

FORCES AND DEFORMATIONS IN FOLDED PLATE STRUCTURES

by *632*

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I. INTRODUCTION

Folded plate roof structures have become widely used in recent years because of their simplicity in forming, their appealing appearance and their superior performance. Reinforced concrete, prestressed concrete, steel, wood and plastic have been used to construct folded plate bunkers, cooling towers, staircases, barrel shells, and even foundations¹². In the recent past, the rapid expansion of the literature on folded plate structures indicates the growing interest of the engineering profession in this type of structure.

The elasticity method in analyzing folded plate structures subjected to uniform normal and tangential loads has been presented by Goldberg and Leve¹. To the writer's knowledge, however, no elasticity analysis for folded plates has been published previously which includes effects of other types of load such as concentrated or line loads in a plate, or ridge loads. This report is concerned with the folded plate structures under the action of concentrated plate loads (normal loads moments, and tangential loads), and concentrated or line ridge loads. Only single span, single cell, simply supported structures are considered herein.

NOTATION — The letter symbols used in this report are defined in the appendix.

II. REVIEW OF LITERATURE

Numerous methods ^{1,2,3,4,6,7,8,9} for the analysis and design of folded plate structures have been presented and have proved to be satisfactory within the limitations inherent in their assumptions. All methods have the following basic assumptions:

1. The material is homogeneous, isotropic, and linearly elastic.
2. The length of each plate is greater than twice its width, and the thickness is small compared to its width.
3. The longitudinal joints are assumed completely monolithic.
4. All plates are rectangular. Each plate has uniform thickness.
5. The structure is supported on end diaphragms which are assumed to be completely rigid in the in-plane direction and perfectly flexible in the direction normal to the plane.

In 1963, the American Society of Civil Engineers² suggested a method for analyzing simply supported prismatic folded plate structures. It is based on Gaafar's³ original paper, and has the following major assumptions:

1. The longitudinal distribution of all loads on all plates is the same.
2. The structural action is considered as a combination of transverse continuous one-way slab action and longitudinal plate action or beam action. The longitudinal stresses are assumed to vary linearly across the plate width.

3. Displacements due to forces other than bending moments are neglected.

The ASCE recommended method may be divided into two steps. The first step is the primary analysis. The relative joint displacements are obtained by analyzing the slab action and beam action and satisfying stress compatibility at the edges. The second step is the correction analysis. The values for stresses and displacements can be found due to the relative joint displacements created in the primary analysis. The actual stresses and displacements can be determined by combining these two steps.

Another method for solving folded plate structures has been published by Fialkow⁴. The method is based on the minimum energy principle. The following assumptions are made in this method:

1. The applied loads are longitudinally symmetrical.
2. The longitudinal stress varies linearly across the width of each plate.

The deflection curves in the longitudinal (u), transverse (v) and normal direction (w), with respect to each plate must be assumed first. Each deformed shape has an undetermined coefficient. By using the minimum energy method, these deflection coefficients can be obtained. Thus the deflection curves can be found and various stress resultants may be expressed in terms of deflections.

Goldberg and Leve¹ also developed a solution for the stresses in a folded plate structure with uniform normal and tangential loads applied. Additional assumptions are implicit in this

method:

1. The structure system is assumed to act as a combination of two-way slabs on elastic supports at the ridges for "out of plane " deflections and as plates or beams for "in-plane" deflections.

2. The applied loading is also expanded approximately by a Fourier series.

Equations are derived which relate each joint force to a linear combination of the joint displacements. In this manner, compatibility is automatically satisfied at each joint.

III. ANALYSIS

Fig. 1 shows a typical plate element and its coordinate system. For the i th plate, the relationship of the internal forces and deformations for a mode m of the trigonometric series are ^{1,6}:

$$M_{yi} = M_{Fyi} + D_{1i} \frac{m\pi}{a} \sin \frac{m\pi x}{a} \left[(A_{1i} \frac{m\pi y}{a} + A_{2i}) \sinh \frac{m\pi y}{a} + (A_{3i} \frac{m\pi y}{a} + A_{4i}) \cosh \frac{m\pi y}{a} \right] \quad (3-1a)$$

$$N_{yi} = N_{Fyi} + D_{2i} \frac{m\pi}{a} \sin \frac{m\pi x}{a} \left[(A_{5i} \frac{m\pi y}{a} + A_{6i}) \sinh \frac{m\pi y}{a} + (A_{7i} \frac{m\pi y}{a} + A_{8i}) \cosh \frac{m\pi y}{a} \right] \quad (3-1b)$$

$$N_{xi} = N_{Fxi} + D_{2i} \frac{m\pi}{a} \sin \frac{m\pi x}{a} \left[(A_{9i} \frac{m\pi y}{a} + A_{10i}) \sinh \frac{m\pi y}{a} + (A_{11i} \frac{m\pi y}{a} + A_{12i}) \cosh \frac{m\pi y}{a} \right] \quad (3-1c)$$

$$N_{xyi} = N_{Fxyi} + D_{2i} \frac{m\pi}{a} \cos \frac{m\pi x}{a} \left[(A_{13i} \frac{m\pi y}{a} + A_{14i}) \sinh \frac{m\pi y}{a} + (A_{15i} \frac{m\pi y}{a} + A_{16i}) \cosh \frac{m\pi y}{a} \right] \quad (3-1d)$$

$$w_i = \frac{1}{2} \sin \frac{m\pi x}{a} \left[(-A_{1i} y + A_{17i}) \sinh \frac{m\pi y}{a} + (-A_{3i} y + A_{18i}) \cosh \frac{m\pi y}{a} \right] \quad (3-1e)$$

$$\begin{aligned} \theta_i = \frac{1}{2} \sin \frac{m\pi x}{a} & \left[(-A_{1i} - A_{3i} \frac{m\pi y}{a} + \frac{m\pi}{a} A_{18i} \right. \\ & - \frac{8a^3 P_{ni} \lambda_{3m} K_{cm}}{m^4 D_i}) \sinh \frac{m\pi y}{a} + (-A_{1i} \frac{m\pi y}{a} + \frac{m\pi}{a} A_{17i} \\ & \left. - A_{3i} + \frac{8a^2 P_{ni} \lambda_{3m} y}{m^3 D_i}) \cosh \frac{m\pi y}{a} \right] \quad (3-1f) \end{aligned}$$

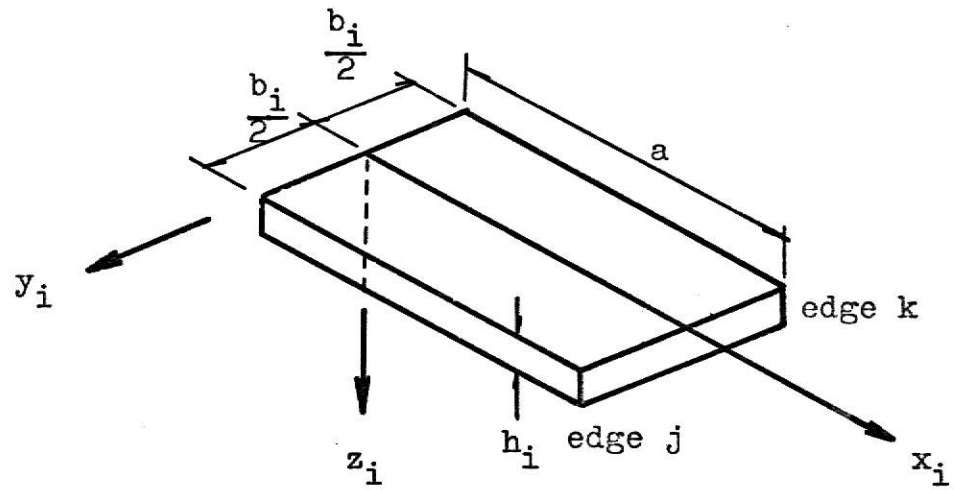


Fig. 1-a Slab Coordinate System for i th Plate

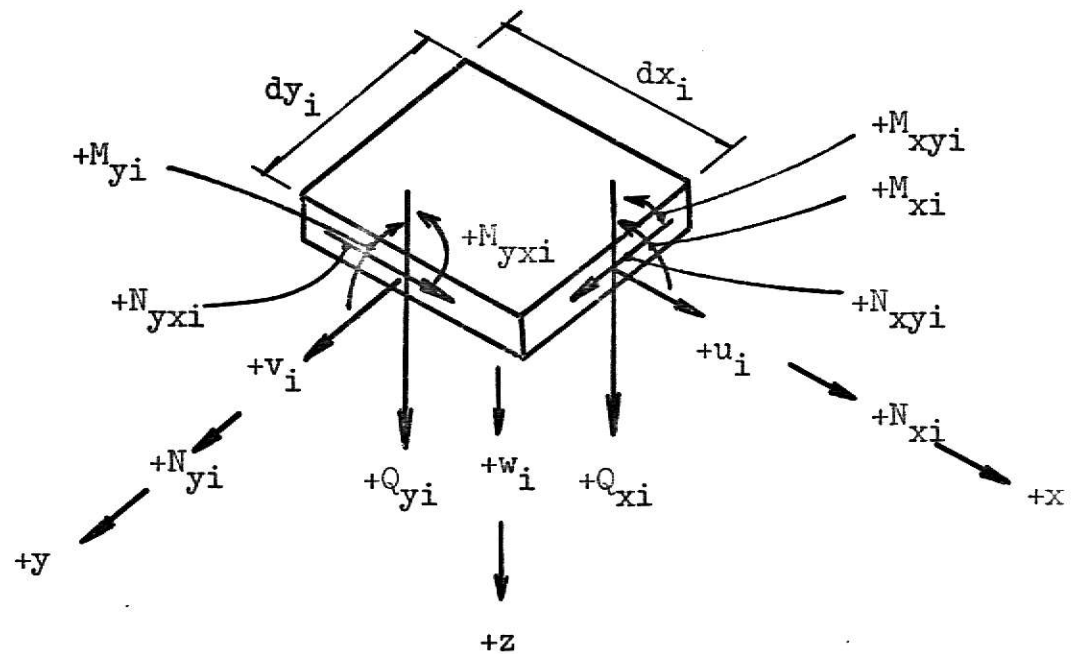


Fig. 1-b Internal Forces and Displacements

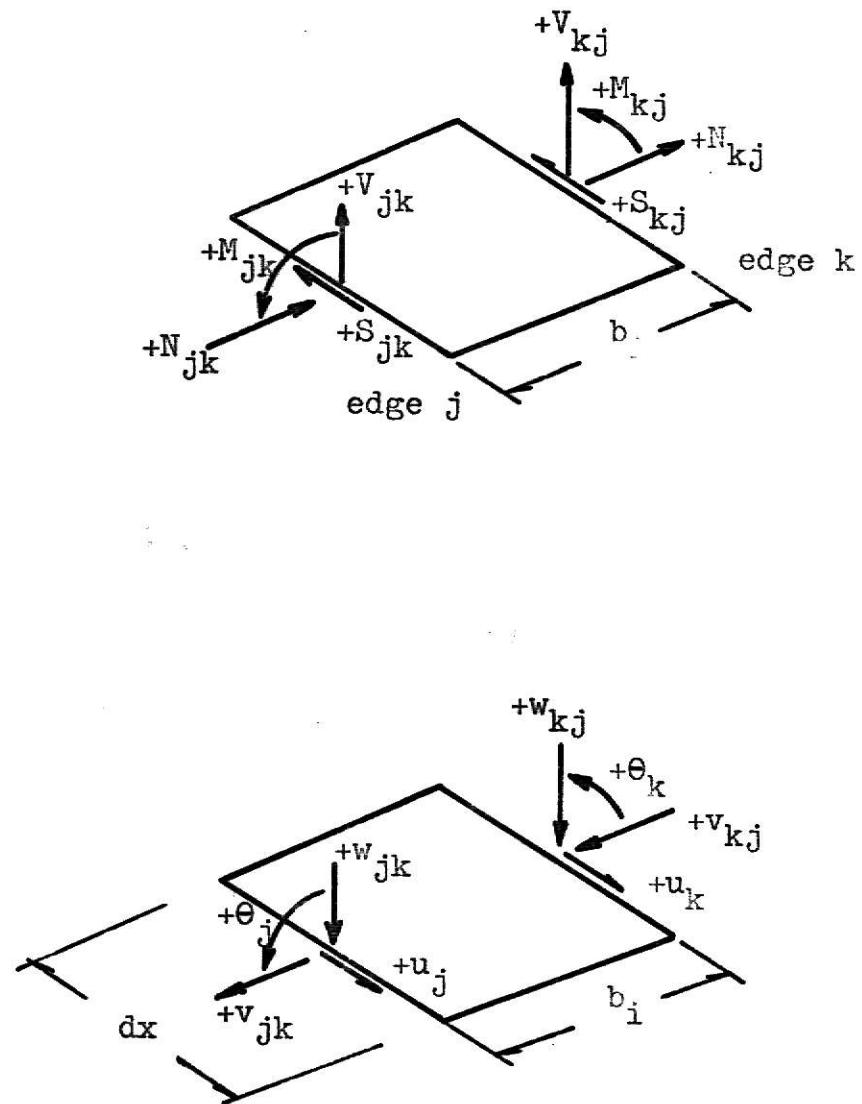


Fig. 1-c Transverse Strip through ith Plate
with Edge Forces and Deformations

The internal forces at the j and k ridges are

$$M_{jk} = M_{Fjk} + \frac{E_i h_i^3}{12(1-\nu_i^2)} \frac{m}{a} \sin \frac{m\pi x}{a} \left[c_{1i} \bar{\theta}_j + c_{2i} \bar{\theta}_k - c_{3i} \bar{w}_{jk} + c_{4i} \bar{w}_{kj} \right] \quad (3-2a)$$

$$V_{jk} = V_{Fjk} + \frac{E_i h_i^3}{12(1-\nu_i^2)} \frac{m^2}{a^2} \sin \frac{m\pi x}{a} \left[c_{5i} \bar{\theta}_j + c_{6i} \bar{\theta}_k - c_{7i} \bar{w}_{jk} + c_{8i} \bar{w}_{kj} \right] \quad (3-2b)$$

$$N_{jk} = N_{Fjk} + \frac{E_i h_i}{(1+\nu_i)^2} \frac{m}{a} \sin \frac{m\pi x}{a} \left[-c_{9i} \bar{v}_{jk} + c_{10i} \bar{v}_{kj} - c_{11i} \bar{u}_j + c_{12i} \bar{u}_k \right] \quad (3-2c)$$

$$S_{jk} = S_{Fjk} + \frac{E_i h_i}{(1+\nu_i)^2} \frac{m}{a} \cos \frac{m\pi x}{a} \left[-c_{13i} \bar{v}_{jk} - c_{14i} \bar{v}_{kj} - c_{15i} \bar{u}_j + c_{16i} \bar{u}_k \right] \quad (3-2d)$$

$$M_{kj} = M_{Fkj} + \frac{E_i h_i^3}{12(1-\nu_i^2)} \frac{m}{a} \sin \frac{m\pi x}{a} \left[c_{1i} \bar{\theta}_k + c_{2i} \bar{\theta}_j + c_{3i} \bar{w}_{kj} - c_{4i} \bar{w}_{jk} \right] \quad (3-2e)$$

$$V_{kj} = V_{Fkj} + \frac{E_i h_i^3}{12(1-\nu_i^2)} \frac{m^2}{a^2} \sin \frac{m\pi x}{a} \left[-C_{5i} \bar{\theta}_k - C_{6i} \bar{\theta}_j - C_{7i} \bar{w}_{kj} + C_{8i} \bar{w}_{jk} \right] \quad (3-2f)$$

$$N_{kj} = N_{Fkj} + \frac{E_i h_i}{(1+\nu_i)^2} \frac{m}{a} \sin \frac{m\pi x}{a} \left[-C_{9i} \bar{v}_{kj} + C_{10i} \bar{v}_{jk} + C_{11i} \bar{u}_k - C_{12i} \bar{u}_j \right] \quad (3-2g)$$

$$S_{kj} = S_{Fkj} + \frac{E_i h_i}{(1+\nu_i)^2} \frac{m}{a} \cos \frac{m\pi x}{a} \left[C_{13i} \bar{v}_{kj} + C_{14i} \bar{v}_{jk} - C_{15i} \bar{u}_k + C_{16i} \bar{u}_j \right] \quad (3-2h)$$

In the above, each stress equation contain two parts: the first part is the stress due to the fixed edge forces; the second part is the stress due to the edge displacements. M_{Fyi} , N_{Fyi} , N_{Fxi} , and N_{Fxyi} are the internal forces induced by the applied plate loads under the condition of fixed edges, while M_{Fjk} , M_{Fkj} , N_{Fkj} , N_{Fjk} , V_{Fjk} , V_{Fkj} , S_{Fjk} , S_{Fkj} are the fixed edge forces at the edges of plate. The edge deformations are defined as:

$\bar{\theta}_j$, $\bar{\theta}_k$ = the maximum values of the rotation at edges j and k respectively of the i th plate for mode m ;

\bar{w}_{jk} , \bar{w}_{kj} = the maximum values of the displacement in the z_i direction at edges j and k respectively, of the i th plate for mode m ;

$\bar{v}_{jk}, \bar{v}_{kj}$ = the maximum values of the displacement in the y_i direction at edges j and k respectively of the i th plate for mode m ;

\bar{u}_j, \bar{u}_k = the maximum values of the displacement in the x_i direction at edges j and k respectively of the i th plate for mode m .

The equations of the internal forces, with concentrated loads applied on the plates, and with the condition of fixity at the edges, will be derived in the following.

Plate Loads and Fixed Edge Force Equations

Consider a rectangular plate with edges simply supported and bent by moments along the edges $y = \pm b/2$. The deflection w has been presented by Timoshenko⁵ for mode m of the longitudinal Fourier series

$$w = \frac{a^2}{2\pi^2 D} \frac{\sin \frac{m\pi x}{a}}{m^2} \left[\frac{E'_m}{\cosh \alpha_m} (\alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a}) + \frac{E''_m}{\sinh \alpha_m} (\alpha_m \coth \alpha_m \sinh \frac{m\pi y}{a} - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a}) \right] \quad (3-3)$$

$$\text{From which, } w_y = \frac{a}{2\pi D} \frac{\sin \frac{m\pi x}{a}}{m} \left[\frac{E'_m}{\cosh \alpha_m} (\alpha_m \tanh \alpha_m \sinh \frac{m\pi y}{a} - \cosh \frac{m\pi y}{a}) + \frac{E''_m}{\sinh \alpha_m} (\alpha_m \coth \alpha_m \cosh \frac{m\pi y}{a} - \sinh \frac{m\pi y}{a}) \right]$$

$$- \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \sinh \frac{m\pi y}{a} + \frac{E_m''}{\sinh \alpha_m} (\alpha_m \coth \alpha_m \cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} - \cosh \frac{m\pi y}{a}) \quad (3-4)$$

In the above, the subscript y indicates $\frac{\partial}{\partial y}$.

The slopes at edges $y = \pm b/2$ are

$$(w_y)_{y=b/2} = \frac{a}{2\pi D} \frac{\sin \frac{m\pi x}{a}}{m} \left[-E_m' (\alpha_m \operatorname{sech}^2 \alpha_m + \tanh \alpha_m) + E_m'' (\alpha_m \operatorname{csch}^2 \alpha_m - \coth \alpha_m) \right] \quad (3-4a)$$

$$(w_y)_{y=-b/2} = \frac{a}{2\pi D} \frac{\sin \frac{m\pi x}{a}}{m} \left[E_m' (\alpha_m \operatorname{sech}^2 \alpha_m + \tanh \alpha_m) + E_m'' (\alpha_m \operatorname{csch}^2 \alpha_m - \coth \alpha_m) \right] \quad (3-4b)$$

where E_m' and E_m'' depend on the loading conditions.

Case a. Plate With Normal Concentrated Load Applied (Fig. 2)

The deflection for a simply supported plate is⁵

$$w = \frac{4P_n}{\pi^4 abd} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} a_{mn}^w \sin \frac{n\pi}{b} (y + b/2) \quad (3-5)$$

where

$$a_{mn}^w = \frac{\sin \frac{m\pi s}{a} \sin \frac{n\pi}{b} (t + b/2)}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2} \right]^{1/2}} \quad (3-5a)$$

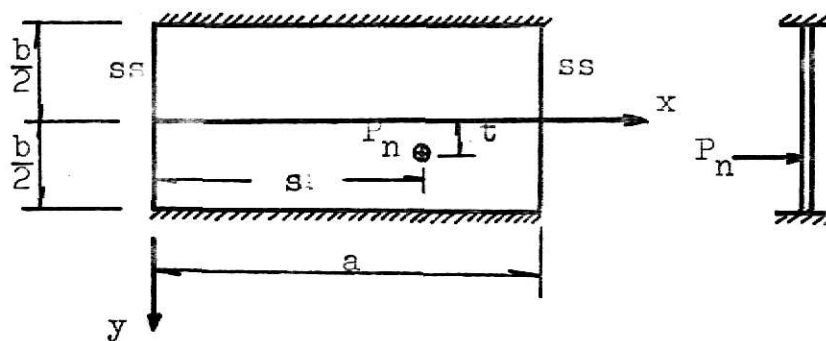


Fig. 2 Case a, Concentrated Normal Load

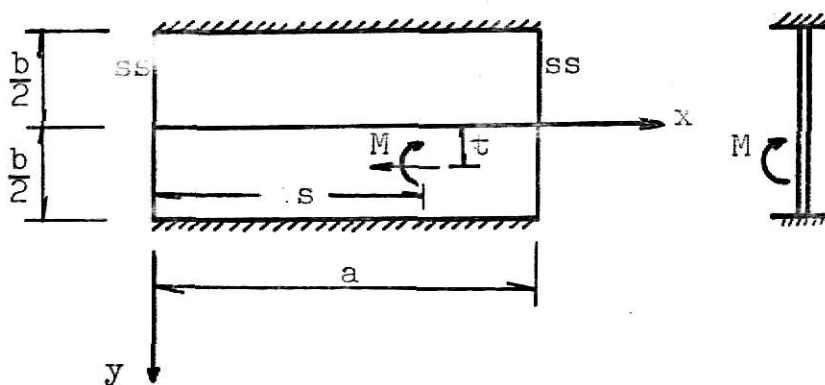


Fig. 3 Case b, Concentrated Couple

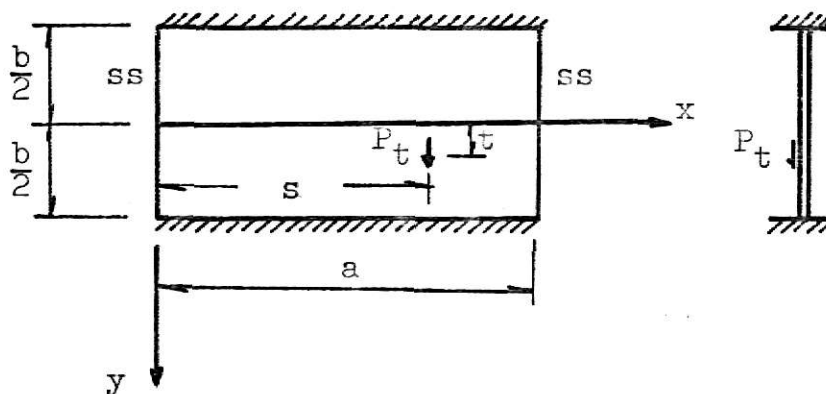


Fig. 4 Case c, Concentrated Tangential Load

Then

$$s_y^w = \frac{4P_n}{3ab^2D} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} n a W_{mn} \cos \frac{n}{b} \left(y + \frac{b}{2}\right) \quad (3-6)$$

$$(s_y^w)_{y=\frac{b}{2}} = \frac{4P_n}{3ab^2D} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} n a W_{mn} \cos n\pi \quad (3-6a)$$

$$(s_y^w)_{y=-\frac{b}{2}} = \frac{4P_n}{3ab^2D} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} n a W_{mn} \quad (3-6b)$$

From Eqs. (3-4a), (3-4b), (3-6a), (3-6b) and applied boundary conditions,

$$(w_y)_{y=\frac{b}{2}} + (s_y^w)_{y=\frac{b}{2}} = 0$$

$$(w_y)_{y=-\frac{b}{2}} + (s_y^w)_{y=-\frac{b}{2}} = 0$$

$a_m^{E'}$ and $a_m^{E''}$ are given as following

$$a_m^{E'} = \frac{-4P_n m}{\pi^2 a^2 b^2 B_1} \sum_{n=1}^{\infty} n a W_{mn} (1 - \cos n\pi) \quad (3-7a)$$

$$a_m^{E''} = \frac{-4P_n m}{\pi^2 a^2 b^2 B_2} \sum_{n=1}^{\infty} n a W_{mn} (1 + \cos n\pi) \quad (3-7b)$$

where

$$B_1 = \alpha_m \operatorname{sech}^2 \alpha_m + \tanh \alpha_m$$

$$B_2 = \alpha_m \operatorname{csch}^2 \alpha_m - \coth \alpha_m$$

Upon combining Eqs. (3-3) and (3-5), the deflection function is

$$a^w = \left\{ \frac{a^2}{2\pi^2 D m^2} \left[\frac{a E'_m}{\cosh \alpha_m} \left(\alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right) + \frac{a E''_m}{\sinh \alpha_m} \left(\alpha_m \coth \alpha_m \sinh \frac{m\pi y}{a} - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \right) \right] + \frac{4P_n}{\pi^4 abD} \sum_{n=1}^{\infty} a^{w_{mn}} \sin \frac{n\pi}{b} \left(y + \frac{b}{2} \right) \right\} \sin \frac{m\pi x}{a} \quad (3-8)$$

The fixed edge plate moment is

$$a^{M_{Fy}} = -D (w_{yy} + \nu w_{xx}) = \left\{ \frac{a E'_m}{2 \cosh \alpha_m} \left[(1-\nu) \left(\alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right) - 2 \cosh \frac{m\pi y}{a} \right] + \frac{a E''_m}{2 \sinh \alpha_m} \left[(1-\nu) \left(\alpha_m \coth \alpha_m \sinh \frac{m\pi y}{a} - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \right) - 2 \sinh \frac{m\pi y}{a} \right] + \frac{4P_n}{\pi^2 ab} \sum_{n=1}^{\infty} a^{w_{mn}} \left(\frac{m^2 \nu}{a^2} + \frac{n^2}{b^2} \right) \sin \frac{n\pi}{b} \left(y + \frac{b}{2} \right) \right\} \sin \frac{m\pi x}{a} \quad (3-9)$$

The effective plate shear is

$$a^{V_{Fy}} = -D \left[w_{yyy} + (2-\nu) w_{xxy} \right] = \left\{ + \frac{m\pi a E'_m}{2a \cosh \alpha_m} \left[+ (1-\nu) \alpha_m \tanh \alpha_m \sinh \frac{m\pi y}{a} + (1+\nu) \sinh \frac{m\pi y}{a} - (1-\nu) \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \right] \right.$$

$$\begin{aligned}
& + \frac{m \pi a E'_m}{2a \sinh \alpha_m} \left[(1-\nu) \alpha_m \coth \alpha_m \cosh \frac{m \pi y}{a} \right. \\
& \left. + (1+\nu) \cosh \frac{m \pi y}{a} - (1-\nu) \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \right] \\
& + \frac{4P_n}{ab^2 \pi} \sum_{n=1}^{\infty} \left[\frac{n^2}{b^2} + (2-\nu) \frac{m^2}{a^2} \right] n a W_{mn} \cos \frac{n \pi}{b} \\
& \left. \left(y + \frac{b}{2} \right) \right\} \sin \frac{m \pi x}{a} \quad (3-10)
\end{aligned}$$

At edges j and k

$$\begin{aligned}
a^M_{Fjk} &= -(a^M_{Fy})_{y=b/2} \\
&= -(a^{E'}_m + a^{E''}_m) \sin \frac{m \pi x}{a} \quad (3-11a)
\end{aligned}$$

$$a^M_{Fkj} = (a^M_{Fy})_{y=-b/2} = (a^{E'}_m - a^{E''}_m) \sin \frac{m \pi x}{a} \quad (3-11b)$$

$$\begin{aligned}
a^V_{Fjk} &= -(a^V_{Fy})_{y=b/2} \\
&= \left\{ \frac{m \pi a E'_m}{2a} \left[(1-\nu) \alpha_m (\tanh^2 \alpha_m - 1) \right. \right. \\
& \quad \left. \left. + (1+\nu) \tanh \alpha_m \right] + \frac{m \pi a E''_m}{2a} \left[(1-\nu) \alpha_m (1 - \coth^2 \alpha_m) \right. \right. \\
& \quad \left. \left. - (1+\nu) \coth \alpha_m \right] - \frac{4P_n}{\pi ab^2} \sum_{n=1}^{\infty} \left[\frac{n^2}{b^2} + (2-\nu) \frac{m^2}{a^2} \right] \right. \\
& \quad \left. n a W_{mn} \cos n \pi \right\} \sin \frac{m \pi x}{a} \quad (3-12a)
\end{aligned}$$

$$\begin{aligned}
a^V_{Fkj} &= (a^V_{Fy})_{y=-b/2} \\
&= \left\{ - \frac{m \pi a E'_m}{2a} \left[(1-\nu) \alpha_m (\tanh^2 \alpha_m - 1) + (1+\nu) \tanh \alpha_m \right] \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{m \pi a E''}{2a} \left[(1-\nu) \alpha_m (1 - \coth^2 \alpha_m) - (1+\nu) \coth \alpha_m \right] \\
& + \frac{4P_n}{\pi ab^2} \sum_{n=1}^{\infty} \left[\frac{n^2}{b^2} + (2-\nu) \frac{m^2}{a^2} \right] n a W_{mn} \left\} \sin \frac{m \pi x}{a} \quad (3-12b)
\end{aligned}$$

Case b. Plate With Transverse Moment
Applied (Fig. 3)

The deflection function for a simply supported plate with single transverse moment applied can be derived from (3-5) in Case a by adding a load $-P_n$ at a distance $u+d$ from x-axis, or

$$\begin{aligned}
b s^w &= \frac{4P_n}{\pi^4 abD} \sin \frac{m \pi x}{a} \left(\sin \frac{m \pi s}{a} - \sin \frac{m \pi (s+d)}{a} \right) \\
& \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{b} (t+b/2)}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2} \right]^2} \sin \frac{n \pi}{b} (y + b/2) \\
&= \frac{4P_n}{\pi^4 abD} \sin \frac{m \pi x}{a} \left(\sin \frac{m \pi s}{a} - \sin \frac{m \pi s}{a} \cos \frac{m \pi d}{a} \right. \\
& \quad \left. - \cos \frac{m \pi s}{a} \sin \frac{m \pi d}{a} \right) \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{b} (t+b/2)}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2} \right]^2} \sin \frac{n \pi}{b} (y + \frac{b}{2}) \\
&= \frac{4P_n}{\pi^4 abD} \sin \frac{m \pi x}{a} \frac{m \pi d}{a} \left[\frac{\sin \frac{m \pi s}{a} (1 - \cos \frac{m \pi d}{a})}{\frac{m \pi d}{a}} \right]
\end{aligned}$$

$$- \frac{\cos \frac{m\pi s}{a} \sin \frac{m\pi d}{a}}{\frac{m\pi d}{a}} \left] \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{b} (t + \frac{b}{2})}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2} \right]^2} \sin \frac{n\pi}{b} (y + \frac{b}{2}) \right.$$

$$= \frac{4P_n d m}{\pi^3 a^2 b D} \sin \frac{m\pi x}{a} \left[\frac{\sin \frac{m\pi s}{a}}{\frac{m\pi d}{a}} (1 - \cos \frac{m\pi d}{a}) \right.$$

$$\left. - \cos \frac{m\pi s}{a} \frac{\sin \frac{m\pi d}{a}}{\frac{m\pi d}{a}} \right] \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{b} (t + \frac{b}{2})}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2} \right]^2} \sin \frac{n\pi}{b} (y + \frac{b}{2})$$

But as $d \rightarrow 0$, $P_n d \rightarrow M$, $1 - \cos \frac{m\pi d}{a} \rightarrow 0$, $\frac{\sin \frac{m\pi d}{a}}{\frac{m\pi d}{a}} \rightarrow 1$

Therefore

$$b s^W = - \frac{4M}{\pi^3 a^2 b D} m \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} b^W_{mn} \sin \frac{n\pi}{b} (y + \frac{b}{2}) \quad (3-13)$$

where

$$b^W_{mn} = \frac{\cos \frac{m\pi s}{a} \sin \frac{n\pi}{b} (t + \frac{b}{2})}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2} \right]^2} \quad (3-13a)$$

Then

$$b s^W_y = - \frac{4M}{\pi^2 a b^2 D} m \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} n b^W_{mn} \cos \frac{n\pi}{b} (y + \frac{b}{2}) \quad (3-14)$$

$$(b s^W_y)_{y=\frac{b}{2}} = - \frac{4M}{\pi^2 a b^2 D} m \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} n b^W_{mn} \cos n\pi \quad (3-14a)$$

$$(b s^w_y)_{y=\frac{b}{2}} = - \frac{4M}{\pi^2 a b^2 D} m \sin \frac{m \pi x}{a} \sum_{n=1}^{\infty} n b^w_{mn} \quad (3-14b)$$

Again, applying the boundary conditions of fixity, and using (3-4a), (3-4b), (3-14a), (3-14b), give

$$b^{E'}_m = \frac{4Mm^2}{B_1 \pi^2 a^3 b^2} \sum_{n=1}^{\infty} n b^w_{mn} (1 - \cos n\pi) \quad (3-15a)$$

$$b^{E''}_m = \frac{4Mm^2}{B_2 \pi^2 a^3 b^2} \sum_{n=1}^{\infty} n b^w_{mn} (1 + \cos n\pi) \quad (3-15b)$$

Combining (3-3) and (3-13) yields the deflection function

$$\begin{aligned} b^w = & \left\{ \frac{a^2}{2 \pi^2 D m^2} \left[\frac{b^{E'}_m}{\cosh \alpha_m} \left(\alpha_m \tanh \alpha_m \cosh \frac{m \pi y}{a} \right. \right. \right. \\ & \left. \left. - \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \right) + \frac{b^{E''}_m}{\sinh \alpha_m} \left(\alpha_m \coth \alpha_m \sinh \frac{m \pi y}{a} \right. \right. \\ & \left. \left. - \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} \right) \right] - \\ & \left. \frac{4Mm}{\pi^3 a^2 b D} \sum_{n=1}^{\infty} b^w_{mn} \sin \frac{n \pi}{b} \left(y + \frac{b}{2} \right) \right\} \sin \frac{m \pi x}{a} \quad (3-16) \end{aligned}$$

The plate moment and effective shear are

$$b^M_{Fy} = \left\{ \frac{b^{E'}_m}{2 \cosh \alpha_m} \left[(1 - \nu) \left(\alpha_m \tanh \alpha_m \cosh \frac{m \pi y}{a} \right. \right. \right.$$

$$\begin{aligned}
& - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a}) - 2 \cosh \frac{m\pi y}{a} \Big] \\
& + \frac{b E_m''}{2 \sinh \alpha_m} \left[(1 - \nu)(\alpha_m \coth \alpha_m \sinh \frac{m\pi y}{a} \right. \\
& \left. - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a}) - 2 \sinh \frac{m\pi y}{a} \right] \\
& - \frac{4Mm}{\pi a^2 b} \sum_{n=1}^{\infty} b W_{mn} \left(\frac{m^2 \nu}{a^2} + \frac{n^2}{b^2} \right) \sin \frac{n\pi}{b} \left(y + \frac{b}{2} \right) \Big\} \sin \frac{m\pi x}{a}
\end{aligned}
\tag{3-17}$$

$$\begin{aligned}
b V_{Fy} = & \left\{ \frac{m\pi b E_m'}{2a \cosh \alpha_m} \left[(1 - \nu) \alpha_m \tanh \alpha_m \sinh \frac{m\pi y}{a} \right. \right. \\
& \left. \left. + (1 + \nu) \sinh \frac{m\pi y}{a} - (1 - \nu) \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \right] \right. \\
& + \frac{m\pi b E_m''}{2a \sinh \alpha_m} \left[(1 - \nu) \alpha_m \coth \alpha_m \cosh \frac{m\pi y}{a} \right. \\
& \left. \left. + (1 + \nu) \cosh \frac{m\pi y}{a} - (1 - \nu) \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right] \right. \\
& \left. - \frac{4Mm}{a^2 b^2} \sum_{n=1}^{\infty} \left[\frac{n^2}{b^2} + (2 - \nu) \frac{m^2}{a^2} \right] n b W_{mn} \cos \frac{n\pi}{b} \left(y + \frac{b}{2} \right) \right\} \\
& \sin \frac{m\pi x}{a}
\end{aligned}
\tag{3-18}$$

At edges j and k

$$b_{Fjk}^M = - (b_{Fy}^M)_y = \frac{b}{2}$$

$$b_{Fjk}^M = - (b_m^{E'} + b_m^{E''}) \sin \frac{m \pi x}{a} \quad (3-19a)$$

$$b_{Fkj}^M = (b_{Fy}^M)_y = - \frac{b}{2}$$

$$b_{Fkj}^M = (b_m^{E'} - b_m^{E''}) \sin \frac{m \pi x}{a} \quad (3-19b)$$

$$b_{Fjk}^V = - (b_{Fy}^V)_y = \frac{b}{2}$$

$$\begin{aligned} b_{Fjk}^V = \sin \frac{m \pi x}{a} & \left\{ \frac{m \pi b_m^{E'}}{2a} \left[(1 - \nu) \alpha_m (\tanh^2 \alpha_m - 1) \right. \right. \\ & + (1 + \nu) \tanh \alpha_m \left. \right] + \frac{m \pi b_m^{E''}}{2a} \left[(1 - \nu) \alpha_m (1 - \right. \\ & \left. \coth^2 \alpha_m) - (1 + \nu) \coth \alpha_m \right] \\ & \left. + \frac{4Mm}{a^2 b^2} \sum_{n=1}^{\infty} \left[\frac{n^2}{b^2} + (2 - \nu) \frac{m^2}{a^2} \right] n b_{mn}^W \cos n \pi \right\} \end{aligned} \quad (3-20a)$$

$$b_{Fkj}^V = (b_{Fy}^V)_y = - \frac{b}{2}$$

$$\begin{aligned} b_{Fkj}^V = & \left\{ - \frac{m \pi b_m^{E'}}{2a} \left[(1 - \nu) \alpha_m (\tanh^2 \alpha_m - 1) \right. \right. \\ & + (1 + \nu) \tanh \alpha_m \left. \right] - \frac{m \pi b_m^{E''}}{2a} \left[(1 - \nu) \alpha_m (1 - \right. \\ & \left. \coth^2 \alpha_m) - (1 + \nu) \coth \alpha_m \right] - \frac{4Mm}{a^2 b^2} \sum_{n=1}^{\infty} \left[\frac{n^2}{b^2} + \right. \end{aligned}$$

$$\left. (2 - \nu) \frac{m^2}{a^2} \right] n b W_{mn} \left. \right\} \sin \frac{m\pi x}{a} \quad (3-20b)$$

Implicitly, the equations for Case b are similar to those for Case a.

Case c. Plate With Concentrated Tangential
Load Applied (Fig. 4)

Let a concentrated tangential load P_t be expanded into Fourier series as

$$P_t(x, y) = \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} W_{pt} \sin \frac{n\pi}{b} (y + \frac{b}{2}) \quad (3-21)$$

where

$$W_{pt} = \frac{4P_t}{ab} \sin \frac{m\pi s}{a} \sum_{n=1}^{\infty} \sin \frac{n\pi}{b} (t + \frac{b}{2}) \quad (3-21a)$$

To determine the fixed edge forces due to P_t , associated deflection function u_F and v_F , the general differential equations must be applied⁵

$$u_{xx} + \frac{1-\nu}{2} u_{yy} + \frac{1+\nu}{2} v_{xy} = 0 \quad (3-22a)$$

$$v_{yy} + \frac{1-\nu}{2} v_{xx} + \frac{1+\nu}{2} u_{xy} = \frac{\nu^2 - 1}{Eh} P_t(x, y) \quad (3-22b)$$

Let $u_F = u_p + u_H$ and $v_F = v_p + v_H$, where u_p and v_p must satisfy (3-22a) and (3-22b) but not necessarily the boundary condition of zero displacements and u_H and v_H must satisfy (3-22a) and the homogeneous part of (3-22b) and also give the nece-

ssary correction at the boundaries such that u_F , v_F are zero.

For a particular solution, assume

$$u_P = \bar{u}_m \cos \frac{m\pi x}{a} \sum_{n=1}^{\infty} \cos \frac{n\pi}{b} (y + \frac{b}{2}) \quad (3-23a)$$

$$v_P = \bar{v}_m \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} \sin \frac{n\pi}{b} (y + \frac{b}{2}) \quad (3-23b)$$

Substituting into (3-22a) yields

$$\begin{aligned} -\bar{u}_m \left(\frac{m\pi}{a}\right)^2 - \frac{1-\nu}{2} \bar{u}_m \left(\frac{n\pi}{b}\right)^2 + \frac{1+\nu}{2} \bar{v}_m \frac{mn\pi^2}{ab} &= 0 \\ \therefore \bar{v}_m &= \sum_{n=1}^{\infty} \bar{u}_m \frac{2b^2 m^2 + (1-\nu)a^2 n^2}{abmn(1+\nu)} \end{aligned} \quad (3-24)$$

From (3-22b)

$$-\bar{v}_m \frac{n^2 \pi^2}{b} - \frac{1-\nu}{2} \bar{v}_m \frac{m^2 \pi^2}{a^2} + \frac{1+\nu}{2} \bar{u}_m \frac{mn\pi^2}{ab} = \frac{\nu^2 - 1}{Eh} W_{pt}$$

Substituting the value of \bar{v}_m yields

$$\bar{u}_m = \frac{m(1+\nu)^2}{abEh\pi^2} \sum_{n=1}^{\infty} \frac{n W_{pt}}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]^2}$$

or

$$\bar{u}_m = \frac{4P_t(1+\nu)^2}{ab\pi^2 Eh} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} \frac{n \sin \frac{n\pi}{b} (t + \frac{b}{2})}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]^2} \quad (3-26)$$

Substituting into (3-24) yields

$$\bar{v}_m = \frac{4P_t(1+\nu)}{ab\pi^2 Eh} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} \frac{\frac{2m^2}{a^2} + (1-\nu)\frac{n^2}{b^2}}{\left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]^2} \sin \frac{n\pi}{b} (t + \frac{b}{2})$$

(3-27)

For the homogeneous solution, let

$$u_H = u^*(y) \cos \frac{m \pi x}{a} \quad (3-28a)$$

$$v_H = v^*(y) \sin \frac{m \pi x}{a} \quad (3-28b)$$

Substituting into (3-22a) and the homogeneous part of (3-22b) yields the solutions¹

$$\begin{aligned} u^*(y) = & A_{5m}^{**} \cosh \frac{m \pi y}{a} + A_{6m}^{**} \sinh \frac{m \pi y}{a} \\ & + A_{7m}^{**} \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} + A_{8m}^{**} \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \end{aligned} \quad (3-29a)$$

$$\begin{aligned} v^*(y) = & (A_{6m}^{**} - \mu A_{7m}^{**}) \cosh \frac{m \pi y}{a} + (A_{5m}^{**} - \mu A_{8m}^{**}) \sinh \frac{m \pi y}{a} \\ & + A_{8m}^{**} \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} + A_{7m}^{**} \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \end{aligned} \quad (3-29b)$$

By combining (3-28), (3-29), and (3-23), the in-plane deflection functions are

$$\begin{aligned} u_F = \sin \frac{m \pi x}{a} \sum_{n=1}^{\infty} \left[\bar{u}_m \cos \frac{n \pi}{b} \left(y + \frac{b}{2} \right) + A_{5m}^{**} \cosh \frac{m \pi y}{a} \right. \\ \left. + A_{6m}^{**} \sinh \frac{m \pi y}{a} + A_{7m}^{**} \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} \right. \\ \left. + A_{8m}^{**} \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \right] \end{aligned} \quad (3-30a)$$

$$\begin{aligned} v_F = \sin \frac{m \pi x}{a} \sum_{n=1}^{\infty} \left[\bar{v}_m \sin \frac{n \pi}{b} \left(y + \frac{b}{2} \right) \right. \\ \left. + (A_{6m}^{**} - \mu A_{7m}^{**}) \cosh \frac{m \pi y}{a} + (A_{5m}^{**} - \mu A_{8m}^{**}) \sinh \frac{m \pi y}{a} \right. \\ \left. + A_{8m}^{**} \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} + A_{7m}^{**} \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \right] \end{aligned}$$

$$\left. + A_{8m}^{**} \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} + A_{7m}^{**} \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right] \quad (3-30b)$$

Applying boundary conditions

$$(u_F)_{y=\pm \frac{b}{2}} = 0$$

$$(\dot{v}_F)_{y=\pm \frac{b}{2}} = 0$$

Yields

$$\begin{aligned} A_{5m}^{**} &= - \frac{\alpha_m \cosh \alpha_m - \mu \sinh \alpha_m}{2(\alpha_m - \mu \sinh \alpha_m \cosh \alpha_m)} \sum_{n=1}^{\infty} \bar{u}_m (1 + \cos n\pi) \\ A_{6m}^{**} &= - \frac{\alpha_m \sinh \alpha_m - \mu \cosh \alpha_m}{2(\alpha_m + \mu \sinh \alpha_m \cosh \alpha_m)} \sum_{n=1}^{\infty} \bar{u}_m (1 - \cos n\pi) \\ A_{7m}^{**} &= \frac{\cosh \alpha_m}{2(\alpha_m + \mu \sinh \alpha_m \cosh \alpha_m)} \sum_{n=1}^{\infty} \bar{u}_m (1 - \cos n\pi) \\ A_{8m}^{**} &= \frac{\sinh \alpha_m}{2(\alpha_m - \mu \sinh \alpha_m \cosh \alpha_m)} \sum_{n=1}^{\infty} \bar{u}_m (1 + \cos n\pi) \end{aligned}$$

Applying the stress-strain equations

$$N_x = \frac{Eh}{1-\nu^2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

$$N_y = \frac{Eh}{1-\nu^2} \left(\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \right)$$

$$N_{xy} = \frac{Eh}{2(1+\nu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

gives

$$N_{Fx} = \sin \frac{m\pi x}{a} \frac{Eh\pi}{1-\nu^2} \sum_{n=1}^{\infty} \left\{ \left(-\bar{u}_m \frac{m}{a} + \frac{\nu n}{b} \bar{v}_m \right) \cos \frac{n\pi}{b} y \right.$$

$$\begin{aligned}
& + \frac{b}{2}) + \frac{m}{a} \left[\left((-1 + \nu) A_{5m}^{**} + \nu(1 - \mu) A_{8m}^{**} \right) \cosh \frac{m \pi y}{a} \right. \\
& + \left((-1 + \nu) A_{6m}^{**} + \nu(1 - \mu) A_{7m}^{**} \right) \sinh \frac{m \pi y}{a} \\
& + (-1 + \nu) A_{7m}^{**} \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} \\
& \left. + (-1 + \nu) A_{8m}^{**} \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \right] \Bigg\} \quad (3-31)
\end{aligned}$$

$$\begin{aligned}
N_{Fy} &= \frac{Eh\pi}{1 - \nu^2} \sin \frac{m \pi x}{a} \sum_{n=1}^{\infty} \left\{ \left(\frac{n}{b} \bar{v}_m - \nu \frac{m}{a} \bar{u}_m \right) \cos \frac{n \pi}{b} \left(y + \frac{b}{2} \right) \right. \\
& + \frac{m}{a} \left[\left((1 - \nu) A_{6m}^{**} + (1 - \mu) A_{7m}^{**} \right) \sinh \frac{m \pi y}{a} \right. \\
& + \left((1 - \nu) A_{5m}^{**} + (1 - \mu) A_{8m}^{**} \right) \cosh \frac{m \pi y}{a} \\
& + (1 - \nu) A_{7m}^{**} \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} \\
& \left. \left. + (1 - \nu) A_{8m}^{**} \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \right] \right\} \quad (3-32)
\end{aligned}$$

$$\begin{aligned}
N_{Fxy} &= \frac{Eh\pi}{2(1 + \nu)} \cos \frac{m \pi x}{a} \sum_{n=1}^{\infty} \left\{ \left(-\frac{n}{b} \bar{u}_m + \frac{m}{a} \bar{v}_m \right) \sin \frac{n \pi}{b} \left(y + \frac{b}{2} \right) \right. \\
& + \frac{m}{a} \left[\left(2A_{5m}^{**} + (1 - \mu) A_{8m}^{**} \right) \sinh \frac{m \pi y}{a} \right. \\
& + \left(2A_{6m}^{**} + (1 - \mu) A_{7m}^{**} \right) \cosh \frac{m \pi y}{a} \\
& + 2A_{7m}^{**} \frac{m \pi y}{a} \sinh \frac{m \pi y}{a} \\
& \left. \left. + 2A_{8m}^{**} \frac{m \pi y}{a} \cosh \frac{m \pi y}{a} \right] \right\} \quad (3-33)
\end{aligned}$$

The fixed edge forces are

$$N_{Fjk} = - (N_{Fy})_y = \frac{b}{2}$$

$$N_{Fjk} = - \frac{Eh\pi}{1-\nu^2} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} \left\{ \left(\frac{n}{b} \bar{v}_m - \nu \frac{m}{a} \bar{u}_m \right) \cos n\pi \right. \\ \left. + \frac{m}{a} \left[\left((1-\nu)A_{6m}^{**} + (1-\mu)A_{7m}^{**} \right) \sinh \alpha_m \right. \right. \\ \left. + \left((1-\nu)A_{5m}^{**} + (1-\mu)A_{8m}^{**} \right) \cosh \alpha_m \right. \\ \left. + (1-\nu)A_{7m}^{**} \alpha_m \cosh \alpha_m + (1-\nu)A_{8m}^{**} \alpha_m \sinh \alpha_m \right] \left. \right\} \quad (3-34a)$$

$$N_{Fkj} = (N_{Fy})_y = - \frac{b}{2}$$

$$N_{Fkj} = \frac{Eh\pi}{1-\nu^2} \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} \left\{ \frac{n}{b} \bar{v}_m - \nu \frac{m}{a} \bar{u}_m \right. \\ \left. + \frac{m}{a} \left[- \left((1-\nu)A_{6m}^{**} + (1-\mu)A_{7m}^{**} \right) \sinh \alpha_m \right. \right. \\ \left. + \left((1-\nu)A_{5m}^{**} + (1-\mu)A_{8m}^{**} \right) \cosh \alpha_m \right. \\ \left. - (1-\nu)A_{7m}^{**} \alpha_m \cosh \alpha_m + (1-\nu)A_{8m}^{**} \alpha_m \sinh \alpha_m \right] \left. \right\} \quad (3-34b)$$

$$S_{Fjk} = - (N_{Fxy})_y = \frac{b}{2}$$

$$S_{Fjk} = - \frac{Eh\pi m}{2a(1+\nu)} \cos \frac{m\pi x}{a} \sum_{n=1}^{\infty} \left[\left(2A_{5m}^{**} + (1-\mu)A_{8m}^{**} \right) \right. \\ \left. \sinh \alpha_m + \left(2A_{6m}^{**} + (1-\mu)A_{7m}^{**} \right) \cosh \alpha_m \right. \\ \left. + 2A_{7m}^{**} \alpha_m \sinh \alpha_m + 2A_{8m}^{**} \alpha_m \cosh \alpha_m \right] \quad (3-35a)$$

$$S_{Fkj} = (N_{Fxy})_y = -\frac{b}{2}$$

$$S_{Fkj} = \frac{Eh\pi m}{2a(1+\nu)} \cos \frac{m\pi x}{a} \sum_{n=1}^{\infty} \left[- \left(2A_{5m}^{**} + (1-\nu)A_{8m}^{**} \right) \right. \\ \left. \sinh \alpha_m + \left(2A_{6m}^{**} + (1-\nu)A_{7m}^{**} \right) \cosh \alpha_m \right. \\ \left. + 2A_{7m}^{**} \alpha_m \sinh \alpha_m - 2A_{8m}^{**} \alpha_m \cosh \alpha_m \right] \quad (3-35b)$$

Edge Loads Expanded into Fourier Series

A concentrated load acting on a ridge (Fig. 5-a) may be generally expressed, for mode m , as

$$Q_c(x) = \frac{2Q_c}{a} \sin \frac{m\pi a_1}{a} \sin \frac{m\pi x}{a} \quad (3-36)$$

In the case of a uniform load acting along the ridge, the expression is

$$Q_u(x) = \frac{2Q_u}{a} (1 - \cos m\pi) \sin \frac{m\pi x}{a} \quad (3-37)$$

In the above, Q_c and Q_u can be a vertical, horizontal load or bending moment.

Matrix Formulation and Solution

Equations (3-2) may be written in matrix form

$$\begin{bmatrix} F_i^j \\ F_i^k \end{bmatrix} = \begin{bmatrix} F_{Fi}^j \\ F_{Fi}^k \end{bmatrix} + \begin{bmatrix} C_i^{11} & C_i^{12} \\ C_i^{21} & C_i^{22} \end{bmatrix} \begin{bmatrix} \delta_i^j \\ \delta_i^k \end{bmatrix}$$

where

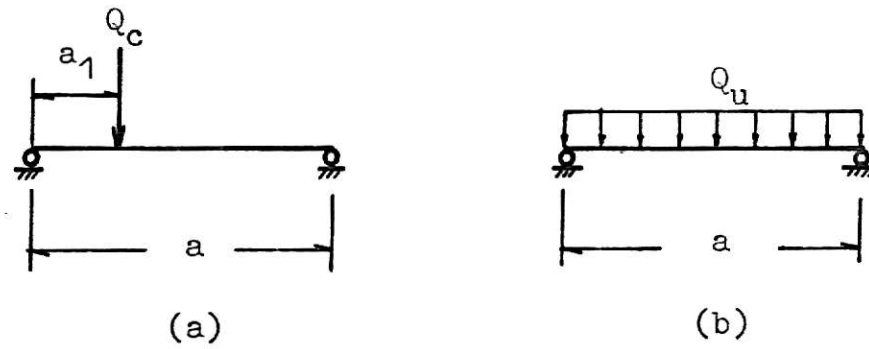


Fig. 5 Edge Loads

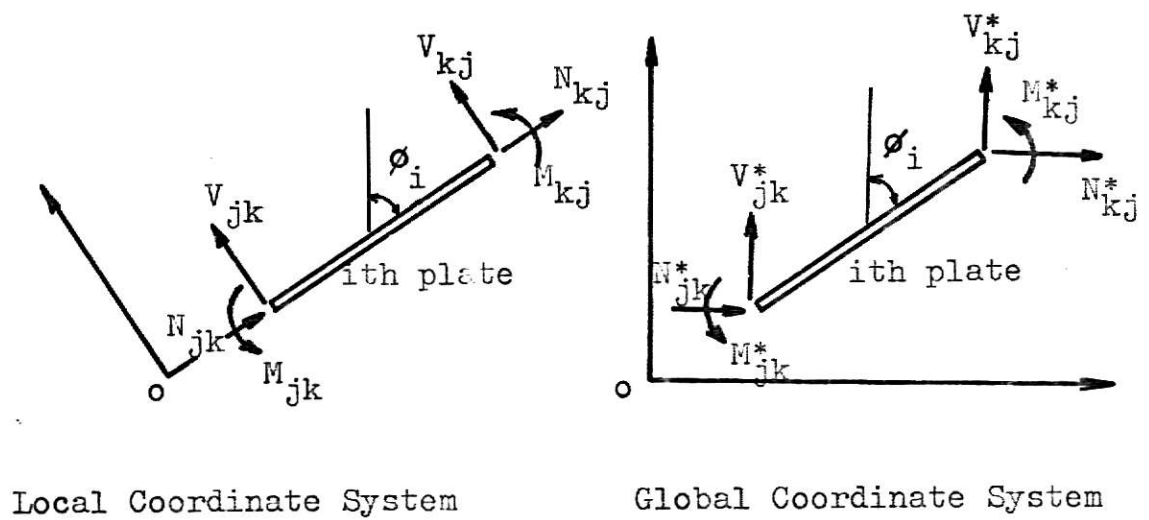


Fig. 6 Plate Edge Forces in Local and Global Coordinate Systems

$$\begin{bmatrix} \frac{F_{Fi}^j}{F_{Fi}^k} \end{bmatrix} = \begin{bmatrix} M_{Fjk} \\ V_{Fjk} \\ N_{Fjk} \\ S_{Fjk} \\ M_{Fkj} \\ V_{Fkj} \\ N_{Fkj} \\ S_{Fkj} \end{bmatrix} \quad \begin{bmatrix} \frac{\delta_i^j}{\delta_i^k} \end{bmatrix} = \begin{bmatrix} \bar{\theta}_j \\ \bar{w}_{jk} \\ \bar{v}_{jk} \\ \bar{u}_j \\ \bar{\theta}_k \\ \bar{w}_{kj} \\ \bar{v}_{kj} \\ \bar{u}_k \end{bmatrix}$$

$$[C_i] = K_i \begin{bmatrix} c_{1i} & -c_{3i} & 0 & 0 & | & c_{2i} & c_{4i} & 0 & 0 \\ c_{5i} & -c_{7i} & 0 & 0 & | & c_{6i} & c_{8i} & 0 & 0 \\ 0 & 0 & -c_{9i} & -c_{11i} & | & 0 & 0 & c_{10i} & c_{12i} \\ 0 & 0 & -c_{13i} & -c_{15i} & | & 0 & 0 & -c_{14i} & c_{16i} \\ \hline c_{2i} & -c_{4i} & 0 & 0 & | & c_{1i} & c_{3i} & 0 & 0 \\ -c_{6i} & c_{8i} & 0 & 0 & | & -c_{5i} & -c_{7i} & 0 & 0 \\ 0 & 0 & c_{10i} & -c_{12i} & | & 0 & 0 & -c_{9i} & c_{11i} \\ 0 & 0 & c_{14i} & c_{16i} & | & 0 & 0 & c_{13i} & -c_{15i} \end{bmatrix}$$

in which, K_i is an 8 x 8 diagonal matrix. The non-zero elements are

$$K_{1i} = \frac{E_i h_i^3 m}{12a(1-\nu_i^2)} ; K_{2i} = K_{1i} \frac{m}{a} ; K_{3i} = \frac{E_i h_i m}{a(1+\nu_i)^2}$$

$$K_{4i} = K_{3i} ; K_{5i} = K_{1i} ; K_{6i} = K_{2i} ; K_{7i} = K_{8i} = K_{3i}$$

As indicated in Fig. 6, the transformation from local coordinates to a global coordinate system yields

$$\begin{aligned} F_i^* &= F_i T_i & \text{or} & & F_i &= T_i^{-1} F_i^* \\ F_{Fi}^* &= F_{Fi} T_i & \text{or} & & F_{Fi} &= T_i^{-1} F_{Fi}^* \\ \delta_i &= \delta_i^* T_i \end{aligned}$$

where

$$T_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \phi_i & -\sin \phi_i & 0 \\ 0 & -\sin \phi_i & -\cos \phi_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and F_{Fi}^* represents

$$F_{Fi}^{j*} = \begin{bmatrix} M_{jk}^* \\ N_{jk}^* \\ V_{jk}^* \\ S_{jk}^* \end{bmatrix} \quad \text{or} \quad F_{Fi}^{k*} = \begin{bmatrix} M_{kj}^* \\ N_{kj}^* \\ V_{kj}^* \\ S_{kj}^* \end{bmatrix}$$

Therefore

$$\begin{bmatrix} T_i^{-1} F_i^{j*} \\ T_i^{-1} F_i^{k*} \end{bmatrix} = \begin{bmatrix} T_i^{-1} F_i^{j*} \\ T_i^{-1} F_i^{k*} \end{bmatrix} + \begin{bmatrix} C_i^{11} & C_i^{12} \\ C_i^{21} & C_i^{22} \end{bmatrix} \begin{bmatrix} T_i \delta_i^{j*} \\ T_i \delta_i^{k*} \end{bmatrix} \quad (3-38)$$

At edge 1, applying free edge boundary conditions yields

$$F_1^{1*} + F_e^1 = 0 \quad \text{where}$$

$$F_e^1 = \begin{bmatrix} M_e^1 \\ N_e^1 \\ V_e^1 \\ S_e^1 \end{bmatrix} \quad \text{or} \quad F_e^i = \begin{bmatrix} M_e^i \\ N_e^i \\ V_e^i \\ S_e^i \end{bmatrix}$$

in which, M_e^i , N_e^i , V_e^i , S_e^i are the applied moments, horizontal forces, vertical forces, longitudinal forces respectively on the i th edge. Since

$$F_1^{1*} = F_{F1}^{1*} + T_1 C_1^{11} T_1 \delta_1^{1*} + T_1 C_1^{12} T_1 \delta_1^{2*}$$

Therefore

$$T_1 C_1^{11} \delta_1^{1*} + T_1 C_1^{12} T_1 \delta_1^{2*} = - (F_{F1}^{1*} + F_e^1) \quad (3-39a)$$

Similarly, at free edge $np+1$

$$T_{np} C_{np}^{21} T_{np} \delta_{np}^{np*} + T_{np} C_{np}^{22} T_{np} \delta_{np}^{np+1*} = - (F_{Fnp}^{np+1*} + F_{Fnp}^{np+1}) \quad (3-39b)$$

Applying force equilibrium at edge k (or edge $i+1$) of i th plate

$$F_i^{i+1*} + F_{i+1}^{i+1*} + F_e^{i+1} = 0 \quad (3-39c)$$

From (3-38)

$$T_i^{-1} F_i^{i+1*} = T_i^{-1} F_{Fi}^{i+1} + C_i^{21} T_i \delta_i^{i*} + C_i^{22} T_i \delta_i^{i+1*}$$

or

$$F_i^{i+1*} = F_{Fi}^{i+1} + T_i C_i^{21} T_i \delta_i^{i*} + T_i C_i^{22} T_i \delta_i^{i+1*} \quad (3-40a)$$

Similarly, at edge j (edge $i+1$) of $i+1$ th plate

$$F_{i+1}^{i+1*} = F_{Fi+1}^{i+1*} + T_{i+1} C_{i+1}^{11} T_{i+1} \delta_{i+1}^{i+1*} + T_{i+1} C_{i+1}^{12} T_{i+1} \delta_{i+1}^{i+2*}$$

$$[C] = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & . & . & . & . & . & 0 \\ 0 & c_2 & 0 & 0 & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & c_i & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & . & c_{np} \end{bmatrix}$$

an $8np \times 8np$ matrix;

$$[T] = \begin{bmatrix} T_1 & 0 & 0 & 0 & 0 & . & . & . & . & . & 0 \\ 0 & T_1 & T_2 & 0 & 0 & . & . & . & . & . & 0 \\ 0 & 0 & 0 & T_2 & T_3 & 0 & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & . & . & T_{np-1} & T_{np} & 0 \\ 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & . & T_{np} \end{bmatrix}$$

a $4(np+1) \times 8np$ transformation matrix.

IV. DESCRIPTION OF COMPUTER PROGRAM

Goldberg and Leve had suggested a general procedure for solving the basic folded plate structures. This procedure can be utilized in computer programming. However, some changes must be made when using the computer to solve the problems. The stiffness method for analysis is preferred instead of the algebraic solution which was used in Goldberg and Leve suggested procedure. The matrix solution is used in the computer program. The general procedures are

1. Read in geometry data, plate's width, thickness, angle, length, load data, and maximum mode number of Fourier series.
2. From plates' inclination angles, calculate transformation matrices.
3. For each mode m of Fourier series
 - a. From geometry data compute stiffness matrices.
 - b. From stiffness matrices compute structure stiffness matrix.
 - c. Compute structure flexibility matrix by inverting structure stiffness matrix.
 - d. Compute fixed edge forces due to plate normal and tangential concentrated loads and moments.
 - e. From the fixed edge forces, computed from d, and the transformation matrix calculate fixed edge forces in the global system.
 - f. Calculate the fixed edge forces matrix by combining the line loads and concentrated loads at the edge with the

fixed edge forces computed in e.

- g. From the structure flexibility matrix and the fixed edge force matrix calculate edge displacements and store.
- h. From the edge displacements calculate internal forces and store.
- i. Calculate internal forces with edges fixed.
- j. Combining internal forces due to edge displacements (procedure h), and those due to edges fixity (procedure i), yields the final internal stresses for mode m.

4. Sum up forces and displacements for all modes of Fourier series.

The IBM 360-50 computer at Kansas State University was used for the solution of the examples in this report. This computer program is a modification of a program written by Dr. Stuart E. Swartz, Civil Engineering Department, Kansas State University, in FORTRAN IV, which deals with the Goldberg solution to simply supported folded plate structures subjected to uniform plate loads.

V. NUMERICAL EXAMPLES

Example 1. This example is chosen from Mr. Shih-Ying Chang's "A Model Study of A Folded Plate Structure"¹⁰. The structure and loading are shown in Fig. 7, other data are given as below :

Plate No.	b, in.	h, in.	ϕ , deg.
1	1.1250	0.1875	0
2	3.7518	0.1250	60
3	3.7762	0.1250	80
4	3.7762	0.1250	100
5	3.7518	0.1250	120
6	1.1250	0.1875	180

Length of the structure $a=26.25$ in.

Poisson's ratio $\nu=0.375$

Elastic Modulus $E=477.5$ ksi

The longitudinal stresses at midspan and near the end diaphragms are found and listed in Table 1. Those results were obtained by the following methods :

- a. ASCE recommended method¹⁰.
- b. Fialkow's minimum energy method¹⁰.
- c. Goldberg's method (loading is considered as a equivalent line load and use Fourier series maximum mode number 7)
- d. Goldberg's method (concentrated load, use Fourier series maximum mode number 15)

Example 2. Same as example 1 except that the eight concentrated loads acted on edge 2. (unsymmetrical loading)

Results and comparison are listed in Table 2.

Example 3. An example previously solved by Fialkow⁴ and Gaafar³ will be used to compare the results between Gaafar, Fialkow and Goldberg method. The model structure and loading condition are shown in Fig. 8. Other given data are

Plate No.	b, in.	h, in.	ϕ , deg.
1	2.5	0.13	0.0
2	3.5	0.13	57.5
3	3.5	0.13	90.0

$$E = 10.5 \times 10^6 \text{ psi}$$

$$\nu = 0.0$$

Results and comparison are listed in Table 3.

Edge concentrated loads

$$8Q = 50 \text{ lbs.}$$

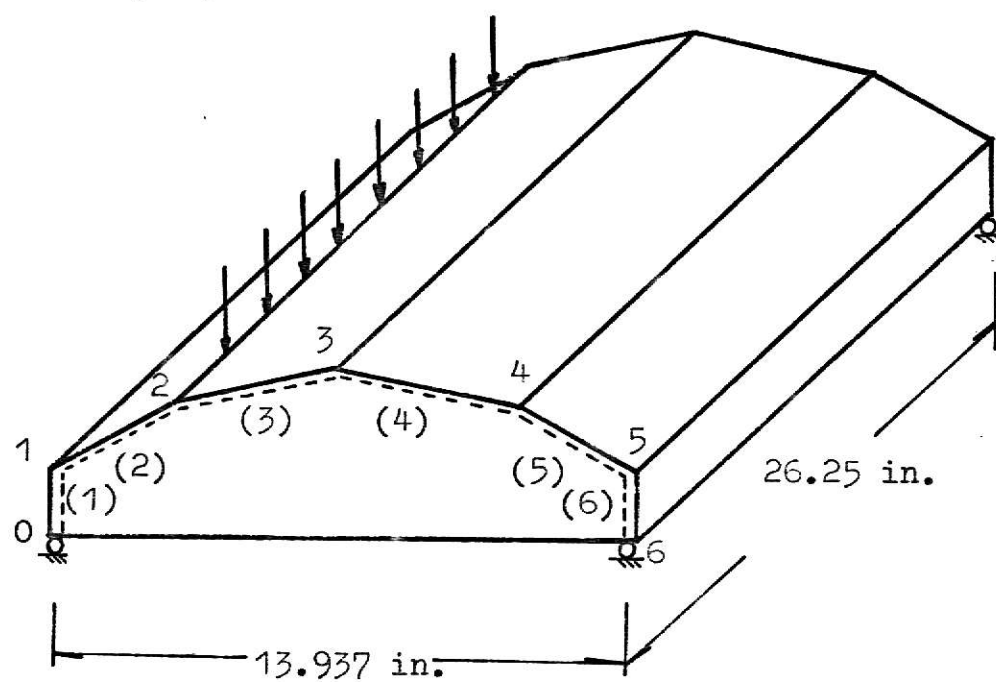


Fig. 7 Model Analyzed by Chang

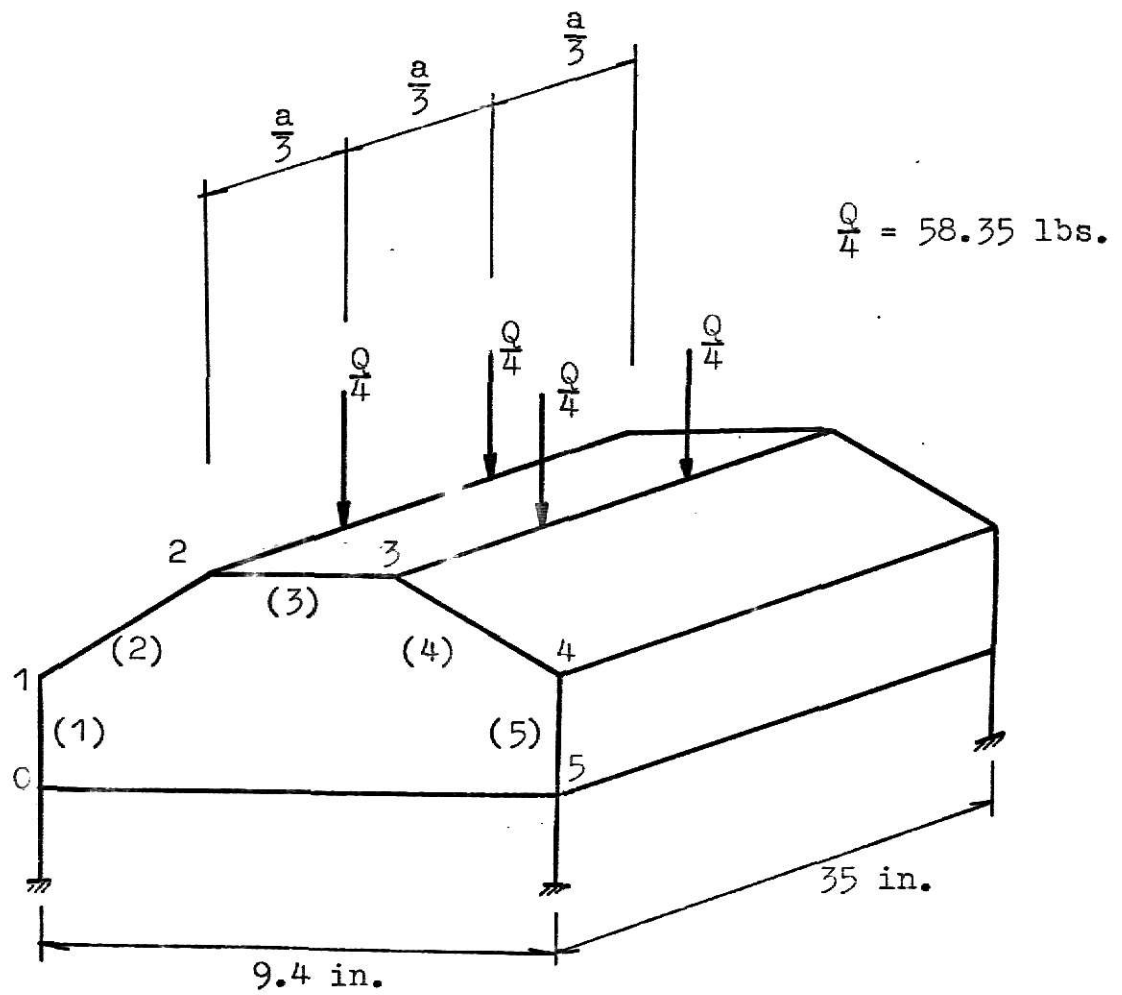


Fig. 8 Model Analyzed by Fialkow and Gaafar

Source method	Gage point stresses, psi							
	1	2	3	4	5	6	7	8
Experimental value ¹⁰	+241.0	-245.0	-245.0	+241.0	+48.0	-58.4	-58.4	+48.9
ASCE recommended method ¹⁰	+211.8	-188.2	-188.2	+211.8	+40.9	-79.3	-79.3	+40.9
Energy method ¹⁰	+215.0	-195.0	-195.0	+215.0	+42.8	-80.2	-80.2	+42.8
Goldberg method	Line load m = 7	+183.0	-170.0	+183.0	+56.4	-72.2	-72.2	+56.4
	Conc. load m = 15	+207.0	-200.0	+207.0	+48.4	-75.2	-75.2	-48.4

Table 1. Comparison of Theoretical and Experimental Stresses for Symmetrical Loading, Example 1.

Source method		Gage point stresses, psi							
		1	2	3	4	5	6	7	8
Experimental value ¹⁰		+125.1	-112.1	-16.4	+30.4	+61.2	-27.8	+10.7	+9.5
ASCE recommended method ¹⁰		+132.3	-108.0	-20.8	+ 3.2	+68.5	-11.5	+18.1	+2.8
Energy method ¹⁰		+138.6	- 98.6	-20.2	+ 1.6	+66.4	-12.5	+16.8	+2.1
Goldberg method	Line load m = 7	+ 94.8	- 99.4	-24.5	+31.8	+51.5	-24.8	+ 6.9	+7.4
	Conc. load m = 15	+106.0	-111.7	-29.0	+34.8	+54.5	-29.8	+ 6.3	+8.4

Table 2. Comparison of Theoretical and Experimental Stresses
for Unsymmetrical Loading, Example 2

Deflection or stress resultant	Gaafar		Minimum Principle		Goldberg method m= 7
	Suggested procedure	Experimental results	Ordinary procedure	Special procedure	
δ_0 in.	- .00497	- .00375	- .005	- .00385	- .003181
δ_5 in.	- .00497	- .00412	- .005	- .00385	- .003181
δ_1 in.	- .00497	- .00372	- .005	- .00385	- .003179
δ_4 in.	- .00497	- .00412	- .005	- .00385	- .003179
δ_2 in.	+ .01330	+ .01312	+ .0134	+ .01305	+ .012495
δ_3 in.	+ .01330	+ .01175	+ .0134	+ .01305	+ .012495
M_1 in. lb. per in.	0.0		0.0	- .39	- .3839
M_2 in. lb. per in.	+4.065		+4.13	+3.86	+3.59
N_1 lb.	+186.0		+194.0	+188.0	+186.63
N_2 lb.	+ 21.6		+ 32.0	+ 20.7	+ 10.5
N_3 lb.	+211.0		-226.0	-209.0	20.2

Table 3. Results of Example 3.

VI. DISCUSSION

1. In examples 1 and 2, the results of the Goldberg method agree very well with those of the ASCE recommended method and energy method. It is seen that the results from the Goldberg method agree more closely to the experimental values than the other methods. The ASCE recommended method and energy method are comparatively not so exact for the following reasons :
 - a. Poisson's ratio has been assumed to be zero in the ASCE recommended method and energy method, while the Goldberg method takes the effects of Poisson's ratio into consideration.
 - b. The ASCE recommended method neglects the effects of slab bending in the longitudinal direction, slab twisting, and the effect of membrane stresses in the transverse direction. The energy method also neglects the strain energy due to normal membrane stresses in the transverse direction. In developing the strain energy due to slab bending, the energy method considers one-way bending in each direction by excluding the effect of Poisson's ratio. The Goldberg method is an elasticity method which includes the effect of all membrane stresses and the two-way bending of the slab.
2. From Table 4, it is clear that for a simply supported folded plate structure with concentrated loads applied along the ridges, the Fourier series converges quickly. A desirable result can be obtained by setting the maximum mode number

equal to 7. If the concentrated loads are expanded into a uniform line load, the series converges more rapidly. This can be seen from eq. (3-3). Since only odd terms are need for computations, the computer time is reduced although there is less than 10% difference. However, this discrepancy can be corrected by using a modified equivalent line load.

3. Formulas have been derived herein for a simple span structure. The Goldberg method and formulas are applicable to general loading conditions.

Loading condition	Longitudinal stress at midspan, psi		Vertical deflection at midspan in.		Maximum mode	Computer time min.*
	m-2th mode	mth mode	m-2th mode	mth mode		
At edge 0 (midspan)						
Line load	410.43	411.25	.033559	.033588	7	6.7
Concentrated load	458.18	458.85	.037429	.037432	7	11.8
	458.85	458.85	.037432	.037432	15	17.0
At edge 5 (midspan)						
Line load	15.685	15.775	.0181221	.0181224	7	6.7
Concentrated load	21.832	21.833	.0202120	.0202121	7	11.8
	21.835	21.835	.0186419	.0186419	15	17.0

*Computer system time is for analyzing a folded plate structure with 6 plate elements. Each plate element is subdivided into 80 equal rectangular areas, we need, because of longitudinal symmetry, to consider only 40 partial areas, or 55 points. At every point, longitudinal stress, transverse stress, shear stress, transverse moment, in-plane deflection, out-of-plane deflection are solved.

Table 4. Accuracy and Computer Running Time for Example 2

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REFERENCES

1. Goldberg, J. E., and Leve, H. L., "Theory of Prismatic Folded Plate Structure", Publication, IABSE, No. 17, 1957.
2. "Phase I Report on Folded Plate Construction", Report of The Task Committee on Folded Plate Construction, ASCE. Vol. 89, No. ST6. December 1963.
3. Gaafar I, "Hipped Plate Considering Joint Displacements", Transaction, ASCE, Vol.119, 1954.
4. Fialkow, M. N., "Folded Plate Analysis by Minimum Energy Principle", Journal of the Structural Division, ASCE, Vol.88, June 1962.
5. S. Timoshenko, "Theory of Plates and Shells", McGraw-Hill Book Company, New York, 1940.
6. Swartz, S. E., and Guralnick, S. A., "Approximate Analysis of the Stability of Folded Plate Structures", International Association for Bridge and Structural Engineering, Vol.29-II, Publication, 1969.
7. Borg, Sidney F., "Advanced Structural Analysis", McGraw-Hill Book Company, New York, 1958.
8. Winter, G. and Pei, M., "Hipped Plate Construction", Journal, ACI, Vol.43, No.5, 1947.
9. Yitshaki, D., and Reiss, Max, "Analysis of Folded Plates", Journal of the Structure Division, ASCE, Vol.88, No.ST5, October 1962.
10. Shih-ying Chang and Peter B. Cooper, "A Model Study of A Folded Plate Structure", Master's Thesis, 1969, Kansas State University.
11. Martin, "Introduction to Matrix Method of Structural Analysis" McGraw-Hill Book Co., New York, 1966.
12. Martin, Ignacio and Ruiz, Sixto, "Folded Plate Raft Foundation for 24 story Building", Journal, American Institute, August 1959.

APPENDIX—NOTATION

$A_{1i} - A_{18i}$ = force-displacement coefficients for i th plate^{1,6}

$$A_{1i} = -\lambda_{1m}(\bar{\theta}_j - \bar{\theta}_k) + \lambda_{3m} \frac{m}{a} (\bar{w}_{jk} + \bar{w}_{kj})$$

$$A_{2i} = \lambda_{2m}(\mu_{1i} - k_{cm})(\bar{\theta}_j + \bar{\theta}_k) + \lambda_{4m} \frac{m\pi}{a}(\mu_{3i} - k_{tm})(-\bar{w}_{jk} + \bar{w}_{kj})$$

$$A_{3i} = \lambda_{2m}(\bar{\theta}_j + \bar{\theta}_k) - \lambda_{4m} \frac{m\pi}{a}(\bar{w}_{jk} - \bar{w}_{kj})$$

$$A_{4i} = \lambda_{1m}(\mu_{1i} - k_{tm})(-\bar{\theta}_j + \bar{\theta}_k) + \frac{m\pi}{a} \lambda_{3m}(\mu_{3i} - k_{cm})(\bar{w}_{jk} + \bar{w}_{kj})$$

$$A_{5i} = \lambda_{5m}(\bar{v}_{jk} - \bar{v}_{kj}) - \lambda_{7m}(\bar{u}_j + \bar{u}_k)$$

$$A_{6i} = \lambda_{6m}(\mu_{5i} + k_{cm})(\bar{v}_{jk} + \bar{v}_{kj}) + \lambda_{8m}(\mu_{6i} - k_{tm})(\bar{u}_j - \bar{u}_k)$$

$$A_{7i} = -\lambda_{6m}(\bar{v}_{jk} + \bar{v}_{kj}) + \lambda_{8m}(\bar{u}_j - \bar{u}_k)$$

$$A_{8i} = \lambda_{5m}(\mu_{5i} + k_{tm})(-\bar{v}_{jk} + \bar{v}_{kj}) - \lambda_{7m}(\mu_{6i} - k_{cm})(\bar{u}_j + \bar{u}_k)$$

$$A_{9i} = -A_{5i}$$

$$A_{10i} = \lambda_{6m}(\mu_{4i} - k_{cm})(\bar{v}_{jk} + \bar{v}_{kj}) + \lambda_{8m}(\mu_{7i} - k_{tm})(-\bar{u}_j + \bar{u}_k)$$

$$A_{11i} = -A_{7i}$$

$$A_{12i} = \lambda_{5m}(\mu_{4i} - k_{tm})(-\bar{v}_{jk} + \bar{v}_{kj}) + \lambda_{7m}(\mu_{7i} - k_{cm})(\bar{u}_j + \bar{u}_k)$$

$$A_{13i} = A_{7i}$$

$$A_{14i} = \lambda_{5m}(\mu_{6i} + k_{tm})(-\bar{v}_{jk} + \bar{v}_{kj}) - \lambda_{7m}(\mu_{5i} - k_{cm})(\bar{u}_j + \bar{u}_k)$$

$$A_{15i} = A_{5i}$$

$$A_{16i} = \lambda_{6m}(\mu_{6i} + k_{cm})(\bar{v}_{jk} + \bar{v}_{kj}) + \lambda_{8m}(\mu_{5i} - k_{tm})(\bar{u}_j - \bar{u}_k)$$

$$A_{17i} = \frac{a}{m\pi} k_{cm} \lambda_{2m}(\bar{\theta}_j + \bar{\theta}_k) + (1 + k_{tm}) \lambda_{4m}(-\bar{w}_{jk} + \bar{w}_{kj})$$

$$A_{18i} = \frac{a}{m\pi} \lambda_{1m} k_{tm}(-\bar{\theta}_j + \bar{\theta}_k) + (1 + k_{cm}) \lambda_{3m}(\bar{w}_{jk} + \bar{w}_{kj})$$

$$a = \text{Span length of structure between supports} \\ (\text{i.e. between end diaphragms})$$

$$b_i = \text{Width of } i\text{th plate.}$$

$$C_{1i} = \pi \left[\frac{\cosh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m + \sinh \alpha_m} - \frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m - \cosh \alpha_m} \right]$$

$$C_{2i} = -\pi \left[\frac{\cosh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m - \sinh \alpha_m} + \frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m - \cosh \alpha_m} \right]$$

$$C_{3i} = \frac{m\pi^2}{a} \left[\frac{\cosh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m + \cosh \alpha_m} - \frac{\sinh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m - \sinh \alpha_m} - (1 - \nu_i) \right]$$

$$C_{4i} = -\frac{m\pi^2}{a} \left[\frac{\cosh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m + \cosh \alpha_m} + \right]$$

$$\left[\frac{\sinh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m - \sinh \alpha_m} \right]$$

$$c_{5i} = \frac{a}{m} c_{3i}$$

$$c_{6i} = \frac{a}{m} c_{4i}$$

$$c_{7i} = \frac{m\pi^3}{a} \left[\frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m + \cosh \alpha_m} - \frac{\sinh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m - \sinh \alpha_m} \right]$$

$$c_{8i} = -\frac{m\pi^3}{a} \left[\frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m + \cosh \alpha_m} + \frac{\cosh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m - \sinh \alpha_m} \right]$$

$$c_{9i} = \pi \left[-\frac{\cosh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m - \frac{3-\nu_i}{1+\nu_i} \sinh \alpha_m} + \frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m + \frac{3-\nu_i}{1+\nu_i} \cosh \alpha_m} \right]$$

$$c_{10i} = -\pi \left[\frac{\cosh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m - \frac{3-\nu_i}{1+\nu_i} \sinh \alpha_m} + \frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m + \frac{3-\nu_i}{1+\nu_i} \cosh \alpha_m} \right]$$

$$C_{11i} = \pi \left[\frac{\cosh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m - \frac{3-\nu_i}{1+\nu_i} \cosh \alpha_m} - \frac{\sinh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m + \frac{3-\nu_i}{1+\nu_i} \sinh \alpha_m} + (1+\nu_i) \right]$$

$$C_{12i} = - \left[\frac{\cosh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m - \frac{3-\nu_i}{1+\nu_i} \cosh \alpha_m} + \frac{\sinh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m + \frac{3-\nu_i}{1+\nu_i} \sinh \alpha_m} \right]$$

$$C_{13i} = C_{11i}$$

$$C_{14i} = C_{12i}$$

$$C_{15i} = \pi \left[- \frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m - \frac{3-\nu_i}{1+\nu_i} \cosh \alpha_m} + \frac{\cosh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m + \frac{3-\nu_i}{1+\nu_i} \sinh \alpha_m} \right]$$

$$C_{16i} = \pi \left[\frac{\sinh \alpha_m}{\alpha_m \operatorname{csch} \alpha_m - \frac{3-\nu_i}{1+\nu_i} \cosh \alpha_m} + \frac{\cosh \alpha_m}{\alpha_m \operatorname{sech} \alpha_m + \frac{3-\nu_i}{1+\nu_i} \sinh \alpha_m} \right]$$

$$D_i = \frac{E_i h_i^3}{12(1 - \nu_i^2)}$$

$$D_{1i} = \frac{E_i h_i^3}{24(1 + \nu_i)}$$

$$D_{2i} = \frac{E_i h_i}{2(1 + \nu_i)}$$

E_i, ν_i = Young's Modulus and Poisson's Ratio for the i th plate

E'_m, E''_m = Coefficients of deflection⁵.

$a^{E'}, a^{E''}_m, b^{E'}, b^{E''}_m$ = Coefficients of deflection for Case a and Case b respectively.

h_i = Thickness of i th plate.

$k_{cm} = \alpha_m \coth \alpha_m$

$M_e^i, N_e^i, V_e^i, S_e^i$ = Applied edge moments, horizontal forces, vertical forces, longitudinal forces on the i th edge.

$M_{xi}, M_{yi}, N_{xi}, N_{yi}, Q_{xi}, Q_{yi}$ = Internal moments, forces, and shears in the i th plate.

$M_{Fxi}, M_{Fyi}, N_{Fxi}, N_{Fyi}$ = Internal moments, forces, and shears in the i th plate corresponding to fixity at the edges.

N_{Fxyi}

m, n = Trigonometric mode numbers.

- P_{ni}, P_{ti} = Loads acting on the i th plate in the z and y directions respectively.
- Q_c, Q_u = Concentrated load, and uniform line load acting along the ridges.
- w_i, θ_i = The z component of displacement and the rotation in the i th plate respectively.
- $\bar{u}_j, \bar{u}_k, \bar{v}_{jk}, \bar{v}_{kj}, \bar{w}_{jk}, \bar{w}_{kj}, \theta_j, \theta_k$ = The maximum values of the x, y, z components of displacement and the rotation at edges j and k respectively of the i th plate for mode m .
- x, y, z = Coordinate directions in the i th plate.
- $\alpha_m = \frac{m \pi b_i}{2a}$
- ϕ_i = The vertical angle of the i th plate.
- $\lambda_{1m} = (\alpha_m \operatorname{sech} \alpha_m + \sinh \alpha_m)^{-1}$
- $\lambda_{2m} = (\alpha_m \operatorname{csch} \alpha_m - \cosh \alpha_m)^{-1}$
- $\lambda_{3m} = (\alpha_m \operatorname{csch} \alpha_m + \cosh \alpha_m)^{-1}$
- $\lambda_{4m} = (\alpha_m \operatorname{sech} \alpha_m + \sinh \alpha_m)^{-1}$
- $\lambda_{5m} = (\alpha_m \operatorname{sech} \alpha_m - \mu_i \sinh \alpha_m)^{-1}$
- $\lambda_{6m} = (\alpha_m \operatorname{csch} \alpha_m + \mu_i \cosh \alpha_m)^{-1}$
- $\lambda_{7m} = (\alpha_m \operatorname{csch} \alpha_m - \mu_i \cosh \alpha_m)^{-1}$
- $\lambda_{8m} = (\alpha_m \operatorname{sech} \alpha_m + \mu_i \sinh \alpha_m)^{-1}$

$$\mu_i = \frac{3 - \nu_i}{1 + \nu_i}$$

$$\mu_{1i} = \frac{2}{1 - \nu_i}$$

$$\mu_{3i} = \frac{1 + \nu_i}{1 - \nu_i}$$

$$\mu_{4i} = \frac{2 \nu_i}{1 + \nu_i}$$

$$\mu_{5i} = \frac{2}{1 + \nu_i}$$

$$\mu_{6i} = \frac{1 - \nu_i}{1 + \nu_i}$$

$$\mu_{7i} = \frac{3 + \nu_i}{1 + \nu_i}$$

s^w, bs^w = The deflection functions of a simply supported rectangle plate with normal concentrated loads, and transverse moment applied respectively.

a^w, b^w = The deflection functions for Case a and Case b respectively.

FORCES AND DEFORMATIONS IN FOLDED PLATE STRUCTURES

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ABSTRACT

This report deals with the analysis of prismatic, simply supported folded plate structures. The purpose of the study may be summarized as follows :

1. To develop equations for fixed end forces and moments at edges and throughout each plate due to concentrated normal and tangential loads and moments in the plate.

2. To develop a matrix solution for folded plate structures subjected to uniform and concentrated plate loads and moments, and edge line loads and concentrated loads and moments.

3. Numerical examples are presented to provide a comparison between the results of the proposed method and other methods previously published.

4. The comparisons between the results of the proposed method and the model tests previously published are presented in this report.