STATIC ANALYSIS

OF PRESTRESSED CABLE NETWORKS

by

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Diploma, Taipei Institute of Technology, 1961

A MASTER'S REPORT

Submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1973

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LIST OF SYMBOLS

A	cross section of cable
C	correction
$\bar{d}$	displacement vector
E	Young's modulus
e	strain expression
F	force
$\bar{F}$	external force vector
$\bar{i}$	unit vector in x direction
j	joint notation
$\bar{j}$	unit vector in y direction
k	joint notation
$\bar{k}$	unit vector in z direction
L	length of member
$\bar{p}$	position vector of joint
T	tension
t	tension coefficient
u	component of displacement in x direction
v	component of displacement in y direction
w	component of displacement in z direction
X	component of external force in x direction
x	Cartesian coordinate
Y	component of external force in y direction
y	Cartesian coordinate

Z	component of external force in z direction
z	Cartesian coordinate
Δ	increment in a variable
ε	strain
σ	stress
λ	direction cosine of x component
μ	direction cosine of y component
ν	direction cosine of z component
α	$\alpha = \frac{\Delta u}{L}$
β	$\beta = \frac{\Delta v}{L}$
γ	$\gamma = \frac{\Delta w}{L}$

I. INTRODUCTION

While the theory of suspension bridges has now reached a fairly complete state, suspended roof structures are still under serious experimental and theoretical investigation. Many papers on this subject have been published. But the approaches to the analysis and the methods of solution are very different among the papers published.

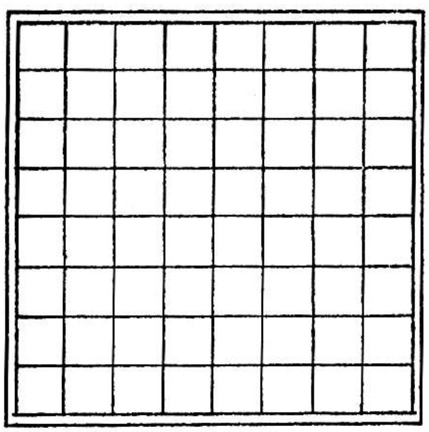
The purpose of this report is to present an effective method for the analysis of static prestressed cable nets. The method will consider nonlinear behavior of the cable nets, yet it will involve only simple mathematical equations and basic physical concepts.

Cable nets are usually designed to support roofs covering large areas such as stadiums, arenas, and shopping centers. A cable net may be formed by intersecting two or more sets of parallel cables as shown in Fig. 1. It may be an orthogonal net, as shown in Fig. 1(a), in which two sets of cables intersect at right angles. It may just as well be an oblique net, as shown in Fig. 1(b), in which two or more sets of cables intersect at specified angles other than right angles.

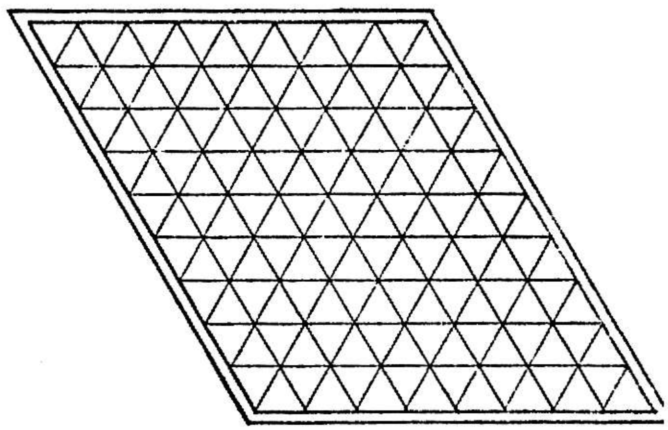
Architects and engineers have a strong interest in utilizing suspension systems for supporting roofs covering large spaces. There are several advantages in using suspension systems for large space structures. The first factor is economy. Suspension systems are usually less expensive than other structural systems for supporting long span roofs. The second factor is esthetics. The variety of roof forms and building shapes possible with suspension systems presents further opportunities for architectural expression. The third factor is stability. Cables work in pure tension so that very large spacings may be achieved with no stability problem.

Though the flexibility of the cables provides the advantages mentioned above, disadvantages also arise from the same characteristics. When the loading condition on suspension structures changes, it causes large movements which complicate the analysis, design, erection and maintenance of such structures in both static and dynamic aspects.

Suspension structures may be treated mathematically as discrete or continuous systems. In the discrete approach, the real structure is idealized into an assemblage of elements interconnected at a finite number of node points at which the loading is assumed concentrated as shown in Fig. 2(a). At each node, after deformation, equilibrium of forces and compatibility of displacements must be satisfied. The



(a) Orthogonal Net



(b) Oblique Net

Fig. 1 Cable Nets

mathematical model consists of a set of simultaneous algebraic equations.

In the continuous approach, simultaneous ordinary differential equations or partial differential equations are set up to represent the real structure. It is assumed that the cables are curved and continuous throughout the whole span as shown in Fig. 2(b). Physically, this situation can exist only if the loadings are distributed uniformly along the cable.

This report is intended to analyze prestressed cable nets by the discrete approach. The study will be limited to elastic theory, and will not include dynamic analysis.

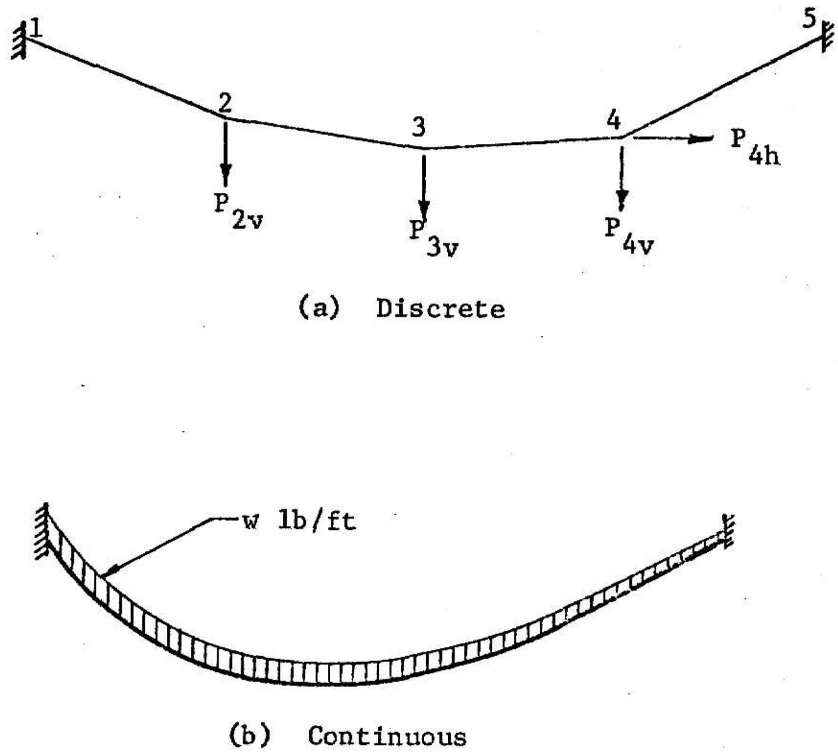


Fig. 2 Discrete and Continuous Approach to Cable Analysis

II LITERATURE REVIEW

According to Siev and Eidelman [1], a comprehensive survey of the existing knowledge with regard to suspended roofs up to 1955, has been compiled by Frei Otto [2]. The first article on the calculation of two-directional networks was presented by H. K. Bandel in 1959 [3].

In 1964, Siev and Eidelman [1] presented a mathematical treatment of stresses in prestressed suspension roof nets. The method followed the discrete approach. It was assumed that the slope of the two-directional net was small and, therefore, that the horizontal displacement components were negligible. Based on this assumption, the equilibrium equation for each joint was established. The relations thus obtained between loads and deflections were non-linear. The analysis further assumed that for small deflections, the non-linear terms, that is the cross terms or square terms of the unknown displacement components, may be disregarded. The simultaneous equations then become linear, and the results yielded an approximate solution. For more accurate results, a method of non-linear correction by an iterative process was also investigated.

In 1965, H. Mollmann [4] presented a study on the theory of suspension structures. The book covered the analysis of isolated single cables and the continuous and discrete approach to cable networks.

In the continuous approach to the cable net problem, a membrane theory was developed, in which cable spacings and the cross section of the cables were assumed to approach zero. In the discrete approach, a linear stress-strain relation was assumed, and the nonlinear simultaneous algebraic equations were established, based on the equilibrium conditions at each joint after the deformation of the structure had taken place. The method considered all three components of displacement at each joint. The mathematical system consisted of 3n equations for a network of n joints. The nonlinear simultaneous equations were solved in two steps. In the first step, the set of equations were linearized, temporarily, by neglecting the second order terms. The solution thus obtained was used, in the second step, to compute the second order terms. The corrections were then carried out iteratively until the differences between two consecutive results were negligible. The method dealt with all three components of displacement and thus yields a complete solution, although there were some minor terms which were neglected.

In 1971, Krishna and Agarwal [5] conducted a model study on a hypar shaped suspension roof net. A 12 ft. square plan was choosen. The network was anchored into a rigid frame. Solid high tensile steel wires were used instead of stranded cables. The results of the model test were compared with the theoretical values [1,6] which were obtained by neglecting nonlinearity. The comparison indicated that the difference was small for the smaller values of load, but it increased with increasing magnitude of loading. A general conclusion possible from this study was that the approximate linear theory could be used for the preliminary analysis of the behavior of a cable network. For some loading conditions, however, the differences between the measured and the computed results were rather large and the use of this approximate theory for final analysis would not generally be satisfactory.

In 1971, the Subcommittee on Cable-Suspended Structures of the Task Committee on Spacial Structures of the Committee on Metals, of the Structural Division of ASCE published a state-of-the-art paper [7]. The main purpose, as stated in that paper, was to aid engineers in locating information on the analysis, design, and errection of cable-suspended structures. The shapes of suspension systems, the structural analysis, the manufacture of wire cables and their physical properties, the design and erection of such structures were each presented in separate parts. In part II of that paper, the general, basic concepts of the structural analysis for continuous systems as well as for discrete systems were discussed. Isolated cables, orthogonal nets, and oblique nets were each presented. The discussion dealt with the initial shapes of suspension structures and the displacements resulting from changes of loading. The counterstressed double-layer suspension system was also presented in great detail. The last section of this part dealt with the dynamic response of suspension systems.

As to the structural strand and rope, the article covered some experimental results in addition to the general material properties.

In the section on the design and errection of such structures, selection of suspension system, loading conditions, cable selection, cable anchorage, fire proofing, watertightness, errection sequence, placement and tensioning of cables were all briefly discussed.

Finally the article encouraged further studies: (1) to develope more sophisticated procedures for the static and the dynamic analysis; (2) to investigate the mechanical properties of structural strand and rope; (3) to investigate the stress-strain relationship above the proportional limit and at elevated temperatures. Research on protection of cables and fittings against corrosion and fire was also urged.

The article referred to 92 papers which should be most valuable to interested engineers.

III METHOD OF ANALYSIS

A. Introduction

Cable roof structures are very flexible while the cables are hanging freely. When such structures are subjected to a small external load, they will deflect tremendously [8]. However, when the structure is prestressed in a proper manner and then subjected to external loads, its deflections will be significantly reduced. As a result, hypar shaped cable roof structures are widely used since the nature of the opposite curvature in their orthogonal axes makes possible the prestressing of the networks. In the hypar shaped cable net system, the direction in which the curvature is concave upward is considered to be the main axis. The direction orthorgonal to the main axis is considered to be the auxilliary axis. The cables along the main axis are the hanging cables; whereas those along the auxiliary axis are the bracing cables [9]. While the cables in the same family are parallel to one another, two cables from different families intersect at a specified angle. For an orthorgonal net, the angle is 90 degrees.

Before the structure is subjected to external loads, both families of cables are prestressed to give the structure some degree of stiffness. When the structure is loaded, the stresses in the hanging cables increase while those in the bracing cables decrease. Thus, the hanging cables are the load carrying elements in such structures.

The stress analysis of a hypar shaped cable net can be carried out by two different approaches, namely, the continuous approach and the discrete approach. The discrete method of analysis presented by Mollmann [4], and Mollman and Mortensen [9] will be followed in the following discussions.

B. General Assumptions

When a cable net system is treated as a discrete system, the following conditions are generally assumed:

(1). The network is made up of perfectly straight tension members.

(2). The tension members are connected by frictionless hinges.

(3). The centerlines of the tension members connected by one hinge intersect at one point.

(4). The tension members are made of Hookean material, thus

$$\sigma = E\varepsilon$$

is assumed throughout the analysis.

(5). The external loads can be applied only at the joints.

The general assumptions are very similar to those for a truss system. However, there is a difference between the analyses of conventional trusses and prestressed cable nets. In conventional truss systems, linear behavior is defined as the case where the change in geometry is so small as to have a negligible effect on the stresses. In prestressed cable nets, the situation is different: the stress in each member and the positions of the related joints are interdependent so as to satisfy the equilibrium conditions. Any displacement upsets the equilibrium and thus affects the load carrying capacity of the nets [10]. Therefore, the analysis of prestressed cable nets cannot be done without considering the effect of the changes in geometry.

Irrespective of the difference between the analyses of trusses and cable nets, since the structural elements are very similar, the conventional truss terminology will be used in this report.

C. Initial State

Let the internal force in the member connecting joints j and k be T^{jk}, with tension assumed to be positive. The tension coefficient is defined as

$$t^{jk} = \frac{T^{jk}}{L^{jk}}$$

where L^{jk} is the length of the member. Cartesian coordinates x^k, y^k and z^k will be used to denote the position of joint k. The analysis will be done in two steps, namely, the initial state and the final state. The initial state is the equilibrium configuration of the cable net, in other words, the positions of the joints, under the action of the prestress and a given external load. The external load is usually the weight of the cables themselves.

Using vector notation, as shown in Fig. 3, the position vector of joint j is expressed by

$$\bar{p}^j = \begin{Bmatrix} x^j \\ y^j \\ z^j \end{Bmatrix}$$

which is a directed line segment from the origin of the Cartesian coordinates to the joint. The vector representing member jk is

$$\Delta\bar{p}^{jk} = \bar{p}^k - \bar{p}^j = \begin{Bmatrix} x^k - x^j \\ y^k - y^j \\ z^k - z^j \end{Bmatrix} = \begin{Bmatrix} \Delta x^{jk} \\ \Delta y^{jk} \\ \Delta z^{jk} \end{Bmatrix}$$

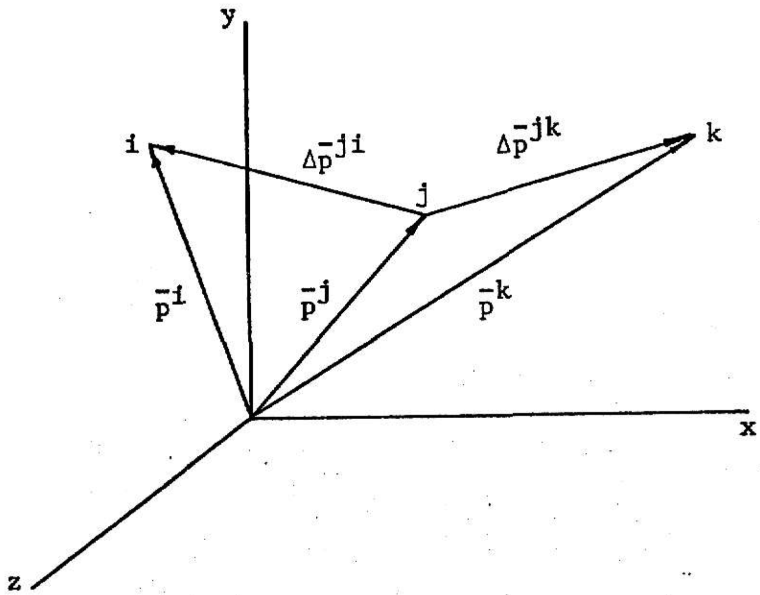


Fig. 3 Vector notation

The equilibrium condition of joint j requires that

$$\sum_k \{t\Delta\bar{p}\}^{jk} + \bar{F}_0^j = 0 \tag{1}$$

If equilibrium of the joint is considered in the direction of each of the three coordinate axes respectively, then

$$\sum F_x = 0, \qquad \sum_k \{t(x^k - x^j)\}^{jk} + X_0^j = 0 \tag{1.a}$$

$$\sum F_y = 0, \qquad \sum_k \{t(y^k - y^j)\}^{jk} + Y_0^j = 0 \tag{1.b}$$

$$\sum F_z = 0, \qquad \sum_k \{t(z^k - z^j)\}^{jk} + Z_0^j = 0 \tag{1.c}$$

where, $\bar{F}_0^j$ is the resultant load vector and X_0^j, Y_0^j, Z_0^j are its components at joint j; k is in turn, each of the four joints connected to joint j, including the boundary joints.

Since the weight of the cables is generally small when compared with the other external loads, the initial state is usually assumed to be the equilibrium configuration due to prestress only. Moreover, the x and y coordinates of each joint, whether interior or boundary, are specified before the analysis is performed. As a result, there is only one equation of equilibrium, e.g., $\sum F_z = 0$, at each interior joint, or

$$\sum_k \{t(z^k - z^j)\}^{jk} = 0 \tag{2}$$

In the case when the tension coefficients are identical for each member connected to joint j, Eq. (2) becomes

$$\sum_k (z^k - z^j) = 0 \tag{2.a}$$

The whole set of simultaneous linear algebraic equations consists of n equations with n unknowns for a network with n interior joints. The solution provides the z coordinates of the interior joints which, together with the x and y coordinates and the boundary joints, form the equilibrium configuration of the cable net under prestress.

D. Final State

When a prestressed cable net is subjected to additional external loads, it deforms into a new equilibrium configuration which will be referred to as the final state. If several loading conditions are to be investigated, as is often encountered at different construction stages during erection, they will each be treated in the same manner, but will be treated separately.

Let $\bar{d}^j$ represent the displacement vector of joint j from the initial state to the final state. Then the new position vector of joint j is

$$\bar{p}^j + \bar{d}^j = \begin{Bmatrix} x^j + u^j \\ y^j + v^j \\ z^j + w^j \end{Bmatrix}$$

where, u^j, v^j and w^j represent the components of displacement of joint j in the x, y and z directions. The vector representing member jk after deformation is

$$\Delta\bar{p}^{jk} + \Delta\bar{d}^{jk} = \begin{Bmatrix} \Delta x + \Delta u \\ \Delta y + \Delta v \\ \Delta z + \Delta w \end{Bmatrix}^{jk}$$

The lengths of the member in both states are

Initial state: L^{jk}

Final state : $L^{jk} + \Delta L^{jk}$

Equilibrium Equations

The equilibrium equation at joint j in the final state is

$$\sum_k \left\{\frac{T+\Delta T}{L+\Delta L}\,(\Delta\bar{p}+\Delta\bar{d})\right\}^{jk} + \bar{F}_0^j + \bar{F}^j = 0 \tag{3}$$

or in x, y and z components:

$$\sum_k \left\{\frac{T+\Delta T}{L+\Delta L}\,(\Delta x+\Delta u)\right\}^{jk} + X_0^j + X^j = 0 \tag{3.a}$$

$$\sum_k \left\{\frac{T+\Delta T}{L+\Delta L}\,(\Delta y+\Delta v)\right\}^{jk} + Y_0^j + Y^j = 0 \tag{3.b}$$

$$\sum_k \left\{\frac{T+\Delta T}{L+\Delta L}\,(\Delta z+\Delta w)\right\}^{jk} + Z_0^j + Z^j = 0 \tag{3.c}$$

The initial state values of x,y, z and thus Δx, Δy, Δz are substituted into Eqs. (3.a), (3.b), and (3.c), the problem in the final state is limited to solving for u,v,w, ΔT and ΔL. If ΔT and ΔL can further be expressed in terms of x,y,z,T,L, and u,v,w, then the mathematical system will be a set of 3n simultaneous algebraic equations in 3n unknown components of joint displacements for a network with n interior joints. Based on that argument, such expressions for ΔT and ΔL will be discussed next.

Elimination of ΔL

The length of member jk can be expressed in terms of the coordinates and the displacement components of joints j and k as

$$L^{jk} = |\Delta\bar{p}^{jk}| = (\Delta\bar{p}^{jk} \cdot \Delta\bar{p}^{jk})^{1/2}$$

$$L^{jk} + \Delta L^{jk} = |\Delta\bar{p}^{jk} + \Delta\bar{d}^{jk}| = \{(\Delta\bar{p}^{jk} + \Delta\bar{d}^{jk}) \cdot (\Delta\bar{p}^{jk} + \Delta\bar{d}^{jk})\}^{1/2}$$

Temporarily dropping superscripts,

$$(L+\Delta L)^2 = (\Delta\bar{p}+\Delta\bar{d}) \cdot (\Delta\bar{p}+\Delta\bar{d}) = \Delta\bar{p} \cdot \Delta\bar{p} + 2\Delta\bar{p} \cdot \Delta\bar{d} + \Delta\bar{d} \cdot \Delta\bar{d}$$

Dividing both sides by L^2,

$$\frac{(L+\Delta L)^2}{L^2} = (1 + \varepsilon)^2 = \frac{\Delta\bar{p}\cdot\Delta\bar{p}}{L^2} + \frac{2\Delta\bar{p}\cdot\Delta\bar{d}}{L^2} + \frac{\Delta\bar{d}\cdot\Delta\bar{d}}{L^2} \tag{4}$$

where ε is the strain of the member based on the length of the cable under prestress, e.g., $\varepsilon = 0$ at the initial state. Let

$$e_1 = \frac{1}{L^2} (\Delta\bar{p}.\Delta\bar{d})$$

$$e_2 = \frac{1}{L^2} (\Delta\bar{d}\cdot\Delta\bar{d})$$

Since $\Delta\bar{p}\cdot\Delta\bar{p} = L^2$, Eq. (4) becomes

$$(1 + \varepsilon)^2 = 1 + 2\varepsilon + \varepsilon^2 = 1 + 2e_1 + e_2$$

or

$$2\varepsilon(1 + \frac{\varepsilon}{2}) = 2(e_1 + \frac{1}{2} e_2)$$

Although the displacements of the joints in a network under load are not necessarily small, the strain is usually small when compared to the length of the member. Therefore, it would be reasonable to assume

$$1 + \frac{\varepsilon}{2} \simeq 1.$$

Then $$\varepsilon = e_1 + \frac{1}{2} e_2 \qquad (5)$$

and $$L + \Delta L = L(1 + \varepsilon) \qquad (6)$$

Stress-Strain Relation (Elimination of ΔT)

Let ε_0 and L_0 be the strain and the length of the member corresponding to the cable before prestress. As shown in Fig. 4, and defined in the previous section,

$$|\varepsilon_0| = \frac{L-L_0}{L}, \qquad L > L_0$$

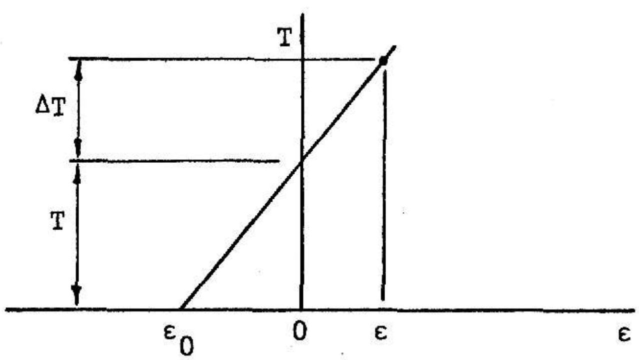


Fig. 4 Stress-Strain Curve

If sign is considered, then

$$\varepsilon_0 = \frac{L_0 - L}{L} < 0$$

Elastic theory requires that

$$T + \Delta T = EA \frac{L + \Delta L - L_0}{L_0} = EA \frac{L}{L_0} + EA \frac{\Delta L}{L_0} - EA \tag{7}$$

Since $\quad T = EA \dfrac{L - L_0}{L_0} = EA \dfrac{L}{L_0} - EA$

$$\Delta T = EA \frac{\Delta L}{L_0} \cdot \frac{L}{L} = EA\varepsilon \frac{L}{L_0}$$

$$\frac{L}{L_0} = \frac{L_0 + L - L_0}{L_0} = 1 + \frac{T}{EA}$$

Thus, $\quad \Delta T = \varepsilon(EA+T) \quad (8)$

E. Solution I

Substituting Eq. (6) and Eq. (8) into Eq. (3), the equilibrium equation at joint j becomes

$$\sum_k \left\{ \frac{T+\varepsilon(EA+T)}{L(1+\varepsilon)} (\Delta\bar{p}+\Delta\bar{d}) \right\}^{jk} + \bar{F}_0^j + \bar{F}^j = 0$$

$$\sum_k \left\{ \frac{T(1+\varepsilon)+\varepsilon EA}{L(1+\varepsilon)} (\Delta\bar{p}+\Delta\bar{d}) \right\}^{jk} + \bar{F}_0^j + \bar{F}^j = 0$$

or

$$\sum_k \left\{ \frac{T}{L}\Delta\bar{p} \right\}^{jk} + \bar{F}_0^j + \sum_k \left\{ \frac{T}{L}\Delta\bar{d} + \frac{\varepsilon EA}{L(1+\varepsilon)} (\Delta\bar{p}+\Delta\bar{d}) \right\}^{jk} + \bar{F}^j = 0$$

Substituting Eq. (1) and Eq. (5) into the last equation, and letting $1+\varepsilon \simeq 1$, then

$$\sum_k \left\{ \frac{T}{L}\Delta\bar{d} + \frac{EA}{L} (e_1 + \frac{1}{2} e_2)(\Delta\bar{p} + \Delta\bar{d}) \right\}^{jk} + F^j = 0 \tag{9}$$

Since $e_1 = \dfrac{\Delta\bar{p}\cdot\Delta\bar{d}}{L^2}$

$$e_2 = \frac{\Delta\bar{d}\cdot\Delta\bar{d}}{L^2}$$

the term $\frac{EA}{L} e_1 \Delta\bar{p}$ is linear in $\Delta\bar{d}$; but $\frac{EA}{2L} e_2(\Delta\bar{p}+\Delta\bar{d})$ and $\frac{EA}{L} e_1 \Delta\bar{d}$ are nonlinear in $\bar{d}$. The summation term in Eq. (9) can be broken up and rearranged into two parts such that the nonlinear terms are separated from the linear terms as follows:

$$\sum_k \left\{ \frac{T}{L} \Delta\bar{d} + \frac{EA}{L} e_1 \Delta\bar{p} \right\}^{jk} + \sum_k \left\{ \frac{EA}{2L} e_2 \Delta\bar{p} + \frac{EA}{L} (e_1 + \frac{1}{2} e_2) \Delta\bar{d} \right\}^{jk} + \bar{F}^j = 0 \qquad (10)$$

In Eq. (10), the first summation consists only of the linear terms in the displacement components, whereas the second summation consists only of the nonlinear terms in the same joint displacement components. Treating the nonlinear summation as the correction element in a successive iteration scheme, the solution can be carried out in the following manner:

(1) Neglecting the correction term temporarily, let

$$- \sum_k \left\{ \frac{T}{L} \Delta\bar{d} + \frac{EA}{L} e_1 \Delta\bar{p} \right\}^{jk} = \bar{F}^j \qquad (11)$$

Expanding e_1 in terms of displacement components,

$$e_1 = \frac{1}{L^2} (\Delta\bar{p} \cdot \Delta\bar{d})$$

$$= \frac{1}{L^2} (\Delta x\bar{i} + \Delta y\bar{j} + \Delta z\bar{k}) \cdot (\Delta u\bar{i} + \Delta v\bar{j} + \Delta w\bar{k})$$

$$= \frac{1}{L^2} (\Delta x\Delta u + \Delta y\Delta v + \Delta z\Delta w)$$

In which, $\bar{i}$, $\bar{j}$, $\bar{k}$ are unit vectors in the x, y and z directions respectivel and

$$\bar{i} \cdot \bar{i} = \bar{j} \cdot \bar{j} = \bar{k} \cdot \bar{k} = 1$$

$$\bar{i} \cdot \bar{j} = \bar{j} \cdot \bar{k} = \bar{k} \cdot \bar{i} = 0$$

Let $\lambda = \frac{\Delta x}{L}, \quad \mu = \frac{\Delta y}{L}, \quad \nu = \frac{\Delta z}{L}$

Then Eq. (11) can be expressed in the x component as

$$- \sum_k \left\{ \frac{T}{L}\Delta u + \frac{EA}{L^3} (\Delta x \Delta u + \Delta y \Delta v + \Delta z \Delta w) \Delta x \right\}^{jk} = X^j$$

or

$$\sum_k \left\{ (\frac{T}{L} + \frac{EA}{L} \lambda^2) u^j + \frac{EA}{L} \lambda\mu v^j + \frac{EA}{L} \lambda\nu w^j \right.$$

$$\left. - (\frac{T}{L} + \frac{EA}{L} \lambda^2) u^k - \frac{EA}{L} \lambda\mu v^k - \frac{EA}{L} \lambda\nu w^k \right. = X^j \qquad (12.a)$$

Accordingly,

$$\sum_k \left\{ \frac{EA}{L} \mu\lambda u^j + (\frac{T}{L} + \frac{EA}{L} \mu^2) v^j + \frac{EA}{L} \mu\nu w^j \right.$$

$$\left. - \frac{EA}{L} \mu\lambda u^k - (\frac{T}{L} + \frac{EA}{L} \mu^2) v^k - \frac{EA}{L} \mu\nu w^k \right. = Y^j \qquad (12.b)$$

$$\sum_k \left\{ \frac{EA}{L} \nu\lambda u^j + \frac{EA}{L} \nu\mu v^j + \left(\frac{T}{L} + \frac{EA}{L} \nu^2\right) w^j \right.$$

$$\left. - \frac{EA}{L} \nu\lambda u^k - \frac{EA}{L} \nu\mu v^k - \left(\frac{T}{L} + \frac{EA}{L} \nu^2\right) w^k \right\} = Z^j \tag{12.c}$$

The set of 3n simultaneous linear equations in 3n displacement components yields the approximate values of the components of displacement of the n interior joints.

(2) With the displacement components, the correction term can be computed from the second summation from Eq. (10) as

$$C^j = \sum_k \left\{ \frac{EA}{2L} e_2 \Delta\bar{p} + \frac{EA}{L} \left(e_1 + \frac{1}{2} e_2\right) \Delta\bar{d} \right\}^{jk} \tag{13}$$

Let $\alpha = \frac{\Delta u}{L}, \quad \beta = \frac{\Delta v}{L}, \quad \gamma = \frac{\Delta w}{L}$

Then, $e_2 = \frac{1}{L^2} (\Delta\bar{d} \cdot \Delta\bar{d})$

$$= \frac{1}{L^2} (\Delta u\bar{i} + \Delta v\bar{j} + \Delta w\bar{k}) \cdot (\Delta u\bar{i} + \Delta v\bar{j} + \Delta w\bar{k})$$

$$= \frac{1}{L^2} (\Delta u^2 + \Delta v^2 + \Delta w^2)$$

$$= \alpha^2 + \beta^2 + \gamma^2$$

and $$e_1 = \frac{1}{L^2} (\Delta x \Delta u + \Delta y \Delta v + \Delta z \Delta w)$$

$$= \lambda\alpha + \mu\beta + \nu\gamma$$

With the solution of the first step, e_1 and e_2 of any member can be computed. In x, y and z components respectively, Eq. (13) can be re-written as

$$C_x^j = \sum_k \left\{ \frac{EA}{2L} e_2 \Delta x + \frac{EA}{L} (e_1 + \frac{1}{2} e_2) \Delta u \right\}^{jk}$$

or

$$C_x^j = \sum_k \left\{ \frac{EA}{2} e_2 \lambda + EA(e_1 + \frac{1}{2} e_2) \alpha \right\}^{jk} \tag{14.a}$$

and

$$C_y^j = \sum_k \left\{ \frac{EA}{2} e_2 \mu + EA(e_1 + \frac{1}{2} e_2) \beta \right\}^{jk} \tag{14.b}$$

$$C_z^j = \sum_k \left\{ \frac{EA}{2} e_2 \nu + EA(e_1 + \frac{1}{2} e_2) \gamma \right\}^{jk} \tag{14.c}$$

The final equations after correction are

$$\sum_k \Big\{ (\frac{T}{L} + \frac{EA}{L}\lambda^2)u^j + \frac{EA}{L}\lambda\mu v^j + \frac{EA}{L}\lambda\nu w^j$$

$$- (\frac{T}{L} + \frac{EA}{L}\lambda^2)u^k - \frac{EA}{L}\lambda\mu v^k - \frac{EA}{L}\lambda\nu w^k = X^j + C_x^j \quad (15.a)$$

$$\sum_k \Big\{ \frac{EA}{L}\mu\lambda u^j + (\frac{T}{L} + \frac{EA}{L}\mu^2)v^j + \frac{EA}{L}\mu\nu w^j$$

$$- \frac{EA}{L}\mu\lambda u^k - (\frac{T}{L} + \frac{EA}{L}\mu^2)v^k - \frac{EA}{L}\mu\nu w^k = Y^j + C_y^j \quad (15.b)$$

$$\sum_k \Big\{ \frac{EA}{L}\nu\lambda u^j + \frac{EA}{L}\nu\mu v^j + (\frac{T}{L} + \frac{EA}{L}\nu^2)w^j$$

$$- \frac{EA}{L}\nu\lambda u^k - \frac{EA}{L}\nu\mu v^k - (\frac{T}{L} + \frac{EA}{L}\nu^2)w^k = Z^j + C_z^j \quad (15.c)$$

As a result of the correction, a set of refined displacement components is obtained.

(3) Step 2 is repeated until the desired accuracy is obtained.

(4) The final stress in a member is then computed by

$$T + \Delta T = T + \varepsilon(EA + T) \quad (16)$$

F. Solution II

According to Mollmann and Mortensen [9], convergence can be achieved faster when Eq. (10) is rewritten by adding and subtracting $\frac{\Delta T}{L} \Delta\bar{d}$ in the linear and the nonlinear terms respectively, as

$$\sum_k \left\{ \frac{T+\Delta T}{L} \Delta\bar{d} + \frac{EA}{L} e_1 \Delta\bar{p} \right\}^{jk} + \sum_k \left\{ \frac{EA}{2L} e_2 \Delta\bar{p} + [\frac{EA}{L} (e_1 + \frac{1}{2} e_2) - \frac{\Delta T}{L}] \Delta\bar{d} \right\}^{jk} + \bar{F}^j = 0 \qquad (17)$$

where ΔT is the predicted change of tension in each member. Since the changes in the tension in the cables of the same family are fairly uniform, they may be predicted by a percentage of the initial prestresses within some acceptable range of accuracy. According to Mollmann and Mortensen [9], it is acceptable if they do not differ by more than about 30 percent from the true ΔT values.

The final equations thus modified are

$$\sum_k \left\{ (\frac{T+\Delta T}{L} + \frac{EA}{L} \lambda^2) u^j + \frac{EA}{L} \lambda\mu v^j + \frac{EA}{L} \lambda\nu w^j \right.$$

$$\left. - (\frac{T+\Delta T}{L} + \frac{EA}{L} \lambda^2) u^k - \frac{EA}{L} \lambda\mu v^k - \frac{EA}{L} \lambda\nu w^k = X^j + C_x^j \qquad (18.a)\right.$$

$$\sum_k \left\{ \frac{EA}{L} \mu\lambda u^j + (\frac{T+\Delta T}{L} + \frac{EA}{L} \mu^2) v^j + \frac{EA}{L} \mu\nu w^j \right.$$

$$\left. - \frac{EA}{L} \mu\lambda u^k - (\frac{T+\Delta T}{L} + \frac{EA}{L} \mu^2) v^k - \frac{EA}{L} \mu\nu w^k = Y^j + C_y^j \qquad (18.b)\right.$$

$$\sum_k \frac{EA}{L} \nu\lambda u^j + \frac{EA}{L} \nu\mu v^j + (\frac{T+\Delta T}{L} + \frac{EA}{L} \nu^2) w^j$$

$$- \frac{EA}{L} \nu\lambda u^k - \frac{EA}{L} \nu\mu v^k = (\frac{T+\Delta T}{L} + \frac{EA}{L} \nu^2) w^k = Z^j + C_z^j \qquad (18.c)$$

where

$$C_x^j = \sum_k \left\{ \frac{EA}{2} e_2\lambda + [EA(e_1 + \frac{1}{2} e_2) - \Delta T]\alpha \right\}^{jk} \qquad (19.a)$$

$$C_y^j = \sum_k \left\{ \frac{EA}{2} e_2\mu + [EA(e_1 + \frac{1}{2} e_2) - \Delta T]\beta \right\}^{jk} \qquad (19.b)$$

$$C_z^j = \sum_k \left\{ \frac{EA}{2} e_2\nu + [EA(e_1 + \frac{1}{2} e_2) - \Delta T]\gamma \right\}^{jk} \qquad (19.c)$$

The method of solution follows the same steps as described in Solution I.

IV NUMERICAL EXAMPLE

As a practical matter, a hypar shaped prestressed cable net problem has to be solved by the use of a computer. There are three times as many equations as there are interior joints. Therefore, considerable computer time is needed to set up the equations and to solve for the displacements and the stresses in a real structure. As an example, Mollmann and Mortensen [9] solved a system of 252 interior joints. There were 3x252=756 equations. For each loading condition, with five iterations, the computer time was about one hour. Thus for the purpose of developing a computer program, a simple example structure has been selected. The program thus developed was checked, in Example 1, by long hand calculation using the same method. In Example 2, Solution I and Solution II are performed in order to compare the convergence.

Example 1:

A cable net, shown in Fig. 5, is fixed at the boundary joints. The initial state corresponds to the prestress loading only. The cross sections of the cables for both families of cables are 1.0 sq. in. and Young's modulus is $E = 24 \times 10^6$ psi. The horizontal component of cable prestress is 20,000 lb each for all cables. The loading is P = 10,000 lb in the z direction at each interior joint.

A WATFIV program, as shown in Appendix I, has been written to solve diamond hypar shaped cable net problems following Solution I discussed in Section III. The example problem is solved using the program. It is further checked by calculations using a desk calculator to set up the 15 simultaneous equations and then solving them by the use of the NOVA 1200 digital mini computer. The results are listed in Table 1 to Table 7 and Eq. (20) and (21).

The computer results were checked by long hand calculation because there are many summation terms performed by do-loops in the computer program and it is very difficult to find an error by checking the statements of the program itself. Therefore, the computer results were checked, up to the correction terms. There after, the stresses are computed by Eq. (16), which is a simple operation and the program can be observed to be logically correct. The computer results for ΔT are listed in Table 7. The table shows that ΔT ranges from 18,320 to 22,360 lb for the hanging cables and from -17,440 to -21,570 lb for the bracing cables. These values would tend to indicate compression in some of the cables in the final state. This result would necessitate the redesign of the cable net in a real design problem. It can be seen that the maximum stress in the hanging cables occurs in the final state, but the maximum stress in the bracing cables occurs in the initial state. For the case of equal horizontal prestress in both families of cables, the cross section of the hanging cables has to be designed to resist the tension in the final state, while that of the bracing cables has to be selected to resist the prestress in the initial state. For the example, the cross sections for both families of cables are 1.0 sq. in. If they are satisfactory for bracing cables, they will not be satisfactory for the hanging cables. Furthermore, the final stresses in the hanging cables are about two times their prestresses. For this reason, the cross section for the hanging cables is revised to 2.0 sq. in. in Example 2.

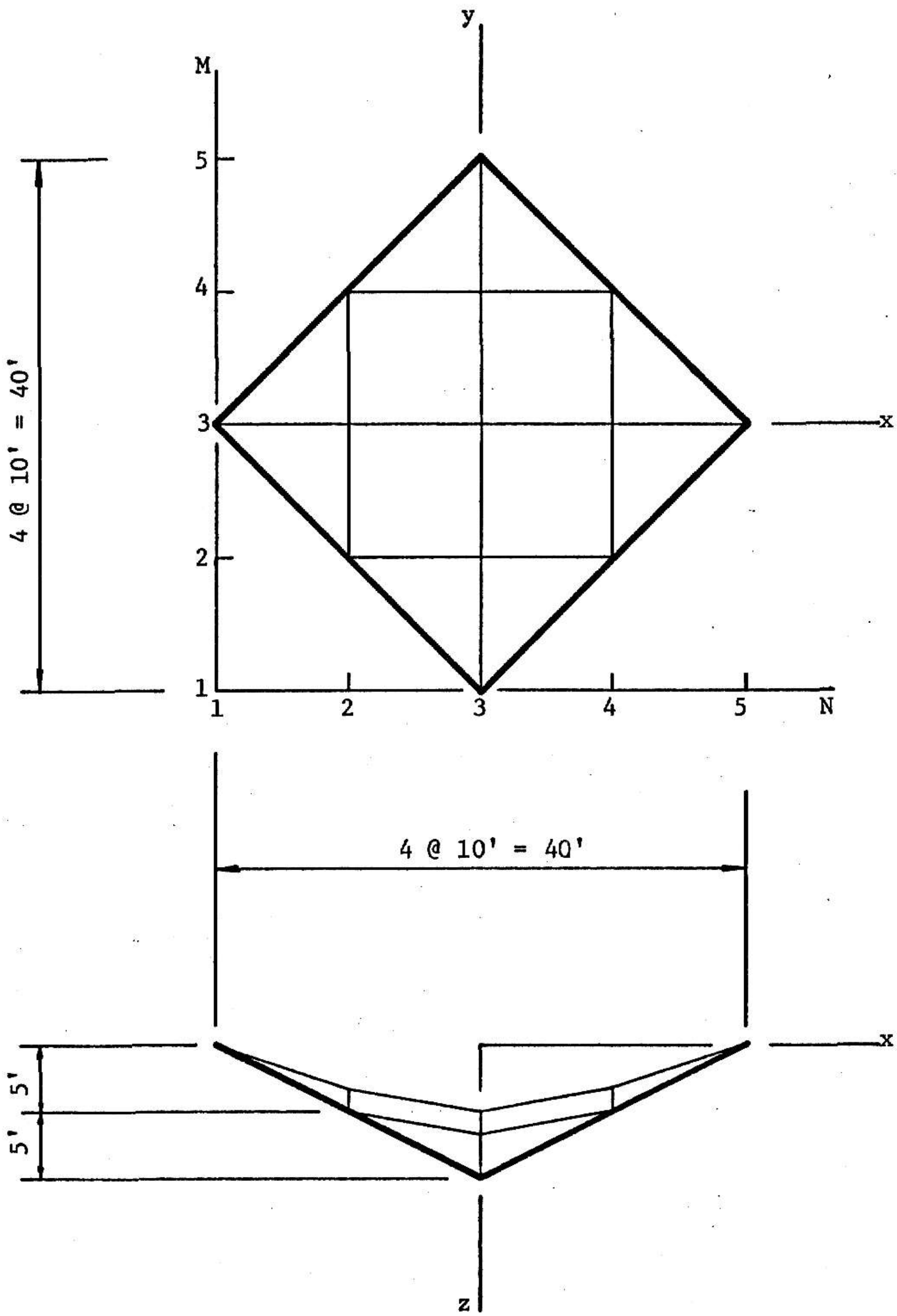


Fig. 5 Example Structure

Table 1: Coordinates of the Joints in Feet

Joint	Type	x	y	z
13[(1)]	B[(2)]	0	-20.0	10.0
22	B	-10.0	-10.0	5.0
23	I[(3)]	0	-10.0	*[(4)]
24	B	10.0	-10.0	5.0
31	B	-20.0	0	0
32	I	-10.0	0	*
33	I	0	0	*
34	I	10.0	0	*
35	B	20.0	0	0
42	B	-10.0	10.0	5.0
43	I	0	10.0	*
44	B	10.0	10.0	5.0
43	B	0	20.0	10.0

(1) Joint 13 represents Joint M,N, as shown in Fig. 5.

(2) B represents boundary joint.

(3) I represents interior joint.

(4) * means the coordinate is to be determined.

Eq. (20) Initial State Matrix Equation

$$\begin{bmatrix} 4 & 0 & -1 & 0 & 0 \\ 0 & 4 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 4 \end{bmatrix} \begin{Bmatrix} z_{23} \\ z_{32} \\ z_{33} \\ z_{34} \\ z_{43} \end{Bmatrix} = \begin{Bmatrix} 20 \\ 10 \\ 0 \\ 10 \\ 20 \end{Bmatrix}$$

Table 2: Initial State Solution in Feet

Joint	z
23	6.25[1]
32	3.75
33	5.00
34	3.75
43	6.25

(1) The computer results and the long hand results were essentially identical.

Table 3: Direction Cosines

Near Joint	Far Joint	L	λ	μ	ν
23[(1)]	22	10.0778	-0.9923[(2)]	0	-0.1240
	24	10.0778	0.9923	0	-0.1240
	13	10.6800	0	-0.9363	0.3511
	33	10.0778	0	0.9923	-0.1240
32	31	10.6800	-0.9363	0	-0.3511
	33	10.0778	0.9923	0	0.1240
	22	10.0778	0	-0.9923	0.1240
	42	10.0778	0	0.9923	0.1240
33	32	10.0778	-0.9923	0	-0.1240
	34	10.0778	0.9923	0	-0.1240
	23	10.0778	0	-0.9923	0.1240
	43	10.0778	0	0.9923	0.1240
34	33	10.0778	-0.9923	0	0.1240
	35	10.6800	0.9923	0	-0.3511
	24	10.0778	0	-0.9923	0.1240
	44	10.0778	0	0.9923	0.1240
43	42	10.0778	-0.9923	0	-0.1240
	44	10.0778	0.9923	0	-0.1240
	33	10.0778	0	-0.9923	-0.1240
	53	10.6800	0	0.9363	0.3511

(1) The joint where equilibrium is considered.

(2) The computer results and the long hand results are essentially identical.

Eq. (21) Final State Linearized Matrix Equation[1]

$$
\begin{bmatrix}
4698.0 & 0 & 0 & 0 & 0 & 0 & -2.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4323.0 & -1032.0 & 0 & 0 & 0 & 0 & -2347.0 & 293.0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1032.0 & 395.0 & 0 & 0 & 0 & 0 & 293.0 & -38.6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4323.0 & 0 & 1032.0 & -2347.0 & 0 & -293.0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4698.0 & 0 & 0 & -2.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1032.0 & 0 & 395.0 & -293.0 & 0 & -38.6 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.0 & 0 & 0 & -2347.0 & 0 & -293.0 & 4698.0 & 0 & 0 & -2347.0 & 0 & 293.0 & -2.0 & 0 & 0 \\
0 & -2347.0 & 293.0 & 0 & -2.0 & 0 & 0 & 4698.0 & 0 & 0 & -2.0 & 0 & 0 & -2347.0 & -293.0 \\
0 & 293.0 & -38.6 & -293.0 & 0 & -38.6 & 0 & 0 & 154.5 & 293.0 & 0 & -38.6 & 0 & -293.0 & -38.6 \\
0 & 0 & 0 & 0 & 0 & 0 & -2347.0 & 0 & 293.0 & 4323.0 & 0 & -1032.0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.0 & 0 & 0 & 4698.0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 293.0 & 0 & -38.6 & -1032.0 & 0 & 395.0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -2.0 & 0 & 0 & 0 & 0 & 0 & 4698.0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2347.0 & -293.0 & 0 & 0 & 0 & 0 & 4323.0 & 1032.0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -293.0 & -38.6 & 0 & 0 & 0 & 0 & 1032.0 & 395.0
\end{bmatrix}
\begin{Bmatrix} U_{23} \\ V_{23} \\ W_{23} \\ U_{32} \\ V_{32} \\ W_{32} \\ U_{33} \\ V_{33} \\ W_{33} \\ U_{34} \\ V_{34} \\ W_{34} \\ U_{43} \\ V_{43} \\ W_{43} \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 10.0 \\ 0 \\ 0 \\ 10.0 \\ 0 \\ 0 \\ 10.0 \\ 0 \\ 0 \\ 10.0 \\ 0 \\ 0 \\ 10.0 \end{Bmatrix}
$$

[1] This is the equation developed by long hand calculation. The computer developed equation compared closely.

Table 4: Final State Solution in Feet

Joint	U		V		W	
	Computer	Hand	Computer	Hand	Computer	Hand
23	0	0	0.01225	0.01225	0.06047	0.06048
32	-0.01225	-0.01225	0	0	0.06047	0.06048
33	0	0	0	0	0.03227	0.03229
34	0.01225	0.01225	0	0	0.06047	0.06048
43	0	0	-0.01225	-0.01225	0.06047	0.06048

Table 5: α, β and γ Values

Near Joint	Far Joint	α		β		γ	
		Computer	Hand	Computer	Hand	Computer	Hand
23	22	0	0	-0.001215	-0.001216	-0.006001	-0.006000
	24	0	0	-0.001215	-0.001216	-0.006001	-0.006000
	13	0	0	-0.001147	-0.001147	-0.005662	-0.005663
	33	0	0	-0.001215	-0.001216	-0.002798	-0.002797
32	31	0.001147	0.001147	0	0	-0.005662	-0.005663
	33	0.001215	0.001216	0	0	-0.002798	-0.002797
	22	0.001215	0.001216	0	0	-0.006001	-0.006000
	42	0.001215	0.001216	0	0	-0.006001	-0.006000
33	32	-0.001215	-0.001216	0	0	0.002798	0.002797
	34	0.001215	0.001216	0	0	0.002798	0.002797
	23	0	0	0.001215	0.001216	0.002798	0.002797
	43	0	0	-0.001215	-0.001216	0.002798	0.002797
34	33	-0.001215	-0.001216	0	0	-0.002798	-0.002797
	35	-0.001147	-0.001147	0	0	-0.005662	-0.005663
	24	-0.001215	-0.001216	0	0	-0.006001	-0.006000
	44	-0.001215	-0.001216	0	0	-0.006001	-0.006000
43	42	0	0	0.001215	0.001216	-0.006001	-0.006000
	44	0	0	0.001215	0.001216	-0.006001	-0.006000
	33	0	0	0.001215	0.001216	-0.002798	-0.002797
	53	0	0	0.001147	0.001147	-0.005662	-0.005663

Table 6: Corrections

Joint	C_x		C_y		C_z	
	Computer	Hand	Computer	Hand	Computer	Hand
23	0	0	-259.100	-259.060	-25.220	-25.259
32	-255.700	-255.684	0	0	9.261	9.255
33	0.002	0	0.003	0	1.250	1.250
34	255.700	255.684	0	0	9.261	9.255
43	0	0	259.100	259.060	25.220	-25.259

Table 7: ΔT in Pounds

Member	ΔT
13 - 23	- 21570
22 - 23	18320
23 - 24	18320
22 - 32	- 17440
23 - 33	- 20530
24 - 34	- 17440
31 - 32	22360
32 - 33	20730
33 - 34	20730
34 - 35	22360
32 - 42	- 17440
33 - 43	- 20530
34 - 44	- 17440
42 - 43	18320
43 - 44	18320
43 - 53	- 21570

Example 2

The problem of Example 1 is used in this example to test the convergence of Solutions I and II. The cross section of the hanging cables is revised to 2.0 sq. in.. A modified program for solution II is shown in Appendix II. The results are tabulated in Tables (8), (9), (10), (11), (12) and Fig. 6.

Table 8: Initial State

Joint	z value in ft	
	Solution I	Solution II
23	6.25	6.25
32	3.75	3.75
33	5.00	5.00
34	3.75	3.75
43	6.25	6.25

Table 9: Final State Displacements in ft., Solution I

Joint	Components	1st Iteration	2nd Iteration	3rd Iteration	4th Iteration	5th Iteration
	U	0	0	0	0	(1)
23	V	0.00858	0.00834	0.00835	0.00835	
	W	0.04175	0.04114	0.04117	0.04117	
	U	-0.00820	-0.00803	-0.00804	-0.00804	
32	V	0	0	0	0	
	W	0.04013	0.03959	0.03962	0.03962	
	U	0	0	0	0	
33	V	0	0	0	0	
	W	0.02042	0.02136	0.02132	0.02132	
	U	0.00820	0.00803	0.00804	0.00804	
34	V	0	0	0	0	
	W	0.04013	0.03959	0.03962	0.03961	
	U	0	0	0	0	
43	V	-0.00858	-0.00834	-0.00835	-0.00835	
	W	0.04175	0.04114	0.04117	0.04117	

(1) Convergence achieved at the 4th iteration. The desired accuracy between iterations was 0.00001 ft.

Table 10: Final State Displacements in ft., Solution II

Joint	Components	1st Iteration	2nd Iteration	3rd Iteration	4th Iteration	5th Iteration [1]
23	U	0	0	0	0	0
	V	0.00881	0.00832	0.00835	0.00835	0.00835
	W	0.04235	0.04110	0.04117	0.04117	0.04117
32	U	-0.00840	-0.00800	-0.00805	-0.00804	-0.00804
	V	0	0	0	0	0
	W	0.04062	0.03950	0.03963	0.03961	0.03961
33	U	0	0	0	0	0
	V	0	0	0	0	0
	W	0.01920	0.02155	0.02130	0.02133	0.02132
34	U	0.00840	0.00800	0.00805	0.00804	0.00804
	V	0	0	0	0	0
	W	0.04062	0.03950	0.03963	0.03961	0.03961
43	U	0	0	0	0	0
	V	-0.00881	-0.00832	-0.00835	-0.00835	-0.00835
	W	0.04235	0.04110	0.04117	0.04117	0.04117

(1) Convergence achieved at the 5th iteration. The desired accuracy between iterations was 0.00001 ft.

Table 11: Differences in the w Components Between Iterations:

Joint	Solution	1st-2nd	2nd-3rd	3rd-4th	4th-5th
23	I	0.00061	- 0.00003	0	
	II	0.00125	- 0.00007	0	0
32	I	0.00054	- 0.00003	0	
	II	0.00112	- 0.00013	0.00002	0
33	I	- 0.00094	0.00004	0	
	II	- 0.00235	0.00025	0.00003	0.00001

Table 12: Stress

Member	ε	Tension in lb		Stress in psi	
		Initial	Final	Initial	Final
13-23	-0.000614	21360	6613	21360	6613
22-23	0.000515	20160	44890	10080	22450
23-24	0.000515	20160	44890	10080	22450
22-32	-0.000480	20160	8634	20160	8634
23-33	-0.000576	20160	6322	20160	6322
24-34	-0.000480	20160	8634	20160	8634
31-32	0.000604	21360	50380	10680	25190
32-33	0.000568	20160	47450	10080	23720
33-34	0.000568	20160	47450	10080	23720
34-35	0.000604	21360	50380	10680	25190
32-42	-0.000480	20160	8634	20160	8634
33-43	-0.000576	20160	6324	20160	6324
34-44	-0.000480	20160	8634	20160	8634
42-43	0.000515	20160	44890	10080	22450
43-44	0.000515	20160	44890	10080	22450
43-53	-0.000614	21360	6613	21360	6613

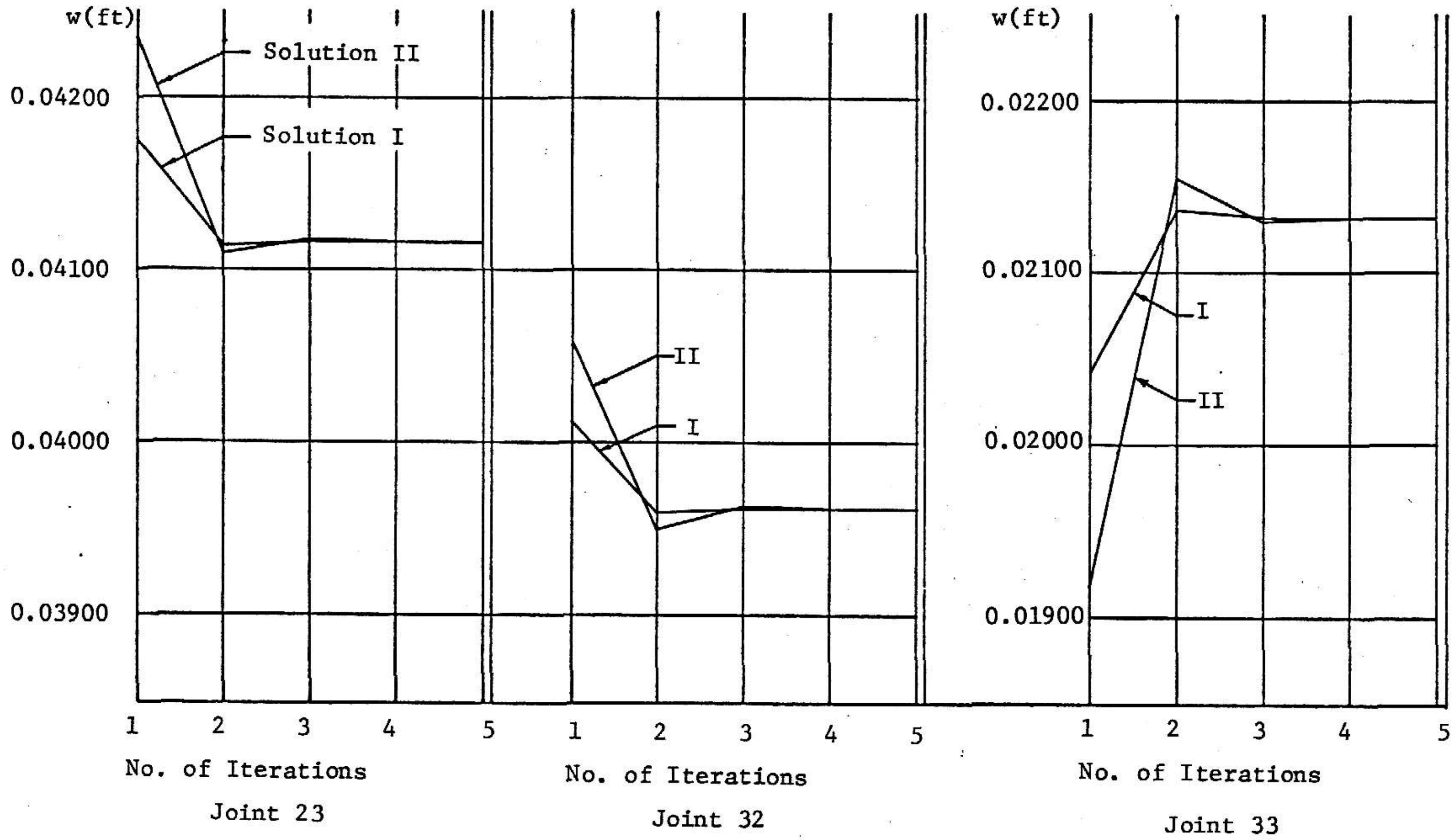


Fig. 6 Comparison of convergence for w components of displacement at Joints 23, 32, 33.

V. CONCLUSIONS

In conventional truss systems, the equilibrium condition of mechanics is based on the original unloaded geometric configuration under the assumption that the deformation of the system under load has a negligible effect on the stresses. However, the analysis of prestressed cable nets is based on equilibrium after the deformation has taken place. Since the deformation has to be considered in the analysis of a cable net, in the discrete method, the equilibrium conditions at a given joint yield 3 functions in terms of the displacements of all of the joints connected by members intersecting at that joint. Moreover, the equations are nonlinear. As a result, a set of nonlinear simultaneous algebraic equations in the displacements of all the interior joints has to be solved. The physical concepts of equilibrium involved are fairly familiar to every engineer, but the solution or even the approximation of the solution is complicated. Thus the method of solution becomes the center of interest to the investigator.

The method of analysis followed in this report converges fairly well. In Example 2, both solutions introduced were performed and the results of convergence are almost identical for that particular problem. In Solution I, as shown in Fig. 6, almost complete convergence was achieved at the 2nd iteration. In Solution II, the first iteration results were fairly poor. However, the 2nd iteration converged extremely fast, such that on the 3rd iteration the results converged almost to the same degree as Solution I did.

Though the example did not show clearly that Solution II converged better than Solution I as Mollmann and Mortensen [9] had stated, the different shapes of the convergence curves tended to show that the rate of convergence for Solution II was greater.

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ACKNOWLEDGMENTS

The sincere appreciation I feel, and I will remember, toward my major professor Dr. Robert R. Snell, Head of the Department of Civil Engineering, for his advice, guidance, patient correction and valuable suggestions on this report is beyond expression.

I am also grateful to Dr. Peter B. Cooper, Associate Professor of Civil Engineering, Dr. Edwin C. Lindly, Associate Professor of Applied Mechanics, and Dr. Stuart E. Swartz, Associate Professor of Civil Engineering, for their enthusiastic assistance which made my career as a graduate student at KSU fruitful and successful.

Thanks are extended to my wife Den-May Lai for her encouragement and her support of our family of three children in Taiwan, Rep. of China during the period of my study at KSU.

APPENDIX I

This is a program for performing the static analysis of diamond hypar shaped prestressed cable networks following Solution I. The dimensions of the arrays are described below:

X(M,N,J3) = the coordinates of joint M,N, where J3 = 1,2,3 is for x,y,z respectively.

DCSX(M,N,I4,J3) = the direction cosines, where I4 = 1 to 4 is for each of the four members intersecting at joint M,N, as shown in Fig. 7, J3 = 1,2,3 for λ,μ,ν.

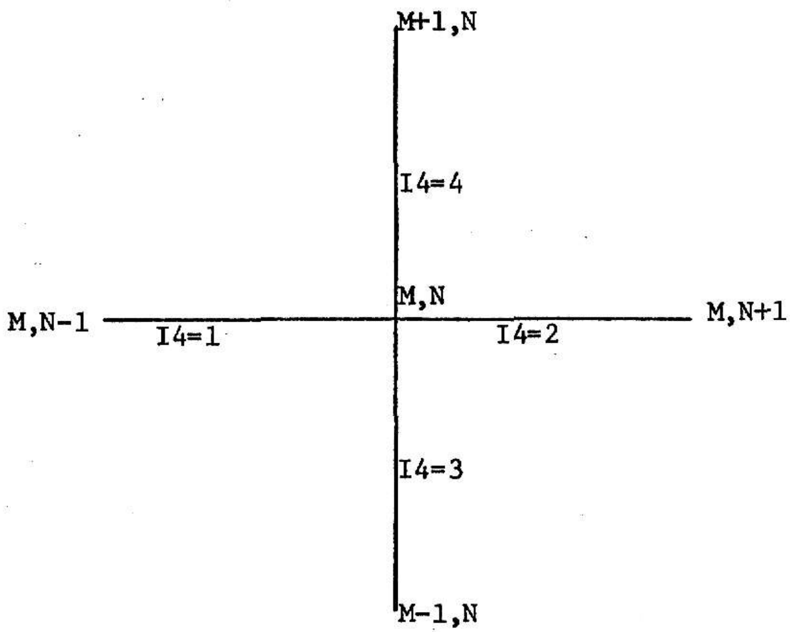

Fig. 7 Member Arrangement

A(I,J,M,N,J3) = the coefficients of Eq. (15.a), (15.b), and (15.c), where I = 1 to I1 is the sequence of the interior joints where equilibrium is considered, I1 is the total number of the interior joints, J = 1,2,3 represents Eq. (15.a), (15.b), and (15.c) respectively and J3 = 1,2,3 represents the u,v,w displacements respectively.

KEY(M,N) = -1,0,1 denote the boundary joints, out-of-net joints, and interior joints respectively.

DX(J3) = $\Delta x, \Delta y, \Delta z$ respectively for J3 = 1,2,3.

S(M,N,I4) = the length of the member identified by I4, as shown in Fig. 7.

F(I4) = EA/L.

C(IT,K) = the results of each iteration in turn, where IT = 1 to IT1 is the number of iterations, K = 1 to K1 is the number of equations.

U(M,N,J3) = the joint displacement components u,v, and w.

COR(I,J): the correction terms to be added to the loading components. For I,J, see A(I,J,M,N,J3).

DU(I4,J3): Δu, Δv, Δw.

E2H(I4): $e_2/2$.

DCSU(I4,J3): α, β, γ.

B(K,L) = the coefficients of the matrix equations at the final state before the operation of Gauss Reduction, where K = 1 to 3xI1, L = 1 to 3xI1+1.

BP(K) = the components of external loads.

E12(I4) = $e_1 + e_2/2$.

P(K,L) = the elements of the stiffness matrix, where L = 1 to 3xI1, I1 is the total number of the interior joints.

SS(J3) = the square of the components of the length of a member at the final state.

DT(M,N,I4) = the change in the tension in a member.

G(K,L) = the coefficients of the matrix equation at the initial state, where K = 1 to I1 and L = 1 to I1+1, I1 is the total number of the interior joints.

GAMA(M,N,I4): the strain in a member identified by I4, as shown in Fig. 7.

The program follows.

ILLEGIBLE DOCUMENT

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```
$JOB              CCL,TIME=15,PAGES=90
C       THIS PROGRAM PERFORMS THE ANALYSIS OF THE DIAMOND HYPAR SHAPED CABLE NET
C       OF EQUAL NO. OF CABLES IN BOTH DIRECTIONS.
C       MN IS THE NO. OF JOINTS IN EACH OF THE DIAGONAL CABLES INCLUDING THE
C       BOUNDARY JOINTS.
C       I1 IS THE TOTAL NO. OF INTERIOR JOINTS.
C       IT1 IS THE MAX. NO. OF ITERATIONS TO TERMINATE THE EXECUTION IF THE
C       ITERATION IS NOT CONVERGENT.
C       SPCM, HM, AND AM ARE THE SPACING, THE HORIZ. COMPONENT OF THE PRETENSION
C       AND CROSS SECTION OF THE CABLES PARALLEL TO THE M AXIS IN FT., LB., IN**
C       SPCN, HN, AND AN ARE THOSE PARALLEL TO THE N AXIS.
C       E IS THE YOUNG'S MODULUS IN PSI.
C       EPSI IS THE DESIRED ACCURACY IN FT..
        DIMENSION X(5,5,3),A(5,3,5,5,3),KEY(5,5),DX(3),S(5,5,4),F(4),
       1DCSX(5,5,4,3),C(9,15),U(5,5,3),COR(5,3),DU(4,3),F2H(4),DCSU(4,3),
       1B(15,16),IP(15),F12(4),P(15,15),SS(3),DT(5,5,4),G(5,6),GAMA(5,5,4)
  100 FORMAT(3I5,6E10.3)
  101 FORMAT(1H1,'   SPACING-M    SPACING-N   PRETEN-H-M   PRETEN-H-N
       1E          AREA-M        AREA-N'//7E12.4/)
  102 FORMAT(6E10.3)
        READ(5,100) MN,I1,IT1,SPCM,SPCN,HM,HN,AM,AN
        READ(5,102) E
        WRITE(6,101) SPCM,SPCN,HM,HN,E,AM,AN
        READ(5,102) EPSI
        READ(5,102) (((X(M,N,J3),J3=1,3),N=1,MN),M=1,MN)
        EAM=E*AM
        EAN=E*AN
        TSM=HM/SPCM
        TSN=HN/SPCN
        DO 160 M=1,MN
        DO 161 N=1,MN
        M2=M+N
        M3=(MN+3)/2
        IF(M2-M3) 162,163,180
  180 M4=N-M
        M5=(MN-1)/2
        IF(M4-M5) 181,163,162
  181 M6=M-N
        IF(M6-M5) 182,163,162
  182 M7=(MN*3+1)/2
        IF(M2-M7) 183,163,162
  183 KEY(M,N)=1
        GO TO 161
  162 KEY(M,N)=0
        GO TO 161
  163 KEY(M,N)=-1
  161 CONTINUE
  160 CONTINUE
        DO 170 M=1,MN
        DO 171 N=1,MN
        DO 172 J3=1,3
        U(M,N,J3)=0
        DO 173 I=1,I1
        DO 174 J=1,3
        A(I,J,M,N,J3)=0
        COR(I,J)=0
  174 CONTINUE
  173 CONTINUE
  172 CONTINUE
  171 CONTINUE
```

```
  170 CONTINUE
      I=0
      K=0
      DO 600 M=1,MN
      DO 601 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 601
      M1=M-1
      N1=N-1
      I=I+1
      DO 602 I4=1,4
      IF(I4 .GT. 2) GO TO 603
      A(I,3,M,N,3)=A(I,3,M,N,3)+1*TSM
      IF(KEY(M,N1) .EQ. -1) GO TO 604
      A(I,3,M,N1,3)=A(I,3,M,N1,3)-1*TSN
      N1=N1+2
      GO TO 602
  604 COR(I,3)=COR(I,3)+X(M,N1,3)*TSN
      N1=N1+2
      GO TO 602
  603 A(I,3,M,N,3)=A(I,3,M,N,3)+1*TSM
      IF(KEY(M1,N) .EQ. -1) GO TO 605
      A(I,3,M1,N,3)=A(I,3,M1,N,3)-1*TSM
      M1=M1+2
      GO TO 602
  605 COR(I,3)=COR(I,3)+X(M1,N,3)*TSM
      M1=M1+2
      GO TO 602
  602 CONTINUE
  601 CONTINUE
  600 CONTINUE
      K1=I1
      L1=K1+1
      L=0
      DO 610 M=1,MN
      DO 611 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 611
      L=L+1
      DO 612 K=1,K1
      G(K,L)=A(K,3,M,N,3)
  612 CONTINUE
  611 CONTINUE
  610 CONTINUE
      DO 613 K=1,K1
      G(K,L1)=COR(K,3)
  613 CONTINUE
      CALL GAUSR(G,K1,L1)
  109 FORMAT(//'     M     N          Z     '/)
  111 FORMAT(2I5,E12.4)
      WRITE(6,109)
      K=0
      DO 620 M=1,MN
      DO 621 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 621
      K=K+1
      X(M,N,3)=G(K,L1)
      WRITE(6,111) M,N,X(M,N,3)
  621 CONTINUE
  620 CONTINUE
      DO 622 I=1,I1
      DO 623 M=1,MN
```

```
      DO 624 N=1,MN
      A(I,3,M,N,3)=0
  624 CONTINUE
  623 CONTINUE
  622 CONTINUE
      K1=I1*3
      L1=K1+1
      I=0
      DO 200 M=1,MN
      DO 201 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 201
      M1=M-1
      N1=N-1
      DO 202 I4=1,4
      IF(I4 .GT. 2) GO TO 204
      DO 203 J3=1,3
      DX(J3)=X(M,N1,J3)-X(M,N,J3)
  203 CONTINUE
      N1=N1+2
      GO TO 206
  204 DO 205 J3=1,3
      DX(J3)=X(M1,N,J3)-X(M,N,J3)
  205 CONTINUE
      M1=M1+2
  206 S(M,N,I4)=SQRT(DX(1)**2+DX(2)**2+DX(3)**2)
      IF(I4 .GT. 2) GO TO 208
      F(I4)=EAN/S(M,N,I4)
      GO TO 209
  208 F(I4)=EAM/S(M,N,I4)
  209 DO 207 J3=1,3
      DCSX(M,N,I4,J3)=DX(J3)/S(M,N,I4)
  207 CONTINUE
  202 CONTINUE
      I=I+1
      DO 220 J=1,3
      DO 221 I4=1,4
      DO 222 J3=1,3
      A(I,J,M,N,J3)=A(I,J,M,N,J3)+F(I4)*DCSX(M,N,I4,J)*DCSX(M,N,I4,
      IF(I4 .GT. 2) GO TO 223
      IF(J .EQ. J3) A(I,J,M,N,J3)=A(I,J,M,N,J3)+TSN
      GO TO 222
  223 IF(J .EQ. J3) A(I,J,M,N,J3)=A(I,J,M,N,J3)+TSM
  222 CONTINUE
  221 CONTINUE
      N1=N-1
      M1=M-1
      DO 224 I4=1,4
      IF(I4 .GT. 2) GO TO 225
      IF(KEY(M,N1) .LE. 0) GO TO 228
      DO 226 J3=1,3
      A(I,J,M,N1,J3)=A(I,J,M,N1,J3)-F(I4)*DCSX(M,N,I4,J)*DCSX(M,N,I
      IF(J .EQ. J3) A(I,J,M,N1,J3)=A(I,J,M,N1,J3)-TSN
  226 CONTINUE
  228 N1=N1+2
      GO TO 224
  225 IF(KEY(M1,N) .LE. 0) GO TO 229
      DO 227 J3=1,3
      A(I,J,M1,N,J3)=A(I,J,M1,N,J3)-F(I4)*DCSX(M,N,I4,J)*DCSX(M,N,I
      IF(J .EQ. J3) A(I,J,M1,N,J3)=A(I,J,M1,N,J3)-TSM
  227 CONTINUE
```

```
  229 M1=M1+2
  224 CONTINUE
  220 CONTINUE
  201 CONTINUE
  200 CONTINUE
      L=0
      K=0
      DO 230 M=1,MN
      DO 231 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 231
      DO 232 J3=1,3
      L=L+1
      DO 233 I=1,I1
      DO 234 J=1,3
      K=K+1
      B(K,L)=A(I,J,M,N,J3)
      P(K,L)=B(K,L)
      IF(K .EQ. K1) K=K-K1
  234 CONTINUE
  233 CONTINUE
  232 CONTINUE
  231 CONTINUE
  230 CONTINUE
      READ(5,102) (BP(K),K=1,K1)
      DO 240 K=1,K1
      B(K,L1)=BP(K)
  240 CONTINUE
  114 FORMAT(//' ((B(K,L),L=1,L1),K=1,K1) ARE'//(5X,10E12.4))
      WRITE(6,114) ((B(K,L),L=1,L1),K=1,K1)
      DO 300 IT=1,IT1
      CALL GAUSR(B,K1,L1)
  105 FORMAT(1H1,11H ITERATION:I2)
  106 FORMAT(//'       U             V             W     '//(3E12.4))
  117 FORMAT(///////////11H ITERATION:I2)
      IF(IT .GT. 1) GO TO 241
      WRITE(6,105) IT
      GO TO 242
  241 WRITE(6,117) IT
  242 WRITE(6,106) (B(K,L1),K=1,K1)
      DO 301 K=1,K1
      C(IT,K)=B(K,L1)
  301 CONTINUE
      IF(IT .EQ. 1) GO TO 302
      DO 303 K=1,K1
      D=C(IT,K)-C((IT-1),K)
      IF(ABS(D) .GT. EPSI) GO TO 302
  303 CONTINUE
      GO TO 999
  302 K=0
      DO 311 M=1,MN
      DO 312 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 312
      DO 313 J3=1,3
      K=K+1
      U(M,N,J3)=B(K,L1)
  313 CONTINUE
  312 CONTINUE
  311 CONTINUE
      I=0
      DO 320 M=1,MN
```

```
      DO 321 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 321
      M1=M-1
      N1=N-1
      DO 322 I4=1,4
      IF(I4 .GT. 2) GO TO 323
      DO 324 J3=1,3
      DU(I4,J3)=U(M,N1,J3)-U(M,N,J3)
  324 CONTINUE
      N1=N1+2
      GO TO 325
  323 DO 326 J3=1,3
      DU(I4,J3)=U(M1,N,J3)-U(M,N,J3)
  326 CONTINUE
      M1=M1+2
  325 E1=0
      E2=0
      DO 327 J3=1,3
      DCSU(I4,J3)=DU(I4,J3)/S(M,N,I4)
      E1=E1+DCSX(M,N,I4,J3)*DCSU(I4,J3)
      E2=E2+DCSU(I4,J3)**2
  327 CONTINUE
      E2H(I4)=E2/2
      E12(I4)=E1+E2H(I4)
  322 CONTINUE
      I=I+1
      DO 332 J=1,3
      COR(I,J)=0
      DO 335 I4=1,4
      IF(I4 .GT. 2) GO TO 336
      COR(I,J)=COR(I,J)+GAM*(E2H(I4)*DCSX(M,N,I4,J)+E12(I4)*DCSU(I4,J))
      GO TO 335
  336 COR(I,J)=COR(I,J)+GAM*(E2H(I4)*DCSX(M,N,I4,J)+E12(I4)*DCSU(I4,J))
  335 CONTINUE
  332 CONTINUE
  321 CONTINUE
  320 CONTINUE
  116 FORMAT(//' COR(I,J) ARE:'//(5X,3E12.4))
      WRITE(6,116) ((COR(I,J),J=1,3),I=1,I1)
      K=0
      DO 340 I=1,I1
      DO 341 J=1,3
      K=K+1
      B(K,L1)=BP(K)+COR(I,J)
  341 CONTINUE
  340 CONTINUE
      DO 350 K=1,K1
      DO 351 L=1,K1
      B(K,L)=P(K,L)
  351 CONTINUE
  350 CONTINUE
  300 CONTINUE
  999 CONTINUE
  107 FORMAT(1H1,//'  M    N   I4      GAMA              T         DELTA T
     1FINAL T       STRESS  FINAL STRESS'//)
  108 FORMAT(I3,2I5,6E12.4)
      WRITE(6,107)
      DO 400 M=1,MM
      DO 401 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 401
```

```
      M1=M-1
      N1=N-1
      DO 402 I4=1,4
      IF(I4 .GT. 2) GO TO 404
      DO 403 J3=1,3
      SS(J3)=(X(M,N,J3)+U(M,N,J3)-X(M,N1,J3)-U(M,N1,J3))**2
  403 CONTINUE
      N1=N1+2
      GO TO 406
  404 DO 405 J3=1,3
      SS(J3)=(X(M,N,J3)+U(M,N,J3)-X(M1,N,J3)-U(M1,N,J3))**2
  405 CONTINUE
      M1=M1+2
  406 DS=SQRT(SS(1)+SS(2)+SS(3))-S(M,N,I4)
      GAMA(M,N,I4)=DS/S(M,N,I4)
      IF(I4 .GT. 2) GO TO 407
      TN=TSN*S(M,N,I4)
      DT(M,N,I4)=(FAN+TN)*DS/S(M,N,I4)
      TFN=TN+DT(M,N,I4)
      STN=TN/AN
      STFN=TFN/AN
      WRITE(6,108) M,N,I4,GAMA(M,N,I4),TN,DT(M,N,I4),TFN,STN,STFN
      GO TO 402
  407 TM=TSM*S(M,N,I4)
      DT(M,N,I4)=(FAM+TM)*DS/S(M,N,I4)
      TFM=TM+DT(M,N,I4)
      STM=TM/AM
      STFM=TFM/AM
      WRITE(6,108) M,N,I4,GAMA(M,N,I4),TM,DT(M,N,I4),TFM,STM,STFM
  402 CONTINUE
  401 CONTINUE
  400 CONTINUE
      STOP
      END

      SUBROUTINE GAUSR(A,N,N1)
      DIMENSION A(N,N1)
      DO 500 J=1,N
      DIV=A(J,J)
      S=1.0/DIV
      DO 501 K=J,N1
  501 A(J,K)=A(J,K)*S
      DO 502 I=1,N
      IF(I-J) 503,502,503
  503 AIJ=-A(I,J)
      DO 504 K=J,N1
  504 A(I,K)=A(I,K)+AIJ*A(J,K)
  502 CONTINUE
  500 CONTINUE
      RETURN
      END

$ENTRY
```

APPENDIX II

This is a program to perform the static analysis of diamond hypar shaped prestressed cable networks following Solution II.

The program follows.

```
$JOB            CCL,TIME=15,PAGES=90
C      THIS PROGRAM PERFORMS THE ANALYSIS OF THE DIAMOND HYPAR SHAPED CABLE N
C      OF EQUAL NO. OF CABLES IN BOTH DIRECTIONS.
C      MN IS THE NO. OF JOINTS IN EACH OF THE DIAGONAL CABLES INCLUDING THE
C      BOUNDARY JOINTS.
C      I1 IS THE TOTAL NO. OF INTERIOR JOINTS.
C      IT1 IS THE MAX. NO. OF ITERATIONS TO TERMINATE THE EXECUTION IF THE
C      ITERATION IS NOT CONVERGENT.
C      SPCM, HM, AND AM ARE THE SPACING, THE HORIZ. COMPONENT OF THE PRETENSI
C      AND CROSS SECTION OF THE CABLES PARALLEL TO THE M AXIS IN FT., LB., IN
C      SPCN, HN, AND AN ARE THOSE PARALLEL TO THE N AXIS.
C      E IS THE YOUNG'S MODULUS IN PSI.
C      EPSI IS THE DESIRED ACCURACY IN FT..
       DIMENSION X(5,5,3),A(5,3,5,5,3),KEY(5,5),DX(3),S(5,5,4),F(4),
      1DCSX(5,5,4,3),C(9,15),U(5,5,3),COR(5,3),DU(4,3),S2H(4),DCSU(4,3),
      1B(15,16),BP(15),E12(4),P(15,15),SS(3),OT(5,5,4),G(5,6),GAMA(5,5,4)
   100 FORMAT(3I5,6F10.3)
   101 FORMAT(1H1,'  SPACING-M   SPACING-N  PRETEN-H-M  PRETEN-H-N
      1E          AREA-M        AREA-N'//7E12.4/)
   102 FORMAT(6F10.3)
       READ(5,100) MN,I1,IT1,SPCM,SPCN,HM,HN,AM,AN
       READ(5,102) E
       WRITE(6,101) SPCM,SPCN,HM,HN,E,AM,AN
       READ(5,102) EPSI
       READ(5,102) (((X(M,N,J3),J3=1,3),N=1,MN),M=1,MN)
       EAM=E*AM
       EAN=E*AN
       TSM=HM/SPCM
       TSN=HN/SPCN
       DO 160 M=1,MN
       DO 161 N=1,MN
       M2=M+N
       M3=(MN+3)/2
       IF(M2-M3) 162,163,180
   180 M4=N-M
       M5=(MN-1)/2
       IF(M4-M5) 181,163,162
   181 M6=M-N
       IF(M6-M5) 182,163,162
   182 M7=(MN*3+1)/2
       IF(M2-M7) 183,163,162
   183 KEY(M,N)=1
       GO TO 161
   162 KEY(M,N)=0
       GO TO 161
   163 KEY(M,N)=-1
   161 CONTINUE
   160 CONTINUE
       DO 170 M=1,MN
       DO 171 N=1,MN
       DO 172 J3=1,3
       U(M,N,J3)=0
       DO 173 I=1,I1
       DO 174 J=1,3
       A(I,J,M,N,J3)=0
       COR(I,J)=0
   174 CONTINUE
   173 CONTINUE
   172 CONTINUE
   171 CONTINUE
```

```
170 CONTINUE
    I=0
    K=0
    DO 600 M=1,MN
    DO 601 N=1,MN
    IF(KEY(M,N) .LE. 0) GO TO 601
    M1=M-1
    N1=N-1
    I=I+1
    DO 602 I4=1,4
    IF(I4 .GT. 2) GO TO 603
    A(I,3,M,N,3)=A(I,3,M,N,3)+1*TSN
    IF(KEY(M,N1) .EQ. -1) GO TO 604
    A(I,3,M,N1,3)=A(I,3,M,N1,3)-1*TSN
    N1=N1+2
    GO TO 602
604 COR(I,3)=COR(I,3)+X(M,N1,3)*TSN
    N1=N1+2
    GO TO 602
603 A(I,3,M,N,3)=A(I,3,M,N,3)+1*TSM
    IF(KEY(M1,N) .EQ. -1) GO TO 605
    A(I,3,M1,N,3)=A(I,3,M1,N,3)-1*TSM
    M1=M1+2
    GO TO 602
605 COR(I,3)=COR(I,3)+X(M1,N,3)*TSM
    M1=M1+2
    GO TO 602
602 CONTINUE
601 CONTINUE
600 CONTINUE
    K1=I1
    L1=K1+1
    L=0
    DO 610 M=1,MN
    DO 611 N=1,MN
    IF(KEY(M,N) .LE. 0) GO TO 611
    L=L+1
    DO 612 K=1,K1
    G(K,L)=A(K,3,M,N,3)
612 CONTINUE
611 CONTINUE
610 CONTINUE
    DO 613 K=1,K1
    G(K,L1)=COR(K,3)
613 CONTINUE
    CALL GAUSR(G,K1,L1)
109 FORMAT(//'     M     N          Z     '/)
111 FORMAT(2I5,E12.4)
    WRITE(6,109)
    K=0
    DO 620 M=1,MN
    DO 621 N=1,MN
    IF(KEY(M,N) .LE. 0) GO TO 621
    K=K+1
    X(M,N,3)=G(K,L1)
    WRITE(6,111) M,N,X(M,N,3)
621 CONTINUE
620 CONTINUE
    DO 622 I=1,I1
    DO 623 M=1,MN
```

```
      DO 624 N=1,MN
      A(I,3,M,N,3)=0
  624 CONTINUE
  623 CONTINUE
  622 CONTINUE
      K1=I1*3
      L1=K1+1
      I=0
      DO 200 M=1,MN
      DO 201 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 201
      M1=M-1
      N1=N-1
      DO 202 I4=1,4
      IF(I4 .GT. 2) GO TO 204
      DO 203 J3=1,3
      DX(J3)=X(M,N1,J3)-X(M,N,J3)
  203 CONTINUE
      N1=N1+2
      GO TO 206
  204 DO 205 J3=1,3
      DX(J3)=X(M1,N,J3)-X(M,N,J3)
  205 CONTINUE
      M1=M1+2
  206 S(M,N,I4)=SQRT(DX(1)**2+DX(2)**2+DX(3)**2)
      IF(I4 .GT. 2) GO TO 208
      F(I4)=EAN/S(M,N,I4)
      GO TO 209
  208 F(I4)=EAM/S(M,N,I4)
  209 DO 207 J3=1,3
      DCSX(M,N,I4,J3)=DX(J3)/S(M,N,I4)
  207 CONTINUE
  202 CONTINUE
      I=I+1
      DO 220 J=1,3
      DO 221 I4=1,4
      DO 222 J3=1,3
      A(I,J,M,N,J3)=A(I,J,M,N,J3)+F(I4)*DCSX(M,N,I4,J)*DCSX(M,N,I4,J3)
      IF(I4 .GT. 2) GO TO 223
      IF(J .EQ. J3) A(I,J,M,N,J3)=A(I,J,M,N,J3)+2.0*TSN
      GO TO 222
  223 IF(J .EQ. J3) A(I,J,M,N,J3)=A(I,J,M,N,J3)+0.4*TSM
  222 CONTINUE
  221 CONTINUE
      N1=N-1
      M1=M-1
      DO 224 I4=1,4
      IF(I4 .GT. 2) GO TO 225
      IF(KEY(M,N1) .LE. 0) GO TO 228
      DO 226 J3=1,3
      A(I,J,M,N1,J3)=A(I,J,M,N1,J3)-F(I4)*DCSX(M,N,I4,J)*DCSX(M,N,I4,J3)
      IF(J .EQ. J3) A(I,J,M,N1,J3)=A(I,J,M,N1,J3)-2.0*TSN
  226 CONTINUE
  228 N1=N1+2
      GO TO 224
  225 IF(KEY(M1,N) .LE. 0) GO TO 229
      DO 227 J3=1,3
      A(I,J,M1,N,J3)=A(I,J,M1,N,J3)-F(I4)*DCSX(M,N,I4,J)*DCSX(M,N,I4,J3)
      IF(J .EQ. J3) A(I,J,M1,N,J3)=A(I,J,M1,N,J3)-0.4*TSM
  227 CONTINUE
```

```
  229 M1=M1+2
  224 CONTINUE
  220 CONTINUE
  201 CONTINUE
  200 CONTINUE
      L=0
      K=0
      DO 230 M=1,MN
      DO 231 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 231
      DO 232 J3=1,3
      L=L+1
      DO 233 I=1,I1
      DO 234 J=1,3
      K=K+1
      B(K,L)=A(I,J,M,N,J3)
      P(K,L)=B(K,L)
      IF(K .EQ. K1) K=K-K1
  234 CONTINUE
  233 CONTINUE
  232 CONTINUE
  231 CONTINUE
  230 CONTINUE
      READ(5,102) (BP(K),K=1,K1)
      DO 240 K=1,K1
      B(K,L1)=BP(K)
  240 CONTINUE
  114 FORMAT(//' ((B(K,L),L=1,L1),K=1,K1) ARE'//(5X,10E12.4))
      WRITE(6,114) ((B(K,L),L=1,L1),K=1,K1)
      DO 300 IT=1,ITI
      CALL GAUSR(B,K1,L1)
  105 FORMAT(1H1,11H ITERATION:I2)
  106 FORMAT(//'          U              V               W      '//(3E12.4))
  117 FORMAT(////////////11H ITERATION:I2)
      IF(IT .GT. 1) GO TO 241
      WRITE(6,105) IT
      GO TO 242
  241 WRITE(6,117) IT
  242 WRITE(6,106) (B(K,L1),K=1,K1)
      DO 301 K=1,K1
      C(IT,K)=B(K,L1)
  301 CONTINUE
      IF(IT .EQ. 1) GO TO 302
      DO 303 K=1,K1
      D=C(IT,K)-C((IT-1),K)
      IF(ABS(D) .GT. EPSI) GO TO 302
  303 CONTINUE
      GO TO 999
  302 K=0
      DO 311 M=1,MN
      DO 312 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 312
      DO 313 J3=1,3
      K=K+1
      U(M,N,J3)=B(K,L1)
  313 CONTINUE
  312 CONTINUE
  311 CONTINUE
      I=0
      DO 320 M=1,MN
```

```
      DO 321 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 321
      M1=M-1
      N1=N-1
      DO 322 I4=1,4
      IF(I4 .GT. 2) GO TO 323
      DO 324 J3=1,3
      DU(I4,J3)=U(M,N1,J3)-U(M,N,J3)
  324 CONTINUE
      N1=N1+2
      GO TO 325
  323 DO 326 J3=1,3
      DU(I4,J3)=U(M1,N,J3)-U(M,N,J3)
  326 CONTINUE
      M1=M1+2
  325 E1=0
      E2=0
      DO 327 J3=1,3
      DCSU(I4,J3)=DU(I4,J3)/S(M,N,I4)
      E1=E1+DCSX(M,N,I4,J3)*DCSU(I4,J3)
      E2=E2+DCSU(I4,J3)**2
  327 CONTINUE
      E2H(I4)=E2/2
      E12(I4)=E1+E2H(I4)
  322 CONTINUE
      I=I+1
      DO 332 J=1,3
      COR(I,J)=0
      DO 335 I4=1,4
      IF(I4 .GT. 2) GO TO 336
      COR(I,J)=COR(I,J)+FAN*(E2H(I4)*DCSX(M,N,I4,J)+E12(I4)*DCSU(I4,J))-
     11.0*TSN*DCSU(I4,J)*S(M,N,I4)
      GO TO 335
  336 COR(I,J)=COR(I,J)+FAN*(E2H(I4)*DCSX(M,N,I4,J)+E12(I4)*DCSU(I4,J))-
     10.6*TSM*DCSU(I4,J)*S(M,N,I4)
  335 CONTINUE
  332 CONTINUE
  321 CONTINUE
  320 CONTINUE
  116 FORMAT(//' COR(I,J) ARE:'//(5X,3E12.4))
      WRITE(6,116) ((COR(I,J),J=1,3),I=1,I1)
      K=0
      DO 340 I=1,I1
      DO 341 J=1,3
      K=K+1
      B(K,L1)=BP(K)+COR(I,J)
  341 CONTINUE
  340 CONTINUE
      DO 350 K=1,K1
      DO 351 L=1,K1
      B(K,L)=P(K,L)
  351 CONTINUE
  350 CONTINUE
  300 CONTINUE
  999 CONTINUE
  107 FORMAT(1H1,//'    M     N    I4      GAMA              T            DELTA T
     1FINAL T          STRESS   FINAL STRESS'//)
  108 FORMAT(I3,2I5,6E12.4)
      WRITE(6,107)
      DO 400 M=1,MN
```

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      DO 401 N=1,MN
      IF(KEY(M,N) .LE. 0) GO TO 401
      M1=M-1
      N1=N-1
      DO 402 I4=1,4
      IF(I4 .GT. 2) GO TO 404
      DO 403 J3=1,3
      SS(J3)=(X(M,N,J3)+U(M,N,J3)-X(M,N1,J3)-U(M,N1,J3))**2
  403 CONTINUE
      N1=N1+2
      GO TO 406
  404 DO 405 J3=1,3
      SS(J3)=(X(M,N,J3)+U(M,N,J3)-X(M1,N,J3)-U(M1,N,J3))**2
  405 CONTINUE
      M1=M1+2
  406 DS=SQRT(SS(1)+SS(2)+SS(3))-S(M,N,I4)
      GAMA(M,N,I4)=DS/S(M,N,I4)
      IF(I4 .GT. 2) GO TO 407
      TN=TSN*S(M,N,I4)
      DT(M,N,I4)=(EAN+TN)*DS/S(M,N,I4)
      TEN=TN+DT(M,N,I4)
      STN=TN/AN
      STEN=TEN/AN
      WRITE(6,108) M,N,I4,GAMA(M,N,I4),TN,DT(M,N,I4),TEN,STN,STEN
      GO TO 402
  407 TM=TSM*S(M,N,I4)
      DT(M,N,I4)=(EAM+TM)*DS/S(M,N,I4)
      TEM=TM+DT(M,N,I4)
      STM=TM/AM
      STEM=TEM/AM
      WRITE(6,108) M,N,I4,GAMA(M,N,I4),TM,DT(M,N,I4),TEM,STM,STEM
  402 CONTINUE
  401 CONTINUE
  400 CONTINUE
      STOP
      END

      SUBROUTINE GAUSR(A,N,N1)
      DIMENSION A(N,N1)
      DO 500 J=1,N
      DIV=A(J,J)
      S=1.0/DIV
      DO 501 K=J,N1
  501 A(J,K)=A(J,K)*S
      DO 502 I=1,N
      IF(I-J) 503,502,503
  503 AIJ=-A(I,J)
      DO 504 K=J,N1
  504 A(I,K)=A(I,K)+AIJ*A(J,K)
  502 CONTINUE
  500 CONTINUE
      RETURN
      END

$ENTRY
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STATIC ANALYSIS

OF PRESTRESSED CABLE NETWORKS

by

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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

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1973

ABSTRACT

A discrete method of analysis for prestressed cable networks is studied in this report. The method includes a basic solution and a modified solution following the same procedure. As the loading is applied to the structure, it deforms. Static equilibrium conditions at the joints of the deformed configuration provide a set of nonlinear simultaneous algebraic equations in displacement components of the interior joints for the fixed boundary case. The set of equations thus obtained is linearized for the first iteration. The solution is subsequently corrected to the desired accuracy. Some error is introduced by neglecting the strain effect in some minor terms, and thus it is an approximate method. However, since the strain is extremely small, for practical purposes, the error is negligible. The method considers the horizontal displacement as well as the vertical displacement and therefore, is a complete solution.