THE APPLICATION OF QUEUEING THEORY TO VEHICULAR PARKING

by

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INTRODUCTION

Large sums of money are currently being invested in new facilities and equipment by industries. Much of this vast investment is founded on decisions based on intuition, judgment, and hunches. Substantial improvement in the efficiency and economy obtained with this money can be achieved by employing many scientific techniques and tools that are presently available. One of these techniques - "queueing theory" - can be used to determine the quantity of facilities required for certain types of processes or systems for which output requirements can be specified.

The congested conditions observed at bus stops, market counters, ticket booths, and often at cafeterias are examples of waiting lines. These waiting lines of people are called "queues" in England and from this "queueing theory" derives its name. The terms waiting line theory or queueing theory are used interchangeably.

Though this problem was so obvious at cinema ticket windows, it was not so apparent in industry and machine shops. Theoretical research into the properties of queues began in connection with problems of telephone operation. A. K. Erlang, an engineer with the Copenhagen telephone exchange, analyzed the situation.[1] In the design of automatic telephone exchanges, one had to know the effect of fluctuations of service demand as

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[1]Philip M. Morse, Queues, Inventories and Maintenance, p. 2.

varying numbers of customers began dialing numbers utilizing the automatic equipment in the telephone exchange. Erlang's work began in 1905, and until about 15 years ago, most of the work on the theory of queues was done in connection with telephone problems. Only recently was it realized that the theory has many applications in connection with a wide variety of operations and systems.

This thesis deals with the theory and its application to vehicular traffic. In particular, the subject to be considered is the automobile parking lot.

In the typical problem for which queueing theory is useful, a sequence of units arrives at some facility which services each unit and eventually discharges it. For example, in a Naval Dock Yard, the units arriving for service are the ships, the facility is the dock and the service is the loading or the unloading operations. Another example is the case of machine maintenance where the units are the individual machines, which "arrive" for repair. The service facility is the repair crew, and the service operation is the work performed in getting the machines in working order again.

A waiting line (queue) results from one of two types of conditions:

1. Units that require service (machines, for example) must wait for service because there is shortage of service facilities (insufficient repair crews) to take care of them. The shortage may be due to lack of facilities (too few repair men), or it may be due to lack of proper scheduling (too much time spent on one

machine).

2. The other situation is where the service facility remains idle (repairmen wait for the machines to break down, i.e., remain idle). This idle time may be caused not only by lack of quantity, but also by the nature of machine maintenance. A simple graphical representation of a queueing system is shown in

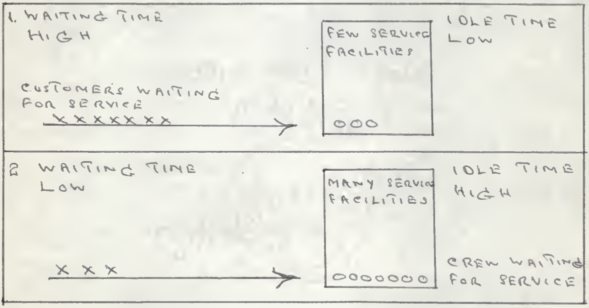

Fig. 1

Fig. 1.[1] In the case of too much demand on the facilities, we may say either that there is an excess of waiting time or that there are not enough service facilities. In the case of too little demand, there is either too much idle facility time or

[1]Thomas L. Saaty, "Resume of Useful Formulas in Queueing Theory," Journal of Operations Research Society of America, Vol. 5, No. 2, April 1957.

too many facilities. One would like to obtain an optimum balance between the costs associated with waiting time and idle time. Using machine maintenance as an example, management must decide how many repairmen there should be in order to have minimum idle time as well as minimum waiting time for a machine.

CHARACTERISTIC OF A QUEUEING PROBLEM

There are four main characteristics which describe a queueing problem.

1. The manner in which units (e.g., customers at a counter, cars at a parking lot, raw materials at a machine center) arrive and become a part of the waiting line. This is the system's "<u>input</u>".

2. The number of service units or <u>stations</u> (e.g., number of toll booths on a turnpike, number of ticket windows at a movie theater, or number of parking places in a lot) operating on the units requiring service. The service policy, which sets the amount of service that can be rendered or is allowed, is a determining factor here. (E.g., a minute car wash, where the service time is constant for all cars, limiting of parking time, etc.)

3. The order in which the units are serviced, which forms the <u>queue</u> <u>discipline</u>.

4. The service provided and its duration: the system's "<u>output</u>". Customers served in a restaurant, trucks loaded, packages wrapped for a shopper, and service rendered to a car at a service station, are typical examples.

One must have a careful account of the following when analyzing any waiting line situation:

(a) The Input Process.

(b) The Service Machanism, or Output Process.

(c) The Queue Discipline.

The Input Process

This is the customer population, more often referred to as "arrivals" in the literature. The two extremes in arrival patterns are the <u>constant</u> and the <u>exponential</u>. With constant arrivals, the customers arrive at exactly time "t" after the arrival of the previous customer. In the exponential case, the customers arrive at "random" intervals, (usually stated as a "Poisson Process"). This Poisson assumption is the simplest hypothesis about the input process and is the most useful in practical applications. Other types of arrival distributions will be discussed briefly at a later point in this thesis.

The Service Mechanism, or Output Process

This is usually referred to as "service time" or, in the case of telephone traffic, as "holding time". It is the length of time required to provide service. Here too, as for the arrival distributions, there are two extreme categories. The <u>constant</u> service rate case may be illustrated by the automatic car wash, where every car comes off the line after it has been in the process for a fixed time. <u>Exponential</u> service would be applicable in situations such as the serviceman's repairing the down machines, where each job requires a different amount of time for completion, so that service time may be considered to vary at random around some mean time.

The Queue Discipline

Queue discipline is the order in which the customers are served, and there are many variations. All customers are classified as either "patient" or "impatient". A machine waiting for service will wait until repaired, and is an example of a "patient" customer. Some human beings also form queues of patient customers. Many human customers, however, are apt to be impatient. A person not finding a parking place will often leave the parking lot; another similar case occurs when a person finds a number of people waiting ahead of him in a barber shop. Such types are referred to as "impatient" customers. Impatient customers may be further divided into two classes: Those who depart immediately (that is, never join the queue), and those who join the queue but become restless and leave the queue when not served. The latter type is known as "reneging".

After determining the customers' patience, four common types of queue discipline may be identified:[1]

1. First come first served.
2. Random.
3. Priority.

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[1]James M. Moore, "To Queue or Not to Queue," The Jour. of Ind. Engr., Vol. XII, No. 2, March-April 1961.

4. Bulk.

"First come first served" is the simplest of the four queue disciplines. The most common examples are those of restaurants and toll booths, where the first customer to arrive will be served first. Throughout the discussions which follow, this type of queue discipline will be assumed.

When the selection of customers for service is random, it is a case of "random" queue discipline. For example, in automatic telephone switchboard operation, there might be many people waiting to place a call, and it is not known which one tried to place his call first. As a line becomes available for a new call, any one of the waiting customers is picked at random.

The "priority" discipline comes in when one type of customer has preference over the other. In the machine repair example, a more expensive machine, or a machine on which labor cost is more, will be repaired before a less important or expensive one.

In case of "bulk" service, the customers arrive or are served in groups. For example, passengers disembarking from an airplane will arrive in a group. This would be considered bulk arrival. On the other hand, some problems involve bulk service but still deal with individual arrivals - such as on an elevator where passengers may arrive individually but are serviced in a group.

ACTIVITIES TO WHICH QUEUEING THEORY HAS BEEN APPLIED[1]

In commerce today, where queueing theory has been applied, many congestion problems have been reduced to a manageable scale. Among the recent applications of queueing theory are the following:

Landing of Aircraft.[2,3] A plane approaching an airport can land on prescribed runways, if there is more than a single runway. If these runways are being used by other aircraft, the planes are "stacked" over the airport at prescribed altitudes until runways become free. A stacked plane is in a queue.

The Scheduling of Patients in Clinics.[4] Patient arrival may be random in the time during which the clinic is open. The holding or the service time required to treat each patient varies from one patient to the next. Therefore it is usually a model represented by Poisson arrivals and exponential service time.

Scheduling of Personnel.[5] Boeing Airplane Company obtained

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[1]Saaty, loc. cit.

[2] E. G. Browen and T. Pearcy, "Delay in Air Traffic Control", Journal of the Royal Aeronautical Society, pp. 251-258, 1941.

[3]T. Pearcy, "Delays in Landing of Air Traffic", Journal of the Royal Aeronautical Society, pp. 799-812, 1948.

[4]L. Bailey, "A Study of Queues and Appointment Systems in Hospital Out-Patient Departments, With Special Reference to Waiting Times," Journal of the Royal Statistical Society, Series B, Vol. 14, p. 185, 1952.

[5]G. Bringham, "On a Congestion Problem in An Aircraft Factory," Journal of Operations Research Society of America, Vol. 3, pp. 412-428, 1955.

the optimum number of clerks to be assigned to tool crib counters in the factory area. The cribs stored a variety of tools required by mechanics in shops and assembly line. The problem resulted from complaints of foremen who felt their mechanics were waiting too long in line. Queueing analysis helped solve the problem.

Telephone Conversation. In the case of a telephone trunking system, calls are initiated by individuals. The frequency of initiation or attempted initiation of calls to a given trunk line may be characterized by a frequency distribution (input distribution). Calls are not identical in length, and so a distribution is again needed to characterize the length of a call or its "holding time". There is a maximum number of calls that may be handled at one time, and there is a fixed number of channels. Because of statistical fluctuations in input and holding times, at times all channels will be occupied and a queue will form. The service system used has been that of random selection for service.

Vehicular Traffic.[1] Though queueing theory has been applied widely and in different situations, in its application to vehicular traffic, the work of L. C. Eddie of New York Port Authority is outstanding. He analyzed traffic delays at toll booths at Port Authority tunnels and bridges. The result of his study was the recommendation of an optimum number and a schedule

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[1] L. C. Eddie, "Traffic Delays at Toll Booths," Journal of Operations Research Society of America, Vol. 2, No. 2, May 1954.

for the toll collectors and the number of toll booths required at any time of day.

Parking Lots. Among the various activities to which the theory is applicable or has been applied are queueing of automobiles at supermarket parking lots, shopping centers' parking areas, and other parking lots, such as those on the campus at Kansas State University. There has, however, been very little published work on the application of the theory to automobile parking lots.

The importance of parking lots must not in any way be underestimated. Theaters, supermarkets, department stores, hotels, and banks all advertise that they have free parking areas for their customers. The reason why so much importance is given to automobile parking is very clear. A large majority of car owners do consider the parking situation before making a selection of a supermarket, theater, hotel, or a bank. The management of these establishments therefore has to consider how much parking space must be provided to prevent disagreeable congestion.

In a parking lot, the arriving units are the automobiles, the service channels are the available parking spaces[1], and the "service process" is the storage of the car (parking) until it is taken out again.

It is well known from experience that cars arriving in such lots do not often join a queue if they find the lot full (case

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[1]Morse, op. cit., p. 3.

of impatient customers). Therefore the service facility (number of parking places) must be adequate enough so that a waiting line or queue seldom forms, and thus few customers are lost. If the number of units arriving at a lot is very large, a balance must be achieved between the cost of service facility (parking area) and the cost of losing a customer.

Morse has developed mathematical models in the case of service channels in parallel, (case of parking lots), and also for the optimizing the number of channels.[1] But in these models, Poisson arrivals and exponential service times have been assumed. Morse points out that in case of parking lots near shopping centers, it is likely that short stays may be prevalent enough to make the exponential distribution a good approximation. This might very well be true, as the arrivals in a parking lot, such as that of a supermarket, are random. This assumption of random arrivals does not hold in case of university parking lots. As it will be shown later, arrivals at such lots are not random.

The construction of models of waiting line processes usually involves relatively complex mathematics, though the models with Poisson arrivals and exponential holding times are considered to be the simplest.[2] In the case of parking lots such as on the Kansas State campus, the situation is severely complicated

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[1] Ibid., p. 30.

[2] C. West Churchman, Russell L. Ackoff, and E. Leonard Arnoff, Introduction to Operations Research, p. 389.

by the fact that we not only have to deal with non-Poisson distribution of arrivals and service times, but also with the case of "impatient customers".

QUEUEING MODELS

There is much in the literature of the past decade about queueing theory. A number of queueing models have been developed and presented by various authors. Figure 2,[1] below, is a graphical summary of the major queueing models developed to date.

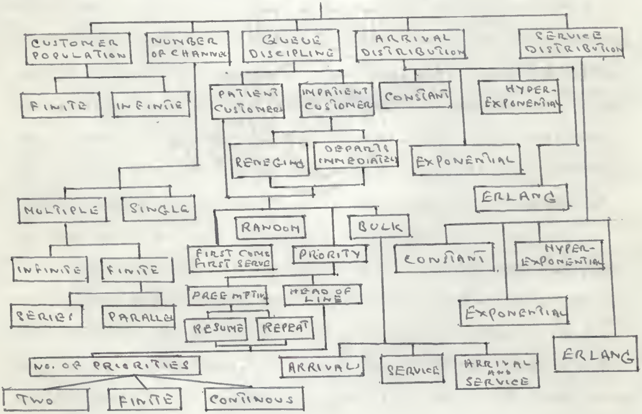

Fig. 2

[1]Moore, op. cit., p. 119.

Explanation of Queueing Solutions

Throughout the discussion to follow, it will be assumed that the units arriving observe the "strict queue discipline", that is, "first come first served". No arriving unit enters the service channel unless and until the channel has finished with the previous unit.

If service is busy, the arriving units wait in a queue, in the order of arrival, until service has finished with all previous arrivals, at which time a unit immediately enters the channel, is serviced, and departs.

The various properties of a waiting line, such as the number in line at any instant or the waiting experienced by a particular arrival, are random variables. The reason that these variables are random rather than functionally dependent on time, is that arrivals are, in general, random events in time, and service times are random variables as well. Thus, in the case of random arrivals and exponential holding times, one is concerned with estimating only the average arrival rate and average service rate.[1]

Actually, arrivals do not often occur at regular intervals in time but tend to be clustered or scattered in some fashion. The "Poisson assumption" specifies the behavior of arrivals, by postulating the existence of a constant " λ " which is independent of time, queue length, or any other random property of the

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[1]Maurice Sasieni, A. Yaspan, and L. Friedman, Operations Research, Methods and Problems, p. 126.

queue such that the probability:

P (an arrival occurs between time t and time $t + \Delta t$) $= \lambda \Delta t$[1]

---------- (1)

If the interval Δt is sufficiently small, a waiting line for which arrivals occur in accordance with equation 1 is called a queue with Poisson arrivals. The <u>mean</u> <u>arrival</u> <u>rate</u> in a waiting line situation is defined as the expected number of arrivals occurring in a time interval of length unity. If arrivals are Poisson, we see from the equation that the expected number of arrivals in a time interval of length 'T' is λT. Setting $T = 1$, it follows that the mean arrival rate for Poisson arrivals is just λ.

The mean arrival rate is a dimensional number, whose units are arrivals per time unit.

In case of service facility, we have the relationship P (a service unit is turned out in interval t to $t + \Delta t$, given that a unit is being serviced at time t) $= \mu t$,[2]

---------- (2)

where μ is a constant. Here also, as in the case of arrival rate λ, it is assumed that the constant of proportionality is independent of time, of queue length, and of any other random characteristic of the waiting line system. The mean servicing rate for a particular station is defined as the conditional expectation of the number of services completed in one time unit.

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[1] <u>Loc. cit.</u>

[2] <u>Ibid.</u>, p. 127.

If servicing times are exponential, it turns out the mean servicing rate μ has the dimensions of services per time unit. The assumption in the case of exponential service rate is analogous to radioactive decay where the chance of survival of an individual nucleus is independent of the length of time it has already survived. In other words, the service performed is independent of time.

Single Station

Consider the problem of determining the probability of a given queue length for the case of a single station for which both input and output are assumed to be random. It is further assumed that the servicing rate is independent of the number of units in the line.

A set of differential equations from which $P_n(t)$, and subsequently n, may be obtained. The equation is formulated by using the fundamental properties of probability.[1]

The equations representing the detailed balancing of transition between states for a statistically steady state are given by,

$$\mu P_1 - \lambda P_o = 0 \qquad (n = 0) \tag{3}$$

$$\mu P_{n+1} + \lambda P_{n-1} - (\lambda + \mu) P_n = 0 \qquad (n > 0) \tag{4}$$

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[1]Churchman, Ackoff, and Arnoff, <u>op. cit.</u>, p. 394.

We have,[1]

$$P_0 = P_0$$

$P_1 = (\frac{\lambda}{\mu})\, P_0$ — From equation 1

$P_2 = (\frac{\lambda}{\mu})^2\, P_0$ — From equation 2, by letting $n = 1$

$$P_n = (\frac{\lambda}{\mu})^n \cdot P_0$$

Summing corresponding members of these equations, we have,[2]

$$P_i = P_0 \sum_0^\infty (\frac{\lambda}{\mu})^n \qquad (5)$$

The sum of probability $= \sum_0^\infty P_i = 1.$

Also by the equation for the sum of an infinite geometric series, we have,[3]

$$\sum_0^\infty (\frac{\lambda}{\mu})^n = \frac{1}{1 - \lambda/\mu} \qquad (6)$$

$$1 = P_0 \left[1 - \frac{1}{\lambda/\mu} \right] \qquad (7)$$

The ratio $\frac{\lambda}{\mu}$ is sometimes called "traffic intensity" or, more often, "utilization factor", and is represented by "ρ". Substituting the value of P in equation 2, we have the probability of a waiting line of length 'n' is given by[4]

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[1] *Ibid.*, p. 396.

[2] *Loc. cit.*

[3] *Ibid.*, p. 397

[4] *Loc. cit.*

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) \quad \text{if } \frac{\lambda}{\mu} < 1 \tag{8}$$

The mean length of the waiting line is given by the equation[1]

$$\bar{n} = \frac{\rho}{1-\rho} \quad \text{if} \quad \frac{\lambda}{\mu} < 1 \tag{9}$$

Multiple Channels

Let us consider the case of a service facility with 'M' equal exponential channels, each of mean service rate 'μ', arranged in parallel. Examples of this nature are restaurants with counter seats and automobile parking facilities.

The equations of detailed balance for steady state operation are given by[2]

$$\mu P_1 - \lambda P_o = 0 \qquad (n = 0) \tag{10}$$

$$(n+1)\mu P_{n+1} + \lambda P_{n-1} - (n\mu + \lambda)\, P_n = 0 \quad (0 < n < M) \tag{11}$$

$$M\mu P_{n+1} + \lambda P_{n-1} - (M\mu + \lambda)\, P_n = 0 \quad (M \leq n) \tag{12}$$

Where 'n' number of units are present in the line. Let 'N' be the maximum number of units allowed. There is a maximum length, equal to N - M, such that arrivals will not join if n = N, but will join if n < N. Equations for n = N is given by,[3]

$$\lambda P_{n-1} - (M\mu + \lambda)\, P_N = 0 \qquad (N \geq M) \tag{13}$$

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[1] Ibid., p. 398.

[2] Morse, op. cit., p. 30.

[3] Loc. cit.

Also we have

$$P_n = \left(\frac{M\rho}{n!}\right)^n P_o \qquad (o \leqslant n \leqslant M) \qquad (14)$$

$$P_n = \left[\frac{M^M}{M!}\rho^n\right] P_o \qquad (M \leqslant n \leqslant N) \qquad (15)$$

$$P_n = \frac{\rho^n}{M!M^{n-M}} \qquad (n \geqslant M) \qquad (16)$$

Where, $\rho = \frac{\lambda}{\mu}$

The utilization factor in this case will be $\frac{\lambda}{M\mu}$.

The above model holds only for the conditions of n, N, and M, specified. If "saturation effects" show up before n = M, another model should be used. This model corresponds to a restaurant operation when it is staffed for all M places, if the service rate per customer is independent of the number of customers seated. The model will also correspond to a parking lot situation, with the restrictions specified above.

The probability of having to wait in the line is given by the formula,[1]

$$P(w) = \left(\frac{\lambda}{\mu}\right)^M \frac{P_o}{M!(1 - \frac{\lambda}{\mu M})}$$

The average time spent waiting in the line is given by the formula,[2]

$$\bar{t}_w = \frac{P_o}{\mu M (M!)[1 - \frac{\lambda}{\mu} M]^2} \left(\frac{\lambda}{\mu}\right)^M$$

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[1]Churchman, Ackoff, and Arnoff, loc. cit.

[2]Ibid., p. 406.

in which P_0 may be determined from the condition $P_n = 1$. It turns out that[1]

$$P_0 = \frac{1}{\sum_{n=0}^{m-1} \left(\frac{\lambda}{\mu}\right)^n / n! + \left[\left[\frac{\lambda}{\mu}\right]^m / m! \left(1 - \frac{\lambda}{\mu m}\right) \right]}$$

Infinite Queues

In the above discussion it was assumed that $\rho < 1$. Thus the equations for the various situations are applicable only when this condition is true. When the maximum queue length is very large the steady state solutions differ radically in character depending on whether ρ is a little less than 1 or a little greater than 1 . In the first case, the probability P_N that a maximum length queue occurs is extremely small, while in the second case it is the largest of all P's. When ρ is less than 1 the mean length is independent of N , if N is very large, when ρ is greater than 1 , the mean length is large and roughly proportional to N .

There are few operational situations which can give rise to very long lines (a ticket window for a popular play, a toll booth on a toll bridge, etc.). In these cases, arriving units are either willing to join, or must join the queue, no matter how long it is. If service rate μ is greater than arrival rate

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[1] Loc. cit.

λ ($\rho < 1$), steady state corresponds to a mean queue length much smaller than N , and the systems' properties are independent of the value of N , as long as it is large enough. On the other hand, when λ is larger than μ, that is $\rho > 1$, the larger N is the less likely we are to find the system in a steady state situation. The steady state solutions for infinite possible queue are valid only for $\rho < 1$, there are no steady state solutions for $\rho > 1$.

DISCUSSION OF THE KANSAS STATE UNIVERSITY PARKING LOT PROBLEM

The purpose of this part of the research was to study the application of "Queueing Theory" to vehicular traffic. As there exists a congestion problem at the Kansas State University parking lots, and because they were conveniently located for such a study, it was decided to examine several of the most used lots to determine whether theory could be applied here and if so, what would be necessary for the solution of the queueing problem involved.

Parking lots such as those on the Kansas State University campus come under the category of "Multiple Channels" facility. Each parking place in the lot represents a single channel. It is a common knowledge, and this was confirmed by questionnaires (see Appendix IV), that cars arriving in a lot do not wait for a parking place if they find the lot full. Therefore, we are dealing with the case of "Impatient Customers".

The customer arrivals or input process in our case, is the number of cars arriving at the entrance of a particular parking lot. The service rate is the duration of parking time of the cars in the lot. A service is started as soon as a car is parked in the parking place in a lot. The service continues as long as the car is parked in that place. The service is completed as soon as the car leaves the lot. Each parking lot (see Appendix III) is considered as a single service facility, with various channels in parallel. If, for instance, a parking lot has 60 parking places, each parking place in that lot is considered to be a channel. In that particular service facility there are 60 channels in parallel. The parking places are considered as channels in parallel due to the fact that service in each channel takes place independent of the other.

The parking lots studied on the campus were the "Union" lots: two student lots, and two faculty-and-staff lots. The object was to study these four services facilities independantly so that the input process and service process could be determined separately. This approach was also necessitated because of the layout of the parking lots, and because of the restrictions placed on the lots. For instance, students' lots are meant for students only, while in faculty-and-staff lots students are not allowed. Visitor parking was neglected. The intention was also to find out whether there was a difference in arrival and service distributions between the two types of lots (student and faculty).

The input to each service facility was obtained with the help of traffic counters, and the service rate of each lot was determined by using the technique of work sampling (see Appendix III for the detailed discussion of the arrival rate and for the service rate determination).

Analysis of the Data

The traffic arrival pattern was analyzed by forming frequency distributions of the number of vehicles arriving in an hour. Observations were formed into one-hour groups, and the frequency of occurrence of each arrival class was computed as a percentage of the total number of intervals observed. These percentages were then plotted against the time intervals, as shown in Fig. 3 through Fig. 13 (see Appendix I).

The data for arrivals of different lots were condensed from data sheets into Tables 1 through 4 (see Appendix II). Adding up the total arrivals for a period and dividing it by the time intervals observed gave us the average arrival rate for that period. For instance, in Table 1, for Lot A_1, looking for the morning period of Monday, we have the arrivals 64, 10, 27, 27, and 8 during the hours 6:45 to 7:44, 7:45 to 8:44, ----------, 10:45 to 11:44. Therefore the total arrivals for the five-hour period is 136 cars. Thus, the average number of cars arriving per hour is $136/5 = 27.2$ cars per hour. The frequency of occurrence for each period will be 64/136, 10/136, and so on.

The data for Monday morning for Lot A_1 are plotted in Fig. 3 and for afternoon in Fig. 5. A word of warning may be in order here. The frequency diagram must be actually in the form of a bar chart, as shown in Fig. 4 for the same data. This figure is realistic as the observations (meter readings) were obtained on the hour. 0, 1, 2, ----- 5, representing 6:45, 7:44, 8:44, ---------- 11:44 for the morning period, or 12:44, 1:44, ------- 4:44 for the afternoon period. The points were joined just to give the reader some concept of how the distribution looks. The frequency distributions obtained from the data have no resemblance to distributions one would expect with pure chance traffic. With the data available presently, it is not possible to know the trend of the curve in between hours, as the shortest interval taken was for one hour. If the interval of one hour, 0 to 1, 1 to 2, ----- , and so on, had been divided into very short time periods of, say, 30 seconds each, we would have obtained a smooth curve. But in the author's opinion it would have made no difference in the conclusions.

By having shorter intervals and a smoother curve, it might have been possible to see whether the arrivals "within one hour" were Poisson or not, but it seems it would not serve any purpose, as it has already been established that arrivals for a whole period of one day do not conform to "Poisson arrivals".

When the chance of occurrence of the next arrival is independent of the time since the last arrival, it is called "Poisson arrival", or exponential arrival. The probability demonstrates that this distribution corresponds to completely random arrivals. But it is common knowledge that cars come in nearly "on the hour", that is, just before the start of next class period. Thus, arrivals of cars are not random in a period of time.

Looking at Tables 1, 2, 3, and 4 for student and faculty lots, we observe that for student lots the heaviest arrival rate is for the first interval 6:44 to 7:45, while in the case of faculty lots it is for the second interval 7:44 to 8:45, and there is a remarkable consistency about these arrivals for the different days of the week. Students arrive earlier to get a parking place as there is a limited parking place for them. As there seems to be enough place for the faculty and staff cars, they do not have to come too early for a place. Thus, it seems that the number of cars arriving at a lot (arrivals) does depend upon time instead of being independent, as should have been in the case of Poisson arrivals. Therefore, in the author's opinion, even an assumption of Poisson input will be unrealistic in this case. The conclusions drawn from the data, that the input process or arrivals on the campus is not Poisson, certainly should not be surprising.

From the Tables 1 through 4, it may be noted that the average arrival rate per hour 'λ' for the different days is independent of the days, that is for MWF group and TTS group. The 'λ' for the afternoons are larger than that of morning periods. This is true for both Lots A_1 and A_2. Also a point of interest is that the 'λ' is directly related to the capacity of the lot.

We have shown that 'λ' is an average figure in case of Poisson arrivals, but as the distribution we got is not Poisson, the parameter 'λ' will not have the same meaning. But it is used here for convenience, instead for the notation, $\bar{X}$.

The difference of the traffic behavior of the student and faculty lots is also of interest. The 'λ' for student lot is much higher than that of faculty. This clearly points to the much heavier student arrival rates than that of faculty. Also, in case of student lots, the heaviest arrival rate is for the first hour period, with the exception of Lot A_1, where the arrivals seem to be heavy near five o'clock also. The explanation for this behavior can be that many students may be coming in for the use of recreational facilities in the Kansas State Union, as well as for the evening meal. Also, since after five o'clock there are no parking restrictions, it seems that everyone would like to park nearest to the Union building.

As mentioned earlier, in the case of the faculty lots, it may be noted that in the mornings the heaviest arrival rate is not for first hour, 6:45 to 7:44, (as was the case with the student lots), but it is for the second hour period, 7:45 to 8:44. This fact was verified from the questionnaire, that students have to be in by 7:30 to get a parking place. 'λ' values for the faculty lot are uniform, the range being $\lambda_H - \lambda_L = 5.4$, while for students the same calculations show a result of 26.

Service Rate

The observations from the data sheets were tabulated as shown in Tables 5 through 8. The reason for the division of each day into morning and afternoon periods was mainly to aid in service rate determination. Cars parked in the mornings leave during the midday break, 12:00 noon to 1:00 p.m. Cars coming back in the afternoon are almost always parked at a different place from their morning space (see Appendix III). Therefore, if the whole day, 8:00 a.m. to 5:00 p.m., had been considered, the value of 'μ' would not have been realistic. By dividing the day into the two periods of morning and afternoon, the maximum service allowed was reduced to four hours for each period. The average parking time in each lot, obtained this way, seems to provide a reasonably realistic picture of the traffic behavior. A car parked in the morning usually stays there until noon, or if it leaves before noon, it rarely comes back in the same period, to the same lot. The percentage of such occurrences is very small and so negligible as not to affect the data. It was found during the collection of the data that more than 85 per cent of all cars leave in the period 12:00 noon to 1:00 p.m. (see Forms 2A and 2B, Appendix III). License plates of the cars are different for the first hour of the afternoon period for the same day, Form 2B).

The service rate of the lots was analyzed in the same manner as that of arrival rates, that is by drawing frequency distribution diagrams. Referring to Table 9, we have in the first column a heading "duration of service" which refers to the time of parking: one hour, two hours, three hours, and four hours. From the above table, we have for Monday morning under the column "no. of cars", the figures 5, 4, 2, and 11. That means five cars were parked for one hour, four cars for two hours, two cars for three hours, and 11 cars for four hours. Therefore, the average parking time for the lot for this day is shown as 2.866 hours. It was assumed here that the minimum service time (parking time) is one hour (see Appendix III).

Observations were formed into hour groups, and the frequency of occurrence of each group was computed as the percentage of the total number of cars. The percentages were then plotted against the service times as shown in Figs. 14 through 17.

Here too, as in the case of frequency distribution diagrams for arrival rates 'λ', the joining of points by straight lines is not intended to show a continuous time distribution. The same arguments hold here as in the case of the arrival rates. It is quite evident from the diagrams that these are far from being exponential distributions. The data analyzed for different days and for different lots show that service distributions are also irregular and do not conform to exponential or random distributions.

Though total parking time (average utilization) for a week of student parking lot is less than that of faculty lots, the difference is not significant, and this shows that students and faculty on the average use a lot for the same duration of time, though we have noted that arrival rates do differ for the two types of lots.

CONCLUSIONS

It has been established that nature of arrivals as well as service distributions regarding the traffic at the parking lots on the campus, are non-Poisson. The mathematical models developed to date in the field of "Queueing Theory" pertain to the following types of distributions, for arrivals and service.

(See Fig. 2.)

1. Constant

2. Exponential

3. Hyper-exponential

4. Erlang

The distribution of arrivals and service of the traffic on the campus does not conform to any of the distributions mentioned. Therefore, the queueing models and formulas discussed in the review of literature are not applicable to the data obtained. Unfortunately, no work has yet appeared discussing the effect upon predicted quantities of departures of model from reality.[1]

Philip M. Morse of M. I. T. pointed out that detailed solutions obtained to date are for cases fed by purely random arrivals from an infinite population. In order to apply the theory to more complicated cases than the ones mentioned, additional investigation and development needs to be done. Morse says,

> To obtain complete solution of these more complicated waiting line problems, will require mathematical ability of a high order. Such solutions will not be achieved in a few months, as casual by-products of work on an immediate practical problem. They probably will only be obtained by using a slower, more fundamental approach, by concentrating on the underlying mathematical relationships, by disregarding for the time being the urgencies and extraneous details of specific applications of the theory. This is the usual way that basic theoretical advances are made in science after all.[2]

---

[1]Thomas L. Saaty, Mathematical Methods of Operations Research, p. 368.

[2]Philip M. Morse, "Where is the Young Blood", Operations Research Society of America, Vol. 2, No. 2, April 1955, p. 35.

The information gained by studying the Kansas State parking situation in the light of queueing theory, certainly seems to bear out what Dr. Morse has said.

GLOSSARY OF SYMBOLS USED

M = Number of service channels.

N = Maximum number allowed in the system.

n = Number of units in the waiting line at time t.

$\bar{n}$ = Mean length of the waiting line.

P = Probability.

$P_n(t)$ = Probability of n units in the queue at time t.

t = Time.

λ = Probability of a new unit entering the line in the time interval t to t+ t, which implies that is mean arrival rate.

μ = Probability that a unit being serviced is completed in the time interval t to t+ t, which implies that is mean service rate.

λ = Mean arrival rate.

$\mu, \bar{x}$ = Mean service rate.

ρ = Utilization factor = .

ACKNOWLEDGMENTS

The writer gratefully acknowledges the invaluable assistance rendered by faculty and staff members in the Department of Industrial Engineering. In particular he would like to thank Dr. Irvin L. Reis, head of the Department of Industrial Engineering, for his suggestions, and directions in pursuing the work presented in this thesis, and Dr. Sammy E. G. Elias, his major professor, for the valuable counsel offered in the construction of the thesis. He would also like to thank Professor J. J. Smaltz for the valuable assistance given in obtaining the information about the parking arrangements of the Kansas State University, and also in obtaining the data on the campus.

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APPENDIX I: FIGURES

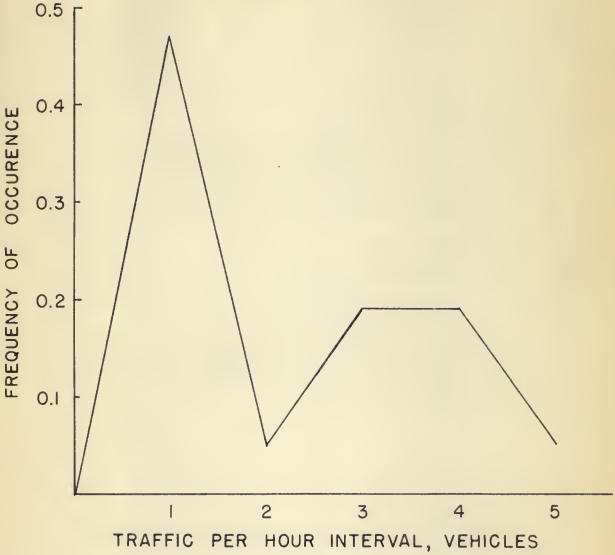

FIG. 3 ARRIVAL RATE,
MONDAY MORNING,
LOT A_1.

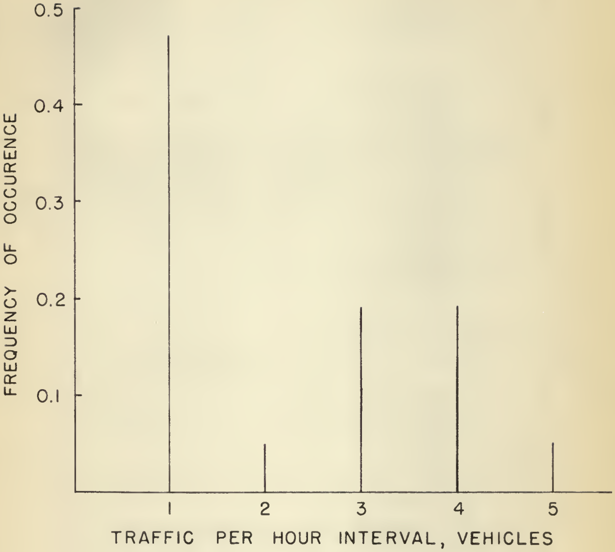

FIG. 4 ARRIVAL RATE,
MONDAY MORNING,
LOT A_1.

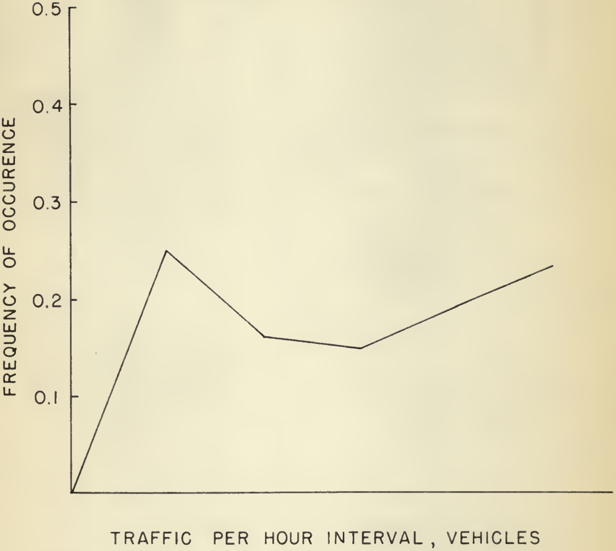

FIG. 5 ARRIVAL RATE,
MONDAY AFTERNOON,
LOT A_1.

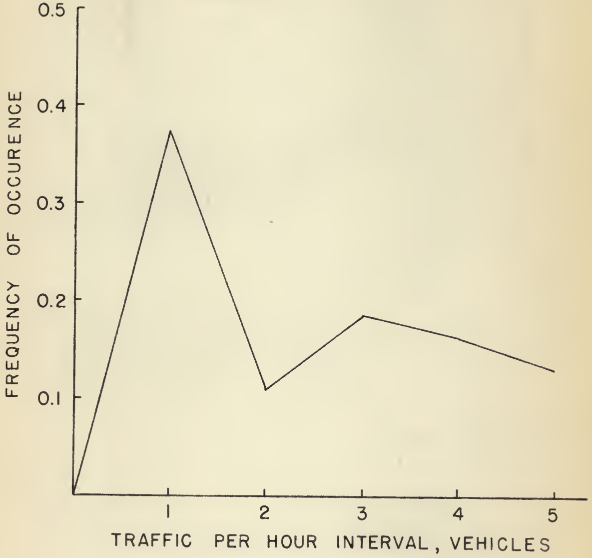

FIG. 6 ARRIVAL RATE,
TUESDAY MORNING,
LOT A_1.

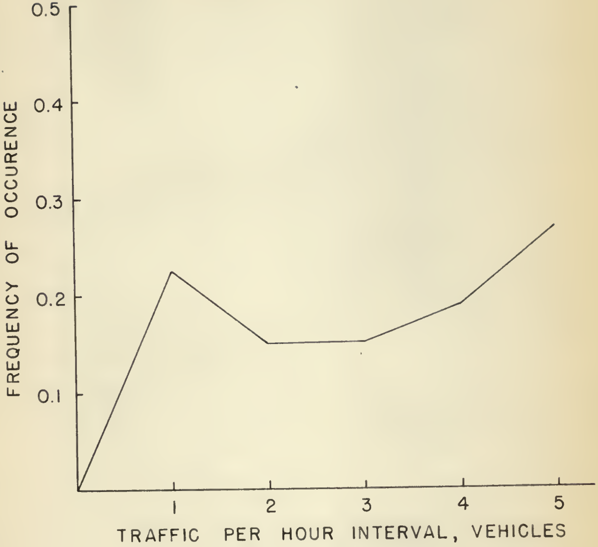

FIG. 7 ARRIVAL RATE,
TUESDAY AFTERNOON,
LOT A_1.

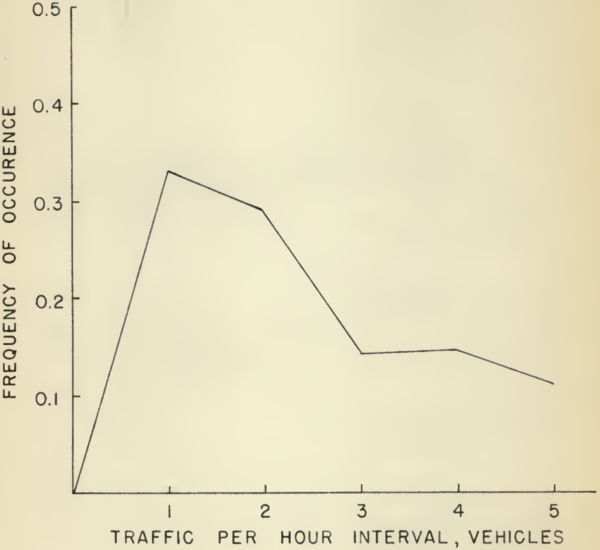

FIG. 8 ARRIVAL RATE,
THURSDAY MORNING,
LOT A_1.

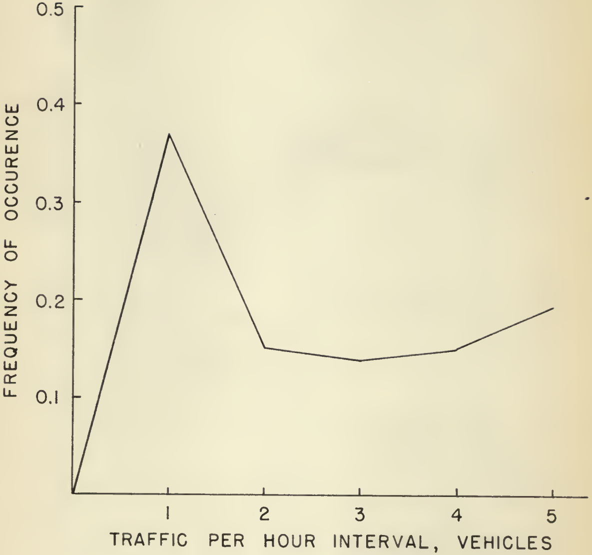

FIG. 9 ARRIVAL RATE,
THURSDAY AFTERNOON,
LOT A_2.

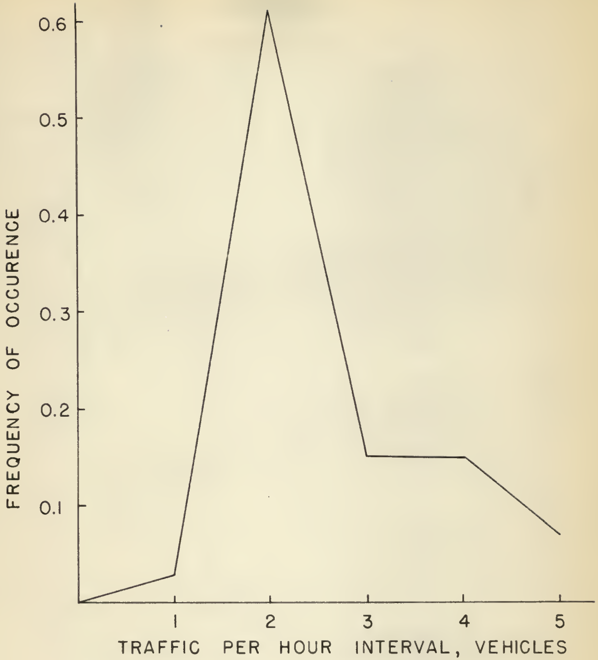

FIG. 10 ARRIVAL RATE,
WEDNESDAY MORNING,
LOT B_1.

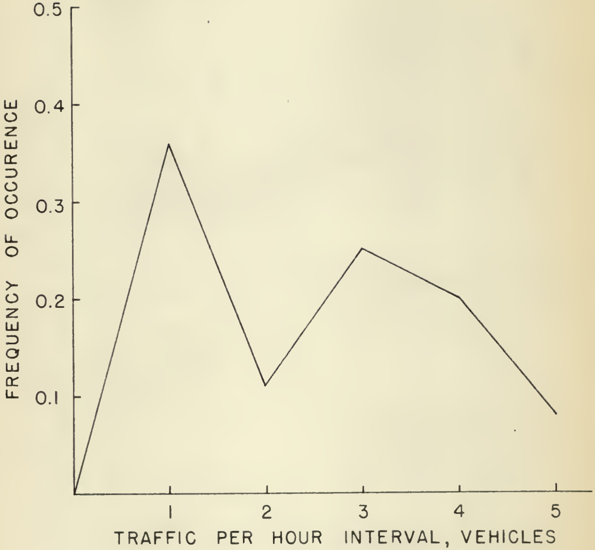

FIG. 11 ARRIVAL RATE, WEDNESDAY AFTERNOON, LOT B_1.

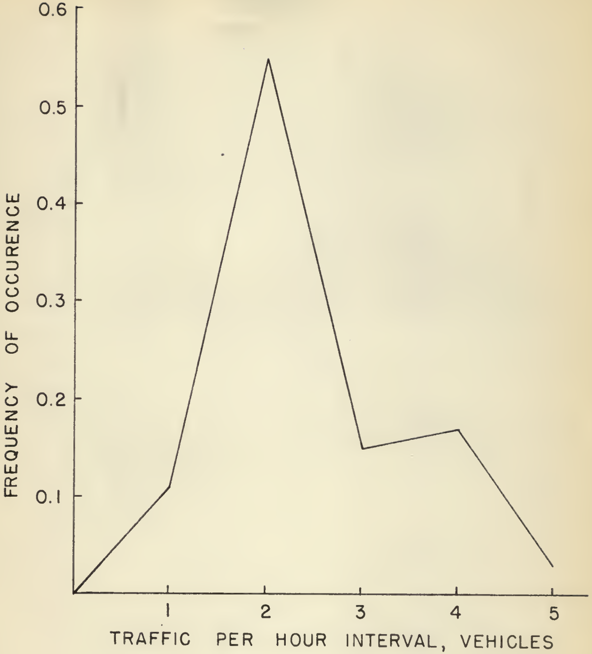

FIG. 12 ARRIVAL RATE,
FRIDAY MORNING,
LOT B_2.

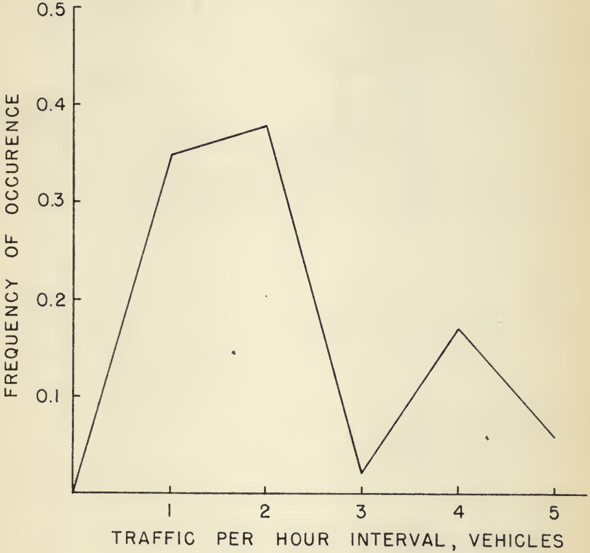

FIG. 13 ARRIVAL RATE,
FRIDAY AFTERNOON,
LOT B_2.

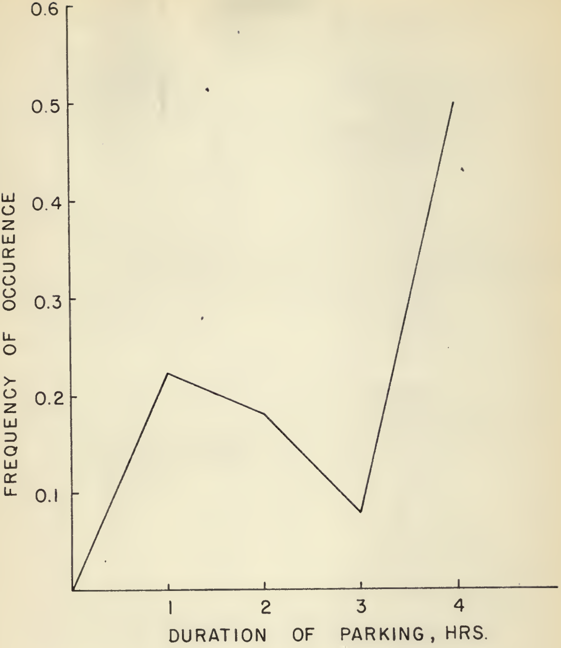

FIG: 14 SERVICE RATE,
MONDAY MORNING,
LOT A_1.

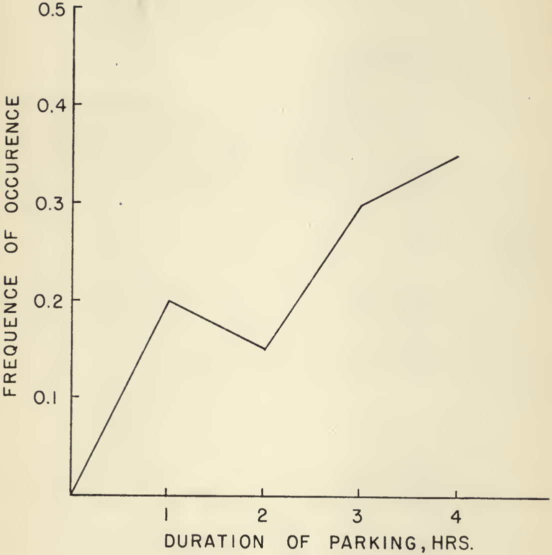

FIG. 15 SERVICE RATE, TUESDAY AFTERNOON, LOT A_2.

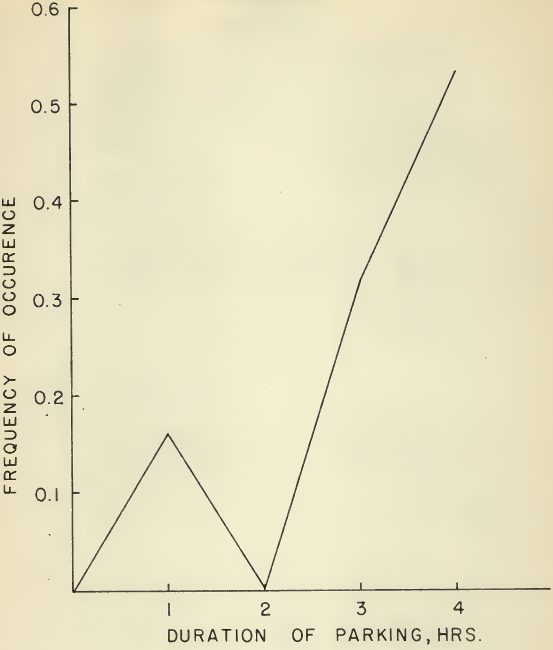

FIG. 16 SERVICE RATE, WEDNESDAY MORNING, LOT B_1.

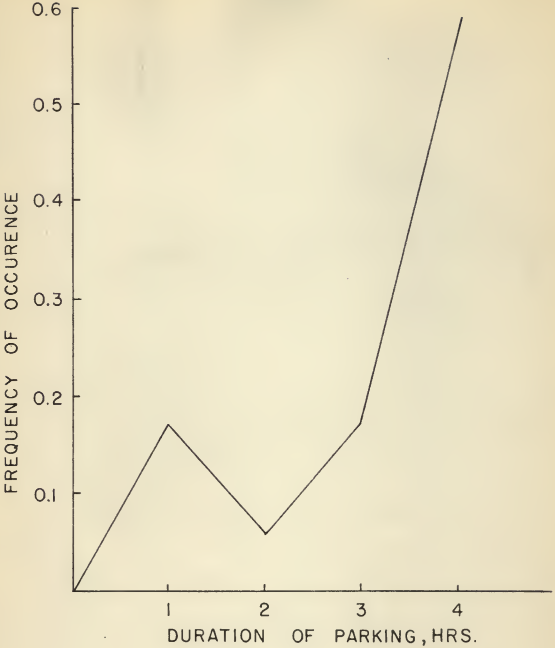

FIG. 17 SERVICE RATE,
THURSDAY MORNING,
LOT B_2.

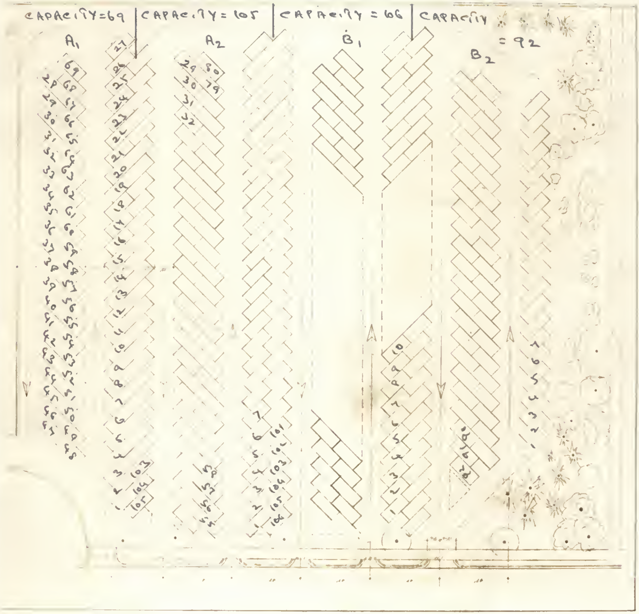

Fig. 18

APPENDIX II: TABLES

Table 1. Arrival rate (number of cars arriving per hour), ' λ ' for Lot A_1 (student).

Hour	Monday No.	Monday %	Wednesday No.	Wednesday %	Friday No.	Friday %	Tuesday No.	Tuesday %	Thursday No.	Thursday %	Saturday No.	Saturday %
Morning												
6:45- 7:44	64	.470	70	.436	64	.329	66	.372	60	.363	88	.437
7:45- 8:44	10	.073	31	.191	33	.170	20	.112	22	.133	22	.109
8:45- 9:44	27	.198	32	.197	19	.097	33	.186	27	.163	21	.104
9:45-10:44	27	.198	19	.117	45	.231	30	.169	32	.193	49	.243
10:45-11:44	8	.058	10	.061	33	.170	28	.158	24	.145	21	.104
Total	136		162		194		177		165		201	
λ	27.2		32.4		38.8		35.4		31.0		40.0	
Average for the five days = 32.96 cars per hour												
Afternoon												
11:45-12:44	62	.285	3	.015	98	.291	65	.229	73	.282		
12:45- 1:44	35	.161	9	.046	40	.119	44	.155	47	.182		
1:45- 2:44	31	.142	30	.153	54	.160	44	.155	18	.069		
2:45- 3.44	43	.198	53	.271	56	.166	54	.190	9	.034		
3:45- 4:44	46	.211	100	.512	88	.261	76	.268	111	.430		
Total	217		195		336		283		258			
λ	43.4		39.0		67.2		56.6		51.6			
Average for the five days = 51.56 cars per hour												

Table 2. Arrival rate (number of cars arriving per hour), ' λ ' for Lot A_2 (student).

Hour	Monday No.	Monday %	Wednesday No.	Wednesday %	Friday No.	Friday %	Tuesday No.	Tuesday %	Thursday No.	Thursday %	Saturday No.	Saturday %
					Morning							
6:45- 7:44	70	.294	60	.236	127	.460	108	.450	80	.327	24	.082
7:45- 8:44	92	.387	94	.370	53	.192	25	.104	70	.286	79	.271
8:45- 9:44	31	.130	48	.189	28	.101	42	.175	34	.139	85	.291
9:45-10:44	41	.172	31	.122	44	.159	37	.154	35	.143	63	.216
10:45-11:44	4	.017	21	.083	24	.087	28	.117	26	.106	41	.140
Total	238		254		276		240		245		292	
λ	47.6		50.8		55.2		48.0		49.0		58.4	
	Average for the five days = 50.12 cars per hour											
					Afternoon							
11:45-12:44	122	.482	157	.428	111	.453	105	.358	128	.369		
12:45- 1:44	33	.130	37	.101	44	.180	36	.123	52	.150		
1:45- 2:44	36	.142	56	.153	30	.122	44	.150	47	.135		
2:45- 3:44	39	.154	44	.120	26	.106	50	.171	52	.150		
3:45- 4:44	23	.091	73	.199	34	.139	58	.198	68	.196		
Total	253		367		245		293		347			
λ	50.6		73.2		49.0		58.6		69.4			
	Average for the five days = 60.16 cars per hour											

Table 3. Arrival rate (number of cars arriving per hour), 'λ' for Lot B_1 (faculty and staff.

Hour	Monday No.	Monday %	Wednesday No.	Wednesday %	Friday No.	Friday %	Tuesday No.	Tuesday %	Thursday No.	Thursday %	Saturday No.	Saturday %
					Morning							
6:45- 7:44	4	.027	4	.026	34	.213	37	.258	17	.113	21	.159
7:45- 8:44	94	.648	93	.607	80	.503	59	.412	89	.593	36	.272
8:45- 9:44	19	.131	23	.150	14	.088	25	.174	19	.126	32	.242
9:45-10:44	27	.186	23	.150	19	.119	14	.097	14	.093	16	.121
10:45-11:44	1	.006	10	.065	12	.075	8	.055	11	.073	27	.204
Total	145		153		159		143		150		132	
λ	29.0		30.6		31.8		28.6		30.0		26.4	
	Average for the five days = 30.0 cars per hour											
					Afternoon							
11:45-12:44	55	.352	73	.356	72	.500	51	.354	56	.354		
12:45- 1:44	48	.307	23	.112	36	.250	46	.319	38	.240		
1:45- 2:44	26	.166	51	.248	25	.173	23	.159	21	.132		
2:45- 3:44	18	.115	41	.200	6	.041	15	.104	17	.107		
3:45- 4:44	9	.057	17	.082	5	.034	9	.062	26	.164		
Total	156		205		144		144		158			
λ	31.2		41.0		28.8		28.8		31.6			
	Average for the five days = 32.28 cars per hour											

Table 4. Arrival rate (number of cars arriving per hour), 'λ' for Lot B_2 (faculty and staff.

Hour	Monday No.	Monday %	Wednesday No.	Wednesday %	Friday No.	Friday %	Tuesday No.	Tuesday %	Thursday No.	Thursday %	Saturday No.	Saturday %
					Morning							
6:45- 7:44	-	-	9	.060	15	.108	23	.167	15	.111	5	.051
7:45- 8:44	76	.563	81	.544	76	.547	49	.355	76	.563	39	.402
8:45- 9:44	28	.207	31	.208	21	.151	45	.326	35	.259	30	.309
9:45-10:44	18	.133	26	.174	23	.165	13	.094	8	.059	19	.196
10:45-11:44	13	.096	2	.013	4	.029	8	.058	1	.010	4	.041
Total	135		149		139		138		135		97	
λ	27.0		29.8		27.8		27.6		27.0		19.4	
Average for the five days = 27.84 cars per hour												
					Afternoon							
11:45-12:44	34	.276	41	.281	40	.351	32	.352	28	.257		
12:45- 1:44	58	.472	45	.308	43	.377	38	.333	54	.495		
1:45- 2:44	15	.122	36	.247	5	.044	5	.055	11	.101		
2:45- 3:44	10	.081	15	.103	19	.167	13	.114	3	.028		
3:45- 4:44	6	.049	9	.062	7	.061	3	.026	13	.119		
Total	123		146		114		91		109			
λ	24.6		29.2		22.8		18.2		21.8			
Average for the five days = 23.32 cars per hour												

Table 5. Service time (parking time), $\bar{X}$ hours, for Lot A_1 (student).

Duration of service	Monday No. of cars	Monday Freq. %	Wednesday No. of cars	Wednesday Freq. %	Friday No. of cars	Friday Freq. %	Tuesday No. of cars	Tuesday Freq. %	Thursday No. of cars	Thursday Freq. %
Morning										
1	5	.227	12	.444	3	.157	5	.294	-	-
2	4	.181	1	.037	1	.052	3	.176	8	.400
3	2	.090	6	.222	2	.105	5	.294	1	.050
4	11	.500	8	.296	13	.684	9	.529	11	.550
Total	22		27		19		17		20	
$\bar{X}$	2.86 hrs.		2.36 hrs.		3.31 hrs.		2.82 hrs.		3.15 hrs.	
Average for the five days = 2.91 hours										
Afternoon										
1	13	.448	17	.515	5	.238	12	.444	17	.500
2	5	.172	7	.212	3	.143	2	.074	11	.324
3	5	.172	7	.212	2	.095	7	.259	2	.059
4	6	.206	2	.061	11	.524	6	.222	4	.118
Total	29		33		21		27		34	
$\bar{X}$	2.14 hrs.		1.82 hrs.		2.91 hrs.		2.25 hrs.		1.80 hrs.	
Average for the five days = 2.18 hours										

Table 6. Service time (parking time), $\overline{X}$ hours, for Lot A_2 (student).

Duration of service	Monday No. of cars	Monday Freq. %	Wednesday No. of cars	Wednesday Freq. %	Friday No. of cars	Friday Freq. %	Tuesday No. of cars	Tuesday Freq. %	Thursday No. of cars	Thursday Freq. %
Morning										
1	9	.346	5	.227	4	.174	12	.444	10	.370
2	5	.192	4	.182	7	.304	4	.148	5	.185
3	4	.154	1	.045	3	.134	4	.148	4	.148
4	8	.308	12	.545	9	.391	7	.259	8	.296
Total	26		22		23		27		27	
$\overline{X}$	2.42 hrs.		2.91 hrs.		2.74 hrs.		2.20 hrs.		2.38 hrs.	
Average for the five days = 2.53 hours										
Afternoon										
1	10	.385	4	.182	5	.227	4	.200	14	.500
2	6	.231	6	.273	3	.136	3	.150	5	.179
3	3	.115	7	.318	11	.500	6	.300	5	.179
4	7	.269	5	.227	3	.136	7	.350	6	.214
Total	26		22		22		20		28	
$\overline{X}$	2.27 hrs.		2.59 hrs.		2.55 hrs.		2.80 hrs.		2.24 hrs.	
Average for five days = 2.49 hours										

Table 7. Service time (parking time), $\bar{X}$ hours, for Lot B_1 (faculty and staff).

Duration of service	Monday		Wednesday		Friday		Tuesday		Thursday	
	No. of cars	Freq. %	No. of cars	Freq. %	No. of cars	Freq. %	No. of cars	Freq. %	No. of cars	Freq. %
					Morning					
1	6	.300	3	.158	2	.100	1	.059	2	.111
2	2	.100	-	-	5	.250	2	.118	1	.056
3	5	.250	6	.316	3	.150	3	.176	4	.222
4	7	.350	10	.526	10	.500	11	.647	11	.611
Total	20		19		20		17		18	
$\bar{X}$	2.65 hrs.		3.21 hrs.		3.05 hrs.		3.40 hrs.		3.33 hrs.	
			Average for the five days = 3.128 hours							
					Afternoon					
1	3	.136	4	.190	5	.238	3	.158	3	.150
2	7	.318	4	.190	3	.143	-	-	4	.200
3	7	.318	4	.190	3	.143	10	.526	8	.400
4	5	.227	9	.429	10	.476	6	.316	5	.250
Total	22		21		21		19		20	
$\bar{X}$	2.62 hrs.		2.76 hrs.		2.86 hrs.		3.00 hrs.		2.75 hrs.	
			Average for the five days = 2.79 hours							

Table 8. Service time (parking time), $\overline{X}$ hours, for Lot B_2 (faculty and staff).

Duration of service	Monday No. of cars	Monday Freq. %	Wednesday No. of cars	Wednesday Freq. %	Friday No. of cars	Friday Freq. %	Tuesday No. of cars	Tuesday Freq. %	Thursday No. of cars	Thursday Freq. %
Morning										
1	8	.333	2	.111	15	.517	1	.063	3	.176
2	6	.250	4	.222	4	.138	6	.375	1	.059
3	2	.083	3	.167	2	.069	1	.063	3	.176
4	8	.333	9	.500	8	.276	8	.500	10	.588
Total	24		18		29		16		17	
$\overline{X}$	2.42 hrs.		3.05 hrs.		2.11 hrs.		3.00 hrs.		3.18 hrs.	
Average for the five days = 2.75 hours										
Afternoon										
1	2	.118	1	.056	6	.261	1	.067	2	.111
2	1	.059	3	.167	5	.217	3	.200	2	.111
3	3	.176	7	.389	4	.174	6	.400	2	.111
4	11	.647	7	.389	8	.348	5	.333	12	.667
Total	17		18		23		15		18	
$\overline{X}$	3.35 hrs.		3.11 hrs.		2.61 hrs.		3.66 hrs.		3.32 hrs.	
Average for the five days = 3.21 hours										

APPENDIX III

FORM I

DATA SHEET

ARRIVAL RATES

LOCATION Lot B_1

DATE 3-6-61

DAY Monday

TIME	METER READING BEGINNING	METER READING ENDING	TOTAL NO. OF CARS
6:45 - 7:44	34538	34542	4
7:45 - 8:44		34636	94
8:45 - 9:44		34655	19
9:45 - 10:44		34682	27
10:45 - 11:44		34683	1
11:45 - 12:44		34738	55
12:45 - 1:44		34786	48
1:45 - 2:44		34812	26
2:45 - 3:44		34830	18
3:45 - 4:44		34839	9
4:45 - 5:44			

DATA SHEET

PARKING TIME

LOCATION Lot A_1

DATE 4-19-61

DAY Wednesday A.M.

CODE ✔ = Car Still There X = Left the Lot
P = Car Parked L = Left

RANDOM PLACES	8:15		9:15		10:15		11:15		12:15	
	P	L	P	L	P	L	P	L	P	L
5	1446									
10	688-173									
11	2929									
15	662									
22	7837									
23	3251									
25	7693									
32	266									
37	9510									
39	5509									
40	84-34									
41	3624									
42	299									
46	1096									
50	6849									
56	6352									

DATA SHEET

PARKING TIME

LOCATION Lot A_1

DATE 4-19-61

DAY Wednesday P.M.

CODE ✔ = Car Still There X = Left the Lot
P = Car Parked L = Left

RANDOM PLACES	1:15		2:15		3:15		4:15		5:15	
	P	L	P	L	P	L	P	L	P	L
5	2971									
10	9342									
11	6254									
15	10600				8322					
22	8842		8003							
23	712									
25	3933									
32	925-263									
37	9510		6193							
39	5157									
40	84520									
41	3624				2219					
42	299									
46	1096									
50	562									
56	1318									

APPENDIX III (cont'd)

For convenience, the student portion of the lot south of the Union was designated by the letters A_1 and A_2 and the faculty portion was designated by the letters B_1 and B_2 as shown in plate, Fig. 18.

Determination of Arrival Rate 'λ'

Traffic counters were set up at entrances of each lot to record the number of cars coming into the lots. A data sheet, Form 1, was prepared to record the meter readings.

The day was divided into ten hourly intervals, 6:45 a.m. to 7:44 a.m., 7:45 a.m. to 8:44 a.m., and similarly through 4:44 p.m. The first meter reading was obtained at 6:45; this was the "meter reading beginning". After that, meter readings were recorded at intervals of 60 minutes, that is at 7:44, 8:44, ------ and 4:44. Successive subtractions gave the number of cars arriving during these intervals. The results of each data sheet were then summarized in Tables 1, 2, 3, and 4 for the different lots.

Determination of Service Rate 'μ'

To determine the service rate of a lot, the method of work sampling was used. Work sampling is a measurement technique for the quantitative analysis, in terms of time, of the state of activity of men, machines, and/or systems.

The underlying idea of work sampling is that by making numerous "spot" checks or observations of the activity being considered, conclusions may be drawn about the manner in which the entire time system is spent. That is, by making observations recording a man or machine as idle, working, or in some other state, conclusions may be drawn about the distribution of these states. The percentages found in the recorded observations reflect, to a known degree of accuracy, the percentages present in the system as a whole.

To determine the service rate of the lots, it was necessary to know how long the various cars were parked in one place. To do this, one would have had to observe a car from the time it came into a lot until it left. To do this for 360 parking places and for the whole day would have been a tremendous problem without the work sampling technique.

The first problem in applying work sampling was to determine a sample size. To arrive at a reasonable figure, the questionnaires (see Appendix IV) returned from Student Union lots were first analyzed. From the answers to question one, it was found that the average parking time (mean value) for the whole Student Union lot, faculty as well as students, is 3.26 hours, with a standard deviation of 1.44 hours. Then calculations were made using the formula $\bar{X} \pm S_D t_{.05}/\sqrt{N}$, where $\bar{X}$ is the mean value of the parking time, S_D is the standard deviation, N is the sample size, and $t_{.05}$ is the table value, for 95 per cent confidence limits.[1] By taking different values of N and in each case

[1] George W. Snedecor, Statistical Methods, p. 46, Table 2.7.1.

finding the 95 per cent confidence interval (the interval estimate), a feasible sample size of 16 was selected. The confidence interval in this case was $2.65 < \bar{X} < 3.87$ hours. An interval of this sort means that when actual work sampling is performed, 95 per cent of the time the mean value of parking times can be expected to be covered by the interval 2.65 hours to 3.87 hours; that is, the actual value will lie between these two limits 19 times out of 20, in the long run. The parking places in each of the lots were then numbered as shown in plate, Fig. 18. With the help of a random digit table, a random sample of parking places was obtained for each lot. Different random numbers were used for different lots and for different days, to assure an unbiased sample.

A data sheet, as shown in Forms 2A and 2B, was employed to record the observations. The random sample parking places obtained were recorded on the data sheet in serial order, to help facilitate observations. A parking lot was then visited at 8:15, and the license plate numbers of cars parked in the places picked in the sample were recorded under the column 'P'. The column was left blank if no car was at that particular place number. The lot was then visited with an interval of one hour, that is, at 9:15, 10:15, and so on, till 4:15. A check mark (✓) was made in the column 'P', under a particular time column, if the car was still parked at the place. But if the car had left the place and if the place were vacant, an 'X' mark was made under the column marked 'L'. If another car occupied the same place, its license number was recorded in the sub-column 'P' of the particular time column. The minimum time a car was parked was assumed to be one hour. If a car was parked at a place at 8:15, 9:15, and was not there at 10:15, it was assumed that the car was parked for two hours. Since the car was parked at 9:15, it is reasonable to assume that it left at the end of the hour. Also, since classes are closed at 50 minutes past the hour, cars usually leave at the end of the hour. The data for the service rate for the different lots were then arranged in Tables 5, 6, 7, and 8.

APPENDIX IV

FORM 3

QUESTIONNAIRE

This information is required in an attempt to solve the parking problem on the campus. Please drop this letter in the Mail box (Campus Mail), or leave it at the Student Union information desk.

Your cooperation will be much appreciated.

1. How long did you park in this parking place today?

   From: ____________

   To: ____________

2. How many times a week do you park in this lot?

   Forenoon ________ Afternoon ________

3. Did you find this parking place in your first attempt to park?

   Yes ________ No ________

4. Do you make more than one round of a lot or do you try another one if space is not available?

   More than one round _____ Try another lot _____

5. Suggestions:

Thank you.

NOTE: <u>DISREGARD</u> <u>IF</u> <u>PREVIOUSLY</u> <u>ANSWERED</u>

APPENDIX IV (cont'd)

A questionnaire, as shown in Form 3, was prepared. The purpose of this questionnaire was multifold. First and foremost, the intention was to let the faculty, staff, and students know what was going on, in order to obtain their cooperation and to eliminate suspicion. An article was also published in the Collegian (daily newspaper of Kansas State University) prior to the appearance of these questionnaires, in which their purpose and use were outlined.

Another important reason for these questionnaires was to get opinions and suggestions from the people who use the parking lots. To know how everyone, faculty as well as students, feels about the present parking situation and to find out what improvements there should be and how. It was very encouraging to note that many good suggestions were made.

A final reason was to determine the general parking behavior. To know whether we are dealing with patient customers or impatient ones, as discussed earlier in the main part of the thesis. Also, it was hoped that an approximation of the average parking time could be found, so that a suitable sample size could be taken for the work sampling discussed in Appendix III. It was also desired to learn how many times a week a student or a member of the faculty uses a parking lot. In other words, the object was to find what percentage of people use cars six times a week, five times a week, and so on, and to know whether there is any relation between the time schedule of classes and the use of cars on the campus.

These questionnaires were put on the windshields of cars parked on the various lots of the campus. The questionnaires were put at random times of each day on the various lots, to cover as many cars as possible. Questionnaires were marked A, B, etc. according to lot designations. This was done so as to make sure from which lot a questionnaire was returned.

Approximately 3,500 questionnaires were distributed during a period of two weeks. A total of 616 were returned, that is about 17.6 per cent. This was lower than the expectations of about 25 per cent.

Only the questionnaires for the Union lots, students and faculty, were analyzed as for the questions one through four. This was due to the fact that the Union lots were selected to test the application of Queueing Theory to Vehicular Parking on the Kansas State University campus, and thus these were for immediate use.

It was found that in case of faculty, 40 per cent use the lot five times a week and about 45.5 per cent use it six times a week. In other words, about 85 per cent of faculty members use cars throughout the week to come to the campus. Thirty-five per cent of the student questionnaires analyzed said they use the lot five times a week and 28.8 per cent use the lot six times a week. That is, 64.3 per cent use their cars almost all round the week. Ten per cent use cars four times a week and 17.7 per cent three times a week. This shows that there is not much difference between the faculty and students as far as the use of cars on the campus is concerned. It also points out the fact that class scheduling, MWF or TTS, has not much effect on the use of cars. Students having cars use them more or less every day to come to the campus.

In the case of students, 62.7 per cent replied that they try another lot if space is not available, and 37.3 per cent said that they make more than one round, against 72 per cent and 28 per cent in the case of faculty and staff. This shows that in both cases we are dealing with impatient customers, as the majority of car drivers do not wait for a parking place to become available.

Of the 616 questionnaires returned, about 15 per cent made comments or suggestions; this does not include absurd suggestions, uncalled for comments, and complaints. It is interesting to note that some faculty members and a few students do feel that in reality there is no parking problem at Kansas State University, as compared with some other campuses across the country, where either no cars are allowed on campus, or only a limited number is allowed to operate. One must realize, however, that there may not be a parking problem right now, but no one can doubt that there is going to be one in the very near future. Only last semester (Spring, 1961), a parking lot west of the Veterinary Medicine building, with a capacity of 119, was removed to make way for a new building. Also, a part of the Union lot will be taken away to make room for the extension of the Student Union. It is very clear that with the present trend of building construction, parking lots have to be moved further away, and some alternatives must be found. One student aptly remarked, "Let us face the fact that the cars are here to stay and we must do something to park them."

It is not possible to list here all genuine and worth-while comments and suggestions; therefore, only a few which are considered to be of general interest are listed. The answers are divided into two categories, one given by the faculty and staff, and the other by the student body.

Suggestions and Comments from Students

1. Do not issue parking permits to students living two or three blocks from campus.
2. More space for student body, as there are often empty spaces in all faculty and staff lots.
3. The traffic department is more than happy to take $3.00 as a fee, and nothing is said about the difficulty of finding a place. I feel that they are taking money under a misrepresentation of fact. Why sell parking permits for more cars than you have parking space? Parking permits only for seniors and juniors should be issued.
4. The Student Union lot should be one lot predominently for the students.
5. Since this is the Student Union parking lot, I do not think the faculty should have such a large portion of it.
6. Make parking for small cars also, two small cars can be parked in place of one big car.

7. Install parking meters in the Union lot and make it a general parking area.
8. Move tennis courts and football practice field and make parking lots.
9. Permits only to those living more than four or five blocks from campus.
10. Don't allow sophomores to drive cars on the campus.
11. Make Union lot an automatic parking facility, with a parking charge of 25 cents.
12. Put in parking meters with no discrimination between faculty and students. This will provide additional source of revenue, or limit parking permits on a distance basis among students as well as faculty.

Suggestions and Comments from Faculty and Staff

1. Allot space for students and faculty on basis of distance from campus. At Pennsylvania State this system is used whereby anyone living closer than a mile has to walk. Faculty has to use car pools, a certain number of parking spaces allowed per department.
2. Limit parking permits to persons living more than three blocks from campus.
3. Encourage the use of bicycles.
4. Issue no parking permits to those living in student housing.
5. Make three kinds of parking stickers:

A. Faculty - white $5.00 fee
(Authorized to park anywhere)

B. Student - purple and white $2.00 fee
(Authorized to park anywhere, issued only to those living more than one mile)

C. Student - purple $1.00 fee
(Not authorized to park between 8:00 and 5:00, and issued to those living within one mile)

6. Make a distinction between faculty and staff, separate lots for staff, or more rigid requirements for holders of "Staff" stickers.

7. Do not issue staff stickers to secretaries having less than three years working experience.

8. Prohibit all freshmen and sophomores from having or driving cars on the campus.

9. Student parking must be operated on "pay as you park" plan. Automobiles are more of a luxury than a necessity in case of students and they should definitely pay for their parking. Automatic gates should be provided in case of student parking lots, where entrance should be by a deposit of ten cents. Also, only students with parking permits (obtained by a payment of $3.00, and excluding freshmen) should be allowed in these automatic parking facilities.

10. (a) Parking on campus should be prohibited for all students, except for physically handicapped.

(b) Reserved parking stalls should be provided for all faculty members (instructors to professors) who drive regularly and are willing to pay a fee for this privilege, $9.00 per year. This fee would also provide a nice income needed for improvement.

(c) Parking lot classifications should include: (1) Faculty, (2) Visitors and Staff, and (3) Graduate Students.

11. All faculty members should have reserved parking spaces. Other staff members should have reserved lots in which visitors can park. Students should <u>not</u> be permitted to park cars on campus unless they live an inordinate distance from campus.
12. Prohibit issuance of staff stickers to those whose husbands (or wives) are students. These should have student permits. Staff permit holders seem to take up a lot of faculty parking space.
13. Each permanent faculty member should have a reserved parking space in a lot nearest to his office.
14. Too many students use the staff parking lots simply because their wives have part time jobs. Unless these wives work full time, they should not be issued staff stickers.
15. Full time faculty and staff members might be given reserved spaces if they drive more than six blocks to work.
16. By more efficient marking of certain parking lots, better utilization of available space could be made.
17. Limit parking to faculty, staff, and to those students who live a certain distance from campus. Students living within a six block radius would be denied parking permits.
18. Many students spend more time hunting for a parking place than it would have taken them to walk from their rooming

places and end up west of the stadium. It is ridiculous to give parking permits to those who live within two or three blocks of the campus, except physically incapacitated persons.

19. Kansas State University has no parking problem and will not have one until West Stadium lot is full.

THE APPLICATION OF QUEUEING THEORY TO VEHICULAR PARKING

by

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Queues - or Waiting Lines, as they are commonly called - are everyday phenomena. The congested conditions observed at bus stops, market counters, ticket booths, and restaurants, are a few examples of waiting lines known to nearly everyone. A similar situation exists in many places in industry, although until very recently not much attention was paid to it by the engineering profession nor by management. One of the most usual examples of industrial waiting lines is found where one or more machines is "waiting" to be repaired while the repairmen are doing something else.

A. K. Erlang, an engineer with the Copenhagen telephone exchange, is usually credited with doing the first theoretical research into the properties of queues. Although Erlang's work began in 1905, only during the past 15 years has it been realized that the theory has many industrial applications. Among the recent uses of Queueing Theory are problem solutions in Aircraft Landing Fields, Patient Scheduling in Clinics, Scheduling Personnel, and Vehicular Traffic Models.

This thesis deals with the theory and its application to vehicular traffic, in particular with the parking of automobiles. For the sake of convenience, the four Student Union parking lots at Kansas State University were taken as a sample of campus parking behavior to study.

The main characteristics which describe a queueing problem are the following:

1. The Input Process
2. The Service Mechanism

3. The Queue Discipline

The Input Process is referred to as "arrivals" in most of the literature - e.g., the number of people arriving at a ticket window. The Service Mechanism is represented by the service performed, e.g., the sale of the ticket. The manner in which the service is performed, with special reference to time, is called the Queue Discipline. That is, in the above example, the first person at the ticket window is served or attended to first.

The mathematical models developed to date in the field of Queueing Theory pertains to the following types of distributions for arrivals and service:

1. Constant
2. Exponential
3. Hyperexponential
4. Erlang

The most common and simplest hypothesis about input and service mechanism is to assume the exponential distribution. In this case, the customers or arrivals arrive at random intervals, and the service time varies at random around some mean value.

This research determined that neither the distribution of arrivals nor the distribution of service for the parking of campus traffic, conformed to any of the common distributions for which theoretical solutions have been devised.