DETERMINATION OF THE FLOW OVER A BLUNT BODY IN A SUPERSONIC STREAM BY THE METHOD OF UCHIDA AND YASUHARA

by 533

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NOMENCLATURE

English alphabets:

x, y, z	Cartesian coordinates
s, n, t	Arc length of α, β, and γ axes, respectively
h_α, h_β, h_γ	Parameters in curvilinear coordinates.
P	Pressure
T	Temperature (absolute)
S	Entropy, energy per degree per unit mass
i	Enthalpy, energy per degree per unit mass
R	Gas constant, length per degree
C_p, C_v	Specific heat at constant pressure and volume, respectively
q	Velocity
q_α, q_β, q_γ	α, β, and γ components of q, respectively
u, v, w	x, y, and z components of q, respectively
q_n, q_t	Normal and tangential velocity component on a curved shock wave, respectively
a	Velocity of sound
E	Total energy, energy per unit mass
w	Angle between velocity vector and tangent to shock wave
K	Connection factor for boundary condition at shock wave
d	Diameter of sphere
b	Detachment distance of bow shock wave from nose of body
P_0	Total pressure in uniform upstream

Greek letters:

α, β, γ	Orthogonal curvilinear coordinates
ρ	Density
γ	C_p/C_v = ratio of specific heat
ψ	Stream function
ξ	γ component of rot q
ε	Constant (ε=0 in two-dimensional flow and ε=1 in axisymmetric flow)
δ	Deflection angle of streamline through shock wave
θ	Angle of flow direction
ℓ	Reference length

Subscripts:

0	Quantities in state at rest of undisturbed flow
*	Quantities at state of locally sonic
b	Value along a certain boundary, which will be replaced by s in the present condition
a	Quantities along the fixed surface
s	Quantities immediately behind the shock wave
1, 2	Quantities before and behind the shock wave, respectively
m	Grade of approximation

CHAPTER I

INTRODUCTION

The problem of the high-speed flow past a blunt-nosed body has been a subject of considerable interest. In this problem, the bow shock wave is detached from the body surface. The difficulty encountered in attempting to analyze the entire flow field seems to be caused by an inability to find proper means of quasi-linearization. By the fact that the flow around the body is a non-linear, mixed subsonic-supersonic flow and the location of the bow shock wave in front of the body is not known in advance, neither the small-disturbance theory nor the analytical method of the hodograph plane can be applied in the general case of a detached, curved shock wave.

Except for a wedge or cone of small apex angle, the effects of rotation (i.e. vorticity) cannot be neglected. In order to retain the non-linearity of the flow pattern as well as the effects of rotationality, numerical methods have been studied by Uchida and Yasuhara (3) and several other authors (5, 8, 10). Many of these authors treated the present special problem by the relaxation method, starting with the observed form of shock wave or requiring the pressure distribution on the entire body and the shock shape. As a result, these methods require some definite, prior knowledge of the flow and, thus, do not represent complete solutions to the problem. On the other hand, although the method of Uchida and Yasuhara requires iterating both the shock shape and streamline pattern, it requires no fixed assumptions regarding values of the flow variables, either at the surface or in the flow field.

The principle of this method is to reduce the partial differential equation to the form of an ordinary one by taking one of the coordinate axes to be nearly coincident with the integral curves of solution. In this case, streamlines and their orthogonals are chosen as a suitable set of curvilinear coordinates. The equation of vorticity for isoenergetic, rotational flow has been given by Uchida, Equation (42) of Ref. (3), and involves only derivatives with respect to a single, independent variable. Approximate solution for flow speed, direction, and entropy gradient are obtained by integration as shown in Equation (34) of this report. A streamline will have the properties of a characteristic, because, in this form, the equations of motion do not determine the rate of change of velocity, q, and entropy, S, in the direction perpendicular to the streamlines.

For hypersonic flow past a blunt body, the shock wave lies close enough to the surface that the streamlines are roughly parallel to the body except in a small region near the stagnation point. This fact immediately suggests the possibility of utilizing an assumed streamline pattern and shock shape for calculating the flow over the nose of a blunt body in a high-speed flow. Computations are started by assuming the flow pattern and the corresponding form of the bow shock wave; the distribution of entropy behind the shock wave can then be determined. With the aid of the continuity, momentum, and energy equations; Equations (34), (37), and (38), these initial assumptions are corrected by a convergent iterative procedure until a final solution is found. It is easy to satisfy the boundary conditions for a flow with an attached shock wave, because the location of the shock is fixed at the point of attachment to the body in this case. If the shock wave is detached, Uchida and Yasuhara use the ratio of the maximum value of two flow deflection angles, before and behind the shock, as a parameter to define the incompleteness of the solution at the shock boundary for a fixed shock-stand-off distance. The correct shock location then can be determined by interpolating for the shock standoff distance which makes $K = \frac{\theta_{s,max}}{\delta_{max}} = 1$. A double iteration technique is carried out whereby both the streamlines and the shock are readjusted until a consistent solution is obtained.

As an example, the flow about a sphere in a uniform supersonic free stream (M=1.5) is presented herein.

CHAPTER II

FUNDAMENTAL EQUATIONS

2-1. Fundamental Equations for Steady Rotational Flow

The steady rotational flow of a nonviscous and non-heat-conductive compressible fluid is governed by the following relations—i.e.,

Equation of state for the ideal gas:

$$P/\rho = RT \tag{1}$$

with the aid of the first law of thermodynamics

$$dQ = C_v dT + Pdv = C_p dT - vdP$$

and the second law of thermodynamics

$$dS = \frac{dQ}{T} = C_p \frac{dT}{T} - \frac{v}{T}\, dP$$

we can find

$$\begin{aligned} dS &= (C_p)(\frac{dP}{P} - \frac{d\rho}{\rho}) - \frac{R}{P}\, dP \\ &= (C_v + R)\frac{dP}{P} - C_p \frac{d\rho}{\rho} - \frac{R}{P}\, dP \\ &= C_v \frac{dP}{P} - C_v \gamma\frac{d\rho}{\rho} \\ &= C_v(\frac{dP}{P} - \gamma\frac{d\rho}{\rho}) \end{aligned}$$

integrating

$$S - S_0 = C_v \ln \frac{P}{P_0}\left(\frac{\rho_0}{\rho}\right)^{\gamma}$$

or

$$\frac{P}{\rho^{\gamma}} = \frac{P_0}{\rho_0^{\gamma}} e^{S-S_0/C_v} \qquad \text{if let } \frac{P_0}{\rho_0^{\gamma}} e^{-S_0/C_v} = 1$$

hence $P = e^{S/C_v} \rho^{\gamma}$ (2)

Equation of continuity:

$$\operatorname{div}(\rho q) = 0 \tag{3}$$

Definition of $\frac{D}{Dt}(q)$:

$$\frac{Dq}{Dt} = \frac{\partial q}{\partial t} + \operatorname{grad}\left(\frac{1}{2} q^2\right) - q \wedge \operatorname{rot} q$$

If it is steady flow, $\frac{\partial q}{\partial t} = 0$, and from momentum equation and the definition of $\frac{D}{Dt}(q)$,

$$\frac{Dq}{Dt} = \operatorname{grad}\left(\frac{1}{2}q^2\right) - q \wedge \operatorname{rot} q = -\frac{1}{\rho} \operatorname{grad} P. \tag{4}$$

Equation of energy in steady flow without dissipation and heat conduction:

$$\frac{DQ}{Dt} = T \frac{DS}{Dt} = 0$$

or

$$T\left(\frac{\partial S}{\partial t} + q \times \operatorname{grad} S\right) = 0.$$

Steady flow: $\frac{\partial S}{\partial t} = 0$;

then

$$T \frac{DS}{Dt} = 0 = q \times \text{grad } S, \tag{5}$$

i.e., the streamlines are perpendiculars to grad S;

then

$$S = f(\psi) \tag{6}$$

where ψ is the stream function.

The equation of energy or enthalpy, i, is given by

$$\frac{Di}{Dt} = T \frac{DS}{Dt} + \frac{1}{\rho} \frac{DP}{Dt} \tag{7}$$

or

$$\frac{Di}{Dt} = \frac{1}{\rho} \frac{DP}{Dt} \ . \tag{8}$$

Since $i = C_p T = \frac{C_p}{R} \frac{P}{\rho} = \frac{\gamma}{\gamma-1} \frac{P}{\rho}$

and since the total energy is defined by

$$E = \frac{1}{2} q^2 + i = \frac{1}{2} q^2 + \frac{\gamma}{\gamma-1} \frac{P}{\rho} \ , \tag{9}$$

$$\frac{DE}{Dt} = \frac{Di}{Dt} + \frac{D(\frac{1}{2}q^2)}{Dt}$$

$$= \frac{1}{\rho} q \times \text{grad } P + q \times \text{grad } (\tfrac{1}{2}q^2)$$

$$= q \times [\frac{1}{\rho} \text{ grad } P + \text{grad } (\tfrac{1}{2}q^2)]$$

$$= q \times (q \wedge \text{rot } q) = 0.$$

and

$$\frac{DE}{Dt} = \frac{\partial E}{\partial t} + q \times \text{grad } E$$

$$= q \times \text{grad } E,$$

hence $\frac{DE}{Dt} = q \times (q \wedge \text{rot } q) = q \times \text{grad } E = 0,$ (10)

i.e., the total energy does not vary along the streamlines.

Therefore,

$$E = G(\psi). \tag{11}$$

Equation for vorticity:

From

$$\nabla(\frac{1}{2} q^2) - q \wedge \text{rot } q = -\frac{1}{\rho} \nabla P,$$

$$\nabla i = T\nabla S + \frac{1}{\rho} \nabla P,$$

and

$$\nabla E = \nabla i + \nabla(\frac{1}{2} q^2),$$

it is easily seen that

$$\nabla(\frac{1}{2} q^2) - q \wedge \text{rot } q = T\nabla S - \nabla i$$

or

$$- q \wedge \text{rot } q - T\nabla S = - \nabla i - \nabla(\frac{1}{2} q^2);$$

therefore,

$$q \wedge \text{rot } q + T\nabla S = \nabla E.$$

For isoenergetic flow, $\nabla E = 0$; then,

$$q \wedge \text{rot } q = - T\nabla S = -T \text{ grad } S. \tag{12}$$

2-2. Fundamental Equations for Geometrical Relations General Curvilinear System:

$$x = f_1(\alpha, \beta, \gamma), \ y = f_2(\alpha, \beta, \gamma), \ z = f_3(\alpha, \beta, \gamma)$$

parameters of coordinates are defined by

$$h_\alpha = \lim_{\delta_\alpha \to 0} \frac{\partial s_\alpha}{\partial \alpha} = \sqrt{(\frac{\partial x}{\partial \alpha})^2 + (\frac{\partial y}{\partial \alpha})^2 + (\frac{\partial z}{\partial \alpha})^2} \tag{13a}$$

$$h_\beta = \lim_{\delta_\beta \to 0} \frac{\partial s_\beta}{\partial \beta} = \sqrt{(\frac{\partial x}{\partial \beta})^2 + (\frac{\partial y}{\partial \beta})^2 + (\frac{\partial z}{\partial \beta})^2} \tag{13b}$$

$$h_\gamma = \lim_{\delta_\gamma \to 0} \frac{\partial s_\gamma}{\partial \gamma} = \sqrt{(\frac{\partial x}{\partial \gamma})^2 + (\frac{\partial y}{\partial \gamma})^2 + (\frac{\partial z}{\partial \gamma})^2} \tag{13c}$$

In the new coordinate system, a vector is given by

$$A = A_\alpha \bar{\alpha} + A_\beta \bar{\beta} + A_\gamma \bar{\gamma} .$$

The operator ∇ is given by

$$\nabla = \frac{\partial}{\partial s_\alpha} \bar{\alpha} + \frac{\partial}{\partial s_\beta} \bar{\beta} + \frac{\partial}{\partial s_\gamma} \bar{\gamma}$$

or

$$\nabla = \frac{1}{h_\alpha}\frac{\partial}{\partial\alpha}\bar{\alpha} + \frac{1}{h_\beta}\frac{\partial}{\partial\beta}\bar{\beta} + \frac{1}{h_\gamma}\frac{\partial}{\partial\gamma}\bar{\gamma} .$$

The divergence is given by

$$\nabla x\ A = \frac{1}{h_\alpha h_\beta h_\gamma}\left[\frac{\partial(A_\alpha h_\beta h_\gamma)}{\partial\alpha} + \frac{\partial(A_\beta h_\alpha h_\gamma)}{\partial\beta} + \frac{\partial(A_\gamma h_\alpha h_\beta)}{\partial\gamma}\right].$$

The rotation is given by

$$\nabla\Lambda A = \begin{vmatrix} \bar{\alpha} & \bar{\beta} & \bar{\gamma} \\ \frac{\partial}{\partial s_\alpha} & \frac{\partial}{\partial s_\beta} & \frac{\partial}{\partial s_\gamma} \\ A_\alpha & A_\beta & A_\gamma \end{vmatrix} = \begin{vmatrix} \bar{\alpha} & \bar{\beta} & \bar{\gamma} \\ \frac{1}{h_\alpha}\frac{\partial}{\partial\alpha} & \frac{1}{h_\beta}\frac{\partial}{\partial\beta} & \frac{1}{h_\gamma}\frac{\partial}{\partial\gamma} \\ A_\alpha & A_\beta & A_\gamma \end{vmatrix}$$

or

$$\nabla\Lambda A = \frac{1}{h_\alpha h_\beta h_\gamma}\begin{vmatrix} h_\alpha\bar{\alpha} & h_\beta\bar{\beta} & h_\gamma\bar{\gamma} \\ \frac{\partial}{\partial\alpha} & \frac{\partial}{\partial\beta} & \frac{\partial}{\partial\gamma} \\ A_\alpha h_\alpha & A_\beta h_\beta & A_\gamma h_\gamma \end{vmatrix} .$$

In this way, we can write

$$\mathrm{div}(\rho q) = \nabla x\ (\rho q) = \frac{1}{h_\alpha h_\beta h}\left[\frac{\partial(\rho q_\alpha h_\beta h_\gamma)}{\partial\alpha} + \frac{\partial(\rho q_\beta h_\alpha h_\gamma)}{\partial\beta}\right.$$

$$\left. + \frac{\partial(\rho q_\gamma h_\alpha h_\beta)}{\partial\gamma}\right]. \tag{14}$$

For two-dimensional flow in the reference plane, X-Y, $q_\gamma = 0$ and

$$\operatorname{div}(\rho q) = \frac{1}{h_\alpha h_\beta} \left[\frac{\partial(\rho q_\alpha h_\beta)}{\partial\alpha} + \frac{\partial(\rho q_\beta h_\alpha)}{\partial\beta}\right] .$$

For axially symmetric flow in the meridian plane, X-Y, where the X-axis is identified as the axis of symmetry,

$$\operatorname{div}(\rho q) = \frac{1}{h_\alpha h_\beta y} \left[\frac{\partial(\rho q_\alpha h_\beta y)}{\partial\alpha} + \frac{\partial(\rho q_\beta h_\alpha y)}{\partial\beta}\right] .$$

Alternatively, the equation of continuity can be written as follows:

$$\operatorname{div}(\rho q) = \frac{1}{h_\alpha h_\beta y^\varepsilon} \left[\frac{\partial(\rho q_\alpha h_\beta y^\varepsilon)}{\partial\alpha} + \frac{\partial(\rho q_\beta h_\alpha y^\varepsilon)}{\partial\beta}\right] = 0, \tag{15}$$

where $\varepsilon = 0$ in two-dimensional flow

$\varepsilon = 1$ in axially symmetric flow.

From the equation of continuity, the stream function can be defined by the velocity components as

$$q_\alpha = \frac{1}{\rho h_\beta y^\varepsilon} \frac{\partial\psi}{\partial\beta} \text{ and } q_\beta = - \frac{1}{\rho h_\alpha y^\varepsilon} \frac{\partial\psi}{\partial\alpha} . \tag{16}$$

The vorticity is given by

$$\nabla\Lambda q = \begin{vmatrix} \bar{\alpha} & \bar{\beta} & \bar{\gamma} \\ \frac{\partial}{\partial s_\alpha} & \frac{\partial}{\partial s_\beta} & \frac{\partial}{\partial s_\gamma} \\ q_\alpha & q_\beta & q_\gamma \end{vmatrix} = \bar{\alpha} \left(\frac{\partial q_\gamma}{\partial s_\beta} - \frac{\partial q_\beta}{\partial s_\gamma}\right) + \bar{\beta} \left(\frac{\partial q_\alpha}{\partial s_\gamma} - \frac{\partial q_\gamma}{\partial s_\alpha}\right) + \bar{\gamma} \left(\frac{\partial q_\beta}{\partial s_\alpha} - \frac{\partial q_\alpha}{\partial s_\beta}\right)$$

or

$$\nabla \Lambda q = \xi_\alpha \bar{\alpha} + \xi_\beta \bar{\beta} + \xi_\gamma \bar{\gamma} .$$

In the special case that the streamlines are parallel to one of the orthogonal curvilinear coordinates. For example, let β = constant (see Fig. 1(b)) so that the streamlines are parallel to the α-axis, then $q_\beta = -\frac{1}{\rho h_\alpha y^\varepsilon} \frac{\partial \psi}{\partial \alpha} = 0$. Then in the two-dimensional case or axi-symmetrical case,

$$q_\gamma = 0, \qquad q_\alpha = f(\alpha, \beta);$$

hence,

$$\text{rot } q = \begin{vmatrix} \bar{\alpha} & \bar{\beta} & \bar{\gamma} \\ \frac{\partial}{\partial s_\alpha} & \frac{\partial}{\partial s_\beta} & \frac{\partial}{\partial s_\gamma} \\ q_\alpha & 0 & 0 \end{vmatrix} \begin{matrix} = \bar{\beta} \left(\frac{\partial q_\alpha}{\partial s_\gamma}\right) + \bar{\gamma} \left(\frac{-\partial q_\alpha}{\partial s_\beta}\right) \\ \\ = \xi_\beta \bar{\beta} + \xi_\gamma \bar{\gamma} \end{matrix}$$

or

$$\text{rot } q = \bar{\gamma} \left(- \frac{\partial q_\alpha}{\partial s_\beta}\right) .$$

because

$$\xi_\beta = \frac{\partial q_\alpha}{\partial s_\gamma} = \frac{\partial q_\alpha}{h_\gamma \partial_\gamma} = 0 .$$

Therefore,

$$\text{rot } q\Lambda q = \begin{vmatrix} \bar{\alpha} & \bar{\beta} & \bar{\gamma} \\ 0 & 0 & \xi_\gamma \\ q_\alpha & 0 & 0 \end{vmatrix} = \bar{\beta}\ (\xi_\gamma\ q_\alpha)$$

Since rot $q\Lambda q = T\Delta S$,

$$\xi_\gamma q_\alpha = T\,\frac{\partial S}{\partial s_\beta}\ .$$

And, because $q_\alpha = \dfrac{1}{\rho h_\beta y^\varepsilon}\dfrac{\partial\psi}{\partial\beta}$ and $ds_\beta = h_\beta\ d\beta$,

$$\xi_\gamma = \frac{T}{\dfrac{1}{\rho h_\beta y^\varepsilon}\dfrac{\partial\psi}{\partial\beta}}\,\frac{\partial S}{\partial s_\beta} = \rho T y^\varepsilon\,\frac{\partial S}{\partial\psi}$$

or

$$\xi_\gamma = \frac{P}{R}\,y^\varepsilon\,\frac{\partial S}{\partial\psi}\ . \tag{17}$$

Also,

$$\frac{1}{h_\alpha h_\beta}\left[\frac{\partial(q_\beta h_\beta)}{\partial\alpha} - \frac{\partial(q_\alpha h_\alpha)}{\partial\beta}\right] = \frac{P}{R}\,y^\varepsilon\,\frac{\partial S}{\partial\psi}\ . \tag{18}$$

2-3. Fundamental Energy Equation in Steady, Isoenergetic Flow of an Ideal Gas.

The velocity of sound is defined by

$$a = \sqrt{(\partial P/\partial\rho)_{s=\text{const}}} = \gamma(P/\rho). \tag{19}$$

The energy equation is given by

$$E = \frac{1}{2} q^2 + \frac{\gamma}{\gamma-1} \frac{P}{\rho} = \frac{1}{2} q_1^2 + \frac{\gamma}{\gamma-1} \frac{P_1}{\rho_1} .$$

If $q = 0$,

$$E = \frac{\gamma}{\gamma-1} \frac{P_0}{\rho_0} = \frac{a_0^2}{\gamma-1} ,$$

where a_0 is the sound speed at stagnation conditions.

If $P = 0$,

$$E = \frac{1}{2} q_1^2 + \frac{\gamma}{\gamma-1} \frac{P_1}{\rho_1} = \frac{1}{2} q_{max}^2 , \tag{20}$$

where q_{max} is the constant maximum speed the fluid can achieve and is determined by the upstream conditions. If $q = a = a_*$, where a_* is defined to be the critical speed of sound,

$$\frac{1}{2} q^2 + \frac{1}{\gamma-1} a^2 = \frac{\gamma + 1}{2(\gamma-1)} a_*^2 = \frac{a_0^2}{\gamma-1} ;$$

hence,

$$a_0^2 = \frac{\gamma+1}{2} a_*^2 .$$

The formula for density can be obtained in the following way:

From

$$\frac{1}{2} q^2 + \frac{\gamma}{\gamma-1} \frac{P}{\rho} = \frac{\gamma}{\gamma-1} \frac{P_0}{\rho_0} ,$$

$$\frac{P}{P_0} \frac{\rho_0}{\rho} = 1 - \frac{\gamma-1}{2} \frac{q^2}{a_0^2} .$$

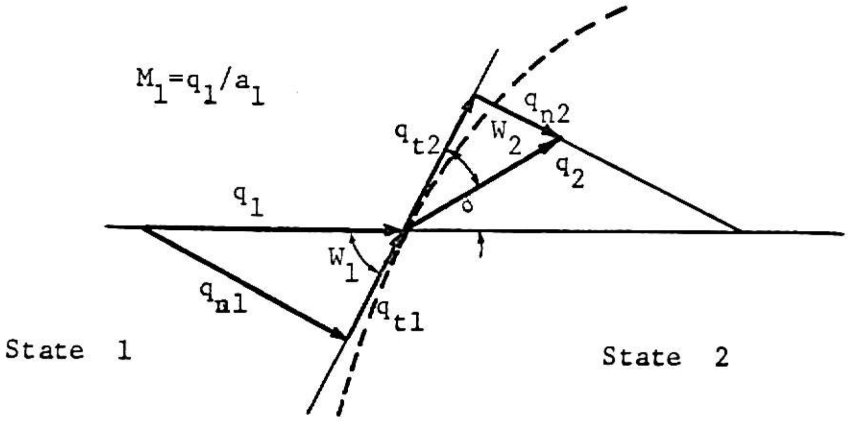

Fig. 1(a). Velocity components before and behind a shock wave.

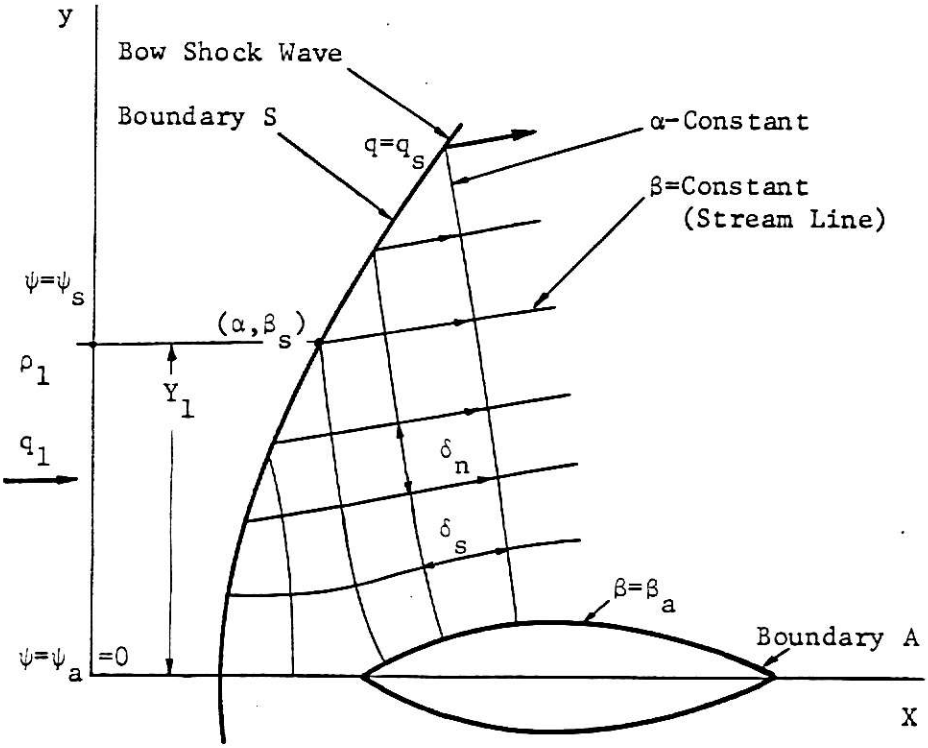

Fig. 1(b). Scheme of the flow pattern.

Since $\frac{P}{P_0} = \frac{\rho^\gamma}{\rho_0{}^\gamma} e^{S-S_0/C_v}$, by substitution into the previous equation,

$$\frac{\rho}{\rho_0} = e^{-[(S-S_0)/R]} \left[1 - \frac{\gamma-1}{2} \frac{q^2}{a_0^2}\right]^{\frac{1}{\gamma-1}}$$

$$= e^{-[(S-S_0)/R]} \left[1 - \frac{\gamma-1}{\gamma+1} \frac{q^2}{a_*^2}\right]^{\frac{1}{\gamma-1}}$$

or

$$\frac{\rho}{\rho_0} = e^{-[(S-S_0)/R]} \left[\left(\frac{\rho}{\rho_0}\right)_{\text{isentropic}}\right] . \tag{21}$$

2-4. Boundary Conditions

(a). Fixed boundary along the surface of a body

Along the solid surface of a body, the stream function must be constant. Thus, a fixed, solid boundary is usually represented by $\beta = \beta_a = 0$ so that $\psi = \psi_a = 0$ when we choose one of the two curvilinear coordinates to be the streamlines.

(b). Variable boundary along the bow shock wave

Introducing the notation shown in Fig. 1(a), fundamental equations are given by

Low of continuity: $\rho_1 q_{n1} = \rho_2 q_{n2}$ (22)

Momentum theorem: $\rho_1 q_{n1}^2 + P_1 = \rho_2 q_{n2}^2 + P_2$ (23a)

$$\rho_1 q_{n1} q_{t1} = \rho_2 q_{n2} q_{t2} \tag{23b}$$

Conservation of energy:

$$\frac{1}{2} q_1^{\ 2} + \frac{\gamma}{\gamma-1} \frac{P_1}{\rho_1} = \frac{1}{2} q_2^{\ 2} + \frac{\gamma}{\gamma-1} \frac{P_2}{\rho_2} \tag{24}$$

In solving these equations, several relations are obtained (6), as follows:

Deflection angle:

$$\frac{1}{\tan\delta} = \left[\frac{\gamma+1}{2} \frac{M_1^{\ 2}}{M_1^{\ 2} \operatorname{Sin}^2 w_1 - 1} - 1\right] \tan w_1 \tag{25}$$

Pressure ratio:

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1} \left[M_1^{\ 2} \operatorname{Sin}^2 W_1 - \frac{\gamma-1}{2\gamma}\right] \tag{26}$$

Density ratio:

$$\frac{\rho_1}{\rho_2} = \frac{2}{\gamma+1} \left[\frac{1}{M_1^{\ 2} \operatorname{Sin}^2 W_1} + \frac{\gamma-1}{2}\right] \tag{27}$$

Entropy change:

$$e^{(S_2-S_1)/C_v} = \frac{P_2/P_1}{(\rho_2/\rho_1)^{\gamma}} \tag{28a}$$

$$e^{(S_2-S_1)/C_p} = \frac{(P_2/P_1)^{1/\gamma}}{\rho_2/\rho_1} \tag{28b}$$

$$e^{(S_2-S_1)/R} = \frac{(P_2/P_1)^{1/\gamma-1}}{(\rho_2/\rho_1)^{\gamma/\gamma-1}} \tag{28c}$$

The form of the shock wave can be expressed by

$$\beta = \beta_s(\alpha) \qquad \text{in curvilinear coordinates}$$

or by

$$y = y_s(x) \qquad \text{in Cartesian coordinates,}$$

as shown in Fig. 1(b).

Since

$$q = \frac{1}{\rho h_\beta y^\varepsilon} \frac{\partial \psi}{\partial \beta} ,$$

integration gives

$$\int_0^\psi d\psi = \int_0^\beta \rho q \, h_\beta \, y^\varepsilon \, d\beta .$$

In the free stream, upstream of the shock,

$$h_\beta = 1 \qquad \text{and} \qquad \beta = y .$$

(i) For the 2-dimensional case, $\varepsilon = 0$, and

$$\psi = \int_0^{y_1} \rho_1 q_1 y^0 dy = \rho_1 q_1 y_1 \ ;$$

(ii) For the axi-symmetrical case, $\varepsilon = 1$,

$$\psi = \int_0^{y_1} \rho_1 q_1 y dy = \rho_1 q_1 y_1 (\frac{y_1}{2}) .$$

Therefore, we can write the shock boundary conditions in the following way:

$$\psi = \psi_s = \rho_1 q_1 y_1 \left(\frac{y_1}{2}\right)^{\varepsilon} \qquad \text{at} \qquad \beta = \beta_s(\alpha)$$

and

$$\psi = 0 \qquad \text{at} \qquad \beta = 0.$$

CHAPTER III

PRINCIPLE OF THE METHOD

3-1. Characteristic Equation Referred to the Streamline Coordinates

A partial differential equation can be formally transformed into an ordinary one in the general case, and then it can be integrated along the coordinate curve (or surface) perpendicular to the reference coordinate if we are careful to choose curvilinear coordinates so that the reference coordinate coincides with the curve of solution about to be analyzed.

In the present case, we assume that β = const. coincides with the streamline ψ = const., as shown in Fig. 1(b). On such streamline coordinates, we find:

$$q_\alpha = q, \qquad q_\beta = 0$$

Since $\dfrac{\partial\psi}{\partial\alpha} = 0$ and $\psi = \psi(\beta)$,

$$\frac{\partial\psi}{\partial\beta} = \frac{d\psi}{d\beta} = \rho q h_\beta y^\varepsilon$$

or

$$\frac{d\beta}{d\psi} = \frac{1}{\rho q h_\beta y^\varepsilon} .$$

The equation of vorticity is reduced from Eq. (18) to

$$-\frac{1}{h_\alpha h_\beta}\frac{\rho(h_\alpha q)}{\partial\beta} = \frac{P}{R}\,y^\varepsilon\,\frac{dS}{d\psi}\ . \tag{29}$$

Because $h_\beta \partial\beta = \frac{1}{\rho q y^\varepsilon}\,\partial\psi$, by substitution into Eq. (29),

$$\frac{\partial(qh_\alpha)}{\partial\psi} = -\frac{Ph_\alpha}{R\rho q}\frac{\partial S}{\partial\psi}\ , \tag{30}$$

and, because

$$\frac{\gamma}{\gamma-1}\frac{P}{\rho} + \frac{1}{2}q^2 = \frac{1}{\gamma-1}a_0^{\ 2} = \frac{\gamma}{\gamma-1}\frac{P_0}{\rho_0}$$

or

$$\frac{P}{\rho} = \frac{P_0}{\rho_0}\left[1 - \frac{\gamma-1}{2}\left(\frac{q}{a_0}\right)^2\right]\ , \tag{31}$$

Eq. (30) can be changed to another expression:

$$\frac{\partial(qh_\alpha)}{\partial\psi} = -\frac{P_0}{\rho_0}\left[1 - \frac{\gamma-1}{2}\left(\frac{q}{a_0}\right)^2\right]\frac{h_\alpha}{Rq}\frac{\partial S}{\partial\psi}\ .$$

Furthermore, because

$$\frac{P_0}{R\rho_0} = \frac{a_0^{\ 2}}{\gamma R} = \frac{a_0^{\ 2}}{(C_p-C_v)\left(\frac{C_p}{C_v}\right)} = \frac{a_0^{\ 2}}{(\gamma-1)C_p}\ ,$$

$$\frac{\partial(qh_\alpha)}{\partial\psi} = -\frac{a_0^{\ 2}}{\gamma-1}\left[1 - \frac{\gamma-1}{2}\left(\frac{q}{a_0}\right)^2\right]\frac{h_\alpha}{qC_p}\frac{\partial S}{\partial\psi}$$

or, reduced to dimensionless form,

$$\frac{\partial(h_\alpha \frac{q}{a_0})}{\partial\psi} = -\frac{h_\alpha}{\gamma-1}\left[1-\frac{\gamma-1}{2}\left(\frac{q}{a_0}\right)^2\right]\frac{\frac{\partial(S/C_p)}{\partial\psi}}{q/a_0}. \qquad (32)$$

Under the known distribution of entropy S, this differential equation is the characteristic equation referred to the streamline coordinates.

In different form,

$$\frac{\partial(h_\alpha q/a_0)}{\partial\psi} = h_\alpha \frac{\partial(q/a_0)}{\partial\psi} + q/a_0 \frac{\partial h_\alpha}{\partial\psi} ;$$

Then,

$$h_\alpha \frac{\partial(q/a_0)}{\partial\psi} + q/a_0 \frac{\partial h_\alpha}{\partial\psi} = -\frac{h_\alpha}{\gamma-1}\left[1-\frac{\gamma-1}{2}(q/a_0)^2\right]\frac{\frac{\partial S/C_p}{\partial\psi}}{q/a_0}$$

or

$$\left(\frac{q}{a_0}\right)\frac{\partial(q/a_0)}{\partial\psi} + (q/a_0)^2\frac{1}{h_\alpha}\frac{\partial h_\alpha}{\partial\psi} = -\frac{1}{\gamma-1}\left[1-\frac{\gamma-1}{2}(q/a_0)^2\right]\frac{\partial S/C_p}{\partial\psi}.$$

But

$$\frac{q}{a_0}\frac{\partial(q/a_0)}{\partial\psi} = \frac{1}{2}\frac{\partial(q/a_0)^2}{\partial\psi} \quad \text{and} \quad \frac{1}{h_\alpha}\frac{\partial h_\alpha}{\partial\psi} = \frac{1}{2}\frac{(\ln h_\alpha^2)}{\partial\psi} ;$$

hence,

$$\frac{\partial(q/a_0)^2}{\partial\psi} + (q/a_0)^2\frac{\partial(\ln h_\alpha^2)}{\partial\psi} = -\frac{2}{\gamma-1}\left[1-\frac{\gamma-1}{2}(q/a_0)^2\right]\frac{\partial S/C_p}{\partial\psi}$$

or

$$\frac{\partial (q/a_0)^2}{\partial \psi} + \left[\frac{\partial (\ln h_\alpha^2)}{\partial \psi} - \frac{d(S/C_p)}{d\psi}\right] (q/a_0)^2 = - \frac{2}{\gamma-1} \frac{d(S/C_p)}{d\psi} . \quad (33)$$

This is the differential equation of Uchida (3). To obtain the integration, because

$$\ln h_\alpha^2 - \frac{S}{C_p} = \ln \frac{h_\alpha^2}{e^{S/C_p}}$$

and

$$\frac{\partial}{\partial \psi} [\ln h_\alpha^2 - \frac{S}{C_p}] = \frac{e^{S/C_p}}{h_\alpha^2} \frac{\partial (h_\alpha^2/e^{S/C_p})}{\partial \psi} ,$$

by substitution,

$$\frac{\partial (q/a_0)^2}{\partial \psi} + (\frac{q}{a_0})^2 \frac{e^{S/C_p}}{h_\alpha^2} \frac{\partial (h_\alpha^2/e^{S/C_p})}{\partial \psi} = \frac{-2}{\gamma-1} \frac{d(S/C_p)}{d\psi}$$

or

$$\frac{h_\alpha^2}{e^{S/C_p}} \frac{\partial (q/a_0)^2}{\partial \psi} + (\frac{q}{a_0})^2 \frac{\partial (h_\alpha^2/e^{S/C_p})}{\partial \psi} = \frac{-2}{\gamma-1} \frac{h_\alpha^2}{e^{S/C_p}} \frac{d(S/C_p)}{d\psi} .$$

Now

$$\frac{\partial}{\partial \psi} [\frac{h_\alpha^2}{e^{S/C_p}} (q/a_0)^2] = \frac{h_\alpha^2}{e^{S/C_p}} \frac{\partial (q/a_0)^2}{\partial \psi} + (q/a_0)^2 \frac{\partial (h_\alpha^2/e^{S/C_p})}{\partial \psi} ;$$

therefore,

$$\frac{\partial}{\partial\psi}\left[\frac{h_\alpha^2}{e^{S/C_p}}(q/a_0)^2\right] = \frac{h_\alpha^2}{e^{S/C_p}}\frac{-2}{\gamma-1}\frac{d(S/C_p)}{d\psi} .$$

Integrating along the normal n or β to the streamline ψ_1,

$$\int \frac{\partial}{\partial\psi}\left[\frac{h_\alpha^2}{e^{S/C_p}}(q/a_0)^2\right]d\psi = \int \frac{-2}{\gamma-1}h_\alpha^2 e^{-S/C_p}\frac{d(S/C_p)}{d\psi}d\psi + F(\alpha),$$

where $F(\alpha)$ is an arbitrary function of α only, or

$$(q/a_0)^2 = \frac{1}{h_\alpha^2}e^{S/C_p}\left[\int \frac{-2}{\gamma-1}h_\alpha^2 e^{-S/C_p}\frac{d(S/C_p)}{d\psi}\,d\psi + F(\alpha)\right].$$

Integration between the streamlines, ψ_b and ψ, along an orthogonal, α = constant, eliminates $F(\alpha)$:

$$(q/a_0)^2 h_\alpha^2 e^{-S/C_p} - (q_b/a_0)^2 h_{\alpha b}^2 e^{-S_b/C_p}$$
$$= \int_{\psi_b}^{\psi} \frac{-2}{\gamma-1}h_\alpha^2 e^{-S/C_p}\frac{d(S/C_p)}{d\psi}\,d\psi.$$

Since

$$\frac{de^{-S/C_p}}{d\psi} = -\,e^{-S/C_p}\frac{d(S/C_p)}{d\psi} ,$$

$$(q/a_0)^2 h_\alpha^2 e^{-S/C_p} = (q_b/a_0)^2 h_{\alpha b}^2 e^{-S_b/C_p}$$
$$+ \frac{2}{\gamma-1}\int_{\psi_b}^{\psi} h_\alpha^2 \frac{d(e^{-S/C_p})}{d\psi}\,d\psi$$

or

$$q^2 h_\alpha^2 e^{-S/C_p} = q_b^2 h_{\alpha b}^2 e^{-S_b/C_p} + \frac{2a_0^2}{\gamma-1} \int_{\psi_b}^{\psi} h_\alpha^2 \frac{de^{-S/C_p}}{d\psi} d\psi. \tag{34}$$

This is the integral equation of Uchida (3). Also, because

$$\int_{\psi_b}^{\psi} h_\alpha^2 \frac{de^{-S/C_p}}{d\psi} d\psi = [h_\alpha^2 e^{-S/C_p}]_{\psi_b}^{\psi} - \int_{\psi_b}^{\psi} e^{-S/C_p} \frac{d(h_\alpha^2)}{d\psi} d\psi,$$

Eq. (34) can be written as

$$(q/a_0)^2 h_\alpha^2 e^{-S/C_p} = (q_b/a_0)^2 h_{\alpha b}^2 e^{-S_b/C_p} + \frac{2}{\gamma-1} [h_\alpha^2 e^{-S/C_p}]_{\psi_b}^{\psi} - \frac{2}{\gamma-1} \int_{\psi_b}^{\psi} e^{-S/C_p} \frac{d(h_\alpha^2)}{d\psi} d\psi$$

or

$$[(q/a_0)^2 - \frac{2}{\gamma-1}] h_\alpha^2 e^{-S/C_p} = [(q_b/a_0)^2 - \frac{2}{\gamma-1}] h_{\alpha b}^2 e^{-S_b/C_p} - \frac{2}{\gamma-1} \int_{\psi_b}^{\psi} e^{-S/C_p} \frac{d(h_\alpha^2)}{d\psi} d\psi$$

or

$$(q/a_0)^2 = \frac{h_{\alpha b}^2}{h_\alpha^2} e^{\frac{(S-S_b)}{C_p}} (q_b/a_0)^2 + \frac{2}{\gamma-1} e^{S/C_p} (\frac{h_{\alpha b}}{h_\alpha})^2 \int_{\psi_b}^{\psi} (\frac{h_\alpha}{h_{\alpha b}})^2 \frac{de^{-S/C_p}}{d\psi} d\psi. \tag{35}$$

In the fully nondimensional form, this equation can be written as

$$(q/a_0)^2 = \left(\frac{h_{\alpha b}/\ell}{h_\alpha/\ell}\right)^2 e^{(S-S_b)/C_p}(q_b/a_0)^2$$

$$+ \frac{2}{\gamma-1} e^{S/C_p}\left(\frac{h_{\alpha b}/\ell}{h_\alpha/\ell}\right)^2 \int_{\psi_b/\bar{\psi}_u}^{\psi/\bar{\psi}_u} \left(\frac{h_\alpha/\ell}{h_{\alpha b}/\ell}\right)^2 \frac{de^{-S/C_p}}{d(\frac{\psi}{\bar{\psi}_u})} d\left(\frac{\psi}{\bar{\psi}_u}\right), \qquad (36)$$

where $\bar{\psi}_u = \rho_0 a_0 \ell (\frac{\ell}{2})^\varepsilon$, unit flux.

The stream function can be calculated by integrating the equation of continuity as follows:

$$\psi - \psi_0 = \int_{\beta_a}^{\beta} \rho q y^\varepsilon h_\beta d\beta \; .$$

Boundary conditions are given by

$$\psi = \psi_a = 0 \quad \text{at} \quad \beta = \beta_a = 0$$

and

$$\psi = \psi_s = \rho_1 q_1 y_1 (y_1/2)^\varepsilon \quad \text{at} \quad \beta = \beta_s(\alpha)$$

or

$$\frac{\psi}{\psi_s} = \frac{\psi}{\rho_0 a_0 \ell (\ell/2)^\varepsilon} = \int_0^\beta \frac{\rho q}{\rho_0 a_0}\left(2\frac{y}{\ell}\right)^\varepsilon \frac{h_\beta}{\ell} d\beta$$

$$= \int_0^{n/\ell} \frac{\rho q}{\rho_0 a_0}\left(2\frac{y}{\ell}\right)^\varepsilon d\left(\frac{n}{\ell}\right) \; , \qquad (37)$$

where $h_\beta d\beta = dn$ and

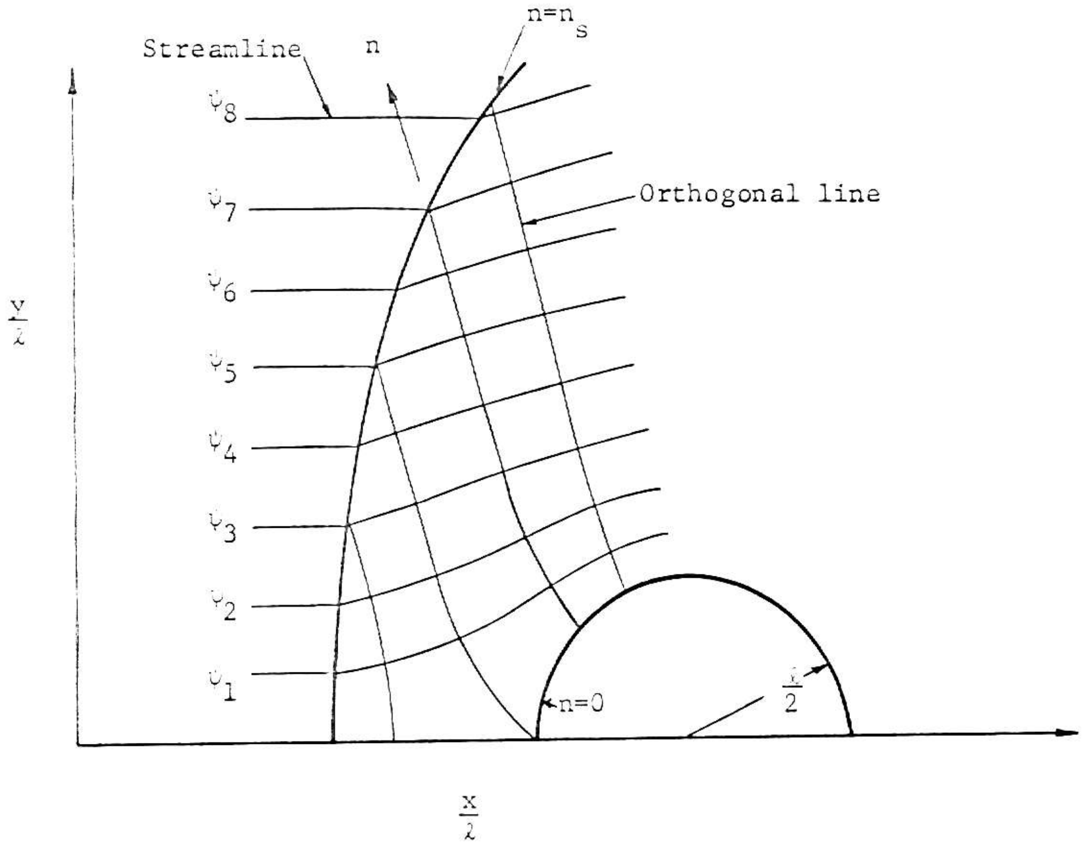

Fig. 2. Approximation of flow pattern.

$$\frac{\rho q}{\rho_0 a_0} = e^{-[(S-S_0)/R]} \left[\left(1 - \frac{\gamma-1}{2} \frac{q^2}{a_0^2}\right)^{1/(\gamma-1)} \frac{q}{a_0} \right] . \tag{38}$$

Hence,

$$\frac{\psi}{\rho_0 a_0 \ell (\ell/2)^{\varepsilon}} = f\left(\frac{q}{a_0}\right) = G\left(\frac{q_s}{a_0}\right) , \tag{39}$$

but $\psi = \psi_s = \rho_1 q_1 y_1 (y_1/2)^{\varepsilon}$ at $n = n_s$;

therefore,

$$\frac{\psi_s}{\rho_0 a_0 \ell (\ell/2)^{\varepsilon}} = \frac{\rho_1 q_1 y_1 (y_1/2)^{\varepsilon}}{\rho_0 a_0 \ell (\ell/2)^{\varepsilon}} = G\left(\frac{q_s}{a_0}\right) . \tag{40}$$

The quantity, q_s/a_0, is obtained by numerical calculation of Eq. (40).

3-2. Approximate Method of Flux Analysis by Uchida and Yasuhara

Consider now that the conditions of a free stream before shock wave are given and that the body shape is known. The method of flux analysis by Uchida and Yasuhara can be reduced to the following steps:

STEP 1 --- Assume the flow pattern.

In the first step of the procedure, an approximate pattern for the streamlines around the body and an approximate detached shock shape are assumed as shown in Fig. 2. Such approximate patterns can be obtained from approximate theories; even incompressible flow patterns have been successfully employed in several cases.

STEP 2 --- Iteration of streamlines.

For the given free stream Mach number and the assumed streamline pattern and shock shape, the entropy distribution and the distribution of the

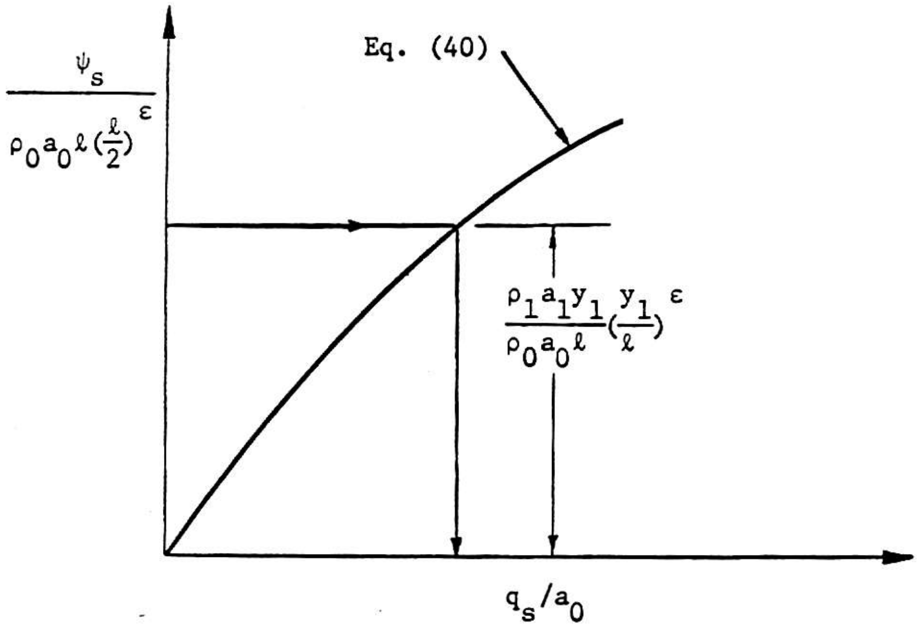

Fig. 3. Representation of Eq. (40).

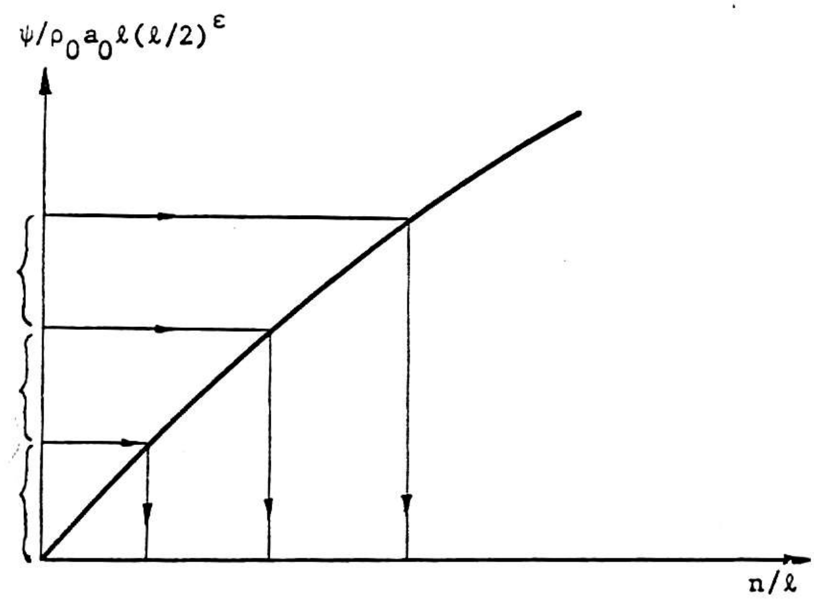

Fig. 4. Flux distribution along an orthogonal curve (α=constant).

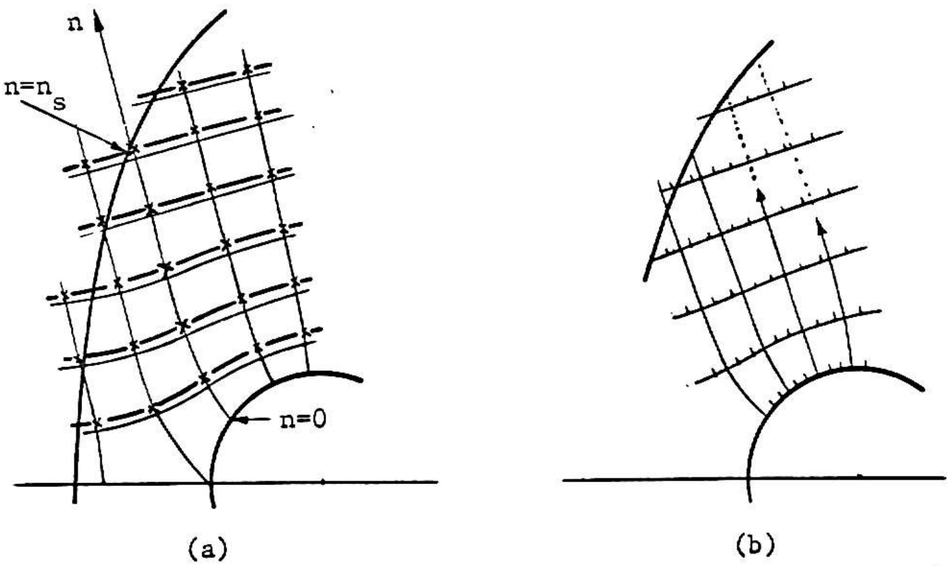

Fig. 5. Process of correction of flow patterns.

approximate values for the geometric parameter, $h_\alpha/h_{\alpha s} = \delta s_s/\delta s$, can be established. In the formula for $h_\alpha/h_{\alpha s}$, δs is the are length of a streamline segment between two adjacent orthogonal curves.

We can obtain the value of flow speed at an arbitrary point, q/a_0, expressed in terms of the boundary value of velocity, q_s/a_0, can be obtained from the following equation:

$$(q/a_0)^2_m = (h_{\alpha s}/h_\alpha)^2_{m-1} e^{[(S-S_s)_{m-1}/C_p]} (q_s/a_0)^2_m +$$

$$\frac{2}{\gamma-1}(h_{\alpha s}/h_\alpha)^2_{m-1} e^{S_{m-1}/C_p} \int_{\beta_{s,m-1}}^{\beta_{m-1}} (h_\alpha/h_{\alpha s})^2_{m-1} \frac{de^{\frac{-S_{m-1}}{C_p}}}{d\beta_{m-1}} d\beta_{m-1} , \tag{41}$$

where m represents the degree of the approximation, i.e. the iteration number.

Using Eqs. (38) and (40), the boundary value of dimensionless speed of flow, q_s/a_0, is obtained as shown in Fig. 3. Then, substituting q_s/a_0 into Eq. (39), a distribution of the stream function along an orthogonal curve (α=constant) can be obtained (see Fig. 4). From the ψ-n relation, values of β or n corresponding to the stream function can be corrected and, accordingly, the intersection of the corrected streamline with the orthogonal curve concerned, α= constant, can be determined as shown schematically in Fig. 5.

STEP 3 --- Correction of the form of the bow shock wave

The value of W_1 corresponding to the δ_{max}, viz., W_{1max}, can be obtained by differentiating Eq. (25), and setting $\frac{d\delta}{dW} = 0$; the following result is obtained:

$$\sin^2 W_{1max} = \frac{1}{\gamma M_1^2}\{\frac{\gamma+1}{4} M_1^2 - 1$$

$$+ \sqrt{[(\gamma+1)(1 + \frac{\gamma-1}{2} M_1^2 + \frac{\gamma+1}{16} M_1^4)]}\} \tag{42}$$

Then, substituting W_{1max} into Eq. (25), δ_{max} can be obtained.

In the initially assumed patterns, the streamline deflection, θ_s, in general does not coincide with the flow deflection angle immediately behind the shock δ obtained from the oblique shock relations for the assumed shock angle. Uchida and Yasuhara use $K = \frac{\theta_s}{\delta} = \frac{\theta_{smax}}{\delta_{max}}$ as a parameter to define the incompleteness of the solution at the shock boundary for a fixed detachment distance.

The first step in their calculation is to find a shock shape which is consistent with K = 1 approximately. They set the value of the detachment distance b on the axis. With a given value of the detachment distance, they find an appropriate value of K and the corresponding shock shape by a method of successive correction.

If the calculations for various values of b are made by starting with identical patterns of flow as the zeroth approximation, the correction factor, K, will be given as a continuous function of detachment distance b. The correct detachment distance for the nose of the bow shock wave (corresponding to K=1) will be determined from the K-b relation as obtained above. A final solution is obtained by determining the flow field corresponding to this value of b.

STEP 4 --- Double iteration

Using the new value of δ from Step 3 (which will give the readjusted distribution of entropy [Eqs. (26), (27), and (28a)] and the streamlines (corrected by the method described in Step 2), the shock shape is once again adjusted to the new streamline pattern. The double iteration technique is carried out by repeating Steps 2 and 3 until the streamline pattern and the shock shape and location provide a completely consistent solution.

CHAPTER IV

FLOW AROUND A SPHERE IN A SUPERSONIC FREE STREAM WITH A DETACHED BOW SHOCK WAVE

As an illustrative example, the flow about a sphere will be determined for the case of an M=1.5 uniform, supersonic free stream:

1. Assume flow pattern and shock form.

A flow pattern for starting the calculation can be assumed arbitrarily. The more nearly the initially chosen flow pattern approximates the final result, the more easily a convergent solution for a fixed detachment distance of the shock wave may be found. In the present problem, the flow pattern corresponding to the incompressible, irrotational flow of a fluid around a sphere is initially chosen for starting the calculation. The stream function corresponding to incompressible, irrotational flow around a sphere is

$$\psi = -\frac{Ud^3}{16r}\,\mathrm{Sin}^2\theta + \frac{Ur^2}{2}\,\mathrm{Sin}^2\theta ,$$

where d = the diameter of the sphere,

r = the distance from the center to streamline,

U = the free stream velocity,

and

θ = angle of polar coordinate.

For simplicity, the value of the detachment distance, b, can be selected from the experimental values of Heberle, Wood and Gooderum's or from

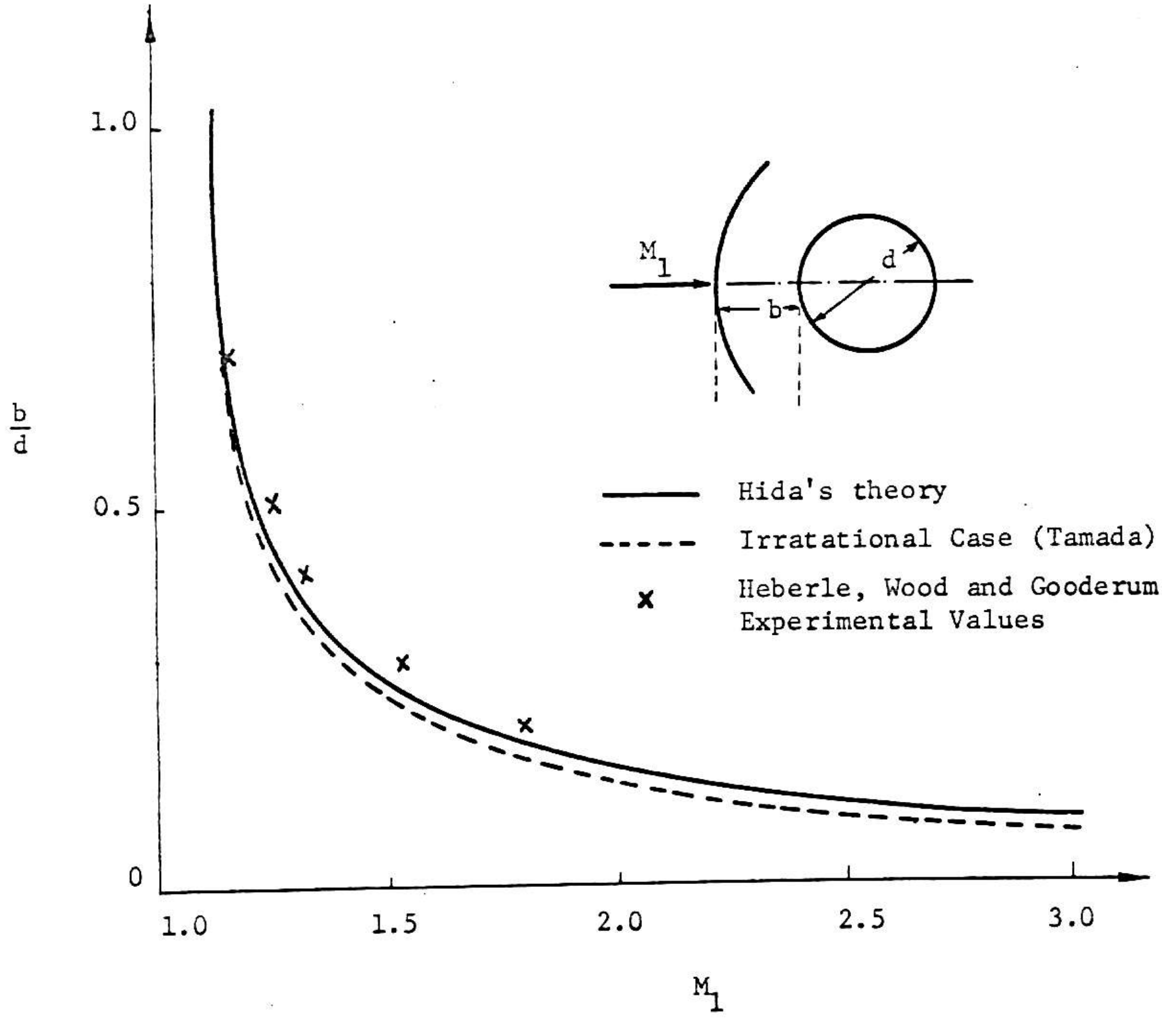

Fig. 6. Case of a Sphere.

the theoretical values of Hida shown in Fig. 6. In the present case, $M_1 = 1.5$; therefore, b/d - 0.287. It is assumed that the entropy in the free stream stagnation state is $S_0 = 0$.

The assumed flow pattern and shock shape are shown in Fig. 7(a).

2. Correction of streamlines.

Given $M_1 = 1.5$, it can be shown for $\gamma = 1.4$, that

$$\rho_1/\rho_0 = 0.395,$$

$$P_1/P_0 = 0.2724,$$

$$M^* = 1.365,$$

and

$$q_1/a_0 = q_1/(\frac{\gamma+1}{2})^{\frac{1}{2}}a^* = (\frac{2}{\gamma+1})^{\frac{1}{2}}M^* = (\frac{2}{1.4+1})^{\frac{1}{2}} \times 1.365 = 1.246.$$

For the sphere, $\varepsilon=1$, and, if $\ell=d$, the shock wave boundary can be represented by

$$\frac{\psi_s}{\rho_0 a_0 (\ell)(\frac{\ell}{2})^\varepsilon} = \frac{\rho_1}{\rho_0}\frac{q_1}{a_0}(\frac{y_1}{d})^2$$

or

$$\frac{\psi_s}{\rho_0 a_0 (d)(\frac{d}{2})} = 0.395 \times 1.246(\frac{y_1}{d})^2 = 0.4922(\frac{y_1}{d})^2 .$$

From Eqs. (26) and (27),

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}(M_1^2 \sin^2 W_1 - \frac{\gamma-1}{2\gamma}) = 2.625 \sin^2 W_1 - 0.1667$$

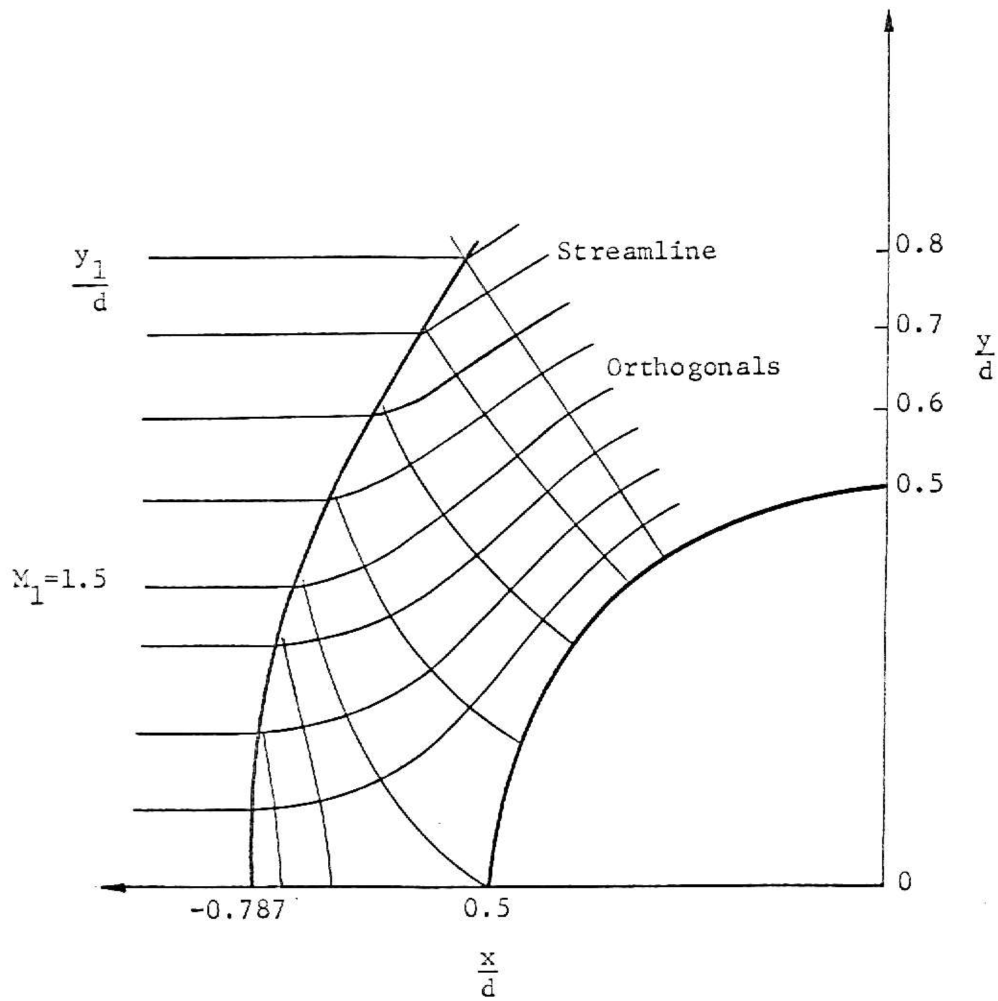

Fig. 7(a). Assumed flow pattern.

and

$$\frac{\rho_1}{\rho_2} = \frac{2}{\gamma+1}\left(\frac{1}{M_1^2 \operatorname{Sin}^2 W_1} + \frac{\gamma-1}{2}\right) = \frac{0.3704}{\operatorname{Sin}^2 W_1} + 0.1667.$$

From Eq. (2),

$$e^{\frac{S_1-S_0}{C_v}} = \left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{\rho_1}\right)^{\gamma} = (0.2724) \times \left(\frac{1}{0.395}\right)^{1.4} = 1.0007.$$

If

$$S_0 = 0,$$

$$e^{-S_1/C_v} = 0.9993,$$

$$e^{-S_1/C_p} = 0.9994,$$

and

$$e^{-S_1/R} = 0.9997.$$

From Eqs. (28a), (28b), and (28c),

$$e^{-S_2/C_v} = e^{-S_1/C_v}\left(\frac{\rho_2}{\rho_1}\right)^{\gamma}\frac{1}{P_2/P_1} = \frac{0.9993}{(P_2/P_1) \times \left(\frac{\rho_1}{\rho_2}\right)^{1.4}},$$

$$e^{-S_2/C_p} = e^{-S_1/C_p}\left(\frac{\rho_2}{\rho_1}\right)\frac{1}{(P_2/P_1)^{1/\gamma}} = \frac{0.9994}{(P_2/P_1)^{1/1.4} \times (\rho_1/\rho_2)},$$

and

$$e^{-S_2/R} = e^{-S_1/R} \left(\frac{\rho_2}{\rho_1}\right)^{\frac{\gamma}{\gamma-1}} \frac{1}{(P_2/P_1)^{1/\gamma-1}}$$

$$= \frac{0.9997}{(P_2/P_1)^{1/0.4} \times (\rho_1/\rho_2)^{1.4/0.4}} .$$

Measuring W_1 values from the initial assumed flow pattern shown in Fig. 7(a) and substituting into the above equations, distribution of the entropy and the entropy gradient behind the shock wave can be obtained as shown in Table II. The approximate value of the parameter, $h_\alpha/h_{\alpha s}$, can be measured from the assumed flow pattern since

$$h_\alpha/h_{\alpha s} = \delta s_s/\delta s.$$

Thus, using Eqs. (36), (38), and (40) and the given data, the corrected streamline distribution can be determined. The ψ-n/d relation along an orthogonal curve (α= const.), for example, the 8-g line shown in Fig. 7(b), can be calculated as follows:

$$W_1 = 66° \text{ (from Fig. 7(a))}$$

and

$$\frac{y_1}{d} = 0.8 \text{ (from Fig. 7(a))}.$$

Values of $\frac{h_\alpha}{h_{\alpha s}}$ and $\frac{y}{d}$ corresponding to each intersection on this orthogonal curve (i.e., the 8-g line shown in Fig. 7(b)) are

$$\frac{h_\alpha}{h_{\alpha s}} = 0.9,\ 0.92,\ 0.95,\ 0.97,\ 0.98,\ 0.99,\ 1,\ 1$$

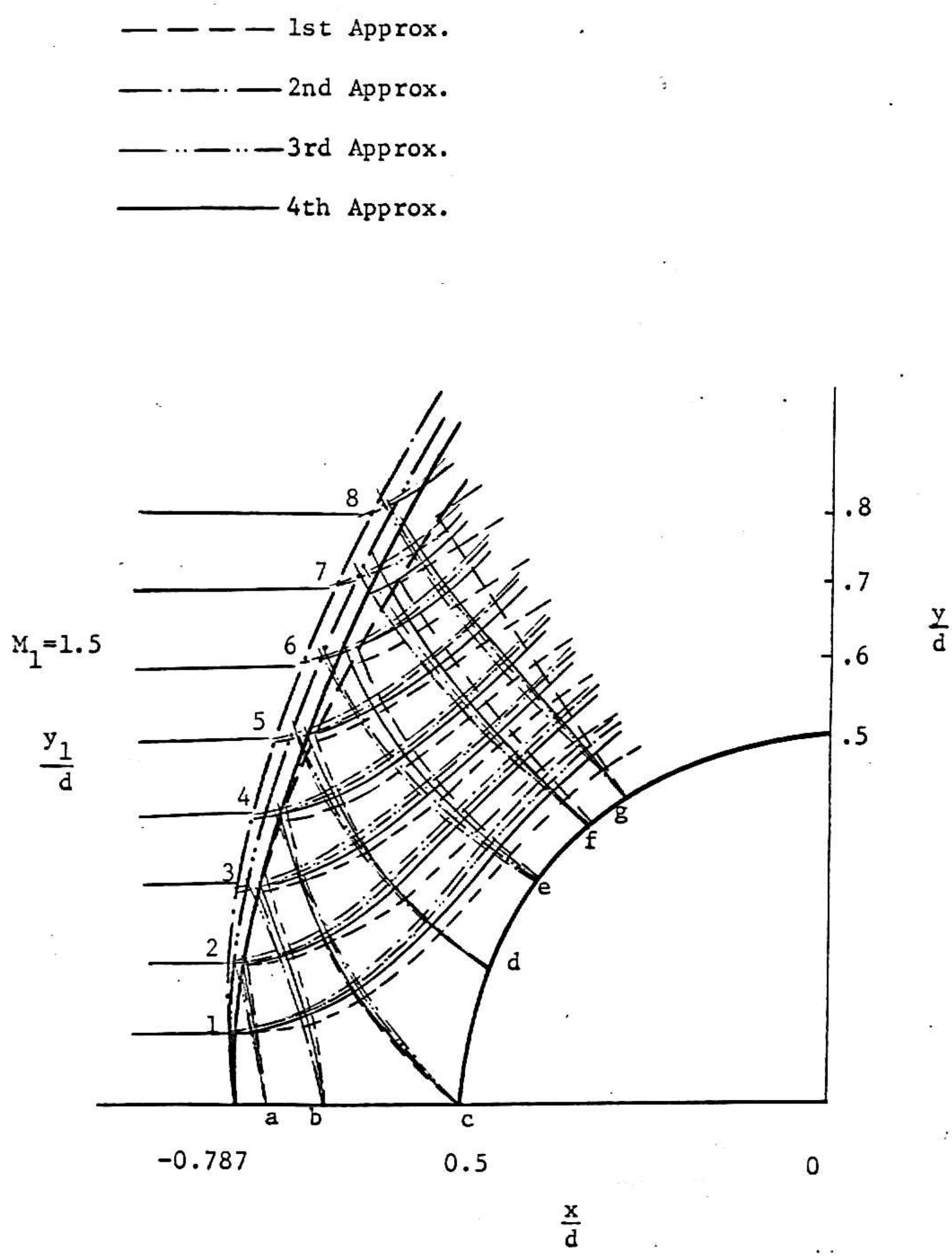

Fig. 7(b). Streamlines and form of bow shock waves in every stage approximation for b/d = 0.287.

and

$$\frac{y}{d} = 0.48,\ 0.525,\ 0.56,\ 0.625,\ 0.662,\ 0.712,\ 0.75,\ 0.80;$$

$$\frac{\psi_s}{\rho_0 a_0 \frac{d^2}{2}} = 0.4922 \times (0.8)^2 = 0.3136;$$

$$\frac{P_2}{P_1} = 2.625 \times \mathrm{Sin}^2(66°) - 0.1667 = 2.0241;$$

$$\frac{\rho_1}{\rho_2} = \frac{0.3704}{\mathrm{Sin}^2(66°)} + 0.1667 = 0.6105;$$

$$e^{-S_2/C_p} = \frac{0.9994}{(2.0241)^{1/1.4} \times (0.6105)} = 0.9969.$$

Similarly, other lines such as 7-f, 6-e (see Fig. 7(b)) and so forth are calculated. The quantities $e^{-S_2/R}$ and $\frac{de^{-S2/C_p}}{d\psi/\rho_0 a_0 d}$ are presented in Table II. Then, substituting the above data into Eqs. (36), (38), and (40), the value of q_s/a_0 corresponding to this orthogonal curve (i.e., the 8-g line shown in Fig. 7(b)) can be found. Thus,

$$q_s/a_0 = 0.90$$

in the first approximation, and the ψ vs α relation can now be established as shown in Fig. 9(a). From the ψ vs α relationship shown in Figures 9(a) and 9(b), values of α or n corresponding to the various streamlines, ψ = constant, can be corrected; then the orthogonal curves, α = constant, can be constructed as shown in Fig. 7(b).

New orthogonal curves and, consequently, new values of the parameters, $h_\alpha/h_{\alpha s}$ and y/d, are used in the succeeding calculation.

3. Correction of the detached-shock-form.

(a) Using Eq. (42), the maximum value of W_1 is obtained:

$$\sin^2 W_{1,max} = \frac{1}{\gamma M_1^2}[\frac{\gamma+1}{4}M_1^2 - 1 + \sqrt{(\gamma+1)(1 + \frac{\gamma-1}{2}M_1^2 + \frac{\gamma+1}{16}M_1^4)}\ ]$$

$$= \frac{1}{1.4x1.5^2}[\frac{1.4+1}{4} \text{ x } 1.5^2 - 1$$

$$+ \sqrt{(2.4)(1 + \frac{0.4}{2} \text{ x } 1.5^2 + \frac{2.4}{16} \text{ x } 1.5^4)}\ ] = 0.842$$

hence $W_{1,max} = 66°36'$.

Using Eq. (25), the corresponding deflection angle,

$$\delta_{max} = 12°6',$$

is then found.

(b) Using Hida's theory or the experimental values of Heberle, Wood and Gooderum (see Fig. 6)

$$b/d = 0.287$$

therefore,

$$\frac{\delta_{max}}{\theta_{s,max}} = K = 0.95 \approx 1 .$$

Since the correction factor is very close to unity, the value of b/d, 0.287, will be assumed as the final solution of the detachment distance.

(c) Then, the form of the shock wave is drawn by tracing the points,

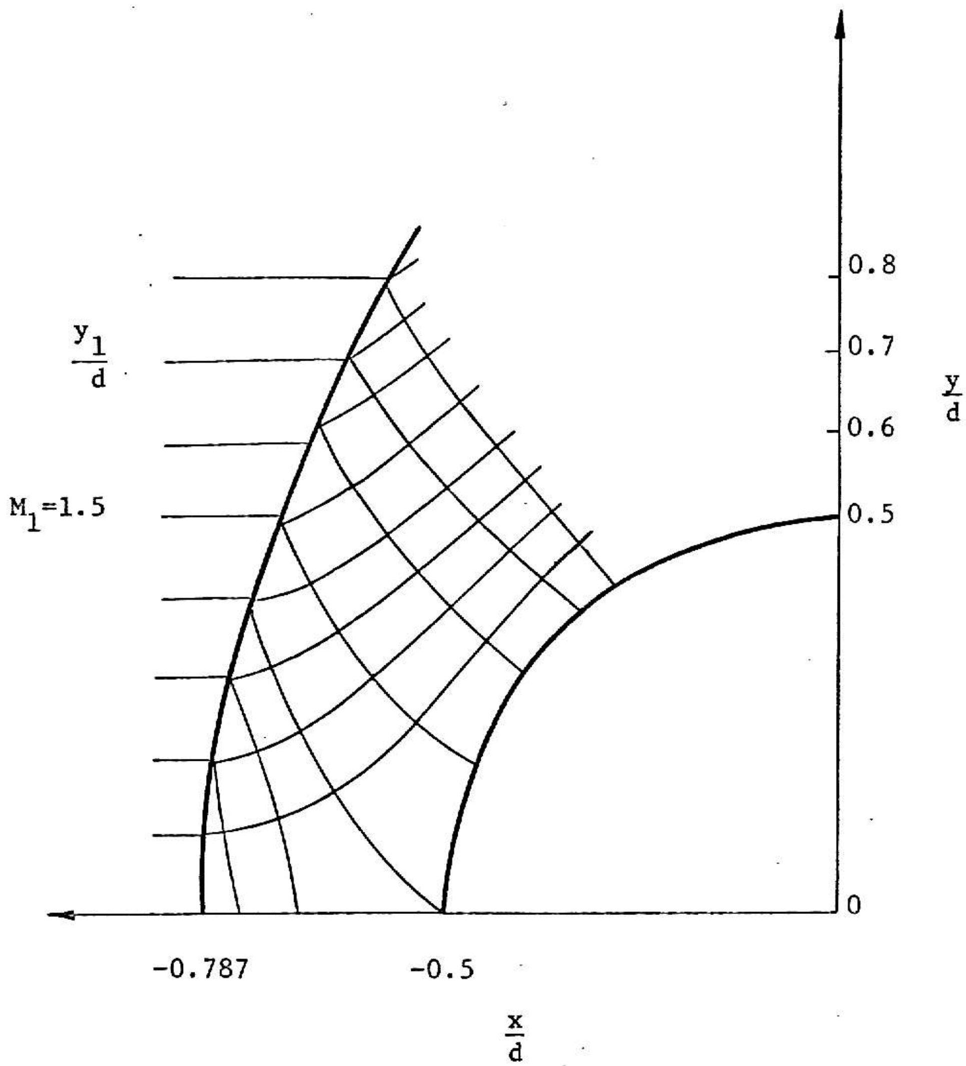

Fig. 7(c). Flow pattern.

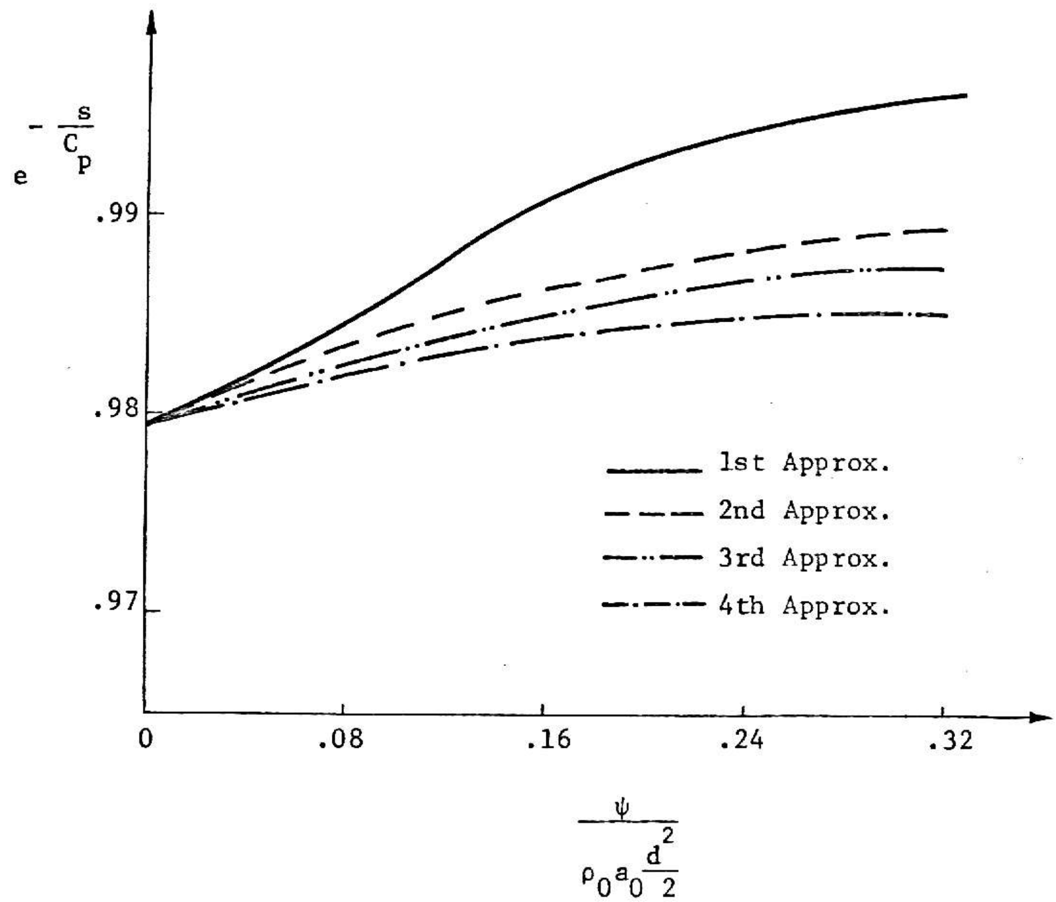

Fig. 8. Distribution of the entropy (sphere).

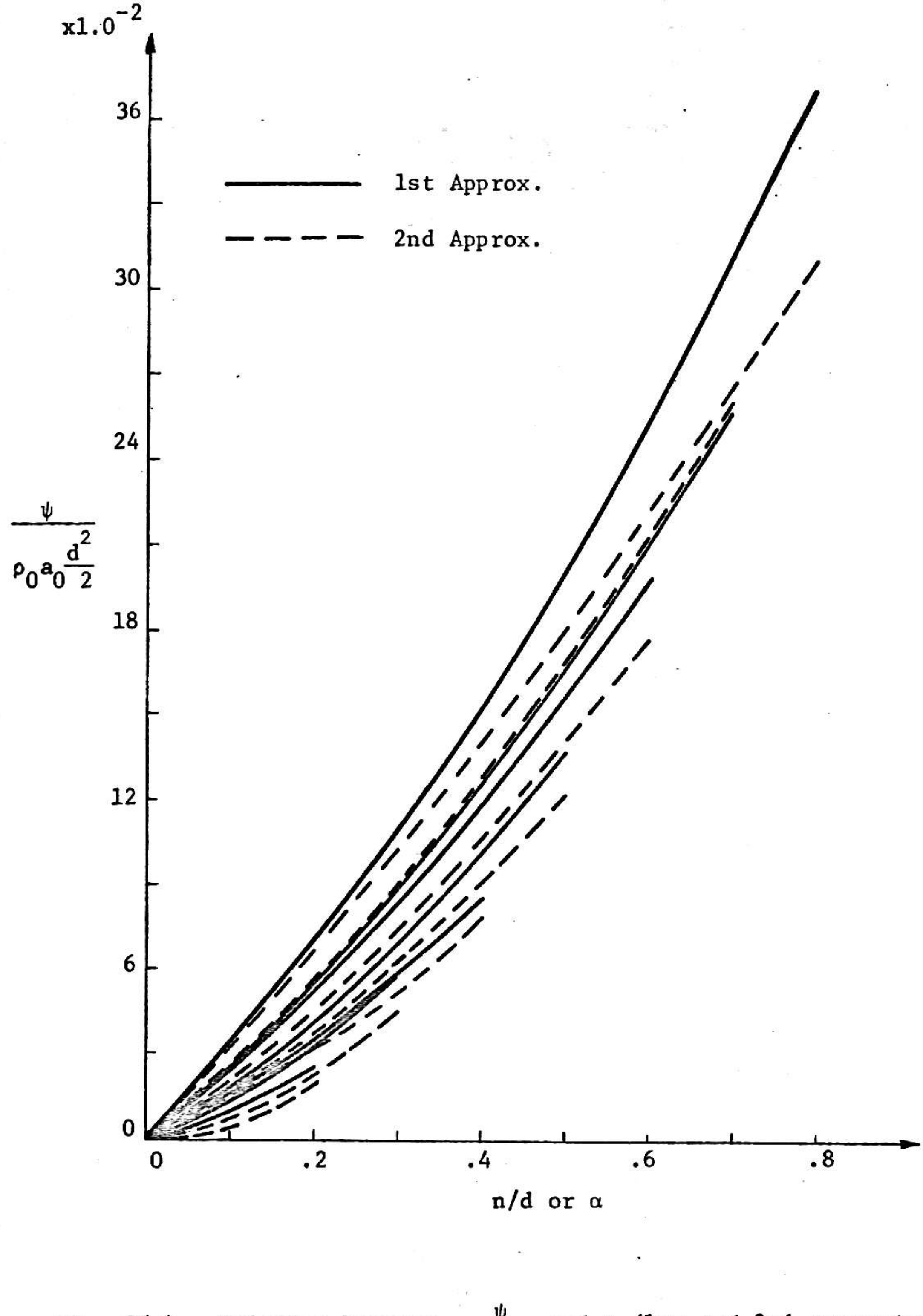

Fig. 9(a). Relation between $\frac{\psi}{\rho_0 a_0 \frac{d^2}{2}}$ and α (1st and 2nd approximations).

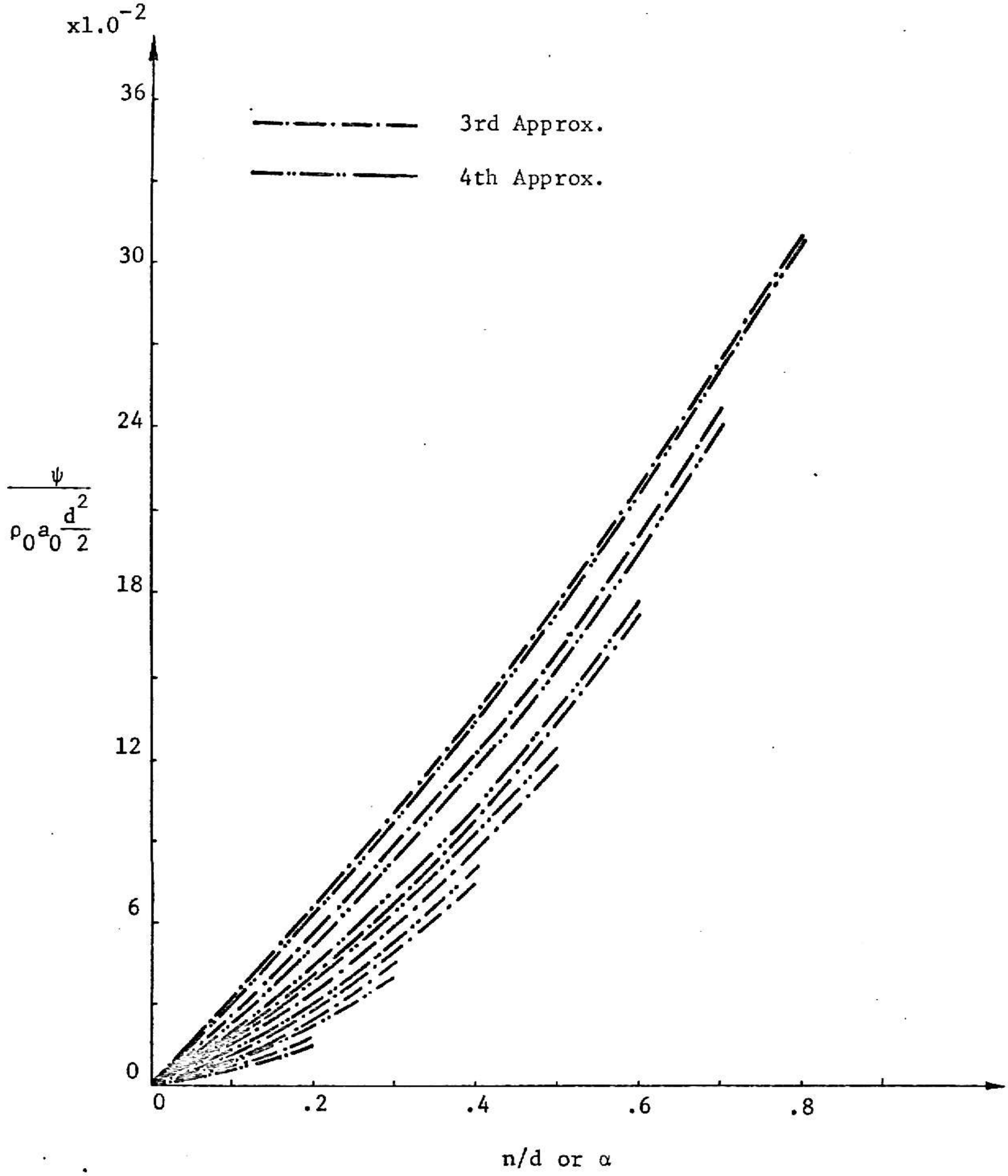

Fig. 9(b). Relation between $\frac{\psi}{\rho_0 a_0 \frac{d^2}{2}}$ and α (3rd and 4th approximations).

on which the angle of flow, θ_s, coincides with the corresponding deflection angle, δ, as given by Eq. (25).

4. Double iteration.

Substituting the new values of W_1 into Eqs. (26), (27), and (28), the new distribution of entropy is determined as shown in Fig. 8 or Table II. Correction of the flow field established by the method outlined above can now be corrected by iteration.

Four iterations have been calculated with the aid of IBM 1620 and IBM 360 computers. Since the linear relation on each α-constant line is not like that on any other α=constant line, a trial-and-error method for finding the values of q_s/a_0 that satisfied Eq. (40) seemed necessary. However, there are always two roots in Eq. (40), especially in the vicinity of the sonic line.

All the results are shown in Table III and Fig. 7(c).

CHAPTER V

DISCUSSION

At present, no one of the existing analytical treatments or numerical methods for solving the supersonic or hypersonic blunt-body problem is entirely adequate for predicting the details of the flow field past general blunt shapes. This is particularly true if an adequate solution of this problem is defined as one which can establish the flow field geometry when non-equilibrium effects are present in the flowing fluid.

The problem which is considered here to be the most interesting one is what is termed the "direct problem", in which the body shape is given and the details of the flow field are unknown. Existing numerical attacks on the direct problem fall mainly into three categories: (1) Steamtube-continuity methods, (2) Methods of integral relations and polynomial approximation, and (3) Relaxation techniques and the unsteady approach method.

In this present report, the method of flux analysis developed by Uchida and Yasuhara, a streamtube-continuity technique, is used. Other such iteration schemes have been reported by Maslen and Moeckel, Gravalos (5), and Gravalos, Edelfelt, and Emmons (11). Among such iterative approaches, the method of Uchida and Yasuhara is, in principle, more nearly exact than the other methods. No fixed assumptions are required in the scheme given by Uchida and Yasuhara. If no difficulties in convergence occur, this method provides a way of completely determining the blunt-body flow under consideration. Furthermore, many of the difficulties associated with the transonic character of the flow in the neighborhood of the sonic line can be minimized.

However, Uchida and Yasuhara have given a very tedious method of successive approximations which does not seem to be easily adapted to computer computation. Such an approach seems to involve procedures of trial and error, and some errors grow with the number of iterations as geometric or arithmetic progressions. To increase the accuracy achieved, one must perform many iterations and, in making these iterations, must carry many significant figures.

Uchida and Yasuhara have published the results of such a calculation for the flow over a circular cylinder at $M = 2$ ($\gamma = 1.4$). Their published results agree fairly well with corresponding experimental results.

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TABLE I

NUMERICAL RELATION BETWEEN FLOW QUANTITIES IN FRONT AND BEHIND INCLINED SHOCK WAVES AT M_1 = 1.50 (from reference 10)

W_1 (deg.)	W_1 (min.)	δ (deg.)	δ (min.)	M_2	$\frac{P_2}{P_1}$	$\frac{P_1}{P_2}$	$\frac{\Delta S}{ft^2 sec^2 / °F}$
41	49	0	0	1.500	1.000	1.000	0
45		2	47	1.405	1.146	.9074	.3339
48		5	5	1.322	1.283	.8373	2.643
51		7	5	1.246	1.419	.7799	7.361
54		8	46	1.174	1.551	.7325	14.752
57		10	8	1.107	1.680	.6932	24.227
60		11	9	1.045	1.802	.6605	35.484
63		11	49	.986	1.917	.6332	47.745
66		12	6	.932	2.024	.6105	60.597
69		11	59	.882	2.121	.5916	73.519
72		11	25	.837	2.208	.5761	85.625
75		10	26	.797	2.282	.5636	96.758
78		9	0	.764	2.345	.5538	106.52
81		7	10	.737	2.394	.5463	114.27
84		5	0	.717	2.430	.5411	120.18
87		2	34	.705	2.451	.5381	123.69
90		0	0	.701	2.458	.5370	124.917

TABLE II

DISTRIBUTION OF THE ENTROPY AND ITS DERIVATIVE
(SPHERE, M_1=1.5, γ=1.4, S_0=0)

y_1/d	$\psi/\rho_0 a_0 d^2/2$	e^{-S/C_p}	$\dfrac{de^{-S/C_p}}{d\psi/\rho_0 a_0 d^2/2}$
		1st Approximation	
0.1	0.0049	0.9799	0.1219
0.2	0.0196	0.9815	0.0397
0.3	0.0441	0.9834	0.0489
0.4	0.0784	0.9871	0.0501
0.5	0.1224	0.9906	0.0235
0.6	0.1764	0.9920	0.0221
0.7	0.3136	0.9941	0.0226
		2nd Approximation	
0.1	0.0049	0.9799	0.1219
0.2	0.0196	0.9808	0.0305
0.3	0.0441	0.9811	0.0271
0.4	0.0784	0.9824	0.0291
0.5	0.1224	0.9834	0.0163
0.6	0.1764	0.9840	0.0102
0.7	0.3136	0.9846	0.0087

TABLE II (Continued)

y_1/d	$\psi/\rho_0 a_0 d^2/2$	e^{-S/C_p}	$\dfrac{de^{-S/C_p}}{d\psi/\rho_0 a_0 d^2/2}$
		3rd Approximation	
0.1	0.0049	0.9799	0.1219
0.2	0.0196	0.9811	0.0483
0.3	0.0441	0.9819	0.0491
0.4	0.0784	0.9840	0.0508
0.5	0.1224	0.9858	0.0254
0.6	0.1764	0.9865	0.0229
0.7	0.3136	0.9885	0.0196
		4th Approximation	
0.1	0.0049	0.9799	0.1219
0.2	0.0196	0.9810	0.0406
0.3	0.0441	0.9815	0.0322
0.4	0.0784	0.9829	0.0356
0.5	0.1224	0.9843	0.0234
0.6	0.1764	0.9852	0.0152
0.7	0.3136	0.9861	0.0131

TABLE III

SOLUTION FOR SPHERE AT M=1.5, γ=1.4

First Approximation

Line No.	n/ℓ	q_s/a_0	y/d	$h_{\alpha s}/h_\alpha$	$\frac{\psi}{\rho_0 a_0 \frac{d^2}{2}}$	C*	P**
8-g	0.47	0.90	0.470	0.90	0.0329	0.0590	0.9111
			0.525	0.92	0.0687	0.0468	0.9683
			0.560	0.95	0.1084	0.0403	0.9874
			0.625	0.97	0.1526	0.0296	0.9853
			0.662	0.98	0.2038	0.0208	1.0034
			0.710	0.99	0.2572	0.0136	1.0027
			0.750	1.00	0.3139	0.0068	1.0007
			0.800	1.00	0.3145	0.0000	1.0000
7-f	0.40	0.77	0.40	0.92	0.0267	0.0521	0.9297
			0.46	0.93	0.0568	0.0395	0.9676
			0.50	0.95	0.0905	0.0333	0.9867
			0.537	0.97	0.1265	0.0229	0.9846
			0.60	0.98	0.1665	0.0138	0.9828
			0.65	1.00	0.2106	0.0068	1.0020
			0.70	1.00	0.2580	0.0000	1.0000
6-e	0.375	0.63	0.355	0.96	0.0232	0.0443	0.9669
			0.425	0.98	0.0516	0.0318	0.9855
			0.460	0.99	0.0830	0.0259	1.0047
			0.500	0.99	0.1176	0.0161	1.0025
			0.560	1.00	0.1567	0.0072	1.0007
			0.600	1.00	0.1983	0.0000	1.0000

$$* \quad C = \frac{-z}{\gamma-1}\left(\frac{h_{\alpha s}}{h_\alpha}\right)^2 e^{S/C_p} \int_{\beta_s}^{\beta} \left(\frac{h_\alpha}{h_{\alpha s}}\right)^2 \frac{de^{-S/C_p}}{d\beta} d\beta$$

$$** \quad P = \left(\frac{h_s}{h}\right)^2 e^{\frac{S-S_s}{C_p}} \left(\frac{q_s}{a_0}\right)^2$$

TABLE III (Continued)

Line No.	n/ℓ	q_s/a_0	y/d	$h_{\alpha s}/h_\alpha$	$\frac{\psi}{\rho_0 a_0 \frac{d^2}{2}}$	C	P
5-d	0.34	0.585	0.245	1.10	0.0174	0.0390	1.2173
			0.340	1.05	0.0414	0.0250	1.1078
			0.410	1.00	0.0695	0.0171	1.0040
			0.450	1.00	0.1008	0.0083	1.0018
			0.500	1.00	0.1360	0.0000	1.0000
4-c	0.34	0.510	0.200	1.30	0.0163	0.0277	1.2151
			0.245	1.20	0.0354	0.0153	1.1058
			0.290	1.05	0.0577	0.0085	1.0021
			0.375	1.00	0.0862	0.0000	1.0000
3-6	0.40	0.382	0.140	1.00	0.0129	0.0161	1.0020
			0.210	1.00	0.0326	0.0058	1.0008
			0.280	1.00	0.0581	0.0000	1.000
2-a	0.22	0.475	0.110	1.10	0.0101	0.0107	1.2115
			0.190	1.00	0.0266	0.0000	1.0000

TABLE III (Continued)

Second Approximation

Line No.	n/ℓ	q_s/a_0	y/d	$h_{\alpha s}/h_\alpha$	$\frac{\psi}{\rho_0 a_0 \frac{d^2}{2}}$	C	P	$(\frac{q}{a_0})^2$
8-g	0.54	1.32	0.50	0.90	.0329	.0389	.8144	1.3800
			0.54	0.92	.0677	.0254	.8502	1.4560
			0.57	0.95	.1030	.0215	.9063	1.5576
			0.63	0.97	.1411	.0158	.9436	1.6283
			0.67	0.98	.1811	.0100	.9622	1.6665
			0.71	0.99	.2228	.0063	.9813	1.7035
			0.73	1.00	.2651	.0033	1.0006	1.7402
			0.80	1.00	.3114	.0000	1.0000	1.7424
7-f	0.48	0.58	0.41	0.90	.0258	.0328	.8139	.3501
			0.47	0.92	.0561	.0207	.8497	.3785
			0.52	0.97	.0909	.0179	.9443	.4213
			0.54	0.98	.1270	.0123	.9626	.4405
			0.60	0.99	.1682	.0067	.9813	.4549
			0.64	1.00	.2129	.0031	1.0006	.4677
			0.70	1.00	.2615	.0000	1.0000	.4705
6-e	0.42	0.58	0.355	0.95	.0211	.0284	.9063	.2772
			0.41	0.97	.0462	.0180	.9440	.3408
			0.44	0.98	.0735	.0140	.9632	.3108
			0.48	0.99	.1037	.0089	.9817	.3223
			0.53	1.00	.1367	.0036	1.0006	.3355
			0.60	1.00	.1763	0.0000	1.0000	.3373
5-d	0.38	0.505	0.25	1.06	.0157	.0249	1.1276	.2570
			0.34	1.04	.0372	.0149	1.0845	.2562
			0.41	1.00	.0624	.0102	1.0023	.2404
			0.45	1.00	.0905	.0051	1.0010	.2451
			0.50	1.00	.1220	0.0000	1.0000	.2500
4-c	0.38	0.40	0.21	1.20	.0153	.0196	1.4437	.2114
			0.25	1.10	.0327	.0097	1.2120	.1842
			0.28	1.02	.0511	.0050	1.0418	.1616
			0.40	1.00	.0774	0.0000	1.0000	.1600
3-b	0.40	0.25	0.15	1.30	.0109	.0116	1.6921	.0941
			0.20	1.20	.0249	.0039	1.4404	.0861
			0.30	1.10	.0447	0.0000	1.2100	.0756
2-a	0.22	0.20	0.11	1.20	.0045	.0060	1.4413	.0465
			0.20	1.10	.0187	0.0000	1.2100	.0492

TABLE III (Continued)

Third Approximation

Line No.	n/ℓ	q_s/a_0	y/d	$h_{\alpha s}/h_\alpha$	$\frac{\psi}{\rho_0 a_0 \frac{d^2}{2}}$	C	P	$(\frac{q}{a_0})^2$
8-g	0.50	1.20	0.505	0.98	.0312	.0504	.9674	1.3426
			0.53	0.99	.0636	.0364	.9861	1.3836
			0.56	0.99	.0978	.0300	.9856	1.3893
			0.62	0.99	.1359	.0225	.9842	1.3948
			0.67	1.00	.1767	.0151	1.0027	1.4288
			0.71	1.00	.2201	.0095	1.0018	1.4331
			0.74	1.00	.2653	.0049	1.0009	1.4364
			0.80	1.00	.3144	0.0000	1.0000	1.4400
7-f	0.45	0.65	0.40	0.95	.0235	.0409	.9082	.3428
			0.46	0.97	.0513	.0290	.9458	.3706
			0.51	0.98	.0824	.0236	.9649	.3840
			0.53	0.99	.1153	.0171	.9833	.3984
			0.60	1.00	.1531	.0100	1.0018	.4132
			0.65	1.00	.1944	.0046	1.0009	.4183
			0.73	1.00	.2410	.0000	1.0000	.4225
6-e	0.39	0.65	0.35	0.98	.0213	.0346	.9656	.3734
			0.41	0.99	.0468	.0238	.9843	.3921
			0.44	0.99	.0742	.0185	.9838	.3971
			0.47	0.99	.1038	.0121	.9824	.4030
			0.53	1.00	.1377	.0053	1.0009	.4176
			0.60	1.00	.1763	0.0000	1.0000	.4225
5-d	0.36	0.56	0.24	1.05	.0154	.0283	1.1075	.3190
			0.33	1.04	.0368	.0183	1.0852	.3220
			0.40	1.03	.0626	.0133	1.0639	.3204
			0.45	1.00	.0915	.0066	1.0014	.3074
			0.50	1.00	.1240	.0000	1.0000	.3136
4-c	0.36	0.44	0.20	1.20	.0149	.0228	1.4444	.2569
			0.24	1.10	.0319	.0122	1.2123	.2226
			0.29	1.00	.0512	.0059	1.0014	.1879
			0.40	1.00	.0782	.0000	1.0000	.1936
3-b	0.40	0.30	0.14	1.20	.0108	.0133	1.4424	.1080
			0.20	1.10	.0257	.0050	1.2106	.0968
			0.30	1.00	.0450	.0000	1.0000	.0841
2-a	0.23	0.25	0.10	1.20	.0090	.0068	1.4416	.0201
			0.20	1.00	.0182	.0000	1.0000	.0354

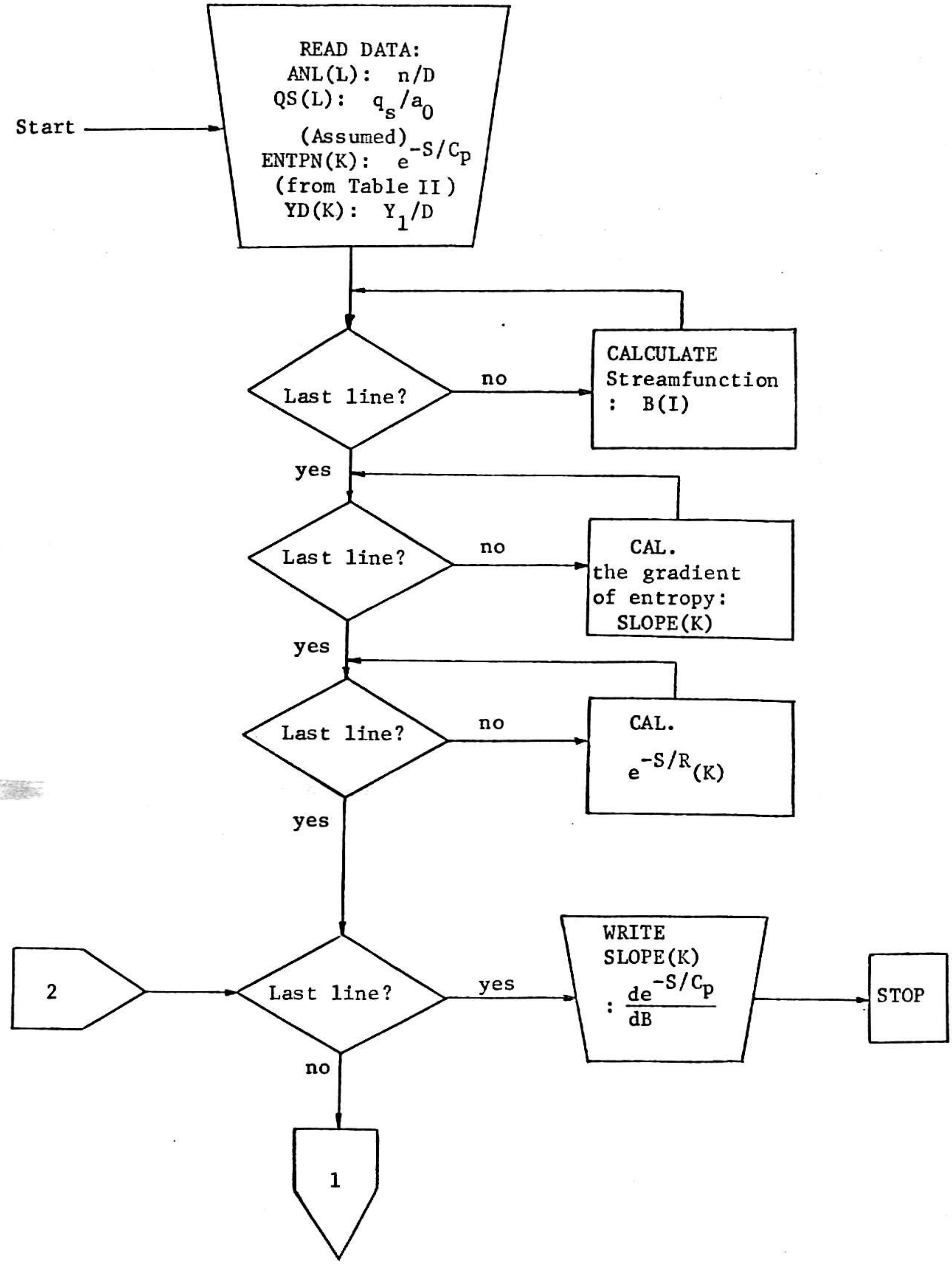
TABLE III (Continued)

Fourth Approximation

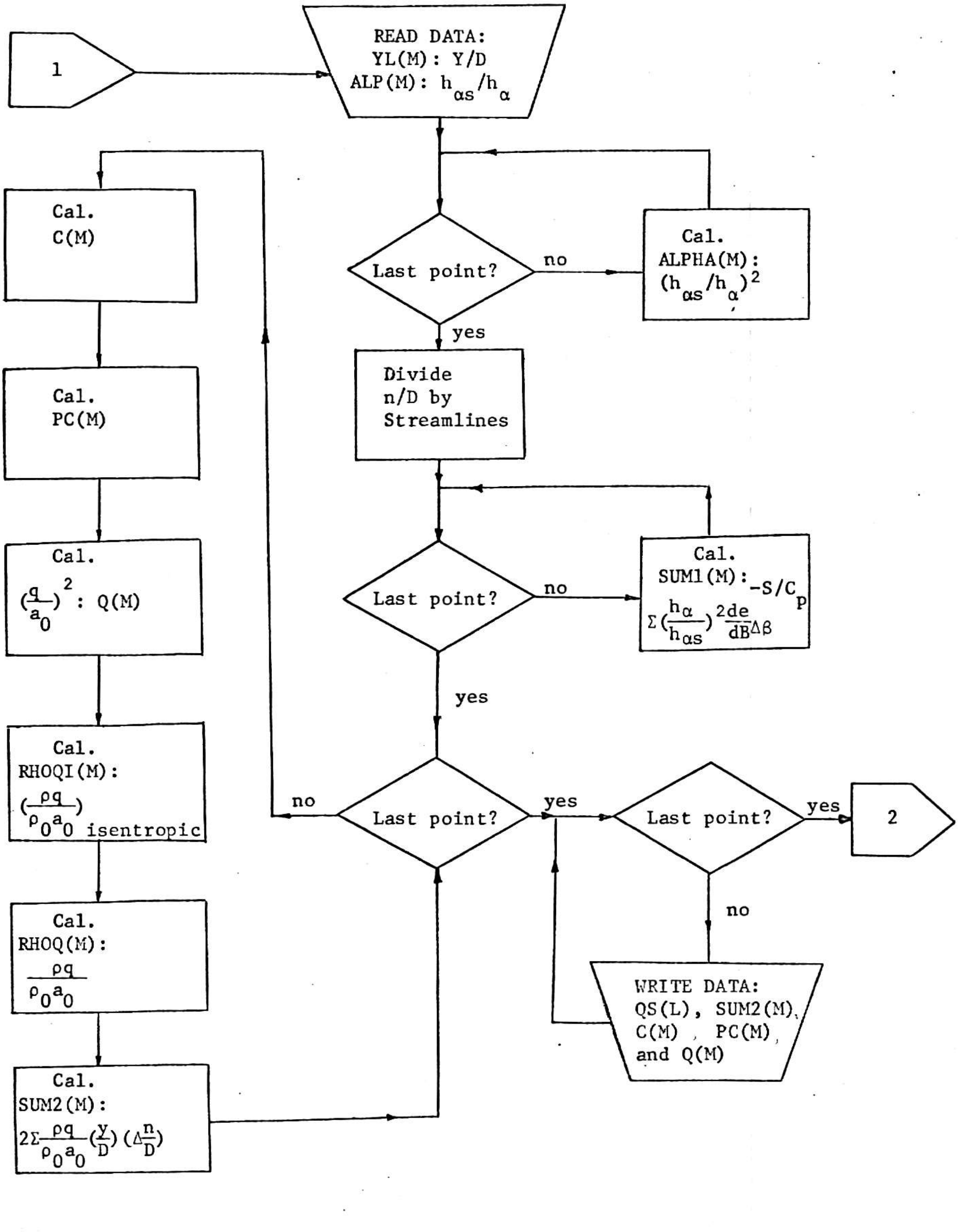
Line No.	n/ℓ	q_s/a_0	y/d	$h_{\alpha s}/h_\alpha$	$\frac{\psi}{\rho_0 a_0 \frac{d^2}{2}}$	C	P	$(\frac{q}{a_0})^2$
8-g	0.52	1.25	0.485	0.95	.0310	.0618	.9111	1.3617
			0.53	0.96	.0645	.0484	.9292	1.4035
			0.56	0.97	.0995	.0408	.9480	1.4405
			0.62	0.98	.1381	.0301	.9655	1.4784
			0.67	0.99	.1794	.0207	.9835	1.5159
			0.71	1.00	.2227	.0143	1.0027	1.5525
			0.74	1.00	.2681	.0073	1.0087	1.5563
			0.80	1.00	.3171	.0000	1.0000	1.5625
7-f	0.46	0.62	0.415	0.95	.0259	.0506	.9104	.3156
			0.46	0.96	.0553	.0385	.9286	.3262
			0.51	0.99	.0856	.0329	.9868	.3405
			0.54	1.00	.1223	.0230	1.0046	.3592
			0.61	1.00	.1596	.0136	1.0027	.3678
			0.645	1.00	.2080	.0069	1.0020	.3762
			0.70	1.00	.2457	0.0000	1.0000	.3844
6-e	0.40	0.61	0.35	0.99	.0207	.0428	.9867	.3125
			0.40	0.99	.0447	.0317	.9855	.3230
			0.43	1.00	.0709	.0253	1.0048	.3364
			0.48	1.00	.1006	.0156	1.0025	.3454
			0.54	1.00	.1345	.0066	1.0007	.3536
			0.60	1.00	.1725	.0000	1.0000	.3600
5-d	0.37	0.53	0.24	1.08	.0150	.0367	1.1734	.2568
			0.33	1.05	.0352	.0260	1.1078	.2601
			0.40	1.00	.0592	.0177	1.0041	.2662
			0.46	1.00	.0868	.0086	1.0018	.2703
			0.50	1.00	.1201	0.0000	1.0000	.2809
4-c	0.37	0.425	0.20	1.30	.0149	.0319	1.6971	.2396
			0.24	1.10	.0309	.0172	1.2136	.1770
			0.29	1.00	.0490	.0088	1.0022	.1516
			0.40	1.00	.0748	0.0000	1.0000	.1600
3-b	0.40	0.27	0.14	1.20	.0107	.0154	1.4428	.1059
			0.21	1.10	.0261	.0068	1.2109	.0951
			0.30	1.00	.0470	.0000	1.0000	.0841
2-a	0.22	0.24	0.11	1.10	.0049	.0068	1.2115	.0511
			0.20	1.00	.0189	0.0000	1.0000	.0602

APPENDIX A

FLOW CHART FOR TABLE III FORTRAN PROGRAM



APPENDIX A (Continued)



APPENDIX B

FORTRAN PROGRAM FOR CALCULATION OF THE FLOW PATTERN ABOUT THE BLUNT BODY ($M=1.5$, $\gamma=1.4$, $S_0=0$)

```
      DIMENSION ANL(10),QS(10),YD(10),ENTPN(10),SLOPE(10)
      DIMENSION ENTRN(10),YL(10),ALP(10),ALPHA(10),HL(10)
      DIMENSION H(10),BETA(10),YY(10),SUM1(10),C(10),PC(10)
      DIMENSION Q(10),RHOQI(10),RHOQ(10),XX(10),SUM2(10)
  100 FORMAT(5X,'QS(L)',6X,'SUM2(M)',6X,'C(M)',6X,'PC(M)',6X,'Q(M)'/)
  300 FORMAT(8F7.4)
  400 FORMAT(7F7.4)
  500 FORMAT(3X,F7.4,4X,F7.4,4X,F7.4,4X,F7.4,4X,F7.4)
  600 FORMAT(6X,'SLOPE'/)
      READ(1,400) (ANL(L),L=1,7)
      READ(1,400) (QS(L),L=1,7)
      READ(1,300)(YD(K),K=1,8)
      READ(1,300)(ENTPN(K),K=1,8)
      BETA(1)=0.4922*(YD(1)**2)
      SLOPE(1)=(ENTPN(1)-.9793)/BETA(1)
      DO 6 J=2,8
    6 BETA(J)=0.4922*(YD(J)**2)
      DO 8 K=2,7
    8 SLOPE(K)=(ENTPN(K+1)-ENTPN(K-1))/(BETA(K+1)-BETA(K-1))
      SLOPE(8)=SLOPE(7)
      R=1.4
      DO 9 K=1,8
    9 ENTRN(K)=ENTPN(K)**(R/(R-1.))
      DO 32 L=1,7
      N=9-L
      READ(1,300)(YL(M),M=1,N)
      READ(1,300)(ALP(M),M=1,N)
      DO 10 M=1,N
   10 ALPHA(M)=ALP(M)**2
      XN=N
      HL(L)=ANL(L)/XN
   18 SUMX2=0.0
      K1=N-1
      DO 17 M=1,K1
      XO=N-M
```

APPENDIX B (Continued)

```
      H(M)=(BETA(N)-BETA(M))/XD
      SUMX1=0.0
      DO 17 I=M,K1
      YY(I)=(1./ALPHA(I))*SLOPE(I)
      YY(I+1)=(1./ALPHA(I+1))*SLOPE(I+1)
      YYM=(YY(I)+YY(I+1))/2.
      SUMX1=SUMX1+YYM
      SUM1(M)=H(M)*SUMX1
   17 CONTINUE
      SUM1(N)=0.0
      DO 21 M=1,N
      C(M)=(2./(R-1.))*ALPHA(M)*(1./ENTPN(M)*SUM1(M))
      PC(M)=ALPHA(M)*(ENTPN(N)/ENTPN(M))
      Q(M)=PC(M)*(QS(L)**2)-C(M)
      RHOQI(M)=((1.-((R-1.)/2.)*Q(M))**(1./(R-1.)))*(Q(M)**.5)
      RHOQ(M)=ENTRN(M)*RHOQI(M)
      XX(M)=RHOQ(M)*YL(M)
      SUMX2=SUMX2+XX(M)
      SUM2(M)=2.*HL(L)*SUMX2
   21 CONTINUE
      SUMX2=2.*HL(L)*SUMX2
      IF(ABS(BETA(N)-SUMX2)-0.003) 30,30,24
   24 IF(BETA(N)-SUMX2) 26,30,28
   26 QS(L)=QS(L)-0.0025
      GO TO 18
   28 QS(L)=QS(L)+0.0025
      GO TO 18
   30 CONTINUE
      WRITE(3,100)
      DO 31 M=1,N
   31 WRITE(3,500)QS(L),SUM2(M),C(M),PC(M),Q(M)
   32 CONTINUE
      WRITE(3,600)
      WRITE(3,300) (SLOPE(J),J=1,8)
      STOP
      END
```

APPENDIX C

ENTROPY DISTRIBUTION IN DETACHED-SHOCK SUPERSONIC FLOW

```
C     ENTROPY DISTRIBUTION IN DETACHED-SHOCK SUPERSONIC FLOW
C     TZONG-NAN LIN
      DIMENSION W(10),X(10),P(10),RHO(10),ENTPN(10),ENTRN(10)
  100 FORMAT (9F7.4)
  200 FORMAT (5X,8HENTPN(K),10X,8HENTRN(K)/)
  300 FORMAT (3X,F7.4,13X,F7.4)
      READ 100,(W(K), K=1,9)
      R=1.4
      F1=1.5
      ENTPO=(0.2724**(1./R))/0.395
      DO 12 K=1,9
      W(K)=W(K)*3.14/180.0
      IF (K-1)5,8,5
    5 X(K)=F1*F1*SIN(W(K))*SIN(W(K))
      P(K)=(2.*R/(R+1.))*(X(K)-(R-1.)/(2.*R))
      RHO(K)=2./(R+1.)*(1./X(K)+(R-1.)/2.)
      GO TO 10
    8 P(K)=(2.*R/(R+1.))*F1*F1-((R-1.)/(R+1.))
      RHO(K)=2./((R+1.)*F1*F1)+(R-1.)/(R+1.)
   10 ENTPN(K)=ENTPO/(RHO(K)*(P(K)**(1./R)))
      ENTRN(K)=ENTPN(K)**(R/(R-1.))
   12 CONTINUE
      PUNCH 200
      DO 14 K=1,9
      PUNCH 300 ,ENTPN(K), ENTRN(K)
   14 PUNCH 300, ENTPN(K), ENTRN(K)
      STOP
      END
```

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DETERMINATION OF THE FLOW OVER A BLUNT BODY IN A SUPERSONIC STREAM BY THE METHOD OF UCHIDA AND YASUHARA

by

TZONG-NAN LIN

B.S., National Taiwan University, Taipei, Taiwan, 1966

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

An iteration technique for the numerical solution of a partial differential equation, based on the conception that the partial differential equation expressed in terms of suitably chosen curvilinear coordinates may be replaced by an ordinary differential equation in a single variable has been studied. This technique, originally proposed by Uchida and Yasuhara and termed by them the "method of flux analysis", has been applied to the steady, rotational flow of a nonviscous and non-heat-conductive compressible fluid by utilizing the corresponding fundamental equations (including the equation of state, and those for the conservation of mass, momentum, and energy) to obtain a relationship between the local speed of flow and the entropy which is valid along orthogonals to the streamlines.

A unified graphical method for analyzing the problem of flow over a sphere at a free stream Mach number of 1.5 has been presented. In the calculations of isoenergetic rotational flow behind a detachment shock wave, the stand-off distance was assumed to be obtained in advance and the upstream stagnation entropy to be constant.

The solution was started by assuming an approximation for both the shock shape and the streamline pattern, and a double-iteration method was carried out whereby both the streamlines and the shock were readjusted until a consistent solution was obtained. By this method of solution, properties of the flow between the shock wave and the sphere were finally obtained.

Because of the tediousness of the computations involved in this method it has been examined with the aid of an IBM 360 computer only in case of a particular symmetric flow problem. An example of a sphere at $M=1.5$ ($\gamma=1.4$) is presented.

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