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Finite Element Analysis of the Laminar Flow in the Inlet Section Between Two Infinite Parallel Plate.

by

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Nomenclature

ℓ_E: inlet length.

D: Depth between plates.

D_H: hydraulic diameter, $D_H = 4Yo$

Yo: duct half width.

η: function of x, $\eta = 1 - \frac{1}{1+cx^+}$

c: parameter in η transformation, $\eta = 1 - \frac{1}{1+cx^+}$.

R: Reynold number, $R = \frac{4YoU\rho}{\mu}$

ρ: fluid density.

W: Width of element.

ℓ: Length of element.

M: Mass of cell.

ΔY: Thickness of a cell in Y direction.

Y_j: Location of a cell in Y direction.

x_i: Location of a element in x direction.

A(1): Interface area between cells.

A(2): End area of cell.

A^e: End area of the control volume.

H_e: Momentum entering first control volume.

H_i: Momentum entering ith control volume.

H_i': Fictitious momentum leaving ith control volume.

H_i'': Actual momentum leaving ith control volume.

ΔH: Difference in momentum.

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μ: Coefficient of absolute viscosity.

$\tau_{i,j+1}$: shear stress at interface of cell j and j+1, in element i.

$F_{i,j+1}$: Shear force at interface between cells j and j+1, in element i.

ΔV_1: Change in velocity due to the shear force.

ΔV_2: Change in velocity due to the pressure.

ΔP: pressure difference between two faces of cell; dynamic pressure difference.

Δt: Incremental time.

$V_{i,j}$: Velocity of cell i at element j in x-direction.

$VP_{i,j}$: Velocity of cell i at element j in y-direction.

$\frac{dP}{dx}$: pressure gradient in x-direction.

$\frac{dP}{dy}$: pressure gradient in Y-direction.

x^+: dimensionless Cartesian coordinate, $x^+ = x/Yo$.

Y^+: dimensionless Cartesian coordinate, $Y^+ = Y/Yo$.

V^+: dimensionless x-component of velocity $V^+ = \frac{V}{U}$.

VP^+: dimensionless Y-component of velocity. $VP^+ = \frac{VP}{U}$.

P^+: dimensionless pressure, $P^+ = \frac{P}{\rho U^2}$

$\tau_{i'+1,j}$: shear stress at end area of cell j, between elements i and i+1.

$F_{i'+1,j}$: shear stress at end area of cell j between elements i and i+ 1.

Chapter I

Introduction

The problem to be studied in this report was that of inlet flow for an incompressible and Newtonian fluid flowing between infinite parallel plates. The inlet region is often described as the downstream distance required for the constant velocity profile, assumed at inlet, to be changed to the parabolic distribution typical of the fully developed laminar flow.

A number of approximate methods have been used to attack this problem, these include:

(1) the matching method

(2) the momentum integral method.

(3) the linearization method.

(4) the finite difference method.

In each case the Navier Stokes Equations are subjected to a numerical technique to obtain a solution.

It is intended to obtain a solution to this problem using finite element analysis. The finite element method of analysis differs in that the governing differential equations are by-passed, and hence the consideration of the behavior of a differential element is by-passed, in favor of consideration of arbitrarily selected elements of fluid used to represent the system, each of which is forced to satisfy physical laws such as conservation of mass, conservation of energy, and conservation of momentum, as well as boundary conditions and constitutive equations, during any incremental time being considered. While finite element analysis has been used effectively in solid mechanics for some while, only recently has it been applied to problems in fluid mechanics. Figure 1 compares the more traditional numerical techniques which have been used in the inlet flow problem with the finite element analysis to be used in this report.

Since finite element analysis is concerned only with the sequence of physical events directly, "transient" and "steady state" are not differentiated in the basic formulation. Rather, a description of the system for each small time interval is developed and is observed from some initial time, for a finite time finally reaching a state where solution for additional time intervals shows no change in the solution obtained, signifying that a steady state has been reached.

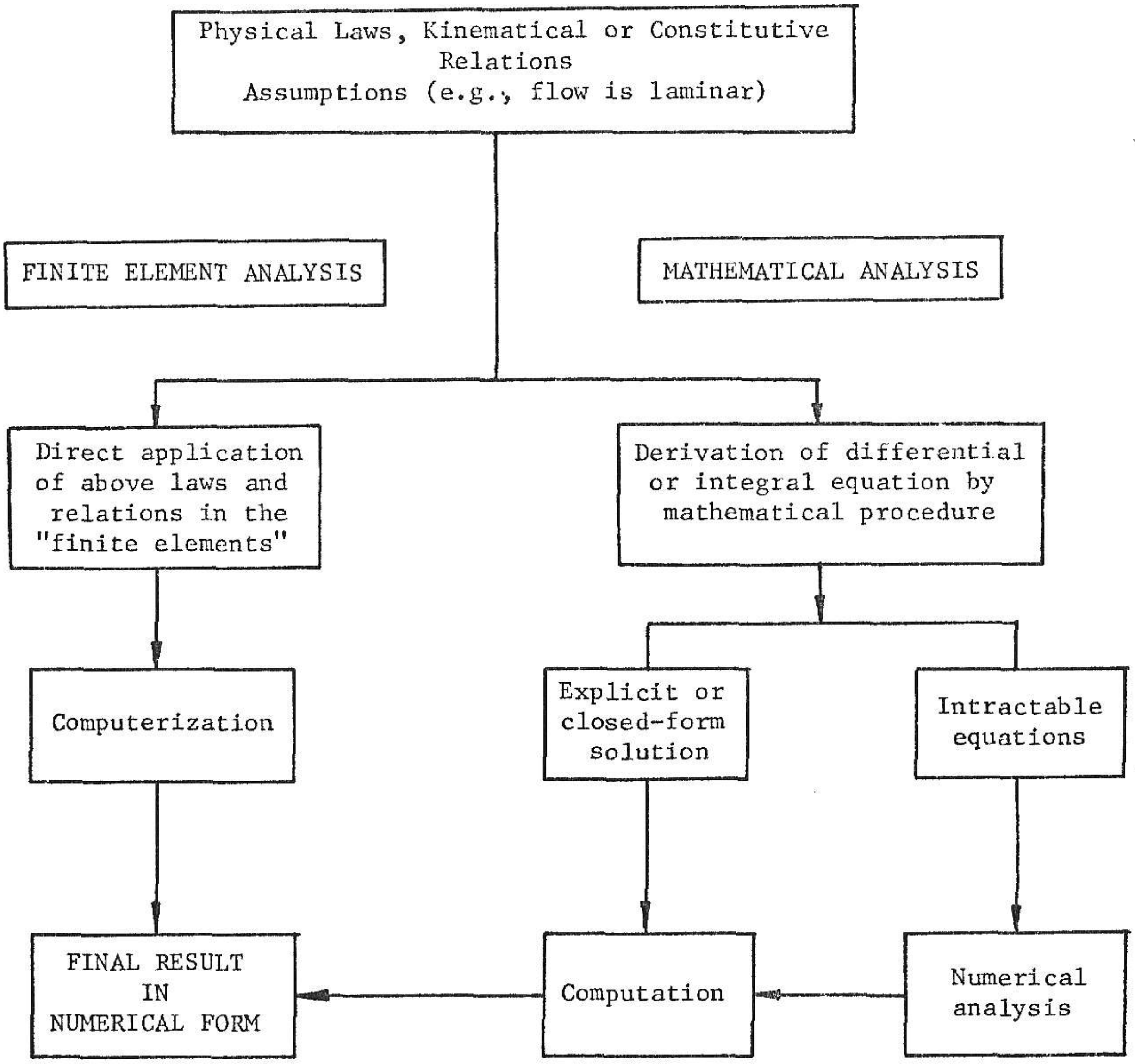


FIG. 1. Comparison of finite element and mathematical analytic schemes.

Chapter II

Literature Survey

Many studies of steady laminar inlet flow of an incompressible Newtonian fluid between parallel plates have been carried out using approximate numerical techniques to obtain a solution.

In general, there are two different approaches to the problem. One uses boundary layer theory which neglects the derivatives $\frac{\partial^2 U}{\partial x^2}$ and $\frac{\partial P}{\partial y}$. The other does not use the boundary layer theory assumption, so that no terms in the Navier Stokes Equations are neglected.

If boundary layer theory is applied, several methods are available for obtaining a solution to the problem.

In 1922, Schiller (3) employed a method based on conservation of momentum. This approach was devised and applied to flow in a circular tube and was similar to Karmen-Pohlhausen momentum integral method commonly applied to the flat plate problem.

In 1934, Schlichting (4) studied laminar flow in the inlet section between parallel plates and found that the boundary layer close to the inlet developed much the same as did the boundary layer over a flat plate which had a pressure gradient in the direction of flow. In 1961, Bodoia and Osterle (6) numerically integrated the boundary layer equation, and in 1962, Colluin and Schowalter (7,8), used Schlichting's method with refinements.

In 1940, Langhaar (9) assumed the inertia terms of boundary layer equation could be linearized. Based on this assumption, a solution was obtained for inlet flow in a straight tube. Although the solution derived from this method was more accurate than that of Schiller's (3), the computational procedure was much more complicated.

Although boundary-layer theory is a powerful tool, it is well known that its assumptions are not valid in the vicinity of the leading edge of a plate such as is found in this case. In this region the derivative $\frac{\partial^2 U}{\partial x^2}$ is not negligible in relation to $\frac{\partial^2 U}{\partial y^2}$, and the pressure gradient in the y direction is not necessary small, so that momentum equation for V is not negligible.

In 1964, Wang and Longwell, (10) using a finite-difference technique, solved the case of laminar flow in the inlet section between parallel plates without using the usual boundary layer assumptions (i.e. neglecting $\frac{\partial^2 U}{\partial x^2}$ and $\frac{\partial P}{\partial y}$). Their analysis was therefore exact in the sense that no terms in the momentum equation, which were not identically zero, were neglected. Their solution indicated that $\frac{\partial^2 U}{\partial x^2}$ was not negligible relative to $\frac{\partial^2 U}{\partial y^2}$ and the velocity normal to the plates was not negligible in the vicinity of the leading edge.

In 1968, a new approach was set forth by David and Ray, in that they proposed solving fluid flow problems using finite element analysis.

Based upon their work, a finite element analysis was employed to solve the inlet flow problem.

Chapter III

Finite Element Analysis

The basic assumptions regarding this problem were:

1. The parallel plates were infinitely wide, straight and rigid.
2. The fluid was incompressible and viscous.
3. The flow was laminar and isothermal.
4. The system was conservative, energy dissipation in the form of heat was neglected, as well as other thermodynamics effects.
5. R = 300, and the material properties were known.
6. The fluid was Newtonian.
7. The flow approaching the inlet section was uniformly distributed over its width, $V_{i,j} = U_o$, at $x = 0$, $J = 2, \ldots, N$.
8. Pressure acting on a cross section was constant.

It was then necessary to subdivide the section into elements of the proper geometric shape. Fig 4. shows a section of arbitrary length, ℓ, in the direction of the flow, the X direction, while the plates were assumed to be infinitely long in the Z direction, without variation along the Z-axis. Therefore, an arbitrary width W was chosen in that direction. The height, which is the gap between the two parallel plates was designated by D. This section with dimension ℓ.W.D. was divided into a number of cells (elements), as shown in Fig.5 , by passing planes parallel to the plate at equal intervals ΔY in the Y direction. Starting with the bottom cell in Fig.4, each cell of size $\ell W \Delta Y$ was labeled for identification in successive positions of the cell interface, as

$Y_j = 0, \qquad Y_{j+1} = Y_j + \Delta Y$.

The interface area between cells was given by

$A(1) = \ell W$,

and the end area by

$A(2) = W\Delta Y$,

while the mass of the cell was given by

$M = (A(1) \cdot \Delta Y)\rho$.

If it is known how the system changes from time t to a new time value, expressed as $t' = t + \Delta t$, this in effect provides an inductive procedure that, when repeated, will solve the problem from $t = 0$ to a specified time, $t' = t_m$.

The boundary conditions and initial conditions must be considered also.

(a) Initially, the total momentum at entry section is

$$H_e = \sum_{J=1}^{J_m} M V_{1,J} , \tag{1}$$

where H_e is the net momentum entering the first element. Only when i=1 can H_e be expressed by

$$H_i = H_e . \tag{2}$$

(b) The presence of boundary layer at the wall poses the boundary conditions

$$V_{i,1} = 0, \qquad V_{i,J_{N+1}} = 0, \qquad VP_{i,1} = VP_{i,N+1} = 0 . \tag{3}$$

The initial conditions of the system are expressed as

$$V_{1,J} = U_o, \qquad J = 2,3, \ldots, J_N . \tag{4}$$

Because the flow is symmetrical to the center line, there can be no flow across the center line, so the control volume ABCD is as shown in Fig. 7.

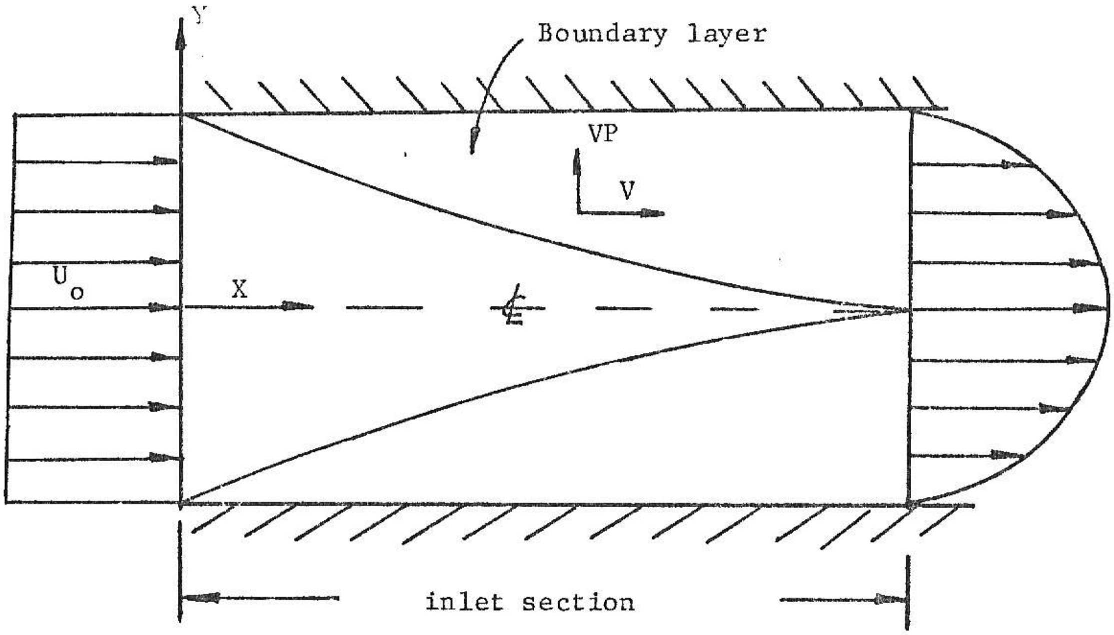


Fig 2. Geometry of duct-entrance region

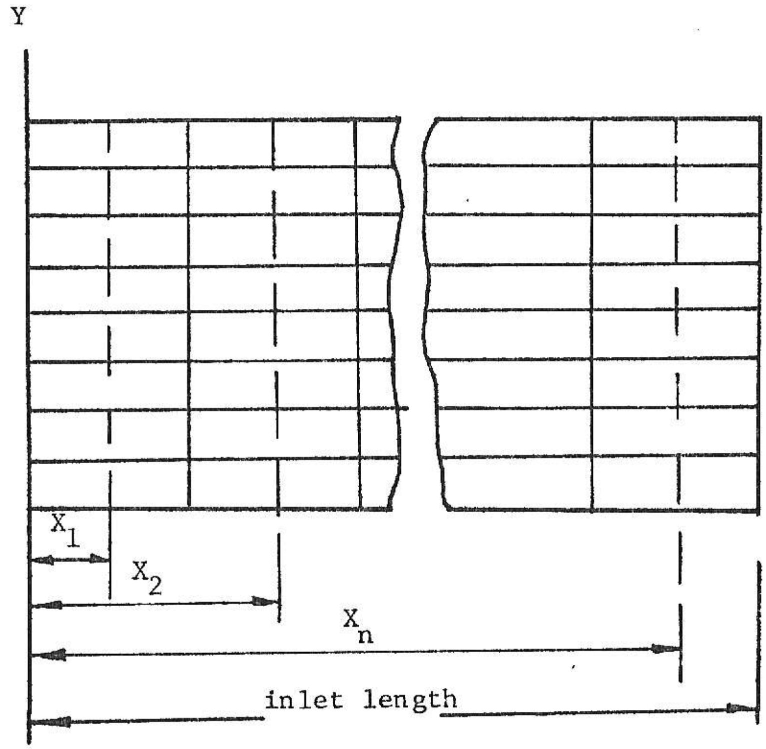


Fig 3. Finite element analysis

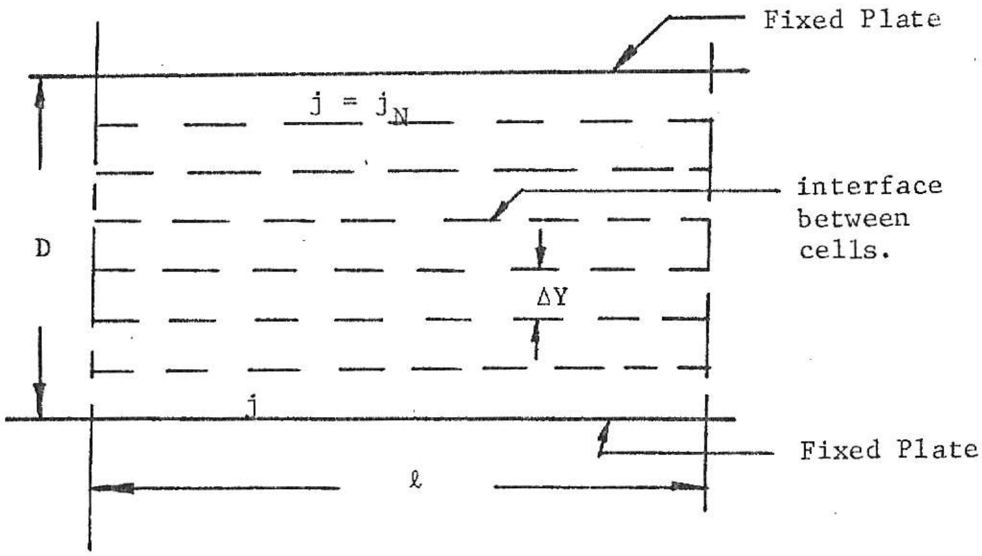


Fig 4. Finite element subdivisions

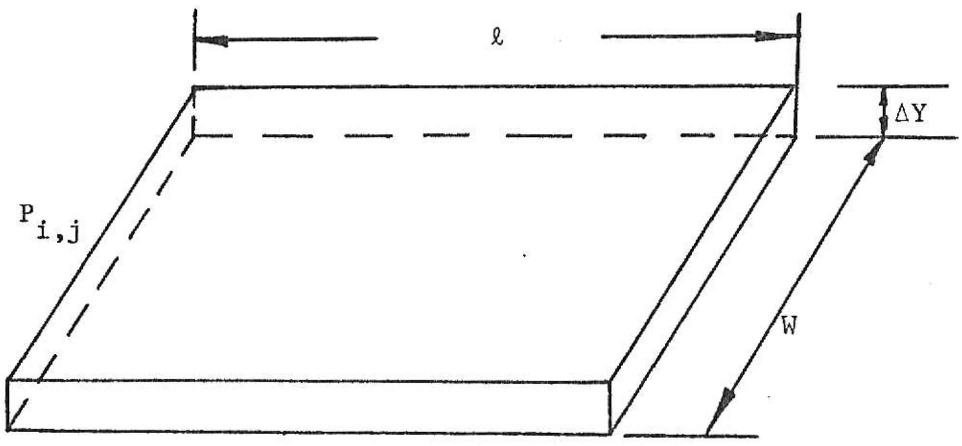


Fig 5. Cell geometry

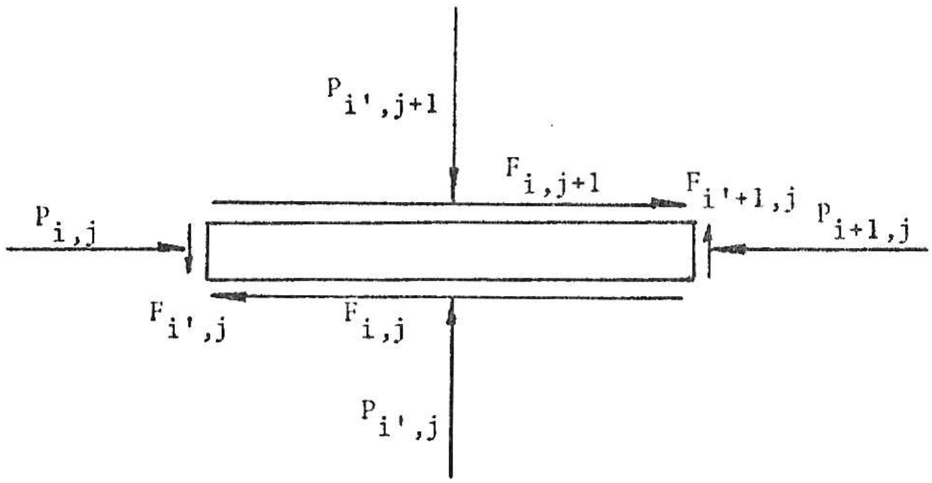


Fig 6. Typical element showing shear stress and pressure forces

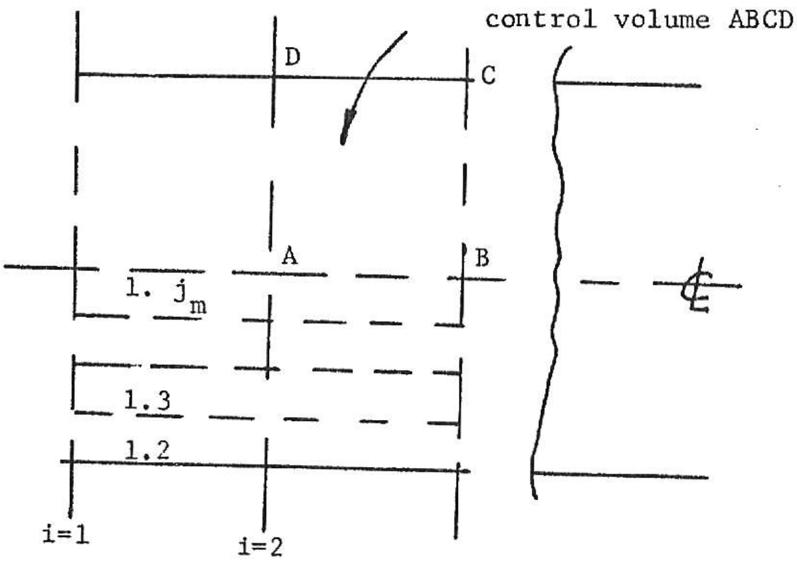


Fig 7. Entrance Problem, finite element subdivision of transition region.

A control volume which has a fixed volume which has fixed volume in space and through which fluid flows.

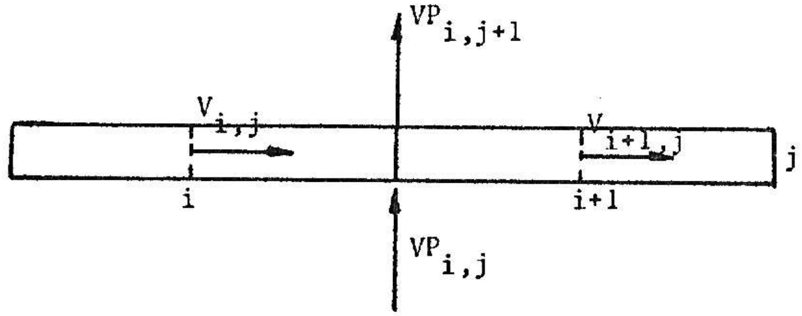


Fig. 8

Consider Figs. 6 & 8, in which $V_{i,j}$, $VP_{i,j}$, $P_{i,j}$ were known. Then, analysing this element, it is required to find

$F_{i',j}$, $F_{i'+1,j}$, $P_{i',j+1}$, $P_{i',j}$, $VP_{i,j+1}$,

$F_{i,j}$, $F_{i,j+1}$, $P_{i+1,j}$, $V_{i+1,j}$.

There are nine unknowns, hence nine equations are needed to find those unknowns. These nine equations are as follows:

(1) Newton's Law of Viscosity

$$\tau_{i,j+1} = \mu \frac{(V_{i,j+1} - V_{i,j})}{\Delta Y} ,$$

$$F_{i,j+1} = \tau_{i,j+1} \cdot A(1) , \tag{1'}$$

$$F_{i,j} = \tau_{i,j} \cdot A(1) . \tag{2'}$$

(2) Conservation of Momentum

Consider a control volume ABCD in Fig. 7, a control volume which has a fixed volume in space and through which fluid flows.

$P_{i,j}$ is known.

$$P_{i+1,j} - P_{i,j} = \frac{\sum_{j=1}^{m} MV'_{i,j} - \sum_{j=1}^{m} MV_{i,j}}{A^e \cdot \Delta t} \quad . \tag{3'}$$

(3) Conservation of Energy

Since the system is isothermal frictional losses owing to the action of viscosity and the formation of the boundary layer will cause an additional pressure drop, thereby increasing the kinetic energy of the fluid as it passes downstream.

$$V_{i+1,j} = V_{i,j} + \Delta V_1 + \Delta V_2 \quad . \tag{4'}$$

(4) Conservation of Mass.

$$VP_{i,j+1} = \frac{(V_{i,j} - V_{i+1,j}) \cdot A(2)}{A(1)} + VP_{i,j} \quad . \tag{5'}$$

(5) Newton's Law of Viscosity

$$\tau_{i',j} = \mu \frac{(VP_{i,j} - VP_{i-1,j})}{\ell} \quad ,$$

$$F_{i',j} = \tau_{i',j} \cdot A(2) \quad , \tag{6'}$$

$$F_{i'+1.j} = \tau_{i'+1,j} \cdot A(2) \quad . \tag{7'}$$

(6) The property of the pressure

$$P_{i',j} = \frac{1}{2}(P_{i,j} + P_{i+1,j}) \; . \tag{8'}$$

(7) Force Equilibrium in Cell

$$P_{i',j+1} = P_{i',j} + \frac{F_{i',j} - F_{i'+1,j}}{A(1)} \; . \tag{9'}$$

The computational procedure was carried out in sequence from j=1, to $j=j_{m+1}$.

The constitutive equation to be satisfied, Newton's Law of Viscosity, expresses the shear stress generated at the interface between two fluid cells in terms of the velocity gradient and the absolute viscosity of the fluid, μ. Therefore,

$$\tau_{i,j+1} = \mu(V_{i,j+1} - V_{i,j})/\Delta Y. \tag{5}$$

The total shearing force F over the interface is given by

$$F_{i,j+1} = \tau_{i,j+1} \cdot A(1). \tag{6}$$

Note that there is no interface at j = 1, therefore, $\tau_{i,1}$ and $F_{i,1}$, if initialized, will quite properly remain zero, and the expression for $\tau_{i,2}$, will give the shear stress on the upper and bottom plate respectively.

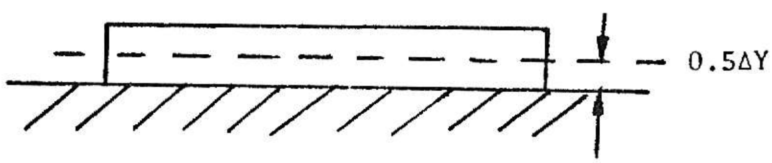


Fig. 9 cell near the plate

this is

$$\tau_{i,2} = \mu\left(\frac{V_{i,2} - 0}{0.5 \cdot \Delta Y}\right).$$

The Impulse — Momentum relationships must be satisfied.

The internal viscous forces calculated with equations (5) and (6) give rise to retardation of the fluid, that is, a change of the momentum of the fluid currently in the element. Any ith element may be considered as a control volume.

The change in velocity caused by viscous forces alone is given by

$$V_1 = \frac{(F_{i,j+1} - F_{i,j})\Delta t}{M}, \tag{7}$$

and the velocity is temporarily updated by

$$V'_{i,j} = V_{i,j} + \Delta V_1. \tag{8}$$

For i = 1 only, however, the entrance condition is inserted by using equation (1).

Since the system is isothermal, frictional losses, caused by the action of viscosity and the formation of the boundry layer, will cause an additional pressure drop, thereby increasing the kinetic energy of the fluid as it passes downstream. To determine the dynamic pressure drop, as well as its effect on the velocity, consider a control volume, shown in Fig. 7, consisting of a space bounded by the cross-section planes, one plane at the left end and the other at the right end of an ith element. The momentum flowing in and out of the control volume must be balanced.

H_i', representing a fictitious momentum that would have left the ith control volume if there had been no dynamic pressure drop, based on the velocity recently updated with equation (8), will be given by

$$H_i' = \sum_{J=1}^{J_m} M\, V_{i,j}. \tag{9}$$

Then the net momentum build-up in the control volume as the result of internal losses to viscous action is given by

$$\Delta H = H_i' - H_i.$$

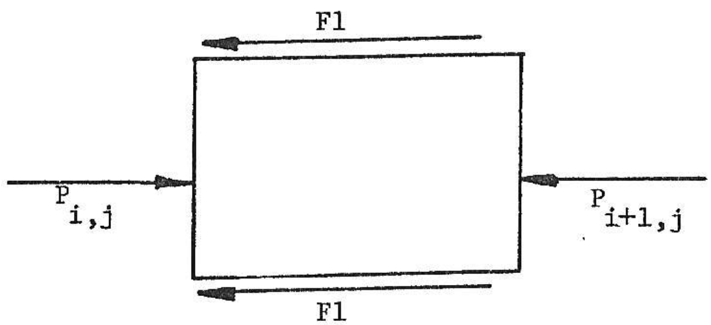
The value of H_i, the entering momentum, has been calculated previously for i = 1 by Eq. 2. For subsequent values of i, it is calculated progressively by Eq. 13, which yields the actual momentum leaving the right plane of the i control volume under consideration. Eventually, this becomes the momentum entering the left plane of the next control volume in the downstream direction, and can be used for H_i in equation (9) when the sequence of operations is carried out for the next element.

Because the fluid is incompressible, the mass in the control volume remains constant, and the dynamic pressure drop along the element is found by

$$\Delta P = - \frac{\Delta H}{(A^e \Delta t)}, \tag{10}$$

in which A^e is the cross-sectional area of the control volume, and ΔP is considered positive along the increasing direction of i.

The force equilibrium is applied to determine the dynamic pressure drop,



$$\tau_{i,2} = \mu(V_{i,2} - 0)/(0.5 \cdot \Delta Y) \ ,$$

$$F1 = \tau_{i,2} = A(1) \ ,$$

$$P_{i,j} - P_{i+1,j} = 2F1 = 2\tau_{i.2}\ A(1) \ .$$

$$\Delta P = 2\tau_{i,2}\ A(1) \ .$$

This pressure drop causes an acceleration of the fluid cells in the control volume, and the change in velocity in the direction of increasing i is given by

$$\Delta V_2 = (A^e\ \Delta P)\ \Delta t/M_i . \tag{11}$$

The final updated velocity of the fluid leaving a particular cell is given by

$$V''_{i,j} = V'_{i,j} + \Delta V_2 . \tag{12}$$

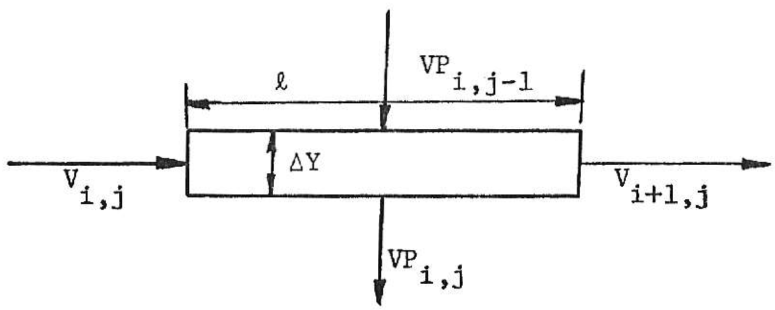
Equations (10) and (11) are repeated for cells j = 1 through $j = j_m$. The same value of ΔP is used for all cells because of the assumption that the pressure acting on the cross section is constant.

Since $V''_{i,j}$ is the final velocity of fluid as it leaves the jth-cell of the ith element, the actual momentum leaving the right side of the ith control volume is updated by

$$H''_i = \sum_{j=1}^{j_m} M_j \; V''_{i,j}, \tag{13}$$

and the value so obtained becomes the value of H_i for the next element, i+1. When the sequence of equations for the i element is completed, i is increased to the next value i+1. The steps from (2) to (13) are repeated till the terminating value i_m is reached.

To determine the velocity in the y direction, one applies conservation of mass. $VP_{i,j-1}$, $VP_{i,j}$, are the velocities in the y direction entering and leaving the cell.



Conservation of mass, requires that

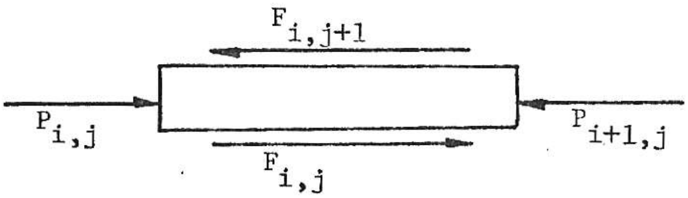
$$\rho V_{i,j} \cdot \Delta Y \cdot W + \rho VP_{i,j-1} \cdot \ell \cdot W$$

$$= \rho V_{i+1,j} \cdot \Delta Y \cdot W + \rho VP_{i,j} \cdot \ell \cdot W .$$

Therefore, $VP_{i,j} = (V_{i,j} - V_{i+1,j}) \, \Delta Y/\ell + VP_{i,j-1}$.

Note that at the wall there is no flow in y-direction, so $VP_{i,1} = 0$.

The following is applied to determine the pressure gradient in X direction.



$$F_{i,j+1} = \tau_{i,j+1}\ A(1) \ ,$$

$$F_{i,j} = \tau_{i,j}\ A(1) \ ,$$

$$F_{i,j+1} + P_{i+1,j} \cdot A(2) = P_{i,j} \cdot A(2) + F_{i,j} \ ,$$

$$\tau_{i,j+1}\ A(1) + P_{i+1,j}\ A(2) = P_{i,j} \cdot A(2) + \tau_{i,j}\ A(1) \ ,$$

and

$$P_{i+1,j} - P_{i,j} = (\tau_{i,j+1} - \tau_{2,j}) \cdot A(1)/A(2) \ .$$

where

$$A(1) = W \cdot \ell \ ,$$

$$A(2) = W \cdot \Delta Y \ ,$$

$$\Delta P = P_{i+1} - P_i = \frac{(\tau_{i,j+1} - \tau_{i,j}) \cdot W \cdot \ell}{W \ . \ \Delta Y} \ ,$$

$$\frac{dP}{dx} \doteq \frac{\Delta P}{\ell} = \frac{(\tau_{i,j+1} - \tau_{i,j}) \cdot W \cdot \ell}{W \cdot \ell \cdot \Delta Y} = \frac{(\tau_{i,j+1} - \tau_{i,j})}{\Delta Y} \ .$$

The following analysis yields the pressure gradient in y direction:

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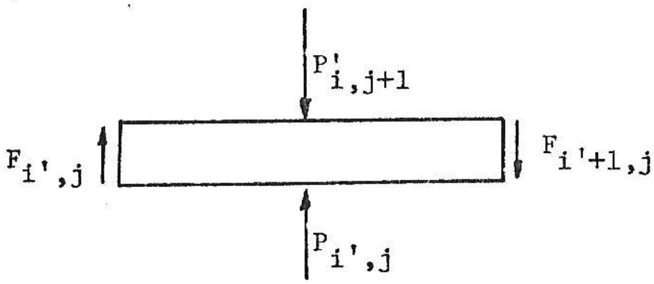

$$\tau_{i',j+1} = \mu(VP_{i+1,j} - VP_{i,j})/\ell \ ,$$

$$F_{i'+1,j} = \tau_{i',j+1} \cdot A(2) \ ,$$

$$F_{i',j} = \tau_{i',j} \ A(2) \ ,$$

$$P_{i',j+1} \cdot A(1) + \tau_{i'+1,j} \ A(2) = P_{i',j+1} \cdot A(1) + \tau_{i',j} \ A(2) \ ,$$

$$P_{i',j} - P_{i',j+1} = (\tau_{i'+1,j} - \tau_{i',j}) \cdot A(2)/A(1) \ ,$$

and

$$\frac{dP}{dy} \doteq \frac{\Delta P}{\Delta Y} = \frac{(\tau_{i'+1,j} - \tau_{i',j}) \cdot W \cdot \Delta Y}{W \cdot \ell \cdot \Delta Y}$$

$$= (\tau_{i'+1,j} - \tau_{i',j})/\ell \ .$$

The velocity obtained from the above procedure is at the center of the element.

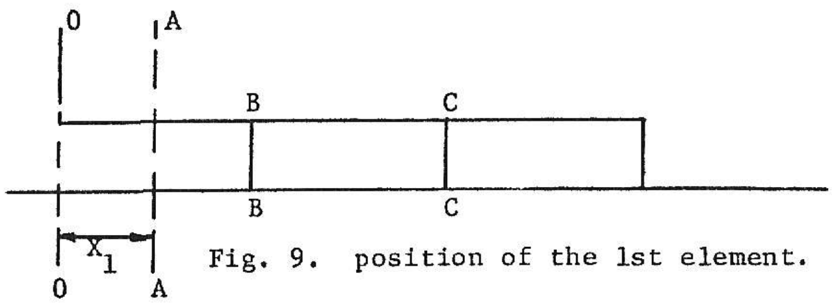

Fig. 9. position of the 1st element.

The velocity at section A-A can be found using the above procedure.

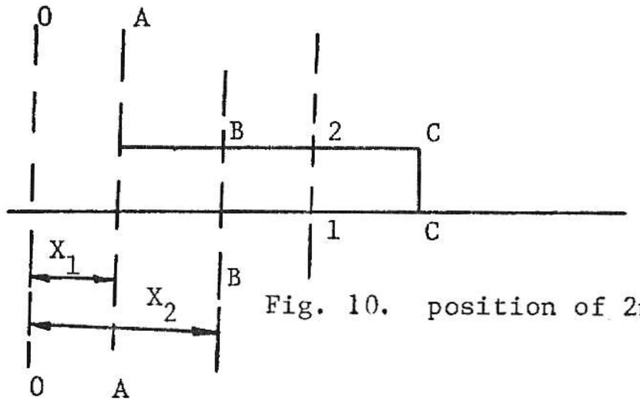

Fig. 10. position of 2nd element

Because the velocity at section A-A was applied to find the velocity for the next element, that element should be started at section A-A thus, the velocity at section B-B was obtained in similar manner as that at section A-A. The positions of the element

$$x_2 = x_1 + \frac{\ell}{2} \quad ,$$

A general equation can be obtained, which is

$$x_i = x_{i-1} + \frac{\ell}{2} .$$

For convenience, ℓ is equal to

$$U\Delta t$$

To facilitate comparison of this result with that of Y. L. Wang and P. A. Longwell, the following dimensionless variables are introduced:

$$U^+ = \frac{V}{U}, \; V^+ = \frac{VP}{U}, \; x^+ = \frac{x}{Yo},$$

$$Y^+ = \frac{Y}{Yo}, \; P^+ = \frac{P}{\rho U^2}, \; Yo = \frac{D}{2},$$

and $D_H = 4Yo$.

Also, $$R = \frac{UD_H}{\gamma} = \frac{\rho U \cdot 4Yo}{\mu}$$

and

$$\eta = 1 - \frac{1}{1 + cx^+} ,$$

where c is a constant equal to 1.2. The value c = 1.2 was chosen so that the results of this analysis could be directly compared with the results of Y. L. Wang and P. A. Longwell (10).

Numerical Results

The velocity profiles and pressure gradients for Reynold number 300 are shown in Figs. 11 through 26. In addition, the results of Y. L. Wang and P. A. Longwell, (5), are shown.

In the computations performed for this study, the numerical values of ΔY and Δt were arbitrarily chosen. Using smaller numerical values, the effect of these values upon the calculations are repeated until further reduction had no effect. For this problem, a small number of cells proved to be adequate. A satisfactory range for Δt, the increment interval, was 1/500 part of the total time, t_m, required for the system to reach a steady state. This is the time beyond which further time increments will not change the value of the variables in the system, and is the time required to be reached steady state. It will be affected, of course, by physical properties of fluid, such as density and viscosity, as well as initial and boundary conditions.

The results obtained compare favorably with the results of Y. L. Wang and P. A. Longwell. At the center line, the velocity is littler higher than that they obtained because of the assumption that the pressure acting on the cross section was constant (i.e. The effects of pressure variation and flow in y-direction were neglected). The results were a little lower than the results of Schlichting (5). At the leading edge the velocity gradient is very large, but the time increment is very short, so it is difficult to define the ΔY and Δt adequately. In this report, this effect was neglected. From the results obtained, this effect is very small, and is estimated to be 0.3%.

The flow studied was symmetrical about the center line. Theoretically, the VP at centerline must be zero, and there can be no flow across the centerline. From the results, it appears that VP approaches zero, but is not exactly zero. This results from numerical error.

The pressure gradient was derived by the force equilibrium in the cell. Because of the difference in the velocity profile from that of Y. L. Wang and P. A. Longwell, the calculated pressure gradient in X, Y direction was different from the results of Y. L. Wang and P. A. Longwell. When the flow become fully developed, $\frac{dP}{dx}$ is constant, and $\frac{dP}{dy}$ is equal zero.

Initially, it was assumed the pressure acting on a cross section was constant. Finally, it was found that the pressure gradient in Y direction were different from zero. At $\eta = 0.1$, the maximum error in the pressure gradient was 0.7602, but its effect on velocity profile was very small, and estimated to be 0.03. As most errors were below 0.03, the assumption seems to be reasonable.

Conclusion

The result of this study demonstrated excellent agreement between finite element analysis and the results of others who have used different techniques in solving this problem.

The finite element analysis yields more realistic velocity profile than will a finite difference solution and it will do so more quickly and at less cost.

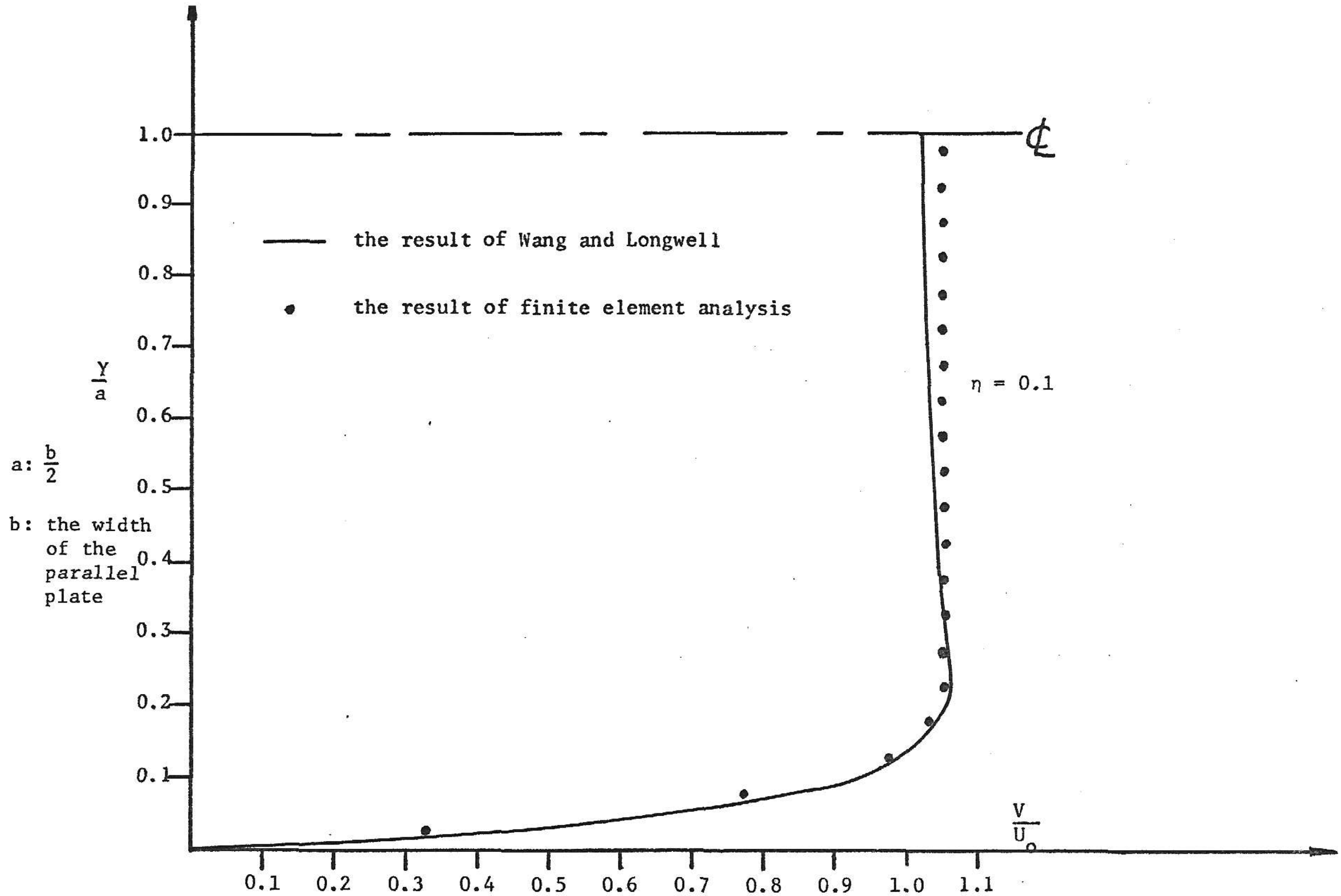

Fig. 11 Velocity Profile at $\eta = 0.1$

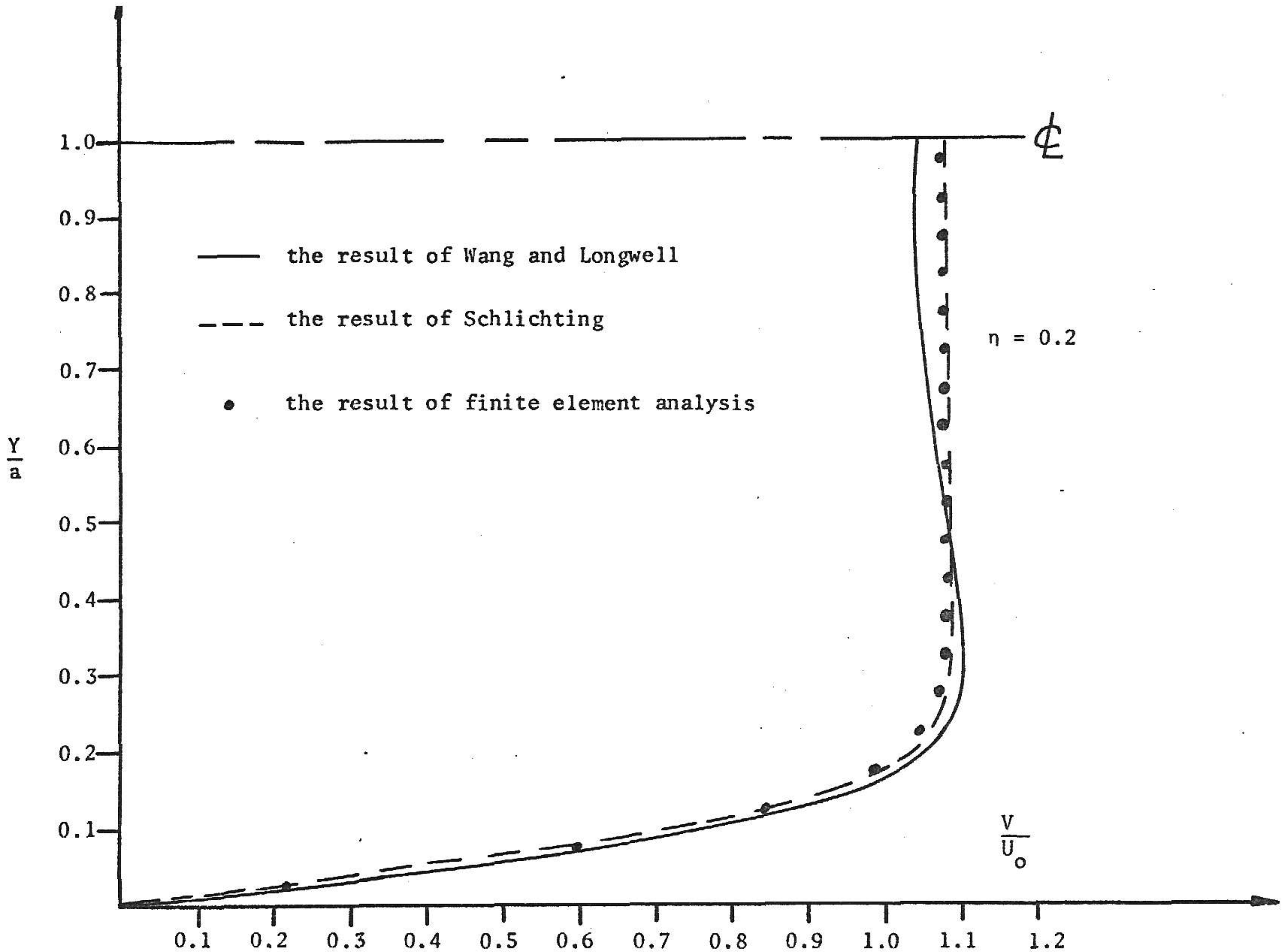

Fig. 12 Velocity Profile at $\eta = 0.2$

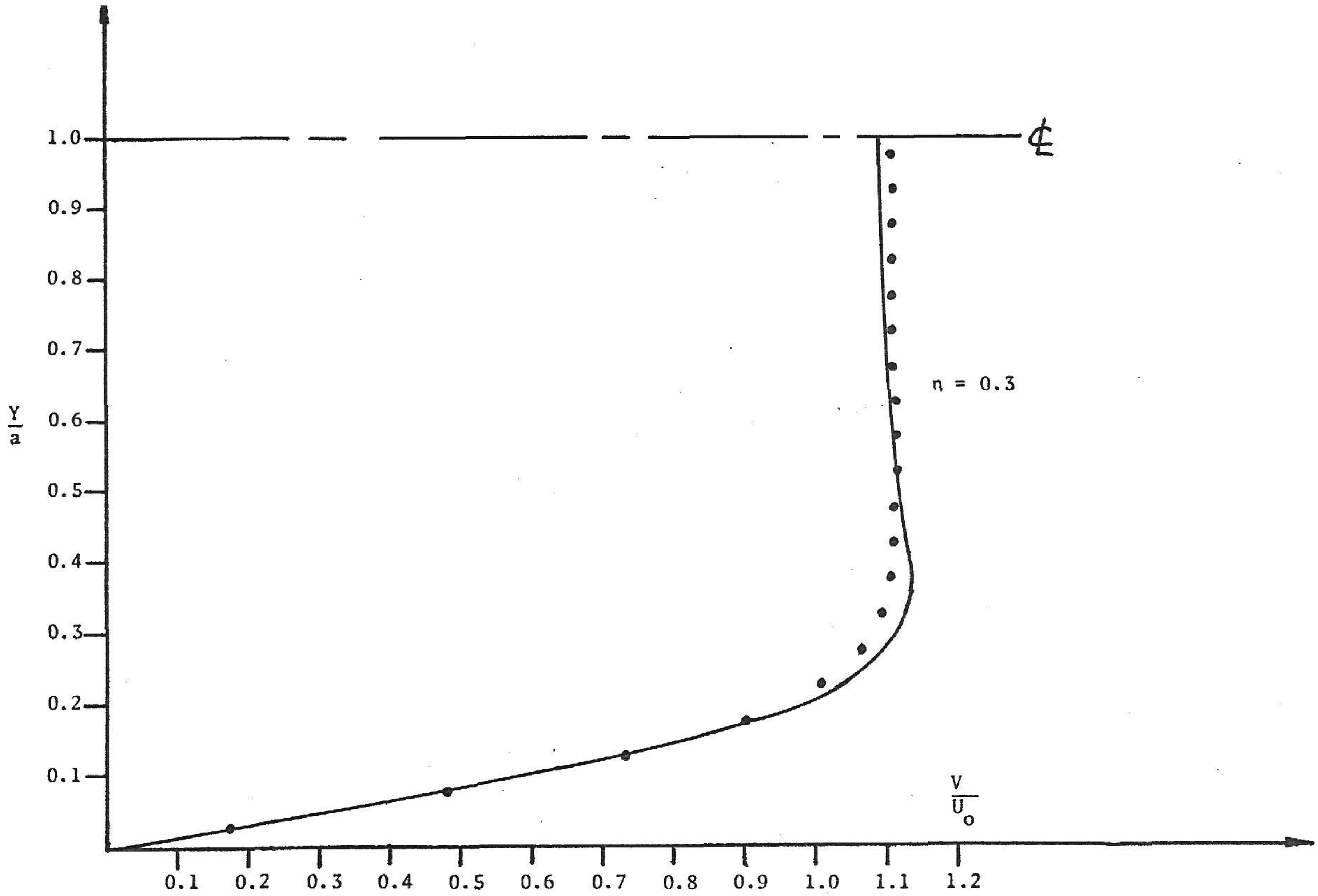

Fig. 13 Velocity Profile at $n = 0.3$

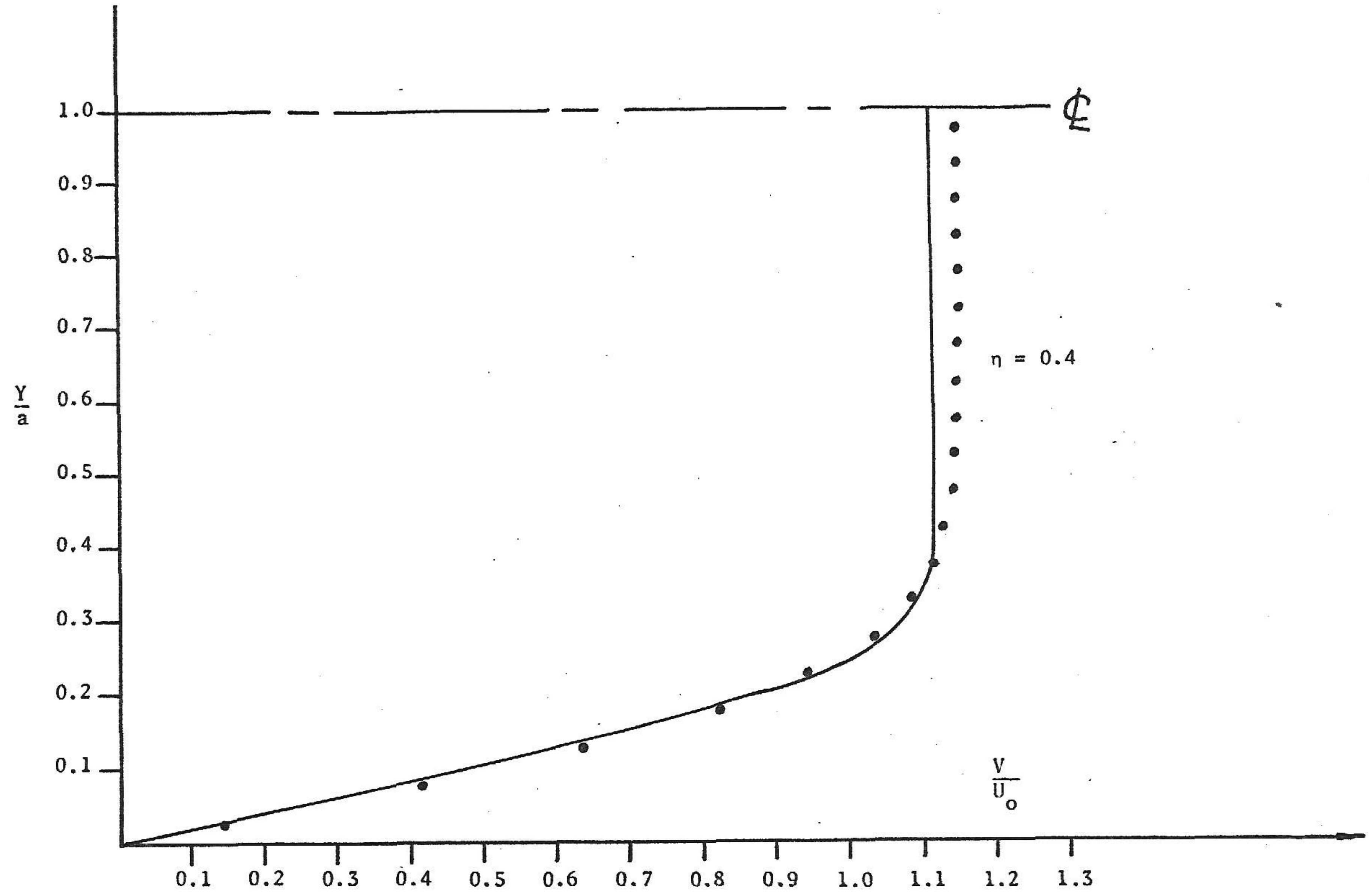

Fig. 14 Velocity Profile at $\eta = 0.4$

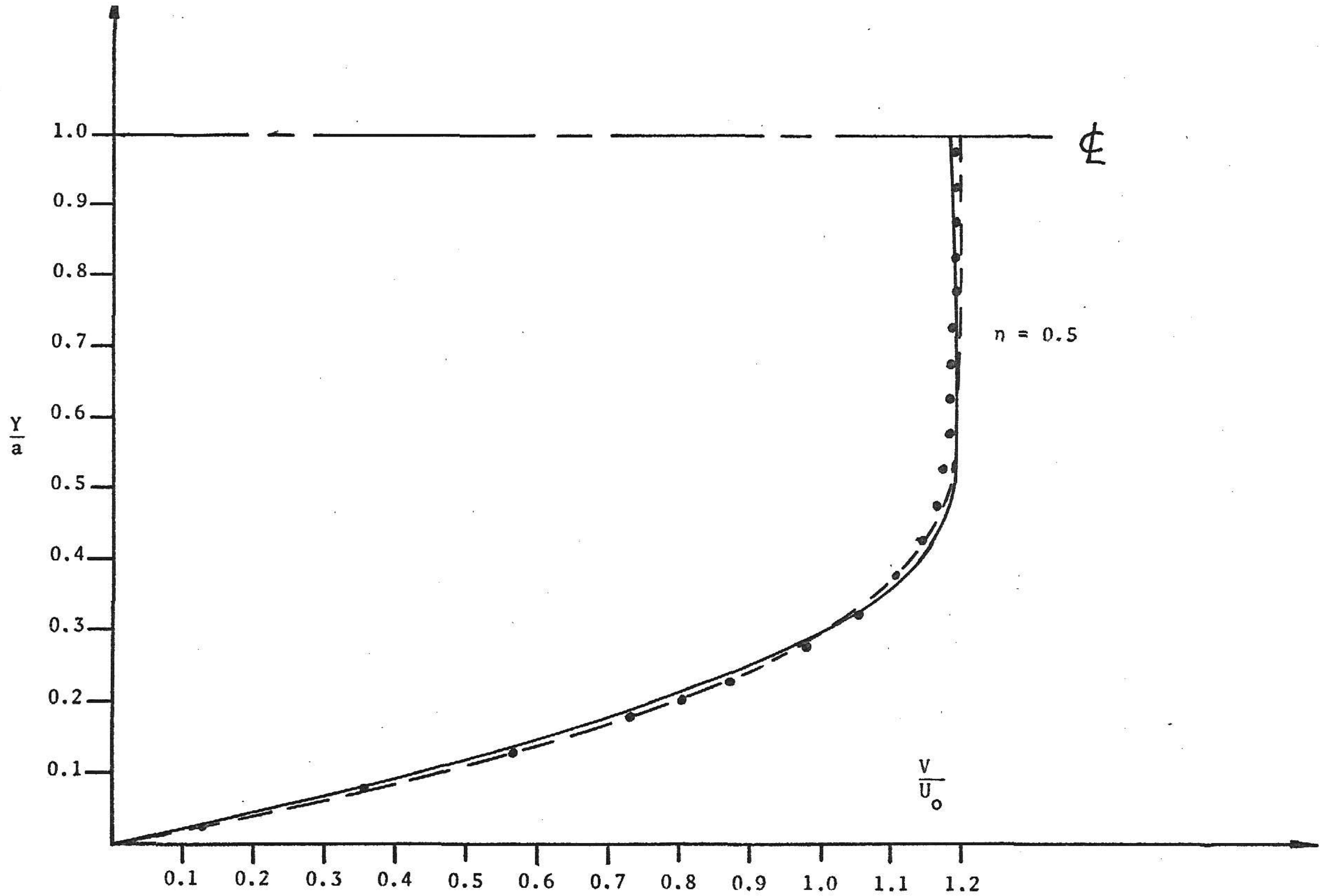

Fig. 15 Velocity Profile at η = 0.5

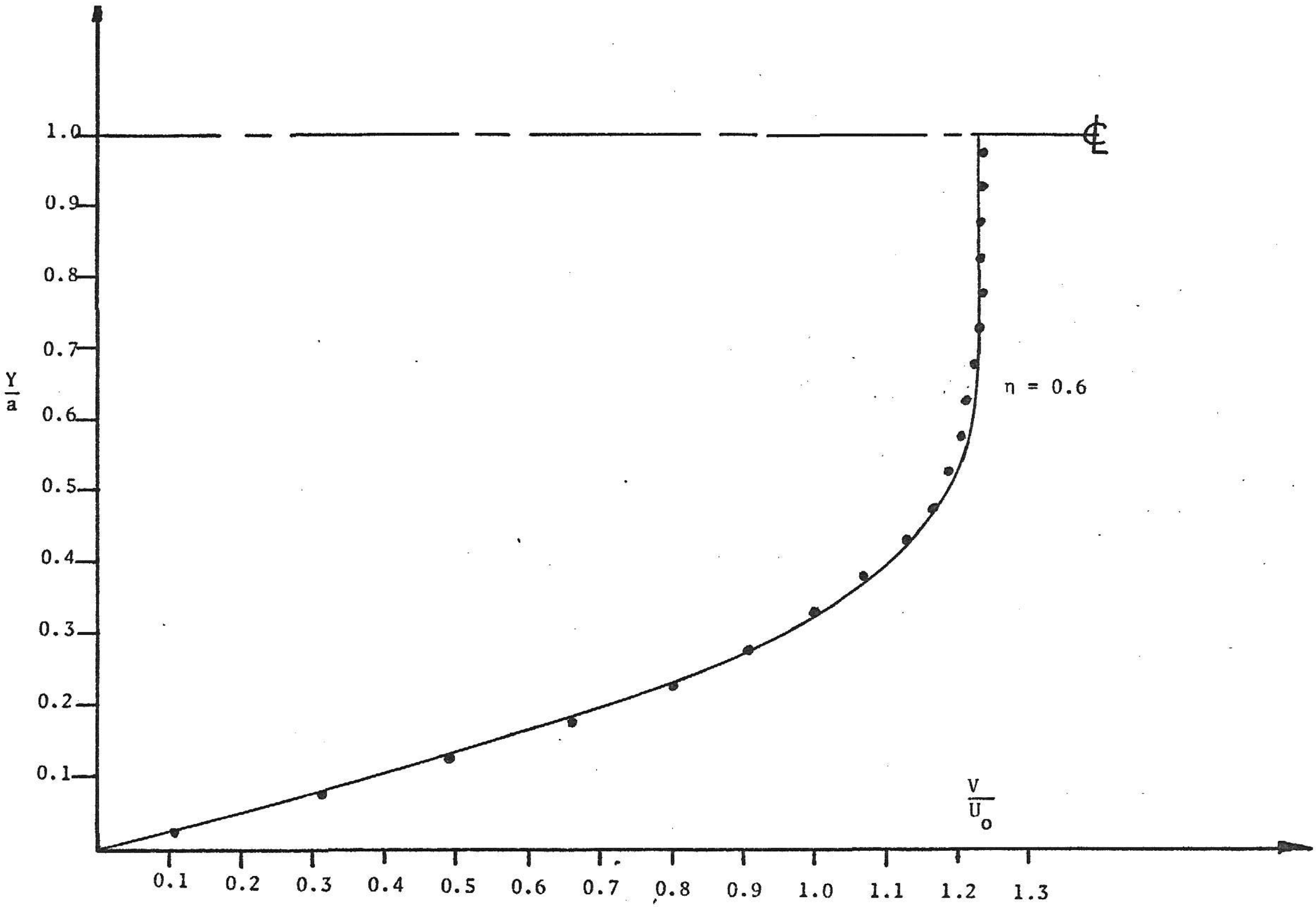

Fig. 16 Velocity Profile at $\eta = 0.6$

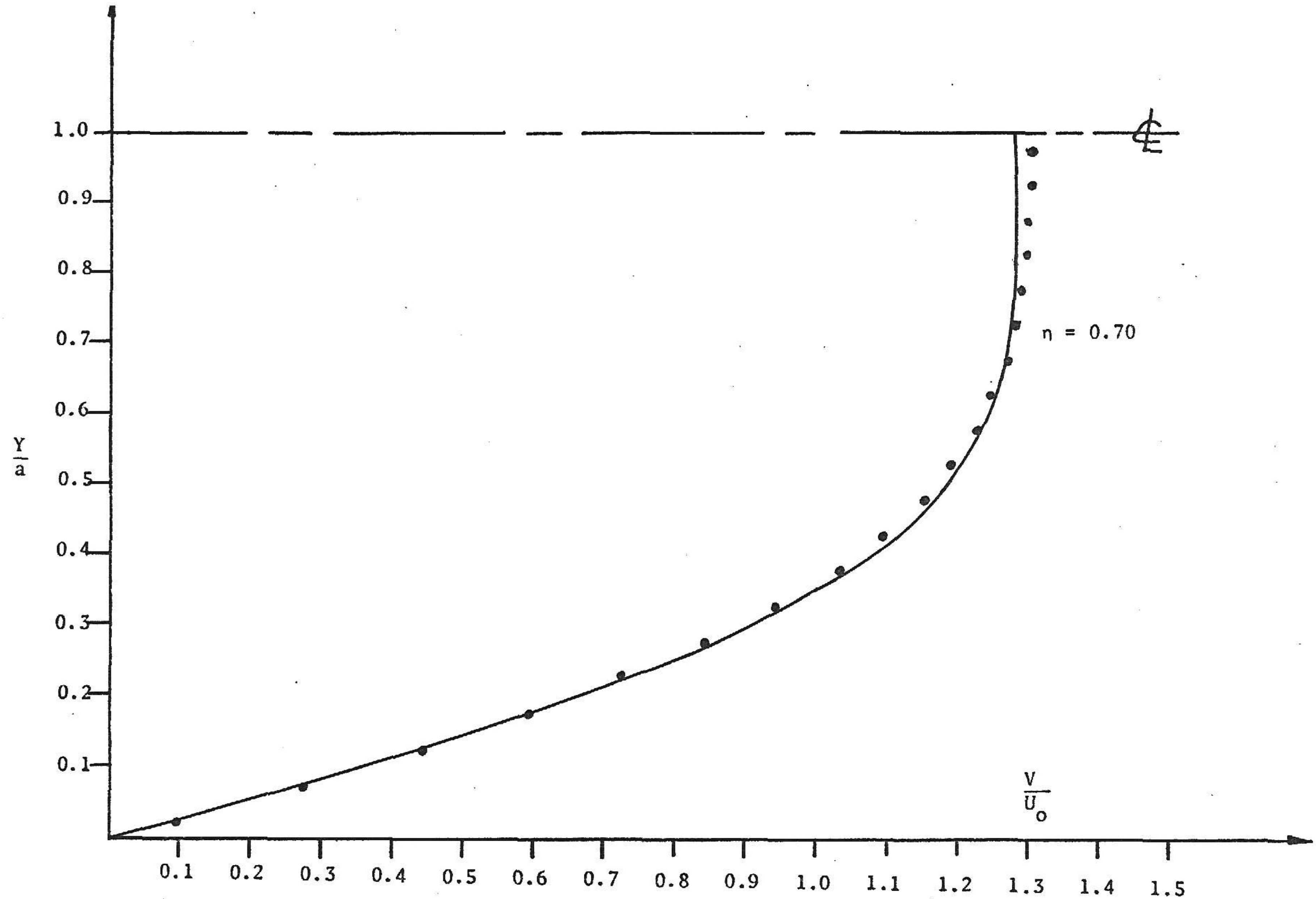

Fig. 17 Velocity Profile at $\eta = 0.70$

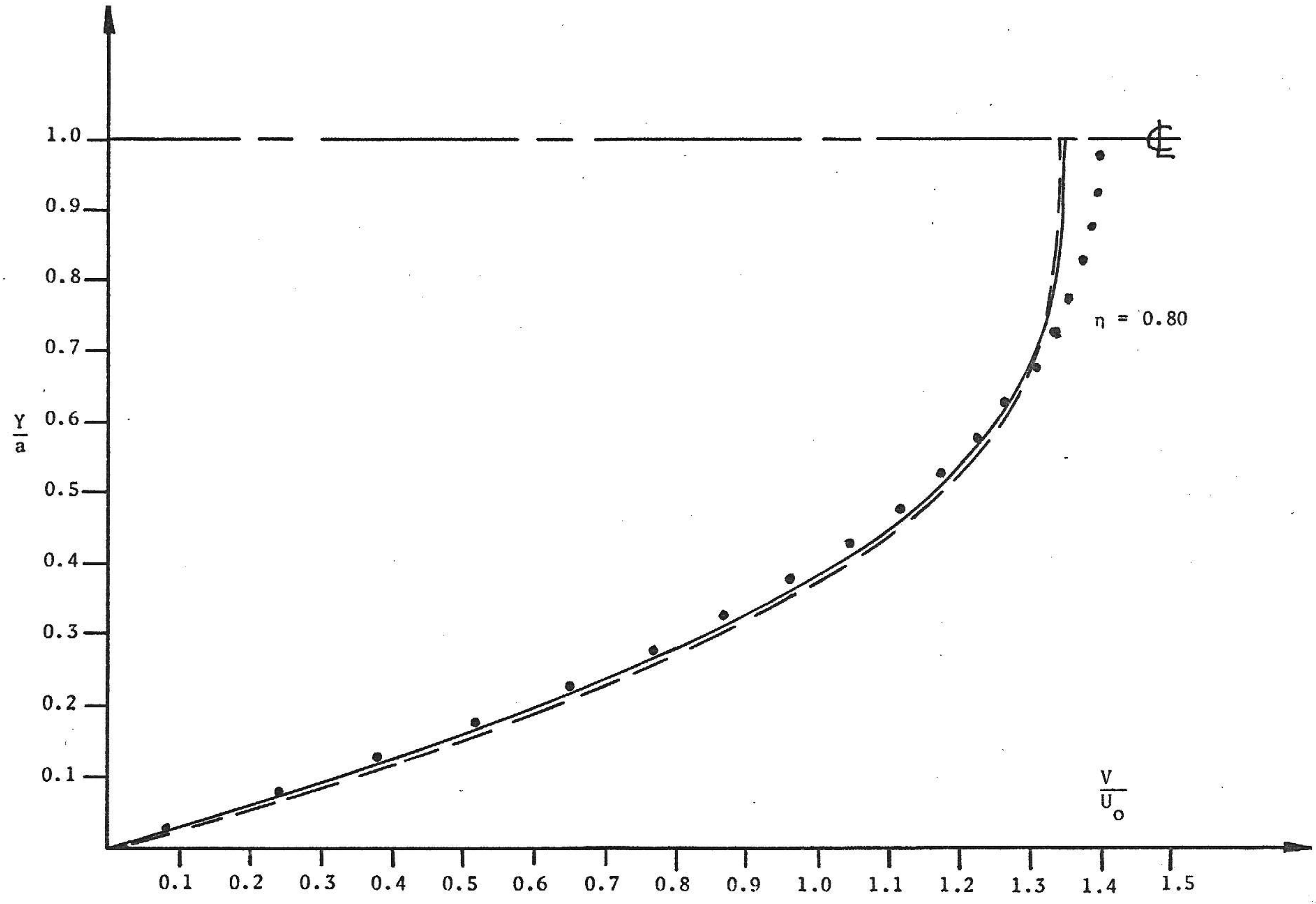

Fig. 18 Velocity Profile at η = 0.80

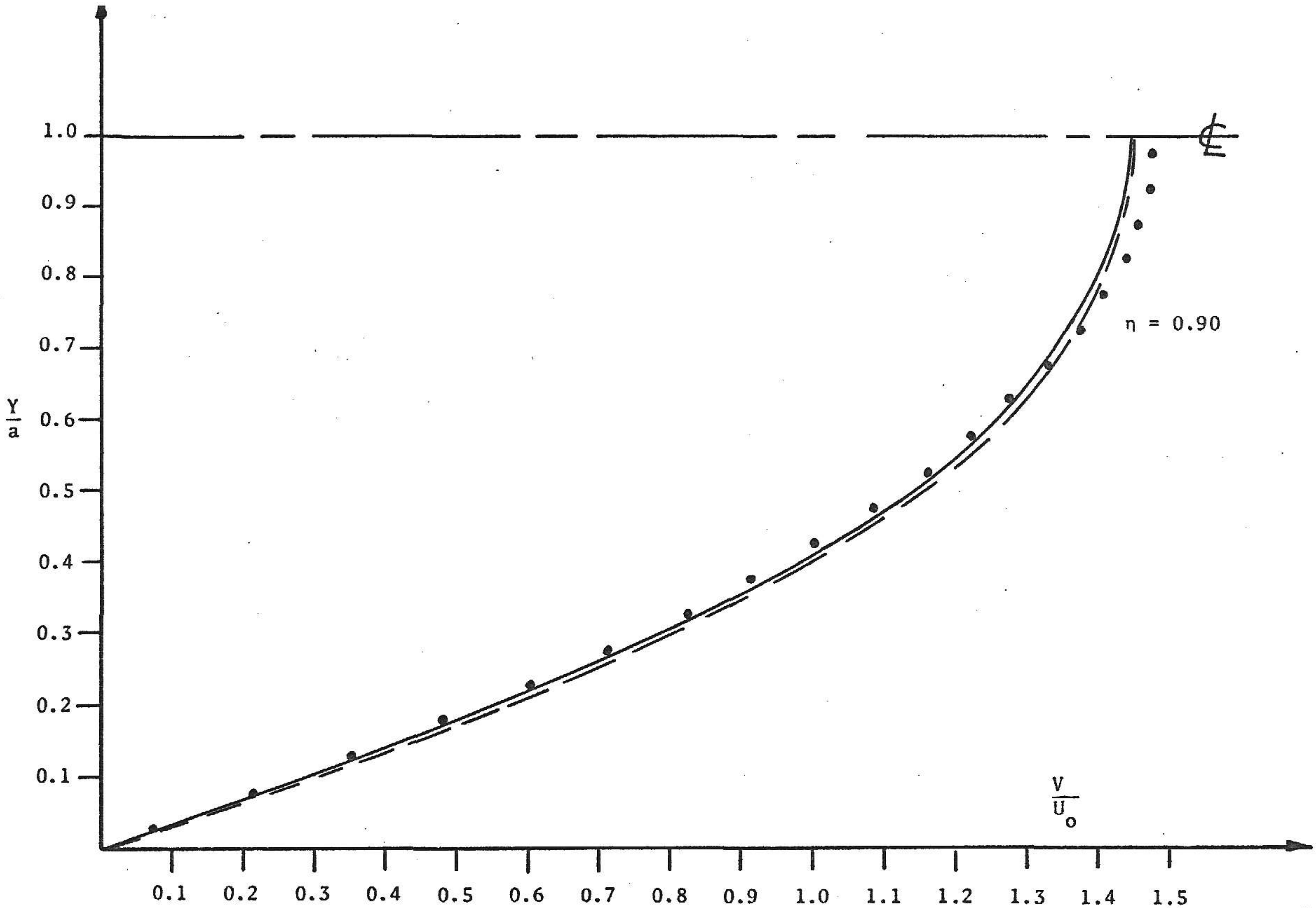

Fig. 19 Velocity Profile at $\eta = 0.90$

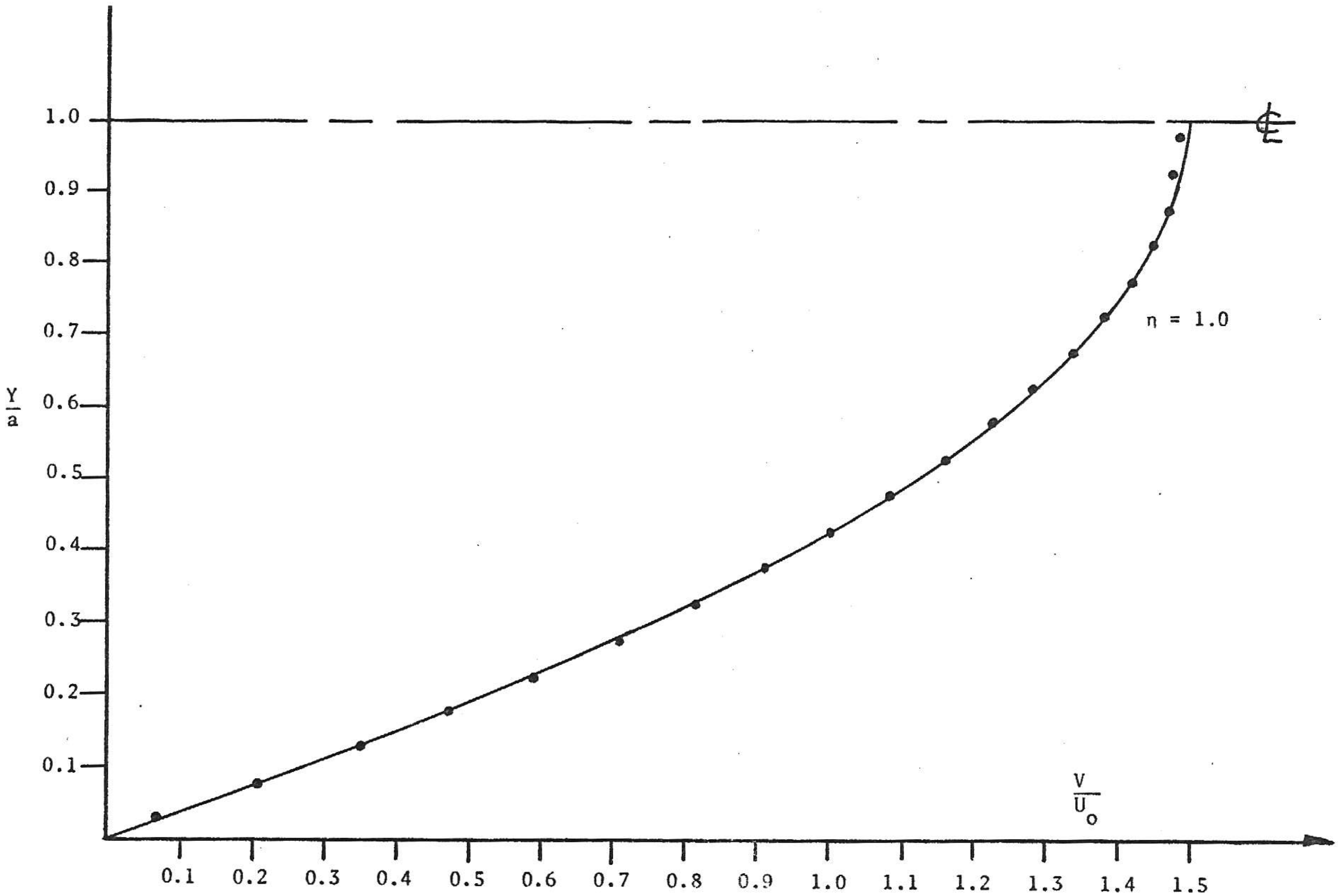

Fig. 20 Velocity Profile at $\eta = 1.0$

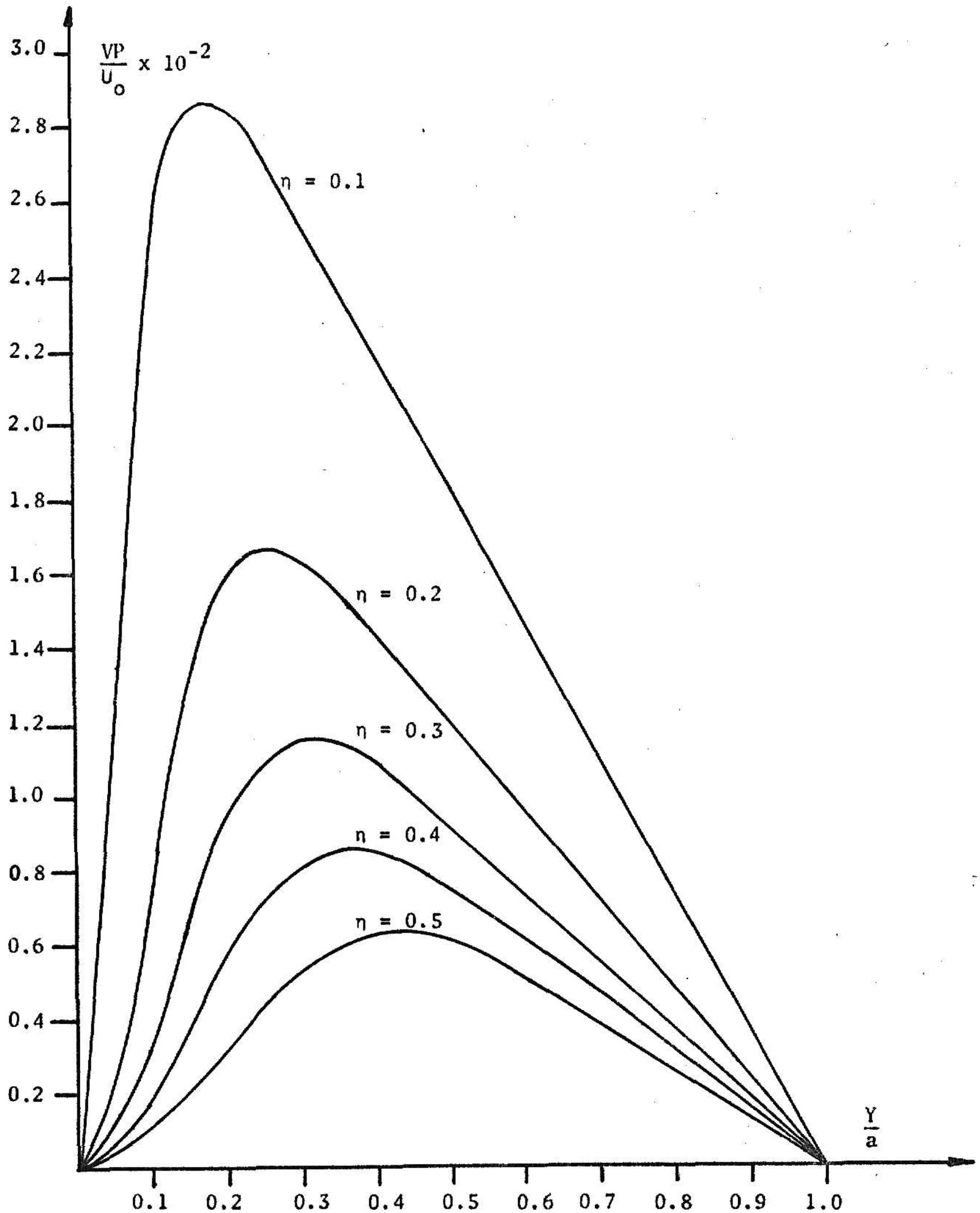

Fig. 21. Velocity Profile by finite element analysis.

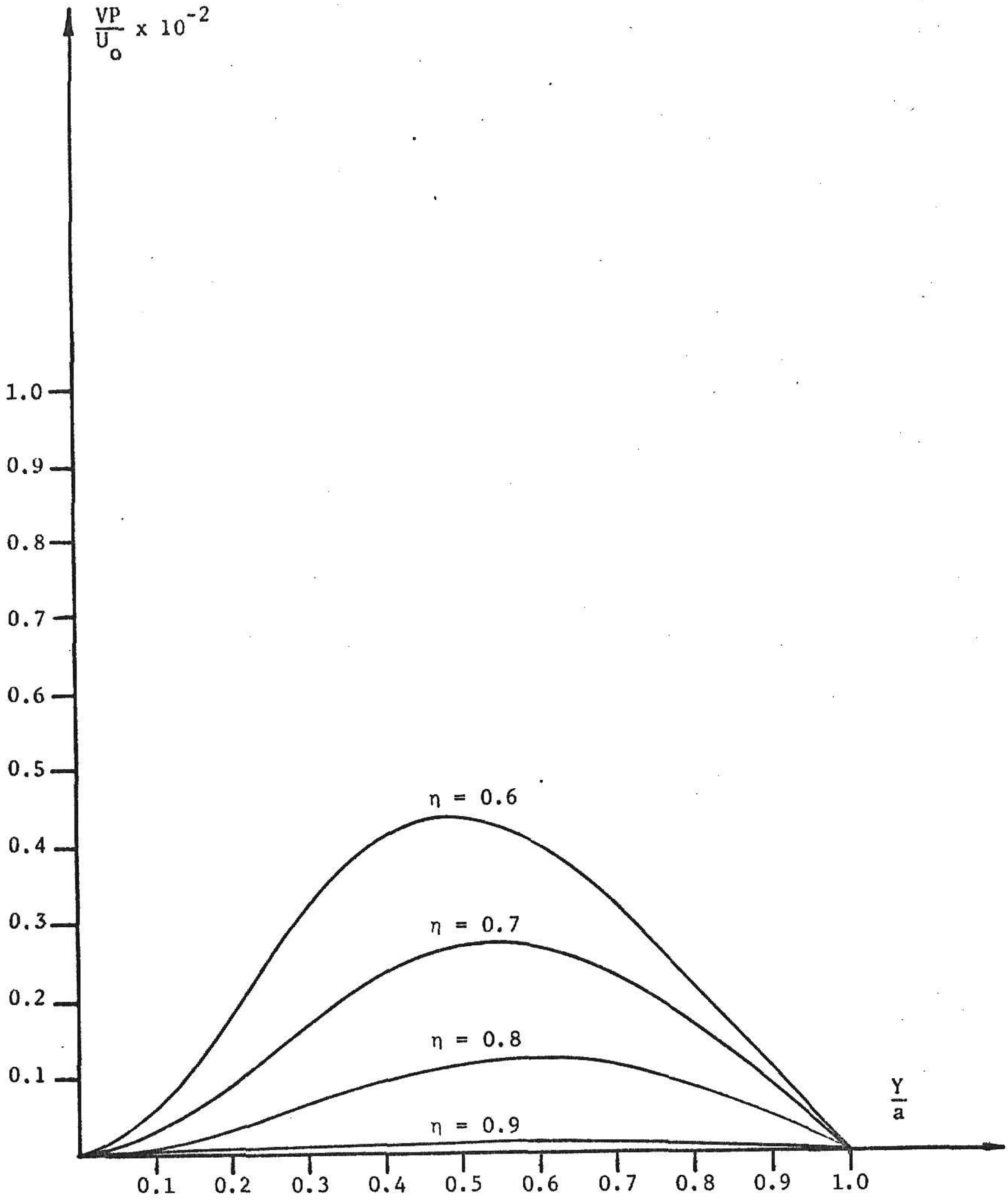

Fig. 22 Velocity Profile by finite element analysis

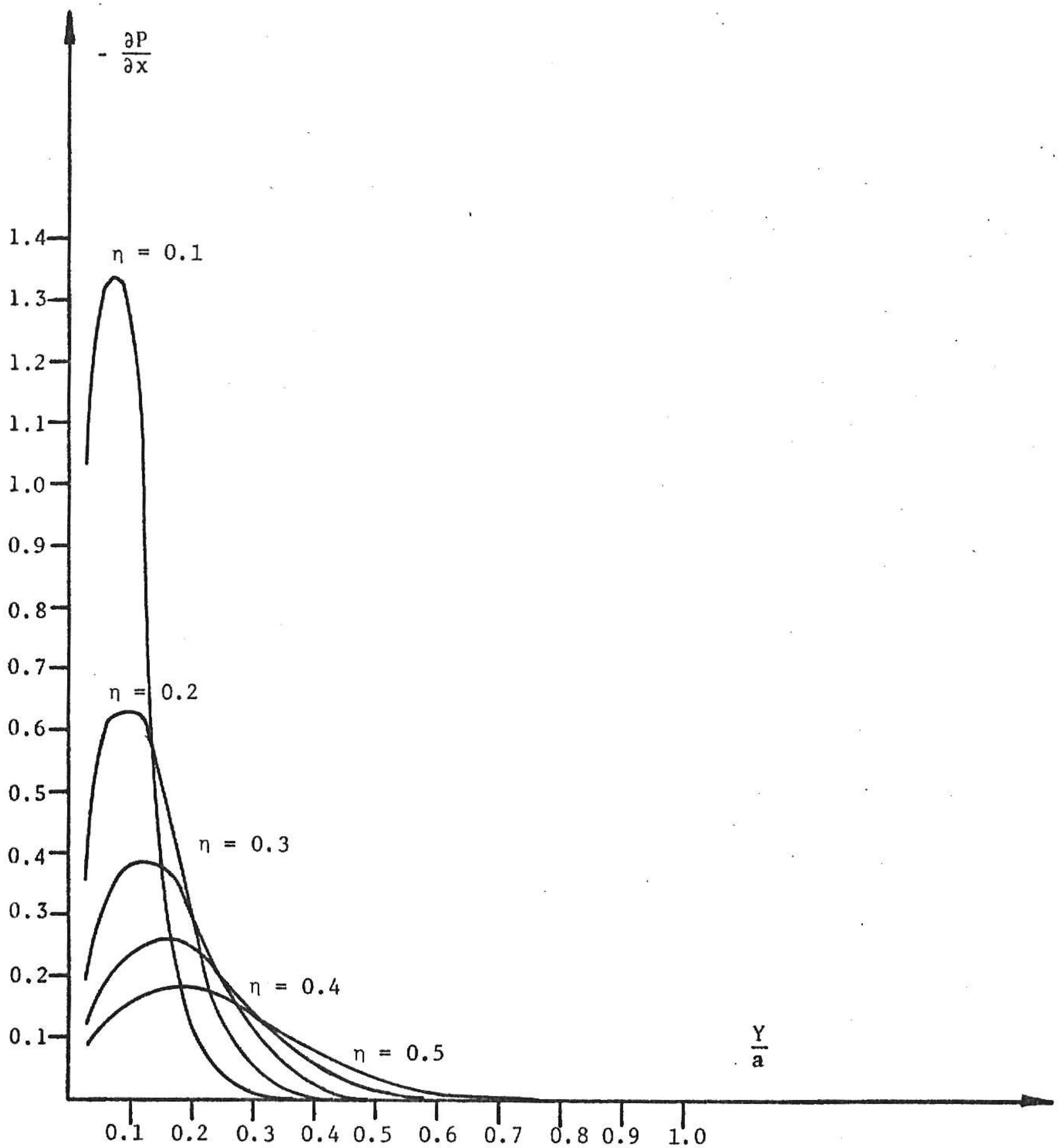

Fig. 23 Pressure gradient by finite element analysis

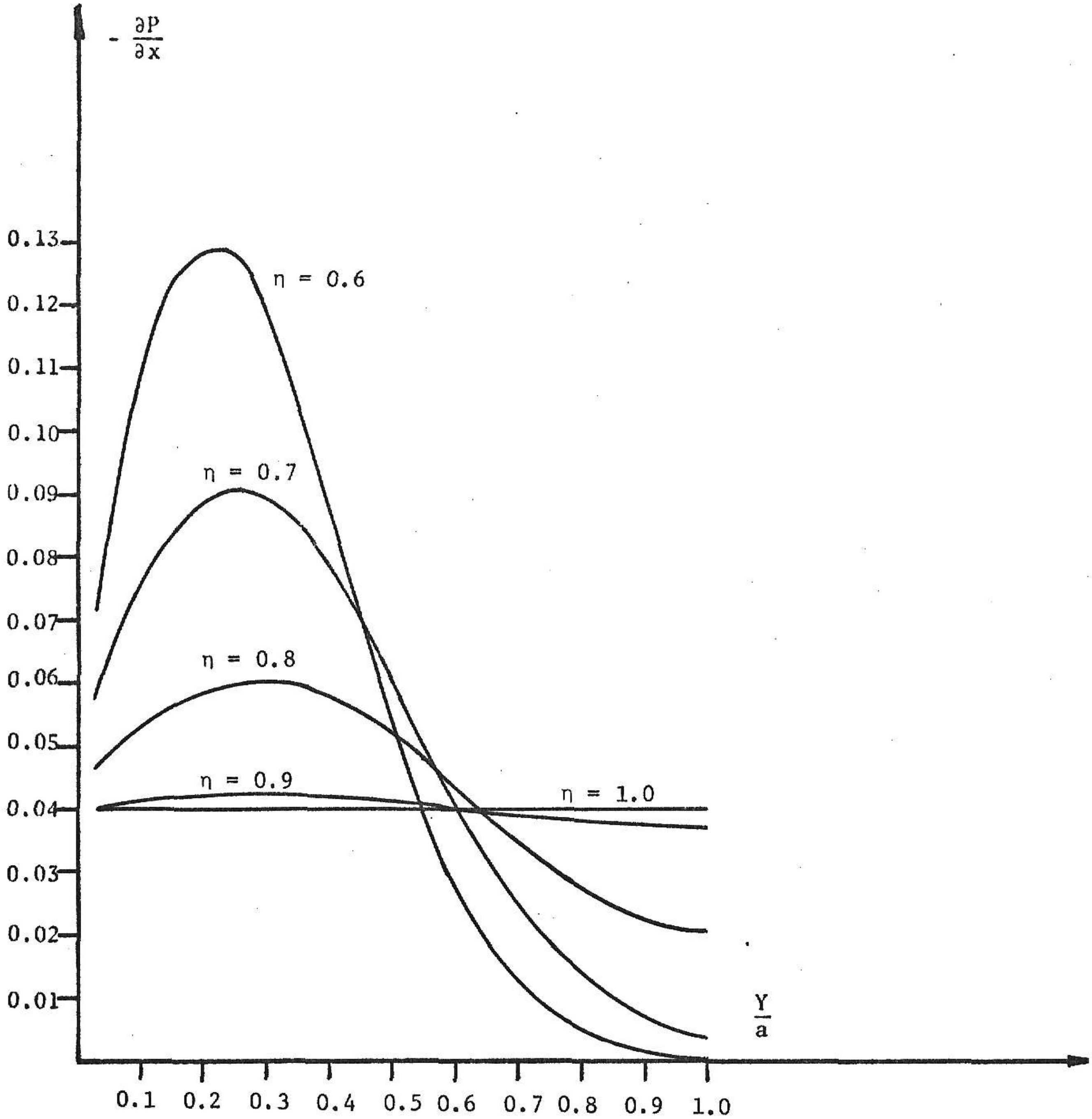

Fig. 24 Pressure gradient by finite element analysis

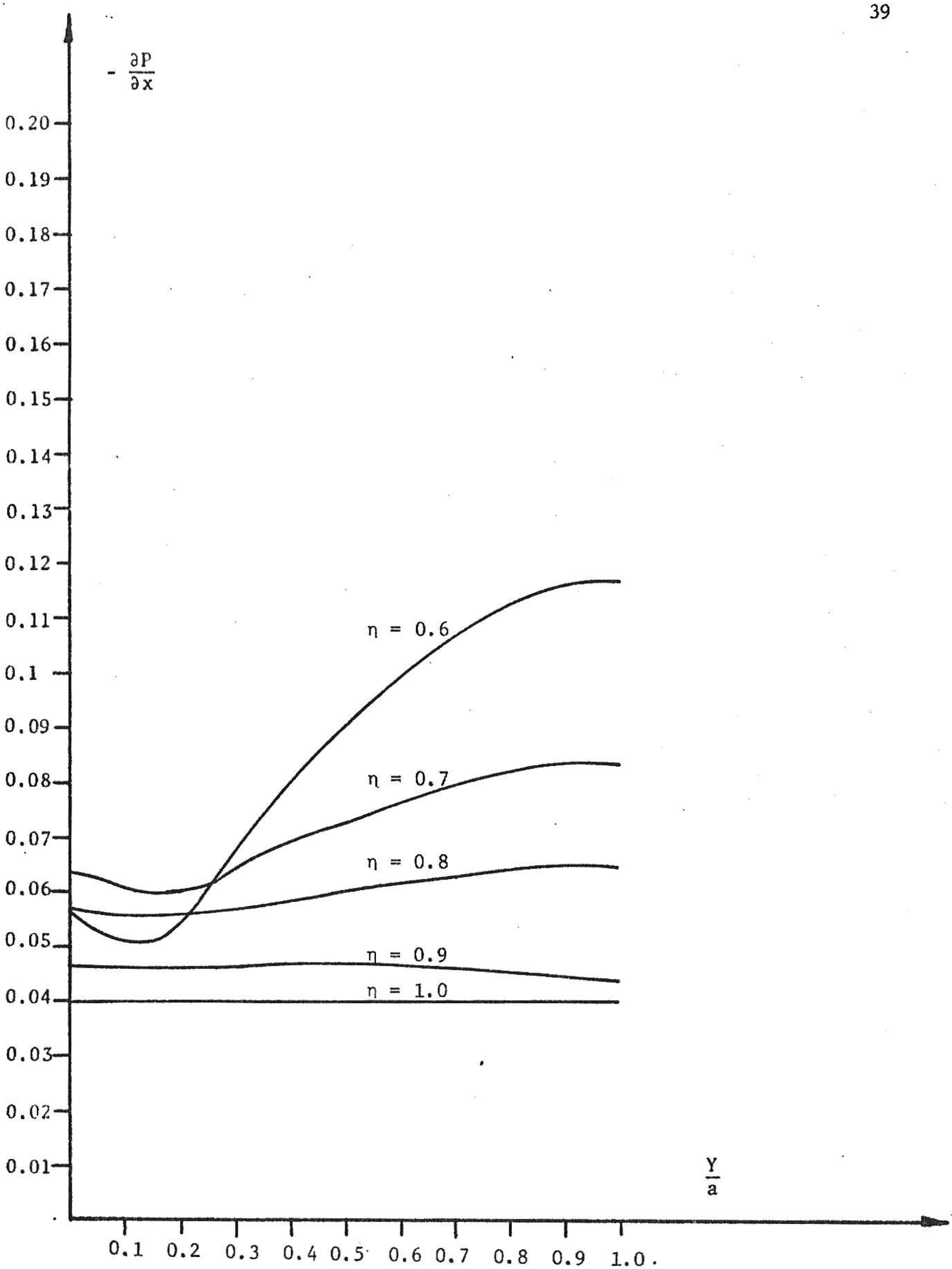

Fig. 25 Pressure gradient by finite difference analysis

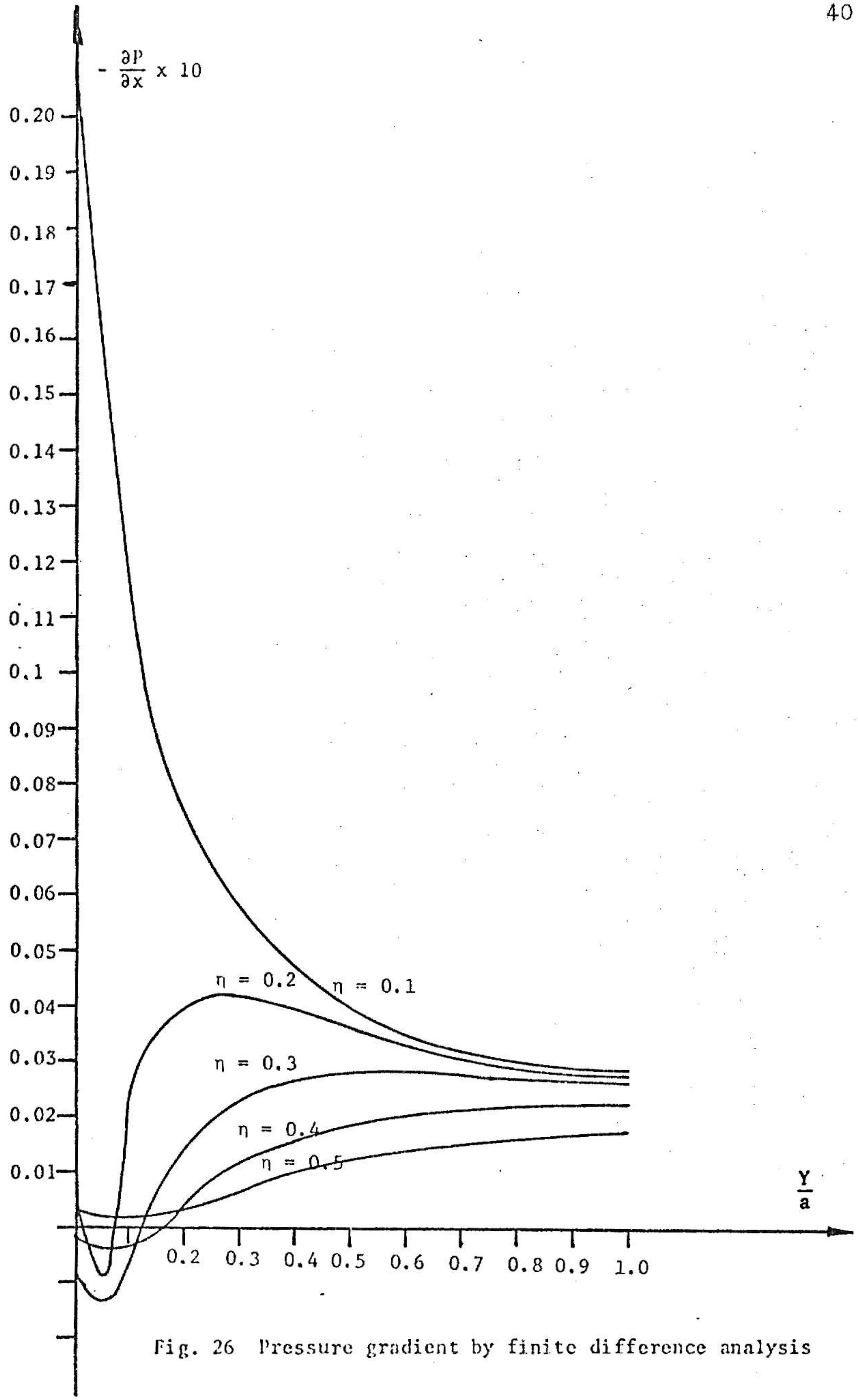

Fig. 26 Pressure gradient by finite difference analysis

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Finite Element Analysis of the Laminar

Flow in the Inlet Section Between Two

Infinite Parallel Plates.

by

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and Oceanic Technology.

Rep. of China. 1968

An Abstract of a Master's Report

submitted in partial fulfillment of the

requirement for the degree.

Master of Science

Department of Applied Mechanics

Kansas State University

Manhattan, Kansas

1973

Abstract

The purpose of this report was to investigate, using finite element analysis, the laminar flow of an incompressible fluid within the inlet region when the flow was constrained between two infinite parallel plates.

The results of this study are shown to be consistent with solutions which were obtained by others working in this area, but the finite element analysis requires much less computer time to obtain these results.

In addition, inflection points in velocity profiles, which would indicate points of potential instability in the laminar flow, are not present in the profiles developed in this report. In this sense, then, the finite element solution would seem to yield more reasonable results.