# Modeling and Forecasting Secondary User Activity Considering Bulk Arrival and Bulk Departure Traffic Model

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***Abstract*— In this paper, we consider an Erlangian process model for secondary user (SU) traffic in a cognitive radio network (CRN). Unlike prior efforts that focused on Poisson traffic models, the model captures bulk arrival and departure scenarios in a frequency band that is open for opportunistic use by multiple secondary users. Specifically, we develop a Kalman filter based estimation and forecasting strategy for the number of secondary users. Using simulated data, we demonstrate that the proposed approach provides a robust upper bound prediction on the number of secondary users.**

## I. Introduction

Several recent measurements of spectrum usage indicate that many licensed spectrum bands remain relatively unused for most of the time [1]- [2]. Therefore, there exists an opportunity for radios with cognitive capabilities to use these licensed bands when primary users are absent and hence improve overall spectrum usage efficiency. The first task for any SU in a CRN is sensing its intended spectrum to identify absence or presence of primary user (PU). Then secondary users start transmission with appropriate transmission parameters if PU is absent.

Prior research efforts in cognitive radio networks have focused on sensing primary users [3]-[4], allocating optimal channels for transmission [5], finding optimal transmit power [6] and joint allocation of channel and transmit power [7]. However, there has been very little emphasis on how multiple secondary users compete for available spectrum. Whenever multiple secondary users coexist in a channel, quality of service (QoS) or bit-error-rate (BER) performance of the secondary users may degrade due to high level of interference. Therefore, it is desirable to develop strategies to sense and predict the behavior of all competing secondary users in a frequency band of interest.

In this paper, we present an integrated modeling and forecasting strategy that can be used to predict the number of secondary users accessing a spectral band of interest. Specifically, we consider a cognitive radio network traffic model where, (1) the PU follows a continuous time Markov chain; (2) multiple secondary users simultaneously use a spectral band when the PU is absent; (3) secondary users can arrive or depart in bulk resulting in an Erlangian traffic model. Unlike prior efforts in modeling and forecasting that are limited to Poisson traffic [8], we propose a Kalman filter based estimate for the number of secondary users from power level measurements for a more general Erlangian traffic model. We then use this estimate to determine an upper bound predictor of the number of secondary users in a spectral band. Having an idea of the number of secondary users that may occupy a given spectral band at a future time instant is critical information from a QoS standpoint. This is because, the predicted number of secondary users quantifies the level of interference a SU may experience if the band is used for transmission. So, while a spectral band may be available for use because the PU is absent, it may not be truly suitable for access by a SU from a QoS standpoint if the predicted number of secondary users is large. Therefore, the predictive capability provided by our proposed approach is a desirable feature when multiple secondary users compete for the same spectral band. We characterize the performance of our proposed forecasting strategy using simulated data and also evaluate its robustness to errors in traffic parameter estimates. Simulation results demonstrate that the proposed strategy provides a robust upper bound prediction of the number of secondary users in a spectral band.

## II. System Model

As in [8], we assume that each channel in a CRN can be used by either a PU or one or more secondary users (once it is determined that the channel will not be used by a PU). We also assume that the PU follows Poisson arrival process with arrival and departure rates, $\lambda_p$ and $\mu_p$, respectively. The maximum number of PU is $N_p$. For ease in presentation, $N_p$ is assumed to be equal to 1. In other words, PU follows a two-state ON-OFF Markov process. In this work, unlike in [8], we assume that secondary users can arrive and leave the network as a bulk or group. This implies that non-nearest neighbor transitions are allowed. The maximum number of the secondary users is $N_s$ and the acceptable maximum number of the secondary users in a bulk is $N_b$ ($N_b < N_s$). Each of the secondary users in a bulk irrespective of bulk size has exponential inter-arrival and service time distributions with rates $\lambda_s$ and $\mu_s$, respectively. Based on these assumptions, the state-transition-rate diagram for $k$-th state of secondary users is shown in Fig. 1.

Our objective is to develop a prediction strategy for secondary users. To accomplish this, we need a dynamic model of

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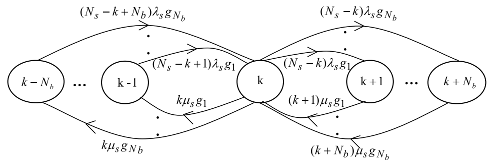


Fig. 1. State-transition-rate diagram of $k$-th state of secondary users.

secondary users activity. From the state-transition-rate diagram in Fig. 1 and concepts from Queueing theory [9], the differential equations for the state probabilities $p_{s,k}(t)$ is evaluated for secondary users. The state probability is defined as

$$p_{s,k}(t) \triangleq \text{prob}\{x_s(t) = k\}, \tag{1}$$

where, $x_s(t)$ is the number of SUs at time $t$ and $k$ indicates that number. In general, the differential equations for the state probabilities for a system shown in Fig. 1 correspond to

$$\begin{aligned} \frac{dp_{s,0}(t)}{dt} &= \mu_s \sum_{j=1}^{\min(N_b,\ N_s)} p_{s,j}(t) j g_j \\ &\quad -N_s\lambda_s p_{s,0}(t) \sum_{j=1}^{\min(N_b,\ N_s)} g_j \end{aligned} \tag{2}$$

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$$\begin{aligned} \frac{dp_{s,k}(t)}{dt} &= \lambda_s \sum_{j=1}^{\min(N_b,\ k)} p_{s,(k-j)}(t)(N_s-k+j)g_j \\ &\quad +\mu_i \sum_{j=1}^{\min(N_b,\ (N_s-k))} p_{s,(k+j)}(t)(k+j)g_j \\ &\quad -p_{s,k}(t)\left(k\mu_s \sum_{j=1}^{\min(N_b,\ k)} g_j + (N_s-k)\lambda_s \right. \\ &\quad \left. \sum_{j=1}^{\min(N_b,\ (N_s-k))} g_j\right), \quad 1 \le k < N_s, \end{aligned} \tag{3}$$

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$$\begin{aligned} \frac{dp_{s,N_s}(t)}{dt} &= \lambda_s \sum_{j=1}^{\min(N_b,\ N_s)} p_{s,(N_s-j)}(t) j g_j \\ &\quad -N_s\mu_i p_{s,N_s}(t) \sum_{j=1}^{\min(N_b,\ N_s)} g_j. \end{aligned} \tag{4}$$

Here,

$$g_j \triangleq \text{P[group size of users} = j\}]. \tag{5}$$

$E\{x_s(t)\}$ can be written as

$$E\{x_s(t)\} = \sum_{k=0}^{N_s} k p_{s,k}(t). \tag{6}$$

Hence,

$$\frac{dE\{x_s(t)\}}{dt} = \sum_{k=0}^{N_s} k \frac{dp_{s,k}(t)}{dt}. \tag{7}$$

Let $\mathbf{L} = (0, 1, 2, \ldots N_s)'$. From equations (2)-(4) and (7), we can write

$$\frac{dE\{x_s(t)\}}{dt} = \mathbf{L}'\dot{\mathbf{P}}_s = \mathbf{L}'\mathbf{Q}_s\mathbf{P}_s, \tag{8}$$

where,

$$\mathbf{Q}_s = \begin{pmatrix} -N_s\lambda_s\sum_{j=1}^{N_b} g_j & e_{1,2} & . & . \\ N_s\lambda_s g_1 & e_{2,2} & . & . \\ N_s\lambda_s g_2 & e_{3,2} & . & . \\ . & . & . & . \\ . & . & . & -N_s\mu_s\sum_{j=1}^{N_b} g_j \end{pmatrix}$$

with $e_{1,2} = \mu_s g_1$, $e_{2,2} = -\left((N_s-1)\lambda_s\sum_{j=1}^{N_b} g_j + \mu_s g_1\right)$ and $e_{3,2} = (N_s-1)\lambda_s g_1$; and

$$\mathbf{P}_s = \begin{pmatrix} p_{s,0}(t) \\ p_{s,1}(t) \\ . \\ . \\ . \\ p_{s,N_s}(t) \end{pmatrix}.$$

In equation (8), $(\cdot)'$ indicates matrix or vector transpose operator. It is easy to show that $\mathbf{L}'\mathbf{Q}_s\mathbf{P}_s$ corresponds to $-(\lambda_s+\mu_s)E\{x_s(t)\} + f_s(t)$. Therefore,

$$\frac{dE\{x_s(t)\}}{dt} = -(\lambda_s+\mu_s)E\{x_s(t)\} + f_s(t), \tag{9}$$

where,

$$\begin{aligned} f_s(t) &= \sum_{k=0}^{N_s} p_{s,k}(t)\left[(\lambda_s+\mu_s)k + k\mu_s \sum_{j=1}^{\min(N_b,\ k)} (-jg_j)\right. \\ &\quad \left. +(N_s-k)\lambda_s \sum_{j=1}^{\min(N_b,\ (N_s-k))} jg_j\right]. \end{aligned} \tag{10}$$

We assume that measurements are performed at discrete time instants $mT$, $m = 1, 2, 3\ldots$ for a given value $T$. Using the initial condition that the number of users at time $t = (m-1)T$ is $x_s(m-1)$, the solution of equation (9) is obtained as

$$\begin{aligned} E\left[x_s(m)|x_s(m-1)\right] &= e^{-T(\lambda_s+\mu_s)}x_s(m-1) \\ &\quad + e^{-T(\lambda_s+\mu_s)}\left[\int_{(m-1)T}^{mT} f_s(t)\right. \\ &\quad \left. e^{(\lambda_s+\mu_s)(t-(m-1)T)}dt\right]. \end{aligned} \tag{11}$$

Therefore, it is possible to express the number of secondary users at time $mT$ in terms of the number of secondary users at time $(m-1)T$ as

$$x_s(m) = A_s x_s(m-1) + B_s(m), \tag{12}$$

where,

$$A_s = e^{-T(\lambda_s+\mu_s)} \tag{13}$$

and

$$B_s(m) = e^{-T(\lambda_s+\mu_s)} \left[ \int_{(m-1)T}^{mT} f_s(t) \right. \left. e^{(\lambda_s+\mu_s)(t-(m-1)T)} dt \right]. \quad (14)$$

Equation (12) establishes the relationship between the number of users at two successive measurement instants and in the most general case corresponds to

$$x_s(m) = A_s x_s(m-1) + B_s(m) u_s(m) + w_s(m). \quad (15)$$

Equation (15) can be considered the state equation where, $x_s(m)$ represents the number of secondary users using the spectrum at the measurement instant $m$. The parameter $B_s(m)$ relates the optional control input, $u_s(m)$ to state. Equation (12) suggests that $u_s(m)$ is equal to 1. $w_s(m)$ is the process noise and assumed to be zero mean Gaussian noise with variance $\sigma_s^2$. The parameter $A_s$ relates the state at previous and current measurement instants, in the absence of either a driving function or process noise. $A_s$ is assumed to be constant over the analysis or varies very slowly.

The received power at a secondary user terminal during the measurement instant $m$ consists of relative power level increments caused by secondary users and in the most general case corresponds to,

$$y(m) = C_s x_s(m) + D + v(m), \quad (16)$$

where, $y(m)$ is received power in dBm; $C_s$ represent the relative increase in power level (in dB) due to the presence of one secondary user; $D$ represents the background thermal noise and $v(m)$ denotes the measurement noise which may arise due to miscalculation, misalignment of timings and is assumed to be zero mean Gaussian noise with variance $\sigma_v^2$. $y(m)$ is the only measurable variable in the system.

It is important to note that the parameter $B_s(m)$ relating optional control input, $u_s(m)$ to the state $x_s(m)$ is not a constant and is time-dependent. The computation of time-dependent $B_s(m)$ is discussed in Sec. III.

## III. Estimation of Spectrum Usage

In this section, we develop a Kalman filter based state estimation technique based on the model from Sec. II. An opportunistic SU or a central controller can use this technique to estimate the number of secondary users (once the traffic parameters are determined in learning phase).

The state estimation based on Kalman filter is summarized below:

*State Equation:* The state equation is

$$x_s(m) = A_s x_s(m-1) + B_s(m) + w_s(m), \quad (17)$$

where, $w_s(m)$ is a white Gaussian noise with mean 0 and variance, $\sigma_s^2$.

*Measurement Equation:*

$$y(m) = C_s x_s(m) + D + v(m), \quad (18)$$

where, $v(m)$ is a white Gaussian noise with mean 0 and variance, $\sigma_v^2$. Based on equations (17) and (18), the Kalman filtering steps are given below:

Step 1: Initialization

$$\widehat{x}_s(0|0) = E\{x_s(0)\} \quad (19)$$

$$M_s(0|0) = \sigma_s^2(0) \quad (20)$$

Step 2: Prediction

$$\widehat{x}_s(m|m-1) = A_s \widehat{x}_s(m-1|m-1) + B_s(m|m-1), \quad (21)$$

$$M_s(m|m-1) = A_s M_s(m-1|m-1) A_s{}' + \sigma_s^2, \ \forall\ m \quad (22)$$

Step 3: Kalman gain calculation

$$k_s(m) = M_s(m|m-1) C_s \left( C_s{}' M_s(m|m-1) C_s + \sigma_v^2 \right)^{-1}, \ \forall\ m \quad (23)$$

Step 4: Correction

$$\widehat{x}_s(m|m) = \widehat{x}_s(m|m-1) + k_s(m) \left( y(m) - C_s{}' \widehat{x}_s(m|m-1) - D \right), \quad (24)$$

$$M_s(m|m) = \{1 - k_s(m) C_s{}'\} M_s(m|m-1), \ \forall\ m \quad (25)$$

Step 5: Computation of $B_s(m), \ \forall\ m$

From equation (14), we observe that $B_s(m)$ is $e^{-T(\lambda_s+\mu_s)}$ multiplied by the integration of $f_s(t) e^{(\lambda_s+\mu_s)(t-(m-1)T)}$ between $(m-1)T$ to $mT$. $f_s(t)$ in turn can be written as $\mathbf{a}^T \mathbf{P}_s$, where, $\mathbf{P}_s$ is as defined in Sec. II and $\mathbf{a}^T$ is equal to

$$\left[ N_s \lambda_s \sum_{j=1}^{N_b} j g_j, \cdots, (\lambda_s + \mu_s) N_s + k\mu_s \sum_{j=1}^{N_b} (-j g_j) \right]^T. \quad (26)$$

The state probabilities $\mathbf{P}_s$ can be computed from the differential equation

$$\dot{\mathbf{P}}_s = \mathbf{Q}_s \mathbf{P}_s. \quad (27)$$

If the matrix $\mathbf{Q}_s$ has unique eigenvalues then the solution of equation (27) for $t \in ((m-1)T, mT]$ is given by

$$\mathbf{P}_s = \mathbf{E}_s e^{\mathbf{\Gamma}_s t} \mathbf{F}_s. \quad (28)$$

Here, $\mathbf{\Gamma}_s$ is a diagonal matrix with eigenvalues of $\mathbf{Q}_s$; $\mathbf{E}_s$ is the matrix of corresponding right eigenvectors, and $\mathbf{F}_s$ is a constant vector determined from the initial condition $(i.e, \ \ \widehat{x}_s(m-1|m-1))$ as

$$\mathbf{F}_s = (e^{\mathbf{\Gamma}_s (m-1)T})^{-1} \mathbf{E}_s^{-1} \mathbf{P}_{(m-1)T(s)}. \quad (29)$$

Here, $\mathbf{P}_{(m-1)T(s)}$ is a vector with all zeros except the $\widehat{x}_s(m-1|m-1))$ th element which is 1. It is very easy to show that at $m$th instant,

$$B_s(m) = e^{-T(\lambda_s+\mu_s)} \mathbf{a}^T \mathbf{I}_s, \quad (30)$$

where

$$\mathbf{I}_s = \mathbf{E}_s \left( \int_{(m-1)T}^{mT} e^{\mathbf{\Gamma}_{sb} t} dt \right) \mathbf{F}_s. \quad (31)$$

Here, $\mathbf{\Gamma}_{sb} = (\mathbf{\Gamma}_s + (\lambda_s + \mu_s))$ and $\mathbf{F}_s$ is computed as

$$\mathbf{F}_s = (e^{\mathbf{\Gamma}_{sb}(m-1)T})^{-1} \mathbf{E}_s^{-1} \mathbf{P}_{(m-1)T(s)}. \quad (32)$$

In equation (31), we have used the fact that the integral of a matrix is the integral of each element of the matrix.

## IV. Forecasting Spectrum Usage

Here, we briefly describe the forecasting tool that can be used by an opportunistic SU for forecasting the number of secondary users at a future time instant.

The approach for forecasting is to determine the most probable state at the next time instant given that we have the current instant state estimate. To do this, we need to calculate the probability of transitioning to another state at time $(m+1)T$. The state transitioning probability values for the instant $(m+1)T$ are computed by integrating the time varying state transitioning probability expressions (i.e., equation (28)) as

$$\begin{aligned} \widetilde{\mathbf{P}}_s &= \frac{1}{T} \int_{mT}^{(m+1)T} \mathbf{P}_s dt \\ &= \frac{1}{T} \mathbf{E}_s \left( \int_{mT}^{(m+1)T} e^{\mathbf{\Gamma}_s t} dt \right) \mathbf{F}_s \quad (33) \\ &= \frac{1}{T} [\widetilde{p}_{s,0} \ \widetilde{p}_{s,1} \ \cdots \ \widetilde{p}_{s,N_s}]'. \quad (34) \end{aligned}$$

Here, $\mathbf{F}_s$ is a constant vector determined from the initial condition $(i.e, \ \widehat{x}_s(m|m))$ as

$$\mathbf{F}_s = (e^{\mathbf{\Gamma}_s mT})^{-1} \mathbf{E}_s^{-1} \mathbf{P}_{mT(s)}. \quad (35)$$

Here, $\mathbf{P}_{mT(s)}$ is a vector with all zeros except the $\widehat{x}_s(m|m))$ th element which is 1. The elements $\widetilde{p}_{s,k}$ of the vector $\widetilde{\mathbf{P}}_s$ denote the probabilities of transitioning to state $k$ at instant $(m+1)T$.

Based on estimated number of secondary users, $\widehat{x}_s(m|m)$ at time $mT$, state transitioning probability values are computed from equation (34) and then prediction for $(m+1)$ th instant is done. The predicted state of SU for $(m+1)$ th instant at time $mT$ corresponds to

$$\widetilde{x}_s(m) = \min_{x_s \in [\widehat{x}_s(m|m), \ N_s]} x_s \ \text{ s.t. } \widetilde{p}_{s,k} < \beta. \quad (36)$$

Here, $\beta$ is a threshold similar to that defined in [8].

## V. Experimental results

We illustrate the performance of the proposed Kalman filter based estimate and upper bound predictor on simulated CRN data.

We consider a CRN during the time when PU is absent and SU starts to use channel opportunistically. As mentioned before, secondary users follow Erlangian process. The maximum number of secondary users, $N_s$ is taken as 20. The arrival and departure rates, $\lambda_s$ and $\mu_s$ of each of the secondary users are taken as, 0.0019 sec$^{-1}$ and 0.0025 sec$^{-1}$, respectively. The state noise variance $\sigma_s^2$ is set to 1. $N_b$ is considered as 4. The group probabilities $g_1$, $g_2$, $g_3$ and $g_4$ are set as 0.65, 0.20, 0.10 and 0.05, respectively. This choice of probabilities reflects a reasonable assumption that a group with 1 secondary user has the maximum probability and a group with $N_b$ number secondary users has the minimum probability to arrive or to depart the network.

The evolution of secondary users, $x_s(m)$ with measurement instant are shown in Fig. 2(a). The number of measurement instants is 3001. The measurement interval, $T$ is 10 sec. At the terminal of a SU, attempting to use this channel, the received power, $y(m)$ is shown in Fig. 2(b). Background noise level $D$ and measurement noise variance, $\sigma_v^2$ are assumed as $-135$ dBm and 3, respectively.

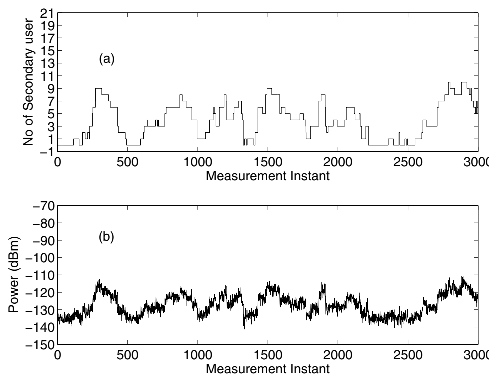


Fig. 2. Evolution of secondary users, $x_s(m)$ and power level variation, $y(m)$ with time.

From $y(m)$, we first estimate the number of secondary users, $\widehat{x}_s(m|m)$ from (19)-(25) and then use this estimate for forecasting. The Kalman filter initialization parameters are set as $\widehat{x}_s(0|0) = B_s(0)/(1-A_s)$ and $M_s(0|0) = \sigma_s^2/(1-A_s^2)$. $B_s(0)$ is evaluated (using equation (30)) assuming $x(-1) = 0$. After estimation, prediction for the number of secondary users is done.

As in [8], the forecast process only involves a table-lookup to determine the next state at each instant from the current state estimate based on (36). $\beta$ is fixed at 0.006 for this simulation. This value of $\beta$ indicates that the system has less than 0.6% chance to exceed the predicted state. Fig. 3 shows the predicted upper bound number of secondary users, $\widetilde{x}_s(m)$ with true number of secondary users, $x_s(m)$. For clarity, only 1200 to 1500 measurement instants are shown in this figure. From Fig. 3, it is evident that the prediction tool provides a good upper bound for the number of secondary users.

In the analysis and simulation thus far, we assumed that secondary users have an accurate estimate of traffic parameters $\lambda_s$ and $\mu_s$. In practice, this may not be possible. Figures 4 and 5 show the robustness of the predictor with erroneous estimate of traffic parameters $\lambda_s$ and $\mu_s$. In Fig. 4, the parameters $\lambda_s$ and $\mu_s$ are assumed to have been overestimated by 10%.

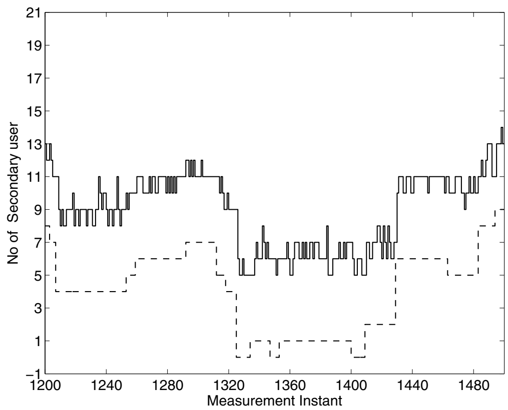


Fig. 3. Performance of the predictor; where $(--)$ and $(-)$ indicate the true and predicted upper bound number of secondary users, respectively; $\beta = 0.006$.

These over estimated $\lambda_s$ and $\mu_s$ values are used in Kalman filter estimator and then in predictor. In Fig. 5, the parameter estimates are assumed to be 10% smaller than their true values. In each case, the proposed upper bound predictor still performs satisfactorily. The predictor shows relatively low sensitivity to erroneous estimate of traffic parameters.

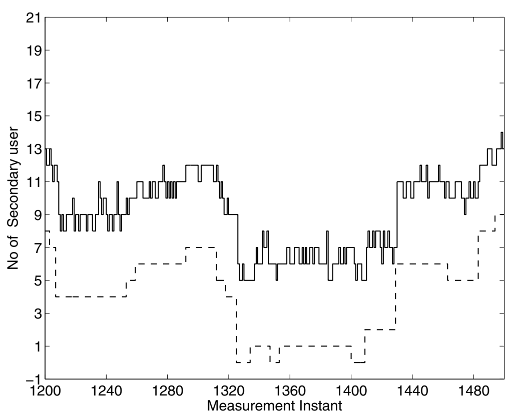


Fig. 4. Sensitivity of the predictor (both $\lambda_s$ and $\mu_s$ are overestimated by 10%); where $(--)$ and $(-)$ indicate the true and predicted upper bound number of secondary users, respectively; $\beta = 0.006$.

## VI. Conclusion

In this paper, we assume that secondary user traffic in a CRN is governed by Erlangian process, i.e., the traffic model incorporates bulk arrival or bulk departure scenarios. Assuming that the PU follows a two-state ON-OFF continuous time Markov process, we develop a Kalman filter based state estimation technique to estimate the number of secondary users based on power level measurements. This estimate is used for upper bound prediction of the number of secondary users at future time instant. Simulation results show that the proposed forecasting strategy provides robust upper bound predictor for the number of secondary users.

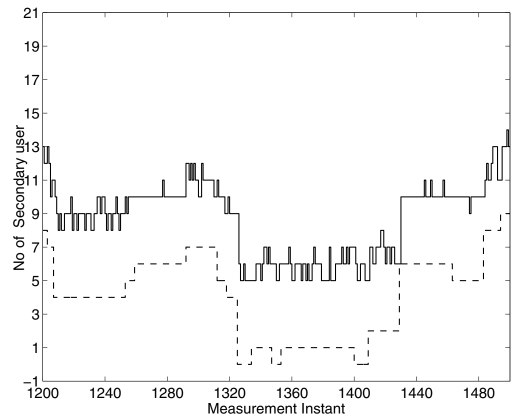


Fig. 5. Sensitivity of the predictor (both $\lambda_s$ and $\mu_s$ are underestimated by 10%); where $(--)$ and $(-)$ indicate the true and predicted upper bound number of secondary users, respectively; $\beta = 0.006$.

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