LEAST SQUARE PARABOLA APPLIED TO BUCKLING OF CONCRETE PLATES

by

ANWAR A. MERCHANT

B.E. (Civil), N.E.D. University of Engineering
and Technology, Pakistan, 1981

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1986

Approved by:

Harry D. Knostman
Major Professor

TABLE OF CONTENTS

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

THIS IS AS RECEIVED FROM CUSTOMER.

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Rectangular concrete panels with either ordinary or prestressed reinforcement are regularly used as components in box girders bridges, folded plate roofs and principally, as shear bearing walls in buildings. These components may be cast in place or may be precast.

To date, the necessity for considering buckling in a plate component has not been great simply because practical limitation in fabrication have precluded the use of plates of such overall dimension and thickness as to allow the possibility of buckling instead of some other mode of failure. However due to the great use of precasting, with a much greater quality control on mix design and fabrication; in conjunction with the availability of high-strength concretes and the necessity for thinner sections (particularly in high-rise buildings); the importance of obtaining information concerning the buckling characteristics of concrete plates with different types of supports and loadings is apparent(7). It is also a well known fact that by employing thinner section there is a large reduction in the dead weight of the structure. These reduction in weight passing down to foundations are directly reflected in the cost of construction.

This report presents results of a study to try and improve the agreement of experimental and theoretical buckling loads in the master's thesis "Buckling Behaviour of reinforced concrete plate models", by Seck (5). Mr. Seck fabricated and tested twenty plates in the KSU structures laboratory. The difference between theoritical and experimental results were sufficient to indicate the need for further study.

Since the plate did not turn out to be of uniform thickness one adjustment was to use thickness of the plate at the section where buckling occured. However the biggest part of new investigation dealt withimproving the equation used to represent the stress-strain relationship. To obtain the concrete stress-strain relationship for use in the theoretical equation for buckling load, Seck ran compression tests on 3 in. by 6 in. cylinders. These theoretical equations are based on the tangent modulus E_T which is determined from the Hognestad parabola. The Hognestad parabola is a function of the compressive strength of the concrete fc' and the corresponding strain ε_0. To find these values Seck drew a curve through the plotted data points and selected the values at the peak. It is felt that a more accurate approach would be to obtain the parabolic equation for the stress-strain relationship by using the least square method. Once the parabolic equation is obtained the values of fc' and ε_0 can be easily determined. This later approach was used in this report. A comparison of the resulting values of fc' and ε_0 for the concrete cylinders are presented in Table 1.1.

It should be noted that the time of test of the last cylinder corresponded to the test of the last plate of that batch. Other plates of the same batch were tested on earlier dates, see Tables 1.2 & 1.3. The attachment of the strain gages to the plates also required that they be removed from the moist room several days before testing. Also by comparing the stress-strain curve for the cylinders and plates,Fig.1.1, it can be seen that the slopes are quite different hence the modulus of elasticity, E, for a given stress will be different. Since the buckling load depends on modulus of elasticity, further investigation of E found from the stress-strain data of the plates was felt desirable. In addition the stress-strain curve are different for different points on the same plate,

TABLE 1.1 CYLINDER COMPRESSIVE LOAD-STRAIN DATA FROM FIVE BATCHES

CYLINDER	PLATE	ULTIMATE CYLINDER STRENGTH f'_c psi		STRAIN AT ULTIMATE STRENGTH $\varepsilon_0 x10^{-6}$	
		Least Squares	Seck	Least Squares	Seck
I	1 to 4	7,412	7,420	3,146	3,100
II	10	6,971	7,100	2,921	3,200
III	16	6,450	6,300	3,202	2,600
IV	5 and 17	7,883	7,300	3,343	2,900
V	15	6,357	6,400	2,804	2,800
	MEAN	7,015	6,900	3,083	2,900

TABLE 1.2 CYLINDER TEST LOG

CYLINDER	CASTING DATE	TESTING DATE	FAILURE STRESS psi (MPa)	AGE AT TEST, DAYS
I	21 July, 82	9 Dec., 82	7,700 (53.10)	141
II	6 Oct., 82	27 Jan., 83	7,240 (49.80)	113
III	25 Oct., 82	11 Feb., 83	6,730 (46.30)	109
IV	1 Sep., 82	15 Feb., 83	7,780 (53.51)	167
V	1 Nov., 82	9 May, 83	6,370 (43.93)	190

TABLE 1.3 PANEL SUPPORT CONDITION TOP AND BOTTOM OF THE PLATES ALWAYS SIMPLY-SUPPORTED

PANEL NUMBER	LONG SIDE (VERTICAL) SUPPORT CONDITION	SPACING (in.)	POURING DATE	TESTING DATE	# OF STRAIN GAGES/PLATE	AGE AT TEST DAYS
1(I)*	SS	1	07/21/82	11/04/82	6	106
2(I)	SS	1	07/21/82	11/19/82	6	121
3(I)	2CA	1	07/21/82	12/02/82	6	134
10(II)	2CA	½	10/06/82	01/27/83	6	113
16(III)	SS	½	10/25/82	02/11/83	6	109
5(IV)	Free	½	09/01/82	12/08/82	6	98
17(IV)	Free	1	09/01/82	02/15/83	6	167
15(V)	SS	2	11/01/82	02/11/83	6	102

*The term within parenthesis indicate cylinder no. corresponding to panel number.

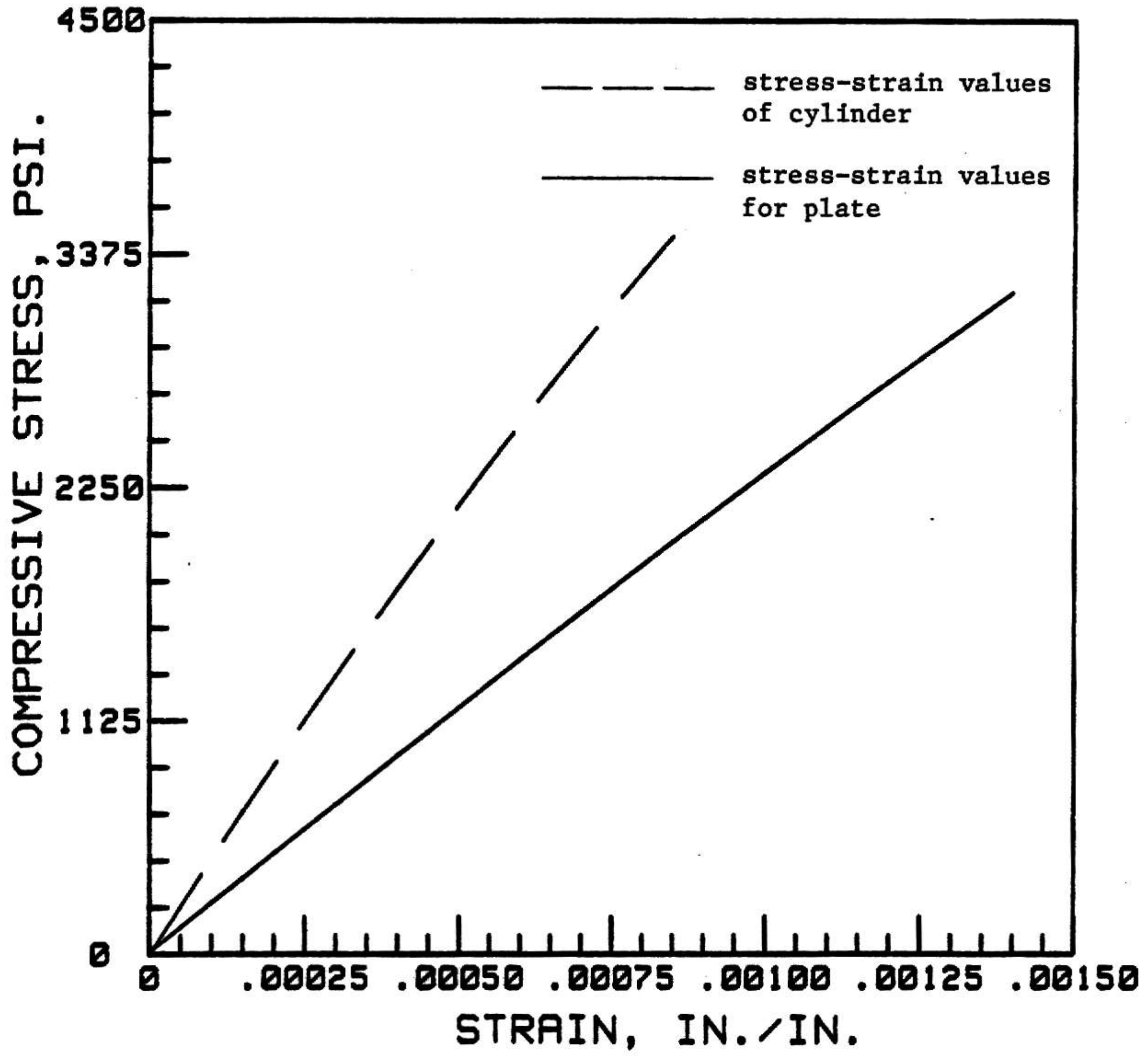

Figure 1.1 Stress-Strain Curves for Plate 1 and Cylinder 1

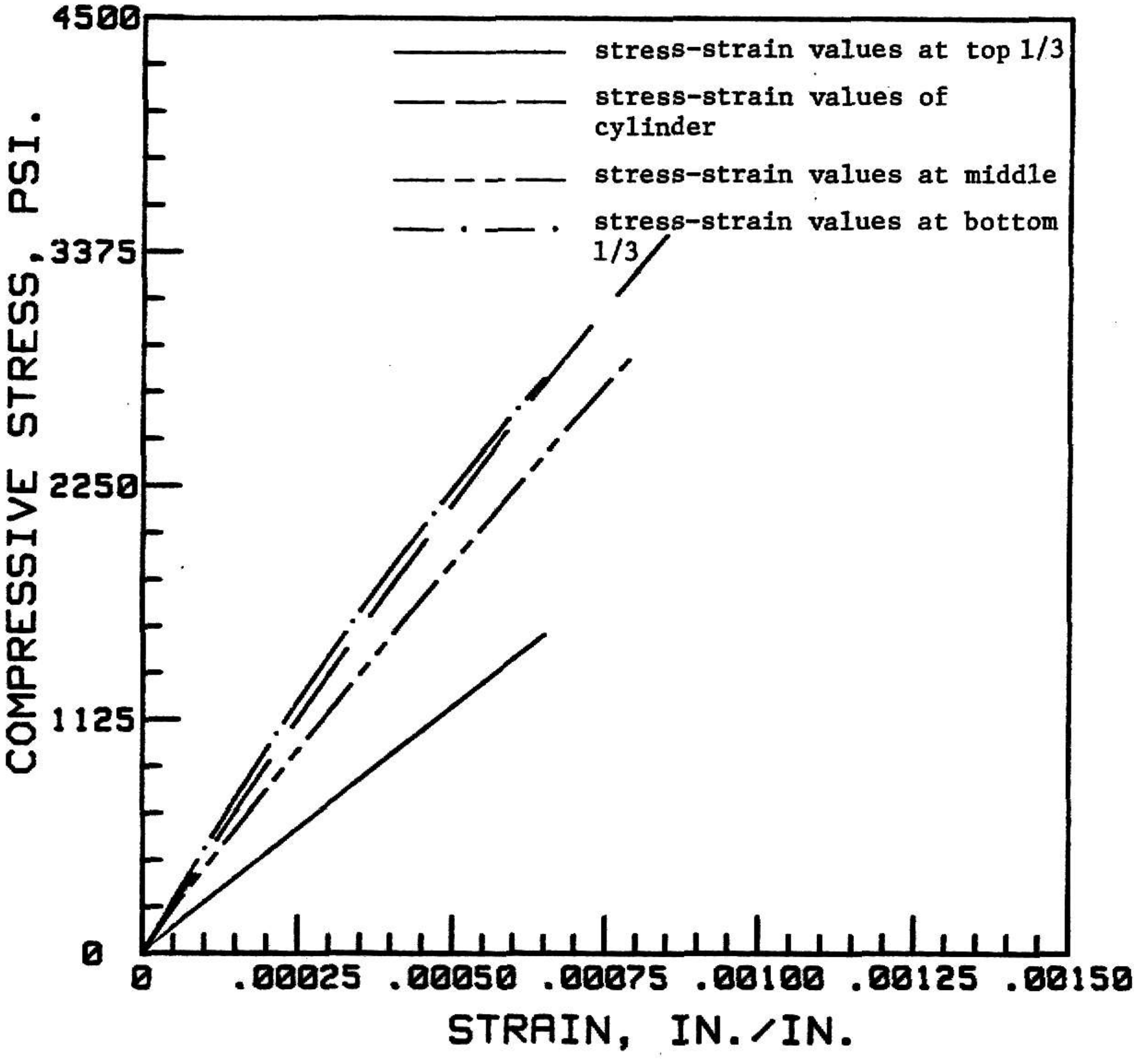

Figure 1.2 Stress-Strain Curves at Different Locations of Plate 1 and Cylinder 1

i.e. the top 1/3, the middle 1/3, and the bottom 1/3 as can be seen in Fig.1.2. The main reason for the difference in the curve is the way these plates are poured. Since the plates were pumped from bottom, the top part was thinner then the middle and the bottom part where the pressure was the greatest, was the thickest. Hence it can be seen that the use of the cylinder data for fc' and ε_0 could result in error. Therefore in an attempt to improve the results it was decided to calculate theoretical buckling load using plate data.

Since the plates buckled below fc', ε_0 and fc' are not available, therefore a parabola was fitted to the data using method of least square. This parabola then could be used to represent the stress-strain and obtain the modulus of elasticity.

The values of the constants of the parabolic equations obtained from the cylinder data and plate data were then used in equations proposed by Swartz et. al, which were modified for this report, to find the theoretical buckling load, Pcr. Finally comparison were made of the new values of Pcr and the values obtained by Seck.

CHAPTER 2

REVIEW OF EXPERIMENTAL PROCEDURE

The experimental procedure for forming the 1/4 inch concrete plates, which was originated by Munoz (3) and used by Seck (5), employed a sheet of plywood on which are bolted two plexiglass plates of the same size. These plates were separated along the exterior edges by 1/4 inch by 1 inch plastic strips with holes drilled at 0.5 in. intervals at the center of the 1/4 in. thickness to accept different reinforcement spacings. An opening was designed on the top plexiglass plate to receive a rectangular funnel that is connected to the hose from the concrete pump.

The size of the structural plate model was determined to be 24 in. (610 mm) long, 12 in (305 mm.) wide, and 0.25 in. (6.4 mm.) thick. The final mix design had a water-cement ratio of 0.5, an aggregate cement ratio of 2, a unit weight of 135 lbs. per cu. ft. (21.2 KN/m), and an average slump of 8.6 in. (218.44 mm.).

The plate was reinforced in perpendicular direction with galvanised wire of 3/64 in. diameter (0.047 in. = 1.19 mm.) and having an average yield strength of 65,896 psi (454.48 MPa). These reinforcement were spaced at 2.0, 1.0, and 0.5 in. (50.8, 25.4, 12.7 mm.) intervals which gives steel ratios of 0.0035, 0.0070 and 0.0140 respectively.

Formwork was constructed very carefully. The inner surfaces of plexiglass were properly greased. Reinforcement was placed in correct position taking care that it did not contact oil from the bottom of the plexiglass. Finally the upper plexiglass plate was positioned and fastened to the plywood by bolts which had been previously dipped into grease.

Once the form was ready, the concrete mixture of sand, cement and water was pumped into the form from the bottom. Pumping was stopped when the form was full and concrete began to spill out at the top.

Each test specimen was cured initially for 24 hours in the molds. After this initial curing period the reinforcement wires were cut and bolts loosened. The plates were removed from the molds and the specimen along with the bottom plexiglass plate and the edge strips were marked and put in the moist room. The three edge strips and the bottom plexiglass were removed on the next day. The test specimen were placed in the moist room with long edges on floor for at least 28 days before they could be tested. A total of 24 test plates were made in this manner.

Prior to being tested in compression, twenty-one thickness readings were taken to determine the average thickness of each plate. Figure 2.1 displays the values of the average thickness of each plate considered in this report. The plates are also shown with the position of the opening, i.e., the trapezoid, in the same position as when it was tested in compression.

Six strain gages were mounted on the twelve plates following standard techniques. They were placed in the same location on opposite faces of the plate, see Fig. 2.2, so that the corresponding surface tension or compression strain could be measured. The strain gages were oriented parallel to the long direction at 0.5 in. (12.7 mm.) from the centerline in the direction of the applied load. The plate was then placed into the test frame designed and built by Munoz (3), as shown in Figs. 2.3a and 2.3b.

The top and bottom edge of the test plate were always simply supported, whereas three different support conditions were used on the

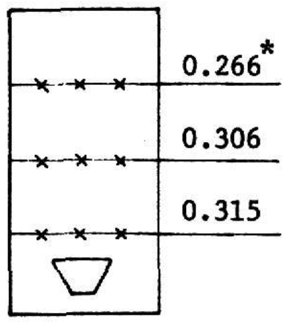

PLATE #1, Opening at B.
S = 1.0 in.

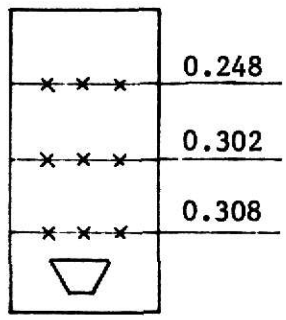

PLATE #2, Opening at B.
S = 1.0 in.

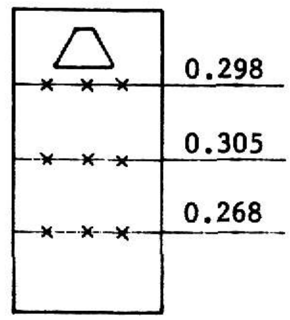

PLATE #3, Opening at T.
S = 1.0 in.

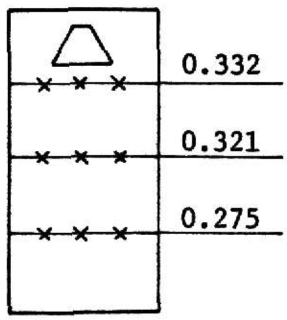

PLATE #5, Opening at T.
S = 1/2 in.

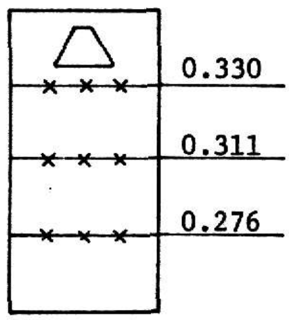

PLATE #10, Opening at T.
S = 1/2 in.

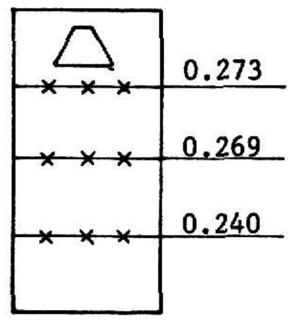

PLATE #15, Opening at T.
S = 2.0 in.

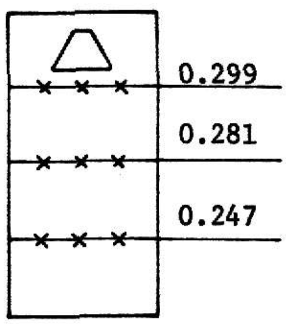

PLATE #16, Opening at T.
S = 1/2 in.

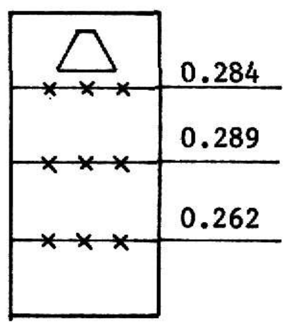

PLATE #17, Opening at T.
S = 1.0 in.

All dimensions in inch, 1 in. = 25.4 mm.

*Average thickness used in the evaluation of the theoretical buckling load, P_{crT}.

T = Top

B = Bottom

Figure 2.1 Plate Average Thicknesses

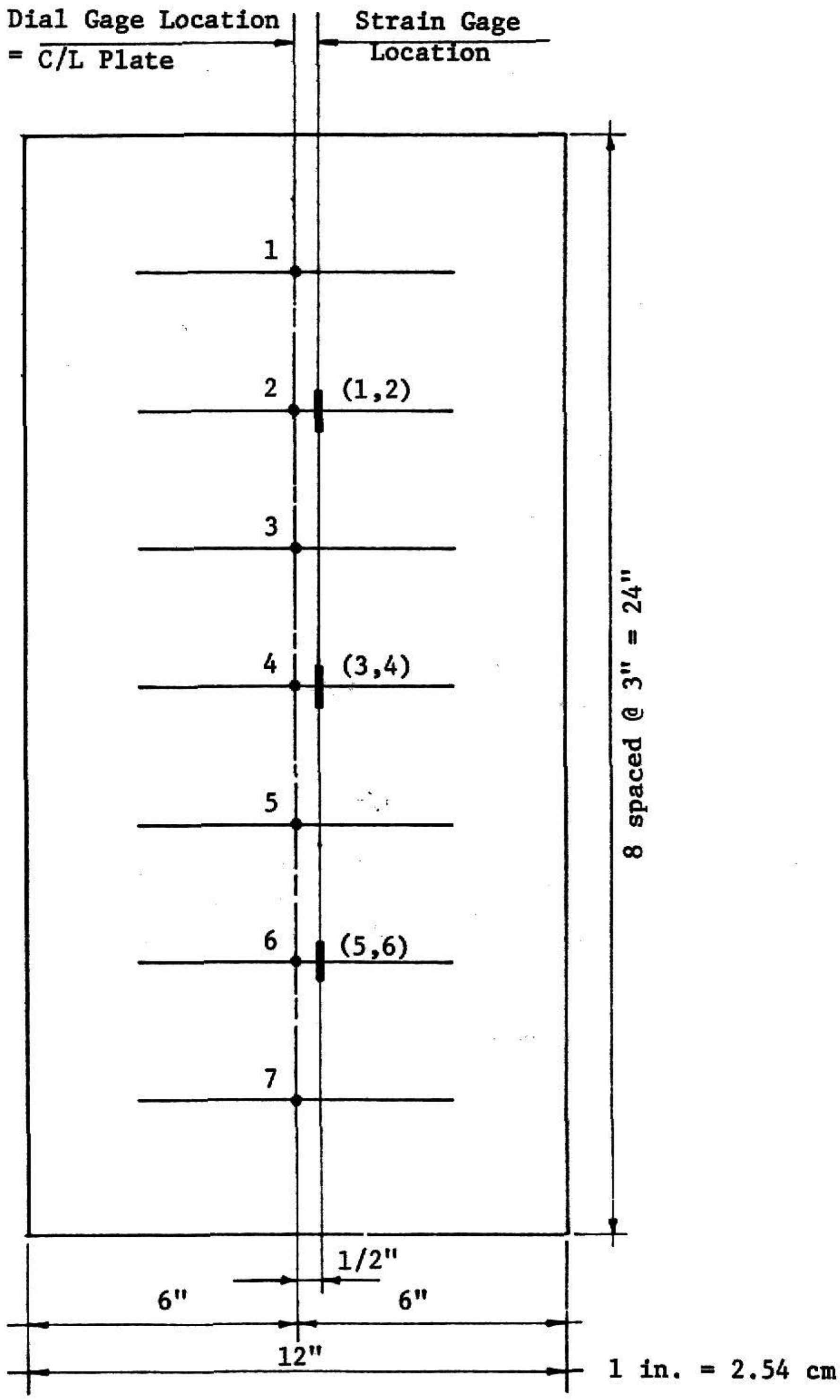

Figure 2.2 Gage Location on Test Plates

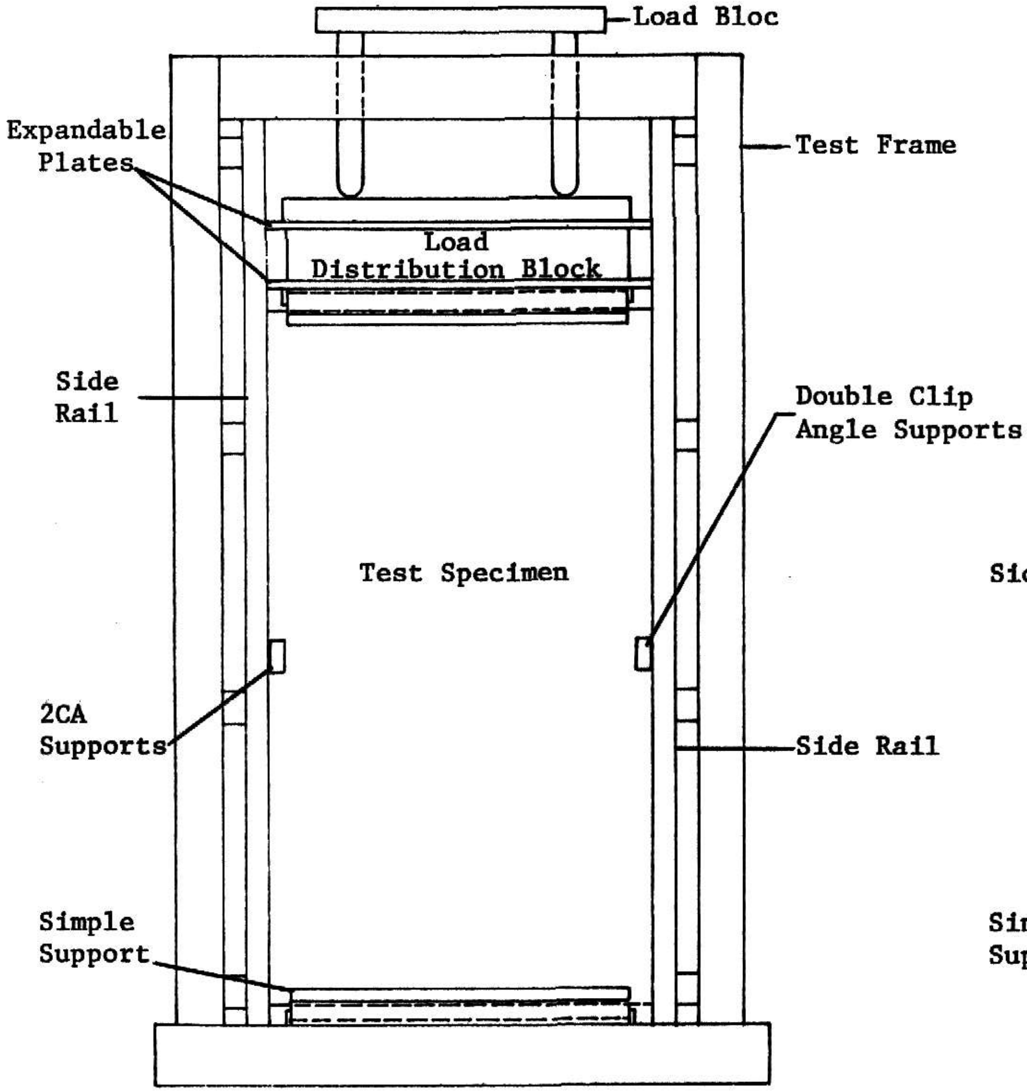

Figure 2.3a Test Plate with Two Double Angles

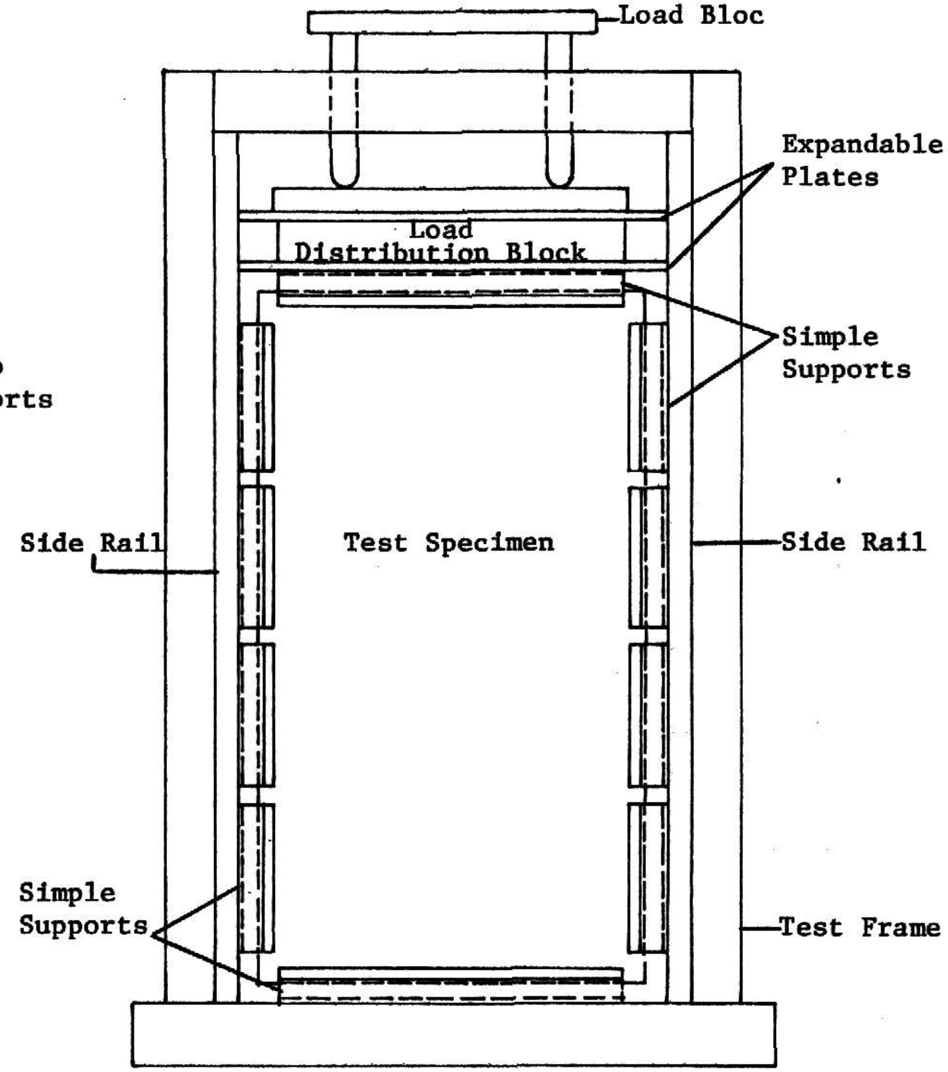

Figure 2.3b Simply-Supported Test Plate

long side. Some were simply supported others had double clip angles at middepth and still others had the long edge free. Figures 2.3a, 2.3b and Table 1.3 show the support conditions for each plate.

In order to measure deflections seven soil test LC-10 dial gages were fixed on the test frame and placed in contact with the center line of the test plate. The location of deflection gages is shown in Fig. 2.2.

After the plate strains gages were connected to the strain indicator, testing was begun by initializing strain and dial gages, loading and unloading the plate several times and then rebalancing the gages to zero. Next a load was applied, strains and dial gage data was read and recorded. The load was then increased and again readings were taken, this procedure was repeated until failure.

The stress-strain data was obtained for both cylinders and plates. Strain gages were attached to five cylinders, one per batch. Four strain gages were used per cylinder, two measured longitudinal strains and two measured transverse strains so that Poisson's ratio could be determined.

CHAPTER 3

CURVE-FITTING

Introduction

In situations where plotted data between two variables is such that plotted points do not fall along a straight line, the problem of analysis becomes complicated (2). The difficulty of drawing a line or a curve through a scattering of points occur if several people should try to repeat this operation on the same set of points, quite different results would be obtained. So we clearly need some objective way of finding the most likely probable curve, i.e., the one that provides the "best fit".

There are various mathematical models which can be used for curve fitting - a linear model, a polynomial model, a exponential model, a lograthimic model etc. To make an intelligent decision about the type of model which should be used an attempt must be made to find out where the original data points came from, since we can guess the gereral shape of the curve from this information. In the case of concrete it is generally taken that the shape of the stress-strain curve will be a second order parabola and therefore the Hognestad curve which is a parabola is often fitted through the data in order to obtain the stress-strain curve.

Least-square curve fitting

A set of observed data points can be fitted quite well with a polynomial(6). Let us assume that we have n+1 tabulated values X_0, $X_1,\dots,X_n$ and their corresponding function $f(X_0)$, $f(X_1)\dots$, $f(X_n)$ and that we decide to fit a curve of polynomial degree m through it, where $m < n$.

Let us assume the form of the polynomial model as

$$Y_m(X) = A_mX^m+A_{m-1}X^{m-1}+\dots+A_2X^2+A_1X+A_0 \quad \text{----Eq. (3.1)}$$

At each of the n+1 tabulated points, the resulting $Y_m(X_i)$ will be approximately equal to tabulated $f(X_i)$, with an error which we shall call δ_i, we might call this the deviation of the fitted curve from the tabulated point,

$$\delta_0 = Y_m(X_0)-f(X_0)$$

$$\delta_1 = Y_m(X_1)-f(X_1)$$

$$\delta_2 = Y_m(X_2)-f(X_2)$$

$$\vdots$$

$$\delta_n = Y_m(X_n)-f(X_n)$$

or, in general terms, the set of deviations

$$\delta_i = Y_m(X_i)-f(X_i) \qquad \text{for } i = 0,1,2,\ldots,n \text{ ---------------Eq.(3.2)}$$

then the sum of the squares of these n+1 deviations should be a minimum:

$$\sum_{i=0}^{n} (\delta_i)^2 = \text{minimum} \text{ ------------------------------------Eq.(3.3)}$$

Above equation can also be written as

$$\sum_{i=0}^{n} (\delta_i)^2 = \sum_{i=0}^{n} \{Y_m(X_i)-f(X_i)\}^2$$

or,

$$\sum_{i=0}^{n} (\delta_i)^2 = \sum_{i=0}^{n} \{A_m(X_i)^m + A_{m-1}(X_i)^{m-1} + \ldots + A_1(X_i)$$

$$+ A_0 - f(X_i)\}^2 \text{ --------------------Eq.(3.4)}$$

Above equation is a function of m+1 unknown variables A_i. In order to minimize equation (3.4) set the partial derivative of the summation with respect to each of the m+1 variables A_i equal to zero.

$$\frac{\partial}{\partial A_0} \sum_{i=0}^{n} (\delta_i)^2 = 0$$

$$\frac{\partial}{\partial A_1} \sum_{i=0}^{n} (\delta_i)^2 = 0$$

$$\vdots$$

$$\frac{\partial}{\partial A_m} \sum_{i=0}^{n} (\delta_i)^2 = 0,$$

or, in general terms we can write

$$\frac{\partial}{\partial A_j} \sum_{i=0}^{n} (\delta_i)^2 = 0 \qquad \text{for } j = 1,2,\ldots,m \text{ ------------------Eq. (3.5)}$$

The series has a finite number of terms, and so derivatives of the sum is equal to the sum of the derivatives of the terms. Thus we can write Eq.(3.5) as

$$\frac{\partial}{\partial A_j} \sum_{i=0}^{n} (\delta_i)^2 = \sum_{i=0}^{n} \frac{\partial}{\partial A_j} (\delta_i)^2$$

or,

$$\sum_{i=0}^{n} 2\delta_i \frac{\partial \delta_i}{\partial A_j} = 0$$

$$\sum_{i=0}^{n} \delta_i \frac{\partial \delta_i}{\partial A_j} = 0 \qquad \text{for } j = 1,2,\ldots,m \text{ ------------------Eq.(3.6)}$$

Now $\delta_i = Y_m(X_i)-f(X_i) = A_m(X_i)^m + A_{m-1}(X_i)^{m-1} + \ldots$

$$+ A_j(X_i)^j + \ldots + A_1X_i + A_0 - f(X_i) \text{ --------------- Eq.(3.7)}$$

When we differentiate with respect to A_j, we assume every other term in Eq.(7) is constant except the term containing A_j, so that all other terms drop out on differentiating and

$$\frac{\partial\delta_i}{\partial A_j} = 0 + 0 +\ldots+ \frac{\partial}{\partial A_j} A_j(X_i)^j +\ldots+ 0 + 0$$

or,

$$\frac{\partial\delta_i}{\partial A_j} = (X_i)^j$$

Substituting above values into Eq.(3.6) we obtain m+1 equations

$$\sum_{i=0}^{n} \delta_i\ (X_i)^j = 0 \qquad \text{for } j = 1,2,\ldots,m \text{ ----------------- Eq.(3.8)}$$

Substituting Eq.(3.2) into Eq.(3.8)

$$\sum_{i=0}^{n} \{Y_m(X_i)-f(X_i)\}(X_i)^j = 0 \qquad \text{for } j = 0,1,2,\ldots,m$$

or,

$$\sum_{i=0}^{n} \{A_m(X_i)^m + A_{m-1}(X_i)^{m-1} +\ldots+ A_1(X_i) + A_0$$

$$- f(X_i)\}(X_i)^j = 0 \qquad \text{for } j = 1,2,\ldots,m$$

or,

$$A_m \sum_{i=0}^{n} (X_i)^m(X_i)^j + A_{m-1} \sum_{i=0}^{n} (X_i)^{m-1}(X_i)^j +\ldots+ A_1 \sum_{i=0}^{n} (X_i)(X_i)^j$$

$$+ A_0 \sum_{i=0}^{n} (X_i)^j = \sum_{i=0}^{n} f(X_i)(X_i)^j$$

$$\text{for } j = 0,1,2,\ldots,m \text{ ------------------ Eq.(3.9)}$$

Let $\alpha_{kj} = \sum_{i=0}^{n} (X_i)^k (X_i)^j$. for $k = 0,1,2,\ldots,m$

$j = 0,1,2,\ldots,m$ ------------Eq.(3.10)

Since j and k in Eq.(3.10) can be interchanged, we also have the result $\alpha_{kj} = \alpha_{jk}$ which reduces the work of evaluating these coefficients to half.

We now have a workable scheme for finding the least-square fit. Using α's we have the system of simultaneous equations

$$\alpha_{m0}A_m + \alpha_{m-10}A_{m-1} + \ldots + \alpha_{00}A_0 = \sum_{i=0}^{n} f(X_i)(X_i)^0$$

$$\alpha_{m1}A_m + \alpha_{m-11}A_{m-1} + \ldots + \alpha_{01}A_0 = \sum_{i=0}^{n} f(X_i)(X_i)$$

$$\alpha_{m2}A_m + \alpha_{m-12}A_{m-1} + \ldots + \alpha_{02}A_0 = \sum_{i=0}^{n} f(X_i)(X_i)^2$$

$$\begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

$$\alpha_{mm}A_m + \alpha_{m-1m}A_{m-1} + \ldots + \alpha_{0m}A_0 = \sum_{i=0}^{n} f(X_i)(X_i)^m$$

which can be solved to find coefficients A_0, A_1, $A_2\ldots$, A_m. Then substituting these values of A's into equation (3.1) we obtain the desired polynomial model.

Above concepts were incorporated in the development of H.P. utility library software program. The use of that program was made to obtain plates and cylinder curves shown in figures 5.1 thru 5.13.

The input data in the above program consisted of concrete strain ε_c and concrete stress σ_c which for the plate is given by

$$\sigma_c = \frac{P_T - E_s \varepsilon_s A_s}{A_g - A_s}$$

where,

P_T = Total load carried by reinforced concrete

E_s = Young's modulus of steel

ε_s = Strain in steel

A_s = Area of steel in plate

A_g = Gross area of loaded edge of plate

CHAPTER 4

ANALYTICAL DETERMINATION OF THE BUCKLING LOAD

Buckling load depends upon the support conditions and therefore two different formulas were used depending upon the manner in which the plates are supported.

Whenever the support conditions at the long edge were two clip angles or free, the panel behaved as a column and the buckling load was calculated from Euler's Equation

$$P_{cr} = \frac{n^2 \pi^2 EI}{L^2} \qquad \text{-------------------------Eq.(4.1)}$$

E = Tangent modulus for concrete,

I = Moment of inertia about bending axis,

L = Unsupported length of panel,

n = Number of waves panel deflects.

The critical load is also given by

$$P_{cr} = A_c f_{cr} + A_s E_s \varepsilon_{cr} \qquad \text{------------------Eq.(4.2)}$$

f_{cr} = Critical stress for concrete,

A_c = Area of concrete,

A_s = Area of steel,

E_s = Modulus of elasticity of steel,

= Critical strain.

Figure 4.1 shows plate proportions and direction of the applied loads.

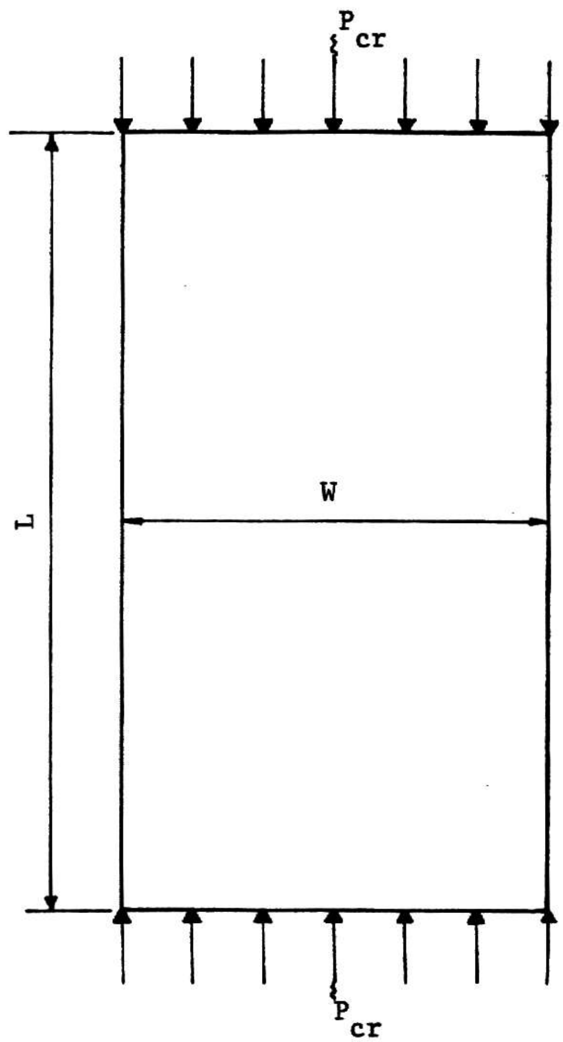

Figure 4.1 Applied Load on Test Panel

<u>Calculation of Critical Stress and Strain</u>

The stress-strain curve for microconcrete is assumed to be a second order polynomial equation of the form,

i. $f_c = A_0 + A_1\varepsilon_c + A_2\varepsilon_c^2$ ----------------------------Eq.(4.3a)

The critical stress for concrete is

ii. $f_{cr} = A_0 + A_1\varepsilon_{cr} + A_2\varepsilon_{cr}^2$ ------------------------Eq.(4.3b)

The Tangent Modulus for concrete is the slope of stress-strain curve at any point,

iii. $E_T = df_c/d\varepsilon_c = A_1 + 2A_2\varepsilon_c$ ------------------------Eq.(4.3c)

Substituting E_T and f_{cr} into Eq.(4.1) and Eq.(4.2) and equating one obtains,

$$n^2\pi^2 I(A_1 + 2A_2\varepsilon_{cr})/L^2 = A_c(A_0 + A_1\varepsilon_{cr} + A_2\varepsilon_{cr}^2) + A_sE_s\varepsilon_{cr}$$

or,

$$\varepsilon_{cr}^2 + (A_1 - 2\pi^2n^2IA_2/L^2A_c + A_sE_s/A_c)\ \varepsilon_{cr}/A_2 - \pi^2n^2IA_1/L^2A_cA_2$$
$$+ A_0/A_2 = 0 \text{ ----------------Eq.(4.4)}$$

Equations (4.3) and (4.4) apply when the plate strains are being used. When the cylinder data is being used the above constants, i.e., A_0, A_1 and A_2 need to be modified by 0.85. Therefore, Eq.(4.3)-Eq.(4.4) become,

$f_c = 0.85(A_0 + A_1\varepsilon_c + A_2\varepsilon_c^2)$ -------------------------Eq.4.5a)

$f_{cr} = 0.85(A_0 + A_1\varepsilon_{cr} + A_2\varepsilon_{cr}^2)$ -----------------------Eq.(4.5b)

$E_T = df_c/d\varepsilon_c = 0.85(A_1 + 2A_2\varepsilon_{cr})$ ---------------------Eq.(4.5c)

$$\varepsilon_{cr}^2 + (A_1 - 2n^2\pi^2I/L^2A_c + A_sE_s/0.85A_c)\ \varepsilon_{cr}/A_2$$
$$- \pi^2n^2IA_1/L^2A_cA_2 + A_0/A_2 = 0 \text{ ----------------Eq.(4.6)}$$

The value of ε_{cr} is then obtained from Eq.(4.4) or (4.6) and substituted in Eq.(4.4b)(or Eq. 4.5b) to find the critical stress f_{cr}. With these values, the buckling load of the panel can be determined by Eq.(4.2). This is the critical load of the panel due to column action.

When the support conditions are such that the panel is simply supported along all edges it will buckle with plate bending action (biaxial curvature). The formulas presented herein have been proposed by Swartz, et. al (7) with the modification, that the stress-strain relationship has been defined in terms of the constants A_0, A_1 and A_2.

The concrete buckling stress is defined as

$$f_{cr} = A_0 + A_1 \varepsilon_{cr} + A_2 \varepsilon_{cr}^2 \quad \text{------------------------------Eq.(4.7)}$$

hence

$$E_{cT} = \frac{df_{cr}}{d\varepsilon_{cr}} = A_1 + 2A_2 \varepsilon_{cr} \quad \text{--------------------------------Eq.(4.8)}$$

According to Ref. (7) if the reinforced concrete is considered as an isotropic material and the tangent modulus theory is applied, the result will be conservative when compared to those obtained experimentally. The differential equation for the plate with an in-plane load may be expressed as

$$D_1 \frac{\partial^4 w}{\partial x^4} + D_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_1 \frac{\partial^4 w}{\partial y^4} = -Nx \frac{\partial^2 w}{\partial x^2} \quad \text{------------------Eq.(4.9)}$$

where,

$$D_1 = \frac{h^3 E_{cT}}{12(1-\mu^2)} + D_s$$

$$D_2 = D_1 - D_s$$

$D_s = E_s h \sum_{i=1}^{j} \rho_1 z_1^2$ which is zero since reinforcement is in the center of the plate.

Hence $D_2 = D_1 = \dfrac{h^3 E_{cT}}{12(1-\mu^2)}$ ------------------------------------Eq.(4.10)

A deflection function satisfying the assumed boundary condition is

$$w = \sum_{m=1}^{\infty} Am \operatorname{Sin} \frac{m\pi x}{a} \operatorname{Sin} \frac{\pi y}{b} \qquad \text{------------------------------Eq.(4.11)}$$

Substituting Eq.(4.10) and Eq.(4.11) into Eq.(4.9) leads to

$$Nx = \frac{\pi^2}{b^2} D_1 \left[\left(\frac{mb}{a}\right)^2 + 2 + \left(\frac{a}{mb}\right)^2 \right] \qquad \text{-------------------------Eq.(4.12)}$$

Considering the minimum value of f_{cr} for a given $\frac{a}{b}$ leads to a unique solution

$$\frac{a}{b} = m \qquad \text{---Eq.(4.13)}$$

Substituting Eqs.(4.8, 4.10 and 4.13) into Eq.(4.12) gives

$$N_x = \frac{4\pi^2}{b^2} \frac{(A_1 + 2A_2\, \varepsilon_{cr})h^3}{12(1-\mu^2)}$$

The critical buckling load is given as

$$P_{cr} = bN_x = \frac{4\pi^2}{b} \frac{(A_1 + 2A_2\, \varepsilon_{cr})h^3}{12(1-\mu^2)} \qquad \text{----------------------Eq.(4.14)}$$

and also $P_{cr} = f_{cr}(1-\rho)bh + E_s \varepsilon_{cr}\, \rho bh = (A_0 + A_1\, \varepsilon_{cr} + A_2\, \varepsilon_{cr}{}^2)(1-\rho)bh$

$$+ E_s\, \varepsilon_{cr}\, \rho bh \qquad \text{---------------------Eq.(4.15)}$$

Equating Eqs.(4.14 and 4.15) leads to

$$\varepsilon_{cr}{}^2 + B\varepsilon_{cr} + C = 0 \qquad \text{--------------------------------------Eq.(4.16)}$$

or $\quad \varepsilon_{cr} = -\tfrac{1}{2}\{B + (B^2 - 4C)^{\frac{1}{2}}\}$ ----------------------------Eq.(4.17)

where

$$B = \frac{\dfrac{A_1 + E_s\rho}{1-\rho} - 2KA_2}{A_2} \qquad \text{------------------------------------Eq.(4.18)}$$

$$C = \frac{A_0 - KA_1}{A_2}$$

$$K = \frac{\pi^2}{3(1-\rho)} \left(\frac{h}{b}\right)^2$$

Equations (4.7-4.16) hold good for plate data. If we are using cylinder data, then above equations need to be modified by 0.85, i.e.,

$$\varepsilon_{cr} = -\tfrac{1}{2} \{B_1 + (B_1^2 - 4C)^{\frac{1}{2}}\} \qquad \text{Eq.(4.19)}$$

where

$$B_1 = \left(A_1 + \frac{E_s \rho}{0.85(1-\rho)} - 2KA_2\right)/A_2 \qquad \text{Eq.(4.20)}$$

$$K = \{\pi^2/3(1-\rho)\}(h/b)^2$$

$$C = (A_0 - KA_1)/A_2$$

and, $f_{cr} = 0.85(A_0 + A_1\varepsilon_{cr} + A_2\varepsilon_{cr}^2)$ Eq.(4.21)

The plate buckling load is then defined as

$$P_{cr} = C_s b(h)\{f_{cr}(1-\rho) + E_s \varepsilon_{cr} \rho\}; \text{ for } \varepsilon_{cr} < \varepsilon_y \qquad \text{Eq.(4.22)}$$

or, $P_{cr} = C_s b(h)\{f_{cr}(1-\rho) + f_y \rho\}$; for $\varepsilon_{cr} > \varepsilon_y$ Eq.(4.23)

where C_s, factor of safety = 1 for laboratory experiments,

h = thickness of panel,

b = width of the panel,

ρ = steel ratio.

The value of B determined from Eq.(4.18)(or Eq.(4.20) if using cylinder data) is substituted in Eq.(4.17)(or Eq. 4.19) to calculate the value of ε_{cr}. This value of ε_{cr} is used in Eq.(4.7)(or Eq. 4.21) to calculate f_{cr}. The critical load due to plate action is then evaluated using Eq.(4.22) or Eq.(4.23).

CHAPTER 5

SUMMARY AND CONCLUSION

The principal objective of this report was to improve the theoretical results obtained by Seck. It was thought that the method employed by Seck to fit the Hognestad parabola to the cylinder data was not accurate enough, as it involved guessing at the ultimate strain ε_0 and ultimate stress fc'. Therefore another method was used to obtain an equation for the stress-strain curve. In this method a parabola was fitted through all the experimental stress-strain points by the method of least square. Since it was found that fitting a parabola to the data by long hand calculations was a tedious job, a software computer programme from the library of Hewlett Packard for the microcomputers was used to obtain the equation for the curve. This method gave excellent agreement between the experimental data and the parabolic equation as can be seen in Figs. 5.1-5.5. The peak of this curve corresponds to the ultimate stress fc' and ultimate strain ε_0. The new values of fc' and ε_0 for the test cylinders were compared with the values obtained obtained by Seck in Table 1.1. It was seen that for most cases there was a slight improvement in the agreement between the theoretical and experimental results. The average ratio of experimental buckling load, to the theoretical buckling load ($\frac{P_{cr_E}}{P_{cr_{T_3}}}$) with the exception of plate 10, for simply supported, two clip angles and free support conditions on long sides was found to be 0.781, 1.135 and 1.083 respectively as compared to Seck who obtained 0.738, 1.109 and 1.349 for the above support conditions, see Table 5.2.

It was thought from reasons given earlier in this report that the use of plate stress-strain data instead of cylinder stress-strain data could yield better results. Since the plate failed by buckling rather than crushing, no values of ultimate stress fc' and ultimate strain ε_0 are available. Therefore the computer program from the Hewlett Packard library was again used to fit a parabola, by the least sqare method, to the stress-strain data of the plate. The resulting curves can be seen in Figs. 5.6-5.13. Since to obtain the critical load, only an equation for stress-strain curve is required it is not necessary to obtain fc' and ε_0 for the plate. Therefore the equations developed by Swartz and used by Seck were rewritten in terms of the parabolic constants A_0, A_1 and A_2. These modified equations were then used to find the theoritical buckling load. These results are presented in Table 5.3.

It can be seen from Table 5.2 that the use of plate data further improved the agreement between theoritical and experimental buckling load. The average ratios ($\frac{P_{cr_E}}{P_{cr_{T_2}}}$) were .959, 1.168 and 1.126 for simply supported , two clip angles and free support conditions respectively.

TABLE 5.1 EXPERIMENTAL BUCKLING LOAD

PLATE NUMBER	LONG SIDE SUPPORT	STEEL SPACING (in)	P_{crE}*	MAXIMUM LOAD	BUCKLING LOCATION
1	S.S.	1	10,800	11,350	1/3 from top
2	S.S.	1	8,700	9,000	1/4 from top
15	S.S.	2	5,800	5,960	Bottom support (no indication of buckling)
16	S.S.	1/2	9,000	10,600	4" from bottom (trapezoidal yield line)
3	2CA	1	4,500	6,400	1/4 from top and 1/4 bottom (with cracks parallel to edges)
10	2CA	1/2	8,000	9,500	1/4 from top, 1/4 from bottom (with cracks)
5	Free	1/2	2,000	3,460	1 wave largest deflection at 1/4 from bottom (thinner thickness)
17	Free	1	1,200	1,450	Middle toward bottom edge

*P_{crE} = Evaluated from Mikhail and Guralnick Method.

1 in = 25.4 m

1 lb = 4.5 N

TABLE 5.2 BUCKLING LOADS AND THEIR RATIOS

PLATE NUMBER	SUPPORT TYPE & STEEL SPACING (in)	THEORETICAL BUCKLING LOAD USING PLATE DATA IN PARABOLIC EQ. P_{crT2}	THEORETICAL BUCKLING LOAD USING CYLINDER DATA IN PARABOLIC EQ. P_{crT3}	EXPERIMENTAL BUCKLING LOAD P_{crE} (lb)	MAXIMUM LOAD P_M (lb)	$\frac{P_{crE}}{P_{crT2}}$	$\frac{P_{crE}}{P_{crT3}}$	$\left(\frac{P_{crE}}{P_{crT1}}\right)_0^*$
1	S.S.(1)	11805$^+$	13302$^+$	10800	11350	0.915	0.812	0.806
2	S.S.(1)	9533	11489	8700	9000	0.913	0.757	0.751
15	S.S.(2)	5791	9387	5800	5960	1.002	0.618	0.602
16	S.S.(1/2)	8963	9626	9000	10600	1.004	0.935	0.793
3	2CA(1)	3854^{++}	3966^{++}	4500	6400	1.168	1.135	1.109
10	2CA(1/2)	4651	4858	8000	9500	1.720	1.647	1.946
5	Free(1/2)	1611	1547	2000	3460	1.241	1.293	1.594
17	Free(1)	1187	1375	1200	1450	1.011	0.873	1.104

$^+$Theoretical buckling load based on plate buckling equation.

$^{++}$Theoretical buckling load based on column buckling equation.

*Values obtained by Seck.

TABLE 5.3 THEORETICAL VALUES OF CRITICAL LOAD (P_{crT}) USING ACTUAL PANEL THICKNESS & PLATE STRAINS

PLATE NUMBER	LONG SUPPORT TYPE	STEEL SPACING (in)	n	THICKNESS t (in)*	I ($in^4 \times 10^{-3}$)	ε_{cr} ($\mu\varepsilon$)	f_{cr} (psi)	P_{crT} (lb)
1	S.S.	1	-	0.266	18.82	1519.07	3443.07	11805
2	S.S.	1	-	0.248	15.25	1042.16	3018.66	9533
15	S.S.	2	-	0.240	13.82	627.61	1953.97	5791
16	S.S.	1/2	-	0.247	15.07	832.794	2733.29	8963
3	2CA	1	2	0.268	19.25	303.614	1150.65	3854
10	2CA	1/2	2	0.276	21.02	385.921	1284.34	4651
5	Free	1/2	1	0.274	20.80	82.6405	464.849	1611
17	Free	1	1	0.262	17.98	69.7332	367.158	1187

*Average thickness measured at bulged face of each plate.

1 in = 25.4 m

1 in^4 = 416,230 mm^4

1 psi = 6.9 KPa

1 lb = 4.5 N

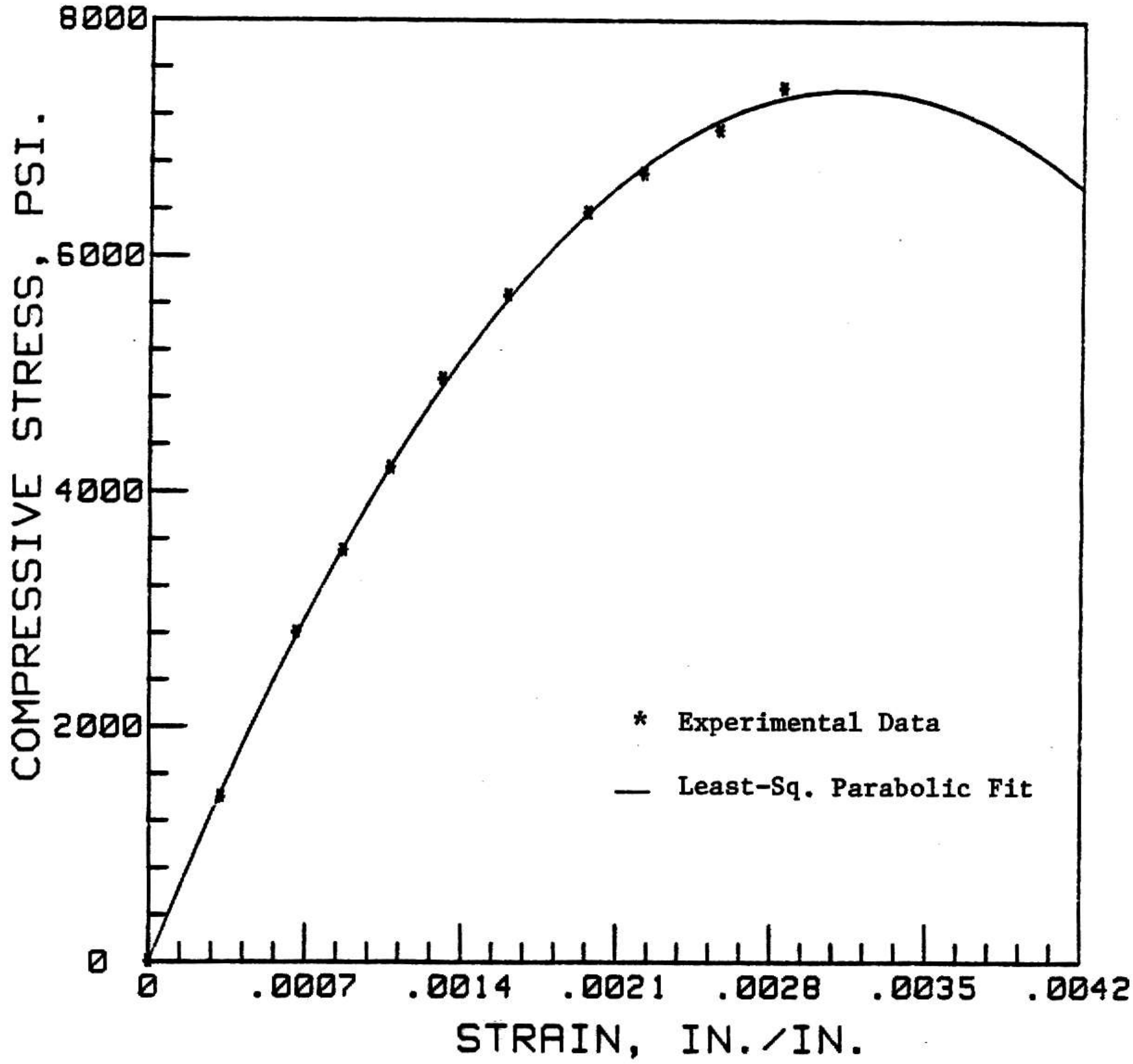

Figure 5.1 Stress-Strain Curve for Cylinder 1

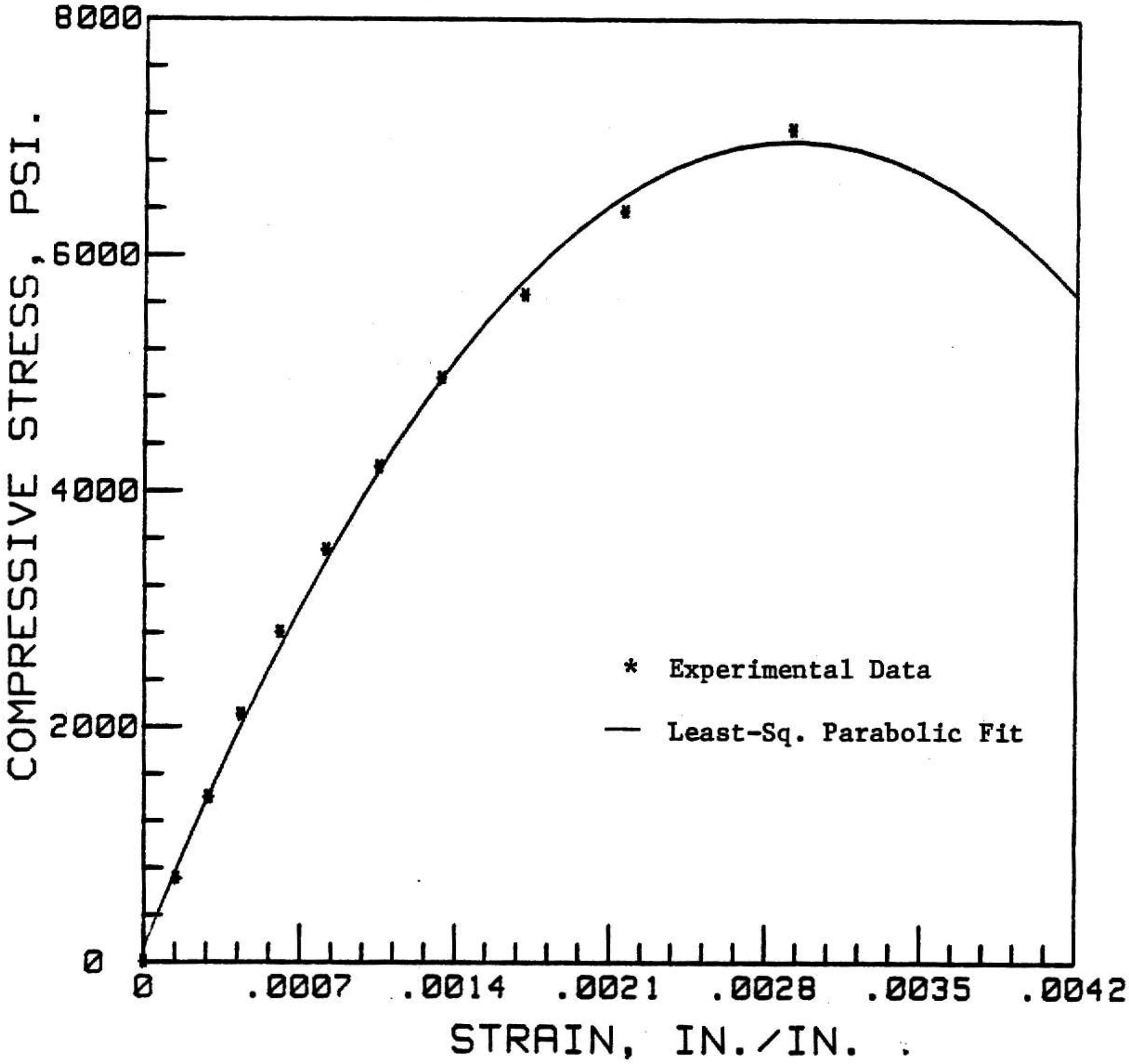

Figure 5.2 Stress-Strain Curve for Cylinder 2

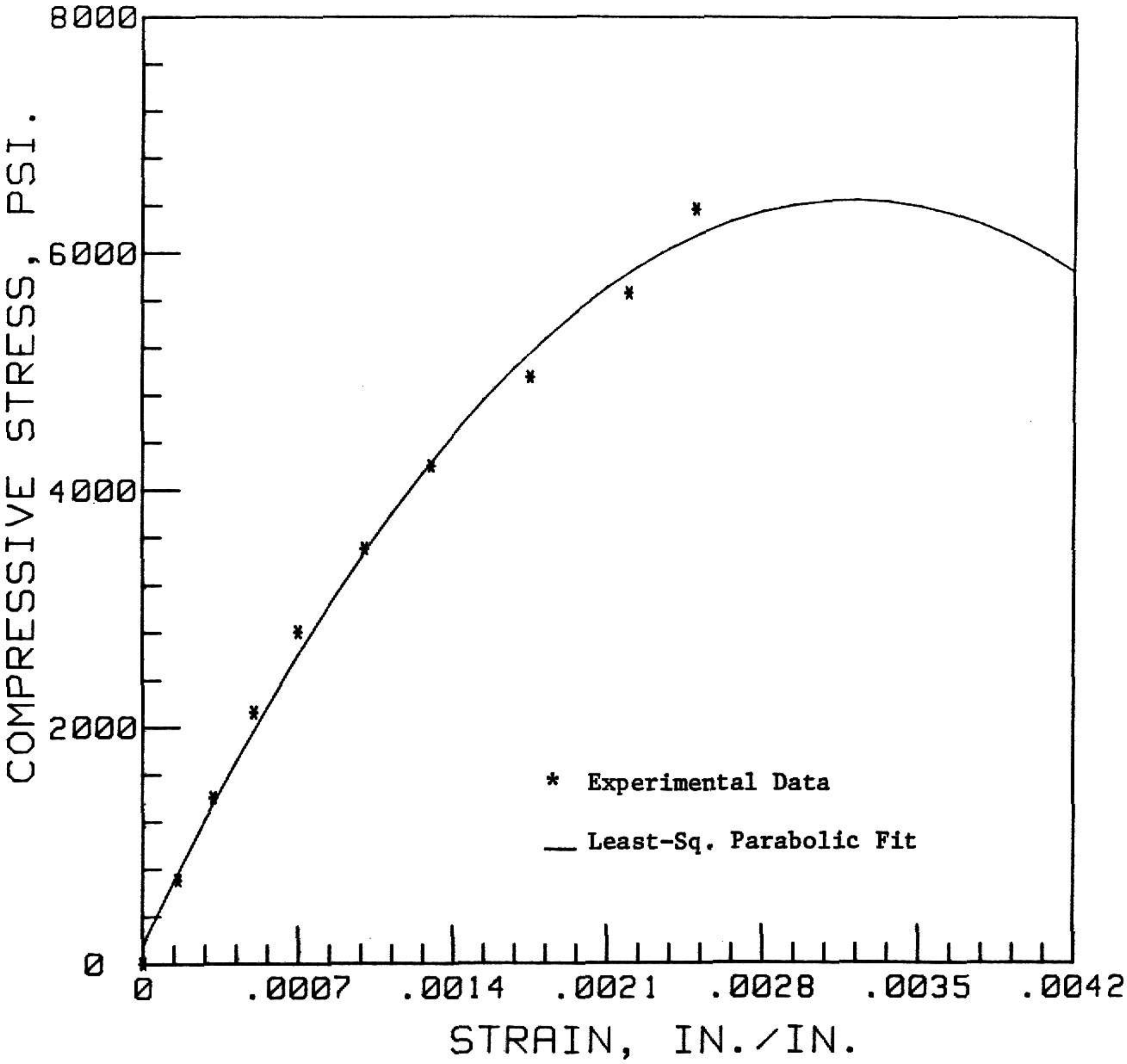

Figure 5.3 Stress-Strain Curve for Cylinder 3

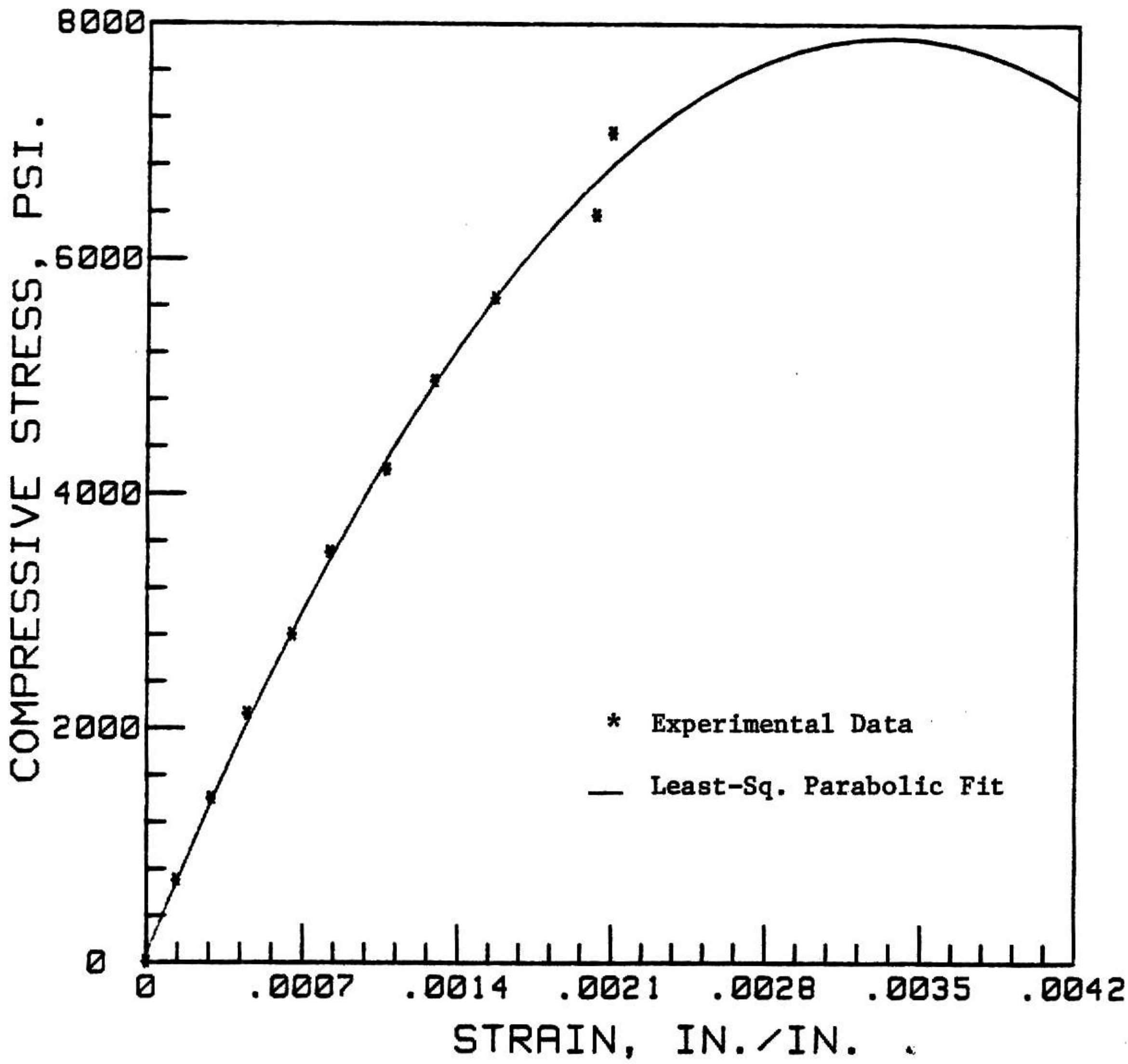

Figure 5.4 Stress-Strain Curve for Cylinder 4

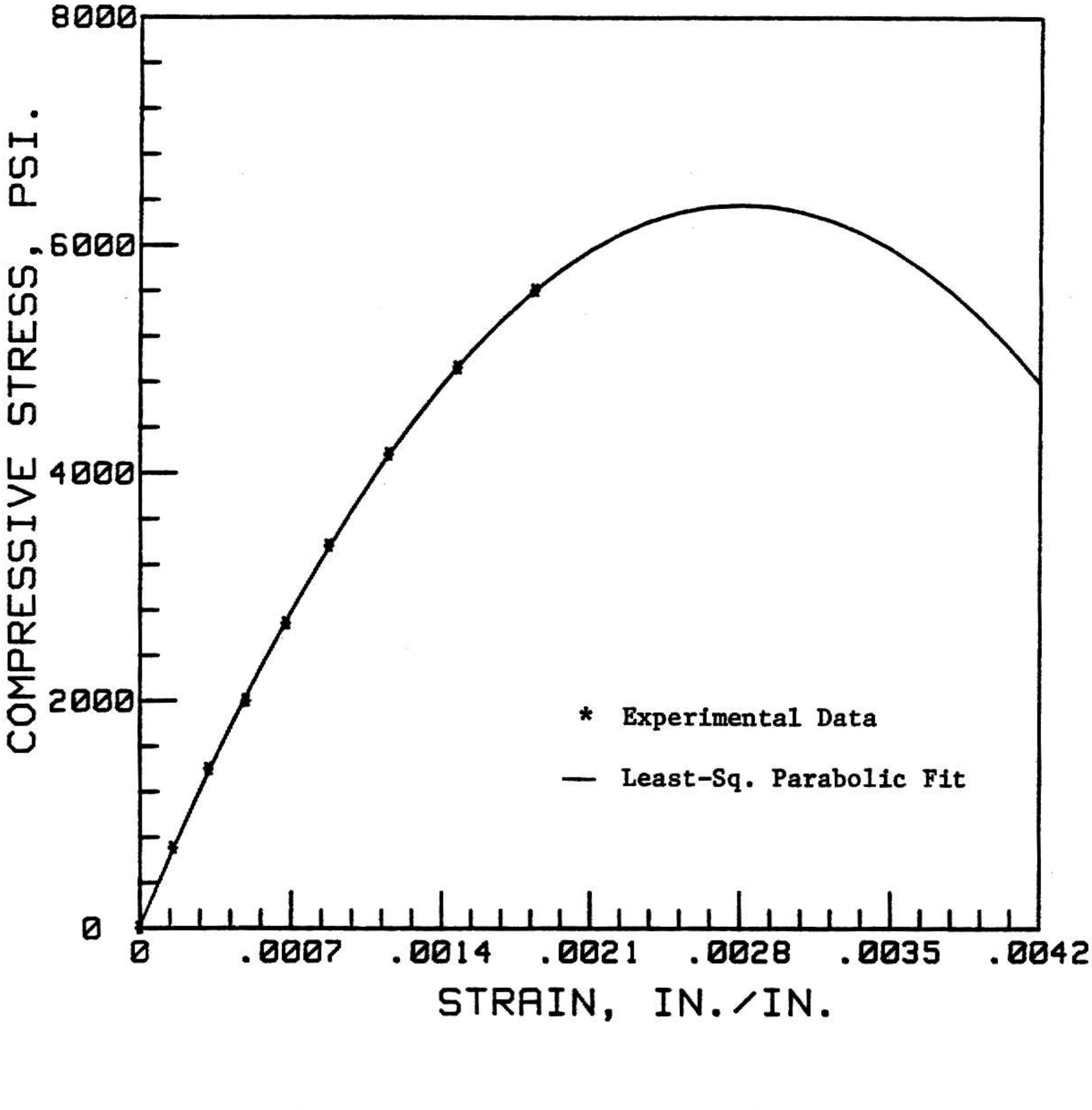

Figure 5.5 Stress-Strain Curve for Cylinder 5

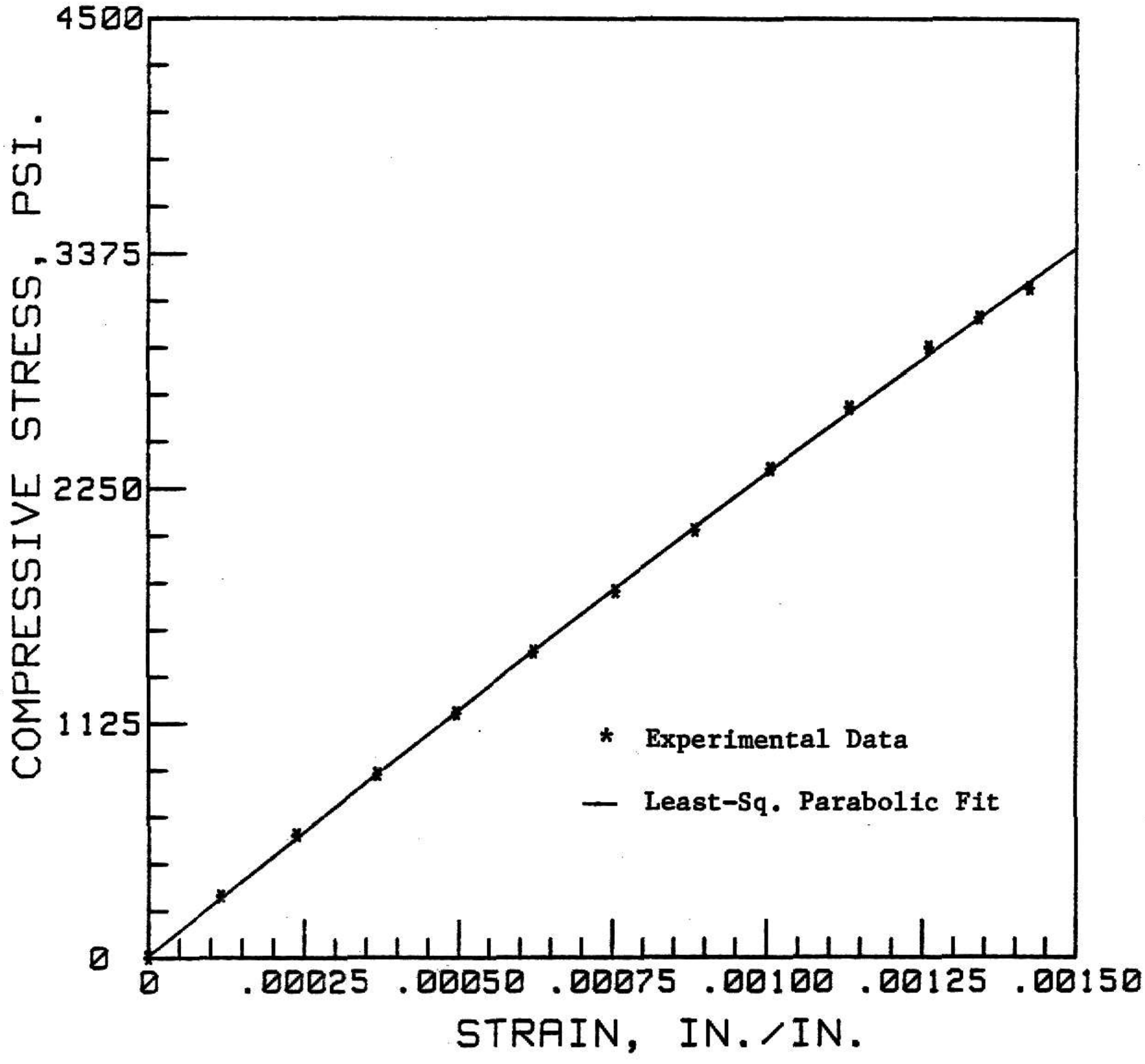

Figure 5.6 Stress-Strain Curve for Plate 1

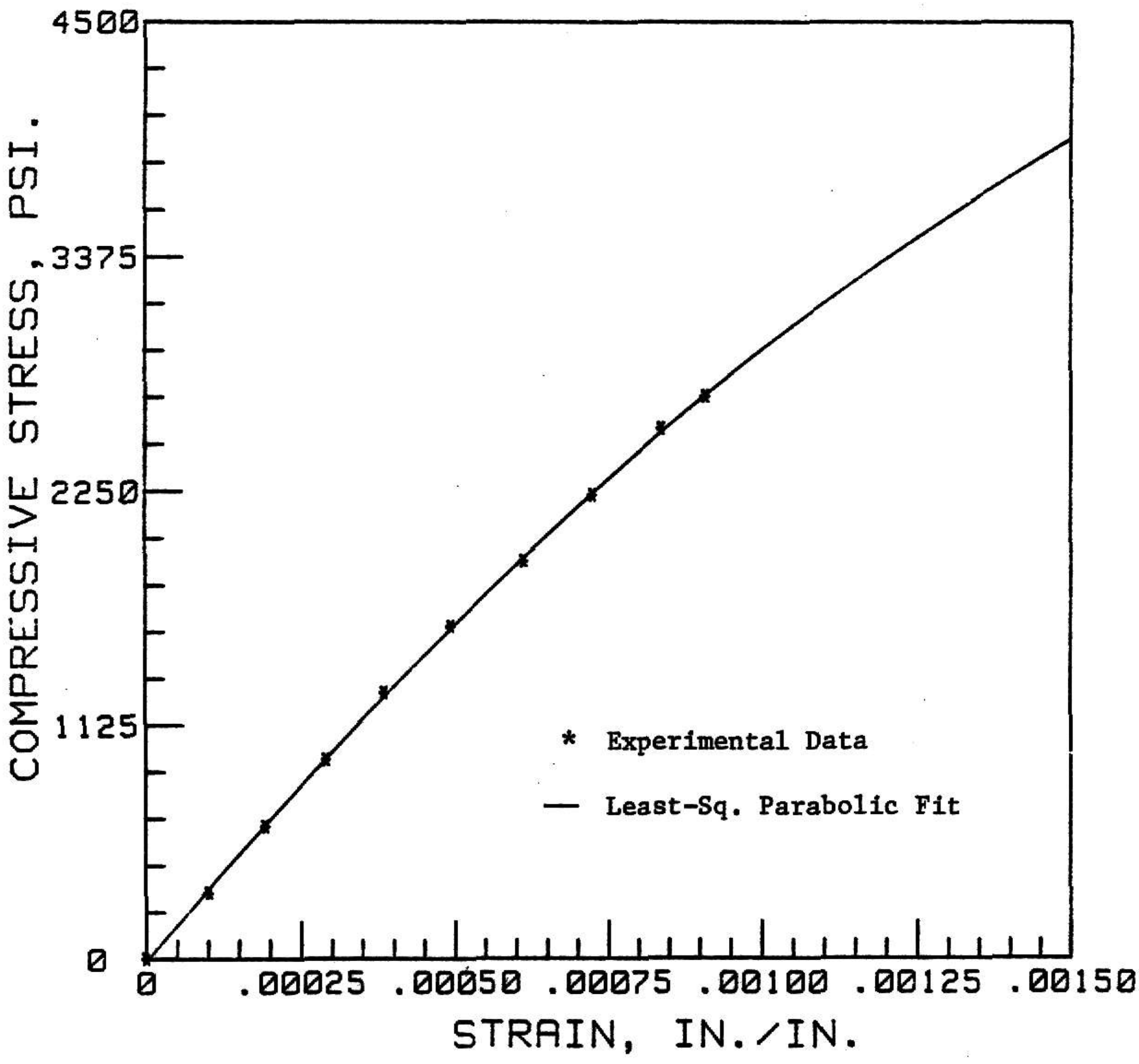

Figure 5.7 Stress-Strain Curve for Plate 2

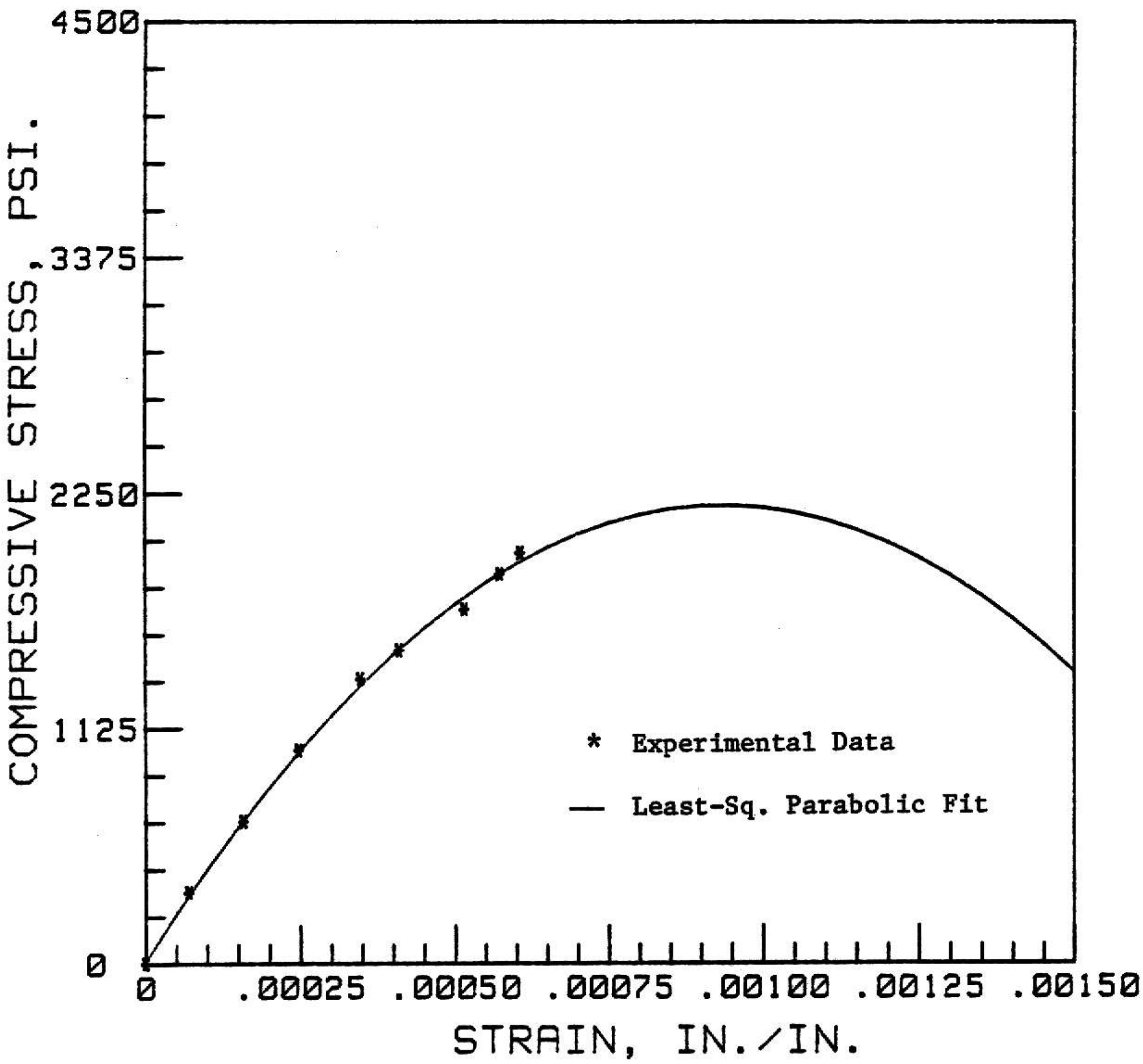

Figure 5.8 Stress-Strain Curve for Plate 15

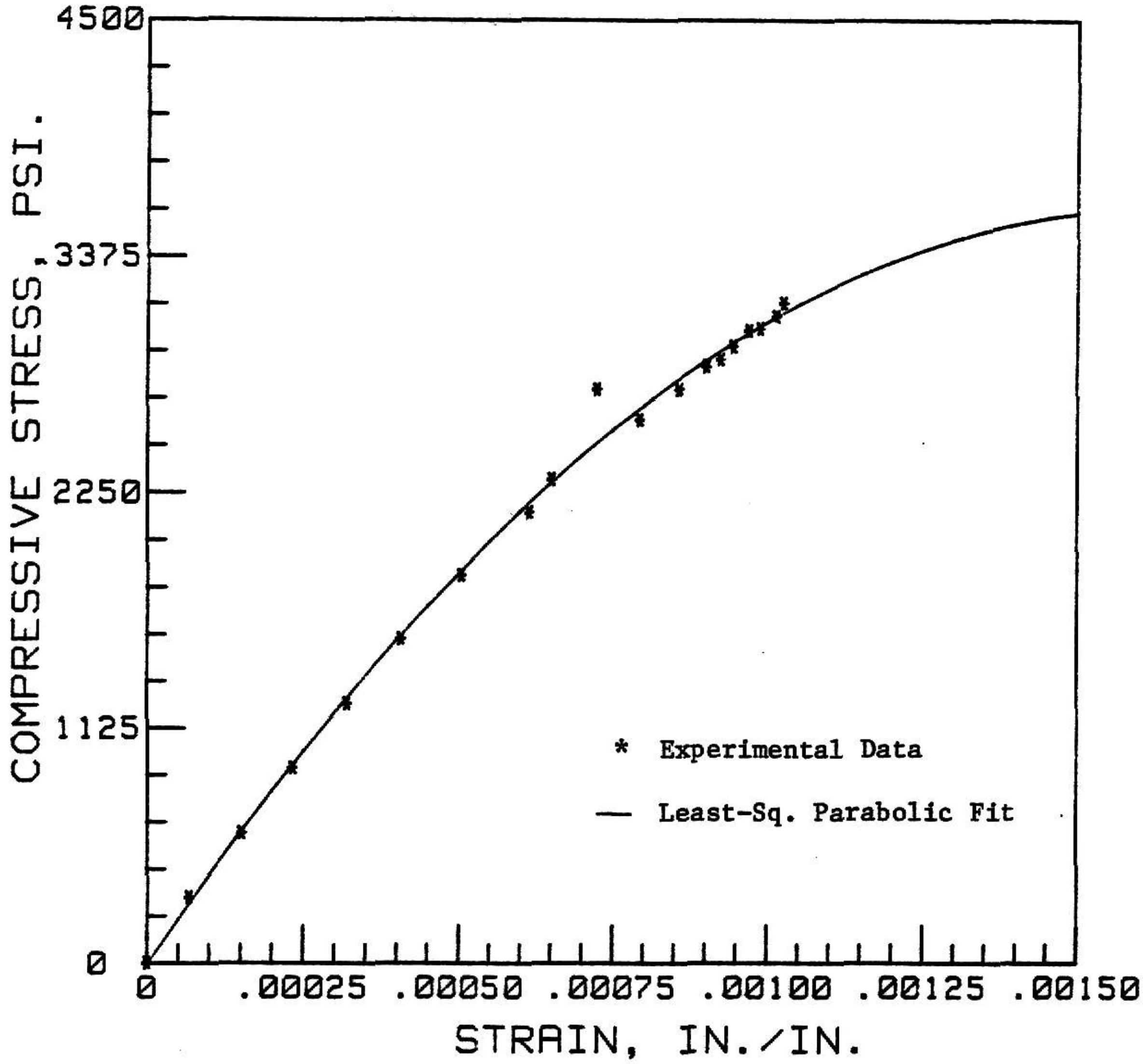

Figure 5.9 Stress-Strain Curve for Plate 16

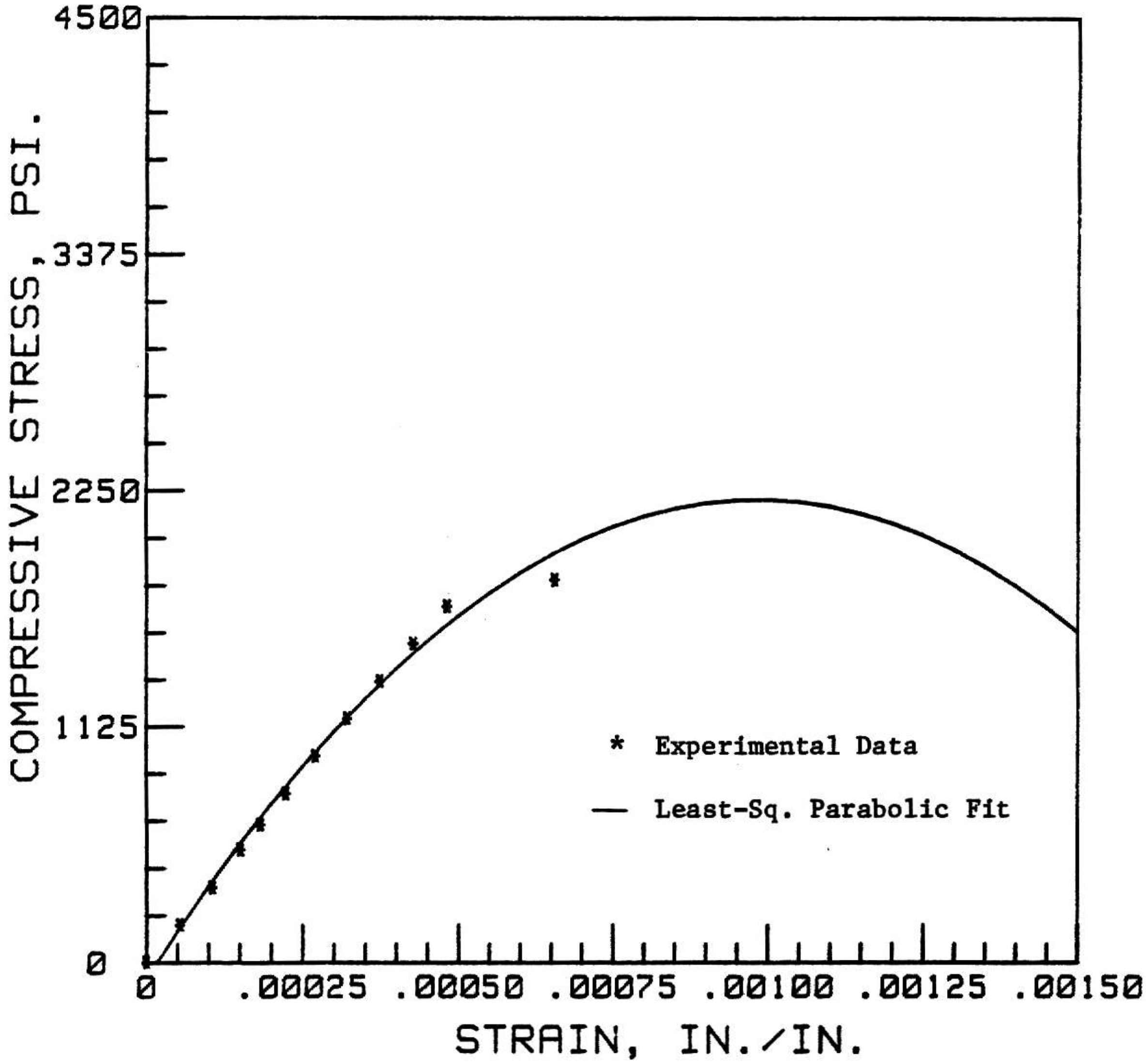

Figure 5.10 Stress-Strain Curve for Plate 3

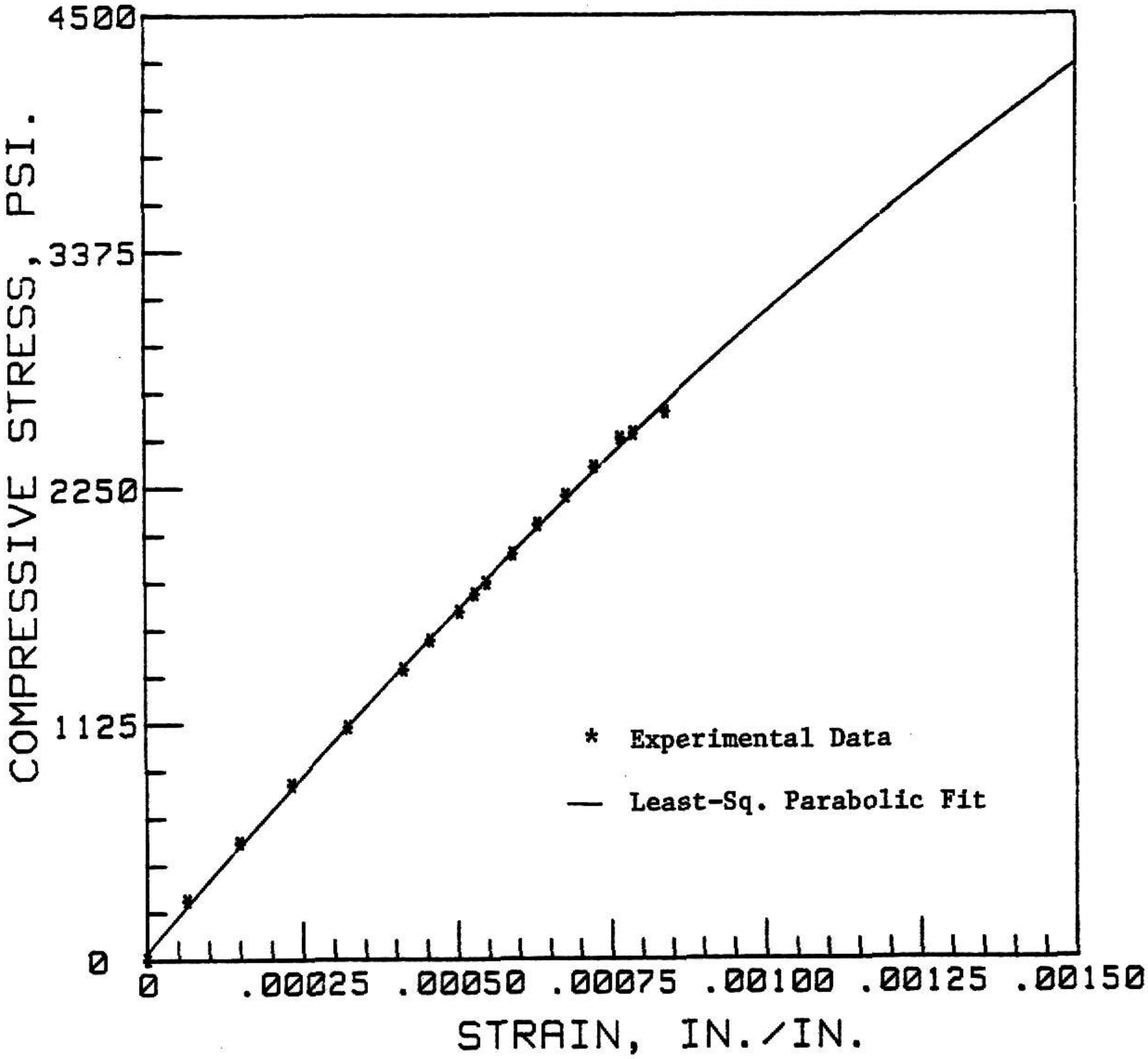

Figure 5.11 Stress-Strain Curve for Plate 10

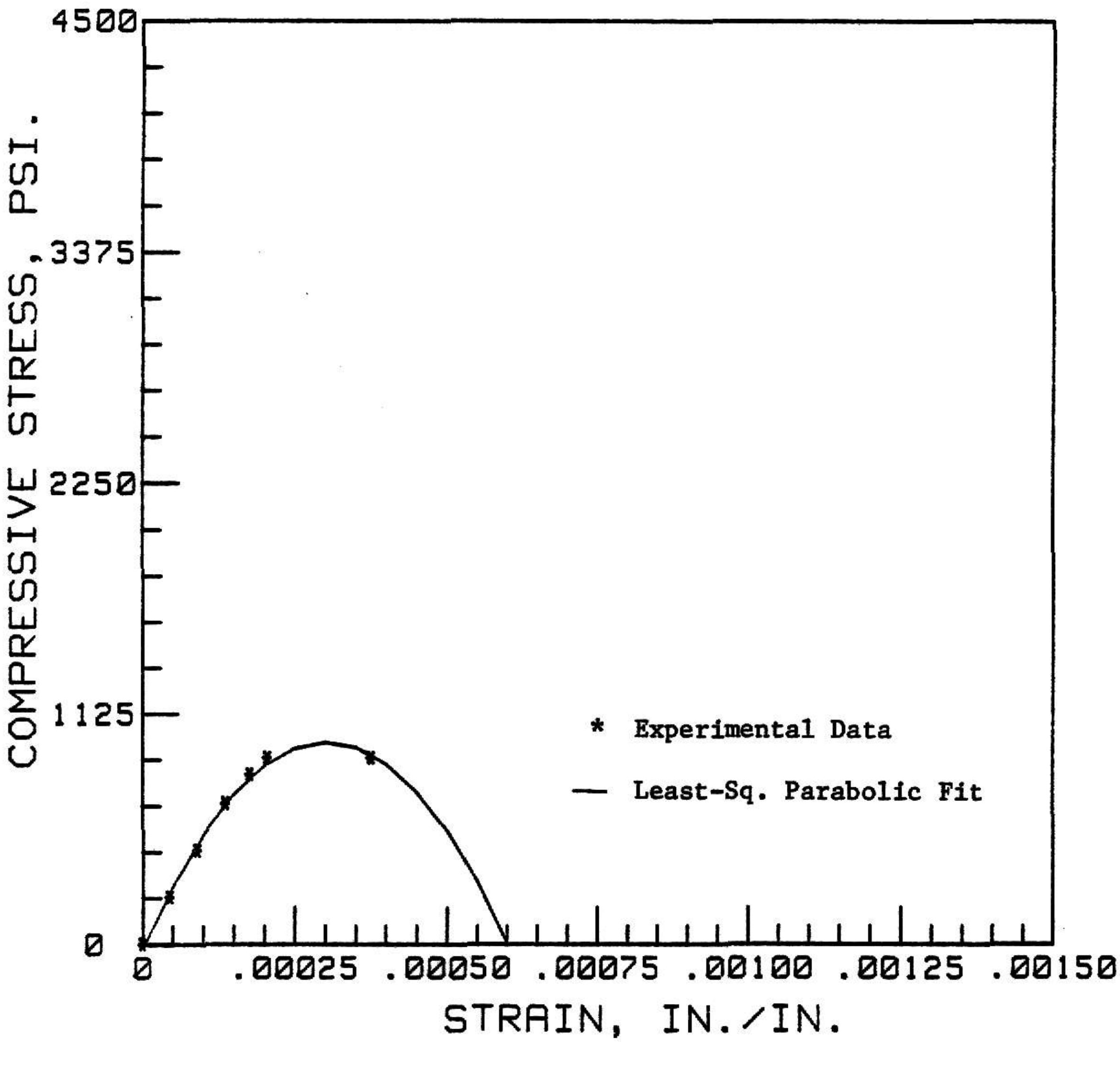

Figure 5.12 Stress-Strain Curve for Plate 5

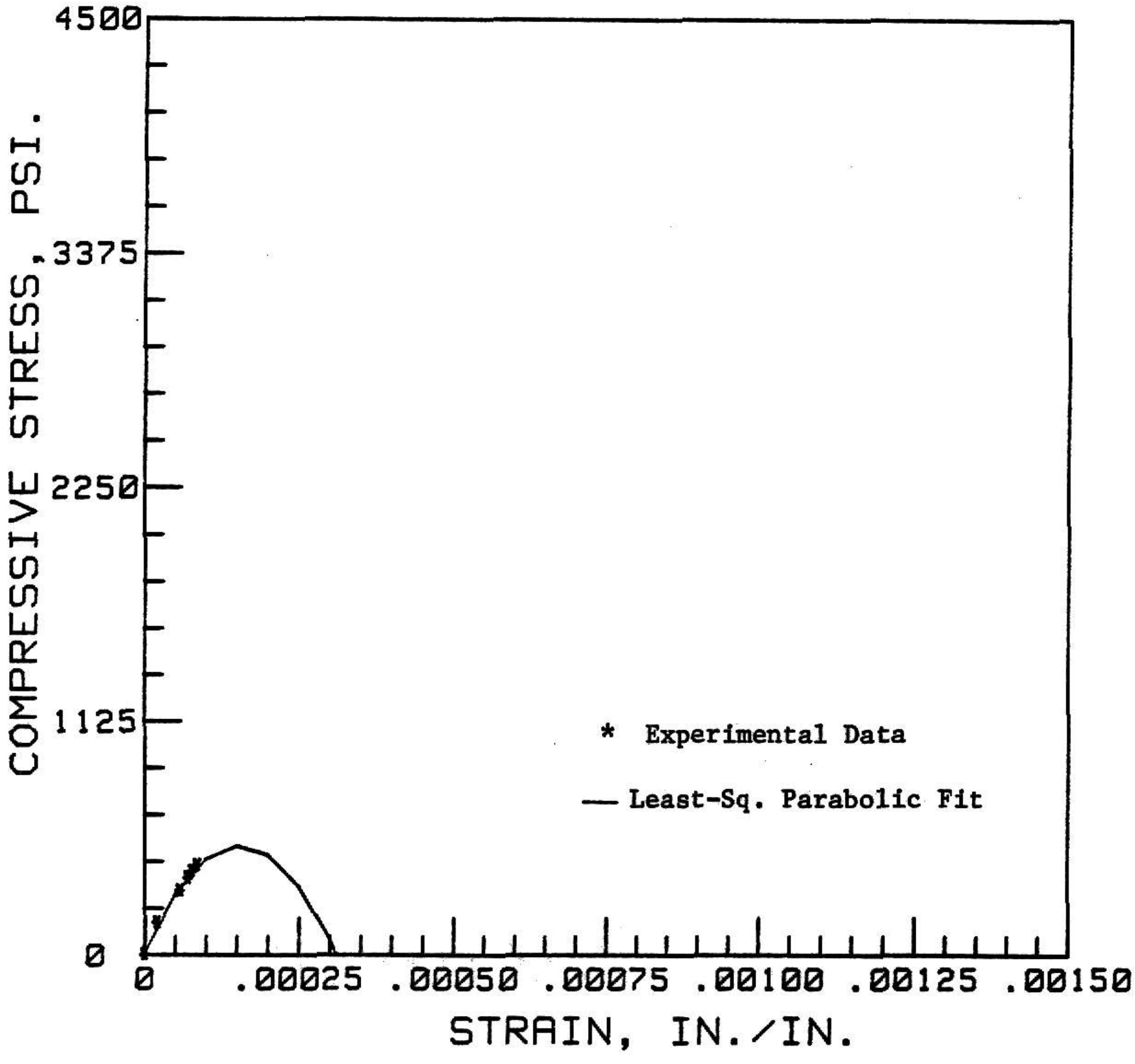

Figure 5.13 Stress-Strain Curve for Plate 17

ACKNOWLEDGMENTS

The writer wishes to express his genuine thanks and deepest appreciation to Dr. H. D. Knostman for his assistance, help and invaluable suggestions during the preparation of this report.

Sincere gratitude is due to Dr. S. E. Swartz for his guidance and help.

Appreciation is put forward to Dr. E. C. Lindly for serving as committee member.

Special thanks is extended to Dr. Robert R. Snell for extending me the financial support.

BIBLIOGRAPHY

1. Bleich, F., Buckling Strength of Metal Structures, McGraw-Hill Book Co., Inc., New York, 1952.

2. Guttman, I., Wilks, S. S., Hunter, J. S., Introductory Engineering Statistics, 2nd ed., John Wiley and Sons, Inc., New York, 1965.

3. Munoz, A., "Buckling Behavior of Reinforced Concrete Wall Panel Models," KSU, Manhattan, Master's Thesis, 1981.

4. Timoshenko, S. P. and Gere, J. M., Theory of Elastic Stability, 2nd ed., McGraw-Hill Book Co., Inc., New York, 1961.

5. Seck. A., "Buckling Behavior of Reinforced Concrete Plate Models," KSU, Manhattan, Master's Thesis, 1983.

6. Stark, A. P., Introduction to Numerical Methods, The MacMillan Company, Collier-MacMillan Limited, London, 1970.

7. Swartz, S. E., Rosebraugh, V. H., Rogacki, S. A., "A Method for Determining the Buckling Strength of Concrete Panels," presented at Third International Congress on Experimental Mechanics held in Los Angeles, CA on May 13-18, 1973, pp. 138-144.

8. Swartz, S. E. and Rosebraugh, V. H., "Buckling of Reinforced Concrete Plates," Journal of the Structural Division, ASCE, January 1974, pp. 195-208.

9. Swartz, S. E., Rosebraugh, V. H., Berman, M. Y., "Buckling Tests on Rectangular Concrete Panels," Title No. 71-5, ACI Journal, January 1974, pp. 33-39.

APPENDIX A

NOTATIONS

NOTATION

Ac, Ag	=	Gross cross-sectional area of concrete panel, sq. in.
As	=	Area of steel reinforcement, sq. in.
A_0, A_1, A_2	=	Known constants
a	=	Plate length
B	=	Parameter for buckling stress-strain calculations
Cs	=	Factor of safety
C	=	Parameter for buckling stress-strain calculations
E	=	Modulus of elasticity for concrete, psi.
D_I	=	Constant in elastic buckling equation,
D_s	=	Plate stiffness coefficient
E_T	=	Tangent modulus for concrete, psi.
f_c	=	Compressive concrete stress, psi.
f_c'	=	Specified compressive strength of concrete, psi.
f_{cr}	=	Critical stress of concrete, psi.
f_y	=	Yield strength of reinforcement, psi.
I	=	Moment of inertial about bending axis of panel, in^4
K	=	Parameter for buckling stress-strain calculations
L	=	Unsupported length of panel
n	=	Number of waves panel deflects
P_{cr}	=	Critical axial load on panel, lbs.
P_{crT}	=	Theoretical critical panel load using actual thickness
P_{crE}	=	Experimentally obtained panel buckling load
h	=	Thickness of panel, in.
b	=	Width of panel, in.
Z_1	=	Distance of ith steel layer from middle surface.

ε_c = Concrete strain, in. per in.

ε_{cr} = Critical strain, buckling strain, in. per in.

ε_0 = Strain at ultimate stress, in. per in.

ε_y = Yield strain of reinforcing steel, in. per in.

μ = Poisson's ratio

ρ = Ratio of reinforcing steel

δ_i = Deviation of the fitted curve from the tabulated point.

σ_c = Stress in concrete

ε_s = Strain in steel

APPENDIX B

COMPUTER PROGRAMS

```
5 REM"THIS PROGRAM IS SAVED UNDER THE NAME AM1..."
10 REM "THIS PROGRAM CALCULATES THE CRITICAL LOAD FOR PLATE SUBJECTED TO
13 REM UNIFORM LOAD ON OPPOSITE SIDES.  THE CALCULATION IS BASED ON COLUMN
15 REM TYPE OR PLATE-TYPE BUCKLING FORMULAS DISCUSSED IN THIS REPORT
17 REM FOR PLATE DATA
20 GOTO 50
30 INPUT   "ARE YOU DONE WITH THE CALCULATION (Y/N)?";Y$
40 IF Y$="Y" THEN 125
50 INPUT "Plate number=?";IJ
80 INPUT "IS THE BUCKLING OF COLUMN TYPE (Y/N)?";A$
90 IF A$="Y" THEN GOSUB 1000
100  REM IF A$=N, THEN THE BUCKLING IS PLATE TYPE
110  IF A$="N" THEN GOSUB 2000
120 GOTO 30
125 PRINT"                                                                        "
130 PRINT "***************Good bye . Have a good day sir!***************"
140 END
1000 REM COLUMN TYPE BUCKLING FORMULA
1010 INPUT "IS THE THICKNESS USED THEORETICAL(Y/N)";B$
1020 LPRINT"Column type buckling - Plate # ";IJ
1030 IF B$="Y" THEN 1040
1040 IF B$="N" THEN 1060
1050 LPRINT "  (USING THEORETICAL THICKNESS)  "
1055 GOTO 1065
1060 LPRINT "  (USING ACTUAL THICKNESS)  "
1065 LPRINT " "
1070 INPUT "AREA OF STEEL = ";AS
1080 INPUT "AREA OF CONCRETE = ";AC
1090 INPUT "NUMBER OF WAVES = ";N
1100 INPUT "MOMENT OF INERTIA = ";I
1102 INPUT "A(0)=";A(0)
1104 INPUT "A(1)=";A(1)
1106 INPUT "A(2)=";A(2)
1110 P=.03427
1120 ES=28.20E6
1130 REM: EC^2+(A(1)-P*(N^2)*I*A(2)/AC+AS*ES/AC)*EC/A(2)-P*(N^2)*I*A(1)/(2*AC*A(2))
1140 D = (A(1)-P*(N^2)*I*A(2)/AC+AS*ES/AC)/A(2)
1150 C = P*(N^2)*I*A(1)/(2*AC*A(2))+A(0)/A(2)
1160 EC=(-D-((D^2)+4*C)^0.5)/2
1170 FC=A(0)+A(1)*EC+A(2)*EC^2
1180 PC=AC*FC + AS*ES*EC
1190 LPRINT "AREA OF STEEL *************************** AS(Sq.in.)=";AS
1200 LPRINT "AREA OF CONCRETE *********************** AC(Sq.in.)=";AC
1210 LPRINT "NUMBER OF WAVES *********************** N         =";N
1220 LPRINT "MOMENT OF INERTIA ********************* I (in ^4) =";I
1230 LPRINT "A(0) *****************************************    =";A(0)
1240 LPRINT "A(1) *****************************************    =";A(1)
```

```
1245 LPRINT "A(2) *******************************************  =";A(2)
1250 LPRINT "CRITICAL STRAIN IN CONCRETE *********** EC(in.)   =";EC
1260 LPRINT "CRITICAL STRESS IN CONCRETE ************ FC(psi)   =";FC
1270 LPRINT "CRITICAL BUCKLING LOAD ***************** PC(lbs.)  =";PC
1275 LPRINT " "
1280 RETURN
2000 REM PLATE TYPE FORMULA
2020 INPUT "IS THE THICKNESS USED THEORETICAL(Y/N)";B$
2030 LPRINT"Plate type buckling - Plate # ";IJ
2040 IF B$="Y" THEN 2060
2050 IF B$="N" THEN 2070
2060 LPRINT "  (USING THEORETICAL THICKNESS)  "
2065 GOTO 2080
2070 LPRINT "  (USING ACTUAL THICKNESS)  "
2080 LPRINT" "
2090 INPUT "PLATE THICKNESS = ";TH
2100 INPUT "STEEL RATIO = ";SR
2102 INPUT "A(0) = ";A(0)
2104 INPUT "A(1) = ";A(1)
2106 INPUT "A(2) = ";A(2)
2110 WH=12
2115 PI = 3.141592654
2120 K = (PI^2*(TH/WH)^2)/(3*(1-SR))
2125 B = (A(1)+((ES*SR)/(1-SR))-2*K*A(2))/A(2)
2130 ES=28.20E6
2140 C = (A(0)-K*A(1))/A(2)
2150 EC = -0.5*(B+(B^2-4*C)^0.5)
2160 FC = A(0)+A(1)*EC+A(2)*(EC^2)


2170 PC=1*WH*TH*(FC*(1-SR)+ES*EC*SR)
2180 LPRINT "PLATE THICKNESS ******************** TH(in.)      = ";TH
2190 LPRINT "STEEL RATIO ************************ SR(in./in.) = ";SR
2200 GOTO 1230
2210 RETURN
```

```
Plate type buckling - Plate #  1
  (USING ACTUAL THICKNESS)

PLATE THICKNESS ******************** TH(in.)        =  .266
STEEL RATIO ************************* SR(in./in.) =  6.48E-03
A(0) *******************************************    = 7.69629
A(1) *******************************************    = 2.40687E+06
A(2) *******************************************    =-9.56955E+07
CRITICAL STRAIN IN CONCRETE *********** EC(in.)     = 1.51907E-03
CRITICAL STRESS IN CONCRETE ************ FC(psi)     = 3443.07
CRITICAL BUCKLING LOAD ***************** PC(lbs.)    = 11805.1

Plate type buckling - Plate #  2
  (USING ACTUAL THICKNESS)

PLATE THICKNESS ******************** TH(in.)        =  .248
STEEL RATIO ************************* SR(in./in.) =  7E-03
A(0) *******************************************    =-14.6704
A(1) *******************************************    = 3.54155E+06
A(2) *******************************************    =-6.05415E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.)     = 1.04216E-03
CRITICAL STRESS IN CONCRETE ************ FC(psi)     = 3018.66
CRITICAL BUCKLING LOAD ***************** PC(lbs.)    = 9532.89

Column type buckling - Plate #  3
  (USING ACTUAL THICKNESS)

AREA OF STEEL ************************** AS(Sq.in.)= .02071
AREA OF CONCRETE ********************** AC(Sq.in.)= 3.1953
NUMBER OF WAVES ********************** N          = 2
MOMENT OF INERTIA ********************* I (in ^4) = .01925
A(0) *******************************************    =-77.6932
A(1) *******************************************    = 4.63957E+06
A(2) *******************************************    =-2.35843E+09
CRITICAL STRAIN IN CONCRETE *********** EC(in.)     = 3.03614E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi)     = 1150.65
CRITICAL BUCKLING LOAD ***************** PC(lbs.)    = 3853.97

Plate type buckling - Plate #  15
  (USING ACTUAL THICKNESS)


PLATE THICKNESS ******************** TH(in.)        =  .24
STEEL RATIO ************************* SR(in./in.) =  3.6E-03
A(0) *******************************************    = 12.2421
A(1) *******************************************    = 4.65978E+06
A(2) *******************************************    =-2.49507E+09
CRITICAL STRAIN IN CONCRETE *********** EC(in.)     = 6.35508E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi)     = 1953.97
CRITICAL BUCKLING LOAD ***************** PC(lbs.)    = 5790.68
```

```
Plate type buckling - Plate #  16
  (USING ACTUAL THICKNESS)

PLATE THICKNESS ******************** TH(in.)        =  .247



STEEL RATIO ************************* SR(in./in.) =  .014
A(0) ******************************************   =-6.97265
A(1) ******************************************   = 4.41146E+06
A(2) ******************************************   =-1.34609E+09
CRITICAL STRAIN IN CONCRETE *********** EC(in.)   = 8.32794E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi)   = 2733.29
CRITICAL BUCKLING LOAD ***************** PC(lbs.)  = 8962.57

Column type buckling - Plate #  10
  (USING ACTUAL THICKNESS)

AREA OF STEEL ************************** AS(Sq.in.)= .04142
AREA OF CONCRETE ********************** AC(Sq.in.)= 3.2706
NUMBER OF WAVES *********************** N          = 2
MOMENT OF INERTIA ********************* I (in ^4) = .02102
A(0) ******************************************   = 31.9684
A(1) ******************************************   = 3.48245E+06
A(2) ******************************************   =-4.40768E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.)   = 3.85921E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi)   = 1284.34
CRITICAL BUCKLING LOAD ***************** PC(lbs.)  = 4651.33

Column type buckling - Plate #  5
  (USING ACTUAL THICKNESS)

AREA OF STEEL ************************** AS(Sq.in.)= .04142
AREA OF CONCRETE ********************** AC(Sq.in.)= 3.25858
NUMBER OF WAVES *********************** N          = 1
MOMENT OF INERTIA ********************* I (in ^4) = .0208
A(0) ******************************************   =-24.5257
A(1) ******************************************   = 6.67664E+06
A(2) ******************************************   =-1.10551E+10
CRITICAL STRAIN IN CONCRETE *********** EC(in.)   = 8.26405E-05
CRITICAL STRESS IN CONCRETE ************ FC(psi)   = 464.849
CRITICAL BUCKLING LOAD ***************** PC(lbs.)  = 1611.27
```

Column type buckling - Plate # 17
(USING ACTUAL THICKNESS)

```
AREA OF STEEL *************************** AS(Sq.in.)= .02071
AREA OF CONCRETE *********************** AC(Sq.in.)= 3.12329
NUMBER OF WAVES ************************ N          = 1
MOMENT OF INERTIA ********************** I (in ^4) = .01798
A(0) *****************************************      = 11.785
A(1) *****************************************      = 6.64691E+06
A(2) *****************************************      =-2.14625E+10
CRITICAL STRAIN IN CONCRETE *********** EC(in.)     = 6.97332E-05
CRITICAL STRESS IN CONCRETE ************ FC(psi)    = 367.158
CRITICAL BUCKLING LOAD ***************** PC(lbs.)   = 1187.47
```

```
5  REM "THIS PROGRAM IS SAVED UNDER THE NAME MY1..."
10 REM "THIS PROGRAM CALCULATES THE CRITICAL LOAD FOR PLATE
12 REM SUBJECTED TO UNIFORM LOAD ON OPPOSITE SIDES.  THE
14 REM CALCULATON IS BASED ON COLUMN-TYPE OR PLATE-TYPE BUCKLING
16 REM FORMULAS PREVIOUSLY DISCUSSED IN THIS REPORT FOR CYLINDER DATA"
20 GOTO 50
30 INPUT   "ARE YOU DONE WITH THE CALCULATION (Y/N)?";Y$
40 IF Y$="Y" THEN 125
50 INPUT "Plate number=?";IJ
80 INPUT "IS THE BUCKLING OF COLUMN TYPE (Y/N)?";A$
90 IF A$="Y" THEN GOSUB 1000
100  REM IF A$=N, THEN THE BUCKLING IS PLATE TYPE
110  IF A$="N" THEN GOSUB 2000
120 GOTO 30
125 PRINT"                                                                    "
130 PRINT "***************Good bye . Have a good day sir!***************"
140 END
1000 REM COLUMN TYPE BUCKLING FORMULA
1010 INPUT "IS THE THICKNESS USED THEORETICAL(Y/N)";B$
1020 LPRINT"Column type buckling - Plate # ";IJ
1030 IF B$="Y" THEN 1040
1040 IF B$="N" THEN 1060
1050 LPRINT "  (USING THEORETICAL THICKNESS)  "
1055 GOTO 1065
1060 LPRINT "  (USING ACTUAL THICKNESS)  "
1065 LPRINT " "

1070 INPUT "AREA OF STEEL = ";AS
1080 INPUT "AREA OF CONCRETE = ";AC
1090 INPUT "NUMBER OF WAVES = ";N
1100 INPUT "MOMENT OF INERTIA = ";I
1102 INPUT "A(0) = ";A(0)
1104 INPUT "A(1) = ";A(1)
1106 INPUT "A(2) = ";A(2)
1110 P=29.129E-3
1120 ES=28.20E6
1130 REM : 0.85*EC^2+(0.85*A(1)-P*(N^2)*I*A(2)/AC+AS*ES/AC)*EC/A(2)-P*(N^2)
1135 REM *I*A(1)/2*AC*A(2))+0.85*A(0)/A(2) = 0
1140 D = (0.85*A(1)-P*(N^2)*I*A(2)/AC+AS*ES/AC)*1/A(2)
1150 C = P*(N^2)*I*A(1)/(2*AC*A(2))+0.85*A(0)/A(2)
1160 EC=(-D-((D^2)+4*0.85*C)^0.5)/2
1170 FC = 0.85*(A(0)+A(1)*EC+A(2)*EC^2)
1180 PC=AC*FC + AS*ES*EC
1190 LPRINT "AREA OF STEEL *************************** AS(Sq.in.)=";AS
1200 LPRINT "AREA OF CONCRETE ********************** AC(Sq.in.)=";AC
1210 LPRINT "NUMBER OF WAVES *********************** N         =";N
1220 LPRINT "MOMENT OF INERTIA ********************* I (in ^4) =";I
1230 LPRINT "A(0) ****************************************      =";A(0)
1240 LPRINT "A(1) ****************************************      =";A(1)
```

```
1245 LPRINT "A(2) ******************************************     =";A(2)
1250 LPRINT "CRITICAL STRAIN IN CONCRETE *********** EC(in.)    =";EC
1260 LPRINT "CRITICAL STRESS IN CONCRETE ************ FC(psi)   =";FC
1270 LPRINT "CRITICAL BUCKLING LOAD ***************** PC(lbs.)  =";PC
1275 LPRINT " "
1280 RETURN
2000 REM PLATE TYPE FORMULA
2020 INPUT "IS THE THICKNESS USED THEORETICAL(Y/N)";B$
2030 LPRINT"Plate type buckling - Plate # ";IJ
2040 IF B$="Y" THEN 2060
2050 IF B$="N" THEN 2070
2060 LPRINT "  (USING THEORETICAL THICKNESS)  "
2065 GOTO 2080
2070 LPRINT "  (USING ACTUAL THICKNESS)  "
2080 LPRINT" "
2090 INPUT "PLATE THICKNESS = ";TH
2100 INPUT "STEEL RATIO = ";SR
2102 INPUT "A(0) = ";A(0)
2104 INPUT "A(1) = ";A(1)
2106 INPUT "A(2) = ";A(2)
2110 WH=12
2115 PI = 3.141592654
2120 K = (PI^2*(TH/WH)^2)/(3*(1-SR))
2125 B = (A(1)+ES*SR/0.85*(1-SR)-2*K*A(2))/A(2)
2130 ES=28.20E6
2140 C = (A(0)-K*A(1))/A(2)
2150 EC = -0.5*(B+(B^2-4*C)^0.5)
2160 FC = 0.85*(A(0)+A(1)*EC+A(2)*EC^2)
2170 PC=1*WH*TH*(FC*(1-SR)+ES*EC*SR)
2180 LPRINT "PLATE THICKNESS ******************* TH(in.)       = ";TH
2190 LPRINT "STEEL RATIO ************************ SR(in./in.) = ";SR
2200 GOTO 1230
2210 RETURN
```

Plate type buckling - Plate # 1
(USING ACTUAL THICKNESS)

PLATE THICKNESS ******************** TH(in.) = .266
STEEL RATIO ************************** SR(in./in.) = 6.48E-03
A(0) ** =-7.91634
A(1) ** = 4.71701E+06
A(2) ** =-7.49597E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.) = 1.23275E-03
CRITICAL STRESS IN CONCRETE ************ FC(psi) = 3967.65
CRITICAL BUCKLING LOAD ***************** PC(lbs.) = 13301.7

Plate type buckling - Plate # 2
(USING ACTUAL THICKNESS)

PLATE THICKNESS ******************** TH(in.) = .248
STEEL RATIO ************************** SR(in./in.) = 7E-03
A(0) ** =-7.91634
A(1) ** = 4.71701E+06
A(2) ** =-7.49597E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.) = 1.0658E-03
CRITICAL STRESS IN CONCRETE ************ FC(psi) = 3666.54
CRITICAL BUCKLING LOAD ***************** PC(lbs.) = 11489.1

Plate type buckling - Plate # 15
(USING ACTUAL THICKNESS)

PLATE THICKNESS ******************** TH(in.) = .24
STEEL RATIO ************************** SR(in./in.) = 3.6E-03
A(0) ** = 10.5793
A(1) ** = 4.52713E+06
A(2) ** =-8.07317E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.) = 9.9945E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi) = 3169.47
CRITICAL BUCKLING LOAD ***************** PC(lbs.) = 9387.44

Plate type buckling - Plate # 16
(USING ACTUAL THICKNESS)

PLATE THICKNESS ******************** TH(in.) = .247
STEEL RATIO ************************** SR(in./in.) = .014
A(0) ** = 156.16
A(1) ** = 3.93372E+06
A(2) ** =-6.14613E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.) = 9.77594E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi) = 2902.21
CRITICAL BUCKLING LOAD ***************** PC(lbs.) = 9625.69

```
Column type buckling - Plate #  3
  (USING ACTUAL THICKNESS)

AREA OF STEEL *************************** AS(Sq.in.)= .02071
AREA OF CONCRETE *********************** AC(Sq.in.)= 3.1953
NUMBER OF WAVES ************************ N          = 2
MOMENT OF INERTIA ********************** I (in ^4) = .01925
A(0) ******************************************      =-7.91634
A(1) ******************************************      = 4.71701E+06
A(2) ******************************************      =-7.49597E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.)    = 3.12547E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi)    = 1184.17
CRITICAL BUCKLING LOAD ***************** PC(lbs.)   = 3966.32

Column type buckling - Plate #  10
  (USING ACTUAL THICKNESS)

AREA OF STEEL *************************** AS(Sq.in.)= .04142
AREA OF CONCRETE *********************** AC(Sq.in.)= 3.2706
NUMBER OF WAVES ************************ N          = 2
MOMENT OF INERTIA ********************** I (in ^4) = .02102
A(0) ******************************************      = 114.761
A(1) ******************************************      = 4.694E+06
A(2) ******************************************      =-8.03391E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.)    = 3.37083E-04
CRITICAL STRESS IN CONCRETE ************ FC(psi)    = 1364.88
CRITICAL BUCKLING LOAD ***************** PC(lbs.)   = 4857.72

Column type buckling - Plate #  5
  (USING ACTUAL THICKNESS)

AREA OF STEEL *************************** AS(Sq.in.)= .04142
AREA OF CONCRETE *********************** AC(Sq.in.)= 3.25858
NUMBER OF WAVES ************************ N          = 1
MOMENT OF INERTIA ********************** I (in ^4) = .0208
A(0) ******************************************      = 74.4836
A(1) ******************************************      = 4.67215E+06
A(2) ******************************************      =-6.98806E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.)    = 9.63055E-05
CRITICAL STRESS IN CONCRETE ************ FC(psi)    = 440.263
CRITICAL BUCKLING LOAD ***************** PC(lbs.)   = 1547.12


Column type buckling - Plate #  17
  (USING ACTUAL THICKNESS)

AREA OF STEEL *************************** AS(Sq.in.)= .02071
AREA OF CONCRETE *********************** AC(Sq.in.)= 3.12329
NUMBER OF WAVES ************************ N          = 1
MOMENT OF INERTIA ********************** I (in ^4) = .01798
A(0) ******************************************      = 74.4836
A(1) ******************************************      = 4.67215E+06
A(2) ******************************************      =-6.98806E+08
CRITICAL STRAIN IN CONCRETE *********** EC(in.)    = 9.18449E-05
CRITICAL STRESS IN CONCRETE ************ FC(psi)    = 423.047
CRITICAL BUCKLING LOAD ***************** PC(lbs.)   = 1374.94
```

LEAST SQUARE PARABOLA APPLIED TO BUCKLING OF CONCRETE PLATES

by

ANWAR A. MERCHANT

B.E. (Civil), N.E.D. University of Engineering and Technology, Pakistan, 1981

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1986

ABSTRACT

The principal objective of this report was to improve the results presented by Abdoulaye Seck in his thesis, "Buckling of Reinforced Concrete Plate Models." The difference between experimental and theoretical results presented in his report were sufficient to indicate further study.

The theoretical buckling loads obtained by Seck were dependent upon Hognestad parabola which is a function of the ultimate strength f_c', and ultimate strain ε_0, of the concrete test cylinders. Seck obtained f_c' and ε_0 by graphically putting a parabola through a plot of stress-strain data.

In this report, a least square parabolic fit of the cylinder data was tryed to obtain improved values of f_c' and ε_0. This procedure was repeated for five cylinders. The result obtained showed good agreement between the experimental and theoretical buckling loads and showed some improvement over the results obtained by Seck. The ratios of predicted and experimental buckling load together with Seck's result have been presented for comparison.

By comparing the stress-strain curves for the cylinders and the plates it can be seen that the slopes are different and hence the modulus of elasticity will be different. Since buckling load depends upon modulus of elasticity, a further investigation into the use of plate stress-strain data was felt desirable. Because these plates buckled at stresses well below the ultimate, no values of f_c' and ε_0 were available to use in the Hognestad parabola.

However, a least square parabola could be fit to the stress-strain data. This parabola could then be used to represent the modulus of elasticity E. This method was applied to eight of the strain gaged panels tested by Seck. It was found that this method further improved the result, except for plate 10, as can be seen from the ratios of experimental to the calculated buckling load which is presented in this report.

The plate equation proposed by Swartz, S. E., was modified to use constants from parabolic fit rather than f_c' and ε_0.