STATISTICAL INFERENCES FOR A PURE BIRTH PROCESS

by

JYH-PING HSU

B. S., National Taiwan University, 1977

M. S., Kansas State University, 1980

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Approved by:

James J. Higgins

Major Professor

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I. Introduction

The Poisson process is a popular model for reliability and queuing applications. Its assumptions are met often enough and sufficiently well in practice to make it useful, and its mathematical simplicity makes it attractive to the practitioner. However, there are important cases where the model is overly simplistic. As a failure model in reliability applications, the Poisson process has the unrealistic property of having a constant failure rate. Therefore, it cannot be used to model the wear out or reliability growth that characterize many real systems. In queuing applications, the constant transition intensity of the Poisson process is not appropriate when the rate at which customers enter the queue is affected by the number already in the queue. In this study a 2-parameter pure birth model is investigated that has many of the desirable mathematical and statistical properties of the Poisson process but is more flexible to account for realistic deviations from a strictly Poisson model.

Specifically, models are proposed for system which change noticeably in their characteristics after the first event of interest has occurred. Of course, systems may change after other events have occurred, but an adjustment in the probability model to account for changes in the system after the first event has occurred can greatly improve the goodness of fit of the model. This is especially true if relatively few events are observed and the bulk of the probability is concentrated near zero. In such cases, a simple Poisson-like model can reasonably describe the non-zero events after adjusting for observing a zero. Conventional parameter estimation methods have been used with the goodness of a specified method being judged by the estimated mean square error. Particularly, small sample behavior of the system is studied.

II. Mathematical Model

1. Pure Birth Process

A natural generalization of the Poisson process is to permit the chance of an event occurring at a given instant of time to depend upon the number of events which have already occurred. An example of this phenomena is the reproduction of living organisms in which under certain conditions (sufficient food, no mortality, no migration, etc.) the probability of a birth at a given instant is proportional to the population size at that time. This example is known as the Yule process.

Let

$N(t)$ = number of events which occurred in time interval $(0,t)$

$P_i(t) = \text{Prob}(N(t)=i)$

k_i = transition intensity at state i

Define a pure birth process as a Markov process satisfying the postulates:

i) $\Pr(N(t+h)-N(t)=1|N(t)=i) = k_i h + o(h)$

ii) $\Pr(N(t+h)-N(t)=0|N(t)=i) = 1 - k_i h + o(h)$

iii) $\Pr(N(t+h)-N(t)<0|N(t)=i) = o(h)$

where $o(h)$ is such that $\lim\limits_{h\to 0} o(h)/h = 0$

iv) $N(0) = 0$

With these postulates $N(t)$ does not denote the population size but, rather, the number of births in the time interval $(0,t)$. For $h>0$, $i>0$, by invoking the law of total probabilities, the Markov property, and iii) it can be obtained that

$$
\begin{aligned}
P_i(t+h) &= \sum_{j=0}^{\infty} P_j(t)\Pr(N(t+h)=i \mid N(t)=j) \\
&= \sum_{j=0}^{\infty} P_j(t)\Pr(N(t+h)-N(t)=i-j \mid N(t)=j) \\
&= \sum_{j=0}^{i} P_j(t)\Pr(N(t+h)-N(t)=i-j \mid N(t)=j) \qquad (1)
\end{aligned}
$$

Now for $j = 0, 1, 2, \ldots\ldots\ldots, i-2$

$$
\begin{aligned}
&\Pr(N(t+h)-N(t)=i-j \mid N(t)=j) \\
&\qquad < \Pr(N(t+h)-N(t)>2 \mid N(t)=j) = o(h)
\end{aligned}
$$

Thus

$$
\begin{aligned}
P_i(t+h) &= P_i(t)(1 - k_i h + o(h)) \\
&\qquad + P_{i-1}(t)(k_{i-1}h + o(h)) + \sum_{j=0}^{i-2} P_j(t)o(h)
\end{aligned}
$$

or

$$
\begin{aligned}
P_i(t+h)-P_i(t) &= P_i(t)(-k_i h+o(h)) + \\
&\qquad P_{i-1}(t)(k_{i-1}h + o(h)) + o(h) \qquad (2)
\end{aligned}
$$

Dividing by h and passing to the $\lim h \to 0$, one gets

$$P_i'(t) = -k_iP_i(t) + k_{i-1}P_{i-1}(t) \text{ for } i > 1 \tag{3}$$

Clearly

$$P_0'(t) = -k_0P(t) \tag{4}$$

with boundary conditions

$$P_0(0) = 1, \text{ and } P_i(0) = 0, \ i > 0$$

The set of differential equations (3) and (4) can then be solved to get the time dependence of the probability of each state. Equation (4) can be solved without difficulty. In solving equation (3) define

$$Q_i(t) = \exp(k_it)P_i(t) \tag{5}$$

then

$$Q_i'(t) = \exp(k_it)P_i'(t) + k_i\exp(k_it)P_i(t)$$

$$= k_{i-1}\exp((k_i - k_{i-1})t)Q_{i-1}(t) \tag{6}$$

hence,

$$Q_i(t) = \int_0^t k_{i-1}\exp((k_i-k_{i-1})s)Q_{i-1}(s)ds \tag{7}$$

it follows that:

$$P_i(t) = k_{i-1}\exp(-k_it)\int_0^t \exp(k_is)P_{i-1}(s)ds \tag{8}$$

For a 2-parameter pure birth process, that, is,

$$k_0 \neq k_1$$

$$k_1 = k_2 = \ldots\ldots\ldots = k$$

It can be shown that

$$P_0(t) = \exp(-k_0 t) \tag{9}$$

$$P_1(t) = \frac{k_0}{k-k_0}(\exp(-k_0 t)-\exp(-kt)) \tag{10}$$

$$P_2(t) = \frac{kk_0}{(k-k_0)^2}\exp(-k_0 t)(1-\exp(-(k-k_0)t)-t(k-k_0)$$

$$\exp(-(k-k_0)t) \tag{11}$$

and in general

$$P_n(t) = \frac{k_0\exp(-k_0 t)k^{n-1}}{(k-k_0)^n}(1-\sum_{j=0}^{n-1}\frac{(t(k-k_0))^j}{j!}$$

$$\exp(-(k-k_0)t)) \tag{12}$$

The moment generating function is found to be equal to:

$$M(s) = \exp(-k_0t) + \frac{k_0\exp(s)}{k\exp(s)-k+k_0}(\exp(kt(\exp(s)-1))$$

$$-\exp(-k_0t)) \tag{13}$$

The first two moments can be found through M(s) and are as shown below:

$$E(N(t)) = M'(0) = kt + \frac{(k_0-k)}{k_0}(1-\exp(-k_0t)) \tag{14}$$

$$\mathrm{Var}(N(t)) = M''(0)-(M'(0))^2 = kt+((k_0-k)(k_0-2k)(1-\exp(-k_0t)$$

$$-(k_0-k)^2(1-\exp(-k_0t))^2/k_0^2+2k(k_0-k)\exp(-k_0t)/k_0$$

$$\tag{15}$$

2. Waiting Time Distribution

For the pure birth process define T_i to be the waiting time between the i-1 th occurrence and the ith occurrence, then,

T_1 = waiting time for the first occurrence

and

$\Pr(T_1 > t) = P(N(t)=0) = \exp(-k_0t)$

Since

$$Pr(T_1 > t) = 1 - F_{T_1}(t)$$

where F_{T_1} is the cumulative distribution function of T_1, hence,

$$1 - F_{T_1}(t) = \exp(-k_0 t) \qquad (16)$$

and

$$f_{T_1}(t) = \frac{dF_{T_1}(t)}{dt} = k_0 \exp(-k_0 t) \qquad (17)$$

That is, the waiting time distribution for the first occurrence is exponential distribution parameterized by k_0. Similarly, the waiting time distribution for the i+1 th occurrence is exponential distribution parameterized by k_i. Thus for a 2-parameter pure birth process the waiting time distributions are:

T_1 is exponentially distributed with parameter k_0

T_2, T_3,... are all exponentially distributed with parameter k.

III Estimating Methods

1. Introduction

Five estimation methods have been used to estimate the parameters k_0 and k in the model previously discussed and the results compared with the experimental data and other distinct methods.

2. Method of $E(N_0)$ and $E(S)$

Higgins and Tsokos (1978) preposed an easy estimating method by matching the empirical probability of zero to the theoretical probability to estimate k_0t and matching the expected value to the sample mean to estimate kt. With notation as in the previous section and by putting N_0/N in for $P_0(t)$ in equation (9) and S/N in for E(N(t)) in equation (14) this can be expressed as:

$$k_0t = -\log(N_0/N) \tag{18}$$

and

$$kt = \left(\frac{S}{N-N_0} - 1\right)\left(\frac{N}{N-N_0} + \frac{1}{\log(N_0/N)}\right)^{-1} \tag{19}$$

where

$Y_1, Y_2, \ldots\ldots, Y_N$ = random sample from $P(Y;k_0,k)$

N = sample size

N_n = number of Y_i's equal to n

$$S = \sum_{n=1}^{\infty} n.N_n$$

The estimated values of k_0t and kt can be obtained directly from the above two equations by substituting the experimental data.

3. Method of $E(N_0)$ and $E(N_1)$

Similar to above method, two equations obtained by equating the observed frequencies of state 0 and 1 to the corresponding expected values can be used to estimate k_0t and kt. It can be shown that

$$E(N_0) = N(\exp(-k_0t)) \qquad (20)$$

$$E(N_1) = N(\exp(-kt)-\exp(-k_0t))/(1-k/k_0) \qquad (21)$$

Equation (20) can be solved directly to get the estimated value of k_0t which substituted into equation (21) and then solved numerically to get kt.

4. Minimum Chi-Square Method

The objective of this method is to find the optimum estimated value of k_0 and k which minimizes the following function:

$$X^2(k_0t,kt) = \sum_{j=0}^{M} \frac{(N_j-NP_j(k_0t,kt))^2}{NP_j(k_0t,kt)} \tag{22}$$

where M is the maximum observed state of the process. This is a two-dimensional parameter searching problem. A number of searching techniques are available, the one used in this study will be discussed later.

5. Maximum Likelihood Method

The likelihood function of the pure birth process can be written as:

$$L(k_0,k) = \prod_{n=0}^{\infty} P(N;k_0,k)^{N_n} = \prod_{n=0}^{\infty} (P_n)^{N_n} \tag{23}$$

Define

$$L^*(k_0,k) = \log(L(k_0,k)) = \sum_{n=0}^{\infty} N_n\log(P_n)$$

$$= \sum_{n=0}^{M} N_n\log(P_n) \tag{24}$$

The maximum likelihood estimator of k_0 and k is then the roots of the following two equations:

$$\frac{\partial L^*}{\partial k_0}(k_0, k) = 0$$

$$\frac{\partial L^*}{\partial k}(k_0, k) = 0$$

or

$$\sum_{n=0}^{M} \frac{N_n}{P_n} \frac{\partial P_n}{\partial k_0} = 0 \qquad (25)$$

and

$$\sum_{n=0}^{M} \frac{N_n}{P_n} \frac{\partial P_n}{\partial k} = 0 \qquad (26)$$

Now, $\frac{\partial P_0}{\partial k_0}$ and $\frac{\partial P_0}{\partial k}$ can be obtained easily; for $n > 0$ it can be shown that

$$\frac{\partial P_n}{\partial k_0} = \frac{((1-k_0)(k-k_0)+nk_0)k^{n-1}}{(k-k_0)^{n+1}} \exp(-k_0)\left(1-\sum_{j=0}^{n-1} \frac{(k-k_0)^j}{j!}\right.$$

$$\left.\exp(-(k-k_0))\right) - \frac{k_0 k^{n-1} \exp(-k)}{(n-1)!(k-k_0)} \qquad (27)$$

$$\frac{\partial P_n}{\partial k} = \frac{-(k+(n-1)k_0)k_0k^{n-2}\exp(-k_0)}{(k-k_0)^{n+1}}(1-\sum_{j=0}^{n-1}\frac{(k-k_0)^j}{j!}$$

$$\exp(-(k-k_0)))) + \frac{k_0k^{n-1}\exp(-k)}{(n-1)!(k-k_0)} \qquad (28)$$

Equation (27) and (28) must be solved simultaneously to obtain the estimated value of k_0 and k. Due to the complexity of these equations direct solving it is quite impractical, however, an alternative approach can be applied. Let $(\widehat{k_0},\widehat{k})$ be the roots of equations (27) and (28), that is,

$$\frac{N_n}{P_n(k_0,k)} \frac{\partial P_n(\widehat{k_0},\widehat{k})}{\partial k_0} = 0 \qquad (29)$$

$$\frac{N_n}{P_n(k_0,k)} \frac{\partial P_n(\widehat{k_0},\widehat{k})}{\partial k} = 0 \qquad (30)$$

Now define

$$F = \left|\frac{N_n}{P_n} \frac{P_n}{k_0}\right| + \left|\frac{N_n}{P_n} \frac{P_n}{k}\right| \qquad (31)$$

then F possesses a minimum value of 0 which occurs when

$$(k_0,k) = (\widehat{k_0},\widehat{k})$$

A root finding problem is now converted to a two-dimensional extremum searching problem, and this can easily be done by applying optimization techniques.

6. Method of Moments

Equations (14) and (15) can be equated to the sample mean and variance respectively and then solved simultaneously to obtain the estimated value of k_0 and k. Again, solving these equation directly is tedious and some parameter searching technique are applied in this case also.

7. Optimization Method

The searching technique used here is derived by Hooke and Jeeves (1961). It is among the simplest and most efficient methods for solving the unconstrained non-linear minimization problems. The technique consists of searching the local nature of the objective function in the space and then moving in a favorable direction for reducing the functional value. The direct search method of Hooke and Jeeves is a sequential search routine for minimizing a function f(x) of more than one variable $X=(x_1, x_2, \ldots, x_r)$. The argument X is varied until the minimum of f(x) is obtained. The search routine determines the sequence of values for X. The successive values of X can be interpreted as points in an r-dimensional space. The procedure consists of two types of moves:

exploratory and pattern.

A move is defined as the procedure of going from a given point to the following point. A move is a success if the value of $f(x)$ decreases; otherwise, it is a failure. The first type of move is the exploratory move which is designed to explore the local behavior of the objective function, $f(x)$. The success or failure of the exploratory moves is utilized by combining it into a pattern which indicates a probable direction for a successful move.

The exploratory move is performed as follows:

1. Introduce a starting point X with a prescribed step length s_i in each of the independent variables x_i, $i=1,2,\ldots,r$.
2. Compute the objective function, $f(x)$ where $X = (x_1,x_2,\ldots,x_r)$, set $i = 1$
3. Compute $f_i(x)$ at the trial point $X = (x_1,x_2,\ldots,x_i+s_i,x_{i+1},\ldots,x_r)$
4. Compute $f_i(x)$ with $f(x)$
   (i) If $f_i(x) < f(x)$, set $f(x) = f_i(x)$, $X = (x_1,x_2,\ldots,x_i+s_i,\ldots,x_r)$ and $i=i+1$. Consider this trial point as a starting point, and repeat from step 3.
   (ii) If $f_i(x) > f(x)$, set $X = (x_1,x_2,\ldots,x_i-2s_i,\ldots,x_r)$. Compute $f_i(x)$, and see if $f_i(x) < f(x)$. If this move is a success the new trial point is retained. Set $f(x) = f_i(x)$, and $X = (x_1,x_2,\ldots,x_i-2s_i,\ldots,x_r)$, and

$i=i+1$, and repeat from step 3. If again $f_i(x) > f(x)$, then move is a failure and X remains unchanged, that is, $X = (x_1, x_2, \ldots, x_i, \ldots, x_r)$. Set $i=i+1$ and repeat from step 3.

The point X_B obtained at the end of the exploratory moves, which is reached by repeating step 3 until $i=r$, is defined as a base point. The starting point introduced in step 1 of the exploratory move is either a starting base point or a point obtained by the pattern move.

The pattern move is designed to utilize the information acquired in the exploratory move, and executes the actual minimization of the function by moving in the direction of the established pattern. The pattern move is a simple step from the current base to the point

$$X = X_B + (X_B - X_B^*) \tag{32}$$

X_B^* is either the starting base point or the preceding base point. Following the pattern move a series of exploratory moves is conducted to further improve the pattern. If the pattern move followed by the exploratory moves brings no improvement, the pattern move is a failure. Then one returns to the last base which becomes a starting base and the process is repeated.

If the exploratory moves from any starting base do not yield a point which is better than this bases, the lengths of all the base are reduced and the moves are repeated. Convergence is assumed when the step lengths, s_i, have been reduced below predetermined limits.

8. Numerical Results

The first set of data is from a reliability study [5]. Failures were recorded for each of 19 PPI consoles which were operated for a period of 8640 hours. The second set of data is taken from a queuing study [1] . The arrivals per unit serving time were recorded at a tool crib counter in a factory. The observed and estimated number of frequencies by five methods metioned above are given in Table 1 and 2. As can be seen every estimating method predicted frequencies reasonably well. In fact, Griffiths (1977) observed that estimation of the parameters k_0 and k is more easily done by using the expressions for $E(N_0)$ and $E(S)$, and he showed that in the case of one set of data from Simmonds (1956) this simpler procedure resulted in values that differ little whether obtained by equating the sample value with $P(N_0)$ and $E(S)$ or $P(N_0)$ and $P(N_1)$, or by maximum likelihood; using the first two moments $E(S)$ and $Var(S)$ gave k_0 and k values of similar order but not so much in agreement as the other estimates.

Table 1

Number of failures	0	1	2	3 or more
Observed	8	4	3	4
$E(N_0)$ and $E(S)$	8.1	4.3	3.4	3.3
$E(N_0)$ and $E(N_1)$	8.0	4.0	3.3	3.7
Min. Chi-Square	8.0	3.7	3.2	4.1
M.L.E.	8.1	3.7	3.2	4.0
Method of Moments	5.9	6.0	4.2	3.0

Table 2

Number of failures	0	1	2	3	4	5 or more*
Observed	272	306	213	117	44	24
$E(N_0)$ and $E(S)$	272.0	292.0	224.0	118.0	38.0	32.0
$E(N_0)$ and $E(N_1)$	272.0	306.0	225.1	112.8	42.8	17.2
Min. Chi-Square	272.7	278.8	223.3	124.0	52.5	24.5
M.L.E.	277.0	240.6	214.5	136.9	67.5	39.5
Method of Moments	208.9	303.7	247.1	135.4	55.8	25.1

* : Observations of failures 5,6,7 and 8 are combined as 5 or more.

IV. Pure Birth Process Simulation

1. Introduction

In this study, pure birth process has been simulated by computer. Primarily, interest will be given to small sample size behavior. Parameters are estimated by the methods discussed in the last chapter. The results are analyzed to compare the goodness of prediction and efficiency of the various methods.

2. Simulation of 2-parameter Pure Birth Process

It has been shown in chapter 1 that the waiting time distribution for the first count of a 2-parameter pure birth process is exponential(k_0). Since the c.d.f. of $T_1 = F(T_1)$ $= 1- \exp(k_0T_1)$ is distributed as uniform (0,1), it is obvious that

$$\ln(1-U) = -k_0T_1 \qquad (33)$$

Consequently, T_1 can be simulated by drawing an random number from (0,1) and substituting in equation (33), provided k_0 is given. Similarly, the following counts can be simulated by the equation below

$$\ln(1-U) = -kT_i \quad i=2,3,\ldots.. \qquad (34)$$

where U is some random number in the interval (0,1), and k is the second parameter of the pure birth process under consideration. The algorithm to generate a two parameter pure birth process is then as listed below:

Let

N = total number of events (observations)

T = ovservation time

NC(i) = number of events observed in state i

1) Set NC(i) = 0 and count = 0

2) A random number within the interval (0,1) is generated and by equation (33) T_1 is calculated. If $T_1 > T$ then count=count+1 and NC(0)=NC(0)+1. Now if $T_1 < T$ then another uniform random number be drawn and by equation (34) T_2 be calculated. If $T_1+T_2 > T$ then count=count+1 and NC(1)=NC(1)+1. If $T_1+T_2 < T$ then repeat drawing random number and by equation (34) to calculate T_i until $T_1+T_2+.....+T_i > T$, then set count=count+1 and NC(i-1)=NC(i-1)+1. Step 2) is repeated until count=N

A 2-parameter pure birth process with N observations can thus be generated and the parameter estimating methods discussed in chapter 3 are then used to estimate the parameters of each simulated data.

3. Random Number Generating

The method most commonly used to generate random number is the linear congruential method [8]. Each number in the sequence, r_j, is calculated from its predecessor, r_{j-1}, using the formula:

$$r_j = (\text{multiplier} \times r_{j-1} + \text{increment}) \text{ MOD modulus} \quad (35)$$

The numbers generated by using this formula repeatedly are not truly random number in the sense that tosses of a coin or throws of a die are random, because we can always predict the value of r_j given the value of r_{j-1}. The sequence generated by this formula is therefore more correctly called a pseudo-random sequence, and its members are called pseudo random numbers.

In this study the following values have been chosen:

modulus $= 2^{16} = 65536$

multiplier $= 25173$

increment $= 13849$

i.e.,

$$r_j = (25173 \times r_{j-1} + 13849) \text{ MOD } 65536 \quad (36)$$

This calculation will not cause overflow on a computer for which

Maximum integer $> 2^{31} - 1$

The equation (36) generates a permutation of the integers

0,1,2,.......,65536

and then repeats itself. The first number generated is the initial value of seed.

Since uniform random numbers are needed equation (36) is modified to get a real random value between 0 and 1, by dividing by 65536.

4. Results and Conclusions

Two different processes have been simulated. The first one comes from [5] with the estimated values of $k_0t = 0.8675$ and $kt = 1.91$. The second is from [1] with the estimated values of $k_0t = 1.273$ and $kt = 1.63$. For each process, 30 sets of data have been generated each containing (i) 20 and (ii) 40 observations. In an attempt to examine the variations of estimated mean square error for each method, two replications of each process have been simulated.

The results of estimated parameters of the first process for five estimating methods are shown in Table 3 and 4 (only estimated means and estimated mean square error are exhibited). Those for the second process are shown in Table 5 and 6. In each table columns one and three show the estimated values of parameters and columns two and four show the corresponding estimated mean square error for each method. Figure 1 and 2 show the box-plots of the distribution of the two estimated parameters for the first process and figures 3 and 4 give the same information for the second process. In each figure the value of horizontal axis represents the following: 1.00, 3.00, 5.00, 7.00, 9.00 stand for frequency distribution of the estimated parameters by using method of $E(N_0)$ & $E(S)$, $E(N_0)$ & $E(N_1)$, Min. Chi-Square, M.L.E., and method of moments respectively with sample size 20 and 2.00, 4.00, 6.00, 8.00, 10.00 are frequency distribution for above five methods with sample size 40.

By examining the results obtained above the following conclusions can be drawn:

(1) The estimation method using either $E(N_0)$ & $E(S)$ or $E(N_0)$ & $E(N_1)$ can give a quick estimation of the system parameters. The second method, however, might result in a large mean square error.

(2) As the number of observations increased the mean square error of each method generally decreased as one would anticipate.

(3) M.L.E. does not give significant improvement over the other methods. In addition, it is a very time consuming technique since it involves solving simultaneous equations involving derivatives.

(4) Minimum Chi-Square method gives reasonable mean square error. Typically, the time consumption of this method is ten times

as much as that of method of moments and one half to two thirds of that of M.L.E..

(5) Method of moments method sometimes gives large mean square error. Another major drawback of this method is that the bias is usually large.

(6) Judging from the above observations it is concluded that method of using $E(N_0)$ and $E(S)$ is generally the most desirable one. The minimum Chi-Square method could also be considered.

(7) The data structure as shown in Table 3, 4, 5 and 6 allows one to test the appropriatness of the simulation process, i.e., if the sample is normally distributed, an F-test can be performed to test the equal sample variance hypothesis. In particular, if certain estimator is unbiased a central F-statistic can be used. Such a test has been done to the simulated data. For those normally distributed sample set the equal sample variance hypothesis is confirmed at 0.05 level.

Table 3

$k_0t=0.8675$ $kt=1.91$ Time=8640 hr number of sets=30 number of observations per set=20				
	replication 1			
	$\hat{k_0t}$	m.$\hat{s}$.e.	$\hat{kt}$	m.$\hat{s}$.e.
$E(N_0)$ and $E(S)$	0.934	0.071	1.827	0.346
$E(N_0)$ and $E(N_1)$	0.934	0.071	2.055	1.664
Min. Chi-Square	0.970	0.078	2.053	0.420
M.L.E.	0.926	0.110	2.268	0.873
Method of Moments	1.253	0.177	1.438	0.299
	replication 2			
	$\hat{k_0t}$	m.$\hat{s}$.e.	$\hat{kt}$	m.$\hat{s}$.e.
$E(N_0)$ and $E(S)$	0.855	0.045	1.836	0.294
$E(N_0)$ and $E(N_1)$	0.855	0.045	2.235	2.453
Min. Chi-Square	0.909	0.054	2.046	0.275
M.L.E.	0.884	0.180	2.316	0.909
Method of Moments	1.358	0.378	1.471	0.313

Table 4

k_0t=0.8675 kt=1.91 Time=8640 hr number of sets=30

number of observations per set=40

replication 1

	$\hat{k_0t}$	m.$\hat{s}$.e.	$\hat{kt}$	m.$\hat{s}$.e.
$E(N_0)$ and E(S)	0.809	0.017	1.948	0.190
$E(N_0)$ and $E(N_1)$	0.809	0.017	2.069	0.612
Min. Chi-Square	0.846	0.015	2.102	0.202
M.L.E.	0.780	0.024	2.202	0.357
Method of Moments	1.146	0.107	1.408	0.305

replication 2

	$\hat{k_0t}$	m.$\hat{s}$.e.	$\hat{kt}$	m.$\hat{s}$.e.
$E(N_0)$ and E(S)	0.894	0.027	1.915	0.239
$E(N_0)$ and $E(N_1)$	0.894	0.027	2.226	1.728
Min. Chi-Square	0.925	0.033	2.060	0.302
M.L.E.	0.874	0.030	2.239	0.867
Method of Moments	1.312	0.272	1.515	0.310

Table 5

k_0t=1.273 kt=1.630 Time=100 hr number of sets=30

number of observations per set=20

replication 1

	$\widehat{k_0t}$	m.$\widehat{s}$.e.	$\widehat{kt}$	m.$\widehat{s}$.e.
$E(N_0)$ and $E(S)$	1.478	0.194	1.500	0.213
$E(N_0)$ and $E(N_1)$	1.478	0.194	1.750	1.708
Min. Chi-Square	1.513	0.195	1.675	0.205
M.L.E.	1.721	0.505	1.742	0.353
Method of Moments	1.603	0.185	1.491	0.087

replication 2

	$\widehat{k_0t}$	m.$\widehat{s}$.e.	$\widehat{kt}$	m.$\widehat{s}$.e.
$E(N_0)$ and $E(S)$	1.298	0.122	1.646	0.234
$E(N_0)$ and $E(N_1)$	1.298	0.122	2.225	3.683
Min. Chi-Square	1.368	0.130	1.798	0.190
M.L.E.	1.283	0.140	2.268	1.364
Method of Moments	1.554	0.225	1.509	0.118

Table 6

$k_0t=1.273$ $kt=1.630$ Time=100 hr number of sets=30

number of observations per set=40

replication 1

	$\hat{k_0t}$	m.$\hat{s}$.e.	$\hat{kt}$	m.$\hat{s}$.e.
$E(N_0)$ and $E(S)$	1.242	0.045	1.614	0.120
$E(N_0)$ and $E(N_1)$	1.242	0.045	1.652	0.268
Min. Chi-Square	1.272	0.042	1.734	0.136
M.L.E.	1.232	0.044	1.841	0.340
Method of Moments	1.388	0.034	1.484	0.065

replication 2

	$\hat{k_0t}$	m.$\hat{s}$.e.	$\hat{kt}$	m.$\hat{s}$.e.
$E(N_0)$ and $E(S)$	1.347	0.058	1.596	0.111
$E(N_0)$ and $E(N_1)$	1.347	0.058	1.656	0.240
Min. Chi-Square	1.378	0.065	1.726	0.160
M.L.E.	1.320	0.077	1.878	0.616
Method of Moments	1.560	0.157	1.550	0.110

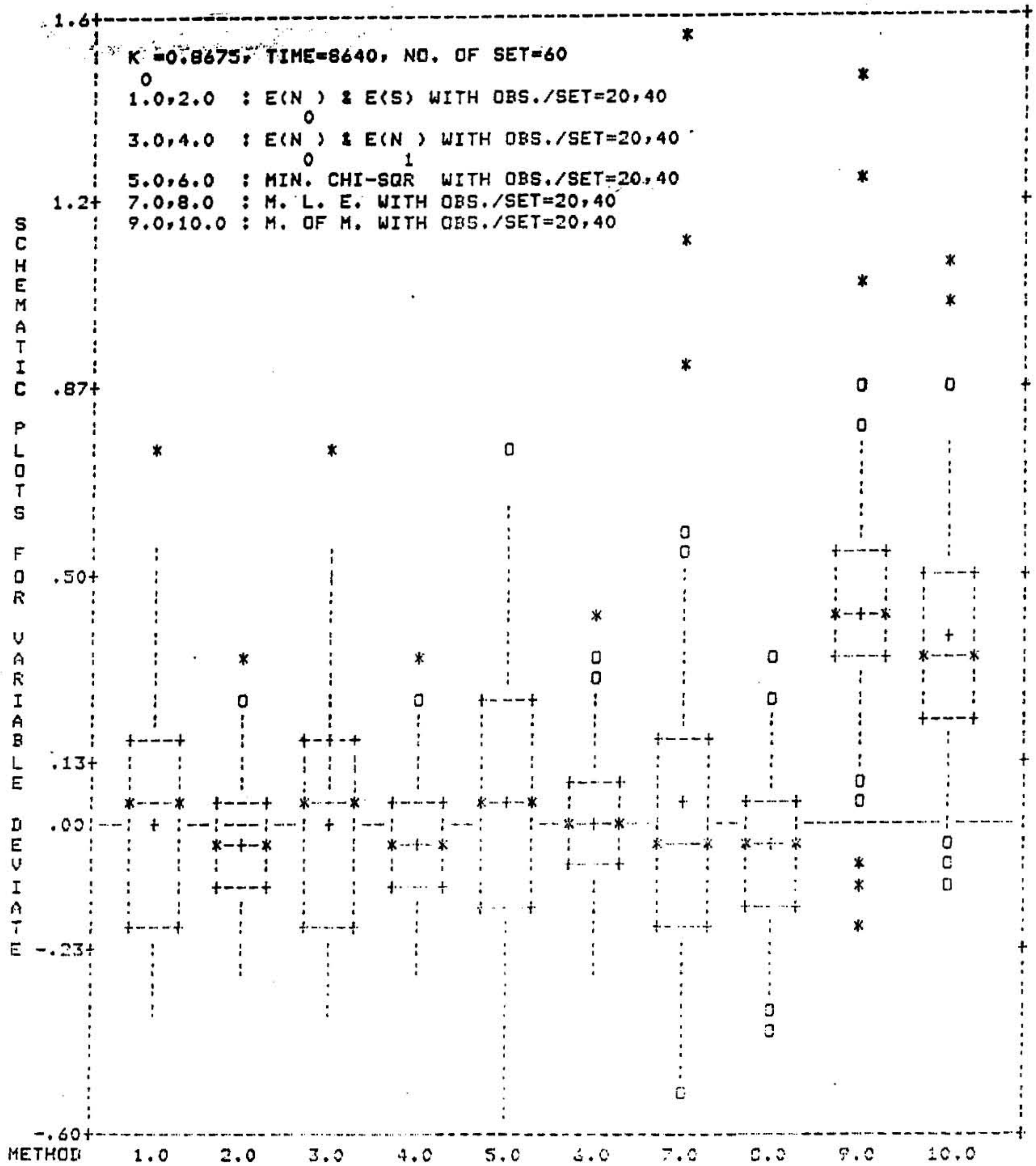

Figure 1. Distribution of $\widehat{k_0} - k_0$.

A Sample Box Plot Is Shown In Figure 5

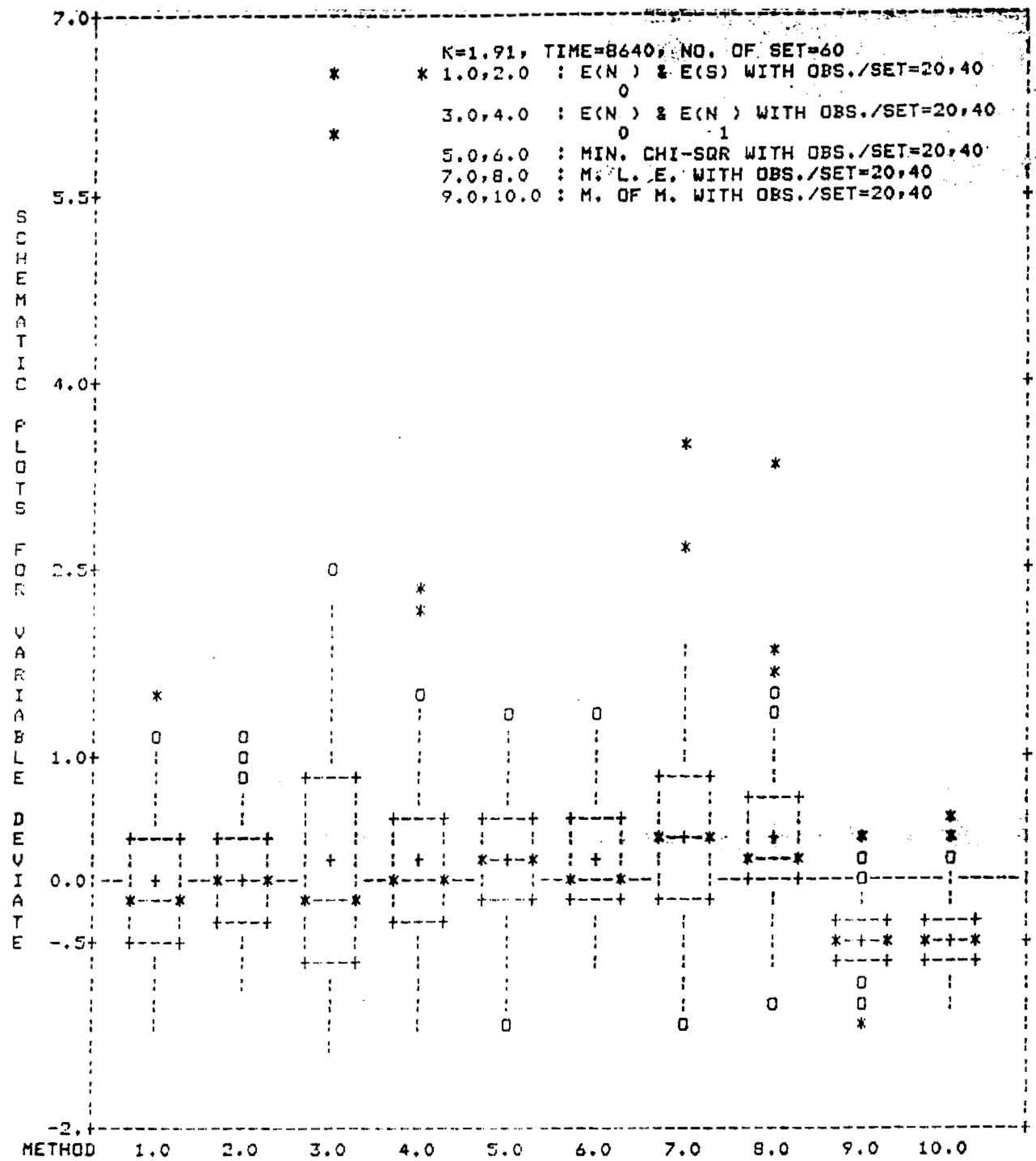

Figure 2. Distribution of $\widehat{k} - k$.

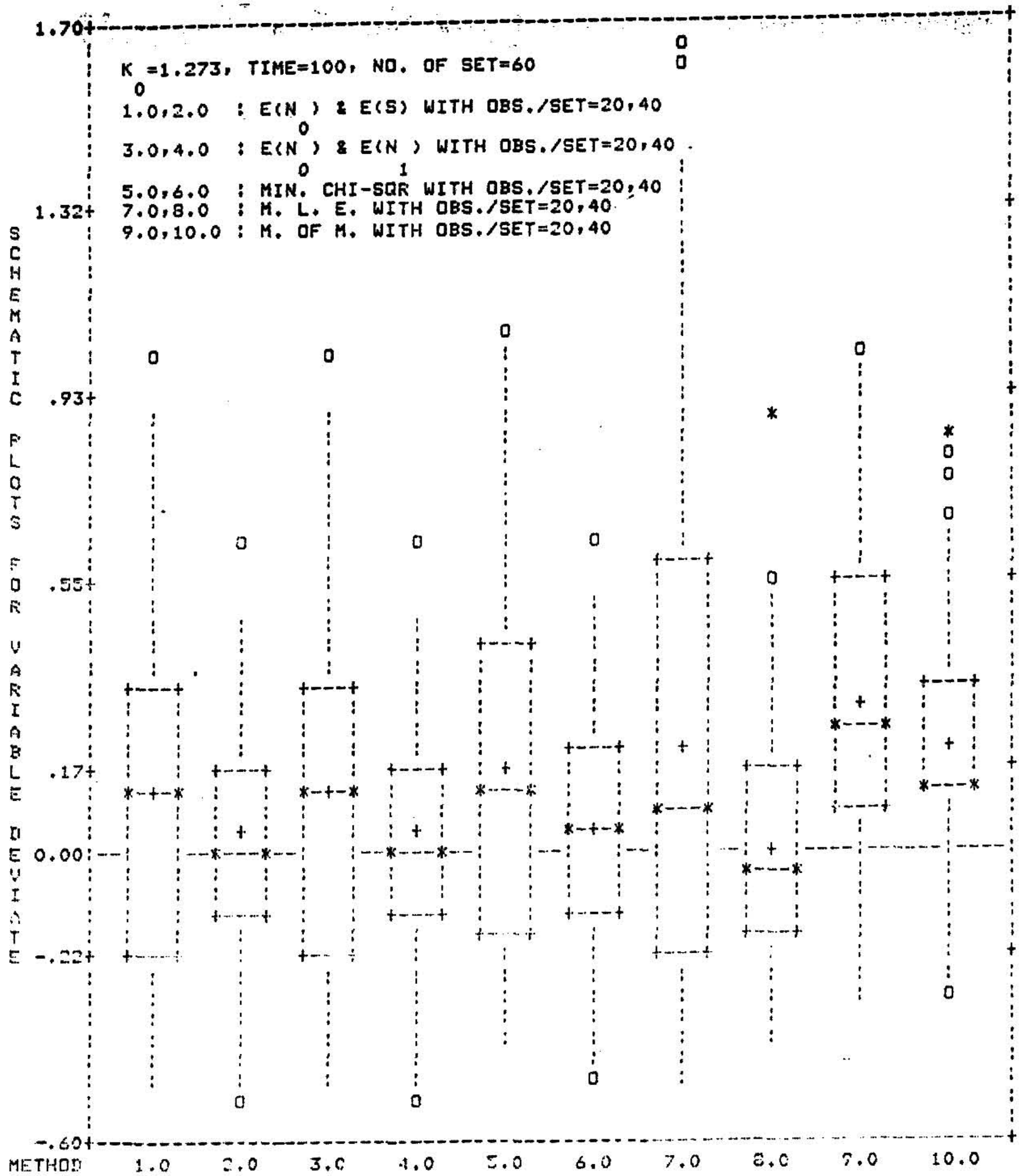

Figure 3. Distribution of $\widehat{k}_0 - k_0$.

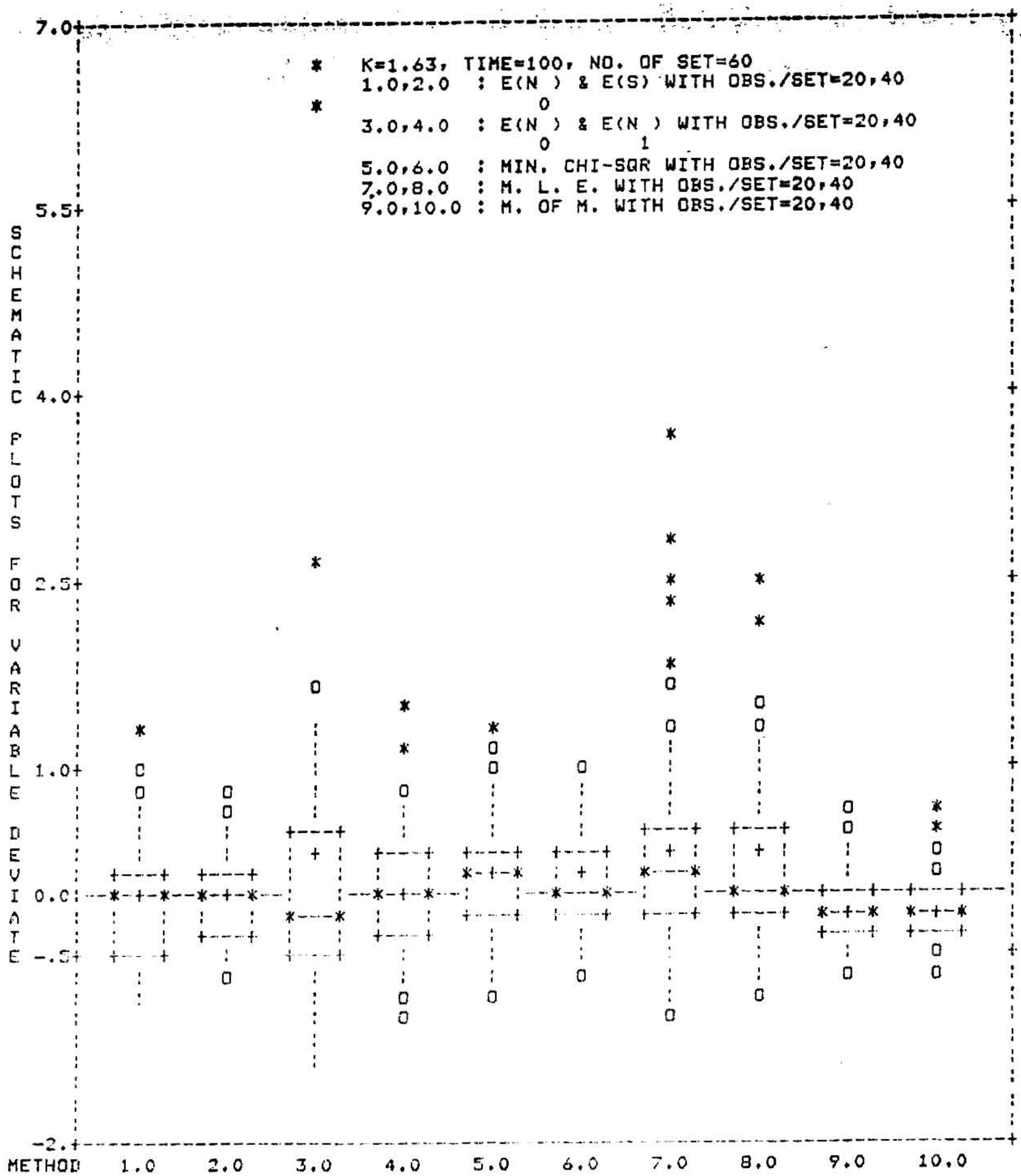

Figure 4. Distribution of $\widehat{k} - k$.

```
+----------------------------------------------------------+
|                                                          |
|     *      UPPER QUARTILE + 1.5 X I.D.                    |
|                                                          |
|     O                                                    |
|                                                          |
|     |      LARGEST VALUE < UPPER QUARTILE + I.D.         |
|     |                                                    |
|     |                                                    |
|     |                                                    |
|     |                                                    |
|   +---+   UPPER QUARTILE                                 |
|   |   |                                                  |
|   |   |                                                  |
|   |   |                                                  |
|   *---*   MEDIUM                                         |
|   |   |                                                  |
|   |   |                                                  |
|   |   |                                                  |
|   +---+   LOWER QUARTILE                                 |
|     |                                                    |
|     |                                                    |
|     |                                                    |
|     |     SMALLEST VALUE > LOWER QUARTILE - I.D.         |
|                                                          |
|     O                                                    |
|                                                          |
|     *     LOWER QUARTILE - 1.5 X I.D.                    |
|     *                                                    |
|                                                          |
|           I.D. : INTERQUARTILE DISTANCE                  |
|                                                          |
+----------------------------------------------------------+
```

Figure 5. Example Box Plot For 50 Data Points

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Appendix

The following is the program used to estimate the parameters of 2-Parameter Poisson process.

```
C -----------------------------------------------------------------------
C    THE PARAMETERS OF A 2-PARAMETER POISSON PROCESS(INTERPRETED AS
C    A PURE BIRTH PRECESS) BE ESTIMATED BY FIVE METHOD(SEE TEXT FOR
C    DETAILS). IN PARTICULAR, MIN-CHISQUARE, M.L.E., AND METHOD OF
C    MOMENTS ARE ALL SOLVED BY HOOKE AND JEEVE OPTIMIZATION METHOD.
C -----------------------------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION EXPXO(5,100),EXPX(5,100),FMSEXO(5),FMSEX(5)
      DIMENSION BSNI(10),DEAI(10),DI(10)
      DIMENSION X(10), BSO(10),BSN(10),DL(10),TITLE(60),NC1(100)
      COMMON XNTOT,DEA(10),NCO,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
   51 FORMAT(6D10.4)
   54 FORMAT(20I4)
  110 FORMAT (10I5)
  444 FORMAT(/5X,20(1X,I4))
  579 FORMAT(3D10.4,I4)
  677 FORMAT (/5X,'METHOD OF MINIMUM CHI-SQR')
  678 FORMAT (/5X,'METHOD OF M.L.E.')
  679 FORMAT (/5X,'METHOD OF MOMENTS')
  779 FORMAT(/5X,5(1X,D12.4))
  888 FORMAT(//5X,'MEAN SQUARE ERROR OF FIRST PARAMETER')
  889 FORMAT(//5X,'MEAN SQUARE ERROR OF SECOND PARAMETER')
  890 FORMAT(/5X,'DATA SET',2X,I4)
  903 FORMAT(/5X,'METHOD OF USING EQ.(17) AND (18)')
  904 FORMAT(/5X,'XO = ',1X,D12.4,3X,'X = ',1X,D12.4)
  905 FORMAT(//5X,'METHOD OF USING E(XO) AND E(X1)')
  906 FORMAT(/5X,'THEOXO=',D12.4,2X,'THEOX=',D12.4,2X,'TIME=',D12.4,
     *2X,'NUMBER OF DATA SET=',I4)
 750  FORMAT ('1 PURE BIRTH PROCESS PARAMETER ESTIMATION ')
 1130 FORMAT (' OBJECTIVE FUNCTION  ',D18.6/' OPTIMAL POINT ',5D18.5
     1/(15X,5D18.5))
C     THEOXO = THEORITICAL VALUE OF XO
C     THEOX = THEORETICAL VALUE OF X
C     TIME = OBSERVING TIME
C     NOP = TOTAL NUMBER OF DATA SET
      READ(5,579) THEOXO,THEOX,TIME,NOP
      WRITE(6,750)
      WRITE(6,906) THEOXO,THEOX,TIME,NOP
C     INPUT SEARCHING DIMENSION
      READ(5,110) ND
C     INPUT INITIAL BASE POINT,INITIAL STEP SIZE AND STOPPING STEP SIZE
      READ(5,51) (BSNI(I),I=1,ND),(DEAI(I),I=1,ND),(DI(I),I=1,ND)
C     M = NUMBER OF STATES(INCLUDING STATE 0)
      DO 1250 LMN=1,NOP
      READ(5,54) M
      IF(M.EQ.0)  GO TO  101
      WRITE(6,890) LMN
      IJ=M-1
C     NM = MAXIMUM STATE NUMBER
      NM=M-2
C     NC = NUMBER OF COUNTS
      READ(5,54) NCO,(NC(LLL),LLL=1,IJ)
      WRITE(6,444) NCO,(NC(LLL),LLL=1,IJ)
C     METHOD OF USING EQ.(17) AND (18)
      NCO1=NCO
C     XNTOT = TOTAL NUMBER OF OBSERVATIONS
      XNTOT=NCO*1.0
      DO 902 L1=1,IJ
  902 XNTOT=XNTOT+NC(L1)*1
      CALL SEMENT(XO1,X1)
```

```
      WRITE(6,903)
      EXPXO(1,LMN)=X01
      EXPX(1,LMN)=X1
      WRITE(6,904) X01,X1
C     METHOD OF USING E(X0) AND E(X1)
      A=0.0
      B=10.0
      DELX=1.0D-05
      XNCO=1.0*NCO
      X02=-DLOG(XNCO/XNTOT)
      EXPXO(2,LMN)=X02
      CALL ROOT(X2,A,B,DELX,IKJ,IJK,X02)
      IF(IJK.EQ.2) X2=A
      WRITE(6,905)
      EXPX(2,LMN)=X2
      WRITE(6,904) X02,X2
C
C     THE FOLLOWING ESTIMATION METHODS USE HOOKE AND JEEVE METHOD
C     METHOD OF MINIMUM CHI-SQUARE (INDEX = 3)
C     METHOD OF M.L.E. (INDEX = 4)
C     METHOD OF MONENT (INDEX = 5)
C
      DO 555 INDEX=3,5
      IF(INDEX.EQ.3) WRITE(6,677)
      IF(INDEX.EQ.4) WRITE(6,678)
      IF(INDEX.EQ.5) WRITE(6,679)
      NN=1
      NE=0
      DO 901 L11=1,ND
      BSN(L11)=BSNI(L11)
      DEA(L11)=DEAI(L11)
      DL(L11)=DI(L11)
  901 CONTINUE
      IF(INDEX.LT.4) GO TO 222
      BSN(1)=ZZZX0
      BSN(2)=ZZZX
      GO TO 700
  222 FXBN=OBJ3(BSN)
      GO TO 701
  700 IF(INDEX.GT.4) GO TO 703
      FXBN=OBJ4(BSN)
      GO TO 701
  703 FXBN=OBJ5(BSN)
  701 CONTINUE
    1 DO 10 I=1,ND
   10 X(I) = BSN(I)
      FX = FXBN
      CALL EPV(FX,X)
      IF(FX.GE.FXBN)   GO TO 3
    2 DO 20 I=1,ND
      BSO(I) = BSN(I)
      BSN(I) = X(I)
   20 CONTINUE
      FXBN = FX
      NNB=NE
      DO 21 I=1,ND
      X(I) = BSN(I)*2.0-BSO(I)
   21 CONTINUE
      IF(INDEX.GT.3) GO TO 500
      FXBN=OBJ3(BSN)
```

```
      GO TO 501
  500 IF(INDEX.GT.4) GO TO 503
      FXBN=OBJ4(BSN)
      GO TO 501
  503 FXBN=OBJ5(BSN)
  501 CONTINUE
      IF(INDEX.GT.3) GO TO 600
      FX=OBJ3(X)
      GO TO 601
  600 IF(INDEX.GT.4) GO TO 603
      FX=OBJ4(X)
      GO TO 601
  603 FX=OBJ5(X)
  601 CONTINUE
      CALL EPV(FX,X)
      IF(FX.LT.FXBN)  GO TO 2
      NN=NNB
      GO TO 1
    3 CONTINUE
      DO 30 I=1,ND
      IF(DEA(I).GE.DL(I))  GO TO 31
   30 CONTINUE
      GO TO 100
   31 DO 35 I=1,ND
      DEA(I) = DEA(I)*0.5
   35 CONTINUE
      GO TO 1
  100 WRITE(6,904)(BSN(I),I=1,ND)
      ZZZXO=BSN(1)
      ZZZX=BSN(2)
      EXPXO(INDEX,LMN)=BSN(1)
      EXPX(INDEX,LMN)=BSN(2)
  555 CONTINUE
 1250 CONTINUE
      DO 777 IND=1,5
      XOMSE=0.0
      XMSE=0.0
      DO 778 LMN=1,NOP
      XOMSE=XOMSE+(EXPXO(IND,LMN)-THEOXO)**2
  778 XMSE=XMSE+(EXPX(IND,LMN)-THEOX)**2
      FMSEXO(IND)=XOMSE/NOP
      FMSEX(IND)=XMSE/NOP
  777 CONTINUE
      WRITE(6,888)
      WRITE(6,779) (FMSEXO(IND),IND=1,5)
      WRITE(6,889)
      WRITE(6,779) (FMSEX(IND),IND=1,5)
  101 STOP
      END
C     -------------- END OF MAIN PROGRAM --------------------------------
C
C     SUBROUTINE EPV PERFORMS EXPLANATORY MOVE
C
      SUBROUTINE EPV (FX,X)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(10), BSO(10),BSN(10),DL(10),TITLE(60)
      COMMON XNTOT,DEA(10),NCO,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      DO 201 I=1,ND
      X(I) = X(I) + DEA(I)
      IF(INDEX.GT.3) GO TO 500
```

```
      FXI=OBJ3(X)
      GO TO 501
  500 IF(INDEX.GT.4) GO TO 503
      FXI=OBJ4(X)
      GO TO 501
  503 FXI=OBJ5(X)
  501 CONTINUE
      NE = N
      IF(FXI-FX)  200,180,180
  180 X(I) = X(I) - 2.*DEA(I)
      IF(INDEX.GT.3) GO TO 600
      FXI=OBJ3(X)
      GO TO 601
  600 IF(INDEX.GT.4) GO TO 603
      FXI=OBJ4(X)
      GO TO 601
  603 FXI=OBJ5(X)
  601 CONTINUE
      NE = N
      IF(FXI-FX) 200,181,181
  181 X(I) = X(I) + DEA(I)
      NE=N-2
      GO TO 202
  200 FX = FXI
  202 CONTINUE
  201 CONTINUE
      RETURN
      END
C     ----------------------------------------------------------------
C
C     SUBROUTINE SEMENT CALCULATE THE ESTIMATED VALUE OF XO AND X BY
C     EQ.(17) AND (18)
C
      SUBROUTINE SEMENT(XO,X)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XNTOT,DEA(10),NCO,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      XNCO=NCO*1.0
      S=0.0
      DO 1 I=1,IJ
      S=S+NC(I)*I*1.0
    1 CONTINUE
      XO=-DLOG(XNCO/XNTOT)
      X=(S/(XNTOT-XNCO)-1.0)/(XNTOT/(XNTOT-XNCO)+1./DLOG(XNCO/XNTOT))
      RETURN
      END
C     ----------------------------------------------------------------
C
C     SUBROUTINE USING BOLZANO METHOD TO FIND ROOT OF F(X) = 0
C
      SUBROUTINE ROOT(X,A,B,DELX,I,K,XO2)
      IMPLICIT REAL*8(A-H,O-Z)
C  A= LEFT LIMIT OF VALUE X
C  B=RIGHT LIMIT OF VALUE X
C  DELX=MINIMUM ERROR
      XO=XO2
      FA=FUNC(A,XO)
      FB=FUNC(B,XO)
      I=0
    7 X=(A+B)/2.
      IF(DABS(X-A).LE.1.0E-09.OR.DABS(X-B).LE.1.0E-09) GO TO 10
```

```
      I=I+1
      F=FUNC(X,X0)
      IF(F) 12,10,11
   12 IF(F+DELX) 3,10,10
   11 IF(F-DELX) 10,10,3
    3 IF(F*FA) 5,8,6
    5 B=X
      FB=F
      GO TO 7
    6 A=X
      FA=F
      GO TO 7
   10 K=1
      GO TO 9
    8 K=2
    9 CONTINUE
      RETURN
      END
C     ----------------------------------------------------------------
C
C     FUNC,FACT,XPOIFC,PROBTY,DPROB ARE USED TO CALCULATE THE
C     PROBABILITY DISTRIBUTION OF PURE BIRTH PROCESS
C
      FUNCTION FUNC(Y,X0)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XNTOT,DEA(10),NCO,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      FUNC=(DEXP(-Y)-DEXP(-X0))/(1.-Y/X0)-NC(1)/XNTOT
      RETURN
      END
C     ----------------------------------------------------------------
      FUNCTION FACT(K)
      IMPLICIT REAL*8(A-H,O-Z)
      FACT=1.0
      IF(K.LE.1) RETURN
      DO 5 J=2,K
      FACT=FACT*J
    5 CONTINUE
      RETURN
      END
C     ----------------------------------------------------------------
      FUNCTION XPOIFC(N,XD)
      IMPLICIT REAL*8(A-H,O-Z)
      L=N-1
      IF(L.EQ.0) GO TO 2
      XPOIFC=1.0
      FACTOR=1.0
      TERM=1.0
      DO 7 K=1,L
      TERM=TERM*XD/FACTOR
      XPOIFC=XPOIFC+TERM
      FACTOR=FACTOR+1.0
    7 CONTINUE
      IF(DABS(XD).LE.1.0D-10) GO TO 10
      XPOIFC=(1.0-XPOIFC*DEXP(-XD))/XD**N
      RETURN
   10 XPOIFC=(1.0-XPOIFC)/XD**N
      RETURN
    2 IF(DABS(XD).LE.1.0D-10) GO TO 11
      XPOIFC=(1.0-DEXP(-XD))/XD
      RETURN
```

```
   11 XPOIFC=1.0/XD
      RETURN
      END
C     -----------------------------------------------------------------
C
C     OBJECTIVE FUNCTION OF MIN-CHISQUARE METHOD
C
      FUNCTION OBJ3(PHI)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION P(100),PHI(10)
      COMMON XNTOT,DEA(10),NCO,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      CALL PROBTY(PHI(1),PHI(2),PO,P)
      OBJ3=(PO*XNTOT-NCO)**2/(PO*XNTOT)
      DO 7 I=1,IJ
      OBJ3=OBJ3+(P(I)*XNTOT-NC(I))**2/(P(I)*XNTOT)
    7 CONTINUE
      RETURN
      END
C     -----------------------------------------------------------------
C
C     OBJECTIVE FUNCTION OF M.L.E. METHOD
C
      FUNCTION OBJ4(PHI)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION P(100),DPO(100),DP1(100),PHI(10)
      COMMON XNTOT,DEA(10),NCO,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      CALL PROBTY(PHI(1),PHI(2),PO,P)
      CALL DPROB(PHI(1),PHI(2),DPOO,DPO,DP10,DP1)
      FO=NCO*DPOO/PO
      F1=NCO*DP10/PO
      DO 5 I=1,IJ
      FO=FO+NC(I)*DPO(I)/P(I)
      F1=F1+NC(I)*DP1(I)/P(I)
    5 CONTINUE
      OBJ4=DABS(FO)+DABS(F1)
      RETURN
      END
C     -----------------------------------------------------------------
      SUBROUTINE PROBTY(XO,X,PO,P)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION P(100)
      COMMON XNTOT,DEA(10),NCO,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      XD=X-XO
      EXO=DEXP(-XO)
      EX=DEXP(-X)
      EXD=DEXP(-XD)
      PO=EXO
      DO 3 I=1,NM
      IF(I.EQ.1) GO TO 4
      P(I)=XO*EXO*X**(I-1)*XPOIFC(I,XD)
      GO TO 3
    4 P(I)=XO*EXO*XPOIFC(I,XD)
    3 CONTINUE
      SUM=PO
      DO 6 I=1,NM
    6 SUM=SUM+P(I)
      P(IJ)=1.0-SUM
      RETURN
      END
C     -----------------------------------------------------------------
```

```
      SUBROUTINE DPROB(X0,X,DP00,DP0,DP10,DP1)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DP0(100),DP1(100)
      COMMON XNTOT,DEA(10),NC0,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      XD=X-X0
      EX0=DEXP(-X0)
      EX=DEXP(-X)
      EXD=DEXP(-XD)
      DP00=-X0*EX0
      DP10=0.0
      DO 3 I=1,NM
      IF(I.EQ.1) GO TO 4
      DP0(I)=((1.-X0)*XD+I*X0)*X**(I-1)*EX0*XPOIFC(I,XD)/XD-
     *X0*X**(I-1)*EX/FACT(I-1)/XD
      IF(I.EQ.2) GO TO 7
      DP1(I)=-(X+(I-1)*X0)*X0*X**(I-2)*EX0*XPOIFC(I,XD)/XD+
     *X0*X**(I-1)*EX/FACT(I-1)/XD
      GO TO 3
    4 DP0(I)=((1.-X0)*XD+X0)*EX0*XPOIFC(I,XD)/XD-X0*EX/XD
      DP1(I)=X0*EX0*XPOIFC(I,XD)/XD+X0*EX/XD
      GO TO 3
    7 DP1(I)=-(X+X0)*X0*EX0*XPOIFC(I,XD)/XD+X0*X*EX/XD
    3 CONTINUE
      SUM2=DP00
      SUM3=DP10
      DO 6 I=1,NM
      SUM2=SUM2+DP0(I)
      SUM3=SUM3+DP1(I)
    6 CONTINUE
      DP0(IJ)=-SUM2
      DP1(IJ)=-SUM3
      RETURN
      END
C     ------------------------------------------------------------------
C
C     OBJECTIVE FUNCTION OF METHOD OF MOMENTS
C
      FUNCTION OBJ5(PHI)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION P(100),PHI(10)
      COMMON XNTOT,DEA(10),NC0,IJ,NB,NC(100),ND,NE,NM,NS,M,N,INDEX
      X0=PHI(1)
      X=PHI(2)
      EX0=DEXP(-X0)
      XD=X-X0
      S=0.
      S2=0.
      DO 301 JJ=1,IJ
      S=S+JJ*NC(JJ)
      S2=S2+NC(JJ)*JJ**2
  301 CONTINUE
      FMEAN=X-XD*(1.-EX0)/X0-S/XNTOT
      FVAR=X+(XD*(XD+X)*(1.-EX0)-(XD*(1.-EX0))**2)/X0/X0+2.*X*EX0*XD/
     *X0-S2/XNTOT+(S/XNTOT)**2
      OBJ5=DABS(FMEAN)+DABS(FVAR)
      RETURN
      END
C     ------------------------------------------------------------------
```

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STATISTICAL INFERENCES FOR A PURE BIRTH PROCESS

by

JYH-PING HSU

B. S., National Taiwan University, 1977

M. S., Kansas State University, 1980

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Abstract

Five parameter estimating methods have been used to estimate a simulated 2-parameter Poisson process namely, method of using $E(N_0)$ and $E(S)$, method of $E(N_0)$ and $E(N_1)$, minimum Chi-Square method, maximum likelihood estimate and method of moments. It has been found that despite of its simplicity the method of using $E(N_0)$ and $E(S)$ is in general the most desirable method. The minimum Chi-Square method is also appropriate and could be considered as to obtain comparable results.