Graphical Representation of Stress and Work.

By Graphical representation of "Stress" and "Work," we mean representation by lines. That is take for instance the different members or pieces which compose a bridge or roof truss. Knowing the total load to which the truss is subjected, a diagram is obtained, showing the direction, magnitude, and kind of stress to which each member is subjected. The answer to the problem in question is obtained in simple lines placed directly before the eye where it can be comprehended in all its relations. This method for the solution of problems is not a new one, but it is almost wholly unknown to the masses. The more prominent Mathematicians and Engineers have labored hard to introduce the method into practice, and at present even have only a fair foot hold.

The ease, accuracy, and speed with which complex problems are solved, and the limited amount of mathematical knowledge and skill required in their solution, are points which specially favor this method and cannot help but give it prominence over the more common method of solving problems by means of long and intricate formulae.

The solving of statical problems "Graphically" consists of an extended use of the principle of the "Parallelogram of Forces". This principle is enunciated in treatises on "Mechanics" as follows. "If two forces be represented in direction and intensity by adjacent sides of a parallelogram their resultant will be represented by that diagonal of the parallelogram which passes through their common point".

To illustrate, if P Fig. 1 be one force represented in direction and intensity by the line OP and Q be the other force similarly represented, then by completing the parallelogram POQR and drawing the line OR, R will be the resultant of the two forces P and Q and is represented in direction and intensity by the line OR. R is a force which acting in the direction indicated in Fig. 1 will produce the same effect as the two forces P and Q, and if acting in the direction indicated by the arrow point in Fig. 2 will hold them in equilibrium.

Now if P were divided into 100 equal divisions each division representing one pound, and Q made equal to 60 pounds by the same scale, then by applying this scale to the line OR we would see at once exactly the number of pounds OR represents.

Suppose we attempt to solve this problem by the more common method. We will proceed something like this, knowing that $P = 100$ Lbs. $Q = 60$ Lbs. and that angle $POQ = 60°$

We have, $R = \sqrt{P^2 + Q^2 + 2PQ \cos. POQ} = \sqrt{10000 + 3600 + (12000 \times \frac{1}{2})} = \sqrt{19600} = 140$ Lbs.

A solution much longer, more complex, and less accurate than if solved graphically.

By refering to Fig. 2 the force represented by the

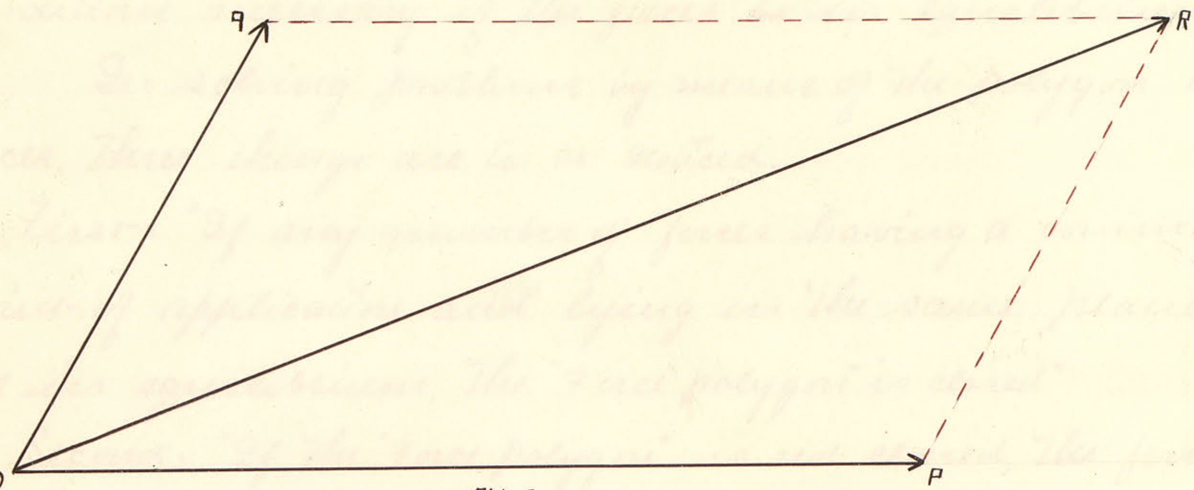

Fig. 1.

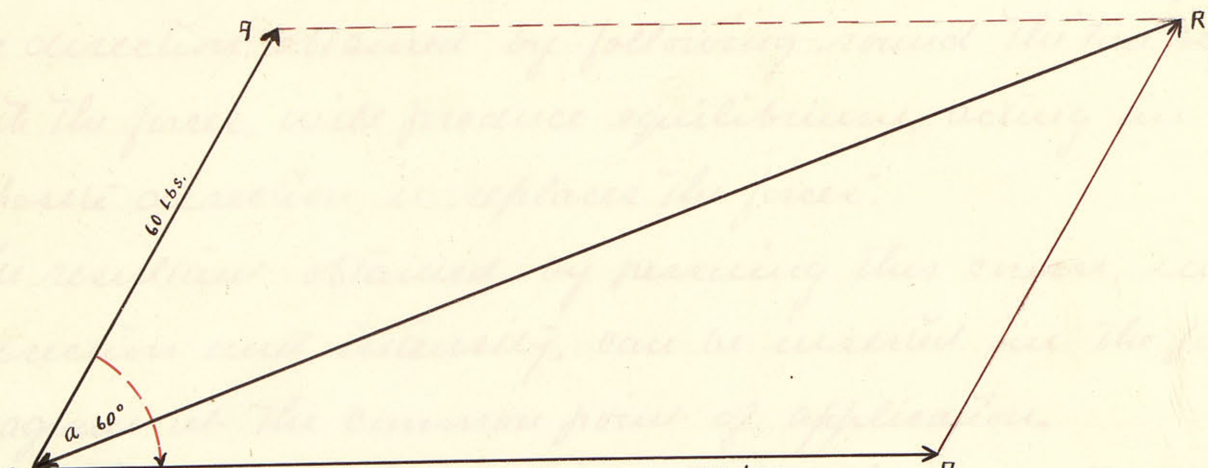

Fig. 2.

line PR is seen to be equal, parallel, and acting in the same direction as the force 9, hence is equal to 60 Lbs. Now we can substitute for the force 9 its equal PR acting from P to R and we have a closed polygon. One of the conditions necessary if the forces be in equilibrium.

In solving problems by means of the polygon of forces, three things are to be noticed.

First = "If any number of forces having a common point of application and lying in the same plane, are in equilibrium, the "Force polygon" is closed"

Second = "If the "Force polygon" is not closed, the forces themselves are not in equilibrium, and the line necessary to close the polygon, gives the resultant in direction and intensity"

Third = "This resultant, if considered as acting in the direction obtained by following round the "Force Polygon" with the forces, will produce equilibrium, acting in the opposite direction, it replaces the forces".

The resultant obtained by pursuing this course, in direction and intensity, can be inserted in the force diagram at the common point of application.

Suppose it be required to find the resultant of a system of forces as P, S, H, G, K, T, lying in the same plane as in Fig. 3 when P = 10 Lbs. S = 20 Lbs H = 25 Lbs. G = 5 Lbs K = 15 Lbs, and T = 20 Lbs. Fig. 3(a) is the diagram polygon, showing by the red line the direction and intensity of the resultant and is obtained as follows. Assume some point as O. and from O draw OP parallel to P Fig. 3 and make it in length by scale equal to 10 lbs. from P draw PS parallel to S in Fig. 3 and by the same scale lay off on it a length equal to 20 lbs. from S draw SH parallel to H in Fig. 3 and make it by the same scale equal to 25 lbs. and continue on round the system in like manner, taking in succession the forces in Fig. 3, each time making the lines drawn of a length by scale equal to the number of pounds represented by the corresponding force in Fig. 3, and when the last one is reached (in this case T) the line necessary then to close the polygon will be the resultant. In the diagram in question it is the red line and by measuring this line with the scale used in constructing the figure the answer is obtained in pounds. This line can be inserted in Fig. 3 and will show exactly the point of application, the direction, and the intensity, of the resultant of the system.

It will be seen by referring to Fig. 3 (b) that it is quite immaterial where we assume the point O, or with which force we begin to construct our "force polygon", the answer will be the same in all cases. The red line in Fig. 3 (b) is equal and parallel to the red line in Fig. 3 (a) and yet the forces with which the polygons were begun and ended were entirely different. All that is necessary is to proceed round the system in a natural order so that a polygon may be formed.

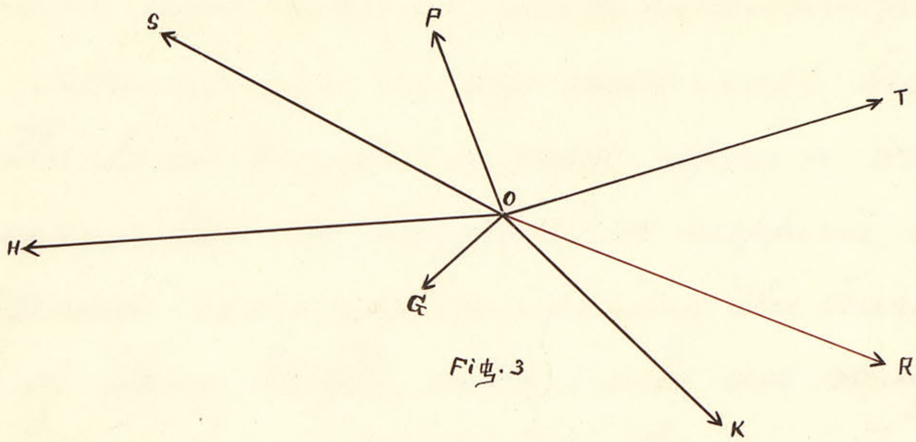

Fig. 3

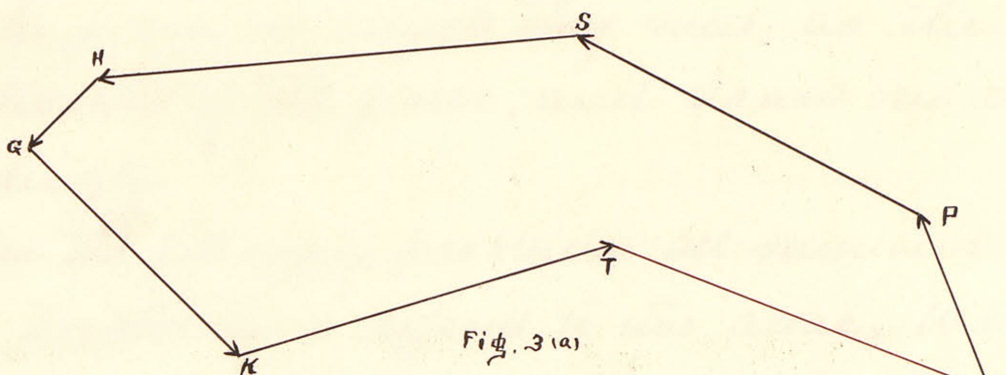

Fig. 3 (a)

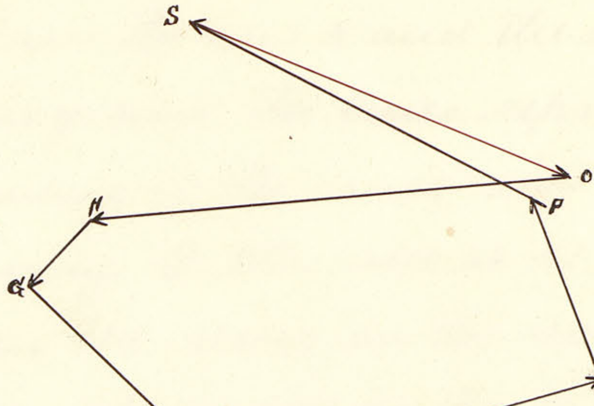

Fig. 3. (b).

We have now considered the "Force polygon" with sufficient completeness to enable us to carry it forward and use it understandingly in determining stresses in bridge and roof trusses.

The load brought to bear upon a bridge is transmitted to the piers or supports, through the different members composing the truss, in the direction of their length, and hence we may consider the members as representing the direction of the forces, and the points at which they meet as the points of application of the forces, and draw our diagram accordingly.

In the following problems all members of a truss will be located by or refered to two areas. When as in Fig. 4 we speak of the line DB we mean the line lying between these two areas. Also if we say line AD we mean the line between the area A and the area D.

In all diagrams the lines representing the stresses in the members of the truss will be determined in the usual way. A letter will be at each end of the stress line. The stress in the member AB of the truss Fig. 4 will be represented by the line AB of the diagram.

We will now consider some of the more common and practical problems of every day life.

Let us take for our first problem a simple "A" or King post truss, such as we meet with every day.

Let it be required to find the stress in the different members when a known load is to be sustained. Fig. 4 represents a common "King post" truss. Let the load to be sustained be 80000 lbs. In practice the load would pass over the bridge on a floor placed on stringers represented by the line IA and is carried to the apex of the triangle by means of a rod as shown in the figure, consequently this rod must be of a strength sufficient to sustain the entire load of 80000 lbs. and our answer for its stress is at once obtained. We will then since this rod serves only to transmit the load to the other members, and its stress is known, consider the load as applied to the apex of the truss as indicated by the arrow. In this position each pier must sustain half of the entire load, hence there will be a reacting force at each pier of 40000 lbs.

To diagram the stress in the members of the truss, draw a line parallel with the direction of the load and lay off on this line a length representing the entire load applied, and divide this distance laid off into two equal parts as half of the load is born by each pier. Beginning now at the pier marked T and knowing that the reacting force, the stress in the member IB and that in the member IA meet at a point we can construct our diagram exactly as we did in case of the simple parallelogram of forces; and since the forces are in equilibrium we know that the polygon must be closed.

In passing from A to B we have included here the reacting force of 40000 lbs. This we have marked off on the line CB which is parallel to the direction of the force, and its magnitude is represented by the length of the line AB. Next pass from B to D. To do so draw from B in the diagram a line BD of infinite length parallel to DB in the truss. Now since these forces are in equilibrium we must close the polygon. In order to do this we must pass from D back to A and in doing this we must pass parallel to DA in the truss. So drawing a line from A parallel to DA of the truss and extending it until it intersects the line BD we determine the magnitude of the stress in both the members DA and DB. The diagram is obtained for the forces acting at the pier marked T′ in exactly the same manner. Again let us consider the forces acting at the apex of the truss. In passing from B to C we have included the load of 80000 lbs. represented by the line BC in the diagram. In passing from C to D we have the line CD in the diagram, and in passing from D to A we have the line DA in the diagram, and also we see we have a closed polygon. By applying our scale to these lines we obtain the answer in pounds. It will be seen that the diagram is an equilateral triangle with sides representing 80000 lbs. This is seen to be correct; since the angle

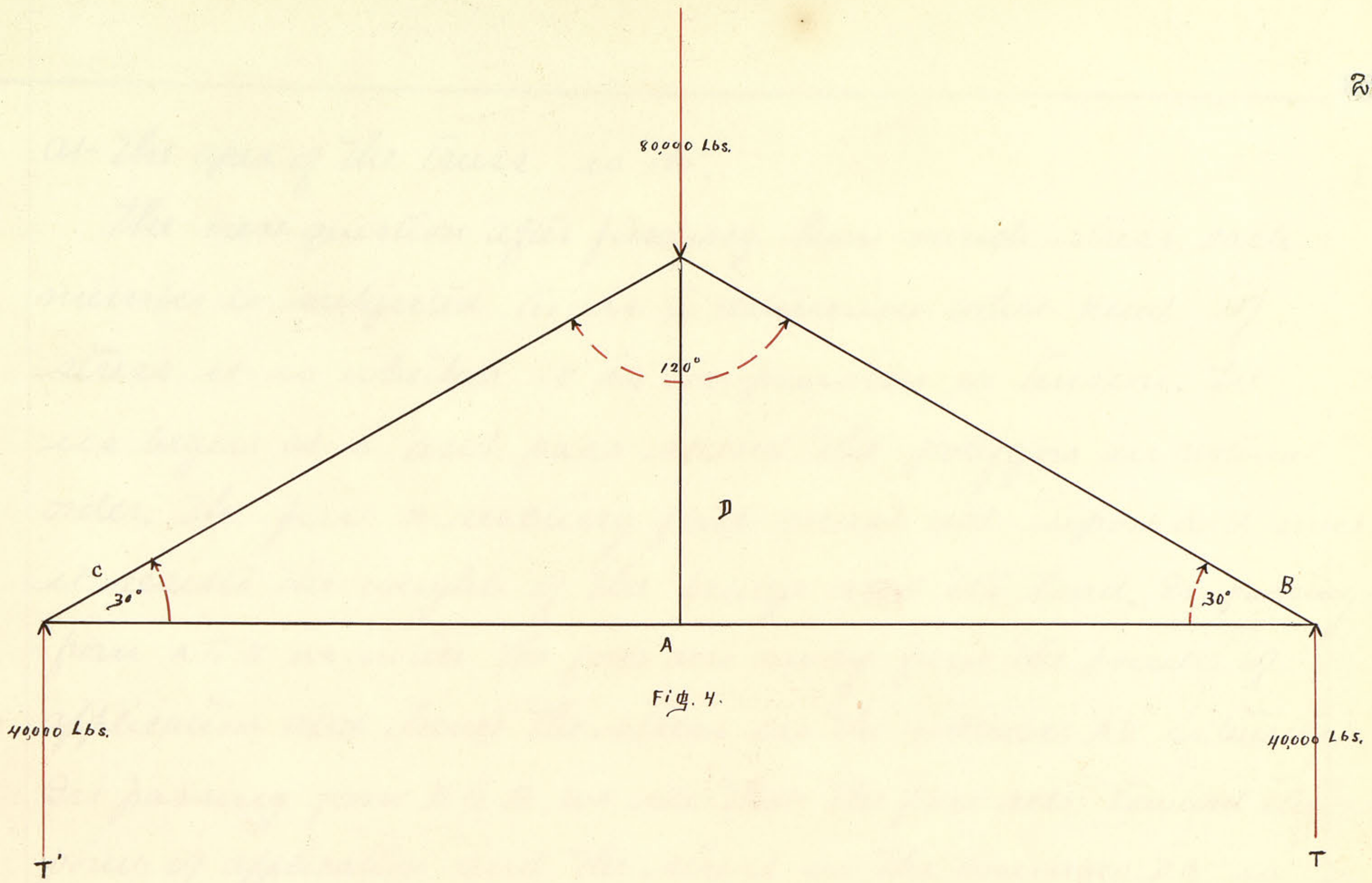
80,000 Lbs.
120°
D
C
30°
30°
B
A
40,000 Lbs.
40,000 Lbs.
T'
T

Fig. 4.

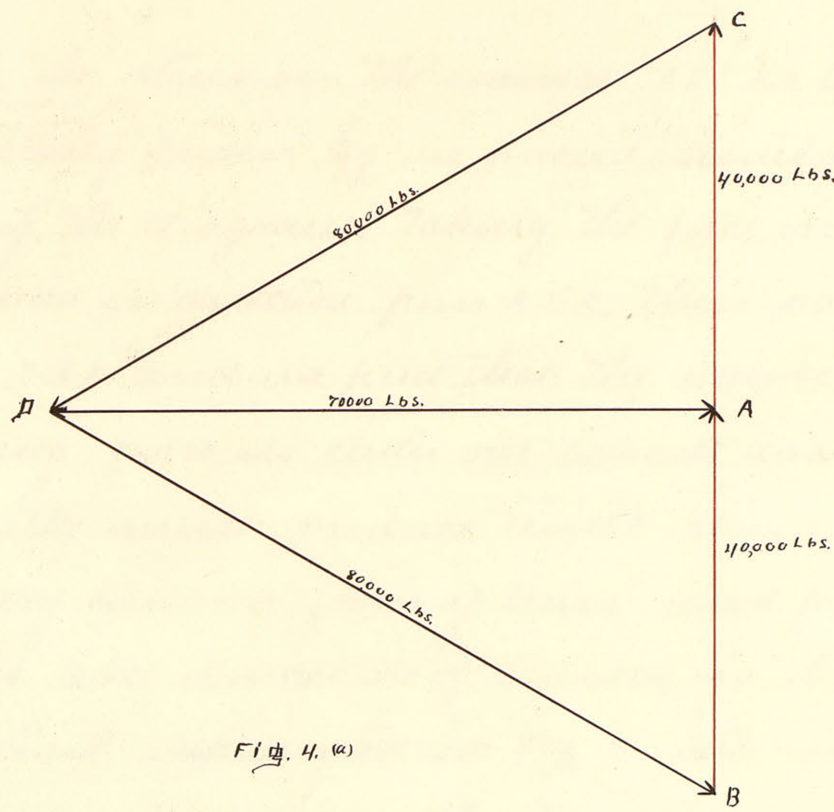
C
40,000 Lbs.
80000 Lbs.
D
70000 Lbs.
A
40,000 Lbs.
80000 Lbs.
B

Fig. 4. (a)

at the apex of the truss is 120°.

The next question after finding how much stress each member is subjected too, is to determine what kind of stress it is, whether it is compression or tension. Let us begin at B and pass round the polygon in natural order. The first or reacting force must act upward since it resists the weight of the bridge and its load. In passing from A to D we notice the force acts away from the point of application and hence the stress in the member AD is tension. In passing from D to B we see that the force acts toward the point of application and the stress in the member DB is compression.

The fact that the stress in the member AD is tension will be conclusively proven if we proceed similarily with the other side of the diagram. Taking the force on the pier T' represented in direction from A to C, then pass from C to D, then from D to A and we find that the member D A has a force acting from its center out toward each end and consequently must produce tensil stress.

Another very common form of truss used for small spans of bridges and small roof trusses is that called the "Queen post truss", represented in Fig. 5 with its corresponding diagram in Fig. 5 (a). The forces to be considered in this case are two, acting as indicated in Fig. 5. They are kept in equilibrium by the members of the truss, which rest upon the two piers. To obtain the diagram for the

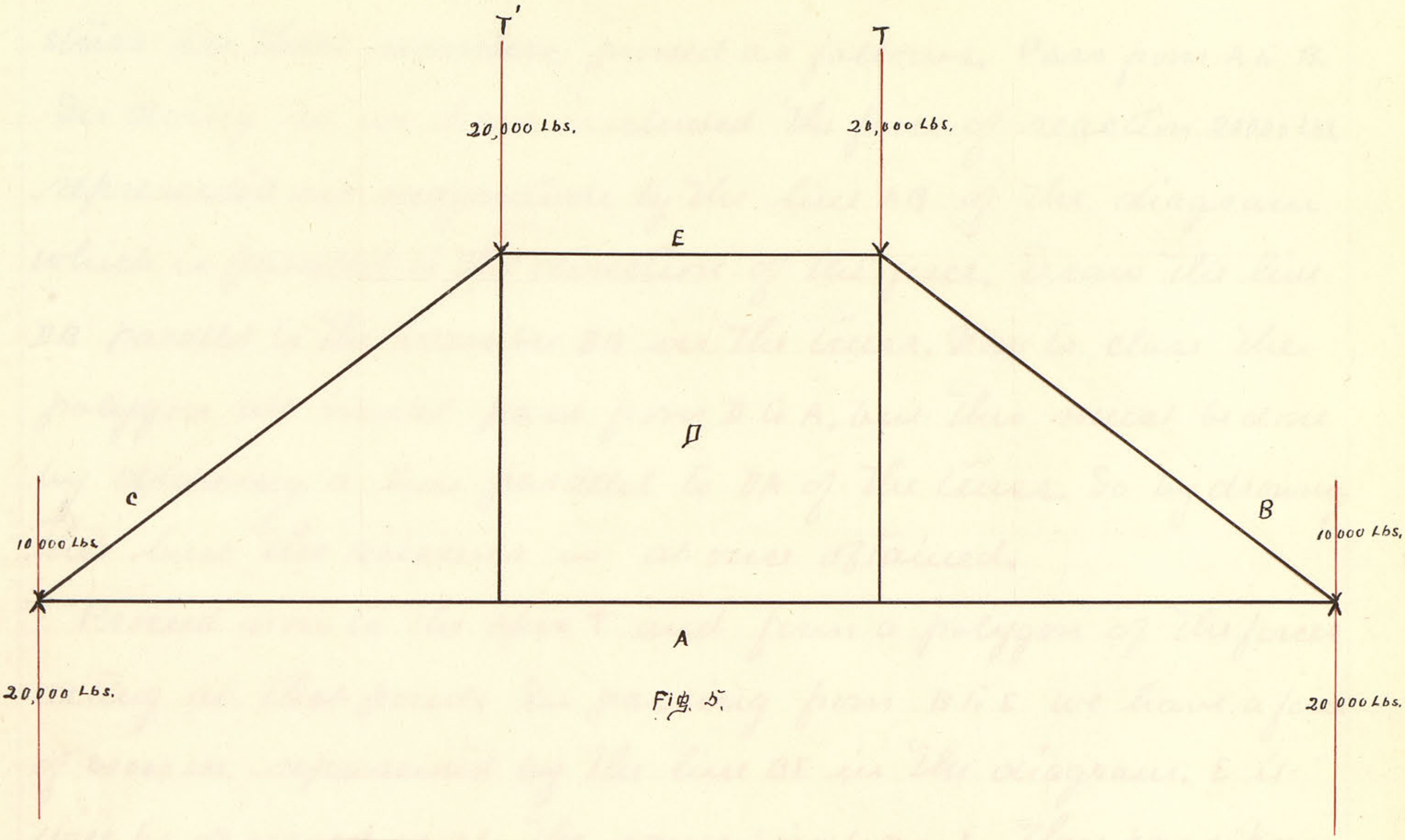
T'
T
20,000 Lbs.
20,000 Lbs.
E
D
C
B
10 000 Lbs.
10 000 Lbs.
A
20,000 Lbs.
20 000 Lbs.

Fig. 5.

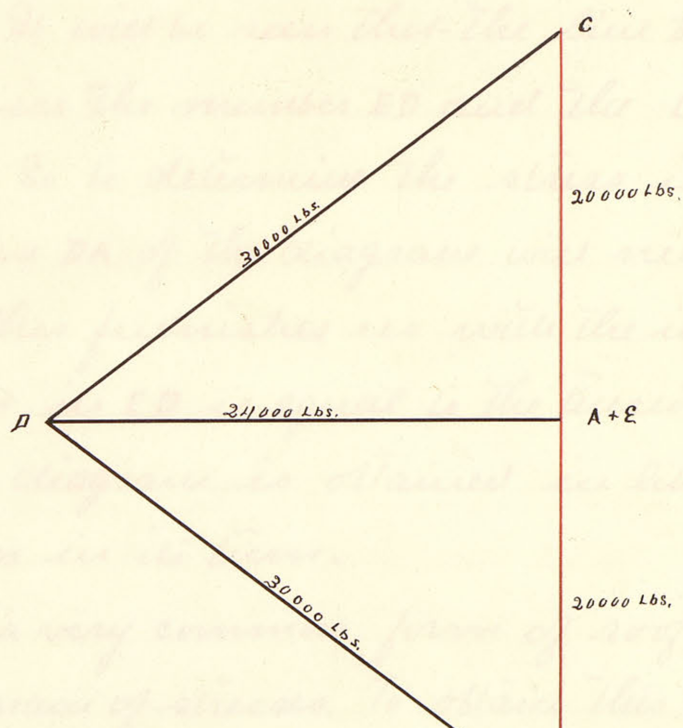
C
20000 Lbs.
30000 Lbs.
D
21,000 Lbs.
A + E
30000 Lbs.
20000 Lbs.
B

Fig. 5. (a)

stress in these members proceed as follows. Pass from A to B. In doing so we have included the force of reaction 20000 Lbs, represented in magnitude by the line AB of the diagram which is parallel to the direction of the force. Draw the line DB parallel to the member DB in the truss. Now to close the polygon we must pass from D to A, but this must be done by drawing a line parallel to DA of the truss. So by drawing this line the answer is at once obtained.

Proceed now to the apex T and form a polygon of the forces acting at that point. In passing from B to E we have a force of 20000 Lbs. represented by the line BE in the diagram. E it will be observed is at the same point as A. Then pass from E to D then from D to B. It will be seen that the line DA represents both the compression in the member ED and the tension in the member DA. So to determine the stress in these two members, the line DA of the diagram will need be measured for each. This furnishes us with the information that the compression in ED is equal to the tension in DA. The remainder of the diagram is obtained in like manner, diagramming each apex in its turn.

Fig. 6 represents a very common form of roof truss and Fig. 6 (a) its diagram of stresses. To obtain this diagram, lay off HB equal to the three loads applied on top and divide the line into three parts in proportion to the size of the weights. In this case they are equal. Bisect this line and letter the point L. Lay off each way from the point L on the line BH

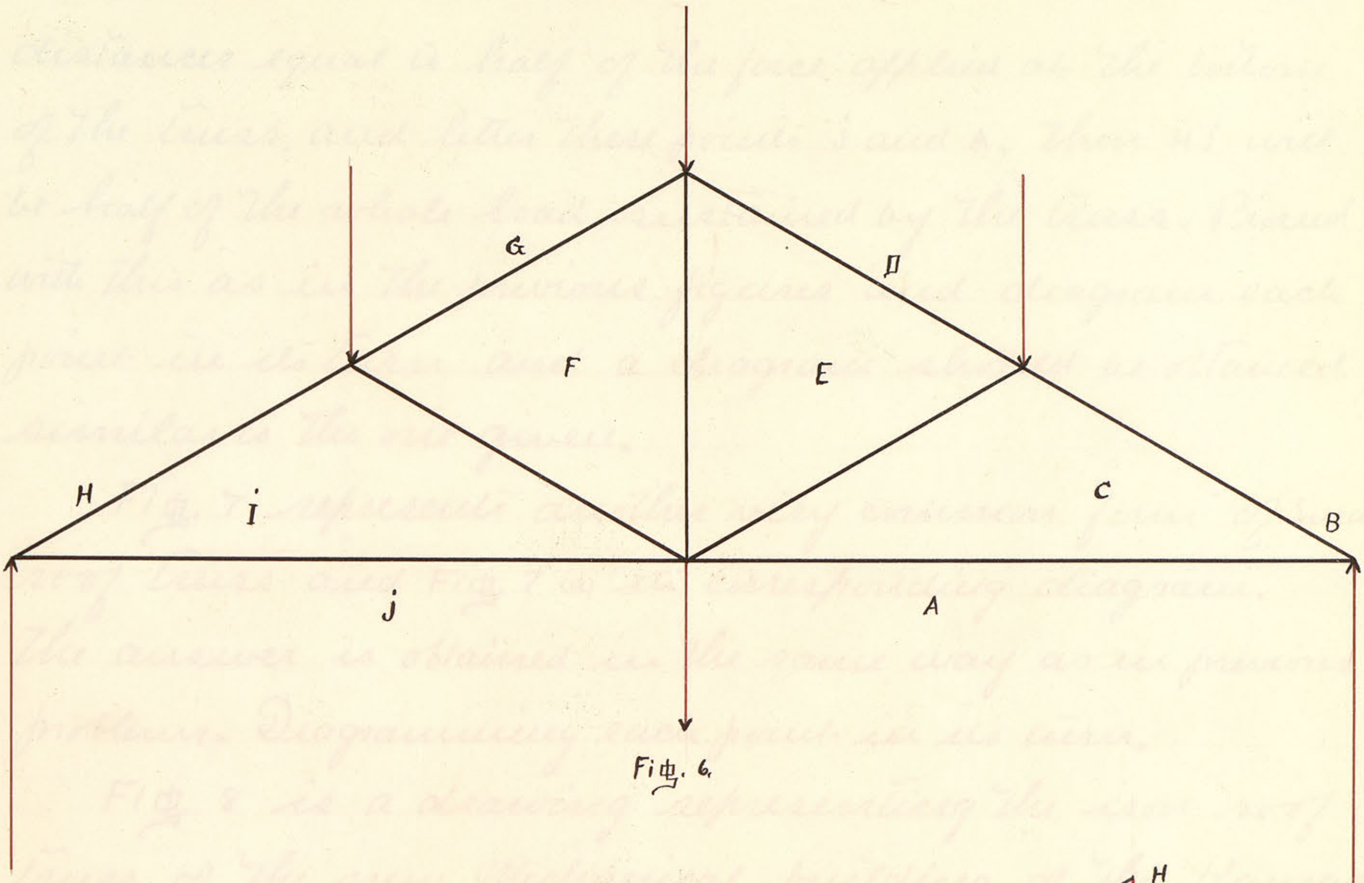

Fig. 6.

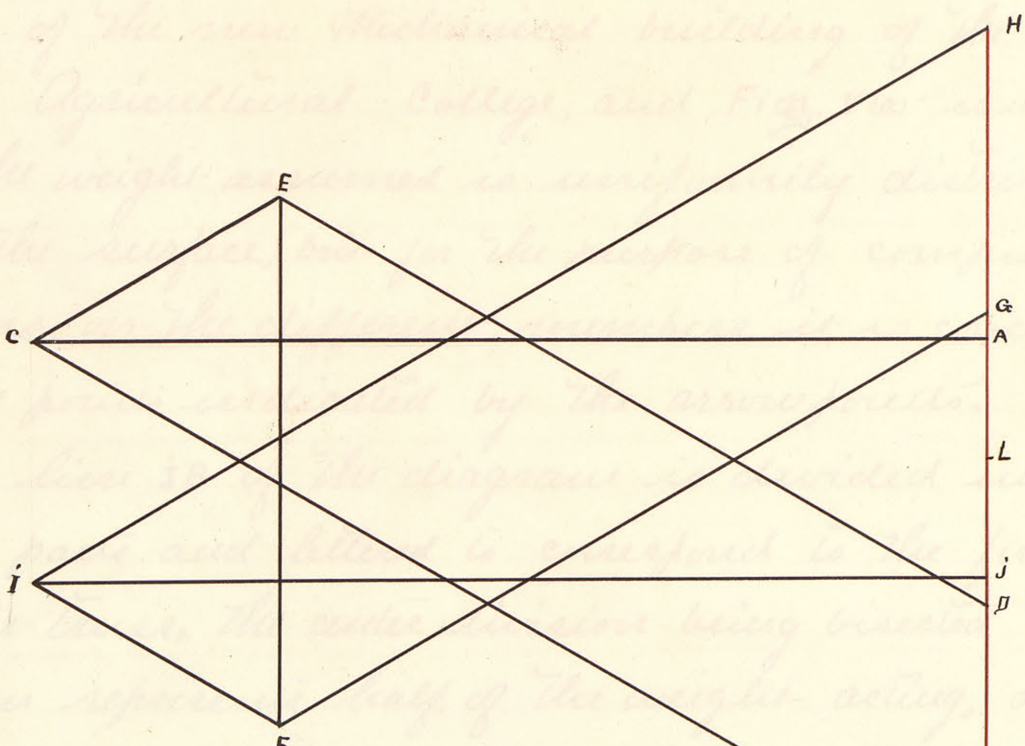

Fig. 6 (a)

distances equal to half of the force applied at the bottom of the truss, and letter these points J and A. Then HJ will be half of the whole load sustained by the truss. Proceed with this as in the previous figures and diagram each point in its turn and a diagram should be obtained similar to the one given.

Fig. 7 represents another very common form of small roof truss and Fig. 7 (a) its corresponding diagram. The answer is obtained in the same way as in previous problems. Diagramming each point in its turn.

Fig. 8 is a drawing representing the iron roof truss of the new Mechanical building of the Kansas State Agricultural College, and Fig. 8 (a) is its diagram.

The weight assumed is uniformily distributed over the surface, but for the purpose of computing the strains on the different members it is concentrated at the points indicated by the arrow points.

The line IB of the diagram is divided into seven equal parts and lettered to correspond to the forces acting on the truss. The center division being bisected at A. AI then represents half of the weight acting, and from this point we proceed as in previous problems.

By following the diagram closely it will be seen that every thing checks until we come to diagram the forces acting at "R". Here it will be found that we have two indeterminate points S and R of the diagram

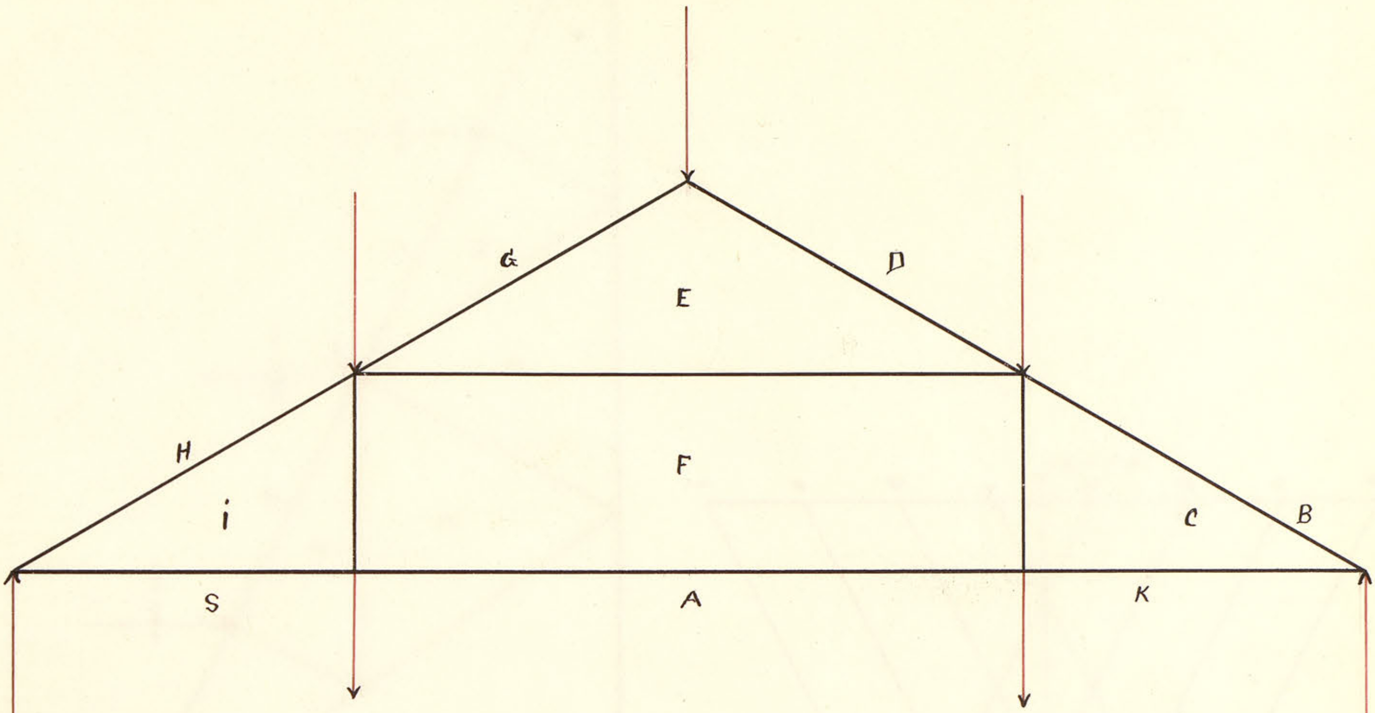

Fig. 7.

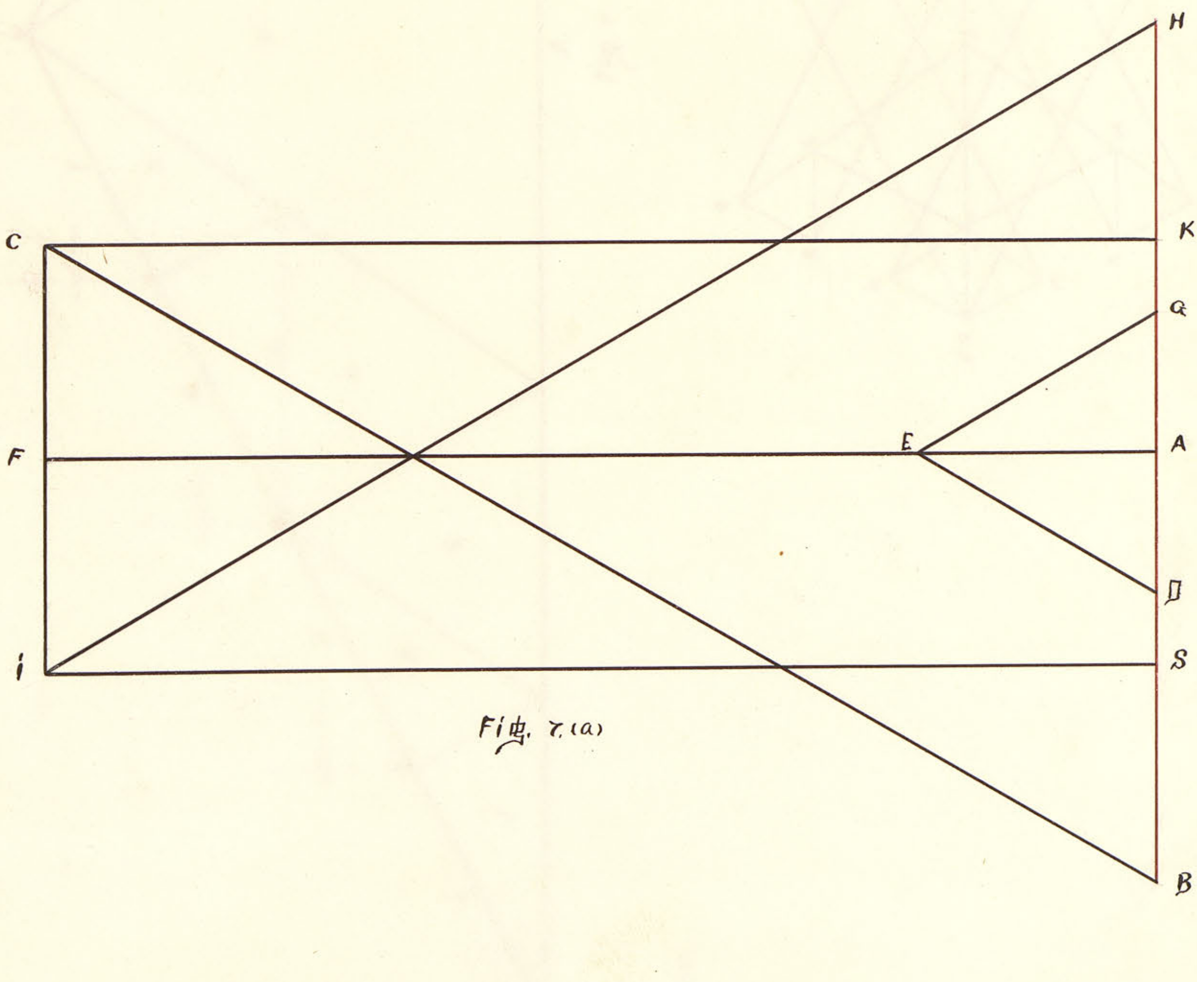

Fig. 7.(a)

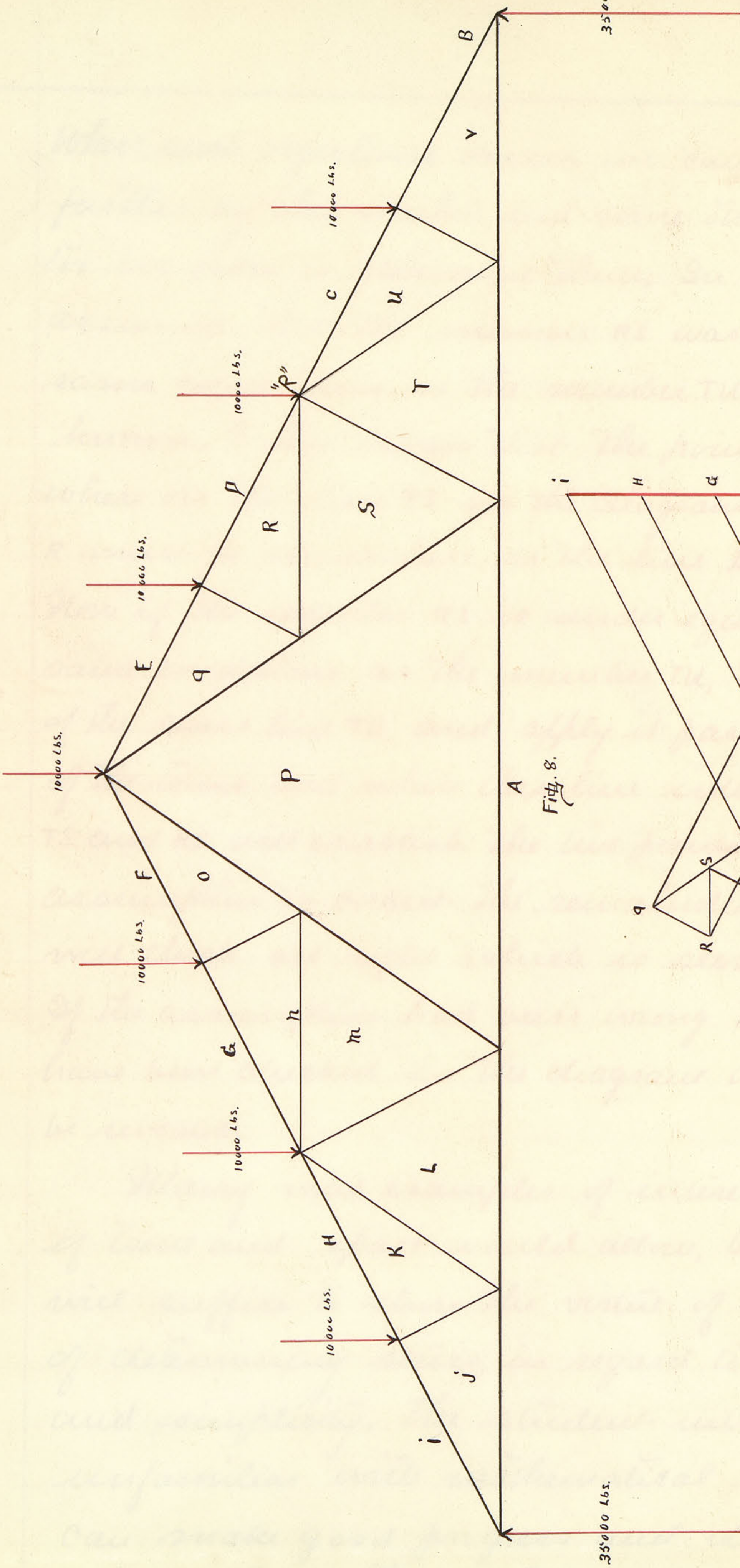
35000 Lbs.
10000 Lbs.
10000 Lbs.
10000 Lbs.
10000 Lbs.
10000 Lbs.
10000 Lbs.
10000 Lbs.
35000 Lbs.
B
V
C
U
T
R
S
E
q
P
A
F
o
G
n
m
L
H
K
j
i

Fig. 8.

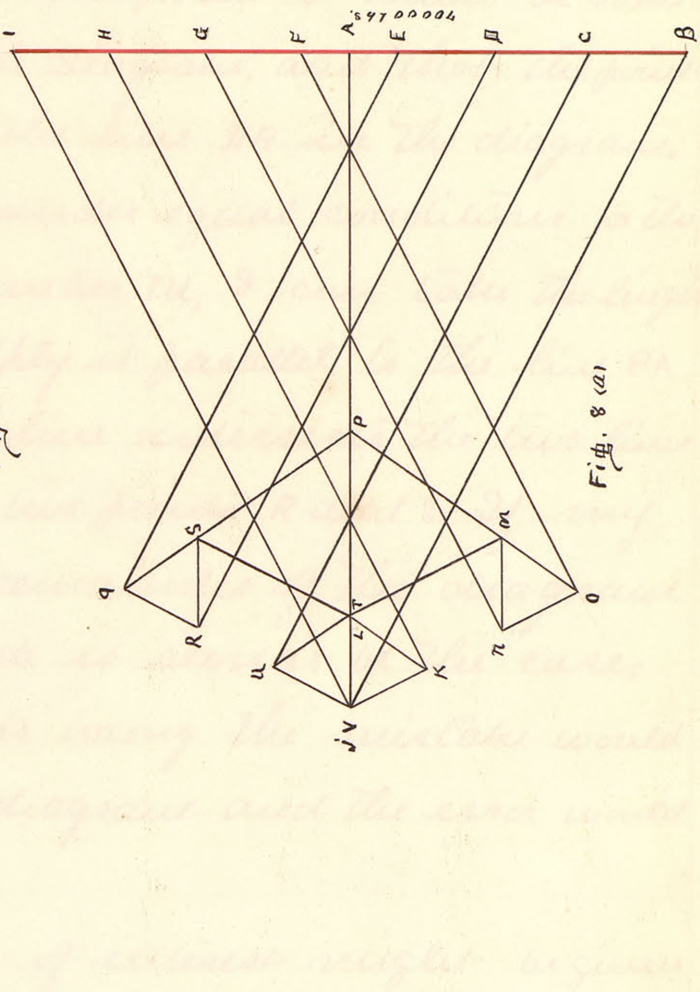
i
H
G
F
A
E
C
B
P
S
M
q
o
R
T
L
n
u
K
jv

Fig. 8 (a)

When such objections occur we cannot proceed any further by this method, and some other must be resorted too in order to determine them. In this example I assumed that the member RS was subjected to the same conditions as the member TU whose stress is known. I also knew that the point S must be somewhere on the line TS in the diagram, and that the point R must be somewhere on the line IIR in the diagram. Now if the member RS is under equal conditions or the same conditions as the member TU, I can take the length of the stress line TU and apply it parallel to the line PA of the truss, and where this line intersects the two lines TS and RII will establish the two points R and S. If my assumption is correct the remainder of the diagram will check all right which is seen to be the case. If the assumption had been wrong the mistake would have been checked in the diagram and the error would be revealed.

Many more examples of interest might be given if time and space would allow, but the few given will suffice to show the virtue of the "Graphic method" of determining stress, in regard to accuracy, rapidity, and simplicity. The student unskilled and unfamiliar with mathematical rules and formulas can make good progress and do good work by mastering this method of solving problems.

Work.

Another very extensive field in which the method of "Graphics" can be used to a great advantage, is that of representing the work done by various machines in performing certain operations.

"Work is the effect produced by a force in overcoming a resistance". In other words the moving of a body of a certain weight through a certain distance represents so much work having been done.

The amount of work, done is measured by the product of the force, by the distance through which it is moved.

Since work is the product of two units, weight and distance, it can be represented by means of areas, one dimension of which represents weight or pressure and the other distance, just as accurately as stress was represented by lines in previous pages.

For example let us refer to Fig. 9. If we represent the weight to be moved by the line BC and the distance through which it is to be moved by the line AB, will not the area ABCD Fig. 9(a) which is the product of AB by BC, represent the work done?

Suppose the line BC represented the total resistance of a train of cars and the line AB the distance between two stations. Then will not the area ABCD Fig. 9(a) represent the work done in moving the train from Station to Station?

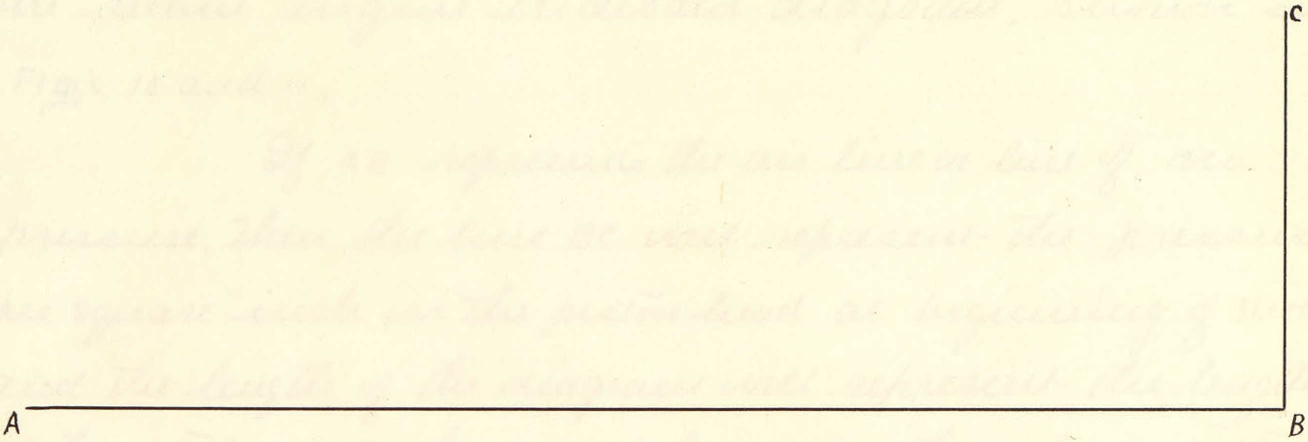

Fig. 9.

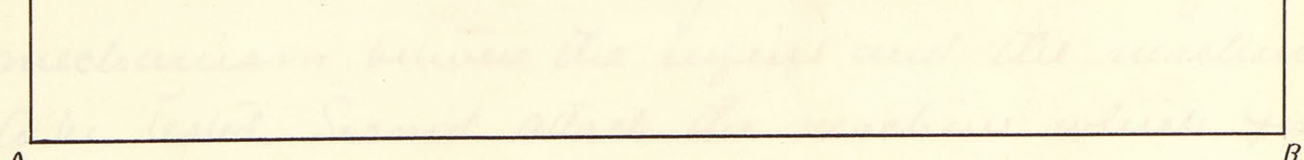

Fig. 9 (a)

Another good example is furnished us by the steam engine indicator diagram, shown in Fig's. 10 and 11.

If AB represents the air line or line of no pressure, then the line BC will represent the pressure per square inch on the piston head at beginning of stroke, and the length of the diagram will represent the length of the stroke, and hence we have for the representation of the work done the area CEFGH. This represents the work done during one stroke of the piston. To find the work done per minute or in any given time we have only to multiply this area by the number of strokes of the piston in that given time. The work necessary to run any machine is readily ascertained by this method. First take a diagram of the steam engine with no machinery attached, calculate the area of the card obtained. This area represents the work done in overcoming the resistance of the mechanism between the engine and the machine to be tested. Second attach the machine which you wish to test and while this is in gear take another indicator card. The difference between the area of these two cards will represent the work required to run the machine which was in gear when the diagram was taken. In this manner the work necessary to run a system of machinery can be readily calculated.

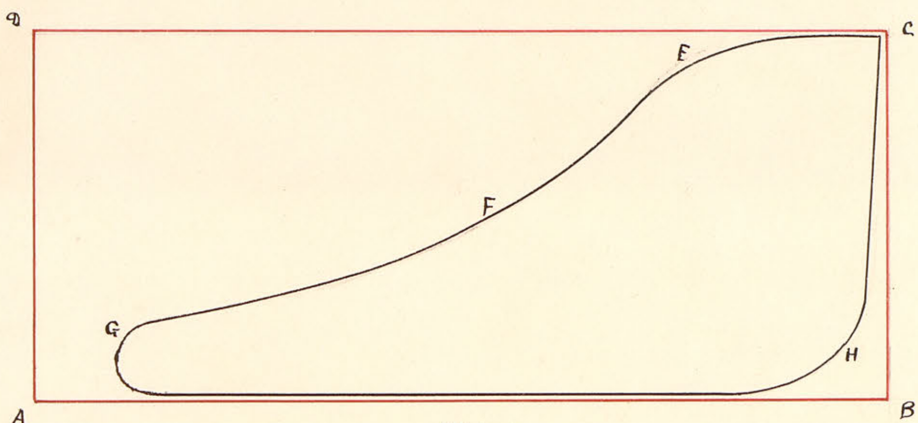

Fig. 10

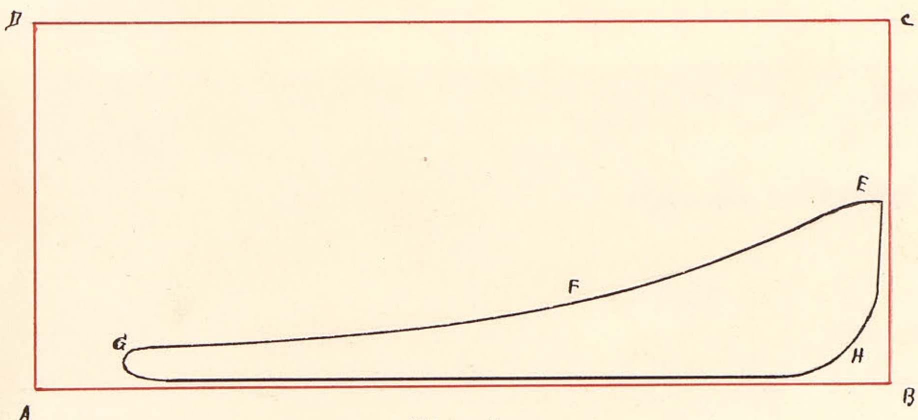

Fig. 11

No. 1.

Two Small Lathes attached

No. 2

Planer attached

On page 23 is found two diagrams taken from the Atlas engine of the Kansas State Agricultural College shops. The difference in area of the two cards represents the difference in work necessary in running the two machines mentioned on the card.

Nearly all kinds of work can be represented in this manner, whether it be walking, hauling or lifting. Areas will represent the work done, and to determine the exact amount of work done, we have only to take the data given and calculate results, but such work is not a part of this thesis.

P. W. Wildin
Class of '92