A COMPARISON OF DESIGN USING STRUT-AND-TIE MODELING AND DEEP BEAM METHOD FOR TRANSFER GIRDERS IN BUILDING STRUCTURES

by

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Abstract

Strut-and-Tie models are useful in designing reinforced concrete structures with discontinuity regions where linear stress distribution is not valid. Deep beams are typically short girders with a large point load or multiple point loads. These point loads, in conjunction with the depth and length of the members, contribute to a member with primarily discontinuity regions. ACI 318-08 *Building Code Requirements for Structural Concrete* provides a method for designing deep beams using either Strut-and-Tie models (STM) or Deep Beam Method (DBM). This report compares dimension requirements, concrete quantities, steel quantities, and constructability of the two methods through the design of three different deep beams. The three designs consider the same single span deep beam with varying height and loading patterns. The first design is a single span deep beam with a large point load at the center girder. The second design is the deep beam with the same large point load at a quarter point of the girder. The last design is the deep beam with half the load at the midpoint and the other half at the quarter point. These three designs allow consideration of different shear and STM model geometry and design considerations.

Comparing the two different designs shows the shear or cracking control reinforcement reduces by an average 13% because the STM considers the extra shear capacity through arching action. The tension steel used for either flexure or the tension tie increases by an average of 16% from deep beam in STM design. This is due to STM taking shear force through tension in the tension reinforcement through arching action. The main advantage of the STM is the ability to decreased member depth without decreasing shear reinforcement spacing. If the member depth is not a concern in the design, the preferred method is DBM unless the designer is familiar with STMs due to the similarity of deep beam and regular beam design theory.

Table of Contents

List of Figures

List of Tables

Acknowledgements

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I also thank **Professor Asad Esmaeily** for his explanations of the Strut-and-Tie model extended nodal zones. He helped me understand the differences in geometry between STM and Deep Beam Design.

1.0 Introduction

Transfer girders, deep beams, are commonly used in construction; thus, understanding the design process and choosing an appropriate design process can only enhance the safety of building design. *American Concrete Institute Building Code Requirements for Structural Concrete* (ACI) 318-08 provides two methods for the design of deep beams, Strut-and-Tie Modeling (STM) or Deep Beam Method (DBM). A deep beam is defined by ACI 318-08 as having a clear span equal to or less than four times the overall depth of the beam or the regions with concentrated loads within twice the member depth from the face of the support. The truss analogy was first introduced during the late 1890's and early 1900's by W. Ritter and E. Morsch (Schlaich, Schafer, & Jennewein, 1987). This method was introduced as the appropriate and rational way to design cracked reinforced concrete through testing data by researchers. The STM is a modified version of the truss analogy which includes the concrete contribution through the concept of equivalent stirrup reinforcement. Once the concrete has cracked, the stresses are transferred to the horizontal and vertical steel across the crack and back into the concrete. This method, however, cannot be applied where geometrical or statical discontinuity occurred. In 1987, the Pre-stressed Concrete Institute Journal, PCI Journal, published a four part article on the truss analogy, "Towards a Consistent Design of Structural Concrete" by Jorg Schlaich, Kurt Schafer, and Mattias Jennewein, which generalized the truss analogy by proposing an analysis method in the form of STMs that are valid in all regions of the structure (Schlaich, Schafer, & Jennewein, 1987). The STM is included in the ACI code, ACI 318-08, found in Appendix A.

The more widely used approach by design professionals in the design of deep beams is through a nonlinear distribution of the strain, DBM, which is covered in ACI 318, Sections 10.2.2, 10.2.6, 10.7 and 11.7. Actual stresses of a deep beam are non-linear. Typically, a reinforced concrete beam is designed by a linear-elastic method of calculating the redistributed stresses after cracking. Applying the linear-elastic method to a deep beam revealed that the stresses determined were less than the actual stresses near the center of the span (Task Committee 426, 1973).

This report analyzes the behavior of transfer girders using both DBM and the STM, compares design results based on shear and flexure for both DBM and STM, and gives recommendations based on economical considerations, technical background, and constructability. The parametric study consists of three transfer girders with different loading designed using DBM and STM.

2.0 Background Information of Deep Beam Design

Definitions are provided for reference. These definitions can be found in the ACI 318-08 (Committee 318, 2008). After the definitions, a brief history of deep beam design is given to provide the reader a time line of design philosophies. Currently, ACI 318 does not provide equations for the design of non-linear stress distribution. The ACI code design assumptions "The strength of a member computed by the strength design method of the Code requires that two basic conditions be satisfied: (1) static equilibrium and (2) compatibility of strains." (Committee 318, 2008) For deep beams "an analysis that considers a nonlinear distribution of strain be used." The commentary references the user to three references for design of non-linear strain distribution: (1) "Design of Deep Girders", Portland Cement Association; (2) "Stresses in Deep Beams", ASCE; and (3) "Reinforced Concrete Structures", Park, R and Paulay, T. This report uses recommendations established by the Euro-International Concrete Committee which are in agreement with the cited references in the ACI 318-08.

2.1 Definitions for Deep Beams in ACI 318-08

B-region: "A portion of a member in which the plane sections assumption of flexural theory can be applied."

Discontinuity: "An abrupt change in geometry or loading."

D-region: "The portion of a member within a distance, h, from a force discontinuity or geometric discontinuity."

Deep Beam: "Deep beams are members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and supports and meet dimensional requirements."

Nodal Region: "The volume of concrete around a node that is assumed to transfer Strut-and-Tie forces through the node."

Node: "The point in a joint in a Strut-and-Tie model where the axes of the struts, ties, and concentrated forces acting on the joint intersect."

Strut: "A compression member in a Strut-and-Tie model. A strut represents the resultant of a parallel or a fan-shaped compression field."

Bottle Shaped Strut: "A strut that is wider at mid-length that at its end."

Strut-and-Tie Model: "A truss model of a structural member, or of a D-region in such a member, made up of struts and ties connected at nodes, capable of transferring the factored loads in the supports or to adjacent B-regions."

Tie: "A tension member in a Strut-and-Tie model."

2.2 A Brief History of Deep Beam Design

The truss analysis, STM theory began in the late 19th century. Wilhelm Ritter developed a truss mechanism to explain the contribution of stirrups to the shear strength of the beam in 1899. Ritter's mechanism did produce over conservative estimates because it neglected the tensile strength within the concrete. In 1927, Richart proposed that the concrete shear strength and the contribution of the steel stirrups be calculated independently then summed to determine the total shear strength, much like what is currently in the code (Brown & Oguzhan, 2008).

In 1962, tests determined the shear strength of reinforced concrete deep beams (Committee 326, 1962). The shear limits of reinforced concrete deep beams were proposed and are found in this report in Section 4.1 in Equations 4.5 and 4.7. From 1962-1973, major contributions in designing for shear were developed through numerous tests. Equations 4.10 and 4.11 in Section 4.1 are two equations developed for designing of deep beams for shear (Task

Committee 426, 1973). After shear design applications, researchers started studying other regions in reinforced concrete structures where STM theory could be used.

In the 1980's, STMs became very popular in the United States in academics and research. Professors J. Schlaich and P. Marti proposed modeling techniques around discontinuity regions (D-regions) where shear stresses and deformations are prominent (Brown & Oguzhan, 2008). "For many years, D-region design has been by "good practice," by rule of thumb or empirical. Three landmark papers by Professor Schlaich of the University of Stuttgard and his coworkers have changed this" (MacGregor & Wight, 2005).

Following their work, additional research was conducted to determine safe behavior models – design assumptions that provide satisfactory results shown by tests. In 1984, the Canadian code, The Canadian Standards Association, CSA, A23.3 was the first to adopt STM theory in North America. The American Association of State Highway and Transportation Officials, AASHTO, later accepted STM in 1989 for its Segmental Guide Specification and 1994 by the Bridge Design Specification. ACI first introduce STM theory in Appendix A in the ACI 318-02 Building Code Requirements for Structural Concrete, where it remains in the ACI 318-08 Building Code (Brown & Oguzhan, 2008).

3.0 Deep Beam

Transfer girders in structures are typically deep beams. A transfer girder supports the loads from columns above and transfers these loads to other support columns. A common location for a transfer girder is entrances for parking garages or other unique structures where large loads are applied to a structure with an opening at a column location (see Figures 3.1 and 3.2). "In general, deep beams are regarded as members loaded on their extreme fibers in compressions. Examples of this type of member are pile caps and transfer girders. Members loaded through the floor slabs or diaphragms are closer to the conditions that are idealized for shear walls" (Task Committee 426, 1973).

Figure 3.1 - Deep Beam, Brunswick Building, Chicago; picture courtesy of (columbia.edu)

Figure 3.2 - Single Span Deep Beam; picture courtesy of (MacGregor & Wight, 2005)

3.1 Code Requirements of Deep Beams

The American Concrete Institute has developed the *Building Code Requirements for Structural Concrete (ACI 318) and Commentary (ACI 318R)*. Code "does not contain detailed requirements for designing deep beams for flexure except that non-linearity of strain distribution and lateral buckling is to be considered." The code does contain the definition of deep beams, shear requirements which tends to govern the size (depth) of a deep beam, minimum area of flexural reinforcement, and minimum horizontal and vertical reinforcement on each face of deep beams in Sections 10.5, 10.6, 10.7, and 11.7. These sections require that deep beams be designed via nonlinear strain distribution or by using STM theory (Committee 318, 2008).

ACI 318-08, Section 10.7.1 states, "Deep beams are members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports." ACI 318 further defines deep beams as members with one of the following to be valid:

(a) the clear span, l_n, is equal to or less than four times the overall depth

$$\frac{l_n}{h} \leq 4.0 \qquad \text{(EQ'N 3.1)}$$

where:

h = overall member depth;

l_n = the clear span for distributed loads measures from the face of the support.

(b) or the regions with concentrated loads within twice the member depth from the face of the support.

$$\frac{a}{h} \leq 2.0 \qquad \text{(E'QN 3.2)}$$

where:

a = regions loaded with the concentrated loads from the face of the support.

4.0 Deep Beam Method

4.1 Shear Design using Deep Beam Method

While other beams are typically governed by requirements for flexural strength, deep beams are governed by requirements for shear strength. Therefore, the first type of failure that designers should consider when designing a deep beam is shear failure to determine the depth of the beam required for shear strength. Shear failure is “a failure under combined shearing force and bending moment, plus, occasionally, axial load, or torsion, or both” (Task Committee 426, 1973). In designing a shorter member, shear typically sets the minimum depth for the beam.

As the depth of a member increases, inclined cracking from shear or flexure tends to become steeper as shown in Figures 4.1 and 4.2. These steeper inclined cracks mean shear transfer mechanisms and shearing failures differ considerably from typical beams. The most common mode of shear failure is the crushing or shearing of the compression area over an inclined crack. This is typically started by cracking along the tensile reinforcement (Task Committee 426, 1973). Figures 4.1 and 4.2 illustrate cracking patterns for standard beams and deep beams respectively while demonstrating the crushing shear failure that can occur in deep beams.

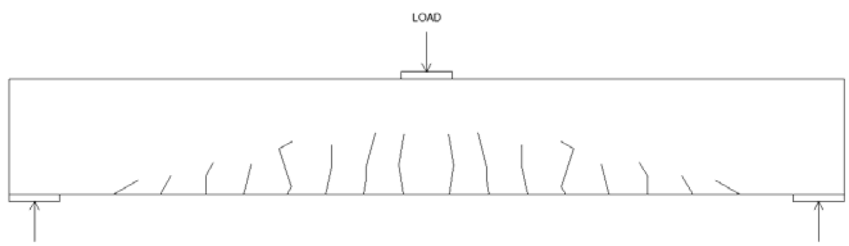


Figure 4.1 - Cracking along Tensile Reinforcement for Standard Beam

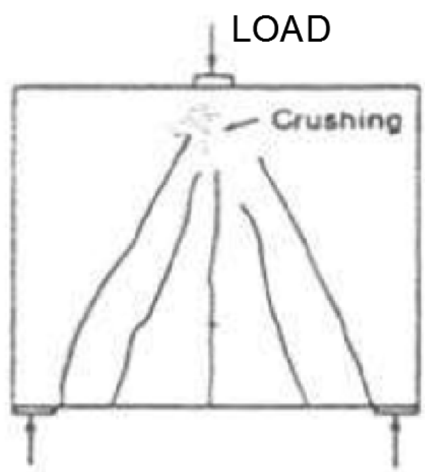


Figure 4.2 - Cracking Causing Crushing of Compression area for a Deep Beam; courtesy of (MacGregor & Wight, 2005)

The most important variable affecting the way a beam loaded with a concentrated load fails in shear is the ratio of *a/d*, the distance from the load to the edge of the support over the effective depth of the member as shown in Figure 4.3. Furthermore, this ratio can be expressed as *M/Vd*, where *M* is the ultimate moment, and *V* is the ultimate shear strength at the critical section of the beam (Task Committee 426, 1973). "This ratio recognizes the fact that a part of the shearing force is carried by the web reinforcement and part by the longitudinal steel. Failure of the beam is considered to occur when a failing stress is reached in the compression zone" (Sheikh, de Paiva, & Neville, 1971). A common characteristic of deep beams is a ratio of *M/V*d less than 2.5 (Task Committee 426, 1973). This is typically attributed to three things common to deep beams: a smaller moment, *M*; a larger effective depth, d; and a higher shear force, *V*.

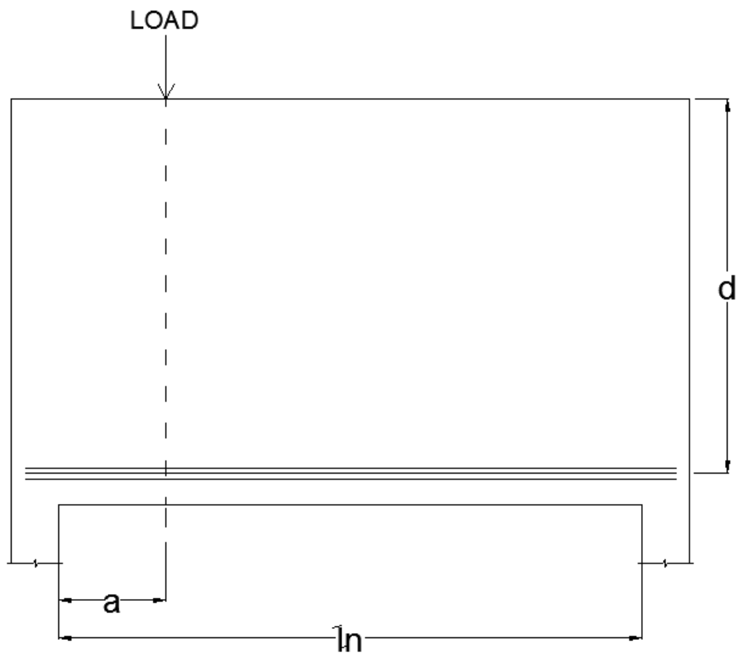


Figure 4.3 - Deep Beam Distances

For deep beams, as the ratio of a/d decreases from about 2.5 to 0, the shear reinforcement parallel to the force is less effective. Similarly, as the ratio a/d decreases to zero, the reinforcement perpendicular to the force being applied to the member increases the shear capacity through shear friction - concrete cracks are jagged and create an interlock between the two sides of the crack creating a friction called shear friction (Task Committee 426, 1973). The ratio of a/d decreases as the depth of the member increases. Thus, the cracks that form become steeper with increasing depth of the beam. Because the angle of the cracks has increased, the forces applied to the vertical shear reinforcement increase cause the vertical reinforcement to become less effective as shown in Figure 4.4b. Figure 4.4a indicates the forces in vertical reinforcement for a standard beam. The cracks form jaggedly, leaving plenty of edges to interlock (aggregate interlock), creating a large coefficient of friction between the two edges of the crack. The horizontal reinforcement holds these cracks together or keeps them from becoming too large, thus increasing the friction between the two edges and the efficiency of the horizontal reinforcement, shown in Figure 4.5.

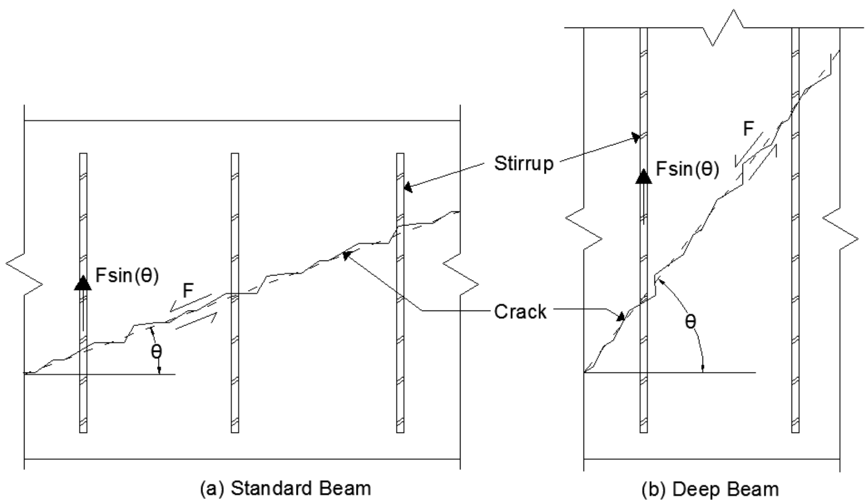


Figure 4.4 - Forces in Vertical Reinforcement Increase with Angle

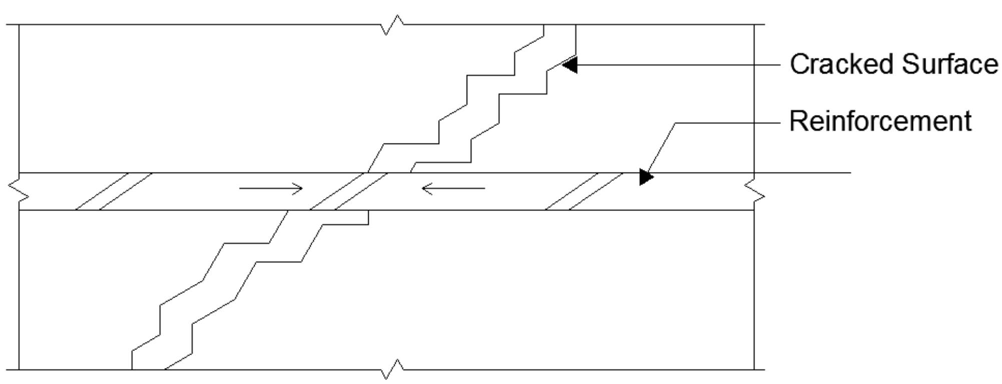


Figure 4.5 – Section of a Deep Beam Showing the Horizontal Reinforcement Resisting Cracking

The first step in determining the required shear capacity is to determine the shear in deep beams at the critical locations. To determine the critical locations of shear, Equations 4.1 and 4.2 are recommended by Hassoun and Al-Manaseer (Hassoun & Al-Manaseer, 2008).

(a) For a uniform load: $x = 0.15l_n \leq d$ (effective depth) (EQ'N 4.1)

(b) For a concentrated load: $x = 0.050a \leq d$ (effective depth) (EQ'N 4.2)

Figure 4.6 represents a deep beam with a distributed load across the beam or a concentrated load at a distance, a, from the edge of the support. The location of the critical section is identified with an x, as calculated using Equation 4.1 or 4.2.

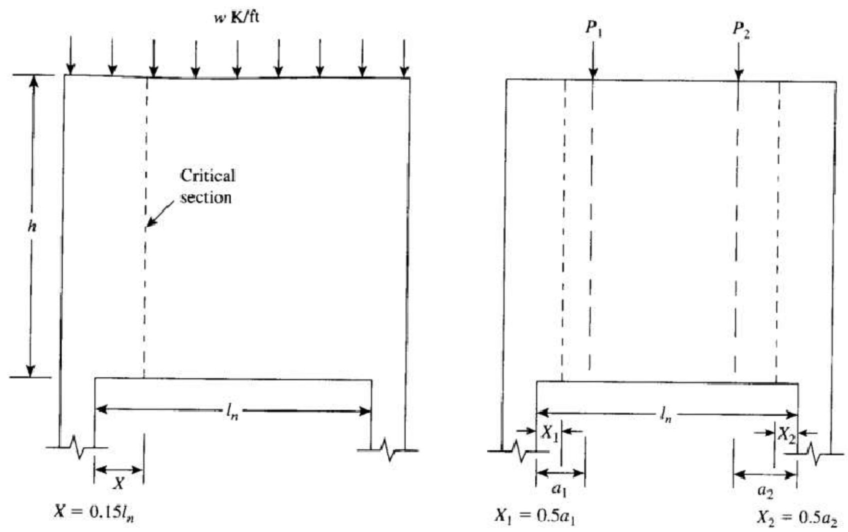


Figure 4.6 - Critical Locations for Shear; courtesy of (Hassoun & Al-Manaseer, 2008)

These locations have been determined to produce reasonable shear values for design and analysis which were determined through numerous tests (Hassoun & Al-Manaseer, 2008). When designing a typical beam, ACI 318-08 Section 11.1.3 allows the shear force used for design to be taken at a distance *d* from the support if it is a non-concentrated force and applied to produce compression at the end regions. "The loads applied to the beam between the face of the column and the point d away from the face are transferred directly to the support by compression in the web above the cracks" (Committee 318, 2008).

The design of concrete sections subject to shear are based on ACI 318-08

Equation 11-1 $\emptyset V_n \geq V_u$ (EQ'N 4.3)

where V_u is the factored shear force and V_n is the nominal shear strength computed by ACI 318-08 Equation 11-2 $\emptyset V_n = \emptyset(V_c + V_s)$ (EQ'N 4.4)

where V_c is the shear strength of the concrete and V_s is the added shear strength of the shear reinforcement.

The maximum ultimate shear limit for deep beams recommendation depends on the referenced code. M. Nadim Hassoun (Hassoun & Al-Manaseer, 2008) recommends the force $\emptyset V_n$ should satisfy either Equation 4.5, 4.6, or 4.7, whichever applies.

(a) For $l_n/d < 2$ $\quad \emptyset V_n \leq \emptyset 8\sqrt{f'_c}b_w d$ (EQ'N 4.5)

(b) For $2 \leq l_n/d \leq 5$ $\quad \emptyset V_n \leq \emptyset\left(\frac{2}{3}\right)\left(10+\frac{l_n}{d}\right)\sqrt{f'_c}b_w d$ (EQ'N 4.6)

(c) For $l_n/d > 5$ $\quad \emptyset V_n \leq \emptyset 10\sqrt{f'_c}b_w d$ (EQ'N 4.7)

where:

d= distance between the extreme compression fiber and centroid of tension reinforcement, taken no less than *0.8h, d > 0.8h* (Hassoun & Al-Manaseer, 2008);

h= overall depth or height of the beam;

b_w= width of web;

f'_c= 28 day compressive strength of the concrete;

$\emptyset$ = 0.75 per ACI 318-08 Section 9.3.2.3.

Older versions of ACI 318 had Equation 4.5 and anything above $l_n/d < 2$, Equation 4.7 was specified. Based on the data collected through beam testing and concrete strength tests, the nominal shear stress, V_n, was limited to $8\sqrt{f'_c}$ for $l_n/d < 2$ and up to $10\sqrt{f'_c}$ for $l_n/d > 5$ (Committee 326, 1962). As the length of the member increases, arching action and shear friction become more efficient because the angle of the transfer of forces through arching action decreases and the increased quantity of shear cracks producing shear friction. However, ACI 318-05 removed the aforementioned criteria and required all beams meeting the deep beam criteria use Equation 4.7, found in ACI 318-08 Section 11.7.3. This criteria is the same for non-deep beams.

ACI 318 provides two equations to use when determining the shear strength of a reinforced concrete beam subject to shear and flexure only: ACI 318-08 Equation 11-3 and 11-5. One equation allows for minor cracking and the other allows for no cracking. To determine the shear strength of concrete for a typical beam, a non-deep beam commonly used in structures, in shear and flexure only, use Equation 4.8 (ACI 318-08 Equation 11-5) if minor cracking is allowed and Equation 4.9 (ACI 318-08 Equation 11-3) if no cracking is allowed.

$$V_c = \left(1.9\lambda\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 3.5\lambda\sqrt{f'_c} b_w d \qquad \text{(EQ'N 4.8)}$$

where:

V_u= factored shear at critical location;

M_u= factored moment at critical location;

$\frac{V_u d}{M_u} \leq 1.0$;

λ= modification factor for weight of concrete;

ρ_w= ratio of A_s to $b_w d$.

$$V_c = 2\lambda\sqrt{f'_c} b_w d \qquad \text{(EQ'N 4.9)}$$

ACI 318 does not specify which equation to use when calculating the shear strength of the concrete for a deep beam. To determine the shear strength of the concrete for a *deep beam* in shear and flexure only, where no cracking is allowed, Equation 4.9 is used. To determine the shear strength where minor cracking is allowed, Equation 4.10 has been developed but is not included in the ACI 318-08. Equation 4.10 takes into account the effect of the factored moment and shear at the critical location into account. This equation is based on the work of Crist (Crist, Shear Behavior of Deep Reinforced Concrete Beams, v2 : Static Tests, October, 1967;Crist, Static and Dynamic Shear Behavior of uniformily Reinforced Concrete Deep Beams, 1971) and dePaiva (dePaiva & Seiss, 1965). "Their work led to the understanding of the reserve shear capacity of a deep beam without web reinforcement and the development of the concrete shear strength equation" (Task Committee 426, 1973).

$$V_c = \left(3.5 - \frac{2.5M_u}{V_u d}\right)\left(1.9\lambda\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 6\lambda\sqrt{f'_c} b_w d \qquad \text{(EQ'N 4.10)}$$

where: $1.0 < 3.5 - \frac{2.5M_u}{V_u d} \leq 2.5$.

In Equation 4.10, the factored shear and the factored moment at the critical location are used because the dead load shears and the moments may interact additively significantly decreasing the overall shear strength of the member at these locations (Task Committee 426, 1973). The second term in brackets,$\left(1.9\lambda\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right)$, which is identical to Equation 4.8, includes the inclined cracking shear while the first term, $\left(3.5 - \frac{2.5M_u}{V_u d}\right)$, represents the increase in the shear over the initial cracking because of the increased shear friction from longer cracks caused by factored shear and factored moment (Task Committee 426, 1973). The actual value for $\frac{V_u d}{M_u}$ is used but is not limited to being less than 1.0 like non-deep beams because the increased length of the crack and the shear reinforcement perpendicular to the force being applied produce higher shear friction capacity. Equation 4.10 is limited to a factor $6\sqrt{f'_c}$ as opposed to $3.5\sqrt{f'_c}$ as in Equation 4.8. The 3.5 factor limits the overall shear strength of the concrete to a reasonable value determined by researchers where cracked concrete will fail. The factor increased to 6 for deep beams because of the increased shear capacity from shear friction produced by increased shear crack length.

According to ACI 318-08 Section 11.4.6.1, where the ultimate shear being applied to the beam is higher than one-half of the design shear capacity of the concrete, steel shear reinforcement is required. This will never be the case for transfer girders because of the large shear forces applied on them. The shear force resisted by the shear reinforcement V_s is not specifically specified in ACI 318 for deep beams; however, ACI 318 does include design parameters for shear friction design method in Section 11.6.4. ASCE Task Committee 426 developed Equation 4.11 which includes the force along a known inclined crack using the shear friction of the concrete and the shear strength of the vertical reinforcement (Task Committee 426, 1973).

$$V_s = \left[\frac{A_v}{S_v}\frac{\left(1+\frac{l_n}{d}\right)}{12} + \frac{A_{vh}}{S_h}\frac{\left(11-\frac{l_n}{d}\right)}{12}\right] f_y d \qquad \text{(EQ'N 4.11)}$$

where:

A_v= total area of vertical shear reinforcement spaced at S_v in the horizontal direction at both faces of the beam;

A_{vh}= total area of horizontal reinforcement spaced at S_h in the vertical direction at both faces of the beam;

s_v= vertical spacing of shear reinforcement;

s_h= horizontal spacing of shear reinforcement.

Vertical reinforcement becomes less effective as the ratio of beam depth to span increases because of the increased angle of the cracks. The effectiveness of the horizontal shear strength increases as the shear friction in the beam increases. This is taken into account by using the relationship of the angle to the ratio of l_n/d in Equation 4.11. The derivation of Equation 4.11 is shown below (Task Committee 426, 1973):

Considering the forces acting along the inclined crack:

$S = F_{DT}tan\emptyset$ (EQ'N 4.12)

where:

F_{DT}= normal force on the inclined crack;

$tan\emptyset$= coefficient of friction (lower bound value of 1.0 is typically sued);

S= shear force along the crack.

Figure 4.7 is a graphical illustration of the shear force along the crack being calculated by the normal force to the crack multiplied by the coefficient of friction. θ represents the angle of the inclined crack to the longitudinal reinforcement.

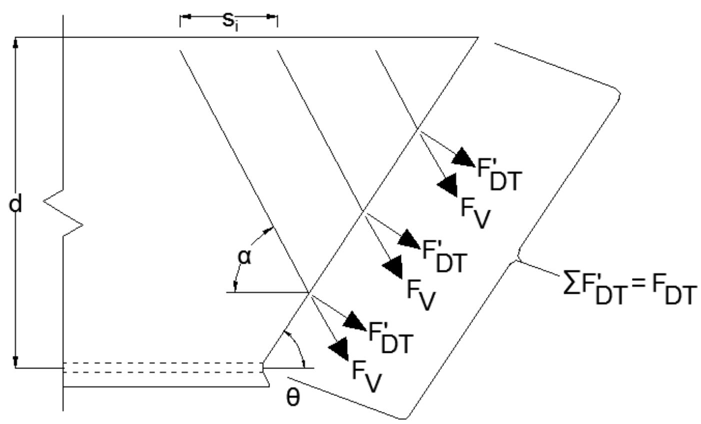


Figure 4.7 - Forces on Inclined Cracking Plane

The total transverse shear force acting at mid-length of the crack, the vertical component of the shear, assuming the shear force along the crack is uniformly distributed, is shown in Equation 4.13.

$V_v = Ssin\emptyset$

where: (EQ'N 4.13)

V_v= transverse resistance of the web reinforcement along the crack.

The normal forces on the inclined crack are assumed to develop through the tension in stirrups. The tension develops in the reinforcing crossing the inclined crack when slip occurs along the crack. When slip occurs, the crack width increases slightly because of the roughness of the crack, thus creating tensile stress in the reinforcing. Assuming that the stirrups are yielded at the ultimate load condition:

$$F_v = A_v f_y \qquad \text{(EQ'N 4.14)}$$

From the geometry of the forces in the stirrups:

$$F_{DT} = \sum(F'_{DT})_i = \sum F_{vi}\, sin(\alpha_i + \theta) \qquad \text{(EQ'N 4.15)}$$

Figure 4.8 represents the summation of the forces in the stirrups to determine the force perpendicular to the inclined crack, F_{DT}. α represents the angle of the stirrups to the longitudinal reinforcement and θ represents the angle of the inclined crack to the longitudinal reinforcement.

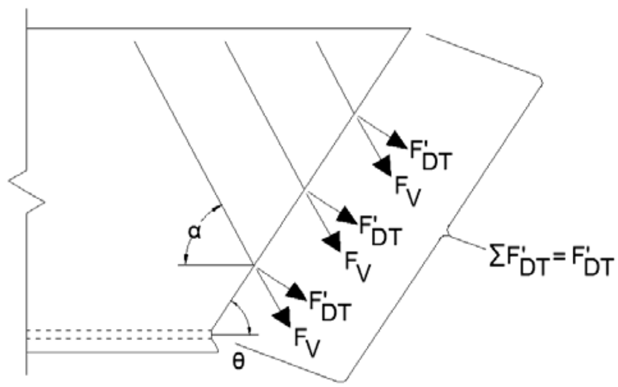


Figure 4.8 - Forces in Stirrups along inclined Crack

Equation 4.15 leads to Equations 4.16 and 4.17.

$$V_{sv} = \sum F_{vi} sin(\alpha_i + \theta) tan\emptyset sin\theta \qquad \text{(EQ'N 4.16)}$$

$$V_{sv} = \frac{A_{vi} f_{yi} d}{s_i} sin^2(\alpha_i + \theta) tan\emptyset \qquad \text{(EQ'N 4.17)}$$

Equation 4.17 represents the transverse capacity of a set of parallel web reinforcing crossing an inclined crack. Considering an arbitrary number of parallel sets of web reinforcing crossing an inclined crack, the transverse capacity is given by Equation 4.18.

$$V_v = d\ tan\theta \sum_{i=1}^{n} \left[\frac{A_{vi} f_{yi}}{s_i} sin^2(\alpha_i + \theta)\right] \qquad \text{(EQ'N 4.18)}$$

The subscript *i* corresponds to each set of parallel web reinforcing designated *i* = 1,2,…,n. In most cases where this equation is used, the shear reinforcement is placed into the member in both a vertical and a longitudinal direction perpendicular to each other.

$$\alpha_1 = \alpha_v = 90° \qquad \text{(EQ'N 4.19)}$$

$$\alpha_2 = \alpha_h = 0° \quad \text{(EQ'N 4.20)}$$

Equation 4.21 is the product of substituting Equation 4.19 and 4.20 into Equation 4.18 and assuming that all sets of web reinforcing have the same yield strength, f_{yi}.

$$V_s = f_y d\ tan\emptyset \left[\frac{A_v}{s} cos^2\theta + \frac{A_{vh}}{s_h} sin^2\theta\right] \quad \text{(EQ'N 4.21)}$$

where:

A_v= area of vertical shear reinforcement;

s= spacing of vertical shear reinforcement;

A_{vh}= area of horizontal shear reinforcemen;

s_h= spacing of horizontal shear reinforcement.

A relationship of θ as a function of l_n/d was determined through experimentation. A lower boundary of the test data is given by $cos^2\theta \frac{1}{12}\left(1 + \frac{l_n}{d}\right)$. Equation 4.22 uses trigonometry identities and the relationship mentioned with Equation 4.21.

$$V_s = f_y d\ tan\emptyset \left[\frac{A_v}{S_v}\frac{\left(1+\frac{l_n}{d}\right)}{12} + \frac{A_{vh}}{S_h}\frac{\left(11-\frac{l_n}{d}\right)}{12}\right] \quad \text{(EQ'N 4.22)}$$

ACI uses this equation assuming that the coefficient of frictions, tanΦ , equals 1.0 while Crist originally suggested that tanΦ =1.5 (Rogowsky & MacGregor, 1983).

$$V_s = \left[\frac{A_v}{S_v}\frac{\left(1+\frac{l_n}{d}\right)}{12} + \frac{A_{vh}}{S_h}\frac{\left(11-\frac{l_n}{d}\right)}{12}\right] f_y d \quad \text{(EQ'N 4.11)}$$

ACI 318-08 also specifies a maximum spacing of vertical and horizontal reinforcement for deep beams. The maximum on center spacing for either is 12 inches or d/5, whichever is smaller, to limit the location of where cracks can occur or restrain the width of the cracks, which is especially important when considering horizontal reinforcement. The shear strength of deep beams relies on the shear friction of the concrete after it cracks. If the cracks become too large, the friction and bearing between the two edges of the crack will reduce significantly thus decreasing the shear strength of the beam considerably.

The vertical shear reinforcement requires keeping a maximum on center spacing of 12 inches or d/5 to help restrain the width of the cracks, but mainly the spacing ensures reinforcement will be present when a crack forms. Cracks become steeper as the ratio of depth to clear span increases, thus reducing how far across the length of the beam a crack will spread;

reducing the spacing reinforcement ensures a crack will be crossed by reinforcement. Figure 4.9 represents a crack that is unreinforced which is these requirements are trying to prevent.

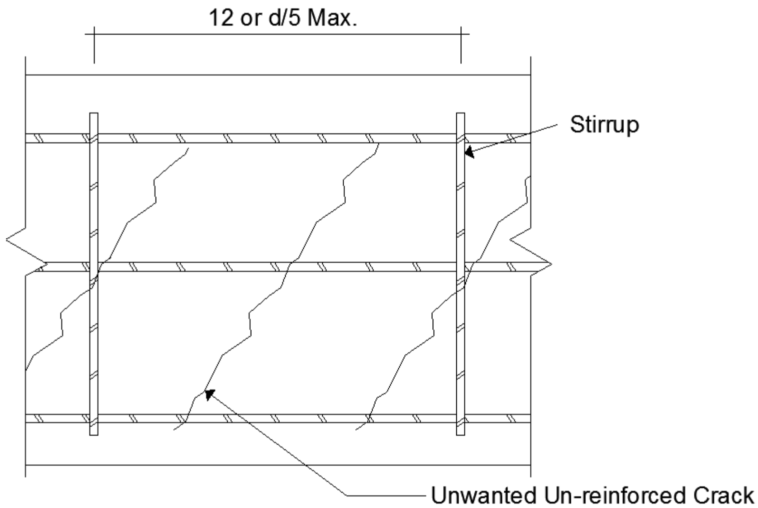


Figure 4.9 - Unwanted Un-Reinforced Crack

These spacing requirements for shear reinforcement can be found in ACI 318-08, Sections 11.7.4 and 11.7.5.

$$S_v \leq d/5 \leq 12in \quad \text{(EQ'N 4.23)}$$

$$S_h \leq d/5 \leq 12in \quad \text{(EQ'N 4.24)}$$

ACI 318-08 specifies a minimum horizontal and vertical shear reinforcement area, A_{vh} and A_v respectively, in Sections 11.7.4 and 11.7.5 which should be used throughout the member as the following:

$$A_{vh} = 0.0015 b_w S_h \quad \text{(EQ'N 4.25)}$$

$$A_v = 0.0025 b_w S_v \quad \text{(EQ'N 4.26)}$$

4.2 Flexure Design using Deep Beam Method

The flexural design of a deep beam is similar to a typical beam with a few changes to the internal moment arm and location of the tension reinforcement. The factored nominal strength, $\emptyset M_n$ must be greater that the factored applied moment, Mu. The design flexural strength is calculated using Equation 4.27.

$$\emptyset M_n = \emptyset A_s F_y jd \qquad \text{(EQ'N 4.27)}$$

where:

j= is a dimensionless ratio used to define the lever arm, jd. It varies because of varying loads;

jd= the modified internal moment arm because of non-linearity of the strain distribution, the distance between the resultant compressive force and the resultant tensile force;

$\emptyset$= 0.9 for tension controlled members per ACI 318-08 Section 9.3.2.1.

Figure 4.10 represents a deep beam and the non-linear stress distribution. *C* is the resultant compression force and *T* is the resultant tensile force. The depth of the compression block is represented by *c* and *y* represents jd which is the internal moment arm.

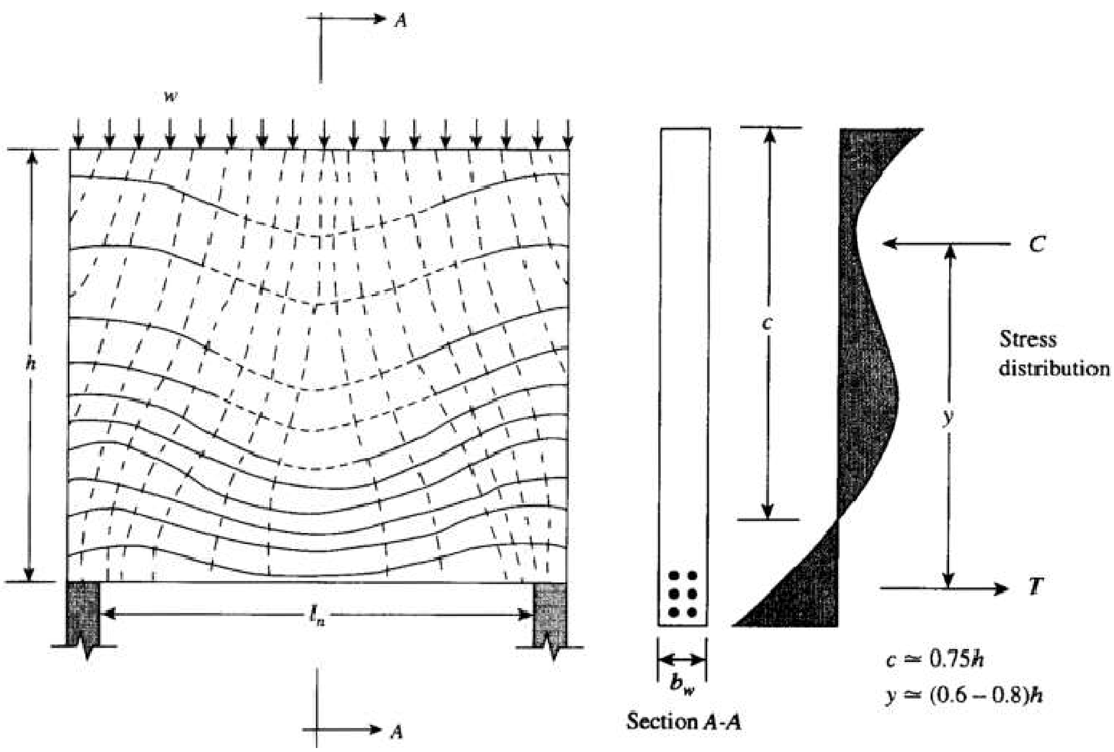


Figure 4.10 - Non-Linear Stress Distribution; courtesy of (Hassoun & Al-Manaseer, 2008)

To determine the amount of flexural steel required, the design flexural strength is set equal to the factored moment, M_u, and Equation 4.27 is rearranged to solve for required area of steel, A_s. ACI 318 limits the amount of steel that can be used to ensure a ductile failure. The minimum steel requirements can be found in ACI 318-08 Equation 10-3, given here as Equation 4.28.

$$A_s = \frac{M_u}{\emptyset f_y jd} \geq \frac{3\sqrt{f'_c} b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \quad \text{(EQ'N 4.28)}$$

The recommended lever arm by CEB (Euro-International Concrete Committee, Comite Euro-International du Beton) is shown in Equations 4.29 and 4.30. These equations take into account the non-linear strain distribution which is required by ACI 318 rather than determining the stresses directly. These values were determined through testing of deep beams.

$$jd = 0.2(l + 2h)\ for\ 1 \leq l/h < 2 \quad \text{(EQ'N 4.29)}$$

$$jd = 0.6l\ for\ l/h < 1 \quad \text{(EQ'N 4.30)}$$

where:

$l =$ effective span measured center to center of supports or $1.15 l_n$, whichever is smaller

Tension reinforcement should be evenly spaced along the face from the base of the beam to the height specified in Equation 4.31, which was determined through testing by the CEB (Kong, Robins, & Sharp, 1975). For a typical beam with a depth greater than 36 inches, skin reinforcement is required to extend to h/2 from the tension face to control cracking per ACI 318-08 Section 10.6.7. The reinforcement distributed on the face helps control cracking. Without this reinforcement the width of the cracks in the web may exceed the allowable crack widths at the flexural tension reinforcement. Prior to 1999, the ACI Code limits for crack control were based on a maximum crack width of 0.016 inch for interior exposure and 0.013 inch for exterior exposure (MacGregor & Wight, 2005). The role cracks have in corrosion of reinforcement is controversial as research has shown that the two do not clearly correlate, thus, the exterior exposure requirement has been eliminated (Committee 318, 2008). ACI 318 has specified a maximum spacing of the flexure reinforcement at the face of the beam to keep cracks within the crack limits. Multiple bars of a smaller diameter are better than one bar in crack control. ACI 318-08 Equation 10-4, given as Equation 4.32, specifies the maximum spacing the flexural reinforcement is allowed

$$y = 0.25h - 0.05l < 0.2h \quad \text{(EQ'N 4.31)}$$

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right) \qquad \text{(EQ'N 4.32)}$$

where:

c_c= least distance from the surface of reinforcement steel to the tension face

f_s= permitted to be taken as 2/3f_y per ACI 318-08 Section 10.6.4

4.3 Deep Beam Method Design Examples

To accurately compare the final design, three simply supported girders with equal clear spans and different loading patterns were designed. The first girder had a clear span of sixteen feet and a width of 24 inches with a column bearing point at the center 1200 kip factored load. The second example was the same as the first except the location of the factored load changed to five feet from the centerline of the right hand support. The third example had the same dimensional constraints with two point loads: a column bearing at the center, with a factored load of 600 kips, and a second column at the quarter point of the girder with a factored load of 600 kips. These two loads equal the total point load applied on the first two examples. Each girder was designed to have #5 bar shear reinforcement spacing of 8 to 10 inches. The maximum allowed shear spacing according to Equation 4.26 is 10.33 inches with #5 bars and a beam width of 24 inches. Normal weight concrete with a 28 day concrete compression strength equal to 4,000 psi and yield strength of the reinforcing bars equal to 60,000 psi is used.

4.3.1 Deep Beam Design Example 1

Design example one is a 24 inch wide transfer girder spanning 16 feet with a column at mid-span with a total factored load of 1,200 kips. The girder is supported by 24 inch square columns. An overall beam depth of 7 feet was determined by design. Figure 4.11 indicates the transfer girder for design.

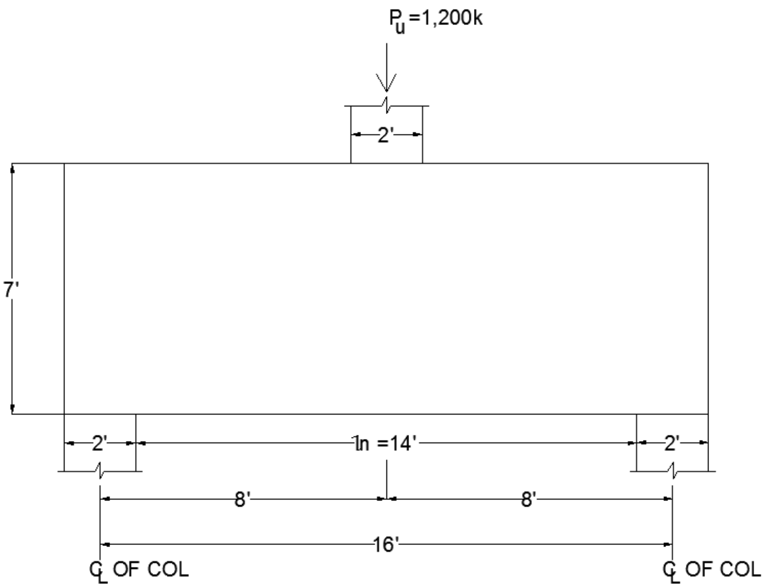


Figure 4.11 – Deep Beam Method Design Example 1

h = 7 ft   f’$_c$ = 4,000 psi F_y = 60,000 psi b_w = 24 inches

Step 1: Check for Deep Beam Criteria

$\frac{l_n}{h} \leq 4.0$ $\frac{14'}{7'} = 2.0 \leq 4.0$ Deep Beam   (EQ’N 3.1)

$\frac{a}{h} < 2.0$ $\frac{7'}{7'} = 1.0 < 2.0$ Deep Beam   (EQ’N 3.2)

Step 2: Determine Flexural Reinforcement

Determine the applied ultimate moment.

Weight of the girder = $w = 150pcf \times 7' \times {}^{24"}/_{12"} = 2{,}100plf$

Factored weight of girder = $w_u = 1.2 \times 2{,}100plf = 2{,}520plf$

$M_u = \frac{Pl}{4} + \frac{w_u l^2}{8}$ $M_u = \frac{(600k+600k)16' \times 12"}{4} + \frac{2.52klf(16' \times 12")^2}{8} = 69{,}212\ k - in$

Determine the area of steel required for a moment capacity higher than the applied moment.

$A_s = \frac{M_u}{\emptyset f_y jd} \geq \frac{3\sqrt{f'_c} b_w d}{f_y} \geq \frac{200 b_w d}{f_y}$ (EQ’N 4.28)

where: $jd = 0.2(l + 2h)\ for\ 1 \leq l/h < 2$ (EQ’N 4.29)

$jd = 0.6l\ for\ l/h < 1$ (EQ’N 4.30)

l = smaller of c/c of supports (16') or $1.15l_n$ $(1.15 \times 14' = 16.1)$

l = 16 ft

$$1 \leq \frac{l}{h} = \frac{16'}{7'} = 2.29 > 2 \qquad \text{(E'QN 4.18)}$$

Use Equation 4.29 to account for non-linear stress distribution, conservatively

$$jd = 0.2(l + 2h) = 0.2(16' + 2 \times 7') \times 12" = 72in$$

$$A_{s,req'd} = \frac{M_u}{\emptyset f_y jd} = \frac{69{,}212\ k\text{-}in}{0.9(60ksi)(72")} = 17.80in^2$$

Try 18 -#9 bars. $A_{s\,(18)\,\#9}$ = 18.0 in^2

Determine the flexural reinforcement location.

$$y = 0.25h - 0.05l < 0.2h$$

$$y = 0.25(84") - 0.05(16' \times 12") = 11.4in < 0.2(84") = 16.8in \qquad \text{(EQ'N 4.31)}$$

y = 11.4in ≈ 12in

Figure 4.12 represents the flexural reinforcement of 3 rows spaced 4.5" on center of 6 #9 bars spaced 4" on center. The maximum allowable spacing allowed by ACI 318-08 is determined from Equation 4.32.

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right) \qquad \text{(EQ'N 4.32)}$$

$$s = 15\left(\frac{40{,}000}{\left(\frac{2}{3}\right)60{,}000psi}\right) - 2.5\left(3" - \frac{1.128"}{2} - 0.625"\right) = 10.4" \leq 12\left(\frac{40{,}000}{\left(\frac{2}{3}\right)60{,}000psi}\right) = 12"$$

10.4" > 4" OK

The minimum allowable spacing by ACI 318-08 Section 7.6 is d_b but no less than 1".

4" > 1.128" OK

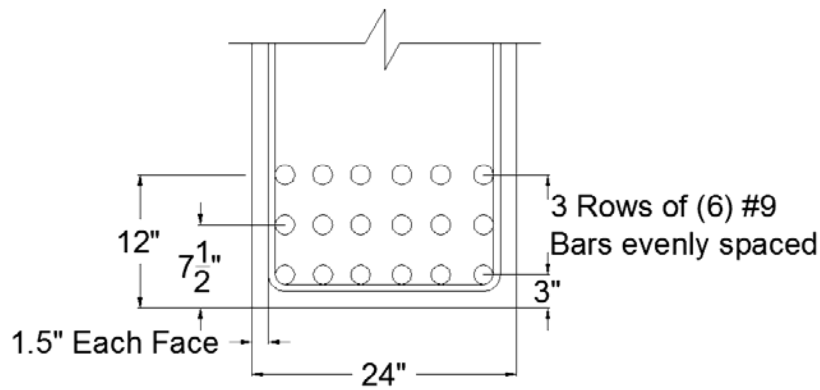


Figure 4.12 - Design Example 1 - Flexural Reinforcement

Determine actual flexural reinforcement depth d.

$d = 84"\text{-}7.5" = 76.5"$

Check the area of steel required against minimum steel requirements.

$\frac{3\sqrt{f'_c}b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \qquad \frac{3\sqrt{4{,}000psi}(24")(76.5")}{60{,}000psi} = 5.81 in^2 \leq \frac{200(24")(76.5")}{60{,}000psi} = 6.12 in^2$

18.0 in^2 > 6.12 in^2 OK

Use 18 -#9 bars. $A_{s\,(18)\,\#9}$ = 18.0 in^2

Step 3: Determine Shear Reinforcement

Draw the ultimate shear diagram shown in Figure 4.13.

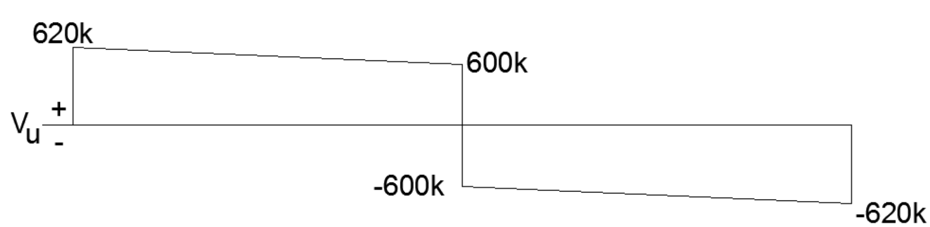


Figure 4.13 – Deep Beam Method Design Example 1 - Shear Diagram

Find critical shear locations.

x = 0.5a ≤ d (effective depth) 0.5(7’x12”) = 42 in ≤ 76.5 in (EQ’N 4.2)

Determine loads at critical section.

$V_{u,x}$= 620k – (2.52klf) x (42"/12") = 611k

$M_{u,x}$= (611k x 42"/12") + 0.5(620k-611k)(42"/12") = 2,154 k-ft

Determine upper limit on shear strength.

Maximum allowable $\emptyset V_n \leq \emptyset 10\sqrt{f'_c} b_w d$ (EQ'N 4.7)

$\emptyset V_n \leq (0.75)10\sqrt{4{,}000psi}(24)(76.5)/1000\# = 870.9k > 611k \quad OK$

Determine Nominal Shear Strength provided by concrete with minor cracking allowed.

$V_c = \left(3.5 - \frac{2.5M_u}{V_u d}\right)\left(1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 6\sqrt{f'_c} b_w d$ (EQ'N 4.10)

$1.0 < 3.5 - \frac{2.5M_u}{V_u d} \leq 2.5 \qquad 1.0 < 3.5 - \frac{2.5(2{,}154k-ft\times 12")}{611k(76.5")} = 2.12 \leq 2.5$

$V_c = (2.12)\left(1.9\sqrt{4{,}000psi} + 2500\frac{18in^2}{(24)(76.5")}\frac{611{,}000^{\#}(76.5")}{2{,}154{,}000^{\#-ft}\times 12"}\right)\frac{(24")(76.5")}{1{,}000} = 640.2k$

$6\sqrt{f'_c} b_w d = 6\sqrt{4{,}000psi}\frac{(24")(76.5")}{1{,}000} = 696.7k$

640.2k < 696.7k OK

Determine Horizontal and Vertical Shear Reinforcement with Minor Cracking Allowed.

$V_u > \frac{\emptyset V_c}{2} \qquad 611k > \frac{0.75(640.2k)}{2} = 240k \quad Shear\ Reinf. Required$ (EQ'N 4.3)

$V_u \leq \emptyset(V_c + V_s)$ (EQ'N 4.4)

$V_s = \frac{V_u}{\emptyset} - V_c \qquad V_s = \frac{611k}{0.75} - 640.2k = 174.5k$

$V_s = \left[\frac{A_v}{S_v}\frac{\left(1+\frac{l_n}{d}\right)}{12} + \frac{A_{vh}}{S_h}\frac{\left(11-\frac{l_n}{d}\right)}{12}\right] f_y d$ (EQ'N 4.11)

Try an S_v = S_h spacing of 10 inches on center with No.5 bars

$V_s = \left[\frac{0.62in^2}{10"}\frac{\left(1+\frac{14'\times 12"}{76.5"}\right)}{12} + \frac{0.62in^2}{10"}\frac{\left(11-\frac{14'\times 12"}{76.5"}\right)}{12}\right](60ksi)(76.5") = 285k > 174.5k \quad OK$

Check minimum shear reinforcement requirement.

$A_{vh} = 0.0015 b_w S_h = 0.0015(24")(10") = 0.36in^2 < 0.62in^2 \quad OK$ (EQ'N 4.25)

$A_v = 0.0025 b_w S_v = 0.0025(24")(10") = 0.60in^2 < 0.62in^2 \quad OK$ (EQ'N 4.26)

$S_v \leq d/5 \leq 12in \qquad \frac{76.5"}{5} = 15.3" > 12" \qquad 10" < 12" \quad OK$ (EQ'N 4.23)

$S_h \leq d/5 \leq 12in \qquad \frac{76.5"}{5} = 15.3" > 12" \qquad 10" < 12" \quad OK$ (EQ'N 4.24)

Use #5 bars at 10 inches on center both vertically and horizontally.

Note: If a 6' girder were used, the flexural reinforcement would be (19) #9 bars and the shear reinforcement would be #5's at 7" vertical and horizontal. The 7' girder was used to keep the shear reinforcement closer to the maximum allowable spacing and to make an easier comparison between the Strut-and-Tie Example 1 design which has a girder height of 7' as well.

Cross sections of the completed design of the girder are shown in Figures 4.14 and 4.15 with dimensions and reinforcement.

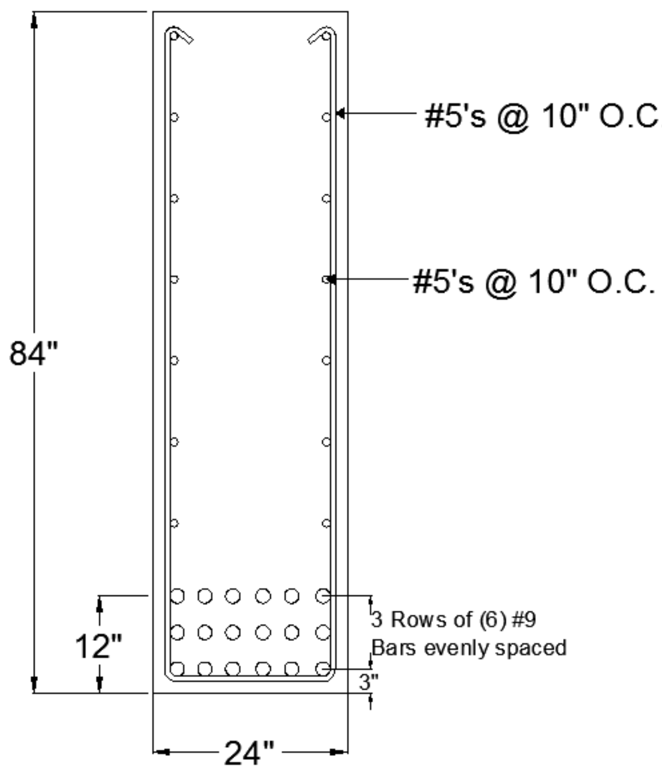


Figure 4.14 – Deep Beam Method Design Example 1 – End Cross Section

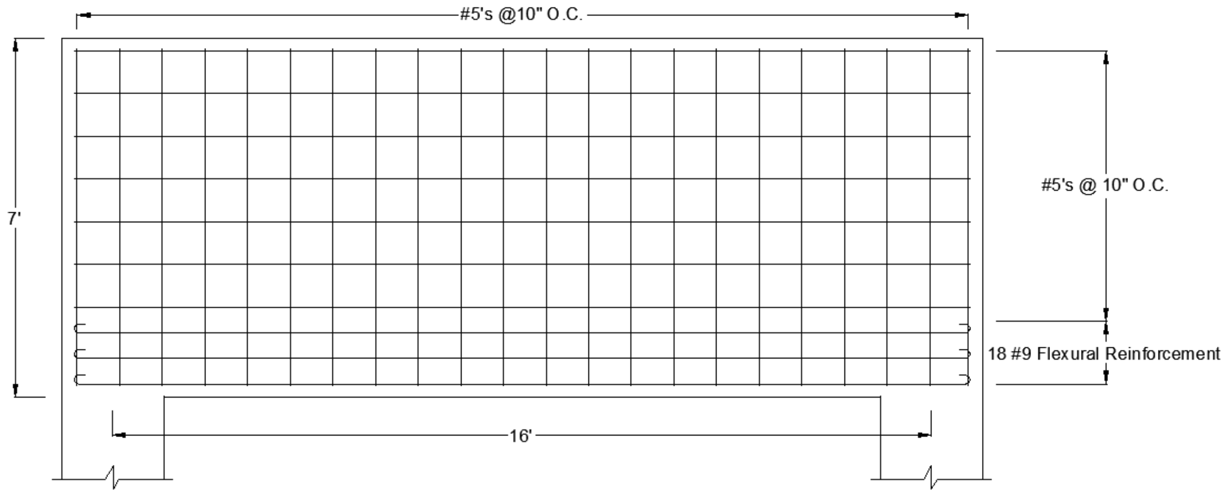


Figure 4.15 – Deep Beam Method Design Example 1 - Longitudinal Section 2

4.3.2 Deep Beam Design Example 2

Design example two is a 24 inch wide transfer girder spanning 16 feet with a column at 5 feet from a support with a factored load of 1,200 kips. The girder is supported by 24 inch square columns. A design height of 8 feet was determined by trial-and-error. Figure 4.16 indicates the transfer girder for design.

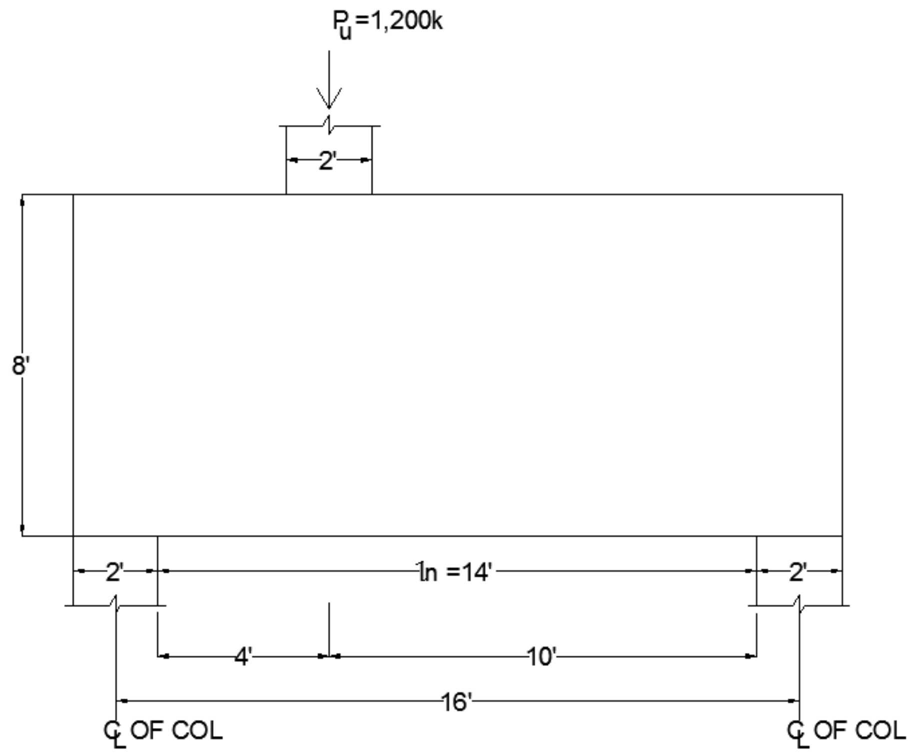


Figure 4.16 – Deep Beam Method Design Example 2

h = 8 ft f'_c = 4,000 psi F_y = 60,000 psi b_w = 24 inches

<u>Step 1: Check for Deep Beam Criteria</u>

$\frac{l_n}{h} \leq 4.0$ $\frac{14'}{8'} = 1.75 \leq 4.0$ Deep Beam   (EQ'N 3.1)

$\frac{a}{h} < 2.0$ $\frac{5'}{8'} = 0.63 < 2.0$ Deep Beam   (EQ'N 3.2)

<u>Step 2: Determine Flexural Reinforcement</u>

Draw the ultimate shear diagrams shown in Figure 4.17.

Weight of the girder = $w = 150pcf \times 8' \times {}^{24"}/_{12"} = 2{,}400plf$

Factored weight of girder = $w_u = 1.2 \times 2{,}400plf = 2{,}880plf$

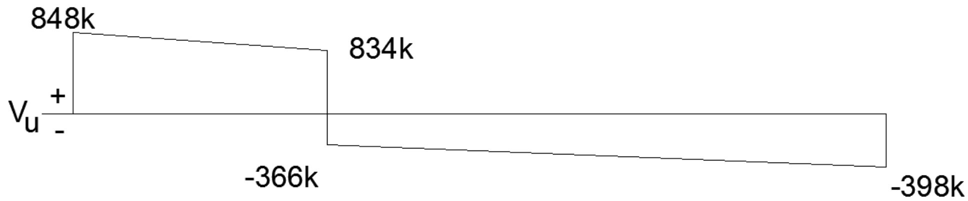


Figure 4.17 – Deep Beam Method Design Example 2 - Shear Diagram

Determine the applied ultimate moment.

$M_u = [834k(5') + 0.5(848k - 834k)(5')] \times 12" = 50{,}460\ k - in$

Determine the area of steel required for a moment capacity higher than the applied ultimate moment.

$A_s = \frac{M_u}{\emptyset f_y jd} \geq \frac{3\sqrt{f'_c} b_w d}{f_y} \geq \frac{200 b_w d}{f_y}$ (EQ'N 4.28)

where: $jd = 0.2(l + 2h)\ for\ 1 \leq l/h < 2$ (EQ'N 4.29)

$jd = 0.6l\ for\ l/h < 1$ (EQ'N 4.30)

l = smaller of c/c of supports (16') or $1.15l_n$ ($1.15 \times 14' = 16.1$)

l = 16 ft

$1 \leq \frac{l}{h} = \frac{16'}{8'} = 2.0 \leq 2$ (E'QN 4.18)

$jd = 0.2(l + 2h) = 0.2(16' + 2 \times 8') \times 12" = 76.8in$

$A_{s,req'd} = \frac{M_u}{\emptyset f_y jd} = \frac{50{,}460\ k-in}{0.9(60ksi)(76.8")} = 12.17in^2$

Try 16 -#8 bars. $A_{s\ (16)\ \#8}$ = 12.64 in^2

Determine the flexural reinforcement location.

$y = 0.25h - 0.05l < 0.2h$

$y = 0.25(96") - 0.05(16' \times 12") = 14.4in < 0.2(96") = 19.2in$ (EQ'N 4.31)

y = 14.4in ≈ 15in

Figure 4.18 represents the flexural reinforcement of 4 rows spaced 3" on center of 4 #8 bars spaced at 6.5". The maximum allowable spacing allowed by ACI 318-08 is determined from Equation 4.32.

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right) \qquad \text{(EQ'N 4.32)}$$

$$s = 15\left(\frac{40{,}000}{\left(\frac{2}{3}\right)60{,}000psi}\right) - 2.5\left(3" - \frac{1.0"}{2} - 0.625"\right) = 10.3" \leq 12\left(\frac{40{,}000}{\left(\frac{2}{3}\right)60{,}000psi}\right) = 12"$$

10.3" > 6.5" OK

The minimum allowable spacing by ACI 318-08 Section 7.6 is d_b but no less than 1".

4" > 1" OK

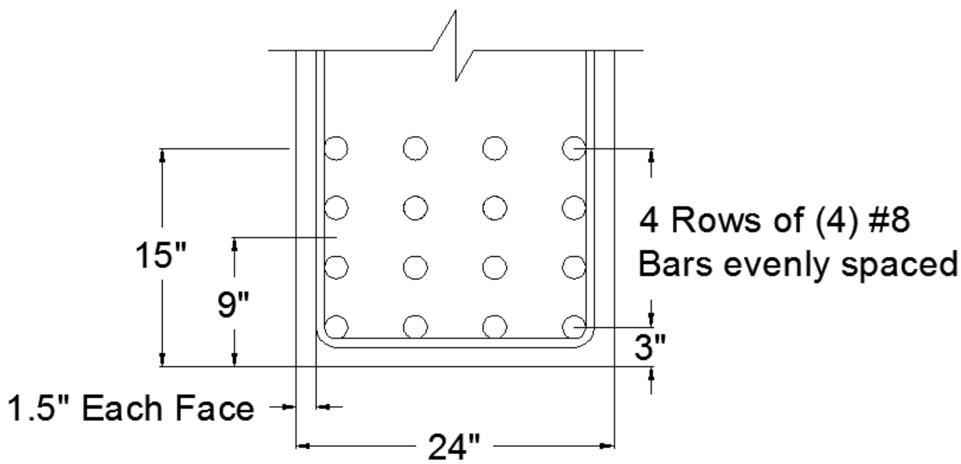


Figure 4.18 – Deep Beam Method Design Example 2 - Flexural Reinforcement

Determine actual flexural reinforcement depth d.

$$d = 96"\text{-}9" = 87"$$

Check the area of steel required against minimum steel requirements.

$$\frac{3\sqrt{f'_c}b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \qquad \frac{3\sqrt{4{,}000psi}(24")(87")}{60{,}000psi} = 6.60in^2 \leq \frac{200(24")(87")}{60{,}000psi} = 6.96in^2$$

12.64 in^2 > 6.96 in^2 OK

Use 16 -#8 bars. $A_{s\,(16)\,\#8}$ = 12.64 in^2

Step 3: Determine Shear Reinforcement

Find critical shear locations.

x = 0.5a ≤ d (effective depth) 0.5(4'x12") = 24 in ≤ 87 in (EQ'N 4.2)

Determine loads at critical section.

$V_{u,x}$= 848k – (2.88klf) x (24"/12") = 842k

$M_{u,x}$= (842k x 24"/12") + 0.5(848k-842k)(24"/12") = 1,690 k-ft

Determine upper limit on shear strength.

Maximum allowable $\emptyset V_n \leq \emptyset 10\sqrt{f'_c} b_w d$ (EQ'N 4.7)

$\emptyset V_n \leq (0.75)10\sqrt{4{,}000 psi}(24)(87")/1000\# = 990.4k > 842k \quad OK$

Determine Nominal Shear Strength provided by concrete with minor cracking allowed.

$V_c = \left(3.5 - \frac{2.5M_u}{V_u d}\right)\left(1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 6\sqrt{f'_c} b_w d$ (EQ'N 4.10)

$1.0 < 3.5 - \frac{2.5M_u}{V_u d} \leq 2.5 \qquad 1.0 < 3.5 - \frac{2.5(1{,}690k - ft \times 12")}{842k(87")} = 2.81 > 2.5$ use 2.5

$$V_c = (2.5)\left(1.9\sqrt{4{,}000psi} + 2500\frac{12.64in^2}{(24)(87")}\frac{842{,}000^{\#}(87")}{1{,}690{,}000^{\#-ft} \times 12"}\right)\frac{(24")(87")}{1{,}000} = 912.6k$$

$$6\sqrt{f'_c} b_w d = 6\sqrt{4{,}000psi}\,\frac{(24")(87")}{1{,}000} = 792.3k$$

912.6k > 792.3k Use 792.3k

Determine Horizontal and Vertical Shear Reinforcement with Minor Cracking Allowed.

$V_u > \frac{\emptyset V_c}{2} \qquad 842k > \frac{0.75(792.3k)}{2} = 297k \quad Shear\ Reinf.Required$ (EQ'N 4.3)

$V_u \leq \emptyset(V_c + V_s)$ (EQ'N 4.4)

$V_s = \frac{V_u}{\emptyset} - V_c \qquad V_s = \frac{842k}{0.75} - 792.3k = 330k$

$V_s = \left[\frac{A_v}{S_v}\frac{\left(1+\frac{l_n}{d}\right)}{12} + \frac{A_{vh}}{S_h}\frac{\left(11-\frac{l_n}{d}\right)}{12}\right] f_y d$ (EQ'N 4.11)

Try an $S_v = S_h$ spacing of 9 inches on center with No.5 bars

$$V_s = \left[\frac{0.62in^2}{9"}\frac{\left(1+\frac{14'\times 12"}{87"}\right)}{12} + \frac{0.62in^2}{9"}\frac{\left(11-\frac{14'\times 12"}{87"}\right)}{12}\right](60ksi)(87") = 360k > 330k \quad OK$$

Check minimum shear reinforcement requirement.

$A_{vh} = 0.0015 b_w S_h = 0.0015(24")(9") = 0.32in^2 < 0.62in^2 \quad OK$ (EQ'N 4.25)

$A_v = 0.0025 b_w S_v = 0.0025(24")(9") = 0.54in^2 < 0.62in^2 \quad OK$ (EQ'N 4.26)

$S_v \leq d/5 \leq 12in \qquad \frac{87"}{5} = 17.4" > 12" \qquad 9" < 12" \quad OK$ (EQ'N 4.23)

$S_h \leq d/5 \leq 12in \qquad \frac{87"}{5} = 17.4" > 12" \qquad 9" < 12" \quad OK$ (EQ'N 4.24)

Use #5 bars at 9 inches on center both vertically and horizontally.

Note: If a 7' girder were used, the flexural reinforcement would be (17) #8 bars and the shear reinforcement would be #5's at 6" vertical and horizontal. The 8' girder was used to keep the shear reinforcement closer to the maximum allowable spacing and to make an easier comparison between the Strut-and-Tie Example 2 design which has a girder height of 8' as well.

Cross sections of the completed design of girder example #2 are shown in Figures 4.19 and 4.20 with dimensions and reinforcement.

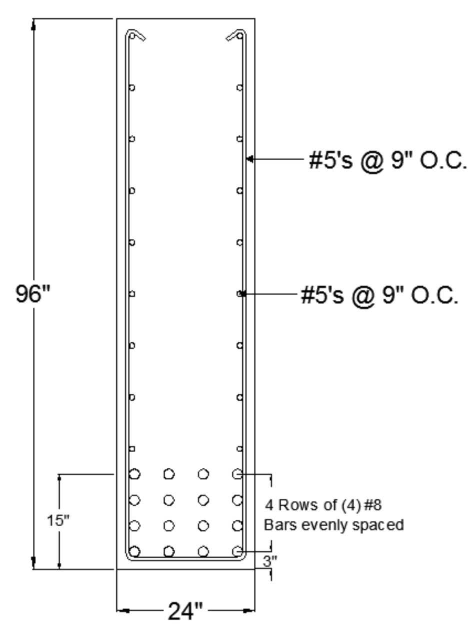


Figure 4.19 – Deep Beam Method Design Example 2 – End Cross Section

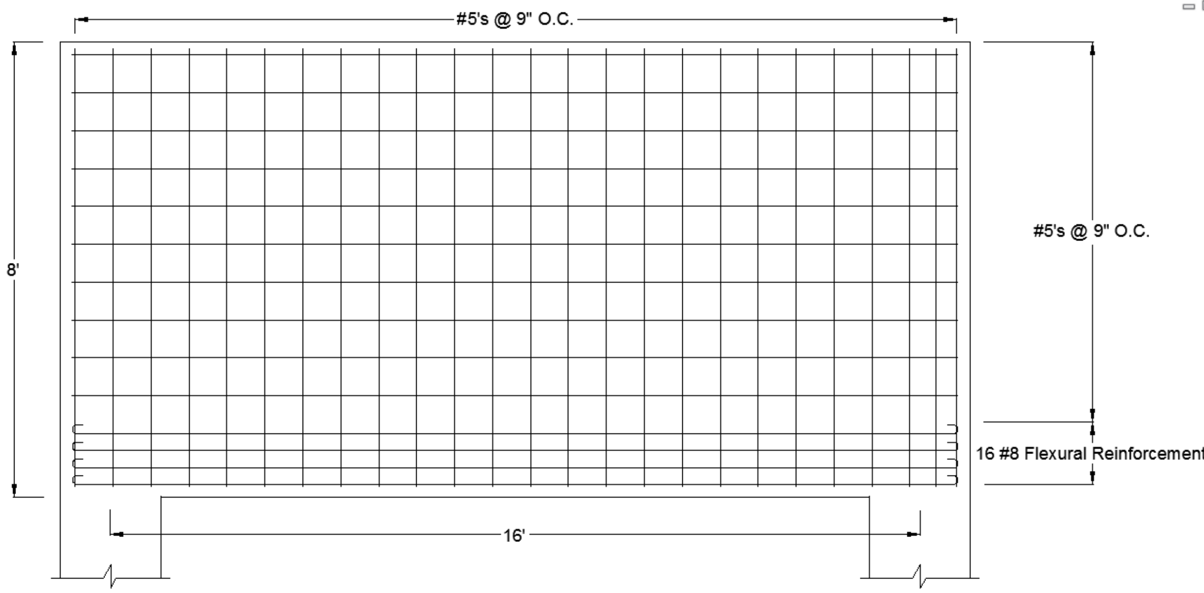


Figure 4.20 – Deep Beam Method Design Example 2 - Longitudinal Section

4.3.3 Deep Beam Design Example 3

Design example three is a 24 inch wide transfer girder spanning 16 feet with a column at midpoint with a factored load of 600 kips; and a second column load at the quarter point with a factored load of 600 kips. The girder is supported by 24 inch square columns. A design height of 7 feet was determined by iteration. Figure 4.21 indicates the transfer girder for design.

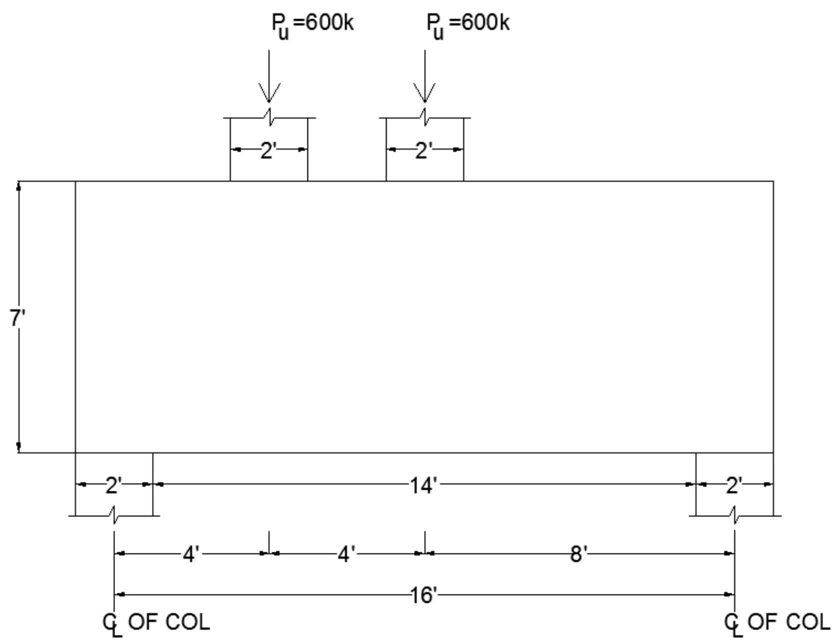


Figure 4.21 – Deep Beam Method Design Example 3

h = 7 ft f'_c = 4,000 psi F_y = 60,000 psi b_w = 24 inches

<u>Step 1: Check for Deep Beam Criteria</u>

$\frac{l_n}{h} \leq 4.0$ $\frac{14'}{7'} = 2.0 \leq 4.0$ Deep Beam   (EQ'N 3.1)

$\frac{a}{h} < 2.0$ $\frac{3'}{7'} = 0.43 < 2.0$ Deep Beam   (EQ'N 3.2)

<u>Step 2: Determine Flexural Reinforcement</u>

Draw the ultimate shear diagram shown in Figure 4.22.

Weight of the girder = $w = 150pcf \times 7' \times {}^{24"}/_{12"} = 2{,}100plf$

Factored weight of girder = $w_u = 1.2 \times 2{,}100plf = 2{,}520plf$

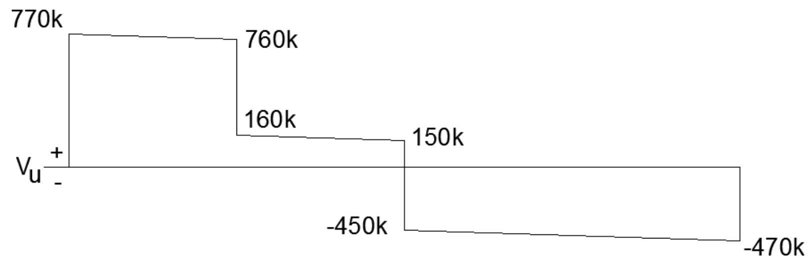


Figure 4.22 – Deep Beam Method Design Example 3 - Shear Diagram

Determine the applied ultimate moment.

$M_u = [760k(4') + 0.5(770k - 760k)(4') + 150k(4') + 0.5(160k - 150k)(4')] \times 12" = 44{,}160\ k - in$

Determine the area of steel required for a moment capacity higher than the applied ultimate moment.

$$A_s = \frac{M_u}{\emptyset f_y jd} \geq \frac{3\sqrt{f'_c} b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \quad \text{(EQ'N 4.28)}$$

where: $jd = 0.2(l + 2h)\ for\ 1 \leq l/h < 2$ (EQ'N 4.29)

$jd = 0.6l\ for\ l/h < 1$ (EQ'N 4.30)

l = smaller of c/c of supports (16') or $1.15l_n$ $(1.15 \times 14' = 16.1)$

l = 16 ft

$1 \leq \frac{l}{h} = \frac{16'}{7'} = 2.3 > 2$ (E'QN 4.18)

Conservatively use Equation 4.29 to account for non-linear stress distribution

$jd = 0.2(l + 2h) = 0.2(16' + 2 \times 7') \times 12" = 72in$

$$A_{s,req'd} = \frac{M_u}{\emptyset f_y jd} = \frac{44{,}160\ k-in}{0.9(60ksi)(72")} = 11.36in^2$$

Try 15-#8 bars. $A_{s\,(15)\,\#8}$ = 11.85 in^2

Determine the flexural reinforcement location.

$y = 0.25h - 0.05l < 0.2h$

$y = 0.25(84") - 0.05(16' \times 12") = 11.4in < 0.2(84") = 16.8in$ (EQ'N 4.31)

y = 11.4in ≈ 12in

Figure 4.23 represents the flexural reinforcement of 3 rows spaced 4.5” on center of 5 #8 bars spaced at 5”. The maximum allowable spacing allowed by ACI 318-08 is determined from Equation 4.32.

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right) \quad \text{(EQ'N 4.32)}$$

$$s = 15\left(\frac{40{,}000}{\left(\frac{2}{3}\right)60{,}000psi}\right) - 2.5\left(3" - \frac{1.0"}{2} - 0.625"\right) = 10.3" \leq 12\left(\frac{40{,}000}{\left(\frac{2}{3}\right)60{,}000psi}\right) = 12"$$

10.3” > 5” OK

The minimum allowable spacing by ACI 318-08 Section 7.6 is d_b but no less than 1”.

4” > 1” OK

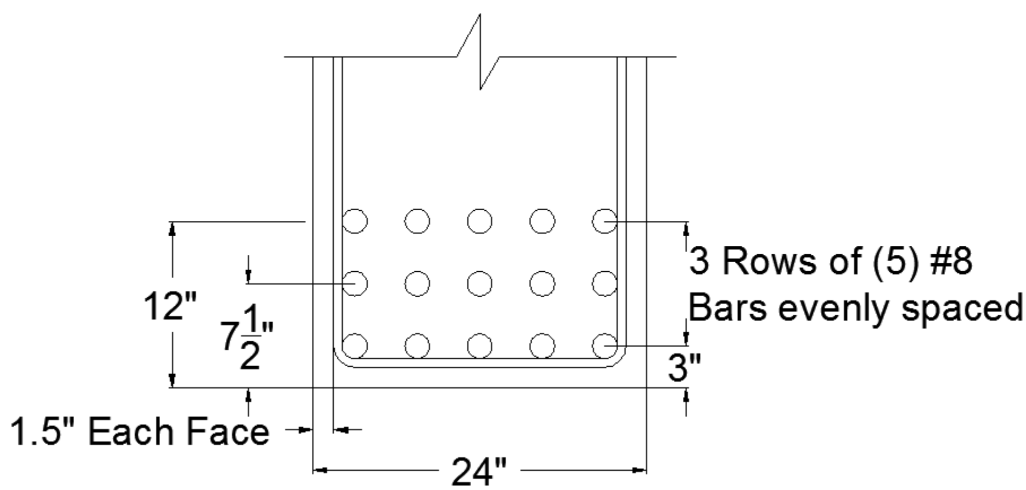


Figure 4.23 – Deep Beam Method Design Example 3 - Flexural Reinforcement

Determine actual flexural reinforcement depth d.

$d = 84"\text{-}7.5" = 76.5"$

Check the area of steel required against minimum steel requirements.

$\frac{3\sqrt{f'_c}b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \qquad \frac{3\sqrt{4{,}000psi}(24")(76.5")}{60{,}000psi} = 5.81in^2 \leq \frac{200(24")(76.5")}{60{,}000psi} = 6.12in^2$

11.85 in^2 > 6.12 in^2 OK

Use 15 -#8 bars. $A_{s\,(15)\,\#8}$ = 11.85 in^2

Step 3: Determine Shear Reinforcement

Find critical shear locations.

x = 0.5a ≤ d (effective depth) 0.5(3'x12") = 18 in ≤ 87 in (EQ'N 4.2)

Determine loads at critical section.

$V_{u,x}$= 770k – (2.52klf) x (18"/12") = 766k

$M_{u,x}$= (766k x 18"/12") + 0.5(770k-766k)(18"/12") = 1,152 k-ft

Determine upper limit on shear strength.

Maximum allowable $\emptyset V_n \leq \emptyset 10\sqrt{f'_c}b_w d$ (EQ'N 4.7)

$$\emptyset V_n \leq (0.75)10\sqrt{4{,}000psi}(24)(76.5")/1000\# = 871k > 766k \quad OK$$

Determine Nominal Shear Strength provided by concrete with minor cracking allowed.

$$V_c = \left(3.5 - \frac{2.5M_u}{V_u d}\right)\left(1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 6\sqrt{f'_c}b_w d \qquad \text{(EQ'N 4.10)}$$

$$1.0 < 3.5 - \frac{2.5M_u}{V_u d} \leq 2.5 \qquad 1.0 < 3.5 - \frac{2.5(1{,}152k-ft \times 12")}{766k(76.5")} = 2.91 > 2.5 \quad \text{use 2.5}$$

$$V_c = (2.5)\left(1.9\sqrt{4{,}000psi} + 2500\frac{11.85in^2}{(24)(76.5")}\frac{766{,}000^{\#}(76.5")}{1{,}152{,}000^{\#-ft} \times 12"}\right)\frac{(24")(76.5")}{1{,}000} = 865.5k$$

$$6\sqrt{f'_c}b_w d = 6\sqrt{4{,}000psi}\frac{(24")(76.5")}{1{,}000} = 696.7k$$

865.5k > 696.7k Use 696.7k

Determine Horizontal and Vertical Shear Reinforcement with Minor Cracking Allowed.

$$V_u > \frac{\emptyset V_c}{2} \qquad 842k > \frac{0.75(696.7k)}{2} = 261k \quad Shear\ Reinf.Required \qquad \text{(EQ'N 4.3)}$$

$$V_u \leq \emptyset(V_c + V_s) \qquad \text{(EQ'N 4.4)}$$

$$V_s = \frac{V_u}{\emptyset} - V_c \qquad V_s = \frac{766k}{0.75} - 696.7k = 325k$$

$$V_s = \left[\frac{A_v}{S_v}\frac{\left(1+\frac{l_n}{d}\right)}{12} + \frac{A_{vh}}{S_h}\frac{\left(11-\frac{l_n}{d}\right)}{12}\right] f_y d \qquad \text{(EQ'N 4.11)}$$

From reiterative design process try an $S_v = S_h$ spacing of 8 inches on center with No.5 bars.

$$V_s = \left[\frac{0.62in^2}{8"}\frac{\left(1+\frac{14'\times12"}{76.5"}\right)}{12} + \frac{0.62in^2}{8"}\frac{\left(11-\frac{14'\times12"}{76.5"}\right)}{12}\right](60ksi)(76.5") = 356k > 325k \quad OK$$

Check minimum shear reinforcement requirement.

$A_{vh} = 0.0015b_wS_h = 0.0015(24")(8") = 0.29in^2 < 0.62in^2 \quad OK$ (EQ'N 4.25)

$A_v = 0.0025b_wS_v = 0.0025(24")(8") = 0.48 < 0.62in^2 \quad OK$ (EQ'N 4.26)

$S_v \leq d/5 \leq 12in \qquad \frac{76.5"}{5} = 15.3" > 12" \qquad 8" < 12" \quad OK$ (EQ'N 4.23)

$S_h \leq d/5 \leq 12in \qquad \frac{76.5"}{5} = 15.3" > 12" \qquad 8" < 12" \quad OK$ (EQ'N 4.24)

Use #5 bars at 8 inches on center both vertically and horizontally.

Note: A 6' girder does not meet the requirements of Equation 4.7. If a 8' girder were used, the flexural reinforcement would be (14) #8 bars and the shear reinforcement would be #5's at 10" vertical and horizontal limited by maximum spacing requirements. The 7' girder was used to keep the shear reinforcement closer to the maximum allowable spacing without having more reinforcement that required for strength and to make an easier comparison between the Strut-and-Tie Example 3 design which has a girder height of 7' as well.

Cross sections of the completed design of the girder are shown in Figures 4.24 and 4.25 with dimensions and reinforcement.

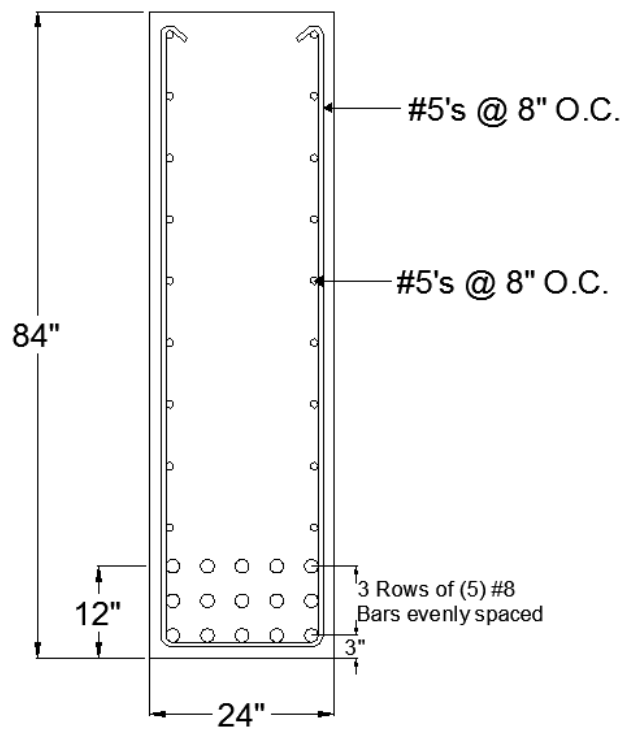


Figure 4.24 – Deep Beam Method Design Example 3 – End Cross Section

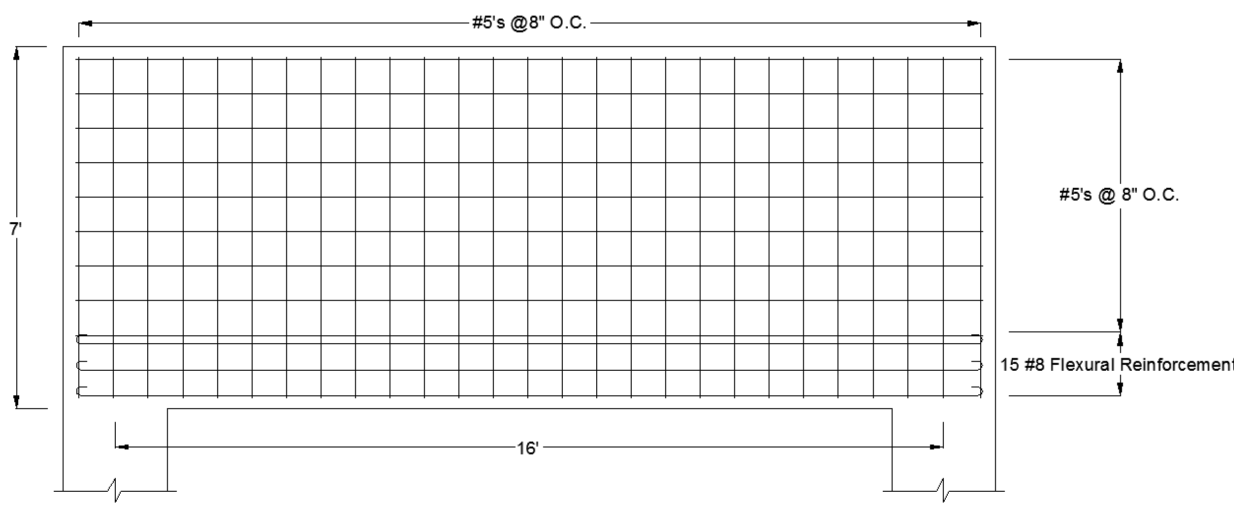


Figure 4.25 – Deep Beam Method Design Example 3 – Longitudinal Cut Section

5.0 Strut-and-Tie Model

The second analysis method allowed by ACI 318 for the design of deep beams is STM. STMs comprise compression struts and tension ties that transfer the forces through the member, through the joints referred to as nodes, and to the supports; as opposed to DBM which transfers the force through shear reinforcement and an internal moment couple with flexural reinforcement. Both design processes have benefits and should be considered when designing deep beams.

Before cracking has occurred in a reinforced concrete beam, an elastic stress field exists. Cracking disturbs the stress field causing the internal forces to alter their path. These reoriented forces can be modeled as an STM (MacGregor & Wight, 2005). The STM analysis evaluates stresses as either compression (struts) or tension members (steel ties) and joins the struts and ties through nodes and nodal regions (Schlaich, Schafer, & Jennewein, 1987). After inclined cracks have formed in deep beams, the beam takes on a "tied arch" behavior allowing the forces to transfer directly to the supports, not vertically through the member until being transferred by the web and flexural reinforcement. This behavior provides some reserve shear capacity in deep beams but not in shallower members. Shallow beams generally fail shortly after inclined cracks form unless flexural reinforcement is provided (Rogowsky & MacGregor, 1983). Figure 5.1 represents a deep beam with a point load applied on the compression face. 5.1(a) illustrates the struts and the ties used for design to transfer a point load to the supports and 5.1(b) represents a uniformly loaded beam with a parabolic STM.

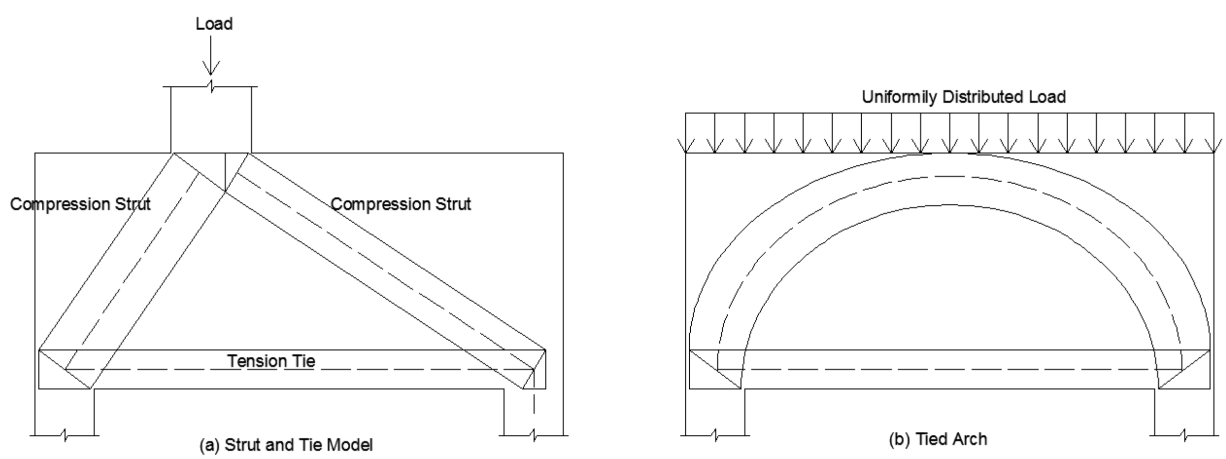


Figure 5.1 - Stut-and-Tie Model and Tied Arch Illustrations

In testing, the stresses in the tension chord reinforcement decreased much less at the ends of the girder, indicating that the steel acts as a tension tie that carries a relatively constant force from one end of the girder to the other, thus confirming the methodology of the STM (Rogowsky & MacGregor, 1983). The STM was developed as a practical way to design for discontinuity regions where non-linear, elastic behavior occurs (commonly referred to as D-Regions). ACI 318-08 Section 11.7.2 allows the use of STM for the design of deep beams. Deep beams typically are used as girders with a discontinuity region caused by a large point load.

5.1 Discontinuity Regions

Members within a structure have discontinuity regions, D-regions, and beam regions also known as Bernoulli regions, B-regions. B-regions are locations where beam theory applies in which linear strain is assumed valid and the internal stress due to bending and torsional moments, shear, and axial forces are easily derived (Schlaich, Schafer, & Jennewein, 1987). D-regions are locations near concentrated loads, adjacent to holes, where abrupt changes in cross section or direction occur, and reactions. At these locations, the distribution of the strain is nonlinear and difficult to calculate (Schlaich, Schafer, & Jennewein, 1987). Figure 5.2 illustrates where D-regions and B-regions occur in members within a structure.

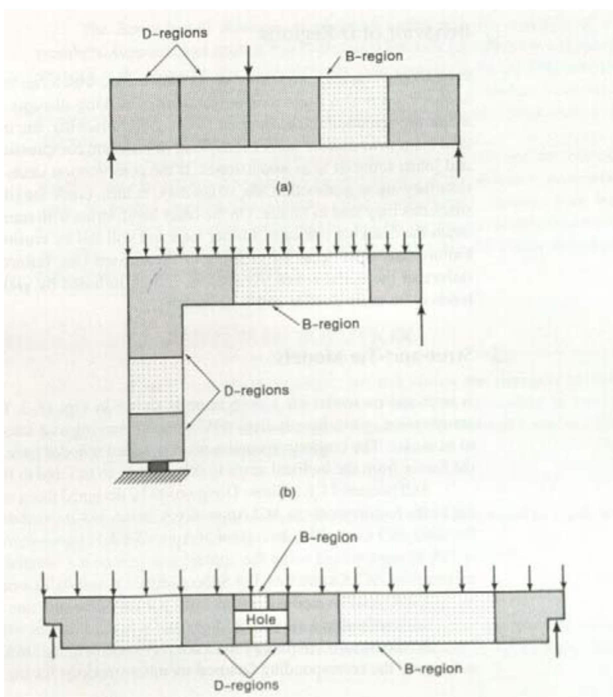


Figure 5.2 - D – Regions; courtesy of (MacGregor & Wight, 2005)

When using the STM approach dividing the structure into B-regions and D-regions is helpful. This specifies where in the structure a non-linear analysis of the stress trajectories is required (Schlaich, Schafer, & Jennewein, 1987). To identify where these regions start and end, Saint Venant's Principle is used. Saint Venant's Principle states that strains produced by a force statically equivalent to zero force and zero couple to a small part of a surface of a body are negligible at distances which are large compared to the small part of the body the force was applied. This suggests that the localized effect of discontinuity dissipates approximately one member depth distance, h, each way from the discontinuity. This principle is not precise; thus, the different stiffness formed by unequal resistance to deformation in different directions due to the unsymmetrical cracks along reinforced concrete members may influence the distance at which the D-regions end is not a concern (Schlaich, Schafer, & Jennewein, 1987). Figure 5.3 illustrates the area D-regions occupy after concentrated loads and reactions.

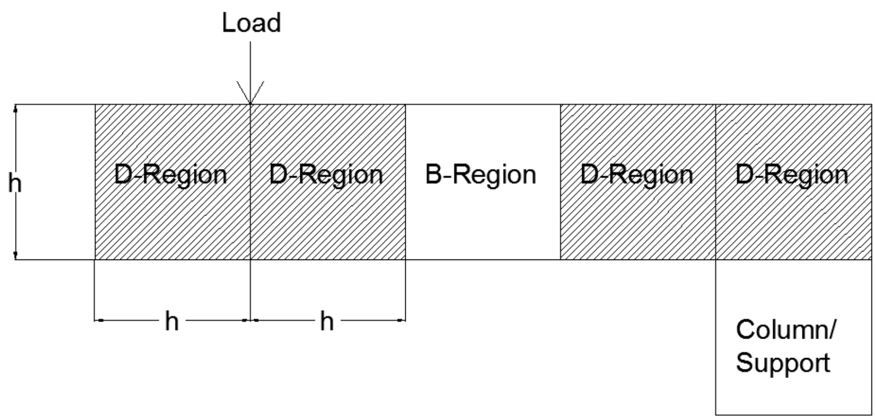


Figure 5.3 - D-Region Distances

5.2 Struts and Ties

A strut represents the compression stress zone within the STM from one nodal zone to the next. The compression stress acts parallel within the strut, which typically follows a load path similar to a force diagram or moment diagram. The struts are typically idealized as a prismatic or linear member within the deep beam even though struts typically vary in cross section throughout the length of the strut to simplify the analysis of STM. As the stresses transfer through the strut, they spread out forming a bottle shaped strut before condensing to enter the nodal zone. As the stresses spread out, transverse tension forces arise that can produce longitudinal cracking. If reinforcement is not provided to transfer the stresses after cracking has occurred or to keep cracking from occurring, the member or structure may fail after cracking. Once cracking has occurred, the internal stresses reorient to transfer to the supports. Without reinforcement to transfer the stresses over the cracks, the stresses could redistribute to a different load paths and consolidate causing concrete crushing and ultimately failing the member. With adequate reinforcement, the strength of the strut directly relates to the crushing strength of the concrete (MacGregor & Wight, 2005). If the crushing strength becomes an issue during design, compression reinforcement can be added to the struts to increase strength allowing smaller nodal regions as well as struts. Figure 5.4 illustrates the struts as bottle shaped struts as well as the idealized prismatic strut transferring the force to the supports directly through the nodes and nodal regions.

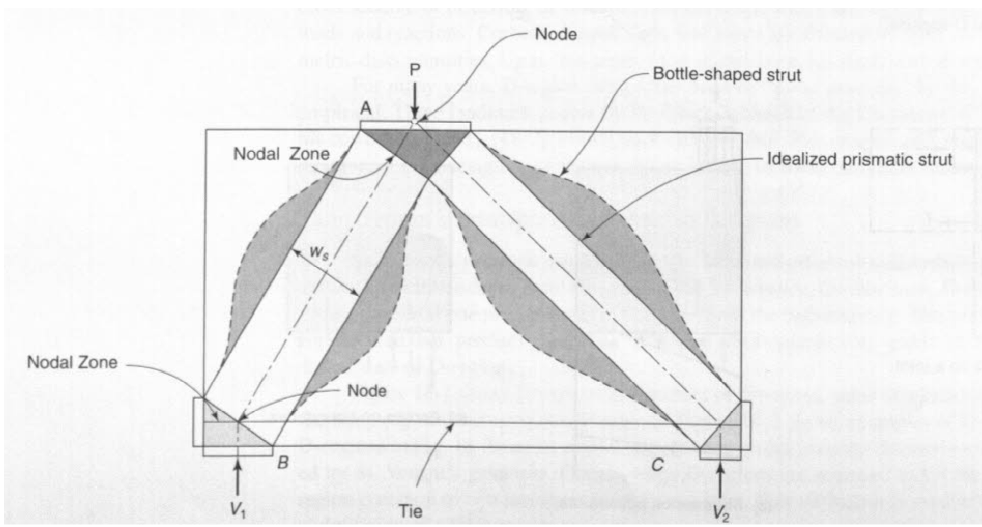


Figure 5.4 - Strut Diagram; courtesy of (MacGregor & Wight, 2005)

The ties consist of reinforcement as well as the surrounding concrete. The concrete does not contribute to the resistance of forces but does increase the axial stiffness of the tie through tension stiffening which is the capacity of the bonded concrete between neighboring cracks to transfer tension through bond slip between the reinforcement and concrete causing the area to act more like an uncracked section by contributing to the flexural stiffness, EI. The concrete helps transfer loads from the struts to the ties or to bearing area by bonding with the reinforcement (MacGregor & Wight, 2005). The most important part of the tie design is the detailing of the end anchorage in the nodal regions. Sufficient anchorage can be produced through bonding/tension splices, hooks, or mechanical anchorage.

5.3 Nodes and Nodal Zones

The nodes are idealized pinned joints where the forces meet from the struts and ties. The nodal zone is the surrounding body of concrete that transfers the load from the struts to the ties or supports. Because these joints are idealized as pinned joints, they must be at static equilibrium. This implies that the forces must pass through a common point, or the forces can be resolved around a certain point to remain in equilibrium. At nodal regions, at least three forces must keep the node at equilibrium because the forces come into the node at different angles. These nodal regions are classified as C-C-C for three compressive forces, C-C-T for two compressive forces and one tensile force, C-T-T for one compressive force and two tensile forces, or T-T-T for three tensile forces (MacGregor & Wight, 2005). Figure 5.5 represents the four nodal regions in static equilibrium specified.

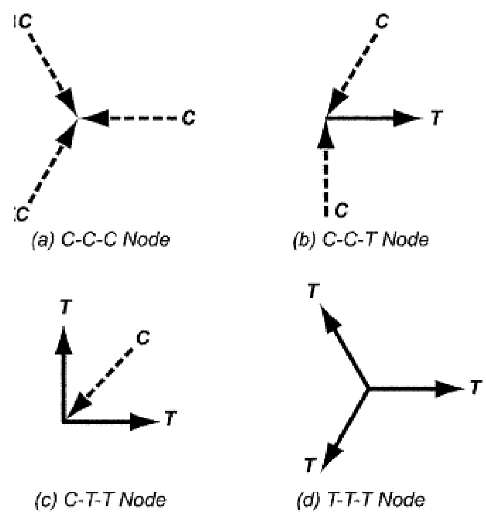


Figure 5.5 - Classifications of Nodes; courtesy of (Committee 318, 2008)

Nodal regions are idealized two different ways: hydrostatic nodal zone and extended nodal zone. To design a hydrostatic nodal region, the nodal region must be perpendicular to the axis of the strut or the tie, producing a uniaxial compression stress instead of a combined compression and shear stress as illustrated in Figure 5.6. For a nodal region to be considered hydrostatic, the region must have the same bearing pressure on all sides of the nodal zone because the in-plane stresses in the node are the same from every direction (MacGregor & Wight, 2005).

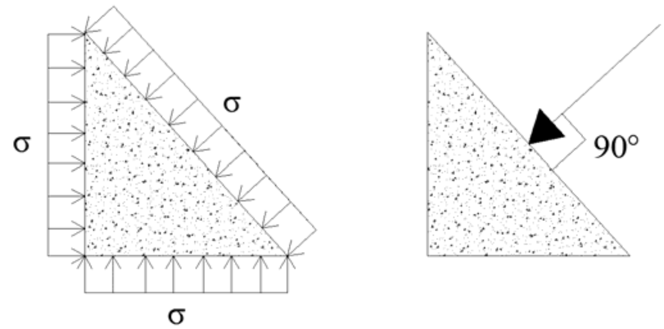


Figure 5.6 - Hydrostatic Nodal Zone

Determining hydrostatic node regions can be very difficult and time consuming if complicated loading is applied to the member. The lengths of the edges of the nodal regions are based on the applied force and the surface area required for the concrete to withstand crushing. When a tension tie is applied to a node, the width of the nodal region is determined using a hypothetical bearing plate on the end of the tie that exerts a bearing pressure on the node equal to the stresses applied from the struts (MacGregor & Wight, 2005). As shown in Figure 5.7, the tension tie reinforcement must be developed past the nodal region before the edge of the bearing, ℓ_{anc}, which could require bent bars unless enough length on the opposite side of the connection exists to develop the required development length.

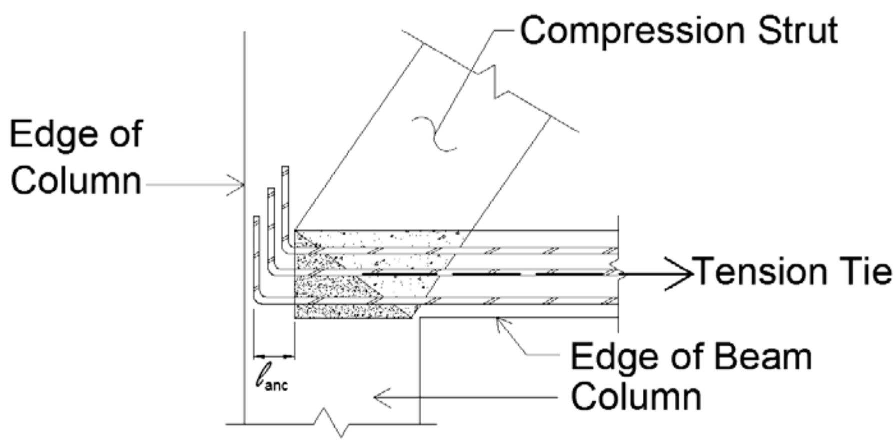


Figure 5.7 - Hydrostatic Nodal Zone Development Length

Designing with an extended nodal zone is much easier when the member is subjected to a more complicated loading pattern. This does not require the axis of the strut to be perpendicular to the face of the nodal zone, and the width of the strut is taken within the strut and not at the node. Figure 5.8 and 5.9 illustrates an extended nodal zone with the axis of the strut at an angle other than perpendicular to the nodal zone and the width of the strut, w_s, taken in compression is $w_s = l_b sin\theta + w_t cos\theta$. Figure 5.9 differentiates the extended nodal zones by a single layer of steel and multiple layers of steel.

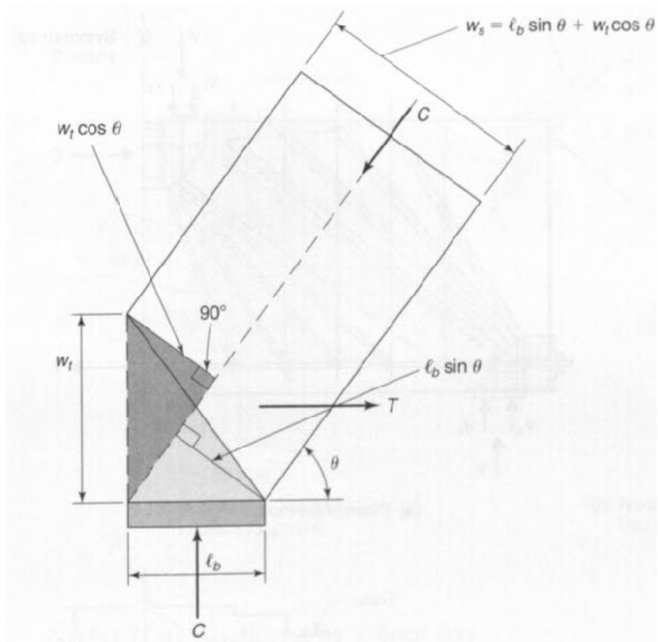


Figure 5.8 - Extended Nodal Zone Strut Width Calculation; courtesy of (MacGregor & Wight, 2005)

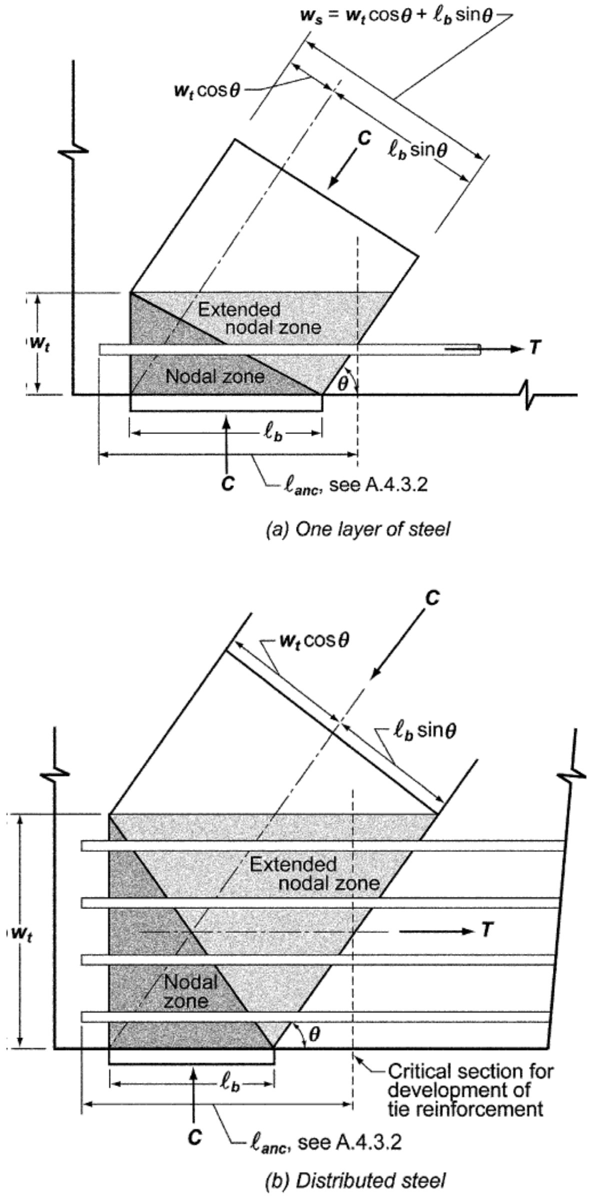


Figure 5.9 - Extended Nodal Zone Geometries; courtesy of (Committee 318, 2008)

An extended nodal zone also allows different stresses to be considered at the different edges of the nodal zone because of different nodal zone widths if (1) the resultants of the three forces coincide, (2) the stresses are within the limits allowed by code determined through testing, and (3) the stress is constant on each of the nodal zone faces (MacGregor & Wight, 2005). One benefit of the extended nodal zones is the tension tie reinforcement must have a development length at the edge of the extended nodal zone, not the end of the bearing illustrated in Figure 5.10. This extra distance provides the benefits of the concrete compressed by the struts increasing the bond between the concrete itself and the tension reinforcement (MacGregor & Wight, 2005).

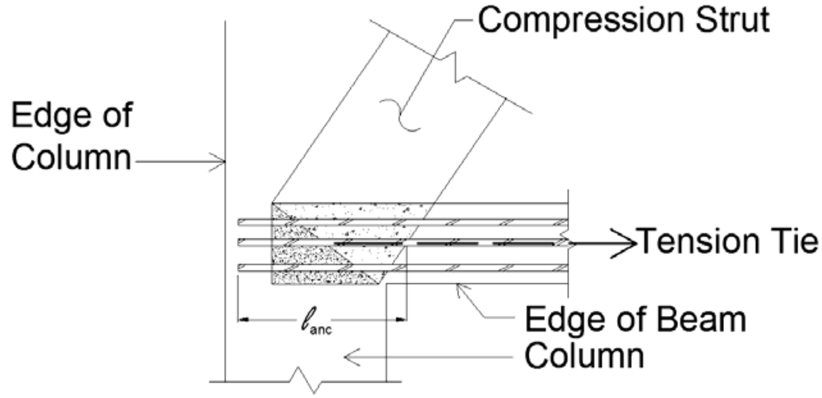


Figure 5.10 - Extended Nodal Zone Development Length

Hydrostatic nodal regions can be used with the extended nodal zones for anchorage based on the work of the Portland Concrete Association (PCA). Designing with hydrostatic nodal regions is conservative when designed nodal regions and will result in if not the same, a very similar area of tension reinforcement. The extended nodal zone anchorage provides the benefits of the concrete compressed by the struts increasing the bond between the concrete itself and the tension reinforcement Geometry of STM, which can be applied to hydrostatic nodal regions which also includes compression struts and tension ties.

Like typical beam design, designing for ductile failure requires the strength of the steel to govern the design. When STM is used in the design of deep beams, four failure modes can occur: (1) the ties can yield, (2) the strut could crush, (3) the node could fail if stresses are higher than was designed, or (4) the anchorage of the tie could fail (MacGregor & Wight, 2005). The following are considerations for the layout of struts and ties (MacGregor & Wight, 2005):

1. A clearly laid out load path keeping the STM in equilibrium must exist.
2. For a simply supported beam with two unequal loads that are not symmetric, the load path and STM should have the same shape as the bending moment. This is the same for a uniformly loaded beam with a parabolic STM.
3. The compressive struts should follow a realistic flow of the compressive forces and stress trajectories. Generally, the strut direction should be within ±15° of the compressive stress direction. It is assumed that the structure will have enough plastic

deformation capacity to adapt to a ±15° change in trajectories. Less restriction occurs within the ties because the ties basically are always placed orthogonally in the member in an absolute arrangement. The must follow, in general, the tensile stress direction.

4. Struts cannot cross or overlap because the width of the individual struts has been determined using their maximum allowable stress.
5. Ties can cross struts because it does not affect the maximum overall compression strength of the strut.
6. An unsuitable location for a compressive strut is over a cracking zone which is why having pictures or diagrams of how the cracking will form is a great way to help in the layout of struts-and-ties.
7. Within a load spreading region, a 2-to-1 strut slope (parallel to load – to – perpendicular to load) is conservative.
8. The width of the struts and nodal zones directly relate to the angles between the struts and the ties. The optimum angle is 45° but should never be less than 25° according to ACI 318. The larger the angles, the less width required for the compression struts.
9. The loads will try to follow the path with the least loads and deformations; therefore, the loads will follow the path that requires the shortest ties because the ties are the most deformable.
10. One of the first steps in designing an STM is determining the location of the nodes. A good starting point would be the axis of tension, which should be about $a/2$ from the tensile side, a being the depth of the rectangular stress block.
11. The angle between the strut and the tie should decrease to include extra web reinforcement when considering ACI 318, Section A.3.3. The European design standards recommend that if no axial load is applied to the beam, and if the ratio a/jd = 2, all the shear should be carried by shear reinforcement, and if a/jd=0.5, all the shear should be resisted by the compression strut.

5.4 Design of STM for Deep Beams

The design of an STM entails laying out a truss that fits within the deep beam with the appropriate cover while being able to transfer the forces without failing. How the beam will react determines the optimum design; one that requires the least amount of steel within a given beam. ACI 318-08, Appendix A, specifies some strength and geometry limitations and design equations. The internal factored forces, Fu, must be less than the design strength represented by ACI 318-08 Equation A-1, given here as Equation 5.1.

$$\emptyset F_n \geq F_u \qquad \text{(EQ'N 5.1)}$$

The first step in the design process is to determine beam dimensions. Typically, the beam width will be governed or equal to the column dimensions to which it is connected. To determine the height of the beam, first determine the ultimate factored shear load applied on the beam must be known. From Equation 4.7, a depth *d* can be determined that is required for the shear force. The angle between the strut and the tie needs to be considered at this time as well. ACI 318-08, Section A.2.5 states that the angle, θ, between any strut and tie must not be less than 25° or greater than 65°in order to "mitigate cracking and to avoid incompatibilities" in the nodal regions due to shortening of the struts and lengthening the ties occurring in the same direction. The optimum angle to keep nodal regions and struts to a reasonable size is 40-45°. As the angle increases, the force in the strut decreases requiring less strut width; however, to increase the angle, the beam depth must increase. As the angle increases past 45°, increasing the angle becomes less effective because the difference in the force in the strut from angle to angle decreases in value.

Once the beam dimensions have been selected, deep beam criteria from Equations 3.1 and 3.2 should be checked to confirm that the member is indeed a deep beam so that ACI 318, Appendix A, can be used for design. If the member is considered a deep beam, node locations should be determined for the tension tie. The nodes should be approximately *a*/2 from the bottom of the beam. A good estimate for this location is 0.05h or approximately 5 inches (MacGregor & Wight, 2005).

5.5.1 Struts

Once the general location of the nodes has been determined, the effective compressive strength of the concrete for both the struts and the nodal regions is determined. According to ACI 318-08, Equation A-2 given here as Equation 5.2, the nominal compressive strength of a strut without longitudinal reinforcement, F_{ns}, shall be taken as the smaller value at the two ends of the strut.

$F_{ns} = f_{ce}A_{cs}$ (EQ'N 5.2)

where:

A_{cs} = cross sectional area of one end of the strut;

f_{ce} = effective compressive strength.

The effective compressive strength of the strut shall be taken as the smaller of the effective compressive strength of the concrete in the strut or the concrete in the nodal zone according to ACI 318-08, Section A.3.1. The compressive strength of the concrete in the strut is determined using ACI 318-08, Equation A-3, and the strength in the nodal zone is determined using Equation A-8, both given respectively below.

$f_{ce} = 0.85\beta_s f'_c$ (EQ'N 5.3)

$f_{ce} = 0.85\beta_n f'_c$ (EQ'N 5.4)

where:

β_s= factor to account for the effect of cracking and confining reinforcement on the effective compression strength of the concrete in a strut;

β_n= factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone.

When cracks form inclined to the axis of the strut, the strut is weakened. The β -factor considers how the forces will be transferred when cracks are formed, or indeed if the transfer is not present. The 0.85 factor is equivalent to the 0.85 used to determine the average stress in the Whitney stress block. The 0.85 takes into account that the strength of the concrete in beams tends to be less than the cylinder strength test, f'_c, due to the sustained loading, vertical migration of bleed water decreasing the strength at the top of the beam, and the different shapes of the compression zones and test cylinders (MacGregor & Wight, 2005).

According to ACI 318-08, Section A.3.2.1, for a uniform cross-section area over the length of the strut, β_s=1.0 which indicates that the strut has an equivalent stress block of depth, a, and a width, b, identical to beams (MacGregor & Wight, 2005).

ACI 318-08, Section A.3.2.2 applies to bottle shaped struts (struts with a midsection larger than the section at the nodes) without reinforcing across the potential cracking or with reinforcing across the potential cracking to resist the transverse tensile force designed according to ACI 318-08, Section A.3.3. When reinforcing is used, β_s=0.75, and without reinforcing, the strut should fail after cracking, giving a much lower value of β_s=0.60λ with λ being the concrete weight factor. When determining the area of steel required to resist transverse tensile cracks with both longitudinal and vertical steel to reinforce against cracking, ACI 318-08, Equation A-4, given as Equation 5.5, gives a minimum area of steel ratio taking into account the angle of the reinforcement and the axis of the strut as long as f'$_c$ is less than 6,000psi.

$$\sum \frac{A_{si}}{b_s s_i} sin\alpha_i \geq 0.003 \qquad \text{(EQ'N 5.5)}$$

where:

A_{si}= total area of surface reinforcement;

s_i= spacing of surface reinforcement;

α_i= angle from the reinforcement to the axis of the strut;

b_s= the effective width, b_w, of the beam.

Figure 5.11 illustrates vertical and horizontal reinforcement with spacing of s_1 and s_2 respectively within the strut boundary, shown in Figure 5.12. The area of steel is multiplied by the angle of the strut to vertical and horizontal reinforcement to get the perpendicular steel area crossing through the strut axis which is divided by the area of concrete to achieve the steel ratio.

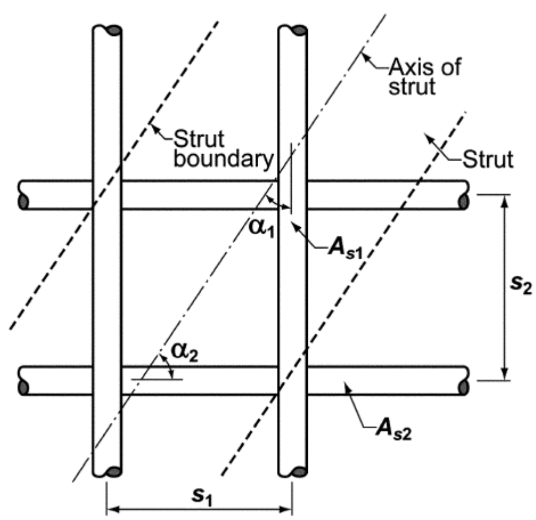
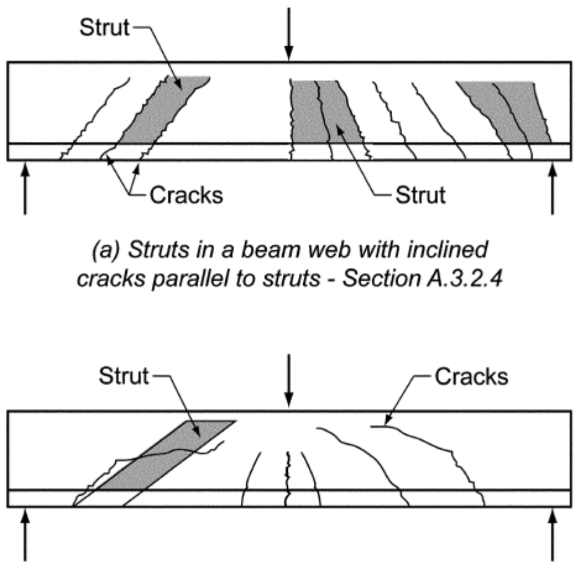


Figure 5.11 - Strut Reinforcement; courtesy of (Committee 318, 2008)



(b) Struts crossed by skew cracks - Section A.3.2.4

Figure 5.12 - Types of Struts; courtesy of (Committee 318, 2008)

As the concrete compressive strength increases, concrete tends to become more brittle, and efficiency of calculating the effective compressive strength tends to decrease. For this reason, the ACI Committee 318 decided that the load spreading to the reinforcement should be calculated when f'_c is higher than 6,000psi. The strength of the reinforcement should be equal to the tension force lost when the concrete cracks. The slope of the load spreading struts is taken as 2 to 1, as permitted by ACI 318-08, Section A.3.3. Equation 5.6 is developed through the geometry presented in Figure 5.13(b) (MacGregor & Wight, 2005).

$$T_n = \frac{C_n}{2}\left(\frac{b_{ef}/4 - a/4}{b_{ef}/2}\right) \qquad \text{(EQ'N 5.6)}$$

where:

T_n= transverse tension force = $A_s f_y$;

C_n= nominal compressive force in the strut;

a= width of the bearing area at the end of the strut;

b_{ef}= effective width of the bottle-shaped strut.

Figure 5.13(a) represents the bottle shaped region based on the effective width of the strut, b_{ef}. Jorg Schlaich and Dieter Weischede in *Detailing of Concrete Structures* recommended that the length of the bottle strut region at one end is the length of $1.5b_{ef}$ (MacGregor & Wight, 2005). Figure 5.13(c) represents the transverse tensile stresses caused by force T in Figure 5.13(b) distributed throughout the bottle shaped region.

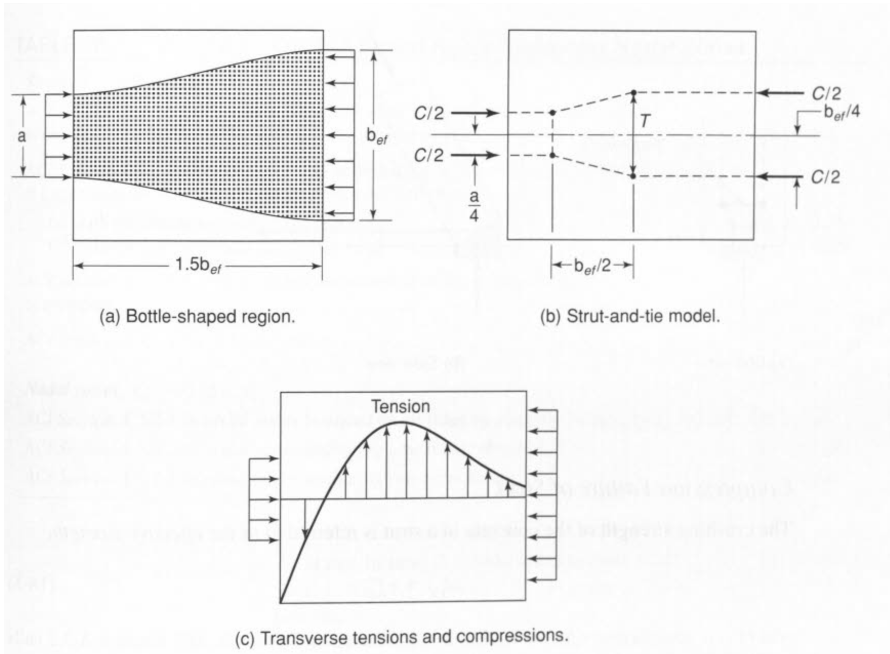


Figure 5.13 - Spread of Stresses and Transverse Tensions in a Strut; courtesy of (MacGregor & Wight, 2005)

The ACI Committee 318 used Equation 5.5 to simplify the design process when the concrete compressive strength is less than 6,000psi but recommends that the actual strains and forces needed to be calculated in reinforcement when 28 day concrete compressive strength extends beyond 6,000 psi because of the increasingly brittle behavior of the high strength concrete.

ACI 318-08, Sections A.3.2.3 gives the value β_s=0.40 for struts in tension members or tension flanges. Concrete is not good in tension, so the tension force will cause cracks to pull apart thus greatly decreasing the strength of the strut. Section A.3.2.4 gives the value of β_s=0.60 for all other situations not mentioned in the previous sections.

If a strut does not have enough strength, compression reinforcement can be added much like a column that includes longitudinal reinforcement along the axis of the strut with ties or spiral reinforcement in accordance with ACI 318-08, Section 7.10. ACI 318-08 Equation A-5, shown here as Equation 5.7, is used to determine the compressive strength of a longitudinally reinforced strut.

$$F_{ns} = f_{ce}A_{cs} + A'_s f'_s \qquad \text{(EQ'N 5.7)}$$

where:

A_{cs}= cross sectional area at one end of a strut normal to the axis of the strut;

A'_s= area of compression reinforcement;

f'_s= stress in compression reinforcement under factored laods.

5.5.2 Nodal Zones

Nodal zones are designed assuming that they will fail by crushing (MacGregor & Wight, 2005). ACI 318-08, Equation A-7, shown as Equation 5.8, sets the limit of the nominal compressive strength of a nodal zone, F_{nn}. As in Section 4.5.1, the compressive strength of the concrete in the node is determined using ACI 318-08 Equation A-8, shown as Equation 5.4.

$$F_{nn} = f_{ce}A_{nz} \qquad \text{(EQ'N 5.8)}$$

$$f_{ce} = 0.85\beta_n f'_c \qquad \text{(EQ'N 5.4)}$$

where:

A_{nz}: smaller of (a) the area of the face of the nodal zone on which F_u acts taken normal to the line of action, or (b) the area of a section through the nodal zone, taken normal to the line of action of the resultant force on the section.

ACI 318-08, Section A.5.2 gives values for β_n based on the geometry of the nodal region. If the nodal zone is bounded by compressive struts, C-C-C, β_n = 1.0. If the nodal zone is bounded by compressive struts with one tension tie, C-C-T, β_n = 0.80; and if the nodal zone is bounded by two or more tension ties, C-T-T or T-T-T, β_n = 0.60. Tension ties decrease nodal strengths because of the increased disruption due to the incompatibility of tension strains and compressive strains (Committee 318, 2008). However, tests have shown that C-C-T and C-T-T nodes develop β_n = 0.95 when properly constructed (MacGregor & Wight, 2005). The values selected are conservative and allow for construction tolerances.

5.5.3 Ties

Ties consist of reinforcement in the tension regions of the element being designed as well as in the surrounding concrete. The concrete does not contribute to the resistance of forces but does increase the axial stiffness of the tie through tension stiffening. The nominal strength of the tie is determined using ACI 318-08 Equation A-6, given as Equation 5.9.

$$F_{nt} = A_{ts}f_y + A_{tp}\left(f_{se} + \Delta f_p\right) \qquad \text{(EQ'N 5.9)}$$

Where:

$\left(f_{se} + \Delta f_p\right) \leq f_{py}$ and A_{tp} is 0 for nonprestressed members.

According to ACI 318, Section A.4.2 and RA.4.2, the axis of the reinforcement in a tie shall coincide with the axis of the tie, and the effective tie width, w_t, is limited depending on the reinforcement geometry and distribution. If the bars are in one layer, w_t can be taken as the diameter of the bar plus twice the cover, which is the lower limit of w_t. The upper limit is determined in accordance with equation 5.10.

$$w_{t,max} = \frac{F_{nt}}{f_{cu}b} \qquad \text{(EQ'N 5.10)}$$

5.5 Design Examples

To accurately compare the design of deep beams through DBM and STM, the three simply supported girders designed using DBM are designed using STM. Each girder's height is calculated to keep the angles of the STM near the optimum 40-45°. Because of the loading geometry in design examples 2 and 3, it is difficult to get all angles near the 40-45°. The girder depths were the same as for the DBM examples to make for an easy comparison of the steel and how the girder transfers the forces. The girders are 24 inches wide with normal weight concrete with 28-day compression strength at 4,000 psi and the yield strength of the reinforcing bars at 60,000 psi. All loads shown are factored for ultimate strength design.

5.6.1 STM Design Example 1

Design example 1 is a 24 inch wide transfer girder spanning 16 feet with a column at mid-span with factored 1,200 Kip load. The girder is supported by 24 inch square columns. A design height of 7 feet was determined by iteration. Figure 5.14 indicates the transfer girder for design. The ultimate shear diagram is shown in Figure 5.15.

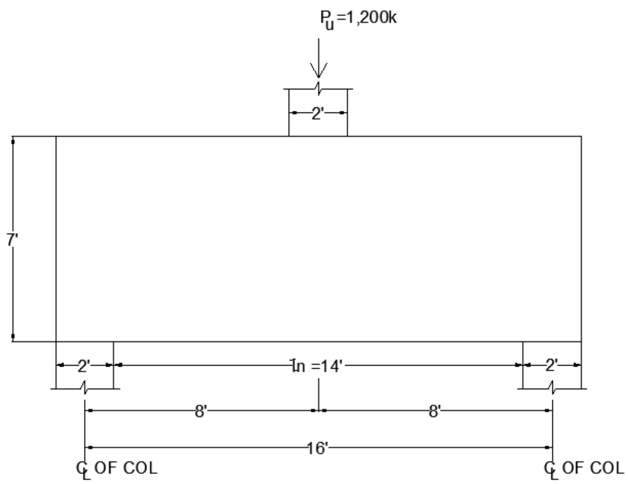


Figure 5.14 - STM Design Example 1

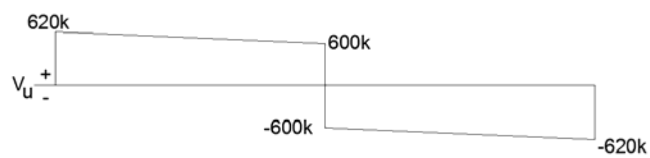


Figure 5.15 – STM Design Example 1 - Shear Diagram

f'_c = 4,000 psi F_y = 60,000 psi b_w = 24 inches

Step 1: Verify Trial Height

The minimum height allowed by code is determined with Equation 4.7 with d assumed to be 0.9h. Solving for h with V_u substituted for $\emptyset V_n$:

$\emptyset V_n \leq \emptyset 10\sqrt{f'_c} b_w d$ $620{,}000\# \leq (0.75)10\sqrt{4000psi}(24")(0.9h)$ (EQ'N 4.7)

h = 54 in Use h = 7 ft

Step 2: Check for Deep Beam Criteria

$\frac{l_n}{h} \leq 4.0$ $\frac{14'}{7'} = 2.0 \leq 4.0$ Deep Beam (EQ'N 3.1)

$\frac{a}{h} < 2.0$ $\frac{7'}{7'} = 1.0 < 2.0$ Deep Beam (EQ'N 3.2)

Step 3: Establish Node Locations

Note: *A good starting point for node locations is 5 inches from the top or bottom face of the girder or 0.05h. Once designed, if the final locations show a difference of roughly 1.5inches or less, the original locations are deemed acceptable because the forces in the strut may increase from 1% to 2%, which should not change the final design. Multiple iterations were performed and acceptable nodal locations were determined. Because of the heavy loads applied on the structure and the minimum height allowable being used, much deeper node locations must be used.*

The node at location C at the loading point is 9 inches from the top of the girder, and the node location at the supports is 10 inches from the bottom of the girder shown in Figure 5.16.

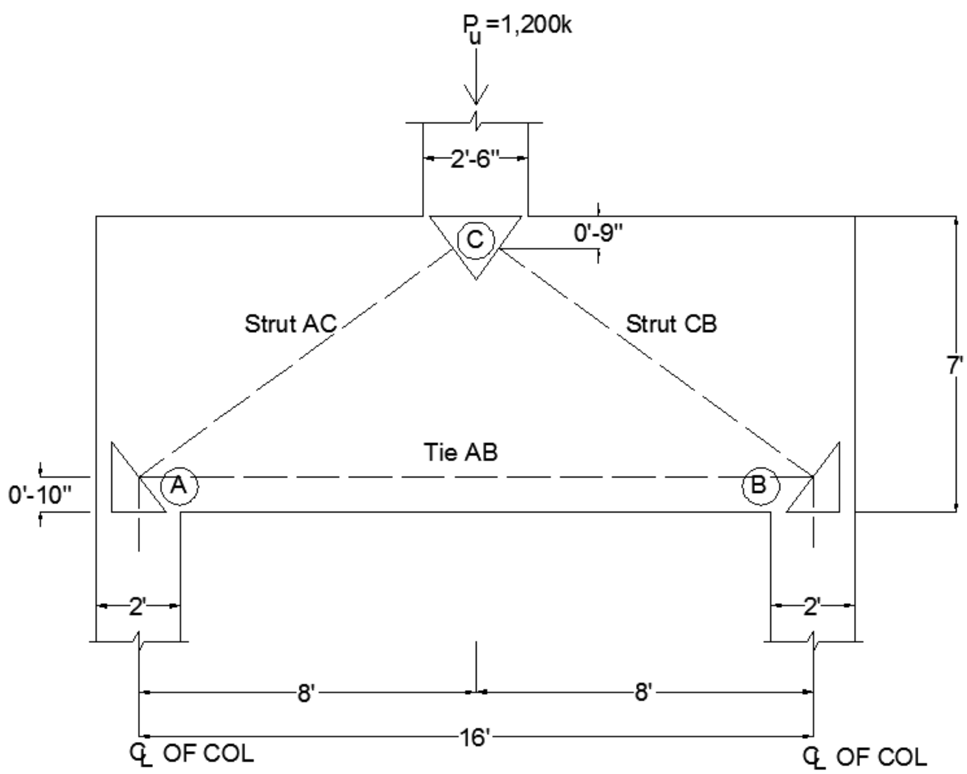


Figure 5.16 - STM Design Example 1 – Node Locations

Angle between Struts and Tie = $tan^{-1}\left(\frac{84"-10"-9"}{8'\times 12"}\right) = 34.1° > 25°$ ACI 318 A.2.5

Step 4: Determine Forces in Struts and Ties

Through Geometry of the Girder:

Length of Strut CA = $\sqrt{(84" - 10" - 9")^2 + (8' \times 12")^2} = 115.9\ in$

Length of Strut CB = $\sqrt{(84" - 10" - 9")^2 + (8' \times 12")^2} = 115.9\ in$

Force in Strut CA = $620k \times \frac{115.9"}{84"-10"-9"} = 1{,}105k$

Force in Strut CB = $620k \times \frac{115.9"}{84"-10"-9"} = 1{,}105k$

Force in Tie AB = $620k \times \frac{8' \times 12"}{84"-10"-9"} = 916k$

Step 5: Determine Effective Concrete Strength in Nodes and Struts

Because enough space within the girder for a bottle shape strut to form in Struts AC and AB exists and steel will be provided to resist cracking, β_s=0.75 using ACI 318 A.3.3.

$f_{ce} = 0.85\beta_s f'_c \qquad f_{ce} = 0.85(0.75)(4{,}000) = 2{,}550\ psi$ (EQ'N 5.3)

The struts within the columns do not have enough space for a bottle shaped strut to form; thus β_s=1.0 using ACI 318 A.3.2.1.

$f_{ce} = 0.85\beta_s f'_c \qquad f_{ce} = 0.85(1.0)(4{,}000) = 3{,}400\ psi$ (EQ'N 5.3)

For the nodal region at C, a C-C-C situation is present; thus β_n=1.0 using ACI 318 A.5.2.1.

$f_{ce} = 0.85\beta_n f'_c \qquad f_{ce} = 0.85(1.0)(4{,}000) = 3{,}400\ psi$ (EQ'N 5.4)

For the nodal region at A and B, a C-C-T situation is present; thus β_n=0.80 using ACI 318 A.5.2.2.

$f_{ce} = 0.85\beta_n f'_c \qquad f_{ce} = 0.85(0.80)(4{,}000) = 2{,}720\ psi$ (EQ'N 5.4)

Step 6: Determine STM Geometry

Note: Hydrostatic nodal regions were used; therefore, the stresses on each face of the region must be identical, and the faces are perpendicular to the axis of the struts. Extended nodal zones could be used, but hydrostatic nodal regions are easy for this type of loading and add some conservatism in the design by requiring a larger nodal zone. Because hydrostatic nodal zones are being used, *the minimum of the above effective concrete strength must be used to ensure a static situation.*

$\emptyset F_n \geq F_u$ with $\emptyset = 0.75$ (EQ'N 5.1)

$f = \frac{P}{A}$; $Width\ of\ the\ strut, w_s = \frac{P}{f \times f_{ce}}$ (EQ'N 5.11)

Width of Strut CA = $w_{s,CA} = \frac{1{,}105{,}000\#}{(0.75)(2{,}550psi)(24")} = 24.1\ in$

Width of Strut CB = $w_{s,CB} = \frac{1{,}105{,}000\#}{(0.75)(2{,}550psi)(24")} = 24.1\ in$

Width of Strut A = $w_{s,A} = \frac{620{,}000\#}{(0.75)(2{,}550psi)(24")} = 13.5\ in$

Width of Strut B = $w_{s,A} = \frac{620{,}000\#}{(0.75)(2{,}550psi)(24")} = 13.5\ in$

Width of Strut C_1 = $w_{s,C1} = \frac{600{,}000\#}{(0.75)(2{,}550psi)(24")} = 13.1\ in$

Width of Strut C_2 = $w_{s,C2} = \frac{600{,}000\#}{(0.75)(2{,}550psi)(24")} = 13.1\ in$

Height of the Tie = $w_T = \frac{916{,}000\#}{(0.75)(2{,}550psi)(24")} = 20.0\ in$ (EQ'N 5.10)

Due to the compression strut width required within the column applying the loads, a 30x24 inch column is required. All other dimensions fit within the girder and supporting columns and follow the STM guidelines, shown in Figure 5.17.

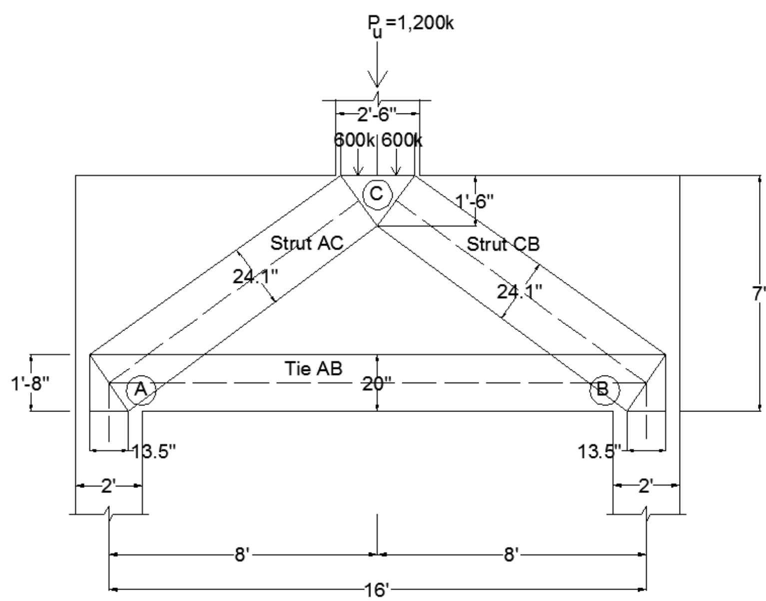


Figure 5.17 - STM Design Example 1 - Geometry

Step 7: Verify Node Locations

Once all geometries were calculated, the design was drawn to scale and actual node locations were determined shown in Figure 5.18. This could also be done using geometry. The node at C is 9 inches from the top of the girder which is what was used for design, and the nodes at A and B are 9.97 inches from the bottom of the girder which is also very close to the 10 inches initially selected. If these nodes were much further apart, new initial node locations would need to be selected and everything recalculated until the differences were appropriate.

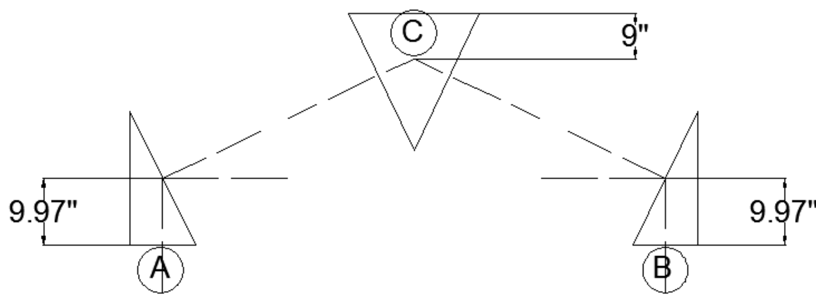


Figure 5.18 - STM Design Example 1 - Actual Node Locations

Step 8: Determine Steel in Tie

$F_{nt} = A_{ts}f_y;\ A_{ts} = \frac{F_{nt}}{\emptyset f_y} = \frac{916{,}000\#}{(0.75)(60{,}000psi)} = 20.3\ in^2$ (EQ'N 5.9)

Try 4 rows of 4 #10 bars

$A_s = (16)(1.27in^2) = 20.32in^2$

Figure 5.19 represents the tension tie reinforcement of 4 rows of 4 #10 bars spaced at 6.5".

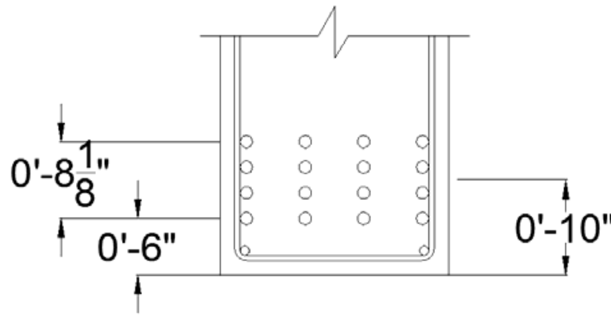


Figure 5.19 - Tension Tie Reinforcement For Design Example 1

Check Tie Location Requirements.

The centroid of the tie should line up with the node location; therefore, the centroid of the bottom tie reinforcement should start 6" above the bottom of the girder.

$y = 10"$ therefore $d=84" - 10" = 74"$

Determine total effective height of reinforcement.

$10" + (2\ rows\ of\ steel)(1.27") + (1.5\ rows\ of\ spaces)(1.41") = 14.66"$

Check against height of tie.

$20">14.66"\ OK$

Check the area of steel required against minimum steel requirements

$\frac{3\sqrt{f'_c}b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \qquad \frac{3\sqrt{4{,}000psi}(24")(74")}{60{,}000psi} = 5.61in^2 < \frac{200(24")(74")}{60{,}000} = 5.92in^2$

20.32 in^2 > 5.92 in^2 OK

Check Development length of #10 Hooked Bars.

Even though the nodal zones were designed using hydrostatic nodal zones, the anchorage length used will fall within the extended nodal zone which is acceptable. Development for a hook can be determined using ACI 318-08 Section 12.5.1.

$\left(\frac{0.02\psi_e f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02(1.0)(60{,}000psi)}{\sqrt{4{,}000psi}}\right) 1.27 = 24.1in$

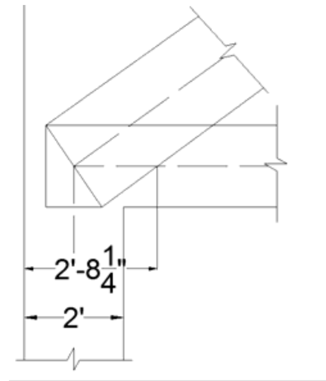


Figure 5.20 - STM Design Example 1 Anchorage Length Available

Available anchorage length: 32.25” – 1.5”cover = 30.75in

30.75in > 24.1in OK

USE 4 Rows of 4 #10 bars.

<u>Step 9: Determine Crack Reinforcement per ACI A.3.3.1</u>

Angle between stirrups and struts = 90°-34.1°= 55.9°

Try #5 stirrups vertically at 10 inches on center and #5 longitudinal bars at 12 inches on center.

$A_{vh} = 0.0015 b_w S_h$ (EQ’N 4.25)

$A_v = 0.0025 b_w S_v$ (EQ’N 4.26)

$S_h = \frac{(2)(0.31in^2)}{.0015(24")} = 17.22" > 12"$ OK

$S_v = \frac{(2)(0.31in^2)}{.0025(24")} = 10.33" > 10"$ OK

$\sum \frac{A_{si}}{b_s s_i} sin\alpha_i \geq 0.003$ (EQ’N 5.5)

$\frac{(2)(0.31)}{(24)(10)} sin(55.9°) = 0.0022$

$\frac{(2)(0.31)}{(24)(12)} sin(34.1°) = 0.0012$

$\sum \frac{A_{si}}{b_s s_i} sin\alpha_i = 0.0022 + 0.0012 = 0.0034 \geq 0.0030 \quad OK$

USE #5 Stirrups at 10 inches O.C. and #5 Longitudinal Reinforcement at 12 inches O.C.

Cut sections of the completed design of the girder are shown in Figure 5.21 with dimensions and reinforcement.

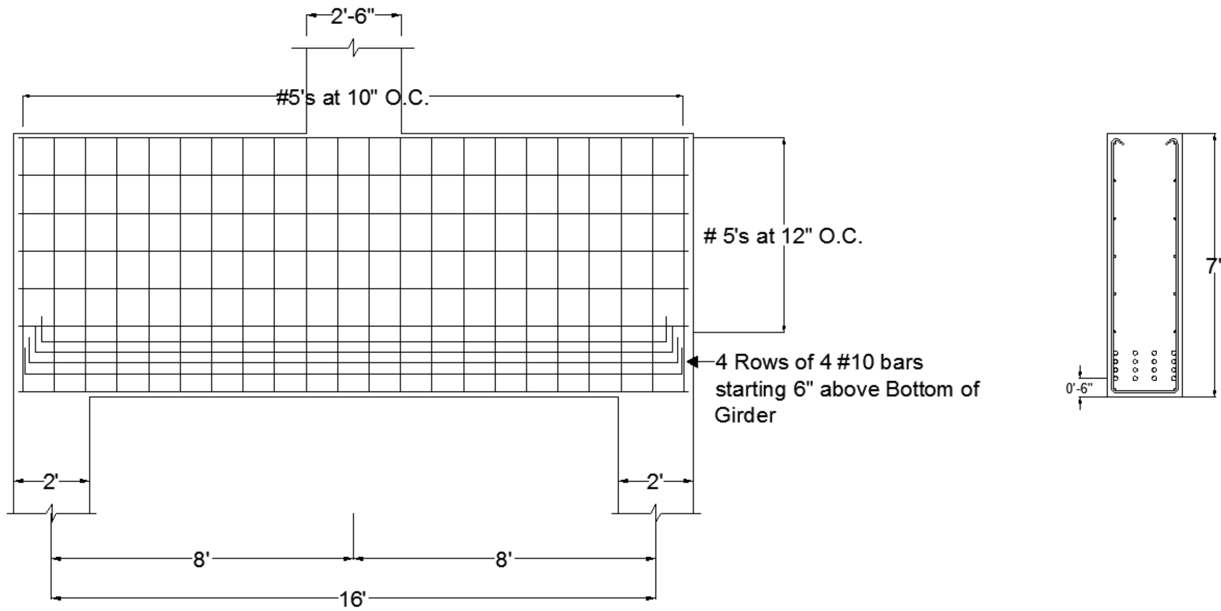


Figure 5.21 - STM Design Example 1 - Final Design Cut Sections

Note: If a 6' girder were used, the Tension reinforcement would be (19) #11 bars and the shear reinforcement would be #5's at 10" vertical and #5's at 12" horizontal. The 7' girder was used to keep the angle between the struts and tie near 40° while making a good comparison to Deep Beam design example 1.

5.6.2 STM Design Example 2

Design example two is a 24 inch wide transfer girder spanning 16 feet with a column at 5 feet from a support with a factored load of 1,200 kips. The girder is supported by 24 inch square columns. A design height of 8 feet was determined by iteration. Figure 5.22 indicates the transfer girder for design. The calculated ultimate shear diagram is illustrated in Figure 5.23.

Note: Hydrostatic nodal regions were used to in the design. To have the forces at each end of the tie equal each other, the weight of the girder is included at the column load location. This is conservative as the struts are designed using the heavier load.

Weight of the girder = $w = 150pcf \times 8' \times {}^{24"}/_{12"} = 2{,}400plf$

Factored weight of girder = $w_u = 1.2 \times 2{,}400plf = 2{,}880plf \times 16' = 46.08k$

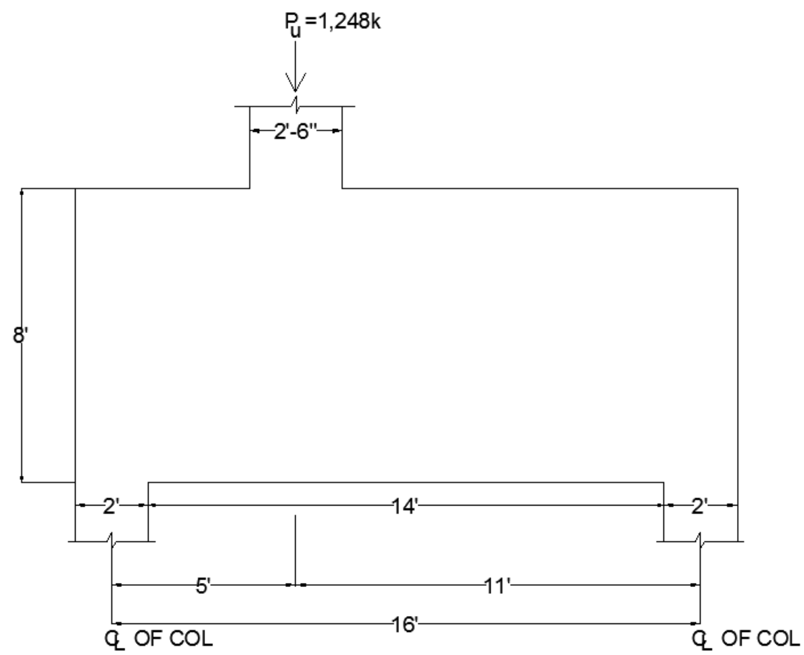


Figure 5.22 - STM Design Example 2

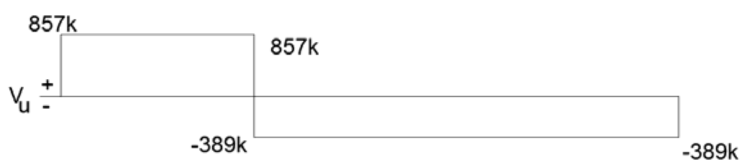


Figure 5.23 – STM Design Example 2 - Shear Diagram

f'_c = 4,000 psi F_y = 60,000 psi b_w = 24 inches

Step 1: Verify Trial Height

The minimum height allowed by code is determined in accordance with Equation 4.7 with *d* assumed to be 0.9h. Solving for *h* with V_u substituted for $\emptyset V_n$:

$\emptyset V_n \leq \emptyset 10\sqrt{f'_c} b_w d$ $857{,}000\# \leq (0.75)10\sqrt{4000psi}(24")(0.9h)$ (EQ'N 4.7)

h = 83.6 in Use h = 8 ft

Step 2: Check for Deep Beam Criteria

$\frac{l_n}{h} \leq 4.0$ $\frac{14'}{8'} = 1.75 \leq 4.0$ Deep Beam (EQ'N 3.1)

$\frac{a}{h} < 2.0$ $\frac{4'}{8'} = 0.5 < 2.0$ Deep Beam (EQ'N 3.2)

Step 3: Establish Node Locations

Note: Multiple iterations were performed and acceptable nodal locations were determined. Because of the heavy loads applied on the structure and the minimum height allowable being used, much deeper node locations must be used.

The node at location C at the loading point is 7 inches from the top of the girder, and the node location at the supports is 7 inches from the bottom of the girder shown in Figure 5.24.

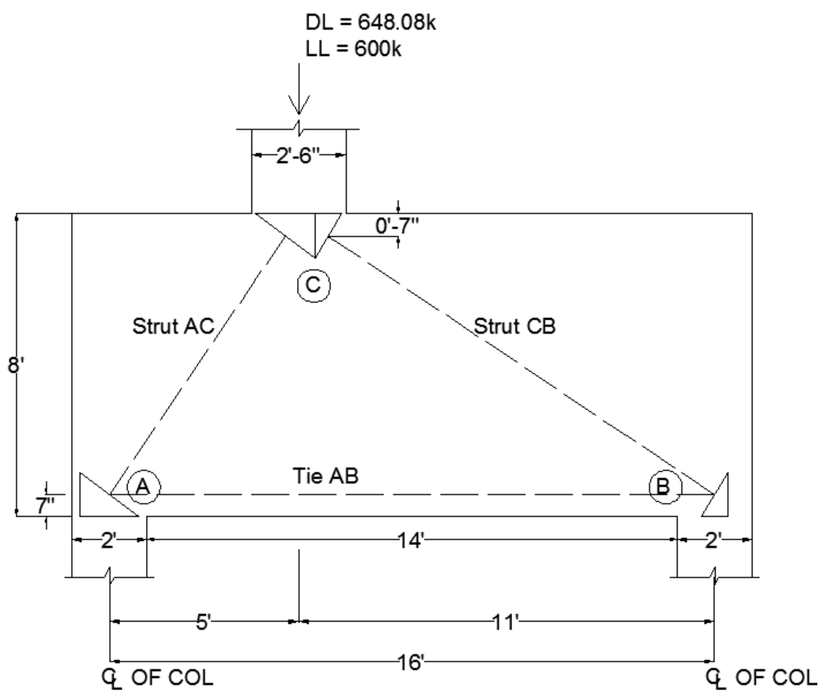


Figure 5.24 - STM Design Example 2 – Node Locations

Angle between Strut CA and Tie = $tan^{-1}\left(\frac{96"-7"-7"}{5'\times12"}\right) = 53.8° > 25°$ ACI 318 A.2.5

Angle between Strut CB and Tie = $tan^{-1}\left(\frac{96"-7"-7"}{11'\times12"}\right) = 31.8° > 25°$ ACI 318 A.2.5

Step 4: Determine Forces in Struts and Ties

Through Geometry of the Girder:

Length of Strut CA = $\sqrt{(96" - 7" - 7")^2 + (5' \times 12")^2} = 101.6\ in$

Length of Strut CB = $\sqrt{(96" - 7" - 7")^2 + (11' \times 12")^2} = 155.4\ in$

Force in Strut CA = $857k \times \frac{101.6"}{96\text{-}7-7"} = 1{,}062k$

Force in Strut CB = $389k \times \frac{155.4"}{96\text{-}7-7"} = 737k$

Force in Tie AB = $857k \times \frac{5' \times 12"}{96\text{-}7-7"} = 627k$ = $388k \times \frac{11' \times 12"}{96\text{-}7-7"} = 627k$

> *Note: If the weight of the girder was not consolidated to the loading point, the forces in the Tie AB from the struts at the supports would not be equal, which will make forming a hydrostatic nodal zone very difficult.*

Step 5: Determine Effective Concrete Strength in Nodes and Struts

Because the girder has enough space for a bottle shaped strut to form in Struts AC and AB and steel will be provided to resist cracking, β_s=0.75 using ACI 318 A.3.3.

$f_{ce} = 0.85\beta_s f'_c \qquad f_{ce} = 0.85(0.75)(4{,}000) = 2{,}550\ psi$ (EQ'N 5.3)

For the struts within the columns, not enough space for a bottle shaped strut to form, thus β_s=1.0 per ACI 318 A.3.2.1.

$f_{ce} = 0.85\beta_s f'_c \qquad f_{ce} = 0.85(1.0)(4{,}000) = 3{,}400\ psi$ (EQ'N 5.3)

For the nodal region at C, a C-C-C situation is present, thus β_n=1.0 per ACI 318 A.5.2.1.

$f_{ce} = 0.85\beta_n f'_c \qquad f_{ce} = 0.85(1.0)(4{,}000) = 3{,}400\ psi$ (EQ'N 5.4)

For the nodal region at A and B, a C-C-T situation is present, thus β_n=0.80 per ACI 318 A.5.2.2.

$f_{ce} = 0.85\beta_n f'_c \qquad f_{ce} = 0.85(0.80)(4{,}000) = 2{,}720\ psi$ (EQ'N 5.4)

Step 6: Determine STM Geometry

> *Note: Hydrostatic nodal regions were determined; therefore, the stresses on each face of the region must be identical, and the faces are perpendicular to the axis of*

the struts. Extended nodal zones could be used, but hydrostatic nodal regions are easy for this type of loading and add some conservatism in the design by requiring a larger nodal zone. Because hydrostatic nodal zones are being used, the minimum of the above effective concrete strength must be used to ensure a static situation.

$\emptyset F_n \geq F_u$ with $\emptyset = 0.75$ (EQ'N 5.1)

$f = \frac{P}{A}$; $Width\ of\ the\ strut, w_s = \frac{P}{f \times f_{ce}}$ (EQ'N 5.11)

Width of Strut CA = $w_{s,CA} = \frac{1{,}062{,}000\#}{(0.75)(2{,}550psi)(24")} = 23.1\ in$

Width of Strut CB = $w_{s,CB} = \frac{737{,}000\#}{(0.75)(2{,}550psi)(24")} = 16.1\ in$

Width of Strut A = $w_{s,A} = \frac{857{,}000\#}{(0.75)(2{,}550psi)(24")} = 18.7\ in$

Width of Strut B = $w_{s,A} = \frac{389{,}000\#}{(0.75)(2{,}550psi)(24")} = 8.5\ in$

Width of Strut C_1 = $w_{s,C1} = \frac{857{,}000\#}{(0.75)(2{,}550psi)(24")} = 18.7\ in$

Width of Strut C_2 = $w_{s,C2} = \frac{389{,}000\#}{(0.75)(2{,}550psi)(24")} = 8.5"\ in$

Height of the Tie = $w_T = \frac{627{,}000\#}{(0.75)(2{,}550psi)(24")} = 13.7\ in$ (EQ'N 5.10)

The compression strut width required within the column applying the loads means a 30x24 inch column is required. All other dimensions fit within the girder and supporting columns and follow the guidelines for STM shown in Figure 5.25.

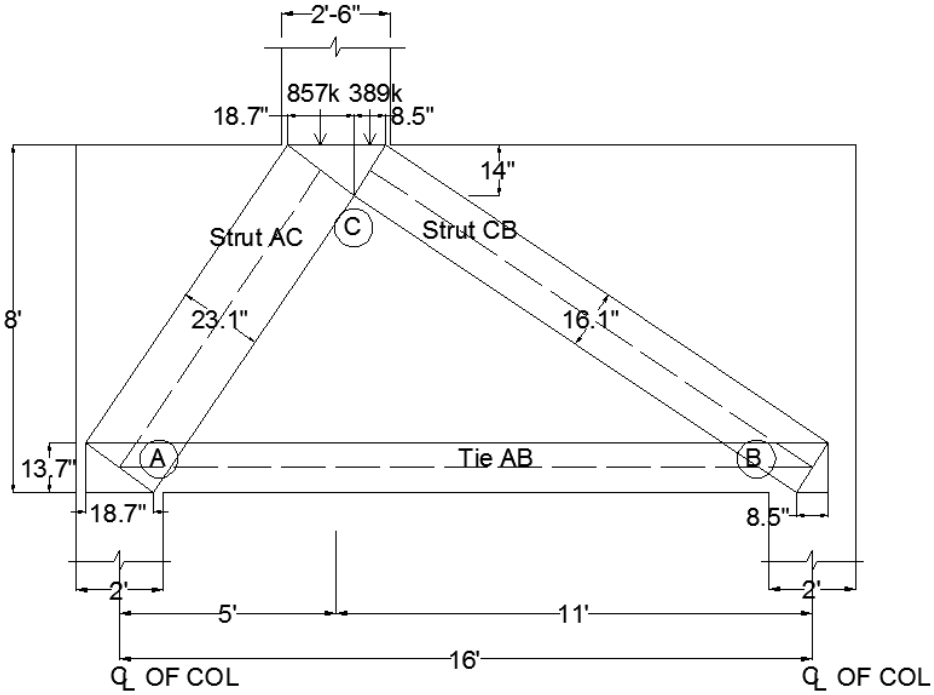


Figure 5.25 - STM Design Example 2 - Geometry

Step 7: Verify Node Locations

Once all geometries were calculated, the design was drawn to scale and actual locations were determined illustrated in Figure 5.26. This could also be done by geometry. The node at C is 7 inches from the top of the girder, which is equal to the 7 inches initially selected, and the nodes at A and B are 6.85 inches from the bottom of the girder, which is very close to the 7 inches initially selected. Initial node selections are considered acceptable.

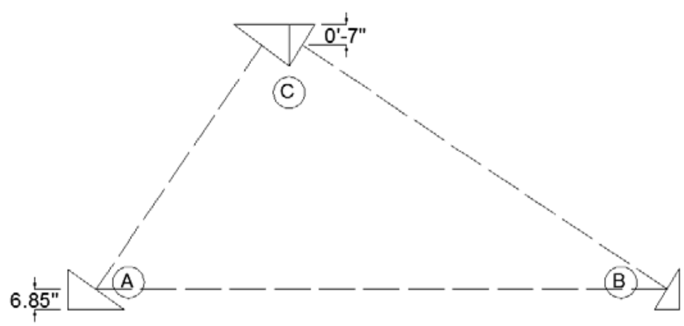


Figure 5.26 - STM Design Example 2 - Actual Node Locations

Step 8: Determine Steel in Tie

$$F_{nt} = A_{ts} f_y;\ A_{ts} = \frac{F_{nt}}{\emptyset f_y} = \frac{627{,}000\#}{(0.75)(60{,}000psi)} = 13.9\ in^2 \qquad \text{(EQ'N 5.9)}$$

Try 3 rows of 3 #11 bars.

$A_s = (9)(1.56in^2) = 14.04in^2$

Figure 5.27 represents the tension tie reinforcement of 3 rows of 3 #11 bars spaced at 9.5".

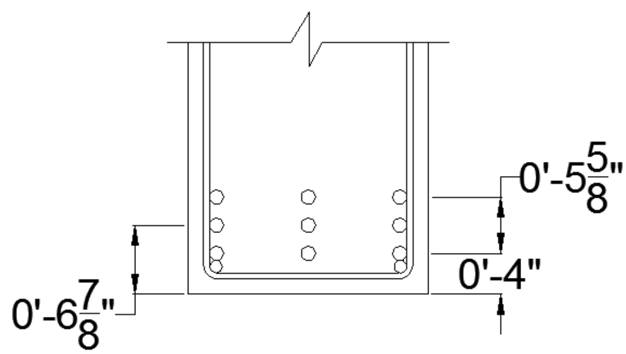


Figure 5.27 - Tension Tie Reinforcement for Design Example 2

Check Tie Location Requirements.

The centroid of the tie should line up with the node location; therefore, the centroid of the bottom tie reinforcement should start 7" above the bottom of the girder.

$y = 6.875"$ therefore d=96" $- 6.875" = 89.125"$

Determine total effective height of reinforcement.

$6.875" + (1.5\ rows\ of\ steel)(1.41") + (1.0\ row\ of\ spaces)(1.41") = 10.4"$

Check against height of tie.

$13.7" > 10.4"\ OK$

Check the area of steel required against minimum steel requirements.

$$\frac{3\sqrt{f'_c}b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \qquad \frac{3\sqrt{4{,}000psi}(24")(89.125")}{60{,}000psi} = 6.76in^2 < \frac{200(24")(89.125")}{60{,}000} = 7.13in^2$$

14.04 in^2 > 7.13 in^2 OK

Check Development length of #11 Hooked Bars.

Even though the nodal zones were designed using hydrostatic nodal zones, the anchorage length used will fall within the extended nodal zone which is acceptable. Development for a hook can be determined using ACI 318-08 Section 12.5.1.

$$\left(\frac{0.02\psi_e f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02(1.0)(60{,}000psi)}{\sqrt{4{,}000psi}}\right) 1.41 = 26.8in$$

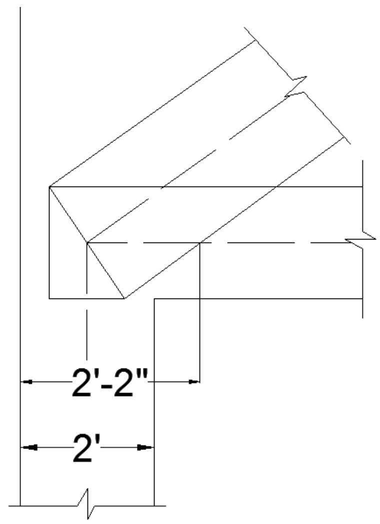


Figure 5.28 - STM Design Example 2 Anchorage Length Available

Available anchorage length: 26” – 1.5”cover = 24.5in

26.8in > 24.5in NG

Some solutions for getting enough development length would be to increase the column width or exchange #11 bars for #10 bars or smaller

Try 3 rows of 6 #8 bars.

$$A_s = (18)(0.79in^2) = 14.22in^2 > 13.9in^2 \quad OK$$

$$\left(\frac{0.02\psi_e f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02(1.0)(60{,}000psi)}{\sqrt{4{,}000psi}}\right) 1.0 = 19.0in < 24.5in \quad OK$$

All other checks OK by inspection

USE 3 Rows of 6 #8 bars.

<u>Step 9: Determine Crack Reinforcement per ACI A.3.3.1</u>

Angle between stirrups and struts = 90°-53.8°= 36.2°

Try #5 stirrups vertically at 10 inches on center and #5 longitudinal bars at 12 inches on center.

$A_{vh} = 0.0015 b_w S_h$ (EQ'N 4.25)

$A_v = 0.0025 b_w S_v$ (EQ'N 4.26)

$S_h = \frac{(2)(0.31in^2)}{.0015(24")} = 17.22" > 12"$ OK

$S_v = \frac{(2)(0.31in^2)}{.0025(24")} = 10.33" > 10"$ OK

$\sum \frac{A_{si}}{b_s s_i} sin\alpha_i \geq 0.003$ (EQ'N 5.5)

$\frac{(2)(0.31)}{(24)(10)} sin(36.2°) = 0.0015$

$\frac{(2)(0.31)}{(24)(12)} sin(53.8°) = 0.0017$

$\sum \frac{A_{si}}{b_s s_i} sin\alpha_i = 0.0015 + 0.0017 = 0.0032 \geq 0.0030 \quad OK$

USE #5 Stirrups at 10 inches O.C. and #5 Longitudinal Reinforcement at 12 inches O.C.

Cut sections of the completed design of the girder are shown in Figure 5.29 with dimensions and reinforcement.

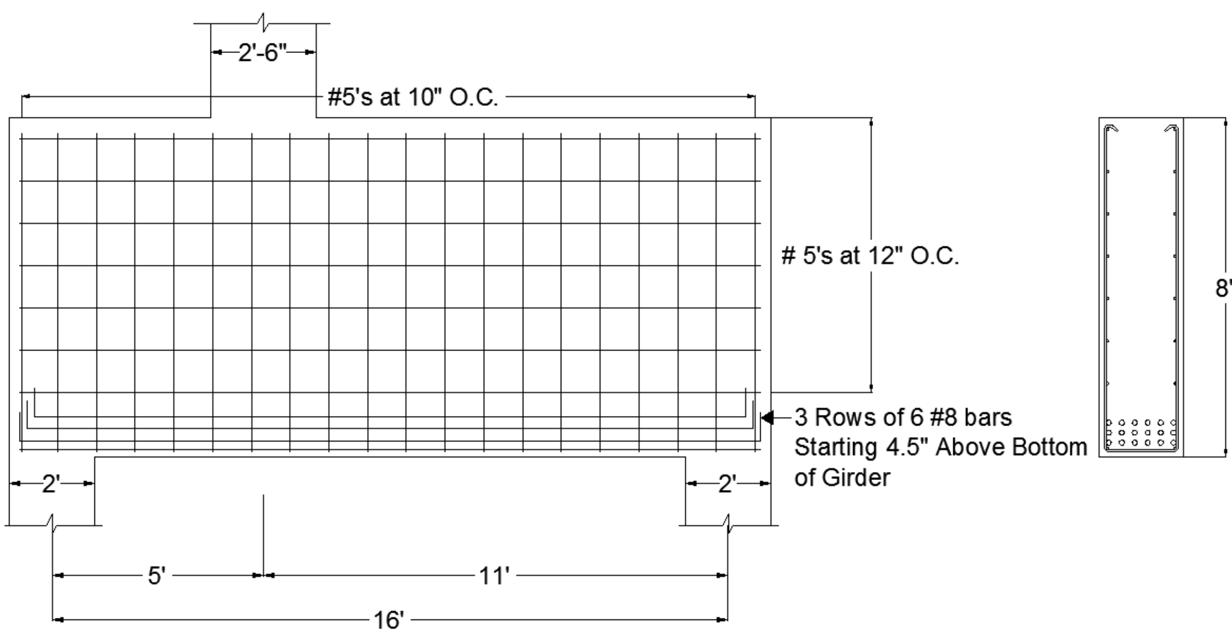


Figure 5.29 - STM Design Example 2 - Final Design Cut Sections

Note: If a 7' girder were used, the Tension reinforcement would be (17) #9 bars and the shear reinforcement would be #5's at 10" vertical and #5's at 12" horizontal. The 8' girder was used to keep the angle between the struts and tie around 45°.

5.6.3 STM Design Example 3

Design example three is a 24 inch wide transfer girder spanning 16 feet with a column at midpoint with a factored load of 600 kips; and a second load at the quarter point with factored load of 600 kips. The girder is supported by 24 inch square columns. A design height of 7 feet was determined by iteration. Figure 5.30 indicates the transfer girder for design. The calculated ultimate shear diagram is illustrated in Figure 5.31.

Note: Due to the offset of the load, the weight of the girder is included in the DL at the column load point. Also, because two loads are applied, getting a hydrostatic nodal zone would be very difficult. The following calculations take advantage of the allowable extended nodal zone according to ACI 318.

Weight of the girder = $w = 150pcf \times 7' \times {}^{24"}/_{12"} = 2{,}100plf$

Factored weight of girder = $w_u = 1.2 \times 2{,}100plf = 2{,}520plf \times 16' = 40.32k$

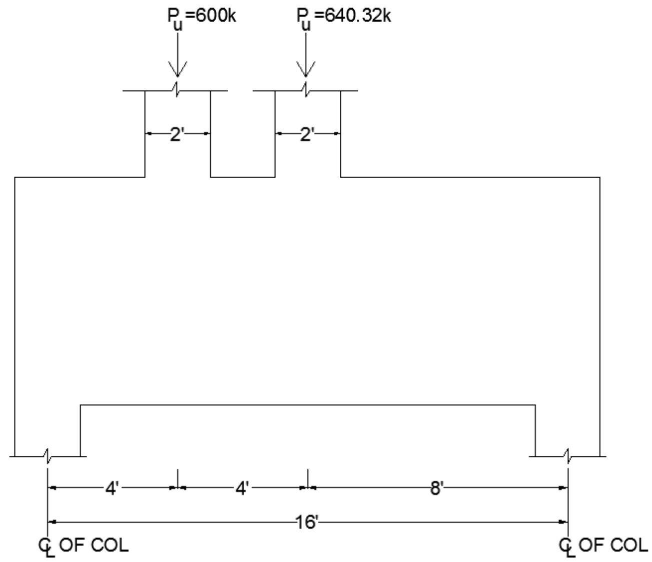


Figure 5.30 – STM Design Example 3

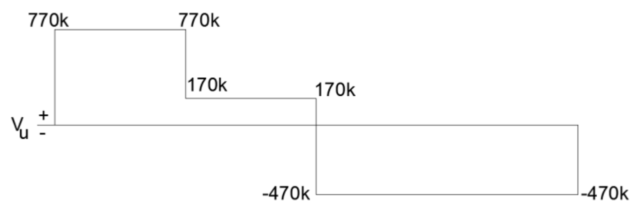


Figure 5.31 – STM Design Example 3 - Shear Diagram

f'_c = 4,000 psi F_y = 60,000 psi b_w = 24 inches

Step 1: Verify Trial Height

The minimum height allowed by code is determined in accordance with Equation 4.7 with *d* assumed to be 0.9h. Solving for *h* with V_u substituted for $\emptyset V_n$:

$\emptyset V_n \leq \emptyset 10\sqrt{f'_c} b_w d$ $770{,}000\# \leq (0.75)10\sqrt{4000psi}(24")(0.9h)$ (EQ'N 4.7)

h = 75.2 in Use h = 7 ft

Step 2: Check for Deep Beam Criteria

$\frac{l_n}{h} \leq 4.0$ $\frac{14'}{7'} = 2.0 \leq 4.0$ Deep Beam (EQ'N 3.1)

$\frac{a}{h} < 2.0$ $\frac{3'}{7'} = 0.4 < 2.0$ Deep Beam (EQ'N 3.2)

Step 3: Establish Node Locations

The node at location C is 9 inches from the top of the girder, the node location at the supports is 8 inches from the bottom of the girder, and the node location at D is 31 inches from the top of the girder shown in Figure 5.32.

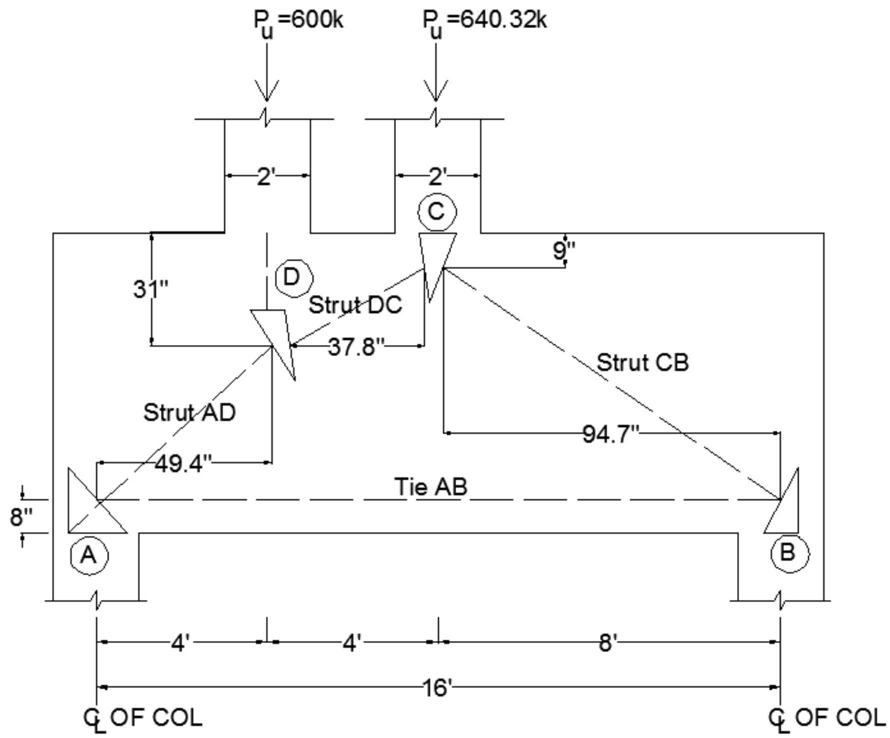


Figure 5.32 - STM Design Example 3 – Node Locations

Angle between Strut AD and Tie = $tan^{-1}\left(\frac{84"-31"-8"}{49.4"}\right) = 42.3° > 25°$ ACI 318 A.2.5

Angle between Strut DC and Tie = $tan^{-1}\left(\frac{31"-9"}{37.8"}\right) = 30.2° > 25°$ ACI 318 A.2.5

Angle between Strut CB and Tie = $tan^{-1}\left(\frac{84\text{-}8-9"}{94.7"}\right) = 35.3° > 25°$ ACI 318 A.2.5

Step 4: Determine Forces in Struts and Ties

Through Geometry of the Girder:

Length of Strut AD = $\sqrt{(84" - 31" - 8")^2 + (49.4")^2} = 66.8\ in$

Length of Strut DC = $\sqrt{(31")^2 + (49.4")^2} = 58.3\ in$

Length of Strut BC = $\sqrt{(84" - 9" - 8")^2 + (94.7")^2} = 116\ in$

Force in Strut BC = $470k \times \frac{116"}{84"-9"-8"} = 813.7k$

Force in Strut DC = $(640.32 - 470)k \times \frac{58.3"}{31"} = 320.3k$

Force in Strut AD =$(600k + 640.32k - 470k) \times \frac{66.8"}{84"-31"-8"} = 1{,}143.5k$

Maximum Force in Tie AB = $770k \times \frac{49.4"}{84"-8"-31"} = 691.6k$

Minimum Force in Tie AB = $470k \times \frac{94.7"}{84"-9"-8"} = 664k$

> *Note: Because the forces on each side are not equal, it is impossible to get a hydrostatic nodal zone with the current geometry. Because this geometry represents the actual path of the forces, this geometry will be used as will extended nodal zones.*

Step 5: Determine Effective Concrete Strength in Nodes and Struts

Because the girder has enough space for a bottle shape strut to form in Struts AD, DC, and BC, and steel will be provided to resist cracking, β_s=0.75 using ACI 318 A.3.3.

$f_{ce} = 0.85\beta_s f'_c \qquad f_{ce} = 0.85(0.75)(4{,}000) = 2{,}550\,psi$ (EQ'N 5.3)

The struts within the columns do not have enough space for a bottle shaped strut to form, so β_s=1.0 using ACI 318 A.3.2.1.

$f_{ce} = 0.85\beta_s f'_c \qquad f_{ce} = 0.85(1.0)(4{,}000) = 3{,}400\,psi$ (EQ'N 5.3)

For the nodal region at C and D, a C-C-C situation is presen;, thus β_n=1.0 according to ACI 318 A.5.2.1.

$f_{ce} = 0.85\beta_n f'_c \qquad f_{ce} = 0.85(1.0)(4{,}000) = 3{,}400\,psi$ (EQ'N 5.4)

For the nodal region at A and B, a C-C-T situation is present, so β_n=0.80 using ACI 318 A.5.2.2.

$f_{ce} = 0.85\beta_n f'_c \qquad f_{ce} = 0.85(0.80)(4{,}000) = 2{,}720\,psi$ (EQ'N 5.4)

Step 6: Determine STM Geometry

> *Note: Extended nodal regions were determined; therefore,, the stresses on each face of the region do not have to be identical, and the faces do not have to be perpendicular to the axis of the struts.*

$\emptyset F_n \geq F_u$ with $\emptyset = 0.75$ (EQ'N 5.1)

$f = \frac{P}{A}; \; Width\ of\ the\ strut, w_s = \frac{P}{f \times f_{ce}}$ (EQ'N 5.11)

Width of Strut A = $w_{s,A} = \frac{770\#,000}{(0.75)(2{,}720psi)(24")} = 15.7\ in$

To get enough strut width in strut AD, use $w_{s,A}$ *= 16.75 in > 15.7 in.*

Note: The 16.75 inch width was determined through geometry because the STM was drawn to scale. The 16.75 inches fits within the column at A.

Width of Strut B = $w_{s,B} = \frac{470{,}000\#}{(0.75)(2{,}720psi)(24")} = 9.6\ in$

Width of Strut C_2 = $w_{s,C2} = \frac{470{,}000\#}{(0.75)(3{,}400psi)(24")} = 7.7"\ in$

Width of Strut C_1 = $w_{s,C1} = \frac{640{,}320\# - 470{,}000\#}{(0.75)(3{,}400psi)(24in)} = 2.8\ in$

Width of Strut D = $w_{s,D} = \frac{600{,}000\#}{(0.75)(3{,}400psi)(24")} = 9.8"\ in$

Required Width of Strut AD = $w_{s,AD} = \frac{1{,}143{,}500\#}{(0.75)(2{,}550psi)(24")} = 24.9\ in$

Available Width of Strut through current geometry = 24.5 in ≈ 24.9 in OK

Required Width of Strut DC = $w_{s,DC} = \frac{320{,}300\#}{(0.75)(2{,}550psi)(24")} = 7\ in$

Available Width of Strut through current geometry = 18.4 in > 7 in OK

Required Width of Strut BC = $w_{s,BC} = \frac{813{,}700\#}{(0.75)(2{,}550psi)(24")} = 17.7\ in$

Available Width of Strut through current geometry = 20.4 in > 17.7 in OK

Height of the Tie = $w_T = \frac{691{,}600\#}{(0.75)(2{,}720psi)(24")} = 14.1\ in$ (EQ'N 5.10)

To get the required Width of Strut in Strut AD, tie height = 18.1"

Because of the extended nodal zone, 24 inch columns still work for the compression struts. Because the geometry determined fits within the girder and follows the rules of STM, this geometry and forces are deemed accurate shown in Figure 5.33.

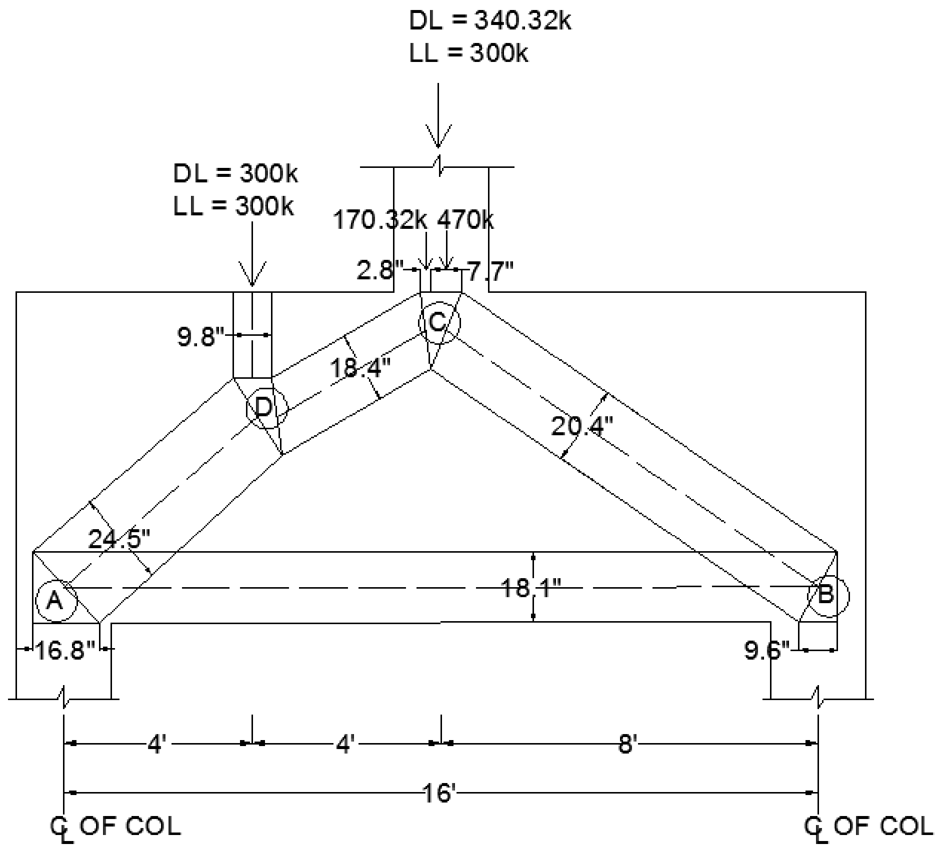


Figure 5.33 - STM Design Example 3 - Geometry

Step 7: Verify Node Locations

Once all geometries were calculated, the design was drawn to scale and actual node locations were determined shown in Figure 5.34. This could also be done through geometry. The node at C is 9.8 inches from the top of the girder, which is very close to the 9 inches initially selected, and the nodes at A and B are 9 inches from the bottom of the girder, which is also very close to the 8 inches initially selected. Node at D was chosen as 31inches and final location was very close at 31.6 inches. Initial node selections are considered acceptable.

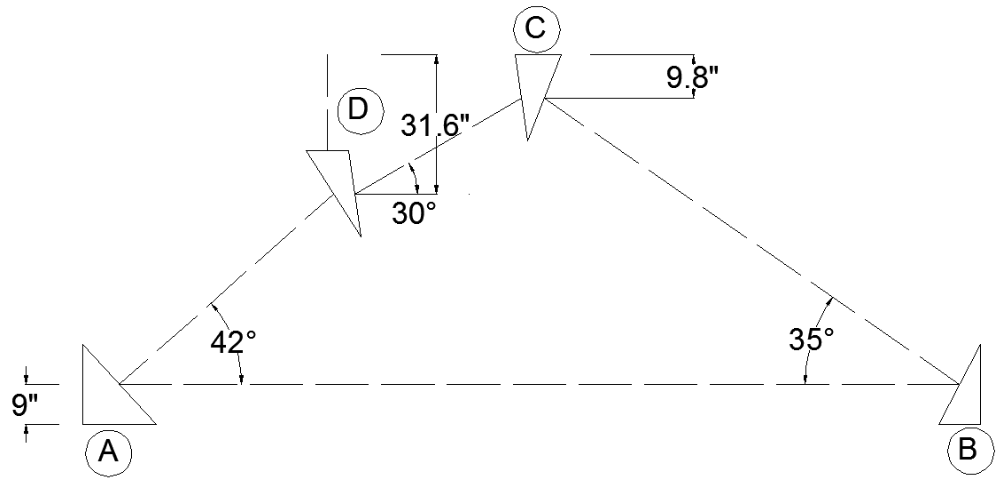


Figure 5.34 - STM Design Example 3 - Actual Node Locations

Step 8: Determine Steel in Tie

$$F_{nt} = A_{ts} f_y;\ A_{ts} = \frac{F_{nt}}{\emptyset f_y} = \frac{691{,}600\#}{(0.75)(60{,}000psi)} = 15.4\ in^2 \qquad \text{(EQ'N 5.9)}$$

Try 4 rows of 4 #9 bars.

$A_s = (16)(1.0in^2) = 16in^2$

Figure 5.35 represents the tension tie reinforcement of 4 rows of 4 #9 bars spaced 6.5".

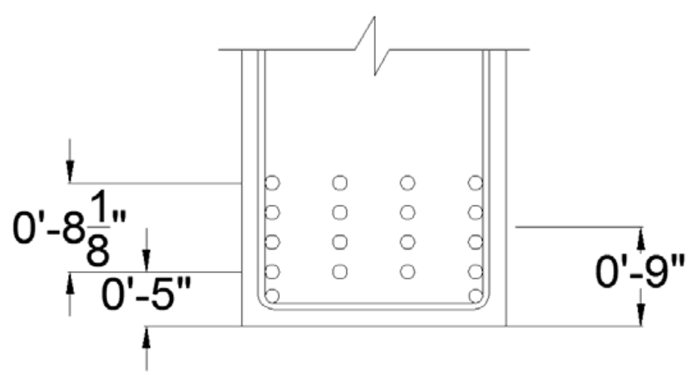


Figure 5.35 - Tension Tie Reinforcement for Design Example 3

Check tie location requirements.

The centroid of the tie should line up with the node location; therefore, the centroid of the bottom tie reinforcement should start 9" above the bottom of the girder.

$y = 9"$ therefore d=84" − 9" = 75"

Determine total effective height of reinforcement.

$$9" + (2\ rows\ of\ steel)(1.128") + (1.5\ rows\ of\ spaces)(1.41") = 13.37"$$

Check against height of Tie

$18.1">13.37"\ OK$

Check the area of steel required against minimum steel requirements.

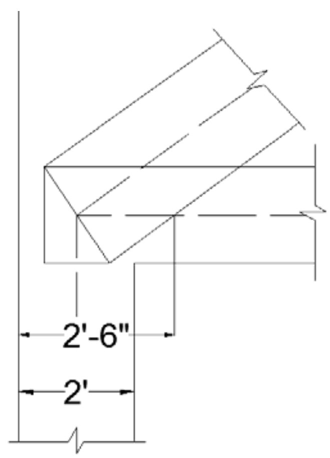
$$\frac{3\sqrt{f'_c}b_w d}{f_y} \geq \frac{200 b_w d}{f_y} \qquad \frac{3\sqrt{4{,}000psi}(24")(75")}{60{,}000psi} = 5.69in^2 < \frac{200(24")(75")}{60{,}000} = 6.0in^2$$

$16.0\ in^2 > 6.0\ in^2$ OK

Check Development length of #9 Hooked Bars.

Development for a hook can be determined using ACI 318-08 Section 12.5.1.

$$\left(\frac{0.02\psi_e f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02(1.0)(60{,}000psi)}{\sqrt{4{,}000psi}}\right) 1.128 = 21.4in$$



5.36 - STM Design Example 3 Anchorage Length Available

Available anchorage length: 30” – 1.5”cover = 28.5in

28.5in > 21.4in OK

USE: 4 Rows of 4 #9 bars.

Step 9: Determine Crack Reinforcement per ACI A.3.3.1

Angle between stirrups and struts = 90°-42°= 48°

Try #5 stirrups vertically at 10 inches on center and #5 longitudinal bars at 12 inches on center.

$A_{vh} = 0.0015 b_w S_h$ (EQ'N 4.25)

$A_v = 0.0025 b_w S_v$ (EQ'N 4.26)

$S_h = \frac{(2)(0.31in^2)}{.0015(24")} = 17.22" > 12"$ OK

$S_v = \frac{(2)(0.31in^2)}{.0025(24")} = 10.33" > 10"$ OK

$\sum \frac{A_{si}}{b_s s_i} sin\alpha_i \geq 0.003$ (EQ'N 5.5)

$\frac{(2)(0.31)}{(24)(10)} sin(48°) = 0.0019$

$\frac{(2)(0.31)}{(24)(12)} sin(42°) = 0.0014$

$\sum \frac{A_{si}}{b_s s_i} sin\alpha_i = 0.0019 + 0.0014 = 0.0033 \geq 0.0030 \quad OK$

USE #5 Stirrups at 10 inches O.C. and #5 Longitudinal Reinforcement at 12 inches O.C.

Cut sections of the completed design of the girder are shown in Figure 5.37 with dimensions and reinforcement.

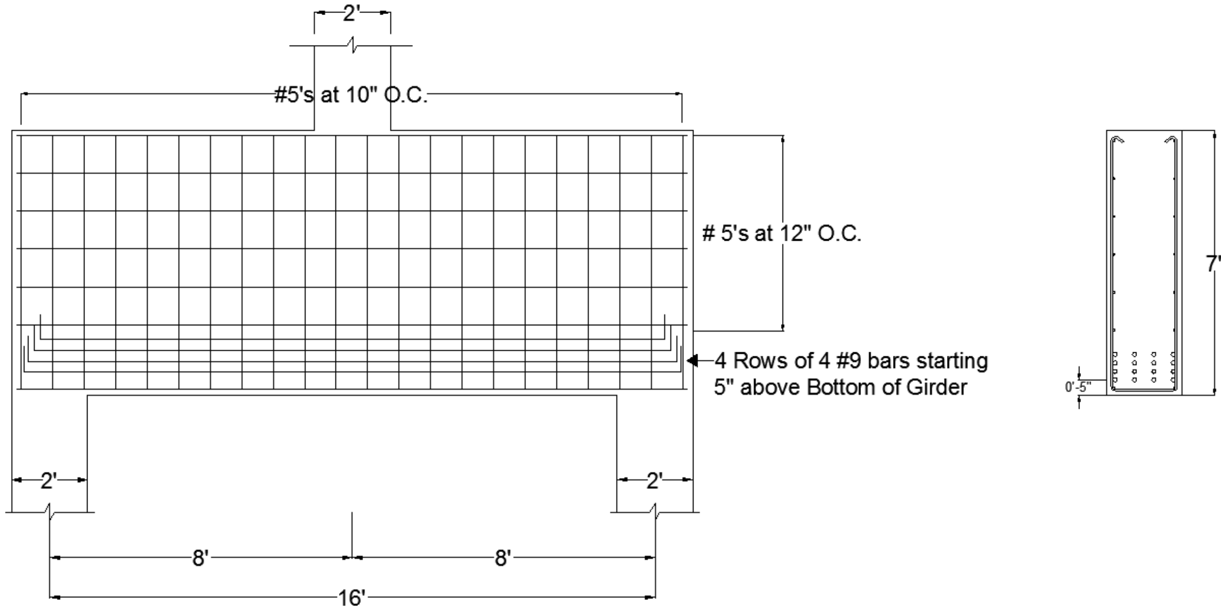


Figure 5.37 - STM Design Example 3 - Final Design Cut Sections

Note: A 6' girder does not meet the requirements of Equation 4.7. If a 8' girder were used, the Tension reinforcement would be (14) #9 bars and the shear reinforcement would be #5's at 10" vertical and #5's at 12". The 7' girder was used to keep the angles between the struts and longitudinal plane around to 40°.

6.0 Results Comparison and Conclusion

The deep beams designed in these examples varied in depth from 7 ft. to 8 ft. with the same amount of loading at varying locations. Table 1 summarizes the deep beams designs specifying concrete and steel quantities.

Table 1 - Deep Beam Summary

Girder	Dimensions	Horizontal Steel	Vertical Steel	Shear Reinf. Volume (in^3)	Flexural Steel	Flexural Steel Area (in^2)
DB 1	7' x 2'	#5's @ 10"	#5's @ 10"	887.2	18 #9 bars	18.0
DB 2	8' x 2'	#5's @ 9"	#5's @ 9"	1,190.40	16 #8 bars	12.6
DB 3	7' x 2'	#5's @ 8"	#5's @ 8"	1,123.40	15 #8 bars	11.9

The depth of the beams was governed by the maximum shear force applied to the structure and the shear reinforcement spacing desired. When the point load was in the center, shear was the lowest and moment was the greatest out of the three, represented by the smaller quantity of shear reinforcement and the greater amount of flexural steel. As the force was moved closer to the supports, the maximum shear became larger, and the moment decrease and represented in Design #2 with a deeper member with less flexural steel and more shear reinforcement than Design #1. For Design #3, the force was split evenly between the two previous locations, which produced the least amount of moment among the three beams and a shear force between the previous two.

Because the STM takes into consideration the extra shear capacity developed through arching action, the shear reinforcement required is decreased. Table 2 shows a design summary of the three girders designed using the Strut-and-Tie method.

Table 2 - STM Summary

Girder	Dimensions	Horizontal Steel	Vertical Steel	Shear Reinf. Volume (in^3)	Tensile Steel	Tensile Steel Area (in^2)
STM 1	7' x 2'	#5's @ 12"	#5's @ 10"	887.2	16 #10 bars	20.3
STM 2	8' x 2'	#5's @ 12"	#5's @ 10"	976.5	18 #8 bars	14.2
STM 3	7' x 2'	#5's @ 12"	#5's @ 10"	887.2	16 #9 bars	16.0

The total steel reinforcement weight calculated using DBM for DB1, DB2, and DB3 are 3,964 lbs, 4,689 lbs, and 4,421 lbs respectively. The total steel reinforcement weight calculated using STM for STM1, STM2, and STM3 are 4,085 lbs, 4,054 lbs, and 3,855 lbs respectively. As the applied load moves towards the supports creating more shear force, the total reinforcement weight decreases from DBM to STM. Comparing the two different designs, the shear or cracking control reinforcement decreases by an average 13% because the STM considers the extra shear capacity through arching action. The tension steel used for either flexure or the tension tie increases by an average of 16% from deep beam to STM design. This is due to STM taking shear force through tension at the nodes in the tension reinforcement to keep the nodes in equilibrium.

In Table 4 and Table 5, examples #1, #2, and #3 were redesigned for both Deep Beam and STM. The first two girders were designed by decreasing the depth by one foot. The third girder was designed by increasing the depth by one foot as the depth could not be decreased by one foot due to allowable shear requirements. The shear reinforcement spacing for the girders designed using DBM decreases as the beam height decreases because there is less concrete shear strength available. The reinforcement for STM stayed the same as the girders height decreased; however, the tension tie reinforcement increased because the shear force is taken through the tension steel instead of vertical and horizontal shear reinforcement.

Table 3 - Re-Designed Deep Beam Summary

Girder	Dimensions	Horizontal Steel	Vertical Steel	Tensile Steel	Tensile Steel Area (in^2)
DP 1	6' x 2'	#5's @ 7"	#5's @ 7"	19 #9 bars	29.6
DP 2	7' x 2'	#5's @ 6"	#5's @ 6"	17 #8 bars	26.5
DP 3	8' x 2'	#5's @10"	#5's @ 10"	14 #8 bars	14.0

Table 4 - Re-Designed STM Summary

Girder	Dimensions	Horizontal Steel	Vertical Steel	Tensile Steel	Tensile Steel Area (in^2)
STM 1	6' x 2'	#5's @ 12"	#5's @ 10"	19 #11 bars	29.6
STM 2	7' x 2'	#5's @ 12"	#5's @ 10"	17 #11 bars	26.5
STM 3	8' x 2'	#5's @ 12"	#5's @ 10"	14 #9 bars	14.0

STM is a method for designing a structure based on how forces are actually transferred to the supports or reactions. The main benefit of this method is the possibility of decreased member depth without increasing vertical and horizontal shear reinforcement; however, tensile reinforcement will increase because of the decreased angle between the struts and tie. If member depth is not an issue, the preferred method is the DBM because it is more widely known and understood. STM takes more time in design, especially if the designer is not familiar with the method. Neither method is more difficult to construct unless compression steel is added along the axis of the struts in an STM design or shear reinforcement at small spacing in DBM design, which could cause some minor constructability issues. Based on this investigation it is recommended that STM be considered when the designer needs to decrease the depth of the member and desires to keep shear reinforcement at reasonable spacing. If this is not required, DBM will produce accurate results within less calculation time.

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