A COMPUTER SIMULATION OF A DUAL REAR WHEELED FARM TRACTOR

by

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TABLE OF CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

I. NOMENCLATURE

X - coordinate in the fore and aft direction, X_1 direction.

Y - coordinate in the transverse direction, X_2 direction.

Z - coordinate in the verticle direction, X_3 direction.

M_1 - mass of the left front wheel and tire.

M_2 - mass of the right front wheel and tire.

M_3 - mass of the inner left rear wheel and tire.

M_4 - mass of the inner right rear wheel and tire.

M_5 - mass of the main chassis.

M_6 - mass of the front axle assembly.

M_7 - mass of the outer left rear wheel and tire.

M_8 - mass of the outer right rear wheel and tire.

J_{ij} - rotational moment of inertia of M_i about the j axis. $i = 1, 2, \ldots 8$; $j = X, Y, Z$.

K_{ij} - spring rate of M_i in the X_j direction. $i = 1, 2, 3, 4, 7, 8$; $j = 1, 2, 3$.

C_{ij} - damping coefficient of M_i in the X_j direction. $i = 1, 2, 3, 4, 7, 8$; $j = 1, 2, 3$.

y_{ij} - ground profile seen by the tire to ground contact point of M_i in the X_j direction. $i = 1, 2, 3, 4, 7, 8$; $j = 1, 2, 3$.

$\dot{y}_{ij}$ - the first derivative of y_{ij} with respect to time.

EI - the product of the modulus of elasticity in tension and the cross sectional moment of inertia of the rear axle.

JG - the product of the cross sectional polar moment of inertia and the modulus of elasticity in shear of the rear axle.

x_i - motion coordinates. $i = 1, 2, 3, \ldots 29$.

$\dot{x}_i$ - the first derivative of x_i with respect to time.

$\ddot{x}_i$ - the second derivative of x_i with respect to time.

AXi - $\ddot{x}_i$

VXi - $\dot{x}_i$

II. INTRODUCTION

The design of large off road vehicles, such as farm tractors, is a process which makes use of many different methods to determine vehicle parameters and predict vehicle performance. One such technique is the use of mathematical modeling and the digital computer to simulate the motion of a vehicle in question.

Computer simulations allow the design engineer to vary certain parameters, and observe the resultant motion. This procedure avoids the cost of building a prototype, and modifying that prototype until the desired performance is achieved. It also alleviates the risk of persons being injured and equipment being damaged while observing a system that may fail.

With a predetermined set of parameters, mathematical modeling may be utilized to observe the performance of a vehicle subject to varying surface conditions. These surface conditions may be due to a certain geometrical configuration, such as a side slope or bump, or different soil types, such as sand or hard packed soil. Vibrations induced by tire tread patterns can also be observed.

Many mathematical models of farm tractors have been developed. Most of these were derived to describe the motion of single rear wheeled vehicles. The rear axles have usually been treated as rigid members, which greatly reduces the degrees of freedom of the system and hence the number of equations of motion.

In this thesis, a dual rear wheeled farm tractor is modeled as eight lumped masses. The rear axles are considered to be elaatic, allowing the vehicle twenty-nine degreea of freedom. The equations of motion for the system are derived by two different methods, allowing one set of equationa to be used as a check for the other.

The first of the sets ia generated uaing Newton's lawa and elementary beam theory. The second set uses the energy method, taking advantage of the Lagrange equation for nonconservative systems. In both cases, the equations are reduced by the Gauss-Jordon method to yield twenty-nine second order differential equations of motion. These differential equations are numerically integrated uaing the fourth order Runge-Kutta-Gill method to give diaplacements as functions of time.

III. REVIEW OF LITERATURE

Among the first work in tractor modeling was that of McKibben [7], done in 1927. In a series of articles, he investigated the kinematics and dynamics of single rear wheeled farm tractors. McKibben described the forces acting on the vehicle due to various causes, such as soil reactions, drawbar pull, and the gyroscopic action of the engine flywheel.

These articles considered the general motion of a tractor as well as a discussion on backward overturning while moving up a slope. Unfortunately, McKibben's work was done with vehicles riding on steel wheels rather than pnumatic tires. He treated the tractor as a single rigid mass, and avoided the use of second order differential equations by considering only constant velocities.

Since McKibben's time, tractors have become somewhat more sophisticated, and with the advent of high speed computers, larger more complex equations can be handled. The more recent work in tractor modeling can be divided into two general categories. The first type is concerned with gross motion of a vehicle, such as backward overturning, or sideward overturns on side slopes. The second type considers small amplitude vibrations for studies in rider comfort or fatigue of tractor components.

A. Studies in Gross Motion

In 1967, Goering and Buchele [2] developed a model to predict backward overturning of an unsprung vehicle. They restricted the motion of the vehicle to a plane normal to the ground, and considered the rear axles to be rigid. The result of their work was a set of nineteen equations that were used to simulate the effect of a sudden clutch engagement with no drawbar load.

Hudson et al. [5] formulated a two dimensional stability model, in 1973, to determine the behavior of single rear wheeled tractors on slopes. They developed two differential equations that could be easily programmed into an analog computer. These equations determined the angle of pitch of a tractor, given the drawbar load.

A model to simulate both backward and sideward overturnings was developed by Davis et al. [1] in 1974. Their model treated a single rear wheeled tractor as five masses with ten degrees of freedom. The rear axles were assumed to be rigid, so that the rear wheels were constrained to have the same motion as the main chassis, except for their ability to rotate about the longitudinal axis of the axle. This model was capable of simulating tractors with either wide front ends, or tricycle type front ends.

B. Studies in Small Amplitude Vibrations

A model to predict small amplitude vibrations of a single rear wheeled farm tractor was developed by Raney et al. [11] in 1961. For their model, the frame and rear axle were assumed to be a single rigid body. This model had only three degrees of freedom, verticle displacement, pitch and roll. A set of three second order differential equations was derived and programmed into an analog computer. The authors also considered the effect of adding a vibration absorber to the system.

In 1964, Huang et al. [4] considered the effect of elastic rims and elastic suspension on the motion of a single rear wheeled vehicle. They derived a model having four degrees of freedom, with which they could compare the effect of elastic rims to rigid rims.

Pershing and Yoerger [9] formulated a model for a single rear wheeled tractor with a side mounted implement. This work, done in 1968, was used to study the motion of a tractor while mowing side slopes. Although the equations were derived for small amplitude vibrations, they were used to determine the size of a bump that would cause a sideward overturn of the vehicle. Pershing and Yoerger considered the tractor as five masses with nine degrees of freedom, and treated the rear axles as rigid members. The Lagrange equation was used to formulate the nine differential equations of motion, which were then integrated numerically using a digital computer.

Mather [6] developed a model in 1970 to determine the stress variations in the rear axle of a dual rear wheeled tractor. He modeled the tractor as eight lumped masses with nineteen degrees of freedom, and treated the rear axles as elastic cantilever beams. In his formulation, the Lagrange equation was used to derive the equations of motion.

The effect of the front wheels being allowed to rotate about the front axle in Pershing and Yoerger's model was neglected in a model developed by Wolken and Yoerger [13] in 1974. Their work considered the dynamic response of a single rear wheeled tractor subject to random inputs. They modeled the tractor as five masses with seven degrees of freedom. The effects of forward velocity, tire spring rate, and frame moments of inertia on the motion of the tractor were all investigated.

During the same year, Smith and Yoerger [12] developed a similar model to predict variations in the forward motion of a single rear wheeled tractor due to a changing drawbar load. They also modeled the tractor as five masses with seven degrees of freedom, but motion was restricted to a plane normal to the ground.

IV. THE SYSTEM BEING MODELED

A. Assumptions

1. A dual rear wheeled tractor is treated as eight discrete masses; four rear wheels, two front wheels, the front axle assembly, and the main chassis. Each mass is assumed to be concentrated at the center of gravity of its respective body.
2. Small angular displacements are assumed, to keep the model linear.
3. The sections of the rear axle between the frame and inner wheels, and between inner and outer wheels are assumed to act as elastic cantilever beams, with negligable mass and internal damping. Longitudinal vibrations of these sections are neglected.
4. Each tire is assumed to act as a set of three mutually perpendicular linear springs with viscous damping.
5. The reaction between a tire and the ground is assumed to act at a single point.
6. The tractor oscillates about its equilibrium position.

B. The Coordinate System and Degrees of Freedom

A right hand coordinate system is used as a reference, with its origin in the plane of the ground and directly below the center of the rear axle. The positive X axis is in the forward direction of travel of the tractor, the positive Y axis is to the right as viewed by an observer riding on the tractor, and the positive Z axis is toward the center of the Earth. Positive rotations are taken according to the right hand rule.

The main chassis is free to translate in any of the three coordinate directions, and to rotate about any of the three coordinate axes. Hence, it has six degrees of freedom. Becauae of the third assumption, the rear wheels are conatrained to move in the Y direction with the main chaasis. They are, however, allowed to translate in the X and Z directions, and to rotate about any of the three axes, giving each rear wheel five degrees of freedom.

The front axle assembly is constrained such that it has no motion with respect to the main chassis, except for rotation about its pin connection with the chassis. The front wheels are constrained such that they have no motion with respect to the front axle assembly, except for rotation about the longitudinal axis of the axle. Each of these three bodies have only one degree of freedom.

As a unit, the tractor then has twenty-nine degrees of freedom. Figure 1 shows these variables along with the reference axes.

x_{18} - rotation of M_3.

x_{19} - rotation of M_4.

x_{20} - rotation of M_7.

x_{21} - rotation of M_8.

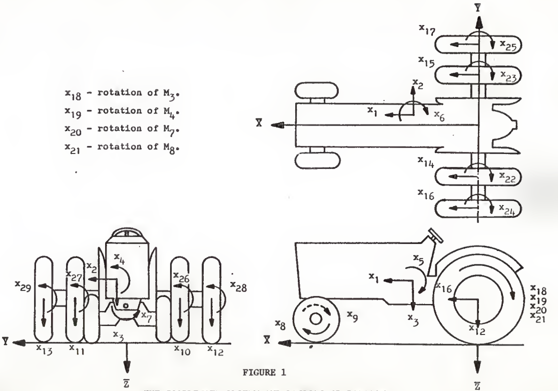


FIGURE 1

THE COORDINATE SYSTEM AND DEGREES OF FREEDOM

V. EQUATIONS OF MOTION

A. Derivation Using Newton's Laws

The equations of motion are first derived using Newton's laws and elementary beam theory. To do this, the forces acting on the system must be considered. These forces are at the tire to ground contact points, and the points at which wheels join axles or axles join the frame. There are seventeen such points, six tire to ground and eleven member to member.

At each point, the force present is broken into components in the X, Y and Z directions, and the moment is broken into components about each coordinate axis. There are three force components at each point accounting for fifty-one unknown forces. The same is not true of the moments at each point, since some members are free to rotate with respect to other members.

The front wheels are free to rotate about the front axle; therefore, no moments are present at these points in the x_8 and x_9 directions. The front axle assembly is allowed to rotate about its pin connection with the frame, in the x_7 direction, eliminating the possibility of a moment at this point in that direction. At the tire to ground contact points, it is assumed that there is no moment present in any direction. There are three moment components at every other point, for a total of thirty unknown moments. Figures 2, 3, 4 and 5 show the forces and moments present at each point, and their assumed directions.

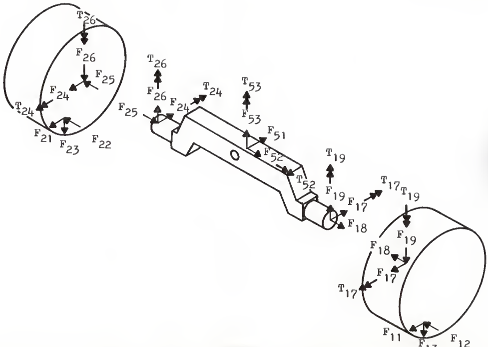


FIGURE 2

FORCES ON THE FRONT WHEELS AND FRONT AXLE ASSEMBLY

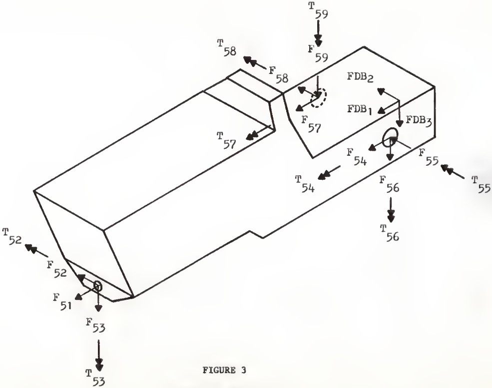


FIGURE 3

FORCES ON THE CHASSIS

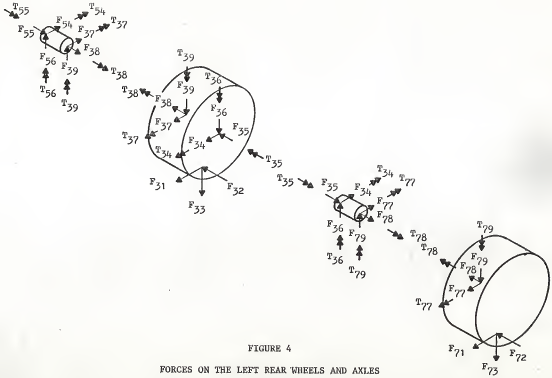


FIGURE 4

FORCES ON THE LEFT REAR WHEELS AND AXLES

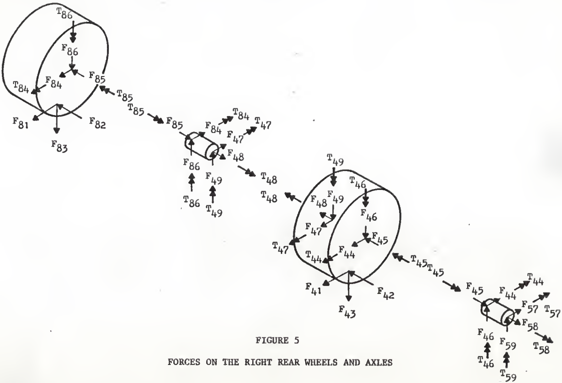


FIGURE 5

FORCES ON THE RIGHT REAR WHEELS AND AXLES

The system may be forced by a drawbar load, or by varying the surface over which the tractor moves. The first type of forcing function is broken into components in the X, Y and Z directions, and input directly into the dynamic equations. The second type is broken into the surface profile that each contact point sees in each coordinate direction. These horizontal, transverse and verticle components are input by subtracting them from the deflection of each contact point in the X, Y and Z directions, respectively. The new deflections are used when applying Hooke's law to the tire ground reaction. When this method is used to force the model, both the functions and their first derivatives with respect to time must be prescribed.

There are twenty-nine unknown displacements, fifty-one unknown forces, and thirty unknown moments for a total of one hundred ten unknowns. It is then necessary to have one hundred ten equations to describe the system. These equations come from four sources, static equations involving the forces on the rear axles, dynamic equations describing the motion of each mass, elastic equations due to the bending of the rear axles, and equations involving the application of Hooke's law to the tires.

a. Static Equations

The sections of the rear axle between the frame and inside wheels, and between inner and outer wheels, are assumed to be massless. Therefore, the following equations must be satisfied for each of these sections, in each coordinate direction

$$\Sigma F_i = 0 \qquad \text{and} \qquad \Sigma T_i = 0 \tag{1a,b}$$

Applying (1a) to the section of axle between the inner and outer left rear wheels, in the X, Y and Z directions, respectively, yields

$$-F_{77} - F_{34} = 0 \quad , \quad -F_{78} - F_{35} = 0 \quad \text{and} \quad -F_{79} - F_{36} = 0 \qquad \text{(2a,b,c}$$

Applying (1b) to the same section about the M_3 end yields

$$-T_{77} - T_{34} - F_{36} \cdot L_{37} = 0 \quad , \quad -T_{78} - T_{35} = 0 \quad \text{and}$$
$$-T_{79} - T_{36} + F_{34} \cdot L_{37} = 0 \qquad \text{(3a,b,c}$$

Equations (2a) through (3c) may be rearranged to give expressions for F_{77}, F_{78}, F_{79}, T_{77}, T_{78} and T_{79}

$$F_{77} = -F_{34} \quad , \quad T_{77} = -T_{34} - F_{36} \cdot L_{37}$$
$$F_{78} = -F_{35} \quad , \quad T_{78} = -T_{35} \qquad \text{(4a-f}$$
$$F_{79} = -F_{36} \quad , \quad T_{79} = -T_{36} + F_{34} \cdot L_{37}$$

In a like manner, the forces and moments acting on the other sections are summed to give

$$F_{54} = -F_{37} \quad , \quad T_{54} = -T_{37} + F_{39} \cdot L_{53}$$
$$F_{55} = -F_{38} \quad , \quad T_{55} = -T_{38} \qquad \text{(5a-f}$$
$$F_{56} = -F_{39} \quad , \quad T_{56} = -T_{39} - F_{37} \cdot L_{53}$$

$$F_{57} = -F_{44} \quad , \quad T_{57} = -T_{44} - F_{46} \cdot L_{54}$$
$$F_{58} = -F_{45} \quad , \quad T_{58} = -T_{45} \qquad \text{(6a-f}$$
$$F_{59} = -F_{46} \quad , \quad T_{59} = -T_{46} + F_{44} \cdot L_{54}$$

$$F_{84} = -F_{47} \quad , \quad T_{84} = -T_{47} + F_{49} \cdot L_{84}$$
$$F_{85} = -F_{48} \quad , \quad T_{85} = -T_{48} \qquad \text{(7a-f}$$
$$F_{86} = -F_{49} \quad , \quad T_{86} = -T_{49} - F_{47} \cdot L_{84}$$

b. Dynamic Equations

Similar equations can be written by summing the forces and moments about the center of mass of each of the masses one through eight. The equations that must now be satisfied are

$$\Sigma F_i = M_i \ddot{x}_i \qquad \text{and} \qquad \Sigma T_i = J_i \ddot{\theta}_i \qquad \text{(8a,b}$$

Application of (8a) and (8b) to the left front wheel yields

$$F_{11} + F_{17} = M_1 \cdot (\ddot{x}_1 + (D_{53} + D_{62}) \cdot \ddot{x}_5 + (D_{99} + D_{13} + \overline{Y}) \cdot \ddot{x}_6) \qquad \text{(9}$$

$$F_{12} + F_{18} = M_1 \cdot (\ddot{x}_2 - D_{53} \cdot \ddot{x}_4 + D_{51} \cdot \ddot{x}_6 - D_{62} \cdot \ddot{x}_7) \qquad \text{(10}$$

$$F_{13} + F_{19} = M_1 \cdot (\ddot{x}_3 - \overline{Y} \cdot \ddot{x}_4 - D_{51} \cdot \ddot{x}_5 - (D_{99} + D_{13}) \cdot \ddot{x}_7) \qquad \text{(11}$$

$$-F_{12} \cdot D_{14} + F_{19} \cdot D_{13} + T_{17} = J_{1X} \cdot \ddot{x}_7 \qquad \text{(12}$$

$$F_{11} \cdot D_{14} = J_{1Y} \cdot \ddot{x}_8 \qquad \text{(13}$$

$$-F_{17} \cdot D_{13} + T_{19} = J_{1Z} \cdot \ddot{x}_6 \qquad \text{(14}$$

Likewise, the forces and moments are summed about the center of mass of masses two through eight, respectively, to give (see Figure 6)

$$F_{21} + F_{24} = M_2 \cdot (\ddot{x}_1 + (D_{53} + D_{62}) \cdot \ddot{x}_5 - (D_{99} + D_{22} - \overline{Y}) \cdot \ddot{x}_6) \qquad \text{(15}$$

$$F_{22} + F_{25} = M_2 \cdot (\ddot{x}_2 - D_{53} \cdot \ddot{x}_4 + D_{51} \cdot \ddot{x}_6 - D_{62} \cdot \ddot{x}_7) \qquad \text{(16}$$

$$F_{23} + F_{26} = M_2 \cdot (\ddot{x}_3 - \overline{Y} \cdot \ddot{x}_4 - D_{51} \cdot \ddot{x}_5 + (D_{99} + D_{22}) \cdot \ddot{x}_7) \qquad \text{(17}$$

$$-F_{22} \cdot D_{24} - F_{26} \cdot D_{22} + T_{24} = J_{2X} \cdot \ddot{x}_7 \qquad \text{(18}$$

$$F_{21} \cdot D_{24} = J_{2Y} \cdot \ddot{x}_9 \qquad \text{(19}$$

$$F_{24} \cdot D_{22} + T_{26} = J_{2Z} \cdot \ddot{x}_6 \qquad \text{(20}$$

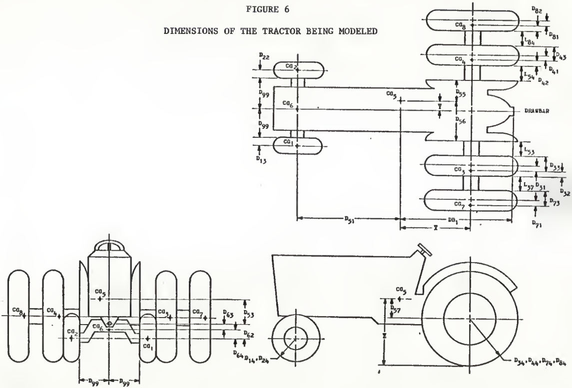
$$F_{31} + F_{34} + F_{37} = M_3 \cdot \ddot{x}_{14} \qquad \text{(21}$$

$$F_{32} + F_{35} + F_{38} = M_3 \cdot (\ddot{x}_2 - D_{57} \cdot \ddot{x}_4 - \overline{X} \cdot \ddot{x}_6) \qquad \text{(22}$$

$$F_{33} + F_{36} + F_{39} = M_3 \cdot \ddot{x}_{10} \qquad \text{(23}$$

FIGURE 6

DIMENSIONS OF THE TRACTOR BEING MODELED



$$-F_{32} \cdot D_{34} + F_{33} \cdot D_{31} - F_{36} \cdot D_{32} + F_{39} \cdot D_{33} + T_{34} + T_{37} = J_{3X} \cdot \ddot{x}_{26}$$ (24

$$F_{31} \cdot D_{34} + T_{35} + T_{38} = J_{3Y} \cdot \ddot{x}_{18}$$ (25

$$-F_{31} \cdot D_{31} + F_{34} \cdot D_{32} - F_{37} \cdot D_{33} + T_{36} + T_{39} = J_{3Z} \cdot \ddot{x}_{22}$$ (26

$$F_{41} + F_{44} + F_{47} = M_4 \cdot \ddot{x}_{15}$$ (27

$$F_{42} + F_{45} + F_{48} = M_4 \cdot (\ddot{x}_2 - D_{57} \cdot \ddot{x}_4 - \overline{X} \cdot \ddot{x}_6)$$ (28

$$F_{43} + F_{46} + F_{49} = M_4 \cdot \ddot{x}_{11}$$ (29

$$-F_{42} \cdot D_{44} + F_{43} \cdot D_{41} - F_{46} \cdot D_{42} + F_{49} \cdot D_{43} + T_{44} + T_{47} = J_{4X} \cdot \ddot{x}_{27}$$ (30

$$F_{41} \cdot D_{44} + T_{45} + T_{48} = J_{4Y} \cdot \ddot{x}_{19}$$ (31

$$-F_{41} \cdot D_{41} + F_{44} \cdot D_{42} - F_{47} \cdot D_{43} + T_{46} + T_{49} = J_{4Z} \cdot \ddot{x}_{23}$$ (32

$$F_{51} + F_{54} + F_{57} + FDB_1 = M_5 \cdot \ddot{x}_1$$ (33

$$F_{52} + F_{55} + F_{58} + FDB_2 = M_5 \cdot \ddot{x}_2$$ (34

$$F_{53} + F_{56} + F_{59} + FDB_3 = M_5 \cdot \ddot{x}_3$$ (35

$$-F_{52} \cdot D_{53} - F_{53} \cdot \overline{Y} - F_{55} \cdot D_{57} - F_{56} \cdot D_{56} - F_{58} \cdot D_{57} + F_{59} \cdot$$
$$D_{55} - FDB_2 \cdot DB_3 - FDB_3 \cdot \overline{Y} + T_{54} + T_{57} = J_{5X} \cdot \ddot{x}_4$$ (36

$$F_{51} \cdot D_{53} - F_{53} \cdot D_{51} + F_{54} \cdot D_{57} + F_{56} \cdot \overline{X} + F_{57} \cdot D_{57} + F_{59} \cdot$$
$$\overline{X} + FDB_1 \cdot DB_3 + FDB_3 \cdot DB_1 + T_{52} + T_{55} + T_{58} = J_{5Y} \cdot \ddot{x}_5$$ (37

$$F_{51} \cdot \overline{Y} + F_{52} \cdot D_{51} + F_{54} \cdot D_{56} - F_{55} \cdot \overline{X} - F_{57} \cdot D_{55} - F_{58} \cdot$$
$$\overline{X} + FDB_1 \cdot \overline{Y} - FDB_2 \cdot DB_1 + T_{53} + T_{56} + T_{59} = J_{5Z} \cdot \ddot{x}_6$$ (38

$$-F_{51} - F_{17} - F_{24} = M_6 \cdot (\ddot{x}_1 + (D_{53} + D_{63}) \cdot \ddot{x}_5 + \overline{Y} \cdot \ddot{x}_6)$$ (39

$$-F_{52} - F_{18} - F_{25} = M_6 \cdot (\ddot{x}_2 - D_{53} \cdot \ddot{x}_4 + D_{51} \cdot \ddot{x}_6 - D_{63} \cdot \ddot{x}_7)$$ (40

$$-F_{53} - F_{19} - F_{26} = M_6 \cdot (\ddot{x}_3 - \overline{Y} \cdot \ddot{x}_4 - D_{51} \cdot \ddot{x}_5)$$ (41

$$F_{18} \cdot D_{64} + F_{19} \cdot D_{99} + F_{25} \cdot D_{64} - F_{26} \cdot D_{99} - F_{52} \cdot D_{63} - T_{17} - T_{24} = J_{6X} \cdot \ddot{x}_7 \quad (42$$

$$-F_{17} \cdot D_{64} - F_{24} \cdot D_{64} + F_{51} \cdot D_{63} - T_{52} = J_{6Y} \cdot \ddot{x}_5 \quad (43$$

$$-F_{17} \cdot D_{99} + F_{24} \cdot D_{99} - T_{19} - T_{26} - T_{53} = J_{6Z} \cdot \ddot{x}_6 \quad (44$$

$$F_{71} + F_{77} = M_7 \cdot \ddot{x}_{16} \quad (45$$

$$F_{72} + F_{78} = M_7 \cdot (\ddot{x}_2 - D_{57} \cdot \ddot{x}_4 - \bar{X} \cdot \ddot{x}_6) \quad (46$$

$$F_{73} + F_{79} = M_7 \cdot \ddot{x}_{12} \quad (47$$

$$-F_{72} \cdot D_{74} + F_{73} \cdot D_{71} + F_{79} \cdot D_{73} + T_{77} = J_{7X} \cdot \ddot{x}_{28} \quad (48$$

$$F_{71} \cdot D_{74} + T_{78} = J_{7Y} \cdot \ddot{x}_{20} \quad (49$$

$$-F_{71} \cdot D_{71} - F_{77} \cdot D_{73} + T_{79} = J_{7Z} \cdot \ddot{x}_{24} \quad (50$$

$$F_{81} + F_{84} = M_8 \cdot \ddot{x}_{17} \quad (51$$

$$F_{82} + F_{85} = M_8 \cdot (\ddot{x}_2 - D_{57} \cdot \ddot{x}_4 - \bar{X} \cdot \ddot{x}_6) \quad (52$$

$$F_{83} + F_{86} = M_8 \cdot \ddot{x}_{13} \quad (53$$

$$-F_{82} \cdot D_{84} + F_{83} \cdot D_{81} - F_{86} \cdot D_{82} + T_{84} = J_{8X} \cdot \ddot{x}_{29} \quad (54$$

$$F_{81} \cdot D_{84} + T_{85} = J_{8Y} \cdot \ddot{x}_{21} \quad (55$$

$$-F_{81} \cdot D_{81} + F_{84} \cdot D_{82} + T_{86} = J_{8Z} \cdot \ddot{x}_{25} \quad (56$$

c. Elastic Equations

Using the rules of elementary beam theory, and the principle of superposition, a set of equations is written to describe the deflection and slope of the rear axles due to the forces and moments applied to them. Consider the deflection of the left rear axle in the vertical direction. Figure 7 shows this case. The equations for the deflections and slopes of the inner and outer wheels in this plane are

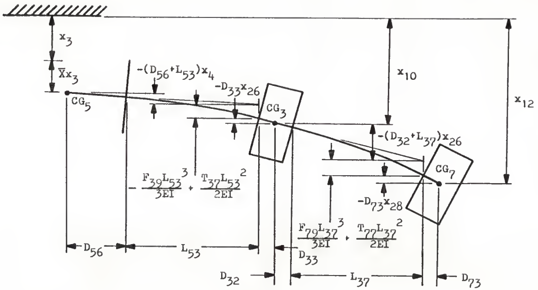


FIGURE 7

DEFLECTED LEFT REAR AXLE

$$-F_{39} \cdot (L_{53}^3/3EI) + T_{37} \cdot (L_{53}^2/2EI) = -x_3 + (D_{56} + L_{53}) \cdot x_4 - \overline{X} \cdot x_5 + x_{10} + D_{33} \cdot x_{26} \quad (57$$

$$F_{39} \cdot (L_{53}^2/2EI) - T_{37} \cdot (L_{53}/EI) = -x_4 + x_{26} \quad (58$$

and

$$-F_{79} \cdot (L_{37}^3/3EI) + T_{77} \cdot (L_{37}^2/2EI) = -x_{10} + x_{12} + (D_{32} + L_{37}) \cdot x_{26} + D_{73} \cdot x_{28} \quad (59$$

$$F_{79} \cdot (L_{37}^2/2EI) - T_{77} \cdot (L_{37}/EI) = -x_{26} + x_{28} \quad (60$$

Similar sets of equations are written for the deflection and slope of the left axle in the horizontal plane, and for the right axle in the verticle and horizontal planes.

$$-F_{37} \cdot (L_{53}^3/3EI) - T_{39} \cdot (L_{53}^2/2EI) = -x_1 - D_{57} \cdot x_5 - (D_{56} + L_{53}) \cdot x_6 + x_{14} - D_{33} \cdot x_{22} \quad (61$$

$$-F_{37} \cdot (L_{53}^2/2EI) - T_{39} \cdot (L_{53}/EI) = -x_6 + x_{22} \quad (62$$

$$-F_{77} \cdot (L_{37}^3/3EI) - T_{79} \cdot (L_{37}^2/2EI) = -x_{14} + x_{16} - (D_{32} + L_{37}) \cdot x_{22} - D_{73} \cdot x_{24} \quad (63$$

$$-F_{77} \cdot (L_{37}^2/2EI) - T_{79} \cdot (L_{37}/EI) = -x_{22} + x_{24} \quad (64$$

$$-F_{46} \cdot (L_{54}^3/3EI) - T_{44} \cdot (L_{54}^2/2EI) = -x_3 - (D_{55} + L_{54}) \cdot x_4 - \overline{X} \cdot x_5 + x_{11} - D_{42} \cdot x_{27} \quad (65$$

$$-F_{46} \cdot (L_{54}^2/2EI) - T_{44} \cdot (L_{54}/EI) = -x_4 + x_{27} \quad (66$$

$$-F_{86} \cdot (L_{84}{}^3/3EI) - T_{84} \cdot (L_{84}{}^2/2EI) = -x_{11} + x_{13} - (D_{43} + L_{84}) \cdot x_{27} - D_{82} \cdot x_{29} \quad (67$$

$$-F_{86} \cdot (L_{84}{}^2/2EI) - T_{84} \cdot (L_{84}/EI) = -x_{27} + x_{29} \quad (68$$

$$-F_{44} \cdot (L_{54}{}^3/3EI) + T_{46} \cdot (L_{54}{}^2/2EI) = -x_1 - D_{57} \cdot x_5 + (D_{55} + L_{54}) \cdot x_6 + x_{15} + D_{42} \cdot x_{23} \quad (69$$

$$F_{44} \cdot (L_{54}{}^2/2EI) - T_{46} \cdot (L_{54}/EI) = -x_6 + x_{23} \quad (70$$

$$-F_{84} \cdot (L_{84}{}^3/3EI) + T_{86} \cdot (L_{84}{}^2/2EI) = -x_{15} + x_{17} + (D_{43} + L_{84}) \cdot x_{23} + D_{82} \cdot x_{25} \quad (71$$

$$F_{84} \cdot (L_{84}{}^2/2EI) - T_{86} \cdot (L_{84}/EI) = -x_{23} + x_{25} \quad (72$$

Elementary beam theory is also used to determine the amount of twiat in the rear axles. The angle of twist in a round bar of length L subject to a moment T is given by

$$\theta = TL/JG \quad (73$$

Applying (73) to the sections of the rear axle yields

$$T_{38} \cdot L_{53}/JG = x_5 - x_{18} \quad (74$$

$$T_{78} \cdot L_{37}/JG = x_{18} - x_{20} \quad (75$$

$$T_{45} \cdot L_{54}/JG = x_5 - x_{19} \quad (76$$

$$T_{85} \cdot L_{84}/JG = x_{19} - x_{21} \quad (77$$

d. Equations Involving Hooke's Law

The tractor tirea are modeled as linear springs with viscous damping as shown in Figure 8. By Hooke's law, the force at the tire to ground contact point of the i^{th} mass in the j direction is given by

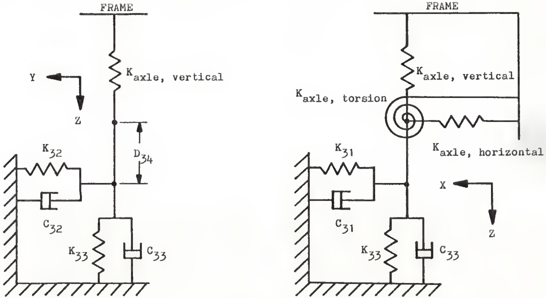


FIGURE 8

SCHEMATIC REPRESENTATION OF THE INSIDE LEFT REAR TIRE

$$F_{ij} = K_{ij} \cdot (x_{ij} - y_{ij}) + C_{ij} \cdot (\dot{x}_{ij} - \dot{y}_{ij}) \quad i=1,2,3,4,7,8;j=1,2,3 \quad (78$$

Applying (78) to the six tire to ground contact points in each coordinate direction yields

$$F_{11} = -(x_1 + (D_{53} + D_{62}) \cdot x_5 + (\overline{Y} + D_{99} + D_{13}) \cdot x_6 + D_{14} \cdot x_8 - y_{11}) \cdot K_{11} -(\dot{x}_1 + (D_{53} + D_{62}) \cdot \dot{x}_5 + (\overline{Y} + D_{99} + D_{13}) \cdot \dot{x}_6 + D_{14} \cdot \dot{x}_8 - \dot{y}_{11}) \cdot C_{11} \quad (79$$

$$F_{12} = -(x_2 - D_{53} \cdot x_4 + D_{51} \cdot x_6 - (D_{62} + D_{14}) \cdot x_7 - y_{12}) \cdot K_{12} -(\dot{x}_2 - D_{53} \cdot \dot{x}_4 + D_{51} \cdot \dot{x}_6 - (D_{62} + D_{14}) \cdot \dot{x}_7 - \dot{y}_{12}) \cdot C_{12} \quad (80$$

$$F_{13} = -(x_3 - \overline{Y} \cdot x_4 - D_{51} \cdot x_5 - (D_{99} + D_{13}) \cdot x_7 - y_{13}) \cdot K_{13} -(\dot{x}_3 - \overline{Y} \cdot \dot{x}_4 - D_{51} \cdot \dot{x}_5 - (D_{99} + D_{13}) \cdot \dot{x}_7 - \dot{y}_{13}) \cdot C_{13} \quad (81$$

$$F_{21} = -(x_1 + (D_{53} + D_{62}) \cdot x_5 - (D_{99} + D_{22} - \overline{Y}) \cdot x_6 + D_{24} \cdot x_9 - y_{21}) \cdot K_{21} -(\dot{x}_1 + (D_{53} + D_{62}) \cdot \dot{x}_5 - (D_{99} + D_{22} - \overline{Y}) \cdot \dot{x}_6 + D_{24} \cdot \dot{x}_9 - \dot{y}_{21}) \cdot C_{21} \quad (82$$

$$F_{22} = -(x_2 - D_{53} \cdot x_4 + D_{51} \cdot x_6 - (D_{62} + D_{24}) \cdot x_7 - y_{22}) \cdot K_{22} -(\dot{x}_2 - D_{53} \cdot \dot{x}_4 + D_{51} \cdot \dot{x}_6 - (D_{62} + D_{24}) \cdot \dot{x}_7 - \dot{y}_{22}) \cdot C_{22} \quad (83$$

$$F_{23} = -(x_3 - \overline{Y} \cdot x_4 - D_{51} \cdot x_5 + (D_{99} + D_{22}) \cdot x_7 - y_{23}) \cdot K_{23} -(\dot{x}_3 - \overline{Y} \cdot \dot{x}_4 - D_{51} \cdot \dot{x}_5 + (D_{99} + D_{22}) \cdot \dot{x}_7 - \dot{y}_{23}) \cdot C_{23} \quad (84$$

$$F_{31} = -(x_{14} + D_{34} \cdot x_{18} - y_{31}) \cdot K_{31} - (\dot{x}_{14} + D_{34} \cdot \dot{x}_{18} - \dot{y}_{31}) \cdot C_{31} \quad (85$$

$$F_{32} = -(x_2 - D_{57} \cdot x_4 - \overline{X} \cdot x_6 - D_{34} \cdot x_{26} - y_{32}) \cdot K_{32}$$
$$-(\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{34} \cdot \dot{x}_{26} - \dot{y}_{32}) \cdot C_{32} \quad (86$$

$$F_{33} = -(x_{10} - y_{33}) \cdot K_{33} - (\dot{x}_{10} - \dot{y}_{33}) \cdot C_{33} \quad (87$$

$$F_{41} = -(x_{15} + D_{44} \cdot x_{19} - y_{41}) \cdot K_{41} - (\dot{x}_{15} + D_{44} \cdot \dot{x}_{19} - \dot{y}_{41}) \cdot C_{41} \quad (88$$

$$F_{42} = -(x_2 - D_{57} \cdot x_4 - \overline{X} \cdot x_6 - D_{44} \cdot x_{27} - y_{42}) \cdot K_{42}$$
$$-(\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{44} \cdot \dot{x}_{27} - \dot{y}_{42}) \cdot C_{42} \quad (89$$

$$F_{43} = -(x_{11} - y_{43}) \cdot K_{43} - (\dot{x}_{11} - \dot{y}_{43}) \cdot C_{43} \quad (90$$

$$F_{71} = -(x_{16} + D_{74} \cdot x_{20} - y_{71}) \cdot K_{71} - (\dot{x}_{16} + D_{74} \cdot \dot{x}_{20} - \dot{y}_{71}) \cdot C_{71} \quad (91$$

$$F_{72} = -(x_2 - D_{57} \cdot x_4 - \overline{X} \cdot x_6 - D_{74} \cdot x_{28} - y_{72}) \cdot K_{72}$$
$$-(\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{74} \cdot \dot{x}_{28} - \dot{y}_{72}) \cdot C_{72} \quad (92$$

$$F_{73} = -(x_{12} - y_{73}) \cdot K_{73} - (\dot{x}_{12} - \dot{y}_{73}) \cdot C_{73} \quad (93$$

$$F_{81} = -(x_{17} + D_{84} \cdot x_{21} - y_{81}) \cdot K_{81} - (\dot{x}_{17} + D_{84} \cdot \dot{x}_{21} - \dot{y}_{81}) \cdot C_{81} \quad (94$$

$$F_{82} = -(x_2 - D_{57} \cdot x_4 - \overline{X} \cdot x_6 - D_{84} \cdot x_{29} - y_{82}) \cdot K_{82}$$
$$-(\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{84} \cdot \dot{x}_{29} - \dot{y}_{82}) \cdot C_{82} \quad (95$$

$$F_{83} = -(x_{13} - y_{83}) \cdot K_{83} - (\dot{x}_{13} - \dot{y}_{83}) \cdot C_{83} \quad (96$$

e. Reduction of the Equations

Equations (4a) through (7f), (9) through (72), (74) through (77), and (79) through (96) constitute one hundred ten equations in one hundred ten unknowns. Because of their simplicity, equations (4a) through (7f) are substituted into equations (9) through (96) where applicable. This reduces the number of equations and unknowns to eighty-six. The resulting equations are cast into matrix form

$$\begin{bmatrix} F \\ 86 \times 57 \end{bmatrix} \begin{Bmatrix} F_{11} \\ \vdots \\ F_{83} \\ T_{17} \\ \vdots \\ T_{53} \end{Bmatrix} = \begin{bmatrix} A \\ 86 \times 29 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \vdots \\ \ddot{x}_{29} \end{Bmatrix} + \begin{bmatrix} B \\ 86 \times 29 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_{29} \end{Bmatrix} + \begin{bmatrix} C \\ 86 \times 29 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{29} \end{Bmatrix} + \begin{bmatrix} D \\ 86 \times 39 \end{bmatrix} \begin{Bmatrix} y_1 \\ \vdots \\ y_{18} \\ \dot{y}_1 \\ \vdots \\ \dot{y}_{18} \\ FDB_1 \\ FDB_2 \\ FDB_3 \end{Bmatrix} \qquad (97$$

Equation (97) is rearranged by subtracting A from either side and combining B, C and D to form

$$\begin{bmatrix} FA \\ 86 \times 86 \end{bmatrix} \begin{Bmatrix} F \\ T \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} CBD \\ 86 \times 97 \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ FDB \end{Bmatrix} \qquad (98$$

In this form, the eighty-six unknowns are solved for using the Gauss-Jordan reduction. This method for solving simultaneous equations is outlined in reference [3], and a listing of the computer program which performs the procedure is in Appendix A.

After reduction, the equations are of the form

$$\begin{bmatrix} I \end{bmatrix} \begin{Bmatrix} F \\ T \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} \; CBD' \; \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ FDB \end{Bmatrix} \tag{99}$$

where I is the identity matrix and CBD' is a recombination of CBD.

The last twenty-nine rows of (99) are the differential equations of motion for the system, and may be cast into the form

$$\begin{bmatrix} I \end{bmatrix} \begin{Bmatrix} \ddot{x} \end{Bmatrix} = \begin{bmatrix} G \\ 29 \times 57 \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ FDB \end{Bmatrix} \tag{100}$$

where G is the last twenty-nine rows of CBD', and is in a form which may be integrated numerically.

B. Derivation Using the Energy Method

The equations of motion are derived by the energy method, to be used as a check for the equations found in part A. With this method, expressions for the kinetic, potential, and dissipative energy are formulated and used to determine the equations of motion with the Lagrange equation for non-conservative systems.

As in part A, the system may be forced by either a drawbar load or by the surface being traversed. The drawbar load is input using the principle of virtual work, and the surface is input by subtracting the profile seen by each tire in each coordinate direction from the applicable displacement in the potential energy term.

a. The Lagrange Equation

For a non-conservative system with n degrees of freedom, the Lagrange equation takes on the form

$$d/dt\ (\partial T/\partial \dot{x}_r) - \partial T/\partial x_r + \partial F/\partial \dot{x}_r + \partial V/\partial x_r = Q_r \qquad r=1,2,3\ \ldots\ n \qquad (101$$

where T is kinetic energy, V is potential energy, and F is the dissipation function due to damping in the tires. Q_r are generalized forces acting on the system that are not included in V or F. For the system being modeled here, they are the drawbar forces.

Application of equation (101) yields a set of n second order differential equations. Reference [8] outlines the formulation of the Lagrange equation and its application to both conservative and non-conservative systems.

b. Kinetic Energy

The kinetic energy of the system is given by

$$T = \tfrac{1}{2} \cdot \Sigma\, M_i \cdot \dot{x}_i^{\,2} \qquad (102)$$

$$T = \tfrac{1}{2} \cdot [M_1 \cdot [(\dot{x}_1 + (\overline{Y} + D_{99} + D_{13}) \cdot \dot{x}_6 + (D_{53} + D_{62}) \cdot \dot{x}_5)^2 + (\dot{x}_2 + D_{51} \cdot \dot{x}_6 - D_{53} \cdot \dot{x}_4 - D_{62} \cdot \dot{x}_7)^2 + (\dot{x}_3 - \overline{Y} \cdot \dot{x}_4 - D_{51} \cdot \dot{x}_5 - (D_{99} + D_{13}) \cdot \dot{x}_7)^2] + J_{1X} \cdot \dot{x}_7^2 + J_{1Y} \cdot \dot{x}_8^2 + J_{1Z} \cdot \dot{x}_6^2 + M_2 \cdot [(\dot{x}_1 + (D_{53} + D_{62}) \cdot \dot{x}_5 - (D_{99} + D_{22} - \overline{Y}) \cdot \dot{x}_6)^2 + (\dot{x}_2 + D_{51} \cdot \dot{x}_6 - D_{53} \cdot \dot{x}_4 - D_{62} \cdot \dot{x}_7)^2 + (x_3 - \overline{Y} \cdot \dot{x}_4 - D_{51} \cdot \dot{x}_5 + (D_{99} + D_{22}) \cdot \dot{x}_7)^2] + J_{2X} \cdot \dot{x}_7^2 + J_{2Y} \cdot \dot{x}_9^2 + J_{2Z} \cdot \dot{x}_6^2 + M_3 \cdot [\dot{x}_{14}^2 + (\dot{x}_2 - \overline{X} \cdot \dot{x}_6 - D_{57} \cdot \dot{x}_4)^2 + \dot{x}_{10}^2] + J_{3X} \cdot \dot{x}_{26}^2 + J_{3Y} \cdot \dot{x}_{18}^2 + J_{3Z} \cdot \dot{x}_{22}^2 + M_4 \cdot [\dot{x}_{15}^2 + (\dot{x}_2 - \overline{X} \cdot \dot{x}_6 - D_{57} \cdot \dot{x}_4)^2 + \dot{x}_{11}^2] + J_{4X} \cdot \dot{x}_{27}^2 + J_{4Y} \cdot \dot{x}_{19}^2 + J_{4Z} \cdot \dot{x}_{23}^2 + M_5 \cdot [\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2] + J_{5X} \cdot \dot{x}_4^2 + J_{5Y} \cdot \dot{x}_5^2 + J_{5Z} \cdot \dot{x}_6^2 + M_6 \cdot [(\dot{x}_1 + (D_{53} + D_{63}) \cdot \dot{x}_5 + \overline{Y} \cdot \dot{x}_6)^2 + (\dot{x}_2 - D_{53} \cdot \dot{x}_4 + D_{51} \cdot \dot{x}_6 - D_{63} \cdot \dot{x}_7)^2 + (\dot{x}_3 - \overline{Y} \cdot \dot{x}_4 - D_{51} \cdot \dot{x}_5)^2] + J_{6X} \cdot \dot{x}_7^2 + J_{6Y} \cdot \dot{x}_5^2 + J_{6Z} \cdot \dot{x}_6^2 + M_7 \cdot [\dot{x}_{16}^2 + (\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6)^2 + \dot{x}_{12}^2] + J_{7X} \cdot \dot{x}_{28}^2 + J_{7Y} \cdot \dot{x}_{20}^2 + J_{7Z} \cdot \dot{x}_{24}^2 + M_8 \cdot [\dot{x}_{17}^2 + (\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6)^2 + \dot{x}_{13}^2] + J_{8X} \cdot \dot{x}_{29}^2 + J_{8Y} \cdot \dot{x}_{21}^2 + J_{8Z} \cdot \dot{x}_{25}^2] \qquad (103$$

c. Potential Energy

The potential energy of the system comes from two sources, the deflection of the tires, and the deflection of the axles. The potential energy of the deflected tires is given by

$$V_1 = \tfrac{1}{2} \cdot \Sigma K_i \quad x_i^2 \tag{104}$$

$$\begin{aligned}
V_1 = \tfrac{1}{2} \cdot [&K_{11} \cdot [x_1 + (D_{53} + D_{62}) \cdot x_5 + (\overline{Y} + D_{99} + D_{13}) \cdot x_6 + D_{14} \cdot \\
&x_8 - y_{11}]^2 + K_{12} \cdot [x_2 - D_{53} \cdot x_4 + D_{51} \cdot x_6 - (D_{62} + D_{14}) \cdot x_7 - \\
&y_{12}]^2 + K_{13} \cdot [x_3 - \overline{Y} \cdot x_4 - D_{51} \cdot x_5 - (D_{99} + D_{13}) \cdot x_7 - y_{13}]^2 \\
&+ K_{21} \cdot [x_1 + (D_{53} + D_{62}) \cdot x_5 - (D_{99} + D_{22} - \overline{Y}) \cdot x_6 + D_{24} \cdot x_9 \\
&- y_{21}]^2 + K_{22} \cdot [x_2 - D_{53} \cdot x_4 + D_{51} \cdot x_6 - (D_{62} + D_{24}) \cdot x_7 - y_{22}]^2 \\
&+ K_{23} \cdot [x_3 - \overline{Y} \cdot x_4 - D_{51} \cdot x_5 + (D_{99} + D_{22}) \cdot x_7 - y_{23}]^2 + K_{31} \cdot \\
&[x_{14} + D_{34} \cdot x_{18} - y_{31}]^2 + K_{32} \cdot [x_2 - D_{57} \cdot x_4 - \overline{X} \cdot x_6 - D_{34} \cdot \\
&x_{26} - y_{32}]^2 + K_{33} \cdot [x_{10} - y_{33}]^2 + K_{41} \cdot [x_{15} + D_{44} \cdot x_{19} - y_{41}]^2 \\
&+ K_{42} \cdot [x_2 - D_{57} \cdot x_4 - \overline{X} \cdot x_6 - D_{44} \cdot x_{27} - y_{42}]^2 + K_{43} \cdot [x_{11} - \\
&y_{43}]^2 + K_{71} \cdot [x_{16} + D_{74} \cdot x_{20} - y_{71}]^2 + K_{72} \cdot [x_2 - D_{57} \cdot x_4 - \overline{X} \cdot \\
&x_6 - D_{74} \cdot x_{28} - y_{72}]^2 + K_{73} \cdot [x_{12} - y_{73}]^2 + K_{81} \cdot [x_{17} + D_{84} \cdot \\
&x_{21} - y_{81}]^2 + K_{82} \cdot [x_2 - D_{57} \cdot x_4 - \overline{X} \cdot x_6 - D_{84} \cdot x_{29} - y_{82}]^2 \\
&+ K_{83} \cdot [x_{13} - y_{83}]^2]
\end{aligned} \tag{105}$$

A discussion of the potential energy in the deflected axle is given in Appendix B. This energy is given by

$$V_2 = (6EI/L_{53}^3) \cdot (-x_3 + (D_{56} + L_{53}) \cdot x_4 - \overline{X} \cdot x_5 + x_{10} + D_{33} \cdot x_{26})^2$$

$$+ (6EI/L_{53}^2) \cdot (-x_3 + (D_{56} + L_{53}) \cdot x_4 - \overline{X} \cdot x_5 + x_{10} + D_{33} \cdot x_{20}) \cdot$$

$$(-x_4 + x_{26}) + (2EI/L_{53}) \cdot (-x_4 + x_{26})^2 + (6EI/L_{37}^3) \cdot (-x_{10} +$$

$$x_{12} + (D_{32} + L_{37}) \cdot x_{26} + D_{73} \cdot x_{28})^2 + (6EI/L_{37}^2) \cdot (-x_{10} + x_{12}$$

$$+ (D_{32} + L_{37}) \cdot x_{26} + D_{73} \cdot x_{28}) \cdot (-x_{26} + x_{28}) + (2EI/L_{37}) \cdot (-x_{26}$$

$$+ x_{28})^2 + (6EI/L_{54}^3) \cdot (-x_3 - (D_{55} + L_{54}) \cdot x_4 - \overline{X} \cdot x_5 + x_{11} -$$

$$D_{42} \cdot x_{27})^2 - (6EI/L_{54}^2) \cdot (-x_3 - (D_{55} + L_{54}) \cdot x_4 - \overline{X} \cdot x_5 + x_{11}$$

$$- D_{42} \cdot x_{27}) \cdot (-x_4 + x_{27}) + (2EI/L_{54}) \cdot (-x_4 + x_{27})^2 + (6EI/L_{84}^3)$$

$$\cdot (-x_{11} + x_{13} - (D_{43} + L_{84}) \cdot x_{27} - D_{82} \cdot x_{29})^2 - (6EI/L_{84}^2) \cdot$$

$$(-x_{11} + x_{13} - (D_{43} + L_{84}) \cdot x_{27} - D_{82} \cdot x_{29}) \cdot (-x_{27} + x_{29}) +$$

$$(2EI/L_{84}) \cdot (-x_{27} + x_{29})^2 + (6EI/L_{53}^3) \cdot (-x_1 - D_{57} \cdot x_5 - (D_{56} +$$

$$L_{53}) \cdot x_6 + x_{14} - D_{33} \cdot x_{22})^2 - (6EI/L_{53}^2) \cdot (-x_1 - D_{57} \cdot x_5 -$$

$$(D_{56} + L_{53}) \cdot x_6 + x_{14} - D_{33} \cdot x_{22}) \cdot (-x_6 + x_{22}) + (2EI/L_{53}) \cdot$$

$$(-x_6 + x_{22})^2 + (6EI/L_{37}^3) \cdot (-x_{14} + x_{16} - (D_{32} + L_{37}) \cdot x_{22} -$$

$$D_{73} \cdot x_{24})^2 - (6EI/L_{37}^2) \cdot (-x_{14} + x_{16} - (D_{32} + L_{37}) \cdot x_{22} - D_{73} \cdot$$

$$x_{24}) \cdot (-x_{22} + x_{24}) + (2EI/L_{37}) \cdot (-x_{22} + x_{24})^2 + (6EI/L_{54}^3) \cdot$$

$$(-x_1 - D_{57} \cdot x_5 + (D_{55} + L_{54}) \cdot x_6 + x_{15} + D_{42} \cdot x_{23})^2 + (6EI/L_{54}^2) \cdot$$

$$(-x_1 - D_{57} \cdot x_5 + (D_{55} + L_{54}) \cdot x_6 + x_{15} + D_{42} \cdot x_{23}) \cdot (-x_6 + x_{23})$$

$$+ (2EI/L_{54}) \cdot (-x_6 + x_{23})^2 + (6EI/L_{84}{}^3) \cdot (-x_{15} + x_{17} + (D_{43} + L_{84}) \cdot$$

$$x_{23} + D_{82} \cdot x_{25})^2 + (6EI/L_{84}{}^2) \cdot (-x_{15} + x_{17} + (D_{43} + L_{84}) \cdot x_{23} +$$

$$D_{82} \cdot x_{25}) \cdot (-x_{23} + x_{25}) + (2EI/L_{84}) \cdot (-x_{23} + x_{25})^2 + \tfrac{1}{2} \cdot (JG/L_{53})$$

$$\cdot (-x_5 + x_{18})^2 + \tfrac{1}{2} \cdot (JG/L_{37}) \cdot (-x_{18} + x_{20})^2 + \tfrac{1}{2} \cdot (JG/L_{59}) \cdot (-x_5$$

$$+ x_{19})^2 + \tfrac{1}{2} \cdot (JG/L_{84}) \cdot (-x_{19} + x_{21})^2 \qquad (106$$

The total potential energy is then

$$V_T = V_1 + V_2 \qquad (107$$

d. Dissipative Energy

The dissipative energy due to the damping in the tires is given by

$$F = \tfrac{1}{2} \cdot \Sigma\, C_i \cdot x_i \qquad (108$$

$$F = \tfrac{1}{2} \cdot [C_{11} \cdot [\dot{x}_1 + (D_{53} + D_{62}) \cdot \dot{x}_5 + (\overline{Y} + D_{99} + D_{13}) \cdot \dot{x}_6 + D_{14} \cdot$$

$$\dot{x}_8 - \dot{y}_{11}]^2 + C_{12} \cdot [\dot{x}_2 - D_{53} \cdot \dot{x}_4 + D_{51} \cdot \dot{x}_6 - (D_{62} + D_{14}) \cdot \dot{x}_7 -$$

$$\dot{y}_{12}]^2 + C_{13} \cdot [\dot{x}_3 - \overline{Y} \cdot \dot{x}_4 - D_{51} \cdot \dot{x}_5 - (D_{99} + D_{13}) \cdot \dot{x}_7 - \dot{y}_{13}]^2$$

$$+ C_{21} \cdot [\dot{x}_1 + (D_{53} + D_{62}) \cdot \dot{x}_5 - (D_{99} + D_{22} - \overline{Y}) \cdot \dot{x}_6 + D_{24} \cdot \dot{x}_9 -$$

$$\dot{y}_{21}]^2 + C_{22} \cdot [\dot{x}_2 - D_{53} \cdot \dot{x}_4 + D_{51} \cdot \dot{x}_6 - (D_{62} + D_{24}) \cdot \dot{x}_7 - \dot{y}_{22}]^2$$

$$+ C_{23} \cdot [\dot{x}_3 - D_{51} \cdot \dot{x}_5 - \overline{Y} \cdot \dot{x}_4 + (D_{99} + D_{22}) \cdot \dot{x}_7 - \dot{y}_{23}]^2 + C_{31} \cdot$$

$$[\dot{x}_{14} + D_{34} \cdot \dot{x}_{18} - \dot{y}_{31}]^2 + C_{32} \cdot [\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{34} \cdot \dot{x}_{26} - \dot{y}_{32}]^2 + C_{33} \cdot [\dot{x}_{10} - \dot{y}_{33}]^2 + C_{41} \cdot [\dot{x}_{15} + D_{44} \cdot \dot{x}_{19} - \dot{y}_{41}]^2 + C_{42} \cdot [\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{44} \cdot \dot{x}_{27} - \dot{y}_{42}]^2 + C_{43} \cdot [\dot{x}_{11} - \dot{y}_{43}]^2 + C_{71} \cdot [\dot{x}_{16} + D_{74} \cdot \dot{x}_{20} - \dot{y}_{71}]^2 + C_{72} \cdot [\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{74} \cdot \dot{x}_{28} - \dot{y}_{72}]^2 + C_{73} \cdot [\dot{x}_{12} - \dot{y}_{73}]^2 + C_{81} \cdot [\dot{x}_{17} + D_{84} \cdot \dot{x}_{21} - \dot{y}_{81}]^2 + C_{82} \cdot [\dot{x}_2 - D_{57} \cdot \dot{x}_4 - \overline{X} \cdot \dot{x}_6 - D_{84} \cdot \dot{x}_{29} - \dot{y}_{82}]^2 + C_{83} \cdot [\dot{x}_{13} - \dot{y}_{83}]] \quad (109$$

e. Generalized Forces

The generalized forces acting on the system are due to the components of the drawbar load, namely FDB_1, FDB_2 and FDB_3. Using the principle of virtual work as outlined in reference [8]

$$\delta W = \Sigma\ Q_i \cdot \delta x_i \quad (110$$

$$= FDB_1 \cdot \delta x_1 + FDB_2 \cdot \delta x_2 + FDB_3 \cdot \delta x_3 + (-FDB_2 \cdot DB_3 - FDB_3 \cdot \overline{Y}) \cdot \delta x_4 + (FDB_1 \cdot DB_3 + FDB_3 \cdot DB_1) \cdot \delta x_5 + (FDB_1 \cdot \overline{Y} - FDB_2 \cdot DB_1) \cdot \delta x_6 \quad (111$$

Therefore

$$Q_1 = FDB_1 \qquad Q_5 = FDB_1 \cdot DB_3 + FDB_3 \cdot DB_1$$
$$Q_2 = FDB_2 \qquad Q_6 = FDB_1 \cdot \overline{Y} - FDB_2 \cdot DB_1 \quad (112\ a\text{-}g$$
$$Q_3 = FDB_3 \qquad Q_7 \text{ through } Q_{29} = 0$$
$$Q_4 = FDB_2 \cdot DB_3 - FDB_3 \cdot \overline{Y}$$

f. Reduction of the Equations

Substitution of the expressions for T, V, F and Q_R into equation (101) with n = 29 yield the equations of motion for the system

$$
\begin{aligned}
&(M_1 + M_2 + M_5 + M_6) \cdot \dot{x}_1 + [D_{53} \cdot (M_1 + M_2 + M_6) + D_{62} \cdot (M_1 + M_2) \\
&+ D_{63} \cdot M_6] \cdot \dot{x}_5 + [\overline{Y} \cdot (M_1 - M_2 + M_6) + D_{99} \cdot (M_1 - M_2) + D_{13} \cdot M_1 \\
&- D_{22} \cdot M_2] \cdot \dot{x}_6 + (C_{11} + C_{21}) \cdot \dot{x}_1 + (D_{53} + D_{62}) \cdot (C_{11} + C_{21}) \cdot \dot{x}_5 \\
&+ [C_{11} \cdot (\overline{Y} + D_{99} + D_{13}) + C_{21} \cdot (\overline{Y} - D_{99} - D_{22})] \cdot \dot{x}_6 + C_{11} \cdot D_{14} \cdot \dot{x}_8 \\
&+ C_{21} \cdot D_{24} \cdot \dot{x}_9 - C_{11} \cdot \dot{y}_{11} - C_{21} \cdot \dot{y}_{21} + (K_{11} + K_{21} + KA_1 + KA_3) \cdot x_1 \\
&+ [(D_{53} + D_{62}) \cdot (K_{11} + K_{21}) + D_{57} \cdot (KA_1 + KA_3)] \cdot x_5 + [K_{11} \cdot (\overline{Y} + \\
&D_{99} + D_{13}) + K_{21} \cdot (\overline{Y} - D_{99} - D_{22}) + KA_1 \cdot (D_{56} + L_{53}) - KA_3 \cdot (D_{55} + \\
&L_{54}) - KA_5 + KA_7] \cdot x_6 + K_{11} \cdot D_{14} \cdot x_8 + K_{21} \cdot D_{24} \cdot x_9 - KA_1 \cdot x_{14} \\
&- KA_3 \cdot x_{15} + (KA_1 \cdot D_{53} + KA_5) \cdot x_{22} + (-KA_3 \cdot D_{42} - KA_7) \cdot x_{23} - \\
&K_{11} \cdot y_{11} - K_{21} \cdot y_{21} = FDB_1
\end{aligned}
\tag{113
$$

$$
\begin{aligned}
&(M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8) \cdot \ddot{x}_2 + [- D_{53} \cdot (M_1 + M_2 + M_6) \\
&- D_{57} \cdot (M_3 + M_4 + M_7 + M_8)] \cdot \ddot{x}_4 + [D_{51} \cdot (M_1 + M_2 + M_6) - \overline{X} \cdot (M_3 + \\
&M_4 + M_7 + M_8)] \cdot \ddot{x}_6 + [- D_{62} \cdot (M_1 + M_2) - D_{63} \cdot M_6] \cdot \ddot{x}_7 + (C_{12} + \\
&C_{22} + C_{32} + C_{42} + C_{72} + C_{82}) \cdot \dot{x}_2 + [- D_{53} \cdot (C_{12} + C_{22}) - D_{57} \cdot (C_{32} \\
&+ C_{42} + C_{72} + C_{82})] \cdot \dot{x}_4 + [D_{51} \cdot (C_{12} + C_{22}) - \overline{X} \cdot (C_{32} + C_{42} + C_{72}
\end{aligned}
$$

$$
\begin{aligned}
&+ C_{82})] \cdot \dot{x}_6 + [- C_{12} \cdot (D_{62} + D_{14}) - C_{22} \cdot (D_{62} + D_{24})] \cdot \dot{x}_7 - C_{32} \cdot \\
&D_{34} \cdot \dot{x}_{26} - C_{42} \cdot D_{44} \cdot \dot{x}_{27} - C_{72} \cdot D_{74} \cdot \dot{x}_{28} - C_{82} \cdot D_{84} \cdot \dot{x}_{29} - \\
&C_{12} \cdot \dot{y}_{12} - C_{22} \cdot \dot{y}_{22} - C_{32} \cdot \dot{y}_{32} - C_{42} \cdot \dot{y}_{42} - C_{72} \cdot \dot{y}_{72} - C_{82} \cdot \dot{y}_{82} \\
&+ (K_{12} + K_{22} + K_{32} + K_{42} + K_{72} + K_{82}) \cdot x_2 + [- D_{53} \cdot (K_{12} + K_{22}) - \\
&D_{57} \cdot (K_{32} + K_{42} + K_{72} + K_{82})] \cdot x_4 + [D_{51} \cdot (K_{12} + K_{22}) - \overline{X} \cdot (K_{32} \\
&+ K_{42} + K_{72} + K_{82})] \cdot x_6 + [- K_{12} \cdot (D_{62} + D_{14}) - K_{22} \cdot (D_{62} + D_{24})] \\
&\cdot x_7 - K_{32} \cdot D_{34} \cdot x_{26} - K_{42} \cdot D_{44} \cdot x_{27} - K_{72} \cdot D_{74} \cdot x_{28} - K_{82} \cdot \\
&D_{84} \cdot x_{29} - K_{12} \cdot y_{12} - K_{22} \cdot y_{22} - K_{32} \cdot y_{32} - K_{42} \cdot y_{42} - K_{72} \cdot y_{72} \\
&- K_{82} \cdot y_{82} = FDB_2 \qquad (114
\end{aligned}
$$

$$
\begin{aligned}
&(M_1 + M_2 + M_5 + M_6) \cdot \ddot{x}_3 - \overline{Y} \cdot (M_1 + M_2 + M_6) \cdot \ddot{x}_4 - D_{51} \cdot (M_1 + M_2 \\
&+ M_6) \cdot \ddot{x}_5 + [D_{99} \cdot (M_2 - M_1) - D_{13} \cdot M_1 + D_{22} \cdot M_2] \cdot \ddot{x}_7 + (C_{13} + \\
&C_{23}) \cdot \dot{x}_3 - \overline{Y} \cdot (C_{13} + C_{23}) \cdot \dot{x}_4 - D_{51} \cdot (C_{13} + C_{23}) \cdot \dot{x}_5 + [- C_{13} \cdot \\
&(D_{99} + D_{13}) + C_{23} \cdot (D_{99} + D_{22})] \cdot \dot{x}_7 - C_{13} \cdot \dot{y}_{13} - C_{23} \cdot \dot{y}_{23} + \\
&(K_{13} + K_{23} + KA_1 + KA_3) \cdot x_3 + [- \overline{Y} \cdot (K_{13} + K_{23}) - (D_{56} + L_{53}) \cdot \\
&KA_1 + (D_{55} + L_{54}) \cdot KA_3 + KA_5 - KA_7] \cdot x_4 + [- D_{51} \cdot (K_{13} + K_{23}) + \\
&\overline{X} \cdot (KA_1 + KA_3)] \cdot x_5 + [- K_{13} \cdot (D_{99} + D_{13}) + K_{23} \cdot (D_{99} + D_{22})] \cdot x_7
\end{aligned}
$$

$$- KA_1 \cdot x_{10} - KA_3 \cdot x_{11} + (- KA_1 \cdot D_{33} - KA_5) \cdot x_{26} + (KA_3 \cdot D_{42}$$

$$+ KA_7) \cdot x_{27} - K_{13} \cdot y_{13} - K_{23} \cdot y_{23} = FDB_3 \qquad (115$$

$$[- D_{53} \cdot (M_1 + M_2 + M_6) - D_{57} \cdot (M_3 + M_4 + M_7 + M_8)] \cdot \ddot{x}_2 - \overline{Y} \cdot (M_1$$

$$+ M_2 + M_6) \cdot \ddot{x}_3 + [(D_{53}{}^2 + \overline{Y}^2) \cdot (M_1 + M_2 + M_6) + D_{57}{}^2 \cdot (M_3 + M_4 +$$

$$M_7 + M_8) + J_{5X}] \cdot \ddot{x}_4 + \overline{Y} \cdot D_{51} \cdot (M_1 + M_2 + M_6) \cdot \ddot{x}_5 + [- D_{51} \cdot D_{53} \cdot$$

$$(M_1 + M_2 + M_6) + \overline{X} \cdot D_{57} \cdot (M_3 + M_4 + M_7 + M_8)] \cdot \ddot{x}_6 + [D_{53} \cdot D_{62} \cdot$$

$$(M_1 + M_2) + D_{53} \cdot D_{63} \cdot M_6 + \overline{Y} \cdot D_{99} \cdot (M_1 - M_2) + \overline{Y} \cdot D_{13} \cdot M_1 -$$

$$\overline{Y} \cdot D_{22} \cdot M_2] \cdot \ddot{x}_7 + [- D_{53} \cdot (C_{12} + C_{22}) - D_{57} \cdot (C_{32} + C_{42} + C_{72}$$

$$+ C_{82})] \cdot \dot{x}_2 - \overline{Y} \cdot (C_{13} + C_{23}) \cdot \dot{x}_3 + [D_{53}{}^2 \cdot (C_{12} + C_{22}) + \overline{Y}^2 \cdot$$

$$(C_{13} + C_{23}) + D_{57}{}^2 \cdot (C_{32} + C_{42} + C_{72} + C_{82})] \cdot \dot{x}_4 + D_{51} \cdot \overline{Y} \cdot (C_{13}$$

$$+ C_{23}) \cdot \dot{x}_5 + [- D_{51} \cdot D_{53} \cdot (C_{12} + C_{22}) + \overline{X} \cdot D_{57} \cdot (C_{32} + C_{42} + C_{72}$$

$$C_{82})] \cdot \dot{x}_6 + [D_{53} \cdot ((D_{62} + D_{14}) \cdot C_{12} + (D_{62} + D_{24}) \cdot C_{22}) + \overline{Y} \cdot$$

$$((D_{99} + D_{13}) \cdot C_{13} - (D_{99} + D_{22}) \cdot C_{23})] \cdot \dot{x}_7 + C_{32} \cdot D_{57} \cdot D_{34} \cdot$$

$$\dot{x}_{26} + C_{42} \cdot D_{57} \cdot D_{44} \cdot \dot{x}_{27} + C_{72} \cdot D_{57} \cdot D_{74} \cdot \dot{x}_{28} + C_{82} \cdot D_{57} \cdot$$

$$D_{84} \cdot \dot{x}_{29} + C_{12} \cdot D_{53} \cdot \dot{y}_{12} + C_{13} \cdot \overline{Y} \cdot \dot{y}_{13} + C_{22} \cdot D_{53} \cdot \dot{y}_{22} +$$

$$C_{23} \cdot \overline{Y} \cdot \dot{y}_{23} + C_{32} \cdot D_{57} \cdot \dot{y}_{32} + C_{42} \cdot D_{57} \cdot \dot{y}_{42} + C_{72} \cdot D_{57} \cdot \dot{y}_{72}$$

$$+ C_{82} \cdot D_{57} \cdot \dot{y}_{82} + [- D_{53} \cdot (K_{12} + K_{22}) - D_{57} \cdot (K_{32} + K_{42} + K_{72} +$$

$$K_{82})] \cdot x_2 + [- \overline{Y} \cdot (K_{13} + K_{23}) - (D_{56} + L_{53}) \cdot KA_1 + (D_{55} + L_{54}) \cdot$$

$$KA_3 + KA_5 - KA_7] \cdot x_3 + [D_{53}{}^2 \cdot (K_{12} + K_{22}) + \overline{Y}^2 \cdot (K_{13} + K_{23}) +$$

$$D_{57}{}^2 \cdot (K_{32} + K_{42} + K_{72} + K_{82}) + (D_{56} + L_{53})^2 \cdot KA_1 + (D_{55} + L_{54})^2 \cdot$$

$$KA_3 - 2 \cdot KA_5 \cdot (D_{56} + L_{53}) - 2 \cdot KA_7 \cdot (D_{55} + L_{54}) + KA_9 + KA_{11}] \cdot$$

$$x_4 + [D_{51} \cdot \overline{Y} \cdot (K_{13} + K_{23}) - \overline{X} \cdot ((D_{56} + L_{53}) \cdot KA_1 - (D_{55} + L_{54}) \cdot$$

$$KA_3 - KA_5 + KA_7)] \cdot x_5 + [- D_{51} \cdot D_{53} \cdot (K_{12} + K_{22}) + \overline{X} \cdot D_{57} \cdot$$

$$(K_{32} + K_{42} + K_{72} + K_{82})] \cdot x_6 + [D_{53} \cdot ((D_{62} + D_{14}) \cdot K_{12} + (D_{62} +$$

$$D_{24}) \cdot K_{22}) + \overline{Y} \cdot ((D_{99} + D_{13}) \cdot K_{13} - (D_{99} + D_{22}) \cdot K_{23})] \cdot x_7 +$$

$$[KA_1 \cdot (D_{56} + L_{53}) - KA_5] \cdot x_{10} + [- KA_3 \cdot (D_{55} + L_{54}) + KA_7] \cdot x_{11}$$

$$+ [D_{34} \cdot D_{57} \cdot K_{32} + D_{33} \cdot (D_{56} + L_{53}) \cdot KA_1 - KA_5 \cdot (D_{33} - D_{56} -$$

$$L_{53}) - KA_9] \cdot x_{26} + [D_{44} \cdot D_{57} \cdot K_{42} + D_{42} \cdot (D_{55} + L_{54}) \cdot KA_3 +$$

$$KA_7 \cdot (- D_{42} + D_{55} + L_{54}) - KA_{11}] \cdot x_{27} + D_{74} \cdot D_{57} \cdot K_{72} \cdot x_{28} +$$

$$D_{84} \cdot D_{57} \cdot K_{82} \cdot x_{29} + K_{12} \cdot D_{53} \cdot y_{12} + K_{13} \cdot \overline{Y} \cdot y_{13} + K_{22} \cdot D_{53} \cdot$$

$$y_{22} + K_{23} \cdot \overline{Y} \cdot y_{23} + K_{32} \cdot D_{57} \cdot y_{32} + K_{42} \cdot D_{57} \cdot y_{42} + K_{72} \cdot D_{57} \cdot$$

$$y_{72} + K_{82} \cdot D_{57} \cdot y_{82} = -DB_3 \cdot FDB_2 - \overline{Y} \cdot FDB_3 \qquad (116$$

$[(D_{53} + D_{62}) \cdot (M_1 + M_2) + (D_{53} + D_{63}) \cdot M_6] \cdot \ddot{x}_1 - D_{51} \cdot (M_1 + M_2$
$+ M_6) \cdot \ddot{x}_3 + D_{51} \cdot \overline{Y} \cdot (M_1 + M_2 + M_6) \cdot \ddot{x}_4 + [(D_{53} + D_{62})^2 \cdot (M_1 +$
$M_2) + (D_{53} + D_{63})^2 \cdot M_6 + D_{51}{}^2 \cdot (M_1 + M_2 + M_6) + J_{5Y} + J_{6Y}] \cdot \ddot{x}_5$
$+ [(D_{53} + D_{62}) \cdot ((\overline{Y} + D_{99} + D_{13}) \cdot M_1 + (\overline{Y} - D_{99} - D_{22}) \cdot M_2) +$
$(D_{53} + D_{63}) \cdot \overline{Y} \cdot M_6] \cdot \ddot{x}_6 + D_{51} \cdot ((D_{99} + D_{13}) \cdot M_1 - (D_{99} + D_{22}) \cdot$
$M_2) \cdot \ddot{x}_7 + (D_{53} + D_{62}) \cdot (C_{11} + C_{21}) \cdot \dot{x}_1 - D_{51} \cdot (C_{13} + C_{23}) \cdot \dot{x}_3$
$+ \overline{Y} \cdot D_{51} \cdot (C_{13} + C_{23}) \cdot \dot{x}_4 + [(D_{53} + D_{62})^2 \cdot (C_{11} + C_{21}) + D_{51}{}^2 \cdot$
$(C_{13} + C_{23})] \cdot \dot{x}_5 + (D_{53} + D_{62}) \cdot [(\overline{Y} + D_{99} + D_{13}) \cdot C_{11} + (\overline{Y} - D_{99}$
$- D_{22}) \cdot C_{21}] \cdot \dot{x}_6 + D_{51} \cdot ((D_{99} + D_{13}) \cdot C_{13} - (D_{99} + D_{22}) \cdot C_{23}) \cdot$
$\dot{x}_7 + C_{11} \cdot D_{14} \cdot (D_{53} + D_{62}) \cdot \dot{x}_8 + C_{21} \cdot D_{24} \cdot (D_{53} + D_{62}) \cdot \dot{x}_9 -$
$C_{11} \cdot (D_{53} + D_{62}) \cdot \dot{y}_{11} + C_{13} \cdot D_{51} \cdot \dot{y}_{13} - C_{21} \cdot (D_{53} + D_{62}) \cdot \dot{y}_{21}$
$+ C_{23} \cdot D_{51} \cdot \dot{y}_{23} + [(D_{53} + D_{62}) \cdot (K_{11} + K_{21}) + D_{57} \cdot (KA_1 + KA_3)] \cdot$
$x_1 + [- D_{51} \cdot (K_{13} + K_{23}) + \overline{X} \cdot (KA_1 + KA_3)] \cdot x_3 + [D_{51} \cdot \overline{Y} \cdot (K_{13} +$
$K_{23}) - \overline{X} \cdot ((D_{56} + L_{53}) \cdot KA_1 - (D_{55} + L_{54}) \cdot KA_3 - KA_5 + KA_7)] \cdot x_4$
$+ [(D_{53} + D_{62})^2 \cdot (K_{11} + K_{21}) + D_{51}{}^2 \cdot (K_{13} + K_{23}) + (\overline{X}^2 + D_{57}{}^2) \cdot$
$(KA_1 + KA_3) + KA_{13} + KA_{15}] \cdot x_5 + [(D_{53} + D_{62}) \cdot ((\overline{Y} + D_{99} + D_{13}) \cdot$
$K_{11} + (\overline{Y} - D_{99} - D_{22}) \cdot K_{21}) + D_{57} \cdot ((D_{56} + L_{53}) \cdot KA_1 - (D_{55} + L_{54}) \cdot$

$$KA_3 - KA_5 + KA_7)] \cdot x_6 + D_{51} \cdot ((D_{99} + D_{13}) \cdot K_{13} - (D_{99} + D_{22}) \cdot$$

$$K_{23}) \cdot x_7 + D_{14} \cdot (D_{53} + D_{62}) \cdot K_{11} \cdot x_8 + D_{24} \cdot (D_{53} + D_{62}) \cdot K_{21} \cdot$$

$$x_9 - \overline{X} \cdot KA_1 \cdot x_{10} - \overline{X} \cdot KA_3 \cdot x_{11} - D_{57} \cdot KA_1 \cdot x_{14} - D_{57} \cdot KA_3 \cdot$$

$$x_{15} - KA_{13} \cdot x_{18} - KA_{15} \cdot x_{19} + D_{57} \cdot (KA_1 \cdot D_{33} + KA_5) \cdot x_{22} +$$

$$D_{57} \cdot (- KA_7 - KA_3 \cdot D_{42}) \cdot x_{23} + (- \overline{X} \cdot D_{33} \cdot KA_1 - \overline{X} \cdot KA_5) \cdot x_{26}$$

$$+ (\overline{X} \cdot D_{42} \cdot KA_3 + \overline{X} \cdot KA_7) \cdot x_{27} - K_{11} \cdot (D_{53} + D_{62}) \cdot y_{11} + K_{13} \cdot$$

$$D_{51} \cdot y_{13} - K_{21} \cdot (D_{53} + D_{62}) \cdot y_{21} + K_{23} \cdot D_{51} \cdot y_{23} = DB_3 \cdot FDB_1$$

$$+ DB_1 \cdot FDB_3 \tag{117}$$

$$[(\overline{Y} + D_{99} + D_{13}) \cdot M_1 + (\overline{Y} - D_{99} - D_{22}) \cdot M_2 + \overline{Y} \cdot M_6] \cdot \ddot{x}_1 +$$

$$[D_{51} \cdot (M_1 + M_2 + M_6) - \overline{X} \cdot (M_3 + M_4 + M_7 + M_8)] \cdot \ddot{x}_2 + [- D_{51} \cdot$$

$$D_{53} \cdot (M_1 + M_2) - D_{51} \cdot D_{53} \cdot M_6 + D_{57} \cdot \overline{X} \cdot (M_3 + M_4 + M_7 + M_8)] \cdot$$

$$\ddot{x}_4 + [(D_{53} + D_{62}) \cdot ((\overline{Y} + D_{99} + D_{13}) \cdot M_1 + (\overline{Y} - D_{99} - D_{22}) \cdot M_2)$$

$$+ \overline{Y} \cdot (D_{53} + D_{63}) \cdot M_6] \cdot \ddot{x}_5 + [(\overline{Y} + D_{99} + D_{13})^2 \cdot M_1 + (\overline{Y} - D_{99} -$$

$$D_{22})^2 \cdot M_2 + D_{51}{}^2 \cdot (M_1 + M_2 + M_6) + \overline{X}^2 \cdot (M_3 + M_4 + M_7 + M_8) + \overline{Y}^2 \cdot$$

$$M_6 + J_{1Z} + J_{2Z} + J_{5Z} + J_{6Z}] \cdot \ddot{x}_6 + [- D_{51} \cdot D_{62} \cdot (M_1 + M_2) - D_{51} \cdot$$

$$D_{63} \cdot M_6] \cdot \ddot{x}_7 + [C_{11} \cdot (\overline{Y} + D_{99} + D_{13}) + C_{21} \cdot (\overline{Y} - D_{99} - D_{22})] \cdot \dot{x}_1$$

$$+ [D_{51} \cdot (C_{12} + C_{22}) - \overline{X} \cdot (C_{32} + C_{42} + C_{72} + C_{82})] \cdot \dot{x}_2 + [- D_{51} \cdot$$

$D_{53} \cdot (C_{12} + C_{22}) + \overline{X} \cdot D_{57} \cdot (C_{32} + C_{42} + C_{72} + C_{82})] \cdot \dot{x}_4 + (D_{53} +$

$D_{62}) \cdot (C_{11} \cdot (\overline{Y} + D_{99} + D_{13}) + C_{21} \cdot (\overline{Y} - D_{99} - D_{22})) \cdot \dot{x}_5 + [C_{11} \cdot$

$(\overline{Y} + D_{99} + D_{13})^2 + C_{21} \cdot (\overline{Y} - D_{99} - D_{22})^2 + D_{51}{}^2 \cdot (C_{12} + C_{22}) +$

$\overline{X}^2 \cdot (C_{32} + C_{42} + C_{72} + C_{82})] \cdot \dot{x}_6 - D_{51} \cdot ((D_{62} + D_{14}) \cdot C_{12} +$

$(D_{62} + D_{24}) \cdot C_{22}) \cdot \dot{x}_7 + C_{11} \cdot D_{14} \cdot (\overline{Y} + D_{99} + D_{13}) \cdot \dot{x}_8 + C_{21} \cdot$

$D_{24} \cdot (\overline{Y} - D_{99} - D_{22}) \cdot \dot{x}_9 + C_{32} \cdot \overline{X} \cdot D_{34} \cdot \dot{x}_{26} + C_{42} \cdot \overline{X} \cdot D_{44} \cdot$

$\dot{x}_{27} + C_{72} \cdot \overline{X} \cdot D_{74} \cdot \dot{x}_{28} + C_{82} \cdot \overline{X} \cdot D_{84} \cdot \dot{x}_{29} - C_{11} \cdot (\overline{Y} + D_{99} +$

$D_{13}) \cdot \dot{y}_{11} - C_{12} \cdot D_{51} \cdot \dot{y}_{12} - C_{21} \cdot (\overline{Y} - D_{99} - D_{22}) \cdot \dot{y}_{21} - C_{22} \cdot$

$D_{51} \cdot \dot{y}_{22} + C_{32} \cdot \overline{X} \cdot \dot{y}_{32} + C_{42} \cdot \overline{X} \cdot \dot{y}_{42} + C_{72} \cdot \overline{X} \cdot \dot{y}_{72} + C_{82} \cdot$

$\overline{X} \cdot \dot{y}_{82} + [K_{11} \cdot (\overline{Y} + D_{99} + D_{13}) + K_{21} \cdot (\overline{Y} - D_{99} - D_{22}) + KA_1 \cdot$

$(D_{56} + L_{53}) - KA_3 \cdot (D_{55} + L_{54}) - KA_5 + KA_7] \cdot x_1 + [D_{51} \cdot (K_{12} +$

$K_{22}) - \overline{X} \cdot (K_{32} + K_{42} + K_{72} + K_{82})] \cdot x_2 + [- D_{51} \cdot D_{53} \cdot (K_{12} + K_{22})$

$+ D_{57} \cdot \overline{X} \cdot (K_{32} + K_{42} + K_{72} + K_{82})] \cdot x_4 + [(D_{53} + D_{62}) \cdot ((\overline{Y} + D_{99}$

$+ D_{13}) \cdot K_{11} + (\overline{Y} - D_{99} - D_{22}) \cdot K_{21}) + D_{57} \cdot ((D_{56} + L_{53}) \cdot KA_1 -$

$(D_{55} + L_{54}) \cdot KA_3 - KA_5 + KA_7)] \cdot x_5 + [(\overline{Y} + D_{99} + D_{13})^2 \cdot K_{11} +$

$(\overline{Y} - D_{99} - D_{22})^2 \cdot K_{21} + D_{51}{}^2 \cdot (K_{12} + K_{22}) + \overline{X}^2 \cdot (K_{32} + K_{42} + K_{72}$

$K_{82}) + (D_{56} + L_{53})^2 \cdot KA_1 + (D_{55} + L_{54})^2 \cdot KA_3 - 2 \cdot KA_5 \cdot (D_{56} +$

$L_{53}) - 2 \cdot KA_7 \cdot (D_{55} + L_{54}) + KA_9 + KA_{11}] \cdot x_6 + [- D_{51} \cdot (D_{62} +$

$D_{14}) \cdot K_{12} - D_{51} \cdot (D_{62} + D_{24}) \cdot K_{22}] \cdot x_7 + D_{14} \cdot (\overline{Y} + D_{99} + D_{13}) \cdot$

$K_{11} \cdot x_8 + D_{24} \cdot (\overline{Y} - D_{99} - D_{22}) \cdot K_{21} \cdot x_9 + [- KA_1 \cdot (D_{56} + L_{53})$

$+ KA_5] \cdot x_{14} + [KA_3 \cdot (D_{55} + L_{54}) - KA_7] \cdot x_{15} + [D_{33} \cdot (D_{56} + L_{53})$

$\cdot KA_1 + KA_5 \cdot (- D_{33} + D_{55} + L_{53}) - KA_9] \cdot x_{22} + [D_{42} \cdot (D_{55} + L_{54})$

$\cdot KA_3 + KA_7 \cdot (- D_{42} + D_{56} + L_{54}) - KA_{11}] \cdot x_{23} + \overline{X} \cdot D_{34} \cdot K_{32} \cdot x_{26}$

$+ \overline{X} \cdot D_{44} \cdot K_{42} \cdot x_{27} + \overline{X} \cdot D_{74} \cdot K_{72} \cdot x_{28} + \overline{X} \cdot D_{84} \cdot K_{82} \cdot x_{29} -$

$K_{11} \cdot (\overline{Y} + D_{99} + D_{13}) \cdot y_{11} - K_{12} \cdot D_{51} \cdot y_{12} - K_{21} \cdot (\overline{Y} - D_{99} - D_{22})$

$\cdot y_{21} - K_{22} \cdot D_{51} \cdot y_{22} + K_{32} \cdot \overline{X} \cdot y_{32} + K_{42} \cdot \overline{X} \cdot y_{42} + K_{72} \cdot \overline{X} \cdot$

$y_{72} + K_{82} \cdot \overline{X} \cdot y_{82} = \overline{Y} \cdot FDB_1 - DB_1 \cdot FDB_2$ (118

$[- D_{62} \cdot (M_1 + M_2) - D_{63} \cdot M_6] \cdot \ddot{x}_2 + (D_{99} + D_{13}) \cdot (M_2 - M_1) \cdot \ddot{x}_3$

$+ [D_{53} \cdot D_{62} \cdot (M_1 + M_2) + D_{63} \cdot D_{53} \cdot M_6 + \overline{Y} \cdot (D_{99} + D_{13}) \cdot (M_1 -$

$M_2)] \cdot \ddot{x}_4 + (D_{99} + D_{13}) \cdot D_{51} \cdot (M_1 - M_2) \cdot \ddot{x}_5 + [- D_{62} \cdot D_{51} \cdot (M_1$

$+ M_2) - D_{63} \cdot D_{51} \cdot M_6] \cdot \ddot{x}_6 + [D_{62}{}^2 \cdot (M_1 + M_2) + D_{63}{}^2 \cdot M_6 + (D_{99}$

$+ D_{22})^2 \cdot (M_1 + M_2) + J_{1X} + J_{2X} + J_{6X}] \cdot \ddot{x}_7 + [- C_{12} \cdot (D_{62} + D_{14}) -$

$C_{22} \cdot (D_{62} + D_{24})] \cdot \dot{x}_2 + [- C_{13} \cdot (D_{99} + D_{13}) + C_{23} \cdot (D_{99} + D_{22})] \cdot$

$\dot{x}_3 + [D_{53} \cdot (C_{12} \cdot (D_{62} + D_{14}) + C_{22} \cdot (D_{62} + D_{24})) + \overline{Y} \cdot (C_{13}\ (D_{99}$

$$+ D_{13}) - C_{23} \cdot (D_{99} + D_{22}))] \cdot \dot{x}_4 + D_{51} \cdot (C_{13} \cdot (D_{99} + D_{13}) - C_{23} \cdot$$
$$(D_{99} + D_{22})) \cdot \dot{x}_5 - D_{51} \cdot (C_{12} \cdot (D_{62} + D_{14}) + C_{22} \cdot (D_{62} + D_{24})) \cdot$$
$$\dot{x}_6 + [C_{12} \cdot (D_{62} + D_{14})^2 + C_{13} \cdot (D_{99} + D_{13})^2 + C_{22} \cdot (D_{62} + D_{24})^2 +$$
$$C_{23} \cdot (D_{99} + D_{22})^2] \cdot \dot{x}_7 + C_{12} \cdot (D_{62} + D_{14}) \cdot \dot{y}_{12} + C_{13} \cdot (D_{99} + D_{13})$$
$$\cdot \dot{y}_{13} + C_{22} \cdot (D_{62} + D_{24}) \cdot \dot{y}_{22} - C_{23} \cdot (D_{99} + D_{22}) \cdot \dot{y}_{23} + [- (D_{62} +$$
$$D_{14}) \cdot K_{12} - (D_{62} + D_{24}) \cdot K_{22}] \cdot x_2 + [- (D_{99} + D_{13}) \cdot K_{13} + (D_{99} +$$
$$D_{22}) \cdot K_{23}] \cdot x_3 + [D_{53} \cdot ((D_{62} + D_{14}) \cdot K_{12} + (D_{62} + D_{24}) \cdot K_{22}) +$$
$$\overline{Y} \cdot ((D_{99} + D_{13}) \cdot K_{13} - (D_{99} + D_{22}) \cdot K_{23})] \cdot x_4 + D_{51} \cdot ((D_{99} +$$
$$D_{13}) \cdot K_{13} - (D_{99} + D_{22}) \cdot K_{23}) \cdot x_5 - D_{51} \cdot ((D_{62} + D_{14}) \cdot K_{12} +$$
$$(D_{62} + D_{24}) \cdot K_{22}) \cdot x_6 + [(D_{62} + D_{14})^2 \cdot K_{12} + (D_{99} + D_{13})^2 \cdot K_{13} +$$
$$(D_{62} + D_{24})^2 \cdot K_{22} + (D_{99} + D_{22})^2 \cdot K_{23}] \cdot x_7 + K_{12} \cdot (D_{62} + D_{14}) \cdot$$
$$y_{12} + K_{13} \cdot (D_{99} + D_{13}) \cdot y_{13} + K_{22} \cdot (D_{62} + D_{24}) \cdot y_{22} - K_{23} \cdot (D_{99}$$
$$+ D_{22}) \cdot y_{23} = 0 \qquad (119$$

$$J_{1Y} \cdot \ddot{x}_8 + C_{11} \cdot D_{14} \cdot \dot{x}_1 + C_{11} \cdot D_{14} \cdot (D_{53} + D_{62}) \cdot \dot{x}_5 + C_{11} \cdot$$
$$D_{14} \cdot (\overline{Y} + D_{99} + D_{13}) \cdot \dot{x}_6 + C_{11} \cdot D_{14}{}^2 \cdot \dot{x}_8 - C_{11} \cdot D_{14} \cdot \dot{y}_{11} +$$
$$K_{11} \cdot D_{14} \cdot x_1 + K_{11} \cdot D_{14} \cdot (D_{53} + D_{62}) \cdot x_5 + K_{11} \cdot D_{14} \cdot (\overline{Y} +$$
$$D_{99} + D_{13}) \cdot x_6 + K_{11} \cdot D_{14}{}^2 \cdot x_8 - K_{11} \cdot D_{14} \cdot y_{11} = 0 \qquad (120$$

$$J_{2Y} \cdot \ddot{x}_9 + C_{21} \cdot D_{24} \cdot \dot{x}_1 + C_{21} \cdot D_{24} \cdot (D_{53} + D_{62}) \cdot \dot{x}_5 + C_{21} \cdot D_{24} \cdot (\overline{Y} - D_{99} - D_{22}) \cdot \dot{x}_6 + C_{21} \cdot {D_{24}}^2 \cdot \dot{x}_9 - C_{21} \cdot D_{24} \cdot \dot{y}_{21} + K_{21} \cdot D_{24} \cdot x_1 + K_{21} \cdot D_{24} \cdot (D_{53} + D_{62}) \cdot x_5 + K_{21} \cdot D_{24} \cdot (\overline{Y} - D_{99} - D_{22}) \cdot x_6 + K_{21} \cdot {D_{24}}^2 \cdot x_9 - K_{21} \cdot D_{24} \cdot y_{21} = 0 \tag{121}$$

$$M_3 \cdot \ddot{x}_{10} + C_{33} \cdot \dot{x}_{10} - C_{33} \cdot \dot{y}_{33} - KA_1 \cdot x_3 + [KA_1 \cdot (D_{56} + L_{53}) - KA_5] \cdot x_4 - KA_1 \cdot \overline{X} \cdot x_5 + (K_{33} + KA_1 + KA_2) \cdot x_{10} - KA_2 \cdot x_{12} + [KA_1 \cdot D_{33} - KA_2 \cdot (D_{32} + L_{37}) + KA_5 + KA_6] \cdot x_{26} + (- KA_2 \cdot D_{73} - KA_6) \cdot x_{28} - K_{33} \cdot y_{33} = 0 \tag{122}$$

$$M_4 \cdot \ddot{x}_{11} + C_{43} \cdot \dot{x}_{11} - C_{43} \cdot \dot{y}_{43} - KA_3 \cdot x_3 + [- KA_3 \cdot (D_{55} + L_{54}) + KA_7] \cdot x_4 - KA_3 \cdot \overline{X} \cdot x_5 + (K_{43} + KA_3 + KA_4) \cdot x_{11} - KA_4 \cdot x_{13} + [- KA_3 \cdot D_{42} + KA_4 \cdot (D_{43} + L_{84}) - KA_7 - KA_8] \cdot x_{27} + (KA_4 \cdot D_{82} + KA_8) \cdot x_{29} - K_{43} \cdot y_{43} = 0 \tag{123}$$

$$M_7 \cdot \ddot{x}_{12} + C_{73} \cdot \dot{x}_{12} - C_{73} \cdot \dot{y}_{73} - KA_2 \cdot x_{10} + (K_{73} + KA_2) \cdot x_{12} + [KA_2 \cdot (D_{32} + L_{37}) - KA_6] \cdot x_{26} + (KA_2 \cdot D_{73} + KA_6) \cdot x_{28} - K_{73} \cdot y_{73} = 0 \tag{124}$$

$$M_8 \cdot \ddot{x}_{13} + C_{83} \cdot \dot{x}_{13} - C_{83} \cdot \dot{y}_{83} - KA_4 \cdot x_{11} + (K_{83} + KA_4) \cdot x_{13} +$$
$$[- KA_4 \cdot (D_{43} + L_{84}) + KA_8] \cdot x_{27} + (- KA_4 \cdot D_{82} - KA_8) \cdot x_{29} -$$
$$K_{83} \cdot y_{83} = 0 \tag{125}$$

$$M_3 \cdot \ddot{x}_{14} + C_{31} \cdot \dot{x}_{14} + C_{31} \cdot D_{34} \cdot \dot{x}_{18} - C_{31} \cdot \dot{y}_{31} - KA_1 \cdot x_1 -$$
$$KA_1 \cdot D_{57} \cdot x_5 + [- KA_1 \cdot (D_{56} + L_{53}) + KA_5] \cdot x_6 + (K_{31} + KA_1 +$$
$$KA_2) \cdot \dot{x}_{14} - KA_2 \cdot x_{16} + K_{31} \cdot D_{34} \cdot x_{18} + [- KA_1 \cdot D_{33} + KA_2 \cdot$$
$$(D_{32} + L_{37}) - KA_5 - KA_6] \cdot x_{22} + (KA_2 \cdot D_{73} + KA_6) \cdot x_{24} - K_{31} \cdot$$
$$y_{31} = 0 \tag{126}$$

$$M_4 \cdot \ddot{x}_{15} + C_{41} \cdot \dot{x}_{15} + C_{41} \cdot D_{44} \cdot \dot{x}_{19} - C_{41} \cdot \dot{y}_{41} - KA_3 \cdot x_1 -$$
$$KA_3 \cdot D_{57} \cdot x_5 + [KA_3 \cdot (D_{55} + L_{54}) - KA_7] \cdot x_6 + (K_{41} + KA_3 +$$
$$KA_4) \cdot x_{15} - KA_4 \cdot x_{17} + K_{41} \cdot D_{44} \cdot x_{19} + [KA_3 \cdot D_{42} - KA_4 \cdot$$
$$(D_{43} + L_{84}) + KA_7 + KA_8] \cdot x_{23} + (- KA_4 \cdot D_{82} - KA_8) \cdot x_{25} - K_{41} \cdot$$
$$y_{41} = 0 \tag{127}$$

$$M_7 \cdot \ddot{x}_{16} + C_{71} \cdot \dot{x}_{16} + C_{71} \cdot D_{74} \cdot \dot{x}_{20} - C_{71} \cdot \dot{y}_{71} - KA_2 \cdot x_{14} +$$
$$(K_{71} + KA_2) \cdot x_{16} + K_{71} \cdot D_{74} \cdot x_{20} + [- KA_2 \cdot (D_{32} + L_{37}) + KA_6]$$
$$\cdot x_{22} + (- KA_2 \cdot D_{73} - KA_6) \cdot x_{24} - K_{71} \cdot y_{71} = 0 \tag{128}$$

$$M_8 \cdot \ddot{x}_{17} + C_{81} \cdot \dot{x}_{17} + C_{81} \cdot D_{84} \cdot \dot{x}_{21} - C_{81} \cdot \dot{y}_{81} - KA_4 \cdot x_{15} +$$

$$(K_{81} + KA_4) \cdot x_{17} + K_{81} \cdot D_{84} \cdot x_{21} + [KA_4 \cdot (D_{43} + L_{84}) + KA_8] \cdot$$

$$x_{23} + (KA_4 \cdot D_{82} + KA_8) \cdot x_{25} - K_{81} \cdot y_{81} = 0 \quad (129$$

$$J_{3Y} \cdot \ddot{x}_{18} + C_{31} \cdot D_{34} \cdot \dot{x}_{14} + C_{31} \cdot D_{34}^2 \cdot \dot{x}_{18} - C_{31} \cdot D_{34} \cdot \dot{y}_{31} -$$

$$KA_{13} \cdot x_5 + K_{31} \cdot D_{34} \cdot x_{14} + (K_{31} \cdot D_{34}^2 + KA_{13} + KA_{14}) \cdot x_{18} -$$

$$KA_{14} \cdot x_{20} - K_{31} \cdot D_{34} \cdot y_{31} = 0 \quad (130$$

$$J_{4Y} \cdot \ddot{x}_{19} + C_{41} \cdot D_{44} \cdot \dot{x}_{15} + C_{41} \cdot D_{44}^2 \cdot \dot{x}_{19} - C_{41} \cdot D_{44} \cdot \dot{y}_{41} -$$

$$KA_{15} \cdot x_5 + K_{41} \cdot D_{44} \cdot x_{15} + (K_{41} \cdot D_{44}^2 + KA_{15} + KA_{16}) \cdot x_{19} -$$

$$KA_{16} \cdot x_{21} - K_{41} \cdot D_{44} \cdot y_{41} = 0 \quad (131$$

$$J_{7Y} \cdot \ddot{x}_{20} + C_{71} \cdot D_{74} \cdot \dot{x}_{16} + C_{71} \cdot D_{74}^2 \cdot \dot{x}_{20} - C_{71} \cdot D_{74} \cdot \dot{y}_{71} +$$

$$K_{71} \cdot D_{74} \cdot x_{16} - KA_{14} \cdot x_{18} + (K_{71} \cdot D_{74}^2 + KA_{14}) \cdot x_{20} - K_{71} \cdot D_{74}$$

$$\cdot y_{71} = 0 \quad (132$$

$$J_{8Y} \cdot \ddot{x}_{21} + C_{81} \cdot D_{84} \cdot \dot{x}_{17} + C_{81} \cdot D_{84}^2 \cdot \dot{x}_{21} - C_{81} \cdot D_{84} \cdot \dot{y}_{81} +$$

$$K_{81} \cdot D_{84} \cdot x_{17} - KA_{16} \cdot x_{19} + (K_{81} \cdot D_{84}^2 + KA_{16}) \cdot x_{21} - K_{81} \cdot D_{84}$$

$$\cdot y_{81} = 0 \quad (133$$

$$J_{3Z} \cdot \ddot{x}_{22} + (KA_1 \cdot D_{33} + KA_5) \cdot x_1 + (KA_1 \cdot D_{33} \cdot D_{57} + D_{57} \cdot KA_5) \cdot x_5 + [(D_{56} + L_{53}) (KA_1 \cdot D_{33} + KA_5) - KA_5 \cdot D_{33} - KA_9] \cdot x_6 + [- KA_1 \cdot D_{33} + KA_2 \cdot (D_{32} + L_{37}) - KA_5 - KA_6] \cdot x_{14} + [- KA_2 \cdot (D_{32} + L_{37}) + KA_6] \cdot x_{16} + [KA_1 \cdot D_{33}^2 + KA_2 \cdot (D_{32} + L_{37})^2 + 2 \cdot KA_5 \cdot D_{33} - 2 \cdot KA_6 \cdot (D_{32} + L_{37}) + KA_9 + KA_{10}] \cdot x_{22} + [KA_2 \cdot D_{73} \cdot (D_{32} + L_{37}) + KA_6 \cdot (- D_{73} + D_{32} + L_{37}) - KA_{10}] \cdot x_{24} = 0 \quad (134$$

$$J_{4Z} \cdot \ddot{x}_{23} + (- KA_3 \cdot D_{42} - KA_7) \cdot x_1 + (- KA_3 \cdot D_{42} \cdot D_{57} - D_{57} \cdot KA_7) \cdot x_5 + [(D_{55} + L_{54}) \cdot (KA_3 \cdot D_{42} + KA_7) - KA_7 \cdot D_{42} - KA_{11}] \cdot x_6 + [KA_3 \cdot D_{42} - KA_4 \cdot (D_{43} + L_{84}) + KA_7 + KA_8] \cdot x_{15} + [KA_4 \cdot (D_{43} + L_{84}) - KA_8] \cdot x_{17} + [KA_3 \cdot D_{42}^2 + KA_4 \cdot (D_{43} + L_{84})^2 + 2 \cdot KA_7 \cdot D_{42} - 2 \cdot KA_8 \cdot (D_{43} + L_{84}) + KA_{11} + KA_{12}] \cdot x_{23} + [KA_4 \cdot D_{82} \cdot (D_{43} + L_{84}) - KA_8 \cdot (D_{82} - D_{43} - L_{84}) - KA_{12}] \cdot x_{25} = 0 \quad (135$$

$$J_{7Z} \cdot \ddot{x}_{24} + (KA_2 \cdot D_{73} + KA_6) \cdot x_{14} + (- KA_2 \cdot D_{73} - KA_6) \cdot x_{16} + [(D_{32} + L_{37}) \cdot (KA_2 \cdot D_{73} + KA_6) - KA_6 \cdot D_{73} - KA_{10}] \cdot x_{22} + (KA_2 \cdot D_{73}^2 + 2 \cdot KA_6 \cdot D_{73} + KA_{10}) \cdot x_{24} = 0 \quad (136$$

$$J_{8Z} \cdot \ddot{x}_{25} + (-KA_4 \cdot D_{82} - KA_8) \cdot x_{15} + (KA_4 \cdot D_{82} + KA_8) \cdot x_{17} + [(D_{43} + L_{84}) \cdot (KA_4 \cdot D_{82} + KA_8) - KA_8 \cdot D_{82} - KA_{12}] \cdot x_{23} + (KA_4 \cdot D_{82}^2 + 2 \cdot KA_8 \cdot D_{82} + KA_{12}) \cdot x_{25} = 0 \qquad (137$$

$$J_{3X} \cdot \ddot{x}_{26} - C_{32} \cdot D_{34} \cdot \dot{x}_2 + C_{32} \cdot D_{34} \cdot D_{57} \cdot \dot{x}_4 + C_{32} \cdot D_{34} \cdot \overline{X} \cdot \dot{x}_6 + C_{32} \cdot D_{34}^2 \cdot \dot{x}_{26} + C_{32} \cdot D_{34} \cdot \dot{y}_{32} - K_{32} \cdot D_{34} \cdot x_2 + (-KA_1 \cdot D_{33} - KA_5) \cdot x_3 + [K_{32} \cdot D_{57} \cdot D_{34} + (D_{56} + L_{53}) \cdot (KA_1 \cdot D_{33} + KA_5) - KA_5 \cdot D_{33} - KA_9] \cdot x_4 + (-KA_1 \cdot D_{33} \cdot \overline{X} - \overline{X} \cdot KA_5) \cdot x_5 + K_{32} \cdot D_{34} \cdot \overline{X} \cdot x_6 + [KA_1 \cdot D_{33} - KA_2 \cdot (D_{32} + L_{37}) + KA_5 + KA_6] \cdot x_{10} + [KA_2 \cdot (D_{32} + L_{37}) - KA_6] \cdot x_{12} + [K_{32} \cdot D_{34}^2 + KA_1 \cdot D_{33}^2 + KA_2 \cdot (D_{32} + L_{37})^2 + 2 \cdot KA_5 \cdot D_{33} - 2 \cdot KA_6 \cdot (D_{32} + L_{37}) + KA_9 + KA_{10}] \cdot x_{26} + [KA_2 \cdot (D_{32} + L_{37}) \cdot D_{73} - KA_6 \cdot (D_{73} - D_{32} - L_{37}) - KA_{10}] \cdot x_{28} + K_{32} \cdot D_{34} \cdot y_{32} = 0 \qquad (138$$

$$J_{4X} \cdot \ddot{x}_{27} - C_{42} \cdot D_{44} \cdot \dot{x}_2 + C_{42} \cdot D_{44} \cdot D_{57} \cdot \dot{x}_4 + C_{42} \cdot D_{44} \cdot \overline{X} \cdot \dot{x}_6 + C_{42} \cdot D_{44}^2 \cdot \dot{x}_{27} + C_{42} \cdot D_{44} \cdot \dot{y}_{42} - K_{42} \cdot D_{44} \cdot x_2 + (KA_3 \cdot D_{42} + KA_7) \cdot x_3 + [K_{42} \cdot D_{44} \cdot D_{57} + (D_{55} + L_{54}) \cdot (KA_3 \cdot D_{42} + KA_7) - KA_7 \cdot D_{42} - KA_{11}] \cdot x_4 + (KA_3 \cdot D_{42} \cdot \overline{X} + \overline{X} \cdot KA_7) \cdot x_5 + K_{42} \cdot$$

$$D_{44} \cdot \overline{X} \cdot x_6 + [- KA_3 \cdot D_{42} + KA_4 \cdot (D_{43} + L_{84}) - KA_7 - KA_8] \cdot x_{11} + [- KA_4 \cdot (D_{43} + L_{84}) + KA_8] \cdot x_{13} + [K_{42} \cdot D_{44}^2 + KA_3 \cdot D_{42}^2 + KA_4 \cdot (D_{43} + L_{84})^2 + 2 \cdot KA_7 \cdot D_{42} - 2 \cdot KA_8 \cdot (D_{43} + L_{84}) + KA_{11} + KA_{12}] \cdot x_{27} + [KA_4 \cdot (D_{43} + L_{84}) \cdot D_{82} - KA_8 \cdot (D_{82} - D_{43} - L_{84}) - KA_{12}] \cdot x_{29} + K_{42} \cdot D_{44} \cdot y_{42} = 0 \qquad (139$$

$$J_{7X} \cdot \ddot{x}_{28} - C_{72} \cdot D_{74} \cdot \dot{x}_2 + C_{72} \cdot D_{74} \cdot D_{57} \cdot \dot{x}_4 + C_{72} \cdot D_{74} \cdot \overline{X} \cdot \dot{x}_6 + C_{72} \cdot D_{74}^2 \cdot \dot{x}_{28} + C_{72} \cdot D_{74} \cdot \dot{y}_{72} - K_{72} \cdot D_{74} \cdot x_2 + K_{72} \cdot D_{74} \cdot D_{57} \cdot x_4 + K_{72} \cdot D_{74} \cdot \overline{X} \cdot x_6 + (- KA_2 \cdot D_{73} - KA_6) \cdot x_{10} + (KA_2 \cdot D_{73} + KA_6) \cdot x_{12} + [KA_2 \cdot D_{73} \cdot (D_{32} + L_{37}) + KA_6 \cdot (D_{32} + L_{37} - D_{73}) - KA_{10}] \cdot x_{26} + (K_{72} \cdot D_{74}^2 + KA_2 \cdot D_{73}^2 + 2 \cdot KA_6 \cdot D_{73} + KA_{10}) \cdot x_{28} + K_{72} \cdot D_{74} \cdot y_{72} = 0 \qquad (140$$

$$J_{8X} \cdot \ddot{x}_{29} - C_{82} \cdot D_{84} \cdot \dot{x}_2 + C_{82} \cdot D_{84} \cdot D_{57} \cdot \dot{x}_4 + C_{82} \cdot D_{84} \cdot \overline{X} \cdot \dot{x}_6 + C_{82} \cdot D_{84}^2 \cdot \dot{x}_{29} + C_{82} \cdot D_{84} \cdot \dot{y}_{82} - K_{82} \cdot D_{84} \cdot x_2 + K_{82} \cdot D_{84} \cdot D_{57} \cdot x_4 + K_{82} \cdot D_{84} \cdot \overline{X} \cdot x_6 + (KA_4 \cdot D_{82} + KA_8) \cdot x_{11} + (- KA_4 \cdot D_{82} - KA_8) \cdot x_{13} + [KA_4 \cdot D_{82} \cdot (D_{43} + L_{84}) + KA_8 \cdot (D_{43} + L_{84} - D_{82}) - KA_{12}] \cdot x_{27} + (K_{82} \cdot D_{84}^2 + KA_4 \cdot D_{82}^2 + 2 \cdot KA_8 \cdot D_{82} + KA_{12}) \cdot x_{29} + K_{82} \cdot D_{84} \cdot y_{82} = 0 \qquad (141$$

where

$$
\begin{aligned}
KA_1 &= 12EI/L_{53}{}^3 & KA_9 &= 4EI/L_{53} \\
KA_2 &= 12EI/L_{37}{}^3 & KA_{10} &= 4EI/L_{37} \\
KA_3 &= 12EI/L_{54}{}^3 & KA_{11} &= 4EI/L_{54} \\
KA_4 &= 12EI/L_{84}{}^3 & KA_{12} &= 4EI/L_{84} \\
KA_5 &= 6EI/L_{53}{}^2 & KA_{13} &= JG/L_{53} \\
KA_6 &= 6EI/L_{37}{}^2 & KA_{14} &= JG/L_{37} \\
KA_7 &= 6EI/L_{54}{}^2 & KA_{15} &= JG/L_{54} \\
KA_8 &= 6EI/L_{84}{}^2 & KA_{16} &= JG/L_{84}
\end{aligned}
\qquad (142\ a\text{-}p
$$

Equations (113) through (141) may be cast into the matrix form

$$
\left[\ A\ \right]\left\{\ddot{x}\right\} + \left[\ B\ \right]\left\{\dot{x}\right\} + \left[\ C\ \right]\left\{x\right\} = \left[\ D\ \right]\left\{\begin{matrix} y \\ \dot{y} \\ FDB \end{matrix}\right\} \qquad (143
$$

Equation (143) may be rearranged to the form

$$
\left[\ A\ \right]\left\{\ddot{x}\right\} = \left[\ CBD\ \right]\left\{\begin{matrix} x \\ \dot{x} \\ y \\ \dot{y} \\ FDB \end{matrix}\right\} \qquad (144
$$

This equation is now in a form which allows the Gauss-Jordan method to be used. Upon reduction, equation (144) becomes

$$
\left[\ I\ \right]\left\{\ddot{x}\right\} = \left[\ CBD'\ \right]\left\{\begin{matrix} x \\ \dot{x} \\ y \\ \dot{y} \\ FDB \end{matrix}\right\} \qquad (145
$$

The matrix CBD' is identially the matrix G in equation (101), allowing equation (145) to be rewritten as

$$
\left[\ I\ \right]\left\{\ddot{x}\right\} = \left[\ G\ \right]\left\{\begin{matrix} x \\ \dot{x} \\ y \\ \dot{y} \\ FDB \end{matrix}\right\} \qquad (146
$$

VI. SOLUTION OF THE EQUATIONS OF MOTION

The equations of motion are to be integrated numerically using the fourth order Runge-Kutta-Gill method. This method is used to integrate first order differential equations with known initial conditions. The equations of motion for this system are second order differential equations which may be transformed into a set of fifty-eight first order equations by the transformation

$$\dot{x}_i = x_{i+29} \qquad i = 1,2,3 \ldots 29 \tag{147}$$

Equations (147) and (100) or (146) constitute the set of fifty-eight equations which are combined in the single matrix equation

$$\left[\begin{matrix} I \\ 58 \times 58 \end{matrix}\right] \begin{Bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_{58} \end{Bmatrix} = \left[\begin{matrix} \left[\begin{matrix} 0 \\ 29 \times 29 \end{matrix}\right] \left[\begin{matrix} I \\ 29 \times 29 \end{matrix}\right] \left[\begin{matrix} 0 \\ 29 \times 29 \end{matrix}\right] \\ G \\ 29 \times 97 \end{matrix}\right] \begin{Bmatrix} x_1 \\ \vdots \\ x_{58} \\ y_1 \\ \vdots \\ FDB_3 \end{Bmatrix} \tag{148}$$

This equation is now in a form which may be integrated to give the displacement and velocity of each of the twenty-nine x's as functions of time. Details of the Runge-Kutta-Gill numerical integration process are outlined in reference [10].

It should be noted that a vector x_o is needed to start the Runge-Kutta-Gill procedure. The first twenty-nine elements of this vector are initial displacements, and the last twenty-nine are initial velocities.

VII. THE COMPUTER PROGRAM

The computer program to simulate the motion of a tractor is broken into two parts. The first part compiles the various tractor parameters, formulates the equations that govern the system, and reduces them to the twenty-nine differential equations of motion. The second part integrates the equations derived in Part I.

A. Part I

Programs to derive the equations of motion are developed using both the method of section VA, and the method of section VB. Both programs use the same tractor parameters, and input them in the same manner. These parameters are used to evaluate the coefficients multiplying each variable.

A list of the parameters needed, and the order in which they are read into the computer is shown in Table I. It is important to note that all distances are positive according to Figure 6. For example, the center of gravity of each wheel is assumed to be to the left of the geometric center of the wheel. If the center of gravity is actually to the right of the geometric center, that distance would be input as a negative value.

TABLE I

ORDER IN WHICH THE TRACTOR PARAMETERS ARE INPUT TO THE COMPUTER

Card	Parameters
1	M_1, J_{1X}, J_{1Y}, J_{1Z}, D_{13}, D_{14}
2	M_2, J_{2X}, J_{2Y}, J_{2Z}, D_{22}, D_{24}
3	M_3, J_{3X}, J_{3Y}, J_{3Z}, D_{31}, D_{32}, D_{33}, D_{34}
4	M_4, J_{4X}, J_{4Y}, J_{4Z}, D_{41}, D_{42}, D_{43}, D_{44}
5	M_5, J_{5X}, J_{5Y}, J_{5Z}, D_{51}, D_{53}, D_{55}, D_{56}
6	D_{57}
7	M_6, J_{6X}, J_{6Y}, J_{6Z}, D_{62}, D_{63}, D_{64}, D_{99}
8	M_7, J_{7X}, J_{7Y}, J_{7Z}, D_{71}, D_{73}, D_{74}
9	M_8, J_{8X}, J_{8Y}, J_{8Z}, D_{81}, D_{82}, D_{84}
10	L_{37}, L_{53}, L_{54}, L_{84}
11	EI*, JG*
12	$\overline{X}$, $\overline{Y}$, $\overline{Z}$, DB_1, DB_3
13	K_{11}, K_{12}, K_{13}, K_{21}, K_{22}, K_{23}, K_{31}, K_{32}
14	K_{33}, K_{41}, K_{42}, K_{43}, K_{71}, K_{72}, K_{73}, K_{81}
15	K_{82}, K_{83}
16	C_{11}, C_{12}, C_{13}, C_{21}, C_{22}, C_{23}, C_{31}, C_{32}
17	C_{33}, C_{41}, C_{42}, C_{43}, C_{71}, C_{72}, C_{73}, C_{81}
18	C_{82}, C_{83}

* These values are input in E format; all others are F format.

a. A Program Using the Method of Section VA

In this program, the coefficients are arranged in five matrices as in equation (97). These five matrices are cast into a single matrix E where

$$\begin{bmatrix} E \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} F \end{bmatrix} \begin{bmatrix} -A \end{bmatrix} \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} D \end{bmatrix} \end{bmatrix} \qquad (149$$

E is s coefficient matrix which multiplies a vector consisting of the variables, as shown in Table II.

The E matrix is now passed into the Gauss-Jordan subroutine which reduces it to the equations of motion. The last one hundred twenty-six columns of the last twenty-nine rows of the reduced E matrix sre the actual equations of motion. These rows and columns are put into a matrix G which may be printed out, or stored on magnetic disk for use in the integration program. A listing of this program, labeled TRA, is in Appendix A.

b. A Program Using the Method of Section VB

This program arranges the coefficients in four matrices as in equation (143). These matrices are cast into a matrix G where

$$\begin{bmatrix} G \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -A \end{bmatrix} \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} -D \end{bmatrix} \end{bmatrix} \qquad (150$$

This matrix is passed to the Gauss-Jordan subroutine for reduction. No further manipulation of this matrix is necessary, as it contains only the equations of motion. This G matrix is identical to that of TRA, and may also be printed out or stored on disk for later use. A listing of this program, labeled TRB, is also in Appendix A.

TABLE II

ORDER OF THE VARIABLES IN THE VECTOR MULTIPLYING E

Element	Variable	Element	Variable	Element	Variable
1	F_{11}	23	F_{42}	45	T_{35}
2	F_{12}	24	F_{43}	46	T_{36}
3	F_{13}	25	F_{44}	47	T_{37}
4	F_{17}	26	F_{45}	48	T_{38}
5	F_{18}	27	F_{46}	49	T_{39}
6	F_{19}	28	F_{47}	50	T_{44}
7	F_{21}	29	F_{48}	51	T_{45}
8	F_{22}	30	F_{49}	52	T_{46}
9	F_{23}	31	F_{51}	53	T_{47}
10	F_{24}	32	F_{52}	54	T_{48}
11	F_{25}	33	F_{53}	55	T_{49}
12	F_{26}	34	F_{71}	56	T_{52}
13	F_{31}	35	F_{72}	57	T_{53}
14	F_{32}	36	F_{73}	58	$\ddot{x}_1$
15	F_{33}	37	F_{81}	59	$\ddot{x}_2$
16	F_{34}	38	F_{82}	60	$\ddot{x}_3$
17	F_{35}	39	F_{83}	61	$\ddot{x}_4$
18	F_{36}	40	T_{17}	62	$\ddot{x}_5$
19	F_{37}	41	T_{19}	63	$\ddot{x}_6$
20	F_{38}	42	T_{24}	64	$\ddot{x}_7$
21	F_{39}	43	T_{26}	65	$\ddot{x}_8$
22	F_{41}	44	T_{34}	66	$\ddot{x}_9$

TABLE II (Continued)

Element	Variable	Element	Variable	Element	Variable
67	$\ddot{x}_{10}$	89	x_3	111	x_{25}
68	$\ddot{x}_{11}$	90	x_4	112	x_{26}
69	$\ddot{x}_{12}$	91	x_5	113	x_{27}
70	$\ddot{x}_{13}$	92	x_6	114	x_{28}
71	$\ddot{x}_{14}$	93	x_7	115	x_{29}
72	$\ddot{x}_{15}$	94	x_8	116	$\dot{x}_1$
73	$\ddot{x}_{16}$	95	x_9	117	$\dot{x}_2$
74	$\ddot{x}_{17}$	96	x_{10}	118	$\dot{x}_3$
75	$\ddot{x}_{18}$	97	x_{11}	119	$\dot{x}_4$
76	$\ddot{x}_{19}$	98	x_{12}	120	$\dot{x}_5$
77	$\ddot{x}_{20}$	99	x_{13}	121	$\dot{x}_6$
78	$\ddot{x}_{21}$	100	x_{14}	122	$\dot{x}_7$
79	$\ddot{x}_{22}$	101	x_{15}	123	$\dot{x}_8$
80	$\ddot{x}_{23}$	102	x_{16}	124	$\dot{x}_9$
81	$\ddot{x}_{24}$	103	x_{17}	125	$\dot{x}_{10}$
82	$\ddot{x}_{25}$	104	x_{18}	126	$\dot{x}_{11}$
83	$\ddot{x}_{26}$	105	x_{19}	127	$\dot{x}_{12}$
84	$\ddot{x}_{27}$	106	x_{20}	128	$\dot{x}_{13}$
85	$\ddot{x}_{28}$	107	x_{21}	129	$\dot{x}_{14}$
86	$\ddot{x}_{29}$	108	x_{22}	130	$\dot{x}_{15}$
87	x_1	109	x_{23}	131	$\dot{x}_{16}$
88	x_2	110	x_{24}	132	$\dot{x}_{17}$

TABLE II (Continued)

Element	Variable	Element	Variable	Element	Variable
133	$\dot{x}_{18}$	150	y_{13}	167	$\dot{y}_{12}$
134	$\dot{x}_{19}$	151	y_{21}	168	$\dot{y}_{13}$
135	$\dot{x}_{20}$	152	y_{22}	169	$\dot{y}_{21}$
136	$\dot{x}_{21}$	153	y_{23}	170	$\dot{y}_{22}$
137	$\dot{x}_{22}$	154	y_{31}	171	$\dot{y}_{23}$
138	$\dot{x}_{23}$	155	y_{32}	172	$\dot{y}_{31}$
139	$\dot{x}_{24}$	156	y_{33}	173	$\dot{y}_{32}$
140	$\dot{x}_{25}$	157	y_{41}	174	$\dot{y}_{33}$
141	$\dot{x}_{26}$	158	y_{42}	175	$\dot{y}_{41}$
142	$\dot{x}_{27}$	159	y_{43}	176	$\dot{y}_{42}$
143	$\dot{x}_{28}$	160	y_{71}	177	$\dot{y}_{43}$
144	$\dot{x}_{29}$	161	y_{72}	178	$\dot{y}_{71}$
145	FDB_1	162	y_{73}	179	$\dot{y}_{72}$
146	FDB_2	163	y_{81}	180	$\dot{y}_{73}$
147	FDB_3	164	y_{82}	181	$\dot{y}_{81}$
148	y_{11}	165	y_{83}	182	$\dot{y}_{82}$
149	y_{12}	166	$\dot{y}_{11}$	183	$\dot{y}_{83}$

B. Part II

Integration of the equations of motion requires the use of the G matrix from Part I, and that the initial displacements and velocities of tbe system are known. The G matrix is read from the disk on which it was stored. The assumed initial conditions, step size, and number of steps to be taken are input via the card reader. The order in which these quantities are read is shown in Table III.

The last ninety-seven columns of G are used in a new matrix H which takes on the form of the right hand side of equation (148). The matrix H and the initial conditions are transferred to the Runge-Kutta-Gill subroutine. This subroutine, along with a derivative subroutine, integrates the equations for the prescribed number of steps. The resulting displacements and velocities are either printed out or stored on magnetic disk for later use. Listings of the integration routine, Runge-Kutta-Gill subroutine, and derivative subroutine are in Appendix A.

The y_i's and $\dot{y}_i$'s are defined in the derivative subroutine. For motion over a hard flat surface, all y's and $\dot{y}$'s are set to zero. If, however, the front wheels see a sine wave in the Z direction, all y's and $\dot{y}$'s are zero except y_{13}, $\dot{y}_{13}$, y_{23} and $\dot{y}_{23}$. The expressions for these variables will be

$$y_{13} = y_{23} = \alpha \sin \beta t \qquad \text{and} \qquad \dot{y}_{13} = \dot{y}_{23} = \alpha\beta \cos \beta t \qquad \text{(151 a,b}$$

where α and β are some constants.

The drawbar loads are also defined in the derivative subroutine. If there is no drawbar load, all FDB's are set to zero. If, for example, the load in the Y direction is constant, and the load in the X direction varies sinusoidally, the drawbar forces would be input as

TABLE III

ORDER IN WHICH THE INITIAL CONDITIONS ARE INPUT TO THE COMPUTER

Card	Parameters
1	x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8
2	x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}
3	x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}
4	x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, $\dot{x}_1$, $\dot{x}_2$, $\dot{x}_3$
5	$\dot{x}_4$, $\dot{x}_5$, $\dot{x}_6$, $\dot{x}_7$, $\dot{x}_8$, $\dot{x}_9$, $\dot{x}_{10}$, $\dot{x}_{11}$
6	$\dot{x}_{12}$, $\dot{x}_{13}$, $\dot{x}_{14}$, $\dot{x}_{15}$, $\dot{x}_{16}$, $\dot{x}_{17}$, $\dot{x}_{18}$, $\dot{x}_{19}$
7	$\dot{x}_{20}$, $\dot{x}_{21}$, $\dot{x}_{22}$, $\dot{x}_{23}$, $\dot{x}_{24}$, $\dot{x}_{25}$, $\dot{x}_{26}$, $\dot{x}_{27}$
8	$\dot{x}_{28}$, $\dot{x}_{29}$
9	DELT (Time increment)
10	NSTEP* (Number of steps to be taken)

* This value is input in I3 format, while all other quantities are F10.4 format.

$$FDB_1 = \alpha \sin \beta t \quad , \quad FDB_2 = \gamma \quad \text{and } FDB_3 = 0 \qquad (152\ a,b,c$$

where α, β and γ are constants.

The values of the FDB's, y's and $\dot{y}$'s are arranged in a vector Y. The order of these functions in the Y vector are given in Table IV. Equations (151 a) and (151 b) are input into the derivative subroutine as

$$\begin{aligned} Y(6) &= \alpha \sin \beta t \quad , \quad Y(9) = \alpha \sin \beta t \\ Y(24) &= \alpha\beta \cos \beta t \quad , \quad Y(27) = \alpha\beta \cos \beta t \end{aligned} \qquad (153\ a,b,c,d$$

Y is initialized in the main program, eliminating the need to define it in the derivative subroutine when simulating motion over a hard flat surface with no drawbar load.

TABLE IV

ORDER OF THE FORCING FUNCTIONS IN THE VECTOR Y

Y(1) = FDB_1	Y(14) = y_{42}	Y(27) = $\dot{y}_{23}$
Y(2) = FDB_2	Y(15) = y_{43}	Y(28) = $\dot{y}_{31}$
Y(3) = FDB_3	Y(16) = y_{71}	Y(29) = $\dot{y}_{32}$
Y(4) = y_{11}	Y(17) = y_{72}	Y(30) = $\dot{y}_{33}$
Y(5) = y_{12}	Y(18) = y_{73}	Y(31) = $\dot{y}_{41}$
Y(6) = y_{13}	Y(19) = y_{81}	Y(32) = $\dot{y}_{42}$
Y(7) = y_{21}	Y(20) = y_{82}	Y(33) = $\dot{y}_{43}$
Y(8) = y_{22}	Y(21) = y_{83}	Y(34) = $\dot{y}_{71}$
Y(9) = y_{23}	Y(22) = $\dot{y}_{11}$	Y(35) = $\dot{y}_{72}$
Y(10) = y_{31}	Y(23) = $\dot{y}_{12}$	Y(36) = $\dot{y}_{73}$
Y(11) = y_{32}	Y(24) = $\dot{y}_{13}$	Y(37) = $\dot{y}_{81}$
Y(12) = y_{33}	Y(25) = $\dot{y}_{21}$	Y(38) = $\dot{y}_{82}$
Y(13) = y_{41}	Y(26) = $\dot{y}_{22}$	Y(39) = $\dot{y}_{83}$

VIII. PRESENTATION OF RESULTS

To demonstrate the programs, the motion of a tractor rolling over a half sinusoid bump with its left rear wheels is simulated. The parameters for a hypothetical tractor are listed in Table V. These parameters are input into TRA or TRB which derive the equations of motion for this tractor. A listing of the computer generated equations of motion for this system is given in Appendix C. The equations generated by TRA are identical to those generated by TRB; therefore, only one set of equations is listed in the appendix.

The bump is input via the Y vector in the derivative subroutine. Since only the left rear wheels traverse the bump, y_{33}, $\dot{y}_{33}$, y_{73} and $\dot{y}_{73}$ are the only non-zero elements of Y. The equation of a half sinusoid bump such as that shown in Figure 9 is

$$y = h \sin (\pi vt/1) \tag{154}$$

where h is the height of the bump, 1 is the length of the bump, and v is the forward velocity of the tractor.

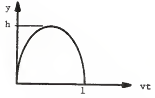


FIGURE 9

THE BUMP BEING TRAVERSED BY THE TRACTOR

TABLE V

LIST OF PARAMETERS FOR A HYPOTHETICAL TRACTOR

Mass (lb-sec/ft and Inertia (lb-ft/sec^2) Values			
M_1 = 1.83	J_{1X} = 0.57	J_{1Y} = 1.14	J_{1Z} = 0.57
M_2 = 1.83	J_{2X} = 0.57	J_{2Y} = 1.14	J_{2Z} = 0.57
M_3 = 11.33	J_{3X} = 11.33	J_{3Y} = 22.66	J_{3Z} = 11.33
M_4 = 11.33	J_{4X} = 11.33	J_{4Y} = 22.66	J_{4Z} = 11.33
M_5 = 128	J_{5X} = 375	J_{5Y} = 900	J_{5Z} = 1050
M_6 = 5.1	J_{6X} = 9.0	J_{6Y} = 2.0	J_{6Z} = 10.0
M_7 = 11.33	J_{7X} = 11.33	J_{7Y} = 22.66	J_{7Z} = 11.33
M_8 = 11.33	J_{8X} = 11.33	J_{8Y} = 22.66	J_{8Z} = 11.33

Dimensions (ft)				
D_{13} = 0.25	D_{14} = 1.12			
D_{22} = 0.25	D_{24} = 1.12			
D_{31} = 0.0	D_{32} = 0.33	D_{33} = 0.33	D_{34} = 2.00	
D_{41} = 0.0	D_{42} = 0.33	D_{43} = 0.33	D_{44} = 2.00	
D_{51} = 3.67	D_{53} = 1.55	D_{55} = 2.83	D_{56} = 2.83	D_{57} = 0.07
D_{62} = 0.0	D_{63} = 0.0	D_{64} = 0.0	D_{99} = 2.21	
D_{71} = 0.0	D_{73} = 0.33	D_{74} = 2.00		
D_{81} = 0.0	D_{82} = 0.33	D_{84} = 2.00		
L_{37} = 1.50	L_{53} = 2.00	L_{54} = 2.00	L_{84} = 1.50	
$\overline{X}$ = 2.33	$\overline{Y}$ = 0.0	$\overline{Z}$ = 2.67	DB_1 = 2.50	DB_3 = 0.67

TABLE V (Continued)

Spring Rates (lb/ft)		
K_{11} = 16,000.0	K_{12} = 10,700.0	K_{13} = 22,600.0
K_{21} = 16,000.0	K_{22} = 10,700.0	K_{23} = 22,600.0
K_{31} = 18,000.0	K_{32} = 11,900.0	K_{33} = 20,500.0
K_{41} = 18,000.0	K_{42} = 11,900.0	K_{43} = 20,500.0
K_{71} = 18,000.0	K_{72} = 11,900.0	K_{73} = 20,500.0
K_{81} = 18,000.0	K_{82} = 11,900.0	K_{83} = 20,500.0

Damping Coefficients (lb-sec/ft)		
C_{11} = 88.0	C_{12} = 25.0	C_{13} = 186.0
C_{21} = 88.0	C_{22} = 25.0	C_{23} = 186.0
C_{31} = 134.0	C_{32} = 32.0	C_{33} = 248.0
C_{41} = 134.0	C_{42} = 32.0	C_{43} = 248.0
C_{71} = 134.0	C_{72} = 32.0	C_{73} = 248.0
C_{81} = 134.0	C_{82} = 32.0	C_{83} = 248.0

Material Properties	
$EI = 1.33 \times 10^6$	$JG = 1.06 \times 10^6$

Using equation (153), the expressions for y_{33} and y_{73} are then

$$Y(12) = Y(18) = y_{33} = y_{73} = -.416 \sin (4.61t) \qquad (155$$

and $\dot{y}_{33}$ and $\dot{y}_{73}$ are given by

$$Y(30) = Y(36) = \dot{y}_{33} = \dot{y}_{73} = -1.914 \cos (4.61t) \qquad (156$$

for the interval $0 \leq t \leq .6815$, and are zero otherwise. These values are negative sinze Z is positive downward. These are the expressions for Y used in the listing of the derivative subroutine in Appendix A.

The assumed initial conditions are such that the tractor is initially at rest, that is all velocities and displacements are zero. A step size of .001 sec. is used, and 1,000 steps are taken. The results of every twentieth step are listed in Appendix C.

To compare the results of this model with those of a model having rigid axles, the programs are run again using the parameter listed in Table V, except for the values of EI and JG, which are increased by a factor of ten. Figures 10 and 11 show the verticle deflections of the centers of gravity of the left rear wheels, for both the soft and stiff cases.

FIGURE 10

MOTION OF THE CENTER OF GRAVITY OF THE INSIDE LEFT REAR WHEEL

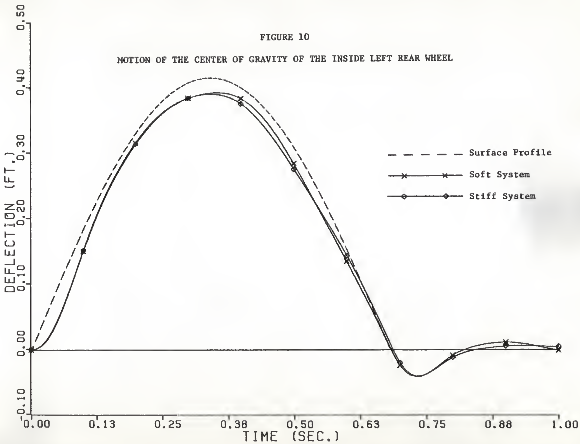
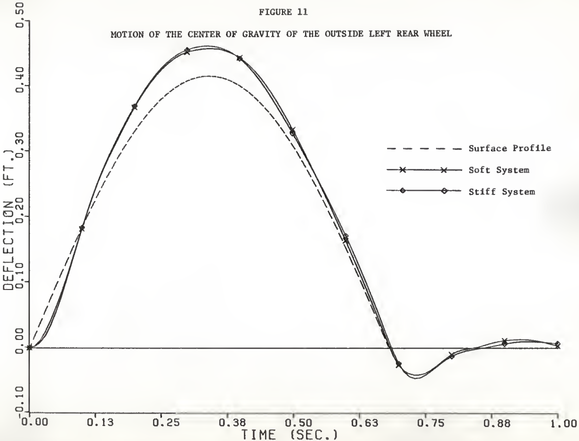


FIGURE 11

MOTION OF THE CENTER OF GRAVITY OF THE OUTSIDE LEFT REAR WHEEL



IX. DISCUSSION AND CONCLUSIONS

As previously mentioned, the equations generated by TRA and TRB are identical, which implies that they are in fact the correct equations of motion for the system provided the original assumptions are valid. Both of these programs have certain advantages as well as drawbacks.

TRA is relatively expensive to use, since it involves the reduction of an eighty-six by one hundred eighty-three matrix. To compile this program, execute it, and print the equations of motion on both paper and magnetic disk requires approximately fifty-five seconds of CPU time on the Itel AS/5 computer. The first fifty-seven rows of the reduced E matrix, in this program, are expressions for the forces and moments acting on the system in terms of x's, $\dot{x}$'s and the forcing functions. These expressions may be used to evaluate the force acting on an axle, for example, at some instant in time.

When simulating a tractor with very stiff axles, TRA requires some equations to be normalized with respect to a very small quantity. This introduces some round off error which may be multiplied through the reduction process. The result is a few unwanted variables being multiplied by quantities of the order of 10^1 or less, while the correct variables are being multiplied by coefficients of order 10^5 or better. These unwanted variables have negligable effect on the integration of the equations of motion.

TRB requires the reduction of a twenty-nine by one hundred twenty-six matrix, which makes it much faster to execute than TRA. To compile, execute, and print the equations of motion with this program requires about twenty-three seconds of CPU time, less than half that of TRA.

This program does not yield expressions for forces or moments acting on the system, but if these expressions are not needed, the savings in CPU time is substantial. Round off and inherited error are also reduced with this program, since it requires fewer mathematical operations.

The CPU time required to execute the integration routine depends on the number of steps taken, which depends on the time increment used. The size of the time increment is not completely arbitrary. Too large a step size will cause the system of equations to oscillate wildly or blow up, and too small a step size requires that excessively many steps be taken. The proper step size must be determined by trial and error. As a rule of thumb, stiffer axles dictate a smaller step size. CPU time may also be saved by not printing values after every step.

For a given system, the amount of time needed to perform a step does not depend on the step size to any noticeable degree. For the example of section VIII, one thousand steps with a step size of .001 sec., printing every tenth step, required approximately twenty-five seconds of computer time.

In deriving the equations of motion by the energy method, it should be noted that the potential energy in the deflected axles is derived by the method given in Appendix B. Most of the previous models have had rigid axles which eliminate this energy term altogether. Mather's model had elastic axles, but his expression for the potential energy of a deflected section of the axle was

$$U = \tfrac{1}{2}\,(3EI/L^3)\;x_1^{\,2} \qquad (157$$

This implies no moment is applied to the rear axle, which is not really a valid assumption. Use of equation (157) produced a set of equations that were radically different from those generated using equation (B 12).

Figures 10 and 11 show that there is noticeable difference between this model and models that treat the rear axles as rigid members. Another simulation was attempted in which the parameters of Table V were used, except that EI and JG were multiplied by one hundred. The resulting system was so stiff that the step size needed to successfully integrate the equations of motion was prohibitively small.

By allowing the rear axles to deflect, the stress in an axle for a given deflection could be determined. The knowledge of this stress level would permit one to predict when the axle would fail.

In conclusion, the computer programs presented here are a very general base, with which most any dual rear wheeled tractor can be simulated. The user is allowed a good deal of freedom in selecting trsctor parameters, and an unlimited variety of forcing functions via the subroutine DERIV. Parameters and initial conditions are easily varied to make comparisons between different systems or different situations.

This work leads to a vast quantity of further research. Among this research is experimental verification of this model, extension of the computer program to determine stress levels in the rear axles, and consideration of the tires as nonlinesr springs.

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APPENDIX A

LISTINGS OF THE COMPUTER PROGRAMS

```
C     THIS PROGRAM GENERATES THE EQUATIONS OF MOTION USING THE EQUATIONS
C     DEVELOPED IN SECTION 5A.
C
      IMPLICIT REAL(A-Z)                                                TRA   1
      INTEGER I,J,K,N,N1                                                TRA   2
      DIMENSION A(86,29)                                                TRA   3
      DIMENSION B(86,29)                                                TRA   4
      DIMENSION C(86,29)                                                TRA   5
      DIMENSION D(86,39)                                                TRA   6
      DIMENSION E(86,183)                                               TRA   7
      DIMENSION F(86,57)                                                TRA   8
      DIMENSION G(29,126)                                               TRA   9
      DIMENSION NAME(126)                                               TRA  10
      REWIND 10                                                         TRA  11
C
C     THE NEXT 15 CARDS READ IN THE VARIOUS TRACTOR PARAMETERS.
C
      READ (5,180) M1,J1X,J1Y,J1Z,D13,D14                               TRA  12
      READ (5,180) M2,J2X,J2Y,J2Z,D22,D24                               TRA  13
      READ (5,180) M3,J3X,J3Y,J3Z,D31,D32,D33,D34                       TRA  14
      READ (5,180) M4,J4X,J4Y,J4Z,D41,D42,D43,D44                       TRA  15
      READ (5,180) M5,J5X,J5Y,J5Z,D51,D53,D55,D56,D57                   TRA  16
      READ (5,180) M6,J6X,J6Y,J6Z,D62,D63,D64,D99                       TRA  17
      READ (5,180) M7,J7X,J7Y,J7Z,D71,D73,C74                           TRA  18
      READ (5,180) M8,J8X,J8Y,J8Z,D81,D82,D84                           TRA  19
      READ (5,180) L37,L53,L54,L84                                      TRA  20
      READ (5,200) EI,GJ                                                TRA  21
      READ (5,180) XBAR,YBAR,ZBAR,DB1,DB3                               TRA  22
      READ (5,180) K11,K12,K13,K21,K22,K23,K31,K32,K33,K41,K42,K43,K71,KTRA  23
     *72,K73,K81,K82,K83                                                TRA  24
      READ (5,180) C11,C12,C13,C21,C22,C23,C31,C32,C33,C41,C42,C43,C71,CTRA  25
     *72,C73,C81,C82,C83                                                TRA  26
      WRITE (6,180) M1,J1X,J1Y,J1Z,D13,D14                              TRA  27
      WRITE (6,180) M2,J2X,J2Y,J2Z,D22,D24                              TRA  28
      WRITE (6,180) M3,J3X,J3Y,J3Z,D31,D32,D33,D34                      TRA  29
      WRITE (6,180) M4,J4X,J4Y,J4Z,D41,D42,D43,D44                      TRA  30
      WRITE (6,180) M5,J5X,J5Y,J5Z,D51,D53,D55,D56,D57                  TRA  31
      WRITE (6,180) M6,J6X,J6Y,J6Z,D62,D63,D64,D99                      TRA  32
      WRITE (6,180) M7,J7X,J7Y,J7Z,D71,D73,D74                          TRA  33
      WRITE (6,180) M8,J8X,J8Y,J8Z,D81,D82,D84                          TRA  34
      WRITE (6,180) L37,L53,L54,L84                                     TRA  35
      WRITE (6,200) EI,GJ                                               TRA  36
      WRITE (6,180) XBAR,YBAR,ZBAR,DB1,DB3                              TRA  37
      WRITE (6,180) K11,K12,K13,K21,K22,K23,K31,K32,K33,K41,K42,K43,K71,TRA  38
     *K72,K73,K81,K82,K83                                               TRA  39
      WRITE (6,180) C11,C12,C13,C21,C22,C23,C31,C32,C33,C41,C42,C43,C71,TRA  40
     *C72,C73,C81,C82,C83                                               TRA  41
      READ (5,170) (NAME(I),I=1,126)                                    TRA  42
C
C     N IS THE NUMBER OF ROWS AND N1 THE NUMBER OF COLUMNS OF THE MATRIX E.
C
      N = 86                                                            TRA  43
      N1 = 183                                                          TRA  44
C
C     THE NEXT 11 CARDS INITIALIZE THE A,B,C,D AND F MATRICIES.
C
      DO 40 I=1,86                                                      TRA  45
      DO 10 J=1,29                                                      TRA  46
      A(I,J) = 0.0                                                      TRA  47
      B(I,J) = 0.0                                                      TRA  48
```

```
      C(I,J) = 0.0
   10 CCNTINUE
      DO 20 J=1,57
   20 F(I,J) = 0.0
      DO 30 J=1,39
   30 D(I,J) = 0.0
   40 CCNTINUE
C
C     THE NEXT 79 CAROS EVALUATE THE NONZERO ELEMENTS CF THE MATRIX A.
C
      A(39,1) = M1
      A(39,5) = M1*(053+062)
      A(39,6) = M1*(099+013+Y8AR)
      A(40,2) = M1
      A(40,4) = -M1*053
      A(40,6) = M1*051
      A(40,7) = -M1*062
      A(41,3) = M1
      A(41,4) = -M1*Y8AR
      A(41,5) = -M1*051
      A(41,7) = -M1*(C99+013)
      A(42,7) = J1X
      A(43,8) = J1Y
      A(44,6) = J1Z
      A(45,1) = M2
      A(45,5) = M2*(053+062).
      A(45,6) = -M2*(D99+022-Y8AR)
      A(46,2) = M2
      A(46,4) = -M2*D53
      A(46,6) = M2*051
      A(46,7) = -M2*062
      A(47,3) = M2
      A(47,4) = -M2*Y8AR
      A(47,5) = -M2*051
      A(47,7) = M2*(D99+022)
      A(48,7) = J2X
      A(49,9) = J2Y
      A(50,6) = J2Z
      A(51,14) = M3
      A(52,2) = M3
      A(52,4) = -M3*057
      A(52,6) = -M3*X8AR
      A(53,10) = M3
      A(54,26) = J3X
      A(55,18) = J3Y
      A(56,22) = J3Z
      A(57,15) = M4
      A(58,2) = M4
      A(58,4) = -M4*057
      A(58,6) = -M4*X8AR
      A(59,11) = M4
      A(60,27) = J4X
      A(61,19) = J4Y
      A(62,23) = J4Z
      A(63,1) = M5
      A(64,2) = M5
      A(65,3) = M5
      A(66,4) = J5X
      A(67,5) = J5Y
      A(68,6) = J5Z
```

```
      A(69,1) = M6                                                      TRA 106
      A(69,5) = M6*(D53+D63)                                            TRA 107
      A(69,6) = M6*YBAR                                                 TRA 108
      A(70,2) = M6                                                      TRA 109
      A(70,4) = -M6*D53                                                 TRA 110
      A(70,6) = M6*D51                                                  TRA 111
      A(70,7) = -M6*D63                                                 TRA 112
      A(71,3) = M6                                                      TRA 113
      A(71,4) = -M6*YBAR                                                TRA 114
      A(71,5) = -M6*D51                                                 TRA 115
      A(72,7) = J6X                                                     TRA 116
      A(73,5) = J6Y                                                     TRA 117
      A(74,6) = J6Z                                                     TRA 118
      A(75,16) = M7                                                     TRA 119
      A(76,2) = M7                                                      TRA 120
      A(76,4) = -M7*D57                                                 TRA 121
      A(76,6) = -M7*XBAR                                                TRA 122
      A(77,12) = M7                                                     TRA 123
      A(78,28) = J7X                                                    TRA 124
      A(79,20) = J7Y                                                    TRA 125
      A(80,24) = J7Z                                                    TRA 126
      A(81,17) = M8                                                     TRA 127
      A(82,2) = M8                                                      TRA 128
      A(82,4) = -M8*D57                                                 TRA 129
      A(82,6) = -M8*XBAR                                                TRA 130
      A(83,13) = M8                                                     TRA 131
      A(84,29) = J8X                                                    TRA 132
      A(85,21) = J8Y                                                    TRA 133
      A(86,25) = J8Z                                                    TRA 134
C
C     THE NEXT 52 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX B.
C
      B(21,1) = C11                                                     TRA 135
      B(21,5) = C11*(D53+D62)                                           TRA 136
      B(21,6) = C11*(D99+D13+YBAR)                                      TRA 137
      B(21,8) = C11*D14                                                 TRA 138
      B(22,2) = C12                                                     TRA 139
      B(22,4) = -C12*D53                                                TRA 140
      B(22,6) = C12*D51                                                 TRA 141
      B(22,7) = -C12*(D62+C14)                                          TRA 142
      B(23,3) = C13                                                     TRA 143
      B(23,4) = -C13*YBAR                                               TRA 144
      B(23,5) = -C13*D51                                                TRA 145
      B(23,7) = -C13*(D99+D13)                                          TRA 146
      B(24,1) = C21                                                     TRA 147
      B(24,5) = C21*(D53+D62)                                           TRA 148
      B(24,6) = -C21*(D99+D22-YBAR)                                     TRA 149
      B(24,9) = C21*D24                                                 TRA 150
      B(25,2) = C22                                                     TRA 151
      B(25,4) = -C22*D53                                                TRA 152
      B(25,6) = C22*D51                                                 TRA 153
      B(25,7) = -C22*(D62+D24)                                          TRA 154
      B(26,3) = C23                                                     TRA 155
      B(26,4) = -C23*YBAR                                               TRA 156
      B(26,5) = -C23*D51                                                TRA 157
      B(26,7) = C23*(D99+D22)                                           TRA 158
      B(27,14) = C31                                                    TRA 159
      B(27,18) = C31*D34                                                TRA 160
      B(28,2) = C32                                                     TRA 161
      B(28,4) = -C32*D57                                                TRA 162
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      B(28,6) = -C32*XBAR                                               TRA 163
      B(28,26) = -C32*D34                                               TRA 164
      B(29,10) = C33                                                    TRA 165
      B(30,15) = C41                                                    TRA 166
      B(30,19) = C41*D44                                                TRA 167
      B(31,2) = C42                                                     TRA 168
      B(31,4) = -C42*D57                                                TRA 169
      B(31,6) = -C42*XBAR                                               TRA 170
      B(31,27) = -C42*D44                                               TRA 171
      B(32,11) = C43                                                    TRA 172
      B(33,16) = C71                                                    TRA 173
      B(33,20) = C71*D74                                                TRA 174
      B(34,2) = C72                                                     TRA 175
      B(34,4) = -C72*D57                                                TRA 176
      B(34,6) = -C72*XBAR                                               TRA 177
      B(34,28) = -C72*D74                                               TRA 178
      B(35,12) = C73                                                    TRA 179
      B(36,17) = C81                                                    TRA 180
      B(36,21) = C81*D84                                                TRA 181
      B(37,2) = C82                                                     TRA 182
      B(37,4) = -C82*D57                                                TRA 183
      B(37,6) = -C82*XBAR                                               TRA 184
      B(37,29) = -C82*D84                                               TRA 185
      B(38,13) = C83                                                    TRA 186
C
C     THE NEXT 112 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX C.
C
      C(1,3) = -1                                                       TRA 187
      C(1,4) = L53+D56                                                  TRA 188
      C(1,5) = -XBAR                                                    TRA 189
      C(1,10) = 1                                                       TRA 190
      C(1,26) = D33                                                     TRA 191
      C(2,4) = -1                                                       TRA 192
      C(2,26) = 1                                                       TRA 193
      C(3,10) = -1                                                      TRA 194
      C(3,12) = 1                                                       TRA 195
      C(3,26) = (L37+D32)                                               TRA 196
      C(3,28) = D73                                                     TRA 197
      C(4,26) = -1                                                      TRA 198
      C(4,28) = 1                                                       TRA 199
      C(5,3) = -1                                                       TRA 200
      C(5,4) = -(L54+D55)                                               TRA 201
      C(5,5) = -XBAR                                                    TRA 202
      C(5,11) = 1                                                       TRA 203
      C(5,27) = -D42                                                    TRA 204
      C(6,4) = -1                                                       TRA 205
      C(6,27) = 1                                                       TRA 206
      C(7,11) = -1                                                      TRA 207
      C(7,13) = 1                                                       TRA 208
      C(7,27) = -(L84+D43)                                              TRA 209
      C(7,29) = -D82                                                    TRA 210
      C(8,27) = -1                                                      TRA 211
      C(8,29) = 1                                                       TRA 212
      C(9,1) = -1                                                       TRA 213
      C(9,5) = -D57                                                     TRA 214
      C(9,6) = -(L53+D56)                                               TRA 215
      C(9,14) = 1                                                       TRA 216
      C(9,22) = -D33                                                    TRA 217
      C(10,6) = -1                                                      TRA 218
      C(10,22) = 1                                                      TRA 219
```

```
C(11,14) = -1
C(11,16) = 1
C(11,22) = -(L37+D32)
C(11,24) = -D73
C(12,22) = -1
C(12,24) = 1
C(13,1) = -1
C(13,5) = -D57
C(13,6) = L54+D55
C(13,15) = 1
C(13,23) = D42
C(14,6) = -1
C(14,23) = 1
C(15,15) = -1
C(15,17) = 1
C(15,23) = L84+D43
C(15,25) = D82
C(16,23) = -1
C(16,25) = 1
C(17,5) = -1
C(17,18) = 1
C(18,18) = -1
C(18,20) = 1
C(19,5) = -1
C(19,19) = 1
C(20,19) = -1
C(20,21) = 1
C(21,1) = K11
C(21,5) = K11*(D53+D62)
C(21,6) = K11*(D99+D13+YBAR)
C(21,8) = K11*D14
C(22,2) = K12
C(22,4) = -K12*D53
C(22,6) = K12*D51
C(22,7) = -K12*(D62+D14)
C(23,3) = K13
C(23,4) = -K13*YBAR
C(23,5) = -K13*D51
C(23,7) = -K13*(D99+C13)
C(24,1) = K21
C(24,5) = K21*(D53+D62)
C(24,6) = -K12*(D99+D22-YBAR)
C(24,9) = K21*D24
C(25,2) = K22
C(25,4) = -K22*D53
C(25,6) = K22*D51
C(25,7) = -K22*(C62+D24)
C(26,3) = K23
C(26,4) = -K23*YBAR
C(26,5) = -K23*D51
C(26,7) = K23*(D99+D22)
C(27,14) = K31
C(27,18) = K31*D34
C(28,2) = K32
C(28,4) = -K32*D57
C(28,6) = -K32*XBAR
C(28,26) = -K32*D34
C(29,10) = K33
C(30,15) = K41
C(30,19) = K41*C44
```

```
      C(31,2) = K42                                                      TRA 280
      C(31,4) = -K42*D57                                                 TRA 281
      C(31,6) = -K42*XBAR                                                TRA 282
      C(31,27) = -K42*D44                                                TRA 283
      C(32,11) = K43                                                     TRA 284
      C(33,16) = K71                                                     TRA 285
      C(33,20) = K71*D74                                                 TRA 286
      C(34,2) = K72                                                      TRA 287
      C(34,4) = -K72*D57                                                 TRA 288
      C(34,6) = -K72*XBAR                                                TRA 289
      C(34,28) = -K72*D74                                                TRA 290
      C(35,12) = K73                                                     TRA 291
      C(36,17) = K81                                                     TRA 292
      C(36,21) = K81*D84                                                 TRA 293
      C(37,2) = K82                                                      TRA 294
      C(37,4) = -K82*D57                                                 TRA 295
      C(37,6) = -K82*XBAR                                                TRA 296
      C(37,29) = -K82*D84                                                TRA 297
      C(38,13) = K83                                                     TRA 298
C
C     THE NEXT 45 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX C.
C
      D(21,4) = K11                                                      TRA 299
      C(21,22) = C11                                                     TRA 300
      D(22,5) = K12                                                      TRA 301
      C(22,23) = C12                                                     TRA 302
      C(23,6) = K13                                                      TRA 303
      D(23,24) = C13                                                     TRA 304
      D(24,7) = K21                                                      TRA 305
      D(24,25) = C21                                                     TRA 306
      D(25,8) = K22                                                      TRA 307
      C(25,26) = C22                                                     TRA 308
      D(26,9) = K23                                                      TRA 309
      D(26,27) = C23                                                     TRA 310
      C(27,10) = K31                                                     TRA 311
      D(27,28) = C31                                                     TRA 312
      C(28,11) = K32                                                     TRA 313
      D(28,29) = C32                                                     TRA 314
      D(29,12) = K33                                                     TRA 315
      C(29,30) = C33                                                     TRA 316
      D(30,13) = K41                                                     TRA 317
      D(30,31) = C41                                                     TRA 318
      C(31,14) = K42                                                     TRA 319
      D(31,32) = C42                                                     TRA 320
      C(32,15) = K43                                                     TRA 321
      D(32,33) = C43                                                     TRA 322
      D(33,16) = K71                                                     TRA 323
      C(33,34) = C71                                                     TRA 324
      D(34,17) = K72                                                     TRA 325
      D(34,35) = C72                                                     TRA 326
      C(35,18) = K73                                                     TRA 327
      D(35,36) = C73                                                     TRA 328
      C(36,19) = K81                                                     TRA 329
      D(36,37) = C81                                                     TRA 330
      D(37,20) = K82                                                     TRA 331
      C(37,38) = C82                                                     TRA 332
      D(38,21) = K83                                                     TRA 333
      C(38,39) = C83                                                     TRA 334
      C(63,1) = 1                                                        TRA 335
      D(64,2) = 1                                                        TRA 336
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      D(65,3) = 1                                                       TRA 337
      D(66,2) = -DB3                                                    TRA 338
      D(66,3) = -YBAR                                                   TRA 339
      D(67,1) = DB3                                                     TRA 340
      D(67,3) = DB1                                                     TRA 341
      D(68,1) = YBAR                                                    TRA 342
      D(68,2) = -DB1                                                    TRA 343
C     THE NEXT 214 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX F.
      F(1,21) = -L53**3/(3*EI)                                          TRA 344
      F(1,47) = L53**2/(2*EI)                                           TRA 345
      F(2,21) = L53**2/(2*EI)                                           TRA 346
      F(2,47) = -L53/EI                                                 TRA 347
      F(3,18) = -L37**3/(6*EI)                                          TRA 348
      F(3,44) = -L37**2/(2*EI)                                          TRA 349
      F(4,18) = L37**2/(2*EI)                                           TRA 350
      F(4,44) = L37/EI                                                  TRA 351
      F(5,27) = -L54**3/(3*EI)                                          TRA 352
      F(5,50) = -L54**2/(2*EI)                                          TRA 353
      F(6,27) = -L54**2/(2*EI)                                          TRA 354
      F(6,50) = -L54/EI                                                 TRA 355
      F(7,30) = -L84**3/(6*EI)                                          TRA 356
      F(7,53) = L84**2/(2*EI)                                           TRA 357
      F(8,30) = -L84**2/(2*EI)                                          TRA 358
      F(8,53) = L84/EI                                                  TRA 359
      F(9,19) = -L53**3/(3*EI)                                          TRA 360
      F(9,49) = -L53**2/(2*EI)                                          TRA 361
      F(10,19) = -L53**2/(2*EI)                                         TRA 362
      F(10,49) = -L53/EI                                                TRA 363
      F(11,16) = -L37**3/(6*EI)                                         TRA 364
      F(11,46) = L37**2/(2*EI)                                          TRA 365
      F(12,16) = -L37**2/(2*EI)                                         TRA 366
      F(12,46) = +L37/EI                                                TRA 367
      F(13,25) = -L54**3/(3*EI)                                         TRA 368
      F(13,52) = L54**2/(2*EI)                                          TRA 369
      F(14,25) = L54**2/(2*EI)                                          TRA 370
      F(14,52) = -L54/EI                                                TRA 371
      F(15,28) = -(L84**3)/(6*EI)                                       TRA 372
      F(15,55) = -L84**2/(2*EI)                                         TRA 373
      F(16,28) = L84**2/(2*EI)                                          TRA 374
      F(16,55) = L84/EI                                                 TRA 375
      F(17,48) = -L53/GJ                                                TRA 376
      F(18,45) = L37/GJ                                                 TRA 377
      F(19,51) = -L54/GJ                                                TRA 378
      F(20,54) = L84/GJ                                                 TRA 379
      F(21,1) = -1.                                                     TRA 380
      F(22,2) = -1.                                                     TRA 381
      F(23,3) = -1.                                                     TRA 382
      F(24,7) = -1.                                                     TRA 383
      F(25,8) = -1.                                                     TRA 384
      F(26,9) = -1.                                                     TRA 385
      F(27,13) = -1.                                                    TRA 386
      F(28,14) = -1.                                                    TRA 387
      F(29,15) = -1.                                                    TRA 388
      F(30,22) = -1.                                                    TRA 389
      F(31,23) = -1.                                                    TRA 390
      F(32,24) = -1.                                                    TRA 391
      F(33,34) = -1.                                                    TRA 392
      F(34,35) = -1.                                                    TRA 393
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      F(35,36) = -1.                                                  TRA 394
      F(36,37) = -1.                                                  TRA 395
      F(37,38) = -1.                                                  TRA 396
      F(38,39) = -1.                                                  TRA 397
      F(39,1) = 1                                                     TRA 398
      F(39,4) = 1                                                     TRA 399
      F(40,2) = 1                                                     TRA 400
      F(40,5) = 1                                                     TRA 401
      F(41,3) = 1                                                     TRA 402
      F(41,6) = 1                                                     TRA 403
      F(42,2) = -D14                                                  TRA 404
      F(42,6) = D13                                                   TRA 405
      F(42,40) = 1                                                    TRA 406
      F(43,1) = D14                                                   TRA 407
      F(44,4) = -D13                                                  TRA 408
      F(44,41) = 1                                                    TRA 409
      F(45,7) = 1                                                     TRA 410
      F(45,10) = 1                                                    TRA 411
      F(46,8) = 1                                                     TRA 412
      F(46,11) = 1                                                    TRA 413
      F(47,9) = 1                                                     TRA 414
      F(47,12) = 1                                                    TRA 415
      F(48,8) = -D24                                                  TRA 416
      F(48,12) = -D22                                                 TRA 417
      F(48,42) = 1                                                    TRA 418
      F(49,7) = D24                                                   TRA 419
      F(50,10) = D22                                                  TRA 420
      F(50,43) = 1                                                    TRA 421
      F(51,13) = 1                                                    TRA 422
      F(51,16) = 1                                                    TRA 423
      F(51,19) = 1                                                    TRA 424
      F(52,14) = 1                                                    TRA 425
      F(52,17) = 1                                                    TRA 426
      F(52,20) = 1                                                    TRA 427
      F(53,15) = 1                                                    TRA 428
      F(53,18) = 1                                                    TRA 429
      F(53,21) = 1                                                    TRA 430
      F(54,14) = -D34                                                 TRA 431
      F(54,15) = D31                                                  TRA 432
      F(54,18) = -D32                                                 TRA 433
      F(54,21) = D33                                                  TRA 434
      F(54,44) = 1                                                    TRA 435
      F(54,47) = 1                                                    TRA 436
      F(55,13) = D34                                                  TRA 437
      F(55,45) = 1                                                    TRA 438
      F(55,48) = 1                                                    TRA 439
      F(56,13) = -D31                                                 TRA 440
      F(56,16) = D32                                                  TRA 441
      F(56,19) = -D33                                                 TRA 442
      F(56,46) = 1                                                    TRA 443
      F(56,49) = 1                                                    TRA 444
      F(57,22) = 1                                                    TRA 445
      F(57,25) = 1                                                    TRA 446
      F(57,28) = 1                                                    TRA 447
      F(58,23) = 1                                                    TRA 448
      F(58,26) = 1                                                    TRA 449
      F(58,29) = 1                                                    TRA 450
      F(59,24) = 1                                                    TRA 451
      F(59,27) = 1                                                    TRA 452
      F(59,30) = 1                                                    TRA 453
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F(60,23) = -D44                         TRA 454
F(60,24) = D41                          TRA 455
F(60,27) = -D42                         TRA 456
F(6D,30) = D43                          TRA 457
F(60,50) = 1                            TRA 458
F(60,53) = 1                            TRA 459
F(61,22) = D44                          TRA 460
F(61,51) = 1                            TRA 461
F(61,54) = 1                            TRA 462
F(62,22) = -D41                         TRA 463
F(62,25) = D42                          TRA 464
F(62,28) = -D43                         TRA 465
F(62,52) = 1                            TRA 466
F(62,55) = 1                            TRA 467
F(63,19) = -1                           TRA 468
F(63,25) = -1                           TRA 469
F(63,31) = 1                            TRA 470
F(64,20) = -1                           TRA 471
F(64,26) = -1                           TRA 472
F(64,32) = 1                            TRA 473
F(65,21) = -1                           TRA 474
F(65,27) = -1                           TRA 475
F(65,33) = 1                            TRA 476
F(66,2D) = 057                          TRA 477
F(66,21) = L53+D56                      TRA 478
F(66,26) = 057                          TRA 479
F(66,27) = -L54-D55                     TRA 480
F(66,32) = -053                         TRA 481
F(66,33) = -YBAR                        TRA 482
F(66,47) = -1                           TRA 483
F(66,50) = -1                           TRA 484
F(67,19) = -057                         TRA 485
F(67,21) = -XBAR                        TRA 486
F(67,25) = -D57                         TRA 487
F(67,27) = -XBAR                        TRA 488
F(67,31) = D53                          TRA 489
F(67,33) = -D51                         TRA 490
F(67,48) = -1                           TRA 491
F(67,51) = -1                           TRA 492
F(67,56) = 1                            TRA 493
F(68,19) = -L53-D56                     TRA 494
F(68,20) = X8AR                         TRA 495
F(68,25) = L54+D55                      TRA 496
F(68,26) = XBAR                         TRA 497
F(68,31) = Y8AR                         TRA 498
F(68,32) = 051                          TRA 499
F(68,49) = -1                           TRA 500
F(68,52) = -1                           TRA 501
F(68,57) = 1                            TRA 502
F(69,4) = -1                            TRA 503
F(69,1D) = -1                           TRA 504
F(69,31) = -1                           TRA 505
F(70,5) = -1                            TRA 506
F(70,11) = -1                           TRA 507
F(70,32) = -1                           TRA 508
F(71,6) = -1                            TRA 509
F(71,12) = -1                           TRA 510
F(71,33) = -1                           TRA 511
F(72,5) = D64                           TRA 512
F(72,6) = D99                           TRA 513
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      F(72,12) = -D99                                              TRA 514
      F(72,32) = -D63                                              TRA 515
      F(72,11) = D64                                               TRA 516
      F(72,40) = -1                                                TRA 517
      F(72,42) = -1                                                TRA 518
      F(73,4) = -D64                                               TRA 519
      F(73,10) = -D64                                              TRA 520
      F(73,31) = D63                                               TRA 521
      F(73,56) = -1                                                TRA 522
      F(74,4) = -D99                                               TRA 523
      F(74,10) = D99                                               TRA 524
      F(74,41) = -1                                                TRA 525
      F(74,43) = -1                                                TRA 526
      F(74,57) = -1                                                TRA 527
      F(75,16) = -1                                                TRA 528
      F(75,34) = 1                                                 TRA 529
      F(76,17) = -1                                                TRA 530
      F(76,35) = 1                                                 TRA 531
      F(77,18) = -1                                                TRA 532
      F(77,36) = 1                                                 TRA 533
      F(78,18) = -(L37+D73)                                        TRA 534
      F(78,35) = -D74                                              TRA 535
      F(78,36) = D71                                               TRA 536
      F(78,44) = -1                                                TRA 537
      F(79,34) = D74                                               TRA 538
      F(79,45) = -1                                                TRA 539
      F(80,16) = L37+D73                                           TRA 540
      F(80,34) = -D71                                              TRA 541
      F(80,46) = -1                                                TRA 542
      F(81,28) = -1                                                TRA 543
      F(81,37) = 1                                                 TRA 544
      F(82,29) = -1                                                TRA 545
      F(82,38) = 1                                                 TRA 546
      F(83,30) = -1                                                TRA 547
      F(83,39) = 1                                                 TRA 548
      F(84,30) = L84+D82                                           TRA 549
      F(84,38) = -D84                                              TRA 550
      F(84,39) = D81                                               TRA 551
      F(84,53) = -1                                                TRA 552
      F(85,37) = D84                                               TRA 553
      F(85,54) = -1                                                TRA 554
      F(86,28) = -(L84+D82)                                        TRA 555
      F(86,37) = -D81                                              TRA 556
      F(86,55) = -1                                                TRA 557
C
C     THE NEXT 10 CARDS CAST THE A,B,C,D AND F MATRICIES INTO THE MATRIX E.
C
      DO 80 I=1,86                                                 TRA 558
      DO 50 J=1,57                                                 TRA 559
   50 E(I,J) = F(I,J)                                              TRA 560
      DO 60 J=1,29                                                 TRA 561
      E(I,J+57) = -A(I,J)                                          TRA 562
      E(I,J+86) = C(I,J)                                           TRA 563
   60 E(I,J+115) = B(I,J)                                          TRA 564
      DO 70 J=1,39                                                 TRA 565
   70 E(I,J+144) = -D(I,J)                                         TRA 566
   80 CONTINUE                                                     TRA 567
      CALL GAUSS (E,N,N1)                                          TRA 568
C
C     THE NEXT 4 CARDS CAST THE LAST 126 COLUMNS OF THE LAST 29 ROWS OF
```

```
C     THE MATRIX E INTO THE MATRIX G.
C
      DO 100 I=1,29
      DO 90 J=1,126
   90 G(I,J) = E(I+57,J+57)
  100 CONTINUE
      WRITE (10) G
C
C     THE NEXT 13 CARDS PRODUCE A FORMATTED COPY OF THE NONZERO ELEMENTS
C     OF THE MATRIX G.
C
      DO 120 I=1,29
      WRITE (6,190) I
      DO 110 J=1,126
      IF (J.EQ.1) ID = 1
      IF (ABS(G(I,J)).LE.0.0001) GO TO 110
      IF (ID.EQ.1) WRITE (6,130) G(I,J),NAME(J)
      IF (ID.EQ.2) WRITE (6,140) G(I,J),NAME(J)
      IF (ID.EQ.3) WRITE (6,150) G(I,J),NAME(J)
      IF (ID.EQ.4) WRITE (6,160) G(I,J),NAME(J)
      IF (ID.EQ.4) ID = 1
      ID = ID+1
  110 CONTINUE
  120 CONTINUE
      STOP
  130 FORMAT (' ',20X,F3.1,1X,A4,'=')
  140 FORMAT ('+',T31,'+(',F13.4,')',A4)
  150 FORMAT ('+',T51,'+(',F13.4,')',A4)
  160 FORMAT ('+',T71,'+(',F13.4,')',A4,/)
  170 FORMAT (20A4)
  180 FORMAT (8F10.2)
  190 FORMAT ('-',47X,'EQUATION',I5)
  200 FORMAT (2E10.4)
      END
```

```
C     THIS PROGRAM GENERATES THE EQUATIONS OF MOTION USING THE EQUATIONS
C     DEVELOPED IN SECTION 5B.
C
      IMPLICIT REAL(A-Z)
      INTEGER I,J,K,N,N1
      DIMENSION A(29,29)
      DIMENSION B(29,29)
      DIMENSION C(29,29)
      DIMENSION D(29,39)
      DIMENSION G(29,126)
      DIMENSION NAME(126)
      REWIND 10
C
C     THE NEXT 15 CARDS READ IN THE VARIOUS TRACTOR PARAMETERS.
C
      READ (5,140) M1,J1X,J1Y,J1Z,D13,C14
      READ (5,140) M2,J2X,J2Y,J2Z,D22,D24
      READ (5,140) M3,J3X,J3Y,J3Z,D31,D32,C33,D34
      READ (5,140) M4,J4X,J4Y,J4Z,D41,D42,D43,D44
      READ (5,140) M5,J5X,J5Y,J5Z,D51,D53,C55,D56,D57
      READ (5,140) M6,J6X,J6Y,J6Z,D62,D63,C64,D99
      READ (5,140) M7,J7X,J7Y,J7Z,D71,D73,D74
      READ (5,140) M8,J8X,J8Y,J8Z,D81,D82,C84
      READ (5,140) L37,L53,L54,L84
      READ (5,160) EI,GJ
      READ (5,140) XBAR,YBAR,ZBAR,DB1,DB3
      READ (5,140) K11,K12,K13,K21,K22,K23,K31,K32,K33,K41,K42,K43,K71,K
     *72,K73,K81,K82,K83
      READ (5,140) C11,C12,C13,C21,C22,C23,C31,C32,C33,C41,C42,C43,C71,C
     *72,C73,C81,C82,C83
      WRITE (6,140) M1,J1X,J1Y,J1Z,D13,D14
      WRITE (6,140) M2,J2X,J2Y,J2Z,D22,D24
      WRITE (6,140) M3,J3X,J3Y,J3Z,D31,D32,D33,C34
      WRITE (6,140) M4,J4X,J4Y,J4Z,D41,D42,D43,C44
      WRITE (6,140) M5,J5X,J5Y,J5Z,D51,D53,D55,D56,D57
      WRITE (6,140) M6,J6X,J6Y,J6Z,D62,D63,D64,C99
      WRITE (6,140) M7,J7X,J7Y,J7Z,D71,D73,D74
      WRITE (6,140) M8,J8X,J8Y,J8Z,D81,D82,D84
      WRITE (6,140) L37,L53,L54,L84
      WRITE (6,160) EI,GJ
      WRITE (6,140) XBAR,YBAR,ZBAR,DB1,DB3
      WRITE (6,140) K11,K12,K13,K21,K22,K23,K31,K32,K33,K41,K42,K43,K71,
     *K72,K73,K81,K82,K83
      WRITE (6,140) C11,C12,C13,C21,C22,C23,C31,C32,C33,C41,C42,C43,C71,
     *C72,C73,C81,C82,C83
      READ (5,130) (NAME(I),I=1,126)
C
C     N IS THE NUMBER OF ROWS AND N1 THE NUMBER OF COLUMNS OF THE MATRIX G.
C
      N = 29
      N1 = 126
C
C     THE NEXT 10 CARDS INITIALIZE THE A,B,C AND D MATRICIES.
C
      DO 30 I=1,29
      DO 10 J=1,29
      A(I,J) = 0.0
      B(I,J) = 0.0
      C(I,J) = 0.0
   10 CONTINUE
```

```
      DO 20 J=1,39
      D(I,J) = 0.0
   20 CONTINUE
   30 CONTINUE
C
C     THE NEXT 16 CARDS EVALUATE CONSTANTS TO BE USED LATER.
C
      KA1 = (12.*EI)/L53**3.
      KA2 = (12.*EI)/L37**3.
      KA3 = (12.*EI)/L54**3.
      KA4 = (12.*EI)/L84**3.
      KA5 = (6.*EI)/L53**2.
      KA6 = (6.*EI)/L37**2.
      KA7 = (6.*EI)/L54**2.
      KA8 = (6.*EI)/L84**2.
      KA9 = (4.*EI)/L53
      KA10 = (4.*EI)/L37
      KA11 = (4.*EI)/L54
      KA12 = (4.*EI)/L84
      KA13 = GJ/L53
      KA14 = GJ/L37
      KA15 = GJ/L54
      KA16 = GJ/L84
C
C     THE NEXT 56 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX A.
C
      A(1,1) = M1+M2+M5+M6
      A(1,5) = D53*(M1+M2+M6)+D62*(M1+M2)+D63*M6
      A(01,06) = YBAR*(M1-M2+M6)+D99*(M1-M2)+D13*M1-D22*M2
      A(02,02) = M1+M2+M3+M4+M5+M6+M7+M8
      A(02,04) = -D53*(M1+M2+M6)-D57*(M3+M4+M7+M8)
      A(02,06) = D51*(M1+M2+M6)-XBAR*(M3+M4+M7+M8)
      A(02,07) = -D62*(M1+M2)-D63*M6
      A(03,03) = M1+M2+M5+M6
      A(03,04) = -YBAR*(M1+M2+M6)
      A(03,05) = -D51*(M1+M2+M6)
      A(03,07) = D99*(M2-M1)-D13*M1+D22*M2
      A(04,02) = -D53*(M1+M2+M6)-D57*(M3+M4+M7+M8)
      A(04,03) = -YBAR*(M1+M2+M6)
      A(04,04) = (D53**2+YBAR**2)*(M1+M2+M6)+D57**2*(M3+M4+M7+M8)+J5X
      A(04,05) = YBAR*D51*(M1+M2+M6)
      A(04,06) = -D51*D53*(M1+M2+M6)+XBAR*D57*(M3+M4+M7+M8)
      A(04,07) = D53*D62*(M1+M2)+D53*D63*M6+YBAR*D99*(M1-M2)+YBAR*D13*M1
     *-YBAR*D22*M2
      A(05,01) = (D53+D62)*(M1+M2)+(D53+D63)*M6
      A(05,03) = -D51*(M1+M2+M6)
      A(05,04) = D51*YBAR*(M1+M2+M6)
      A(05,05) = (D53+D62)**2*(M1+M2)+(D53+D63)**2*M6+D51**2*(M1+M2+M6)+
     *J5Y+J6Y
      A(05,06) = (D53+D62)*((YBAR+D99+D13)*M1+(YBAR-D99-D22)*M2)+(D53+D6
     *3)*YBAR*M6
      A(05,07) = D51*((D99+D13)*M1-(D99+D22)*M2)
      A(06,01) = (YBAR+D99+D13)*M1+(YBAR-D99-D22)*M2+YBAR*M6
      A(06,02) = D51*(M1+M2+M6)-XBAR*(M3+M4+M7+M8)
      A(06,04) = -D51*D53*(M1+M2)-D51*D53*M6+D57*XBAR*(M3+M4+M7+M8)
      A(06,05) = (D53+D62)*((YBAR+D99+D13)*M1+(YBAR-D99-D22)*M2)+YBAR*(D
     *53D53+D63)*M6
      A(06,06) = (YBAR+D99)
      A(06,06) = (YBAR+D99+D13)**2*M1+(YBAR-D99-D22)**2*M2+D51**2*(M1+M2
     *+M6)+XBAR**2*(M3+M4+M7+M8)+YBAR**2*M6+J1Z+J2Z+J5Z+J6Z
```

```
      A(06,07) = -D51*D62*(M1+M2)-D51*C63*M6
      A(07,02) = -D62*(M1+M2)-D63*M6
      A(07,03) = (D99+D13)*(M2-M1)
      A(07,04) = D53*D62*(M1+M2)+D63*D53*M6+YBAR*(C99+D13)*(M1-M2)
      A(07,05) = (D99+D13)*D51*(M1-M2)
      A(07,06) = -D62*D51*(M1+M2)-D63*C51*M6
      A(07,C7) = D62**2*(M1+M2)+D63**2*M6+(D99+D22)**2*(M1+M2)+J1X+J2X+J
     *6X
      A(08,08) = J1Y
      A(09,09) = J2Y
      A(10,10) = M3
      A(11,11) = M4
      A(12,12) = M7
      A(13,13) = M8
      A(14,14) = M3
      A(15,15) = M4
      A(16,16) = M7
      A(17,17) = M8
      A(18,18) = J3Y
      A(19,19) = J4Y
      A(20,20) = J7Y
      A(21,21) = J8Y
      A(22,22) = J3Z
      A(23,23) = J4Z
      A(24,24) = J7Z
      A(25,25) = J8Z
      A(26,26) = J3X
      A(27,27) = J4X
      A(28,28) = J7X
      A(29,29) = J8X
C
C     THE NEXT 97 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX B.
C
      B(01,C1) = C11+C21
      B(01,05) = (C53+D62)*(C11+C21)
      B(01,06) = C11*(YEAR+D99+D13)+C21*(YBAR-D99-D22)
      B(01,08) = C11*D14
      B(01,09) = C21*C24
      B(02,02) = C12+C22+C32+C42+C72+C82
      B(02,04) = -D53*(C12+C22)-D57*(C32+C42+C72+C82)
      B(02,06) = D51*(C12+C22)-XBAR*(C32+C42+C72+C82)
      B(02,C7) = -C12*(D62+D14)-C22*(D62+D24)
      B(02,26) = -C32*C34
      B(02,27) = -C42*D44
      B(02,28) = -C72*D74
      B(02,29) = -C82*C84
      B(03,03) = C13+C23
      B(03,04) = -YEAR*(C13+C23)
      B(03,05) = -D51*(C13+C23)
      B(03,C7) = -C13*(D99+D13)+C23*(D99+D22)
      B(04,02) = -D53*(C12+C22)-D57*(C32+C42+C72+C82)
      B(04,03) = -YBAR*(C13+C23)
      B(04,04) = D53**2*(C12+C22)+YBAR**2*(C13+C23)+D57**2*(C32+C42+C72+
     *C82)
      B(04,C5) = D51*YBAR*(C13+C23)
      B(04,06) = -C51*D53*(C12+C22)+XBAR*D57*(C32+C42+C72+C82)
      B(04,C7) = D53*((D62+D14)*C12+(D62+C24)*C22)+YBAR*((D99+D13)*C13-(
     *C99+D22)*C23)
      B(04,26) = C32*D57*D34
      B(04,27) = C42*C57*D44
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```
      B(D4,28) = C72*C57*D74
      B(D4,29) = C82*D57*DE4
      B(D5,D1) = (D53+D62)*(C11+C21)
      B(D5,D3) = -D51*(C13+C23)
      B(C5,D4) = YBAR*D51*(C13+C23)
      B(D5,D5) = (D53+D62)**2*(C11+C21)+D51**2*(C13+C23)
      B(D5,D6) = (D53+D62)*((YBAR+D99+D13)*C11+(YBAR-D99-D22)*C21)
      B(D5,C7) = D51*((D99+D13)*C13-(D99+D22)*C23)
      B(D5,D8) = C11*C14*(C53+C62)
      B(C5,C9) = C21*C24*(D53+C62)
      B(D6,D1) = C11*(YBAR+D99+D13)+C21*(YBAR-D99-D22)
      B(D6,D2) = D51*(C12+C22)-XBAR*(C32+C42+C72+C82)
      B(D6,D4) = -D51*D53*(C12+C22)+XBAR*C57*(C32+C42+C72+C82)
      B(D6,D5) = (D53+D62)*(C11*(YBAR+C99+C13)+C21*(YBAR-D99-D22))
      B(C6,C6) = C11*(YBAR+D99+D13)**2+C21*(YBAR-D99-C22)**2+D51**2*(C12
     *+C22)+XBAR**2*(C32+C42+C72+C82)
      B(D6,D7) = -D51*((C62+C14)*C12+(D62+C24)*C22)
      B(D6,C8) = C11*D14*(YBAR+D99+D13)
      B(D6,D9) = C21*C24*(YBAR-D99-D22)
      B(D6,26) = C32*XBAR*C34
      B(D6,27) = C42*XBAR*D44
      B(D6,28) = C72*XEAR*C74
      B(D6,29) = C82*XBAR*D84
      B(D7,D2) = -C12*(D62+D14)-C22*(D62+D24)
      B(D7,D3) = -C13*(D99+D13)+C23*(D99+C22)
      B(C7,D4) = D53*(C12*(D62+D14)+C22*(D62+D24))+YBAR*(C13*(D99+D13)-C
     *23*(D99+D22))
      B(C7,C5) = D51*(C13*(D99+D13)-C23*(C99+C22))
      B(D7,D6) = -D51*(C12*(D62+D14)+C22*(D62+D24))
      B(D7,D7) = C12*(D62+C14)**2+C13*(D99+D13)**2+C22*(D62+D24)**2+C23*
     *(D99+D22)**2
      B(D8,D1) = C11*D14
      B(D8,C5) = C11*D14*(C53+C62)
      B(D8,D6) = C11*D14*(YBAR+D99+D13)
      B(D8,D8) = C11*C14**2
      B(D9,C1) = C21*D24
      B(D9,D5) = C21*D24*(C53+C62)
      B(D9,D6) = C21*D24*(YBAR-D99-D22)
      B(D9,D9) = C21*D24**2
      B(10,10) = C33
      B(11,11) = C43
      B(12,12) = C73
      B(13,13) = C83
      B(14,14) = C31
      B(14,18) = C31*C34
      B(15,15) = C41
      B(15,19) = C41*D44
      B(16,16) = C71
      B(16,20) = C71*D74
      B(17,17) = C81
      B(17,21) = C81*D84
      B(18,14) = C31*D34
      B(18,18) = C31*D34**2
      B(19,15) = C41*C44
      B(19,19) = C41*D44**2
      B(20,16) = C71*C74
      B(20,20) = C71*D74**2
      B(21,17) = C81*D84
      B(21,21) = C81*D84**2
      B(26,D2) = -C32*D34
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```
      B(26,04) = C32*C34*D57                                            TRB 220
      B(26,06) = C32*D34*XBAR                                           TRB 221
      B(26,26) = C32*D34**2                                             TRB 222
      B(27,02) = -C42*D44                                               TRB 223
      B(27,04) = C42*D44*D57                                            TRB 224
      B(27,06) = C42*D44*XBAR                                           TRB 225
      B(27,27) = C42*C44**2                                             TRB 226
      B(28,02) = -C72*D74                                               TRB 227
      B(28,04) = C72*C74*D57                                            TRB 228
      B(28,06) = C72*D74*XBAR                                           TRB 229
      B(28,28) = C72*D74**2                                             TRB 230
      B(29,02) = -C82*D84                                               TRB 231
      B(29,04) = C82*D84*D57                                            TRB 232
      B(29,06) = C82*D84*XBAR                                           TRB 233
      B(29,29) = C82*D84**2                                             TRB 234
C
C     THE NEXT 201 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX C.
C
      C(01,01) = K11+K21+KA1+KA3                                        TRB 235
      C(01,05) = (D53+D62)*(K11+K21)+D57*(KA1+KA3)                      TRB 236
      C(01,06) = K11*(YBAR+D99+D13)+K21*(YBAR-D99-D22)+KA1*(D56+L53)-KA3TRB 237
     **(D55+L54)-KA5+KA7                                                TRB 238
      C(01,08) = K11*D14                                                TRB 239
      C(01,09) = K21*D24                                                TRB 240
      C(01,14) = -KA1                                                   TRB 241
      C(01,15) = -KA3                                                   TRB 242
      C(01,22) = KA1*D33+KA5                                            TRB 243
      C(01,23) = -KA3*D42-KA7                                           TRB 244
      C(02,02) = K12+K22+K32+K42+K72+K82                                TRB 245
      C(02,04) = -D53*(K12+K22)-D57*(K32+K42+K72+K82)                   TRB 246
      C(02,06) = D51*(K12+K22)-XBAR*(K32+K42+K72+K82)                   TRB 247
      C(02,07) = -K12*(D62+D14)-K22*(D62+D24)                           TRB 248
      C(02,26) = -K32*D34                                               TRB 249
      C(02,27) = -K42*D44                                               TRB 250
      C(02,28) = -K72*D74                                               TRB 251
      C(02,29) = -K82*D84                                               TRB 252
      C(03,03) = K13+K23+KA1+KA3                                        TRB 253
      C(03,04) = -YBAR*(K13+K23)-(D56+L53)*KA1+(D55+L54)*KA3+KA5-KA7    TRB 254
      C(03,05) = -D51*(K13+K23)+XBAR*(KA1+KA3)                          TRB 255
      C(03,07) = -K13*(D99+D13)+K23*(D99+D22)                           TRB 256
      C(03,10) = -KA1                                                   TRB 257
      C(03,11) = -KA3                                                   TRB 258
      C(03,26) = -KA1*D33-KA5                                           TRB 259
      C(03,27) = KA3*D42+KA7                                            TRB 260
      C(04,02) = -D53*(K12+K22)-D57*(K32+K42+K72+K82)                   TRB 261
      C(04,03) = -YBAR*(K13+K23)-(D56+L53)*KA1+(D55+L54)*KA3+KA5-KA7    TRB 262
      C(04,04) = D53**2*(K12+K22)+YBAR**2*(K13+K23)+D57**2*(K32+K42+K72+TRB 263
     *K82)+(D56+L53)**2*KA1+(D55+L54)**2*KA3-2*KA5*(D56+L53)-2*KA7*(D55+TRB 264
     *L54)+KA9+KA11                                                     TRB 265
      C(04,05) = D51*YBAR*(K13+K23)-XBAR*((D56+L53)*KA1-(D55+L54)*KA3-KATRB 266
     *5+KA7)                                                            TRB 267
      C(04,06) = -D51*D53*(K12+K22)+XBAR*D57*(K32+K42+K72+K82)          TRB 268
      C(04,07) = D53*((D62+D14)*K12+(D62+D24)*K22)+YBAR*((D99+D13)*K13-(TRB 269
     *D99+D22)*K23)                                                     TRB 270
      C(04,10) = KA1*(D56+L53)-KA5                                      TRB 271
      C(04,11) = -KA3*(D55+L54)+KA7                                     TRB 272
      C(04,26) = D34*D57*K32+D33*(D56+L53)*KA1-KA5*(D33-D56-L53)-KA9    TRB 273
      C(04,27) = D44*D57*K42+D42*(D55+L54)*KA3+KA7*(-D42+D55+L54)-KA11  TRB 274
      C(04,28) = D74*D57*K72                                            TRB 275
      C(04,29) = D84*D57*K82                                            TRB 276
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```
      C(05,01) = (D53+D62)*(K11+K21)+D57*(KA1+KA3)
      C(05,03) = -D51*(K13+K23)+XBAR*(KA1+KA3)
      C(05,04) = D51*YBAR*(K13+K23)-XBAR*((D56+L53)*KA1-(D55+L54)*KA3-KA
     *5+KA7)
      C(05,05) = (D53+D62)**2*(K11+K21)+D51**2*(K13+K23)+(XBAR**2+D57**2
     *)*(KA1+KA3)+KA13+KA15
      C(05,06) = (D53+D62)*((YBAR+D99+D13)*K11+(YBAR-D99-D22)*K21)+D57*(
     *(D56+L53)*KA1-(D55+L54)*KA3-KA5+KA7)
      C(05,07) = D51*((D99+D13)*K13-(D99+D22)*K23)
      C(05,08) = D14*(D53+D62)*K11
      C(05,09) = D24*(D53+D62)*K21
      C(05,10) = -XBAR*KA1
      C(05,11) = -XBAR*KA3
      C(05,14) = -D57*KA1
      C(05,15) = -D57*KA3
      C(05,18) = -KA13
      C(05,19) = -KA15
      C(05,22) = D57*(KA1*D33+KA5)
      C(05,23) = D57*(-KA7-KA3*D42)
      C(05,26) = -XBAR*D33*KA1-XBAR*KA5
      C(05,27) = XBAR*D42*KA3+XBAR*KA7
      C(06,01) = K11*(YBAR+D99+D13)+K21*(YBAR-D99-D22)+KA1*(D56+L53)-KA3
     **(D55+L54)-KA5+KA7
      C(06,02) = D51*(K12+K22)-XBAR*(K32+K42+K72+KB2)
      C(06,04) = -D51*D53*(K12+K22)+D57*XBAR*(K32+K42+K72+KB2)
      C(06,05) = (D53+D62)*((YBAR+D99+D13)*K11+(YBAR-D99-D22)*K21)+D57*(
     *(D56+L53)*KA1-(D55+L54)*KA3-KA5+KA7)
      C(06,06) = (YBAR+D99+D13)**2*K11+(YBAR-D99-D22)**2*K21+D51**2*(K12
     *+K22)+XBAR**2*(K32+K42+K72+KB2)+(D56+L53)**2*KA1+(D55+L54)**2*KA3-
     *2*KA5*(D56+L53)-2*KA7*(D55+L54)+KA9+KA11
      C(06,07) = -D51*(D62+D14)*K12-D51*(D62+D24)*K22
      C(06,08) = D14*(YBAR+D99+D13)*K11
      C(06,09) = D24*(YBAR-D99-D22)*K21
      C(06,14) = -KA1*(D56+L53)+KA5
      C(06,15) = KA3*(D55+L54)-KA7
      C(06,22) = D33*(D56+L53)*KA1+KA5*(-D33+D55+L53)-KA9
      C(06,23) = D42*(D55+L54)*KA3+KA7*(-D42+D56+L54)-KA11
      C(06,26) = XBAR*D34*K32
      C(06,27) = XBAR*D44*K42
      C(06,28) = XBAR*D74*K72
      C(06,29) = XBAR*D84*KB2
      C(07,02) = -(D62+D14)*K12-(D62+D24)*K22
      C(07,03) = -(D99+D13)*K13+(D99+D22)*K23
      C(07,04) = D53*((D62+D14)*K12+(D62+D24)*K22)+YBAR*((D99+D13)*K13-(
     *D99+D22)*K23)
      C(07,05) = D51*((D99+D13)*K13-(D99+D22)*K23)
      C(07,06) = -D51*((D62+D14)*K12+(D62+D24)*K22)
      C(07,07) = (D62+D14)**2*K12+(D99+D13)**2*K13+(D62+D24)**2*K22+(D99
     *+D22)**2*K23
      C(08,01) = K11*D14
      C(08,05) = K11*D14*(D53+D62)
      C(08,06) = K11*D14*(YBAR+D99+D13)
      C(08,08) = K11*D14**2
      C(09,01) = K21*D24
      C(09,05) = K21*D24*(D53+D62)
      C(09,06) = K21*D24*(YBAR-D99-D22)
      C(09,09) = K21*D24**2
      C(10,03) = -KA1
      C(10,04) = KA1*(D56+L53)-KA5
      C(10,05) = -KA1*XBAR
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```
C(10,10) = K33+KA1+KA2                                 TR8 337
C(10,12) = -KA2                                        TR8 338
C(10,26) = KA1*D33-KA2*(C32+L37)+KA5+KA6               TR8 339
C(10,28) = -KA2*D73-KA6                                TR8 340
C(11,D3) = -KA3                                        TR8 341
C(11,04) = -KA3*(D55+L54)+KA7                          TR8 342
C(11,05) = -KA3*XBAR                                   TR8 343
C(11,11) = K43+KA3+KA4                                 TR8 344
C(11,13) = -KA4                                        TR8 345
C(11,27) = -KA3*042+KA4*(D43+L84)-KA7-KA8              TR8 346
C(11,29) = KA4*C82+KA8                                 TR8 347
C(12,10) = -KA2                                        TR8 348
C(12,12) = K73+KA2                                     TR8 349
C(12,26) = KA2*(C32+L37)-KA6                           TR8 350
C(12,28) = KA2*D73+KA6                                 TR8 351
C(13,11) = -KA4                                        TR8 352
C(13,13) = K83+KA4                                     TR8 353
C(13,27) = -KA4*(D43+L84)+KA8                          TR8 354
C(13,29) = -KA4*D82-KA8                                TR8 355
C(14,C1) = -KA1                                        TR8 356
C(14,05) = -KA1*D57                                    TR8 357
C(14,D6) = -KA1*(D56+L53)+KA5                          TR8 358
C(14,14) = K31+KA1+KA2                                 TR8 359
C(14,16) = -KA2                                        TR8 360
C(14,18) = K31*D34                                     TR8 361
C(14,22) = -KA1*033+KA2*(D32+L37)-KA5-KA6              TR8 362
C(14,24) = +KA2*073+KA6                                TR8 363
C(15,01) = -KA3                                        TR8 364
C(15,D5) = -KA3*D57                                    TR8 365
C(15,06) = KA3*(C55+L54)-KA7                           TR8 366
C(15,15) = K41+KA3+KA4                                 TR8 367
C(15,17) = -KA4                                        TR8 368
C(15,19) = K41*D44                                     TR8 369
C(15,23) = KA3*D42-KA4*(D43+L84)+KA7+KA8               TR8 370
C(15,25) = -KA4*D82-KA8                                TR8 371
C(16,14) = -KA2                                        TR8 372
C(16,16) = K71+KA2                                     TR8 373
C(16,20) = K71*D74                                     TR8 374
C(16,22) = -KA2*(D32+L37)+KA6                          TR8 375
C(16,24) = -KA2*C73-KA6                                TR8 376
C(17,15) = -KA4                                        TR8 377
C(17,17) = K81+KA4                                     TR8 378
C(17,21) = K81*C84                                     TR8 379
C(17,23) = KA4*(043+L84)-KA8                           TR8 380
C(17,25) = KA4*C82+KA8                                 TR8 381
C(18,05) = -KA13                                       TR8 382
C(18,14) = K31*034                                     TR8 383
C(18,18) = K31*C34**2+KA13+KA14                        TR8 384
C(18,20) = -KA14                                       TR8 385
C(19,05) = -KA15                                       TR8 386
C(19,15) = K41*C44                                     TR8 387
C(19,19) = K41*044**2+KA15+KA16                        TR8 388
C(19,21) = -KA16                                       TR8 389
C(20,16) = K71*D74                                     TR8 390
C(20,18) = -KA14                                       TR8 391
C(20,20) = K71*074**2+KA14                             TR8 392
C(21,17) = K81*D84                                     TR8 393
C(21,19) = -KA16                                       TR8 394
C(21,21) = K81*C84**2+KA16                             TR8 395
C(22,01) = KA1*D33+KA5                                 TR8 396
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      C(22,05) = KA1*C33*D57+D57*KA5
      C(22,06) = (D56+L53)*(KA1*D33+KA5)-KA5*D33-KA9
      C(22,14) = -KA1*C33+KA2*(D32+L37)-KA5-KA6
      C(22,16) = -KA2*(D32+L37)+KA6
      C(22,22) = KA1*D33**2+KA2*(D32+L37)**2+2*KA5*D33-2*KA6*(D32+L37)+K
     *A9+KA10
      C(22,24) = KA2*D73*(D32+L37)+KA6*(-C73+D32+L37)-KA10
      C(23,01) = -KA3*D42-KA7
      C(23,05) = -KA3*C42*C57-C57*KA7
      C(23,06) = (D55+L54)*(KA3*D42+KA7)-KA7*D42-KA11
      C(23,15) = KA3*D42-KA4*(C43+L84)+KA7+KA8
      C(23,17) = KA4*(D43+L84)-KA8
      C(23,23) = KA3*D42**2+KA4*(D43+L84)**2+2*KA7*D42-2*KA8*(D43+L84)+K
     *A11+KA12
      C(23,25) = KA4*D82*(D43+L84)-KA8*(C82-C43-L84)-KA12
      C(24,14) = KA2*C73+KA6
      C(24,16) = -KA2*D73-KA6
      C(24,22) = (D32+L37)*(KA2*D73+KA6)-KA6*D73-KA10
      C(24,24) = KA2*C73**2+2*KA6*D73+KA10
      C(25,15) = -KA4*D82-KA8
      C(25,17) = KA4*D82+KA8
      C(25,23) = (D43+L84)*(KA4*C82+KA8)-KA8*C82-KA12
      C(25,25) = KA4*D82**2+2*KA8*D82+KA12
      C(26,02) = -K32*C34
      C(26,03) = -KA1*D33-KA5
      C(26,04) = K32*D57*D34+(D56+L53)*(KA1*D33+KA5)-KA5*D33-KA9
      C(26,05) = -KA1*D33*XBAR-XBAR*KA5
      C(26,06) = K32*D34*XBAR
      C(26,10) = KA1*C33-KA2*(C32+L37)+KA5+KA6
      C(26,12) = KA2*(C32+L37)-KA6
      C(26,26) = K32*D34**2+KA1*D33**2+KA2*(D32+L37)**2+2*KA5*D33-2*KA6*
     *(D32+L37)+KA9+KA10
      C(26,28) = KA2*(D32+L37)*D73-KA6*(C73-C32-L37)-KA10
      C(27,02) = -K42*D44
      C(27,03) = KA3*C42+KA7
      C(27,04) = K42*D44*D57+(D55+L54)*(KA3*D42+KA7)-KA7*D42-KA11
      C(27,05) = KA3*C42*XBAR+XBAR*KA7
      C(27,06) = K42*D44*XBAR
      C(27,11) = -KA3*D42+KA4*(D43+L84)-KA7-KA8
      C(27,13) = -KA4*(D43+L84)+KA8
      C(27,27) = K42*D44**2+KA3*D42**2+KA4*(D43+L84)**2+2*KA7*D42-2*KA8*
     *(D43+L84)+KA11+KA12
      C(27,29) = KA4*(D43+L84)*D82-KA8*(C82-D43-L84)-KA12
      C(28,02) = -K72*D74
      C(28,04) = K72*C74*D57
      C(28,06) = K72*D74*XBAR
      C(28,10) = -KA2*D73-KA6
      C(28,12) = KA2*D73+KA6
      C(28,26) = KA2*D73*(C32+L37)+KA6*(D32+L37-D73)-KA10
      C(28,28) = K72*C74**2+KA2*D73**2+2*KA6*D73+KA10
      C(29,02) = -K82*D84
      C(29,04) = K82*D84*D57
      C(29,06) = K82*D84*XBAR
      C(29,11) = KA4*D82+KA8
      C(29,13) = -KA4*C82-KA8
      C(29,27) = KA4*C82*(C43+L84)+KA8*(D43+L84-D82)-KA12
      C(29,29) = K82*D84**2+KA4*D82**2+2*KA8*D82+KA12
C
C     THE NEXT 113 CARDS EVALUATE THE NONZERO ELEMENTS OF THE MATRIX D.
C
```

```
D(01,01) = -1.
D(01,04) = -K11
D(01,07) = -K21
D(01,22) = -C11
D(01,25) = -C21
D(02,02) = -1.
D(02,05) = -K12
D(02,08) = -K22
D(02,11) = -K32
D(02,14) = -K42
D(02,17) = -K72
D(02,20) = -K82
D(02,23) = -C12
D(02,26) = -C22
D(02,29) = -C32
D(02,32) = -C42
D(02,35) = -C72
D(02,38) = -C82
D(03,03) = -1.
D(03,06) = -K13
D(03,09) = -K23
D(03,24) = -C13
D(03,27) = -C23
D(04,02) = DB3
D(04,03) = YBAR
D(04,05) = K12*D53
D(04,06) = K13*YBAR
D(04,08) = K22*D53
D(04,09) = K23*YBAR
D(04,11) = K32*D57
D(04,14) = K42*D57
D(04,17) = K72*D57
D(04,20) = K82*D57
D(04,23) = C12*D53
D(04,24) = C13*YBAR
D(04,26) = C22*D53
D(04,27) = C23*YBAR
D(04,29) = C32*D57
D(04,32) = C42*D57
D(04,35) = C72*D57
D(04,38) = C82*D57
D(05,01) = -DB3
D(05,03) = -DB1
D(05,04) = -K11*(D53+D62)
D(05,06) = K13*D51
D(05,07) = -K21*(D53+D62)
D(05,09) = K23*D51
D(05,22) = -C11*(D53+D62)
D(05,24) = C13*D51
D(05,25) = -C21*(D53+D62)
D(05,27) = C23*D51
D(06,01) = -YBAR
D(06,02) = DB1
D(06,04) = -K11*(YBAR+D99+D13)
D(06,05) = -K12*D51
D(06,07) = -K21*(YBAR-D99-D22)
D(06,08) = -K22*D51
D(06,11) = K32*XBAR
D(06,14) = K42*XBAR
D(06,17) = K72*XBAR
```

```
      D(06,20) = K82*XBAR
      D(06,22) = -C11*(YBAR+D99+D13)
      C(06,23) = -C12*D51
      D(06,25) = -C21*(YBAR-D99-D22)
      C(06,26) = -C22*D51
      D(06,29) = C32*XBAR
      D(06,32) = C42*XBAR
      C(06,35) = C72*XBAR
      D(06,38) = C82*XBAR
      D(07,05) = K12*(D62+D14)
      C(07,06) = K13*(D99+C13)
      D(07,08) = K22*(D62+D24)
      C(07,09) = -K23*(D99+D22)
      C(07,23) = C12*(C62+C14)
      D(07,24) = C13*(D99+D13)
      C(07,26) = C22*(D62+D24)
      D(07,27) = -C23*(D99+D22)
      C(08,04) = -K11*D14
      C(08,22) = -C11*D14
      D(09,07) = -K21*D24
      C(09,25) = -C21*D24
      D(10,12) = -K33
      D(10,30) = -C33
      C(11,15) = -K43
      D(11,33) = -C43
      C(12,18) = -K73
      C(12,36) = -C73
      D(13,21) = -K83
      C(13,39) = -C83
      C(14,10) = -K31
      D(14,28) = -C31
      D(15,13) = -K41
      D(15,31) = -C41
      C(16,16) = -K71
      D(16,34) = -C71
      D(17,19) = -K81
      C(17,37) = -C81
      D(18,10) = -K31*D34
      D(18,28) = -C31*D34
      D(19,13) = -K41*D44
      D(19,31) = -C41*D44
      C(20,16) = -K71*D74
      D(20,34) = -C71*D74
      D(21,19) = -K81*D84
      C(21,37) = -C81*C84
      D(26,11) = K32*D34
      D(26,29) = C32*D34
      C(27,14) = K42*C44
      D(27,32) = C42*D44
      D(28,17) = K72*D74
      D(28,35) = C72*D74
      D(29,20) = K82*D84
      C(29,38) = C82*C84
C
C     THE NEXT 10 CARDS CAST THE A,B,C AND D MATRICIES INTO THE MATRIX G.
C
      DO 60 I=1,29
      DO 40 J=1,29
      G(I,J) = -A(I,J)
      G(I,J+29) = C(I,J)
```

```
      G(I,J+58) = B(I,J)
   40 CONTINUE
      DO 50 J=1,39
      G(I,J+87) = O(I,J)
   50 CONTINUE
   60 CONTINUE
      CALL GAUSS (G,N,N1)
      WRITE (10) G
C
C     THE NEXT 13 CARDS PRODUCE A FORMATTED COPY OF THE NONZERO ELEMENTS
C     OF THE MATRIX G.
C
      DO 80 I=1,29
      WRITE (6,150) I
      DO 70 J=1,126
      IF (J.EQ.1) ID = 1
      IF (ABS(G(I,J)).LE.0.0001) GO TO 70
      IF (ID.EQ.1) WRITE (6,90) G(I,J),NAME(J)
      IF (ID.EQ.2) WRITE (6,100) G(I,J),NAME(J)
      IF (ID.EQ.3) WRITE (6,110) G(I,J),NAME(J)
      IF (ID.EQ.4) WRITE (6,120) G(I,J),NAME(J)
      IF (ID.EQ.4) ID = 1
      ID = ID+1
   70 CONTINUE
   80 CONTINUE
      STOP
   90 FORMAT (' ',20X,F3.1,1X,A4,'=')
  100 FORMAT ('+',T31,'+(',F13.4,')',A4)
  110 FORMAT ('+',T51,'+(',F13.4,')',A4)
  120 FORMAT ('+',T71,'+(',F13.4,')',A4,/)
  130 FORMAT (20A4)
  140 FORMAT (8F10.2)
  150 FORMAT ('-',47X,'EQUATION',I5)
  160 FORMAT (2E10.4)
      END
```

```
      SUBROUTINE GAUSS (A,N,N1)
C
C     THIS PROGRAM PERFORMS A GAUSS-JORDAN REDUCTION ON AN N X N1 MATRIX A.
C
      DIMENSION A(N,N1)
      DO 80 J=1,N
      BIG = ABS(A(J,J))
      KSAVE = J
      IF (J.EQ.N) GO TO 30
      KK = J+1
      DO 10 K=KK,N
      IF (ABS(A(K,J)).LE.BIG) GO TO 10
      BIG = ABS(A(K,J))
      KSAVE = K
   10 CONTINUE
      DO 20 M=J,N1
      DUMMY = A(KSAVE,M)
      A(KSAVE,M) = A(J,M)
   20 A(J,M) = DUMMY
   30 DIV = A(J,J)
      S = 1.0/DIV
      DO 40 K=J,N1
   40 A(J,K) = A(J,K)*S
      DO 70 I=1,N
      IF (I-J) 50,70,50
   50 AIJ = -A(I,J)
      DO 60 K=J,N1
   60 A(I,K) = A(I,K)+AIJ*A(J,K)
   70 CONTINUE
   80 CONTINUE
      RETURN
      END
```

```
C     THIS PROGRAM INTEGRATES THE EQUATIONS OF MOTION USING THE RUNGE-KUTTA-
C     GILL METHOD.
C
      DIMENSION G(29,126), H(58,97), Q(58), X(58), DX(58), Y(39)
      COMMON H,Y
      REWIND 10
C
C     THE MATRIX G IS READ FROM DISK.
C
      READ (10) G
C
C     THE NEXT THREE CARDS READ IN THE INITIAL CONDITIONS, INCREMENT SIZE,
C     AND NUMBER OF STEPS TO BE TAKEN.
C
      READ (5,90) (X(I),I=1,58)
      READ (5,90) DELT
      READ (5,100) NSTEP
C
C     THE NEXT 4 CARDS INITIALIZE THE H MATRIX.
C
      DO 20 I=1,58
      DO 10 J=1,97
   10 H(I,J) = 0.0
   20 CONTINUE
C
C     THE NEXT 5 CARDS CAST THE G MATRIX INTO THE H MATRIX. H CONTAINS THE
C     EXPRESSIONS FOR THE DERIVATIVE OF EACH X.
C
      DO 40 I=1,29
      H(I,I+29) = 1.0
      DO 30 J=1,97
   30 H(I+29,J) = G(I,J+29)
   40 CONTINUE
C
C     THE NEXT 4 CARDS INITIALIZE THE WORK VECTOR Q, AND THE SURFACE
C     CONDITION VECTOR Y.
C
      DO 50 I=1,58
   50 Q(I) = 0.0
      DO 60 I=1,39
   60 Y(I) = 0.0
C
C     NEQ IS THE NUMBER OF FIRST ORDER EQUATIONS TO BE INTEGRATED.
C
      NEQ = 58
C
C     THE TIME IS INITIALLY 0.0 .
C
      T = 0.0
C
C     THE NEXT 4 CARDS PRODUCE A HARD COPY OF THE INITIAL TIME AND DISPLACEMENTS
C     , AND WRITE THE VARIABLE T AND VECTOR X ON DISK.
C
      WRITE (6,120) T
      WRITE (6,110) (X(I),I=1,29)
      WRITE (10) T
      WRITE (10) X
C
C     THE NEXT 8 CARDS CALL THE RKG SUBROUTINE.  THE PROCEDURE IS PERFORMED
C     TEN TIMES FOR EVERY STEP REQUESTED BY NSTEP.  T AND X ARE PRINTED
```

```
C     OUT AND STORED ON DISK.
C
      DO 80 J=1,NSTEP
      DO 70 K=1,10
      CALL RKG (NEQ,DELT,T,X,DX,Q)
   70 CONTINUE
      WRITE (10) T
      WRITE (10) X
      WRITE (6,120) T
   80 WRITE (6,110) (X(I),I=1,29)
      STOP
   90 FORMAT (8F10.4)
  100 FORMAT (I3)
  110 FORMAT (' ',12F10.4)
  120 FORMAT ('-',F15.5)
      END
```

```
      SUBROUTINE RKG(NEQ,H,X,Y,DY,Q)
C
C     THE INDEPENDENT VARIABLE X IS INCREMENTED IN THIS PROGRAM
C     Y(I) AND DY(I) ARE THE DEPENDENT VARIABLE AND ITS DERIVATIVE
C     ALL THE Q(I) MUST BE INITIALLY SET TO ZERO IN THE MAIN PROGRAM
C     H = INTERVAL SIZE
C     NEQ = NUMBER OF FIRST ORDER EQUATIONS
C     A SUBROUTINE DERIV(NEQ,X,Y,DY) MUST BE PROVIDED
C
      DIMENSION  A(2)
      DIMENSION Y(NEQ),DY(NEQ),Q(NEQ)
      A(1)=0.292893218813452475
      A(2)=1.70710678118654752
      H2=.5*H
      CALL DERIV(NEQ,X,Y,DY)
      DO 13 I=1,NEQ
      B=H2*DY(I)-Q(I)
      Y(I)=Y(I)+B
   13 Q(I)=Q(I)+3.*B-H2*DY(I)
      X=X+H2
      DO 20 J=1,2
      CALL DERIV(NEQ,X,Y,DY)
      DO 20 I=1,NEQ
      B=A(J)*(H*DY(I)-Q(I))
      Y(I)=Y(I)+B
   20 Q(I)=Q(I)+3.*B-A(J)*H*DY(I)
      X=X+H2
      CALL DERIV(NEQ,X,Y,DY)
      DO 26 I=1,NEQ
      B=0.166666666666666666*(H*DY(I)-2.*Q(I))
      Y(I)=Y(I)+B
   26 Q(I)=Q(I)+3.*B-H2*DY(I)
      RETURN
      END
```

```
      SUBROUTINE DERIV(NEQ,T,X,DX)
C
C     THIS SUBROUTINE EVALUATES THE DERIVATIVE OF EACH X FOR USE IN THE RKG
C     SUBROUTINE.
C
      DIMENSION X(NEQ),DX(NEQ),H(58,97),Y(39)
      COMMON H,Y
C
C     THE NEXT 11 CARDS EVALUATE THE SURFACE CONDITIONS FOR THE LEFT REAR
C     WHEELS TRAVERSING A SINUSOIDAL BUMP.
C
      IF(T.GE.0.6815) GO TO 10
      Y(12)=-0.416*SIN(4.61*T)
      Y(18)=Y(12)
      Y(30)=-1.918*COS(4.61*T)
      Y(36)=Y(30)
      GO TO 20
   10 Y(12)=0.0
      Y(18)=0.0
      Y(30)=0.0
      Y(36)=0.0
   20 CONTINUE
C
C     THE NEXT 7 CARDS EVALUATE THE DERIVATIVES.
C
      DO 50 I=1,58
      DX(I)=0.0
      DO 30 J=1,58
   30 DX(I)=DX(I)+H(I,J)*X(J)
      DO 40 J=1,39
   40 DX(I)=DX(I)+H(I,J+58)*Y(J)
   50 CONTINUE
      RETURN
      END
```

APPENDIX B

DERIVATION OF THE POTENTIAL ENERGY IN THE REAR AXLES

The rear axles are treated as cantilever beams with lumped masses at their ends, as in Figure B1

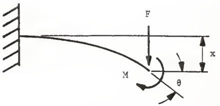

FIGURE B1

The lumped mass causes a force F and moment M to be transferred to the end of the beam. The deflection, x, and slope, θ, at the end of the beam are obtained using elementary beam theory and superposition.

$$x = FL^3/3EI + ML^2/2EI \tag{B1}$$

$$\theta = FL^2/2EI + ML/EI \tag{B2}$$

Equations B1 and B2 may be used to find expressions for F and M in terms of x and θ.

$$F = 12EI/L^3\ x - 6EI/L^2\ \theta \tag{B3}$$

$$M = -\ 6EI/L^2\ x + 4EI/L\ \theta \tag{B4}$$

The differential of energy for the beam is given by

$$dU = Fdx + Md\theta \tag{B5}$$

$dF/d\theta$ is equal to dM/dx, therefore B5 is an exact differential, which implies

$$dU/dx = F \tag{B6}$$

and

$$dU/d\theta = M \tag{B7}$$

Substitution of (B3) into (B6) and integrating yields

$$U = 6EI/L^3 \cdot x^2 - 6EI/L^2 \cdot x \cdot \theta + f(\theta) \tag{B8}$$

Substituting (B8) into (B7) yields

$$dU/d\theta = - 6EI/L^2 \cdot x + f' \cdot (\theta) \tag{B9}$$

which must correspond to (B4). To do this,

$$f'(\theta) = 4EI/L \cdot \theta \tag{B10}$$

Integrating (B10) gives

$$f(\theta) = 2EI/L \cdot \theta^2 + C \tag{B11}$$

where the constant C is a reference level which may be set to zero for this problem. Substituting (B11) into (B8) yields the final expression for the energy in the axle

$$U = 6EI/L^3 \cdot x^2 - 6EI/L^2 \cdot x\theta + 2EI/L \cdot \theta^2 \tag{B12}$$

APPENDIX C

SAMPLE EQUATIONS OF MOTION AND INTEGRATION RESULTS

```
                              EQUATION     1
1.0 AX1 =+(  -29185.6641)X1  +(     969.5659)X3  +(  -17305.6992)X5
         +(    -128.5317)X8  +(    -128.5317)X9  +(    -492.2498)X10
         +(    -492.2498)X11 +(   14478.0703)X14 +(   14478.0703)X15
         +(     -50.9822)X18 +(     -50.9822)X19 +(  -19255.8242)X22
         +(   19255.8242)X23 +(    -654.6924)X26 +(     654.6924)X27
         +(      -1.2624)VX1 +(      -0.1229)VX3 +(      -1.5056)VX5
         +(      -0.7069)VX8 +(      -0.7069)VX9 +(       0.0073)FD81
         +(      -0.0003)FD83+(     114.7605)Y11 +(       7.4674)Y13
         +(     114.7605)Y21 +(       7.4674)Y23 +(       0.6312)YD11
         +(       0.0615)YD13+(       0.6312)YD21+(       0.0615)YD23

                              EQUATION     2
1.0 AX2 =+(    -348.9856)X2  +(  -35122.5234)X4  +(  -16947.1680)X6
         +(     140.6962)X7  +(     -12.4841)X8  +(      12.4841)X9
         +(   -4527.2656)X10 +(    4527.2656)X11 +(    2163.8501)X14
         +(   -2163.8501)X15 +(   -2689.5959)X22 +(   -2689.5959)X23
         +(   -5515.5625)X26 +(   -5515.5625)X27 +(     111.6813)X28
         +(     111.6813)X29 +(      -0.8942)VX2 +(       0.8574)VX4
         +(       0.0207)VX6 +(       0.3287)VX7 +(      -0.0687)VX8
         +(       0.0687)VX9 +(       0.3003)VX26+(       0.3003)VX27
         +(       0.3003)VX28+(       0.3003)VX29+(       0.0046)FD82
         +(      11.1466)Y11 +(      62.8117)Y12 +(     -11.1466)Y21
         +(      62.8117)Y22 +(      55.8406)Y32 +(      55.8406)Y42
         +(      55.8406)Y72 +(      55.8406)Y82 +(       0.0613)YD11
         +(       0.1468)YD12+(      -0.0613)YD21+(       0.1468)YD22
         +(       0.1502)YD32+(       0.1502)YD42+(       0.1502)YD72
         +(       0.1502)YD82

                              EQUATION     3
1.0 AX3 =+(    -529.2183)X1  +(  -31801.3750)X3  +(  -72931.5625)X5
         +(      -5.9210)X8  +(      -5.9210)X9  +(   15753.1133)X10
         +(   15753.1133)X11 +(     259.3223)X14 +(     259.3223)X15
         +(     120.7128)X18 +(     120.7128)X19 +(    -344.8989)X22
         +(     344.8989)X23 +(   20951.6328)X26 +(  -20951.6328)X27
         +(      -0.0582)VX1 +(      -2.4291)VX3 +(       8.8245)VX5
         +(      -0.0326)VX8 +(      -0.0326)VX9 +(       0.0001)FD81
         +(       0.0079)FD83+(       5.2866)Y11 +(     147.5721)Y13
         +(       5.2866)Y21 +(     147.5721)Y23 +(       0.0291)YD11
         +(       1.2145)YD13+(       0.0291)YD21+(       1.2145)YD23

                              EQUATION     4
1.0 AX4 =+(     119.2762)X2  +( -147657.3125)X4  +(     472.3611)X6
         +(     -77.8404)X7  +(       0.2446)X8  +(      -0.2446)X9
         +(  -18830.1289)X10 +(   18830.1289)X11 +(     -42.4043)X14
         +(      42.4043)X15 +(      52.7072)X22 +(      52.7072)X23
         +(  -23430.1250)X26 +(  -23430.1250)X27 +(     -24.8879)X28
```

```
                              EQUATION    9
1.0 AX9 =+(  -15719.3008)X1  +(  -24364.9023)X5  +(   38669.4688)X6
         +(  -17605.6094)X9  +(      -86.4562)VX1 +(     -134.0070)VX5
         +(     212.6821)VX6 +(      -96.8309)VX9 +(   15719.3008)Y21
         +(      86.4562)YD21

                              EQUATION   10
1.0 AX10=+(  176081.1250)X3  +( -674390.8125)X4  +(  410269.0625)X5
         +( -595268.4375)X10 +(  417377.8750)X12 +(  216580.2500)X26
         +(  450767.8125)X28 +(      -21.8888)VX10+(    1809.3555)Y33
         +(      21.8888)YD33

                              EQUATION   11
1.0 AX11=+(  176081.1250)X3  +(  674390.8125)X4  +(  410269.0625)X5
         +( -595268.4375)X11 +(  417377.8750)X13 +( -216580.2500)X27
         +( -450767.8125)X29 +(      -21.8888)VX11+(    1809.3555)Y43
         +(      21.8888)YD43

                              EQUATION   12
1.0 AX12=+(  417377.8750)X10 +( -419187.2500)X12 +( -450768.2500)X26
         +( -450767.8125)X28 +(      -21.8888)VX12+(    1809.3555)Y73
         +(      21.8888)YD73

                              EQUATION   13
1.0 AX13=+(  417377.8750)X11 +( -419187.2500)X13 +(  450768.2500)X27
         +(  450767.8125)X29 +(      -21.8888)VX13+(    1809.3555)Y83
         +(      21.8888)YD83

                              EQUATION   14
1.0 AX14=+(  176081.1250)X1  +(  117974.2500)X5  +(  674390.8125)X6
         +( -595047.7500)X14 +(  417377.8750)X16 +(   -3177.4048)X18
         +( -216580.2500)X22 +( -450767.8125)X24 +(     -11.8270)VX14
         +(     -23.6540)VX18+(    1588.7024)Y31 +(      11.8270)Y031

                              EQUATION   15
1.0 AX15=+(  176081.1250)X1  +(  117974.2500)X5  +( -674390.8125)X6
         +( -595047.7500)X15 +(  417377.8750)X17 +(   -3177.4048)X19
         +(  216580.2500)X23 +(  450767.8125)X25 +(     -11.8270)VX15
         +(     -23.6540)VX19+(    1588.7024)Y41 +(      11.8270)Y041

                              EQUATION   16
1.0 AX16=+(  417377.8750)X14 +( -418966.5625)X16 +(   -3177.4048)X20
         +(  450768.2500)X22 +(  450767.8125)X24 +(     -11.8270)VX16
         +(     -23.6540)VX20+(    1588.7024)Y71 +(      11.8270)Y071

                              EQUATION   17
1.0 AX17=+(  417377.8750)X15 +( -418966.5625)X17 +(   -3177.4048)X21
         +( -450768.2500)X23 +( -450767.8125)X25 +(     -11.8270)VX17
         +(     -23.6540)VX21+(    1588.7024)Y81 +(      11.8270)Y081
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          +(     -24.8879)X29 +(       0.2962)VX2 +(        -0.3414)VX4
          +(       0.2900)VX6 +(      -0.1819)VX7 +(         0.0013)VX8
          +(      -0.0013)VX9 +(      -0.0669)VX26+(        -0.0669)VX27
          +(      -0.0669)VX28+(      -0.0669)VX29+(        -0.0010)F082
          +(      -0.2184)Y11 +(     -34.7502)Y12 +(         0.2184)Y21
          +(     -34.7502)Y22 +(     -12.4439)Y32 +(       -12.4439)Y42
          +(     -12.4439)Y72 +(     -12.4439)Y82 +(        -0.0012)Y011
          +(      -0.0812)Y012+(       0.0012)Y021+(        -0.0812)Y022
          +(      -0.0335)Y032+(      -0.0335)Y042+(        -0.0335)Y072
          +(      -0.0335)Y082

                               EQUATION   5
1.0 AX5 =+(   -2251.2527)X1  +(   -9765.6523)X3  +(  -26231.3945)X5
          +(     -25.1876)X8  +(     -25.1876)X9  +(    4958.0352)X10
          +(    4958.0352)X11 +(    1103.1370)X14 +(    1103.1370)X15
          +(     513.5024)X18 +(     513.5024)X19 +(   -1467.1724)X22
          +(    1467.1724)X23 +(    6594.1875)X26 +(   -6594.1875)X27
          +(      -0.2474)VX1 +(       1.2380)VX3 +(        -4.9269)VX5
          +(      -0.1385)VX8 +(      -0.1385)VX9 +(         0.0006)F081
          +(       0.0026)F083+(      22.4889)Y11 +(       -75.2128)Y13
          +(      22.4889)Y21 +(     -75.2128)Y23 +(         0.1237)Y011
          +(      -0.6190)Y013+(       0.1237)Y021+(        -0.6190)Y023


                               EQUATION   6
1.0 AX6 =+(       2.9318)X2  +(     384.4805)X4  +(  -42736.9102)X6
          +(      69.0413)X7  +(     -31.0957)X8  +(       31.0957)X9
          +(      42.4044)X10 +(     -42.4044)X11 +(     5389.7500)X14
          +(   -5389.7500)X15 +(   -6699.2852)X22 +(    -6699.2852)X23
          +(      20.4193)X26 +(      20.4193)X27 +(      -32.2880)X28
          +(     -32.2880)X29 +(       0.0296)VX2 +(         0.1069)VX4
          +(      -1.6845)VX6 +(       0.1613)VX7 +(        -0.1710)VX8
          +(       0.1710)VX9 +(      -0.0868)VX26+(        -0.0868)VX27
          +(      -0.0868)VX28+(      -0.0868)VX29+(        -0.0015)F082
          +(      27.7640)Y11 +(      30.8220)Y12 +(       -27.7640)Y21
          +(      30.8220)Y22 +(     -16.1440)Y32 +(       -16.1440)Y42
          +(     -16.1440)Y72 +(     -16.1440)Y82 +(         0.1527)Y011
          +(       0.0720)Y012+(      -0.1527)Y021+(         0.0720)Y022
          +(      -0.0434)Y032+(      -0.0434)Y042+(        -0.0434)Y072
          +(      -0.0434)Y082

                               EQUATION   7
1.0 AX7 =+(     742.3000)X2  +(   -1150.5642)X4  +(     2724.2400)X6
          +(   -9302.7891)X7  +(       1.7343)VX2 +(        -2.6882)VX4
          +(       6.3650)VX6 +(     -71.6630)VX7 +(      -371.1499)Y12
          +(   -1721.8335)Y13 +(    -371.1499)Y22 +(     1721.8335)Y23
          +(      -0.8672)Y012+(     -14.1708)Y013+(        -0.8672)Y022
          +(      14.1708)Y023

                               EQUATION   8
1.0 AX8 =+(  -15719.3008)X1  +(  -24364.9023)X5  +(  -38669.4688)X6
          +(  -17605.6094)X8  +(     -86.4562)VX1 +(      -134.0070)VX5
          +(    -212.6821)VX6 +(     -96.8309)VX8 +(    15719.3008)Y11
          +(      86.4562)Y011
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                              EQUATION   18
1.0 AX18=+(    23389.2422)X5  +(    -1588.7031)X14 +(  -57752.2734)X18
          +(    31185.6523)X20 +(      -11.8270)VX14+(      -23.6540)VX18
          +(     1588.7031)Y31 +(       11.8270)Y031


                              EQUATION   19
1.0 AX19=+(    23389.2422)X5  +(    -1588.7031)X15 +(  -57752.2734)X19
          +(    31185.6523)X21 +(      -11.8270)VX15+(      -23.6540)VX19
          +(     1588.7031)Y41 +(       11.8270)Y041


                              EQUATION   20
1.0 AX20=+(    -1588.7031)X16 +(    31185.6523)X18 +(  -34363.0586)X20
          +(      -11.8270)VX16+(      -23.6540)VX20+(     1588.7031)Y71
          +(       11.8270)Y071


                              EQUATION   21
1.0 AX21=+(    -1588.7031)X17 +(    31185.6523)X19 +(  -34363.0586)X21
          +(      -11.8270)VX17+(      -23.6540)VX21+(     1588.7031)Y81
          +(       11.8270)Y081


                              EQUATION   22
1.0 AX22=+( -234187.8750)X1  +( -156905.8125)X5  +( -838245.6875)X6
          +( -216580.2500)X14 +(  450768.2500)X16 +( -935251.1875)X22
          +( -408571.0000)X24


                              EQUATION   23
1.0 AX23=+(  234187.8750)X1  +(  156905.8125)X5  +( -838245.6875)X6
          +(  216580.2500)X15 +( -450768.2500)X17 +( -935251.1875)X23
          +( -408571.0000)X25


                              EQUATION   24
1.0 AX24=+( -450767.8125)X14 +(  450767.8125)X16 +( -408571.0000)X22
          +( -565087.3750)X24


                              EQUATION   25
1.0 AX25=+(  450767.8125)X15 +( -450767.8125)X17 +( -408571.0000)X23
          +( -565087.3750)X25


                              EQUATION   26
1.0 AX26=+(     2100.6174)X2  +(  234187.8750)X3  +( -839653.0625)X4
          +(  545657.8125)X5  +(    -4894.4375)X6  +(  216580.2500)X10
          +( -450768.2500)X12 +( -939452.4375)X26 +( -408571.0000)X28
          +(        5.6487)VX2 +(       -3.7846)VX4 +(      -13.1615)VX6
          +(      -11.2974)VX26+(    -2100.6174)Y32 +(       -5.6487)Y032


                              EQUATION   27
1.0 AX27=+(     2100.6174)X2  +( -234187.8750)X3  +( -839653.0625)X4
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+( -545657.8125)X5  +(   -4894.4375)X6  +( -216580.2500)X11
+(  450768.2500)X13 +( -939452.4375)X27 +( -408571.0000)X29
+(       5.6487)VX2 +(      -3.7846)VX4 +(     -13.1615)VX6
+(     -11.2974)VX27+(   -2100.6174)Y42 +(      -5.6487)YD42


                     EQUATION   28
1.0 AX28=+(    2100.6174)X2  +(   -1407.4133)X4  +(   -4894.4375)X6
         +(  450767.8125)X10 +( -450767.8125)X12 +( -408570.6250)X26
         +( -569288.6250)X28 +(       5.6487)VX2 +(      -3.7846)VX4
         +(     -13.1615)VX6 +(     -11.2974)VX28+(   -2100.6174)Y72
         +(      -5.6487)YD72


                     EQUATION   29
1.0 AX29=+(    2100.6174)X2  +(   -1407.4133)X4  +(   -4894.4375)X6
         +( -450767.8125)X11 +(  450767.8125)X13 +( -408570.6250)X27
         +( -569288.6250)X29 +(       5.6487)VX2 +(      -3.7846)VX4
         +(     -13.1615)VX6 +(     -11.2974)VX29+(   -2100.6174)Y82
         +(      -5.6487)YD82
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time (sec.)	x_1 (ft.)	x_2 (ft.)	x_3 (ft.)	x_4 (rad.)	x_5 (rad.)	x_6 (rad.)	x_7 (rad.)	x_8 (rad.)	x_9 (rad.)
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.020	0.0000	0.0002	-0.0006	0.0006	-0.0002	-0.0000	-0.0000	0.0001	0.0001
0.040	0.0003	0.0011	-0.0042	0.0031	-0.0012	-0.0000	-0.0002	0.0013	0.0012
0.060	0.0011	0.0034	-0.0133	0.0072	-0.0037	-0.0001	-0.0005	0.0042	0.0037
0.080	0.0031	0.0076	-0.0278	0.0115	-0.0074	-0.0002	-0.0009	0.0080	0.0070
0.100	0.0071	0.0141	-0.0455	0.0154	-0.0114	-0.0004	-0.0010	0.0108	0.0089
0.120	0.0136	0.0232	-0.0645	0.0183	-0.0154	-0.0007	-0.0008	0.0111	0.0081
0.140	0.0231	0.0343	-0.0832	0.0204	-0.0192	-0.0009	-0.0003	0.0083	0.0043
0.160	0.0354	0.0467	-0.0994	0.0223	-0.0224	-0.0011	0.0004	0.0023	-0.0025
0.180	0.0499	0.0594	-0.1114	0.0241	-0.0252	-0.0012	0.0012	-0.0066	-0.0119
0.200	0.0660	0.0713	-0.1186	0.0262	-0.0277	-0.0012	0.0019	-0.0177	-0.0230
0.220	0.0827	0.0813	-0.1218	0.0286	-0.0302	-0.0011	0.0025	-0.0295	-0.0346
0.240	0.0989	0.0888	-0.1223	0.0307	-0.0327	-0.0011	0.0029	-0.0409	-0.0455
0.260	0.1136	0.0932	-0.1213	0.0324	-0.0352	-0.0010	0.0032	-0.0509	-0.0552
0.280	0.1261	0.0946	-0.1204	0.0335	-0.0374	-0.0010	0.0032	-0.0590	-0.0633
0.300	0.1358	0.0930	-0.1208	0.0339	-0.0392	-0.0010	0.0030	-0.0650	-0.0694
0.320	0.1421	0.0889	-0.1228	0.0341	-0.0405	-0.0011	0.0027	-0.0687	-0.0736
0.340	0.1447	0.0831	-0.1261	0.0338	-0.0411	-0.0012	0.0022	-0.0700	-0.0754
0.360	0.1436	0.0762	-0.1296	0.0334	-0.0410	-0.0014	0.0017	-0.0689	-0.0749
0.380	0.1390	0.0692	-0.1325	0.0329	-0.0402	-0.0014	0.0012	-0.0656	-0.0719
0.400	0.1312	0.0627	-0.1334	0.0320	-0.0389	-0.0014	0.0007	-0.0604	-0.0668
0.420	0.1206	0.0575	-0.1316	0.0310	-0.0371	-0.0014	0.0004	-0.0535	-0.0556
0.440	0.1078	0.0539	-0.1268	0.0297	-0.0349	-0.0013	0.0002	-0.0453	-0.0509
0.460	0.0937	0.0518	-0.1192	0.0281	-0.0323	-0.0011	0.0003	-0.0365	-0.0414
0.480	0.0790	0.0512	-0.1094	0.0264	-0.0294	-0.0010	0.0004	-0.0276	-0.0319
0.500	0.0644	0.0516	-0.0984	0.0246	-0.0262	-0.0008	0.0007	-0.0193	-0.0229
0.520	0.0505	0.0522	-0.0868	0.0226	-0.0229	-0.0007	0.0011	-0.0117	-0.0149
0.540	0.0377	0.0525	-0.0755	0.0204	-0.0194	-0.0007	0.0014	-0.0051	-0.0080
0.560	0.0265	0.0516	-0.0648	0.0181	-0.0160	-0.0006	0.0017	0.0002	-0.0026
0.580	0.0170	0.0491	-0.0547	0.0156	-0.0128	-0.0006	0.0018	0.0040	0.0015
0.600	0.0094	0.0445	-0.0451	0.0129	-0.0098	-0.0005	0.0019	0.0066	0.0042
0.620	0.0034	0.0377	-0.0356	0.0100	-0.0071	-0.0005	0.0017	0.0079	0.0059
0.640	-0.0009	0.0288	-0.0257	0.0070	-0.0045	-0.0003	0.0015	0.0080	0.0065
0.660	-0.0040	0.0181	-0.0151	0.0038	-0.0021	-0.0002	0.0011	0.0070	0.0062
0.680	-0.0061	0.0061	-0.0038	0.0005	0.0003	-0.0000	0.0006	0.0052	0.0051
0.700	-0.0079	-0.0063	0.0079	-0.0023	0.0026	0.0001	0.0000	0.0030	0.0036
0.720	-0.0095	-0.0178	0.0172	-0.0032	0.0042	0.0002	-0.0007	0.0018	0.0030
0.740	-0.0112	-0.0272	0.0214	-0.0024	0.0045	0.0003	-0.0016	0.0028	0.0042
0.760	-0.0128	-0.0332	0.0199	-0.0012	0.0036	0.0003	-0.0023	0.0057	0.0069
0.780	-0.0139	-0.0350	0.0143	-0.0001	0.0023	0.0001	-0.0028	0.0090	0.0056
0.800	-0.0140	-0.0322	0.0065	0.0002	0.0011	-0.0000	-0.0028	0.0113	0.0111
0.820	-0.0129	-0.0252	-0.0022	-0.0003	0.0001	-0.0002	-0.0023	0.0120	0.0112
0.840	-0.0106	-0.0150	-0.0096	-0.0007	-0.0004	-0.0003	-0.0015	0.0111	0.0096
0.860	-0.0073	-0.0031	-0.0139	-0.0010	-0.0005	-0.0003	-0.0005	0.0082	0.0066
0.880	-0.0032	0.0089	-0.0142	-0.0007	-0.0003	-0.0002	0.0005	0.0039	0.0028
0.900	0.0013	0.0194	-0.0112	0.0000	-0.0002	-0.0001	0.0014	-0.0007	-0.0011
0.920	0.0055	0.0271	-0.0060	0.0008	-0.0002	0.0001	0.0020	-0.0050	-0.0045
0.940	0.0092	0.0312	0.0002	0.0013	-0.0005	0.0003	0.0024	-0.0083	-0.0071
0.960	0.0119	0.0312	0.0056	0.0015	-0.0008	0.0003	0.0025	-0.0105	-0.0089
0.980	0.0137	0.0272	0.0090	0.0013	-0.0011	0.0003	0.0023	-0.0115	-0.0100
1.000	0.0144	0.0200	0.0097	0.0010	-0.0014	0.0002	0.0018	-0.0115	-0.0105

time (sec.)	x_{10} (ft.)	x_{11} (ft.)	x_{12} (ft.)	x_{13} (ft.)	x_{14} (ft.)	x_{15} (ft.)	x_{16} (ft.)	x_{17} (ft.)	x_{18} (rad.)	x_{19} (rad.)
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.020	-0.0062	0.0008	-0.0106	0.0002	-0.0001	-0.0000	0.0000	0.0000	-0.0001	-0.0001
0.040	-0.0275	0.0045	-0.0408	0.0038	-0.0004	-0.0002	-0.0000	0.0001	-0.0007	-0.0007
0.060	-0.0622	0.0095	-0.0820	0.0154	-0.0010	-0.0002	0.0002	0.0009	-0.0025	-0.0026
0.080	-0.1050	0.0104	-0.1301	0.0281	-0.0012	0.0007	0.0013	0.0033	-0.0056	-0.0058
0.100	-0.1509	0.0039	-0.1816	0.0306	-0.0001	0.0036	0.0035	0.0076	-0.0091	-0.0095
0.120	-0.1940	-0.0090	-0.2306	0.0243	0.0030	0.0088	0.0067	0.0135	-0.0126	-0.0133
0.140	-0.2313	-0.0241	-0.2714	0.0164	0.0087	0.0167	0.0116	0.0215	-0.0162	-0.0172
0.160	-0.2634	-0.0370	-0.3053	0.0100	0.0175	0.0272	0.0194	0.0316	-0.0198	-0.0210
0.180	-0.2914	-0.0453	-0.3376	0.0068	0.0291	0.0398	0.0301	0.0436	-0.0233	-0.0246
0.200	-0.3159	-0.0477	-0.3676	0.0085	0.0426	0.0535	0.0423	0.0559	-0.0266	-0.0279
0.220	-0.3368	-0.0450	-0.3933	0.0155	0.0568	0.0672	0.0549	0.0677	-0.0300	-0.0311
0.240	-0.3538	-0.0406	-0.4148	0.0246	0.0709	0.0803	0.0678	0.0792	-0.0334	-0.0344
0.260	-0.3673	-0.0370	-0.4319	0.0310	0.0839	0.0925	0.0800	0.0903	-0.0366	-0.0376
0.280	-0.3774	-0.0360	-0.4445	0.0337	0.0946	0.1029	0.0901	0.1000	-0.0395	-0.0404
0.300	-0.3844	-0.0379	-0.4523	0.0332	0.1023	0.1111	0.0971	0.1075	-0.0416	-0.0426
0.320	-0.3893	-0.0420	-0.4562	0.0297	0.1069	0.1167	0.1010	0.1127	-0.0431	-0.0442
0.340	-0.3923	-0.0474	-0.4579	0.0243	0.1085	0.1195	0.1023	0.1156	-0.0436	-0.0449
0.360	-0.3930	-0.0525	-0.4570	0.0184	0.1072	0.1193	0.1011	0.1161	-0.0434	-0.0448
0.380	-0.3906	-0.0564	-0.4523	0.0134	0.1029	0.1158	0.0972	0.1132	-0.0423	-0.0438
0.400	-0.3841	-0.0584	-0.4436	0.0098	0.0962	0.1093	0.0909	0.1071	-0.0405	-0.0420
0.420	-0.3729	-0.0580	-0.4304	0.0075	0.0876	0.1001	0.0831	0.0986	-0.0382	-0.0396
0.440	-0.3568	-0.0552	-0.4123	0.0069	0.0776	0.0890	0.0744	0.0885	-0.0355	-0.0367
0.460	-0.3360	-0.0500	-0.3896	0.0084	0.0666	0.0766	0.0648	0.0771	-0.0323	-0.0334
0.480	-0.3116	-0.0428	-0.3629	0.0112	0.0551	0.0637	0.0544	0.0649	-0.0289	-0.0298
0.500	-0.2844	-0.0344	-0.3336	0.0150	0.0437	0.0511	0.0439	0.0527	-0.0252	-0.0260
0.520	-0.2556	-0.0257	-0.3021	0.0191	0.0329	0.0394	0.0338	0.0416	-0.0215	-0.0222
0.540	-0.2259	-0.0176	-0.2691	0.0226	0.0231	0.0290	0.0247	0.0317	-0.0178	-0.0184
0.560	-0.1959	-0.0108	-0.2351	0.0248	0.0145	0.0200	0.0164	0.0231	-0.0142	-0.0149
0.580	-0.1657	-0.0059	-0.2001	0.0249	0.0073	0.0126	0.0091	0.0156	-0.0110	-0.0116
0.600	-0.1352	-0.0028	-0.1640	0.0229	0.0018	0.0066	0.0034	0.0095	-0.0080	-0.0086
0.620	-0.1043	-0.0013	-0.1268	0.0191	-0.0021	0.0021	-0.0005	0.0048	-0.0055	-0.0060
0.640	-0.0725	-0.0006	-0.0882	0.0140	-0.0044	-0.0012	-0.0029	0.0010	-0.0033	-0.0036
0.660	-0.0397	-0.0004	-0.0483	0.0081	-0.0055	-0.0037	-0.0045	-0.0021	-0.0012	-0.0014
0.680	-0.0058	0.0002	-0.0072	0.0021	-0.0057	-0.0054	-0.0052	-0.0048	0.0007	0.0007
0.700	0.0239	0.0019	0.0261	-0.0034	-0.0056	-0.0067	-0.0055	-0.0070	0.0026	0.0028
0.720	0.0393	0.0072	0.0395	-0.0054	-0.0058	-0.0081	-0.0058	-0.0089	0.0041	0.0044
0.740	0.0407	0.0149	0.0410	0.0014	-0.0069	-0.0097	-0.0066	-0.0104	0.0045	0.0049
0.760	0.0331	0.0193	0.0344	0.0112	-0.0087	-0.0113	-0.0078	-0.0113	0.0039	0.0042
0.780	0.0206	0.0168	0.0223	0.0118	-0.0107	-0.0122	-0.0090	-0.0111	0.0029	0.0031
0.800	0.0087	0.0078	0.0099	0.0038	-0.0122	-0.0120	-0.0103	-0.0102	0.0021	0.0020
0.820	0.0004	-0.0038	0.0027	-0.0056	-0.0127	-0.0108	-0.0112	-0.0087	0.0014	0.0011
0.840	-0.0049	-0.0132	-0.0000	-0.0130	-0.0115	-0.0084	-0.0105	-0.0064	0.0007	0.0003
0.860	-0.0084	-0.0180	-0.0038	-0.0167	-0.0086	-0.0053	-0.0080	-0.0035	0.0003	-0.0001
0.880	-0.0106	-0.0167	-0.0085	-0.0150	-0.0043	-0.0018	-0.0042	-0.0007	0.0001	-0.0002
0.900	-0.0115	-0.0101	-0.0116	-0.0074	0.0005	0.0014	-0.0000	0.0014	-0.0003	-0.0003
0.920	-0.0106	-0.0013	-0.0130	0.0031	0.0052	0.0043	0.0043	0.0032	-0.0008	-0.0006
0.940	-0.0084	0.0066	-0.0129	0.0112	0.0093	0.0068	0.0085	0.0051	-0.0014	-0.0010
0.960	-0.0053	0.0117	-0.0110	0.0155	0.0123	0.0089	0.0115	0.0071	-0.0019	-0.0015
0.980	-0.0018	0.0131	-0.0074	0.0165	0.0136	0.0105	0.0127	0.0085	-0.0023	-0.0019
1.000	0.0006	0.0114	-0.0032	0.0138	0.0135	0.0114	0.0123	0.0094	-0.0024	-0.0022

time (sec.)	x_{20} (rad.)	x_{21} (rad.)	x_{22} (rad.)	x_{23} (rad.)	x_{24} (rad.)	x_{25} (rad.)	x_{26} (rad.)	x_{27} (rad.)	x_{28} (rad.)	x_{29} (rad.)
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.020	-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0019	-0.0002	0.0021	-0.0003
0.040	-0.0005	-0.0005	0.0002	-0.0001	0.0002	-0.0001	0.0060	0.0001	0.0063	-0.0006
0.060	-0.0021	-0.0022	0.0005	-0.0005	0.0006	-0.0005	0.0092	0.0033	0.0091	0.0022
0.080	-0.0050	-0.0053	0.0010	-0.0011	0.0012	-0.0013	0.0119	0.0087	0.0114	0.0078
0.100	-0.0083	-0.0089	0.0015	-0.0017	0.0018	-0.0019	0.0146	0.0128	0.0139	0.0119
0.120	-0.0117	-0.0127	0.0015	-0.0021	0.0018	-0.0023	0.0173	0.0159	0.0166	0.0150
0.140	-0.0152	-0.0166	0.0012	-0.0021	0.0015	-0.0023	0.0189	0.0191	0.0183	0.0185
0.160	-0.0189	-0.0206	0.0007	-0.0020	0.0010	-0.0021	0.0199	0.0220	0.0191	0.0215
0.180	-0.0226	-0.0244	0.0003	-0.0017	0.0006	-0.0018	0.0218	0.0243	0.0211	0.0240
0.200	-0.0262	-0.0280	-0.0002	-0.0011	-0.0001	-0.0011	0.0243	0.0262	0.0237	0.0259
0.220	-0.0298	-0.0315	-0.0009	-0.0003	-0.0009	-0.0002	0.0265	0.0282	0.0259	0.0279
0.240	-0.0335	-0.0350	-0.0014	0.0004	-0.0015	0.0006	0.0286	0.0304	0.0280	0.0301
0.260	-0.0371	-0.0384	-0.0017	0.0009	-0.0018	0.0011	0.0303	0.0317	0.0297	0.0313
0.280	-0.0401	-0.0414	-0.0020	0.0012	-0.0021	0.0015	0.0315	0.0325	0.0309	0.0321
0.300	-0.0424	-0.0438	-0.0023	0.0014	-0.0025	0.0018	0.0318	0.0332	0.0312	0.0327
0.320	-0.0438	-0.0454	-0.0026	0.0016	-0.0028	0.0020	0.0314	0.0335	0.0307	0.0330
0.340	-0.0444	-0.0462	-0.0028	0.0015	-0.0030	0.0020	0.0309	0.0335	0.0300	0.0329
0.360	-0.0441	-0.0461	-0.0027	0.0013	-0.0029	0.0017	0.0302	0.0332	0.0292	0.0326
0.380	-0.0429	-0.0450	-0.0026	0.0010	-0.0027	0.0014	0.0292	0.0327	0.0281	0.0320
0.400	-0.0410	-0.0431	-0.0024	0.0008	-0.0025	0.0012	0.0282	0.0320	0.0271	0.0312
0.420	-0.0385	-0.0405	-0.0020	0.0005	-0.0021	0.0008	0.0273	0.0308	0.0261	0.0300
0.440	-0.0356	-0.0374	-0.0015	0.0001	-0.0015	0.0003	0.0263	0.0292	0.0252	0.0284
0.460	-0.0323	-0.0339	-0.0009	-0.0003	-0.0008	-0.0002	0.0254	0.0274	0.0244	0.0267
0.480	-0.0287	-0.0300	-0.0004	-0.0006	-0.0003	-0.0005	0.0243	0.0254	0.0234	0.0247
0.500	-0.0249	-0.0260	-0.0000	-0.0008	0.0001	-0.0008	0.0231	0.0232	0.0225	0.0226
0.520	-0.0210	-0.0220	0.0003	-0.0010	0.0005	-0.0010	0.0218	0.0211	0.0213	0.0205
0.540	-0.0173	-0.0182	0.0006	-0.0012	0.0008	-0.0013	0.0202	0.0189	0.0199	0.0184
0.560	-0.0137	-0.0145	0.0007	-0.0014	0.0010	-0.0015	0.0182	0.0167	0.0181	0.0164
0.580	-0.0103	-0.0112	0.0007	-0.0013	0.0009	-0.0014	0.0160	0.0144	0.0159	0.0142
0.600	-0.0074	-0.0082	0.0007	-0.0013	0.0008	-0.0014	0.0133	0.0120	0.0134	0.0119
0.620	-0.0050	-0.0056	0.0006	-0.0012	0.0008	-0.0013	0.0104	0.0095	0.0105	0.0095
0.640	-0.0029	-0.0033	0.0006	-0.0010	0.0007	-0.0011	0.0072	0.0068	0.0073	0.0068
0.660	-0.0010	-0.0012	0.0004	-0.0007	0.0005	-0.0008	0.0039	0.0039	0.0040	0.0040
0.680	0.0008	0.0008	0.0002	-0.0003	0.0002	-0.0003	0.0006	0.0008	0.0007	0.0010
0.700	0.0025	0.0028	0.0000	0.0001	0.0000	0.0001	-0.0011	-0.0025	-0.0009	-0.0024
0.720	0.0040	0.0044	0.0000	0.0003	-0.0000	0.0004	-0.0004	-0.0055	0.0000	-0.0060
0.740	0.0045	0.0050	0.0001	0.0003	0.0001	0.0003	-0.0002	-0.0057	-0.0002	-0.0066
0.760	0.0040	0.0044	0.0004	0.0000	0.0005	-0.0000	-0.0005	-0.0034	-0.0008	-0.0041
0.780	0.0031	0.0033	0.0007	-0.0004	0.0008	-0.0005	-0.0005	-0.0019	-0.0011	-0.0027
0.800	0.0024	0.0023	0.0008	-0.0008	0.0009	-0.0009	-0.0003	-0.0015	-0.0008	-0.0022
0.820	0.0018	0.0014	0.0006	-0.0009	0.0007	-0.0010	-0.0009	-0.0007	-0.0012	-0.0010
0.840	0.0011	0.0005	0.0004	-0.0009	0.0005	-0.0010	-0.0021	0.0001	-0.0024	0.0000
0.860	0.0006	0.0000	0.0002	-0.0008	0.0003	-0.0009	-0.0020	0.0004	-0.0022	0.0007
0.880	0.0002	-0.0002	0.0000	-0.0005	0.0001	-0.0005	-0.0010	0.0006	-0.0009	0.0010
0.900	-0.0003	-0.0004	-0.0002	0.0000	-0.0002	0.0000	-0.0001	0.0010	0.0002	0.0014
0.920	-0.0010	-0.0007	-0.0004	0.0005	-0.0004	0.0005	0.0010	0.0018	0.0013	0.0023
0.940	-0.0017	-0.0012	-0.0003	0.0007	-0.0004	0.0008	0.0019	0.0019	0.0023	0.0024
0.960	-0.0023	-0.0017	-0.0003	0.0008	-0.0004	0.0009	0.0025	0.0016	0.0029	0.0020
0.980	-0.0027	-0.0021	-0.0004	0.0009	-0.0005	0.0009	0.0024	0.0014	0.0028	0.0017
1.000	-0.0028	-0.0025	-0.0005	0.0009	-0.0006	0.0010	0.0016	0.0010	0.0019	0.0013

VITA

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Master of Science

Thesis: A Computer Simulation of a Dual Rear Wheeled Farm Tractor

Major Field: Mechanical Engineering

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A COMPUTER SIMULATION OF A DUAL REAR WHEELED FARM TRACTOR

by

JONATHAN CRAIG GOERING

B.S., Kansas State University, 1978

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

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A dual rear wheeled farm tractor is modeled as eight lumped masses with elastic rear axles. The equations of motion are derived using Newton's laws, and elementary beam theory. These equations are checked by deriving the same equations using the energy method with the Lagrange equation for non-conservative systems. The equations of motion are integrated numerically using the Runge-Kutta-Gill method, to give the motion of the vehicle as a function of time.

Computer programs are presented which generate the equations of motion, using either Newton's laws or the energy method, for a given set of tractor parameters. A program to perform the numerical integration is also presented. This program includes provisions, by which the system may be forced. The forcing may be due to a drawbar load or surface profile.

The computer programs are demonstrated by simulating the motion of a tractor traversing a half sinusoid bump with the left rear wheels. The results of a system with soft rear axles are compared with the results of a system with stiff axles.