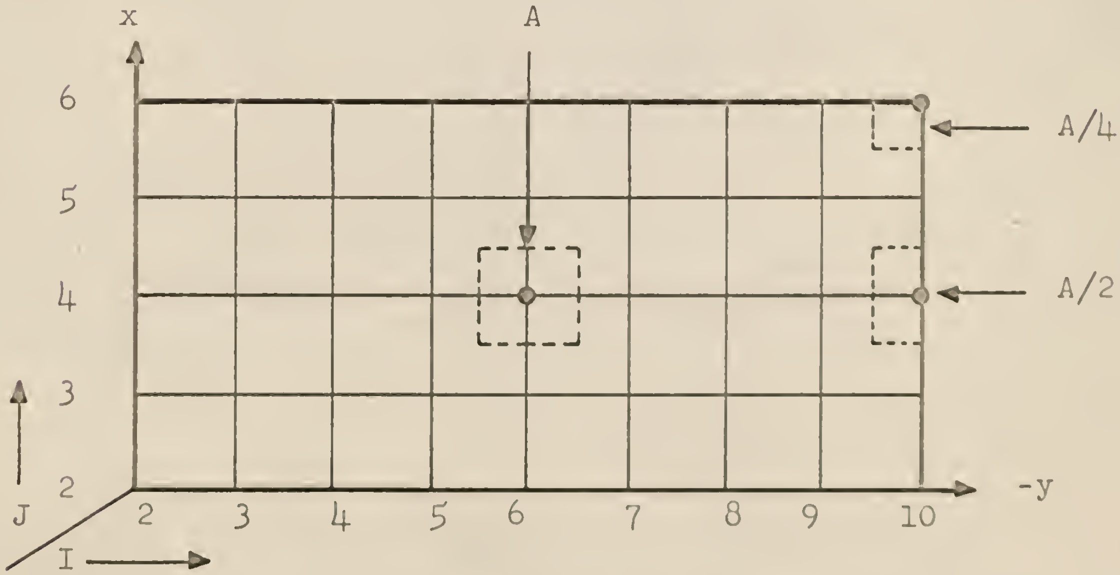


Fig. 19. Correction of area for corners and boundaries.

Region 1.[1] (Decay run, $-H_c < H < \infty$)

Flux density B is computed by a simple substitution of H in equation (2-23).

Region 2. (Decay run, $-\infty < H < -H_c$)

Since Region II of the hysteresis loop is a reflection of Region I about the H axis, we may compute B by evaluating the difference between H and H_c, forming a new variable equal to the sum of $-H_c$ and $|H - H_c|$ to replace H in equation (2-23). Then compute B with H replaced by $|H - H_c| - H_c$. A minus sign is then assigned to B in order to obtain the reflection characteristic.

Region 3. (Buildup run, $-\infty < H < H_c$)

Values of B in this region may be obtained by adding the quantity $2H_c$ to H and computing the negative of B with H replaced by the sum $H + 2H_c$.

Region 4. (Buildup run, $H_c < H < \infty$)

Values of B in this region are computed by subtracting the quantity $2H_c$ from H and computing B with H replaced by the difference $H - 2H_c$.

Subroutine FLUD determines the particular region in which H falls, and computes the corresponding B according to the rules discussed above. A flow diagram of the subroutine is given in Fig. 20.

4.4 Subprogram FORCX

This subroutine computes a fictitious force for each point in the lattice where Force = $B^2A/2\mu_0$. The reason for calculating forces is to obtain a force time relation for the electromechanical system shown in Fig. 2. The rectangular toroid was used

[1]The value H_c is a positive real number.

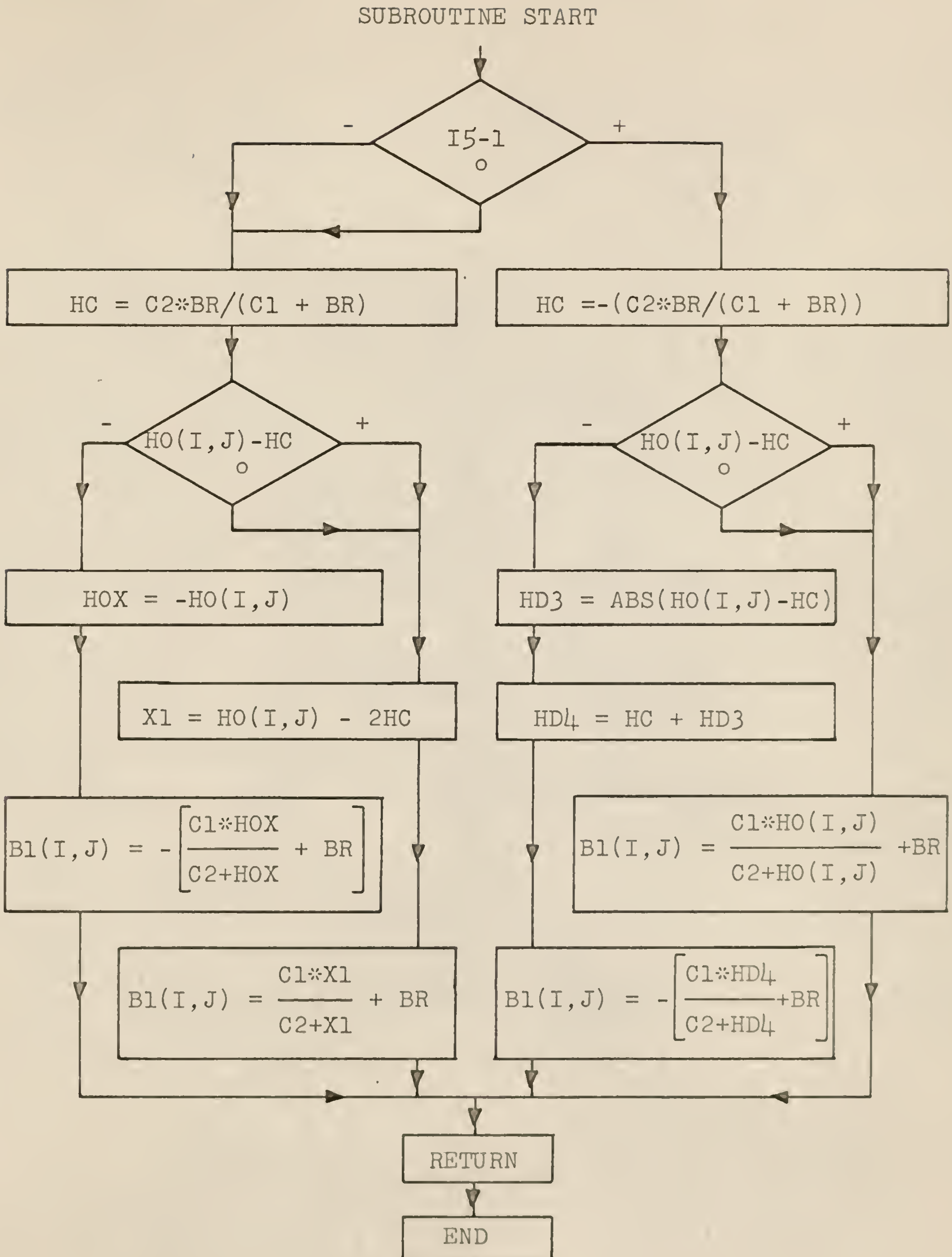


Fig. 20. Flow chart of FLUD.

as an approximation to the system with the assumption that the gap would not increase the reluctance of the flux path.

Total force in the toroid is the summation of forces at each grid point; i.e.,

$$(4\text{-}2) \qquad \text{Total force (FORCT)} = \sum_{I=2}^{NY-1} \sum_{J=2}^{NX-1} \text{FORCE}(I,J)$$

Computation of the force may be approximated by the formula given if the area A is adjusted as follows. Since approximate values of B are known only for points at intersecting grid lines, areas used for computing force values along all boundaries on the lattice should only be one-half of those for the inside points and areas used for the corners should be only one-fourth of the inside areas. (See Fig. 19.)

Cards (MG91466 - MG91480) select the correct formula for computing force depending on values of I and J; (i.e., replace A by A/2 on all boundaries and A by A/4 for all corners). A flow chart of the subroutine is given in Fig. 21.

4.5 Subprogram FLUX

Subroutine FLUX evaluates total flux by forming the product B^2A. Operations performed closely resemble those of subroutine FORCX except only total flux is computed and the flux for each point in the lattice is not. Once B is determined and subroutine FLUX is called, B is multiplied by the appropriate area and added to the existing value of total flux. When all points

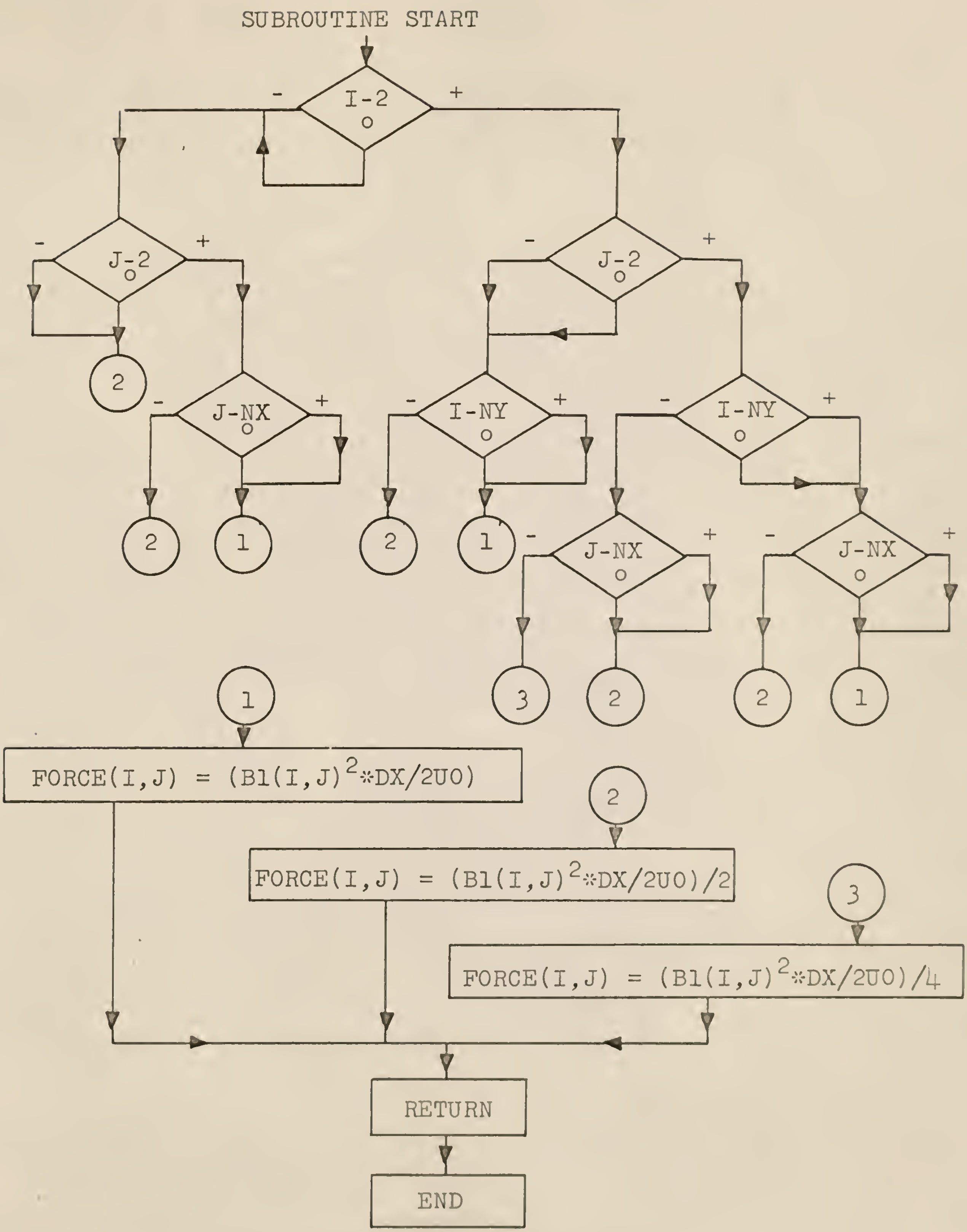


Fig. 21. Flow chart of FORCX.

have been summed, the total flux is known; however, no matrix print-out of the individual flux values can occur since the individual values are not stored. A flow diagram would be a duplication of that for subroutine FORCX with a change of variables.

A listing of the main program and associated subprograms with four sample data sets is given in Appendix C. Further information concerning their use will be considered next.

5.0 USING THE PROGRAM ON A SAMPLE PROBLEM

5.1 Preparing the Input Data

Perhaps the best method of discussing the use of the program is to consider a sample problem of the type outlined in section 2.0. Formulation of an input data set specifying all input parameters discussed in section 4.2B must be obtained before execution. Let us assume we are given a rectangular toroid of ferromagnetic material as illustrated by Fig. 7, with an initial field intensity of HO. We then reverse the applied field such that the boundary at time = 0^+ is at -HO. If HO is sufficient to saturate the core a flux reversal will occur and a flux distribution will exist in the core during the transient state. We can determine the distribution pattern as a function of time by solving for H(I,J), B(I,J), and FORCE(I,J) as a function of time. Let us assume that we desire to investigate the time dependence of the distribution pattern and the relationship between total flux (FLUXT), total force (FORCT), and time. Typical numerical values may be as given below.

Given: A. Material is 2.5 per cent silicon iron with a hysteresis loop given by Fig. 22.

B. Material conductivity (COND) - COND = 2.5×10^{-7} mhos

C. Material dimensions (see Fig. 12) - AX = 0.004 meter
BY = 0.008 meter

D. Initial conditions - HO = 250 amp-t/m
TO = 0.0 sec

G. Maximum field intensity - HMAX = 250 amp-t/m
(see section 3.4)

H. Boundary conditions
1. HO(I,NX), I = 2, NY - HO = -250 amp-t/m
2. HO(NY,J), J = 2, NX

The above information is sufficient to calculate numerical values for the remaining input data variables. (See section 4.2B.) We will use the mks system of units; thus the constant $\mu_o = 4\pi \times 10^{-7}$ webers/amp-t/m = 4πCUO; hence

$$CUO = 1.0 \times 10^{-7} \text{ weber/amp-t/meter.}$$

We obtain BR, B2, H2, B3, and H3 from the hysteresis loop for the given material. An approximation to this loop is shown by Fig. 22. Typical values for 2.5 per cent silicon iron are given below.

BR = 0.71 weber/m^2

H2 = 11.94 amp-t/m

B2 = 0.80 weber/m^2

H3 = 103.50 amp-t/m

B3 = 1.20 webers/m^2

Values of HIO, DHX, and I3 to yield a symmetrical hysteresis loop approximation with a range of H from +400 amp-t/m to -400 amp-t/m are

HIO = 400

DHX = 10

I3 = 81

Note: Maximum value of I3 = 100

For symmetrical loop
I3 = 1 + 2(HIO/DHX)

Grid size is selected by specifying NI, the number of divisions desired in the X-direction. (See Fig. 12.) This also specifies the quantity NJ since the grid is square and NJ = (BY)(AX)/4NI; hence we shall use NI = 4 and NJ = 8. The quantity CDT must be less than 0.5 to insure convergence, hence CDT = 0.2. Remaining variables of the first five cards control the program as discussed in section 4.2B. We desire to begin at T(K) = 0, hence let K = 0; to print-out all data available in both formats, hence I4 = 1; to investigate a flux decay, hence I5 = 2; to plot every value of FORCT and FLUXT versus time T(K), hence KI = 1; to print-out the distribution pattern for all variables H(I,J), B(I,J), and FORCE(I,J) for every fifth value of T(K), hence KG = 5; and to execute this input data set third from the last, hence KZ = 3. Collecting the above quantities in a list we have

CDT = 0.2	K = 0	KI = 1
NI = 4	I4 = 1	KG = 5
NJ = 8	I5 = 2	KZ = 3

Variable E0 is the maximum error norm E, thus if we wish the total error norm for one-fourth of the lattice to be less than or equal to 1.0, we require E0 $<$ 1.0. Variables HS0, TKM, and K0 halt execution. (See section 4.2B.) Let us halt when T(K) becomes $\geq$ 20.0 milliseconds, or when K $\geq$ 100, and not stop for the condition HS $\leq$ HS0, hence we have

EO = 1.0

HSO = -1.0×10^{-7}

TKM = 20.0

KO = 100

Note: 1. T(K) = K Δt

2. $\Delta t = CDT \dfrac{\sigma C_1 C_2 h^2}{(C_2 + HMAX)^2}$

This completes formulation of a typical input data set.

If we collect these values, sequence them according to the "read" instructions (see section 4.2B) and express the numbers in accordance with the format specifications, we have a data set as given by Table 3.

5.2 Output Data

Output data are available on Tapes 6, 9, and 15. The output data set included was generated by execution of the program with input data set 3. I4 = 1 was used to print-out all available data. Due to the voluminous amount of data available only a small number of distribution patterns are included; however, patterns for all increments of time considered may be obtained by specifying KG = 1. Duplications of page headings, etc., were deleted. When the execution is stopped, the halt condition is indicated (see page 101) and all data for the last value of time are printed-out on all tapes.

5.2A Output Tape 6
(Pages 71 - 82)

Tape 6 lists values for the approximate hysteresis loop (pages 71 - 74) and provides a graphical representation of the

TABLE 3. TYPICAL INPUT DATA SET

DATA SET NO 3 (5 X 9 MATRIX)

.10000000E+01	-.10000000E+08	.71000000E-00	.25000000E+03	.10000000E-06
.00000000E+00	.20000000E-01	.25000000E+07	.40000000E-02	.80000000E-02
.20000000E+00	3 4 8	0 100 1	5	
.80000000E-00	.11940000E+02	.12000000E+01	.10350000E+03	
.40000000E+03	.10000000E+02	81 1 2		
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
+.25000000E+03	+.25000000E+03	+.25000000E+03	+.25000000E+03	-.25000000E+03
-.25000000E+03	-.25000000E+03	-.25000000E+03	-.25000000E+03	-.25000000E+03

approximation (page 75). A listing of the total force, total flux, and the corresponding time then follows, (pages 78 and 79). Next is a graphical output illustrating the time dependence of total force and total flux, (pages 80 and 81, respectively). At completion of the run, a print-out of the continuation data available as a punched card output of Tape 16 occurs.

5.2B Output Tape 9
(Pages 83 - 92)

The first five pages of print-out for this tape duplicate pages 71 - 74 and 76 of Tape 6 output and are not included; however, the following page lists initial and boundary conditions at time $T(K) = 0^+$, (page 83). A listing of corresponding values of flux density, force, and time are also given at $T(K) = 0^+$. As time increases in increments of Δt, a complete list specifying distribution patterns at T(K) (K = 0, KG, 2KG, 3KG, . . .) is listed according to format specifications as shown by pages 83 - 92. This tape also contains a listing of iterations and their associated error norm E (pages 84 and 86).

5.2C Output Tape 15
(Pages 93 - 100)

Tape 15 gives a more convenient output format since data are given in a matrix format which eases interpretation by locating values calculated for specific points in the same physical location as they would occupy in the matrix of Fig. 12. All data available on Tape 9 except error norm values are available on Tape 15. Total values of force and flux are listed following each matrix output. Duplicates of pages 71-74 and 76 of Tape 9 precede the output and are not included.

BR	B2	H2	B3	H3
0.70999999E 00	0.80000000E 00	0.11939999E 02	0.12000000E 01	0.10349999E 03

HIO	DHX	I3	I4
0.40000000E 03	0.09999999E 02	81	1

C1 = 0.11655107E 01   C2 = 0.14268442E 03

B(I)	H(I)	I
0.15690707E 01	0.40000000E 03	1
0.15633179E 01	0.39000000E 03	2
0.15573450E 01	0.38000000E 03	3
0.15511392E 01	0.37000000E 03	4
0.15446864E 01	0.36000000E 03	5
0.15379716E 01	0.34999999E 03	6
0.15309787E 01	0.34000000E 03	7
0.15236899E 01	0.33000000E 03	8
0.15160859E 01	0.32000000E 03	9
0.15081461E 01	0.30999999E 03	10
0.14998475E 01	0.30000000E 03	11
0.14911654E 01	0.29000000E 03	12
0.14820724E 01	0.27999999E 03	13
0.14725388E 01	0.27000000E 03	14
0.14625316E 01	0.26000000E 03	15
0.14520148E 01	0.25000000E 03	16
0.14409484E 01	0.23999999E 03	17
0.14292881E 01	0.23000000E 03	18
0.14169847E 01	0.22000000E 03	19
0.14039837E 01	0.20999999E 03	20
0.13902239E 01	0.20000000E 03	21
0.13756369E 01	0.19000000E 03	22
0.13601458E 01	0.18000000E 03	23
0.13436639E 01	0.16999999E 03	24
0.13260929E 01	0.16000000E 03	25
0.13073211E 01	0.15000000E 03	26
0.12872214E 01	0.13999999E 03	27
0.12656473E 01	0.13000000E 03	28
0.12424308E 01	0.12000000E 03	29
0.12173766E 01	0.11000000E 03	30
0.11902577E 01	0.09999999E 03	31
0.11608078E 01	0.90000000E 02	32
0.11287129E 01	0.80000000E 02	33
0.10935999E 01	0.70000000E 02	34
0.10550223E 01	0.59999999E 02	35
0.10124402E 01	0.50000000E 02	36
0.96519651E 00	0.40000000E 02	37
0.91248103E 00	0.30000000E 02	38
0.85328485E 00	0.20000000E 02	39
0.78633461E 00	0.09999999E 02	40
0.70999999E 00	0.	41
0.62215918E 00	-0.09999999E 02	42

0.51999857E 00	-0.20000000E 02	43
0.39970580E-00	-0.30000000E 02	44
0.25598342E-00	-0.40000000E 02	45
0.81247769E-01	-0.50000000E 02	46
-0.11858648E-00	-0.59999999E 02	47
-0.28646551E-00	-0.70000000E 02	48
-0.42506013E-00	-0.80000000E 02	49
-0.54141808E 00	-0.90000000E 02	50
-0.64049356E 00	-0.09999999E 03	51
-0.72587077E 00	-0.11000000E 03	52
-0.80020694E 00	-0.12000000E 03	53
-0.86551375E 00	-0.13000000E 03	54
-0.92334213E 00	-0.13999999E 03	55
-0.97490710E 00	-0.15000000E 03	56
-0.10211739E 01	-0.16000000E 03	57
-0.10629193E 01	-0.16999999E 03	58
-0.11007752E 01	-0.18000000E 03	59
-0.11352608E 01	-0.19000000E 03	60
-0.11668073E 01	-0.20000000E 03	61
-0.11957748E 01	-0.20999999E 03	62
-0.12224673E 01	-0.22000000E 03	63
-0.12471426E 01	-0.23000000E 03	64
-0.12700212E 01	-0.23999999E 03	65
-0.12912922E 01	-0.25000000E 03	66
-0.13111195E 01	-0.26000000E 03	67
-0.13296451E 01	-0.27000000E 03	68
-0.13469932E 01	-0.27999999E 03	69
-0.13632727E 01	-0.29000000E 03	70
-0.13785791E 01	-0.30000000E 03	71
-0.13929974E 01	-0.30999999E 03	72
-0.14066026E 01	-0.32000000E 03	73
-0.14194615E 01	-0.33000000E 03	74
-0.14316341E 01	-0.34000000E 03	75
-0.14431737E 01	-0.34999999E 03	76
-0.14541285E 01	-0.36000000E 03	77
-0.14645419E 01	-0.37000000E 03	78
-0.14744531E 01	-0.38000000E 03	79
-0.14838974E 01	-0.39000000E 03	80
-0.15690707E 01	-0.40000000E 03	81
-0.15633179E 01	-0.39000000E 03	82
-0.15573450E 01	-0.38000000E 03	83
-0.15511392E 01	-0.37000000E 03	84
-0.15446864E 01	-0.36000000E 03	85
-0.15379716E 01	-0.34999999E 03	86
-0.15309787E 01	-0.34000000E 03	87
-0.15236899E 01	-0.33000000E 03	88
-0.15160859E 01	-0.32000000E 03	89
-0.15081461E 01	-0.30999999E 03	90
-0.14998475E 01	-0.30000000E 03	91
-0.14911654E 01	-0.29000000E 03	92
-0.14820724E 01	-0.27999999E 03	93

-0.14725388E 01	-0.27000000E 03	94
-0.14625316E 01	-0.26000000E 03	95
-0.14520148E 01	-0.25000000E 03	96
-0.14409484E 01	-0.23999999E 03	97
-0.14292881E 01	-0.23000000E 03	98
-0.14169847E 01	-0.22000000E 03	99
-0.14039837E 01	-0.20999999E 03	100
-0.13902239E 01	-0.20000000E 03	101
-0.13756369E 01	-0.19000000E 03	102
-0.13601458E 01	-0.18000000E 03	103
-0.13436639E 01	-0.16999999E 03	104
-0.13260929E 01	-0.16000000E 03	105
-0.13073211E 01	-0.15000000E 03	106
-0.12872214E 01	-0.13999999E 03	107
-0.12656473E 01	-0.13000000E 03	108
-0.12424308E 01	-0.12000000E 03	109
-0.12173766E 01	-0.11000000E 03	110
-0.11902577E 01	-0.09999999E 03	111
-0.11608078E 01	-0.90000000E 02	112
-0.11287129E 01	-0.80000000E 02	113
-0.10935999E 01	-0.70000000E 02	114
-0.10550223E 01	-0.59999999E 02	115
-0.10124402E 01	-0.50000000E 02	116
-0.96519651E 00	-0.40000000E 02	117
-0.91248103E 00	-0.30000000E 02	118
-0.85328485E 00	-0.20000000E 02	119
-0.78633461E 00	-0.09999999E 02	120
-0.70999999E 00	-0.	121
-0.62215918E 00	0.09999999E 02	122
-0.51999857E 00	0.20000000E 02	123
-0.39970580E-00	0.30000000E 02	124
-0.25598342E-00	0.40000000E 02	125
-0.81247769E-01	0.50000000E 02	126
0.11858648E-00	0.59999999E 02	127
0.28646551E-00	0.70000000E 02	128
0.42506013E-00	0.80000000E 02	129
0.54141808E 00	0.90000000E 02	130
0.64049356E 00	0.09999999E 03	131
0.72587077E 00	0.11000000E 03	132
0.80020694E 00	0.12000000E 03	133
0.86551375E 00	0.13000000E 03	134
0.92334213E 00	0.13999999E 03	135
0.97490710E 00	0.15000000E 03	136
0.10211739E 01	0.16000000E 03	137
0.10629193E 01	0.16999999E 03	138
0.11007752E 01	0.18000000E 03	139
0.11352608E 01	0.19000000E 03	140
0.11668073E 01	0.20000000E 03	141
0.11957748E 01	0.20999999E 03	142
0.12224673E 01	0.22000000E 03	143
0.12471426E 01	0.23000000E 03	144

0.12700212E 01	0.23999999E 03	145
0.12912922E 01	0.25000000E 03	146
0.13111195E 01	0.26000000E 03	147
0.13296451E 01	0.27000000E 03	148
0.13469932E 01	0.27999999E 03	149
0.13632727E 01	0.29000000E 03	150
0.13785791E 01	0.30000000E 03	151
0.13929974E 01	0.30999999E 03	152
0.14066026E 01	0.32000000E 03	153
0.14194615E 01	0.33000000E 03	154
0.14316341E 01	0.34000000E 03	155
0.14431737E 01	0.34999999E 03	156
0.14541285E 01	0.36000000E 03	157
0.14645419E 01	0.37000000E 03	158
0.14744531E 01	0.38000000E 03	159
0.14838974E 01	0.39000000E 03	160
0.14929072E 01	0.40000000E 03	161
0.15015116E 01	0.41000000E 03	162

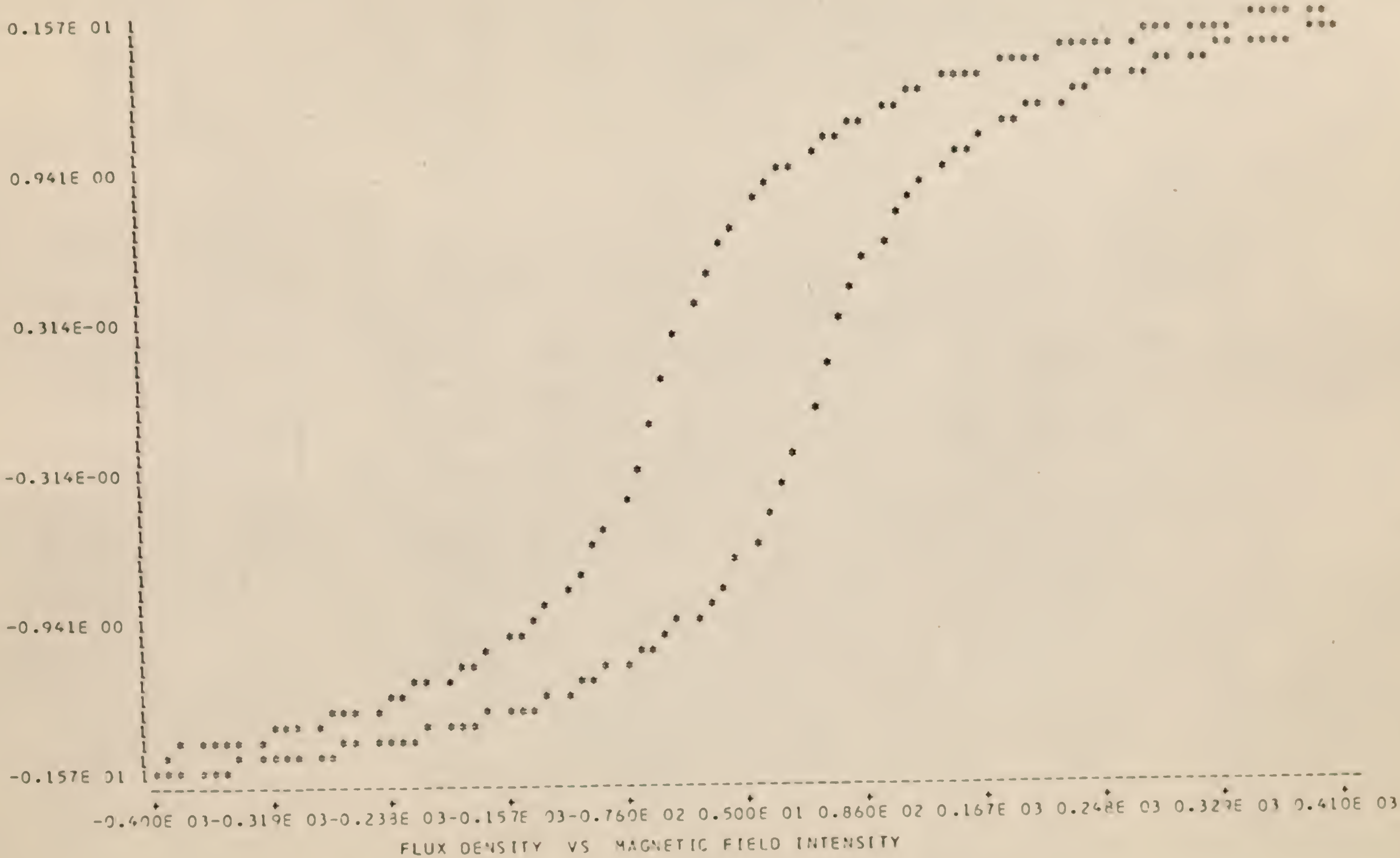


Fig. 22. Printer-plot of approximate hysteresis loop.

```
THIS IS A DECAY RUN
LATTICE COVERS ONLY 1/4 OF THE CROSS SECTION AREA
LATTICE SIZE = NI  BY  NJ =    4 BY    8      (MAX = 48 BY 48)
NI =      4     NJ =       8    ANJ =    0.80000000E 01    CDT =    0.20000000E-00

      CX               C1                C2                DT
  0.49999999E-03  0.11655107E 01   0.14268442E 03   0.13480794E-03
      EO               HSO               BR                HMAX              CUO
  0.09999999E 01 -0.09999999E 08   0.70999999E 00   0.25000000E 03   0.99999999E-07
      TO               TKM               COND              AX                BY
 -0.              0.20000000E-01   0.25000000E 07   0.39999999E-02   0.79999999E-02
      CDT            KZ    NI    NJ     K    KO    KI    KG
  0.20000000E-00      3     4     8     0   100     1     5
      B2               H2                B3                H3
  0.80000000E 00  0.11939999E 02   0.12000000E 01   0.10349999E 03
      HIO              DHX               I3    I4    I5
  0.40000000E 03  0.09999999E 02    81     1     2

EO    = MAXIMUM MATRIX ERROR
HSO   = MINIMUM SUM OF H1(I,J)
BR    = RESIDUAL FLUX DENSITY
HMAX  = MAXIMUM FIELD INTENSITY APPLIED
CUO   = CONSTANT USED TO ALLOW ALL UNITS
TO    = INITIAL VALUE OF TIME IN SECONDS
TKM   = SPECIFIES MAXIMUM TIME
COND  = MATERIAL CONDUCTIVITY
AX    = CORE WIDTH
BY    = CORE HEIGHT
KZ    = NO OF SETS OF DATA
CDT   = CONSTANT MULTIPLIER FOR THE TIME INCREMENT
NI    = NO OF DIVISIONS IN THE X DIRECTION
NJ    = NO OF DIVISIONS IN THE Y DIRECTION
K     = NO OF INCREMENTS OR TIME SUBSCRIPT
KO    = MAXIMUM NO OF INCREMENTS PER RUN
KI    = SPECIFIES INCREMENTS FOR FORCT AND FLUXT PLOTS
KG    = SPECIFIES INCREMENT FOR OUTPUT H1,B1,FORCE
H2    = FIELD INTENSITY CORRESPONDING TO B2 VALUE
B3    = FLUX DENSITY FROM HYSTERESIS CURVE
H3    = FIELD INTENSITY CORRESPONDING TO B3 VALUE
HIO   = MAXIMUM VALUE OF FIELD INTENSITY FOR B VS H
DHX   = INCREMENTS OF FIELD INTENSITY IN B VS H
I3    = MAXIMUM NO OF POINTS ON B VS H CURVE
I4    = USED TO SPECIFY OUTPUT FORMAT
         USE 1 FOR OUTPUT TAPES 9 AND 15  (ALL DATA)
         USE 2 FOR OUTPUT TAPE  9 ONLY    (ALL DATA)
         USE 3 FOR OUTPUT TAPE 15 ONLY    (H1)
         USE 4 FOR OUTPUT TAPE 15 ONLY    (B1)
         USE 5 FOR OUTPUT TAPE 15 ONLY    (FORCE)
         USE 6 FOR OUTPUT TAPE 15 ONLY    (H1,B1,FORCE)
         USE 7 FOR OUTPUT TAPE  6 ONLY    (ITERATIONS ON 9)
I5    = VALUE TO SPECIFY BUILDUP OR DECAY
```

```
         USE 0 OR 1 FOR BUILDUP
         USE 2 FOR DECAY
CX    = DIVISION WIDTHS
C1    = CONSTANT IN MODIFIED FROELICH APPROXIMATION
C2    = CONSTANT IN MODIFIED FROELICH APPROXIMATION
DT    = TIME INCREMENT IN SECONDS
```

K	TOTAL FORCE (FORCT)	TIME(K)	TOTAL FLUX (FLUXT)
0	0.64588968E 01	-0.	0.76726136E-05
1	0.61366858E 01	0.13480794E-03	0.73824580E-05
2	0.58863641E 01	0.26961589E-03	0.71434717E-05
3	0.56693082E 01	0.40442384E-03	0.69269229E-05
4	0.54746731E 01	0.53923178E-03	0.67250592E-05
5	0.52970427E 01	0.67403973E-03	0.65340440E-05
6	0.51331148E 01	0.80884767E-03	0.63515279E-05
7	0.49807078E 01	0.94365562E-03	0.61759755E-05
8	0.48382594E 01	0.10784635E-02	0.60062890E-05
9	0.47045754E 01	0.12132715E-02	0.58416083E-05
10	0.45787790E 01	0.13480794E-02	0.56813462E-05
11	0.44601419E 01	0.14828873E-02	0.55249920E-05
12	0.43480699E 01	0.16176953E-02	0.53721312E-05
13	0.42420702E 01	0.17525032E-02	0.52224213E-05
14	0.41417263E 01	0.18873112E-02	0.50755748E-05
15	0.40466841E 01	0.20221191E-02	0.49313471E-05
16	0.39566297E 01	0.21569270E-02	0.47897567E-05
17	0.38711483E 01	0.22917350E-02	0.46521715E-05
18	0.37897687E 01	0.24265429E-02	0.45183534E-05
19	0.37121330E 01	0.25613508E-02	0.43878131E-05
20	0.36379664E 01	0.26961587E-02	0.42601639E-05
21	0.35670491E 01	0.28309666E-02	0.41350934E-05
22	0.34992010E 01	0.29657745E-02	0.40123452E-05
23	0.34342711E 01	0.31005825E-02	0.38917053E-05
24	0.33721296E 01	0.32353904E-02	0.37729935E-05
25	0.33126634E 01	0.33701983E-02	0.36560558E-05
26	0.32557724E 01	0.35050063E-02	0.35407588E-05
27	0.32013676E 01	0.36398142E-02	0.34269867E-05
28	0.31493678E 01	0.37746221E-02	0.33146381E-05
29	0.30997001E 01	0.39094301E-02	0.32036224E-05
30	0.30522969E 01	0.40442379E-02	0.30938599E-05
31	0.30070964E 01	0.41790458E-02	0.29852786E-05
32	0.29640412E 01	0.43138537E-02	0.28778397E-05
33	0.29230534E 01	0.44486616E-02	0.27722174E-05
34	0.28840140E 01	0.45834695E-02	0.26686006E-05
35	0.28468167E 01	0.47182774E-02	0.25668260E-05
36	0.28113715E 01	0.48530853E-02	0.24667510E-05
37	0.27776012E 01	0.49878932E-02	0.23682502E-05
38	0.27454463E 01	0.51227011E-02	0.22712404E-05
39	0.27148397E 01	0.52575090E-02	0.21756566E-05
40	0.26857109E 01	0.53923169E-02	0.20819284E-05
41	0.26579735E 01	0.55271248E-02	0.19901370E-05
42	0.26315439E 01	0.56619327E-02	0.19002638E-05
43	0.26063437E 01	0.57967406E-02	0.18123583E-05
44	0.25822925E 01	0.59315485E-02	0.17264927E-05
45	0.25593032E 01	0.60663564E-02	0.16430592E-05
46	0.25372707E 01	0.62011642E-02	0.15622683E-05
47	0.25160980E 01	0.63359721E-02	0.14839365E-05
48	0.24957021E 01	0.64707800E-02	0.14078944E-05
49	0.24760111E 01	0.66055879E-02	0.13339885E-05
50	0.24569634E 01	0.67403958E-02	0.12620801E-05

51	0.24385050E 01	0.68752037E-02	0.11920425E-05
52	0.24205893E 01	0.70100116E-02	0.11237606E-05
53	0.24031757E 01	0.71448195E-02	0.10571295E-05
54	0.23862282E 01	0.72796274E-02	0.99205282E-06
55	0.23697157E 01	0.74144353E-02	0.92844202E-06
56	0.23536103E 01	0.75492432E-02	0.86621615E-06
57	0.23378878E 01	0.76840511E-02	0.80530030E-06
58	0.23225264E 01	0.78188589E-02	0.74562551E-06
59	0.23075069E 01	0.79536668E-02	0.68712774E-06
60	0.22928122E 01	0.80884747E-02	0.62974814E-06
61	0.22784270E 01	0.82232826E-02	0.57343163E-06
62	0.22643373E 01	0.83580905E-02	0.51812734E-06
63	0.22505310E 01	0.84928984E-02	0.46378787E-06
64	0.22369968E 01	0.86277063E-02	0.41036884E-06
65	0.22237246E 01	0.87625142E-02	0.35782902E-06
66	0.22107055E 01	0.88973220E-02	0.30613001E-06
67	0.21979311E 01	0.90321299E-02	0.25523542E-06
68	0.21853936E 01	0.91669378E-02	0.20511159E-06
69	0.21730865E 01	0.93017457E-02	0.15572666E-06
70	0.21610032E 01	0.94365536E-02	0.10705083E-06
71	0.21491379E 01	0.95713615E-02	0.59056052E-07
72	0.21374853E 01	0.97061694E-02	0.11715887E-07
73	0.21260404E 01	0.98409773E-02	-0.34994581E-07
74	0.21147087E 01	0.99757852E-02	-0.81098836E-07
75	0.21037559E 01	0.10110593E-01	-0.12661908E-06
76	0.20929031E 01	0.10245401E-01	-0.17157628E-06
77	0.20822515E 01	0.10380208E-01	-0.21599039E-06
78	0.20717827E 01	0.10515016E-01	-0.25988017E-06
79	0.20614985E 01	0.10649824E-01	-0.30326343E-06
80	0.20513958E 01	0.10784632E-01	-0.34615710E-06
81	0.20414717E 01	0.10919441E-01	-0.38857735E-06
82	0.20317236E 01	0.11054248E-01	-0.43053936E-06
83	0.20221487E 01	0.11189056E-01	-0.47205772E-06
84	0.20127450E 01	0.11323864E-01	-0.51314625E-06
85	0.20035098E 01	0.11458672E-01	-0.55381811E-06
86	0.19944410E 01	0.11593480E-01	-0.59408595E-06
87	0.19855365E 01	0.11728287E-01	-0.63396166E-06
88	0.19767943E 01	0.11863095E-01	-0.67345674E-06
89	0.19682126E 01	0.11997903E-01	-0.71258217E-06
90	0.19597393E 01	0.12132711E-01	-0.75134836E-06
91	0.19515228E 01	0.12267519E-01	-0.78976536E-06
92	0.19434114E 01	0.12402327E-01	-0.82784270E-06
93	0.19354533E 01	0.12537135E-01	-0.86558955E-06
94	0.19276469E 01	0.12671943E-01	-0.90299660E-06
95	0.19199897E 01	0.12806751E-01	-0.94003195E-06
96	0.19124786E 01	0.12941559E-01	-0.97670471E-06
97	0.19051109E 01	0.13076366E-01	-0.10130251E-05
98	0.18978339E 01	0.13211174E-01	-0.10490031E-05
99	0.18907955E 01	0.13345982E-01	-0.10846481E-05
100	0.18838430E 01	0.13480790E-01	-0.11199690E-05

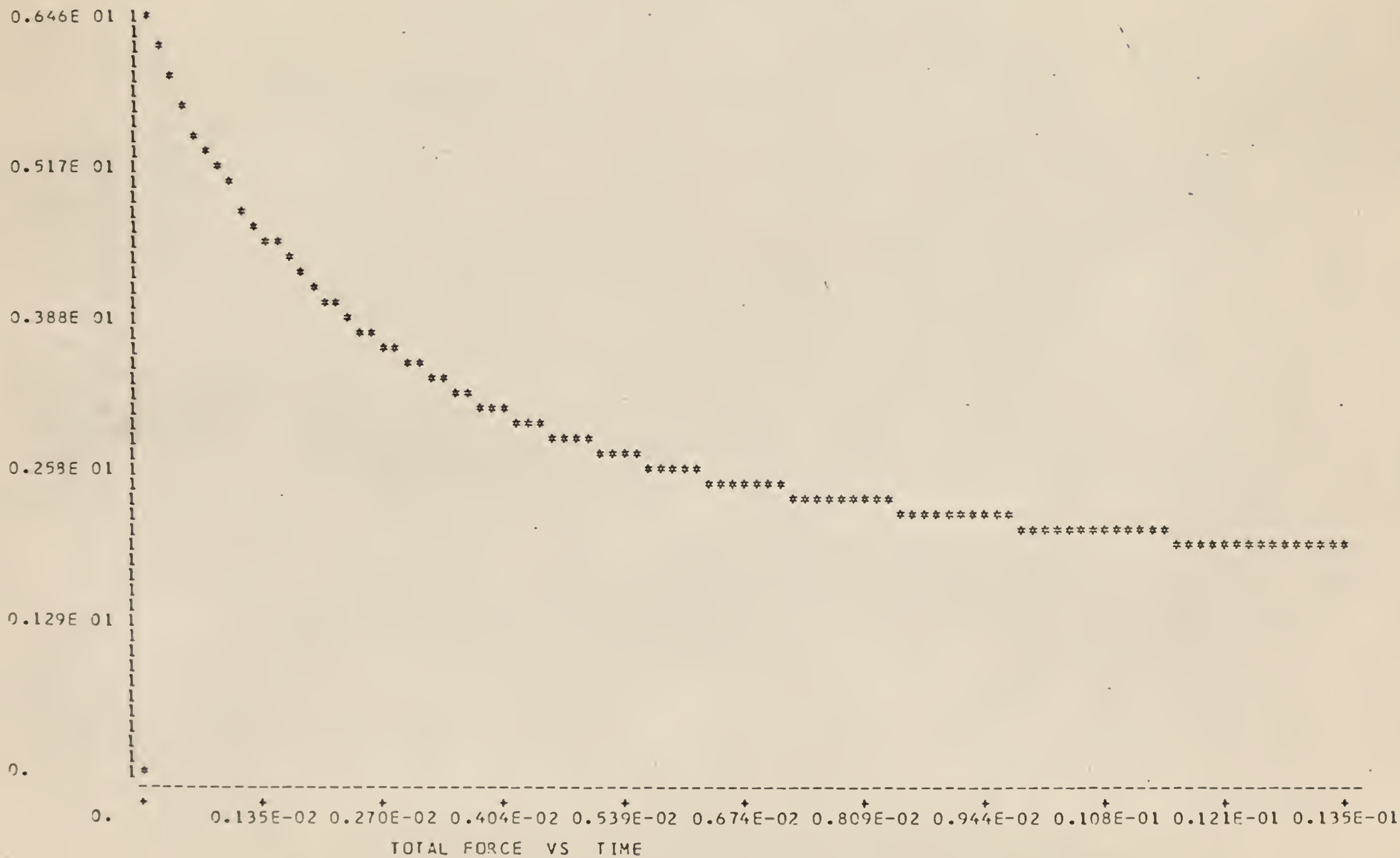


Fig. 23. Printer-plot of total force (newtons) vs time (sec).

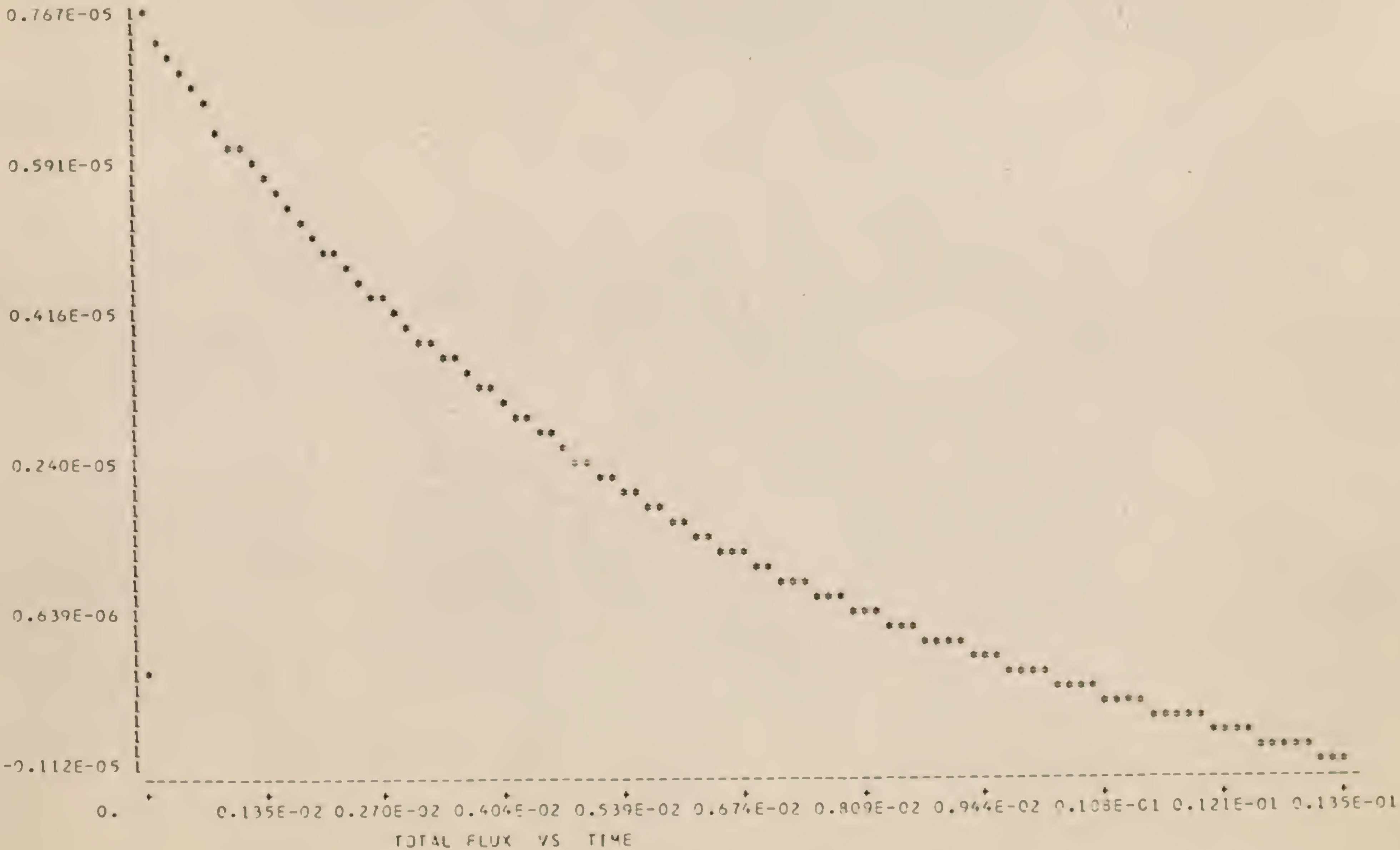


Fig. 24. Printer-plot of total flux (webers) vs time (sec).

```
THIS IS OUTPUT DATA TO BE USED FOR A CONTINUATION  RUN. CHANGE K TO K = 0
  0.09999999E 01 -0.09999999E 08  0.70999999E 00  0.25000000E 03  0.99999999E-07
  0.13480790E-01  0.20000000E-01  0.25000000E 07  0.39999999E-02  0.79999999E-02
  C.20000000E-00      3     4     8  100  100     1     5
  0.80000000E 00  0.11939999E 02  0.12000000E 01  0.10349999E 03
  0.40000000E 03  0.09999999E 02   81     1     2
  0.21705081E 02  0.11066926E 02 -0.22811322E 02 -0.88754325E 02 -0.25000000E 03
  0.21169190E 02  0.10602032E 02 -0.23092266E 02 -0.88822591E 02 -0.25000000E 03
  0.19324502F 02  0.89978344E 01 -0.24069635E 02 -0.89066953E 02 -0.25000000E 03
  0.15366369E 02  0.55395380E 01 -0.26207998E 02 -0.89625838E 02 -0.25000000E 03
  0.75987871E 01 -0.12978579E 01 -0.30529381E 02 -0.90811701E 02 -0.25000000E 03
 -0.72812592E 01 -0.14556853E 02 -0.39210112E 02 -0.93332411E 02 -0.25000000E 03
 -0.35769892E 02 -0.40476813E 02 -0.57302518E 02 -0.99157947E 02 -0.25000000E 03
 -0.92339200E 02 -0.93752974E 02 -0.99211878E 02 -0.11693345E 03 -0.25000000E 03
 -0.25000000E 03 -0.25000000E 03 -0.25000000E 03 -0.25000000E 03 -0.25000000E 03

(KZ =     3   PREVIOUS DATA FOR DATA SET NO.    3)
```

FORCE(I,J)	BO(I,J)	HO(I,J)	TIME(0)	(I	, J)
0.52430335E-01	0.14520148E 01	0.25000000E 03	-0.	2	2
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	2	3
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	2	4
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	-0.	2	6
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	3	2
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	3	3
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	3	4
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	3	6
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	4	2
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	4	3
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	4	4
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	4	6
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	5	2
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	5	3
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	5	4
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	5	6
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	6	2
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	6	3
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	6	4
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	6	6
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	7	2
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	7	3
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	7	4
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	7	6
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	8	2
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	8	3
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	8	4
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	8	6
0.10486067E-00	0.14520148E 01	0.25000000E 03	-0.	9	2
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	9	3
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	9	4
0.20972134E-00	0.14520148E 01	0.25000000E 03	-0.	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	-0.	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	-0.	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	-0.	10	6

TOTAL FORCE (FORCT) = 0.64588968E 01 TIME(0) = -0.
TOTAL FLUX (FLUXT) = 0.767261 36E-05 TIME(0) = -0.

```
NN =      1     TIME(    1) =    0.13480794E-03
NN =      2     E =    0.51259772E 03
NN =      3     E =    0.17505107E 03
NN =      4     E =    0.65356948E 02
NN =      5     E =    0.25517551E 02
NN =      6     E =    0.10395429E 02
NN =      7     E =    0.44650878E 01
NN =      8     E =    0.19566707E 01
NN =      9     E =    0.88535690E 00

NN =      1     TIME(    2) =    0.26961589E-03
NN =      2     E =    0.14928652E 03
NN =      3     E =    0.39733071E 02
NN =      4     E =    0.12150040E 02
NN =      5     E =    0.38595571E 01
NN =      6     E =    0.13126096E 01
NN =      7     E =    0.51683140E 00

NN =      1     TIME(    3) =    0.40442384E-03
NN =      2     E =    0.67984150E 02
NN =      3     E =    0.14060203E 02
NN =      4     E =    0.32708525E 01
NN =      5     E =    0.84317970E 00

NN =      1     TIME(    4) =    0.53923178E-03
NN =      2     E =    0.44885638E 02
NN =      3     E =    0.60331847E 01
NN =      4     E =    0.12207507E 01
NN =      5     E =    0.26661849E-00

NN =      1     TIME(    5) =    0.67403973E-03
NN =      2     E =    0.31600174E 02
NN =      3     E =    0.32818570E 01
NN =      4     E =    0.51847541E 00
```

FORCE(I,J)	B1(I,J)	H1(I,J)	TIME(5)	(I	, J)
0.51868869E-01	0.14442192E 01	0.24290222E 03	0.67403973E-03	2	2
0.10222475E-00	0.14336488E 01	0.23367802E 03	0.67403973E-03	2	3
0.94765411E-01	0.13803514E 01	0.19316754E 03	0.67403973E-03	2	4
0.62529676E-01	0.11212639E 01	0.77800727E 02	0.67403973E-03	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	2	6
0.10373606E-00	0.14442076E 01	0.24289179E 03	0.67403973E-03	3	2
0.20444663E-00	0.14336387E 01	0.23366942E 03	0.67403973E-03	3	3
0.18952918E-00	0.13803454E 01	0.19316351E 03	0.67403973E-03	3	4
0.12505901E-00	0.11212623E 01	0.77800281E 02	0.67403973E-03	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	3	6
0.10372317E-00	0.14441179E 01	0.24281161E 03	0.67403973E-03	4	2
0.20442403E-00	0.14335594E 01	0.23360194E 03	0.67403973E-03	4	3
0.18951543E-00	0.13802953E 01	0.19312953E 03	0.67403973E-03	4	4
0.12505569E-00	0.11212474E 01	0.77795920E 02	0.67403973E-03	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	4	6
0.10364556E-00	0.14435775E 01	0.24232931E 03	0.67403973E-03	5	2
0.20428472E-00	0.14330709E 01	0.23318644E 03	0.67403973E-03	5	3
0.18942284E-00	0.13799581E 01	0.19290102E 03	0.67403973E-03	5	4
0.12502786E-00	0.11211227E 01	0.77759462E 02	0.67403973E-03	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	5	6
0.10323692E-00	0.14407289E 01	0.23980681E 03	0.67403973E-03	6	2
0.20353252E-00	0.14304301E 01	0.23095629E 03	0.67403973E-03	6	3
0.18887325E-00	0.13779548E 01	0.19154983E 03	0.67403973E-03	6	4
0.12481143E-00	0.11201519E 01	0.77476157E 02	0.67403973E-03	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	6	6
0.10140003E-00	0.14278539E 01	0.22880605E 03	0.67403973E-03	7	2
0.20006448E-00	0.14181910E 01	0.22095668E 03	0.67403973E-03	7	3
0.18607759E-00	0.13677187E 01	0.18481231E 03	0.67403973E-03	7	4
0.12331796E-00	0.11134300E 01	0.75534222E 02	0.67403973E-03	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	7	6
0.94122320E-01	0.13756634E 01	0.19001767E 03	0.67403973E-03	8	2
0.18599571E-00	0.13674177E 01	0.18461831E 03	0.67403973E-03	8	3
0.17366058E-00	0.13212968E 01	0.15738061E 03	0.67403973E-03	8	4
0.11450005E-00	0.10728835E 01	0.64510448E 02	0.67403973E-03	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	8	6
0.62283787E-01	0.11190571E 01	0.77157515E 02	0.67403973E-03	9	2
0.12328993E-00	0.11133034E 01	0.75497995E 02	0.67403973E-03	9	3
0.11449326E-00	0.10728751E 01	0.64598279E 02	0.67403973E-03	9	4
0.64406883E-01	0.89466644E 00	0.12613811E 02	0.67403973E-03	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.67403973E-03	10	6

TOTAL FORCE (FORCT) = 0.52970427E 01 TIME(5) = 0.67403973E-03
TOTAL FLUX (FLUXT) = 0.65340440E-05 TIME(5) = 0.67403973E-03

```
NN =     1      TIME(   6) =    0.80884767E-03
NN =     2     E =    0.22873706E 02
NN =     3     E =    0.21933233E 01
NN =     4     E =    0.23584962E-00

NN =     1      TIME(   7) =    0.94365562E-03
NN =     2     E =    0.17104258E 02
NN =     3     E =    0.15590467E 01
NN =     4     E =    0.13956106E-00

NN =     1      TIME(   8) =    0.10784635E-02
NN =     2     E =    0.13121015E 02
NN =     3     E =    0.11226968E 01
NN =     4     E =    0.95485449E-01

NN =     1      TIME(   9) =    0.12132715E-02
NN =     2     E =    0.10537402E 02
NN =     3     E =    0.81881452E 00

NN =     1      TIME(  10) =    0.13480794E-02
NN =     2     E =    0.87489747E 01
NN =     3     E =    0.60457301E 00
```

FORCE (I,J)	B1(I,J)	H1(I,J)	TIME(10)	(I	, J)
0.49851105E-01	0.14158496E 01	0.21910437E 03	0.13480794E-02	2	2
0.96734820E-01	0.13949812E 01	0.20339263E 03	0.13480794E-02	2	3
0.84842205E-01	0.13060828E 01	0.14936348E 03	0.13480794E-02	2	4
0.42267073E-01	0.92186183E 00	0.31698651E 02	0.13480794E-02	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	2	6
0.99680693E-01	0.14155968E 01	0.21898415E 03	0.13480794E-02	3	2
0.19353264E-00	0.13948479E 01	0.20329663E 03	0.13480794E-02	3	3
0.16966254E-00	0.13059986E 01	0.14932033E 03	0.13480794E-02	3	4
0.84528904E-01	0.92183325E 00	0.31693427E 02	0.13480794E-02	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	3	6
0.99569172E-01	0.14149047E 01	0.21836215E 03	0.13480794E-02	4	2
0.19333717E-00	0.13941433E 01	0.20279021E 03	0.13480794E-02	4	3
0.16953912E-00	0.13055235E 01	0.14907693E 03	0.13480794E-02	4	4
0.84494670E-01	0.92164657E 00	0.31659298E 02	0.13480794E-02	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	4	6
0.99159152E-01	0.14119884E 01	0.21609063E 03	0.13480794E-02	5	2
0.19260004E-00	0.13914831E 01	0.20089148E 03	0.13480794E-02	5	3
0.16903107E-00	0.13035659E 01	0.14807831E 03	0.13480794E-02	5	4
0.84321138E-01	0.92069965E 00	0.31486394E 02	0.13480794E-02	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	5	6
0.97831076E-01	0.14025009E 01	0.20889440E 03	0.13480794E-02	6	2
0.19014932E-00	0.13826019E 01	0.19470092E 03	0.13480794E-02	6	3
0.16718078E-00	0.12964116E 01	0.14448617E 03	0.13480794E-02	6	4
0.83518212E-01	0.91630560E 00	0.30688532E 02	0.13480794E-02	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	6	6
0.93912602E-01	0.13741264E 01	0.18899771E 03	0.13480794E-02	7	2
0.18273975E-00	0.13553961E 01	0.17705328E 03	0.13480794E-02	7	3
0.16108289E-00	0.12725488E 01	0.13312110E 03	0.13480794E-02	7	4
0.80141807E-01	0.89759270E 00	0.27370962E 02	0.13480794E-02	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	7	6
0.82485616E-01	0.12878161E 01	0.14023606E 03	0.13480794E-02	8	2
0.16068070E-00	0.12709592E 01	0.13239589E 03	0.13480794E-02	8	3
0.14156511E-00	0.11929654E 01	0.10096276E 03	0.13480794E-02	8	4
0.66944157E-01	0.82036305E 00	0.14924062E 02	0.13480794E-02	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	8	6
0.41281864E-01	0.91105458E 00	0.29744595E 02	0.13480794E-02	9	2
0.79979756E-01	0.89668476E 00	0.27213221E 02	0.13480794E-02	9	3
0.66930976E-01	0.82028227E 00	0.14911997E 02	0.13480794E-02	9	4
0.15441298E-01	0.39399585E-00	-0.30434300E 02	0.13480794E-02	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.13480794E-02	10	6

TOTAL FORCE (FORCT) = 0.45787790E 01 TIME(10) = 0.13480794E-02
TOTAL FLUX (FLUXT) = 0.56813462E-05 TIME(10) = 0.13480794E-02

FORCE(I,J)	B1(I,J)	H1(I,J)	TIME(20)	(I	, J)
0.44763909E-01	0.13416639E 01	0.16882860E 03	0.26961587E-02	2	2
0.85440540E-01	0.13106801E 01	0.15174057E 03	0.26961587E-02	2	3
0.69391819E-01	0.11811876E 01	0.96829750E 02	0.26961587E-02	2	4
0.18530010E-01	0.61038322E 00	-0.11235042E 02	0.26961587E-02	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	2	6
0.89430710E-01	0.13409360E 01	0.16840447E 03	0.26961587E-02	3	2
0.17070544E-00	0.13100064E 01	0.15138979E 03	0.26961587E-02	3	3
0.13866169E-00	0.11806685E 01	0.96650832E 02	0.26961587E-02	3	4
0.37026459E-01	0.61010677E 00	-0.11263760E 02	0.26961587E-02	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	3	6
0.89041477E-01	0.13380148E 01	0.16671370E 03	0.26961587E-02	4	2
0.16999342E-00	0.13072715E 01	0.14997443E 03	0.26961587E-02	4	3
0.13814707E-00	0.11784755E 01	0.95897850E 02	0.26961587E-02	4	4
0.36868624E-01	0.60880501E 00	-0.11398818E 02	0.26961587E-02	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	4	6
0.88003264E-01	0.13301913E 01	0.16227492E 03	0.26961587E-02	5	2
0.16806377E-00	0.12998307E 01	0.14619173E 03	0.26961587E-02	5	3
0.13667153E-00	0.11721651E 01	0.93757255E 02	0.26961587E-02	5	4
0.36339643E-01	0.60442174E 00	-0.11851547E 02	0.26961587E-02	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	5	6
0.85464005E-01	0.13108601E 01	0.15183441E 03	0.26961587E-02	6	2
0.16326898E-00	0.12811548E 01	0.13711461E 03	0.26961587E-02	6	3
0.13279344E-00	0.11554152E 01	0.88257473E 02	0.26961587E-02	6	4
0.34705687E-01	0.59067702E 00	-0.13251153E 02	0.26961587E-02	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	6	6
0.79392660E-01	0.12634408E 01	0.12901696E 03	0.26961587E-02	7	2
0.15164423E-00	0.12347035E 01	0.11683237E 03	0.26961587E-02	7	3
0.12291981E-00	0.11116311E 01	0.75020336E 02	0.26961587E-02	7	4
0.29956402E-01	0.54877533E 00	-0.17338986E 02	0.26961587E-02	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	7	6
0.64003878E-01	0.11344043E 01	0.81710129E 02	0.26961587E-02	8	2
0.12185185E-00	0.11067915E 01	0.73649745E 02	0.26961587E-02	8	3
0.96674807E-01	0.98583994E 00	0.44238906E 02	0.26961587E-02	8	4
0.16887335E-01	0.41203140E-00	-0.29050958E 02	0.26961587E-02	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	8	6
0.16349057E-01	0.57333851E 00	-0.14974566E 02	0.26961587E-02	9	2
0.29524913E-01	0.54480874E 00	-0.17712619E 02	0.26961587E-02	9	3
0.16849285E-01	0.41156694E-00	-0.29087009E 02	0.26961587E-02	9	4
0.48887759E-02	-0.22169189E-00	-0.65901104E 02	0.26961587E-02	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.26961587E-02	10	6

TOTAL FORCE (FORCT) = 0.36379664E 01 TIME(20) = 0.26961587E-02
TOTAL FLUX (FLUXT) = 0.42601639E-05 TIME(20) = 0.26961587E-02

FORCE(I,J)	B1(I,J)	H1(I,J)	TIME(40)	(I	, J)
0.35764449E-01	0.11992383E 01	0.10322275E 03	0.53923169E-02	2	2
0.66466665E-01	0.11560235E 01	0.88452752E 02	0.53923169E-02	2	3
0.47051156E-01	0.97263501E 00	0.41505075E 02	0.53923169E-02	2	4
0.48473524E-03	0.98722704E-01	-0.49088437E 02	0.53923169E-02	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	2	6
0.71327852E-01	0.11975518E 01	0.10261101E 03	0.53923169E-02	3	2
0.13755854E-00	0.11543927E 01	0.87930042E 02	0.53923169E-02	3	3
0.93818438E-01	0.97116685E 00	0.41206053E 02	0.53923169E-02	3	4
0.94599222E-03	0.97519964E-01	-0.49151751E 02	0.53923169E-02	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	3	6
0.70601285E-01	0.11914369E 01	0.10041833E 03	0.53923169E-02	4	2
0.13119754E-00	0.11484513E 01	0.86045487E 02	0.53923169E-02	4	3
0.92769282E-01	0.96572138E 00	0.40105393E 02	0.53923169E-02	4	4
0.85610344E-03	0.92771135E-01	-0.49400901E 02	0.53923169E-02	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	4	6
0.68914851E-01	0.11771211E 01	0.95435143E 02	0.53923169E-02	5	2
0.12801585E-00	0.11344402E 01	0.81720982E 02	0.53923169E-02	5	3
0.90253160E-01	0.95253503E 00	0.37493915E 02	0.53923169E-02	5	4
0.63924025E-03	0.80164447E-01	-0.50055926E 02	0.53923169E-02	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	5	6
0.65268493E-01	0.11455565E 01	0.85138415E 02	0.53923169E-02	6	2
0.12108719E-00	0.11033133E 01	0.72675309E 02	0.53923169E-02	6	3
0.84640805E-01	0.92244322E 00	0.31805028E 02	0.53923169E-02	6	4
0.23541576E-03	0.49648283E-01	-0.51653909E 02	0.53923169E-02	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	6	6
0.57332665E-01	0.10736576E 01	0.64710458E 02	0.53923169E-02	7	2
0.10592006E-00	0.10319030E 01	0.54445374E 02	0.53923169E-02	7	3
0.72158296E-01	0.85171223E 00	0.19750106E 02	0.53923169E-02	7	4
0.90730357E-04	-0.30201413E-01	-0.55466334E 02	0.53923169E-02	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	7	6
0.38818811E-01	0.88345791E 00	0.24948010E 02	0.53923169E-02	8	2
0.70459558E-01	0.84162710E 00	0.18165620E 02	0.53923169E-02	8	3
0.43166781E-01	0.65875600E 00	-0.60091966E 01	0.53923169E-02	8	4
0.41887528E-02	-0.20520698E-00	-0.64908683E 02	0.53923169E-02	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	8	6
0.2548614[illegible]-05	0.71584135E-02	-0.53675393E 02	0.53923169E-02	9	2
0.1637329[illegible]-03	-0.40571190E-01	-0.55975630E 02	0.53923169E-02	9	3
0.42287963E-02	-0.20618552E-00	-0.64967045E 02	0.53923169E-02	9	4
0.32571247E-01	-0.57222515E 00	-0.92946576E 02	0.53923169E-02	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.53923169E-02	10	6

TOTAL FORCE (FORCT) = 0.26857109E 01 TIME(40) = 0.53923169E-02
TOTAL FLUX (FLUXT) = 0.20819284E-05 TIME(40) = 0.53923169E-02

FORCE(I,J)	B1(I,J)	H1(I,J)	TIME(60)	(I	, J)
0.28738201E-01	0.10750026E 01	0.65058924E 02	0.80884747E-02	2	2
0.51942707E-01	0.10219438E 01	0.52145334E 02	0.80884747E-02	2	3
0.31363516E-01	0.79410351E 00	0.11096895E 02	0.80884747E-02	2	4
0.33478174E-02	-0.25944519E-00	-0.68250201E 02	0.80884747E-02	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	2	6
0.57225447E-01	0.10726532E 01	0.64451001E 02	0.80884747E-02	3	2
0.10341742E-00	0.10196393E 01	0.51620746E 02	0.80884747E-02	3	3
0.62381281E-01	0.79191195E 00	0.10785875E 02	0.80884747E-02	3	4
0.67545367E-02	-0.26058387E-00	-0.68322758E 02	0.80884747E-02	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	3	6
0.56341376E-01	0.10643353E 01	0.62327000E 02	0.80884747E-02	4	2
0.10176432E-00	0.10114572E 01	0.49780778E 02	0.80884747E-02	4	3
0.61148413E-01	0.78404745E 00	0.95800489E 01	0.80884747E-02	4	4
0.69754802E-02	-0.26481148E-00	-0.68593042E 02	0.80884747E-02	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	4	6
0.54363807E-01	0.10454895E 01	0.57672172E 02	0.80884747E-02	5	2
0.98052040E-01	0.99283726E 00	0.45720726E 02	0.80884747E-02	5	3
0.58344700E-01	0.76586188E 00	0.71830125E 01	0.80884747E-02	5	4
0.75216380E-02	-0.27498307E-00	-0.69249194E 02	0.80884747E-02	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	5	6
0.50240808E-01	0.10050625E 01	0.48366828E 02	0.80884747E-02	6	2
0.90285081E-01	0.95270347E 00	0.37526801E 02	0.80884747E-02	6	3
0.52432254E-01	0.72602072E 00	0.19886278E 01	0.80884747E-02	6	4
0.88001782E-02	-0.29743733E-00	-0.70727625E 02	0.80884747E-02	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	6	6
0.41581303E-01	0.91435279E 00	0.30336285E 02	0.80884747E-02	7	2
0.73947329E-01	0.86220591E 00	0.21432260E 02	0.80884747E-02	7	3
0.40084353E-01	0.63480040E 00	-0.86481190E 01	0.80884747E-02	7	4
0.11863885E-01	-0.34535298E-00	-0.74027571E 02	0.80884747E-02	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	7	6
0.22579847E-01	0.67379107E 00	-0.42992123E 01	0.80884747E-02	8	2
0.38336109E-01	0.62080295E 00	-0.10143420E 02	0.80884747E-02	8	3
0.14916033E-01	0.38723662E-00	-0.30944101E 02	0.80884747E-02	8	4
0.19975237E-01	-0.44812144E-00	-0.81852385E 02	0.80884747E-02	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	8	6
0.52611569E-02	-0.32524107E-00	-0.72617624E 02	0.80884747E-02	9	2
0.12351447E-01	-0.35237791E-00	-0.74528828E 02	0.80884747E-02	9	3
0.20042253E-01	-0.44887252E-00	-0.81913723E 02	0.80884747E-02	9	4
0.46720230E-01	-0.68533394E 00	-0.10507315E 03	0.80884747E-02	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.80884747E-02	10	6

TOTAL FORCE (FORCT) = 0.22928122E 01 TIME(60) = 0.80884747E-02
TOTAL FLUX (FLUXT) = 0.62974814E-06 TIME(60) = 0.80884747E-02

FORCE(I,J)	B1(I,J)	H1(I,J)	TIME(80)	(I	, J)
0.23130637E-01	0.96443628E 00	0.37847560E 02	0.10784632E-01	2	2
0.40516707E-01	0.90257192E 00	0.28241255E 02	0.10784632E-01	2	3
0.20020494E-01	0.63445693E 00	-0.86852137E 01	0.10784632E-01	2	4
0.91549833E-02	-0.42903579E-00	-0.80315129E 02	0.10784632E-01	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	2	6
0.45986420E-01	0.96156699E 00	0.39274509E 02	0.10784632E-01	3	2
0.80525393E-01	0.89973824E 00	0.27744881E 02	0.10784632E-01	3	3
0.39692173E-01	0.63168737E 00	-0.89835838E 01	0.10784632E-01	3	4
0.18387320E-01	-0.42994110E-00	-0.80387130E 02	0.10784632E-01	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	3	6
0.45028677E-01	0.95150121E 00	0.37292323E 02	0.10784632E-01	4	2
0.78752437E-01	0.88977818E 00	0.26022827E 02	0.10784632E-01	4	3
0.38469558E-01	0.62188252E 00	-0.10029282E 02	0.10784632E-01	4	4
0.18668772E-01	-0.43321913E-00	-0.80648596E 02	0.10784632E-01	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	4	6
0.42922333E-01	0.92898010E 00	0.33010077E 02	0.10784632E-01	5	2
0.74845462E-01	0.86742610E 00	0.22282109E 02	0.10784632E-01	5	3
0.35767052E-01	0.59964100E 00	-0.12341783E 02	0.10784632E-01	5	4
0.19329875E-01	-0.44082303E-00	-0.81259704E 02	0.10784632E-01	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	5	6
0.38615819E-01	0.88114499E 00	0.24558090E 02	0.10784632E-01	6	2
0.66850295E-01	0.81978773E 00	0.14838172E 02	0.10784632E-01	6	3
0.30278623E-01	0.55171884E 00	-0.17060275E 02	0.10784632E-01	6	4
0.20778389E-01	-0.45704155E-00	-0.82585043E 02	0.10784632E-01	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	6	6
0.29810452E-01	0.77419264E 00	0.83166630E 01	0.10784632E-01	7	2
0.50548927E-01	0.71286242E 00	0.35128806E-00	0.10784632E-01	7	3
0.19521182E-01	0.44299906E-00	-0.26594459E 02	0.10784632E-01	7	4
0.23971021E-01	-0.49089999E-00	-0.85451940E 02	0.10784632E-01	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	7	6
0.11871676E-01	0.48856322E-00	-0.22780658E 02	0.10784632E-01	8	2
0.18036178E-01	0.42581602E-00	-0.27970458E 02	0.10784632E-01	8	3
0.21488351E-02	0.14697765E-00	-0.46475545E 02	0.10784632E-01	8	4
0.31623027E-01	-0.55383428E 00	-0.92130306E 02	0.10784632E-01	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	8	6
0.11332951E-01	-0.47734925E-00	-0.84287893E 02	0.10784632E-01	9	2
0.24492793E-01	-0.49621388E-00	-0.85914662E 02	0.10784632E-01	9	3
0.31690815E-01	-0.56443828E 00	-0.92188714E 02	0.10784632E-01	9	4
0.54914397E-01	-0.74300692E 00	-0.11218879E 03	0.10784632E-01	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.10784632E-01	10	6

TOTAL FORCE (FORCT) = 0.20513958E 01 TIME(80) = 0.10784632E-01
TOTAL FLUX (FLUXT) = -0.34615710E-06 TIME(80) = 0.10784632E-01

FORCE(I,J)	B1(I,J)	H1(I,J)	TIME(100)	(I	, J)
0.18559017E-01	0.86388759E 00	0.21705081E 02	0.13480790E-01	2	2
0.31346868E-01	0.79389273E 00	0.11066926E 02	0.13480790E-01	2	3
0.11854441E-01	0.48820844E-00	-0.22811322E 02	0.13480790E-01	2	4
0.13863061E-01	-0.52795187E 00	-0.88754325E 02	0.13480790E-01	2	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	2	6
0.36834265E-01	0.86057902E 00	0.21169190E 02	0.13480790E-01	3	2
0.62176703E-01	0.79061235E 00	0.10602032E 02	0.13480790E-01	3	3
0.23393401E-01	0.48494940E-00	-0.23092266E 02	0.13480790E-01	3	4
0.27804430E-01	-0.52869688E 00	-0.88822591E 02	0.13480790E-01	3	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	3	6
0.35851645E-01	0.84902268E 00	0.19324502E 02	0.13480790E-01	4	2
0.60385093E-01	0.77913842E 00	0.89978344E 01	0.13480790E-01	4	3
0.22301017E-01	0.47349139E-00	-0.24069635E 02	0.13480790E-01	4	4
0.28084927E-01	-0.53135699E 00	-0.89066953E 02	0.13480790E-01	4	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	4	6
0.33713473E-01	0.82331590E 00	0.15366369E 02	0.13480790E-01	5	2
0.56485146E-01	0.75355834E 00	0.55395380E 01	0.13480790E-01	5	3
0.19942318E-01	0.44775204E-00	-0.26207998E 02	0.13480790E-01	5	4
0.28727540E-01	-0.53740162E 00	-0.89625838E 02	0.13480790E-01	5	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	5	6
0.29406694E-01	0.76893184E 00	0.75987371E 01	0.13480790E-01	6	2
0.48643972E-01	0.69930118E 00	-0.12978579E 01	0.13480790E-01	6	3
0.15343007E-01	0.39273987E-00	-0.30529381E 02	0.13480790E-01	6	4
0.30095617E-01	-0.55004900E 00	-0.90811701E 02	0.13480790E-01	6	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	6	6
0.20840845E-01	0.64732505E 00	-0.72812592E 01	0.13480790E-01	7	2
0.33184131E-01	0.57758377E 00	-0.14556853E 02	0.13480790E-01	7	3
0.71629496E-02	0.26834635E-00	-0.39210112E 02	0.13480790E-01	7	4
0.33020057E-01	-0.57615411E 00	-0.93332411E 02	0.13480790E-01	7	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	7	6
0.50948890E-02	0.32006052E-00	-0.35769892E 02	0.13480790E-01	8	2
0.61390605E-02	0.24842809E-00	-0.40476813E 02	0.13480790E-01	8	3
0.44717512E-03	-0.67048423E-01	-0.57302518E 02	0.13480790E-01	8	4
0.39822282E-01	-0.63272183E 00	-0.99157947E 02	0.13480790E-01	8	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	8	6
0.15932756E-01	-0.56599189E 00	-0.92339200E 02	0.13480790E-01	9	2
0.33509656E-01	-0.58040980E 00	-0.93752974E 02	0.13480790E-01	9	3
0.39885331E-01	-0.63322252E 00	-0.99211878E 02	0.13480790E-01	9	4
0.60279059E-01	-0.77845405E 00	-0.11693345E 03	0.13480790E-01	9	5
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	9	6
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	10	2
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	10	3
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	10	4
0.82931516E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	10	5
0.41465758E-01	-0.12912922E 01	-0.25000000E 03	0.13480790E-01	10	6

TOTAL FORCE (FORCT) = 0.18838430E 01 TIME(100) = 0.13480790E-01
TOTAL FLUX (FLUXT) = -0.11199690E-05 TIME(100) = 0.13480790E-01

MAGNETIC FIELD INTENSITY HO(I,J) AT TIME (0) = -0.

	J=2	3	4	5	6
2	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
3	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
4	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
5	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
6	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
7	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
8	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
9	0.25000000E 03	0.25000000E 03	0.25000000E 03	0.25000000E 03	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.64588968E 01 TIME(0) = -0.
TOTAL FLUX (FLUXT) = 0.767726136E-05 TIME(0) = -0.

FLUX DENSITY MATRIX B1(I,J) AT TIME(0) = -0.

	J=2	3	4	5	6
2	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
3	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
4	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
5	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
6	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
7	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
8	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
9	0.14520148E 01	0.14520148E 01	0.14520148E 01	0.14520148E 01	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.64588968E 01 TIME(0) = -0.
TOTAL FLUX (FLUXT) = 0.767726136E-05 TIME(0) = -0.

THEORETICAL FORCE(I,J) AT TIME(0) = -0.

	J=2	3	4	5	6
2	0.52430335E-01	0.10486067E-00	0.10486067E-00	0.10486067E-00	0.41465758E-01
3	0.10486067E-00	0.20972134E-00	0.20972134E-00	0.20972134E-00	0.82931516E-01
4	0.10486067E-00	0.20972134E-00	0.20972134E-00	0.20972134E-00	0.82931516E-01
5	0.10486067E-00	0.20972134E-00	0.20972134E-00	0.20972134E-00	0.82931516E-01
6	0.10486067E-00	0.20972134E-00	0.20972134E-00	0.20972134E-00	0.82931516E-01
7	0.10486067E-00	0.20972134E-00	0.20972134E-00	0.20972134E-00	0.82931516E-01
8	0.10486067E-00	0.20972134E-00	0.20972134E-00	0.20972134E-00	0.82931516E-01
9	0.10486067E-00	0.20972134E-00	0.20972134E-00	0.20972134E-00	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.64588968E 01 TIME(0) = -0.
TOTAL FLUX (FLUXT) = 0.767726136E-05 TIME(0) = -0.

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (5) = 0.67403973E-03

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (5) = 0.67403973E-03

	J=2	3	4	5	6
2	0.24290222E 03	0.23367802E 03	0.19316754E 03	0.77800727E 02	-0.25000000E 03
3	0.24289179E 03	0.23366942E 03	0.19316351E 03	0.77800281E 02	-0.25000000E 03
4	0.24281161E 03	0.23360194E 03	0.19312953E 03	0.77795920E 02	-0.25000000E 03
5	0.24232931E 03	0.23318644E 03	0.19290102E 03	0.77759462E 02	-0.25000000E 03
6	0.23980681E 03	0.23095629E 03	0.19154983E 03	0.77476157E 02	-0.25000000E 03
7	0.22880605E 03	0.22095668E 03	0.18481231E 03	0.75534222E 02	-0.25000000E 03
8	0.19001767E 03	0.18461831E 03	0.15738061E 03	0.64510448E 02	-0.25000000E 03
9	0.77157515E 02	0.75497995E 02	0.64508279E 02	0.12613811E 02	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.52970427E 01 TIME(5) = 0.67403973E-03
TOTAL FLUX (FLUXT) = 0.65340440E-05 TIME(5) = 0.67403973E-03

FLUX DENSITY MATRIX B1(I,J) AT TIME(5) = 0.67403973E-03

	J=2	3	4	5	6
2	0.14442192E 01	0.14336488E 01	0.13803514E 01	0.11212639E 01	-0.12912922E 01
3	0.14442076E 01	0.14336387E 01	0.13803454E 01	0.11212623E 01	-0.12912922E 01
4	0.14441179E 01	0.14335594E 01	0.13802953E 01	0.11212474E 01	-0.12912922E 01
5	0.14435775E 01	0.14330709E 01	0.13799581E 01	0.11211227E 01	-0.12912922E 01
6	0.14407289E 01	0.14304301E 01	0.13779548E 01	0.11201519E 01	-0.12912922E 01
7	0.14278539E 01	0.14181910E 01	0.13677137E 01	0.11134300E 01	-0.12912922E 01
8	0.13756634E 01	0.13674177E 01	0.13212968E 01	0.10728835E 01	-0.12912922E 01
9	0.11190571E 01	0.11133034E 01	0.10728751E 01	0.80466644E 00	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.52970427E 01 TIME(5) = 0.67403973E-03
TOTAL FLUX (FLUXT) = 0.65340440E-05 TIME(5) = 0.67403973E-03

THEORETICAL FORCE(I,J) AT TIME(5) = 0.67403973E-03

	J=2	3	4	5	6
2	0.51868869E-01	0.10222475E-00	0.94765411E-01	0.62529676E-01	0.41465758E-01
3	0.10373606E-00	0.20444663E-00	0.18952918E-00	0.12505901E-00	0.82931516E-01
4	0.10372317E-00	0.20442403E-00	0.18951543E-00	0.12505569E-00	0.82931516E-01
5	0.10364556E-00	0.20428472E-00	0.18942284E-00	0.12502786E-00	0.82931516E-01
6	0.10323692E-00	0.20353252E-00	0.18887325E-00	0.12481143E-00	0.82931516E-01
7	0.10140003E-00	0.20006448E-00	0.18607759E-00	0.12331796E-00	0.82931516E-01
8	0.94122820E-01	0.18599571E-00	0.17366058E-00	0.11450005E-00	0.82931516E-01
9	0.62283787E-01	0.12328993E-00	0.11449826E-00	0.64406883E-01	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.52970427E 01 TIME(5) = 0.67403973E-03
TOTAL FLUX (FLUXT) = 0.65340440E-05 TIME(5) = 0.67403973E-03

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (10) = 0.13480794E-02

	J=2	3	4	5	6
2	0.21910437E 03	0.20339263E 03	0.14936348E 03	0.31698651E 02	-0.25000000E 03
3	0.21898415E 03	0.20329663E 03	0.14932033E 03	0.31693427E 02	-0.25000000E 03
4	0.21836215E 03	0.20279021E 03	0.14907693E 03	0.31659298E 02	-0.25000000E 03
5	0.21609063E 03	0.20089148E 03	0.14807831E 03	0.31486394E 02	-0.25000000E 03
6	0.20889440E 03	0.19470092E 03	0.14448617E 03	0.30688532E 02	-0.25000000E 03
7	0.18899771E 03	0.17705328E 03	0.13312110E 03	0.27370962E 02	-0.25000000E 03
8	0.14028606E 03	0.13239589E 03	0.10096276E 03	0.14924062E 02	-0.25000000E 03
9	0.29744595E 02	0.27213221E 02	0.14911997E 02	-0.30434300E 02	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.45787790E 01 TIME(10) = 0.13480794E-02
TOTAL FLUX (FLUXT) = 0.56813462E-05 TIME(10) = 0.13480794E-02

FLUX DENSITY MATRIX B1(I,J) AT TIME(10) = 0.13480794E-02

	J=2	3	4	5	6
2	0.14158496E 01	0.13949812E 01	0.13060828E 01	0.92186183E 00	-0.12912922E 01
3	0.14156963E 01	0.13948479E 01	0.13059986E 01	0.92183325E 00	-0.12912922E 01
4	0.14149047E 01	0.13941433E 01	0.13055235E 01	0.92164657E 00	-0.12912922E 01
5	0.14119884E 01	0.13914831E 01	0.13035659E 01	0.92069965E 00	-0.12912922E 01
6	0.14025009E 01	0.13826019E 01	0.12964116E 01	0.91630560E 00	-0.12912922E 01
7	0.13741264E 01	0.13553961E 01	0.12725488E 01	0.89759270E 00	-0.12912922E 01
8	0.12878161E 01	0.12709592E 01	0.11929654E 01	0.82036305E 00	-0.12912922E 01
9	0.91105458E 00	0.89668476E 00	0.82028227E 00	0.39399585E-00	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.45787790E 01 TIME(10) = 0.13480794E-02
TOTAL FLUX (FLUXT) = 0.56813462E-05 TIME(10) = 0.13480794E-02

THEORETICAL FORCE(I,J) AT TIME(10) = 0.13480794E-02

	J=2	3	4	5	6
2	0.49851105E-01	0.96784820E-01	0.84842205E-01	0.42267073E-01	0.41465758E-01
3	0.99680693E-01	0.19353264E-00	0.16966254E-00	0.84528904E-01	0.82931516E-01
4	0.99569172E-01	0.19333717E-00	0.16953912E-00	0.84494670E-01	0.82931516E-01
5	0.99159152E-01	0.19260004E-00	0.16903107E-00	0.84321138E-01	0.82931516E-01
6	0.97831076E-01	0.19014932E-00	0.16718078E-00	0.83518212E-01	0.82931516E-01
7	0.93912602E-01	0.18273975E-00	0.16103289E-00	0.80141807E-01	0.82931516E-01
8	0.82485616E-01	0.16068070E-00	0.14156511E-00	0.66944157E-01	0.82931516E-01
9	0.41231864E-01	0.79979756E-01	0.66930976E-01	0.15441298E-01	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.45787790E 01 TIME(10) = 0.13480794E-02
TOTAL FLUX (FLUXT) = 0.56813462E-05 TIME(10) = 0.13480794E-02

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (20) = 0.26961587E-02

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (20) = 0.26961587E-02

	J=2	3	4	5	6
2	0.16882860E 03	0.15174057E 03	0.96829750E 02	-0.11235042E 02	-0.25000000E 03
3	0.16840447E 03	0.15138979E 03	0.96650832E 02	-0.11263760E 02	-0.25000000E 03
4	0.16671370E 03	0.14997443E 03	0.95897850E 02	-0.11398818E 02	-0.25000000E 03
5	0.16227492E 03	0.14619173E 03	0.93757255E 02	-0.11851547E 02	-0.25000000E 03
6	0.15183441E 03	0.13711461E 03	0.88257473E 02	-0.13251153E 02	-0.25000000E 03
7	0.12901696E 03	0.11683237E 03	0.75020336E 02	-0.17338986E 02	-0.25000000E 03
8	0.81710129E 02	0.73649745E 02	0.44238906E 02	-0.29050958E 02	-0.25000000E 03
9	-0.14974566E 02	-0.17712619E 02	-0.29087009E 02	-0.65901104E 02	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.36379664E 01 TIME(20) = 0.26961587E-02
TOTAL FLUX (FLUXT) = 0.42601639E-05 TIME(20) = 0.26961587E-02

FLUX DENSITY MATRIX B1(I,J) AT TIME(20) = 0.26961587E-02

	J=2	3	4	5	6
2	0.13416639E 01	0.13106801E 01	0.11811876E 01	0.61038322E 00	-0.12912922E 01
3	0.13409360E 01	0.13100064E 01	0.11806685E 01	0.61010677E 00	-0.12912922E 01
4	0.13380148E 01	0.13072715E 01	0.11784755E 01	0.60880501E 00	-0.12912922E 01
5	0.13301913E 01	0.12998307E 01	0.11721651E 01	0.60442174E 00	-0.12912922E 01
6	0.13108601E 01	0.12811548E 01	0.11554152E 01	0.59067702E 00	-0.12912922E 01
7	0.12634408E 01	0.12347035E 01	0.11116311E 01	0.54877533E 00	-0.12912922E 01
8	0.11344043E 01	0.11067915E 01	0.98583994E 00	0.41203140E-00	-0.12912922E 01
9	0.57333851E 00	0.54480874E 00	0.41156694E-00	-0.22169189E-00	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.36379664E 01 TIME(20) = 0.26961587E-02
TOTAL FLUX (FLUXT) = 0.42601639E-05 TIME(20) = 0.26961587E-02

THEORETICAL FORCE(I,J) AT TIME(20) = 0.26961587E-02

	J=2	3	4	5	6
2	0.44763909E-01	0.85440540E-01	0.69391819E-01	0.18530010E-01	0.41465758E-01
3	0.89430710E-01	0.17070544E-00	0.13866169E-00	0.37026459E-01	0.82931516E-01
4	0.89041477E-01	0.16999342E-00	0.13814707E-00	0.36868624E-01	0.82931516E-01
5	0.88003264E-01	0.16806377E-00	0.13667153E-00	0.36339643E-01	0.82931516E-01
6	0.85464005E-01	0.16326898E-00	0.13279344E-00	0.34705687E-01	0.82931516E-01
7	0.79392660E-01	0.15164423E-00	0.12291981E-00	0.29956402E-01	0.82931516E-01
8	0.64003878E-01	0.12185185E-00	0.96674807E-01	0.16887335E-01	0.82931516E-01
9	0.16349057E-01	0.29524913E-01	0.16849285E-01	0.48887759E-02	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.36379664E 01 TIME(20) = 0.26961587E-02
TOTAL FLUX (FLUXT) = 0.42601639E-05 TIME(20) = 0.26961587E-02

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (40) = 0.53923169E-02

	J=2	3	4	5	6
2	0.10322275E 03	0.88452752E 02	0.41505075E 02	-0.49088437E 02	-0.25000000E 03
3	0.10261101E 03	0.87930042E 02	0.41206053E 02	-0.49151751E 02	-0.25000000E 03
4	0.10041833E 03	0.86045487E 02	0.40105393E 02	-0.49400901E 02	-0.25000000E 03
5	0.95435143E 02	0.81720982E 02	0.37493915E 02	-0.50055926E 02	-0.25000000E 03
6	0.85138415E 02	0.72675309E 02	0.31805028E 02	-0.51653909E 02	-0.25000000E 03
7	0.64710458E 02	0.54445374E 02	0.19750106E 02	-0.55466334E 02	-0.25000000E 03
8	0.24946010E 02	0.18165620E 02	-0.60091966E 01	-0.64908683E 02	-0.25000000E 03
9	-0.53675393E 02	-0.55975630E 02	-0.64967045E 02	-0.92946576E 02	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.26857109E 01 TIME(40) = 0.53923169E-02
TOTAL FLUX (FLUXT) = 0.20819284E-05 TIME(40) = 0.53923169E-02

FLUX DENSITY MATRIX B1(I,J) AT TIME(40) = 0.53923169E-02

	J=2	3	4	5	6
2	0.11992383E 01	0.11560235E 01	0.97263501E 00	0.98722704E-01	-0.12912922E 01
3	0.11975518E 01	0.11543927E 01	0.97116685E 00	0.97519964E-01	-0.12912922E 01
4	0.11914369E 01	0.11484513E 01	0.96572138E 00	0.92771135E-01	-0.12912922E 01
5	0.11771211E 01	0.11344402E 01	0.95253503E 00	0.80164447E-01	-0.12912922E 01
6	0.11455565E 01	0.11033133E 01	0.92244322E 00	0.48648283E-01	-0.12912922E 01
7	0.10736576E 01	0.10319030E 01	0.85171223E 00	-0.30201413E-01	-0.12912922E 01
8	0.88345791E 00	0.84162710E 00	0.65875600E 00	-0.29520698E-00	-0.12912922E 01
9	0.71584135E-02	-0.40571190E-01	-0.20618552E-00	-0.57222515E 00	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.26857109E 01 TIME(40) = 0.53923169E-02
TOTAL FLUX (FLUXT) = 0.20819284E-05 TIME(40) = 0.53923169E-02

THEORETICAL FORCE(I,J) AT TIME(40) = 0.53923169E-02

	J=2	3	4	5	6
2	0.35764449E-01	0.66466665E-01	0.47051156E-01	0.48473524E-03	0.41465758E-01
3	0.71327852E-01	0.13255854E-00	0.93818438E-01	0.94599222E-03	0.82931516E-01
4	0.70601285E-01	0.13119754E-00	0.92769282E-01	0.85610344E-03	0.82931516E-01
5	0.68914351E-01	0.12801585E-00	0.90253160E-01	0.63924025E-03	0.82931516E-01
6	0.65268493E-01	0.12108719E-00	0.84640805E-01	0.23541576E-03	0.82931516E-01
7	0.57332665E-01	0.10592006E-00	0.72158296E-01	0.90730857E-04	0.82931516E-01
8	0.38819811E-01	0.70459558E-01	0.43156781E-01	0.41887528E-02	0.82931516E-01
9	0.25486140E-05	0.16373292E-03	0.42287968E-02	0.32571247E-01	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.26857109E 01 TIME(40) = 0.53923169E-02
TOTAL FLUX (FLUXT) = 0.20819284E-05 TIME(40) = 0.53923169E-02

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (60) = 0.80884747E-02

	J=2	3	4	5	6
2	0.65058924E 02	0.52145334E 02	0.11096895E 02	-0.68250201E 02	-0.25000000E 03
3	0.64451001E 02	0.51620746E 02	0.10785875E 02	-0.68322758E 02	-0.25000000E 03
4	0.62327000E 02	0.49780778E 02	0.96800489E 01	-0.68593042E 02	-0.25000000E 03
5	0.57672172E 02	0.45720726E 02	0.71830125E 01	-0.69249194E 02	-0.25000000E 03
6	0.48366828E 02	0.37526801E 02	0.19886278E 01	-0.70727625E 02	-0.25000000E 03
7	0.30336285E 02	0.21432260E 02	-0.86481190E 01	-0.74027571E 02	-0.25000000E 03
8	-0.42992123E 01	-0.10143420E 02	-0.30944101E 02	-0.81852385E 02	-0.25000000E 03
9	-0.72617624E 02	-0.74528828E 02	-0.81913723E 02	-0.10507315E 03	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.22928122E 01 TIME(60) = 0.80884747E-02
TOTAL FLUX (FLUXT) = 0.62974814E-06 TIME(60) = 0.80884747E-02

FLUX DENSITY MATRIX B1(I,J) AT TIME(60) = 0.80884747E-02

	J=2	3	4	5	6
2	0.10750026E 01	0.10219438E 01	0.79410351E 00	-0.25944519E-00	-0.12912922E 01
3	0.10726532E 01	0.10196393E 01	0.79191195E 00	-0.26058387E-00	-0.12912922E 01
4	0.10643353E 01	0.10114572E 01	0.78404745E 00	-0.26481148E-00	-0.12912922E 01
5	0.10454895E 01	0.99283726E 00	0.76586188E 00	-0.27498307E-00	-0.12912922E 01
6	0.10050625E 01	0.95270347E 00	0.72602072E 00	-0.29743733E-00	-0.12912922E 01
7	0.91435279E 00	0.86220591E 00	0.63480040E 00	-0.34535298E-00	-0.12912922E 01
8	0.67379107E 00	0.62080295E 00	0.38723662E-00	-0.44812144E-00	-0.12912922E 01
9	-0.32524107E-00	-0.35237791E-00	-0.44887252E-00	-0.68533394E 00	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.22928122E 01 TIME(60) = 0.80884747E-02
TOTAL FLUX (FLUXT) = 0.62974814E-06 TIME(60) = 0.80884747E-02

THEORETICAL FORCE(I,J) AT TIME(60) = 0.80884747E-02

	J=2	3	4	5	6
2	0.28738201E-01	0.51942707E-01	0.31363516E-01	0.33478174E-02	0.41465758E-01
3	0.57225447E-01	0.10341742E-00	0.62381281E-01	0.67545367E-02	0.82931516E-01
4	0.56341376E-01	0.10176432E-00	0.61148413E-01	0.69754802E-02	0.82931516E-01
5	0.54363807E-01	0.98052040E-01	0.58344700E-01	0.75216380E-02	0.82931516E-01
6	0.50249808E-01	0.90285081E-01	0.52432254E-01	0.88001782E-02	0.82931516E-01
7	0.41581303E-01	0.73947329E-01	0.40084353E-01	0.11863885E-01	0.82931516E-01
8	0.22579847E-01	0.38336109E-01	0.14916033E-01	0.19975237E-01	0.82931516E-01
9	0.52611569E-02	0.12351447E-01	0.20042253E-01	0.46720230E-01	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.22928122E 01 TIME(60) = 0.80884747E-02
TOTAL FLUX (FLUXT) = 0.62974814E-06 TIME(60) = 0.80884747E-02

THEORETICAL FORCE(I,J) AT TIME(80) = 0.10784632E-01

	J=2	3	4	5	6
2	0.23130637E-01	0.40516707E-01	0.20020494E-01	0.91549833E-02	0.41465758E-01
3	0.45986420E-01	0.80525393E-01	0.39692173E-01	0.18387320E-01	0.82931516E-01
4	0.45028677E-01	0.78752437E-01	0.38469558E-01	0.18668772E-01	0.82931516E-01
5	0.42922333E-01	0.74845462E-01	0.35767052E-01	0.19329875E-01	0.82931516E-01
6	0.38615819E-01	0.66850295E-01	0.30278623E-01	0.20778389E-01	0.82931516E-01
7	0.29810452E-01	0.50548927E-01	0.19521182E-01	0.23971021E-01	0.82931516E-01
8	0.11871676E-01	0.18036178E-01	0.21488351E-02	0.31623027E-01	0.82931516E-01
9	0.11332951E-01	0.24492793E-01	0.31690815E-01	0.54914397E-01	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.20513958E 01 TIME(80) = 0.10784632E-01
TOTAL FLUX (FLUXT) = -0.34615710E-06 TIME(80) = 0.10784632E-01

FLUX DENSITY MATRIX BI(I,J) AT TIME(80) = 0.10784632E-01

	J=2	3	4	5	6
2	0.96443628E 00	0.90257192E 00	0.63445693E 00	-0.42903579E-00	-0.12912922E 01
3	0.96156699E 00	0.89973824E 00	0.63168737E 00	-0.42994110E-00	-0.12912922E 01
4	0.95150121E 00	0.88977818E 00	0.62188252E 00	-0.43321913E-00	-0.12912922E 01
5	0.92898010E 00	0.86742610E 00	0.59964100E 00	-0.44082303E-00	-0.12912922E 01
6	0.88114499E 00	0.81978773E 00	0.55171884E 00	-0.45704155E-00	-0.12912922E 01
7	0.77419264E 00	0.71286242E 00	0.44299906E-00	-0.49089999E-00	-0.12912922E 01
8	0.48856322E-00	0.42581602E-00	0.14697765E-00	-0.56383428E 00	-0.12912922E 01
9	-0.47734925E-00	-0.49621388E-00	-0.56443828E 00	-0.74300692E 00	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.20513958E 01 TIME(80) = 0.10784632E-01
TOTAL FLUX (FLUXT) = -0.34615710E-06 TIME(80) = 0.10784632E-01

MAGNETIC FIELD INTENSITY HI(I,J) AT TIME (80) = 0.10784632E-01

	J=2	3	4	5	6
2	0.39847560E 02	0.28241255E 02	-0.86852137E 01	-0.80315129E 02	-0.25000000E 03
3	0.39274509E 02	0.27744881E 02	-0.89835838E 01	-0.80387130E 02	-0.25000000E 03
4	0.37292323E 02	0.26022827E 02	-0.10029282E 02	-0.80648596E 02	-0.25000000E 03
5	0.33010077E 02	0.22782109E 02	-0.12341733E 02	-0.81259704E 02	-0.25000000E 03
6	0.24558990E 02	0.14838172E 02	-0.17060275E 02	-0.82585043E 02	-0.25000000E 03
7	0.83166630E 01	0.35128806E-00	-0.26594459E 02	-0.85451940E 02	-0.25000000E 03
8	-0.22780658E 02	-0.27970458E 02	-0.46475545E 02	-0.92130306E 02	-0.25000000E 03
9	-0.84287893E 02	-0.85914662E 02	-0.92188714E 02	-0.11218879E 03	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.20513958E 01 TIME(80) = 0.10784632E-01
TOTAL FLUX (FLUXT) = -0.34615710E-06 TIME(80) = 0.10784632E-01

MAGNETIC FIELD INTENSITY HI(I,J) AT TIME (100) = 0.13480790E-01

MAGNETIC FIELD INTENSITY H1(I,J) AT TIME (100) = 0.13480790E-01

	J=2	3	4	5	6
2	0.21705081E 02	0.11066926E 02	-0.22811322E 02	-0.88754325E 02	-0.25000000E 03
3	0.21169190E 02	0.10602032E 02	-0.23092266E 02	-0.88822591E 02	-0.25000000E 03
4	0.19324502E 02	0.89978344E 01	-0.24069635E 02	-0.89066953E 02	-0.25000000E 03
5	0.15366369E 02	0.55395380E 01	-0.26207998E 02	-0.89625838E 02	-0.25000000E 03
6	0.75987871E 01	-0.12978579E 01	-0.30529381E 02	-0.90811701E 02	-0.25000000E 03
7	-0.72812592E 01	-0.14556853E 02	-0.39210112E 02	-0.93332411E 02	-0.25000000E 03
8	-0.35769892E 02	-0.40476813E 02	-0.57302518E 02	-0.99157947E 02	-0.25000000E 03
9	-0.92339200E 02	-0.93752974E 02	-0.99211878E 02	-0.11693345E 03	-0.25000000E 03
10	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03	-0.25000000E 03

TOTAL FORCE (FORCT) = 0.18838430E 01 TIME(100) = 0.13480790E-01
TOTAL FLUX (FLUXT) = -0.11199690E-05 TIME(100) = 0.13480790E-01

FLUX DENSITY MATRIX B1(I,J) AT TIME(100) = 0.13480790E-01

	J=2	3	4	5	6
2	0.86388759E 00	0.79389273E 00	0.48820844E-00	-0.52795187E 00	-0.12912922E 01
3	0.86057902E 00	0.79061235E 00	0.48494940E-00	-0.52869688E 00	-0.12912922E 01
4	0.84902268E 00	0.77913842E 00	0.47349139E-00	-0.53135699E 00	-0.12912922E 01
5	0.82331590E 00	0.75355834E 00	0.44775204E-00	-0.53740162E 00	-0.12912922E 01
6	0.76893184E 00	0.69930118E 00	0.39273987E-00	-0.55004900E 00	-0.12912922E 01
7	0.64732505E 00	0.57758377E 00	0.26834635E-00	-0.57615411E 00	-0.12912922E 01
8	0.32006052E-00	0.24842809E-00	-0.67048423E-01	-0.63272183E 00	-0.12912922E 01
9	-0.56599189E 00	-0.58040980E 00	-0.63322252E 00	-0.77845405E 00	-0.12912922E 01
10	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01	-0.12912922E 01

TOTAL FORCE (FORCT) = 0.18838430E 01 TIME(100) = 0.13480790E-01
TOTAL FLUX (FLUXT) = -0.11199690E-05 TIME(100) = 0.13480790E-01

THEORETICAL FORCE(I,J) AT TIME(100) = 0.13480790E-01

	J=2	3	4	5	6
2	0.18559017E-01	0.31346868E-01	0.11854441E-01	0.13863061E-01	0.41465758E-01
3	0.36834265E-01	0.62176703E-01	0.23393401E-01	0.27804430E-01	0.82931516E-01
4	0.35851645E-01	0.60385093E-01	0.22301017E-01	0.28084927E-01	0.82931516E-01
5	0.33713473E-01	0.56485146E-01	0.19942318E-01	0.28727540E-01	0.82931516E-01
6	0.29406694E-01	0.48643972E-01	0.15343007E-01	0.30095617E-01	0.82931516E-01
7	0.20840845E-01	0.33184131E-01	0.71629496E-02	0.33020057E-01	0.82931516E-01
8	0.50948890E-02	0.61390605E-02	0.44717512E-03	0.39822282E-01	0.82931516E-01
9	0.15932756E-01	0.33509656E-01	0.39885331E-01	0.60279059E-01	0.82931516E-01
10	0.41465758E-01	0.82931516E-01	0.82931516E-01	0.82931516E-01	0.41465758E-01

TOTAL FORCE (FORCT) = 0.18838430E 01 TIME(100) = 0.13480790E-01
TOTAL FLUX (FLUXT) = -0.11199690E-05 TIME(100) = 0.13480790E-01

(KZ = 3 PREVIOUS DATA FOR DATA SET NO. 3)

```
HS   =   0.11573666E 06      HSO =  -0.09999999E 08
T    =   0.37365199E-05      TKM =   0.99999999E-02
K    =     8                 KO  =     8

STOP BECAUSE K IS GREATER THAN OR = TO KO

THIS IS A DECAY RUN
```

5.3 Discussion of Output Data and Results

For the sample problem considered, a magnetic field intensity of 250 amp-t/m was sufficient to saturate the core. Assuming uniform flux density, initial values of total force and flux can be calculated using equations (2-2) and (2-4); hence

$$(2\text{-}2) \qquad \phi = BA = (1.452)(0.004)(0.008) = 46.464 \times 10^{-6} \text{ webers}$$

$$(2\text{-}4) \qquad F = \frac{B^2A}{2\mu_0} = \frac{(1.452)^2(0.004)(0.008)}{(2)(4\pi \times 10^{-7})} = 26.843 \text{ newtons}$$

Total values computed by summing $B(I,J)h^2$ and FORCE(I,J) compare closely with those given above; thus one may conclude computations of total force and flux specified by subroutines FORCX and FLUX are valid approximations.

Distribution patterns are indicated by the print-out of Tapes 9 and 15. Tape 15 Output Data indicates values of field intensity and flux density for each point in the lattice of Fig. 12 in a convenient format. (Note physical position of H(I,J) in Fig. 12 and matrix output.) Examining these values as time increases, i.e., for TIME(0), TIME(5), TIME(10), . . ., TIME(100), we note that points near the boundary approach boundary conditions much more rapidly than those at the center of the core. Given sufficient time, all points will reach the same values as that on the outer boundary. Let us think of the field intensity at each grid point as a vector pointing in the z direction (see Fig. 7), whose magnitude represents the magnitude of field intensity at that point. If a membrane was stretched over the tips of the vectors, the surface generated as time increases represents the distribution pattern. We may visualize this distribution pattern by citing the following example. Suppose a soap bubble has been partially formed by exerting pressure behind a membrane across a tube of rectangular cross section. If we release the pressure sustaining the bubble, the membrane will return to its initial state. The changing shape of the bubble during the period of time in which it is decaying to its initial state, is analogous to the shape of the surface representing the flux decay distribution patterns. We should also note that force depends only on the magnitude of flux density, thus force is always positive. Furthermore, it will return to a value corresponding to $H = -250$ amp-t/m when all transients have decayed.

It is hoped that the solution given for the sample problem was useful in illustrating how one might obtain a solution for other equations of this type. An explicit expression for H and B was not found; however, the thesis does present details of how the Modified Euler Method of numerical integration may be used to establish a numerical solution which approximates the actual solution.

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The author wishes to express his thanks to Dr. Charles A Halijak for assistance given in preparation of this thesis.

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APPENDICES

APPENDIX A

Terminology and Formulae

I. Terminology

Most symbols are defined in the context of the thesis. Definition of variables in the program are listed in section 4.2B and in the "comments" section of the program listing. (Appendix C, Cards MG90000 - MG90106.) Those variables not defined or frequently used are given in this section for convenience. Overbarred symbols indicate vector quantities.

1. $\bar{i}, \bar{j}, \bar{k}$ - Unit vectors in cartesian co-ordinates
2. $\bar{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ - Del-operator[1]
3. $\bar{E} = (E_x, E_y, E_z)$ - Electric field intensity
4. $\bar{B} = (B_x, B_y, B_z)$ - Flux density
5. $\bar{H} = (H_x, H_y, H_z)$ - Magnetic field intensity
6. $\bar{D} = (D_x, D_y, D_z)$ - Displacement current density
7. $\bar{i} = (i_x, i_y, i_z)$ - Conduction current density
8. $\overline{dS} = (dS_x, dS_y, dS_z)$ - Differential surface
9. $\overline{dl} = (dl_x, dl_y, dl_z)$ - Differential length
10. σ = - Material conductivity
11. μ_o = - Permeability of free space
12. t - Time in seconds

---

[1]Parentheses denote vector components, e.g., $\bar{H} = (H_x, H_y, H_z) = H_x\bar{i} + H_y\bar{j} + H_z\bar{k}$.

13. h - Grid size in sections 3.1 and 3.2

14. D_i - $(dH/dt)_i$

15. P_i - $(\partial H/\partial t)_i$

16. I - Specifies row of matrix shown by Fig. 12

17. J - Specifies column of matrix shown by Fig. 12.

18. B(I,J) - Flux density at grid point I,J

19. H(I,J) - Magnetic field intensity at grid point I,J

20. $P_i(I,J)$ - $\partial H/\partial t$ at grid point I,J

21. FORCE(I,J) - Force at each grid point

22. FLUXT - Total flux

23. FORCT - Total force.

II. Formulae

1. Curl of $\bar{H}$

$$\bar{\nabla} \times \bar{H} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

2. Divergence of $\bar{H}$

$$\bar{\nabla} \cdot \bar{H} = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z}$$

3. Gradient of ϕ

$$\bar{\nabla}(\phi) = \frac{\partial \phi}{\partial x}\bar{i} + \frac{\partial \phi}{\partial y}\bar{j} + \frac{\partial \phi}{\partial z}\bar{k}$$

4. Other useful relations

a. $\overline{\nabla} \cdot \overline{\nabla} = \overline{\nabla}^2$

b. $\overline{\nabla} \times (\overline{\nabla} \cdot \overline{H}) = 0$

c. $\overline{\nabla} \cdot (\overline{\nabla} \times \overline{H}) = 0$

d. $\overline{\nabla} \times (\overline{\nabla} \times \overline{H}) = \quad (\overline{\nabla} \cdot \overline{H}) - \overline{\nabla}^2\overline{H}$

e. $\overline{A} \times (\overline{B} \times \overline{C}) - (\overline{A} \times \overline{B}) \times \overline{C}$

$= \overline{A}(\overline{B} \cdot \overline{C}) - (\overline{A} \cdot \overline{B})\overline{C}$

APPENDIX B

Units and Conversion Factors

Several systems of units used interchangeably are the MKS, CGS, and Mixed English. Conversion factors are given below.

	MKS		CGS		MIXED ENGLISH
Length	meter	x 10^2	= cm	x 0.3937	= inch
Time	second	x 1.0	= second	x 1.0	= second
Force	newton	x 10^3	= dyne	x 2248.0×10^{-6}	= pound
Voltage	volt	x 1.0	= volt	x 1.0	= volt
Current	ampere	x 1.0	= ampere	x 1.0	= ampere
Resistance	ohm	x 1.0	= ohm	x 1.0	= ohm
Capacitance	farad	x 1.0	= farad	x 1.0	= farad
Flux	weber	x 10^8	= maxwell	x 1.0	= maxwell
Flux density	weber/m^2	x 10^4	= gauss	x 6.45	= lines/in^2
MMF	amp-turn	x $.4\pi$	= gilbert	x $1/.4\pi$	= amp-turn
Magnetic intensity	amp-t/m	x $.004\pi$	= oersted	x 2.02	= amp-t/in

B-H Conversion Factors

B (lines/cm^2)	Gauss	x 6.450	= B (lines/in^2)	Maxwells/in^2
B (lines/in^2)		x 0.155	= B (lines/cm^2)	Gauss
H (oersteds)		x 2.020	= H (amp-t/in)	
H (amp-t/in)		x 0.495	= H (oersteds)	

APPENDIX C

Program Listing with Data Sets

The program is written in FORTRAN IV language for use with the IBM 7090/7094 IBSYS operating system. Processing is to be with the IBJOB FORTRAN IV compiler. All cards with a "$" sign in column 1 are system control cards used for all programs to be executed and are not unique to this program. All cards with a "C" in columns are comment cards and are not processed.

The program consists of a main program named EMEX and six subprograms FLUX, FORCX, FLUX, PPLT, INSE, and LOGI. The subprograms PPLT, INSE, and LOGI are provided to obtain graphical results while FLUD, FORCX, and FLUX are subroutines called by EMEX to perform specific calculations. One should also note that organization of the program is sequenced by numbers in columns 76-80 of the listing. These also aided in locating various sections within the program in the discussion of section 4.0.

The data input is on tape unit 5 and data output is on tape units 6, 7, 9, 15, and 16. Definition of symbols used is given in Appendix A, in the "comments" section of the program listing and in section 4.2B. If a larger matrix than 35 x 35 is desired, additional storage is required and the dimension statement must be altered accordingly.

```
$ID    SHURTZ
$EXECUTE        IBJOB                                                           MG90002
$IBJOB          MAP                                                             MG90004
$IBFTC EMEX                                                                     MG90006
C      SOURCE PROGRAM FOR THE 7090                                              MG90008
C      LATTICE COVERS ONLY 1/4 OF THE CROSS SECTION                             MG90010
C      HSO,TKM,AND KO STOP THE PROGRAM AND PRINT OUT THE LAST DATA              MG90012
C      HO    = INITIAL VALUE OF MAGNETIC FIELD INTENSITY                        MG90014
C      DHO   = PARTIAL DERIVATIVE OF HO WITH RESPECT TO TIME                    MG90016
C      H1    = NEXT VALUE OF FIELD INTENSITY AT A LATER TIME                    MG90018
C      DH1   = PARTIAL DERIVATIVE OF H1 WITH  RESPECT TO TIME                   MG90020
C      HS    = THE SUM OF ALL MATRIX TERMS                                      MG90022
C      FLUXT= TOTAL FLUXT                                                       MG90024
C      FORCT= TOTAL FORCE                                                       MG90026
C      E     = SUM OF THE ABSOLUTE VALUE OF EACH TERM ERROR                     MG90028
C      EO    = MAXIMUM MATRIX ERROR                                             MG90030
C      HSO   = MINIMUM SUM OF H1(I,J)                                           MG90032
C      BR    = RESIDUAL FLUX DENSITY                                            MG90034
C      HMAX = MAXIMUM FIELD INTENSITY APPLIED                                   MG90036
C      CUO   = CONSTANT USED TO ALLOW ALL UNITS                                 MG90038
C      TO    = INITIAL VALUE OF TIME IN SECONDS                                 MG90040
C      TKM   = SPECIFIES MAXIMUM TIME IN SECONDS                                MG90042
C      COND = MATERIAL CONDUCTIVITY                                             MG90044
C      AX    = CORE WIDTH                                                       MG90046
C      BY    = CORE HEIGHT                                                      MG90048
C      KZ    = NUMBER OF SETS OF DATA                                           MG90050
C      CDT   = CONSTANT MULTIPLIER FOR THE TIME INCREMENT                       MG90052
C      NI    = NO OF DIVISIONS IN THE X DIRECTION                               MG90054
C      NJ    = NO OF DIVISIONS IN THE Y DIRECTION                               MG90056
C      K     = NO OF INCREMENTS OR TIME SUBSCRIPT                               MG90058
C      KO    = MAXIMUM NO OF INCREMENTS PER RUN                                 MG90060
C      KI    = SPECIFIES INCREMENTS OF FORCT AND FLUXT FOR PPLT PLOTS           MG90062
C      KG    = SPECIFIES TIME INCREMENT FOR OUTPUT OF H1, B1, AND FORCE         MG90063
C      B2    = FLUX DENSITY FROM HYSTERESIS CURVE FOR C1,C2                     MG90064
C      H2    = FIELD INTENSITY CORRESPONDING TO B2 VALUE                        MG90066
C      B3    = FLUX DENSITY FROM HYSTERSIS CURVE FOR C1,C2                      MG90068
C      H3    = FIELD INTENSITY CORRESPONDING TO B3 VALUE                        MG90070
C      HIO   = MAXIMUM VALUE OF FIELD INTENSITY FOR B VS H                      MG90072
C      DHX   = INCREMENTS OF FIELD INTENSITY IN B VS H                          MG90074
C      I3    = MAXIMUM NO OF POINTS ON B VS H CURVE  (MAX I3 = 100)             MG90076
C      I4    = VALUE TO SPECIFY OUTPUT DATA AND FORMAT USED                     MG90078
C              USE 1 FOR TAPE 9 AND TAPE 15 OUTPUT   (ALL DATA)                 MG90080
C              USE 2 FOR TAPE 9 OUTPUT ONLY          (ALL DATA)                 MG90082
C              USE 3 FOR TAPE 15 OUTPUT (H1)                                    MG90084
C              USE 4 FOR TAPE 15 OUTPUT (B1)                                    MG90086
C              USE 5 FOR TAPE 15 OUTPUT (FORCE)                                 MG90088
C              USE 6 FOR TAPE 15 OUTPUT (H1,B1,FORCE)                           MG90090
C              USE 7 FOR TAPE 6 OUTPUT ONLY (ITERATIONS ON 9)                   MG90092
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C      I5    = VALUE USED FOR SPECIFYING BUILDUP OR DECAY
C                USE 0 OR 1 FOR BUILDUP
C                USE 2 FOR DECAY
C      CX    = DIVISION WIDTHS
C      C1    = CONSTANT IN MODIFIED FROELICH APPROXIMATION
C      C2    = CONSTANT IN MODIFIED FROELICH APPROXIMATION
C      DT    = TIME INCREMENT IN SECONDS
       DIMENSION HO(35,35),DHO(35,35),H1(35,35),DH1(35,35),
      1B1(35,35),FORCE(35,35),B(201),H(201),TFORC( 800),TFLUX( 800),
      2TIME( 800)
       COMMON I,J,UO,DX,T2,FLUXT,FORCE,BR,NX,NY,I5,B1,C1,C2
       REWIND 9
       REWIND15
       REWIND16
    48 READ (5,1) EO,HSO,BR,HMAX,CUO
       READ (5,1) TO,TKM,COND,AX,BY
     1 FORMAT (5E16.8)
       READ (5,2)CDT,KZ,NI,NJ,K,KO,KI,KG
     2 FORMAT ( E16.8,7I5)
   221 READ (5,213) B2,H2,B3,H3
   213 FORMAT (4E16.8)
   278 READ (5,279) HIO,DHX,I3,I4,I5
   279 FORMAT (2E16.8,3I5)
       C2=(B3-B2)*H3*H2/(((B2-BR)*H3)-((B3-BR)*H2))
       C1=(B2-BR)*(C2+H2)/H2
       HC=-(C2*BR/(C1+BR))
       H(1)=ABS(HIO)
       DO 624 I=1,I3
       IF (H(I)-HC) 621,622,622
   621 HD=ABS(H(I)-HC)
       HD2=HC+HD
       B(I)=-((C1*HD2/(C2+HD2))+BR)
       GO TO 623
   622 B(I)= (C1*H(I)/(C2+H(I)))+BR
   623 H(I+1)=H(I)-DHX
   624 CONTINUE
       HC=C2*BR/(C1+BR)
       I6=2*I3
       DO 627  I=I3,I6
       IF (H(I)-HC) 625,626,626
   625 HD1=-H(I)
       B(I)=-(C1*HD1/(C2+HD1)+BR)
       GO TO 627
   626 HX=H(I)-2.0*HC
       B(I)=(C1*HX/(C2+HX))+BR
   627 H(I+1)=H(I)+DHX
       WRITE ( 6,215)
       WRITE ( 9,215)
```

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      WRITE (15,215)                                                    MG90190
  215 FORMAT (1H1,8X,2HBR,14X,2HB2,14X,2HH2,14X,2HB3,14X,2HH3)          MG90192
      WRITE ( 6,216) BR,B2,H2,B3,H3                                     MG90194
      WRITE ( 9,216) BR,B2,H2,B3,H3                                     MG90196
      WRITE (15,216) BR,B2,H2,B3,H3                                     MG90198
  216 FORMAT (5E16.8)                                                   MG90200
      WRITE ( 6,280)                                                    MG90202
      WRITE ( 9,280)                                                    MG90204
      WRITE (15,280)                                                    MG90206
  280 FORMAT (1H0,7X,3HHI0,13X,3HDHX,9X,2HI3,3X,2HI4)                   MG90208
      WRITE ( 6,281) HI0,DHX,I3,I4                                      MG90210
      WRITE ( 9,281) HI0,DHX,I3,I4                                      MG90212
      WRITE (15,281) HI0,DHX,I3,I4                                      MG90214
  281 FORMAT (1H ,2E16.8,2I5)                                           MG90216
      WRITE ( 6,222) C1,C2                                              MG90218
      WRITE ( 9,222) C1,C2                                              MG90220
      WRITE (15,222) C1,C2                                              MG90222
  222 FORMAT (1H0,5HC1 = E16.8,6X,5HC2 = E16.8/)                        MG90224
      WRITE ( 6,217)                                                    MG90226
      WRITE ( 9,217)                                                    MG90228
      WRITE (15,217)                                                    MG90230
  217 FORMAT (1H0,8X,4HB(I),12X,4HH(I),11X,1HI/)                        MG90232
      DO 219 I=1,I6                                                     MG90234
      WRITE ( 6,218) B(I),H(I),I                                        MG90236
      WRITE ( 9,218) B(I),H(I),I                                        MG90238
      WRITE (15,218) B(I),H(I),I                                        MG90240
  218 FORMAT (1H ,2E16.8,3X,I5)                                         MG90242
  219 CONTINUE                                                          MG90244
      CALL PPLT(H,B,I6)                                                 MG90246
      WRITE ( 6,610)                                                    MG90248
  610 FORMAT (1H0,40X,45HFLUX DENSITY  VS  MAGNETIC FIELD INTENSITY    ) MG90250
      NN=1                                                              MG90252
      I1=NJ+1                                                           MG90254
      I2=NI+1                                                           MG90256
      NY=NJ+2                                                           MG90258
      NX=NI+2                                                           MG90260
      READ (5,1) ((HO(I,J),J=2,NX),I=2,NY)                              MG90262
      LI4=1                                                             MG90264
      PI=3.14159                                                        MG90266
      U0=4.0*PI*CU0                                                     MG90268
      X=COND*C1*C2                                                      MG90270
      T=T0                                                              MG90272
      ANI=NI                                                            MG90274
      CX=AX/(2.0*ANI)                                                   MG90276
      ANJ=BY/(2.0*CX)                                                   MG90278
      IF (I5-1) 600,600,602                                             MG90280
  600 WRITE ( 6,601)                                                    MG90282
      WRITE ( 9,601)                                                    MG90284
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      WRITE (15,601)                                                    MG90286
  601 FORMAT (1H1,25HTHIS IS A BUILDUP RUN                         )    MG90288
      GO TO 604                                                         MG90290
  602 WRITE ( 6,603)                                                    MG90292
      WRITE ( 9,603)                                                    MG90294
      WRITE (15,603)                                                    MG90296
  603 FORMAT (1H1,25HTHIS IS A DECAY RUN                           )    MG90298
  604 WRITE ( 6,399)                                                    MG90300
      WRITE ( 9,399)                                                    MG90302
      WRITE (15,399)                                                    MG90304
  399 FORMAT (1H ,50HLATTICE COVERS ONLY 1/4 OF THE CROSS SECTION AREA )MG90306
      WRITE (6,56) NI,NJ                                                MG90308
      WRITE (9,56) NI,NJ                                                MG90310
      WRITE (15,56) NI,NJ                                               MG90312
   56 FORMAT (1H ,28HLATTICE SIZE = NI  BY  NJ =  I3,4H BY  I3,         MG90314
     15X,20H(MAX = 48 BY 48)                                       )    MG90316
      WRITE ( 6,51) NI,NJ,ANJ,CDT                                       MG90318
      WRITE (9,51) NI,NJ,ANJ,CDT                                        MG90320
      WRITE (15,51) NI,NJ,ANJ,CDT                                       MG90322
   51 FORMAT (1H , 5HNI =   I5,4X,5HNJ =   I5,4X,6HANJ = E16.8,         MG90324
     14X,6HCDT =  E16.8)                                                MG90326
      ANJ1=ANJ+0.5                                                      MG90328
      NJ1=ANJ1                                                          MG90330
      IX=ABS (NJ1-NJ)                                                   MG90332
      IF(IX-2) 58,58,59                                                 MG90334
   59 WRITE (6,60)                                                      MG90336
      WRITE (9,60)                                                      MG90338
      WRITE (15,60)                                                     MG90340
   60 FORMAT (1H0, 49HTHE VALUE OF NJ READ IN IS NOT CORRECT.  RELOAD  )MG90342
   61 STOP                                                              MG90344
   58 CONTINUE                                                          MG90346
      DX=CX**2                                                          MG90348
      DT=(CDT*X*DX)/((C2*HMAX)**2)                                      MG90350
      DO 49 I=1,NY                                                      MG90352
      DO 49 J=1,NX                                                      MG90354
      DHO(I,J)=0.0                                                      MG90356
      H1 (I,J)=0.0                                                      MG90358
      DH1(I,J)=0.0                                                      MG90360
      B1 (I,J)=0.0                                                      MG90362
      FORCE(I,J)=0.0                                                    MG90364
   49 CONTINUE                                                          MG90366
      WRITE (6,39)                                                      MG90368
      WRITE (9,39)                                                      MG90370
      WRITE (15,39)                                                     MG90372
   39 FORMAT (1H0,7X,2HCX,14X,2HC1,14X,2HC2,14X,2HDT)                   MG90374
      WRITE ( 6,40) CX,C1,C2,DT                                         MG90376
      WRITE ( 9,40) CX,C1,C2,DT                                         MG90378
      WRITE (15,40) CX,C1,C2,DT                                         MG90380
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 40 FORMAT (1H ,4E16.8)                                                MG90382
    WRITE ( 6,400)                                                     MG90384
    WRITE ( 9,400)                                                     MG90386
    WRITE (15,400)                                                     MG90388
400 FORMAT ( 1H ,7X,2HEO,13X,3HHSO,14X,2HBR,13X,4HHMAX,12X,3HCUO)       MG90390
    WRITE ( 6,401)  EO,HSO,BR,HMAX,CUO                                 MG90392
    WRITE ( 9,401)  EO,HSO,BR,HMAX,CUO                                 MG90394
    WRITE (15,401)  EO,HSO,BR,HMAX,CUO                                 MG90396
401 FORMAT (1H ,5E16.8)                                                MG90398
    WRITE ( 6,402)                                                     MG90400
    WRITE ( 9,402)                                                     MG90402
    WRITE (15,402)                                                     MG90404
402 FORMAT (1H ,7X,2HTO,13X,3HTKM,13X,4HCOND,13X,2HAX,14X,2HBY)         MG90406
    WRITE ( 6,401)  TO,TKM,COND,AX,BY                                  MG90408
    WRITE ( 9,401)  TO,TKM,COND,AX,BY                                  MG90410
    WRITE (15,401)  TO,TKM,COND,AX,BY                                  MG90412
    WRITE ( 6,403)                                                     MG90414
    WRITE ( 9,403)                                                     MG90416
    WRITE (15,403)                                                     MG90418
403 FORMAT (1H ,7X,3HCDT,9X,2HKZ,3X,2HNI,3X,2HNJ,4X,1HK,3X,2HKO,3X,     MG90420
   12HKI,3X,2HKG)                                                      MG90422
    WRITE ( 6,404) CDT,KZ,NI,NJ,K,KO,KI,KG                             MG90424
    WRITE ( 9,404) CDT,KZ,NI,NJ,K,KO,KI,KG                             MG90426
    WRITE (15,404) CDT,KZ,NI,NJ,K,KO,KI,KG                             MG90428
404 FORMAT (1H , E16.8,7I5)                                            MG90430
    WRITE ( 6,405)                                                     MG90432
    WRITE ( 9,405)                                                     MG90434
    WRITE (15,405)                                                     MG90436
405 FORMAT (1H ,7X,2HB2,14X,2HH2,14X,2HB3,14X,2HH3)                     MG90438
    WRITE ( 6,406)  B2,H2,B3,H3                                        MG90440
    WRITE ( 9,406)  B2,H2,B3,H3                                        MG90442
    WRITE (15,406)  B2,H2,B3,H3                                        MG90444
406 FORMAT (1H ,4E16.8)                                                MG90446
    WRITE ( 6,407)                                                     MG90448
    WRITE ( 9,407)                                                     MG90450
    WRITE (15,407)                                                     MG90452
407 FORMAT (1H ,6X,3HHIO,13X,3HDHX,10X,2HI3,3X,2HI4,3X,2HI5)            MG90454
    WRITE ( 6,408)  HIO,DHX,I3,I4,I5                                   MG90456
    WRITE ( 9,408)  HIO,DHX,I3,I4,I5                                   MG90458
    WRITE (15,408)  HIO,DHX,I3,I4,I5                                   MG90460
408 FORMAT (1H ,2E16.8,3I5)                                            MG90462
    WRITE ( 6,500)                                                     MG90464
    WRITE ( 9,500)                                                     MG90466
    WRITE (15,500)                                                     MG90468
500 FORMAT (1H0,50HEO    = MAXIMUM MATRIX ERROR                      ) MG90470
    WRITE ( 6,501)                                                     MG90472
    WRITE ( 9,501)                                                     MG90474
    WRITE (15,501)                                                     MG90476
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501 FORMAT (1H ,50HHSO   = MINIMUM SUM OF H1(I,J)                      ) MG90478
    WRITE ( 6,502)                                                       MG90480
    WRITE ( 9,502)                                                       MG90482
    WRITE (15,502)                                                       MG90484
502 FORMAT (1H ,50HBR    = RESIDUAL FLUX DENSITY                       ) MG90486
    WRITE ( 6,503)                                                       MG90488
    WRITE ( 9,503)                                                       MG90490
    WRITE (15,503)                                                       MG90492
503 FORMAT (1H ,50HHMAX = MAXIMUM FIELD INTENSITY APPLIED              ) MG90494
    WRITE ( 6,504)                                                       MG90496
    WRITE ( 9,504)                                                       MG90498
    WRITE (15,504)                                                       MG90500
504 FORMAT (1H ,50HCUO   = CONSTANT USED TO ALLOW ALL UNITS            ) MG90502
    WRITE ( 6,505)                                                       MG90504
    WRITE ( 9,505)                                                       MG90506
    WRITE (15,505)                                                       MG90508
505 FORMAT (1H ,50HTO    = INITIAL VALUE OF TIME IN SECONDS            ) MG90510
    WRITE ( 6,506)                                                       MG90512
    WRITE ( 9,506)                                                       MG90514
    WRITE (15,506)                                                       MG90516
506 FORMAT (1H ,50HTKM   = SPECIFIES MAXIMUM TIME                      ) MG90518
    WRITE ( 6,507)                                                       MG90520
    WRITE ( 9,507)                                                       MG90522
    WRITE (15,507)                                                       MG90524
507 FORMAT (1H ,50HCOND = MATERIAL CONDUCTIVITY                        ) MG90526
    WRITE ( 6,508)                                                       MG90528
    WRITE ( 9,508)                                                       MG90530
    WRITE (15,508)                                                       MG90532
508 FORMAT (1H ,50HAX    = CORE WIDTH                                  ) MG90534
    WRITE ( 6,509)                                                       MG90536
    WRITE ( 9,509)                                                       MG90538
    WRITE (15,509)                                                       MG90540
509 FORMAT (1H ,50HBY    = CORE HEIGHT                                 ) MG90542
    WRITE ( 6,510)                                                       MG90544
    WRITE ( 9,510)                                                       MG90546
    WRITE (15,510)                                                       MG90548
510 FORMAT (1H ,50HKZ    = NO OF SETS OF DATA                          ) MG90550
    WRITE ( 6,511)                                                       MG90552
    WRITE ( 9,511)                                                       MG90554
    WRITE (15,511)                                                       MG90556
511 FORMAT (1H ,50HCDT   = CONSTANT MULTIPLIER FOR THE TIME INCREMENT ) MG90558
    WRITE ( 6,512)                                                       MG90560
    WRITE ( 9,512)                                                       MG90562
    WRITE (15,512)                                                       MG90564
512 FORMAT (1H ,50HMI    = NO OF DIVISIONS IN THE X DIRECTION          ) MG90566
    WRITE ( 6,513)                                                       MG90568
    WRITE ( 9,513)                                                       MG90570
    WRITE (15,513)                                                       MG90572
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 513 FORMAT (1H ,50HNJ    = NO OF DIVISIONS IN THE Y DIRECTION           ) MG90574
     WRITE ( 6,514)                                                        MG90576
     WRITE ( 9,514)                                                        MG90578
     WRITE (15,514)                                                        MG90580
 514 FORMAT (1H ,50HK     = NO OF INCREMENTS OR TIME SUBSCRIPT           ) MG90582
     WRITE ( 6,515)                                                        MG90584
     WRITE ( 9,515)                                                        MG90586
     WRITE (15,515)                                                        MG90588
 515 FORMAT (1H ,50HKO    = MAXIMUM NO OF INCREMENTS PER RUN             ) MG90590
     WRITE ( 6,634)                                                        MG90592
     WRITE ( 9,634)                                                        MG90594
     WRITE (15,634)                                                        MG90596
 634 FORMAT (1H ,60HKI    = SPECIFIES INCREMENTS FOR FORCT AND FLUXT PLO MG90598
    1TS            )                                                       MG90599
     WRITE ( 6,1580)                                                       MG90600
     WRITE ( 9,1580)                                                       MG90601
     WRITE (15,1580)                                                       MG90602
1580 FORMAT(1H ,50HKG   = SPECIFIES INCREMENT FOR OUTPUT H1,B1,FORCE   ) MG90603
     WRITE ( 6,516)                                                        MG90604
     WRITE (15,516)                                                        MG90605
     WRITE (15,516)                                                        MG90606
 516 FORMAT (1H ,50HB2    = FLUX DENSITY FROM HYSTERESIS CURVE           ) MG90608
     WRITE ( 6,517)                                                        MG90610
     WRITE ( 9,517)                                                        MG90612
     WRITE (15,517)                                                        MG90614
 517 FORMAT (1H ,50HH2    = FIELD INTENSITY CORRESPONDING TO B2 VALUE    ) MG90616
     WRITE ( 6,518)                                                        MG90618
     WRITE ( 9,518)                                                        MG90620
     WRITE (15,518)                                                        MG90622
 518 FORMAT (1H ,50HB3    = FLUX DENSITY FROM HYSTERESIS CURVE           ) MG90624
     WRITE ( 6,519)                                                        MG90626
     WRITE ( 9,519)                                                        MG90628
     WRITE (15,519)                                                        MG90630
 519 FORMAT (1H ,50HH3    = FIELD INTENSITY CORRESPONDING TO B3 VALUE    ) MG90632
     WRITE ( 6,520)                                                        MG90634
     WRITE ( 9,520)                                                        MG90636
     WRITE (15,520)                                                        MG90638
 520 FORMAT (1H ,50HHIO   = MAXIMUM VALUE OF FIELD INTENSITY FOR B VS H) MG90640
     WRITE ( 6,521)                                                        MG90642
     WRITE ( 9,521)                                                        MG90644
     WRITE (15,521)                                                        MG90646
 521 FORMAT (1H ,50HDHX   = INCREMENTS OF FIELD INTENSITY IN B VS H      ) MG90648
     WRITE ( 6,522)                                                        MG90650
     WRITE ( 9,522)                                                        MG90652
     WRITE (15,522)                                                        MG90654
 522 FORMAT (1H ,50HI3    = MAXIMUM NO OF POINTS ON B VS H CURVE         ) MG90656
     WRITE ( 6,523)                                                        MG90658
     WRITE ( 9,523)                                                        MG90660
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      WRITE (15,523)                                                    MG90662
  523 FORMAT (1H ,45HI4    = USED TO SPECIFY OUTPUT FORMAT          /8X, MG90664
     150H USE 1 FOR OUTPUT TAPES 9 AND 15  (ALL DATA)               /8X, MG90666
     250H USE 2 FOR OUTPUT TAPE  9 ONLY    (ALL DATA)               /8X, MG90668
     350H USE 3 FOR OUTPUT TAPE 15 ONLY    (H1)                     /8X, MG90670
     450H USE 4 FOR OUTPUT TAPE 15 ONLY    (B1)                     /8X, MG90672
     550H USE 5 FOR OUTPUT TAPE 15 ONLY    (FORCE)                  /8X, MG90674
     650H USE 6 FOR OUTPUT TAPE 15 ONLY    (H1,B1,FORCE)            /8X, MG90676
     751H USE 7 FOR OUTPUT TAPE  6 ONLY    (ITERATIONS ON 9)           ) MG90678
      WRITE ( 6,577)                                                    MG90680
      WRITE ( 9,577)                                                    MG90682
      WRITE (15,577)                                                    MG90684
  577 FORMAT (1H ,45HI5    = VALUE TO SPECIFY BUILDUP OR DECAY      /8X, MG90686
     150H USE 0 OR 1 FOR BUILDUP                                    /8X, MG90688
     250H USE 2 FOR DECAY                                              ) MG90690
      WRITE ( 6,524)                                                    MG90692
      WRITE ( 9,524)                                                    MG90694
      WRITE (15,524)                                                    MG90696
  524 FORMAT (1H ,50HCX    = DIVISION WIDTHS                           ) MG90698
      WRITE ( 6,525)                                                    MG90700
      WRITE ( 9,525)                                                    MG90702
      WRITE (15,525)                                                    MG90704
  525 FORMAT (1H ,50HC1    = CONSTANT IN MODIFIED FROELICH APPROXIMATION) MG90706
      WRITE ( 6,526)                                                    MG90708
      WRITE ( 9,526)                                                    MG90710
      WRITE (15,526)                                                    MG90712
  526 FORMAT (1H ,50HC2    = CONSTANT IN MODIFIED FROELICH APPROXIMATION) MG90714
      WRITE ( 6,527)                                                    MG90716
      WRITE ( 9,527)                                                    MG90718
      WRITE (15,527)                                                    MG90720
  527 FORMAT (1H ,50HDT    = TIME INCREMENT IN SECONDS                 ) MG90722
      WRITE(9,41) K                                                     MG90724
   41 FORMAT (1H1,4X,10HFORCE(I,J),7X,7HBO(I,J),9X,7HHO(I,J),9X,5HTIME( MG90726
     1I4,1H),7X,7H(I , J)/)                                             MG90728
      WRITE ( 6,88)                                                     MG90730
   88 FORMAT (1H1,2X,2H K,2X,19HTOTAL FORCE (FORCT),11X,10HTIME(    K), MG90732
     110X,19HTOTAL FLUX (FLUXT)/)                                       MG90734
      DO 117 I=2,NY                                                     MG90736
      DO 117 J=2,NX                                                     MG90738
      CALL FLUD (HO)                                                    MG90740
      T2=B1(I,J)                                                        MG90742
      CALL FORCX                                                        MG90744
  117 CONTINUE                                                          MG90746
      DO 119 I=2,NY                                                     MG90748
      DO 119 J=2,NX                                                     MG90750
      WRITE (9,42) FORCE(I,J),B1(I,J),HO(I,J),T,I,J                     MG90752
   42 FORMAT (1H ,4E16.8,3X,2I4)                                        MG90754
  119 CONTINUE                                                          MG90756
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  120 FORCT =0.0                                                          MG90758
      FLUXT=0.0                                                           MG90760
      DO 121 I=2,NY                                                       MG90762
      DO 121 J=2,NX                                                       MG90764
      FORCT=FORCT+FORCE(I,J)                                              MG90766
      T2=B1(I,J)                                                          MG90768
      CALL FLUX                                                           MG90770
  121 CONTINUE                                                            MG90772
      WRITE (9,200) FORCT,K,T                                             MG90774
  200 FORMAT (1H0,22HTOTAL FORCE (FORCT) = E16.8,4X,5HTIME( I4,4H) =    E MG90776
     116.8)                                                               MG90778
      WRITE (9,212) FLUXT,K,T                                             MG90780
  212 FORMAT (1H ,22HTOTAL FLUX  (FLUXT) = E16.8,4X,5HTIME(I4,4H) =     E MG90782
     116.8/)                                                              MG90784
      WRITE ( 9,1600)                                                     MG90785
 1600 FORMAT (1H1)                                                        MG90786
      WRITE ( 6,89) K,FORCT,T,FLUXT                                       MG90787
   89 FORMAT (1H ,I4,2X,E16.8,11X,E16.8,8X,E16.8)                         MG90788
      WRITE (15,290) K,T                                                  MG90790
  290 FORMAT (1H1,42HMAGNETIC FIELD INTENSITY HO(I,J) AT TIME (I4,4H) =   MG90792
     1E16.8)                                                              MG90794
      WRITE (15,271)                                                      MG90796
      DO 295 I=2,NY                                                       MG90798
      IF (NX-8) 491,491,492                                               MG90800
  491 WRITE (15,272) I,(HO(I,J),J=2,NX)                                   MG90802
      GO TO 295                                                           MG90804
  492 WRITE (15,272) I,(HO(I,J),J=2,8 )                                   MG90806
      WRITE (15,375) (HO(I,J),J=9,NX)                                     MG90808
  295 CONTINUE                                                            MG90810
      WRITE (15,200) FORCT,K,T                                            MG90812
      WRITE (15,212) FLUXT,K,T                                            MG90814
      WRITE (15,270) K,T                                                  MG90816
      WRITE (15,271)                                                      MG90818
      DO 275 I=2,NY                                                       MG90820
      IF (NX-8) 489,489,490                                               MG90822
  489 WRITE (15,272) I,(B1(I,J),J=2,NX)                                   MG90824
      GO TO 275                                                           MG90826
  490 WRITE (15,272) I,(B1(I,J),J=2,8 )                                   MG90828
      WRITE (15,375) (B1(I,J),J=9,NX)                                     MG90830
  275 CONTINUE                                                            MG90832
      WRITE (15,200) FORCT,K,T                                            MG90834
      WRITE (15,212) FLUXT,K,T                                            MG90836
  535 WRITE (15,536) K,T                                                  MG90838
  536 FORMAT (1H1,31HTHEORETICAL FORCE(I,J) AT TIME( I4, 4H) =    E16.8)  MG90840
      WRITE (15,271)                                                      MG90842
      DO 537 I=2,NY                                                       MG90844
      IF (NX-8) 538,538,539                                               MG90846
  538 WRITE (15,272) I,(FORCE(I,J),J=2,NX)                                MG90848
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      GO TO 537                                                 MG90850
  539 WRITE (15,272) I,(FORCE(I,J),J=2,8)                       MG90852
      WRITE (15,375) (FORCE(I,J),J=9,NX)                        MG90854
  537 CONTINUE                                                  MG90856
      WRITE (15,200) FORCT,K,T                                  MG90858
      WRITE (15,212) FLUXT,K,T                                  MG90860
      IF (K-(K/KI)*KI) 631,630,631                              MG90862
  630 KK=(K+KI)/KI                                              MG90864
      TFORC (KK) = FORCT                                        MG90866
      TFLUX (KK) = FLUXT                                        MG90868
      TIME  (KK) = T                                            MG90870
      K1=K                                                      MG90872
  631 CONTINUE                                                  MG90874
      T=T+DT                                                    MG90876
      K=K+1                                                     MG90878
    4 DO 16  I=2,I1                                             MG90880
      DO 16  J=2,I2                                             MG90882
      IF(I-2) 6,6,5                                             MG90884
    5 IF(J-2) 10,10,30                                          MG90886
    6 IF(J-2) 8,8,7                                             MG90888
    7 HO(I-1,J)=HO(I+1,J)                                       MG90890
      GO TO 30                                                  MG90892
    8 HO(I,J-1)=HO(I,J+1)                                       MG90894
      HO(I-1,J)=HO(I+1,J)                                       MG90896
      GO TO 30                                                  MG90898
   10 HO(I,J-1)=HO(I,J+1)                                       MG90900
   30 X1=(C2*HO(I,J))**2                                        MG90902
      X2=(HO(I,J+1)-2.0*HO(I,J)+HO(I,J-1))/DX                   MG90904
      X3=(HO(I+1,J)-2.0*HO(I,J)+HO(I-1,J))/DX                   MG90906
   16 DHO(I,J)=X1*(X2+X3)/X                                     MG90908
      DO 17 I=2,I1                                              MG90910
      DO 17 J=2,I2                                              MG90912
   17 H1(I,J)=HO(I,J)+DHO(I,J)*DT                               MG90914
      I=NJ+2                                                    MG90916
      DO 9 J=2,NX                                               MG90918
    9 H1(I,J)=HO(I,J)                                           MG90920
      J=NI+2                                                    MG90922
      DO 90 I=2,NY                                              MG90924
   90 H1(I,J)=HO(I,J)                                           MG90926
  557 WRITE ( 9,558) NN,K,T                                     MG90936
  558 FORMAT (1H0,5HNN = I4,5X,5HTIME(I4,4H) =  E16.8)          MG90938
  560 CONTINUE                                                  MG90940
   18 DO 32 I=2,I1                                              MG90942
      DO 32 J=2,I2                                              MG90944
      IF(I-2) 20,20,19                                          MG90946
   19 IF(J-2) 23,23,24                                          MG90948
   20 IF(J-2) 22,22,21                                          MG90950
   21 H1(I-1,J)=H1(I+1,J)                                       MG90952
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```
      GO TO 24                                                         MG90954
   22 H1(I,J-1)=H1(I,J+1)                                              MG90956
      H1(I-1,J)=H1(I+1,J)                                              MG90958
      GO TO 24                                                         MG90960
   23 H1(I,J-1)=H1(I,J+1)                                              MG90962
   24 X4=(C2+H1(I,J))**2                                               MG90964
      X5=(H1(I,J+1)-2.0*H1(I,J)+H1(I,J-1))/DX                          MG90966
      X6=(H1(I+1,J)-2.0*H1(I,J)+H1(I-1,J))/DX                          MG90968
   32 DH1(I,J)=X4*(X5+X6)/X                                            MG90970
      NN=NN+1                                                          MG90972
      E=0.0                                                            MG90974
   33 DO 34  I=2,I1                                                    MG90976
      DO 34  J=2,I2                                                    MG90978
      X7=(DHO(I,J)+DH1(I,J))/2.0                                       MG90980
      T1=HO(I,J)+X7*DT                                                 MG90982
      E=E+ABS (T1-H1(I,J))                                             MG90984
      H1(I,J)=T1                                                       MG90986
      CALL FLUD (H1)                                                   MG90988
      T2=B1(I,J)                                                       MG90990
      CALL FORCX                                                       MG90992
   34 CONTINUE                                                         MG90994
      I=NY                                                             MG90996
      DO 201 J=2,NX                                                    MG90998
      CALL FLUD (HO)                                                   MG91000
      T2=B1(I,J)                                                       MG91002
      CALL FORCX                                                       MG91004
  201 CONTINUE                                                         MG91006
      J=NX                                                             MG91008
      DO 202 I=2,NY                                                    MG91010
      CALL FLUD (HO)                                                   MG91012
      T2=B1(I,J)                                                       MG91014
      CALL FORCX                                                       MG91016
  202 CONTINUE                                                         MG91018
      IF (NN-3) 204,204,211                                            MG91020
  211 DE=E1-E                                                          MG91022
      IF (DE-(0.01*E1)) 205,204,204                                    MG91024
  204 KN=1                                                             MG91026
      GO TO 206                                                        MG91028
  205 KN=2                                                             MG91030
  206 GO TO (106,207),KN                                               MG91032
  207 WRITE (9,208)                                                    MG91034
      WRITE (6,208)                                                    MG91036
      WRITE (15,208)                                                   MG91038
  208 FORMAT (1H0,72HTHE ERROR DOES NOT CONVERGE FOR THIS SET OF DATA. MG91040
     1PROCESSING IS HALTED.)                                           MG91042
      WRITE (6,57) NN,E                                                MG91044
      WRITE (9,57) NN,E                                                MG91046
      WRITE (15,57) NN,E                                               MG91048
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      GO TO 46                                                          MG91050
  106 WRITE (9,224) NN,E                                                MG91052
  224 FORMAT (1H ,5HNN = I4,4X,4HE =  E16.8)                            MG91054
   57 FORMAT (1H ,5HNN = I4,4X,4HE = E16.8/)                            MG91056
      E1=E                                                              MG91058
      IF(E-E0) 81,81,18                                                 MG91060
   81 FORCT=0.0                                                         MG91062
      FLUXT=0.0                                                         MG91064
      DO 83 I=2,NY                                                      MG91066
      DO 83 J=2,NX                                                      MG91068
      FORCT=FORCT+FORCE(I,J)                                            MG91070
      T2=B1(I,J)                                                        MG91072
      CALL FLUX                                                         MG91074
   83 CONTINUE                                                          MG91076
      WRITE ( 6,89) K,FORCT,T,FLUXT                                     MG91077
      IF (I4-2) 31,31,550                                               MG91078
   31 IF (LI4-1) 1578,1578,1579                                         MG91079
 1578 IF(K-(K/KG)*KG) 550,1579,550                                      MG91080
 1579 CONTINUE                                                          MG91081
      WRITE (9,123)  K                                                  MG91082
  123 FORMAT (1H1,4X,10HFORCE(I,J),7X,7HB1(I,J),9X,7HH1(I,J),9X,5HTIME( MG91084
     1I4,1H),7X,7H(I  , J)/)                                            MG91086
   35 DO 80 I=2,NY                                                      MG91088
      DO 80 J=2,NX                                                      MG91090
      WRITE (9,42) FORCE(I,J),B1(I,J),H1(I,J),T,I,J                     MG91092
   80 CONTINUE                                                          MG91094
      WRITE (9,200) FORCT,K,T                                           MG91096
      WRITE (9,212) FLUXT,K,T                                           MG91097
      WRITE ( 9,1600)                                                   MG91098
  550 CONTINUE                                                          MG91099
      IF (LI4-1) 1590,1590,1591                                         MG91100
 1590 IF (K-(K/KG)*KG) 555,1591,555                                     MG91101
 1591 CONTINUE                                                          MG91102
      GO TO (552,555,552,553,554,552,555), I4                           MG91103
  552 WRITE (15,296) K,T                                                MG91104
  296 FORMAT (1H1,42HMAGNETIC FIELD INTENSITY H1(I,J) AT TIME (I4,4H) = MG91106
     1E16.8)                                                            MG91108
      WRITE (15,271)                                                    MG91110
      DO 297 I=2,NY                                                     MG91112
      IF (NX-8  ) 389,389,390                                           MG91114
  389 WRITE (15,272)  I,(H1(I,J),J=2,NX)                                MG91116
      GO TO 297                                                         MG91118
  390 WRITE (15,272) I,(H1(I,J),J=2,8)                                  MG91120
      WRITE (15,375) (H1(I,J),J=9,NX)                                   MG91122
  297 CONTINUE                                                          MG91124
      WRITE (15,200) FORCT,K,T                                          MG91126
      WRITE (15,212) FLUXT,K,T                                          MG91128
      GO TO (553,555,555,553,554,553,555), I4                           MG91130
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553 WRITE (15,270) K,T                                                    MG91132
270 FORMAT (1H1, 36HFLUX DENSITY MATRIX B1(I,J) AT TIME(I4,4H) =         MG91134
   1E16.8)                                                                MG91136
    WRITE (15,271)                                                        MG91138
271 FORMAT (1H0,13X,3HJ=2,15X,1H3,15X,1H4,15X,1H5,15X,1H6)                MG91140
    DO 273 I=2,NY                                                         MG91144
    IF (NX-8  ) 379,379,380                                               MG91146
379 WRITE (15,272) I,(B1(I,J),J=2,NX)                                     MG91148
272 FORMAT(1H ,I4,1X,7E16.8)                                              MG91150
    GO TO 273                                                             MG91152
380 WRITE (15,272)  I,(B1(I,J),J=2,8)                                     MG91154
    WRITE (15,375) (B1(I,J),J=9,NX)                                       MG91156
375 FORMAT (1H ,5X,7E16.8)                                                MG91158
273 CONTINUE                                                              MG91160
    WRITE (15,200) FORCT,K,T                                              MG91162
    WRITE (15,212) FLUXT,K,T                                              MG91164
    GO TO (554,555,555,555,555,554,555) , I4                              MG91166
554 WRITE (15,536) K,T                                                    MG91168
    WRITE (15,271)                                                        MG91170
    DO 547 I=2,NY                                                         MG91172
    IF (NX-8) 548,548,549                                                 MG91174
548 WRITE (15,272) I,(FORCE(I,J),J=2,NX)                                  MG91176
    GO TO 547                                                             MG91178
549 WRITE (15,272) I,(FORCE(I,J),J=2,8)                                   MG91180
    WRITE (15,375) (FORCE(I,J),J=9,NX)                                    MG91182
547 CONTINUE                                                              MG91184
    WRITE (15,200) FORCT,K,T                                              MG91186
    WRITE (15,212) FLUXT,K,T                                              MG91188
    IF(LI4-1)  555,555,635                                                MG91190
555 CONTINUE                                                              MG91192
    IF (K-(K/KI)*KI) 633,632,633                                          MG91194
632 KK=(K +KI)/KI                                                         MG91196
    TFORC (KK) = FORCT                                                    MG91198
    TFLUX (KK) = FLUXT                                                    MG91200
    TIME  (KK) = T                                                        MG91202
633 CONTINUE                                                              MG91204
 86 HS=0.0                                                                MG91206
    DO 43 I=2,NY                                                          MG91208
    DO 43 J=2,NX                                                          MG91210
 43 HS=HS+H1(I,J)                                                         MG91212
    IF (I5-1)  572,572,571                                                MG91214
572 IF (HS-HSO)  44,46,46                                                 MG91216
571 IF (HS-HSO)  46,46,44                                                 MG91218
 44 IF (T-TKM) 276,46,46                                                  MG91220
276 IF (K-KO)  277,46,46                                                  MG91222
277 T=T+DT                                                                MG91224
    K=K+1                                                                 MG91226
    MM=1                                                                  MG91228
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      DO 45 I=2,NY                                                      MG91230
      DO 45 J=2,NX                                                      MG91232
   45 HO(I,J)=H1(I,J)                                                   MG91234
      CALL SSWITCH (1,KKK)                                              MG91236
      GO TO (46,4),KKK                                                  MG91238
   46 WRITE ( 6,561)  HS,HSO,T,TKM,K,KO                                 MG91240
      WRITE ( 9,561)  HS,HSO,T,TKM,K,KO                                 MG91242
      WRITE (15,561)  HS,HSO,T,TKM,K,KO                                 MG91244
  561 FORMAT (1HO,6HHS   = E16.8,5X, 6HHSO = E16.8                    / MG91246
     11H , 6HT    = E16.8,5X, 6HTKM = E16.8                           / MG91248
     21H , 6HK    = I5,16X,6HKO  = I5                                 ) MG91250
      IF (I5-1)  573,573,574                                            MG91252
  573 IF (HS-HSO)  569,575,575                                          MG91254
  574 IF (HS-HSO)  566,566,569                                          MG91256
  569 IF (T-TKM) 570,567,567                                            MG91258
  570 IF (K-KO) 565,568,568                                             MG91260
  575 WRITE ( 6,576)                                                    MG91262
      WRITE ( 9,576)                                                    MG91264
      WRITE (15,576)                                                    MG91266
  576 FORMAT(1HO,60HSTOP BECAUSE HS IS GREATER THAN OR = TO HSO (BUILDUP MG91268
     1)                                                               ) MG91270
      GO TO 1565                                                        MG91272
  566 WRITE ( 6,562)                                                    MG91274
      WRITE ( 9,562)                                                    MG91276
      WRITE (15,562)                                                    MG91278
  562 FORMAT (1HO,50HSTOP BECAUSE HS IS LESS THAN OR = TO HSO  (DECAY) ) MG91280
      GO TO 1565                                                        MG91282
  567 WRITE ( 6,563)                                                    MG91284
      WRITE ( 9,563)                                                    MG91286
      WRITE (15,563)                                                    MG91288
  563 FORMAT (1HO,50HSTOP BECAUSE TIME (T) IS GREATER THAN OR = TO TKM ) MG91290
      GO TO 1565                                                        MG91292
  568 WRITE ( 6,564)                                                    MG91293
      WRITE ( 9,564)                                                    MG91294
      WRITE (15,564)                                                    MG91295
  564 FORMAT (1HO,50HSTOP BECAUSE K IS GREATER THAN OR = TO KO        ) MG91296
      GO TO 1565                                                        MG91297
  565 WRITE ( 6,1566)                                                   MG91298
      WRITE ( 9,1566)                                                   MG91299
      WRITE (15,1566)                                                   MG91300
 1566 FORMAT (1HO, 42HSTOP BECAUSE SENSE SWITCH 1 WAS TURNED ON       ) MG91301
 1565 CONTINUE                                                          MG91302
      IF (I5-1) 605,605,606                                             MG91304
  605 WRITE ( 6,607)                                                    MG91306
      WRITE ( 9,607)                                                    MG91308
      WRITE (15,607)                                                    MG91310
  607 FORMAT (1HO,25HTHIS IS A BUILDUP RUN                            ) MG91312
      GO TO 609                                                         MG91314
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  606 WRITE ( 6,608)                                                        MG91316
      WRITE ( 9,608)                                                        MG91318
      WRITE (15,608)                                                        MG91320
  608 FORMAT (1H0, 25HTHIS IS A DECAY RUN                                 ) MG91322
  609 CONTINUE                                                              MG91324
      K2=KK-K1+1                                                            MG91326
      CALL PPLT (TIME(K1),TFORC(K1),K2)                                     MG91328
      WRITE ( 6,611)                                                        MG91330
  611 FORMAT (1H0,40X,45HTOTAL FORCE  VS  TIME                            ) MG91332
      CALL PPLT (TIME(K1),TFLUX(K1),K2)                                     MG91334
      WRITE ( 6,612)                                                        MG91336
  612 FORMAT (1H0,40X,45HTOTAL FLUX  VS  TIME                             ) MG91338
      WRITE (16,1000) E0,HS0,BR,HMAX,CU0                                    MG91340
      WRITE (16,1000)  T,TKM,COND,AX,BY                                     MG91342
 1000 FORMAT (5E16.8)                                                       MG91344
      WRITE (16,2000) CDT,KZ,NI,NJ,K,K0,KI,KG                               MG91346
 2000 FORMAT ( E16.8,7I5)                                                   MG91348
      WRITE (16,2130) B2,H2,B3,H3                                           MG91350
 2130 FORMAT (4E16.8)                                                       MG91352
      WRITE (16,2790) HI0,DHX,I3,I4,I5                                      MG91354
 2790 FORMAT (2E16.8,3I5)                                                   MG91356
      WRITE (16,1000) ((H1(I,J),J=2,NX),I=2,NY)                             MG91358
      WRITE ( 6,1577)                                                       MG91359
 1577 FORMAT (1H1,90HTHIS IS OUTPUT DATA TO BE USED FOR A CONTINUATION      MG91360
     1RUN. CHANGE K TO K = 0                                            / ) MG91361
      WRITE ( 6,1001) E0,HS0,BR,HMAX,CU0                                    MG91362
 1001 FORMAT (1H ,5E16.8)                                                   MG91363
      WRITE ( 6,1002)  T,TKM,COND,AX,BY                                     MG91364
 1002 FORMAT (1H ,5E16.8)                                                   MG91366
      WRITE ( 6,2001) CDT,KZ,NI,NJ,K,K0,KI,KG                               MG91368
 2001 FORMAT (1H , E16.8,7I5)                                               MG91370
      WRITE ( 6,2131) B2,H2,B3,H3                                           MG91372
 2131 FORMAT (1H ,4E16.8)                                                   MG91374
      WRITE ( 6,2791) HI0,DHX,I3,I4,I5                                      MG91376
 2791 FORMAT (1H ,2E16.8,3I5)                                               MG91378
      WRITE ( 6,1002) ((H1(I,J),J=2,NX),I=2,NY)                             MG91380
      LI4=6                                                                 MG91382
      I4=LI4                                                                MG91384
      GO TO 31                                                              MG91386
  635 WRITE ( 6,636) KZ,KZ                                                  MG91388
      WRITE ( 9,636) KZ,KZ                                                  MG91390
      WRITE (15,636) KZ,KZ                                                  MG91392
  636 FORMAT(1H0,6H(KZ = I4,3X,31HPREVIOUS DATA FOR DATA SET NO. I4,1H)) MG91394
      KZ=KZ-1                                                               MG91396
  285 IF (KZ) 47,47,48                                                      MG91398
   47 CONTINUE                                                              MG91400
      END FILE  9                                                           MG91402
      END FILE 15                                                           MG91404
```

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      END FILE 16
      STOP
      END
$IBFTC FLUD
      SUBROUTINE FLUD (HO)
      DIMENSION HO(35,35),FORCE(35,35),B1(35,35)
      COMMON I,J,U0,DX,T2,FLUXT,FORCE,BR,NX,NY,I5,B1,C1,C2
      IF (I5-1) 614,614,613
  613 HC=-(C2*BR/(C1+BR))
      GO TO 617
  614 HC= (C2*BR/(C1+BR))
      IF (HO(I,J)-HC)  615,616,616
  615 HOX=-HO(I,J)
      B1(I,J)=-(C1*HOX/(C2+HOX)+BR)
      GO TO 620
  616 X1=HO(I,J)-2.0*HC
      B1(I,J)=(C1*X1/(C2+X1))+BR
      GO TO 620
  617 IF (HO(I,J)-HC)  618,619,619
  618 HD3=ABS (HO(I,J)-HC)
      HD4=HC+HD3
      B1(I,J)=-((C1*HD4/(C2+HD4))+BR)
      GO TO 620
  619 B1(I,J)=(C1*HO(I,J)/(C2+HO(I,J)))+BR
  620 RETURN
      END
$IBFTC FORCX
      SUBROUTINE FORCX
      DIMENSION FORCE(35,35),B1(35,35)
      COMMON I,J,U0,DX,T2,FLUXT,FORCE,BR,NX,NY,I5,B1,C1,C2
      IF (I-2)  300,300,301
  300 IF (J-2)  253,253,302
  302 IF (J-NX) 252,253,253
  301 IF (J-2)  303,303,304
  303 IF (I-NY) 252,253,253
  304 IF (I-NY) 305,306,306
  305 IF (J-NX) 254,252,252
  306 IF (J-NX) 252,253,253
  252 FORCE(I,J)=((T2**2)*(DX/2.0))/(2.0*U0)
      GO TO 256
  253 FORCE(I,J)=((T2**2)*(DX/4.0))/(2.0*U0)
      GO TO 256
  254 FORCE(I,J)=((T2**2)*DX)/(2.0*U0)
  256 CONTINUE
      RETURN
      END
$IBFTC FLUX
      SUBROUTINE FLUX
```

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      DIMENSION FORCE(35,35),B1(35,35)                                  MG91502
      COMMON I,J,U0,DX,T2,FLUXT,FORCE,BR,NX,NY,I5,B1,C1,C2              MG91504
      IF (I-2)  310,310,311                                             MG91506
  310 IF (J-2)  263,263,312                                             MG91508
  312 IF (J-NX) 262,263,263                                             MG91510
  311 IF (J-2)  313,313,314                                             MG91512
  313 IF (I-NY) 262,263,263                                             MG91514
  314 IF (I-NY) 315,316,316                                             MG91516
  315 IF (J-NX) 264,262,262                                             MG91518
  316 IF (J-NX) 262,263,263                                             MG91520
  262 FLUXT=FLUXT+(T2   *   DX/2.0)                                     MG91522
      GO TO 266                                                         MG91524
  263 FLUXT=FLUXT+(T2   *   DX/4.0)                                     MG91526
      GO TO 266                                                         MG91528
  264 FLUXT=FLUXT+(T2   *   DX)                                         MG91530
  266 CONTINUE                                                          MG91532
      RETURN                                                            MG91534
      END                                                               MG91536
...............................................................................
C ..... PUT PPLT, INSERT, LOGI  IN THIS POSITION (BINARY DECK)
...............................................................................
$DATA
  .10000000E+02  -.10000000E+08   .71000000E-00   .13400000E+04   .10000000E-06
  .00000000E+00   .10000000E-01   .25000000E+07   .88900000E-03   .67310000E-02
  .20000000E+00    4    4   30    0    8    1    2
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.00000000E+00	.20000000E-01	.25000000E+07	.40000000E-02	.80000000E-02
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.80000000E-00	.11940000E+02	.12000000E+01	.10350000E+03	
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.20000000E+00	2 5 10	0 100 1	5	
.80000000E-00	.11940000E+02	.12000000E+01	.10350000E+03	
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-.25000000E+03				
.10000000E+02	-.10000000E+08	.71000000E-00	.25000000E+03	.10000000E-06
.00000000E+00	.10000000E-01	.25000000E+07	.88900000E-03	.67310000E-02
.20000000E+00	1 4 30	0 1000 5	50	
.80000000E-00	.11940000E+02	.12000000E+01	.10350000E+03	
.40000000E+03	.10000000E+02	81 6 2		

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TIME DEPENDENCE OF THE MAGNETIC FIELD IN A RECTANGULAR TOROID

by

GLEN LEROY SHURTZ

B.S.M.E., Kansas State University, 1963
B.S.E.E., Kansas State University, 1964

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

The purpose of this thesis was to investigate the time dependence of the magnetic field in a rectangular toroid when subjected to an applied magnetomotive force.

According to Lenz's law, eddy currents generated in the transient state induce an mmf in opposition to that applied. This results in a nonuniform time varying distribution of field intensity and flux within the core during a transition between states. An equation describing time dependence of this distribution pattern was derived by applying Maxwell's equations with the assumption that symmetrical boundary conditions were applied along the length of the toroid and no leakage could occur. Combining this equation with another which provided an approximation to the B-H relationship for a given ferromagnetic material, yielded the Hysteretic Diffusion Equation.

$$\frac{\partial H}{\partial t} = \frac{(C_2 + H)^2}{\sigma C_1 C_2} \left[\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} \right]$$

Its solution yields the time dependence of the field intensity as a function of position in the core.

The Modified Euler Method of numerical integration was used to evaluate time variance of the magnetic field intensity and flux distribution patterns from which the time dependence of total force and total flux was calculated.

Discussion of error criteria and selection of an appropriate time increment to insure convergence were discussed and a sample problem illustrating the method was given.