REACTIVITY DETERMINATIONS IN
THE K.S.U. GRAPHITE SUB-
CRITICAL ASSEMBLY

by

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Nomenclature of Terms

a	Extrapolation dimension in the x direction, cm.
a_1	Physical radius of a control rod, cm.
a_o	Physical radius of the control rod, cm.
a_n	Othogonality constants.
Amn	Amplitude coefficient for the mn mode.
b	Extrapolation dimension in the y direction, cm.
B^2	Buckling, cm^{-2}.
B_g^2	Geometric buckling, cm^{-2}.
B_m^2	Material buckling, cm^{-2}.
c	Extrapolated height of a reactor, cm.
C_E	End correction factor.
C_H	Harmonic and end correction factor.
d	Linear extrapolation distance, cm.
d_o	Linear extrapolation distance into the rod, cm.
D	Thermal diffusion coefficient.
f	Thermal utilization factor.
J	Neutron current, $n/sec-cm^2$ (n = neutrons).
Jmn	The mn modal current, $n/sec-cm^2$.
k_e	Effective multiplication factor.
k_∞	Infinite multiplication factor
$\ell*$	Thermal neutron lifetime, sec.
L	Thermal diffusion length, cm.
M^2	Migration area (= $L^2 + \tau$), cm^2.
M	Matrix operator.

n	Neutron density, n/cm^3.
p	Resonance escape probability.
P	Poison matrix operator.
r_1	Outer radius of a control annulus, cm.
r_o	Inner radius of a control annulus, cm.
R_c	Extrapolated critical reactor radius, cm.
R_o	Equivalent subcritical reactor radius, cm.
S	Thermal neutron source term, $n/sec\text{-}cm^3$.
S_{mn}	m, n modal source constant.
v	Average neutron velocity, cm/sec.
α^2	Radial component of the buckling (cylindrical geometry), cm^{-2}.
α_m^2	x component of the buckling, cm^{-2}.
β	Effective delay neutron fraction.
β_n^2	y component of the buckling, cm^{-2}.
$\gamma_{m,n}$	m, n modal separation constant for the z direction, cm^{-1}.
ϵ	Spacing between data points.
η	Average number of fast fission neutrons emitted as a result of the capture of one thermal neutron in the fuel material.
λ	Average decay constant for delay neutrons.
λ_{tr}	Transport mean free path.
ω	Reciprocal reactor period, sec^{-1}.
μ^2	Major buckling (two group), cm^{-2}.
υ^2	Minor buckling (two group), cm^{-2}.
Σ_a	Macroscopic absorption cross section, cm^{-1}.
Σ_R	Macroscopic slowing down cross section, cm^{-1}.
τ	Fermi age, cm^2.

ϕ Neutron flux, $n/sec\text{-}cm^2$.

ϕ_1 Fast group neutron flux, $n/sec\text{-}cm^2$.

ϕ_2 Thermal group neutron flux, $n/sec\text{-}cm^2$.

∇^2 Laplacian operator.

$S_e^2 (b)$ Error variance of the slope.

$S_e^2 (Yi)$ Error variance of the predicted least squares value of Yi.

$\delta (x_i, y_i)$ Dirac delta function.

Δk_e Change in the effective multiplication factor.

ϕ_r^+ Adjoint vector flux set.

ϕ_r' Perturbed vector flux set.

$\mathcal{L}$ Non-leakage probability.

1.0 INTRODUCTION

The worth of a control rod in a given reactor system can be determined experimentally by two methods. These experimental methods are 1) a static method and 2) a kinetic method.

Several references (7,9,11,14,17,23) discuss the theory of calculating the worth of a control rod for a given reactor system by the static method. The kinetic method is generally used to determine control rod worth in a critical reactor system. There are many references (2,4,7,8,10,11,13,14,16,18,23) which discuss the experimental and theoretical procedures used to find the worth of a control rod in a critical reactor system by the kinetic method.

Very little experimental work has been performed to determine control rod worth in a subcritical reactor system. It is the object of this paper to present a study of the various mathematical models which were used to calculate the control rod worth of a cadmium rod in the K.S.U. graphite subcritical reactor.

Methods used to calculate the reactivity worth of a centrally located poison rod were the following:

1. The effective multiplication factor, k_e, of the K.S.U. graphite pile was calculated from experimental data by the method presented by Kaiser (12). The effective multiplication factor of the pile with the poison rod was calculated by finding the theoretical increase in the geometric buckling due to the presence of the poison rod.

2. The method which Wood (25) presented was revised and extended to calculate the worth of a control rod in an

exponential assembly. This was done by experimentally measuring the inverse relaxation lengths with and without the poison rod present. The worth of the rod was calculated from this change in inverse relaxation lengths.

3. A method presented by Murray and Niestlie (18) was revised and extended to include a two group analysis for calculating the worth of a control rod in an exponential assembly. The values calculated for the infinite multiplication factor, k_∞ , by method one were used in these calculations.

4. One group and two group perturbation calculations were made to give an approximation to the reactivity worth of the control rod. In order to better approximate the reactivity worth of a control rod calculated by perturbation theory, the perturbed flux was estimated more accurately by a variational method.

5. The theoretical approximation equations presented by Glasstone and Edlund (7) were used to calculate the reactivity worth of the rod. These calculations were based on one group, corrected one group, and two group theories.

A kinetic study was attempted. Several hundred rod drops were performed. The data were much too random for any clear cut conclusions. A one delay group analog solution to the kinetics equations was attempted for this work. However, problems, such as defining a true source condition, arose which led this author to include the results for only illustrative purposes.

2.0 THEORY

2.1 One Group Diffusion Models Used to Calculate the Reactivity Worth of a Control Rod in a Subcritical Reactor System

2.1.1 <u>A Model Using Experimental Data to Find the Effective Multiplication Factor of a Subcritical Reactor with No Control Rod.</u> Thermal neutron diffusion theory has been studied by many investigators (4,5,6, 7,12,14,17,21,23). The following paragraphs will serve as an outline to the basic theory of neutron diffusion in a multiplying medium with special emphasis on the assumptions involved in this theory.

The rate of change in neutron density in a volume element of a multiplying medium is the result of (a) the net flow of neutrons through the boundary of the element, (b) the number of neutrons absorbed by the medium per unit time, and (c) the production of neutrons by sources within the element. The neutron current, J, defined as the number of neutrons per second flowing through a unit area normal to the direction of flow, can be expressed, according to Fick's law, as

$$J = -D(r) \text{ grad } \phi(r,t) . \tag{1}$$

$D(r)$ is the neutron diffusion coefficient and $\phi(r)$ is the neutron flux which is equal to the average neutron velocity, v, times the neutron density, n. Then the net flow of neutrons into a volume element can be written in terms of the current as

$$- \text{div } J = \text{div } (D(r) \text{ grad } \phi(r,t)). \tag{2}$$

When the diffusion coefficient, $D(r)$, is independent of position, it is also independent of the operator, div. The leakage term can then be written as $D\nabla^2\phi(r,t)$ where $\nabla^2\phi$ is equal to div (grad $\phi(r,t)$). The

neutron balance on the element of volume becomes

$$D\nabla^2\phi(r,t) - \Sigma_a\phi(r,t) + k_\infty\Sigma_a\phi(r,t) + S = \partial n(r,t)/\partial t. \qquad (3)$$

In general only steady state diffusion will be considered in this work. In this case Eq. (3) becomes

$$D\nabla^2\phi(r,t) - \Sigma_a\phi(r,t) + k_\infty\Sigma_a\phi(r,t) + S = 0 . \qquad (4)$$

Eq. (4) will be solved subject to a point source boundary condition. Dividing Eq. (4) by D and defining B_m^2 as $\Sigma_a(k_\infty - 1)/D$ results in the equation

$$\nabla^2\phi(r) + B_m^2\phi(r) = 0 . \qquad (5)$$

Σ_a is the averaged thermal neutron absorbance cross section and k_∞ is the infinite multiplication factor.

The validity of elementary diffusion theory depends on several important considerations which are listed below.

1. There are no collisions between neutrons, which means that each neutron diffuses independently of all other neutrons. The importance of this assumption results in a convenient mathematical simplification discussed by Weinberg and Wigner (23) and Hughes (11). This makes superposition of solutions possible.

2. The neutrons diffuse with a constant (average) energy and no energy is gained or lost (on the average) in a collision with a nucleus. This assumption is supported by Hughes (11) who states that experimental evidence indicates that the Maxwellian distribution is maintained reasonably well throughout the diffusion process, thus, allowing the diffusion equation to be energy independent.

3. The flux is a slowly varying function throughout the pile with no sharp dips or spikes. This assumption is essential to the validity

of Fick's law and means that diffusion theory is not applicable

in close proximity to concentrated sources, absorbers, or the

boundaries of the diffusing medium. It is also necessary that the

medium be only slightly absorbing, since a high absorption cross-

section would cause the neutron density to vary substantially with-

in one mean-free path.

4.   The neutron scattering is spherically symmetric or isotropic,

allowing the velocity vector to be treated as a scalar in the neutron

balance, Eq. (3). Since scattering is not isotropic near sources,

boundaries, and absorbers, this assumption necessitates restrictions

similar to those stated in condition (3).

5.   The diffusion coefficient is independent of position. This

allows D to be extracted from the term for divergence of the current

in Eq. (2), leaving $D \nabla^2 \phi(r,t)$ for div(Dgrad ($\phi(r,t)$)). Eq. (6) is

derived in Appendix (A). The result obtained for rectangular

geometry is

$$\phi(x,y,z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos(m\pi x/a)\cos(n\pi y/b) \sinh \gamma_{mn}(c-z) \quad . \quad (6)$$

The m and n are both odd, and a, b, and c are the extrapolated

dimensions of the pile in the x, y, and z directions. The variables

were assumed separable and the auxiliary separation constants

equation was found to be

$$B_m^2 = \alpha_m^2 + \beta_n^2 - \gamma_{mn}^2 \tag{7}$$

where

$$\alpha_m = m\pi/a \text{ and } \beta_n = n\pi/b \quad .$$

In order to determine the quantity A_{mn}, a source boundary condition

is needed. The source boundary condition used in this work consisted of four point thermal sources, one located at each of the positions $x=\pm x_0$ and $y=\pm y_0$ in the source layer in the z dimension. The source condition was approximated by expanding in a series of cosine functions such that $S\,\delta(x,y)$ at z=o was written as

$$S\,\delta(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} S_{mn} \cos(m\pi x/a)\cos(n\pi y/b) \tag{8}$$

where m and n were both odd.

The quantity S_{mn} was regarded as the sources for each mn mode of the flux. S_{mn} was determined by virtue of the orthogonality property of the cosines. Each member of Eq. (7) was multiplied by $\cos(l\pi x/a)$ and $\cos(h\pi y/b)$ and integrated over the interval of othogonality, from -a/2 to a/2 and -b/2 to b/2. From the property of the Dirac delta function, the left member of Eq. (8) reduced to $S\cos(m\pi x_i/a)\cos(n\pi y_i/b)$. By virtue of the orthogonal properties, the right member of Eq. (8) was found to be S_{mn} times one-half the range of orthogonality in each of the x and y directions. The result was determined to be

$$S_{mn} = \sum_{i=1}^{4} (4S/ab)\cos(m\pi x_i/a)\cos(n\pi y_i/b) \ . \tag{9}$$

The origin was located in the source plane so that only positive values in the z direction were considered. The current density in any one mode was assumed to be equal to only one half of the total source neutrons emitted in that mode. Using Fick's law and the expression for thermal flux which is given by Eq. (1), the mn modal current density was found to be

$$J_{mn} = -D\left[\partial\phi_{mn}/\partial z\right]_{z=o} D\,\gamma_{mn} A_{mn}\cos(m\pi x/a)\cos(n\pi y/b)\cosh\gamma_{mn}c=\tfrac{1}{2}S_{mn}. \tag{10}$$

Substituting from Eq. (9) and solving for A_{mn}, yields Eq. (11).

$$A_{mn} = \sum_{i=1}^{4} 2S \cos(m\pi x_i/a)\cos(n\pi y_i/b) \;/\; ab\, D\, \gamma_{mn} \cosh \gamma_{mn}\, c. \qquad (11)$$

Substituting this result back into Eq. (8), the expression for total thermal neutron flux was found to be

$$\phi(x,y,z) = \sum_{i=1}^{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[2S \cos(m\pi x_i/a)\cos(n\pi y_i/b) \;/\; ab\, D\, \gamma_{mn} \right]$$

$$\times \left[\cos(m\pi x/a)\cos(n\pi y/b)\sinh\gamma_{mn}(c-z) \;/\; \cosh\gamma_{mn}c \right], \qquad (12)$$

where m and n are both odd.

Expressing the hyperbolic functions in terms of exponentials and rearranging Eq. (11) gives

$$\phi(x,y,z) = \sum_{i=1}^{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[2S \cos(m\pi x_i/a)\cos(n\pi y_i/b) \;/\; ab\, D\, \gamma_{mn} \cos(m\pi x/a) \right]$$

$$\times \left\{ \cos(n\pi y/b)\exp(-\gamma_{mn}z)\left[1-\exp{-2\gamma_{mn}(c-z)}\right] \;/\; \left[1+\exp(-2\gamma_{mn}c)\right] \right\}. \qquad (13)$$

In order to determine γ_{11} easily, this equation was rearranged again in order to express the flux simply in terms of an exponential decay in the z direction.

$$\phi(x,y,z) = C_E\, C_H \exp(-\gamma_{11}z) \qquad (14)$$

where

$$C_E = \left[1-\exp{-2\gamma_{11}(c-z)}\right] / \left[1+\exp(-2\gamma_{11}c)\right]$$

and

$$C_H = 1 + \sum_{i=1}^{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \gamma_{11}\cos(m\pi x/a)\cos(n\pi y/b)\cos(m\pi x_i/a)\cos(n\pi y_i/b)$$

$$\times \exp(-\gamma_{mn}z)\left[1-\exp(-2\gamma_{mn}(c-z))\right] \;/\; C_E\,\gamma_{mn}\cos(\pi x/a)\cos(\pi y/b)\cos(\pi x_i/a)$$

$$\times \cos(\pi y_i/b)\exp(-\gamma_{11}z)\left[1+\exp(-2\gamma_{mn}c)\right].$$

The m and n are odd but not simultaneously equal to unity. C_E is called the end correction factor and C_H is called the composite end and harmonic correction factor.

The iteration concept described by Kaiser (12) was used in an effort to calculate γ_{11}, the inverse relaxation length. The following was the procedure used.

1. Assuming $C_E C_H = 1.0$, a least squares analysis was performed on the vertical traverse data taken at $x = y = 0$. The value thus obtained was used as an initial estimate for γ_{11}.

2. The values of C_E and C_H were calculated using this initial estimate of γ_{11}.

3. The modified count rates were then calculated by dividing the original count rates by $C_E C_H$.

4. A least squares analysis was then performed on the modified count rates thus determining an improved estimate of γ_{11}.

5. The new value for γ_{11} was compared with the previous value. When the change in the value of γ_{11} between successive iterations is considered negligible, the iteration process is complete. The final value of γ_{11} was used in the calculation of the material buckling and the effective multiplication factor. An IBM 1620 program designed to find γ_{11} by this method is discussed in Appendix (E).

If the rectangular parallelepiped reactor were critical, with no external source, the boundary conditions impressed on the solution for ϕ (x,y,z) would be that the flux be zero at all extrapolated boundaries.

Thus, for a critical reactor at steady state, Eq. (6) becomes

$$\phi(x,y,z) = A \cos(\pi x/a)\cos(\pi y/b)\cos(\pi z/c) \qquad (15)$$

where a,b, and c are the extrapolated x, y, and z dimensions of the critical assembly. Only one mode was considered because (1) criticality can only be achieved in the first mode, and (2) after criticality has been achieved, the higher modes decay out exponentially. The auxiliary separation constants equation for the critical solution is

$$B_g^2 = (\pi/a)^2 + (\pi/b)^2 + (\pi/c)^2 \quad . \qquad (16)$$

Eq. (16) is the definition for geometric buckling in a rectangular parallelepiped reactor. Therefore, the condition for criticality is

$$B_m^2 = B_g^2 = B^2 . \qquad (17)$$

The effective multiplication factor, k_e, is defined using the Fermi slowing down model by Glasstone and Edlund (7) as

$$k_e = k_\infty \ \exp(-B_g^2 \tau)/[1 + L^2 B_g^2] \qquad (18)$$

where the condition for criticality is $k_e = 1$. Since criticality has been assumed, Eq. (18) can be rearranged to yield k_∞ , the infinite multiplication factor. Equating $k_e = 1$, $B_g^2 = B_m^2 = B^2$ and rearranging Eq. (18) becomes

$$k_\infty = (1 + L^2 B^2) \exp(B^2\tau) . \qquad (19)$$

The best estimate of γ_{11}, found by the previously described iterative technique, is used in Eq. (7) to find B_m^2 . This value is then used in

Eq. (19) to find k_∞. The infinite multiplication factor and the geometric buckling for the subcritical system under experimental consideration, found from Eq. (16), are used in Eq. (18) to find k_e.

2.1.2 A Definition of the Extrapolation Distances of a Subcritical Rectangular Parallelepiped Reactor. Kaiser (12) described an experimental method to determine the effective pile size of a subcritical rectangular parallelepiped reactor system. It was found that the data were much too random for definite conclusions. For this reason, he assumed that the linear extrapolation distance (0.71 λ_{tt}) could be used for determining the effective size in the x, y, and z directions. The quantity λ_{tt} represents the transport mean free path of the reactor core medium.

A study of the extrapolation distance in the z direction was not made by Kaiser (12). Therefore, a method of improving on the estimate of the extrapolation distance in the z direction was sought and finally the following technique for determining this distance was used. This method involved a two parameter iterative procedure.

Three count rate measurements, equally spaced in the z direction and taken close to the top of the pile, were used. These points were given z positions of z, z + ϵ, and z + 2ϵ, where ϵ was the spacing between each measured count rate. These count rates and the extrapolated height, c, were related by Glasstone and Edlund (7) by the equation

$$\tanh \gamma_{11}(c-z-\epsilon) = (\phi_1 + \phi_3) / (\phi_1 - \phi_3) \tanh \gamma_{11} \qquad .(20)$$

Eq. (20) was incorporated with the iterative procedure for finding γ_{11}. An initial guess for c was used in the iterative technique to

compute γ_{11}. This γ_{11} and the newly generated flux values were used to calculate an improved value for c by using Eq. (20). This c was then used to compute a corrected γ_{11} by the iterative procedure. This new value of γ_{11} was used in Eq. (20). A new value for c was determined. This value for c was then used to find a new value for γ_{11}. When the change in the values for γ_{11} and c between successive iterations was considered negligible, the iteration process was stopped.

2.1.3 A One Group Calculational Model for Finding the Effective Multiplication Factor of a Subcritical Reactor with a Control Rod. To find the change in k_e due to the insertion of a cylindrical control rod in a rectangular parallelepiped subcritical system, the rectangular parallelepiped subcritical system is approximated by a cylindrical system which has a k_e equal to the k_e for the rectangular parallelpiped system. Since k_∞, L^2, and τ are equal for both subcritical systems, the geometric bucklings must be equal. In cylindrical geometry, the geometric buckling is given by a similar auxiliary separation constants equation as Eq. (16),

$$B_g^2 = \alpha^2 + (\pi/c)^2 \qquad (21)$$

where $\alpha = (2.4048/R_0)$ for the cylindrical reactor with no control rod, and R_0 is the extrapolated radius of the reactor system. Equating the right members of Eq. (16) and (21), the equivalent radius for the cylindrical system can be found and is given as

$$R_0 = (2.4048) \sqrt{(\pi/a)^2 + (\pi/b)^2} \ . \qquad (22)$$

In cylindrical geometry, the radial component of the solution to

Eq. (5) can be written as

$$R(r) = A\,J_o(\alpha r) + B\,Y_o(\alpha r) \qquad (23)$$

where J_o and Y_o are Bessel functions of the first and second kind of order zero.

For a system with no control rod, Eq. (23) is subject to the following boundary conditions

      i). the flux must be finite and non-negative

      ii). $R(r = R_o) = 0$

Applying the first condition to Eq. (23), the constant, B, is found to be equal to zero. The application of the second condition yields a definition for α. The quantity α is equal to $(2.4048/R_o)$. Therefore, the auxiliary separation constants equation for a subcritical system in cylindrical geometry can be written as

$$B_m^2 = \alpha^2 - \gamma^2 , \qquad (24)$$

where γ is the inverse relaxation length.

However, if a control rod is inserted, the necessary boundary conditions are

      i). $\left[R(r = a) \,/\, R'(r = a) \right] = d$

      ii). $R(r = R_o) = 0,$

where a_o is the physical boundary of the control rod and d_o is linear extrapolation distance into the rod. If these two boundary conditions are applied to Eq. (23) and the result rearranged, the following equation is found.

$$J_0(\alpha_1 R_0) / Y_0(\alpha_1 R_0) = \left[J_0(\alpha_1 a_0) + \alpha_1 d_0 J_1(\alpha_1 a_0) \right] /$$
$$\left[Y_0(\alpha_1 a_0) + \alpha_1 d_0 Y_1(\alpha_1 a_0) \right] \qquad (25)$$

where α_1 is the new radial component of the buckling for the sub-critical system with the control rod inserted.

Eq. (25) can be solved for α_1. This value of α_1 is then used in Eq. (21) to determine the new geometric buckling of the subcritical system with the control rod inserted. Using this geometric buckling and the one determined for the system without the control rod from Eq. (18), the change in k_e due to the control rod can be calculated.

2.1.4 A Model Using Experimental Data to Find the Effective Multiplication Factor of a Subcritical Reactor with a Control Rod. A method to predict the worth of a control rod in a critical reactor by using data from an exponential assembly was developed by Wood (25). The following method was based on a steady-state flux condition in a sub-critical reactor system with an external source condition. A cylindrical exponential assembly with a central control rod was considered. For the assembly without the rod, the material buckling, B_m^2, was given by the separation constants equation

$$B_m^2 = \alpha_0^2 - \gamma_0^2 \qquad (28)$$

where α_0 is equal to 2.4048/R_0 for a cylindrical, steady-state reactor with no control rod, γ_0 is the inverse relaxation length, and R_0 is the extrapolated radius of the reactor.

The control rod was treated as an internal boundary when it was inserted, hence, the material buckling of the assembly was the same with

or without the rod. With the control rod inserted, the radial leakage was increased both inward toward the rod and outward through the boundary. Due to this increased leakage, the radial component of the geometric buckling was changed. Therefore, from Eq. (28), the relaxation length must also change. The material buckling of the assembly in terms of quantities defined with the control rod present becomes

$$B_m^2 = \alpha^2 - \gamma^2 , \tag{29}$$

where α is the new radial component of the buckling and γ is the new inverse relaxation length. The linear extrapolation distance at the outer boundary is assumed to be unchanged when the control is inserted.

If γ_o and γ are calculated by using the iterative technique described before, α can be calculated by using Eq. (28) and (29).

The calculated value for α can then be used to find the change in k_e in a manner as described below. In cylindrical geometry and with no control rod, the geometric buckling is given as

$$B_{g_1}^2 = \alpha_o^2 + (\pi/c)^2 . \tag{30}$$

However, if a control rod is inserted, the geometric buckling becomes

$$B_{g_2}^2 = \alpha^2 + (\pi/c)^2 \tag{31}$$

where α is the same quantity which appears in Eq. (29). Employing the same definition for the effective multiplication factor, k_e, as was used previously, the change in k_e due to the insertion of the rod is

$$\Delta k_e = k_\infty \left[\exp(-B^2_{g_2} \tau) / (1 + L^2 B^2_{g_2}) - \exp(-B^2_{g_1} \tau) / (1 + L^2 B^2_{g_1}) \right]$$

(32)

where k_∞ is determined by the one group iterative method described previously.

In brief review, this method consists of the following main points.

1. Determine γ_o and γ from the vertical traverse data taken without and with the control rod inserted.

2. The material buckling, B^2_m, was determined from Eq. (28).

3. The change in the radial component of the buckling, α^2, was then determined from Eq. (29).

4. Eq's. (30) and (31) were then used to find the geometric bucklings of the subcritical reactior system without and with the control rod inserted.

5. These geometric bucklings, $B^2_{g_1}$ and $B^2_{g_2}$, were then used in Eq. (32) to find the change in the effective multiplication factor, k_e, due to the presence of the control rod.

2.1.5 <u>A Method of Extending the Results of Reactivity Measurements of a Control Rod in a Subcritical Reactor to a Critical Reactor of the Same Core Material</u>. It was stated before that in cylindrical geometry the radial component of the general solution to Eq. (5) could be written as

$$R(r) = A J_o(\alpha r) + B Y_o(\alpha r) .$$

(33)

Also the boundary conditions, which are necessary when the control rod is inserted, were stated before as

i). $\left[R(r) / R'(r) \right]_{r=a_1} = d$

ii). $R(r=R_o) = 0$.

Eq. (29), however, states that the radial component of the buckling is B_o. Applying the above boundary conditions to the above solution and rearranging slightly, an expression for d_o is derived.

$$d_o = \left[Y_o(\alpha R_o) J_o(\alpha a_1) - Y_o(\alpha a_1) J_o(\alpha R_o) \right] \ /$$

$$\alpha \left[Y_1(\alpha a_1) J_o(\alpha R_o) - Y_o(\alpha R_o) J_1(\alpha a_1) \right] . \qquad (34)$$

Assuming that the linear thermal extrapolation distance into the control rod is unchanged for a full-sized reactor, Eq. (34) can be applied to the full-sized reactor. A measure of the worth of a control rod in a critical reactor is then computed by finding the change in bucklings.

If an identical control rod were inserted in the critical reactor, the radius of the critical reactor, R_c, should be inserted in boundary condition ii) for R_o and as a consequence, R_c can be substituted into Eq. (34). Therefore, Eq. (34) becomes

$$d = \left[Y_o(BR_c) J_o(Ba_1) - Y_o(Ba_1) J_o(BR_c) \right] \ /$$
$$B \left[Y_1(Ba_1) J_o(BR_c) - Y_o(BR_c) J_1(Ba_1) \right] \qquad (35)$$

B^2 is the new radial component of the buckling. Using the value obtained for d from Eq. (34), Eq. (35) can be solved for B. Thus, the change in the buckling due to the insertion of the control rod in a critical reactor is

$$\Delta B^2 = B^2 - (2.405/R_c)^2 . \qquad (36)$$

2.1.6 A One Group Approximation Method for Predicting the Reactivity Worth of a Control Rod (Nordheim - Scalettar). Glasstone and Edlund (7)

and Weinberg and Wigner (23) present a method attributed to Scalettar

and Nordheim for predicting the approximate worth of a centrally

located control rod in a large, cylindrical, bare, homogeneous reactor.

Their solution is obtained by solving Eq. (5) in cylindrical geometry

subject to the following boundary conditions:

i). $\phi (r = R_o) = 0$

ii). $\phi (r = a_o) = 0$.

The assumptions made in the derivation of their final equation (Eq.

(11.12.1), Ref. 7)

$$\Delta k_e = 7.5 \ M^2/R_o^2 \left[0.116 + \ln \ (R_o/2.4a_o) \right]^{-1}$$

are as follows:

a. The control rod displaces an equivalent cylinder of the reactor

core and does not leave a hole upon being withdrawn.

b. The modified one-group critical equation is valid, i.e.

$$k_\infty = 1 + M^2 B^2$$

where $M^2 = L^2 + \tau$ (migration area)

B^2 = Buckling

c. The flux goes to zero within the control rod at an extrapolated

distance, d_o.

d. The diameter of the rod is much smaller than the radius of the

reactor. This allows for

$$\alpha = \alpha_o + \Delta\alpha ,$$

where $\Delta\alpha$ is a small value. This is necessary to be able to use the

asymptotic expressions for the Bessel functions.

2.2 Two Group Diffusion Models Used to Calculate the Reactivity Worth of a Control Rod in a Subcritical Reactor System

2.2.1 A Two Group Calculational Model for Finding the Change in Geometric Buckling Due to a Control Rod in a Subcritical Reactor. Meghreblian and Holmes (14) and Murray and Niestlie (18) derived a two group method for calculating the worth of a centrally located control rod in a bare, critical, cylindrical reactor. This method with a few variations can be used to compute the worth of a control rod in a sub-critical assembly.

A derivation of the solutions to the steady-state two group diffusion equations, Eq. (39) and (40), is presented in Appendix B.

$$D_1 \nabla^2 \phi_1 - \Sigma_R \phi_1 + k_\infty \Sigma_a \phi_2 = 0 \qquad (39)$$

$$D_2 \nabla^2 \phi_2 - \Sigma_a \phi_2 + \Sigma_R \phi_1 = 0 . \qquad (40)$$

These solutions were found to be

$$\phi_1 = AX(r) + CY(r) \qquad (41)$$

$$\phi_2 = S_1 AX(r) + S_2 CY(r) . \qquad (42)$$

The flux components X(r) and Y(r) has permissible solutions in cylindrical geometry of

$$X(r) = AJ_0(\alpha_1 r) + A_0 Y_0(\alpha_1 r) \qquad (43)$$

$$Y(r) = C I_0(\alpha_2 r) + C_0 K_0(\alpha_2 r) . \qquad (44)$$

For the cylindrical reactor with a centrally located control rod inserted, the boundary conditions are

$$i). \quad \left\{ [\partial / \partial r] \phi_1 (r) \right\}_{r=a_1} = 0$$

$$ii). \quad \left\{ \phi_2(r) / \phi_2{}'(r) \right\}_{r=a_1} = 0$$

$$iii). \quad \phi_1 (r=R_c) = \phi_2 (r=R_c) = 0 \; .$$

The quantity a_1 is the physical boundary of the control rod and R_c is the extrapolated critical radius of the reactor. Since $\phi_1(r)$, and $\phi_2(r)$ must vanish at the extrapolated radius of the reactor, $X(r)$ and $Y(r)$ must vanish individually at $r = R_c$. Therefore, applying boundary condition iii) to Eq. (43) and (44), ratios of the constants A, A_o, C, and C_o can be formed as

$$A_o/A = - J_o(\alpha_1 R_c) / Y_o(\alpha_1 R_c) = - T_o(\alpha_1 R_c) \tag{45}$$

$$C_o/C = - I_o(\alpha_2 R_c) / K_o(\alpha_2 R_c) = - Q_o(\alpha_2 R_c) \; . \tag{46}$$

Boundary conditions i) and ii) were applied to Eq. (41) and (42). Using Eq. (45) and (46) and rearranging the result, was found to be

$$\left[1 - C_1 \, T_o(\alpha_1 R_c) \right] / C_3 \alpha_1 \, T_o(\alpha_1 R_c) = S_2 \left[1 - C_2 Q_o(\alpha_2 R_c) \right] /$$

$$S_1 \, C_4 \, \alpha_2 \, Q_o(\alpha_2 R_c) \tag{47}$$

where
$$C_1 = Y_o(\alpha_1 a_1) + \alpha_1 \, Y_1(\alpha_1 a_1)$$
$$C_2 = K_o(\alpha_2 a_1) + \alpha_2 \, K_1(\alpha_2 a_1)$$
$$C_3 = Y_1(\alpha_1 a_1)$$
$$C_4 = K_1(\alpha_2 a_1) \; .$$

For a bare reactor with no control rod, the boundary conditions applied
to Eq's. (43) and (44) are

    i) The fluxes must be finite and non-negative

    ii) ϕ_1 $(r=R_c) = \phi_2$ $(r=R_c) = 0$.

Using the first condition, A_0 is zero by noting that Y_0 $(\alpha_1 r)$ goes to
a negative infinity at the origin. If the second boundary condition is
applied to Eq's. (43) and (44), the result is

$$\alpha R_c = 2.4048. \tag{48}$$

Only the first mode is used because (1) the reactor will go critical in
only the first mode and (2) the higher modes will decay out exponentially.

In order to compute the worth of a centrally located control rod in
a cylindrical, bare, subcritical reactor, the following procedure was
used. In this procedure the above two group analysis was used only to
find the increased buckling of the system containing the control rod.
The change in the effective multiplication factor is found by using the
Fermi slowing down model.

1. A critical reactor of the same k_∞ calculated by using the one
group point thermal source model was assumed. This assumption
allowed the quantities μ^2 and ν^2 to be calculated where,

$$\left.\begin{array}{c}\mu^2\\\nu^2\end{array}\right\} = \tfrac{1}{2}\left[-(1/\tau + 1/L^2) \pm \sqrt{(1/\tau + 1/L^2)^2 + 4(k_\infty -1)/\tau L^2} \right]. \tag{49}$$

Using these values of μ^2 and ν^2 to calculate α_1 and α_2 (see
Appendix B), the critical radius, R_c, can be calculated using Eq.(47).

2. With the critical radius of the reactor, having a control rod
inserted, known, the radial component of the increased geometric
buckling can be calculated. Since the subcritical reactor system

was unreflected, the minor two group buckling, ν^2, was not applicable and the major two group buckling, μ^2, was equal to B^2.

3. Using the term α from above, the geometric buckling of the equivalent cylindrical subcritical system used in this work can be calculated. Use is made of the geometric buckling relation given as

$$B^2_{g2} = \alpha^2 + (\pi/c)^2 \ . \tag{50}$$

The geometric buckling of the system without a control rod is given as

$$B^2_{g1} = \alpha^2_o + (\pi/c)^2 \ . \tag{51}$$

4. The change in k_e is now calculated by using the Fermi slowing down definition for k_e (Eq. 18). This change is given by

$$k_e = k_\infty \left\{ \exp(-B^2_{g_1} \tau) \ / \left[1 + L^2 B^2_{g_1} \right] - \exp(-B^2_{g_o} \tau)/\left[1 + L^2 B^2_{g_o} \right] \right\} \tag{52}$$

2.2.2 <u>A Method for Extending the Results of Reactivity Measurements of a Control Rod in a Subcritical Reactor to a Critical Reactor of the Same Core Material</u>. To treat the energy dependence of neutron absorption more accurately, a two group analysis was derived by Wood (25). In section 2.1 a method was described for calculating d_2, the thermal extrapolation distance into the control rod. If vertical traverses are taken in the multiplying medium, a fast extrapolation distance, d_1, can be found.

The solutions, Eq's. (43) and (44), to the two group diffusion equations, Eq's. (39) and (40), are considered valid. A separation of

variables technique is employed to solve the applicable wave equations.
These wave equations are given in Appendix B as

$$\nabla^2 \, X(r) + \mu^2 \, Y(r) = 0 \tag{53}$$

$$\nabla^2 \, X(r) - \nu^2 \, Y(r) = 0 \, . \tag{54}$$

The auxiliary separation constants equations which apply to a subcritical
assembly are given by

$$\gamma_1^2 = \alpha_1^2 - \mu^2 \tag{55}$$

$$\gamma_2^2 = \alpha_2^2 + \nu^2 \, . \tag{56}$$

The quantities γ_1 and γ_2 are inverse relaxation lengths and are equal
if no internal boundaries exist in the reactor core medium. A relation-
ship between μ^2 and ν^2 can be found by using Eq. (49). This
relationship is

$$\mu^2 = \nu^2 - (1/\tau + 1/L^2) \, . \tag{57}$$

The radial flux components are of the same form as shown in Eq's. (43)
and (44).

The two group boundary conditions which are assumed consist of the
following:

$$\text{i). } \phi_1 \, (r=R_o) = \phi_2 \, (r=R_o) = 0$$
$$\text{ii). } \left\{ \phi_1 \, (r) \, / \phi_1{}' \, (r) \right\}_{r=a_1} = d_1 \tag{58}$$

$$\text{iii). } \left\{ \phi_2 \, (r) \, / \, \phi_2{}' \, (r) \right\}_{r=a_1} = d_2 \, .$$

Applying boundary condition i) to Eq's. (43) and (44), equations identical

to Eq's. (45) and (46) are obtained. If boundary conditions ii) and iii) are applied to Eq's. (43) and (44), the following transcendental equation is obtained.

$$\left[L_0(\alpha'_2 a_1, d_1) - Q_0(\alpha_2 R_0) M_0(\alpha_2 a_1, d_1)\right] / \left[L_0(\alpha'_2 a_1, d_2) - Q_0(\alpha_2 R_0)\right.$$

$$\left. \times M_0(\alpha'_2 a_1, d_2)\right] = \left[N_0(\alpha_1 a_1, d_1) - T_0(\alpha_2 R_0) O_0(\alpha_1 a_1, d_1)\right] /$$

$$\left[N_0(\alpha'_1 a_1, d_2) - T_0(\alpha_2 R_0) O_0(\alpha_1 a_1, d_2)\right]. \tag{59}$$

The quantities L_0, M_0, N_0, and O_0 are defined by

$$L_0(\alpha_2 a_1, d_i) = I_0(\alpha'_2 a_1) - d_i \left\{[d/dr] I_0(\alpha'_2 r)\right\}_{r=a_1} , \quad i = 1,2$$

$$M_0(\alpha_2 a_1, d_i) = K_0(\alpha_2 a_1) - d_i \left\{[d/dr] K_0(\alpha_2 r)\right\}_{r=a_1} , \quad i = 1,2$$

$$N_0(\alpha'_1 a_1, d_i) = J_0(\alpha_1 a_1) - d_i \left\{[d/dr] J_0(\alpha_1 r)\right\}_{r=a_1} , \quad i = 1,2$$

$$O_0(\alpha'_1 a_1, d_i) = Y_0(\alpha_1 a_1) - d_i \left\{[d/dr] Y_0(\alpha_1 r)\right\}_{r=a_1} , \quad i = 1,2 .$$

Without the rod Eq. (55) becomes

$$\gamma_{10}^2 = \alpha_{10}^2 - \mu^2 \tag{55'}$$

where γ_{10} is the inverse relaxation length and α_{10} is equal to 2.4048/R_0. If the subcritical reactor is unreflected, Eq. (57) will not apply when the rod is withdrawn. However, if the rod is inserted, Eq. (55) and (56) are given as

$$\gamma_{11}^2 = \alpha'^2_{11} - \mu^2 , \tag{55''}$$

$$\gamma_{21}^2 = \alpha_{21}^2 + \upsilon^2 . \tag{56''}$$

The quantity μ^2 can be determined from Eq. (55') if the inverse relaxation length measured in the multiplying medium with no control rod present is used. With the control rod inserted, however, a new inverse relaxation length, $\gamma_{11} = \gamma_{21}$, can be measured. This value is used in Eq. (56'') to determine α_{11}^2. Also, α_{21}^2 can be calculated using υ^2 and the measured value for γ_{21}.

The values of α_{11}, α_{21}, and d_2, determined in a previous section from measurements on the pure diffusing medium, are used in Eq. (59) to solve for d_1. Knowing d_1, d_2, μ^2 and υ^2, Eq. (59) can be used to solve for the bucklings α_{13} and α_{23} of a critical reactor of radius R_c. The change in major bucklings is found to be

$$\Delta\alpha_1^2 = \alpha_{13}^2 - (2.4048/R_c)^2 . \tag{60}$$

This method can be extended to more groups by considering the appropriate boundary conditions for each group.

2.2.3 A Two Group Approximation Method for Predicting the Reactivity Worth of a Control Rod (Nordheim - Scalettar). To improve the value of Δk_e obtained, using modified one group theory, Glasstone and Edlund (7) present a method, based on a two group model, for predicting the worth of a centrally located rod in a large, cylindrical, bare, homogeneous reactor. Their solution is obtained by solving Eq's. (39) and (40) in cylindrical geometry subject to the following boundary conditions:

i). $\phi_1 (r = R_o) = 0$

ia). $\left\{ [d/dr] \phi_1(r) \right\}_{r=a_1} = 0$

ii). $\phi_2 (r = R_o) = 0$

iia). $\phi_2 (r = a_o) = 0$.

The assumptions made in the derivation of their final equation, (Eq. 11.31.1) reference 7)

$$\Delta k_e = 7.5 \, M^2/R_o^2 \left[0.116 \, (1 + \tau/L^2) + \tau/L^2 \, \ln(L \, \sqrt{\tau}/Ma_o) + \ln(R_o/2.4a_o) \right]^{-1},$$

are listed below.

a.   The control rod displaces an equivalent cylinder of the reactor core and when the control rod is withdrawn, a void is not left.

b.   The thermal flux goes to zero within the control rod at an extrapolated distance d. The first derivative of the fast flux is zero at the surface of the control rod.

c.   The diameter of the control rod is much smaller than the radius of the reactor. This allows estimating $\Delta\mu$ to be small, where $\Delta\mu$ is written as

$$\Delta\mu = \mu - \mu_o .$$

This approximation is necessary to make the Bessel function approximations.

d.   The value of k-1 is considered small. Therefore, the major and minor bucklings can be conveniently approximated.

The modified one group approximation is derived by multiplying Eq. (11.31.1) by L^2/M^2. The result is Eq. (11.33) from Glasstone and Edlund (7).

$$\Delta k_e = 7.5 \, L^2/R^2 \, (0.116 + \ln R/2.4a_o)^{-1}$$

2.3 Perturbation Models for Estimating
the Reactivity Worth of the Control Rod

2.3.1 A One Group Approximation. Perturbation theory can be used
to estimate the worth of a control rod if the flux perturbation is small.
The resultant change in reciprocal reactor period, $\Delta \omega$, derived from
perturbation theory in Appendix C, becomes

$$\Delta \omega = \int_V \phi_r^{Tr} P \phi_r' \, dV \Big/ \int_V \phi_r^{T+} \phi_r' \, dV . \tag{61}$$

The quantity $\Delta \omega$ is given as ($\omega' - \omega$), where ω' is the reciprocal
period of the perturbed reactor and ω is the reciprocal period of the
unperturbed reactor. P is the matrix operator representing the per-
turbation inserted into the reactor system. The quantities ϕ_r^{T+} and
ϕ_r' are spatially dependent vector flux sets representing the adjoint
fluxes and perturbed fluxes, respectively.

The reactivity, $\Delta k_e / k_e$, change due to a perturbation in a reactor
can be found by considering the time dependent diffusion equation in
which the delayed neutron groups have been neglected. This equation
was derived by Glasstone and Edlund (7), the result being

$$\left\{ \Delta k_e / \ell^* \right\} \phi' = d \phi' / dt . \tag{62}$$

The quantity ℓ^* is the thermal neutron lifetime and is given as

$$\ell^* = k_e / k_\infty \Sigma_{a_c} v . \tag{63}$$

Substituting Eq. (63) into Eq. (62), the result is

$$\left\{ k_\infty \Sigma_{a_c} v \, \Delta k_e / k_e \right\} \phi' = d \phi' / dt . \tag{64}$$

If ϕ' has a solution of the type

$$\phi' = \phi_r' \exp(\omega t) \tag{65}$$

where ϕ' is dependent only on space, Eq. (64) can be rewritten as

$$\left\{ k_\infty \mathcal{E}_{a_c} v \, \Delta k_e/k_e \right\} \phi_r' = \omega \, \phi_r' . \tag{66}$$

Since the perturbation in the reactor is small, the unperturbed flux, ϕ , will be approximately equal to the perturbed flux, ϕ' . Making use of this approximation, multiplying Eq. (66) by ϕ , and integrating over the entire reactor volume, an equation relating $\Delta k_e/k_e$ to the reciprocal reactor period, ω , is obtained as

$$\Delta k_e/k_e = \omega \int_V \phi_r^2 \, dV \bigg/ \int_V k_\infty \mathcal{E}_{a_c} v \, \phi_r'^2 \, dV. \tag{67}$$

Considering Eq. (61); if a one group calculation is to be performed, the matrix operator M, is self adjoint, i.e. the flux functions ϕ_r , will form a complete orthogonal set. Therefore, the term $\phi_r^\dagger$ is equal to ϕ . Also for small perturbations, $\phi_r^{\dagger'}$ is approximately equal to ϕ . Suppose further that a pure thermal absorption change is made in the reactor system. This change is due to the insertion of a centrally located control rod into the reactor system. This means that P can be given by

$$P = - \Delta \left[v \, \mathcal{E}_a(r) \right] . \tag{68}$$

Therefore, for the one group approximation with a pure absorption perturbation, Eq. (61) becomes

$$\Delta \omega = \omega' - \omega = - \int_V \Delta \left[v \, \mathcal{E}_a(r) \right] \phi_r^2 \, dV \bigg/ \int_V \phi_r^2 \, dV. \tag{69}$$

If a fictitious critical reactor, $\omega = 0$, is thought of as being a reference reactor, Eq. (69) will yield the $\Delta\omega$ for the equivalent cylindrical subcritical reactor system under consideration in this work. Therefore, $\Delta\omega_1$ is defined as the change in reciprocal reactor period due to the unperturbed reactor system being subcritical by an amount $k_{e_1} - 1$ ($= k_{e_1}$). The term k_{e_1} is the effective multiplication factor for the unperturbed, equivalent, cylindrical, subcritical reactor system. With the control rod inserted, a new $\Delta\omega_2$ can be found which is thought of as the change in the reciprocal reactor period due to the reactor being subcritical and to the addition of the absorber. The change in the effective multiplication factor of the reactor caused by the insertion of the control rod is $k_{e_2} - 1$ ($= \Delta k_{e_2}$).

The quantity $\Delta\omega_1$ ($= \omega_1 - 0$) contains a fictitious $\Delta\left[v\,\Sigma_a(r)\right]_1$, which can be thought of as the amount of absorber which makes the reference reactor subcritical by the amount Δk_{e_1}. The term $\Delta\omega_2$ contains this same $\Delta\left[v\,\Sigma_a(r)\right]_1$ plus the absorption term due to the presence of the control rod $\Delta\left[v\,\Sigma_a(r)\right]_r$. Therefore, $\Delta\omega_1$ and $\Delta\omega_2$ can be written as

$$\Delta\omega = \omega_1 = - \int_V \Delta\left[v\,\Sigma_a(r)\right]_1 \phi_r^2 \, dV \Big/ \int_V \phi_r^2 \, dV \tag{70}$$

$$\Delta\omega = \omega_2 = - \int_V \Delta\left[v\,\Sigma_a(r)\right]_1 \phi_r^2 \, dV - \int_V \Delta\left[v\,\Sigma_a(r)\right]_r \phi_r^2 \, dV \Big/$$

$$\int_V \phi_r^2 \, dV \; . \tag{71}$$

Substituting Eq. (70) and (71) into Eq. (67) and subtracting the quantity $(k_{e_2} - 1) / k_{e_2}$ is found to be

$$(k_{e_2} - 1) / k_{e_2} = (k_{e_1} - 1) / k_{e_1} - \int_V \Delta[v \Sigma_a(r)]_r \phi_r^2 \, dV \; /$$

$$v \, k_\infty \Sigma_{a_c} \int_V \phi_r^2 \, dV . \qquad (72)$$

In the one group model v is a constant representing the average neutron velocity and can be factored. Thus, Eq. (72) can be rewritten as

$$(k_{e_2} - 1) / k_{e_2} = (k_{e_1} - 1) / k_{e_1} - \int_V \Delta[\Sigma_a(r)]_r \phi_r^2 \, dV \; /$$

$$k_\infty \Sigma_{a_c} \int_V \phi_r^2 \, dV . \qquad (73)$$

For a hollow cylindrical annulus control rod, the value of $\Delta[\Sigma_a(r)]_r$ will be zero for $r < r_0$ and $r > r_1$, where r_0 and r_1 are the inner and outer radii, respectively. The quantity $\Delta[\Sigma_a(r)]_r$ will be equal to the value of the absorption cross section of the control rod material if the control rod is inserted in a hole initially containing only air. Therefore, Eq. (73) can be written as

$$(k_{e_2} - 1)/k_{e_2} = (k_{e_1} - 1)/k_{e_1} - (\Delta \Sigma_a)_r \int_{r_0}^{r_1} \phi_r^2 \, r \, dr \; / \; k_\infty \Sigma_{a_c}$$

$$\times \int_V \phi_r^2 \, r \, dr . \qquad (74)$$

Using an average value for k_{e_1} obtained from the method explained in section 2.1 , the quantity $(k_{e_1} - 1) / k_{e_1}$ can be calculated.

2.3.2 A Two Group Approximation. As in the one group model, assume the critical reference reactor to exist so that the equation for becomes

$$\Delta \omega = \omega' = \int_V \bar{\phi}_r^{+} \; P \, \bar{\phi}_r \, dV \; / \; \int_V \bar{\phi}_r^{+} \bar{\phi}_r \, dV . \qquad (75)$$

Here the quanities, $\bar{\phi}_r^{+}$ and $\bar{\phi}_r$, represent matrix operators. In order

to express Eq. (75) in terms of $\Delta k_e / k_e$, consider a perturbed reactor which has been perturbed by only a change in η, the average number of fast fission neutrons emitted as a result of the capture of one thermal neutron in fuel material, a factor in the infinite multiplication factor. Thus, the P operator in Eq. (75) for this reactor becomes

$$P = \left\| \begin{matrix} 0 & v_1 f \Sigma_{a_c} \Delta\eta \\ 0 & 0 \end{matrix} \right\| \tag{76}$$

where f is the thermal utilization factor and $p\epsilon$ has been assumed to be equal to unity. Substituting Eq. (76) into Eq. (75), and performing the necessary matrix multiplication, the following is the result,

$$\omega' = v_1 f \Sigma_{a_c} \Delta\eta \int_V \phi_1^+ \phi_2 \, dV \; / \; \int_V \phi_r^+ \phi_r \, dV . \tag{77}$$

Here ϕ_1^+ is the adjoint fast flux while ϕ_2 is the thermal flux.

The effective multiplication factor is given by

$$k_e = \eta \epsilon p f \mathcal{L}. \tag{78}$$

Where $\mathcal{L}$ is the non-leakage probability. Therefore, a uniform change in η results in an expression such that

$$\Delta k_e / k_e = \Delta\eta / \eta . \tag{79}$$

From the previous section, it is found that $\Delta k_e / k_e$ is equal to $\ell^* \omega'$, so that the thermal neutron lifetime, ℓ^*, can be given by

$$\ell^* = \Delta\eta / \eta \, \omega'. \tag{80}$$

Substituting Eq. (77) for ω' into Eq. (80) produces

$$\ell^* = \int_V \tilde{\phi}_r^+ \, \bar{\phi}_r \, dV \; / \; v_1 \eta f \Sigma_{a_c} \int_V \phi_1^\tau \, \phi_2 \, dV . \tag{81}$$

Using Eq. (81), the reactivity, $\Delta k_e / k_e$, can be expressed as

$$\Delta k_e / k_e = \int_V \tilde{\phi}_r^+ \, P \, \bar{\phi}_r \, dV \; / \; v_1 \eta f \Sigma_{a_c} \int_V \phi_1^+ \, \phi_2 \, dV . \tag{82}$$

Consider now a perturbed reactor, in which the perturbation is a purely absorption change. Thus, the P operator in two group theory can be written as

$$P = \left\| \begin{array}{cc} 0 & 0 \\ 0 & - \Delta \left[v_2 \Sigma_a(r) \right] \end{array} \right\| . \tag{83}$$

The reactivity associated with subcritical reactor is given by Eq. (82) where the P operator has been replaced by Eq. (83) and the necessary matrix multiplication has been performed.

$$\Delta k_{e1} / k_{e1} = - \int_V \phi_2^\tau \, \Delta \left[v_2 \Sigma_a(r) \right]_1 \phi_2 \, dV \; / \; v_1 \eta f \Sigma_{a_c}$$
$$\times \int_V \phi_1^+ \, \phi_2 \, dV . \tag{84}$$

After the control rod is inserted, Eq. (82) becomes

$$\Delta k_{e2} / k_{e2} = - \left\{ \int_V \phi_2^\tau \, \Delta \left[v_2 \Sigma_a(r) \right]_1 \phi_2 \, dV - \int_V \phi_2^+ \, \Delta \left[v_2 \Sigma_a(r) \right]_r \right.$$
$$\left. \times \phi_2 \, dV \right\} \; / \; v_1 \eta f \Sigma_{a_c} \int_V \phi_1^+ \, \phi_2 \, dV . \tag{85}$$

Subtracting Eq. (84) from (85), produces the desired result of

$$\Delta k_{e2} / k_{e2} = \Delta k_{e1} / k_{e1} - \int_V \phi_2^+ \, \Delta \left[v_2 \Sigma_a(r) \right]_r \phi_2 \, dV \; /$$
$$v_1 \eta f \Sigma_{a_c} \int_V \phi_1^+ \, \phi_2 \, dV . \tag{86}$$

The steady-state two group adjoint diffusion equations for a bare reactor can be written as

$$v_1 D_1 \nabla^2 \phi_1^\dagger - v_1 \Sigma_R \phi_1^\dagger + v_2 \Sigma_R \phi_2 = 0 \tag{87}$$

$$v_2 D_2 \nabla^2 \phi_2^\dagger - v_2 \Sigma_a \phi_2^\dagger + v_1 \eta f \Sigma_a \phi_1^\dagger = 0 . \tag{88}$$

Eq's. (87) and (88) have solutions of the form

$$\phi_1^\dagger = S_1^\dagger A^\dagger X(r) + S_2^\dagger C^\dagger Y(r) \tag{89}$$

$$\phi_2^\dagger = A^\dagger X(r) + C^\dagger Y(r) \tag{90}$$

where $X(r)$ and $Y(r)$ are given by Eq's. (43) and (44). By a similar method as was used in Appendix B to determine S_1 and S_2, the adjoint coupling coefficients, S_1 and S_2 , can be determined to be

$$S_1^\dagger = \{v_2 / v_1\} / \{1 + \mu^2 \tau\} \tag{91}$$

$$S_2^\dagger = \{v_2 / v_1\} / \{1 - \upsilon^2 \tau\} . \tag{92}$$

For a bare subcritical reactor, there will be no $Y(r)$ component to the fluxes. Therefore, after substituting Eq's. (89), (90), (91), and (92) into Eq. (86), an expression for the reactivity of the centrally located control rod is given as

$$\Delta k_{e_2} / k_{e_2} = \Delta k_{e_1} / k_{e_1} - (1 + \mu_g^2) \int_V \Delta[\Sigma_a(r)] X^2(r) dV /$$
$$\eta f \Sigma_{a_c} \int_V X^2(r) dV . \tag{93}$$

Since a two group model is being studied, the effective multiplication factor, k_e, can be written as

$$k_e = k_\infty \quad (1 + B_g^2 \tau)(1 \div B_g^2 L^2) \quad . \tag{94}$$

The geometric buckling, B_g^2, for the one group reactor, is equal to the μ_g^2, the geometric buckling for the two group reactor, since the reactor core is bare. Therefore, Eq. (93) is identical with Eq. (74) if $p \epsilon$ is assumed to be unity. No new information is gained from a two group perturbation analysis on a bare reactor.

2.3.3 <u>An Improvement for the Perturbation Models by a Variational Technique</u>. Both one group and two group perturbation theory yield identical results for the reactivity worth of a control rod in a bare reactor. In order to improve these results, the assumption that the perturbed flux is equal to the unperturbed flux must be improved. The following method is proposed to make this improvement (3,14).

In section 2.0, the neutron diffusion equation for a multiplying medium was written as

$$-D_o \nabla^2 \phi_o(r) + \Sigma_a^\circ \phi_o(r) = \upsilon \Sigma_f \phi_o(r) \tag{95}$$

where the superscript o indicates an unperturbed system. Replacing $(\upsilon \Sigma_f - \Sigma_a) / D_o$ by B_o^2, Eq. (95) becomes

$$\nabla^2 \phi_o(r) + B_o^2 \phi_o(r) = 0 \tag{96}$$

where $\phi_o(r)$ is the flux corresponding to the lowest mode of the set of eigenfunctions $\phi_n(r)$. If all the eigenfunctions are considered, Eq. (96) can be written as

$$\nabla^2 \phi_n(r) + B_n^2 \phi_n(r) = 0 \quad , \tag{97}$$

where the functions $\phi_n(r)$ will be assumed to be orthogonal functions.

For the system under consideration, the perturbation will occur in the thermal neutron absorption cross section. For the perturbed system, this cross section can be written as

$$\Sigma_a(r) = \Sigma_a^o + \Delta \Sigma_a(r) \tag{98}$$

where $\Delta \Sigma_a(r)$ is the spatially dependent absorption cross section change. With this change in cross section, the reactor flux becomes

$$\phi(r) = \phi_o(r) + \Delta \phi(r) \tag{99}$$

where $\Delta \phi(r)$ is flux change due to the absorption cross section change. In order to maintain a steady-state operation, a hypothetical change in ν must be assumed.

$$\nu \rightarrow \nu + \Delta \nu$$

With these changes Eq. (95) becomes

$$-D_o \nabla^2 \Delta \phi(r) + \Delta \Sigma_a(r) \phi_o(r) + \Sigma_a^o \Delta \phi(r) = \nu \Sigma_f \Delta \phi(r) +$$

$$\Delta \nu \Sigma_f \phi_o(r), \tag{100}$$

if the products of differentials are ignored and the terms contained in Eq. (95) are factored and replaced by zero.

Using all the eigenfunctions, $\phi(r)$ can be expanded such that

$$\phi(r) = \sum_{n=0}^{\infty} a_n \phi_n(r) = \phi_o(r) + \sum_{n=1}^{\infty} a_n \phi_n(r) \tag{101}$$

where the a_o constant has been defined to be unity. Note that the second term of the right hand member of Eq. (101) is the $\Delta \phi(r)$

presented in Eq. (100).

To evaluate the constants a_n, Eq. (101) should be multiplied by $\phi_n(r)$ where $n \neq 0$, and the result integrated over the whole reactor volume, V. This results in an expression such as

$$\int_V \phi_n(r) \, \phi(r) \, dV = \int_V \phi_n(r) \, \phi_o(r) \, dV + \int_V \phi_n(r) \, \Delta\phi(r) \, dV. \quad (102)$$

Since the eigenfunctions $\phi_n(r)$ are orthogonal and $n \neq 0$ in Eq. (102), the first term of the right hand member is equal to zero. Also by virtue of orthogonality principals, the left hand member becomes

$$\int_V \phi_n^2(r) \, dV .$$

An expression for $\Delta\phi(r)$ in terms of $\phi_o(r)$ is obtained from Eq. (100)

$$\Delta\phi(r) = \phi_o(r) \left[\Delta\upsilon\Sigma_f - \Delta\Sigma_a \right] / -D_o\nabla^2 + \left[\Sigma_a^o - \upsilon\Sigma_f \right]. \quad (103)$$

In Eq. (103) the ∇^2 can be replaced by $(-B_n^2)$ from Eq. (97) and $\left[\Sigma_a^o - \upsilon\Sigma_f \right]$ can be replaced by $(-D_oB_o^2)$. Therefore, Eq. (103) becomes

$$\Delta\phi(r) = \phi_o(r) \left[\Delta\upsilon\Sigma_f - \Delta\Sigma_a(r) \right] / D_o (B_n^2 - B_o^2) . \quad (104)$$

Substituting for $\Delta\phi(r)$ in Eq. (104), the following is the result

$$a_n = - \int_V \phi_n(r) \, \Delta\Sigma_a(r) \, \phi_o(r) \, dV / D_o (B_n^2 - B_o^2) \int_V \phi_n^2(r) \, dV . \quad (105)$$

The expression for the perturbed flux, found by using Eq's. (101) and (105), can now be used in the one group perturbation model, Eq. (74) for the perturbed flux and a better estimate of the worth of the rod is obtained (3).

3.0 EXPERIMENTAL FACILITIES

3.1 General Pile Description

The Kansas State University graphite pile, shown in Fig. (5), consisted of a rectangular parallelepiped, 68 in. square, and 100 in. high, resting on a concrete foundation. The pile was constructed of machined R-IHLM Nuclear Grade Graphite blocks 4 $\pm$ 0.01 inches in cross section and of specified lengths. In stacking, the long dimension of the blocks was alternated 90° from layer to layer. As seen in Fig. (5), holes were drilled through the entire length of certain graphite blocks to accommodate fuel slugs in an eight inch lattice formation. These holes were approximately 1.75 inch in diameter. The Nuclear Grade Graphite used for this pile has a thermal neutron absorption cross section between 3.7 and 4.5 millibarns.

For a portion of this work, each of the drilled fuel ports were filled with graphite cylinders. These graphite cylinders were made out of the same material as the graphite blocks and were 1.625 inches in diameter and 22.68 inches in length. Since the cylinders were of a smaller diameter than the fuel ports, small crescent shaped air gaps of 0.125 inches were left between the tops of the graphite cylinders and the tops of the fuel ports.

The blocks along the central vertical axis of the pile contained horizontal slots, shown in Fig. (5), with a cross section of 1.261 inches by 0.343 inches. These slots were used in this work to accommodate a BF_3 probe. The density of the solid graphite blocks was 1.683 g/cc.
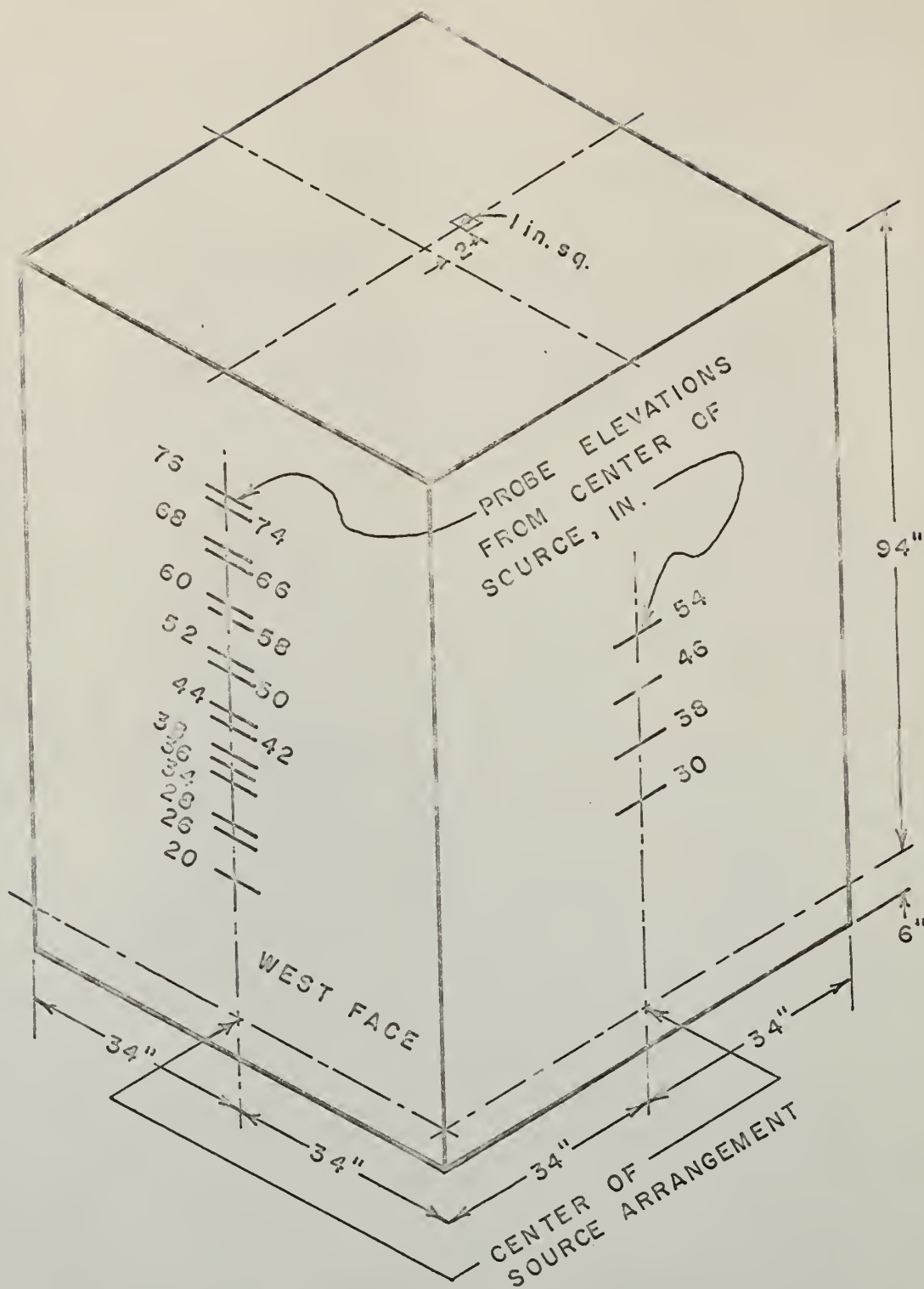
Fig. 1. Schematic Diagram of KSU pile.

Steichen (21) calculated the percentage air void to be approximately 0.3 per cent giving a corrected density for the pile of 1.678 g/cc.

The graphite pile contains a square machined hole with a side of 1.0625 inches extending from the top of the assembly to a distance of 20 inches above the concrete foundation. This hole, as can be seen by Fig. (1), was positioned 2.0 inches in the x dimension from the center of the pile. This hole was used to accommodate a 0.75 inch outside diameter aluminum tube filled with rolled sheet cadimum.

3.2 Non-Multiplying Configuration

Table 1. Summary of Physical Specifications of Plutonium-Beryllium Neutron Sources

Source number	Grams Pu	Grams Be	Neutron Emission rate (n/sec)
365	7.87	15.99	$(1.64 \pm 0.115)10^6$
366	7.87	16.01	$(1.73 \pm 0.121)10^6$
367	7.86	15.89	$(1.82 \pm 0.127)10^6$
368	7.86	15.88	$(1.69 \pm 0.118)10^6$
369	7.86	16.09	$(1.71 \pm 0.120)10^6$

For this portion of the work, experimentation was carried out in a diffusing medium with and without the poison rod inserted. A four-source configuration was used with the sources located in the source plane at positions 2, 4, 10, and 12 as shown in Fig. (2). In this configuration, each of the four sources was located 22 inches from the

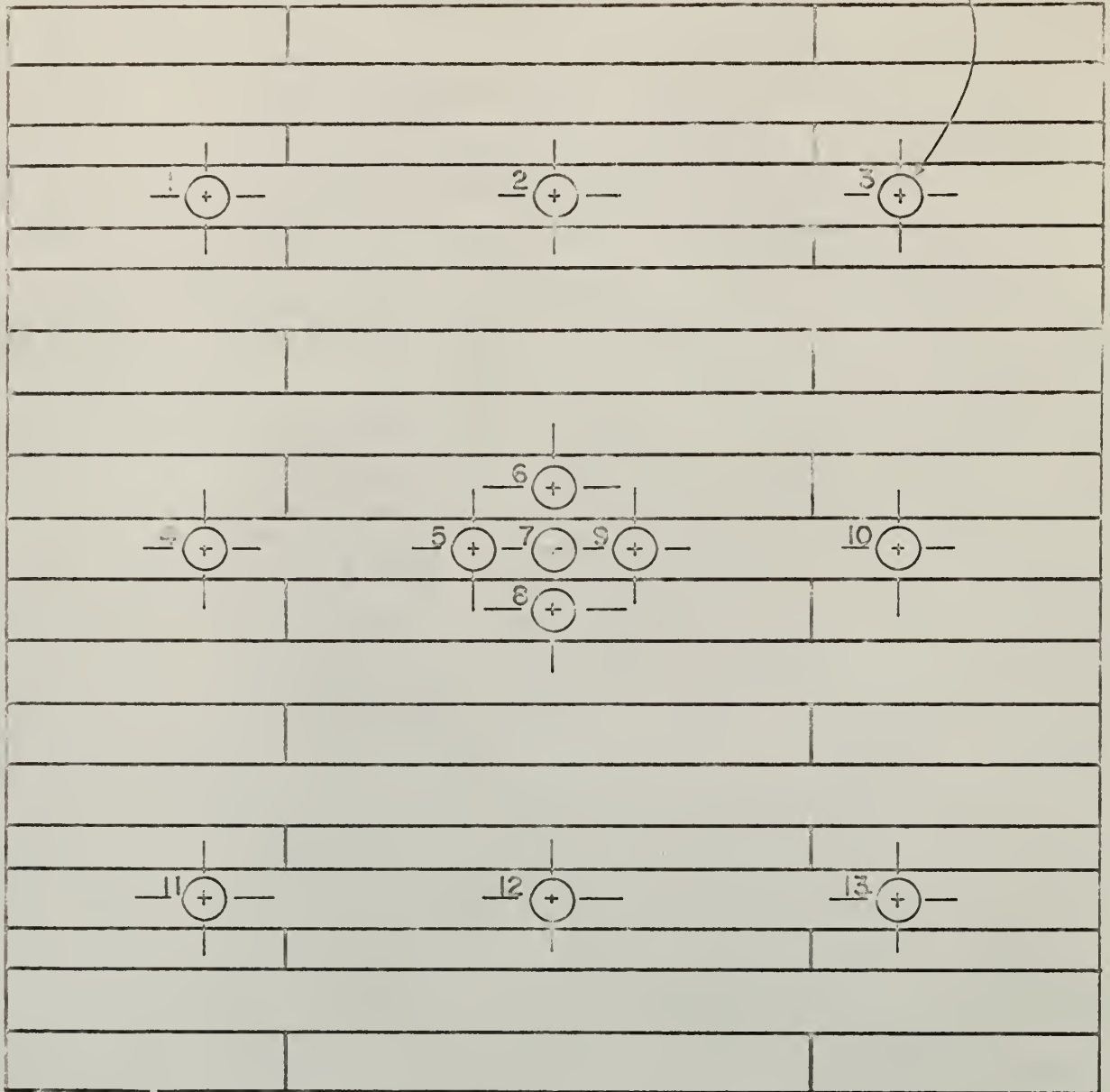ONE OF 13 POSSIBLE SOURCE LOCATIONS

Fig. 2. Source locations in KSU pile.

center of the pile. The sources were located in a plane six inches above the concrete foundation (see Fig. (1)).

The inner container of each source is 0.85 inch in diameter, 0.90 inch in height, and is made of tantalum. The outside container of each source is 1.02 inches in diameter, 1.30 inches in height, and is made of 18-8 stainless steel. All sources are sealed by welding.

3.3 Multiplying Configuration

For this portion of the work, the fuel ports contained aluminum tubes 68 inches long and 1.312 inches outside diameter. Each tube was supported in the fuel port by three aluminum rings, one fixed at the center, and two removable rings at the ends. Each aluminum tube accommodated eight Savannah River Type MK VIIa fuel element rejects. Fig. (5) illustrates a portion of the loaded pile. In order that the poison rod would extend through the total length of the fuel region, the top three layers of the pile were left without fuel. The physical characteristics of the fuel slugs are shown in Figs. (3) and (4).

3.4 Neutron Detection System

Table (2) contains a listing of the various electronic components used throughout this work.

The Model NC-202 BF_3 probe's active volume, a length of 0.50 inches and a diameter of 0.1875 inches, contained $B^{10}F_3$ gas at a pressure of 70 cm Hg and an enrichment of 96 per cent. The model (RCL-10504) BF_3 probe's active volume with a length of 12 inches and a diameter of 1 inch, contained $B^{10}F_3$ gas at a pressure of 12 cm Hg and an
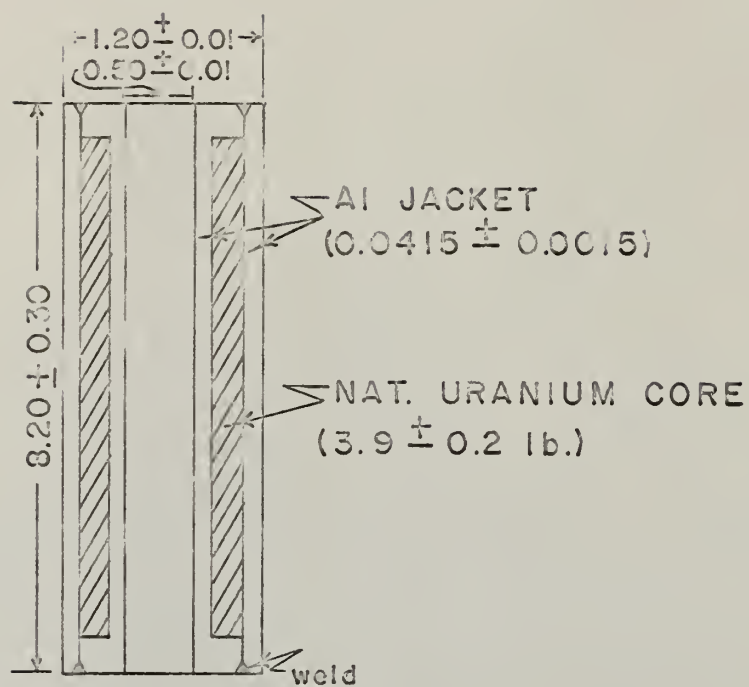
CROSS   SECTION

HOLLOW   SLUG

Fig. 3.   The physical dimensions of the fuel slugs.

Fig. 4. Fuel elements for KSU pile

Fig. 5.   KSU exponential pile.
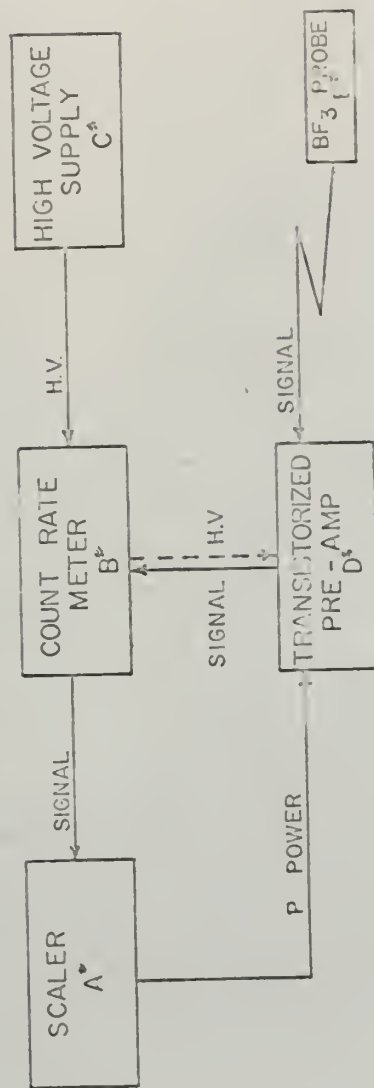
FIG. 6 FLOW DIAGRAM FOR NEUTRON DETECTION SYSTEM
* LETTERS CORRESPOND TO LABELS ON FIG. 8
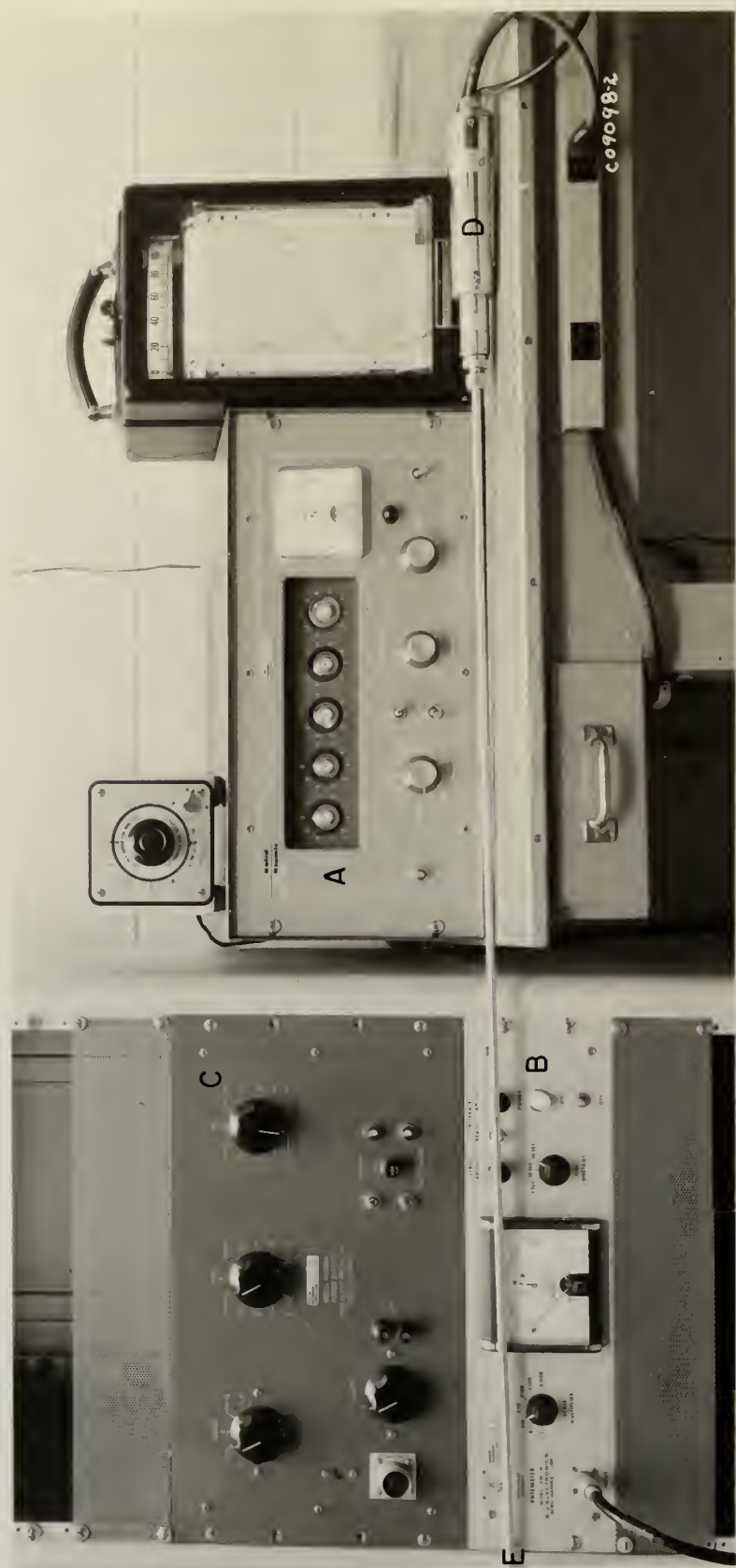
Table 2. A List of the Electronic Equipment Used

Component	Type and Model	Nuclear Engr. Inventory No.
BF_3 Probe	Nuclear-Chicago Model NC-202	No. 97
BF_3 Probe	Radiation Counting Lab. RCL-10504	No. 320
Pre Amplifier	Nuclear-Chicago Model 1062	No. 210
Count Rate Meter	B.J. Electronics Model DM1-D	No. 363
Scaler	Baird Atomics Model 132	No. 144
High Voltage Power Supply	John Fluke Model 400 BDA	No. 188
Timer	Baird Atomic Model 960	No. 147
Traversing Mechanism	K.S.U. Nuclear Engr.	No. 568

enrichment of 96 per cent. The Model NC-202 BF_3 probe with the associated counting system is shown in Fig. (7). Fig. (6) shows a block diagram of the neutron detection system.

The traversing mechanism, shown in Fig. (8), was capable of supporting the Model NC-202 BF_3 probe in a variety of positions in the pile. This mechanism was designed and built at Kansas State University and a detailed description of it is provided by L.R. Foulke (5).
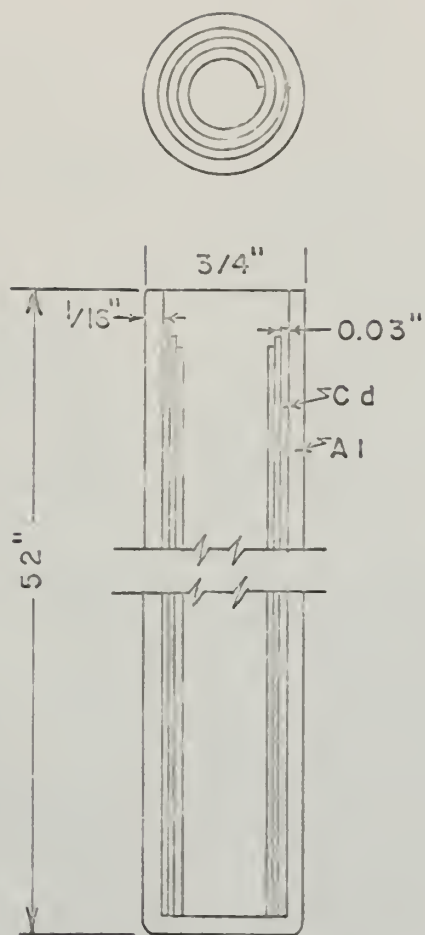
Fig. 9. The physical dimensions of the poison rod.

3.5 Cadmium Poison Rod

Fig. (9) is an illustration of the poison rod used in this work.
The 0.625 inch inside diameter aluminum tube contained a total of 2.4
pounds of 0.03 inch thick sheet cadmium. The sheet cadmium was shaped
into a cylindrical annulus with a thickness of 0.09 inch. Thus, the
actual poison element was a cylindrical annulus of dimensions 0.445
inch inside diameter, 0.625 inch outside diameter and 52 inches in
length. Aluminum was used for the containing tube so that as little poison
other than the actual cadmium was added to the reactor system as possible.

4.0 EXPERIMENTAL PROCEDURE

Vertical Traverse Measurements

All data were collected using the Model NC-202 BF_3 probe and
associated counting equipment which was previously described. The counter
was operated at 1400 volts with a pulse height sensitivity of 0.8.

Two separate vertical traverses were made, one without the poison
rod and one with the poison rod. For the traverse without the poison
rod, the BF_3 probe was positioned horizontally with its active volume
centered on the vertical or z axis. The vertical traverse was made with
the BF_3 probe in this position in order to obtain a maximum count rate.
Also, a mathematical simplification in the iterative analysis procedure
resulted from data obtained in this position.

For a mathematical simplification, the poison rod was assumed to
be located at the center of the reactor system. All the models used in

the analysis of these data assumed that the poison rod was located in this position. However, in the physical system, the poison rod was centered on a point 2.5 inches in the y dimension from the vertical axis. This can be seen from Fig. (1). In order to obtain as close as possible to a maximum count rate and for the mathematical simplification, the data collected with the poison rod inserted were taken at a position 2.5 inches from the center of the pile in the x direction.

Each count rate measurement was taken so that approximately 10,000 counts were collected at each z position. This was done so that each data point would have approximately the same variance. The z positions used for collecting data are shown in Fig. (1).

5.0   DATA PRESENTATION AND ANALYSIS

5.1   Presentation of Raw Data

The data taken for each of the vertical traverses were corrected for background counts. The data for each traverse, both with the rod and without the rod, are presented in Tables (3a) and (3b). The deviation reported is the standard deviation of each measurement.

5.2   Analysis of the Vertical Traverse Data

The data, presented in the previous section, were analysed so that the best estimate of γ_{11} was obtained by a least squares fit of the modified count rates. The IBM 1620 code, presented in Appendix E, was used to calculate these slope values.

Error variances of these slopes were calculated using a method given by Mickley, et. al. (15). The error variance of the slope is given by

Table 3a. The Actual Data (Corrected for Background) Collected With
           No Poison Rod

Distance From Source	Set No. 1	Set No. 2	Set No. 3
Inches	C/m.	C/m.	C/m.
42.156	638.5±6.3	641.1±6.3	643.7±5.7
50.156	401.2±3.7	394.4±3.6	393.0±3.6
58.156	250.1±2.0	252.6±2.1	250.7±2.1
66.156	101.0±1.6	161.1±1.6	156.7±1.6
74.156	103.2±0.91	101.0±0.39	100.5±0.37

Table 3a (continued)

Distance From Source	Set No. 4	Set No. 5	Set No. 6
Inches	C/m.	C/m.	C/m.
42.156	638.7±5.6	640.3±5.7	634.6±6.5
50.156	397.8±3.8	394.9±3.8	342.4±3.8
58.156	249.3±2.2	249.9±2.0	247.5±2.3
66.156	159.8±1.5	160.9±1.1	156.6±1.2
74.156	98.8±0.78	99.0±0.69	100.2±0.84

Table 3b. The Actual Data (Corrected for Background) Collected With
           the Poison Rod Inserted

Distance From Source	Set No. 1	Set No. 2	Set No. 3
Inches	C/m.	C/m.	C/m.
42.156	500.8±4.1	491.3±4.9	503.6±3.3
50.156	291.5±2.5	291.7±2.7	294.4±2.7
58.156	173.5±1.7	175.1±1.8	175.9±1.7
66.156	114.4±0.96	107.7±0.93	108.9±0.4
74.156	68.7±0.68	68.3±0.27	67.1±0.71

Table 3b (continued)

Distance From Source	Set No. 4	Set No. 5	Set No. 6
Inches	C/m.	C/m.	C/m.
42.156	530.4±5.3	500.5±5.0	486.6±2.9
50.156	301.0±2.9	290.0±2.8	304.4±2.3
58.156	188.7±1.9	171.6±1.7	176.8±1.8
66.156	113.7±1.1	105.7±0.81	107.9±1.0
74.156	69.2±0.33	68.3±0.66	67.3±0.77

$$S_e^2 (b) = \sum_{i=1}^{n_i} (Y_i - y_i)^2 \ / \ (n_i - 2) \sum_{i=1}^{n_i} z_i^2 . \qquad (106)$$

This equation was formulated on the basis that the data could be fit by a linear plot. In the present problem, the quantity y_i is the logarithm of the modified count-rate corresponding to position z_i. Y_i is calculated by

$$Y_i = a + b \ z_i \qquad (107)$$

where a and b are the least squares intercept and slope, respectively.

Mickley, et. al. (15) also present a method for finding the confidence limit envelope for a set of data. The estimate of the error variance of Y_i caused by a variance of y_i is given by

$$S_e^2 (Y_i) = \left\{ \sum_{i=1}^{n_i} (Y_i - y_i)^2 \ / \ (n_i - 2) \right\} \cdot \left\{ 1/n_i + z_i / \sum_{i=1}^{n_i} z_i^2 \right\} . \qquad (108)$$

Using the t test, the confidence limits of Y_i are given as

$$\text{Confidence limits of } Y_i = (\pm t) \ S_e (Y_i).$$

The term $(n_i - z)$ represents the number of degrees of freedom associated with $S_e^2 (Y_i)$. The number of degrees of freedom in this work was 3, giving a value of t = 3.182 corresponding to a 95 per cent confidence limit.

All the data, both with the rod and without the rod, are plotted in Figs. (10-17). The values given for γ were calculated from the least squares technique; error variances of the γ values and the 95 per cent confidence limits are presented on these plots.
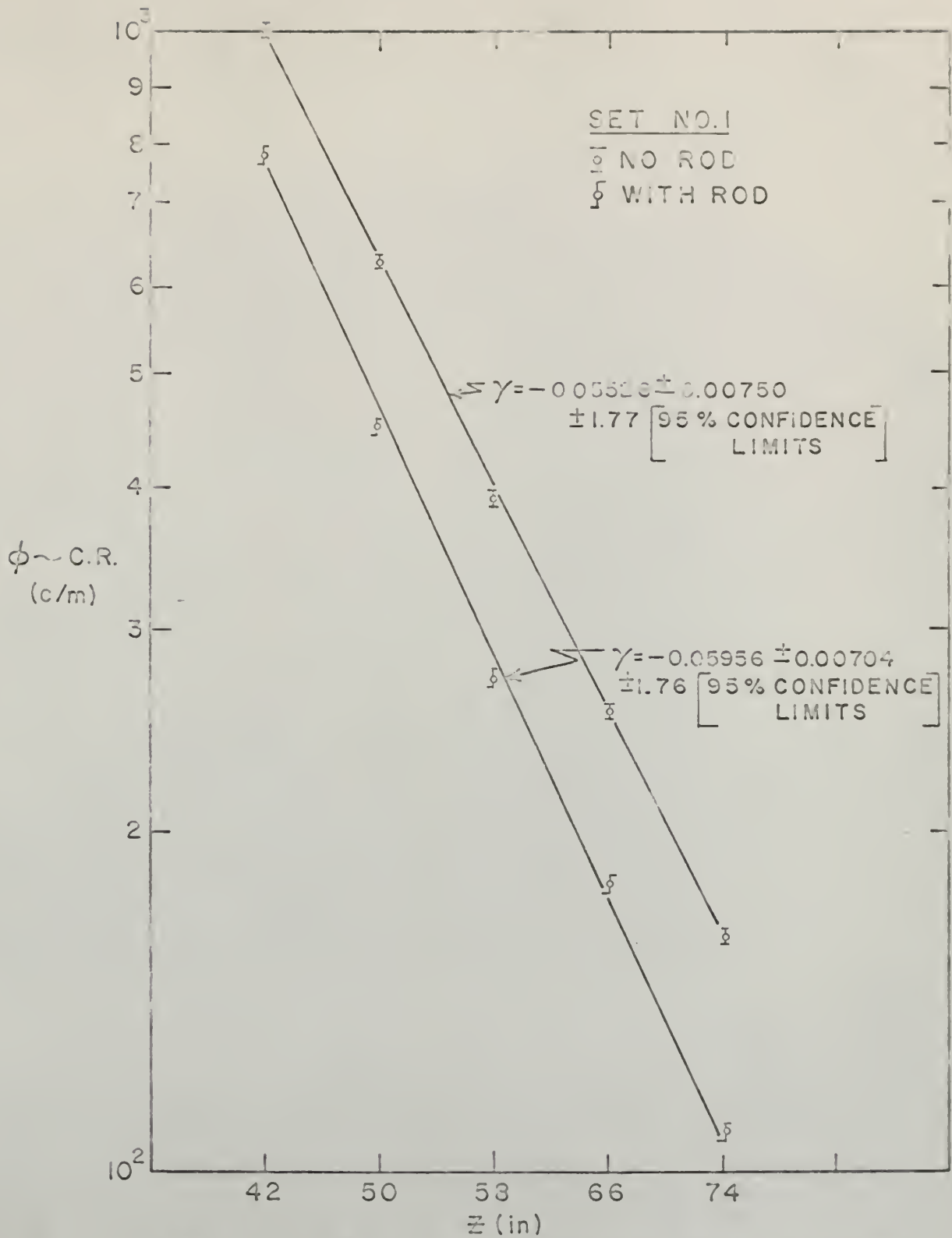
Fig. 10. A plot of the data presented in Tables 3a and 3b.
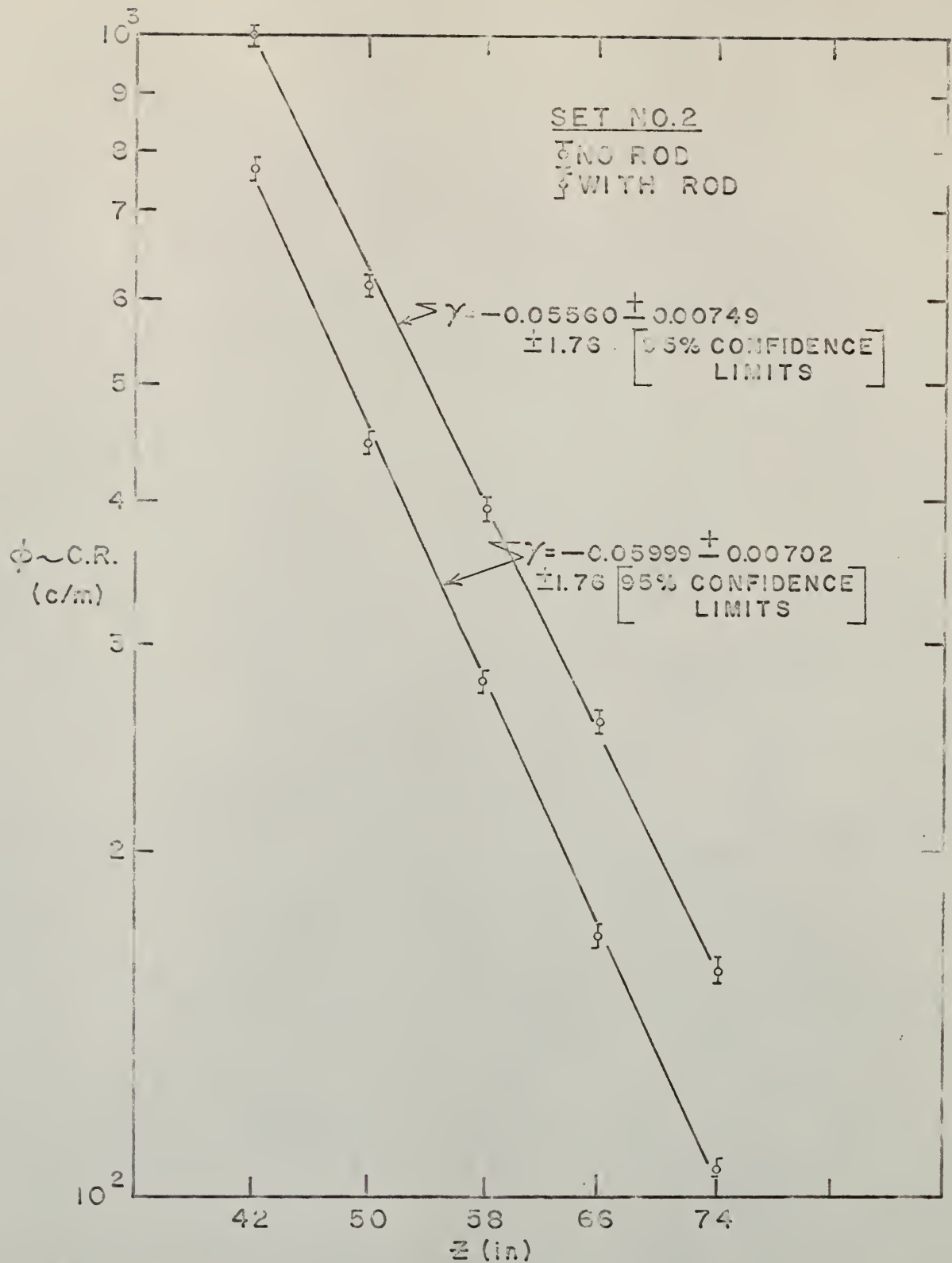
Fig. 11. A plot of the data presented in Tables 3a and 3b.
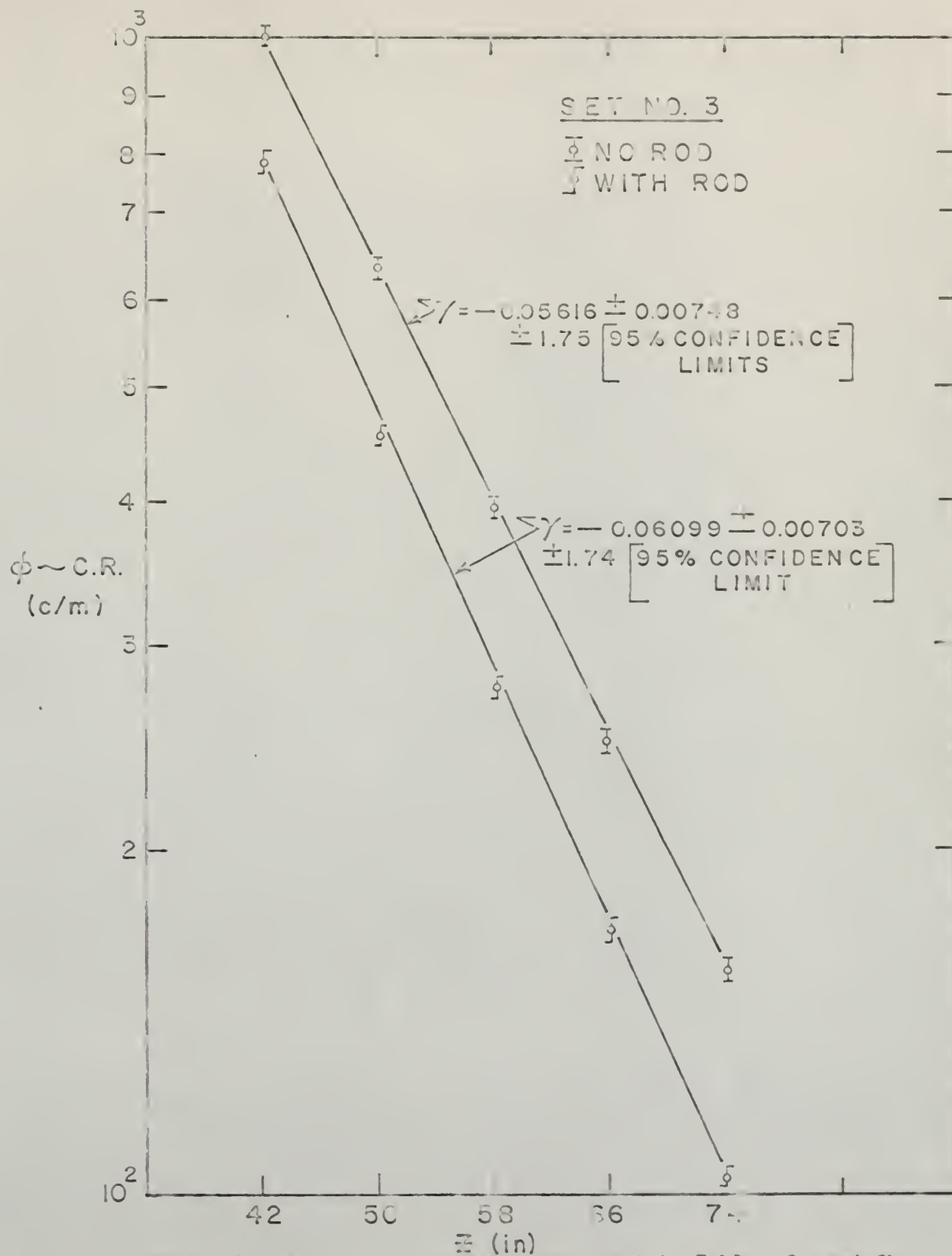
Fig. 12. A plot of the data presented in Tables 3a and 3b.
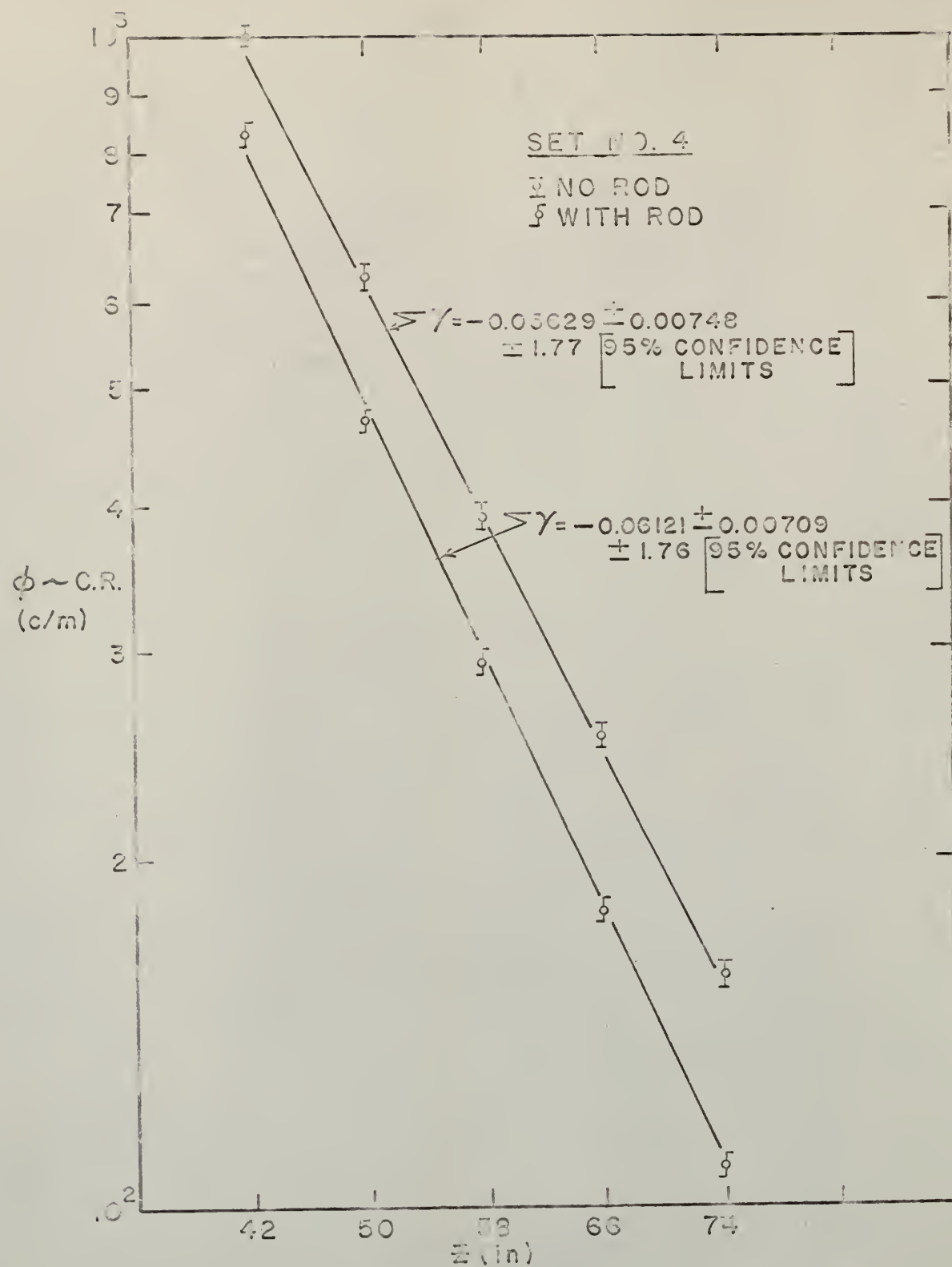
Fig. 13. A plot of the data presented in Tables 3a and 3b.
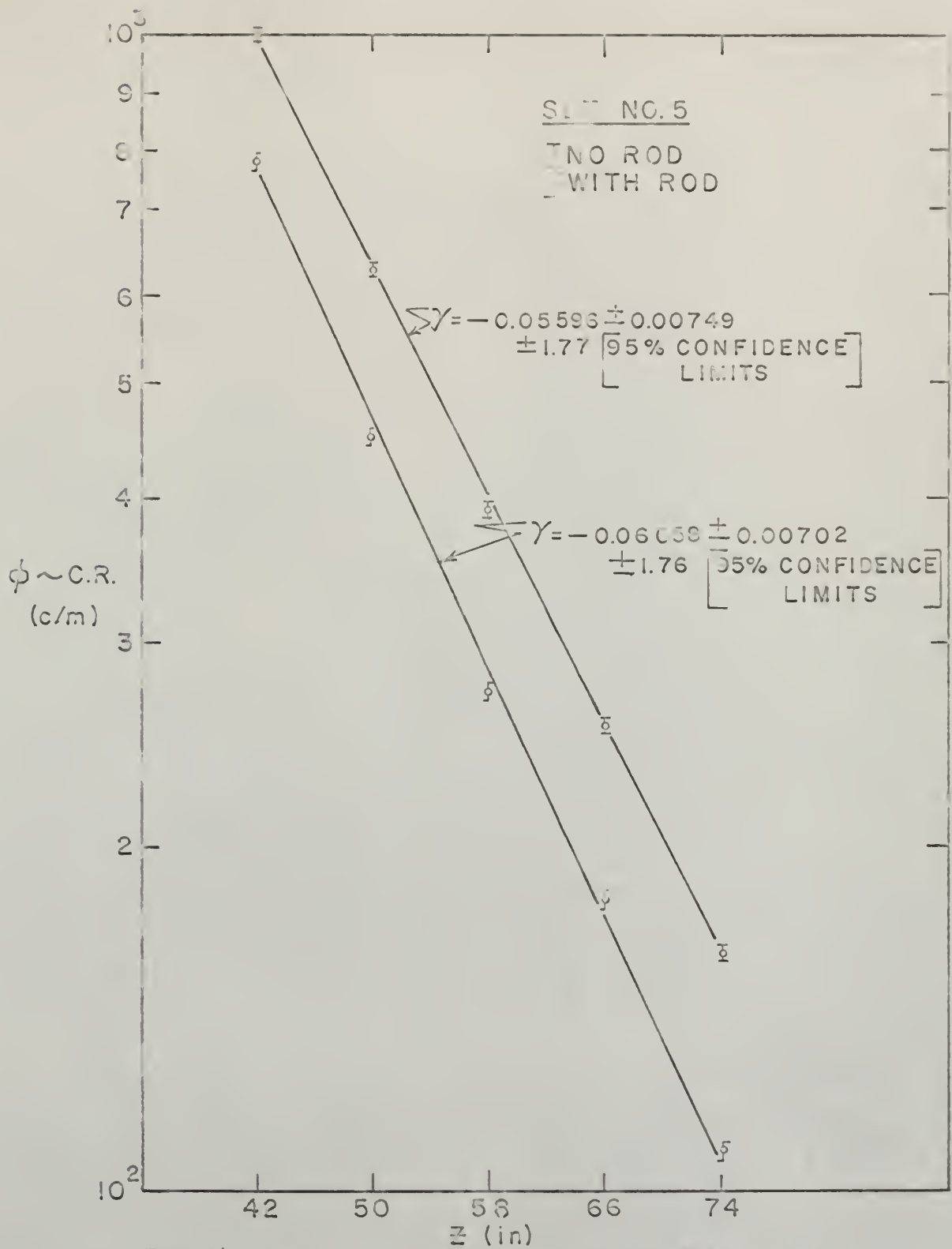
Fig. 14. A plot of the data presented in Tables 3a and 3b.
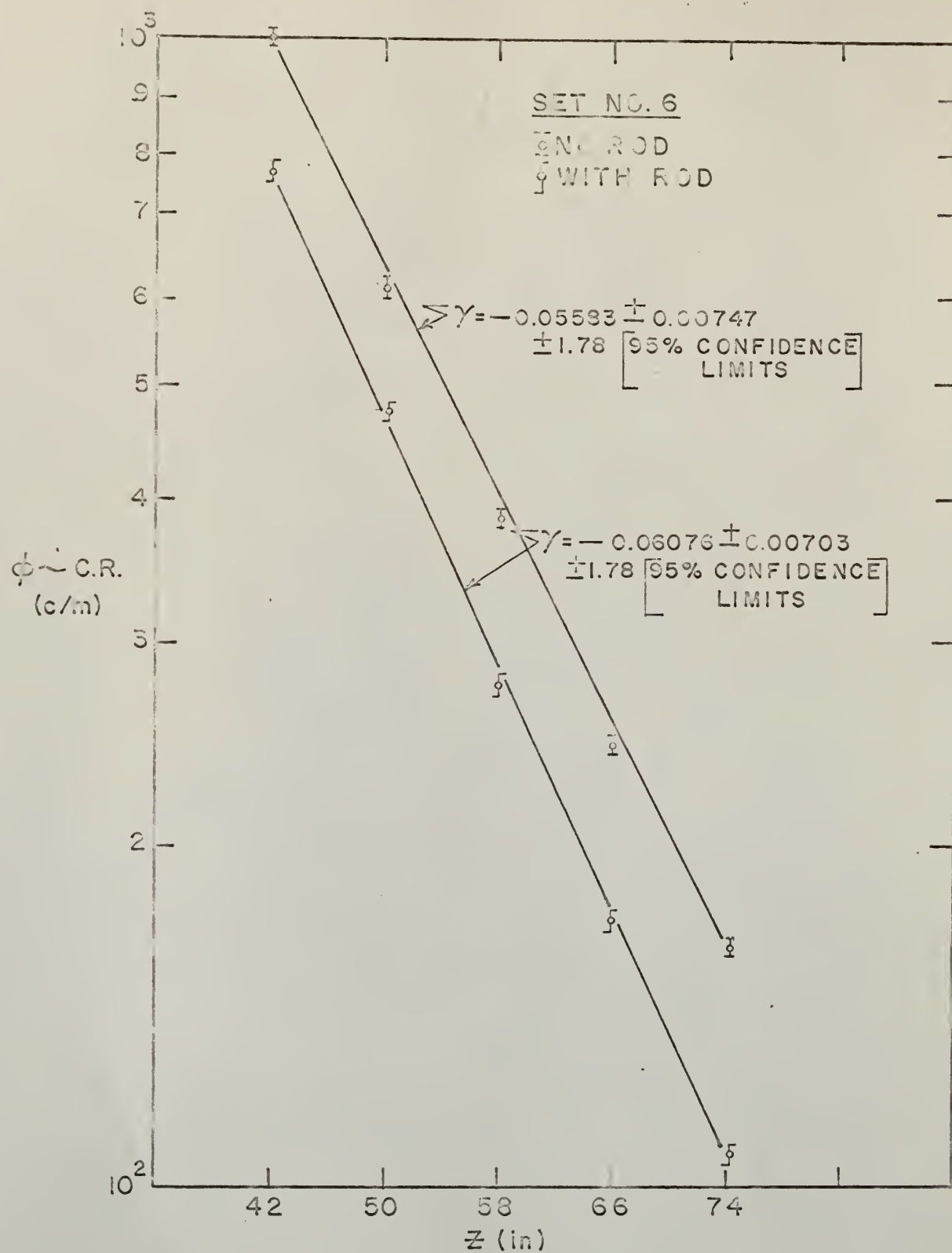
Fig. 15. A plot of the data presented in Tables 3a and 3b.
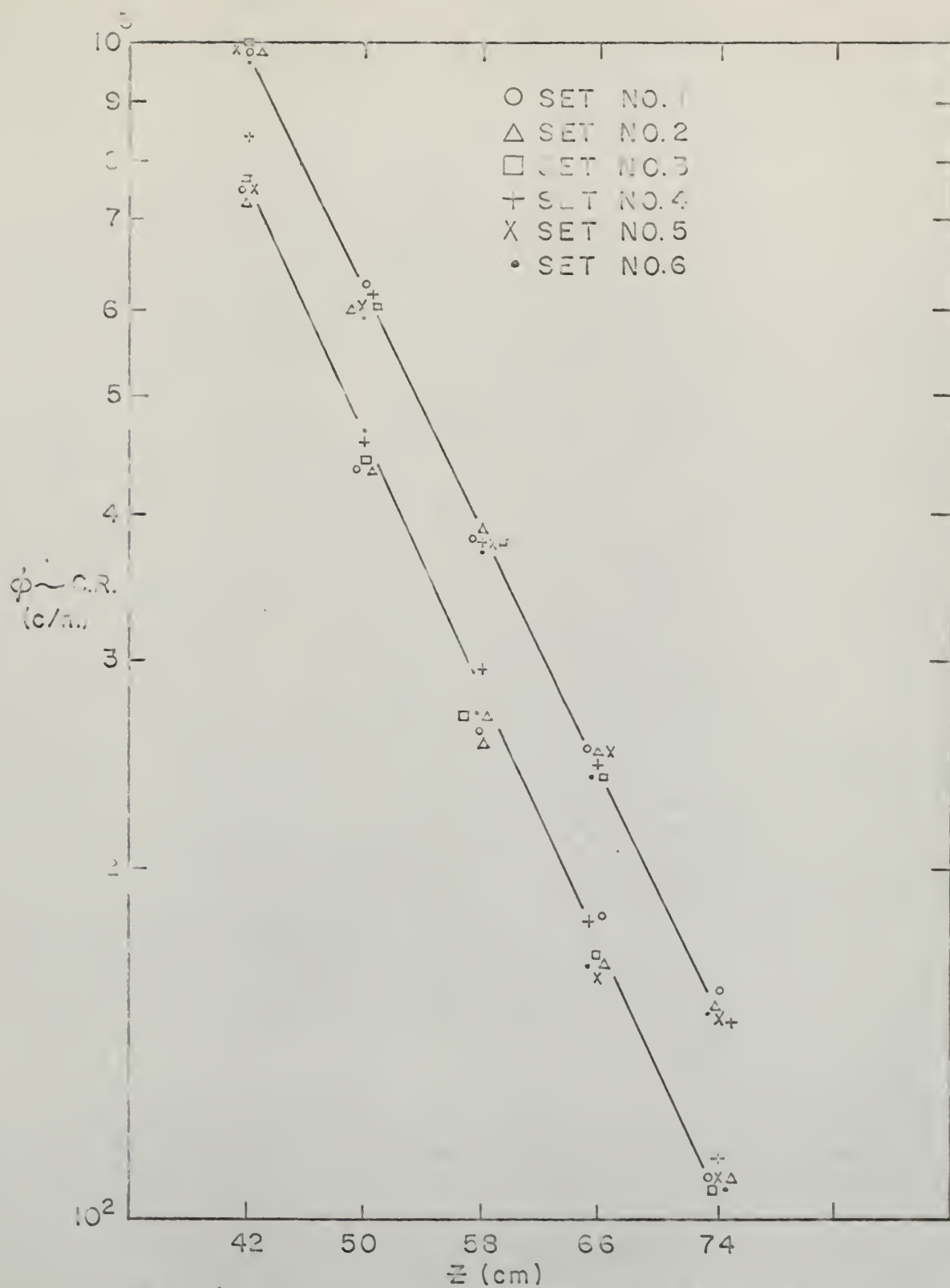
Fig. 16. A plot of the data presented in Tables 3a and 3b.

6.0 RESULTS AND CONCLUSIONS

6.1 Defining the Effective Pile Size and
Equivalent Cylindrical Pile Radius

In most reactor calculations a convenient boundary condition and
one that is often used is that the neutron flux vanishes at an ex-
trapolation distance past the physical boundary of the reactor. This
assumes that the negative neutron current, J_{x-}, equals zero at the
physical boundary of the reactor. For the case of a semi-infinite
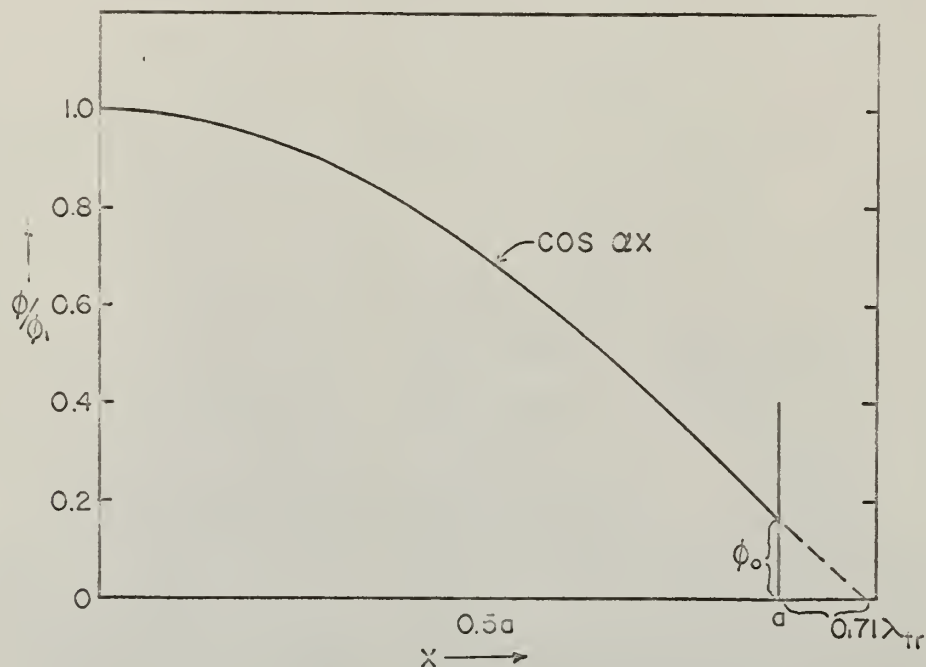
Fig 18. A representative flux plot in the x direction
for a cylindrical or rectangular parallelepiped
reactor.

slab, the flux shape in the x direction is a cosine function (see Fig.
(18)). The flux is also a cosine function in the x and y and r
directions for the rectangular parallelepiped and cylindrical geometries.
However, in the z direction for a cylindrical or rectangular
parallelepiped subcritical reactor, the flux shape is an exponential

function. A question then arises as to the validity of using 0.71 λ_{tr} for the extrapolation distance in the z direction.

Kaiser (12) described an experimental method for determining the extrapolation distance in the x and y dimensions. This method consisted of taking data near the boundary and fitting the data by a least squares technique to a cosine function. Then the cosine curve could actually be extrapolated to zero. Kaiser (12) concluded that his data were much too random for any definite conclusions. His calculations showed that the pile looked like an inverted pyramid. Therefore, he used 0.71 λ_{tr} for the extrapolation distance in both the x and y dimensions.

Based on Kaiser's conclusions, 0.71 λ_{tr} was used for the extrapolation distance, d, in the x and y dimensions in this work. The value used for λ_{tr} was 2.58 $\pm$ 0.09 cm. This was the same value which Foulke (5) and Kaiser (12) used. Thus, the effective x and y dimensions of the rectangular pile are 69.375 inches by 69.375 inches.

A study of the extrapolated height, c, was not made by Kaiser (12). Therefore, a technique to make this study was developed. This technique for determining the extrapolated height, c, consisted of a two parameter, γ_{11} and c, iterative procedure. This iterative procedure consisted of using Eq. (20) to calculate the extrapolated height, c, and the IBM 1620 iterative technique (12) to find the γ_{11} parameter. Using this method, an extrapolated height of 107.36 inches was determined.

An analysis was performed to show that for the external source positioning used in this work the technique described above for finding

the parameter, γ_{11}, will yield a better estimate of this parameter than the value obtained when using 0.71 λ_{tr} plus the physical boundary of the pile for the extrapolated height, c. Table (4) shows the results of this analysis.

The source positions used in this work were one source located at each of the following four positions in the source layer;

1. x = ± 22 inches, y = 0 inches

2. x = 0 inches, y = ± 22 inches.

These positions were approximately one half the distance between the pile's origin and outside boundaries. For this reason, the harmonic content of the parameter, γ_{11}, whould be negligible and consequently both the C_E and C_H factors should be very nearly unity.

Table 4. The Results of the Extrapolated Height Analysis

z, in.	Count Rates	Modified Count Rates Extrap. Height	C_E Extrap. Height	C_H Extrap. Height	Modified Count Rates 0.71 λ_{tr}	C_E 0.71 λ_{tr}	C_H 0.71 λ_{tr}
42.156	638.5	615.4	0.9992	1.038	619.8	0.9960	1.034
50.156	401.2	394.4	0.9982	1.019	398.2	0.9907	1.017
58.156	250.1	248.8	0.9957	1.009	253.4	0.9785	1.008
66.156	161.0	161.9	0.9894	1.005	168.7	0.9503	1.004
74.156	103.2	105.7	0.9744	1.002	116.4	1.8848	1.002

It can be seen from Table (4) that when $0.71 \lambda_{tr}$ is used for the
extrapolation distance in the z direction, the C_E factors are quite
different from unity near the top of the pile. When the iterative
procedure described previously for finding γ_{11} and Eq. (20) is used,
the calculated curve of ϕ versus z position is nearly linear and the
original data points are positioned very close to the calculated curve.
Also, simple least squares fits of linear curves to the data points
on semi-logarithmic plots yield slopes very close to the values
obtained for γ_{11} from the iterative technique. However, when $0.71 \lambda_{tr}$
is used for the extrapolation distance, the calculated curve is quite
different from the linear plot expected because of the source positioning.

As described in section 2.1.3, the equivalent cylindrical radius
was calculated from Eq. (22). The value found for this radius was
37.55 inches.

6.2 Calculation of the Control Rod Worth
of a Centrally Located Control Rod
in a Subcritical Reactor System by One Group Models

6.2.1 The Results of the Semi-Experimental and Calculational
Method for Finding the Change in the Effective Multiplication Factor.
The results of using the IBM 1620 program, which are presented in
Appendix E, to calculate the effective multiplication factor of the
K.S.U. graphite subcritical assembly are presented in Table (5). The
effective multiplication factor of the graphite pile with a centrally
located control rod was calculated by using Eq's. (18) and (24).

The values of Δk_e presented in Table (5) were calculated from six
different sets of count rate versus z position data. As can be seen from

Table 5. One Group Effective Multiplication Factors

Set Number	k_{e_0} No. Rod	k_{e_1} With Rod	$- \Delta k_e$	% Dev. From the Average
1	0.5824	0.5377	0.0447	+1.36
2	0.5787	0.5344	0.0443	-0.45
3	0.5740	0.5302	0.0438	-0.68
4	0.5729	0.5293	0.0436	-1.13
5	0.5757	0.5316	0.0441	0
6	0.5767	0.5325	0.0442	+0.23
Average			0.0441	

Tables (3a and 3b), section 5.1, the lowest data point was taken at a

z position of 42.156 inches from the source and the highest data point

was taken at a z position of 74.156 inches.

6.2.2 The Results of Calculating the Change in the Effective

Multiplication Factor by Experimentally Measuring the Inverse Relaxation

Lengths. The results of finding the change in the effective multiplication

factor, Δk_e, by measuring the change in relaxation length due to the

presence of the control rod, are presented in Table (6). Six sets of

count rate versus z position both with and without the control rod were

analyzed to find the Δk_e.

The first portion of the IBM 1620 program presented in Appendix E

was used to analyze each set of data to find the parameter, γ_{11}.

Table 6. Results of the Difference in Relaxation Lengths

Set	cm.$^{-1}$	cm.$^{-1}$	k_{e_0}	k_{e_1}	$-\Delta k_e$	Per Cent Dev.
1	0.021755	0.02345	0.5324	0.5506	0.0318	-6.74
2	0.021890	0.02362	0.5737	0.5464	0.0323	-5.28
3	0.022110	0.02401	0.5740	0.5387	0.0353	+3.52
4	0.022160	0.02410	0.5729	0.5367	0.0362	+6.16
5	0.022030	0.02385	0.5757	0.5417	0.0340	-0.29
6	0 021980	0.02385	0.5767	0.5418	0.349	2.35
Average					0.0341	

6.2.3 The Results of the One Group Approximation Method for
Finding the Change in the Effective Multiplication Factor. The change
in the effective multiplication factor was calculated using an
approximate method presented by Glasstone and Edlund (7). The re-
sults of these one group calculations are presented in Table (7). These
values are also plotted in Fig. (20).

It can be seen from Table (7) that this approximate method yields
quite high values for Δk_e compared to the results of the other models
used here. These results show that erroneous approximations were made
which make this a poor calculational model.

Table 7. The Calculated Results of the Approximate Methods

a_o/R_o $\times 10^4$	$-\Delta k_e$ One Group	$-\Delta k_e$ Two Group	$-\Delta k_e$ One Group
5	0.1350	0.0880	0.08365
10	0.1503	0.0985	0.09308
20	0.1095	0.1119	0.1049
30	0.1831	0.1215	0.1133
40	0.1941	0.1294	0.1202
50	0.2037	0.1363	0.1261
60	0.2122	0.1425	0.1314
70	0.2200	0.1482	0.1362
80	0.2272	0.1525	0.1407
85	0.2304	0.1560	0.1428
90	0.2340	0.1585	0.1449
100	0.2404	0.1633	0.1488

6.3 Calculation of Control Rod Worth of a Centrally Located Control Rod in a Subcritical Reactor System by Using the Two Group Models

6.3.1 The Results of the Two Group Calculational Method for Finding the Change in the Effective Multiplication Factor. The change in the effective multiplication factor was calculated by first finding the increase in the radial buckling due to the control rod. This increased buckling was then used in Eq. (33) to give Δk_e. The results of these calculations are presented in Table (8).

Table 8. The Results of the Two Group Method

Set Number	k_{e_1} No Rod	k_{e_2} With Rod	$-\Delta k_e$	% Dev. From the Average
1	0.5824	0.5525	0.0299	-0.33
2	0.5787	0.5485	0.0302	+0.67
3	0.5740	0.5442	0.0298	-0.67
4	0.5729	0.5432	0.0297	-1.00
5	0.5757	0.5458	0.0299	-0.33
6	0.5767	0.5463	0.0304	+1.33
Average			0.0300	

The values of k_{e_2} were found by using the k_∞'s calculated by the method of section 2.1. These values of k_∞ were used to calculate the major and minor two group bucklings, μ^2 and υ^2 respectively. Eq. (47) was then used to calculate the radius of a fictitious critical reactor. This radius was used to calculate the increase in the radial component of the buckling. The geometric buckling of the system with the control rod was calculated using these results. Eq. (52) was used to find Δk_e.

4.2.2 The Results of the Two Group Approximation Method for Finding the Change in the Effective Multiplication Factor. The two group approximate and corrected one group approximate equations derived and presented by Glasstone and Edlund ( ) were employed to calculate Δk_e, theoretically. These results are presented in Table (7). Also a comparative plot of the one group, two group, and corrected one group
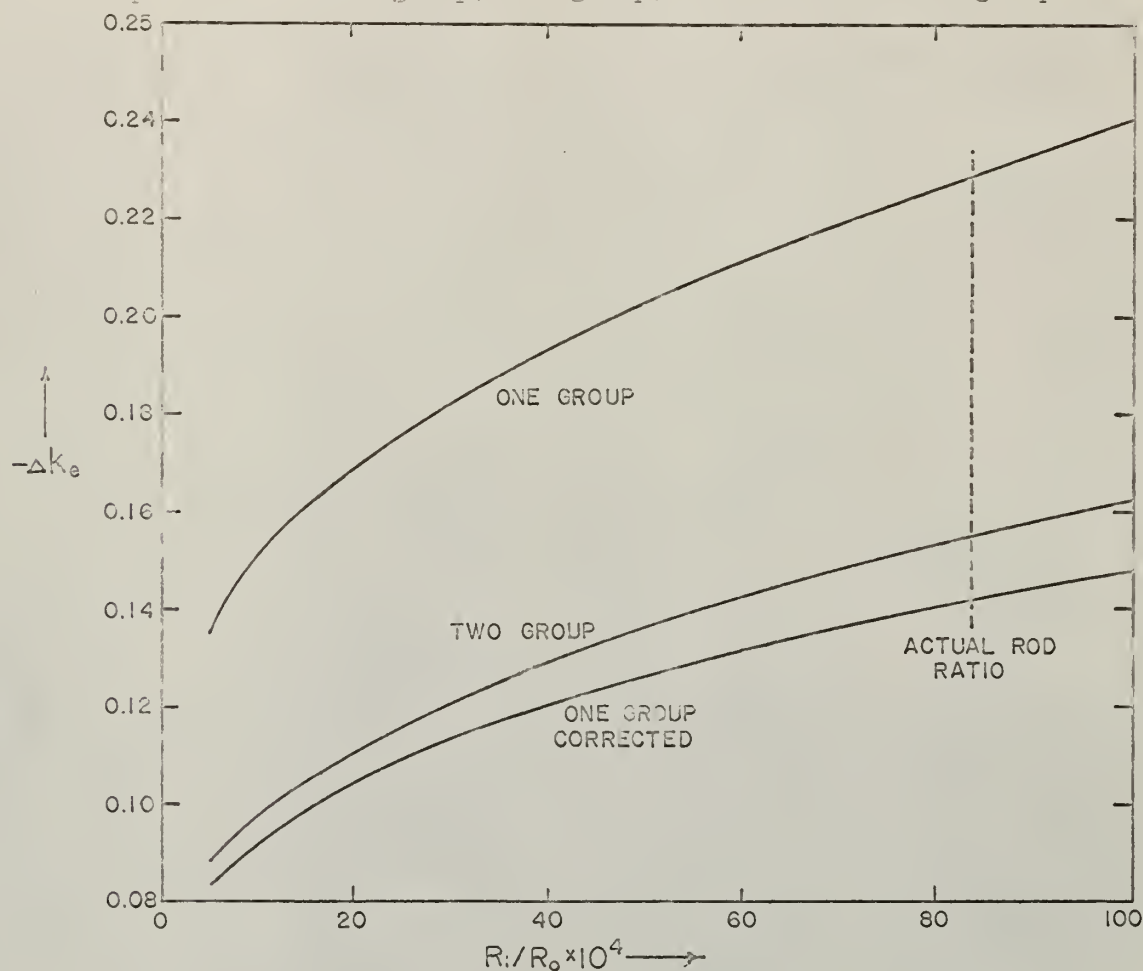

Fig. 20. The results of the approximate methods.

values for $-\Delta k_e$ versus the ratio of the control rod radius to reactor radius is presented in Fig. (20).

The results show the values for Δk_e to be quite high. This would seem to indicate that erroneous assumptions had been made in the derivation.

6.4 Perturbation Calculations

The change in the effective multiplication factor due to the

insertion of the cadmium annulus was calculated by using Eq. (74),

section 2.3. As was explained in section 2.3, a one group or

multigroup calculation yields identical results for the case of a

bare reactor. Since the cadmium annulus was in excess of 0.03 inch

thick, the inner radius, r_o, in Eq. (74) was varied for calculational

purposes. The average value of k_{e_1} calculated by the method of section

2.1 was also used. Therefore, a plot of Δk_e ($= k_{e_2} - k_{e_1}$) versus

Δr ($= r_1 - r_o$) was made. This plot is presented in Fig. (21).

It can be seen that from the first order perturbation assumption

(the perturbed and unperturbed fluxes are approximately equal) Δk_e is

linear with respect to the cadmium thickness. This means that the last

1 mil increment of cadmium is just as effective as the first 1 mil in-

crement. However, most references (7, 14, and 17) state that 0.03

inch is "black" or acts as an absolute sink to thermal neutrons. This

means that the Δk_e versus Δr plot should reach a constant value for

a Δr of 0.03 inch. Because the first order perturbation calculations

don't yield these results, another method of analysis was sought and

is presented below.

In section 2.3 there is presented a method for evaluating the constants

of a flux function synthesized from orthogonal functions. This method

will yield a better approximation to the perturbed flux. Therefore, a

better value for k_{e_2} can be calculated. Eq. (105) was used to evaluate

the orthogonality constants. The orthogonal functions used were zero-

order Bessel functions of the first kind with arguments such as $\alpha_n r$,
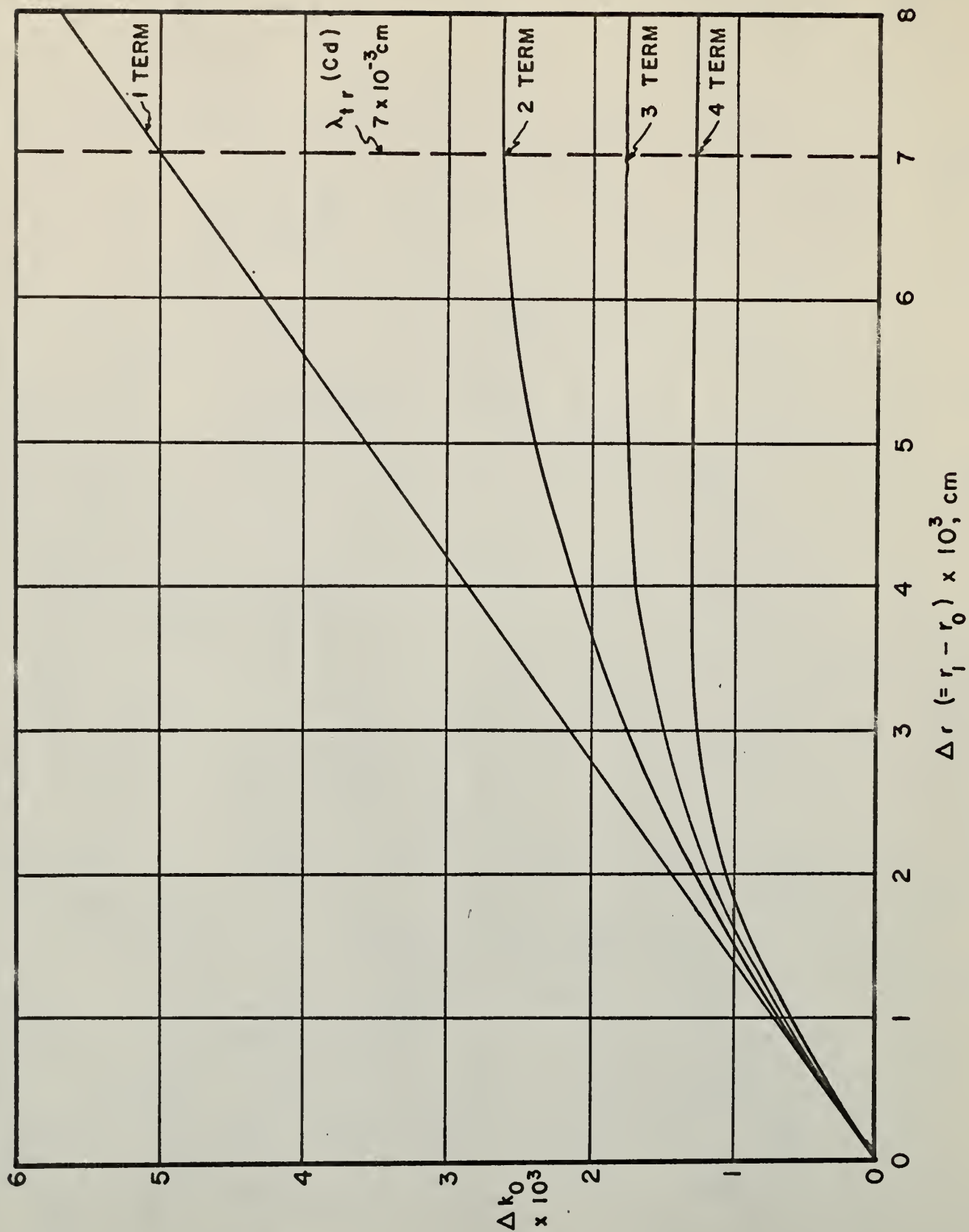
Fig. 21. The Results of the Improved Perturbation Calculations

where α_n is j_n/R_0. The j_n is the n-th zero of the J_0 Bessel function and R_0 is extrapolated equivalent radius. These new flux functions were then used as the perturbed fluxes in Eq. (74). The results of the Δk_e versus Δr calculations are presented in Fig. (21).

It can be seen that the two term flux approximation yields results which are in keeping with theoretical expectations. It can also be seen that the cadmium in excess of one transport mean free path (0.007 cm.) is not effective in decreasing the effective multiplication factor. Fig. (21) shows that for the three and four term flux approximations the maximum Δk_e for the cadmium poison is decreased. Furthermore, the Δr after which further addition of cadmium is in-effective, is also decreased. Actually, the results of using the three and four term flux approximations showed the Δk_e values reached a maximum at Δr values of approximately 0.005 cm. and 0.0035 cm., respectively.

If Fig. (21) is carefully noted, it can be seen that the maximum values of Δk_e is less when more terms are assumed for the flux function. However, the difference in maximum values becomes less as the number of terms increases. If an infinite number of terms is assumed for the flux function, an exact representation of the per-turbed flux is obtained. This flux function will yield an exact curve for Δk_e versus Δr.

6.5 General Comparisons

Table (9) is a comparison of the Δk_e values of the cadmium annulus in the K.S.U. graphite, calculated by using experimental data from three models.

Table 9. Comparison of Three Semi-Experimental Methods

Set Number	$-\Delta k_e$ One Group Sec. 2.1.3	$-\Delta k_e$ Relaxation Length Sec. 2.1.4	$-\Delta k_e$ Two Group Sec. 2.2.1
1	0.0447	0.0318	0.0299
2	0.0443	0.0323	0.0302
3	0.0438	0.0353	0.0298
4	0.0436	0.0362	0.0297
5	0.0441	0.0340	0.0299
6	0.0442	0.0349	0.0304
Average	0.0441	0.0341	0.0300

It can be seen from Table (9) that the one group model yields higher values than the relaxation length model. The two group model gives Δk_e values lower than both the one group and relaxation length models.

Since the one group and relaxation length models assume all neutrons are of thermal energy, these models should yield higher values for Δk_e than the two group model. However, the two group model accounts for the

neutron energy spectrum better, consequently, the Δk_e values calculated by this model are more accurate. The one group model uses a theoretical method to find the increase in radial buckling and hence, the Δk_e value. The relaxation length model makes use of an experimentally determined change in relaxation length to calculate the Δk_e value of the cadmium poison in the K.S.U. graphite pile. Therefore, the relaxation model yields a more accurate value for Δk_e than the one group model.

As was explained in the perturbation section, an exact solution for Δk_e could be calculated if the perturbed flux could be represented exactly. It can be seen from Fig. (21) that the series representation of the perturbed flux yields a Δk_e value which is less than the two group Δk_e value. If an infinite series were used to represent the perturbed flux, an exact value for Δk_e could be calculated based on the improved perturbation model. By comparing Fig. (21) and Table (9), it can be seen that the two group model and the improved perturbation model yields similar values for the Δk_e of the cadmium poison in the Kansas State University graphite subcritical reactor system.

7.0 SUGGESTIONS FOR FURTHER INVESTIGATION

This work has served as a good introduction to the theory and experiments involving control rod statics and kinetics in subcritical assemblies. All of this work can possibly be extended to a different reactor system, such as the water moderated, natural uranium system available at Kansas State University.

In addition to the static methods presented in this work, further refinements could be developed if transport theory calculations were used to determine Δk_e. Different functions could be used for the series representation of the perturbed flux and Δk_e could be calculated using the series representation in the perturbation calculations.

The two kinetic methods presented by Hogan (10) and the method developed by Kimel, et. al. (13) should be investigated further for possible extension to the subcritical reactor. Absorbers ranging from "gray" to "black" could be inserted into the reactor. The methods presented above could be applied to these absorbing rods.

ACKNOWLEDGMENT

LITERATURE CITED

1. Baraff, G. A., Murray, R. L. and Menius, A. C., Jr.
    Perturbation Integrals for Two Group Calculations and
    Applications to Reflector Ducts. Nuc. Sci. and Eng., 4(5)
    623-33 (1958).

2. Drake, M. K.
    Theoretical Reactor Kinetic Models and Experimental Verification.
    M.S. Thesis. Kansas State University (1960).

3. Dogget, W. O.
    Private Communication.

4. Etheringtion, H.
    Nuclear Engineering Handbook. McGraw-Hill (1958).

5. Foulke, L. R.
    K.S.U. Pile Standardization and Study of Slowing Down and Diffusion
    Models. Kansas Engineering Experiment Station. Manhattan, Kansas
    (1961).

6. Glasstone, S.
    Principles of Nuclear Reactor Engineering. Van Nostrand (1955).

7. Glasstone, S. and Edlund, M. E.
    The Elements of Nuclear Reactor Theory. Van Nostrand (1952).

8. Henry, A. F.
    The Application of Reactor Kinetics to the Analysis of Experiments.
    Nuc. Sci. and Eng., 3(1) 52-70 (1958).

9. Hoag, J. B.
    Nuclear Reactor Experiments. Van Nostrand (1958).

10. Hogan, W. S.
    Negative Reactivity Measurements, Nuc. Sci. and Eng., 8(6) 518-22
    (1960).

11. Hughes, D. J.
    Pile Neutron Research. Addison-Wesley (1953).

12. Kaiser, R. E.
    Experimental and Theoretical Investigation of the Determination
    of the Diffusion Length of Thermal Neutrons in Graphite. Kansas
    Engineering Experiment Station. Manhattan, Kansas (1961).

13. Kimel, W. R., Carey, W. E., Prohammer, F. G. and Baldwin, G. C.
    Determination of Time Behavior of Neutron Density and of
    Reactivity on the Argonaut Reactor. Nuc. Sci. and Eng., 6(3) 233-7
    (1959).

14. Meghreblian, R. V. and Holmes, D. K.
    Reactor Analysis. McGraw-Hill (1960).

15. Michley, H. S., Sherwood, T. K. and Reed, C. E.
    Applied Mathematics in Chemical Engineering. McGraw-Hill (1957).

16. Mowery, A. L., Jr. and Murray, R. L.
    A Generalized Variational Method for Reactor Analysis, A Ph.D.
    dissertation, North Carolina State College (1961).

17. Murray, R. L.
    Nuclear Reactor Physics. Prentice-Hall (1959).

18. Murray, R. L. and Niestlie, J. W.
    Reactor Control-Rod Theories. Nucleonics, 13(2) 18-22 (1955).

19. Price, W. J.
    Nuclear Radiation Detection. McGraw-Hill (1958).

20. Snedecor, G. W.
    Statistical Methods. Iowa State College (1959).

21. Steichen, C. U.
    The Experimental Determination and Analytic Verification of the
    Age of Pu-Be Source Neutrons in Graphite. M.S. Thesis. Kansas
    State University (1960).

22. Van Artsdalen, J. W.
    Notes on Electronic Analog Computers-Scaling and Mechanization.
    The Boeing Airplane Company: Wichita Division Analog Computer
    Laboratory (1959).

23. Weinberg, A. M. and Wigner, E. P.
    The Physical Theory of Neutron Chain Reactors. Chicago: University
    of Chicago (1958).

24. Wayland, Harold.
    Differential Equations Applied in Science and Engineering.
    Van Nostrand (1957).

25. Wood, D. E.
    Hanford Progress Report, HW-64866.

APPENDICES

APPENDIX A

Solution to the Thermal Diffusion Equation

The procedure for the solution to the thermal diffusion equation is shown in Fig. (A-1). The equation to be solved in a multiplying medium is

$$\nabla^2 \phi \ + B_m^2 \, \phi = 0. \tag{A-1}$$
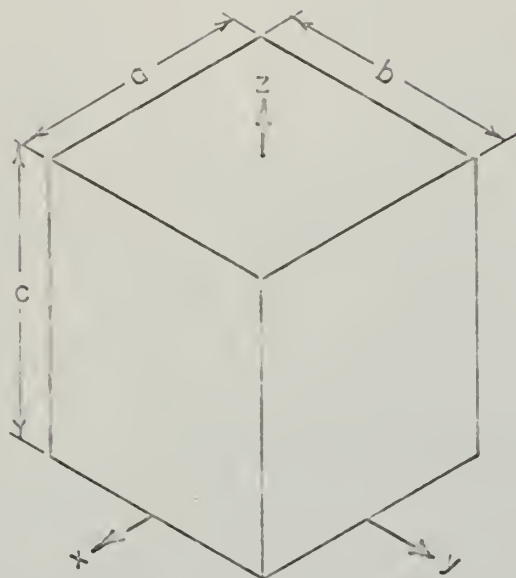
Fig. A-1. The rectangular parallelepiped geometry used in this work.

The solution is subject to the boundary conditions that the flux, ϕ, be zero at all extrapolated boundaries. These boundary conditions may be stated mathematically as

$$\phi (\pm a/2,y,z) = \phi(x,\pm b/2,z) = \phi(x,y,c) = 0$$

where a, b, and c are the extrapolated x, y, and z dimensions of the pile.

Eq. (A-1) may be written in the form

$$(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2) + B_m^2 = 0. \tag{A-2}$$

Assuming the variables are separable, a solution of Eq. (A-2) should exist such that

$$\phi(x,y,z) = X(x) \, Y(y) \, Z(z) \, . \tag{A-3}$$

Substituting this solution into Eq. (A-2), the diffusion equation becomes

$$YZX'' + XZY'' + XYZ'' + B_m^2 XYZ = 0. \tag{A-4}$$

Dividing both sides of this equation by XYZ gives

$$X''/X + Y''/Y + Z''/Z + B_m^2 = 0. \tag{A-5}$$

In order to meet the boundary conditions let each term of Eq. (A-5) be equal to a constant as follows

$$X''/X = -\alpha_m^2 \tag{A-6}$$

$$Y''/Y = -\beta_n^2 \tag{A-7}$$

$$Z''/Z = \gamma_{mn}^2 \, . \tag{A-8}$$

Rewriting Eq. (A-6) gives

$$X'' + \alpha_m^2 \, X = 0 \, . \tag{A-9}$$

A solution of this equation is

$$X(x) = A_m \cos \alpha_m x. \tag{A-10}$$

To meet the boundary condition that $X(a/2) = 0$, α_m must be given by

$$\alpha_m = m\pi/a, \quad m = 1,3,5, \ldots \qquad \text{(A-11)}$$

so that the solution for $X(x)$ is

$$X(x) = A_m \cos m\pi x/a \ , \quad m=1,3,5, \ldots \qquad \text{(A-12)}$$

Similarly for $Y(y)$

$$Y(y) = A_n \cos n\pi y/a \ , \quad n = 1,3,5, \ldots \qquad \text{(A-13)}$$

Rewriting Eq. (A-8) gives

$$Z'' - \gamma_{mn}^2 Z = 0 \ . \qquad \text{(A-14)}$$

Eq. (A-14) has a solution given by

$$Z(z) = C_{mn} \sinh \gamma_{mn} z \ . \qquad \text{(A-15)}$$

This function may be made to fit the boundary condition by altering the argument to give

$$Z(z) = C_{mn} \sinh \gamma_{mn} (c-z) \ . \qquad \text{(A-16)}$$

Thus, the solution for the flux, φ , is found by combining the solutions for $X(x)$, $Y(y)$, and $Z(z)$ as a product solution.

$$\varphi(x,y,z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos m\pi x/a \cos n\pi y/a \sinh \gamma_{mn}(c-z) \quad \text{(A-17)}$$

The quantities m and n are summed over odd integers. Substituting the constants into Eq. (A-5), the auxiliary separation constants equation is obtained and can be written as

$$-\alpha_m^2 - \beta_n^2 + \gamma_{mn}^2 = -B_m^2 \ .$$

Then $n\pi/a$ and $m\pi/b$ can be substituted for β_n and α_m. The result is

$$B_m^2 + \zeta_{mn}^2 - (n\pi/a)^2 - (m\pi/b)^2 = 0. \qquad (A-18)$$

B_m^2 is the material buckling of the pile.

APPENDIX B

Solution to the Two Group Diffusion Equations

The equations to be solved, using cylindrical geometry, are

$$D_1 \nabla^2 \phi_1 - \Sigma_R \phi_1 + k_\infty \Sigma_a \phi_2 = 0 \qquad (B-1)$$

$$D_2 \nabla^2 \phi_2 - \Sigma_a \phi_2 + \Sigma_R \phi_1 = 0 \qquad (B-2)$$

where ϕ_2 is the thermal flux

ϕ_1 is p times the fast flux without resonance capture

D_1, D_2 are the fast and thermal diffusion coefficients

Σ_R is a slowing down cross section for the fast neutrons

Σ_a is the thermal neutron absorption cross section

k_∞ is the infinite multiplication factor.

The homogeneous parts of Eq's. (B-1) and (B-2) will satisfy equations of the type

$$\nabla^2 \phi_1 + B^2 \phi_1 = 0 \qquad (B-3)$$

$$\nabla^2 \phi_2 + B^2 \phi_2 = 0 . \qquad (B-4)$$

The quantity B^2 used in both Eq's. (B-3) and (B-4) is the same term. To illustrate the validity of this statement, Eq. (B-2) may be solved for ϕ_1 in terms of ϕ_2 and this result substituted into Eq. (B-2). The first of these manipulations results in an expression involving only ϕ_2 and the second equation is one involving only ϕ_1. In both equations, the operator parts are identical.

From Eq's. (B-3) and (B-4), the quantities $-B^2 \phi_1$ and $-B^2 \phi_2$ may be inserted in Eq's. (B-1) and (B-2) for $\nabla^2 \phi_1$ and $\nabla^2 \phi_2$, respectively. The results after rearranging are

$$-(D_1 B^2 + \mathcal{E}_R)\, \varphi_1 + k\mathcal{E}_a \varphi_2 = 0 \qquad\qquad (B\text{-}5)$$

$$\mathcal{E}_R \phi_1 - (D_2 B^2 + \mathcal{E}_a)\, \varphi_2 = 0. \qquad\qquad (B\text{-}6)$$

Since Eq's. (B-5) and (B-6) are simultaneous equations, both equal to

zero, the determinant of their coefficients must be equal to zero, if

a non-trivial solution exists. Setting this determinant equal to zero,

expanding the determinant, and rearranging the result, the following

quadratic equation in B^2 is obtained.

$$\left[B^2\right]^2 + (1/\tau + 1/L^2)\, B^2 - \left[k_\infty - 1/\tau L^2\right] = 0 \qquad\qquad (B\text{-}7)$$

The quantities L^2 and τ are the thermal diffusion length $(D_2/\mathcal{E}_a)$ and

Fermi age $(D_1/\mathcal{E}_R)$, respectively.

Using the quadratic formula, Eq. (B-7) can be solved for the two

roots of B^2. These two roots of B^2 are given by the following expression

$$B^2 = \begin{Bmatrix} \mu^2 \\ \upsilon^2 \end{Bmatrix} = \tfrac{1}{2}\left[-(1/\tau + 1/L^2) \pm \sqrt{(1/\tau + 1/L^2) + 4(k_\infty - 1)/\tau L^2}\right] (B\text{-}8)$$

Using these quantities for B^2 in Eq's. (B-3) and (B-4) and defining $X(r)$

and $Y(r)$ to be the flux components corresponding to μ^2 and υ^2,

respectively, these equations become

$$\nabla^2 X(r) + \mu^2 X(r) = 0 \qquad . \qquad\qquad (B\text{-}9)$$

$$\nabla^2 Y(r) - \upsilon^2 Y(r) = 0 \; . \qquad\qquad (B\text{-}10)$$

The solutions to Eq's. (B-1) and (B-2) are linear combinations of

$X(r)$ and $Y(r)$. The permissible solutions for $X(r)$ and $Y(r)$ in cylindrical

geometry are

$$X(r) = A\left[J_0(\alpha_1 r) + N Y_0(\alpha_1 r)\right] \qquad (B\text{-}11)$$

$$Y(r) = C\left[I_0(\alpha_2 r) + MK_0(\alpha_2 r)\right]. \qquad (B\text{-}12)$$

For a critical cylindrical reactor of finite height, the auxiliary separation constants equations define α_1 and α_2 as

$$\alpha_1^2 = \mu^2 - (\pi/c)^2 \qquad (B\text{-}13)$$

$$\alpha_2^2 = \upsilon^2 + (\pi/c)^2. \qquad (B\text{-}14)$$

Since ϕ_1 and ϕ_2 are linear combinations of $X(r)$ and $Y(r)$, the expressions for ϕ_1 and ϕ_2 can be written as

$$\phi_1 = AX(r) + C Y(r) \qquad (B\text{-}15)$$

$$\phi_2 = A'X(r) + C' Y(r). \qquad (B\text{-}16)$$

Eq's. (B-15) and (B-16) represent the most general solutions to Eq's. (B-1) and (B-2). However, permissible solutions exist such that $\phi_1 = AX(r)$ and $\phi_2 = A'X(r)$. If these solutions are substituted into either Eq. (B-1) or (B-2), a relationship between A and A' can be shown. Using only Eq. (B-1) and rearranging the result, the ratio A'/A, the coupling coefficient S_1, is given as

$$S_1 = A'/A = \Sigma_R/[D_2 \mu^2 + \Sigma_a]. \qquad (B\text{-}17)$$

Now if only $\phi_1 = CY$ and $\phi_2 = C'Y$ are substituted into Eq. (B-1), a second coupling coefficient, S_2, is given by

$$S_2 = C'/C = \Sigma_R/[\Sigma_a - D_2 \upsilon^2]. \qquad (B\text{-}18)$$

Therefore, the general solutions of Eq's. (B-1) and (B-2) are

$$\phi_1 = AJ_0(\alpha_1 r) + A_0 Y_0(\alpha_1 r) + CI_0(\alpha_2 r) + C_0 K_0(\alpha_2 r) \qquad \text{(B-19)}$$

$$\phi_2 = S_1 \left[AJ_0(\alpha_1 r) + A_0 Y_0(\alpha_1 r) \right] + S_2 \left[(I_0(\alpha_2 r) + C_0 K_0(\alpha_2 r) \right]. \qquad \text{(B-20)}$$

APPENDIX C

Derivation of the Perturbation Equations

Good discussions of perturbation theory can be found in many references (1,7,14,16,17,23). Glasstone and Edlund (7) present a derivation of the perturbation equations which is general enough to include both one group and two group perturbation theory. Therefore, this derivation will be followed quite closely in the following discussion.

In general, the diffusion equations can be written, using the compact matrix notation, as

$$M \underline{\phi} = \partial \underline{\phi} / \partial t \qquad (C-1)$$

where M is the matrix operator and $\underline{\phi}$ is a vector set of fluxes. If only one group of neutrons is considered, M and $\underline{\phi}$ become

$$M = v \left[\nabla^2 + \Sigma_a / D \, (k_\infty - 1) \right] \qquad (C-2)$$

$$\underline{\phi} = \phi(r,t) \qquad (C-3)$$

whereas, if two groups of neutrons are considered, these quantities are written as

$$M = \left\| \begin{array}{cc} v_1 (D_1 \nabla^2 - \Sigma_R) & v_1 k \Sigma_a \\ v_2 \Sigma_R & v_1 (D_2 \nabla^2 - \Sigma_a) \end{array} \right\| \qquad (C-4)$$

$$\underline{\phi} = \left\| \begin{array}{c} \phi_1(r,t) \\ \phi_2(r,t) \end{array} \right\| . \qquad (C-5)$$

Consider a solution of Eq. (C-1) of the type

$$\bar{\Phi} = \exp(\omega t)\, \bar{\phi}_r$$ (C-6)

where $\bar{\phi}_r$ is a vector flux set depending only on space and ω is the same for every group since the groups are coupled by the fission chain reaction. Using Eq. (C-6), Eq. (C-1) can be written as

$$M\, \bar{\phi}_r = \omega\, \bar{\phi}_r.$$ (C-7)

The quantity ω is actually the reciprocal of the reactor period. Eq. (C-7) will yield a series of eigenvalues, ω_k, and a set of corresponding eigenfunctions, $\phi_k(r)$. The eigenvalues will have only one positive value corresponding to the stable reactor period, and all the others will be negative. All the negative eigenvalues represent transients which decay out with time. Therefore, only the positive eigenvalue is of interest here.

If the reactor is perturbed a small amount, Eq. (C-7) can be written as

$$(M + P)\, \bar{\phi}_r' = \omega'\, \bar{\phi}_r'$$ (C-8)

where P is the matrix operator due to the change in the reactor system. Due to this change, $\bar{\phi}_r$ becomes $\bar{\phi}_r'$ and ω becomes ω'. Based on these considerations, a general equation for $\omega' - \omega$ can now be derived. This equation can then be used to find the worth of a control rod in a particular reactor system.

Consider two particular eigenfunctions, $\bar{\phi}_{r\,k}$ and $\bar{\phi}_{r\,l}$ so that Eq. (C-7) can be written as

$$\mathbf{M} \, \vec{\phi}_{r_k} = \omega_k \, \vec{\phi}_{r_k} \tag{C-9}$$

$$\mathbf{M} \, \vec{\phi}_{r_1} = \omega_1 \, \vec{\phi}_{r_1} . \tag{C-10}$$

Multiply Eq. (C-9) by $\vec{\phi}_{r_1}$ and Eq. (C-10) by $\vec{\phi}_{r_k}$, integrate both equations over the reactor volume, and subtract the result. The final result will be in a form of the type

$$\int_V \vec{\phi}_{r_1} \mathbf{M} \, \vec{\phi}_{r_k} dV - \int_V \vec{\phi}_{r_k} \mathbf{M} \, \vec{\phi}_{r_1} dV = (\omega_k - \omega_1) \int_V \vec{\phi}_{r_1} \vec{\phi}_{r_k} dV. \tag{C-11}$$

If the eigenfunctions, $\vec{\phi}_{r_i}$, are orthogonal, i.e.

$$\int_V \vec{\phi}_{r_k} \vec{\phi}_{r_1} dV = c_k \quad \text{if} \quad k=1$$

$$= 0 \quad \text{if} \quad k \neq 1$$

then it follows that

$$\int_V \vec{\phi}_{r_1} \mathbf{M} \, \vec{\phi}_{r_k} dV = \int_V \vec{\phi}_{r_k} \mathbf{M} \, \vec{\phi}_{r_1} dV . \tag{C-12}$$

In this case the matrix operator $\mathbf{M}$ is said to self-adjoint.

Consider now the case when the eigenfunctions, $\vec{\phi}_{r_i}$, do not form an orthogonal set. For this case an adjoint operator $\mathbf{M}^+$ can be defined such that

$$\int_V \vec{\phi}_{r_k}^+ \mathbf{M} \, \vec{\phi}_{r_1} dV = \int_V \vec{\phi}_{r_1} \mathbf{M}^+ \, \vec{\phi}_{r_k}^+ dV . \tag{C-13}$$

The $\vec{\phi}_{r_i}^+$'s are called the adjoint functions and are the eigenfunctions of the adjoint matrix equation

$$\mathbf{M}^+ \, \vec{\phi}_r^+ = \ell^* \, \vec{\phi}_r^+ \tag{C-14}$$

where ω^* is the complex conjugate of ω. The adjoint matrix operator is, in general, formed by replacing each element of the matrix operator

M by its complex conjugate and interchanging rows and columns. Since all the elements of M are real for a reactor system, M^+ is formed merely by interchanging the rows and columns of M.

A similar proof can be performed to illustrate the validity of Eq. (C-13) as was performed to show Eq. (C-12). Consider now Eq's. (C-8) and (C-14). Multiply Eq. (C-8) by ϕ_r^+ and Eq. (C-14) by ϕ_r', integrate the resulting equations over the whole reactor volume, and subtract the results. The final result will be in a form such as

$$\int_V \phi_r' \; M \; \phi_r' \; dV - \int_V \phi_r' \; M^+ \; \phi_r^+ \; dV + \int_V \phi_r^+ \; P \; \phi_r' \; dV = $$

$$(\omega' - \omega) \int_V \phi_r^+ \; \phi_r' \; dV \qquad (C-15)$$

where ω^* is equal to ω. Using Eq. (C-13), Eq. (C-15) can be put into a form such as

$$\Delta\omega = \int_V \phi_r^+ \; P \; \phi_r' \; dV \; / \; \int_V \phi_r^+ \; \phi_r' \; dV \qquad (C-16)$$

where $\Delta\omega$ is equal to $\omega' - \omega$. Eq. (C-16) is then used in the particular group model to find the worth of a change in the reactor system, denoted by the P matrix operator.

APPENDIX D

An IBM 1620 Program for Finding the
Effective Multiplication Factor in the
K.S.U. Graphite Superitical Pile

This program was written to determine the material buckling, B_m^2, and the effective multiplication factor, keff, in the K.S.U. exponential graphite pile. The program was written in Fortran I. The fortran program print-out and logic diagram are included in this section.

This program is based on a point thermal source solution to the thermal diffusion equation. The best estimate of γ_{11} is determined by a harmonic and end correction iterative analysis which was outlined in section 2.1. B_m^2 is calculated from the equation

$$B_m^2 = (\pi/a)^2 + (\pi/b)^2 - \gamma_{11}^2 . \qquad (D-1)$$

The program then calculates the infinite multiplication factor from

$$k_\infty = \exp(B_m^2 \tau) \cdot (1 + L_s^2 B_m^2) . \qquad (D-2)$$

The effective multiplication factor can then be calculated from

$$keff = k_\infty \exp(-B_g^2 \tau) / [1 + L_t^2 B_g^2] \qquad (D-3)$$

where B_g^2 is the geometric buckling and is formed from

$$B_g^2 = (\pi/a)^2 + (\pi/b)^2 + (\pi/c)^2 . \qquad (D-4)$$

This program will analyze a maximum of six sets of count rate versus z position data. More data sets can be analyzed by changing a few cards in the source deck and recompiling. Or the program can be restarted each time a new series of six sets of data is to be analyzed.

Table D-1. Data Loading Sequence

Card Number	Symbol	Data	Position	Format
1	B	Extrapolated x dimension	1	E10.4
1	H	Extrapolated z dimension	2	E10.4
1	W	Effective Diffusion Length	3	E10.4
1	TAU	Fermi Age	4	E10.4
1	A	Source Position in x direction	5	E10.4
1	WY	Effective Fuel Height	6	E10.4
2-6	Z(I)	Z position	1	E10.4
7-11	C(I)	Count rate Corresponding to Z position	1	E10.4

The input data were loaded in the following manner after the object and subroutine decks had been loaded.

The output will be written out by the typewriter and all output will be labeled. Sense switches 1 and 2 in the on position will give all of the correction factor values used to correct the count rate data.

Fig. D-1. Flow diagram for IBM 1620 program.

LOOP 3

CALC.
CORR. COEFF.
R

PRINT
γ_{11} R

CALCULATE
$B_m^2 = (\frac{\pi}{a})^2 + (\frac{\pi}{c})^2 - \gamma_{11}^2$

I=1

CALCULATE
CE =
$\dfrac{1 - e^{\gamma_{11}(C-Z)}}{1 + e^{2\gamma_{11}C}}$

LOOP 5
CH = 0

CALCULATE
CONST =
$\dfrac{\beta e^{\gamma_{11}Z}}{CE \cos\frac{\pi x}{a} \cos\frac{\pi y}{b}}$

m = 3   n = 2

LOOP 1
CALCULATE
$T_2 = \cos\frac{n\pi y}{b}$

LOOP 2
CALCULATE
$T_3 = \cos\frac{m\pi x}{a}$

CALCULATE
$\gamma_{mn}^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 - B_m^2$

DIMENSION
AND
FORMAT

READ
CONSTANTS
B, H, W, Tau,
A, WY

READ
Z(I)

MN = 1

READ
C(I)

LOOP 4

I = 1

SET
SUMLN = 0. SUMZ2 = 0
SMLN2 = 0. SUMZL = 0
SUMZ = 0.

CALCULATE
SUMLN = $\Sigma\ln C$ SUMZ2 = ΣZ^2
SMLN2 = $\Sigma(\ln C)^2$ SUMZL = $\Sigma Z(\ln C)$
SUMZ = ΣZ

I = I+1

NO   DOES
I = 5   YES

CALCULATE
SLOPE = $-\gamma_{11}$
γ_{11} = FIRS

$\dfrac{1 - e^{-2\gamma_{mn}(C-Z)}}{\gamma_{mn}\, e^{\gamma_{mn}Z}}$

CALC.
CH =
CH + $(T_1 \times T_2 \times T_3)$

DOES
m = 7

NO   YES
m = m+1
LOOP 2

DOES
n = 7

NO   YES
n = n+1
m = 1.
LOOP 1

CALCULATE
CH = 1. + CH × CONST

CALCULATE
CC(I) = $\dfrac{C(I)}{(CE) \times (CH)}$

PRINT
CC(I), CE, CH

A

PROGRAM LISTING - APPENDIX D

```
      DIMENSION C(6),Z(6),CC(6),BET(50)
    1 FORMAT(E10.4,E10.4,E10.4,E10.4,E10.4,E10.4,E10.4,E10.4)
    2 FORMAT(E15.8,E15.8,E15.8,E15.8)
    3 FORMAT(E15.8,E15.8 /)
  118 FORMAT(E15.8, 5H= BET  /)
  161 FORMAT(E15.8, 5H= EN2  /)
  119 FORMAT(E15.8, 6H= EN2N  /)
  162 FORMAT(E15.8, 5H= BCC  /)
  163 FORMAT(E15.8, 7H= AKINF  /)
  166 FORMAT(E15.8, 7H= AKEFF  /)
  919 FORMAT(E15.8, 4H= R  /)
      READ 1, B, H, W, TAU, A, WY
      PRINT 1, B, H, W, TAU, A, WY
      DO 113 I = 1,5,1
  113 READ 1, Z(I)
      MN = 1
  114 DO 5 I = 1,5,1
    5 READ 1, C(I)
      PRINT 235
  235 FORMAT( 16H        C(I) C/M, 17H          Z(I) CM  /)
      DO 231 I = 1,5,1
  231 PRINT 3, C(I),Z(I)
      I = 1
      IN = 1
      SUMLN = 0.
      SMLN2 = 0.
      SUMZ = 0.
      SUMZ2 = 0.
      SUMZL = 0.
   10 TM2 = LOG(C(I))
      TM22 = TM2**2.
      SUMLN = SUMLN + TM2
      SMLN2 = SMLN2 + TM22
      SUMZ = SUMZ + Z(I)
      TM3 = Z(I)**2.
      SUMZ2 = SUMZ2 + TM3
      TM4 = Z(I)*TM2
      SUMZL = SUMZL + TM4
      TEST = I - 5
      IF(TEST) 15,20,20
   15 I = I + 1
      GO TO 10
   20 BET(IN) = ((SUMZ*SUMLN)-(5.*SUMZL))/((SUMZ**2.)-(5.*SUMZ2))
      SRT = SMLN2*SUMZ2
      SSRT = SQRT(SRT)
      AR = SUMZL/SSRT
      PRINT 919, AR
      PRINT 118, BET(IN)
      IF(BET(IN)) 100,105,105
  100 BET(IN) = - BET(IN)
  105 I = 1
```

```
 26 FIRS = BET(IN)
    BM2 = 2.*(3.1416/B)**2.-FIRS**2.
 27 TRMA = -2.*FIRS*(H-Z(I))
    TRMB = -2.*FIRS*H
    TRMC = EXP(TRMA)
    TRMD = EXP(TRMB)
    TRME = 1.-TRMC
    TRMF = 1.-TRMD
    CE = TRML/TRMF
    TRMG = FIRS*Z(I)
    TRMH = EXP(TRMG)
    TRMI = TRMH/CE
    TERA = TRMI*FIRS
    ARGA = 3.1416*A/B
    TERB = COS(ARGA)
    TERC = TERB*2.
    AM = 3.
    AN = 1.
    BCH = 0.
 30 ARGB = AM*3.1416*A/B
    TRD = COS(ARGB)
    IF(TRD) 520,521,521
520 TRD = -TRD
521 ARGC = AN*3.1416*A/B
    TERE = COS(ARGC)
    IF(TERE) 522,523,523
522 TERE = -TERE
523 TERF = TRD + TERE
    BETM2 = (AM*3.1416/B)**2.+(AN*3.1416/B)**2.-BM2
    IF(BATM2) 90,150,150
 90 BETM2 = -BETM2
    GO TO 150
150 TRMM = SQRT(BETM2)
    TM = TRMM*Z(I)
    TRM = EXP(TM)
    TRMN = 1./TRM
    TRMO = -2.*TRMM*(H-Z(I))
    TRMP = EXP(TRMO)
    TRMQ = 1.-TRMP
    TRMR = -2.*TRMM*H
    TRMS = EXP(TRMR)
    TRMT = 1.+TRMS
    F = TRMQ/TRMT
    BCH = BCH+TERA*TRMN*TERF*F/(TRMM*TERC)
    TESI = AM-7.
    IF(TESI) 50,51,51
 50 AM = AM+2.
    GO TO 30
 51 TESJ = AN-7.
    IF(TESJ) 52,53,53
 52 AM = 1.
    AN = AN+2.
    GO TO 30
```

```
 53 CH = 1.+BCH
    CC(I) = C(I)/(CH*CE)
    TES2 = I-5
    IF(TES2) 59,58,58
 59 IF(SENSE SWITCH 1) 60,200
 60 PRINT 2, Z(I),C(I),CC(I),CH
200 I = I+1
    GO TO 27
 58 IF(SENSE SWITCH 2) 61,201
 61 PRINT 2, Z(I),CC(I),CH,CE
201 SMLA = 0.
    SMLA2 = 0.
    SUMZ = 0.
    SMZ2 = 0.
    ZSLA = 0.
    I = 1
 64 TMI = LOG(CC(I))
    TM2 = TMI**2.
    SMLA = SMLA+TMI
    SMLA2 = SMLA2+TM12
    SUMZ = SUMZ + Z(I)
    TMIC = Z(I)**2.
    SMZ2 = SMZ2+TMIC
    TMIB = Z(I)*TMI
    ZSLA = ZSLA+TMIB
    TEST = I-5
    IF(TEST) 65,66,66
 65 I = I+1
    GO TO 64
 66 IN = IN+1
    BET(IN) =(SUMZ*SMLA-5.*ZSLA)/(SUMZ**2.-5.*SMZ2)
    SRT1 = SMLA2*SMZ2
    SSRT1 = SQRT(SRT1)
    AR1 = ZSLA/SSRT1
    IF(BET(IN)) 400,410,410
400 BET(IN) = -BET(IN)
410 DIFF = BET(IN)-FIRS
    IF(DIFF) 900,910,910
900 DIFF = -DIFF
910 TEST = 0.00001-DIFF
    IF(TEST) 105,62,62
 62 BM2N = 2.*(3.1416/B)**2.-BET(IN)**2.
    DRY = BM2N*TAU
    DRN = EXP(DRY)
    AKINF = DRN*(1.+BM2N*W**2.)
    BG2 = 2.*(3.1416/B)**2.+(3.1416/XY)**2.
    TRNU = -BG2*TAU
    TRNV = EXP(TRNU)
    AKEFF = AKINF*TRNV/(1.+W**2.*BG2)
    IF(SENSE SWITCH 3) 1000,1001
1000 PRINT 118, BET(IN)
    PRINT 119, BM2N
```

```
        PRINT 161,DK2
        PRINT 162,DG2
1001    PRINT 162,AKINF
        PRINT 166,AKEFF
        PRINT 919,AD1
        TESTS = MN-6
        IF(TESTS) 117,173,173
117     MN = MN+1
        GO TO 114
173     END
```

APPENDIX E

1. Analog Solution to the One Delay Neutron Group Reactor Kinetic Equations

In order to obtain an estimate of the dynamic characteristics of a subcritical reactor for a step change in reactivity, a one-group kinetic model was set up on the analog computer. The standard neutron kinetic equations for a critical reactor were derived by Drake (2). These equations were modified to give

$$dn/dt = \left\{ k_e \ (1 - \beta) \ / \ \ell^* \right\} n(t) - n(t)/\ell^* + \lambda \ c(t) + S \qquad (E-1)$$

$$dc/dt = \left\{ k_e \ \beta \ / \ \ell^* \right\} n(t) - \lambda \ c(t) \qquad (E-2)$$

$n(t)$ = neutron density

$c(t)$ = precursor density

k_e = effective multiplication constant

β = effective delay neutron fraction

ℓ^* = thermal neutron lifetime

λ = averaged delay neutron decay constant

S = source term .

The only modification needed to obtain Eq's. (E-1) and (E-2) from the critical kinetic equations is the addition of the source term, S, in Eq. (E-1). For a near critical system, the source neutrons become negligible and the source term is dropped from Eq. (E-1). For the reactor system studied in this work, k_e is quite low, (0.5) and the source term is needed to sustain a neutron flux. In the physical problem, the source term was not space independent. However, Eq's.

(E-1) and (E-2) are spatially independent. Therefore, a spatial

independent source term must be approximated in order that a solution

may be found for Eq's. (E-1) and (E-2).

The approximation made was to consider an idealized average unit

volume and writing a neutron balance on this unit volume. In the

physical system, the source neutrons enter the system from the lower

boundary. Thus, the source term actually is spatial dependent. The

unit volume on which the neutron balance was written was assumed to

have a number of source neutrons equal to the total number of source

neutrons entering the system divided by the total reactor core volume.

Eq's. (E-1) and (E-2) can be applied to a critical reactor system

in which a step insertion of reactivity was made. For a critical system

Δk_e was defined to be

$$\Delta k_e = k_e - 1 \tag{E-3}$$

where k_e is the value of the effective multiplication factor for the

system after the step change in reactivity. The effective multiplication

factor for the initially critical system is defined to be unity. If the

physical system which was considered in this work, was thought of as a

critical system containing control elements of enough reactivity value

to give the system an initial effective multiplication factor of k_{e_1};

the actual system under consideration was one in which a negative re-

activity was "built-in". After dropping the poison rod into this system,

the effective multiplication factor was lowered still further to a

value of k_{e_2}. Three Δk_e's can now be defined,

$$\Delta k_{e_1} = k_{e_1} - 1 \tag{E-4}$$

$$\Delta k_{e_2} = k_{e_2} - k_{e_1} \tag{E-5}$$

$$\Delta k_c = \Delta k_{e_1} + \Delta k_{e_2} = k_{e_2} - 1 . \tag{E-6}$$

In view of the above separation of reactivities, Eq's. (E-1) and (E-2) can now be rewritten as

$$dn/dt = \left\{ \left[(1-\beta) k_{e_1} -1 \right] / \ell^* \right\} n(t) + \lambda c(t) + \left\{ k_{c_2} (1-\beta)/\ell^* \right\}$$

$$n(t) + S \tag{E-7}$$

and

$$dc/dt = \left\{ k_{e_1} \beta / \ell^* \right\} n(t) + \left\{ \Delta k_{e_2} \beta / \ell^* \right\} n(t) - \lambda c(t) . \tag{E-8}$$

To derive the machine equations which were used for the analog solution, maximum values of all the variables were estimated. Since the neutron density was operated between two relative levels, an arbitrary value could be assigned to be its maximum value. With this value chosen, the maximum values of all the other quantities were calculated through the use of the following boundary conditions.

a. $t < (0-)$ b. $t \rightarrow \infty$

   $dn/dt = dc/dt = k_{e_2} = 0$ $dn/dt = dc/dt = 0$

   $n(t) = n_0 \quad c(t) = c_0$. $n(t) = n_\infty \quad c(t) = c_\infty$.

Applying these boundary conditions to Eq's. (E-7) and (E-8), the following relations were obtained,

$$S = - \left\{ \Delta k_{e_1} / \ell^* \right\} n_0 \tag{E-9}$$

$$C_0 = \left\{ \beta k_{e_1} / \lambda \ell^* \right\} n_0 . \tag{E-10}$$

All the nuclear parameters (β, ℓ^*, λ) were known. The value for k_{e_1}, calculated by using the vertical traverse data, was assumed to be

correct. The one group approximations for Δk_{e_2} due to the poison rod

ave an upper bound for Δk_{e_2}. A series of curves of the flux versus

time with Δk_{e_2} as a parameter was found.

The problem of parameter and time scaling is discussed in the next

section.

2. Derivation of the Machine Equations for the Analog Solution

As was previously derived, the following are the modified spatially

independent reactor kinetic equations which apply to a subcritical

reactor system with a homogenous source, S;

$$dn/dt = \left\{ \Delta k_{e_1} / \ell^* \right\} n + \left\{ (1-\beta) \Delta k_{e_2} / \ell^* \right\} n + \lambda C + S \qquad (E-11)$$

$$dc/dt = \left\{ k_{e_1} \beta / \ell^* \right\} n + \left\{ \Delta k_{e_2} \beta / \ell^* \right\} n - \lambda C \qquad (E-12)$$

when the boundary condition at $(t = 0-)$ was applied, the following two

defining equations resulted

$$C_o = \left\{ k_{e_1} \beta / \lambda \ell^* \right\} n_o \qquad (E-13)$$

$$S = -\left\{ \Delta k_{e_1} / \ell^* \right\} n_o. \qquad (E-14)$$

Table (E-1) contains the nuclear parameters used in deriving the machine

equations. Hughes' data for delayed neutron fractions were used. The

value taken for thermal neutron lifetime is a typical value for a

graphite moderated thermal reactor.

Table E-1.   Analog Constants

Parameter	Value	Units
λ	0.0802	sec^{-1}
ℓ^*	10^{-3}	sec
β	7.55×10^{-3}	
$1-\beta$	0.99245	
n_0	0.04	r/cc
k_{e_1}	0.5757	
k_{e_2}	0.4757	
Δk_e	-0.4243	
Δk_{e_2}	-0.10	

Using these constants, the following two equations were calculated.

$$(dn/dt)max. = -0.099245 \times 10^3 \, n_0 \qquad (E-15)$$

$$(dc/dt)max. = -0.755 \, n_0 \qquad (E-16)$$

with these equations and the constants of Table (E-1), the method of scaling presented by Van Artsdalen (22) can be followed to derive the following machine equations

$$\left[2.5\frac{dn}{dt}\right] = -(0.4236)(10)\left[250n\right] - (0.99245)\left[250n\right] + (0.0501)\left[4c\right] + (0.4243)\left[1c\right] \qquad (E-17)$$

$$\left[300\frac{dc}{dt}\right] = (0.5216)(10)\left[250n\right] - (0.906)\left[250n\right] - (0.601)$$
$$\times (10)\left[4c\right] \tag{E-18}$$

$$\left[250n\right] = \int\left[2.5\frac{dn}{dt}\right] d\tau + (1)\left[10\right] \tag{E-19}$$

$$\left[4c\right] = (1.33\times10^{-4})\int\left[300\frac{dc}{dt}\right] d\tau + (0.8672)\left[10\right] \tag{E-20}$$

where $t = 10^{-2}\tau$ and τ is the machine time. A possible program for the PACE TR-10 analog computer is presented in Fig. (22). In Table (E-2), a listing of the potentiometer settings is presented.

Table E-2. Analog Potentiometer Settings

$-\Delta k_{e_2}$	Potentiometer No. 2	Potentiometer No. 4
0.1	0.992	0.906
0.08	0.794	0.725
0.055	0.545	0.499
0.04	0.397	0.362
0.015	0.149	0.136

3. Results of the Analog Solution

The one delayed neutron group analog solution was worked on the PACE TR-10 analog computer. Values for control rod worth, Δk_e, were varied from -0.1 to -0.015. A series of transient response curves to the step insertion of these Δk_e values were obtained. These curves are presented in Fig. (E-2).

This analog solution was obtained in order that a comparison between it and an experimentally determined curve could be made. The cadmium poison rod, described in section 3.5, was dropped several hundred times. The data were accumulated in a 256 channel analyzer with a multi-scalar plug-in logic. Definite transient response curves were obtained but the data were not reproducible. No conclusions could be reached from these data.
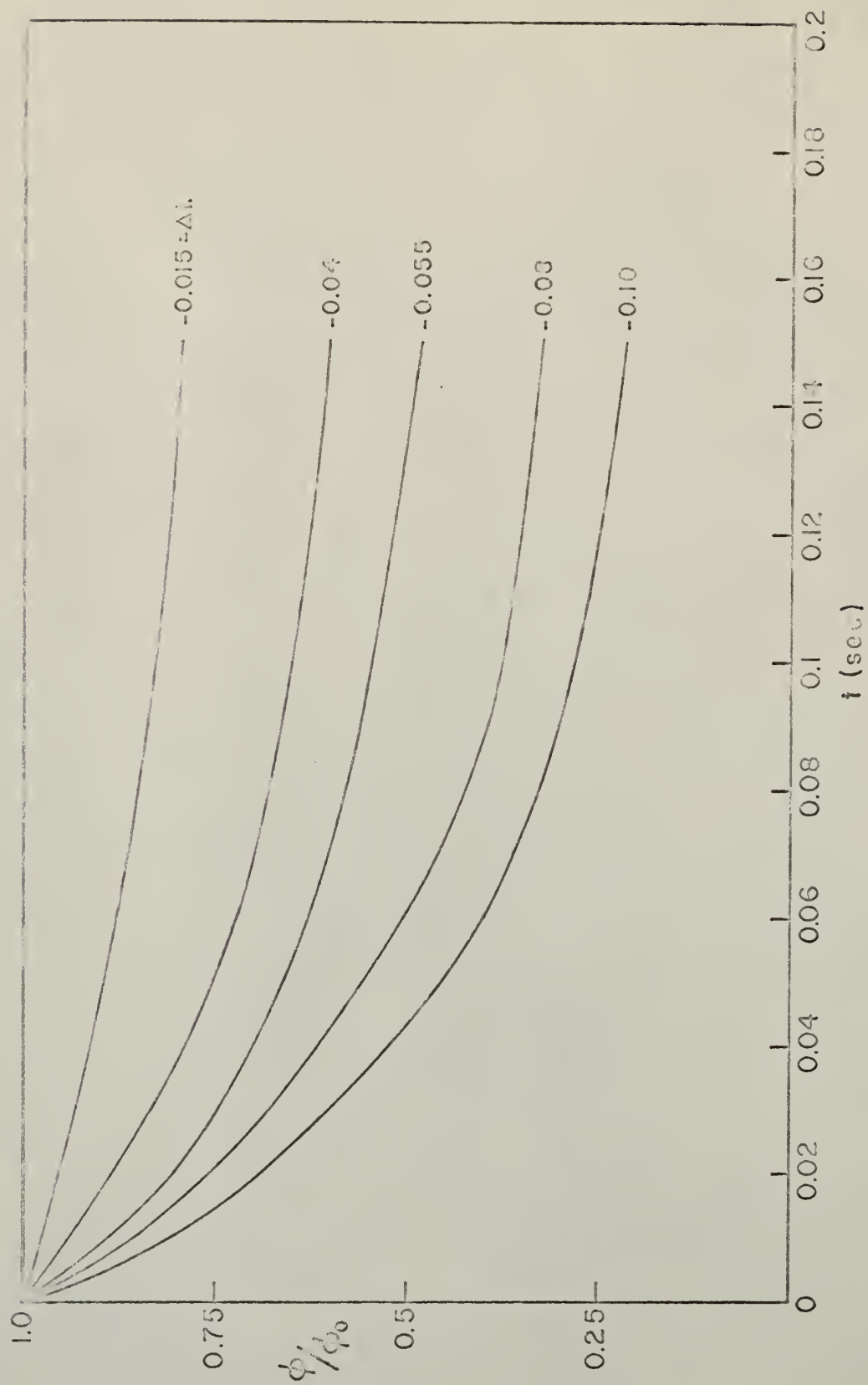
Fig. E-2. The results of the one delay group analog solution.
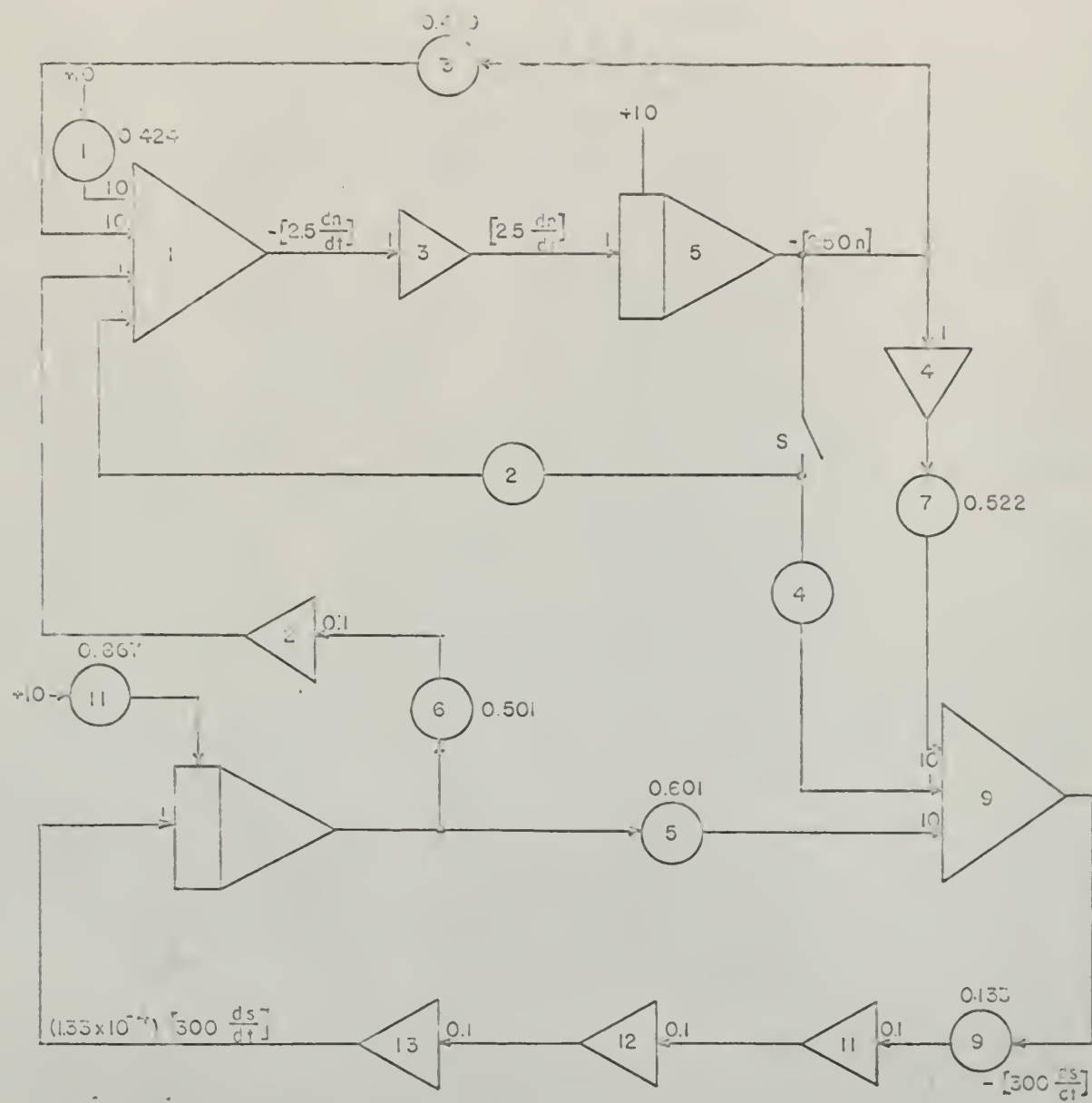
Fig. E-1. Analog solution flow diagram

REACTIVITY DETERMINATIONS IN
THE K.S.U. GRAPHITE SUB-
CRITICAL ASSEMBLY

by

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B.S., Kansas State University, 1961

AN ABSTRACT OF
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The objectives of this thesis were to make reactivity determinations and control rod worth calculations for a neutron poison, a hollow cadmium cylinder, in the Kansas State University graphite subcritical assembly.

To carry out these objectives, several mathematical models were used. The reactivity determinations were made by using the following three methods to analyze the data taken in the graphite subcritical assembly. These methods were: 1. a one group analysis, 2. a relaxation length analysis, and 3. a two group analysis. The theoretical control rod worth calculations were made through the use of the following three methods: 1. an approximate one and two group technique, 2. a perturbation technique, and 3. an improved perturbation technique. These methods were all developed for a subcritical reactor system.

A comparison of the results of the calculations showed that the two group analysis and the improved perturbation technique yielded similar values for the reactivity of the control rod.

Date Due